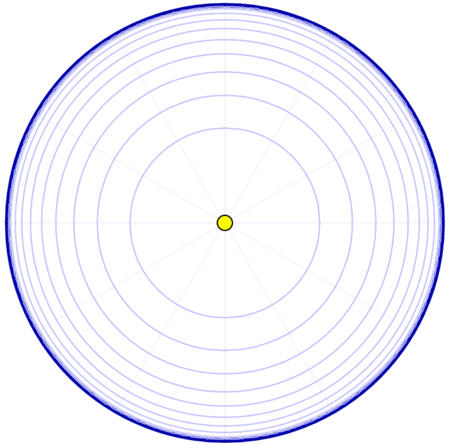
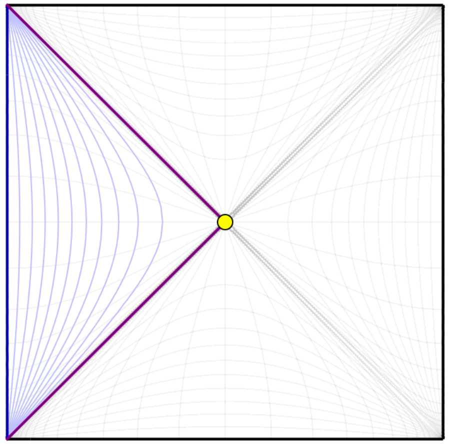
De Sitter horizon edge partition functions

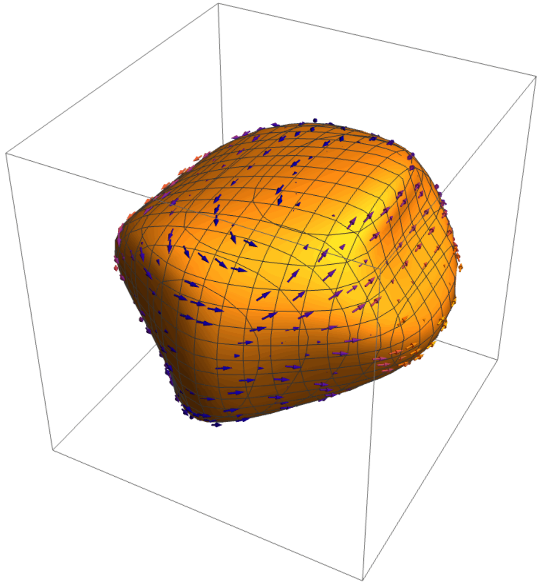




Y. T. Albert Law

Based on 2501.17912

+ work with D. Anninos, F. Denef, Z. Sun, K. Parmentier, M. Grewal, A. Ball, G. Wong



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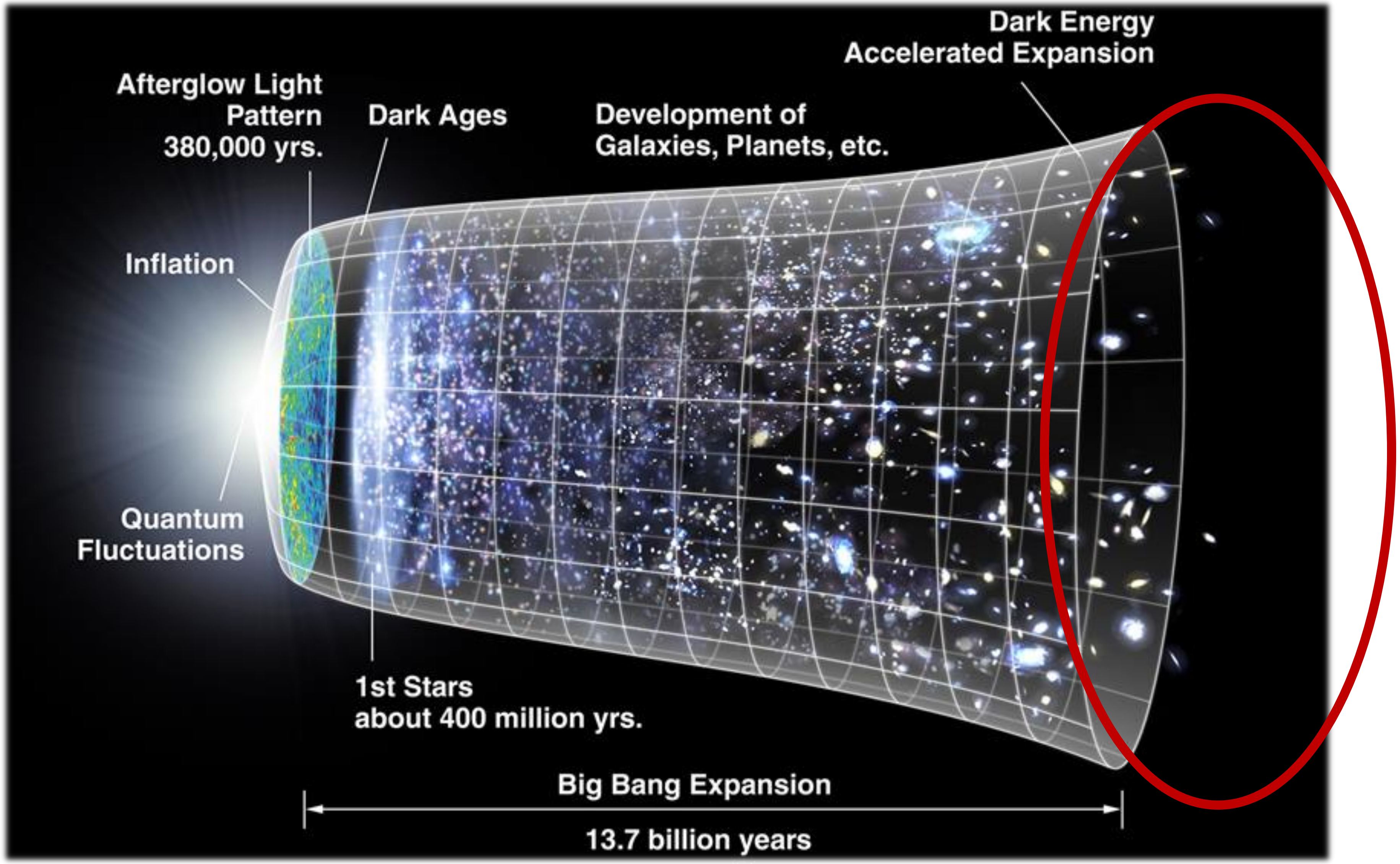
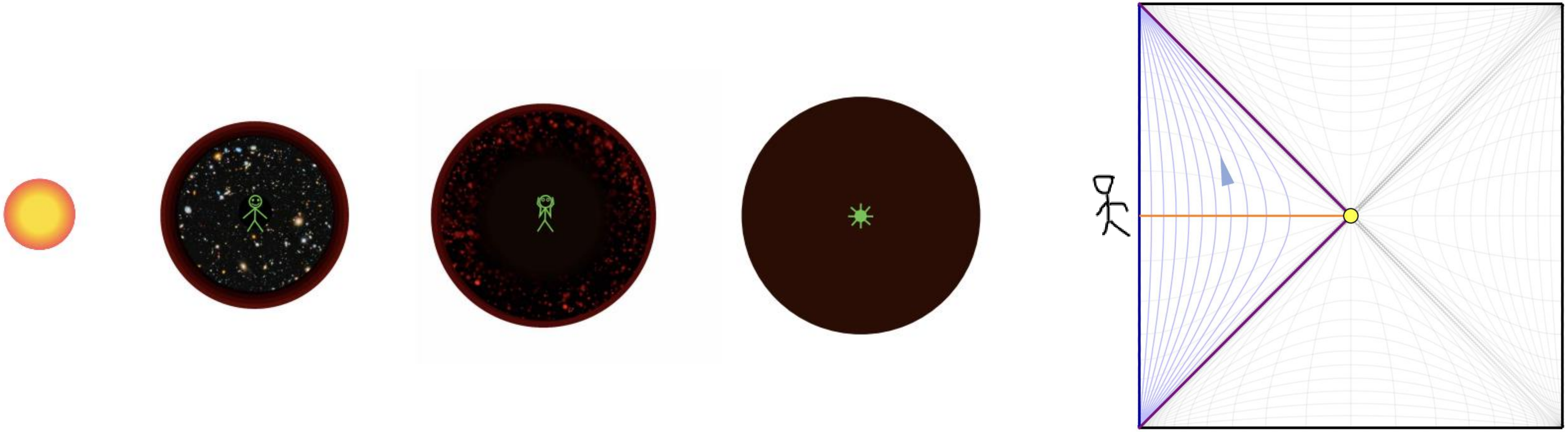

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Extrapolating the current ΛCDM model, our observable universe will become indistinguishable from a static patch of dS_4



[Loeb 01, Krauss, Scherrer 07]

