Exploring stringy gravity

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Limits of our theory

- We know General Relativity must break down at some scale
- Most easily seen through the existence of singularities
- We don’t understand black hole interiors well enough to probe singularities within them
- We certainly don’t understand cosmology well enough to talk about the Big Bang
- So, what *can* we study *right now*?

Limits of our theory

- The question of cosmic censorship---whether or not an observer can see a singularity---is a long and fruitful one
- After 50+ years, we have come to a finite set of exceptions to the censorship conjecture
- These exceptions fall into different categories, although they do share a neat feature: they are all in a suitable sense "small"

Penrose 70s

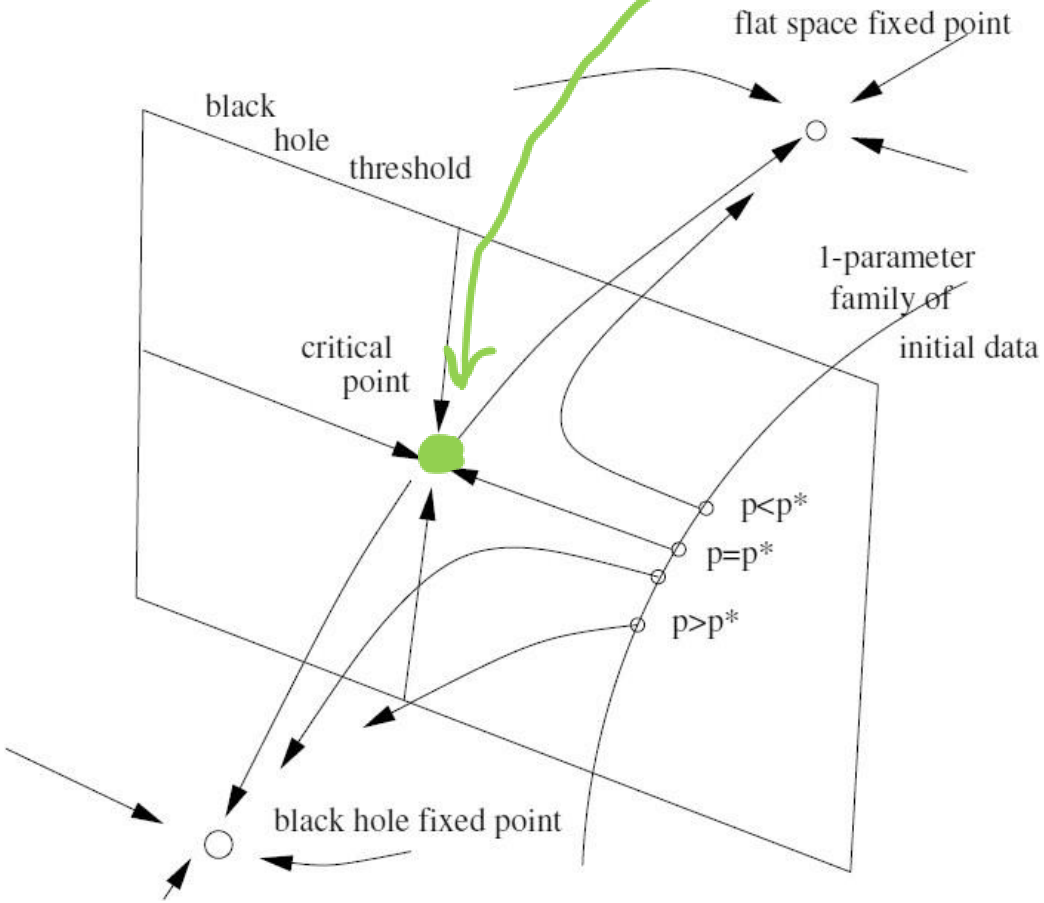
Emparan 2020

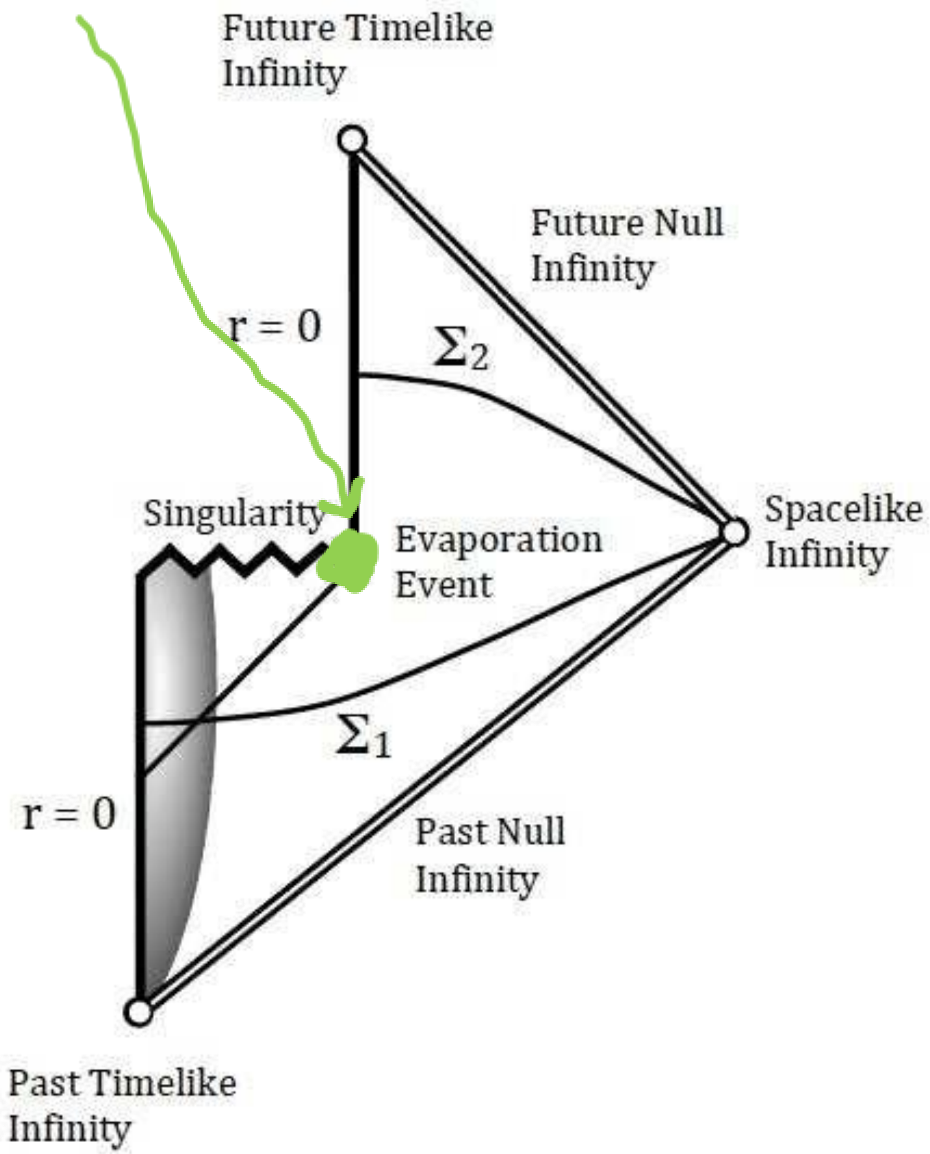
Naked singularities

The three categories can be broadly described as coming from

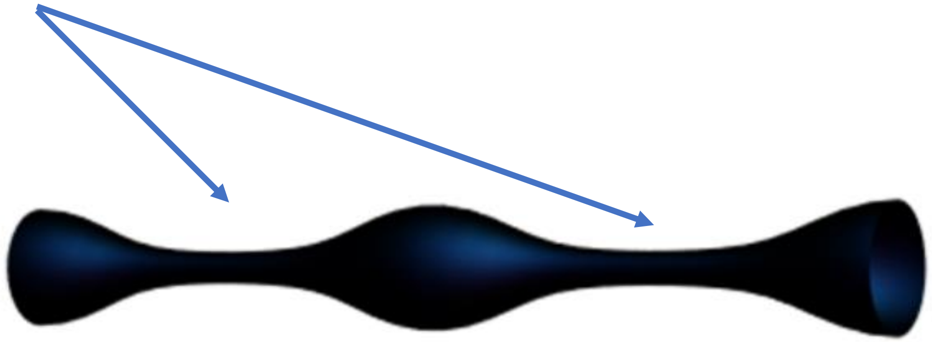
- Critical collapse
- Black hole evaporation
- Gregory-Laflamme instability

naked singularities!

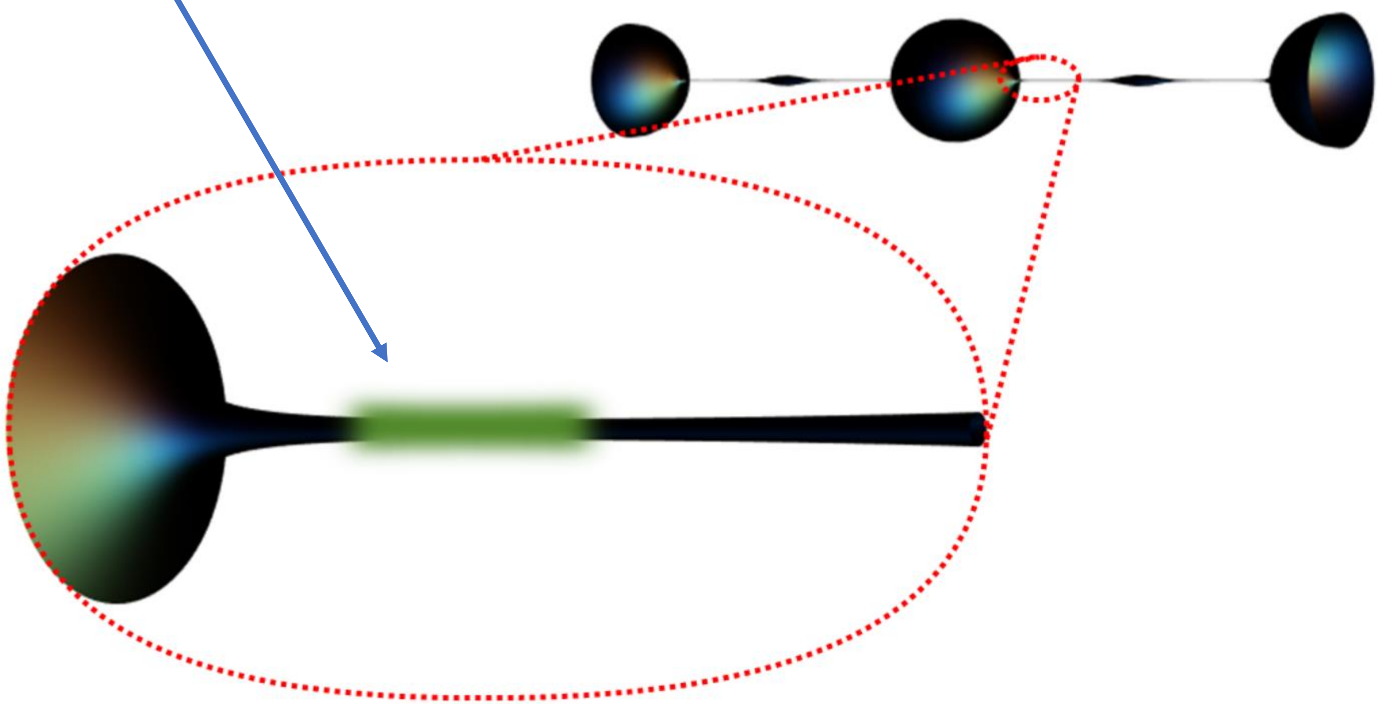




This is where the pinch happens!



