Phenomenology of Electroweak interactions

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Disclaimer

- This talk **is not** about any new result or super-detailed computation for some specific process. A basic knowledge of QFT and Standard Model (SM) will be enough

- What this talk **is** about:

  ▴ Overall review of the Electroweak (EW) sector of the SM

  ▴ General discussion on aspects of EW SM (e.g. GBET, complex-mass scheme, input-schemes, …)

  ▴ Higher-order EW corrections at high-energy (focus of my Ph.D.)

- I won’t adopt an historical approach (i.e. Fermi theory → IVB → EW)

- See “References” slide for more details on the following topics…or just ask!

Framework: notation & conventions (i)

- Conventions:

  → φ_i is a generic **SM** multiplet

  → All incoming particles, i.e. $n \to 0$ process $\varphi_{i_1}(p_1)\ldots\varphi_{i_n}(p_n) \to 0$

  → Corresponding predictions for the physical process

$$\varphi_{i_1}(p_1^{\text{in}})\ldots\varphi_{i_m}(p_m^{\text{in}}) \to \varphi_{j_1}(p_1^{\text{out}})\ldots\varphi_{j_{n-m}}(p_{n-m}^{\text{out}})$$

  are obtained via *crossing symmetries* from

$$\varphi_{i_1}(p_1^{\text{in}})\ldots\varphi_{i_m}(p_m^{\text{in}})\bar{\varphi}_{j_1}(-p_1^{\text{out}})\ldots\bar{\varphi}_{j_{n-m}}(-p_{n-m}^{\text{out}}) \to 0$$

  → *'t Hooft–Feynman gauge*[1]: all propagators with scalar-like pole structure $\sim \dfrac{1}{p^2 - M^2}$

[1] Gauge-fixing and FP (ghost) terms won't be covered in this talk, but have to be included for a fully quantized theory

Review of **EW** Standard Model

The Electroweak Standard Model: gauge group

- **EW Standard Model** described by the $SU(2)_L \otimes U(1)_Y$ *gauge* symmetry group

▲ $SU(2)_L$: *weak isospin group*, 3 generators → 3 **gauge** fields W^i_μ

$$[I_i, I_j] = i\epsilon_{ijk} I_k, \qquad I_i = \frac{\sigma_i}{2}, \qquad i = 1, ..., 3$$

▲ $U(1)_Y$: *weak hypercharge group*, 1 generator → 1 **gauge** field B_μ

$$Y = 2(Q - I_3)$$

Gell-Mann Nishijima relation

The Electroweak Standard Model: gauge group

- **EW Standard Model** described by the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ *gauge* symmetry group

▲ $\mathrm{SU}(2)_L$: *weak isospin group*, 3 generators → 3 **gauge** fields W_μ^i

$$[I_i, I_j] = i\epsilon_{ijk} I_k, \qquad I_i = \frac{\sigma_i}{2}, \qquad i = 1, \ldots, 3$$

$$\begin{cases} \boxed{\begin{array}{l} \psi_f \to \psi'_f = e^{-igI^i\alpha^i(x)}\psi_f \\ \Phi \to \Phi' = e^{-igI^i\alpha^i(x)}\Phi \end{array}} \\ \boxed{W_\mu^i \to W_\mu'^i = W_\mu^i + \partial_\mu \alpha^i(x) + g\epsilon^{ijk}\alpha^j(x) W_\mu^k} \end{cases}$$

▲ $\mathrm{U}(1)_Y$: *weak hypercharge group*, 1 generator → 1 **gauge** field B_μ

Adjoint reps.

$$\begin{cases} \boxed{\begin{array}{l} \psi_f \to \psi'_f = e^{-ig'\frac{Y_\psi}{2}\alpha(x)}\psi_f \\ \Phi \to \Phi' = e^{-ig'\frac{Y_\Phi}{2}\alpha(x)}\Phi \end{array}} \\ \boxed{B_\mu \to B'_\mu = B_\mu + \partial_\mu \alpha(x)} \end{cases}$$

$$Y = 2(Q - I_3)$$

Gell-Mann Nishijima relation

Fundamental reps.

The Electroweak Standard Model (i)

- **EW** sector of **SM** is described by the classical Lagrangian

$$\mathcal{L}_{\mathrm{EW}}^{\mathrm{cl.}} =$$

The Electroweak Standard Model (i)

- **EW** sector of **SM** is described by the classical Lagrangian

$$\mathcal{L}_{\mathrm{EW}}^{\mathrm{cl.}} = \mathcal{L}_{\mathrm{YM}}$$

Yang-Mills

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \longrightarrow B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk} W^j_\mu W^k_\nu$$

- No mass terms are present at this stage as would violate the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ *gauge* symmetry!
- These are dynamically generated through **Higgs** mechanism and spontaneous breaking of *gauge* symmetry (in few slides)

The Electroweak Standard Model (i)

- The *physical* **gauge** fields are not W^i_μ and B_μ but rather a linear combination of them:

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{2}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\mathrm{w} & s_\mathrm{w} \\ -s_\mathrm{w} & c_\mathrm{w} \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

$$c_\mathrm{w} = \cos\theta_\mathrm{w}$$
$$s_\mathrm{w} = \sin\theta_\mathrm{w}$$

θ_w = weak mixing angle

- Rotation from the "*interaction*" to the "*mass*" basis:

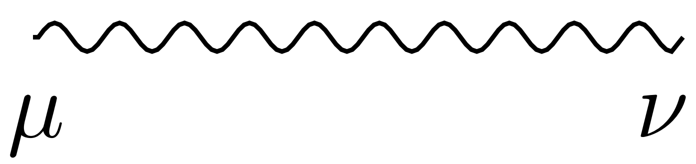
$$\begin{aligned} \mathcal{L}_\mathrm{YM} = & -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\ & -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ & + \text{(trilinear interaction terms involving } AW^+W^-,\ ZW^+W^-) \\ & + \text{(quadrilinear interaction terms involving} \\ & \quad AAW^+W^-,\ AZW^+W^-,\ ZZW^+W^-,\ W^+W^-W^+W^-) \end{aligned}$$

The Electroweak Standard Model (i)

- Rotation from the "*interaction*" to the "*mass*" basis:

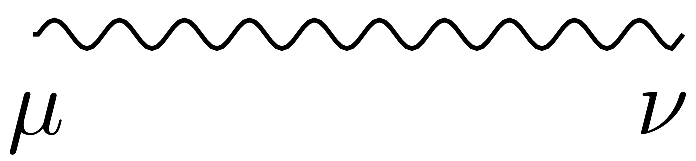
$$
\begin{aligned}
\mathcal{L}_{\mathrm{YM}} \;=\; & -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\
& -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)
\end{aligned}
$$

- Kinetic terms $\longrightarrow$ propagators



$$= \frac{-ig_{\mu\nu}}{p^2 - m_V^2 + i0}$$

in *'t Hooft–Feynman gauge* $(\xi_V = 1)$



$$= \frac{-i}{p^2 - m_V^2 + i0}\left[g_{\mu\nu} - (1-\xi_V)\frac{p_\mu p_\nu}{p^2 - \xi_V m_V^2}\right]$$

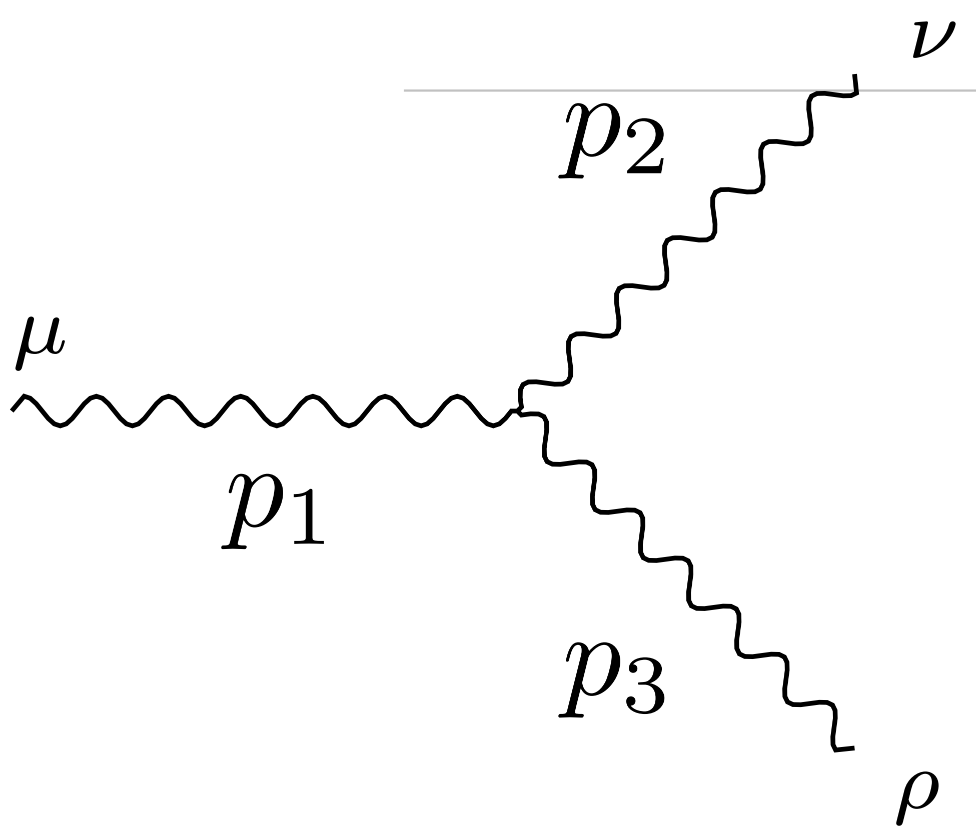
General R_ξ *gauge*

The Electroweak Standard Model (i)

- Rotation from the "*interaction*" to the "*mass*" basis:

$$\mathcal{L}_{\mathrm{YM}} =$$

$$+ \quad \text{(trilinear interaction terms involving } AW^+W^-,\ ZW^+W^-\text{)}$$



$$= -ieC[g_{\mu\nu}(p_2 - p_1)_\rho + g_{\nu\rho}(p_3 - p_2)_\mu + g_{\rho\mu}(p_1 - p_3)_\nu]$$

$$AW^+W^- : \quad C = 1$$

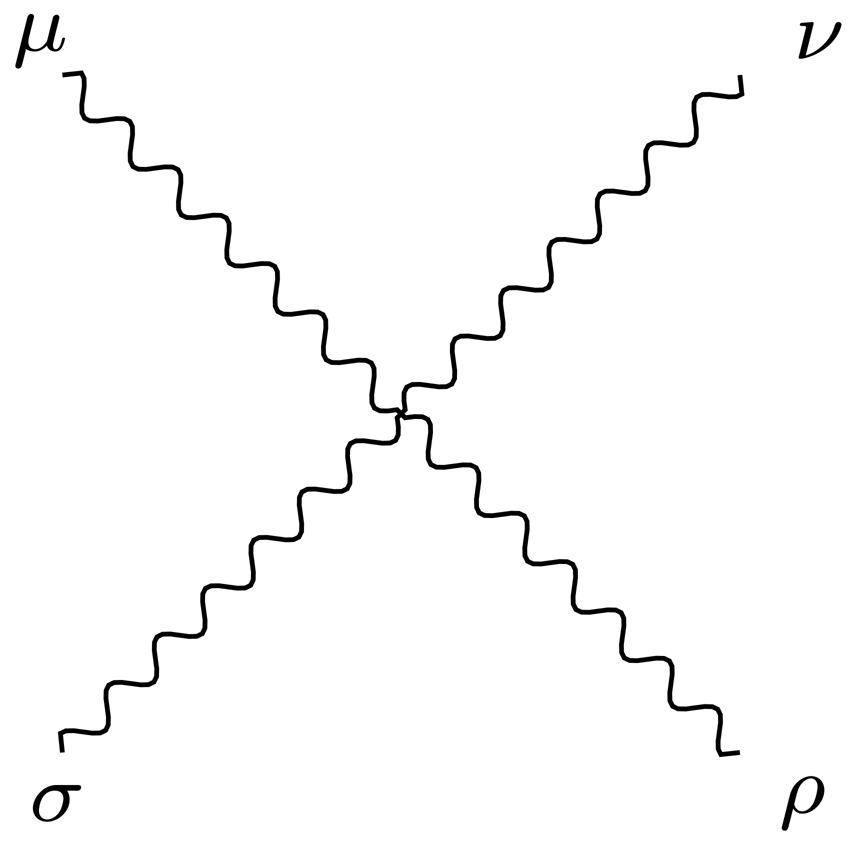
$$ZW^+W^- : \quad C = -\frac{c_{\mathrm{w}}}{s_{\mathrm{w}}}$$

The Electroweak Standard Model (i)

- Rotation from the "*interaction*" to the "*mass*" basis:

$$\mathcal{L}_{\mathrm{YM}} =$$

$$+ \ \text{(quadrilinear interaction terms involving } AAW^+W^-,\ AZW^+W^-,\ ZZW^+W^-,\ W^+W^-W^+W^-\text{)}$$



$$= ie^2 C[2g_{\mu\nu}g_{\sigma\rho} - g_{\nu\rho}g_{\mu\sigma} - g_{\rho\mu}g_{\nu\sigma}]$$

$$\begin{cases} W^+W^+W^-W^- & : \quad C = \frac{1}{s_{\mathrm{w}}^2} \\ W^+W^-ZZ & : \quad C = -\frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2} \\ W^+W^-AZ & : \quad C = \frac{c_{\mathrm{w}}}{s_{\mathrm{w}}} \\ W^+W^-AA & : \quad C = -1 \end{cases}$$

The Electroweak Standard Model (ii)

- **EW** sector of **SM** is described by the classical Lagrangian

$$\mathcal{L}_{\rm EW}^{\rm cl.} = \mathcal{L}_{\rm YM} + \mathcal{L}_{\rm F}$$

No mass terms also for fermions as these would violate the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ *gauge* symmetry. **Higgs** mechanism needed also in this case!

Fermions

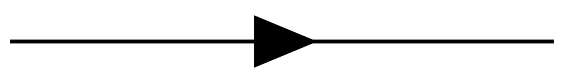
$$\mathcal{L}_{\rm F} = \sum_{\psi_f = q,l} \bar{\psi}_f i \not{D} \psi_f , \qquad \not{D} = \gamma^\mu D_\mu$$

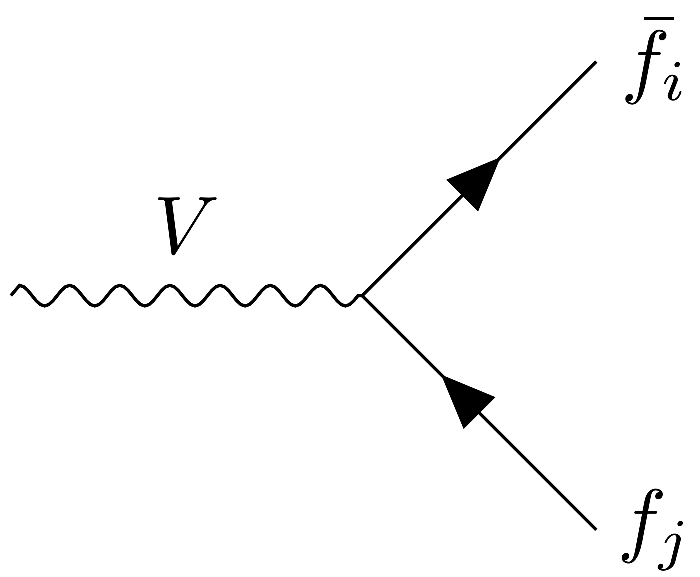
The covariant derivative contains kinetic terms and interactions with **gauge** fields

$$D_\mu \psi_f = \left(\partial_\mu + ig \frac{\sigma_i}{2} W_\mu^i + ig' \frac{Y_{\psi_f}}{2} B_\mu \right) \psi_f$$

The Electroweak Standard Model (ii)

$$\mathcal{L}_{\mathrm{F}} = \sum_{\psi_f = q,l} \bar{\psi}_f i \not{D} \psi_f , \qquad \not{D} = \gamma^\mu D_\mu$$

- Kinetic terms:  $= \dfrac{i(\not{p} + m_f)}{p^2 - m_f^2}$

- Interaction terms: 

$$= ie\gamma^\mu (C^- P_- + C^+ P_+) \quad \begin{cases} \gamma \bar{f}_i f_j & : \quad C^- = C^+ = -Q_f \delta_{ij} \\ Z \bar{f}_i f_j & : \quad C^- = \frac{T_{3L} - s_{\mathrm{w}}^2 Q_f}{s_{\mathrm{w}} c_{\mathrm{w}}} \delta_{ij}, \quad C^+ = -\frac{s_{\mathrm{w}}}{c_{\mathrm{w}}} Q_f \delta_{ij} \\ W^+ \bar{u}_i d_j & : \quad C^- = \frac{1}{\sqrt{2} s_{\mathrm{w}}} \boxed{V_{ij}}, \quad C^+ = 0 \\ W^+ \nu_i \ell_j & : \quad C^- = \frac{1}{\sqrt{2} s_{\mathrm{w}}} \boxed{\delta_{ij}}, \quad C^+ = 0 \end{cases}$$

CKM matrix, see below

PMNS $\overset{m_\nu = 0}{=} \mathbb{1}$

Fermions interact differently with **EW gauge** bosons: **SM** is a ***chiral theory***

$$P_\pm = \frac{1 \pm \gamma^5}{2}$$

The Electroweak Standard Model (iii)

- **EW** sector of **SM** is described by the classical Lagrangian

$$\mathcal{L}_{\mathrm{EW}}^{\mathrm{cl.}} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{H}}$$

Higgs

$$\mathcal{L}_{\mathrm{H}} = \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - V(\Phi)$$

with potential

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

and **Higgs** field

$$\Phi(x) = \begin{pmatrix} \phi_a(x) \\ \phi_b(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{a,1}(x) + i\phi_{a,2}(x) \\ \phi_{b,1}(x) + i\phi_{b,2}(x) \end{pmatrix}$$

Higgs mechanism (i)