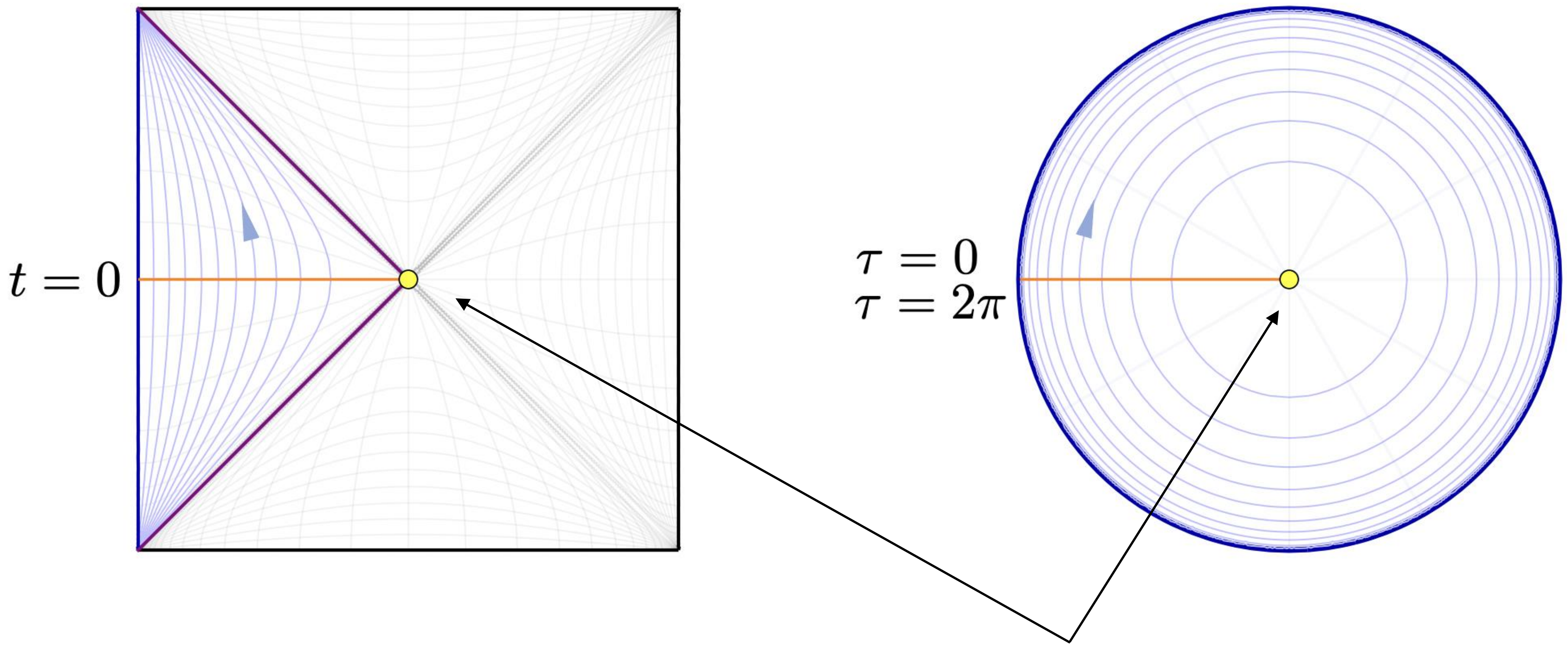
Semiclassically, the de Sitter horizon has an entropy

$$S = \underbrace{\frac{A}{4G_N}}_{\text{Gibbons-Hawking}} + \boxed{\underbrace{\# \log \frac{A}{4G_N} + \text{finite}}_{\text{1-loop}}} \underbrace{+ \cdots}_{\text{Higher-loops + non-pert.}}$$

Quantum corrections provide precision tests for candidate microscopic models

[Anninos, Denef, AL, Sun 20] [AL 20] [David, Mukherjee 21] [Anninos, Harris 21] [Anninos, Bautista, Mühlmann 21] [Anninos, Mühlmann 21] [Grewal, Parmentier 21] [Mühlmann 22] [Bobev, Hertog, Hong, Karlsson, Reys 22] [Castro, Coman, Fliss, Zukowski 23] [Bourne, Castro, Fliss 24] [Bandaru 24] [Shyam 21] [Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang 21]

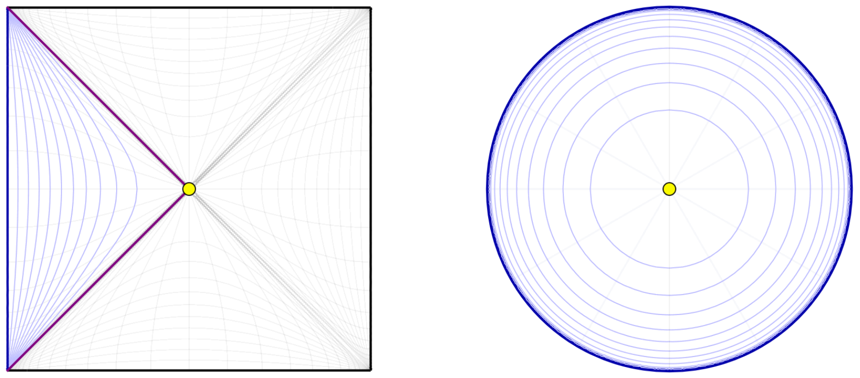
We considered free fields on a dS_{d+1} static patch in both Lorentzian and Euclidean signatures [Anninos, Denef, AL, Sun 20]



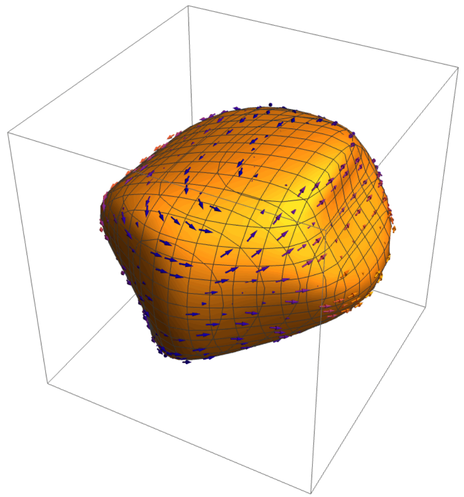
and we found a curious feature related to the horizon (``edge")

I’ll review these calculations and report some progress in understanding these ``edge” degrees of freedom

Plan of the talk



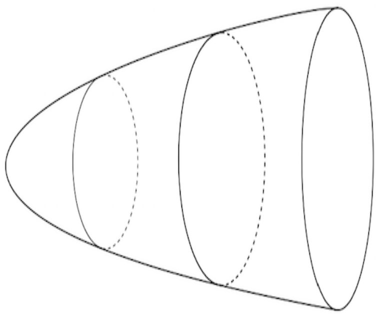
1-loop dS: Lorentzian vs Euclidean



Case study: linearized gravity

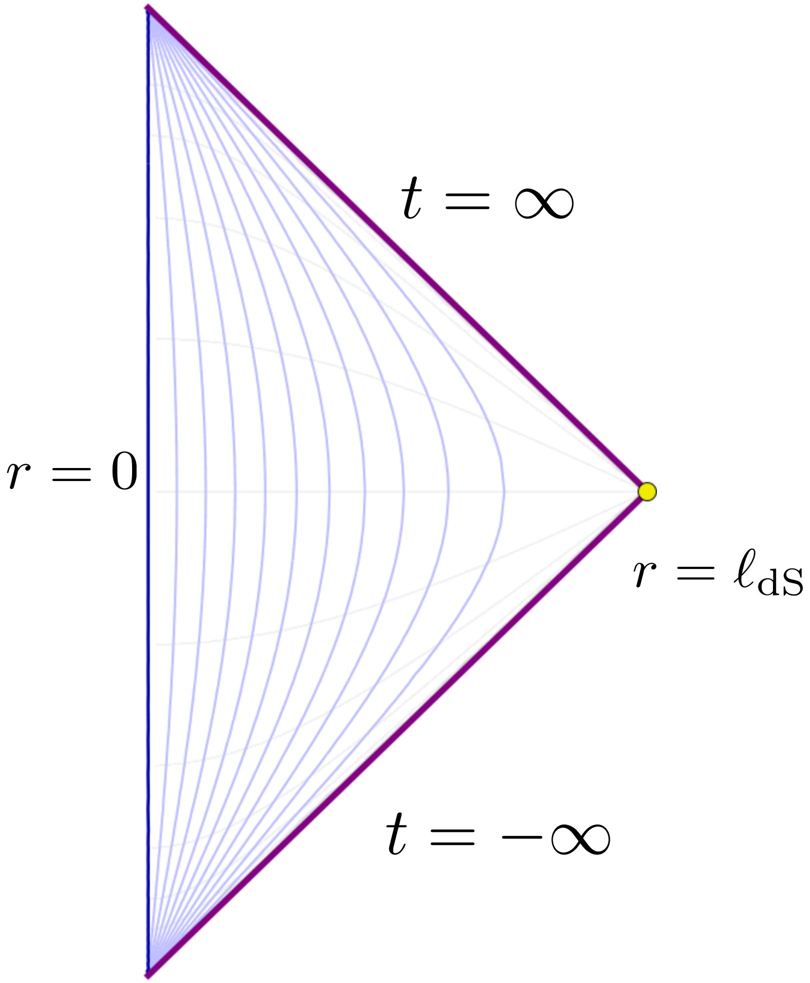
$$\phi_{\mu_1 \cdots \mu_s}$$

Remark 1: Higher-spin



Remark 2: Black holes

We first consider a free field on a dS_{d+1} static patch



round S^{d-1}

$$ds^2 = -\left(\ell_{\text{dS}}^2 - r^2\right) dt^2 + \frac{dr^2}{1-\frac{r^2}{\ell_{\text{dS}}^2}} + r^2 d\Omega^2$$

$$0 \leq r < \ell_{\text{dS}}$$

It is natural to study the ideal gas thermal canonical partition function

Boson

$$\log Z_{\text{bulk}}(\beta) \overset{?}{\equiv} \log \mathrm{Tr}\, e^{-\beta \hat{H}} \overset{?}{=} -\int_0^\infty d\omega\, \rho(\omega) \log\left(e^{\frac{\beta\omega}{2}} - e^{-\frac{\beta\omega}{2}}\right)$$

