Musings on the 3d black string

Workshop on Black Holes, Holography and de Sitter

Milan, 13-15 January 2025

Stéphane Detournay

Based on 2412.16136 with José Figueroa & Alejandro Vilar López

and 1911.12359 with Luis Apolo & Wei Song

Plan

I. The 3d black string: what and why?

II. Boundary conditions through "Asymptotic T-duality"

(see also Spindel 1810.00603
SD, Petropoulos, Zwikel 1812.08764)

III. Black strings from TsT, marginal and $T\bar{T}$ deformations

I. The 3d black string

"Exact Black Strings Solutions in Three dimensions"

[Horne-Horowitz, hep-th/9108001]

The authors introduced charged black strings:

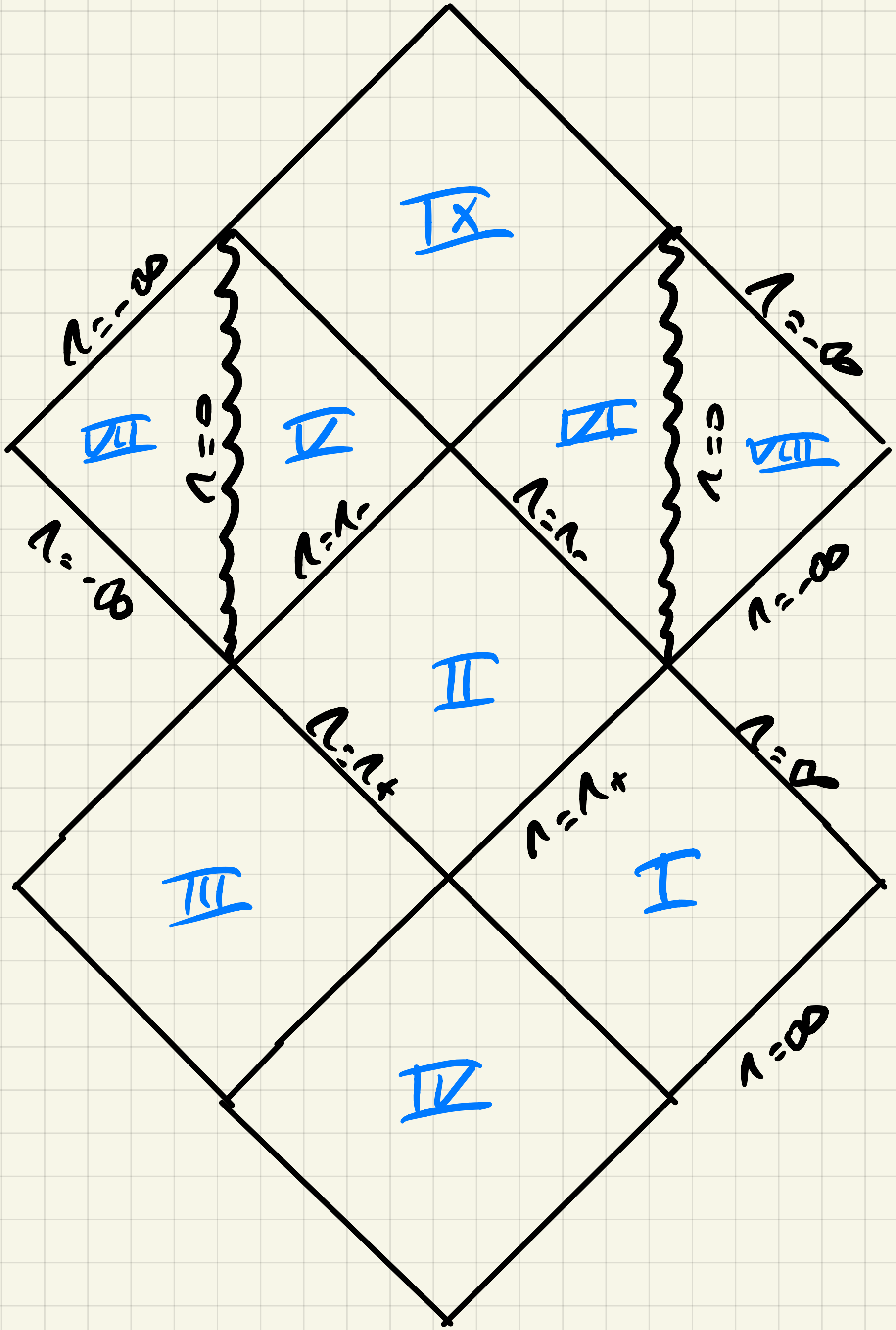
$$
\begin{cases}
ds^2 = -\left(1-\frac{\underbrace{M}_{\text{mass}}}{r}\right)dt^2 + \left(1-\frac{Q^2}{Mr}\right)dx^2 + \left(1-\frac{M}{r}\right)^{-1}\left(1-\frac{\underbrace{Q^2}_{\text{charge}}}{Mr}\right)^{-1}\underbrace{k}_{\text{level}}\,\frac{dr^2}{8r^2} \\
H_{rtx} = \frac{Q}{r^2} \\
\Phi = \ln r + \frac{1}{2}\ln\frac{k}{2}
\end{cases}
$$

Target space of a gauged WZW model

Solution to the low energy string action EOM derived from

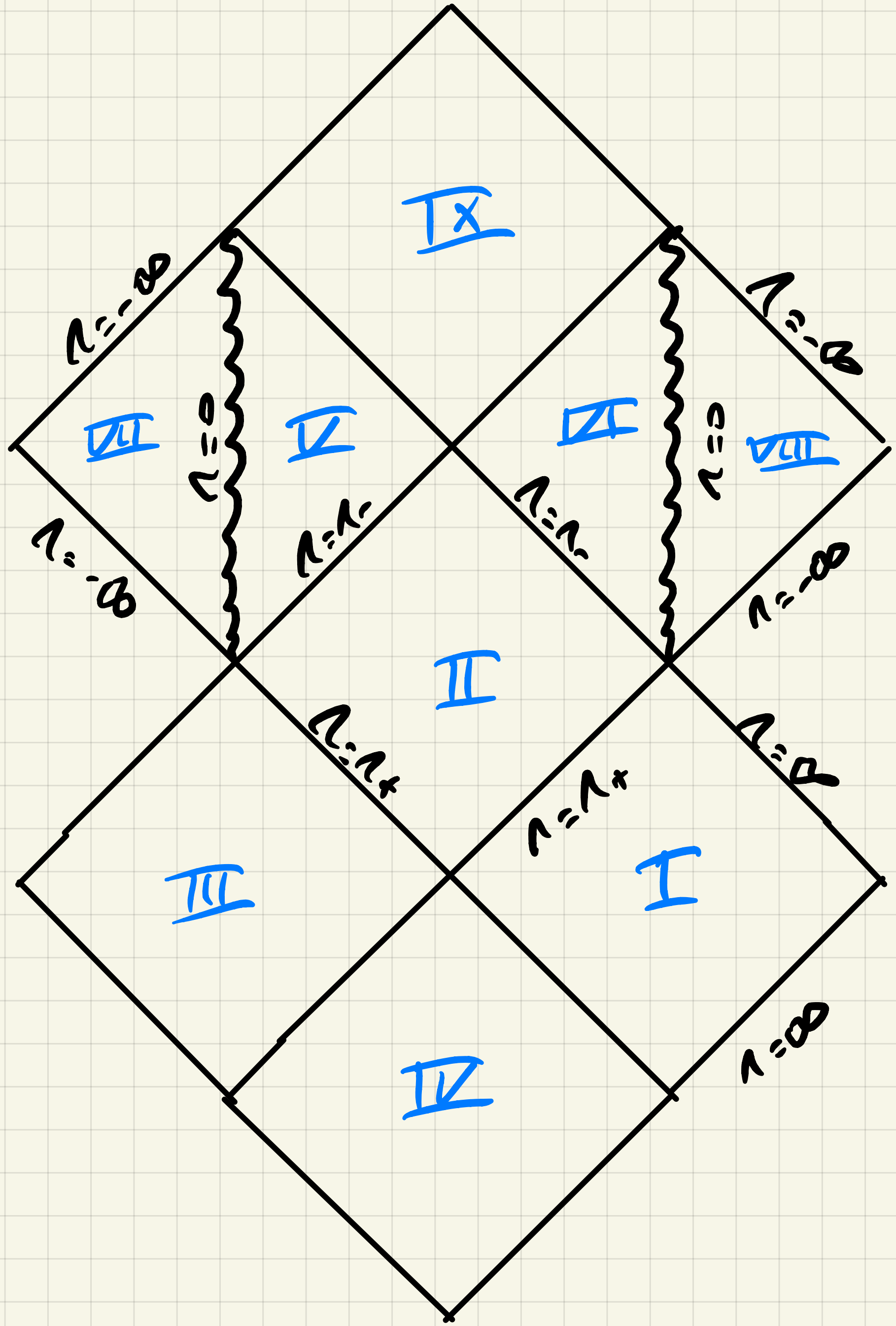
$$S = \int d^3x\sqrt{-g}\, e^{-2\Phi}\left[R + 4(\nabla\Phi)^2 - \frac{1}{12}H^2 + \frac{4}{k}\right]$$