- **Higgs** Lagrangian

$$\mathcal{L}_{\mathrm{H}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

- Interactions with **gauge** fields are contained in the covariant derivative

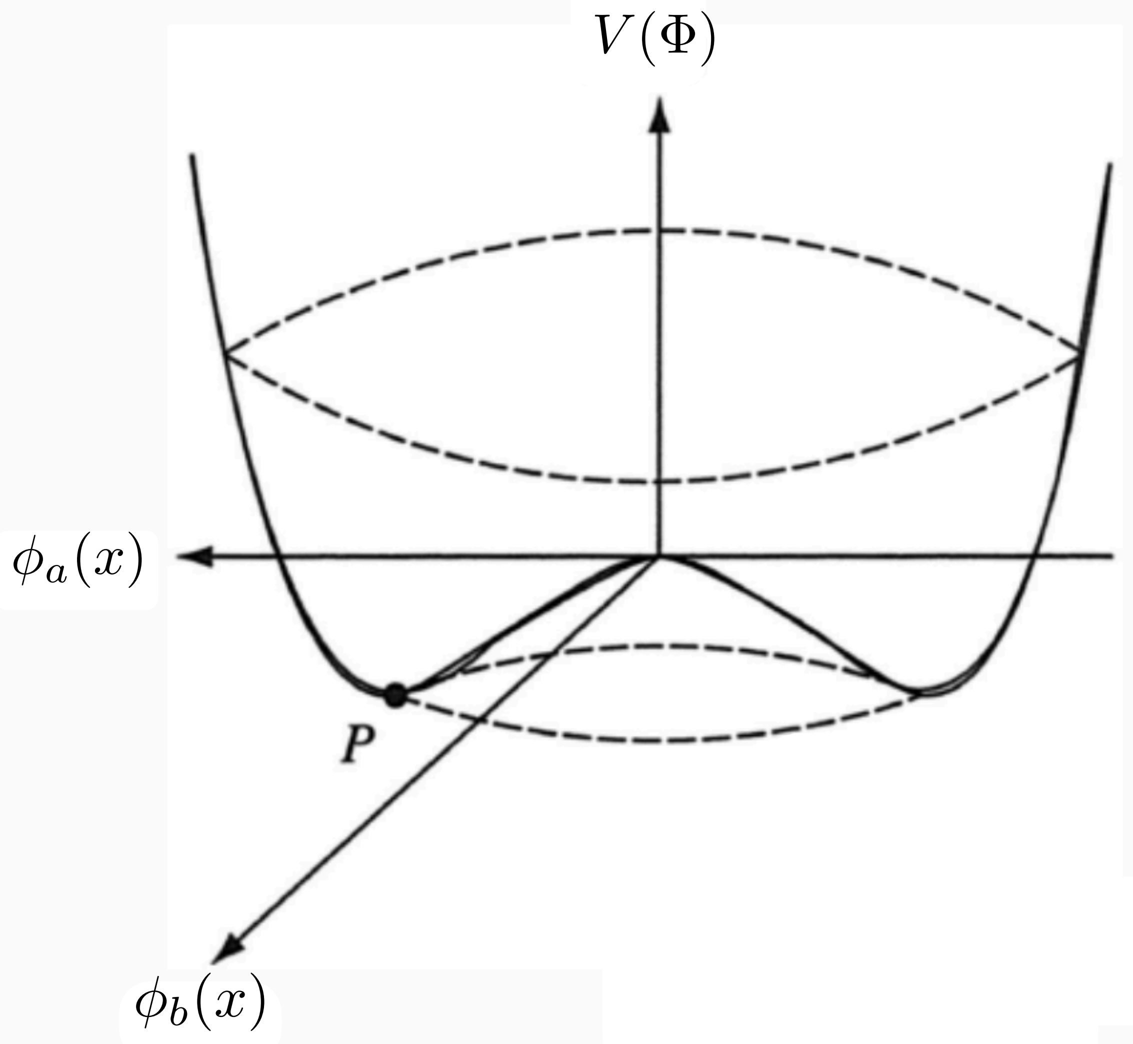
$$D_\mu \Phi = \left(\partial_\mu + ig \frac{\sigma_i}{2} W^i_\mu + ig' \frac{Y_\Phi}{2} B_\mu \right) \Phi, \qquad Y_\Phi = 1$$

- For $\mu^2 < 0$ and $\lambda > 0$ the potential has the well know mexican-hat shape

▲ Minimum at $\Phi_0(x) = \begin{pmatrix} \phi_{a,0} \\ \phi_{b,0} \end{pmatrix}$ | $\Phi_0^\dagger \Phi_0 = \frac{v^2}{2}, \quad v = \sqrt{-\mu^2/(2\lambda)}$

▲ The phase is not determined: the choice of a specific direction spontaneously breaks the symmetry (**SSB**) $\Rightarrow$ **gauge** boson and **fermion** masses allowed!

Vacuum expectation value (**VEV**) of the **Higgs** field



Higgs mechanism (ii)

- Typically one chooses

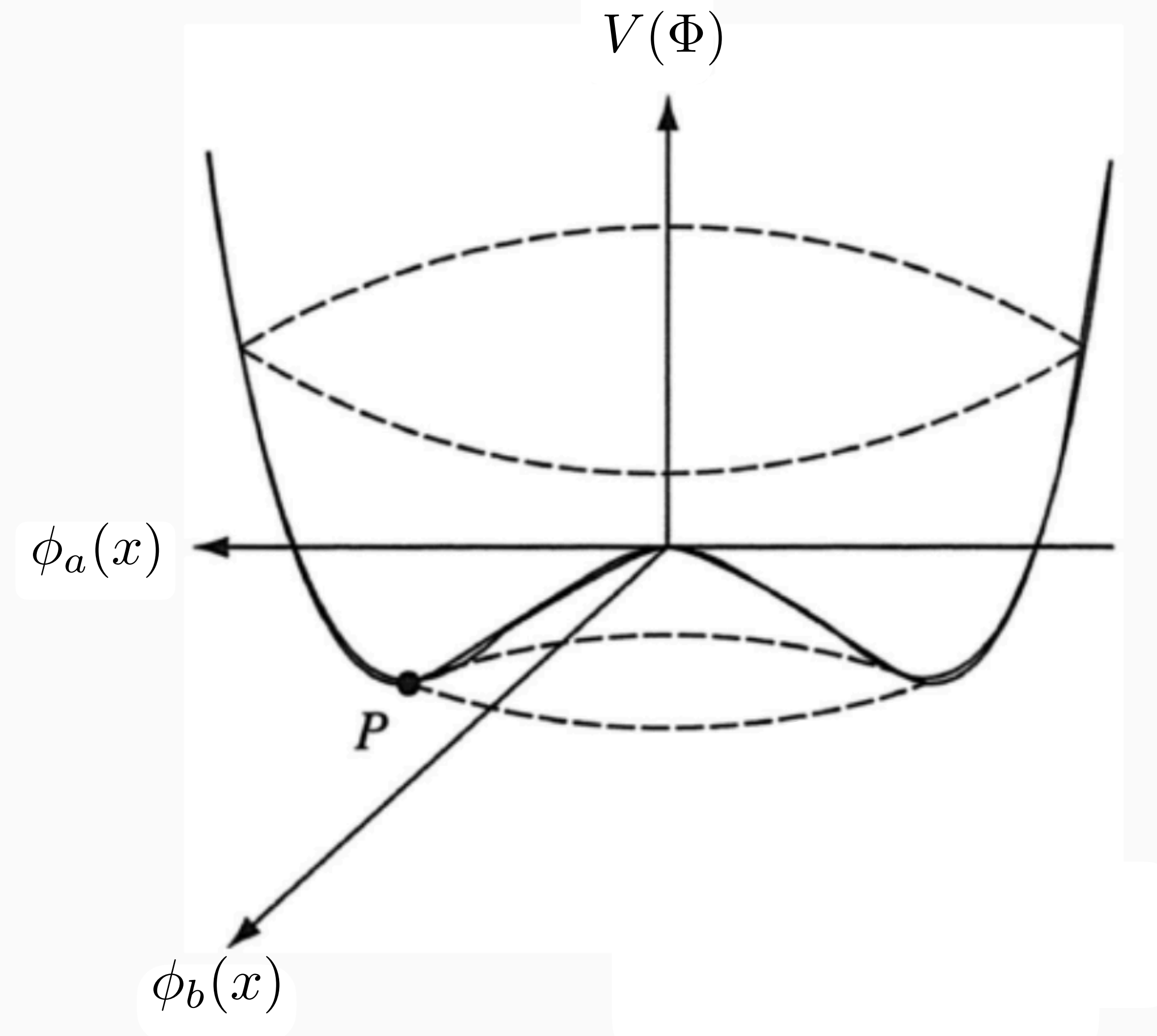
$$\Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

and expands the **Higgs** field as an excitation around the minimum as

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + H(x) + i\chi(x)) \end{pmatrix}, \quad \phi^\pm = \frac{\rho_1(x) \pm i\rho_2(x)}{\sqrt{2}}, \quad \phi^- = (\phi^+)^\dagger$$

physical
Higgs boson

goldstone bosons



- *D.o.f. counting:*
    - **Higgs** boson: 1 real scalar → 1 *d.o.f.*
    - **Goldstone** bosons: 1 real scalar + 1 complex scalar → 3 *d.o.f.*
    - **Gauge** bosons: 4 massless vector fields = (4 x 2 =) 8 *d.o.f.*

Higgs mechanism (iii)

- With this expansion the **Higgs** Lagrangian provides the following explicit terms
  - Kinetic terms: $$(\partial_\mu \phi^+)(\partial^\mu \phi^-) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) + \frac{1}{2}(\partial_\mu H)(\partial^\mu H)$$

Higgs mechanism (iii)

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  - Kinetic terms: $$(\partial_\mu \phi^+)(\partial^\mu \phi^-) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) + \frac{1}{2}(\partial_\mu H)(\partial^\mu H)$$
  - Mass terms: $$\frac{1}{2}\boxed{\frac{v^2 g^2}{4}} W_\mu^+ W^{-\mu} + \frac{1}{2}\boxed{\frac{v^2 (g+g')^2}{4}} Z_\mu Z^\mu + \frac{1}{2}\boxed{(-2\mu^2)} H^2,$$

No **photon** mass term!

$$m_{W^\pm}^2 \qquad m_Z^2 \qquad m_H^2 \qquad m_\gamma = 0$$

Higgs mechanism (iii)

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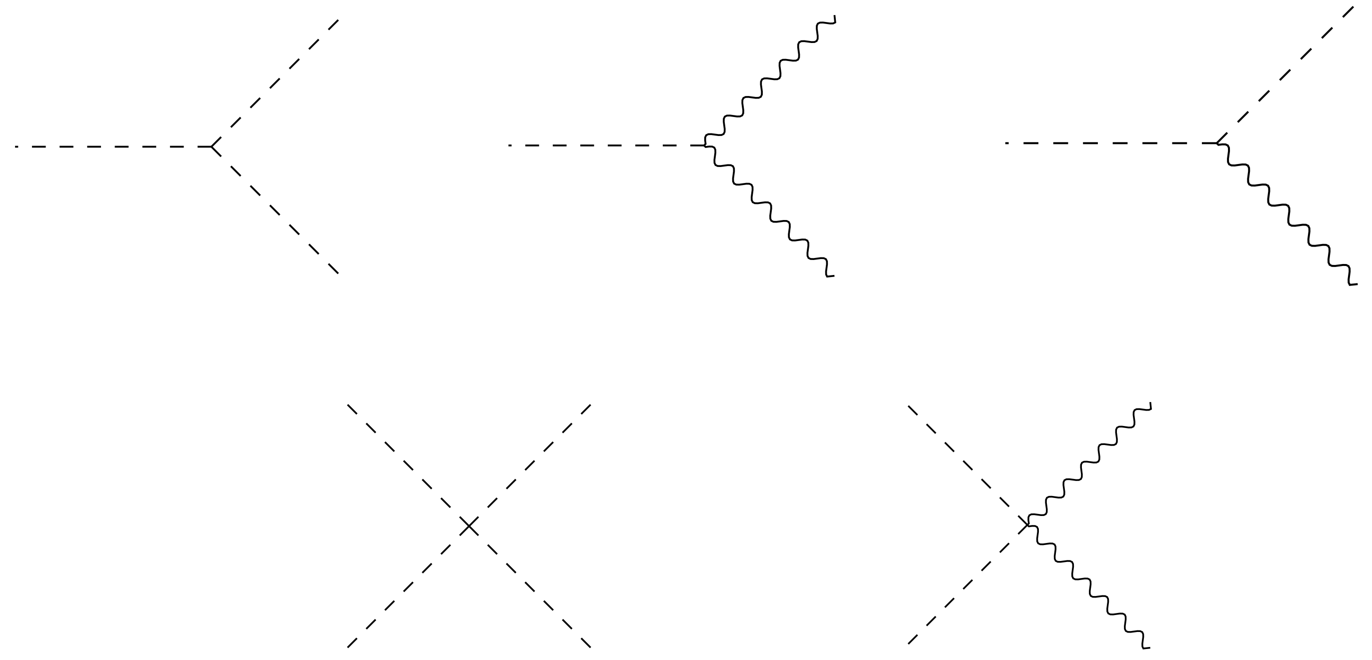
  - Kinetic terms: $$(\partial_\mu\phi^+)(\partial^\mu\phi^-) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi) + \frac{1}{2}(\partial_\mu H)(\partial^\mu H)$$

  - Mass terms: $$\frac{1}{2}\boxed{\frac{v^2g^2}{4}}W^+_\mu W^{-\mu} + \frac{1}{2}\boxed{\frac{v^2(g+g')^2}{4}}Z_\mu Z^\mu + \frac{1}{2}\boxed{(-2\mu^2)}H^2,$$

    No **photon** mass term!

    $m^2_{W^\pm}$ $\qquad$ m^2_Z $\qquad$ m^2_H $\qquad$ $m_\gamma = 0$

  - Interaction terms:
    - Trilinear (SSS, SSV, SVV)
    - Quadrilinear (SSSS, SSVV)



Higgs mechanism (iv)

- *D.o.f. counting*, again:
  - **Higgs** boson: 1 real scalar → 1 *d.o.f.*
  - **Goldstone** bosons: 1 real scalar + 1 complex scalar/2 real scalars → 3 *d.o.f.*
  - **Gauge** bosons:
    - → 1 massless vector field = (1 x 2) 2 *d.o.f.*
    - → 3 massive vector fields = (3 x 3) 9 *d.o.f.*

Total: 15 *d.o.f.* ?!

Higgs mechanism (iv)

- *D.o.f. counting*, again:
  - **Higgs** boson: 1 real scalar → 1 *d.o.f.*
  - **Goldstone** bosons: 1 real scalar + 1 complex scalar/2 real scalars → 3 *d.o.f.*
  - **Gauge** bosons: → 1 massless vector field = (1 x 2) 2 *d.o.f.* ; → 3 massive vector fields = (3 x 3) 9 *d.o.f.*

Total: 15 *d.o.f.* ?!

- **Goldstone** boson **are not** physical *d.o.f.* and can be eliminated by suitable *gauge transformation*
- The *gauge* in which **goldstones** are absent is known as *Unitary gauge.* In this case the **Higgs** field can be parametrised just as

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

but vector bosons' propagators get modified w.r.t. *'t Hooft–Feynman gauge* → $\dfrac{-i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_V^2}\right)}{p^2 - m_V^2 + i0}$

Higgs mechanism (v)

- NOTE: the original $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ symmetry is still there, but not manifest in Φ_0. However, a new symmetry is manifest in Φ_0: $\mathrm{U}(1)_{em}$

- In the **Higgs** mechanism this is ensured assuming $Y_\Phi = 1$, which guarantees

$$\mathrm{SU}(2)_L : \quad \tau^i \Phi_0 \neq 0 \quad \forall i$$

$$\mathrm{U}(1)_Y : \quad Y_\Phi \Phi_0 \neq 0$$

Broken generators

$$\mathrm{U}(1)_{em} : \quad Q\Phi_0 \overset{Y_\Phi = 1}{=} 0$$

Unbroken generator

- More generally (and formally): said $\mathcal{G}$ the symmetry group of the potential V and $\mathcal{H}$ the subgroup of $\mathcal{G}$ which leaves the vacuum invariant, then:

	Goldstone mechanism	**Higgs** mechanism
$\#$ unbroken gener. $= \dim\mathcal{H}$	$\#$ massive scalars	$\#$ massless gauge bosons
$\#$ broken gener. $= \dim\mathcal{G}/\mathcal{H}$	$\#$ massless Gold. bosons	$\#$ massive gauge bosons

Goldstone Boson Equivalence Theorem (i)

- In *Unitary gauge*, unphysical **goldstone** bosons (*1 d.o.f.* each) are "eaten up" by massless **gauge** bosons (*2 d.o.f.* each) $\Rightarrow W^{\pm}, Z$ acquire mass (*3 d.o.f.* each)

- This suggests a relation between longitudinally polarized **gauge** bosons and **goldstone** bosons holding at energy above the **EW** scale

- In the high-energy regime $m_V/E \ll 1$:

$$\epsilon^{\mu}_{\mathrm{L}}(q) = \frac{q^{\mu}}{m_V} + \mathcal{O}\left(\frac{m_V}{q^0}\right), \quad q^0 \sim E = \sqrt{s}$$

Goldstone Boson Equivalence Theorem (ii)

- Consider the process

$$V_{\mathrm{L}}^{a_1}(q_1)...V_{\mathrm{L}}^{a_m}(q_m)\varphi_{i_1}(p_1)...\varphi_{i_n}(p_n) \to 0$$

- By means of **EW** *Ward identities* one can show that

$$\mathcal{M}^{V_{\mathrm{L}}^{a_1}...V_{\mathrm{L}}^{a_m}\varphi_{i_1}...\varphi_{i_n}}(q_1,...,q_m,p_1,...,p_n) = \left[\prod_{k=1}^{m} i^{(1-Q_{V^{a_k}})} A^{V^{a_k}}\right] \mathcal{M}^{\Phi_{a_1}...\Phi_{a_m}\varphi_{i_1}...\varphi_{i_n}}(q_1,...,q_m,p_1,...,p_n) + \mathcal{O}\left(m_{V^{a_k}} E^{d-1}\right)$$

with factors

$$A^{V^a} = 1 + \delta A^{V_a}$$

LO contribution

Higher order contributions

Goldstone Boson Equivalence Theorem (ii)

- Consider the process

$$V_{\rm L}^{a_1}(q_1)...V_{\rm L}^{a_m}(q_m)\varphi_{i_1}(p_1)...\varphi_{i_n}(p_n) \to 0$$