Density of normal mode/single-particle energy spectrum

Continuous

$$\Longrightarrow \rho(\omega) = \infty\ ??$$

We make sense of the single-particle density of states using the Harish-Chandra character of the dS group $SO(1,d+1)$

[Anninos, Denef, AL, Sun 20]

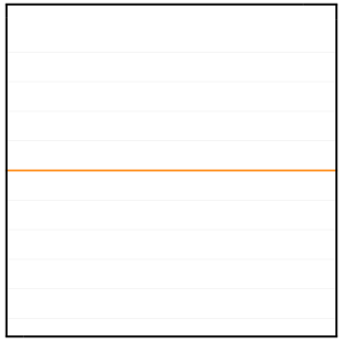
$$\chi_{[\Delta,\mathbf{s}]}(t) = \mathrm{tr}_{[\Delta,\mathbf{s}]}\, e^{-iHt}$$

$\mathfrak{so}(1,1)$-weight (~mass)

spin

dS boost

Can be computed as a trace over the global Hilbert space



See [Sun 21] for a review on the representation theory of $SO(1,d+1)$

Characters for massive fields in dS_{d+1}

Scalar $$\chi(t) = \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1 - e^{-t}|^d} \qquad \bar{\Delta} \equiv d - \Delta$$

p-form

$$\underbrace{\binom{d}{p}} \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1 - e^{-t}|^d}$$

rank-s totally symmetric tensor

$$\underbrace{\frac{2s+d-2}{d-2} \binom{s+d-3}{d-3}} \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1 - e^{-t}|^d}$$

Number of physical polarizations

[Hirai 65] [Basile, X. Bekaert, and N. Boulanger 16]

Character for massless spin-2

$$\chi(t) = \left[\frac{(d+2)(d-1)}{2} \frac{e^{-dt}+1}{|1-e^{-t}|^d} - d \frac{e^{-(d+1)t}+e^{t}}{|1-e^{-t}|^d} \right]_+$$

Number of polarizations

Ghost

Notation: $\left[\sum_k c_k q^k \right]_+ \equiv \sum_{k<0} (-c_k)\, q^{-k} + \sum_{k>0} c_k q^k = \sum_k c_k q^k - c_0 - \sum_{k<0} c_k \left(q^k + q^{-k} \right)$

Quasinormal mode (QNM) expansion

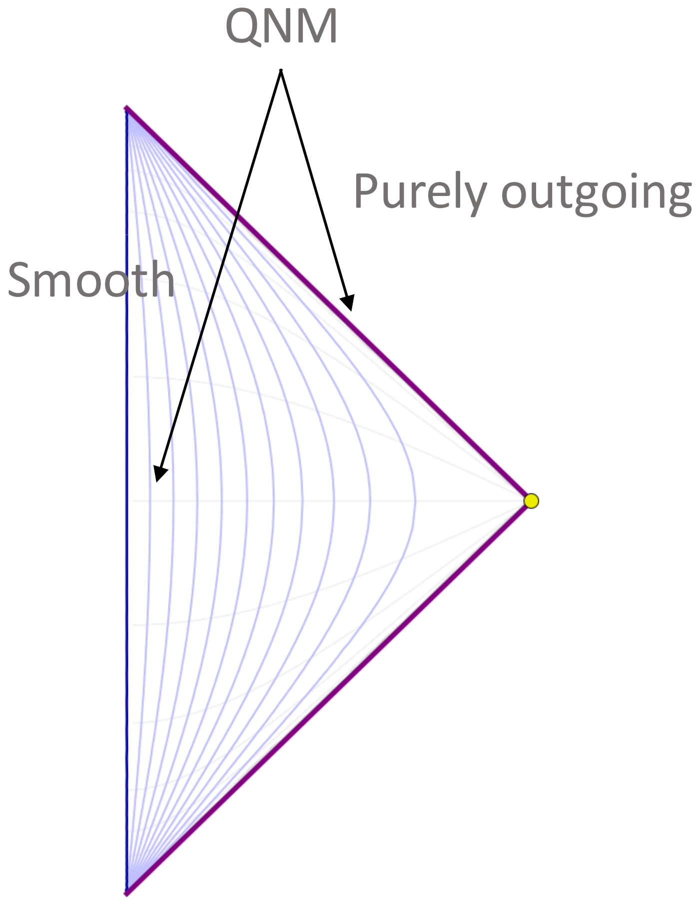
E.g. scalar in dS_4 with $m^2 = \Delta\bar{\Delta} = \Delta(3-\Delta)$

$$\chi(t) = \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1-e^{-t}|^3}$$

$$= \sum_{n=0}^{\infty}\sum_{l=0}^{\infty} \underset{\uparrow \text{ degeneracy}}{(2l+1)}\, e^{-i\overset{\text{QNM frequency}}{(-i(\Delta+2n+l))}|t|} + (\Delta \to \bar{\Delta})$$

degeneracy

QNM frequency



- **Algebraic** construction of QNMs
  [Ng, Strominger 12] [Jafferis, Lupsasca, Lysov, Ng, Strominger 13] [Tanhayi 14] [Sun 20]

- Can be understood as an integrated Green function [Grewal, AL 24]

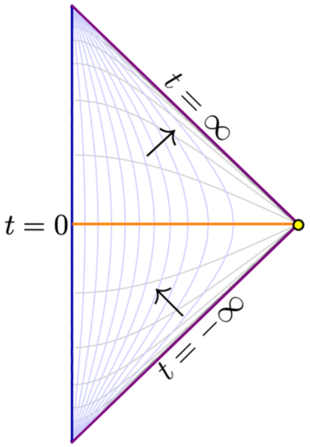
Key observation: The Fourier transform

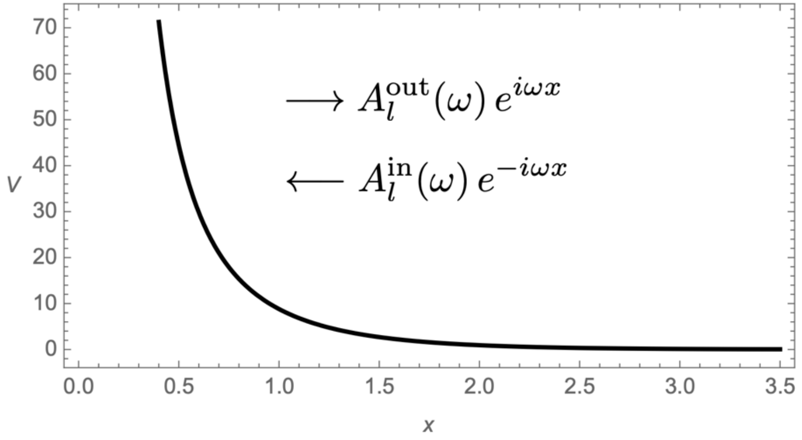
$$\tilde{\rho}(\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-i\omega t} \chi(t)$$

can be interpreted as a single-particle density of states

Can be understood in terms of scattering phases as a relative/renormalized DOS

[AL, Parmentier 22]





We define a ``quasicanonical" partition function uniformly for **any** particles or $SO(1,d+1)$-UIRs

$$\log Z_{\rm bulk}(\beta) \equiv -\int_0^\infty d\omega\, \tilde{\rho}(\omega) \log\left(e^{\frac{\beta\omega}{2}} - e^{-\frac{\beta\omega}{2}}\right)$$

$$= \int_0^\infty \frac{dt}{2t} \frac{1+e^{-\frac{2\pi}{\beta}t}}{1-e^{-\frac{2\pi}{\beta}t}} \chi(t)$$

at any inverse temperature β

[Anninos, Denef, AL, Sun 20]