Rather simple exact CFT producing a background qualitatively similar to Reissner-Nordström in (2+1) dimensions



- Outer and inner horizons at $r_+ = M$, $r_- = \frac{Q^2}{M}$
- Curvature singularity at $r=0$
- Ricci scalar $\xrightarrow{r\to\infty} 0$, "asymptotically flat"
- Hawking $T°$: $T_H = \frac{1}{\pi M}\sqrt{\frac{M^2-Q^2}{2R}}$
- Bekenstein - Hawking entropy :

$$S_{BH} \sim \frac{1}{G}\sqrt{M^2-Q^2}$$



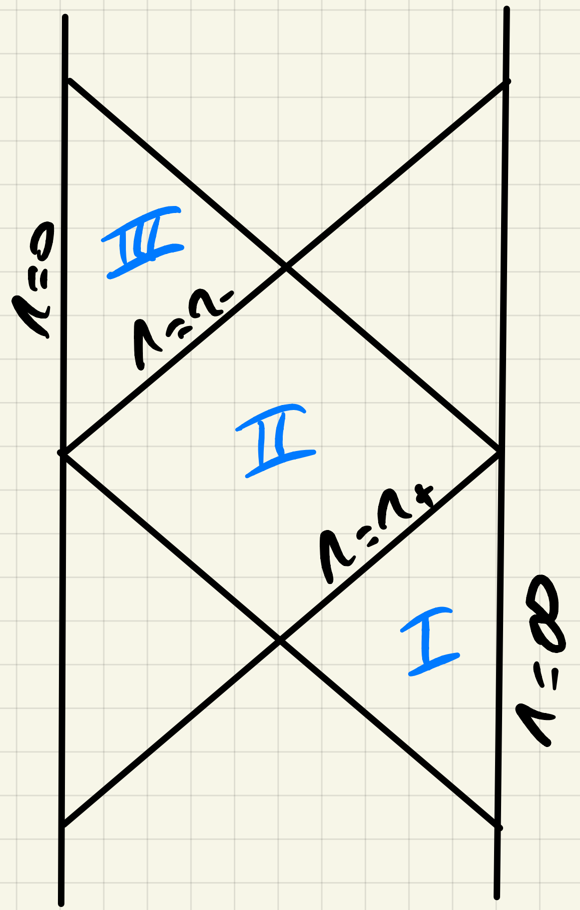
- Outer and inner horizons at $r_+ = M$, $r_- = \frac{Q^2}{M}$
- Curvature singularity at $r = 0$
- Ricci scalar $\xrightarrow{r \to \infty} 0$, "asymptotically flat"
- Hawking $T°$: $T_H = \frac{1}{\pi M}\sqrt{\frac{M^2 - Q^2}{2R}}$
- Bekenstein - Hawking entropy :

$$S_{BH} \sim \frac{1}{G}\sqrt{M^2 - Q^2}$$

$$= \log \rho \; ?$$

BTZ black holes stealing the show

[Bañados, Teitelboim, Zanelli, '92]



AdS/CFT

$$N_{HH} \ll N_{BTZ}$$

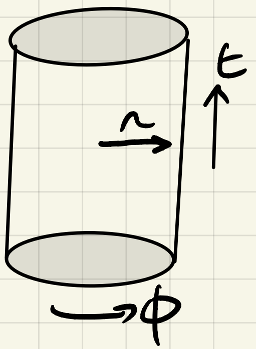
citations to HH = 226

citations to BTZ = 3507

BTZ black holes & AdS_3 gravity

Solutions to pure 3d gravity w/ $\Lambda = -1/\ell^2 < 0$

$$\boxed{ds^2 = \underbrace{\left(M - \frac{r^2}{\ell^2}\right)}_{\text{mass}} dt^2 - \underbrace{J}_{\text{angular momentum}}\, dt\, d\phi + r^2 d\phi^2 + \left(\frac{r^2}{\ell^2} - M + \frac{J^2}{4r^2}\right)^{-1} dr^2}$$



$x^{\pm} = \frac{t}{\ell} \pm \phi$

Asymptotically AdS_3 solutions

BTZ belong to a phase space endowed with 2d conformal symmetry generated by $l_n^{\pm} = e^{inx^{\pm}}(\partial_{\pm} - i n r \partial_r)$

The corresponding charges satisfy a Virasoro algebra with

$$\boxed{c = \frac{3\ell}{2G}}$$

[Brown-Henneaux '86]

Suggests that quantum gravity in AdS_3 would be a CFT_2

Observation that the Bekenstein – Hawking entropy of BTZ matches that of a hot gas of strongly interacting particles in a CFT_2:

$$S_{BH} = \frac{\text{Area}}{4G} = \frac{2\pi r_+}{4G} = 2\pi\sqrt{\frac{c L_0^+}{6}} + 2\pi\sqrt{\frac{c L_0^-}{6}}$$

Cardy formula

Also: BH perturbations/QNM, correlation functions, EE, ...

Suggests that quantum gravity in AdS_3 would be a CFT_2

Observation that the Bekenstein-Hawking entropy of BTZ matches that of a hot gas of strongly interacting particles in a CFT_2:

$$S_{BH} = \frac{\text{Area}}{4G} = \frac{2\pi r_+}{4G} = \underbrace{2\pi\sqrt{\frac{c L_0^+}{6}} + 2\pi\sqrt{\frac{c L_0^-}{6}}}_{\text{Cardy formula}}$$

Also: BH perturbations/QNM, correlation functions, EE, ...

[Comment: Boundary conditions are generically not unique, and finding them is sometimes a bit of an art.

They can depend on the bulk theory at hand. Even within Einstein gravity, there exist alternatives to Brown-Henneaux BCs with different symmetries: $Vir \oplus Vir$, $Vir \oplus u(1)$, $(Vir \oplus u(1))^2$, $u(1) \oplus \hat{u}(1)$, $Vir \oplus \hat{sl}(2)$, $\hat{sl}(2) \oplus \hat{sl}(2)$, ...

[Many authors; see e.g. Grumiller - Riegler 1608.01308]

Suggests that quantum gravity in AdS_3 would be a CFT_2

with Brown-Henneaux BCs

Observation that the Bekenstein-Hawking entropy of BTZ matches that of a hot gas of strongly interacting particles in a CFT_2:

$$S_{BH} = \frac{\text{Area}}{4G} = \frac{2\pi r_+}{4G} = \underbrace{2\pi\sqrt{\frac{c L_0^+}{6}} + 2\pi\sqrt{\frac{c L_0^-}{6}}}_{\text{Cardy formula}}$$

Also: BH perturbations/QNM, correlation functions, EE, ...

[Comment: Boundary conditions are generically not unique, and finding them is sometimes a bit of an art.

They can depend on the bulk theory at hand. Even within Einstein gravity, there exist alternatives to Brown-Henneaux BCs with different symmetries: $Vir \oplus u(1)$, $(Vir \oplus u(1))^2$, $\hat{u}(1) \oplus \hat{u}(1)$, $Vir \oplus \hat{sl}(2)$, $\hat{sl}(2) \oplus \hat{sl}(2)$, ...