- By means of **EW** *Ward identities* one can show that

$$\mathcal{M}^{V_{\rm L}^{a_1}...V_{\rm L}^{a_m}\varphi_{i_1}...\varphi_{i_n}}(q_1,...,q_m,p_1,...,p_n) = \left[\prod_{k=1}^{m} i^{(1-Q_{V^{a_k}})}A^{V^{a_k}}\right]\mathcal{M}^{\Phi_{a_1}...\Phi_{a_m}\varphi_{i_1}...\varphi_{i_n}}(q_1,...,q_m,p_1,...,p_n) + \mathcal{O}\left(m_{V^{a_k}}E^{d-1}\right)$$

with factors

$$A^{V^a} = \underbrace{1}_{\text{LO contribution}} + \underbrace{\delta A^{V_a}}_{\text{Higher order contributions}}$$

This goes under the name "***Goldstone Boson Equivalence Theorem***" (GBET)

Goldstone Boson Equivalence Theorem (iii)

- Example 1.: one longitudinal **gauge** boson

$$V_{\rm L}^{a_1}(q_1)\varphi_{i_1}(p_1)...\varphi_{i_n}(p_n) \to 0$$

- Employing the **GBET** at LO we have

$$\mathcal{M}^{V_{\rm L}^a\varphi_{i_1}\cdots\varphi_{i_n}}\left(q_1,p_1,...,p_n\right) = i^{(1-Q_{V^a})}\mathcal{M}^{\Phi_a\varphi_{i_1}\cdots\varphi_{i_n}}\left(q_1,p_1,...,p_n\right) + \mathcal{O}\left(m_{V^a}E^{d-1}\right)$$

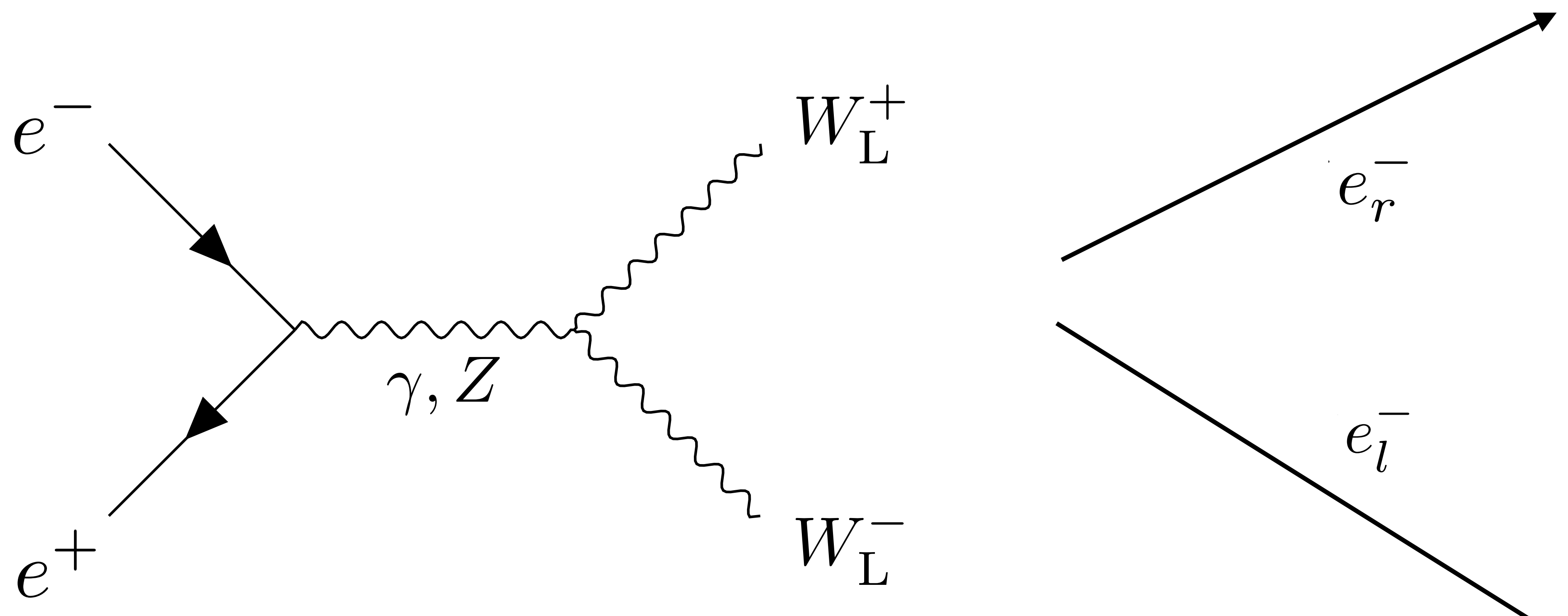
which just mean

$$V_{\rm L}^a = W_{\rm L}^{\pm}\ : \qquad \mathcal{M}^{W_{\rm L}^{\pm}\varphi_{i_1}\cdots\varphi_{i_n}}\left(q_1,p_1,...,p_n\right) = \pm\mathcal{M}^{\phi^{\pm}\varphi_{i_1}\cdots\varphi_{i_n}}\left(q_1,p_1,...,p_n\right)$$

$$V_{\rm L}^a = Z_{\rm L}^{\pm}\ : \qquad \mathcal{M}^{Z_{\rm L}\varphi_{i_1}\cdots\varphi_{i_n}}\left(q_1,p_1,...,p_n\right) = i\mathcal{M}^{\chi\varphi_{i_1}\cdots\varphi_{i_n}}\left(q_1,p_1,...,p_n\right)$$

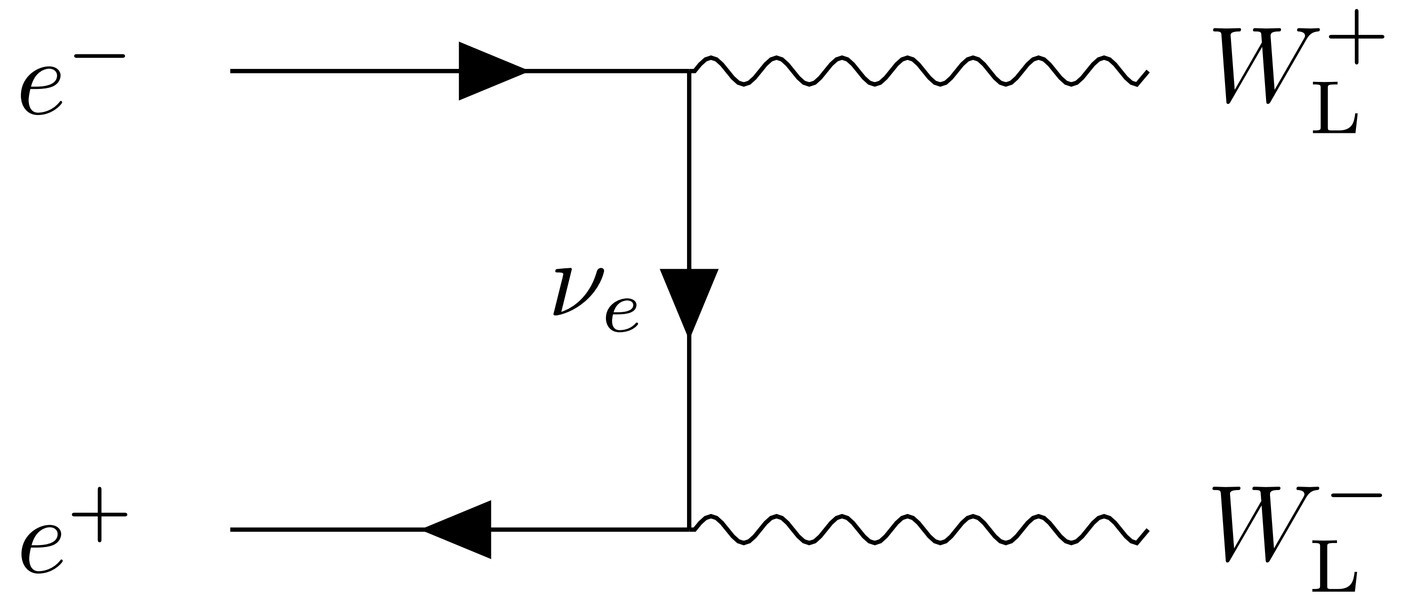
Goldstone Boson Equivalence Theorem (iv)

- Example 2: $e^+e^- \to W_{\rm L}^+W_{\rm L}^-$



$$\mathcal{M}(e_l^+e_r^- \to W_{\rm L}^+W_{\rm L}^-) \overset{s\gg m_W^2}{=} \bar{v}_{\rm R}\gamma^\mu u_{\rm R}\left[\frac{ie^2m_Z^2}{2m_W^2 s}\right](k_+-k_-)^\mu$$

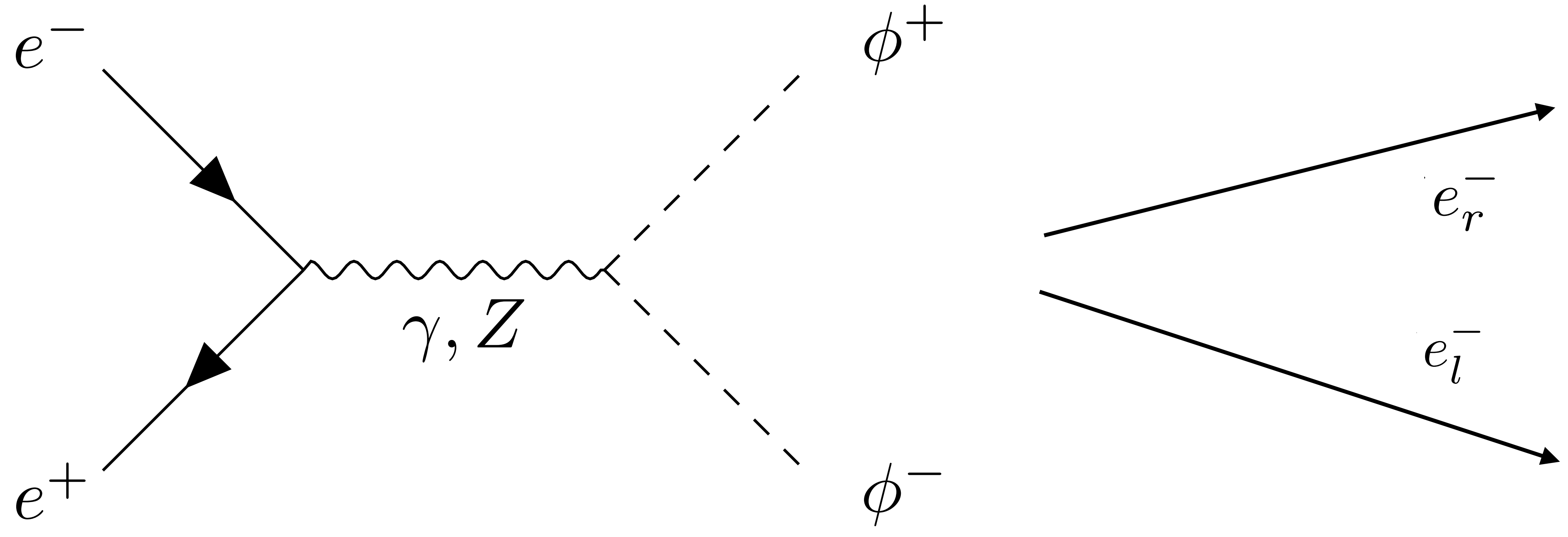
$$\mathcal{M}_1(e_r^+e_l^- \to W_{\rm L}^+W_{\rm L}^-) \overset{s\gg m_W^2}{=} \bar{v}_{\rm L}\gamma^\mu u_{\rm L}\frac{ie^2 s}{2m_W^2}\left[\frac{m_Z^2}{s(s-m_Z^2)}-\frac{1}{2s_{\rm w}^2(s-m_Z^2)}\right](k_+-k_-)^\mu$$



$$\mathcal{M}_2(e_r^+e_l^- \to W_{\rm L}^+W_{\rm L}^-) \overset{s\gg m_W^2}{=} \bar{v}_{\rm L}\gamma^\mu u_{\rm L}\frac{ie^2}{4s_{\rm w}^2m_W^2}(k_+-k_-)^\mu$$

Goldstone Boson Equivalence Theorem (v)

- Example 2: $e^+e^- \to \phi^+\phi^-$
- Remarkably easier than before:
  - No t-channel since $\ell\nu_\ell\phi$ coupling is $\propto m_\ell \overset{s \gg m_W^2}{\approx} 0$
  - No complex Lorentz structure from trilinear VVV coupling
  - SSV coupling $\propto (p - p')^\mu$



$$\mathcal{M}(e_l^+ e_r^- \to \phi^+\phi^-) = -\bar{v}_{\rm R}\gamma^\mu u_{\rm L}\frac{ie^2}{2c_{\rm w}^2}\frac{(k_+ - k_-)^\mu}{s}$$

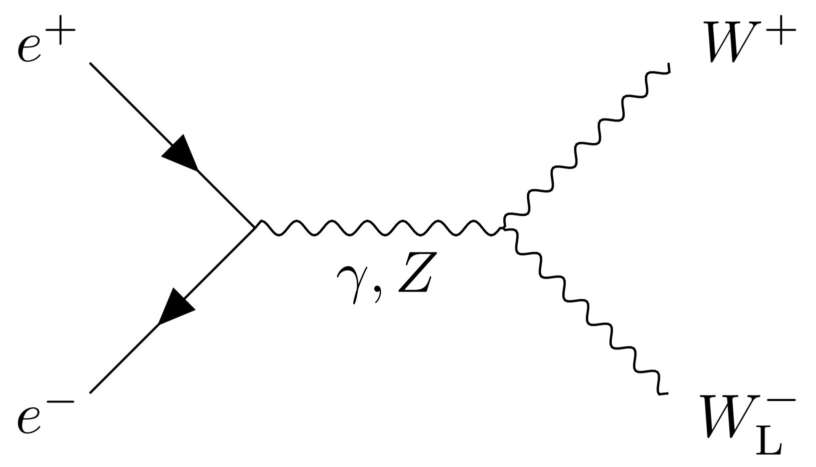
$$\mathcal{M}(e_r^+ e_l^- \to \phi^+\phi^-) = \bar{v}_{\rm L}\gamma^\mu u_{\rm L} i\left[\frac{e^2}{4c_{\rm w}^2} + \frac{e^2}{4s_{\rm w}^2}\right]\frac{(k_+ - k_-)^\mu}{s}$$

Goldstone Boson Equivalence Theorem (vi)

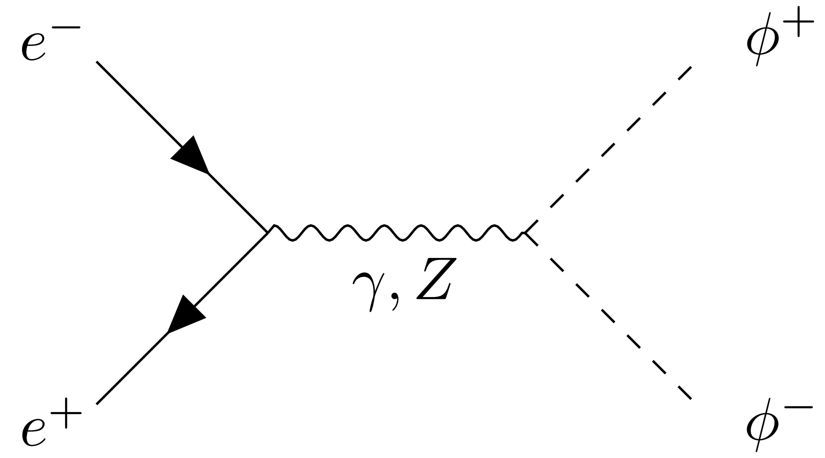
Remark

The GBET **does not** hold diagram by diagram, but only at the **amplitude level** !

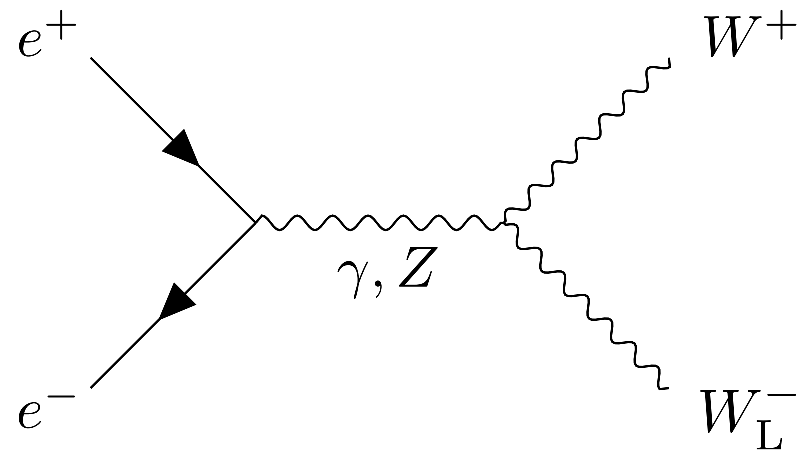
- Right-handed electrons:



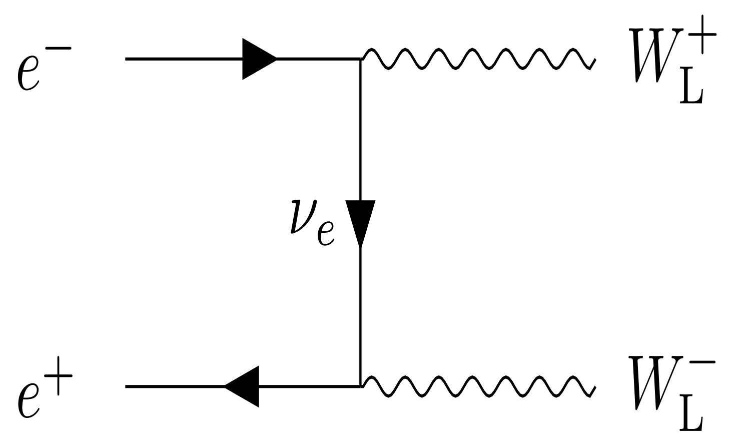
$\propto$



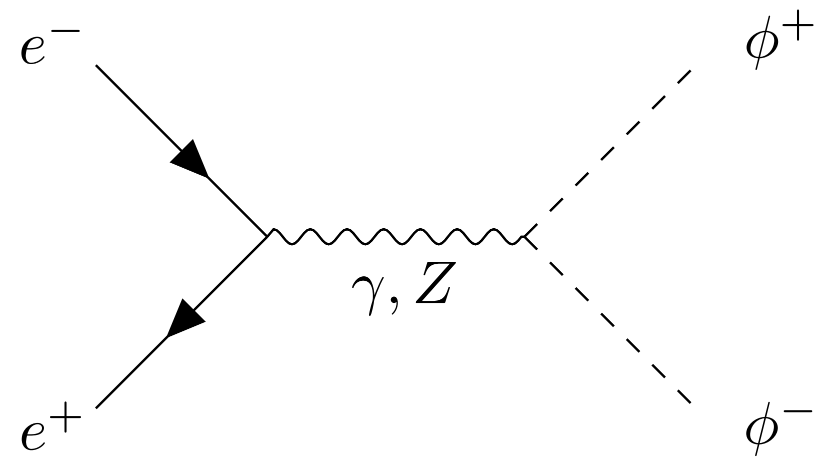
- Left-handed electrons:



$+$



$\propto$



The Electroweak Standard Model (v)

- **EW** sector of **SM** is described by the classical Lagrangian

$$\mathcal{L}_{\mathrm{EW}}^{\mathrm{cl.}} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{Yuk}}$$

Charge-conjugate **Higgs** doublet

$$\tilde{\Phi} = i\left(\Phi^{\dagger}\tau_2\right)^{\mathrm{T}}$$

Yukawa

$$\mathcal{L}_{\mathrm{Yuk}} = \sum_{\psi_f = q,l}\sum_{i,j=1}^{3} \bar{\psi}_{f_i,\mathrm{L}}\left(\Upsilon_{\psi_f}\right)^{ij}\tilde{\Phi}\psi_{f_j,\mathrm{R}} + \mathrm{h.c.}$$

- Yukawa coupling $\left(\Upsilon_{\psi_f}\right)^{ij}$: 3x3 matrices in generation space
- After **SSB** $\Rightarrow m_{ij}^{\psi} = \frac{v}{\sqrt{2}}\Upsilon_{\psi_f}^{ij}$

NB: ordinary Dirac mass terms $\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}}$ **are not** $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge invariant

The Electroweak Standard Model (vi)

- After **SSB** ⇒ $m^{\psi}_{ij} = \frac{v}{\sqrt{2}} \Upsilon^{ij}_{\psi_f}$

- Can be diagonalised through unitary field transformation

$$\hat{\psi}_{f,\mathrm{L/R}} \equiv U_{f,\mathrm{L/R}} \psi_{f,\mathrm{L/R}} \quad \Big| \quad m^{\psi}_{ii} = \frac{v}{\sqrt{2}} \sum_{m,n} U^{im}_{f,\mathrm{L}} \left(\Upsilon_{\psi_f}\right)^{mn} \left(U^{ni}_{f,\mathrm{R}}\right)^{\dagger} \equiv \frac{v}{\sqrt{2}} \lambda_{\psi_{f_i}}$$

- This defines the mass basis for the **SM** and the **Yukawa** Lagrangian takes form

$$\mathcal{L}_{\mathrm{Yuk}} = \sum_{\psi_f = q,l}^{3} m_{\psi_f} \bar{\hat{\psi}}_{f,\mathrm{L}} \hat{\psi}_{f,\mathrm{R}} + \mathrm{h.c.} + \mathcal{L}_{H-\psi\ \mathrm{int.}}$$

- Consequences:

▲ These matrices drop out in NC interactions due to unitarity

▲ Two matrices remain in CC

CKM matrix: $V_{\mathrm{CKM}} = U_{u,\mathrm{L}} U^{\dagger}_{d,\mathrm{L}}$ ⟶ Source of **CP violation** in whole **SM**

PMNS matrix: $U_{\mathrm{PMNS}} = U_{\nu,\mathrm{L}} U^{\dagger}_{e,\mathrm{L}} \overset{m_\nu = 0}{=} \mathbb{1}$

EW input parameters

EW input-parameters (i)

- **EW** sector of **SM** has the following set of parameters

Before **SSB**		After **SSB**	
gauge couplings	g, g'	**gauge** couplings	g, g' or e, s_{w}
Higgs parameters	μ, λ	**Masses**	$m_H, \quad m_{W/Z}, \quad m_\psi$
Yukawa couplings	$\Upsilon^{ij}_{\psi_f}$	**CKM** matrix	V_{CKM}

- **Not all** of them are **independent**!

- At tree-level: plenty of relations between input parameters

$$m_W = \frac{vg}{\sqrt{2}}\,, \qquad m_Z = \frac{v(g+g')}{\sqrt{2}}$$

$$m^{\psi}_{ij} = \frac{v}{\sqrt{2}}\Upsilon^{ij}_{\psi_f}$$

$$c_{\mathrm{w}} = \frac{m_W}{m_Z}$$

- Higher-order corrections modify such relations which in general depend on all input parameters

EW input-parameters (ii)

- Only a *defined set* of **input**-parameters are **independent** and can be regarded as *free* parameters → crucial to guarantee *gauge invariance* and consistency in results!

- Comparisons *data* vs *theoretical predictions*: only **input**-parameters of that calculation can be extracted from data

- The **input**-parameter set should be chosen in a “phenomenological way”, i.e. precisely known parameters should be preferred (e.g. em. coupling)

- The appropriate **input**-parameter set in general differs from one renormalisation scheme to another (in the following we assume on-shell scheme)

On-shell renormalisation (i)

- **On-shell** (**OS**) scheme [Ross & Taylor 83749; 1979]:
  - Renormalisation conditions fixed for **on-shell** external fields
  - Renormalised parameters are the physical parameters to ***all*** orders in PT

In SM: $m_W, \quad m_Z, \quad m_f, \quad m_H, \quad e, \quad V_{ij}$

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- Features:
  - Parameters have clear physical meaning and can be directly measured in experiments
    - ▲ True for most of particle masses but for light-quarks due to strong interactions; other schemes might be better
  - Electric charge e derived from Thomson cross-section which is **exact** to all orders in PT

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    - ▲ True for most of particle masses but for light-quarks due to strong interactions; other schemes might be better
  - Electric charge e derived from Thomson cross-section which is **exact** to all orders in PT
- Renormalisation of parameters ⇒ finite S-matrix but Green functions still divergent. Fields need to be renormalized too!
- Also, a clever renormalisation of fields simplifies the renormalisation conditions for parameters

EW input schemes (iii)

- Most employed **input**-schemes

Additional inputs: $m_H\,,m_\psi$

  - $\alpha(0)$-scheme: $\{\alpha(0), m_W\,, m_Z\}$

    Appropriate for:

    ▲ pure **QED** interactions

    ▲ external on-shell **photons**

    ▲ processes at $Q^2 \ll m_Z^2$

$$e = \sqrt{4\pi\alpha}$$
$$e = g s_\mathrm{w} = g' c_\mathrm{w}$$

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$$e = \sqrt{4\pi\alpha}$$
$$e = g s_w = g' c_w$$

For $Q^2 \geq m_Z^2$ large logs associated to the running of α from $Q^2 = 0$ to $Q^2 \geq m_Z^2$ appear in the light fermion contribution to charge renormalisation

$$\Delta\alpha(Q^2)_{light} = \frac{\alpha}{3\pi}\sum_f N_C^f Q_f^2 \log\frac{Q^2}{m_f^2}$$
$$= \frac{\alpha}{3\pi}\sum_f N_C^f Q_f^2 \log\frac{Q^2}{m_Z^2} + \Delta\alpha(m_Z^2)$$

- The hadronic contribution to $\Delta\alpha(m_Z^2) \approx 6\,\%$ is non-perturbative and is extracted from $e^+e^- \to$ had. measurements + dispersion relations
- In renormalised **QED** amplitudes $\Delta\alpha(m_Z^2)$ drops out due to Ward identities (but remains un-subtracted in full **EW**)

EW input schemes (iii)

- Most employed **input-**schemes

Additional inputs: m_H, m_ψ

  - $\alpha(0)$-scheme: $\{\alpha(0), m_W, m_Z\}$
  - $\alpha(m_Z)$-scheme: $\{\alpha(m_Z), m_W, m_Z\}$

$$e = \sqrt{4\pi\alpha}$$
$$e = g s_\mathrm{w} = g' c_\mathrm{w}$$

Appropriate for:

▲ Hard **EW** interactions at $Q^2 \geq m_Z^2$

▲ All $\Delta\alpha(m_Z^2)$ terms are subtracted and no large logs from light-fermions contributions to charge renormalisation appear. This cancellation takes place at each loop-order

The $\alpha(m_Z^2)$ scheme resums LL effects from the running of α

Removal of light-quark masses dependence

EW input schemes (iii)

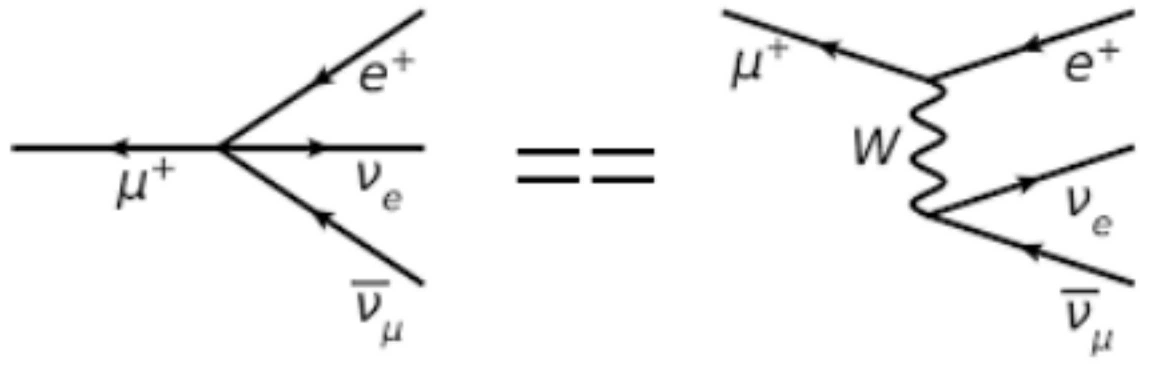
- Most employed **input**-schemes

Additional inputs: m_H, m_ψ

  - $\alpha(0)$-scheme: $\{\alpha(0), m_W, m_Z\}$
  - $\alpha(m_Z)$-scheme: $\{\alpha(m_Z), m_W, m_Z\}$
  - G_μ-scheme: $\{G_\mu, m_W, m_Z\}$

$$e = \sqrt{4\pi\alpha}$$
$$e = g s_{\mathrm{w}} = g' c_{\mathrm{w}}$$

Determined from **muon** decay in **Fermi** theory (low energy)



$$|\mathcal{M}|^2 = \left|\frac{8}{\sqrt{2}}G_\mu\right|^2 = \left|\frac{g^2}{m_W^2}\right|^2$$

$$G_\mu = 1.16637 \times 10^{-5} \quad \mathrm{GeV}^{-2}$$

Appropriate for:

▲ Hard **EW** interactions at $Q^2 \geq m_Z^2$

▲ Beyond LO, it resums large logs from $\Delta\alpha(m_Z^2)$ and m_t^2/m_W^2 enhanced terms from renormalisation of weak mixing angle → preferable to $\alpha(m_Z)$ if s_{w} (c_{w}) factors appear at LO

EW input schemes (iii)

- NB: adopting a different scheme w.r.t. $\alpha(0)$ means that also the CT for electric charge renormalisation changes

  - In $\alpha(m_Z)$ -scheme: $$\delta Z_e\Big|_{\alpha(m_Z^2)} = \delta Z_e\Big|_{\alpha(0)} - \frac{\Delta\alpha(m_Z^2)}{2}$$

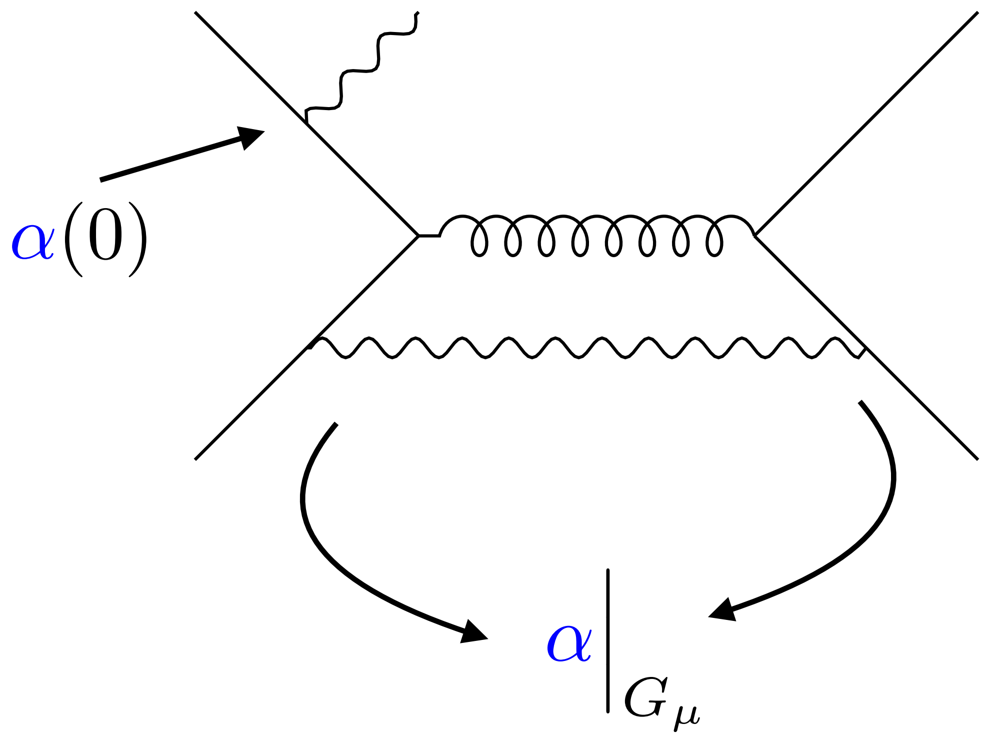
  - In G_μ -scheme: $$\delta Z_e\Big|_{G_\mu} = \delta Z_e\Big|_{\alpha(0)} - \frac{1}{2}\mathrm{Re}(\Delta r)$$

Radiative corrections to **muon** decay

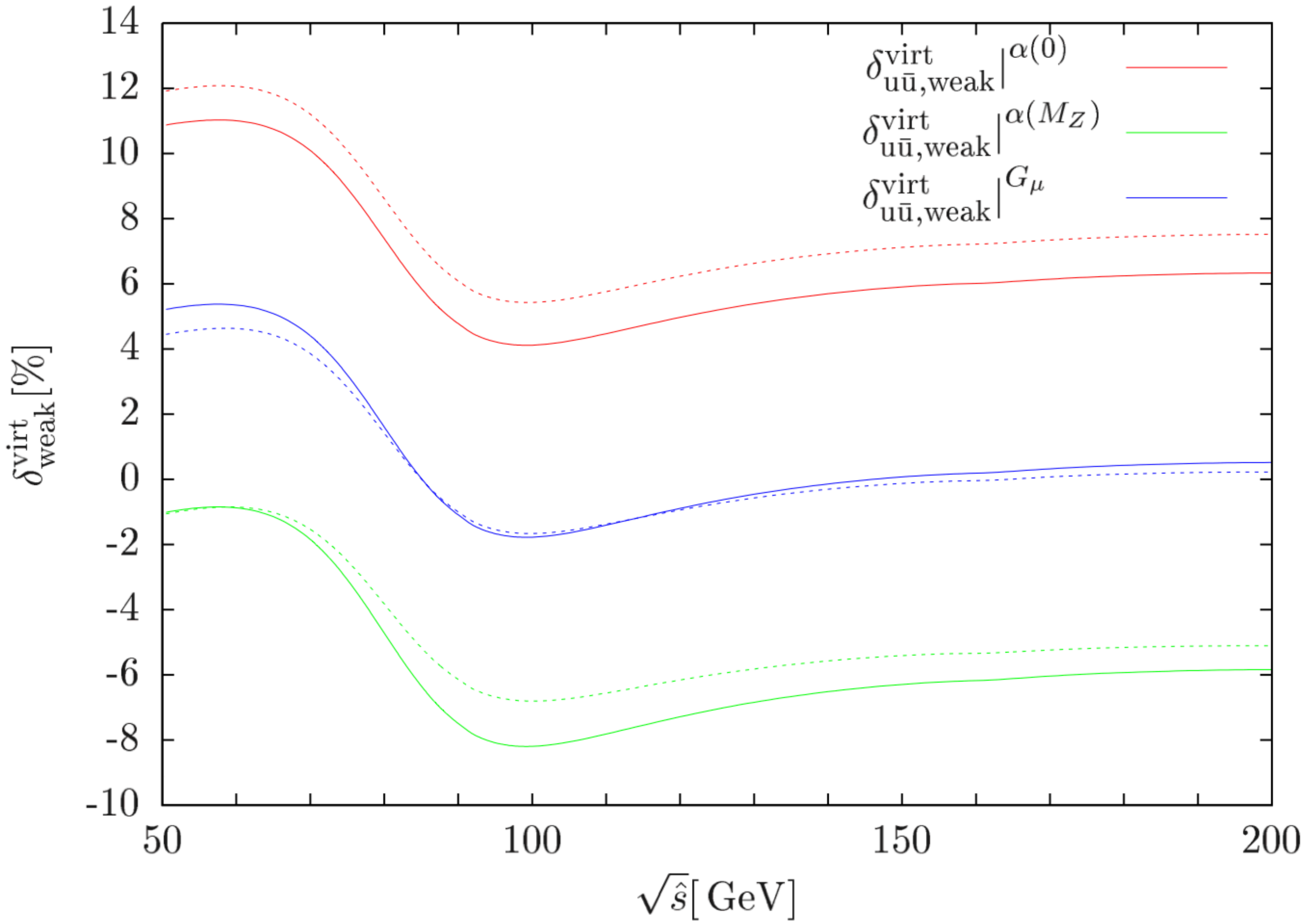
$$\Delta r = \Pi^{AA}(0) - \frac{c_W^2}{s_W^2}\left(\frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Sigma_T^{W}(M_W^2)}{M_W^2}\right) + \frac{\Sigma_T^{W}(0) - \Sigma_T^{W}(M_W^2)}{M_W^2} + 2\frac{c_W}{s_W}\frac{\Sigma_T^{AZ}(0)}{M_Z^2} + \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7-4s_W^2}{2s_W^2}\log c_W^2\right)$$

EW input schemes (iv)

- Other schemes can be employed as, e.g., the s_w -scheme $\{G_\mu, m_Z, s_\mathrm{w}\}$
- Mixed schemes are possible as well if we are consistent, i.e. all factors α are global factors to *gauge invariant* pieces of amplitude, e.g.