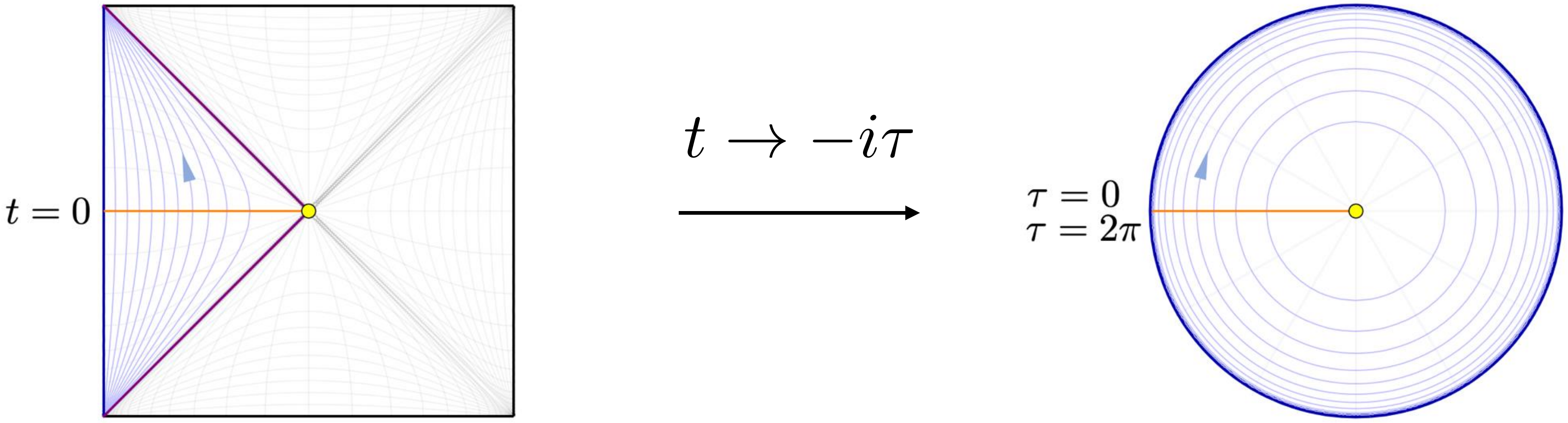
Thermodynamic quantities in gravitational systems were proposed to be computed by a Euclidean path integral

[Gibbons, Hawking 76]

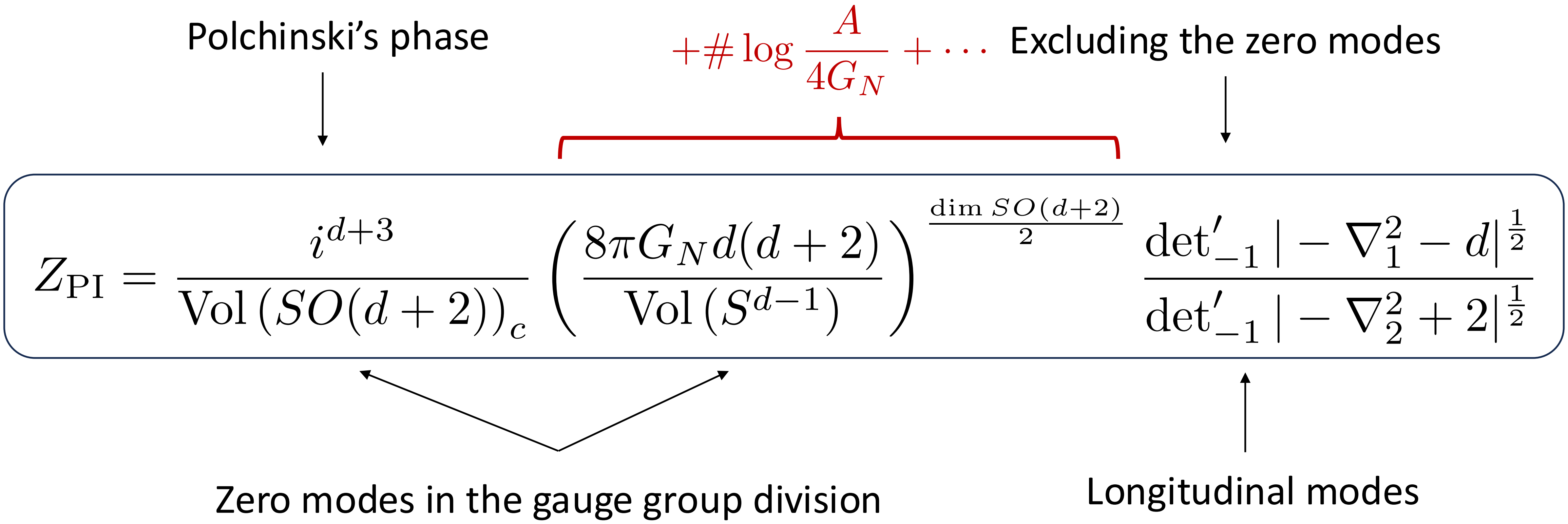
Euclidean gravitational path integral with $\Lambda > 0$

$$\mathcal{Z} = \int \mathcal{D}g\mathcal{D}\Phi\, e^{-S[g,\Phi]} = e^{\frac{A}{4G}} \left(\underbrace{\det(L)}_{\text{e.g. scalar: } \det\left(-\nabla_0^2 + m^2\right)^{-\frac{1}{2}}} + \cdots\right) + \cdots$$

The leading saddle is a round sphere, which is the Euclideanized static patch



1-loop partition function for gravitons on S^{d+1}



Polchinski's phase

Excluding the zero modes

$$\boxed{Z_{\mathrm{PI}} = \frac{i^{d+3}}{\mathrm{Vol}\,(SO(d+2))_c} \overbrace{\left(\frac{8\pi G_N d(d+2)}{\mathrm{Vol}\,(S^{d-1})}\right)^{\frac{\dim SO(d+2)}{2}}}^{+\#\log\frac{A}{4G_N}+\cdots} \frac{\det'_{-1}|-\nabla_1^2 - d|^{\frac{1}{2}}}{\det'_{-1}|-\nabla_2^2 + 2|^{\frac{1}{2}}}}$$

Zero modes in the gauge group division

Longitudinal modes

[Gibbons, Perry 78] [Christensen, Duff 80] [Allen 83] [Fradkin, Tseytlin 84] [Griffin, Kosower 89] [Polchinski 89] [Taylor, Veneziano 90] [Vassilevich 93] [Volkov, Wipf 00] [AL 20]

Comparing with $Z_{\text{bulk}}(\beta)$, we found

Scalar $$Z_{\text{PI}} = Z_{\text{bulk}}(\beta = 2\pi)$$

Spin $s \geq 1$ $$Z_{\text{PI}} = Z_{\text{bulk}}(\beta = 2\pi) Z_{\text{edge}}$$

``Edge”: looks like d.o.f. on S^{d-1}

[Anninos, Denef, AL, Sun 20]

Z_{edge} for Maxwell, Yang-Mills, and p-form gauge theories

$$Z_{\mathrm{edge}}^{\mathrm{U}(1)}\left(S^{d+1}\right) = \frac{1}{Z_{\mathrm{cpt}}(S^{d-1})}$$

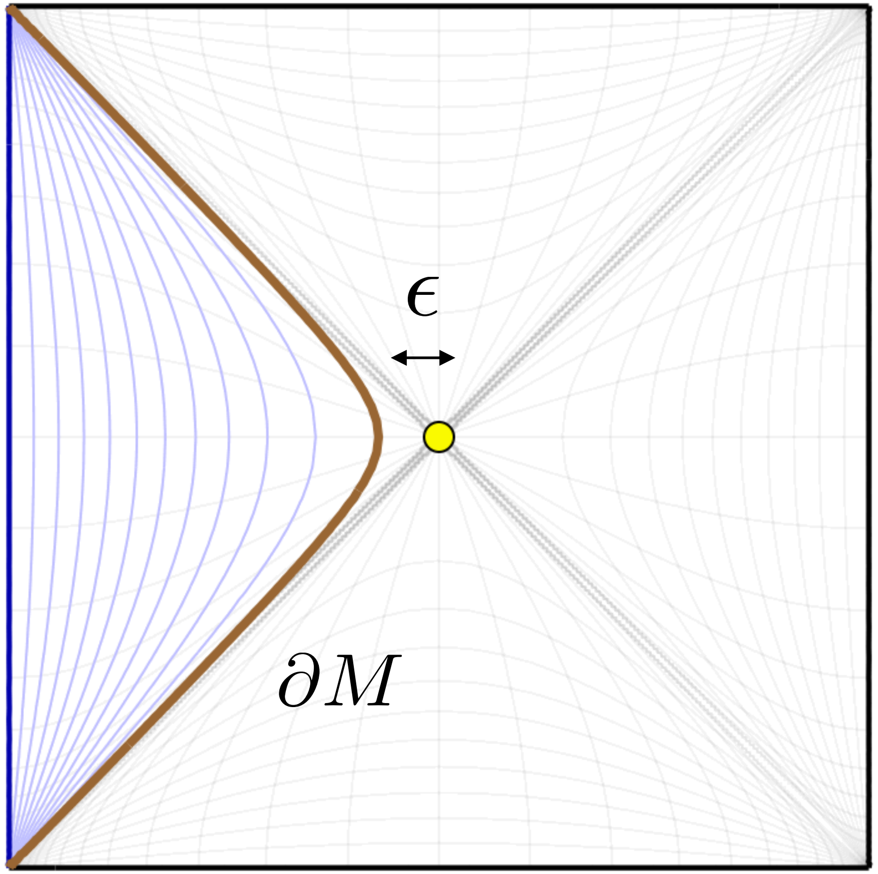
[Fukelman, Sempe, Silva 23] [Ball, AL, Wong 24]

$$Z_{\mathrm{edge}}^{\mathrm{YM,1\text{-}loop}}\left(S^{d+1}\right) = \frac{1}{Z_{\mathrm{SM}}^{\mathrm{1\text{-}loop}}(S^{d-1})}$$

$$Z_{\mathrm{edge}}^{\mathrm{p-form}}\left(S^{d+1}\right) = \frac{1}{Z_{\mathrm{PI}}^{(\mathrm{p}-1)-\mathrm{form}}(S^{d-1})}$$

