Exploring Defects with Degrees of Freedom in Free Scalar CFTs

Jacopo Sisti

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UPPSALA
UNIVERSITET

Based on:

[Vladimir Bashmakov, JS, 2024]

[Bianchi, Chalabi, Prochazka, Robinson, JS, 2021]

[Chalabi, Herzog, O'Bannon, Robinson, JS, 2021]

Defects

Local operators are not enough

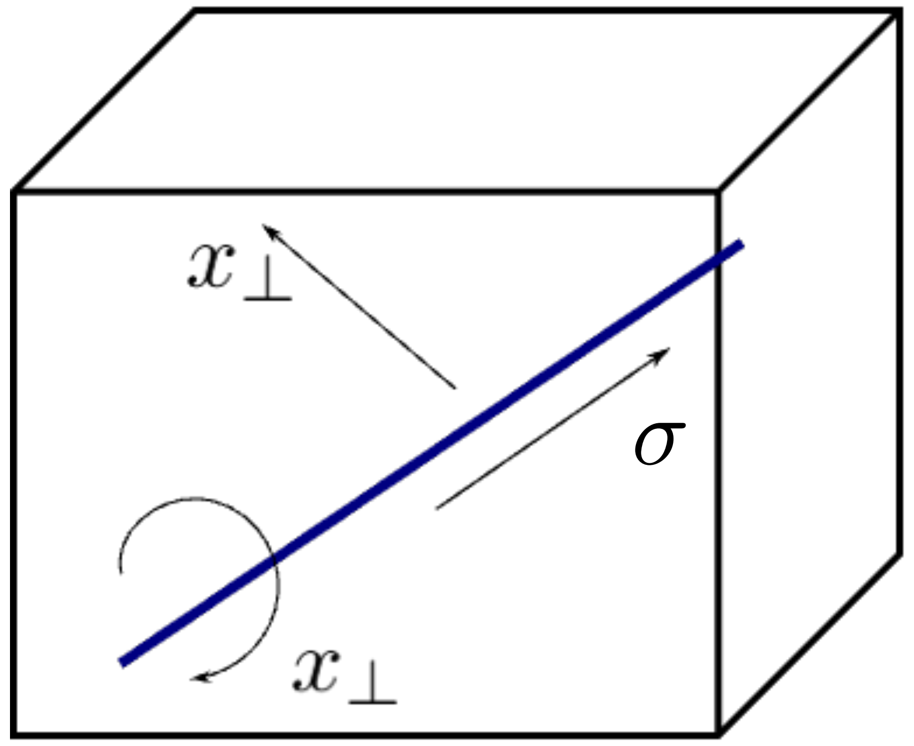
- To distinguish groups with same algebra but different global structure
- SPT phases and phases with topological order
- Strings and Branes

Some Examples:

- Boundaries (space-time ends on them)
- Interfaces (for instance between different phases)
- Line defects: for example Wilson lines
- Topological Defects (symmetries and generalised symmetries)

Defects

Extended objects that break part of the translational symmetry



- Localised on submanifolds of the spacetime
- p=dimension of defect space-time, q=d-p is the codimension
- The stress energy tensor is no longer conserved:

$$\partial_\mu T^{\mu\nu} = \hat{\mathcal{D}}^\nu \, \delta^q\,(\Sigma)$$

$\hat{\mathcal{D}}^\nu$ = displacement operator
protected dimension p+1

- If we ask for maximal symmetry preserved: “flat” defect
- We mainly focus on conformal field theories
- Conformal group is broken: $SO(d+1,1) \longrightarrow SO(d+1-p,1) \times SO(q)$
- New non-trivial correlators: one-point functions
- Localised contribution to anomalies: i.e. defect Weyl anomalies

Correlators in defect CFTs

[Billò, Gonçalves, Lauria, Meineri, 2016]
[Herzog, Shrestha, 2020]

Bulk correlators:

$$\langle \mathcal{O}(x)\rangle = \frac{A_{\mathcal{O}}}{|x_\perp|^{\Delta_{\mathcal{O}}}}$$

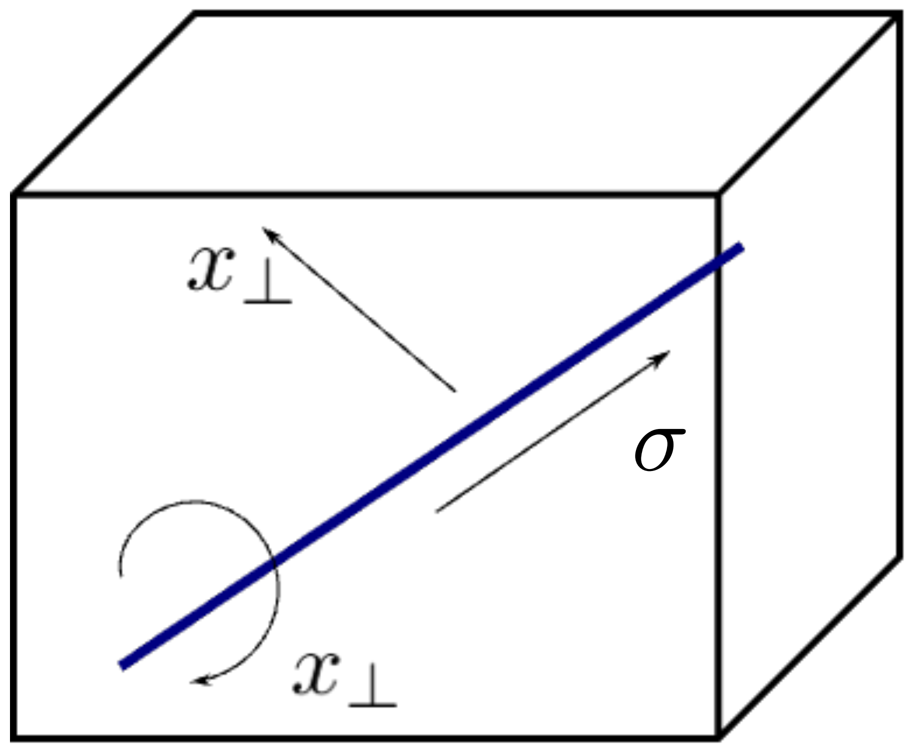
$$\langle T^{ij}\rangle = h\frac{(d-q+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

$$\langle T^{ab}\rangle = -h\frac{(q-1)\delta^{ab}}{|x_\perp|^d}$$

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\rangle = \frac{1}{|x_\perp|^{\Delta_1}|y_\perp|^{\Delta_2}} f(\xi_1,\xi_2)\,, \qquad \xi_1 = \frac{s^2}{4|x_\perp|\,|y_\perp|},\ \xi_2 = \frac{x_\perp\cdot y_\perp}{|x_\perp|\,|y_\perp|},\ s = x-y$$

$i, j, k, \cdots =$ orthogonal

$a, b, c, \cdots =$ parallel



Bulk-defect correlators:

$$\left\langle \hat{\mathcal{O}}_{\hat\Delta}(\sigma_x)\mathcal{O}_\Delta(y)\right\rangle = \frac{C_{\hat{\mathcal{O}}\mathcal{O}}}{|y_\perp|^{\Delta-\hat\Delta}\left(y_\perp^2 + (\sigma_x-\sigma_y)^2\right)^{\hat\Delta}}$$

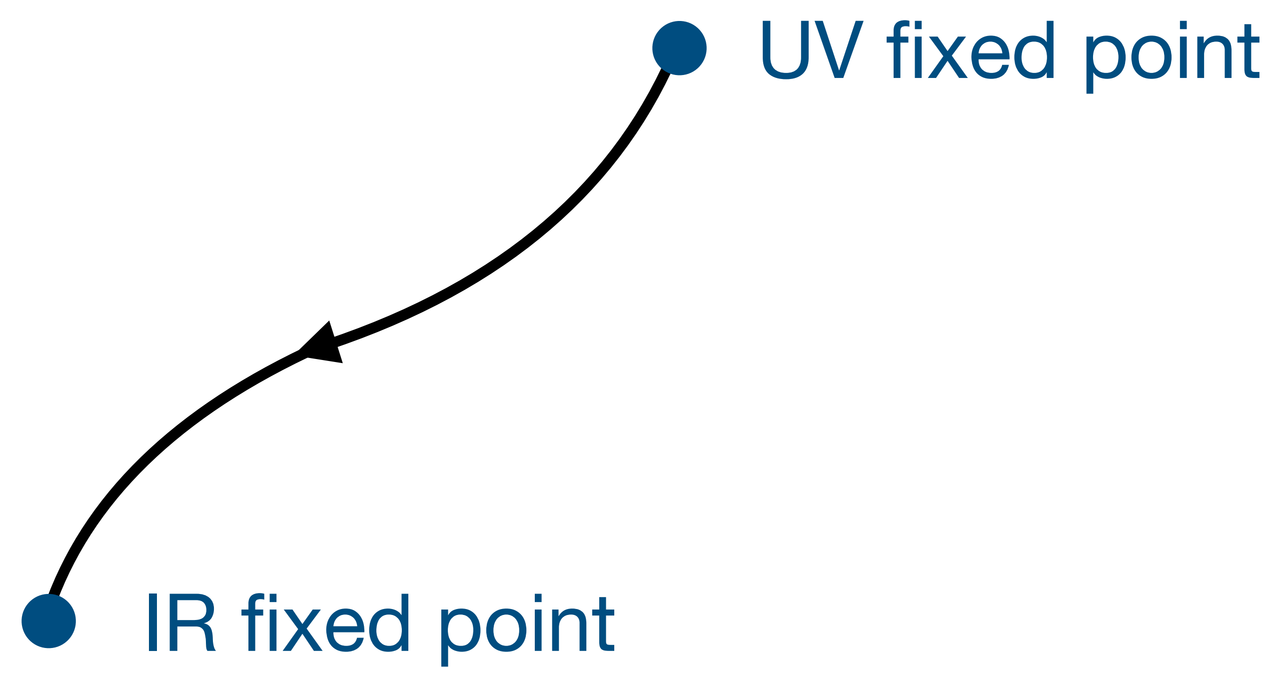
Defect correlators:

$$\left\langle \hat{\mathcal{O}}_1(\sigma_x)\hat{\mathcal{O}}_2(\sigma_y)\right\rangle = \frac{\delta_{\Delta_{\hat{\mathcal{O}}_1},\Delta_{\hat{\mathcal{O}}_2}}}{|\sigma_x-\sigma_y|^{2\Delta_{\hat{\mathcal{O}}_1}}} \qquad \left\langle \hat{\mathcal{D}}^i(\sigma)\hat{\mathcal{D}}^j(0)\right\rangle = \frac{C_{\hat{\mathcal{D}}\hat{\mathcal{D}}}}{|\sigma|^{2p+2}}\delta^{ij}$$

$$\left\langle \hat{\mathcal{O}}_1(\sigma_1)\hat{\mathcal{O}}_2(\sigma_2)\hat{\mathcal{O}}_3(\sigma_3)\right\rangle = \frac{\hat{C}_{123}}{|\sigma_{12}|^{\hat\Delta_1+\hat\Delta_2-\hat\Delta_3}|\sigma_{13}|^{\hat\Delta_1+\hat\Delta_3-\hat\Delta_2}|\sigma_{23}|^{\hat\Delta_2+\hat\Delta_3-\hat\Delta_1}}$$

RG flows

- We can deform the theory by turning on *relevant deformations*



- Two possible deformations:

Bulk deformations:

$$\delta S_{\text{bulk}} = \int d^d x \, \mathcal{O}_{\Delta}$$

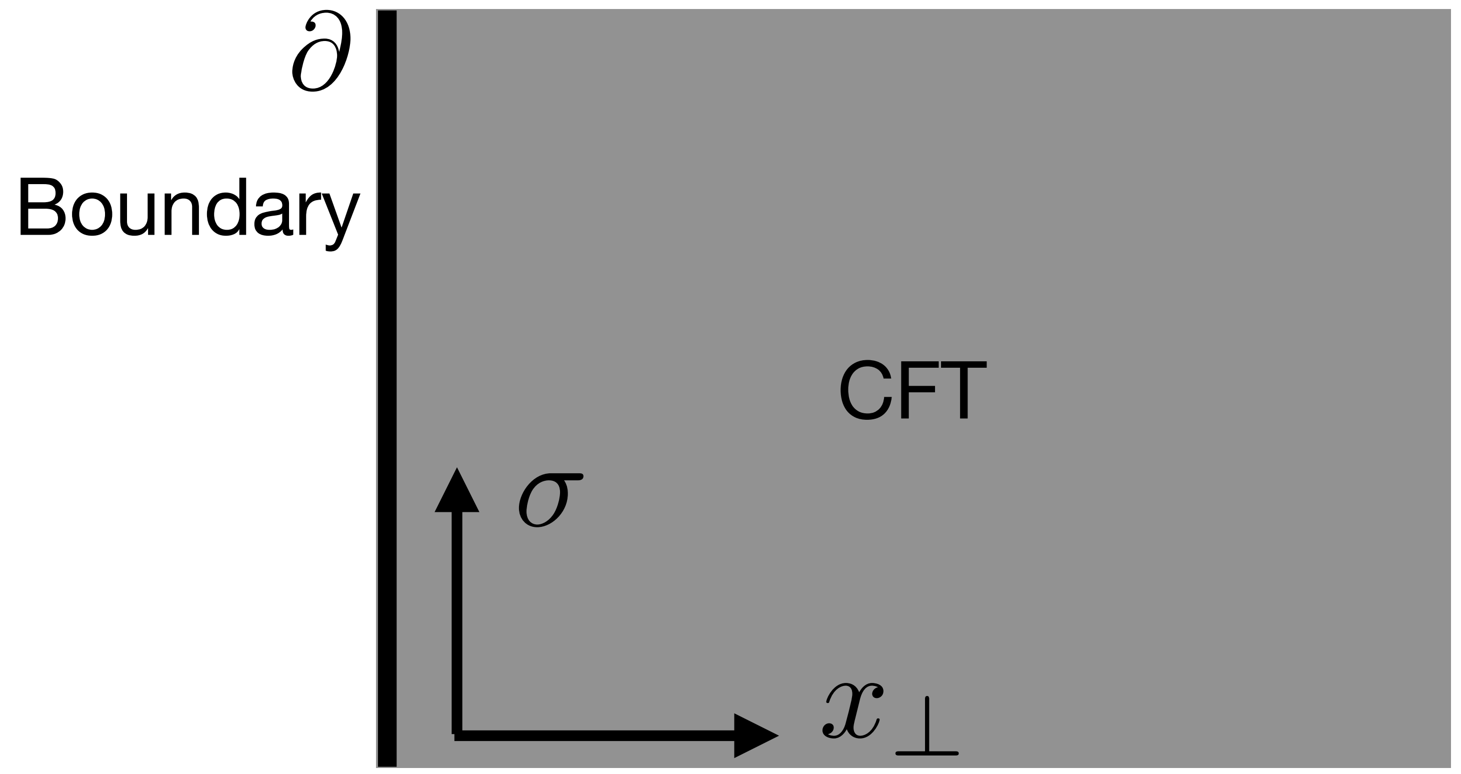
- Modify both the bulk and the defect

Defect-localised deformations:

$$\delta S_{\text{defect}} = \int d^p x \, \hat{\mathcal{O}}_{\hat{\Delta}}$$

- Only affects the defect theory
- Same bulk primary operators
- Defect monotonicity theorems

Example: Boundary CFT



- Action of free massless scalar field:

$$\delta S = \int d^d x \, \delta\phi \, \partial^2 \phi - \int d^{d-1}x \;\; \delta\phi \partial_\perp \phi|_\partial$$

- Conformal Boundary conditions:
  - $\partial_\perp \phi(\sigma, x_\perp = 0)|_\partial = 0$ — Neumann b.c.
  - $\phi(\sigma, x_\perp = 0)|_\partial = 0$ — Dirichlet b.c.

Correlators:

$$\langle \phi(x_1)\phi(x_2)\rangle = C_\phi \left[\frac{1}{[(\sigma_1 - \sigma_2)^2 + (x_{\perp 1} - x_{\perp 2})^2]^{d/2-1}} \pm \frac{1}{[(\sigma_1 - \sigma_2)^2 + (x_{\perp 1} + x_{\perp 2})^2]^{d/2-1}} \right]$$

$$\left\langle \phi(x)^2 \right\rangle = \pm \frac{C_\phi}{|x_\perp|^{d-2}}$$

For the free scalar with a boundary:

$$\delta S_\partial = h \int d^{d-1}\sigma \;\; \hat{\phi}^2 \Big|_\partial$$

Neumann b.c. → Dirichlet b.c.