[Dittmaier *et al.*, 0911.2329; 2009]



- Differences between these schemes amount to 2-6% → *scheme uncertainties*
- Scheme dependence reduced when including higher-order corrections

Unstable particles & complex-mass scheme

Unstable particles (i)

- **EW** gauge bosons **are not** stable particles ⇒ decays need to be taken into account for reliable theoretical predictions

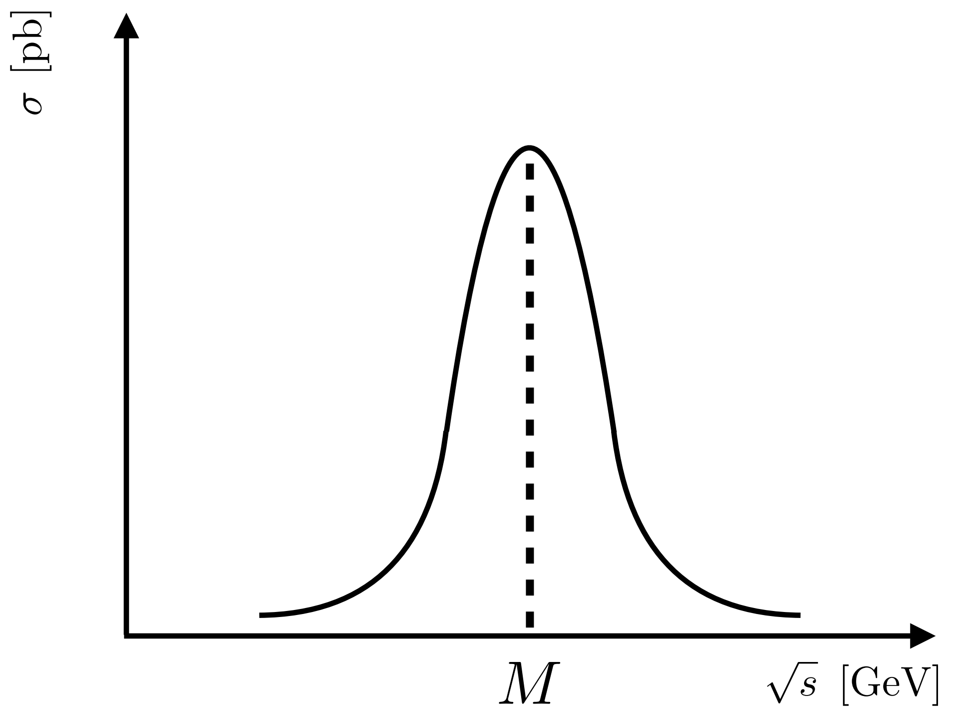
- Not a trivial task and it requires an appropriate treatment

- Processes involving a massive s-channel propagator

$$\sim \frac{1}{p^2 - M^2}$$

  diverge for $p^2 \to M^2$

- Experimentally we observe a peak at the resonance value $p^2 \to M^2$ with a *Breit-Wigner* (BW) shape



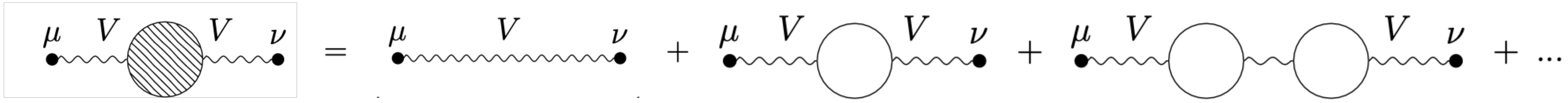
Unstable particles (ii)

- First attempt: _Dyson summation_ → all-order summation of 1-PI self energies

- Steps (e.g. for **gauge** boson propagator):

1. Consider the LO (bare) two-point function $G_0(p) = p^2 - M_0^2$

2. The physical two-point function is **not** $G_0(p)$ but rather



$$G^{-1}(p) = G_0^{-1}(p) + G_0^{-1}(p)\Sigma(p^2)G_0^{-1}(p) + G_0^{-1}(p)\Sigma(p^2)G_0^{-1}(p)\Sigma(p^2)G_0^{-1}(p) + \ldots$$

$$= G_0^{-1}(p)\sum_{n=0}^{\infty}[\Sigma(p^2)G_0^{-1}(p)]^n = \frac{1}{G_0(p) + \Sigma(p^2)}$$

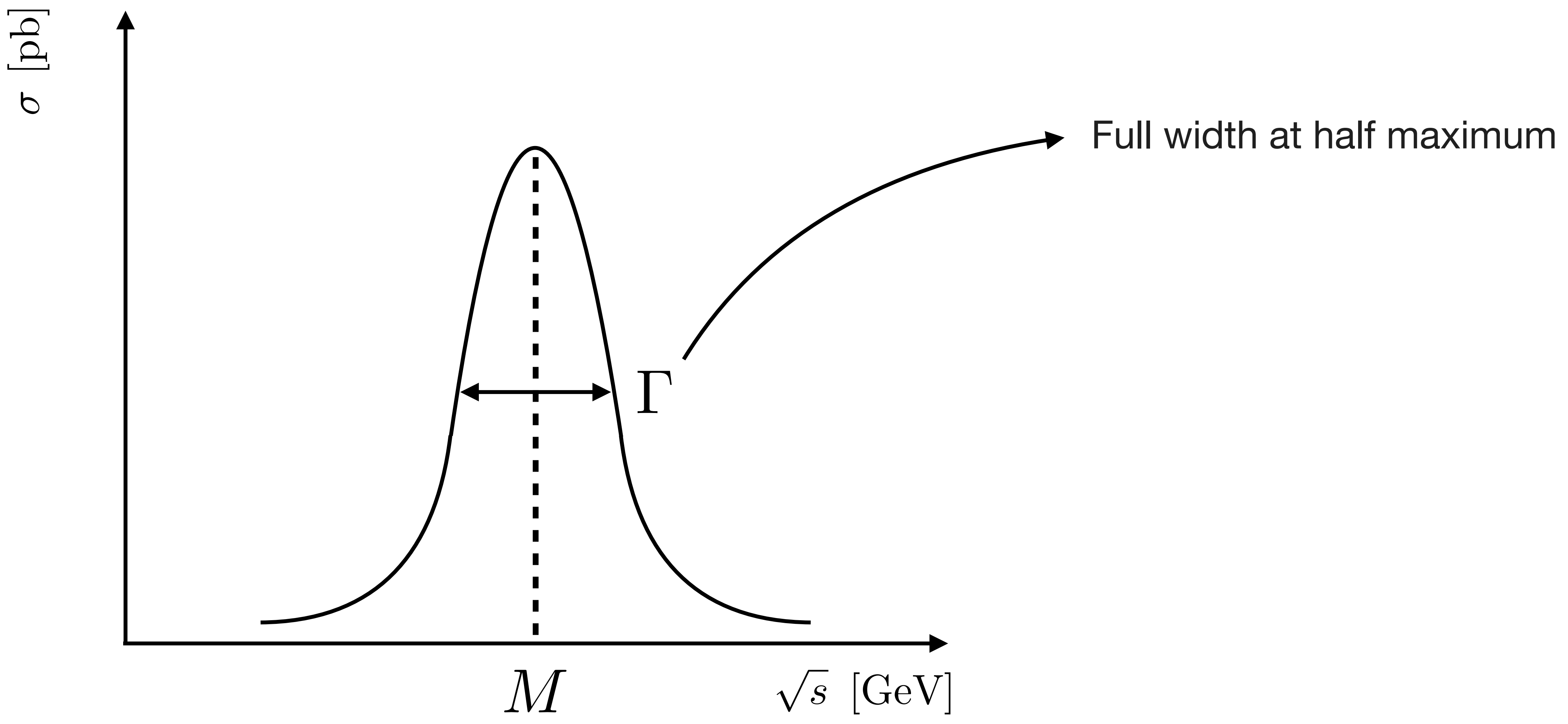
3. $\mathrm{Re}[\Sigma(p^2)]$ defines the physical mass as $M^2 = M_0^2 - \mathrm{Re}[\Sigma(p^2)]$

Unstable particles (iii)

- Steps (e.g. for **gauge** boson propagator):

4. The imaginary part $\mathrm{Im}[\Sigma(p^2)]$ regulates the propagator for $p^2 \to M^2$
5. No need to compute any loop! By means of the *Optical theorem* one gets $\mathrm{Im}[\Sigma(p^2)] = \Gamma M$ and the propagator is

$$\sim \frac{1}{p^2 - M^2 + i\Gamma M}$$



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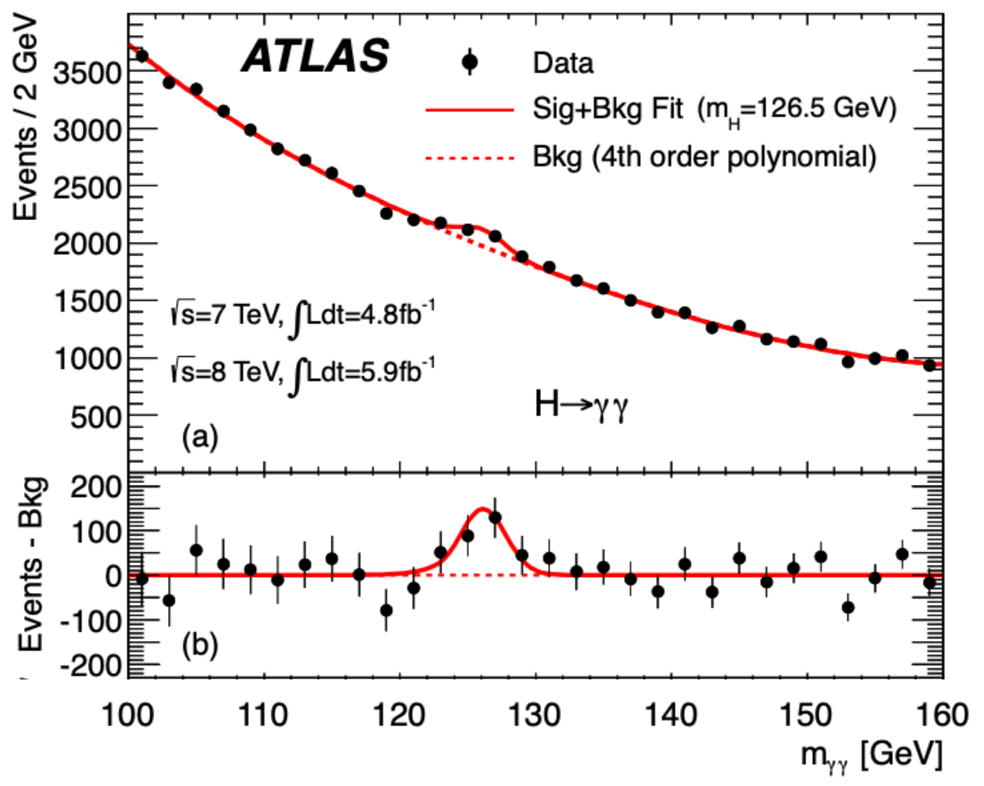
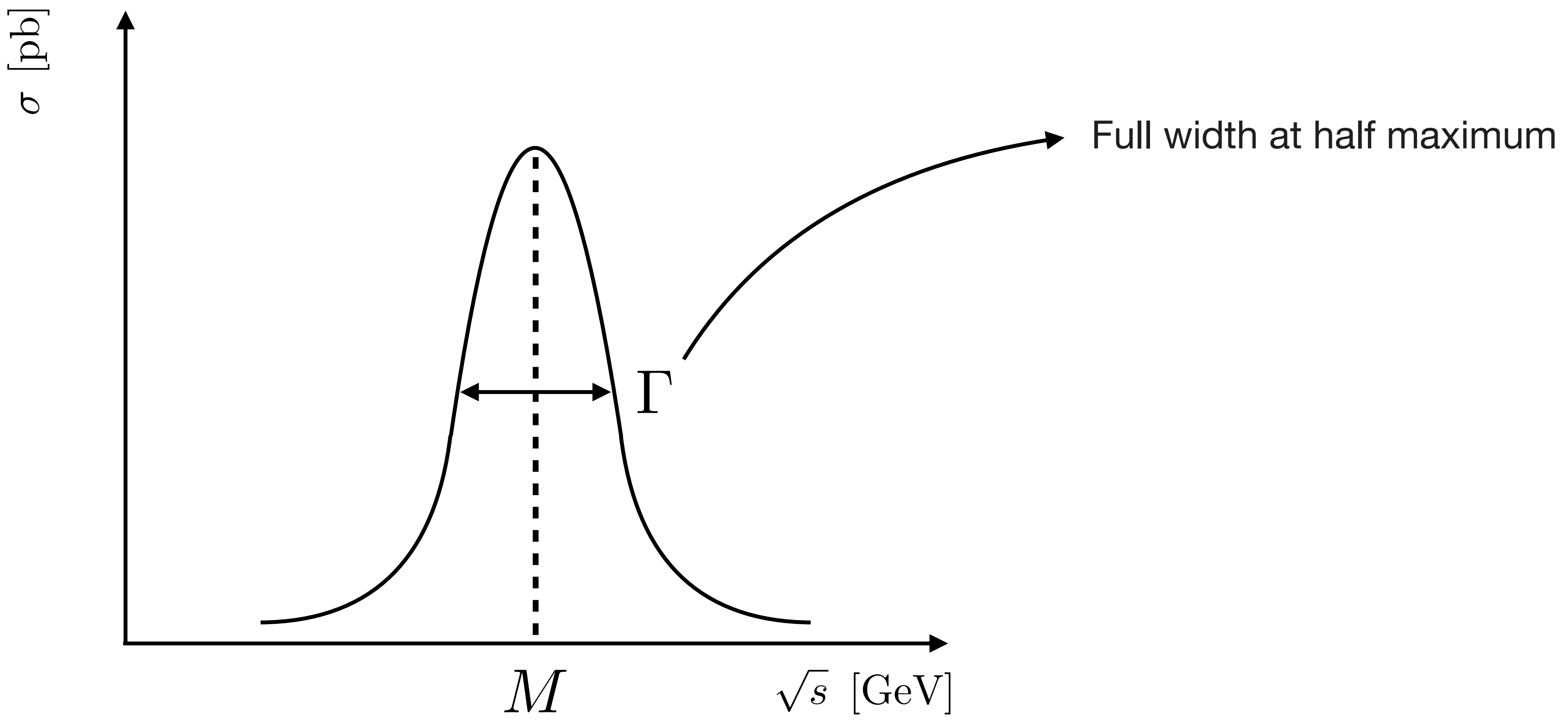
- Way to infer a new particle



[Atlas collaboration 1207.7214; *2012*]



The complex-mass scheme (i)

- However: Dyson summation mixes *different* orders in PT and spoils *gauge invariance* if applied carelessly

- Proposal: *complex-mass scheme* (CMS) [Denner *et al.* *9904472; 1999*]

- In the CMS **all** masses of unstable particles are consistently taken as complex quantities and defined as the location of the poles in the complex plane:

$$\mu_V^2 = m_V^2 - i\Gamma_V m_V , \qquad \mu_H^2 = m_H^2 - i\Gamma_H m_H , \qquad \mu_t^2 = m_t^2 - i\Gamma_t m_t$$

$V = W^{\pm}, Z$

- Parameters defined through masses are redefined as well, e.g. the weak mixing angle and Fermi constant

$$c_{\rm w}^2 = \frac{\mu_W^2}{\mu_Z^2} \qquad \frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8\mu_W^2}$$

The complex-mass scheme (ii)

- Consequences:
  1. *Gauge invariance* automatically preserved if complex masses are introduced everywhere in the Lagrangian
  2. Masses modified just by analytic continuation ⇒ *Ward/Taylor-Slavnov* ids. are still valid
  3. Spurious terms induced in CMS (as in weak mixing angle) are always $\mathcal{O}(\Gamma/m) \sim \mathcal{O}(\alpha)$ beyond the calculation at hand
  4. Scheme generalized for **NLO EW** corrections (on-shell ren. + CMS Denner *et al.* 0605312)
     - Real bare masses squared are split into complex renormalized masses and mass counterterms (and similarly for fields)

$$m_{W,0}^2 = \mu_W^2 + \delta\mu_W^2 \qquad W_0^\pm = \left(1 + \frac{1}{2}\delta\mathcal{Z}_W\right) W^\pm$$

     - Renormalisation condition fixed as usual but for complex masses: $\hat{\Sigma}_W(\mu_W^2) = 0, \quad \hat{\Sigma}'_W(\mu_W^2) = 0$

The complex-mass scheme (iii)

- Consequences:

    1. *Gauge invariance* automatically preserved if complex masses are introduced everywhere in the Lagrangian

    2. Masses modified just by analytic continuation ⇒ *Ward/Taylor-Slavnov* ids. are still valid

    3. Spurious terms induced in CMS (as in weak mixing angle) are always $\mathcal{O}(\Gamma/m) \sim \mathcal{O}(\alpha)$ beyond the calculation at hand

    4. Scheme generalized for **NLO EW** corrections (on-shell ren. + CMS Denner *et al.* 0605312)

    5. Violation of unitarity in CMS? Cutkosky equations no longer valid and unitarity cannot be proved order by order. However:

        - Unitarity-violating terms are **always** one order higher w.r.t. the order od the calculation
        - Violation never enhanced due to exact preservation of *Ward/Taylor-Slavnov* identities
        - Unstable particle **not** external states → only S-matrix connecting stable external states must be unitary

Not always so “weak” corrections

Not always so “weak” corrections (i)

- Higher order **EW** corrections are expected to be always much smaller than corresponding **QCD** ones. Naively: $\alpha \simeq 0.01, \ \alpha_{\mathrm{S}} \simeq 0.1 \Rightarrow$ **NLO EW** $\sim$ **NNLO QCD**