[Mukherjee 23] [Ball, AL 24]

Edge modes in Maxwell [Ball, AL, Wong 24]



Dynamical edge mode boundary condition

$$\boxed{A_t|_{\partial M} = 0 = n^{\mu} F_{\mu i}|_{\partial M}}$$

$$A_i = \tilde{A}_i + \partial_i \alpha \qquad E_i = \tilde{E}_i + \partial_i \beta$$

We showed that Z_{edge} is a thermal trace over the edge mode Hilbert space

This has been generalized to p-form gauge theories [Ball, AL 24]

Brief recap

- We made sense of ideal gas thermal partition functions in Lorentzian dS static patch

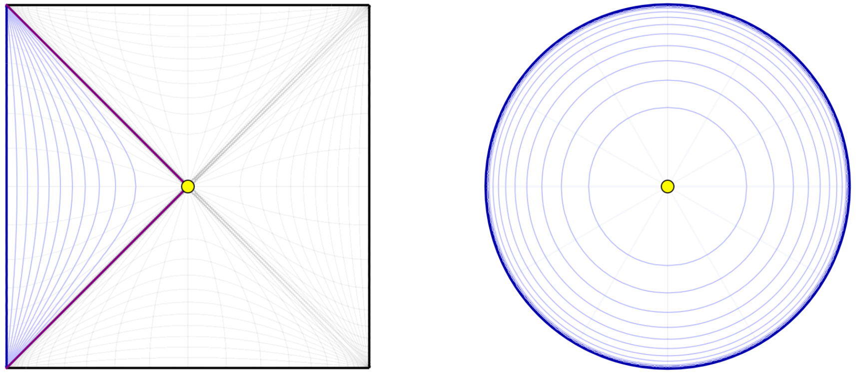
$$\log Z_{\mathrm{bulk}}(\beta) \equiv \int_0^\infty \frac{dt}{2t} \frac{1+e^{-\frac{2\pi}{\beta}t}}{1-e^{-\frac{2\pi}{\beta}t}} \chi(t)$$

- Comparing with 1-loop Euclidean sphere partition functions, we found

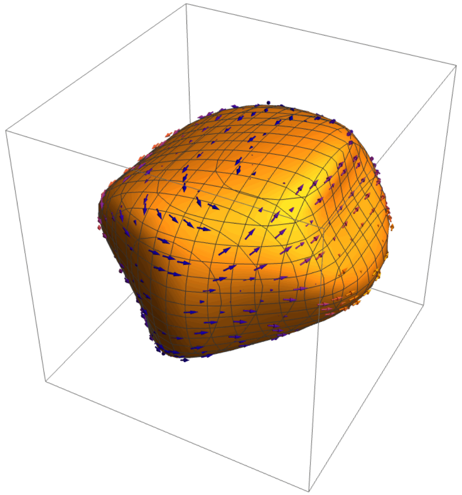
$$Z_{\mathrm{PI}} = \begin{cases} Z_{\mathrm{bulk}}(\beta = 2\pi) & , \quad s = 0 \\ Z_{\mathrm{bulk}}(\beta = 2\pi) Z_{\mathrm{edge}} & , \quad s \geq 1 \end{cases}$$

- For Maxwell and p-form gauge theories, Z_{edge} have been shown to be captured by ``edge modes" residing on the stretched horizon

What about gravity?



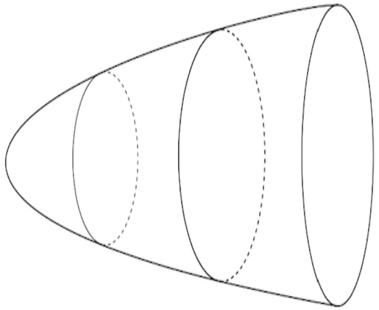
1-loop dS: Lorentzian vs Euclidean



Case study: linearized gravity

$$\phi_{\mu_1 \cdots \mu_s}$$

Remark 1: Higher-spin



Remark 2: Black holes

$Z_{\rm edge}$ for linearized gravity

$$\log Z_{\rm edge} = \log \frac{i^{d+3}}{{\rm Vol}(SO(d+2))_c} \left(\frac{32\pi^3 G_N}{{\rm Vol}(S^{d-1})\ell_{\rm dS}^{d-1}} \right)^{\frac{\dim SO(d+2)}{2}}$$

$$- \int_0^\infty \frac{dt}{2t} \frac{1+e^{-t}}{1-e^{-t}} \left[(d+2) \frac{e^{-(d-1)t}+e^{t}}{(1-e^{-t})^{d-2}} - \frac{e^{-dt}+e^{2t}}{(1-e^{-t})^{d-2}} \right]_+$$

This may have an edge mode interpretation, but it is not clear at this point.

Q: what is the $SO(d)$ or S^{d-1} field content of $Z_{\rm edge}$?

Notation: $\left[\sum_k c_k q^k \right]_+ \equiv \sum_{k<0} (-c_k)\, q^{-k} + \sum_{k>0} c_k q^k = \sum_k c_k q^k - c_0 - \sum_{k<0} c_k \left(q^k + q^{-k} \right)$

To analyze the $SO(d)/S^{d-1}$ field content, we unpack the sphere by applying the $SO(d+2) \to U(1) \times SO(d)$ branching rule

$$\begin{tikzcd}
\text{Spherical harmonics on } S^{d+1} \arrow[d] & \\
Z_{\text{PI}} \sim \prod (SO(d+2)\text{-irreps}) \arrow[r, "\longrightarrow" description, phantom] \arrow[d, "\text{[AL, to appear]}"] & Z_{\text{bulk}}^{\text{old}} Z_{\text{edge}}^{\text{old}} \\
\prod (U(1) \times SO(d)\text{-irreps}) \arrow[r] & Z_{\text{bulk}}^{\text{refined}} Z_{\text{edge}}^{\text{refined}} \\
& \text{Read off the } SO(d)/S^{d-1} \text{ field content} \arrow[u]
\end{tikzcd}$$

For gravity, the main result is the more refined formula

$$Z_{\text{edge}} = Z_{\text{edge}}^{\text{det}} Z_{\text{edge}}^{\text{non-det}}$$

$$\boxed{Z_{\text{edge}}^{\text{det}} = \underbrace{\det{}'_{-1} \left|-\nabla_1^2 - (d-2)\right|^{\frac{1}{2}}}_{\text{vector}} \underbrace{\det{}' \left|-\nabla_0^2 - (d-1)\right| \det{}' \left(-\nabla_0^2\right)^{\frac{1}{2}}}_{\text{scalars}}}$$

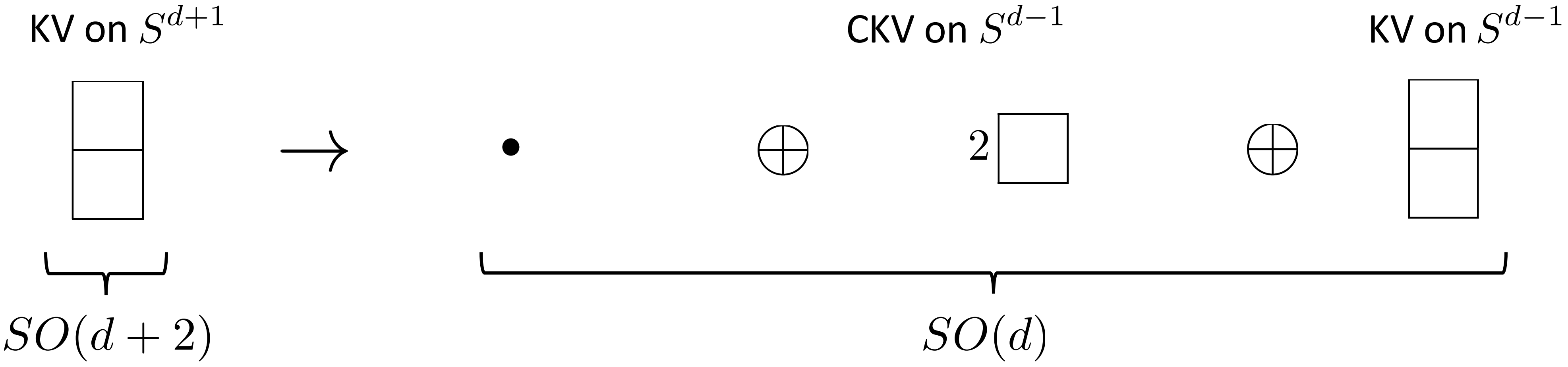
$$Z_{\text{edge}}^{\text{non-det}} = \frac{i^{d+3}}{\mathrm{Vol}(SO(d+2))_c} \left(\frac{16\pi^2 G_N}{\mathrm{Vol}(S^{d-1})}\right)^{\frac{\dim SO(d+2)}{2}} d^{\frac{\dim SO(d)+2d}{2}} (d-2)^{\frac{1}{2}}$$