(Bulk) Weyl Anomaly

- QFT on curved space-time
- Action invariant under Weyl rescaling (classical) $g_{\mu\nu} \longrightarrow e^{2\omega(x)} g_{\mu\nu}$

$$\delta_\omega S = -\int_{\mathcal{M}_d} d^d x \sqrt{g}\, \delta\omega\, T^\mu{}_\mu = 0 \quad \longrightarrow \quad T^\mu{}_\mu = 0$$

- Effective action is not invariant: Weyl anomaly (quantum effect) $\langle T^{\mu\nu} \rangle = -\frac{2}{\sqrt{g}} \frac{\delta W}{\delta g_{\mu\nu}}$

$$\boxed{\delta_\omega W = -\int_{\mathcal{M}_d} d^d x \sqrt{g}\, \delta\omega\, \langle T^\mu{}_\mu \rangle \neq 0} \quad \longrightarrow \quad W = \frac{a_{d-2}}{\epsilon^{d-2}} + \frac{a_{d-4}}{\epsilon^{d-4}} \cdots + \left[\int d^d x \sqrt{g}\, \langle T^\mu{}_\mu \rangle \right] \log \epsilon + \mathcal{O}(1)$$

- Local quantity of dimension d

$$T^\mu{}_\mu = \frac{1}{(4\pi)^{\frac{d}{2}}} \left[(-)^{\frac{d}{2}-1} a_{\mathcal{M}} E_d + \sum_n c_n I_n \right]$$

anomaly coefficients
“central charges”

curvature invariants

- Bulk anomalies absent in odd dimensions

(Bulk) Weyl Anomaly

2 dimensions:

$$T^{\mu}{}_{\mu} = \frac{c}{24\pi} R$$

c = Virasoro central charge

[Zamolodchikov, 1986]

$$c_{\rm UV} \geq c_{\rm IR}$$

$$\langle T(x) T(0) \rangle = \frac{c}{|x|^4}$$

4 dimensions:

$$T^{\mu}{}_{\mu} = \frac{1}{16\pi^2} \Big(- a E_4 + c W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} \Big)$$

$$a_{\rm UV} \geq a_{\rm IR}$$

[Cardy, 1988]

[Komargodski, Schwimmer, 2011]

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle \propto \frac{c}{|x|^4} I_{\mu\nu,\rho\sigma}(x)$$

6 dimensions:

$$T^{\mu}{}_{\mu} = \frac{1}{(4\pi)^3} (a\, E_6 + c_1 I_1 + c_2 I_2 + c_3 I_3)$$

$$I_1 = W_{\mu\lambda\rho\nu} W^{\lambda\sigma\tau\rho} W_{\sigma}{}^{\mu\nu}{}_{\tau} \qquad I_2 = W_{\mu\nu}{}^{\lambda\rho} W_{\lambda\rho}{}^{\sigma\tau} W_{\sigma\tau}{}^{\mu\nu}$$

$$I_3 = W_{\mu\nu\lambda\rho} \left(D^2\, \delta^{\nu}{}_{\sigma} - \frac{6}{5} R\, \delta^{\nu}{}_{\sigma} + 4 R^{\nu}{}_{\sigma} \right) W^{\sigma\nu\lambda\rho}$$

$$a_{\rm UV} \geq a_{\rm IR} \quad \text{(conjectured)}$$

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle \propto \frac{c_3}{|x|^4} I_{\mu\nu,\rho\sigma}(x)$$

$$\langle TTT \rangle \propto c_1 + c_2$$

Defect Weyl Anomaly

- In the presence of a p-dimensional defect:

$$\delta W = -\frac{1}{2}\int_{\mathcal{M}_d} d^d x \sqrt{g}\, \delta g_{\mu\nu}\, \langle T^{\mu\nu}\rangle - \frac{1}{2}\int_{\Sigma_p} d^p y \sqrt{\bar{g}} \left(\delta g_{\mu\nu} \langle T^{\mu\nu}|_{\Sigma_p}\rangle + 2\delta X^i(y^a)\langle \mathcal{D}_i\rangle\right)$$

- Defect-localised contribution to Weyl anomaly

$$T^\mu{}_\mu = T^\mu{}_\mu\big|_{\mathcal{M}_d} + \delta^{(q)}(x_\perp) T^\mu{}_\mu\big|_{\Sigma_p} \qquad D_\mu T^{\mu i} = \delta^{(q)}(x_\perp)\mathcal{D}^i$$

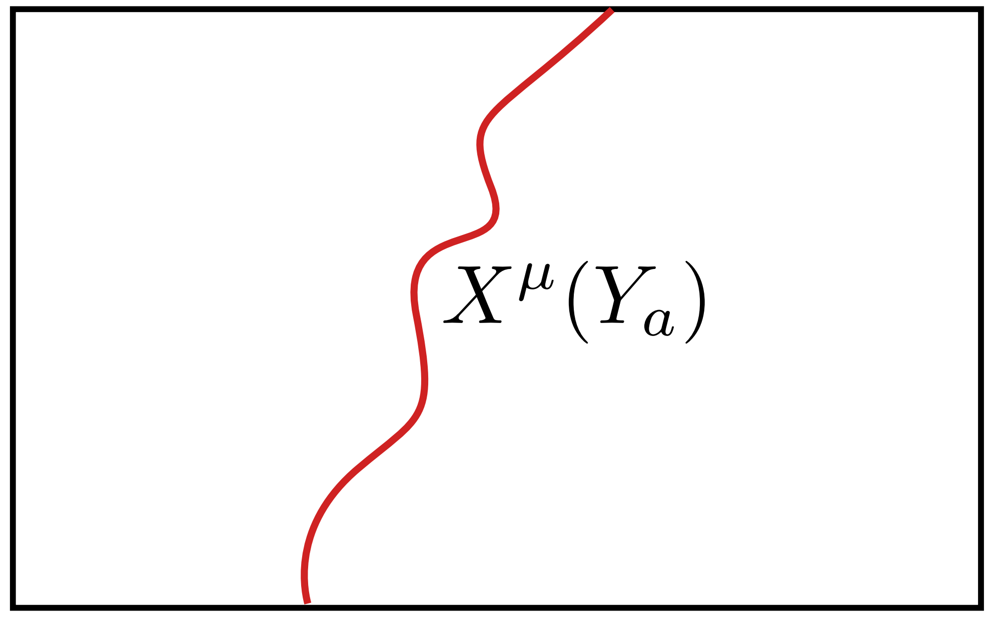
- For one-dimensional boundary is “trivial” $T^\mu{}_\mu|_{\Sigma_1} = \dfrac{c}{12\pi} K$

Defect Weyl Anomaly

$e^\mu_a = \partial_a X^\mu$ $\quad X^\mu(Y_a)$ embedding function

$\bar{g}_{ab} = e^\mu_a e^\nu_b g_{\mu\nu}$ (induced metric)

$\Pi^i_{ab} = \bar{D}_a e^\mu_b$ (second fundamental form)



- For two-dimensional defects [Henningson, Skenderis, 1999] [Schwimmer, Theisen, 2008]

$$T^\mu{}_\mu|_{\Sigma_2} = \frac{1}{24\pi}(b\,\bar{R} + d_1 \mathring{\Pi}^2 + d_2 W^{ab}{}_{ab})$$

$$\left\langle \hat{\mathcal{D}}_i(\sigma)\hat{\mathcal{D}}_j(0)\right\rangle \propto d_1 \frac{\delta_{ij}}{|\sigma|^6} \qquad \langle T_{\mu\nu}\rangle \propto \frac{d_2}{|x_\perp|^d} \qquad b_{\text{UV}} \geq b_{\text{IR}}$$

from defect sphere free energy

[Jensen, O'Bannon, 2015]

- For three-dimensional boundaries [Herzog, Huang, Jensen, 2015 - 2017]

$$T^\mu{}_\mu|_{\Sigma_3} = \frac{1}{16\pi^2}\left(a_{\mathcal{M}} E_4|_{\partial\mathcal{M}} + b_1 \mathring{K}^3 + b_2 \mathring{K}^{ab} W^c{}_{acb}\right)$$

$$E_4|_{\partial\mathcal{M}} = \delta^{abc}_{def}\left(2K^d{}_a R^{ef}{}_{bc} + \frac{8}{3}K^d{}_a K^e{}_b K^f{}_c\right)$$

$$\left\langle \hat{\mathcal{D}}\hat{\mathcal{D}}\hat{\mathcal{D}}\right\rangle \propto b_1 \qquad \left\langle \hat{\mathcal{D}}(\sigma)\hat{\mathcal{D}}(0)\right\rangle \propto b_2 \frac{1}{|\sigma|^8}$$

4d Defect Weyl Anomaly

[Chalabi, Herzog, O'Bannon, Robinson, JS, 2021]

- Full Weyl anomaly for p=4 dimensional defects and codimension q>1

$$
\begin{aligned}
T^{\mu}{}_{\mu}|_{\Sigma_4} = \frac{1}{(4\pi)^2}\Big(& - a_\Sigma \overline{E}_4 + d_1 \mathcal{J}_1 + d_2 \mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2 \\
&+ d_5 W_{aibj}W^{aibj} + d_6 W^{b}{}_{iab} W_{c}{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk} \\
&+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^{c}{}_{acb} W_{d}{}^{adb} + d_{12} W^{a}{}_{iaj} W_{b}^{ibj} \\
&+ d_{13} W_{ab}{}^{ab} \mathring{\Pi}^i_{cd} \mathring{\Pi}^{cd}_i + d_{14} W^{a}{}_{bij} \mathring{\Pi}^i_{ac} \mathring{\Pi}^{jbc} + d_{15} W^{a}{}_{ibj} \mathring{\Pi}^i{}_{ac} \mathring{\Pi}^{jbc} \\
&+ d_{16} W^{abcd} \mathring{\Pi}^i_{ac} \mathring{\Pi}_{ibd} + d_{17} W_{a}{}^{bac} \mathring{\Pi}^i_{bd} \mathring{\Pi}_{ic}{}^{d} + d_{18} W^{c}{}_{icj} \mathring{\Pi}^i_{ab} \mathring{\Pi}^{jab} \\
&+ d_{19} \mathrm{Tr}\, \mathring{\Pi}^i \mathring{\Pi}_i \mathring{\Pi}^j \mathring{\Pi}_j + d_{20} \mathrm{Tr}\, \mathring{\Pi}^i \mathring{\Pi}^j \mathring{\Pi}_i \mathring{\Pi}_j + d_{21} (\mathrm{Tr}\, \mathring{\Pi}^i \mathring{\Pi}_i)^2 + d_{22} (\mathrm{Tr}\, \mathring{\Pi}^i \mathring{\Pi}^j)(\mathrm{Tr}\, \mathring{\Pi}_i \mathring{\Pi}_j)\Big)
\end{aligned}
$$