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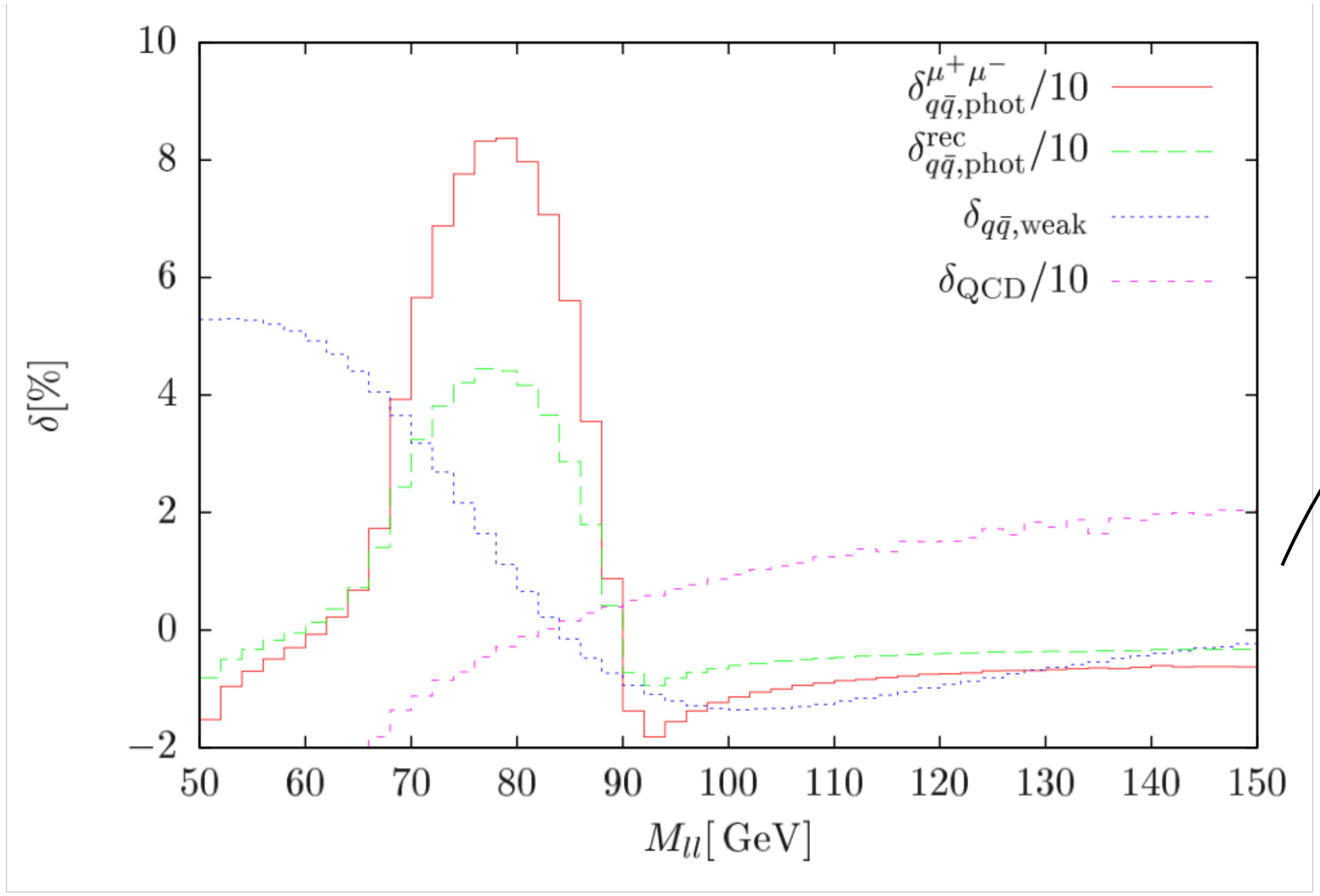
- Higher order **EW** corrections are expected to be always much smaller than corresponding **QCD** ones. Naively: $\alpha \simeq 0.01$, $\alpha_S \simeq 0.1 \Rightarrow$ **NLO EW** $\sim$ **NNLO QCD**
- **NLO EW** not always at the (sub)-percent level: systematic enhancements are possible
  - **Photon** emissions (e.g. collinear **FSR**)



[Dittmaier *et al.*, 0911.2329; 2009]

Large logs… $\sim \alpha \log\left(\frac{m_\ell^2}{Q^2}\right)$

- At **LL** the effect is **universal** → *factorisation*

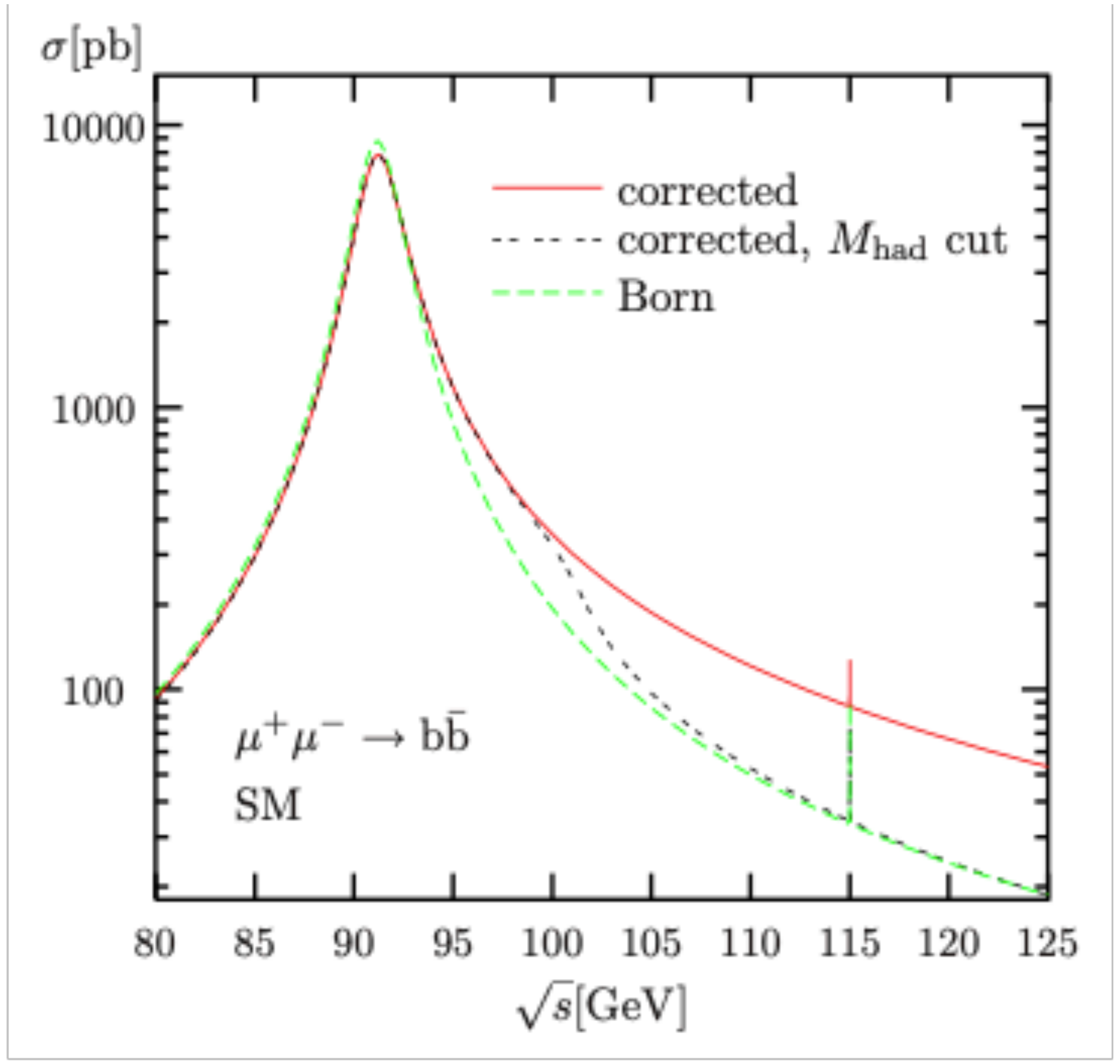
$$\sigma_{\mathrm{LL,\ FSR}} = \int d\sigma^{\mathrm{LO}} \prod_i \int_0^1 dz_i \Gamma^{\mathrm{LL}}_{\ell\ell}(z_i, Q^2)\Theta(z_i p_i)$$

$$\mathcal{O}(\alpha): \quad \Gamma^{\mathrm{LL},1}_{\ell\ell} = \frac{\alpha(0)}{2\pi}\left[\log\frac{Q^2}{m_\ell^2} - 1\right]\left(\frac{1+z^2}{1-z}\right)$$

KLN theorem assures mass-singular **FSR** effects cancel **only** if *fully inclusive*

Not always so “weak” corrections (ii)

- Higher order **EW** corrections are expected to be always much smaller than corresponding **QCD** ones. Naively: $\alpha \simeq 0.01,\ \alpha_S \simeq 0.1 \Rightarrow$ **NLO EW** ~ **NNLO QCD**
- **NLO EW** not always at the (sub)-percent level: systematic enhancements are possible
  - **Photon** emissions (e.g. **ISR** from radiative return)



[Dittmaier *et al.*, 0203120; 2002]

- On the left: LO Xs for $\mu^+\mu^- \to b\bar{b}$ and corresponding **photonic** + **QCD** corrections (solid red)
- Above the **Z- boson** resonance, **ISR** induces very large positive correction (can exceed 100% in relative size)
- Radiative return can be smeared by cuts on minimal value of final state (dashed grey), preventing “too hard” **ISR**. On the left: $\sqrt{s} - M(b\bar{b}) < 10 \quad \mathrm{GeV}$

Not always so “weak” corrections (iii)

- Higher order **EW** corrections are expected to be always much smaller than corresponding **QCD** ones. Naively: $\alpha \simeq 0.01, \ \alpha_S \simeq 0.1 \Rightarrow$ **NLO EW ~ NNLO QCD**

- **NLO EW** not always at the (sub)-percent level: systematic enhancements are possible
    - Radiative corrections enhanced by large mass ratios

Heavy fermions

Light fermions

Corrections to ρ parameter

At TL: $\rho = \dfrac{m_W^2}{c_{\mathrm{w}}^2 m_Z^2} = 1$

Beyond LO: $\rho = 1 + \Delta\rho$

$$\Delta\rho^{\text{1-loop}} = \left(\frac{\Sigma_{\mathrm{T}}^{WW}(m_W^2)}{m_W^2} - \frac{\Sigma_{\mathrm{T}}^{ZZ}(m_Z^2)}{m_Z^2} \right) \overset{m_t \gg m_b}{\sim} \frac{3\alpha}{16\pi c_{\mathrm{w}}^2 s_{\mathrm{w}}^2} \frac{m_t^2}{m_Z^2}$$

Running of electromagnetic coupling

Renormalisation at zero momentum transfer:
In fermions-loop the relevant scale is set by m_f

$$\Delta\alpha(s) = \underbrace{\frac{\alpha}{3\pi} \sum_f N_C^f Q_f^2 \log \frac{s}{m_f^2}}_{\Delta\alpha_{\mathrm{LL}}} + \ldots$$

LL in $\Delta\alpha$ are universal and can be summed to all orders by

$$\alpha = \alpha(0) \to \alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)_{\mathrm{LL}}}$$

Not always so “weak” corrections (iv)

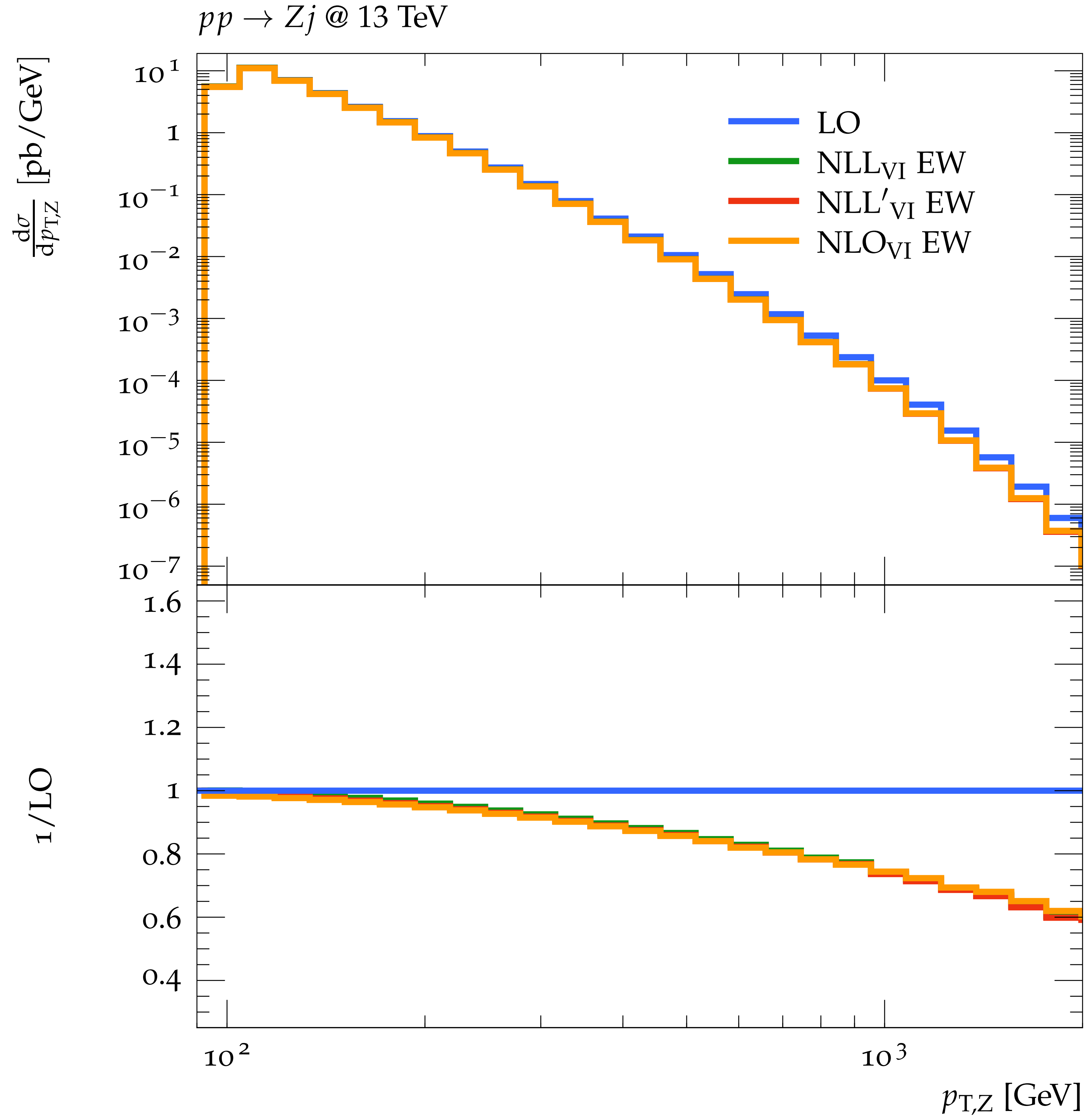
- Higher order **EW** corrections are expected to be always much smaller than corresponding **QCD** ones. Naively: $\alpha \simeq 0.01,\ \alpha_S \simeq 0.1 \Rightarrow$ **NLO EW** ~ **NNLO QCD**

- **NLO EW** not always at the (sub)-percent level: systematic enhancements are possible
  - High-energy regime: <u>*Sudakov*</u> enhancement

At $\sqrt{s} \gg m_W$ **1-loop EW** show a logarithmic enhancement and are typically (but not always) **universal**

$$\mathcal{M}^{\text{1-loop}} \sim \left(A(\alpha, s_\mathrm{w}) \log^2 \frac{|r_{kl}|}{m_W^2} + B(\alpha, s_\mathrm{w}) \log \frac{|r_{kl}|}{m_W^2} \right) \mathcal{M}_0$$

$$|r_{kl}| = |(p_k + p_l)^2| \approx 2|p_k p_l| \sim s \gg m^2_{Z/W/H/t}, \qquad k \neq l$$



[Lindert & **L.M.**, <u>2312.07927</u>; 2023]

EW corrections at high energy

EW corrections at high energy (i)

- In the energy range above the **EW** scale, $\sqrt{s} \gg m_W$, _Sudakov logs_ represent the leading contribution of **EW** radiative corrections

- _Sudakov logarithms_ from **N^{n}LO** **EW** corrections

$$\alpha^n \log^k \frac{|r_{kl}|}{m_i^2}, \qquad 1 \leq k \leq 2n \qquad\qquad |r_{kl}| = |(p_k + p_l)^2| \sim s$$

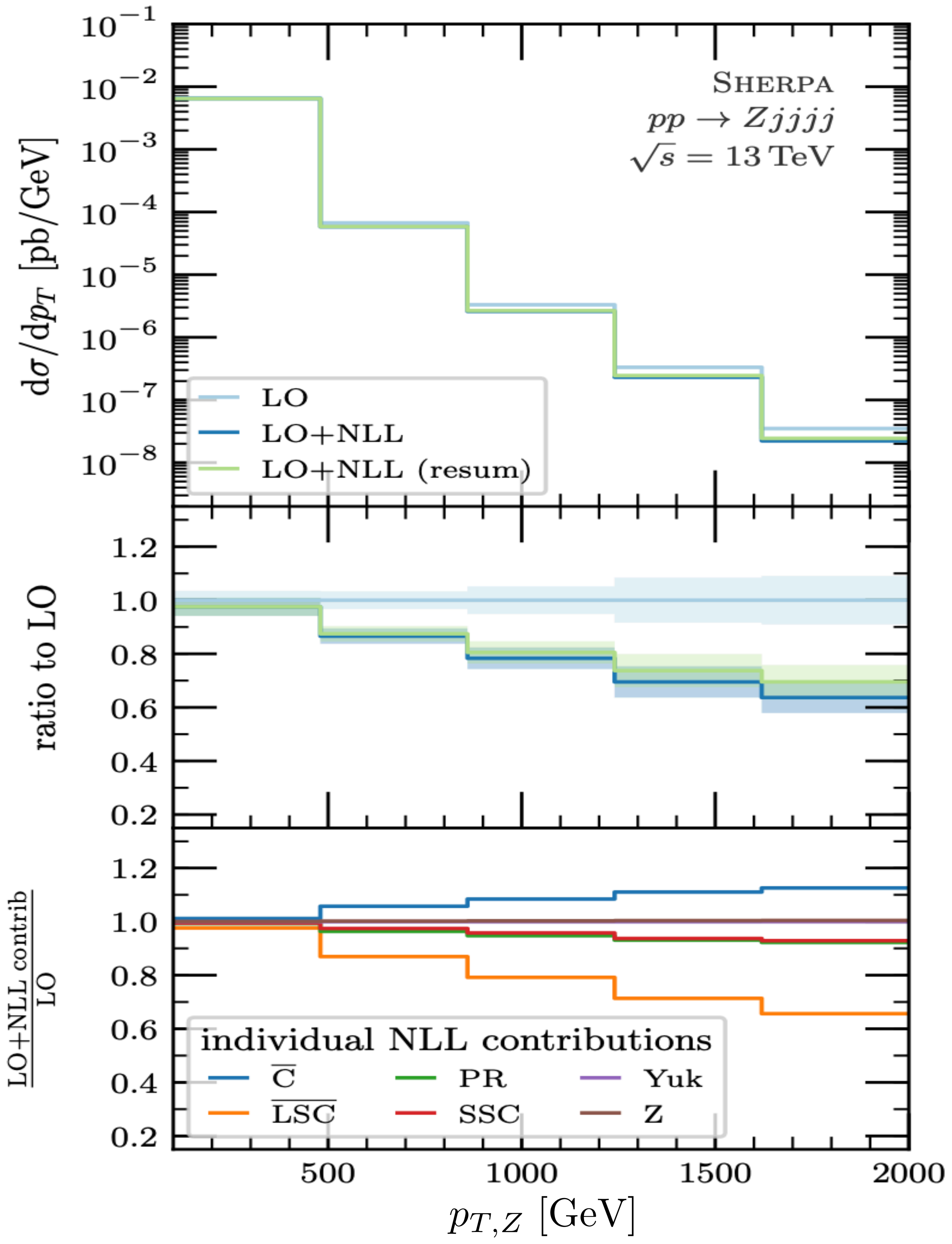
- At **NLO**

Double logs (DL): $$L(|r_{kl}|, m_i^2) = \frac{\alpha}{4\pi} \log^2 \frac{|r_{kl}|}{m_i^2}$$