The forbidden region
that needs censoring



Chih-Hung Wu + MT, in preparation

Stringy gravity

- We expect that all of these naked singularities will be "resolved" in some way by quantum gravity
- Indeed, one can show exact results in 2d for critical collapse
- But for small string coupling, ls < < lp, our gravitational EFT will first break down due to stringy physics

Our goal is then to see if we can understand the resolution of naked singularities using perturbative string theory

Focus today on **Gregory-Laflamme instability**

Tools

To address this question, we will make use of the black hole -- string correspondence principle, and crucially, the Horowitz-Polchinski (HP) solution

We will then construct new HP solutions and see what role they will play in the black string evolution

Black hole – string correspondence

- The BH-S correspondence is a very natural idea, put forth by Susskind in ‘93, and further developed by Horowitz and Polchinski in ‘96 and ’97, and Damour and Veneziano in ’99
- An observation: highly excited strings at the Hagedorn temperature have large entropy---perhaps can connect to a black hole of some sort then
- In fact, one can make a direct analogy with the Strominger-Vafa counting

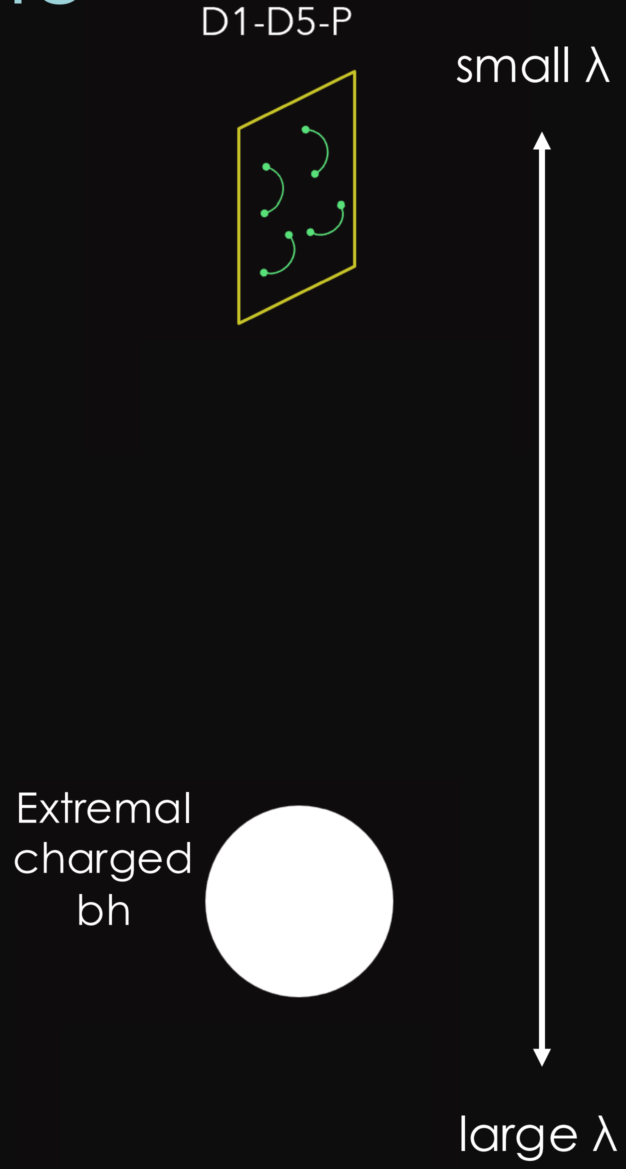
$$\delta\lambda = N\delta g$$

What comprises a (SUSY) black hole

Strominger-Vafa 1996:

- There are objects in string theory (such as D-branes) that we understand well for small string coupling g_s and **small** 't Hooft coupling $g_s*N = \lambda$
- But black holes are non-perturbative objects, with **large** 't Hooft coupling, so their understanding seems pretty much out of reach
- Cue SUSY---we can count the microscopic dofs (the index*) which will give the same answer at any value of the 't Hooft coupling λ
- This is how we can compare the two and obtain a microscopic understanding of black hole entropy

*at large λ, it is equivalent to the entropy



$\delta\lambda = S\delta g^2$

What comprises a (neutral) black hole

Horowitz-Polchinski 1996:

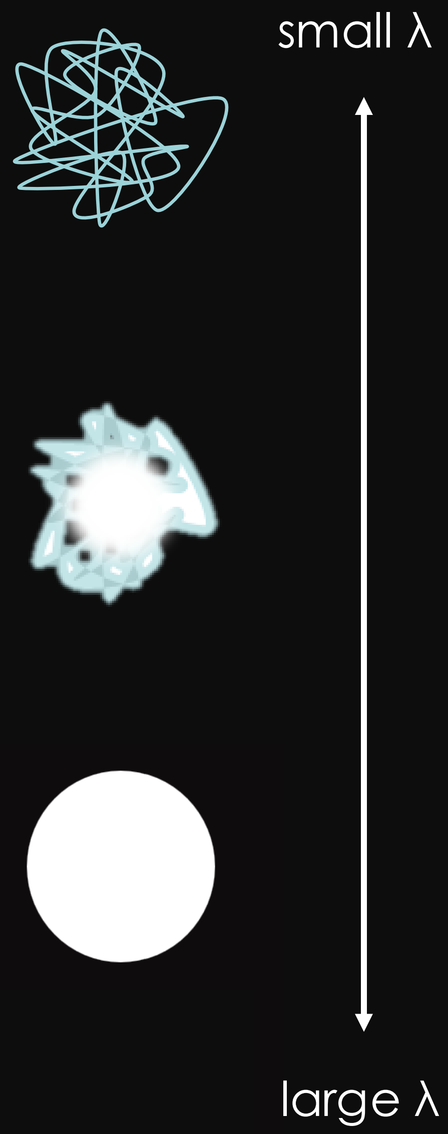
- Black holes are highly degenerate objects with a large entropy, but so are strings when highly excited!

How can we relate them without any SUSY-like protection?

simply **fix the entropy*** while changing the string coupling!