- 2 "non-trivial" invariants

$$
\begin{aligned}
\mathcal{J}_1 = &\frac{1}{d-1} R \mathring{\Pi}^i_{ab} \mathring{\Pi}^{ab}_i - \frac{1}{d-2} N^{\mu\nu} R_{\mu\nu} \mathring{\Pi}^i_{ab} \mathring{\Pi}^{ab}_i - \frac{2}{d-2} R^{a}{}_{b} \mathring{\Pi}^i_{ac} \mathring{\Pi}^{bc}_i - \frac{1}{2} W^{c}{}_{acb} \Pi_i \mathring{\Pi}^{iab} \\
&+ \frac{4}{9} W^{c}{}_{ica} \overline{D}^b \mathring{\Pi}^i_{ab} + \mathring{\Pi}^{iab} D_i W^{c}{}_{acb} - \frac{1}{2} \Pi^i \mathrm{Tr}\, \mathring{\Pi}_i \mathring{\Pi}^j \mathring{\Pi}_j + \frac{1}{16} \Pi^i \Pi_i \mathrm{Tr}\, \mathring{\Pi}^j \mathring{\Pi}_j \\
&+ \frac{2}{9} \overline{D}^b \mathring{\Pi}^i_{ab} \overline{D}^c \mathring{\Pi}_{ic}{}^{a}
\end{aligned}
$$

Related to two-point function of displacement operator

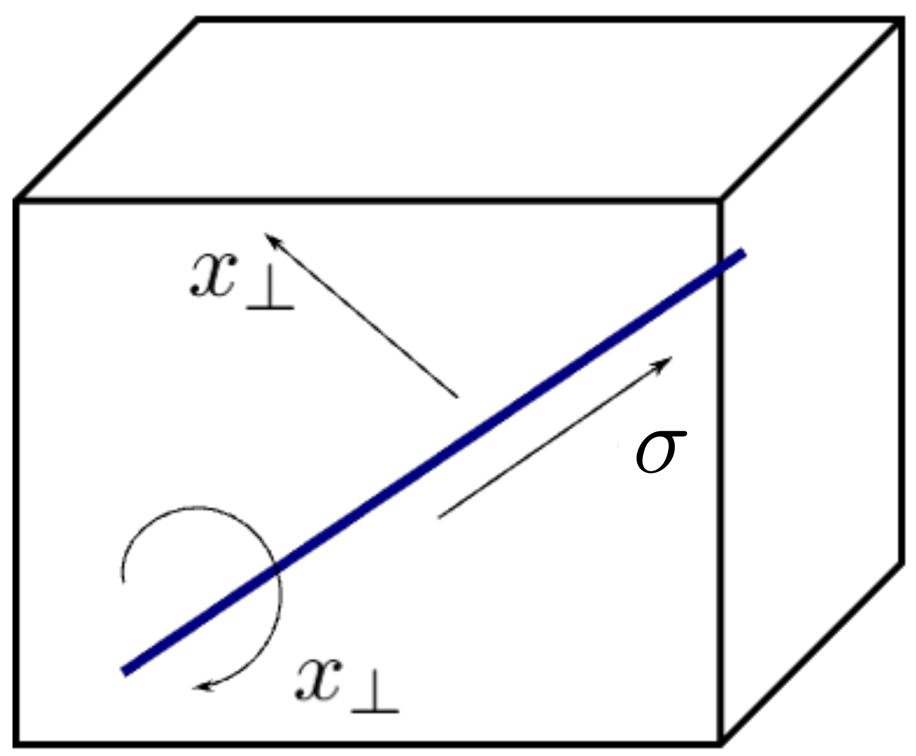
$$
\begin{aligned}
\mathcal{J}_2 = &\frac{d-4}{d-2} W_{ab}{}^{ab} N^{\mu\nu} R_{\mu\nu} - \frac{d-4}{d-1} R W_{ab}{}^{ab} + \frac{4(d-5)}{3(d-2)} R_{ab} W_{c}{}^{acb} \\
&- \frac{5(d-4)}{48} W_{ab}{}^{ab} \Pi^i \Pi_i + \frac{2(d-5)}{3} W^{c}{}_{ica} \overline{D}^b \mathring{\Pi}^i_{ab} + \frac{4(d+1)}{9} \mathring{\Pi}^{iab} D_i W^{c}{}_{acb} \\
&- \frac{1}{3} W_{ic}{}^{ac} \overline{D}_a \Pi^i - \frac{2(d-5)}{3} \Pi^i \mathrm{Tr}\, \mathring{\Pi}_i \mathring{\Pi}^j \mathring{\Pi}_j + \frac{(d-10)}{12} \Pi^i D_i W_{ab}{}^{ab} + \frac{1}{3} D^i D_i W_{ab}{}^{ab},
\end{aligned}
$$

Related to one-point function of the stress tensor

$a_{\Sigma \mathrm{UV}} \geq a_{\Sigma \mathrm{IR}}$ from defect sphere free energy defect a-theorem [Wang, 2021]

Free Scalar Fields with defects

[Bashmakov, JS, 2024]
(see also [Lauria, Liendo, van Rees, Zhao, 2021])



- Action of the free scalar field: $S = \int d^d x \, \frac{1}{2} (\partial \phi)^2$
- We use cylindrical coordinates: $ds^2 = d\sigma^2 + dx_\perp^2 + x_\perp^2 d\Omega_{q-1}^2$ $\qquad x_\perp = \sqrt{x_{\perp i} x_\perp^i}$
- Equation of motion: $-\partial^2 \phi = -\left(\partial_{||}^2 + \frac{1}{x_\perp^{q-1}} \partial_{x_\perp} (x_\perp^{q-1} \partial_{x_\perp}) + \frac{1}{x_\perp^2} \partial_{\Omega_{q-1}}^2 \right) \phi = 0$
- Ansatz: $\phi \sim e^{-i\omega\sigma_0} e^{i\vec{k}\vec{\sigma}} Y_{\{l\}}(\{\vec{\theta}\}) f_l(k_\perp x_\perp)$
- Solution in terms of Bessel functions:

$$\phi \sim e^{-i\omega\sigma_0} e^{i\vec{k}\vec{\sigma}} Y_{\{l\}}(\{\vec{\theta}\}) \left(c^+_{k,\{l\}} \frac{J_{l-1+q/2}(k_\perp x_\perp)}{x_\perp^{q/2-1}} + c^-_{k,\{l\}} \frac{J_{-(l-1+q/2)}(k_\perp x_\perp)}{x_\perp^{q/2-1}} \right) + c.c.$$

regular modes $\qquad$ singular modes

- Regular or singular in the $x_\perp \longrightarrow 0$ limit

Free Scalar Fields with defects

- We can recast the solution in terms of defect operators, which are *defect primaries*

$$\phi = \sum_{\{l\}} c_{\phi\hat{O}^+_{\{l\}}} x_\perp^l Y_{\{l\}}(\{\theta\}) \mathcal{C}_l^+ (x_\perp^2 \partial_\sigma^2) \boxed{\hat{O}^+_{\{l\}}(\sigma)} + c_{\phi\hat{O}^-_{\{l\}}} x_\perp^{2-l-q} Y_{\{l\}}(\{\theta\}) \mathcal{C}_l^- (x_\perp^2 \partial_\sigma^2) \boxed{\hat{O}^-_{\{l\}}(\sigma)}$$

with conformal dimensions

$$\boxed{\hat{\Delta}^+_{\{l\}} = d/2 - 1 + l} \qquad \boxed{\hat{\Delta}^-_{\{l\}} = d/2 + 1 - l - q}$$

where the operators

$$\mathcal{C}_l^+ (x_\perp^2 \partial_\sigma^2) \equiv \sum_{k=0}^{+\infty} \frac{(-4)^{-k} (x_\perp^2 \partial_\sigma^2)^k}{k!(1 + \hat{\Delta}^+_{\{l\}} - \frac{p}{2})_k}\,, \qquad \mathcal{C}_l^- (x_\perp^2 \partial_\sigma^2) \equiv \sum_{k=0}^{+\infty} \frac{(-4)^{-k} (x_\perp^2 \partial_\sigma^2)^k}{k!(1 + \hat{\Delta}^-_{\{l\}} - \frac{p}{2})_k}$$

resum the descendant

Unitarity constraint: [Lauria, Liendo, van Rees, Zhao, 2021]

$$\hat{\Delta} \geq \frac{p}{2} - 1\,, \qquad p \geq 2$$

$$\hat{\Delta} \geq 0\,, \qquad p < 2$$

→

“ $+$ ” are always allowed

“ $-$ ” are allowed if:

- $l = 0 \quad q < 4$; $l = 1 \quad q < 2$ if $p \geq 2$
- $l = 0 \quad q < 3$ if $p = 1$

Free Scalar Fields with defects

- We consider only the $l = 0$ minus mode

- The propagator:

$$\langle\phi(x_1)\phi(x_2)\rangle = \frac{C_\phi}{|x_1 - x_2|^{d-2}} - \frac{\xi}{4\pi^{\frac{p}{2}+1}}\frac{\Gamma\left(\frac{q}{2}\right)}{\pi^{\frac{q}{2}-1}}\left(\frac{1}{x_{\perp 1}x_{\perp 2}}\right)^{\frac{d}{2}-1}\left[\frac{\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{q}{2}\right)}F_{\frac{d}{2}-1}(\eta) - \frac{\Gamma\left(\frac{p-q}{2}+1\right)}{\Gamma\left(2-\frac{q}{2}\right)}F_{\frac{p-q}{2}+1}(\eta)\right] \qquad C_\phi = \frac{\Gamma\left(\frac{d}{2}-1\right)}{4\pi^{\frac{d}{2}}}$$

where $$F_{\hat\Delta}(\eta) = \left(\frac{\eta}{2}\right)^{\hat\Delta} {}_2F_1\left(\frac{\hat\Delta}{2}, \frac{\hat\Delta+1}{2}; \hat\Delta + 1 - \frac{p}{2}; \eta^2\right)$$

cross-ratio: $$\eta \equiv \frac{2x_\perp x'_\perp}{x_\perp^2 + x'^2_\perp + |\sigma^a|^2}$$

- For quadratic theories: $\xi = 0, 1$

- With interactions, unitarity gives: $\xi \in [0, 1]$

- Stress-tensor one-point function:

$$\langle T_{\perp\perp}\rangle = \frac{(1-q)(2-q)\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{p-q}{2}+1\right)}{\pi^{\frac{d-1}{2}}2^{p+3}(d-1)\Gamma\left(\frac{p+3}{2}\right)\Gamma\left(1-\frac{q}{2}\right)}\xi\,\frac{1}{x_\perp^d}$$

For $p = 2$

$$T^\mu{}_\mu|_{\Sigma_2} \sim \frac{1}{24\pi}d_2 W^{ab}{}_{ab}$$

$$d_2 = -\frac{1}{8}(d-2)(d-4)^2\xi$$

Scalar Free Fields with defects

- The **displacement operator**: $\partial_\mu T^{\mu\nu} = \hat{\mathcal{D}}^\nu \, \delta^q(\Sigma)$

$$\hat{\hat{O}}^-_{l=0} \equiv \lim_{x_\perp \to 0} x_\perp^{q-2}\phi\,, \qquad \left(\hat{\hat{O}}^+_{l=1}\right)^i \equiv \lim_{x_\perp \to 0} \partial^i \phi$$

$$\hat{D}^i \equiv \kappa_{\hat{D}}\, \hat{\hat{O}}^-_{l=0} \left(\hat{\hat{O}}^+_{l=1}\right)^i \qquad \kappa_{\hat{D}} = -4\pi^{q/2-1} \sin(\pi q/2)\, \Gamma(2-q/2)$$

- The free coefficient is fixed by a Ward identity

- The two-point function reads:

$$\langle \hat{D}^i(\sigma) \hat{D}^j(0) \rangle = \frac{C_{\hat{D}}}{|\sigma|^{2p+2}} \delta^{ij} \qquad C_{\hat{D}} = \frac{1}{\pi^{p+1}} (2-q) \Gamma\left(\frac{d}{2}\right) \sin\left(\pi \frac{q}{2}\right) \Gamma\left(\frac{p-q}{2}+1\right) \xi$$

- From this we can extract the anomaly coefficient:

$$T^\mu{}_\mu|_{\Sigma_2} \sim \frac{1}{24\pi} d_1 \mathring{\Pi}^2 \qquad d_1 = 3\pi \frac{(d-2)(d-4)^2}{16} \xi \qquad p = 2$$

Scalar Free Fields with defects

$$\delta S_{\text{pert}} = h_c \int d^p\sigma \left(\hat{O}_{\hat{\Delta}}\right)^2 \qquad \begin{cases} \left(\hat{O}^{+}_{l=0}\right)^2 & \text{relevant if} \quad q<2 \\ \left(\hat{O}^{-}_{l=0}\right)^2 & \text{relevant if} \quad 2<q\leq 4 \end{cases}$$

$\xi = 0 \longrightarrow \xi = 1$

$\xi = 1 \longrightarrow \xi = 0$

- Long-distance limit of the defect two-point function:

$$\left\langle \hat{O}_{\hat{\Delta}}(\sigma_1)\hat{O}_{\hat{\Delta}}(\sigma_2)\right\rangle_{h_c} = \frac{\Gamma(\hat{\Delta})(p-2\hat{\Delta})\sin\left(\pi\left(\frac{p}{2}-\hat{\Delta}\right)\right)\Gamma(p-\hat{\Delta})}{2\pi^{p+1}}\frac{1}{h_c^2}\frac{1}{|\sigma|^{2(p-\hat{\Delta})}}+\mathcal{O}\left(\frac{1}{h_c^3}\right) \longrightarrow \hat{\Delta}_{\text{IR}} = p-\hat{\Delta}$$

- Defect sphere free energy and Euler Anomalies:

$p=2$

$$\Delta b = (\hat{\Delta}-1)^3 = \begin{cases} -\frac{(4-d)^3}{8} & \text{if} \quad 2<d<4 \\ \frac{(4-d)^3}{8} & \text{if} \quad 4<d\leq 6 \end{cases}$$

$p=4$

$$\Delta a_{\Sigma} = \begin{cases} -\frac{(d-6)^3(3(d-12)d+88)}{23040} & \text{if} \quad 4<d<6 \\ \frac{(d-6)^3(3(d-12)d+88)}{23040} & \text{if} \quad 6<d\leq 8 \end{cases}$$

Scalar Free Fields with defects interactions

- We can consider a generic deformation localised on the defect:

$$S = S_{\mathrm{DCFT}} + \int d^p\sigma \sum_k g_{0,k}\, \hat{\mathcal{O}}_k$$

$p - \hat{\Delta}_k = 0$
marginal operator

$\delta_k\, \epsilon = p - \hat{\Delta}_k$
slightly relevant operator

- We can apply conformal perturbation theory:

$$\beta_{g_i} = -\epsilon\, \delta_i g_i + \frac{\pi^{p/2}}{\Gamma\left(\frac{p}{2}\right)} \frac{1}{N_i} \sum_{k,l} C_{ikl}\, g_k g_l$$

$\langle \hat{\mathcal{O}}_i \hat{\mathcal{O}}_j \rangle \sim N_i \delta_{ij}$
operator normalisation

$\langle \hat{\mathcal{O}}_i \hat{\mathcal{O}}_j \hat{\mathcal{O}}_k \rangle \sim C_{ijk}$
3-point function

Example: Coupling to a lower dimensional CFT

$$\delta S_{\mathrm{D}} = \int d^p x\, g_{0,1}\, \hat{\phi}\, \hat{\mathcal{O}}_1 + \int d^p x\, g_{0,2}\, \hat{\mathcal{O}}_2$$

- Solution to the beta functions:

$g_{1*} = 0 \qquad g_{2*} = 0$ Trivial fixed point

$g_{1*} = 0\,, \qquad g_{2*} = \dfrac{C_{\hat{\mathcal{O}}_2}\Gamma\left(\frac{p}{2}\right)}{\pi^{p/2} C_{\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2}} \delta_2\, \epsilon$ Decoupled fixed point

$$g_{1*} = \pm \frac{\sqrt{\delta_1 C_{\hat{\mathcal{O}}_1}}\Gamma\left(\frac{p}{2}\right)\sqrt{2C_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2}\delta_2 C_2 - C_{\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2}\delta_1 C_{\hat{\mathcal{O}}_1}}}{2\pi^{p/2} C^{1/2}_{\phi/\partial\phi} C^{3/2}_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2}}\, \epsilon, \qquad g_{2*} = \frac{C_{\hat{\mathcal{O}}_1}\Gamma\left(\frac{p}{2}\right)\delta_1\, \epsilon}{2\pi^{p/2} C_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2}}$$

Coupled fixed point

$$\frac{2C_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2}\delta_2 C_{\hat{\mathcal{O}}_2}}{C_{\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2}\delta_1 C_{\hat{\mathcal{O}}_1}} > 1$$