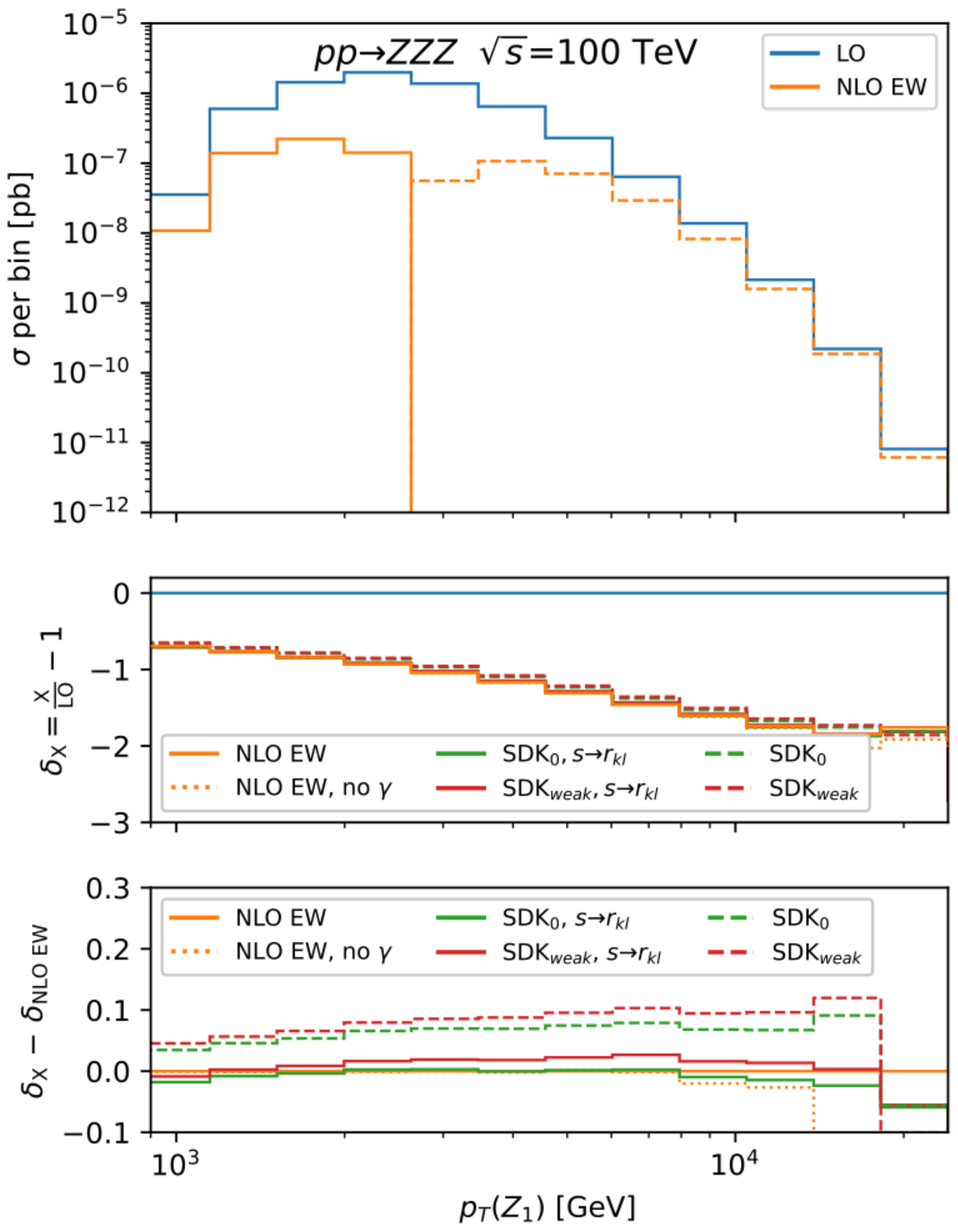
Single logs (SL): $$l(|r_{kl}|, m_i^2) = \frac{\alpha}{4\pi} \log \frac{|r_{kl}|}{m_i^2}$$

EW corrections at high energy (ii)

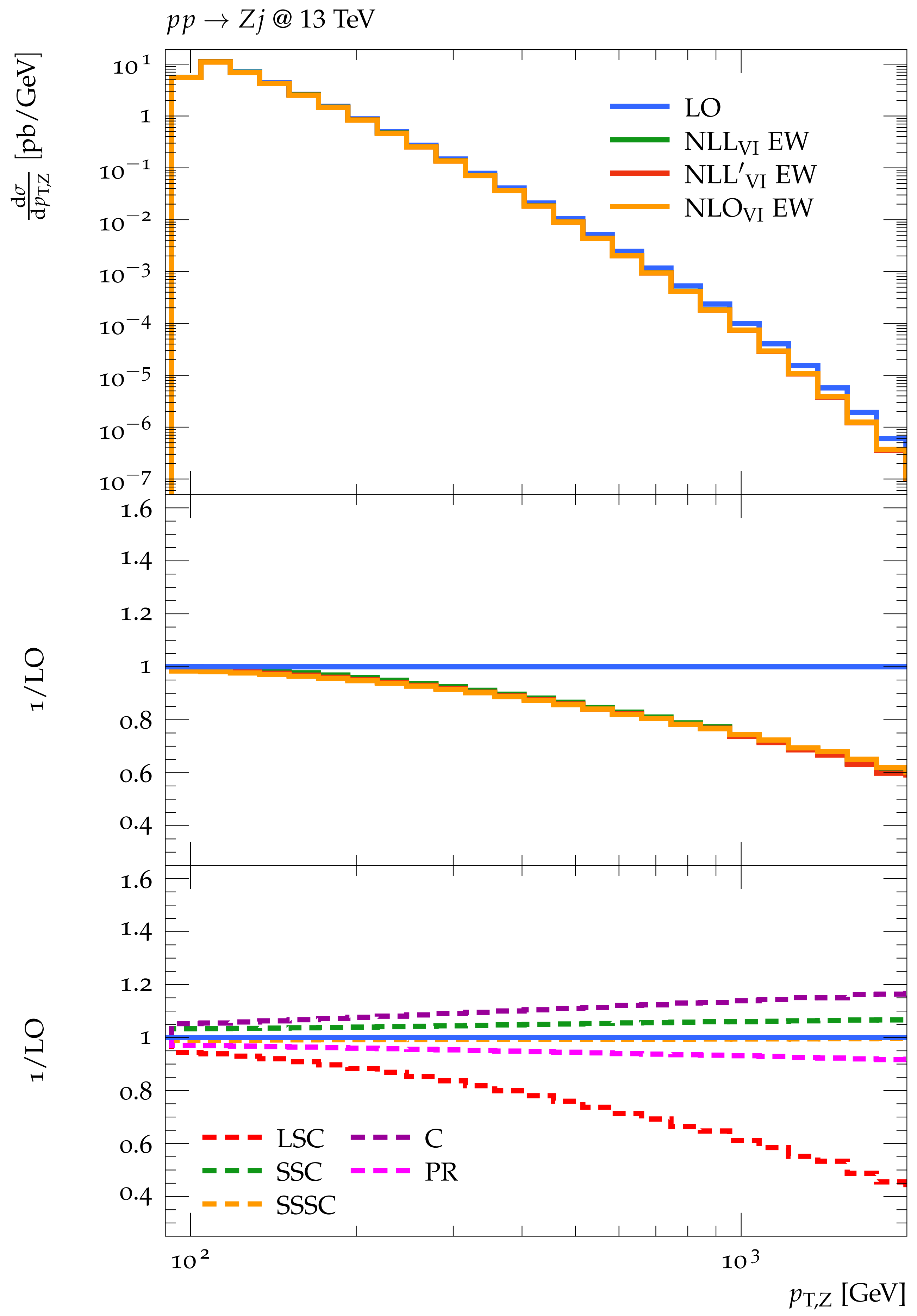
- Without clear signs of NP as resonances, small deviations in tails of kinematic distributions are under scrutiny
- **NLO EW** corrections and their *Sudakov approximation* are crucial as they can provide several ten % effects in tails



Sherpa: Bothmann, Napoletano 2006.14635; *2020*



MadGraph: Pagani, Zaro 2110.03714; *2021*



OpenLoops: Lindert & **L.M.**, 2312.07927; 2023

- Still relevant at **2-loop**: $\alpha^2 \log^4 \left(s/m_i^2\right) \simeq 3\%$ at $s = 1$ TeV

EW corrections at high energy (iii)

- DP algorithm based on ***Logarithmic approximation*** (**LA**):

→ On-shell external momenta $p_k^2 = m_{\varphi_k}^2$ and all r_{kl} much larger than W/Z masses

$$\boxed{|r_{kl}| = |(p_k + p_l)^2| \approx 2|p_k p_l| \sim s \gg m^2_{Z/W/H/t}, \qquad k \neq l}$$

→ Not mass-suppressed Born matrix element, i.e. $\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d$

→ At one-loop keep only leading and universal DL and SL corrections

$$\delta^{\rm DL}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim L\,\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}} \qquad \delta^{\rm SL}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim l\,\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}}$$

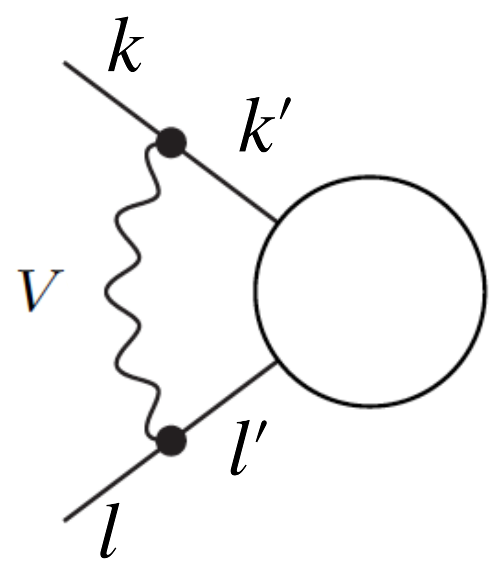
neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim m^n E^{d-n} L)$ contributions

[Denner and Pozzorini 0010201; *2001*]

Origin of DL and SL in a nutshell (i)

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** **gauge** boson V

$$\delta^{\mathrm{DL}}\mathcal{M} \sim \frac{\alpha}{4\pi}\, I^{V}_{kk'} I^{\bar{V}}_{ll'}\, \log^2\frac{|r_{kl}|}{m_V^2}\, \mathcal{M}_0$$



LA: $\boxed{|r_{kl}| = |(p_k + p_l)^2| \approx 2|p_k p_l| \sim s \gg m^2_{Z/W/H/t}, \qquad k \neq l}$

- DL usually split into:

  ➜ **LSC** (angular independent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{s}{m_V^2}\right)$

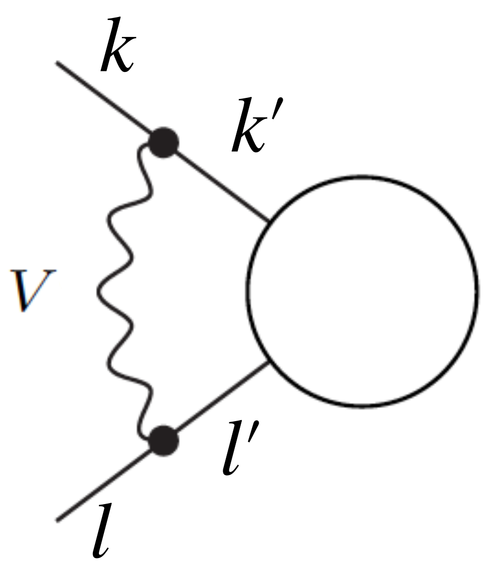
  ➜ **SSC** (angular dependent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$

  ➜ **Sub-SSC** (angular dependent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$

Origin of DL and SL in a nutshell (ii)

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC) *gauge*** boson V

$$\delta^{\rm DL}\mathcal{M} \sim \frac{\alpha}{4\pi}\, I^{V}_{kk'} I^{\bar{V}}_{ll'}\ \log^2\frac{|r_{kl}|}{m_V^2}\, \mathcal{M}_0$$



LA: $|r_{kl}| = |(p_k+p_l)^2| \approx 2|p_k p_l| \sim s \gg m^2_{Z/W/H/t}, \qquad k \neq l$

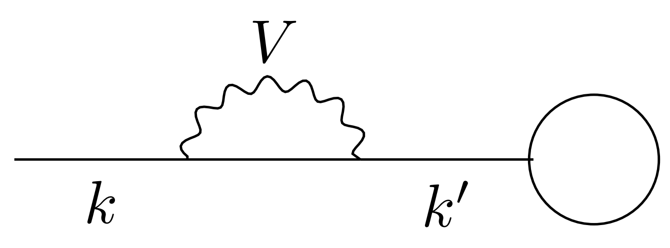
- DL usually split into:
  - ➔ **LSC** (angular independent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{s}{m_V^2}\right)$
  - ➔ **SSC** (angular dependent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$
  - ➔ **Sub-SSC** (angular dependent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$

- SL have a triple origin:
  - ➔ **PR**: UV renormalisation of **EW** dimensionless parameters

$$\boxed{g_{i,0} = g_i + \delta g_i = (1+\delta Z_{g_i})\, g_i}$$

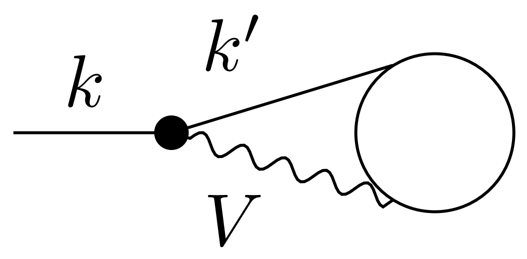
  - ➔ **WF**: wave-function renormalisation of external fields

$$\delta^{\rm WF}\mathcal{M} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right)\mathcal{M}_0$$



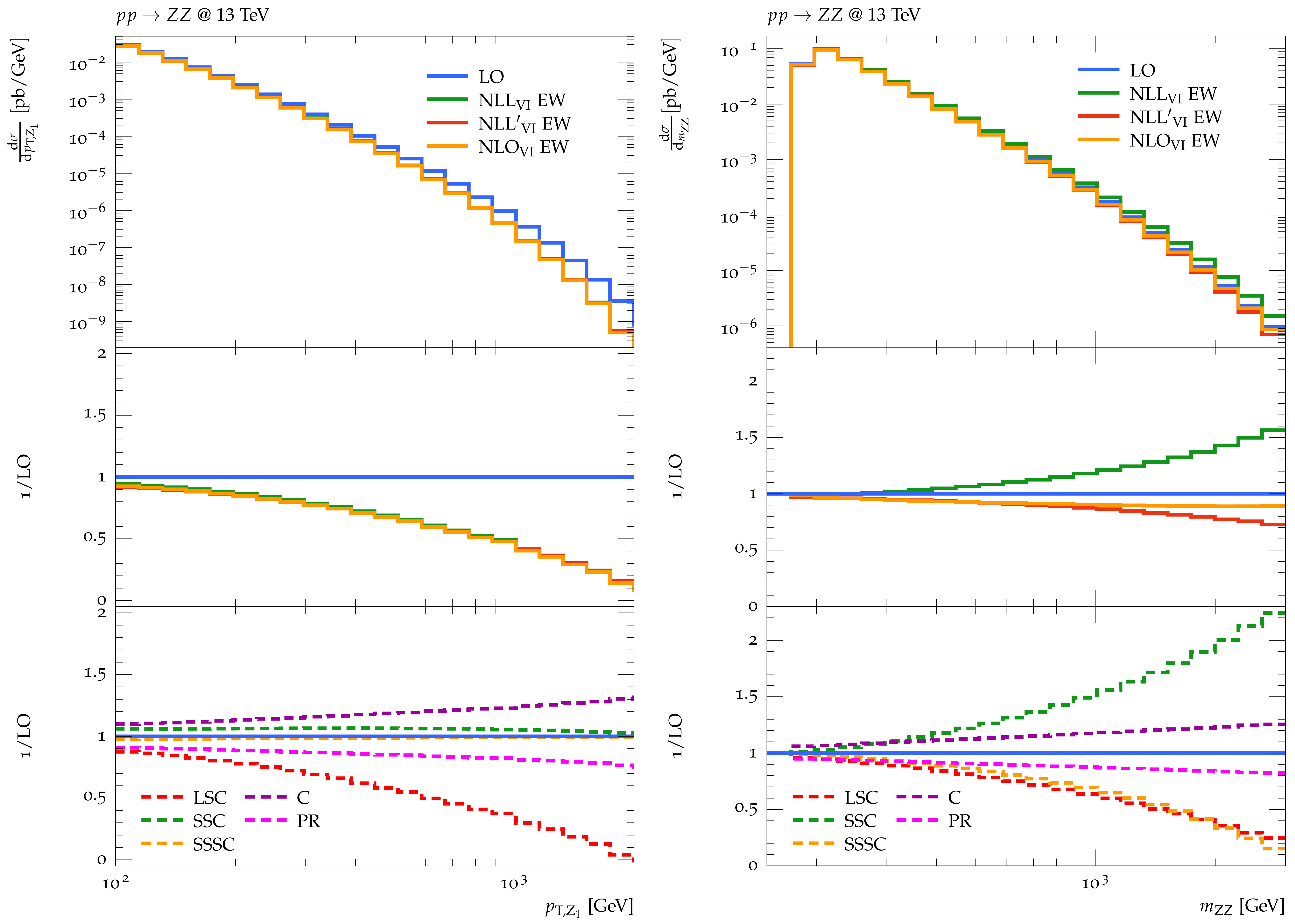
  - ➔ **Coll**: external leg emission of a collinear gauge boson

$$\delta^{\rm coll}\mathcal{M} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right)\mathcal{M}_0$$