- Of course, without SUSY-like protection, the mass will get renormalized as we change the coupling---need to check that the mass changes *adiabatically* from one side to another

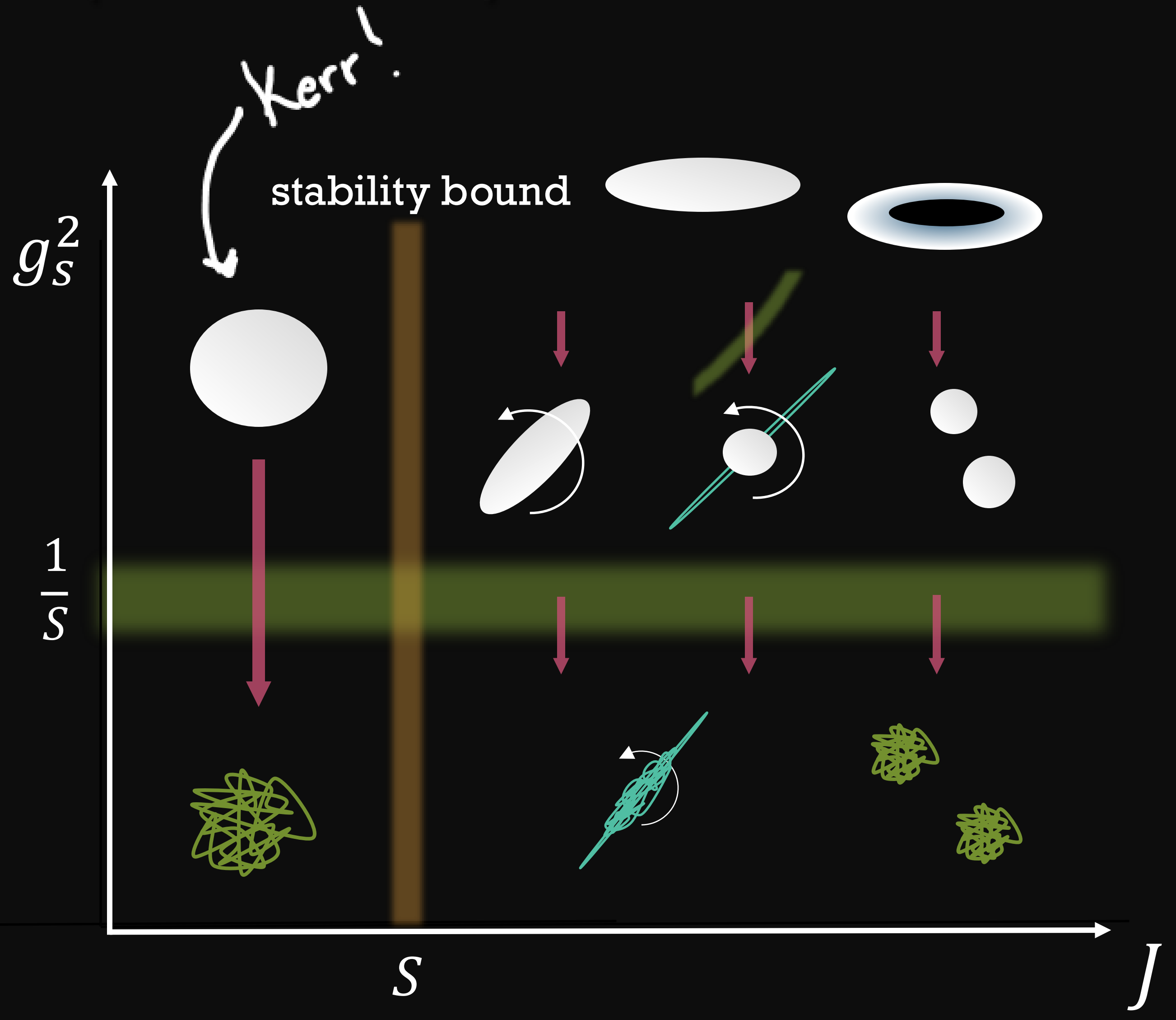
*recall, we are trying to give a description of bh microstates in terms of known states



WHAT COMPRISES A (ROTATING) BLACK HOLE

Note: we can establish the correspondence principle also for rotating black holes

Not only do their thermodynamic properties match, but the shapes and sizes do as well!



Čeplak, Emparan, Puhm, MT '23, '24

The black hole-string transition

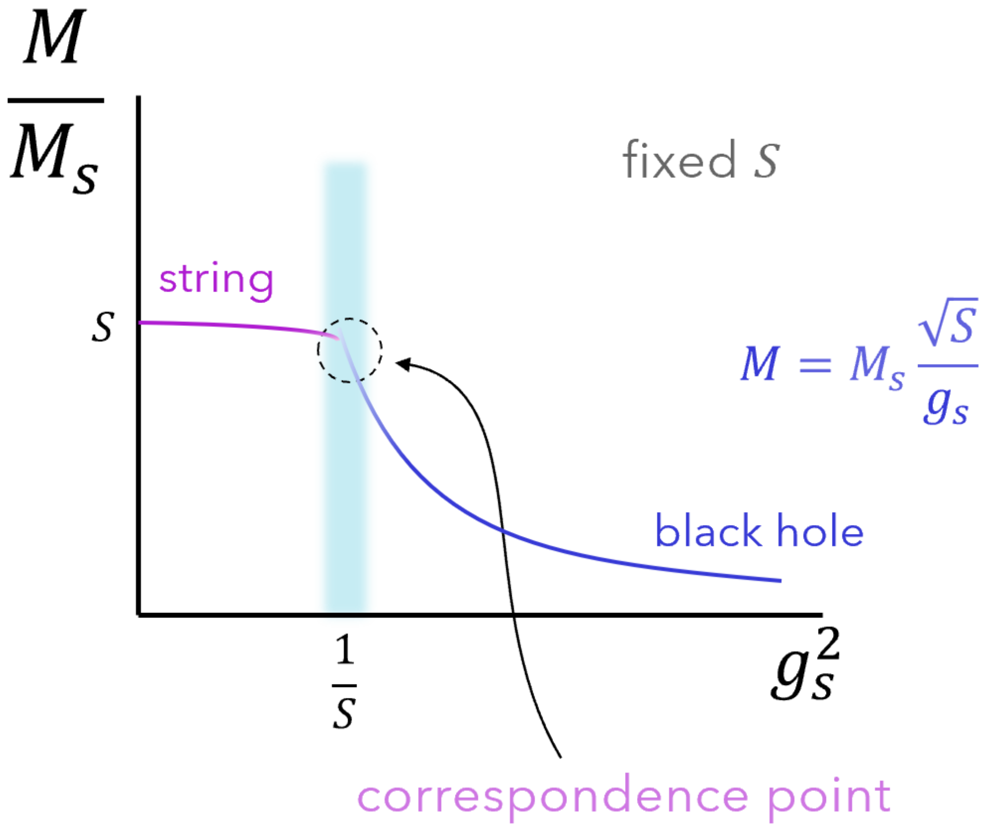
Key: adiabats of the string and the black hole meet at the correspondence point