[AL, to appear]

Z_{edge} and shift symmetries

[Bonifacio, Hinterbichler, Joyce, Rosen 18] [Bonifacio, Hinterbichler, Johnson, Joyce 19]

Field	χ	ϕ^a	A_μ
multiplicity	1	2	1
Mass²	0	$-(d-1)$	$-2(d-2)$
Shift symmetry	$\chi \to \chi + c$	$\phi^a \to \phi^a + Y_1$	$A_\mu \to A_\mu + Y_{1,\mu}$



KV on S^{d+1}

CKV on S^{d-1}

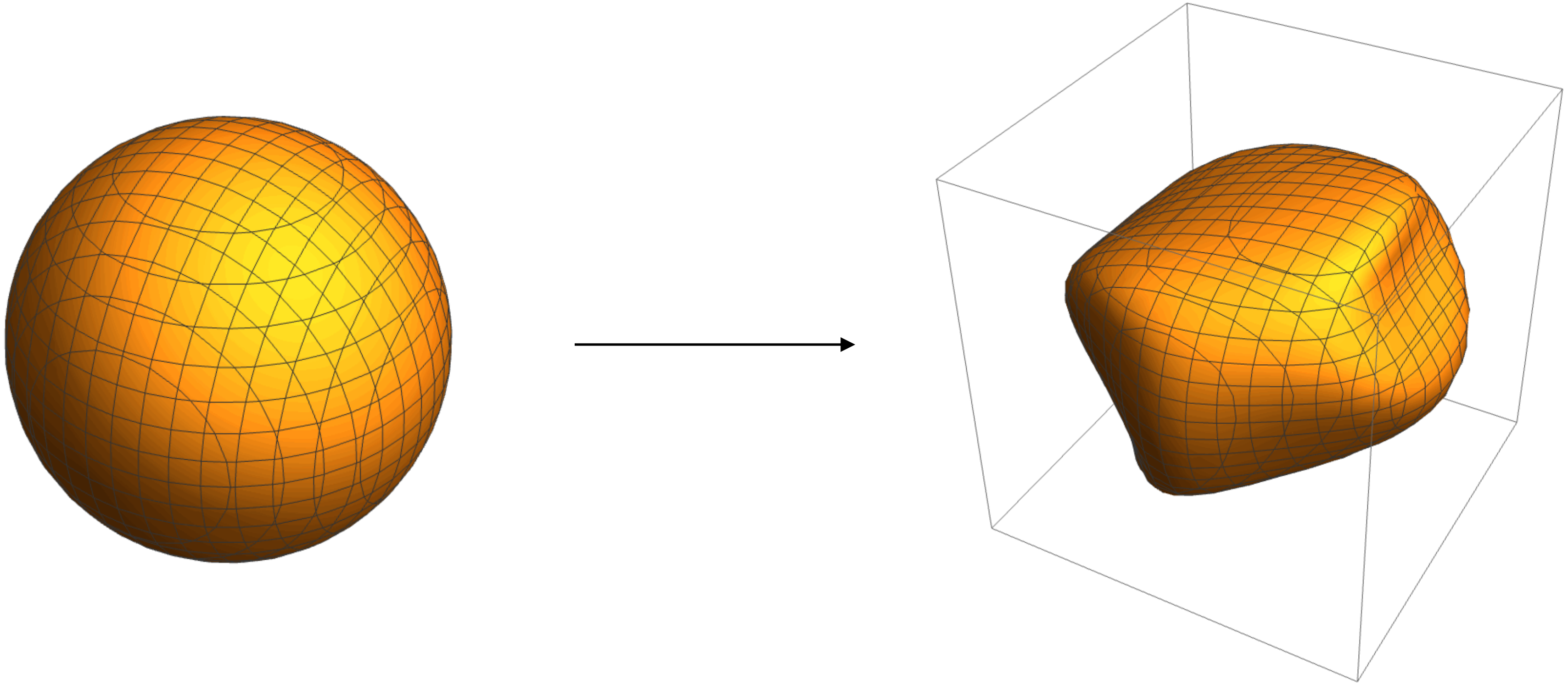
KV on S^{d-1}

$SO(d+2)$

$SO(d)$

What is a possible interpretation?

The two tachyonic scalars ϕ^a are known to describe small deformations of a spherical brane embedded in some ambient space



We are then led to consider a S^{d-1} brane embedded in a rigid round S^{d+1}

[Goon, Hinterbichler, Trodden 11, Burrage, de Rham, Heisenberg 11]

Recall a round S^{d+1} can be described as a hypersurface in $\mathbb{R}^{d+2}$

$$S^{d+1}: \quad \left(X^1\right)^2 + \left(X^2\right)^2 + \cdots + \left(X^{d+2}\right)^2 = 1$$

An interior S^{d-1} brane can be described by the (non-unique) parametrization

$$X^i(x) = \sqrt{1 - \phi^a(x)\phi^a(x)}\,\underset{\substack{\uparrow \\ \text{Rigid round } S^{d-1}\,:\; \Omega^2 = 1}}{\Omega^i(x)}\,, \quad X^a(x) = \phi^a(x)$$

[Clark, Love, Nitta, ter Veldhuis 05, Clark, Love, Nitta, ter Veldhuis, Xiong 07]

With the induced metric $G_{\mu\nu}[\phi^a] \equiv \delta_{AB}\partial_\mu X^A \partial_\nu X^B$ **,**

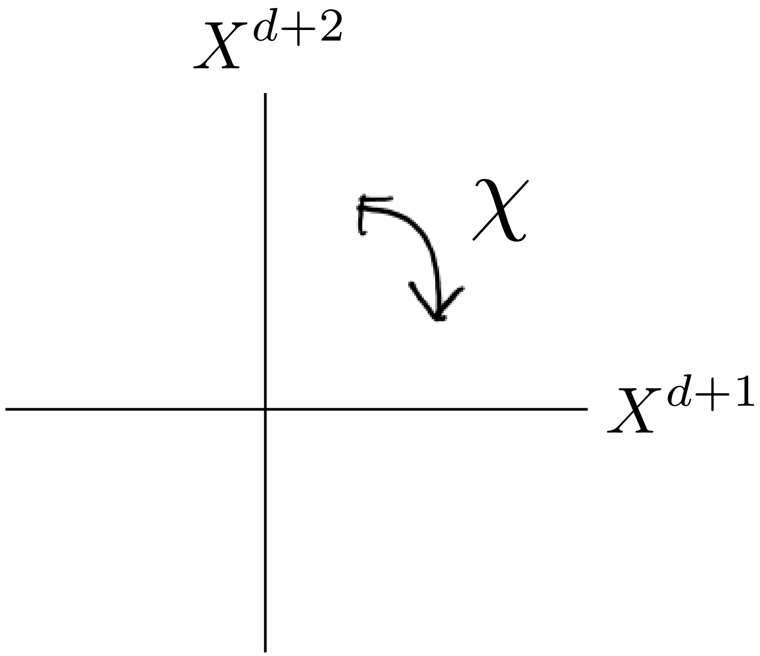
we write down the simplest worldvolume action for the S^{d-1} **brane**

$$S^{\mathrm{brane}}[\phi^a] = \frac{1}{8\pi G_N}\int_{S^{d-1}} d^{d-1}x\sqrt{G[\phi^a]}$$

$$\approx \frac{1}{8\pi G_N}\int_{S^{d-1}} \sqrt{\bar{g}}d^{d-1}x\left(1+\frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a \underset{\uparrow}{-} \frac{d-1}{2}\phi^a\phi^a+\cdots\right)$$

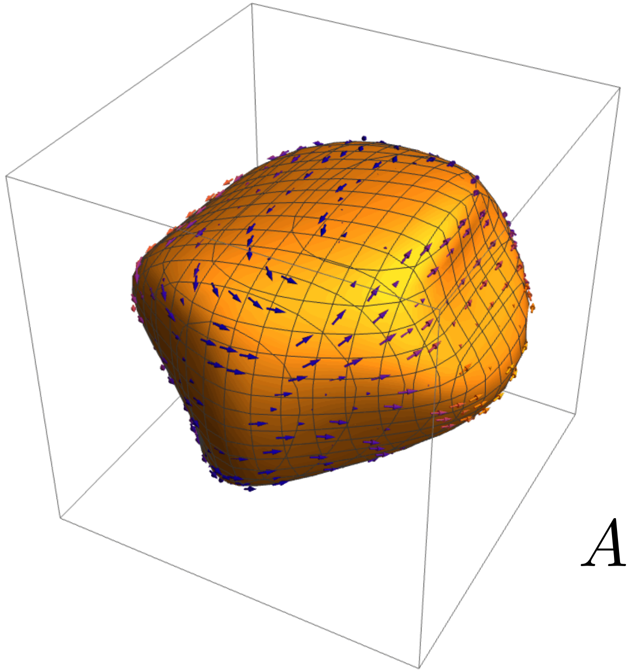
The deformations always decrease the size of the brane