Scalar Free Fields with defects interactions

- Perturbative one-point function:

$$\langle \phi^2(x_\perp)\rangle_{\rm D} = \frac{g_1^2}{2}\int d^p\sigma_1 d^p\sigma_2 \left\langle \phi^2(x_\perp)\hat{\mathcal{O}}_1(\sigma_1)\hat{\phi}(\sigma_1)\hat{\mathcal{O}}_1(\sigma_2)\hat{\phi}(\sigma_2)\right\rangle_{0,c} + \hat{\mathcal{O}}(g_1^2 g_2)$$

- We obtain at the first non-trivial order:

$$\langle \phi^2(x_\perp)\rangle_{\rm D} = g_{1*}^2 C_{\hat{\mathcal{O}}_1}\frac{\Gamma(\frac{d-p-2}{2})\Gamma(\frac{p}{2})\Gamma(\frac{d-2}{2})}{16\pi^{d-p}\Gamma(p)}\frac{1}{|x_\perp|^{d-2}} + \hat{\mathcal{O}}(\epsilon^2) \quad \longrightarrow \quad \xi = g_{1*}^2 C_{\hat{\mathcal{O}}_1}\frac{\pi^{\frac{d}{2}-q+1}}{2(2-q)\Gamma\left(\frac{d}{2}-q+1\right)\sin\left(\frac{\pi q}{2}\right)}$$

- Anomalous dimensions of the operator $\hat{\mathcal{O}}_1$? No, it is protected by $\Box\phi = g_{1*}\hat{\mathcal{O}}_1\ \delta^{d-p}(y)$

- Anomalous dimensions of the operator $\hat{\mathcal{O}}_2$:

$$\left\langle \hat{\mathcal{O}}_2(\sigma)^{\rm ren}\hat{\mathcal{O}}_2(0)^{\rm ren}\right\rangle_{\rm D} = \frac{C_{\hat{\mathcal{O}}_2}}{|\sigma|^{2\Delta_2^{(0)}}}\left(1+2\delta_2\log|\sigma|\right) - \frac{4\pi^{p/2}g_{2*}C_{\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2}}{\Gamma\left(\frac{p}{2}\right)|\sigma|^{2\hat{\Delta}_2^{(0)}}}\log(\mu|\sigma|)$$

$$\Delta_2^{\rm ren} = \hat{\Delta}_2 + \hat{\gamma}_2 g_{2*} = \hat{\Delta}_2 + \frac{C_{\hat{\mathcal{O}}_1}C_{\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2\hat{\mathcal{O}}_2}}{C_{\hat{\mathcal{O}}_2}C_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2}}\delta_1\,\epsilon$$

Scalar Free Fields with defects interactions

- Deformation:

$$S = S_{\mathrm{DCFT}} + \int d^p\,\sigma \sum_k g_k\,\hat{\mathcal{O}}_k$$

- Perturbative correction to the defect sphere free energy:

$$\delta F = -\log\left|\frac{Z(g_{i,0})}{Z(0)}\right| = \sum_{n=1}^{\infty}\sum_{i_1,\ldots,i_n}\frac{(-1)^n g_{i_1}\ldots g_{i_n}}{n!}\int d^p\sigma_1\sqrt{G}\ldots\int d^p\sigma_N\sqrt{G}\left\langle\hat{\mathcal{O}}_{i_1}(\sigma_1)\ldots\hat{\mathcal{O}}_{i_n}(\sigma_n)\right\rangle$$

$$\delta F = -\frac{\pi^{p+1}}{\sin\left(\frac{\pi p}{2}\right)}\frac{1}{3\,\Gamma\,(p+1)}\sum_i N_i\,\delta_i\,\epsilon\,\left(g_*^i\right)^2 + \mathcal{O}(\epsilon^4)$$

- Anomaly when the defect dimension is p *even*

$$\Delta F|_{\mathrm{anom}} = (-1)^{\frac{p}{2}+1}\frac{2\pi^p}{3\,\Gamma\,(p+1)}\sum_i N_i\,\delta_i\,\epsilon\,\left(g_*^i\right)^2$$

$$\Delta b = -\pi^2\sum_i N_i\,\delta_i\,\epsilon\,\left(g_*^i\right)^2 \leq 0\,, \qquad p=2$$

$$\Delta a = -\frac{\pi^4}{9}\sum_i N_i\,\delta_i\,\epsilon\,\left(g_*^i\right)^2 \leq 0\,, \qquad p=4$$

Free scalar: $\Delta F|_{\mathrm{anom}} = (-1)^{\frac{p}{2}+1}\frac{\Gamma\left(\frac{p}{2}\right)}{12\,\Gamma(p+1)}\epsilon^3$

Scalar Free Field coupled to minimal models

Example with d=3 and p=2:

Minimal models $\mathcal{M}(p,p+1)$ $\qquad S \sim \frac{1}{2}\int d^3x\,(\partial\phi)^2 + \int d^2\sigma\,\hat{\Phi}_{(m,m+1)}\partial_\perp\hat{\phi}$

- Scaling dimension $\quad h = \bar{h} = \dfrac{((p+1)m - pn)^2 - 1}{4p(p+1)}$ $\qquad m = 0, \ldots, p-1$, $\; n = 0, \ldots, p$
- We take the limit $\quad p \to \infty$ $\;\longrightarrow\;$ $\Delta = h + \bar{h} = \dfrac{(m-n)^2}{2} + \mathcal{O}(1/p)$, $\; n = m+1$ $\;\longrightarrow\;$ $\Delta = \dfrac{1}{2} + \mathcal{O}(1/p)$
- We choose: $\quad m = 1 \qquad \hat{\Delta} = 2 - \epsilon \qquad \epsilon = \dfrac{3}{2p} + o(1/p) \ll 1$

$$\delta S_{d=3} = \int d^2x\, g_1\hat{\Phi}_{(1,2)}\partial_y\hat{\phi} \; + \; g_2\hat{\Phi}_{(1,3)}$$

$$\hat{\Delta}_{(1,2)} = \frac{1}{2} - \frac{3}{2p} + o(1/p) \qquad\qquad \hat{\Delta}_{(1,3)} = 2 - \frac{4}{p+1}$$

Scalar Free Fields coupled to minimal models

$$\delta S_{d=3} = \int d^2x\, g_1 \hat{\Phi}_{(1,2)} \partial_y \hat{\phi} \; + \; g_2 \hat{\Phi}_{(1,3)}$$

- System of beta functions:

$$g_1 = \pm \frac{1}{\pi p} \sqrt{\frac{2}{C_{\partial_y \hat{\phi}}}} + \mathcal{O}(1/p), \quad g_2 = -\frac{\sqrt{3}}{2\pi p} + \mathcal{O}(1/p^2)$$ Coupled fixed point

$$g_1 = 0 + \mathcal{O}(1/p^2), \quad g_2 = -\frac{\sqrt{3}}{\pi p} + \mathcal{O}(1/p^2)$$ Decoupled fixed point

- Anomalous dimension of the g_1 operator $\Delta_\pm = 2 \pm \frac{\sqrt{6}}{p}$

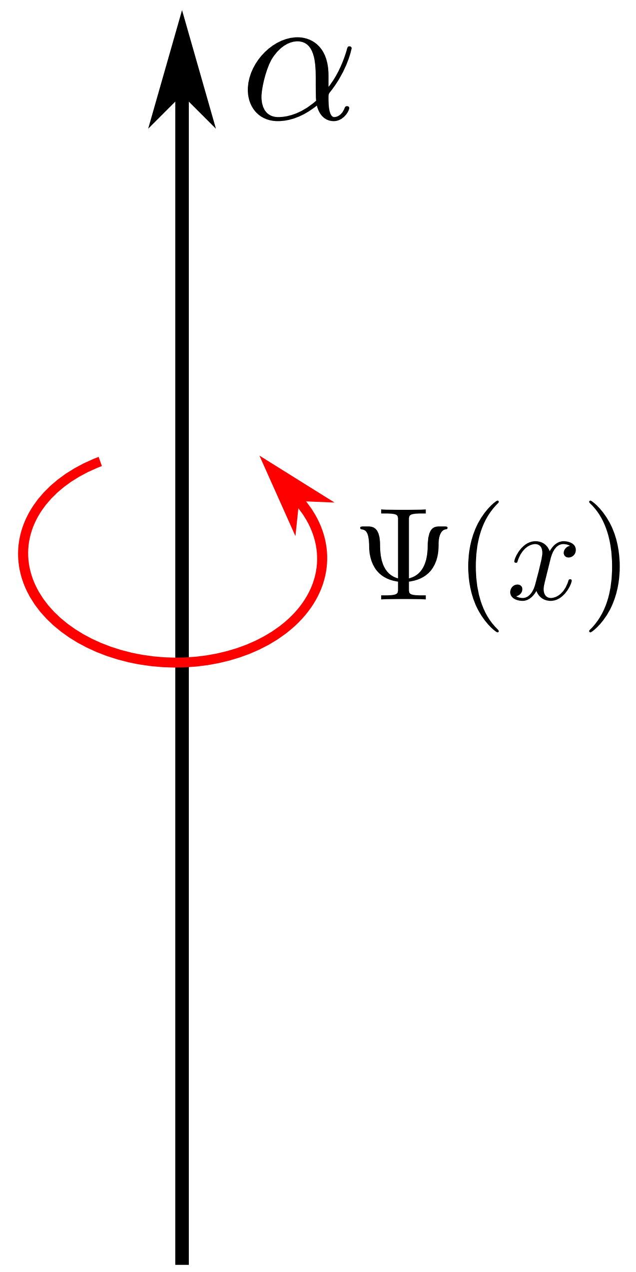
- Similar analysis for $d = 5$

$$\delta S_{d=5} \int d^2x\, g_1 \hat{\Phi}_{(1,2)} \hat{\phi} \; + \; g_2 \hat{\Phi}_{(1,3)}$$