The blue line is the line of fixed black hole mass in Mp

The pink line is the line of fixed string mass in Ms

They match up to O(1) factors for the coupling constant ~ 1/S

We can adiabatically* switch between the black hole and the fundamental string



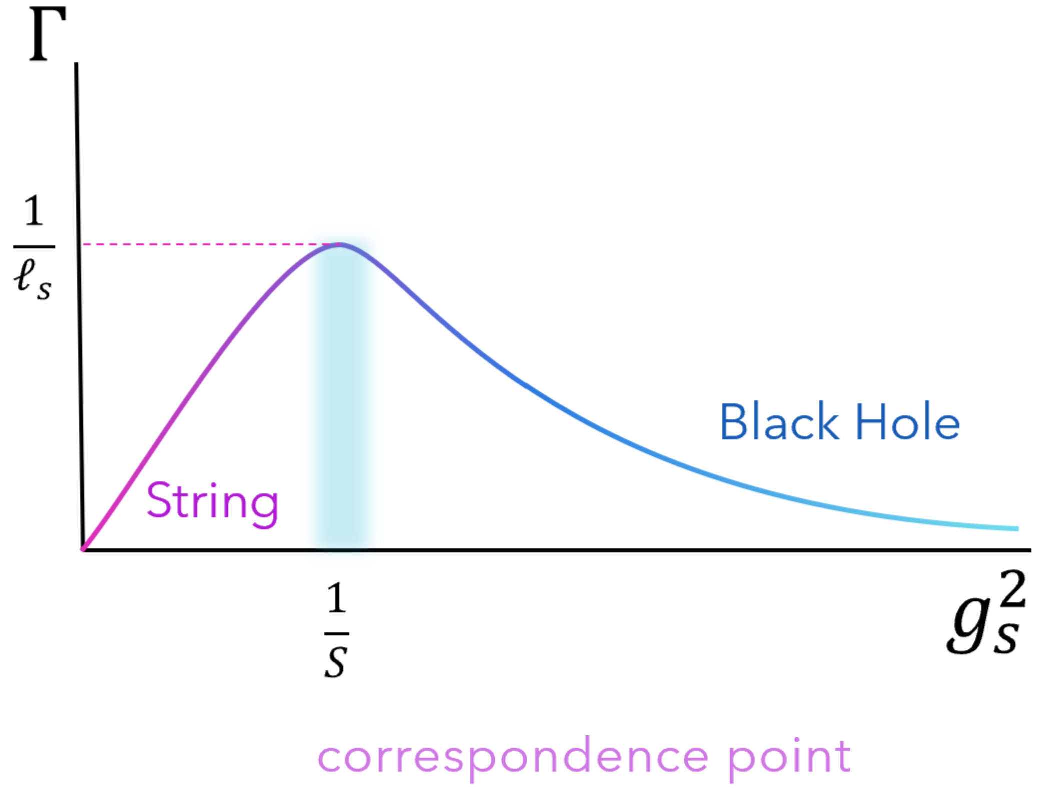
The black hole-string transition

We can adiabatically* switch between the black hole and the fundamental string

while taking quantum dynamics into account

String decay $\Gamma \sim g_s^2 M \sim g_s^2 \frac{S}{\ell_s}$

Black hole decay $\Gamma \sim T_{BH} \sim \frac{1}{g_s \sqrt{S}\, \ell_s}$



What do we need

- We want to study stringy GL
- This means that we want to study string configurations that include gravitational backreaction
- In order to compare different phases of the string, we need to perform either a dynamical evolution (hard) or a thermodynamic analysis (less hard)

- Both of these can be addressed within the **thermal scalar formalism**, which captures the essential mean-field features of a highly-excited string

Self-gravitating strings

The highly excited states of string theory near the Hagedorn temperature $T = \beta - \beta_H$ can be collectively described, in the Euclidean time formalism, by an effective mean-field χ.

This is the **winding mode** of the string around the Euclidean time circle, which becomes almost massless when its length is $\beta \simeq \beta_H$.

Being light, *this field must be added to the effective action* of string theory at low energies, which also contains the graviton and dilaton.

The coupling between the latter and χ allows to describe self-gravitating configurations of highly-excited strings, often called *string balls or string stars*.

Polchinski 86
Atick, Witten 88

Self-gravitating strings

After integrating the Euclidean time circle, the effective action in the d non-compact spatial directions becomes

$$I_d = \frac{1}{16\pi G_N} \int d^d x \sqrt{g} e^{-2\phi_d} \left(-\mathcal{R} - 4(\nabla\phi_d)^2 + (\nabla\varphi)^2 + |\nabla\chi|^2 + m(\varphi)^2|\chi|^2\right)$$

Here φ_d and g_{ab} are the d-dimensional dilaton and spatial metric. The field φ measures the length $\beta\exp(\varphi)$ of the Euclidean time circle, so φ is the gravitational potential in d dimensions.