$Vir \oplus Vir$

[Many authors; see e.g. Grumiller-Riegler 1608.01308]
]

II. Boundary conditions for the black string :

Asymptotic T-duality

Back to black string: relation to BT2

T-duality / Buscher's rules is/are a symmetry that maps any solution $(g_{\mu\nu}, B_{\mu\nu}, \Phi)$ of the low energy string equations with a translational symmetry $\hookrightarrow \partial_x$ to another solution $(\tilde{g}_{\mu\nu}, \tilde{B}_{\mu\nu}, \tilde{\Phi})$ with

$$\begin{cases} \tilde{g}_{xx} = 1/g_{xx} \,, \quad \tilde{g}_{x\alpha} = B_{x\alpha}/g_{xx} \\ \tilde{g}_{\alpha\beta} = g_{\alpha\beta} - (g_{x\alpha} g_{x\beta} - B_{x\alpha} B_{x\beta})/g_{xx} \\ \tilde{B}_{x\alpha} = g_{x\alpha}/g_{xx} \qquad \tilde{B}_{\alpha\beta} = B_{\alpha\beta} - 2 g_{x[\alpha} B_{\beta]x}/g_{xx} \\ \tilde{\Phi} = \Phi - \frac{1}{2} \ln g_{xx} \end{cases}$$

Disclaimer: in what follows, "T-duality" will stand for its low-energy manifestation, the Buscher rules

Back to black string: relation to BTZ

Horowitz and Welch showed that T-dualizing a BTZ BH with mass M and angular momentum J along ∂_ϕ resulted in a 3d black string with mass $\mathcal{M} = \frac{r_+^2}{\ell}$ and charge $Q = \frac{J}{2}$.

$M = \frac{r_+^2 + r_-^2}{\ell}$

$J = \frac{2 r_+ r_-}{\ell}$

$$ds^2 = \left(M - \frac{r^2}{\ell^2}\right) dt^2 - J\, dt\, d\phi + r^2 d\phi^2 + \left(\frac{r^2}{\ell^2} - M + \frac{J}{4r^2}\right)^{-1} dr^2$$

$\downarrow$ T-duality

$$ds^2 = -\left(1 - \frac{\mathcal{M}}{\rho}\right) dt^2 + \left(1 - \frac{Q^2}{\mathcal{M}\rho}\right) dx^2 + \left(1 - \frac{\mathcal{M}}{\rho}\right)^{-1} \left(1 - \frac{Q^2}{\mathcal{M}\rho}\right)^{-1} \frac{\ell^2 d\rho^2}{4\rho^2}$$

Looking for boundary conditions encompassing the 3d black string, is there a sense in which we could T-dualize the Brown-Henneaux boundary conditions?

zero modes

bdry condition

$$
\begin{tikzcd}
\boxed{\text{BTZ BH}} \arrow[rr, leftrightarrow, "\text{T-dual}"] & & \boxed{\text{Black string}} \\
\boxed{\text{Brown-Henneaux}} \arrow[u, "\cap"'] \arrow[rr, leftrightarrow, "?"] & & \boxed{?} \arrow[u, "\cap"']
\end{tikzcd}
$$

$\boxed{\text{T-duality}}$ requires the existence of an exact KV to map one solution onto another.

Here, we will look for a generalization, $\boxed{\text{asymptotic T-duality}}$, using an asymptotic KV to map a set of boundary conditions onto another.

Warm-up: the dual of chiral Brown-Henneaux

The family of Bañados metrics constitute a gauge-fixed, on-shell version of Brown-Henneaux: $(g_{rr} = \frac{\ell^2}{r^2}, g_{ra} = 0)$

$\{x^a\} = x^{\pm} = t \pm \phi$

$$ds^2 = \ell^2 \frac{dr^2}{r^2} - r^2 \left(dx^+ - \frac{\ell^2}{r^2} Y_{--}(x^-) dx^-\right)\left(dx^- - \frac{\ell^2}{r^2} Y_{++}(x^+) dx^+\right)$$

They solve the low energy string EOM when supplemented by

$$\begin{cases} B = \left(\frac{r^2}{c_0} + b(x^a) + \frac{\ell^2}{c_0 r^2} Y_{++} Y_{--}\right) dx^+ \wedge dx^- \\ \Phi = \frac{1}{4} \log \frac{2}{c_0} = \text{cst.} \end{cases}$$

[see also Du, Liu, Song 2403]

The corresponding asymptotic symmetries are the usual Brown-Henneaux diffeos:

$$\xi[T^+, T^-] = T^a \partial_a - \frac{r}{2} \partial_a T^a \partial_r + O(1/r^2)$$

with Virasoro charges

$$Q_\xi = \frac{\ell}{8\pi G} \int d\phi \left[Y_{++}(x^+) T^+(x^+) + Y_{--}(x^-) T^-(x^-)\right]$$

and modes $L_n^{\pm} \equiv Q_{l_n^{\pm}}$ with $l_n^{\pm} = \xi[e^{inx^{\pm}}]$ (+ global B-charge)

Warm-up: the dual of chiral Brown-Henneaux

To be able to T-dualize, we restrict the phase space to

$$\begin{cases} ds^2 = \ell^2 \dfrac{dr^2}{r^2} - r^2\left(dx^+ - \dfrac{\ell^2}{r^2}\, \overset{\text{cst.}}{y_{--}}\, dx^-\right)\left(dx^- - \dfrac{\ell^2}{r^2} Y_{++}(x^+)\, dx^+\right) \\ B = \left(\dfrac{r^2}{c_0} + b(x^+) + \dfrac{\ell^4}{c_0 r^2} Y_{++}(x^+)\, y_{--}\right) dx^+ \wedge dx^- \\ \Phi = \dfrac{1}{4} \log \dfrac{2}{c_0} \end{cases}$$

ASG generated by L_n^+
chiral Virasoro

and apply Buscher along ∂_-. One gets

$$\begin{cases} ds^2 = \dfrac{\tilde\ell^2}{\rho^2} d\rho^2 - \rho^2 dx^+\left(dx^- - P(x^+)\, dx^+\right) + \hat\ell^2\left[L(x^+)(dx^+)^2 + \Delta\left(dx^- - P(x^+)\, dx^+\right)^2\right] + O(1/\rho^2) \\ B = \left(\dfrac{\rho^2}{\hat c_0} + O(1/\rho^2)\right) dx^+ \wedge dx^- \\ \Phi = \dfrac{1}{4} \log \dfrac{2}{\hat c_0} \end{cases}$$

This no longer satisfies Brown-Henneaux (e.g. $g_{++} \sim \rho^2 P(x^+) \Leftrightarrow O(1)$)

These are the Compère-Song-Strominger BCs preserved by

$$\xi[\epsilon, \sigma] = \overset{L_n\ \text{Vir}}{\epsilon(x^+)\, \partial_+} - \frac{\rho}{2}\epsilon'(x^+)\, \partial_\rho + \overset{P_n\ u(1)}{\sigma(x^+)\, \partial_-}$$

ASG generated by L_n
P_n, Vir $\ltimes$ $u(1)$

Lessons:

- T-duality has changed the asymptotic behaviour
- T-duality produces an exchange between gauge separating transformations and diffeos
- Obtained different ASG!

Dual boundary conditions: a phase space for the black string

Start with Brown–Henneaux BCs:

$$\begin{cases} ds^2 = \frac{\ell^2}{r^2} dr^2 + r^2 \left(\eta_{ab} + \frac{\ell^2}{r^2} \gamma_{ab}(x^a) + \dots \right) dx^a dx^b \\ B = \left(\frac{r^2}{c_0} + b(x^a) + \dots \right) dx^+ \wedge dx^- \qquad \left(x^\pm = \frac{t}{\ell} \pm \phi \right) \\ \Phi = \frac{1}{4} \log\left(\frac{r}{c_0}\right) \end{cases}$$

Procedure: pick an exact KV of the leading components. Here, we pick

$$\eta = \frac{\partial_+ + \partial_-}{\ell} = \frac{\partial_\phi}{\ell}$$

because we know that for the BTZ contained above, this will give the black string. Applying Buscher rules, one obtains:

$$\begin{cases} ds^2 = \left(1+\frac{F(x^a)}{\hat{r}}+\dots\right)d\hat{r}^2+\hat{r}^2\left(M_{ab}(x^a)+\frac{1}{\hat{r}}Z_{ab}(x^a)+\dots\right)dx^a dx^b \\ B = O(1/\hat{r})\, dy\wedge dw \\ \Phi = \frac{\hat{r}_0}{\hat{r}^n}\left(1+\frac{\varphi(x^a)}{\hat{r}}+\dots\right) \end{cases}$$

$(\hat{r}\sim r^2,\ y\sim t,\ w\sim\phi)$

with

$$(M_{ab}) = \begin{pmatrix} A(x^a) & -1/2 \\ -1/2 & 0 \end{pmatrix} \qquad (Z_{ab}) = \begin{pmatrix} Z_{yy}(x^a) & Z_{yw}(x^a) \\ Z_{yw}(x^a) & \ell/4 \end{pmatrix}$$

Some comments:

$\nearrow Y_{ab},\ b$

- Some subleading components in the original BCs become leading after dualizing ($M_{ab}\leftarrow A(x^a)$), so the EOM will impose restrictions on them