dim	Δb_{Int}	a_{ϕ^2}	a_T	$b_{\mathcal{O}_1 \phi}$	$C_{\hat{D}}$
$d=3$	$-\frac{6}{p^3}$	$\frac{1}{\pi p^2}$	$-$	$\pm\sqrt{\frac{2}{\pi}\frac{1}{p}}$	$\frac{18}{\pi^2 p^2}$
$d=5$	$-\frac{6}{p^3}$	$\frac{1}{2\pi^2 p^2}$	$\frac{1}{32\pi^2 p^2}$	$\pm\frac{1}{\pi p}$	$\frac{6}{\pi^2 p^2}$

Monodromy defects

[Bianchi, Chalabi, Prochazka, Robinson, JS, 2021]
[Giombi, Helfenderer, Ji, Khanchandani, 2021]



- Co-dimension 2 operator which implements a flavour symmetry rotation

- Simplest case: U(1) rotation $\Psi(x) \to e^{-2\pi i\alpha}\Psi(x)$ $\qquad \alpha \in [0,1]$

- In a Lagrangian theory it can be achieved by “gauging” the global symmetry by coupling to an external potential

$$S \longrightarrow S + \int d^d x\, J^\mu A_\mu \qquad\qquad A = \alpha\, d\theta$$

- This is a pure gauge everywhere but at the origin $\qquad F_{xy} = 2\pi\,\alpha\,\delta^{(2)}(x,y)$

Monodromy defects

- Complex Scalar free theory

$$I_{\text{scalar}} = \int d^d x \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} (\nabla_\mu - ieA_\mu)\varphi(\nabla_\nu + ieA_\nu)\varphi^\dagger + \frac{d-2}{8(d-1)} \mathcal{R} |\varphi|^2 \right]$$

$$A = \alpha \, d\theta \qquad \alpha \in (0,1)$$

- Mode expansion and regularity conditions

$$\varphi = \varphi_0 + \sum_{m=1}^{\infty} \varphi_{m-\alpha} z^{m-\alpha} + \sum_{m=1}^{\infty} \varphi_{m+\alpha} \bar{z}^{m+\alpha}$$

$$\varphi_0 = \begin{cases} \varphi_\alpha \, \bar{z}^\alpha & \text{regular mode} \quad \xi = 0 \\ \varphi_{-\alpha} \, z^{-\alpha} & \text{singular mode} \quad \xi = 1 \end{cases}$$

$$z = x_\perp \, e^{i\theta}$$

- Propagator as sum of hypergeometric functions

$$\langle \phi(x) \phi^\dagger(0, x'_\perp) \rangle = \left(\frac{1}{x_\perp x'_\perp} \right)^{\frac{d}{2}-1} \left(\sum_{s \in \mathbb{Z}-\alpha} c_s^+ \, F_{\hat{\Delta}^+,s}(\eta,\theta) + c_{-\alpha}^- \, F_{\hat{\Delta}^-,-\alpha}(\eta,\theta) \right)$$

$$c_{\phi \hat{O}_s^+} = \sqrt{\frac{\Gamma\left(\frac{d}{2} - 1 + |s|\right)}{4\pi^{d/2}\Gamma(1+|s|)}} \qquad c_{\phi \hat{O}_{-\alpha}^-} = \sqrt{\xi \frac{\Gamma\left(\frac{d}{2} - 1 - \alpha\right)}{4\pi^{d/2}\Gamma(1-\alpha)}} \qquad c_{\phi \hat{O}_{-\alpha}^+} = \sqrt{(1-\xi) \frac{\Gamma\left(\frac{d}{2} - 1 + \alpha\right)}{4\pi^{d/2}\Gamma(1+\alpha)}}$$

$$c_s^\pm = |c_{\phi \hat{O}_s^\pm}|^2$$

$$F_{\hat{\Delta},s}(\eta,\theta) = \left(\frac{\eta}{2} \right)^{\hat{\Delta}_s} {}_2F_1 \left(\frac{\hat{\Delta}_s}{2}, \frac{\hat{\Delta}+1}{2}; \hat{\Delta}_s + 2 - \frac{d}{2}; \eta^2 \right) e^{is\theta}$$

$$\eta \equiv \frac{2 x_\perp x'_\perp}{x_\perp^2 + x'^2_\perp + |\sigma^a|^2}$$

- Similar discussion for fermions

Monodromy defects

- One-point functions:

$$\langle|\phi(x)|^2\rangle = \frac{\Gamma(\frac{d}{2}-\alpha)\Gamma(\frac{d}{2}+\alpha-1)\sin(\pi\alpha)}{2^{d-1}\pi^{\frac{d+1}{2}}\Gamma\left(\frac{d-1}{2}\right)}\frac{1}{x_\perp^{d-2}}\left(-\frac{1}{d-2}+\frac{\xi}{\frac{d}{2}-\alpha-1}\right)$$

- Stress-energy tensor

$$\langle T_{\perp\perp}(x)\rangle = -\frac{a_T}{x_\perp^d} \quad \longrightarrow \quad a_T = \frac{\Gamma\left(\frac{d}{2}-\alpha\right)\Gamma\left(\frac{d}{2}+\alpha-1\right)\sin(\pi\alpha)\left(\frac{\alpha(1-\alpha)}{d}+\frac{\alpha^2\xi}{\frac{d}{2}-\alpha-1}\right)}{2^{d-1}\pi^{\frac{d+1}{2}}\Gamma\left(\frac{d+1}{2}\right)}$$

- Displacement operator

$$\hat{D}_z = \hat{O}^-_{-\alpha}\hat{O}^{+\dagger}_{1+\alpha}\,, \qquad \hat{D}_{\bar{z}} = \hat{O}^{-\dagger}_{-\alpha}\hat{O}^{+}_{1+\alpha}$$

$$d_1 = d_2 = \frac{3}{2}\left[(1-\alpha)^2\alpha^2+4\xi\alpha^3\right]$$ d=4

$$d_1 = 2d_2 = -\frac{\alpha(1-\alpha^2)(2-\alpha)}{36}\left[\alpha(1-\alpha)+\frac{6\alpha^2\xi}{2-\alpha}\right]$$ d=6

- Current and defect sphere free energy:

$$b = \frac{(1-\alpha)^2\alpha^2+4\xi\alpha^3}{2}$$ d=4

$$a_\Sigma = \frac{\alpha^2}{720}(1-\alpha)^2(3+\alpha-\alpha^2)+\frac{\alpha^3}{360}(5-3\alpha^2)\xi$$ d=6

Monodromy defects

$$\varphi_0 = \begin{cases} \varphi_\alpha \, \bar{z}^\alpha & \text{regular mode} \\ \varphi_{-\alpha} \, z^{-\alpha} & \text{singular mode} \end{cases} \quad \longrightarrow \quad \hat{\Delta}^+_\alpha = \frac{d-2}{2} + \alpha, \quad \hat{\Delta}^-_\alpha = \frac{d-2}{2} - \alpha$$