The mass of the thermal scalar χ depends on φ and takes the value m_∞^2 at large distances where $\varphi \to 0$.

$$m(\varphi)^2 = m_\infty^2 + \frac{\kappa}{\alpha'}\varphi + \mathcal{O}(\varphi^2), \qquad m_\infty^2 = \frac{\kappa}{\alpha'}\frac{\beta - \beta_H}{\beta_H}$$

Self-gravitating strings

Very close to the Hagedorn temperature, when the winding scalar is very light, $m_\infty^2 \ll \kappa/\alpha'$, the dominant interaction is the one between φ and χ.

The dilaton φ_d and the spatial metric g_{ab} can consistently remain fixed and the field equations for φ and χ are

$$\nabla^2\chi - (\Delta_\beta + \varphi)\,\chi = 0,$$
$$\nabla^2\varphi - \frac{1}{2}\chi^2 = 0\,.$$

$$\Delta_\beta \equiv \frac{\beta - \beta_H}{\beta_H} = \frac{T_H}{T} - 1$$

These equations are the *same as for a non-relativistic boson star* where a boson condensate χ is coupled to the Newtonian potential φ.

One important difference is that our formalism is purely Euclidean, and we cannot study time-dependent fluctuations (e.g., quasi-normal modes) of the string ball.

Self-gravitating HP balls

- Let us first construct HP string balls. These correspond to spherically symmetric configurations where φ and χ vanish asymptotically and the condensate is regular at the origin,

$$\varphi(r)\,,\chi(r)\to 0 \quad \text{as } r\to\infty\,, \qquad \partial_r\chi(0)=0$$

- Our EoMs also allow a rescaling of variables: this will help us later on

$$(x^i,\chi,\varphi,\Delta_\beta)\to(\lambda^{-1/2}x^i,\lambda\chi,\lambda\varphi,\lambda\Delta_\beta)$$

- This implies that we can arbitrarily fix the overall amplitude of the condensate, e.g., by selecting a value for χ(0), and if we find a solution, then a simple rescaling gives a solution for any other amplitude.

Self-gravitating HP balls

- From the solution we can extract its mass from the asymptotic fall-off of φ,

$$\varphi(r) = -\frac{8\pi}{(d-2)\omega_{d-1}}\frac{G_N M}{r^{d-2}} + O(r^{1-d})$$

- Thus, we obtain the temperature and mass of a solution of a given amplitude. *By rescaling it, we can find the relation β(M) for the string ball states in d dimensions.* Observe that the combination

$$\frac{G_N M}{\Delta_\beta^{(4-d)/2}} \equiv \mathrm{g}_d^{-(4-d)/2}$$

is a pure number (in string units) that is invariant under the rescaling of EoMs

Self-gravitating HP balls

*at the leading order of χ interactions

- If we compute it in some (arbitrary) reference solution $(\varphi_0(r), \chi_0(r))$ with mass M_0 and temperature $\Delta_{\beta 0}$, then the mass and temperature of any other solution are related by

$$G_N M = \left(\frac{\Delta_\beta}{\Delta_{\beta 0}}\right)^{\frac{4-d}{2}} G_N M_0 = \left(\frac{\Delta_\beta}{\mathrm{g}_d}\right)^{\frac{4-d}{2}} \longrightarrow \frac{\beta}{\beta_H} = 1 + \mathrm{g}_d (G_N M)^{\frac{2}{4-d}}$$

- From here, we can integrate the first law to obtain the entropy

Note that HP balls exist only* for d = 3, 4, 5!

$$S(M) = \int \beta(M) dM \longrightarrow S_b = \beta_H M + \mathrm{g}_d \frac{d-4}{d-6} M^{\frac{d-6}{d-4}}$$

- Let us also restore units for a second

$$\frac{S}{\beta_H M} - 1 \propto \frac{d-4}{d-6}\left(g^2 \frac{M}{M_s}\right)^{\frac{2}{4-d}}$$

The size of the corrections is measured by the 't Hooft-like coupling $g^2 S$

Self-gravitating HP balls

- Notice a peculiar feature: we obtained this entropy from a purely classical analysis
- Another way to obtain is directly from the effective action $S = (1 - \beta\partial_\beta)$ (-I)
- This is emphasizing that this entropy is a *classical* entropy---just like the entropy of a black hole!
- This represents some compelling evidence that the thermal scalar formalism and the string star have something in common with black hole physics
- Of course, we don't know all the details of black hole microscopics, but in the thermal scalar formalism, it is clear that the entropy is a result of a mean-field theory approach---perhaps gravity is doing something similar?

Chen, Maldacena, Witten 21

Self-gravitating HP strings

- To construct the stringy string solutions, we proceed in a similar manner as before, with the assumption that our solutions are cylindrical, not spherical, with $z \sim z + L$
- One writes down the scaling again and obtains the solutions for uniform strings as a translationally invariant ball solution in one dimension less.

- We can already make some heuristic predictions just based on the *uniform* string solution, similar to the original GL argument for black strings.

see also Chu '24, '25

Black string thermodynamics

- The entropy of a black hole in D=d+1 dimensions takes the form

$$S_{BH} = c_d M^{\frac{d-1}{d-2}}$$

- Now we imagine that this black hole is localized in a circle of length L and neglect the finite-size distortions that would modify the entropy formula above.
- For a black string in the same number of dimensions, the formula above applies after replacing d → d-1and scaling S → S/L and M → M/L, so the entropy is

$$S_{BS} = c_{d-1} M^{\frac{d-2}{d-3}} L^{-\frac{1}{d-3}}$$

- It is now clear that if we compare a black hole and a black string of the same mass, then for L sufficiently larger than M, it will be entropically favorable for a *black string to transition into a localized black hole*.