χ and A_μ have natural geometric interpretations too



Can write $S[A]$ in terms of $\mathcal{L}_A \bar{g}_{\mu\nu}$

Round S^{d-1} metric



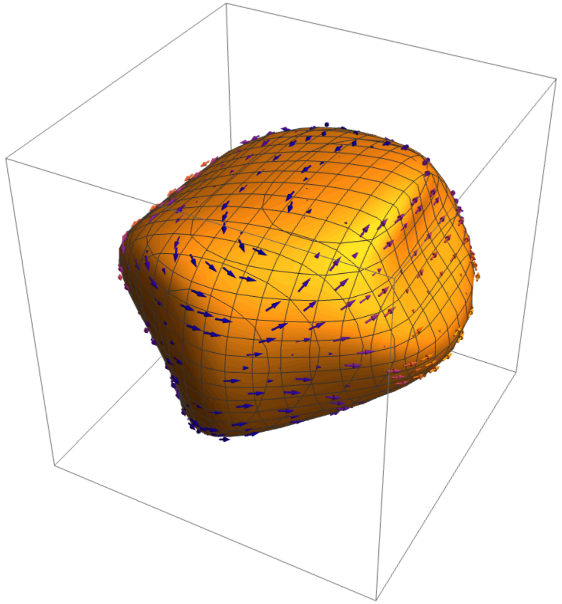
A_μ = diffeos on S^{d-1}

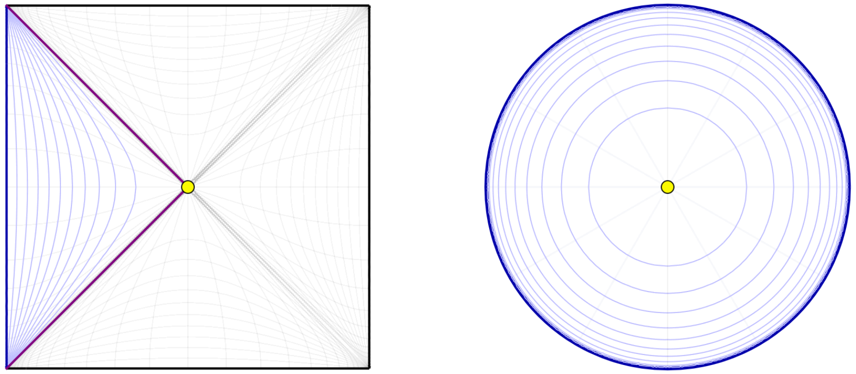
An interesting problem: incorporate these into $S^{\rm brane}$ to fully nonlinearize $SO(d+2)$

Brief recap

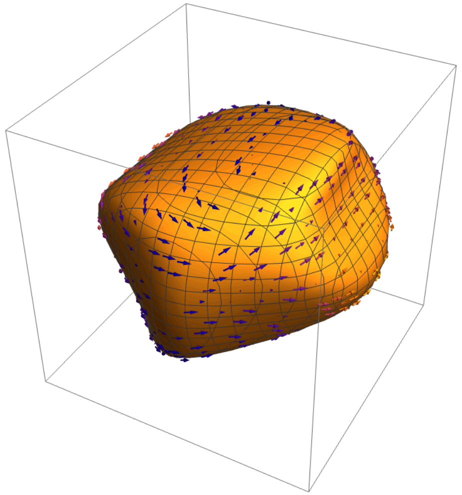
- Applying the $SO(d+2) \to U(1) \times SO(d)$ branching rule, we identified the $SO(d)$ or S^{d-1} field content for Z_{edge} of linearized gravity on S^{d+1}

- The field content suggests that Z_{edge} describes a S^{d-1} brane embedded in an ambient round S^{d+1}





1-loop dS: Lorentzian vs Euclidean



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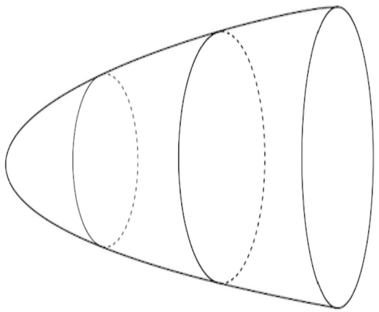


Remark 2: Black holes

Massless higher-spin gauge fields

$$\phi_{\mu_1\cdots\mu_s} \to \phi_{\mu_1\cdots\mu_s} + \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

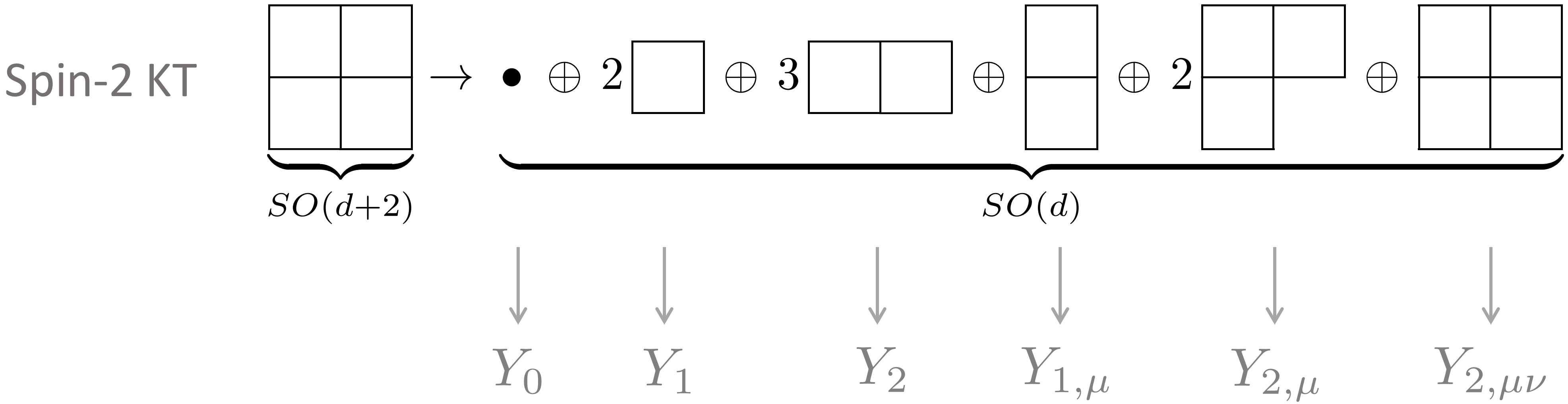
The global part corresponds to higher-spin symmetries generated by spin-(s-1) Killing tensors

$$\nabla_{(\mu_1}\bar{\xi}_{\mu_2\cdots\mu_s)} = 0$$

We have studied their $Z_{\rm PI}$ and found their $Z_{\rm edge}$ too [Anninos, Denef, AL, Sun 20] [AL 20]

We can **guess** the field content of Z_{edge} by demanding the **nonlinear realization** of the **global HS symmetries**

e.g. s=3



Including all (generically tachyonic) fields invariant under these shift symmetries recover (the kinematic part of) Z_{edge} found in [Anninos, Denef, AL, Sun 20] !

We have a wealth of structures that invite further investigation!

	p-form	**Spin-s totally symmetric tensor**	**Mixed-symmetry tensor (d>3)**
Massive	Massive (p-1)-form	Massive spin ≤ s-1	??
Partially massless	╳	Massive + shift-symmetric spin ≤ s-1	??
Massless	Massless (p-1)-form	Shift-symmetric spin ≤ s-1	??