  $\hookrightarrow$ fluctuating boundary metric

- By construction, the black strings are included and correspond to $Y_{++}=L_+=\text{cst},\ Y_{--}=L_-=\text{cst}$. (which translates into values for $F, M, Z, \dots$)

Dual Asymptotic Symmetry Group

The transformations preserving the previous boundary conditions are

$$\xi[R,Q,T,S] = \left[R(z) + w\,Q(z) + \frac{w^2}{8}T''(z) + \dots\right]\partial_{\hat{r}} + \left[T(z) + \dots\right]\partial_z + \left(S(z) - w\,T'(z) + \dots\right)\partial_w$$

Expanding in modes, one gets 4 towers of generators t_n, r_n, s_n, q_n whose corresponding charges are non-trivial, finite, integrable on-shell:

$$\mathcal{R} = Q_{\xi[R,0,0,0]} \sim \int dz\, A_1(z)\, R(z)$$

$$[A(z,w) \simeq A_0(z) + w A_1(z)]$$

$$\mathcal{Q} = Q_{\xi[0,Q,0,0]} \sim \int dz\, A_0(z)\, Q(z)$$

$$\mathcal{T} = Q_{\xi[0,0,T,0]} \sim \int dz \left[(Z_{zz} + 2A\,Z_{zw})T - \frac{w}{2}(\dots)T'\right]$$

$$\mathcal{S} = Q_{\xi[0,0,0,S]} \sim \int dz\, \left(F + \varphi - 2Z_{zw} + \frac{l}{2}A\right) S(z)$$

Expanding in modes, we get the following algebra:

$$\left(\bar{S}_m = S_m - \frac{Q_m}{8} \right)$$

$$\left([L_m, O_n] = [(h-1)m - n]\, O_{m+n} \right) \qquad h =$$

$$\begin{cases} [T_n, T_m] = (n-m)\, T_{n+m} & \\ [T_m, R_n] = -n R_{n+m} + \frac{2\pi\ell}{G} m^2 \delta_{m+n} & \text{WCFT, Vir} \ltimes \hat{u}(1) \qquad 1 \\ [T_m, Q_n] = (m-n)\, Q_{m+n} & \text{BMS}_3 \qquad 2 \\ [T_m, \bar{S}_n] = -(m+n)\, \bar{S}_{m+n} & \text{BMS}_2 \qquad 0 \\ [\bar{S}_m, Q_n] = i R_{m+n} - \frac{2\pi\ell}{G} m\, \delta_{m+n,0} & \end{cases}$$

↳ loop algebra of the Heisenberg algebra $[q,p] = i\hbar$

Outlook & some open questions

- Outlined a procedure to generate new BCs from existing ones
  "Asymptotic T-duality"
- Applied it to Brown-Henneaux BCs in AdS_3. Relation to CSS.
  Identified new BCs for the black string with a (surprisingly) large ASG

Outlook & some open questions

- Outlined a procedure to generate new BCs from existing ones "Asymptotic T-duality"
- Applied it to Brown-Henneaux BCs in AdS_3. Relation to CSS. Identified new BCs for the black string with a (surprisingly) large ASG
- What is this algebra? QFTs where it is realized?

Outlook & some open questions

- Outlined a procedure to generate new BCs from existing ones "Asymptotic T-duality"
- Applied it to Brown-Henneaux BCs in AdS_3. Relation to CSS. Identified new BCs for the black string with a (surprisingly) large ASG
- What is this algebra? QFTs where it is realized?
- The black string has a Bekenstein-Hawking entropy given by

$$S_{BH} = \frac{A_H}{4} = \sqrt{T_0}\left[\sqrt{S_0 - \sqrt{S_0^2 - \alpha T_0}} + \sqrt{S_0 + \sqrt{S_0^2 - \alpha T_0}}\right]$$

$\alpha = 4l$

Cardy-type?

III. Black strings from deformations

($T\bar{T}$, marginal and $T\bar{T}$)

AdS_3/CFT_2 and String theory

A classic statement in AdS/CFT [Maldacena] is that

$$\boxed{\text{IIB String theory on } AdS_5\times S^5 \xleftrightarrow{\text{dual}} SU(N)\ \mathcal{N}=4 \text{ SYM}}$$

Derived from a stack of D3 branes + decoupling limit, AdS_5/CFT_4

A lower-dimensional counterpart is

$$\boxed{\text{IIB String theory on } AdS_3\times S^3\times T^4 \xleftrightarrow{\text{dual}} \mathrm{Sym}^N(X)\ \text{CFT } (+\text{ deform?})}$$

Derived from a configuration of D1/D5 or $\underset{Q_5}{\text{NS5}}/\underset{Q_1}{\text{F1}}$, $AdS_3/\underbrace{CFT_2}_{\text{with } c=6Q_1Q_5}$

Identifying the precise CFT_2 dual is still an open question

[see work by Eberhardt, Gaberdiel, Gopakumar; Balthazar, Giveon, Kutasov, Martinec, ...]

The NS5-F1 setup is particularly interesting because then only NS-NS fluxes are present, and string theory is (in principle) exactly solvable: it is described by a

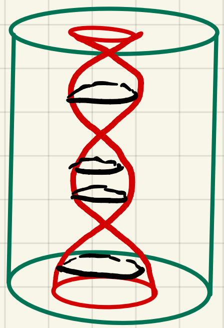
$SL(2,\mathbb{R})$ WZW model

[Many authors since '89: Balog et al., Petropoulos, ..., Maldacena-Ooguri, ..., Eberhardt et al, ...]

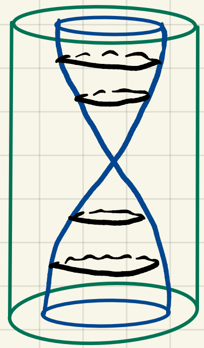
CFT with affine symmetry generated by $\widehat{SL(2)} \times \widehat{SL(2)}$ currents $J^a(z), \bar{J}^a(\bar{z})$

More intricate than $SU(2)$ because of spectrally flowed representations.

Spectrum has a discrete and a continuous component corresponding classically to short and long strings.



bound states



scattering states

Away from AdS_3: marginal deformations of WZW models

The $SL(2,\mathbb{R})$ WZW describes strings on AdS_3 (and BTZ).

How to describe more general backgrounds?

Away from AdS_3: marginal deformations of WZW models

The $SL(2,\mathbb{R})$ WZW describes strings on AdS_3 (and BTZ).

How to describe more general backgrounds?

WZW models admit integrable marginal deformations, thus allowing to reach new exact string backgrounds:

$$\boxed{S_{\lambda,WZW} = S_{WZW} + \lambda \int d^2z \, \underbrace{\mathcal{O}(z,\bar{z})}_{(1,1)\text{ operator (necessary)}}}$$

For instance, in the $SL(2,\mathbb{R})$ WZW model,

$$\mathcal{O} = J^1 \bar{J}^1$$

is exactly marginal. [Chaudhuri - Schwartz; Hassan - Sen; Giveon - Kiritsis; Forste - Roggenkamp; ...]

Black strings from $J_1\bar{J}_1$ deformation

[see also Horne-Horowitz; Israël-Kounnas, Petropoulos, Orlando, Spindel, SD's]

Start with WZW group element

$$g = e^{2T_u u T^1}\, e^{g(r,T_u,T_v)T^3}\, e^{2T_v v T^1} \in SL(2,\mathbb{R})$$

The WZW action will correspond to string propagation on BTZ with:

$$\begin{cases} \dfrac{ds^2}{\ell^2} = \dfrac{dr^2}{4(r^2 - T_u^2 T_v^2)} + r\,du\,dv + T_u^2\,du^2 + T_v^2\,dv^2 \\ B = \ldots \\ e^{2\Phi} = e^{2\Phi_0} \end{cases}$$

Now, turn on marginal operator $\mathcal{O} = J^1\bar{J}^1$.
The deformed action satisfies

$$\boxed{\frac{\partial S_{\lambda,WZW}}{\partial\lambda} \sim \int d^2z\, J^1\bar{J}^1}$$

The backgrounds after deformation can be read off, with metric

$$ds^2 = \frac{dr^2}{4(r^2 - 4T_u^2 T_v^2)} + \frac{r\, du\, dv + T_u^2\, du^2 + T_v^2\, dv^2}{1 + 2\lambda r + 4\lambda^2 T_u^2 T_v^2}$$

After a change of radial coordinate $\rho = \left[\frac{(M+Q)^2 - J^2}{4M}\right] r + \frac{M^2 + Q^2 - J^2}{2M}$

and setting $\lambda = 1/2$, the metric becomes a rotating generalization of HH:

$$\frac{ds^2}{\ell^2} = \frac{d\rho^2}{4\left[\rho^2 - \left(\frac{M^2+Q^2}{M}\right)\rho + Q^2\right]} - \left(1 - \frac{M}{\rho}\right) dt^2 + \left(1 - \frac{Q^2 - J^2}{M\rho}\right) d\varphi^2 + \frac{2J}{\rho}\, dt\, d\varphi$$

$$T_u^2 = \frac{M+J-Q}{M-J+Q}\,, \qquad T_v^2 = \frac{M-J-Q}{M+J+Q}\,, \qquad e^{2\Phi_0} = \overset{\text{fixed}}{Q}$$

$J = 0$: this precisely the Horne-Horowitz black string (at fixed Q).