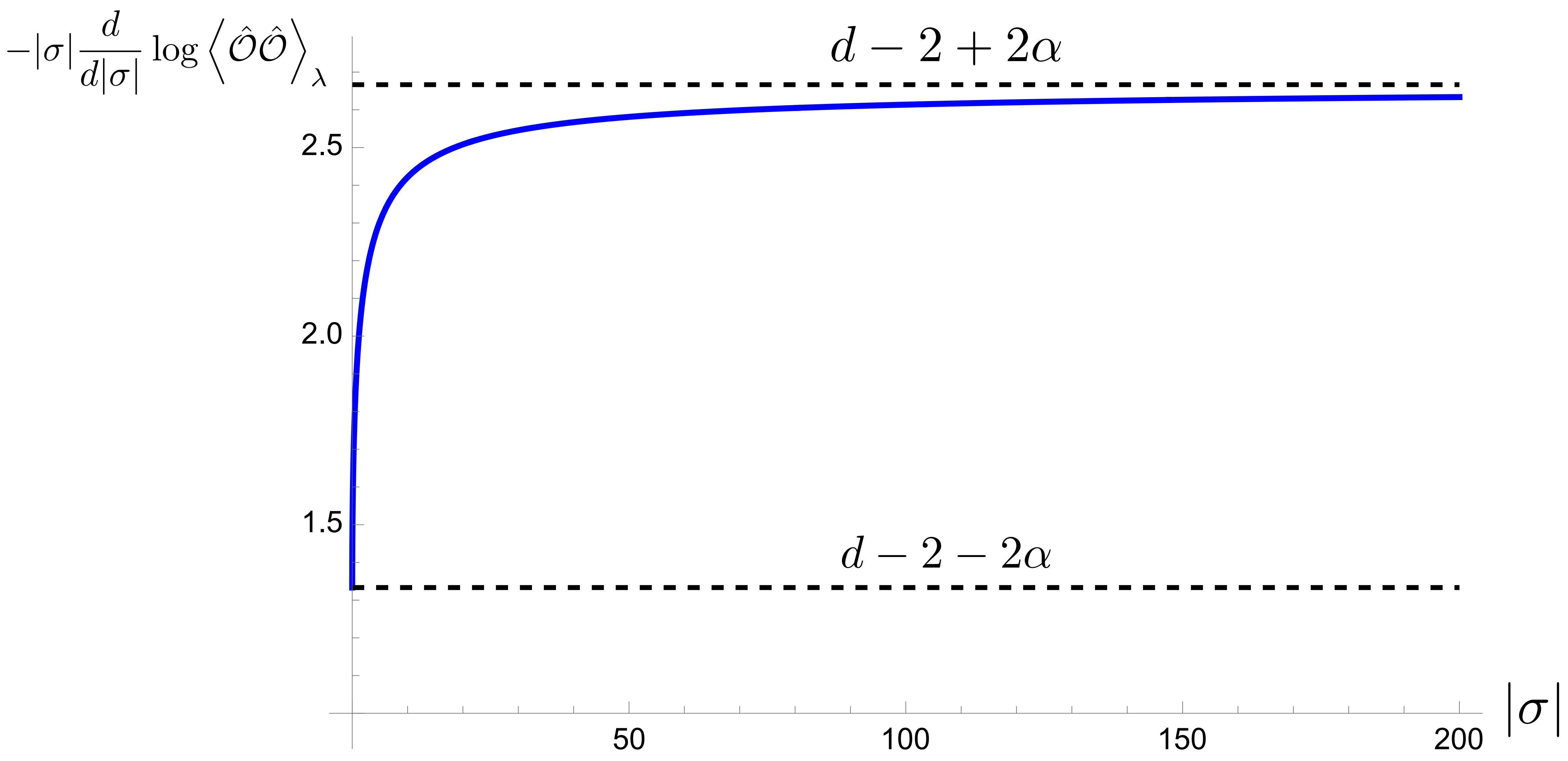
- The singular and regular modes are connected by RG flow:

$$S_{def} = \lambda \int d^{d-2}\sigma \, \hat{O}_{-\alpha}(\sigma)\hat{O}^\dagger_{-\alpha}(\sigma) \qquad \lambda = \Lambda^{2\alpha}\bar{\lambda}$$

relevant deformation

$$\left\langle \hat{\mathcal{O}}_{-\alpha}(\sigma)\hat{\mathcal{O}}_{-\alpha}(0) \right\rangle_\lambda \longrightarrow \frac{1}{|\sigma|^{d-2-2\alpha}} \qquad \sigma \to 0$$

$$\left\langle \hat{\mathcal{O}}_{-\alpha}(\sigma)\hat{\mathcal{O}}_{-\alpha}(0) \right\rangle_\lambda \longrightarrow \frac{\Lambda^{-4\alpha}}{|\sigma|^{d-2+2\alpha}} \qquad |\sigma| \to \infty$$



- The Euler anomaly decreases

$$a_\Sigma = \frac{\alpha^2}{720}(1-\alpha)^2(3+\alpha-\alpha^2) + \frac{\alpha^3}{360}(5-3\alpha^2)\xi \quad \longrightarrow \quad a_\Sigma^{(\mathrm{UV})} \geq a_\Sigma^{(\mathrm{IR})}$$

$$\xi = 1 \qquad \xi = 0$$

defect a-theorem

[Wang, 2021]

Monodromy defects with self interactions

[Bashmakov, JS, 2024]

- Starting from $\xi = 1$ we can deform the monodromy defect through the coupling

$$S_{\lambda_n} = \lambda_n \int d^{d-2}\sigma \left(\hat{O}^-_{-\alpha}(\sigma)\hat{O}^{\dagger -}_{-\alpha}(\sigma)\right)^n \qquad \Delta^-_{-\alpha} = d/2 - 1 - \alpha$$

Relevant if: $\alpha > \bar{\alpha}$, $\bar{\alpha} \equiv \dfrac{(n-1)(d-2)}{2n}$ $\qquad \alpha \in (0,1)$

- We take $\alpha = \bar{\alpha} + \epsilon$ with $0 < \epsilon \ll 1$

- The beta function reads

$$\beta = -2n\,\epsilon\,\lambda_n + \frac{\pi^{d/2-1}}{\Gamma\left(\frac{d}{2}-1\right)}\frac{C_{nnn}}{(n!)^2}\lambda_n^2 \qquad C_{nnn} = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!}\frac{n!}{(n-k)!}\frac{n!}{k!}\frac{n!}{k!}\frac{n!}{(n-k)}\frac{n!}{k!}k! = (n!)^3 \mathrm{Fr}_n$$

$$\lambda_{n*} = \frac{2n}{(n!)\mathrm{Fr}_n}\frac{\Gamma\left(\frac{d}{2}-1\right)}{\pi^{d/2-1}}\epsilon$$

- We get the same propagator but with

$$\xi = 1 - \frac{4\pi\, n^3\, \Gamma^2\left(\frac{d}{2}-1\right)\csc\left(\pi\,\bar{\alpha}\right)}{\mathrm{Fr}_n^2\,(n-1)\,\Gamma\left(\frac{d-2}{2n}+1\right)\Gamma\left(\bar{\alpha}+\frac{d}{2}-1\right)}\epsilon^2 \qquad \xi \in [0,1]$$

Monodromy defects coupled to Minimal Models

- We can couple to Minimal Models:

$$S_{\text{int}} = g \int d^2\sigma \, \left(\hat{O}^{-}_{-\alpha}\hat{O}^{-\dagger}_{-\alpha}\right)^n \hat{\Phi}$$

- Slightly relevant if $\alpha = \bar{\alpha} + \epsilon$ $\qquad \bar{\alpha} = \frac{n-1}{n} + \frac{\Delta_{\hat{\Phi}}}{2n}$

- Beta function and fixed point:

$$\beta_g = -2n\,\epsilon\, g + \pi \frac{C_{nnn}}{(n!)^2} C_{\hat{\Phi}\hat{\Phi}\hat{\Phi}}\, g^2 \qquad g_* = \frac{2n}{\pi\, n!\, \text{Fr}_n\, C_{\hat{\Phi}\hat{\Phi}\hat{\Phi}}}\,\epsilon$$

$$C_{\hat{\Phi}\hat{\Phi}\hat{\Phi}} \neq 0$$

- Propagator:

$$\xi = 1 - \frac{16\, n^5}{\text{Fr}_n^2\, C^2_{\hat{\Phi}\hat{\Phi}\hat{\Phi}} (\Delta_{\hat{\Phi}} + 2n - 2)^2}\,\epsilon^2$$

- An anomalous dimension:

$$\left\langle \left(O^{-}_{-\alpha}\right)^m (\sigma)\hat{\Phi}(\sigma) \left(O^{-\dagger}_{-\alpha}\right)^m (0)\hat{\Phi}(0)\right\rangle = \frac{m!}{|\sigma|^{2m(1-\alpha)+2\Delta_{\hat{\Phi}}}} \left[1 - \frac{4\pi C_{\hat{\Phi}\hat{\Phi}\hat{\Phi}}\, m!}{((m-n)!)^2} \log\left(\mu|\sigma|\right)\, g_*\right]$$

$$\gamma_{\hat{O}^m\hat{\Phi}} = \frac{2\pi C_{\hat{\Phi}\hat{\Phi}\hat{\Phi}}\, m!}{((m-n)!)^2}\, g_*$$

- Examples:

$\hat{\Phi} = \varepsilon'$	$\Delta_{\varepsilon'} = 6/5$	$\bar{\alpha} = 3/15$	$n = 1$	Tricritical Ising Model
$\hat{\Phi} = \sigma$	$\Delta_{\hat{\Phi}} = 2/15$	$\bar{\alpha} = 1/15$	$n = 1$	Three-State Potts Model

Conclusion and Outlook

Summary

- We discussed the possible defect CFTs in the case of bulk free scalar fields
- In the quadratic case we can admit singularities leading to non-trivial DCFTs
- We found perturbative fixed points where the bulk field is coupled to lower dimensional CFTs
- Similarly, we studied the case of monodromy defects with defect interactions

Outlook

- Extension to theories with both bulk and defect interactions
- Fermionic theories
- Relaxing the assumption of unitarity (higher-derivative theories)

Thank you for your attention!