Stringy string thermodynamics

- The entropies of an HP ball and HP string are given by

$$S_b = \beta_H M + g_d \frac{d-4}{d-6} M^{\frac{d-6}{d-4}}$$

$$S_s = \beta_H M + g_{d-1} \frac{d-5}{d-7} M^{\frac{d-7}{d-5}} L^{\frac{2}{d-5}}$$

Now we have a dimension-dependent situation:

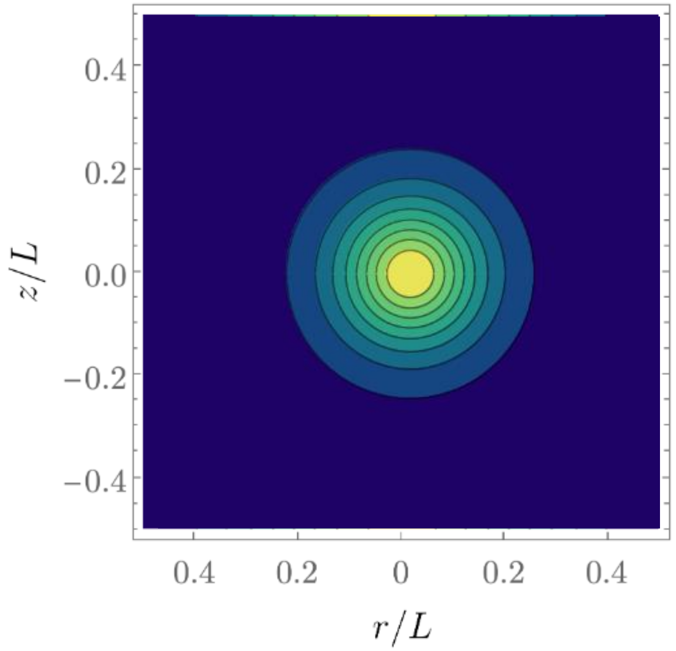
- **d = 4:** the correction for a string is $\sim +1/L^2$ while the ball receives no correction. So, for any nonzero length L the *string* is always more entropic than the ball and therefore *will be thermodynamically preferred*
- **d = 5:** the ball entropy is now corrected by a negative term, while the string receives no correction. So, again, *the string is thermodynamically favored*.
- **d = 6:** the string entropy is now reduced by $\sim - L^2$. The approximations should break down for large enough L, but, in any case, in this dimension, the string cannot evolve into a localized string ball since the latter does not exist.

Self-gravitating HP strings

- So, from the thermodynamic argument we see that the strings dominate over ball solutions!
- Bear in mind that this is valid for very large L's, and also, it cannot tell us if there are any stable non-uniform solutions
- So, we must (numerically) construct the non-uniform solutions
- We start by finding the *zero mode* of the uniform string.
- This is a small, linearized perturbation, which signals the appearance of a family of non-uniform string configurations, namely the non-linear extensions of the zero mode.

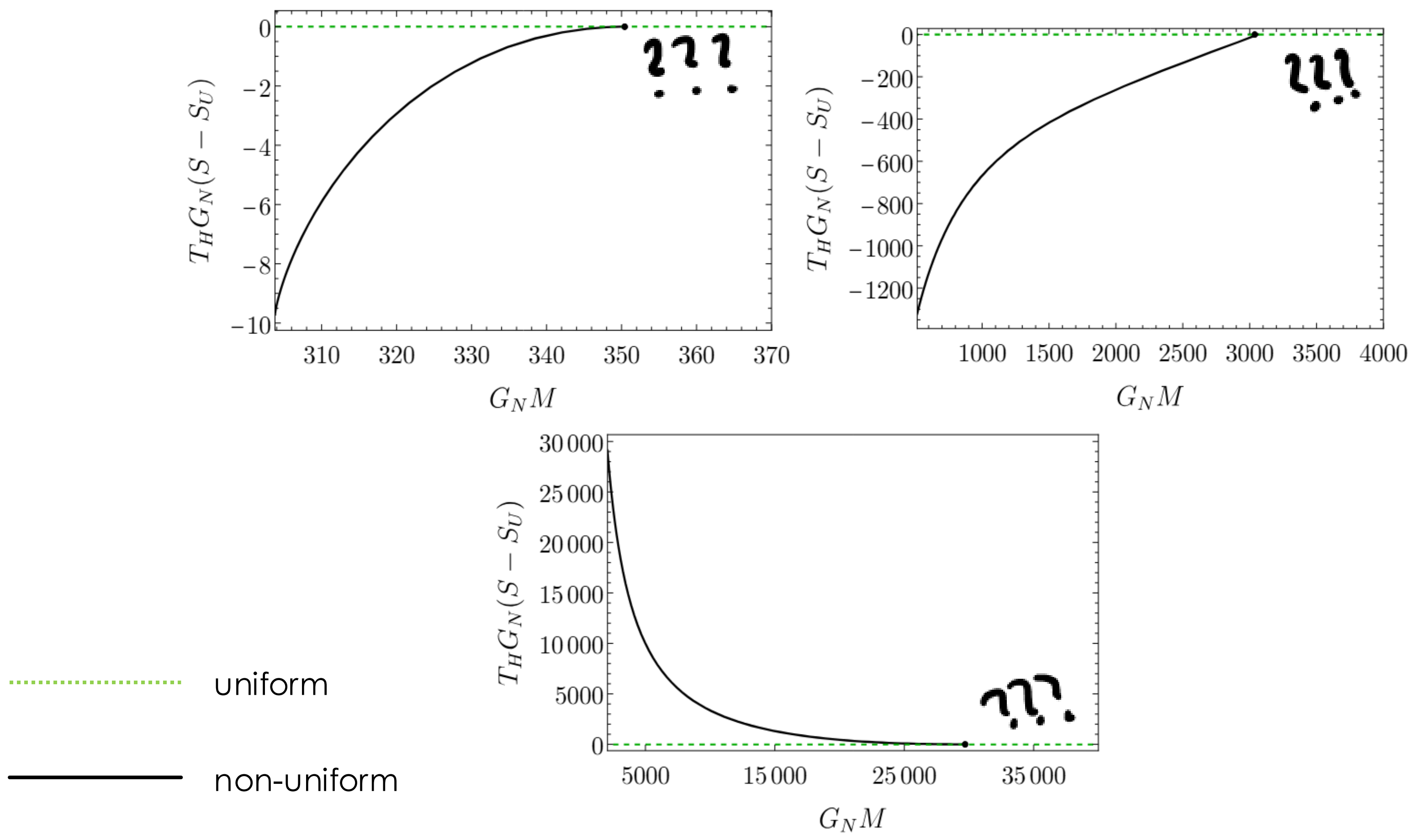
Self-gravitating HP strings

Using relaxation methods, we obtain a family of non-uniform stringy strings, which tend to a higher-dimensional ball solution for large enough L