<u>Observation</u>: the deformed background can be obtained from the undeformed one by a

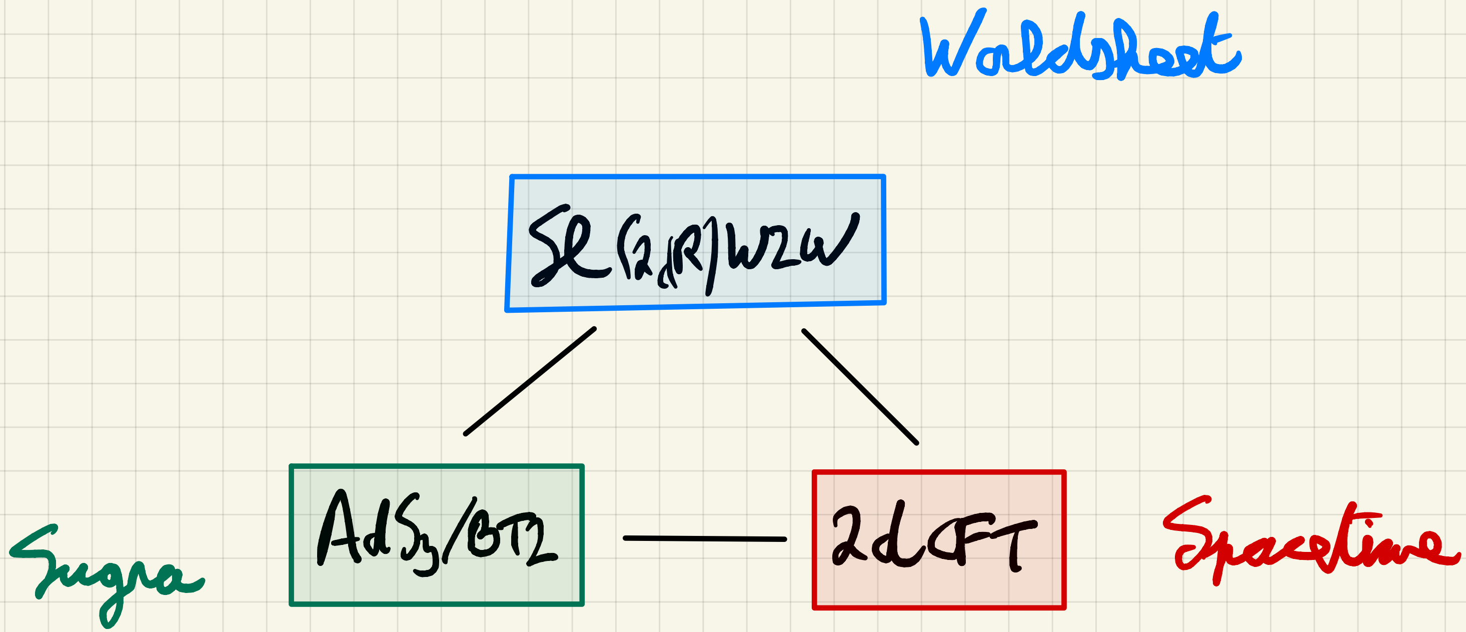
TsT transformation

T - duality along u

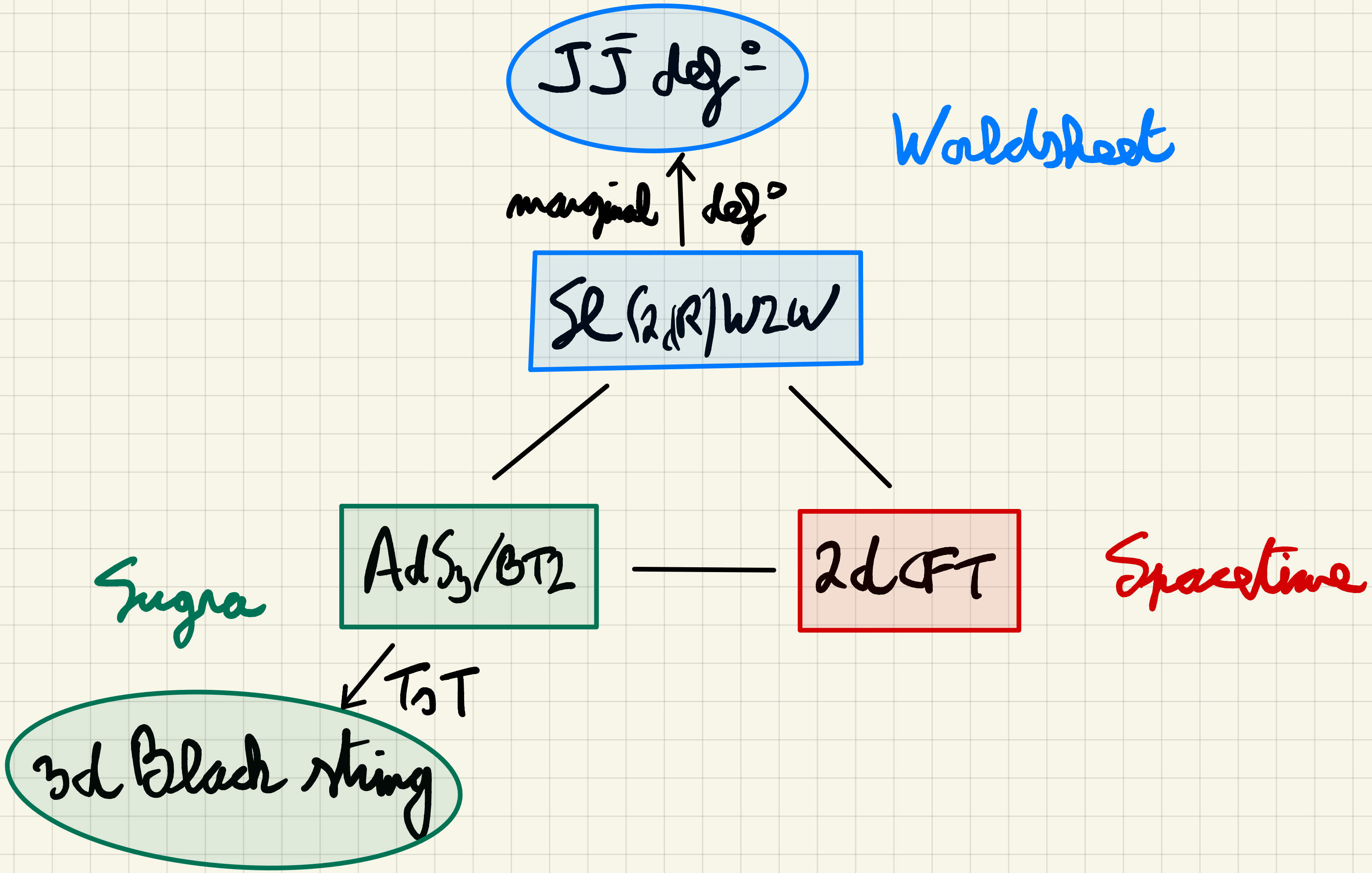
shift along $v \to v - \frac{2\lambda}{R} u$

$\underbrace{\text{T - duality}}_{\text{understood: Buscher's rules}}$ along u once more