In the case of dS_4, the edge theory lives on S^2

For higher-spin gauge fields, the edge theory consists of fields with integer Δ

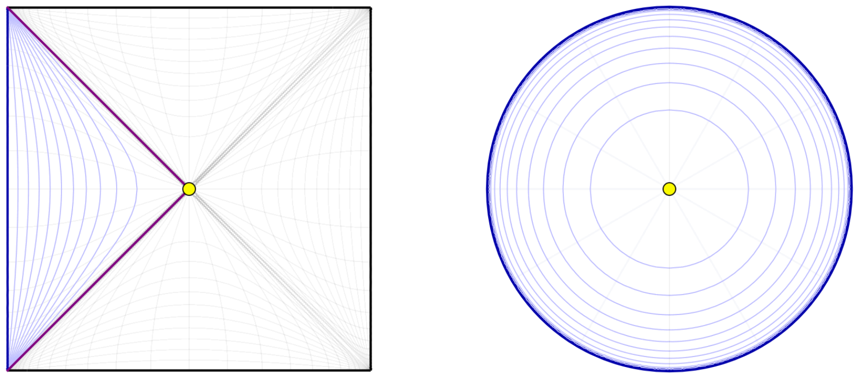
Discrete series representation of $SO(1,2)$

[Anninos, Anous, Pethybridge, Şengör 23]

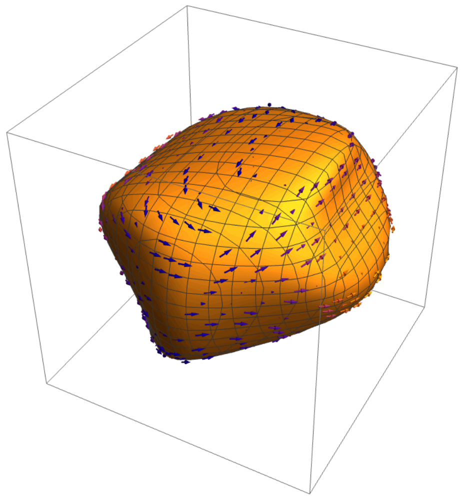
E.g. gravity: the edge theory consists of one $\Delta = 1$ and three $\Delta = 2$

Appear in the bosonic part of dS_2 supergravity

[Anninos, Benetti Genolini, Mühlmann 23]



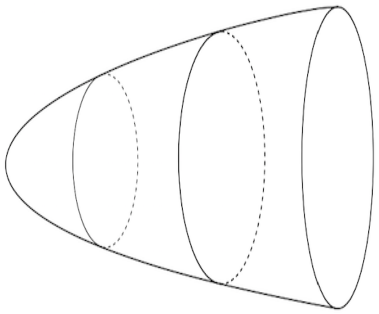
1-loop dS: Lorentzian vs Euclidean



Case study: linearized gravity

$$\phi_{\mu_1\cdots\mu_s}$$

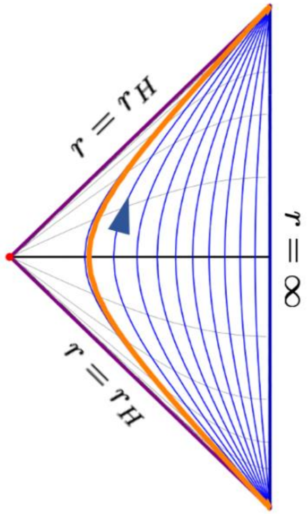
Remark 1: Higher-spin

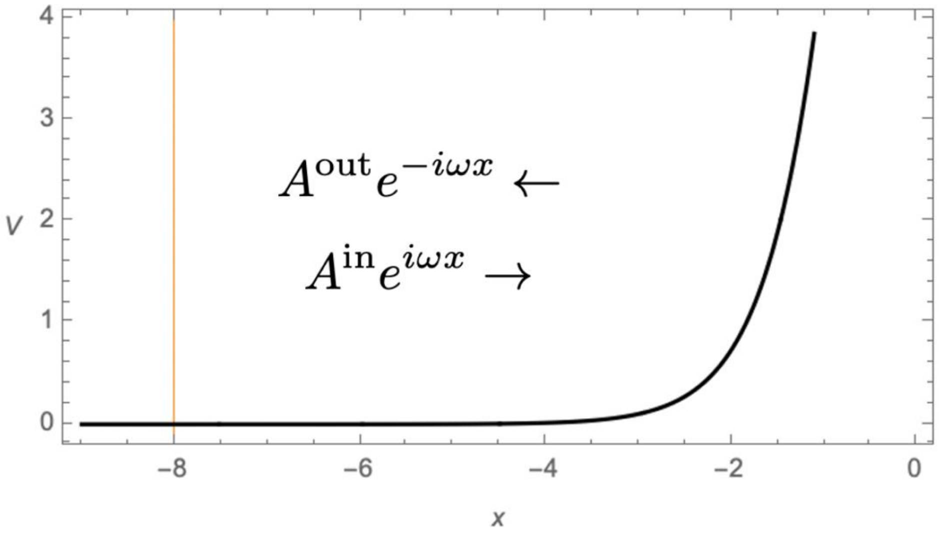


Remark 2: Black holes

The issue of continuous spectrum for free fields is not new ['t Hooft 84]

E.g. static BTZ





$$S(\omega) \equiv \frac{A^{\text{out}}(\omega)}{A^{\text{in}}(\omega)}$$

QNMs = scattering poles

Regulated DOS

$$\rho^R(\omega) = \frac{R}{\pi} + \boxed{\frac{1}{2\pi i} \log S(\omega)} + O\left(\frac{1}{R}\right)$$

↑

Brick wall/Weyl term $\rightarrow \infty$ as $R \rightarrow \infty$

[AL, Parmentier 22] [Grewal, AL, Parmentier 22]

Can define a relative/renormalized DOS in terms of scattering phases

Krein-Friedel-Lloyd formula

$$\tilde{\rho}(\omega) = \frac{1}{2\pi i}\partial_\omega \log \frac{S(\omega)}{\bar{S}(\omega)}$$

← Reference scattering phase

For an appropriate reference, we have a quasicanonical partition function

$$\log Z_{\rm bulk}(\beta) \equiv \int_0^\infty \frac{dt}{2t}\frac{1+e^{-\frac{2\pi}{\beta}t}}{1-e^{-\frac{2\pi}{\beta}t}}\chi(t)$$

$$\boxed{\chi(t) = \sum_z N_z\, e^{-izt}}$$

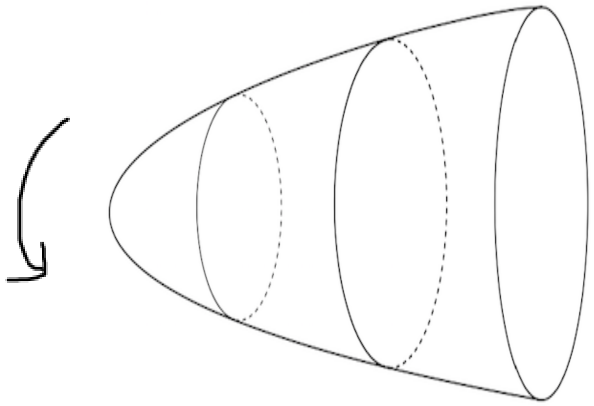
QNM

degeneracy

[AL, Parmentier 22] [Grewal, AL, Parmentier 22]

We compared this with the 1-loop Euclidean black hole partition functions

$$\tau \simeq \tau + \beta_H$$



Generally,

Scalar $$Z_{\mathrm{PI}} = Z_{\mathrm{bulk}}(\beta = \beta_H)$$

Equivalent to the QNM formula by [Denef, Hartnoll, Sachdev 09], argued based on analyticity argument around the origin

[AL, Parmentier 22]

On the other hand,

Spin $s \geq 1$ $$Z_{\mathrm{PI}} = \overbrace{Z_{\mathrm{bulk}}(\beta = \beta_H)}^{\text{Naïve DHS}}\, Z_{\mathrm{edge}}$$

From the DHS point of view, it is related to QNMs that do not have good smoothness properties under analytic continuation

[Castro, Keeler, Szepietowski 17] [Keeler, Martin, Svesko 18, 19] [Grewal, AL, Parmentier 22]

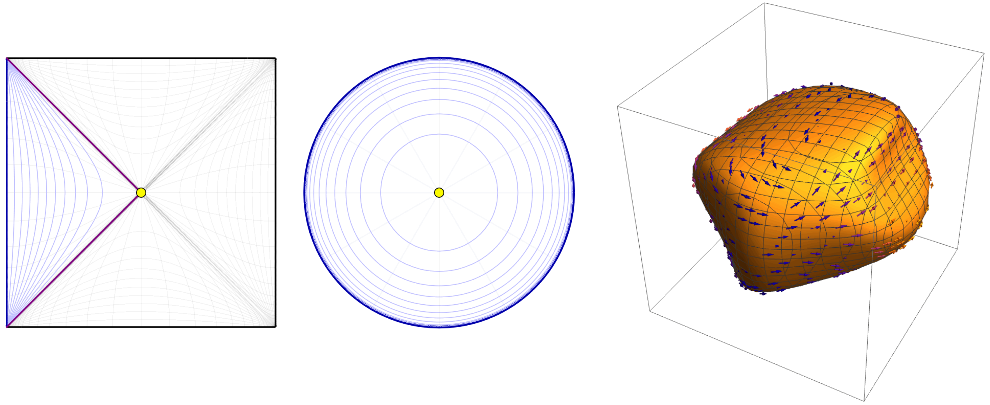
Additional comment: Near-extremal BTZ

Z_{edge} is crucial for the $T^{3/2}$ correction of the graviton partition function [Kapec, AL, Toldo 24]

and seems to be related to the `` Schwarzian modes'' [Kolanowski, Marolf, Rakic, Rangamani, Turiacic 24]

Summary

The discrepancy of Lorentzian and Euclidean calculations of 1-loop dS thermodynamics reveals co-dimension-2 degrees of freedom



	p-form	Spin-s totally symmetric
Massive	Massive (p-1)-form	Massive spin ≤ s-1
Massless	Massless (p-1)-form	Shift-symmetric spin ≤ s-1

Future directions:

1. Connection with literature on gravitational edge modes

2. Edge modes as Goldstones?

3. $Z^{\mathrm{BH}}_{\mathrm{edge}}$ vs $Z^{\mathrm{dS}}_{\mathrm{edge}}$?