Contour plots of χ of the non-uniform string configurations in d = 5 for increasing L

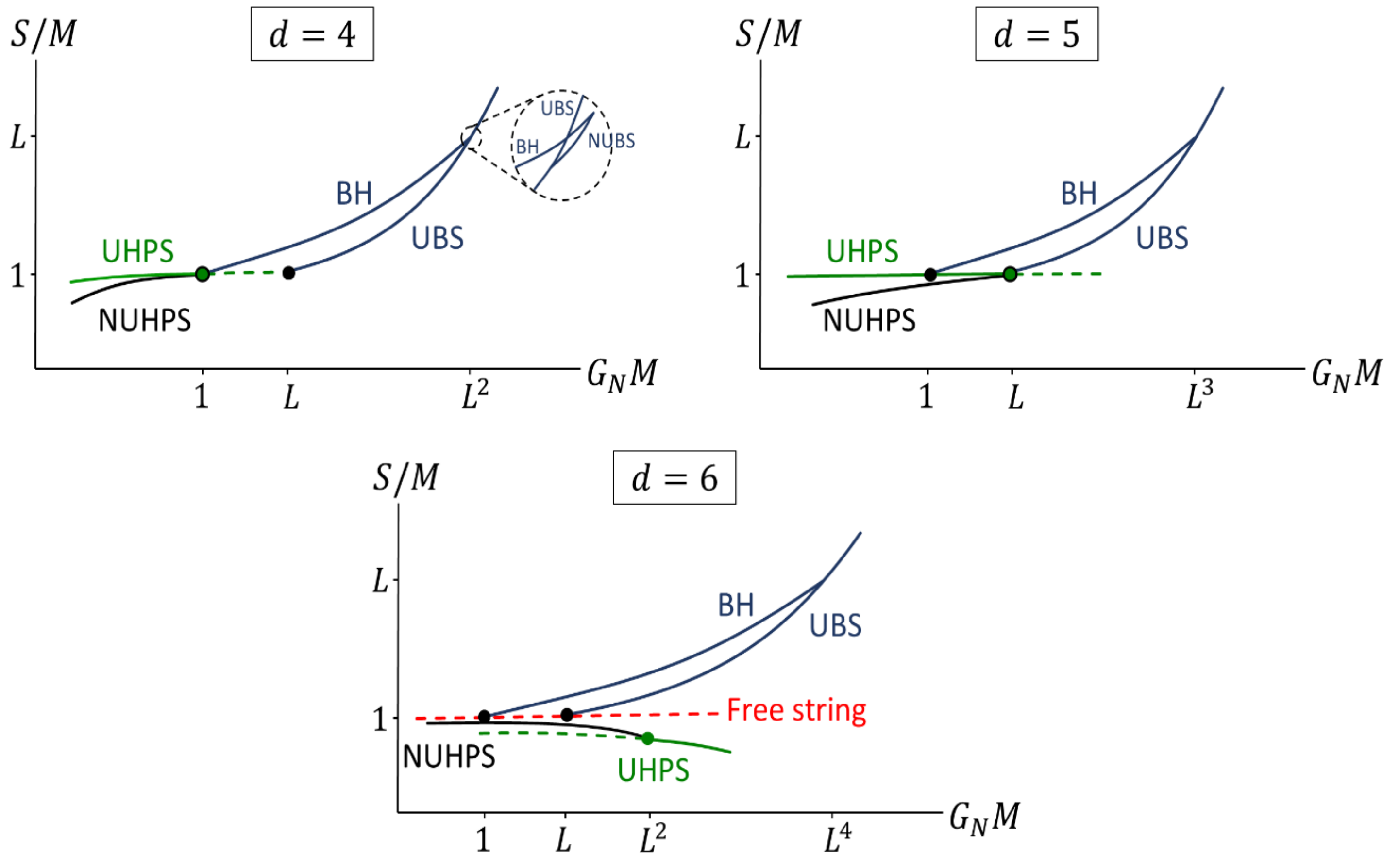
Self-gravitating HP strings



- Let’s see what does the thermodynamics tell us
- <u>For the masses</u>: in d = 4, 5 the non-uniform string tends to the ball solution, whereas for d = 6, the approximation breaks down at L = 1
- <u>For the entropies</u>: in d = 4, 5 the uniform string dominates over the non-uniform ones, while this behavior is reversed for d = 6

Adding black holes to the mix

- Note that this is a fairly strange scenario: the UHP evolves into a black object again!
- Nevertheless, it takes care of the singularity---UHP smoothed out the transition between the black string and a black hole



Figueras, Kovacs, Yao xx

Conclusion and a proposal

So what can we say about the endpoint of the GL instability of black strings then?

- Our proposal is that

Stringy physics slows down the GL instability, such that the string star simply evaporates at the Hagedorn temperature and fizzles out

- In fact, there is some preliminary evidence from Figueras et al. that this might be the case from the black hole side as well:

They do a full non-linear evolution of a black string with higher curvature corrections (EGB in 5d) and see that the GL instability switches off!

- **Key point:** the higher curvature corrections are of the type found in string theory

Summary and outlook

We studied a stringy version of the Gregory-Laflamme instability

We constructed stringy strings using the thermal scalar formalism and we obtain the relevant thermodynamic phases of this object

Unlike black strings, stringy strings do not lead to a pinch-off; instead, they settle on a string star: either spherical or extended

We propose this string star will evaporate away, providing a smoothening of the GL naked singularity

Outlook: given that the critical collapse singularity and the GL one share a host of similarities (crucially, the same symmetry) one should be able to see a slow-down of the critical collapse as well.

This way, we will have a resolution of all known classical naked singularities

Thank you!