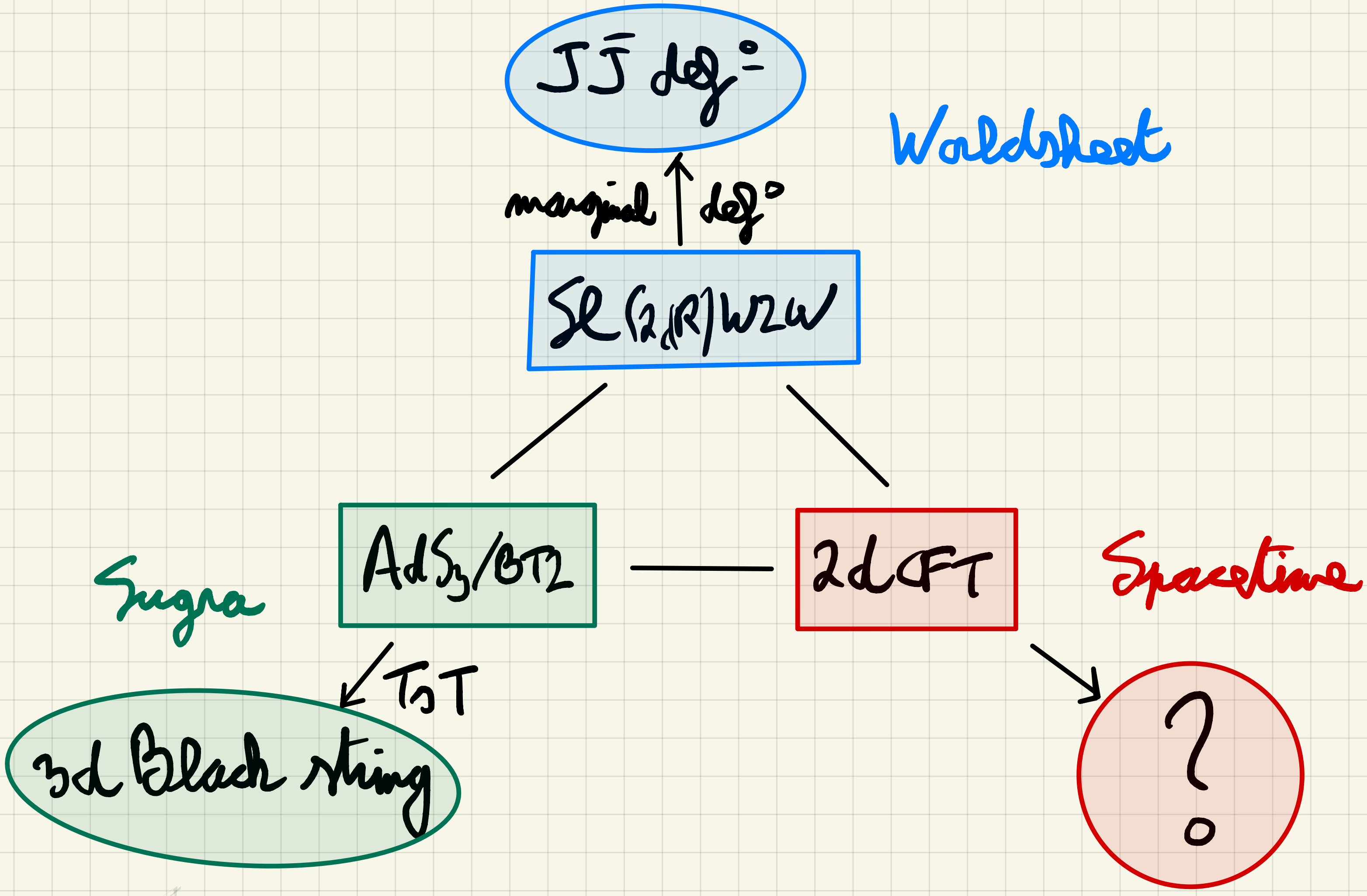
KG.



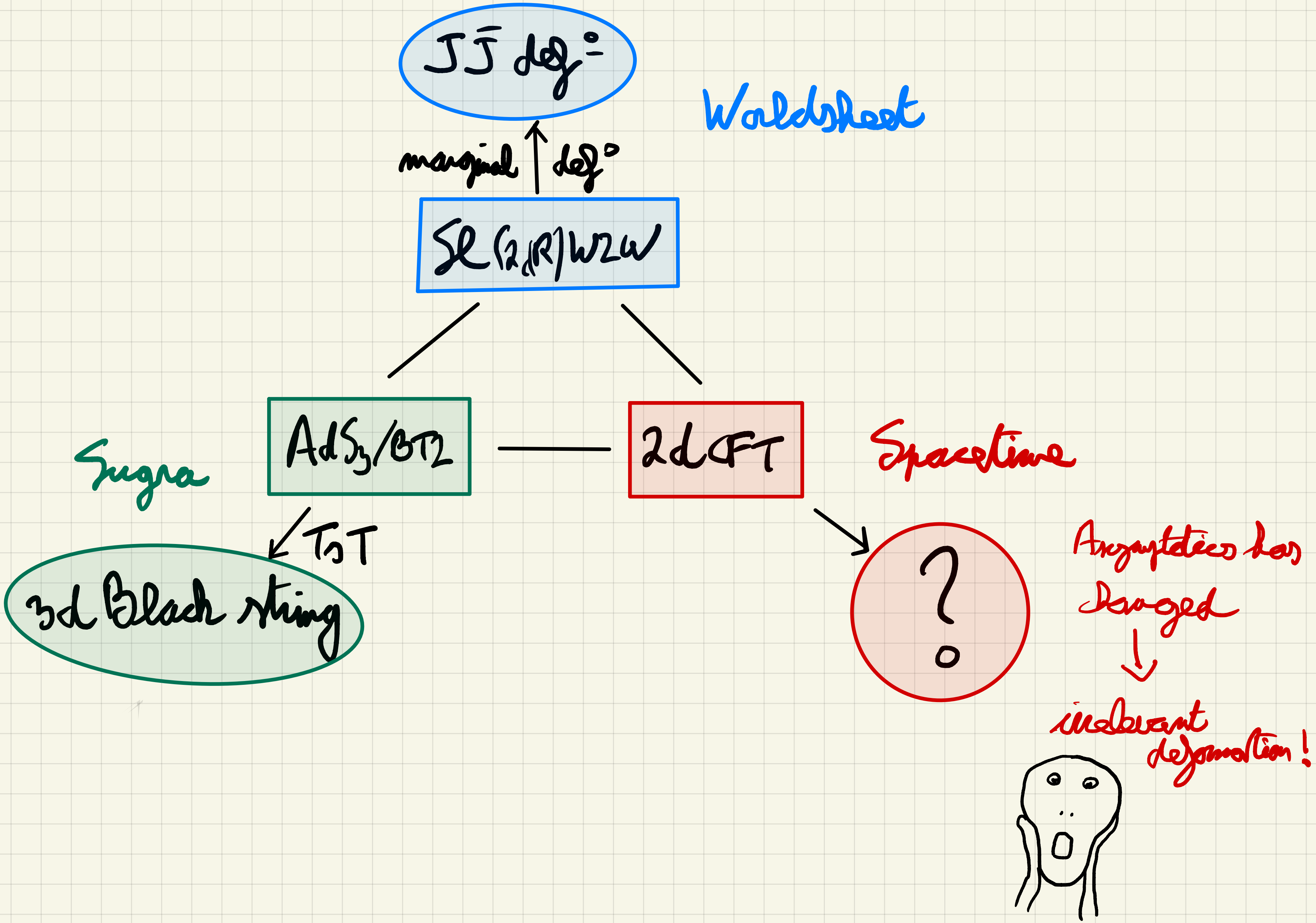
KG.



$J\bar{J}$ def$^{\underline{o}}$
Worldsheet
marginal
def^{o}
$SL(2,\mathbb{R})$ WZW
AdS_3/BTZ
2d CFT
Sugra
Spacetime
$T\bar{T}$
3d Black string


$J\bar{J}$ def°
Worldsheet
marginal def°
$SL(2,\mathbb{R})$ WZW
AdS_3/BTZ
2d CFT
Sugra
Spacetime
$T\bar{T}$
3d Black string
?

TG.



JJ̄ def°
Worldsheet
marginal def°
SL(2,R) WZW
AdS3/BTZ
2d CFT
Sugra
Spacetime
TsT
3d Black string
?
Asymptotics has changed
irrelevant deformation!

$T\bar{T}$ deformations to the rescue

[Smirnov, Zamolodchikov; Cavaglia, Negro, Szecsenyi, Tateo '16]

Irrelevant def$^{=}$ of 2d QFTs triggered by

$$T\bar{T} = -\pi^2 \det(T_{\mu\nu})$$

The corresponding deformed action then satisfies

$$\frac{\partial S_{QFT}}{\partial \mu} = \int d^2x \left(T_{xx} T_{\bar{x}\bar{x}} - T_{x\bar{x}} T_{\bar{x}x}\right)$$

Many physically interesting quantities can be computed exactly and explicitly in terms of the data of the undeformed theory

For instance, the finite volume spectra $\left((x,\bar{x}) \sim (x+2\pi R, \bar{x}+2\pi R)\right)$ are related by

$$\boxed{E_{L/R} = E_{L/R}(\mu) + \frac{2\mu}{R} E_L(\mu) E_R(\mu)}$$

Proposal:

String theory on a TsT transformation of $AdS_3 \times M$ w/ NSNS fluxes

$\xleftrightarrow[\text{to}]{\text{dual}}$

Single trace $T\bar{T}$ deformation of a CFT_2

[see also Giveon, Itzhaki, Kutasov]

1) Thermodynamics

[Datta, Jiang, Aharony, Giveon, Kutasov; Apolo, Song, Yu; ...]

The total entropy in the deformed CFT is

$$\frac{1}{2\pi} S_{T\bar{T}}(E_L, E_R) = \sqrt{\frac{c}{6} R E_L \left(1 + \frac{2\mu}{R_\mu} E_R\right)} + \sqrt{\frac{c}{6} R E_L \left(1 + \frac{2\mu}{R_\mu} E_L\right)}$$

$\uparrow \mathrm{sign}^{\mu}(x)$

The Bekenstein - Hawking entropy of the TsT black string

$$\frac{1}{2\pi} S_{BH} = \sqrt{Q_L \left(Q_e Q_m + 2\lambda Q_R\right)} + \sqrt{Q_R \left(Q_e Q_m + 2\lambda Q_L\right)}$$

Q_{m} $\quad$ Q_{w}

$\longrightarrow \boxed{S_{T\bar{T}} = S_{BS}}$!

with identifications

$\boxed{\begin{array}{l} R E_L \leftrightarrow Q_L \\ R E_R \leftrightarrow Q_R \end{array}}$

$p = \# F_1$ = el. charge

$\boxed{c = 6 Q_e Q_m}$

magn. charge = $k = Q_5$ = $\# NS5$

$\boxed{\lambda = \frac{\mu k}{R^2}}$

($\mu = \lambda = 0$: this is just the usual Cardy matching for BTZ)

2) String spectrum

[Alday, Arutyunov, Frolov '05] showed that it is possible to determine the string spectrum on a $T\bar{T}$ transformed background in terms of that of the undeformed one.

Observation:

$$\begin{array}{c}\text{Spectrum of strings}\\ \text{on } AdS_3 \ (E,J)\end{array} \xleftrightarrow[\text{match}]{\text{should}} \begin{array}{c}\text{Spectrum of dual}\\ \text{CFT } (E,J)\end{array}$$

2) String spectrum

[Alday, Arutyunov, Frolov '05] showed that it is possible to determine the string spectrum on a TsT transformed background in terms of that of the undeformed one.

Observation:

$$\begin{tikzcd}
\text{Spectrum of strings on } AdS_3 \ (E,J) \arrow[r, leftrightarrow, "\text{should}", "\text{match}"'] \arrow[d, leftrightarrow, "TsT"] & \text{Spectrum of dual CFT } (E,J) \\
\text{Spectrum of strings on TsT } (E(\lambda), J(\lambda)) &
\end{tikzcd}$$

2) String spectrum

[Alday, Arutyunov, Frolov '05] showed that it is possible to determine the string spectrum on a TsT transformed background in terms of that of the undeformed one.

Observation:

$$\begin{tikzcd}
\text{Spectrum of strings on } AdS_3 \ (E, J) \arrow[rr, leftrightarrow, "\text{should}", "\text{match}"'] \arrow[d, leftrightarrow, "TsT"] & & \text{Spectrum of dual CFT } (E, J) \\
\text{Spectrum of strings on TsT } (E(\lambda), J(\lambda)) \arrow[rr, leftrightarrow, "\text{should}", "\text{match}"'] & & \text{Spectrum of dual to TsT } (E(\lambda), J(\lambda))
\end{tikzcd}$$

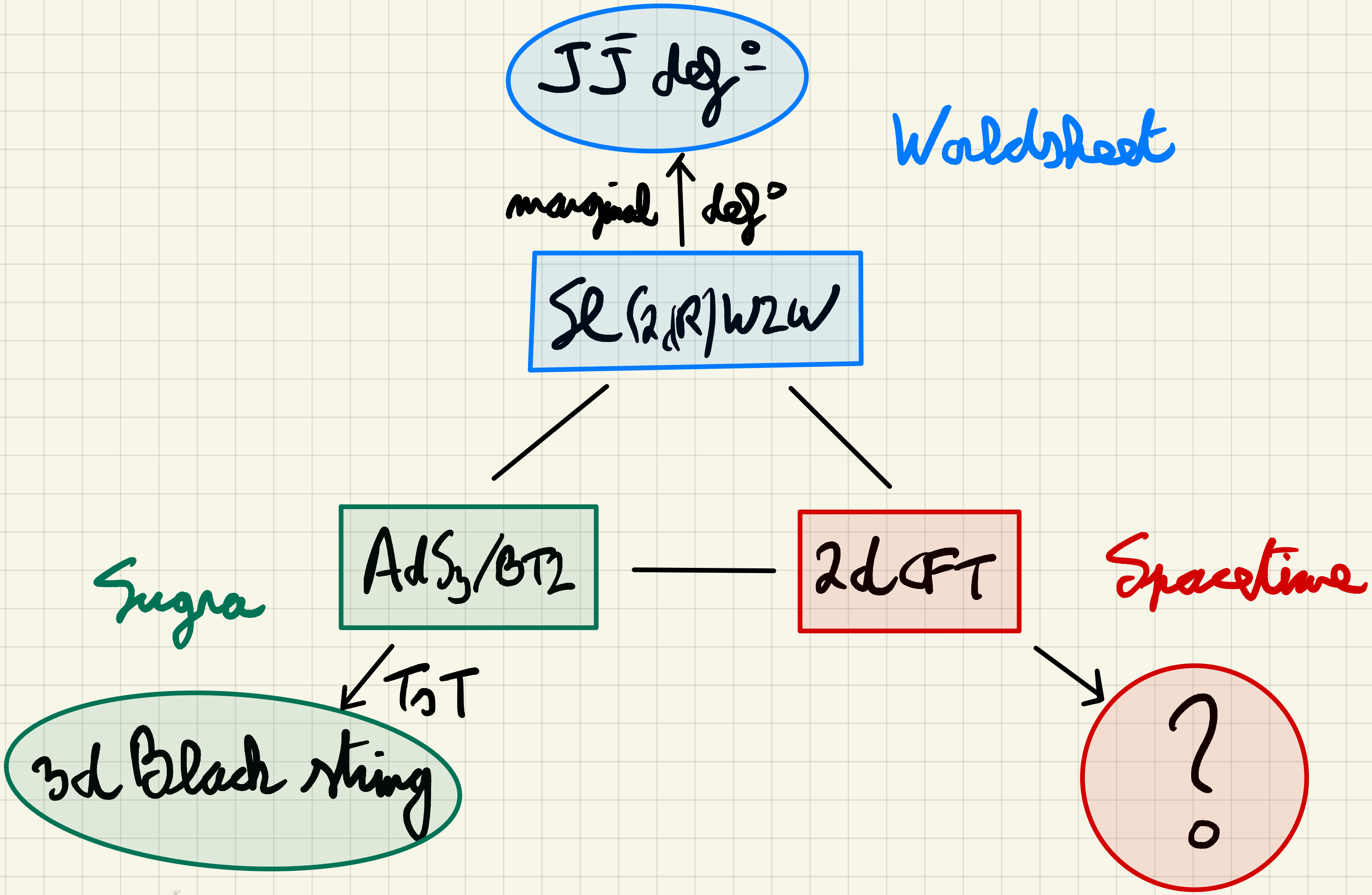
2) String spectrum

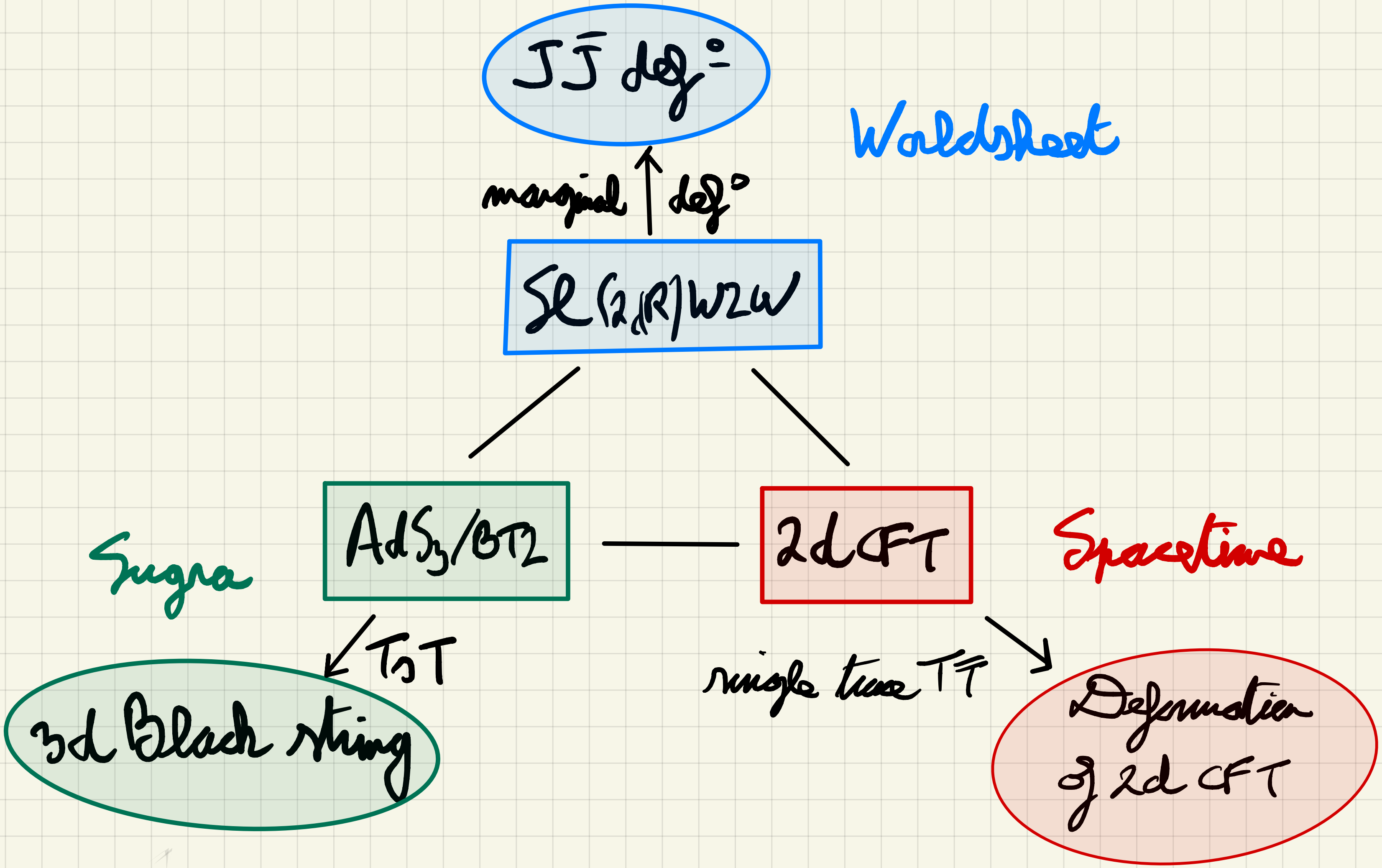
[Alday, Arutyunov, Frolov '05] showed that it is possible to determine the string spectrum on a TsT transformed background in terms of that of the undeformed one.

Observation:

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\text{Spectrum of strings on } AdS_3 \ (E, J) \arrow[r, leftrightarrow, "\text{should}", "\text{match}"'] \arrow[d, leftrightarrow, "TsT"] & \text{Spectrum of dual CFT } (E, J) \arrow[d, leftrightarrow, "T\bar{T}\,!"] \\
\text{Spectrum of strings on } TsT \ (E(\lambda), J(\lambda)) \arrow[r, leftrightarrow, "\text{should}", "\text{match}"'] & \text{Spectrum of dual to } TsT \ \underbrace{(E(\lambda), J(\lambda))}_{\text{precisely } (E(\mu), J(\mu)) \text{ expected for a } T\bar{T} \text{ def}}
\end{tikzcd}
$$

for $\boxed{\lambda = \frac{\mu R}{R^2}}$


JJ̄ def:
Worldsheet
marginal def°
SL(2,R) WZW
AdS3/BTZ
Sugra
2d CFT
Spacetime
TT̄
3d Black string
?



$J\bar{J}$ def.
Worldsheet
marginal def.
$SL(2,\mathbb{R})$ WZW
Sugra
AdS_3/BTZ
2d CFT
Spacetime
$T\bar{T}$
3d Black string
single trace $T\bar{T}$
Deformation of 2d CFT

More:

- other observables (e.g. EE, QNM)? partition function, correlation functions
- gravitational phase space? (in the first part, we discussed the T-dual of Brown-Henneaux; what is the $T\bar{T}$ dual?)
  [see also Du, Lai, Liu, Song 2407.19495]
- Near-extremal black strings (does a Schwarzian sector subsist after $T\bar{T}$?)
- ...

More:

partition function, correlation functions

- other observables (e.g. EE, QNM)?
- gravitational phase space? (in the first part, we discussed the T-dual of Brown-Henneaux; what is the TsT dual?)
  [see also Du, Lai, Liu, Song 2407.19495]
- Near-extremal black strings (does a Schwarzian sector subsist after $T\bar{T}$?)
- ...

Thank you!

BTZ (T_u, T_v, ϕ_0) $\xrightarrow{T\bar{T}}$ $(T_u, T_v, \phi_0, \lambda)$

$$\lambda = \frac{1}{2} \to (T_u, T_v, \phi_0) \leftrightarrow (M, J, Q)$$

In $T\bar{T}$, Q comes from the cst in the dilaton → is a constant !

$\downarrow J=0$

(M, Q) HH

Rem: $(Q_e ; Q_f) \leftrightarrow (\lambda, \mu)$ = cts in the sol?

We keep Q cst. What we match is the $(T_u, T_v, \phi_0 = \text{cst.})$

$= (M, J, Q = \text{cst.})$

entropy rel/ a $T\bar{T}$-def CFT

Not quite HH that has $J=0$ and Q allowed to vary!

* didn't compare / write the grand charge, rotating black string entropy
* didn't check whether it belongs to our BC : to go from (r, z, w) to (R, T, X) requires a non-trivial (charge-dependent) change of coordinates !

$T\bar{T}$ black string is related to HH but not quite the same

For $\lambda = 1/2$, $J = 0$, it is the fixed Q charge of HH

Cannot have Q using just $T\bar{T}$.

HH: $(\Phi_0, \lambda) \leftrightarrow (M, Q)$

$T\bar{T}$: $(T_L, T_R, \Phi_0, \lambda) \xrightarrow[\substack{\lambda = 1/2 \\ J = 0\ (T_L = T_R}]{} (T_L, \Phi_0) \leftrightarrow (M, Q\text{ fixed})$

transition limit $k = 1$

free boson deformation T^4
Liouville (see Eberhardt)

need CFT w/ $c = 6k$

Why single trace?

CFT is a SCO of $\mathcal{O}$ for $\mathrm{Sym}^N(X)$

2 types of def: $\mathrm{Sym}^N(T\bar{T}$ def of seed CFT $X) \to$ single trace

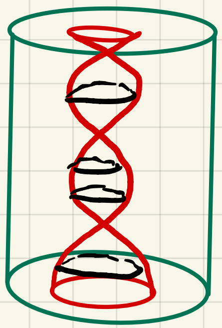
or

$T\bar{T}$ def of $\mathrm{Sym}^N \to$ double trace

$\begin{cases} J_{(1)} = T_{zz}\,dz + T_{z\bar{z}}\,d\bar{z} \\ J_{(2)} = T_{\bar{z}z}\,dz + T_{\bar{z}\bar{z}}\,d\bar{z} \end{cases} \to$ single trace

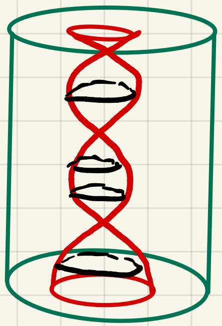
$\sum_{i=1}^{N} \int J^{(i)}_{(1)} \wedge J^{(i)}_{(2)}$

- Sundar Apropos of TsT, Theme of $T\bar{T}$ Wei/Davis
- Review Sugra pert., parameters in AdS_3/CFT_2 λ, μ, Φ_5, Φ_1, ...



short strings

(bound states)



long strings

(scattering states)