- **C**=**WF**+**Coll**: Full gauge-invariant SL associated to external fields

EW corrections at high energy (iv)



OpenLoops + Sherpa

NLL = standard **LA** prediction

$$\boxed{\mathrm{NLL_{VI}\ EW} = (1 + \mathrm{LSC} + \mathrm{SSC} + \mathrm{C} + \mathrm{PR} + \mathbf{I})\mathrm{LO}}$$

NLL′ = **LA** prediction + **SSSC**

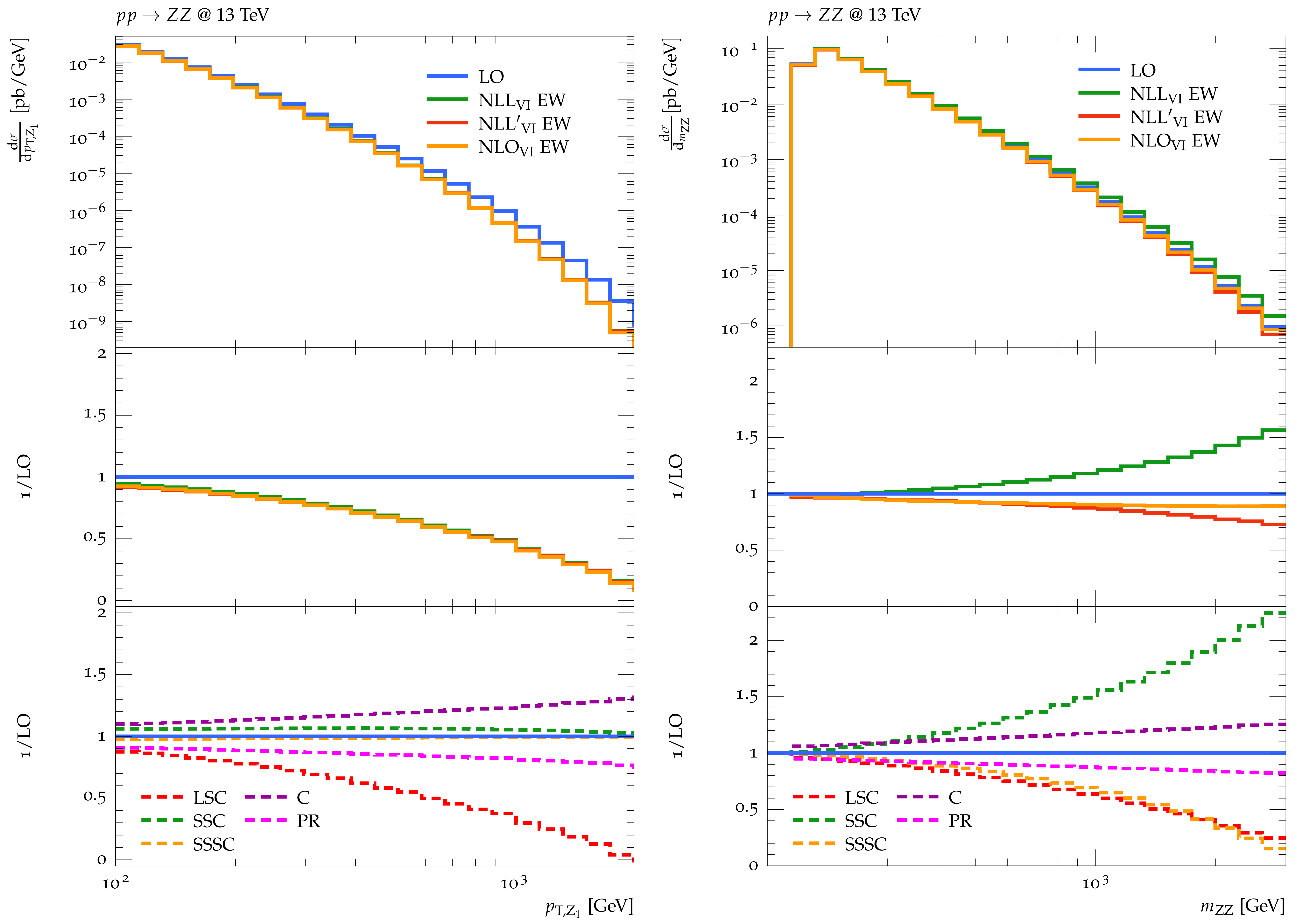
$$\boxed{\mathrm{NLL'_{VI}\ EW} = (1 + \mathrm{LSC} + \mathrm{SSC} + \mathrm{SSSC} + \mathrm{C} + \mathrm{PR} + \mathbf{I})\mathrm{LO}}$$

[Lindert & **L.M.**, 2312.07927; 2023]

NNLO QCD+NLO EW: [Grazzini *et al*, 1912.00068; 2020]

NLO EW vs NLL EW: [Bothmann *et al*, 2111.13453; 2021]

EW corrections at high energy (iv)



[Lindert & **L.M.**, 2312.07927; 2023]

SSC and **SSSC** become very large for PS regions where **LA** condition is violated, with hierarchy among invariants

$$s \sim |r_{kl}| = |(p_k + p_l)^2| \gg |r_{k'l'}| = |(p_{k'} + p_{l'})^2| \gg m_W^2$$

E.g. in a $2 \to 2$ process in CM:

$$s = (p_1 + p_2)^2 = 4E^2 \qquad t = (p_1 - p_3)^2 = -4E^2 \sin^2\left(\frac{\theta}{2}\right)$$

$$p_{3\mathrm{T}} = E \sin\theta \qquad M_{34} = 2E$$

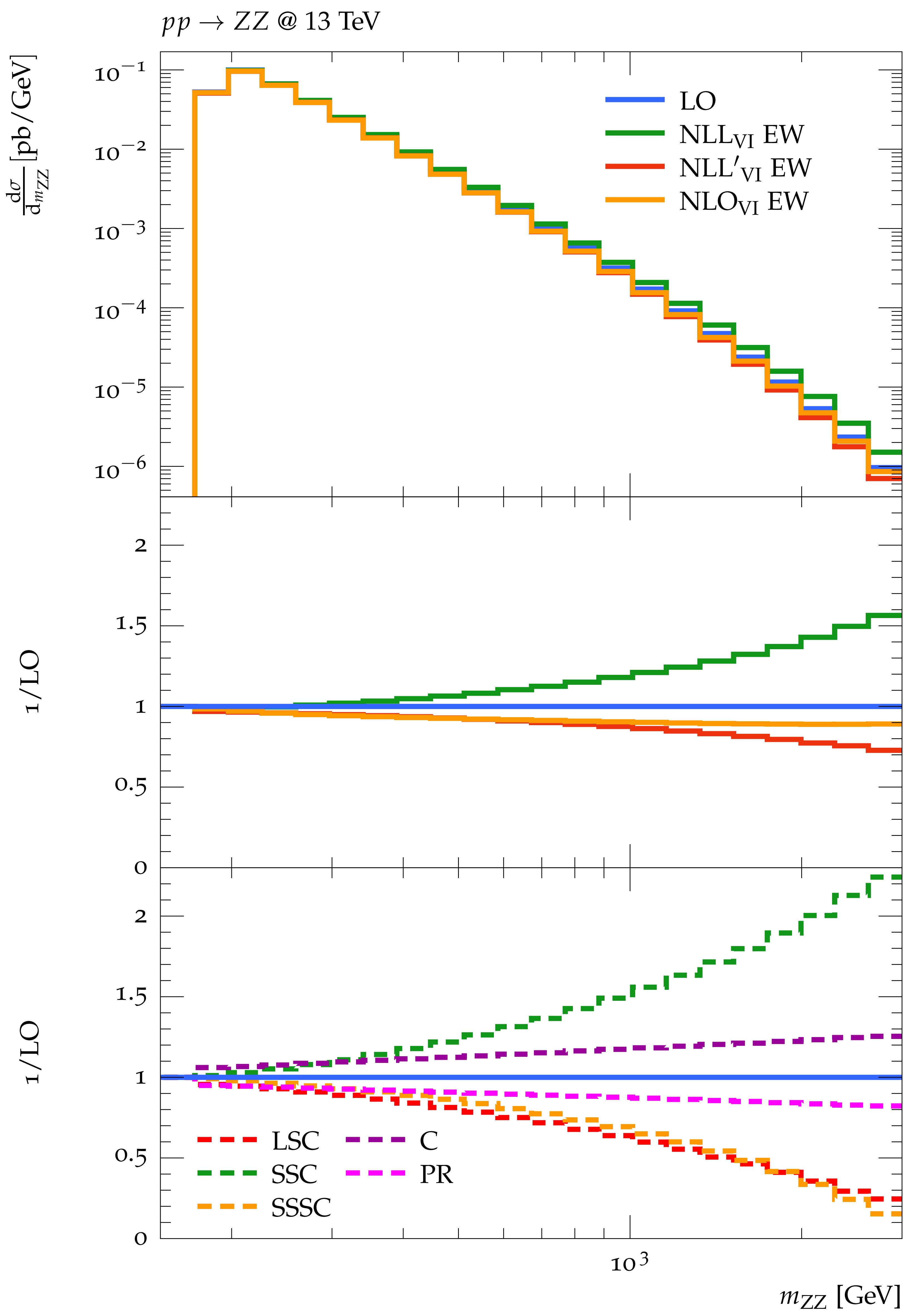
High $p_{\mathrm{T}} \to$ not to small $\theta \to s \sim t$

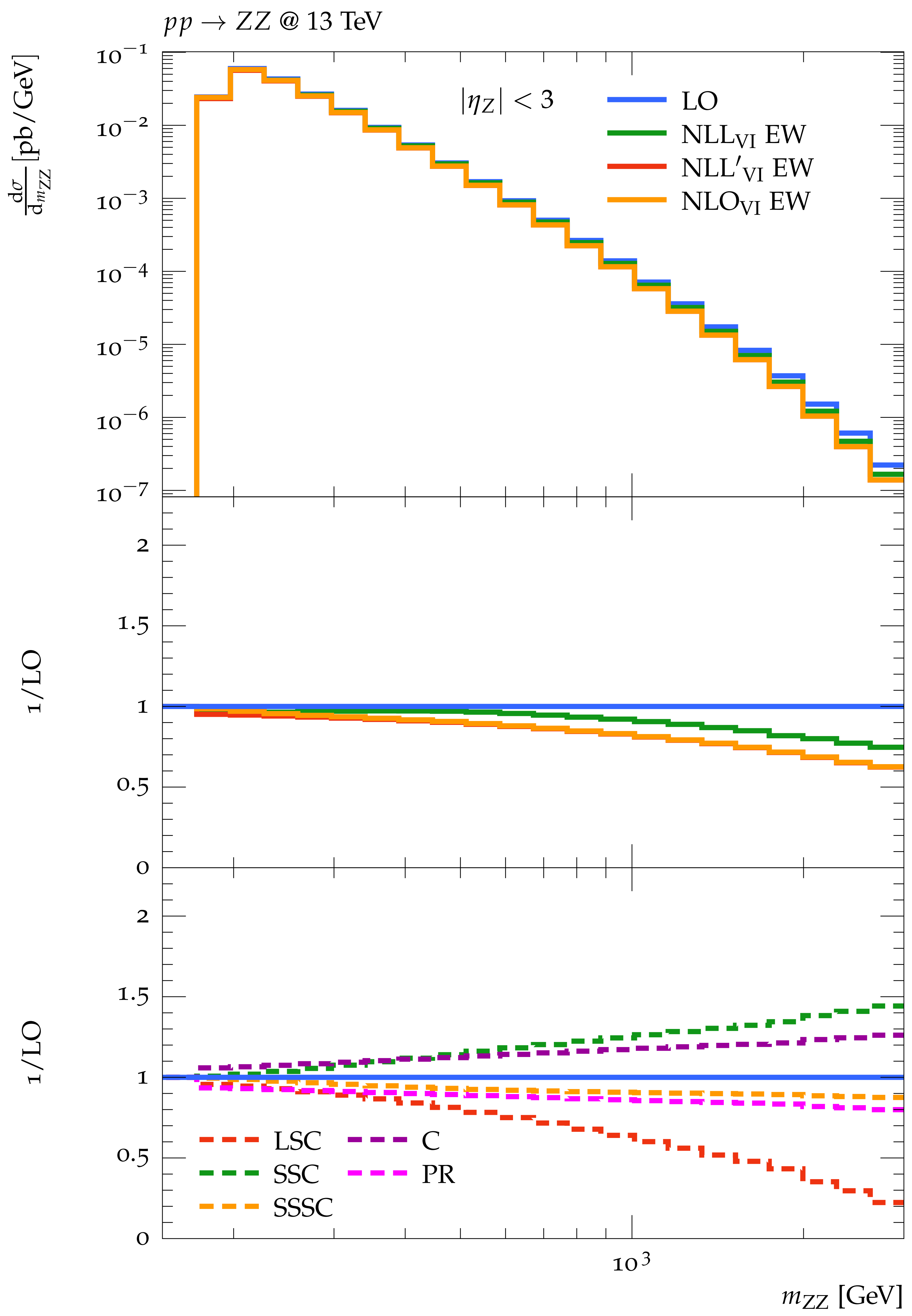
High $m_{ZZ} \to$ no constraint on θ

$$\delta^{\mathrm{SSC,\,V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$$

$$\delta^{\mathrm{S\text{-}SSC,\,V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$$

EW corrections at high energy (v)



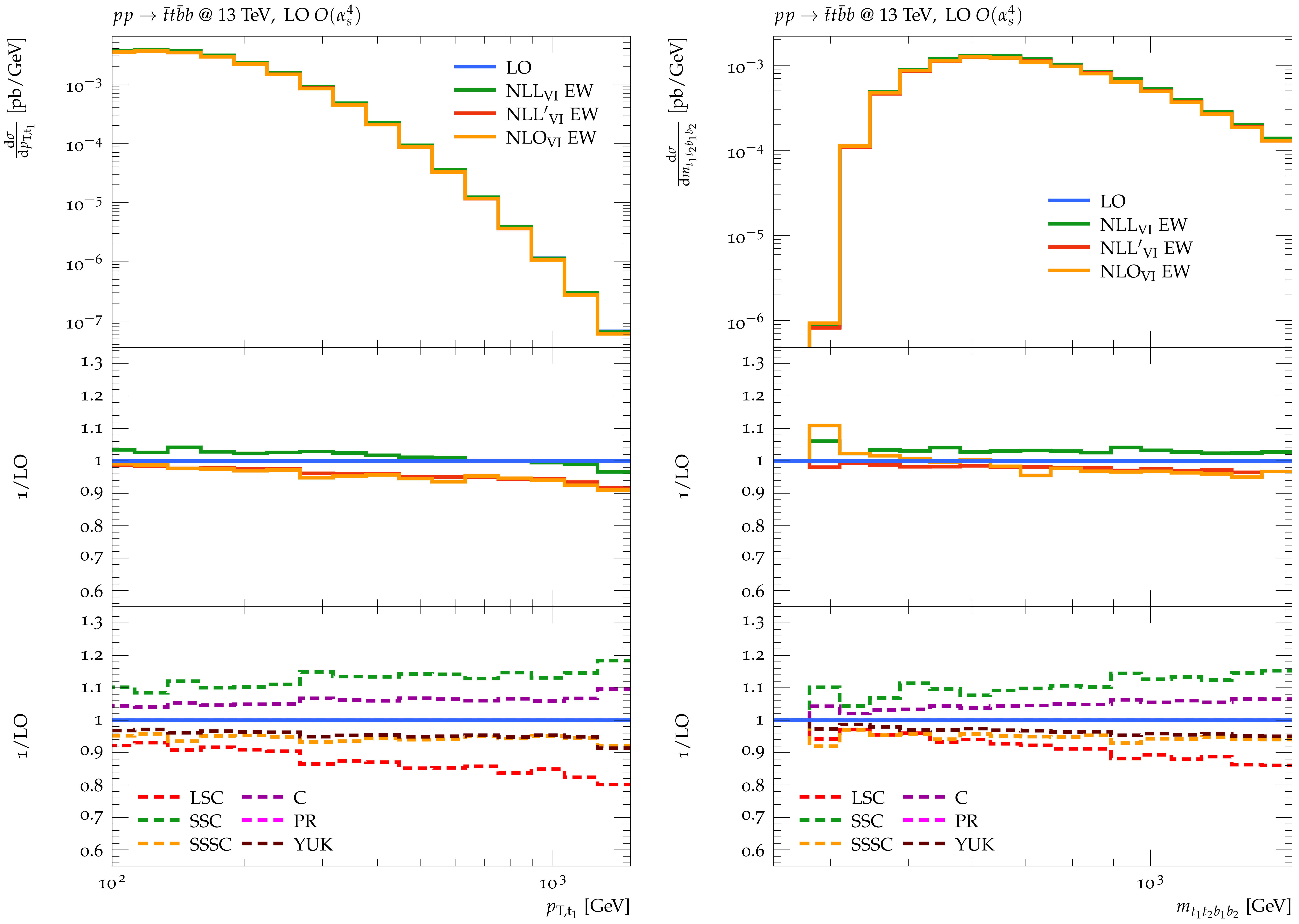


Pseudo-rapidity cut $|\eta_Z| < 3$ avoids pathological very forward configurations which violate LA; such cuts are anyway applied in any realistic analysis

Again, the inclusion of **SSSC** provides more accurate predictions. However, no full control on it as there are *non-universal* **SSSC**-like terms arising from high-energy expansion of 4-point functions. These angular contributions *cannot* be reliably controlled in LA

[Lindert & **L.M.**, 2312.07927; 2023]

EW corrections at high energy (vii)



[Lindert & **L.M.**, 2312.07927; 2023]

- **NLO EW** never computed before and expected to be small. We explicitly checked and verified it, observing $\sim 6-7\,\%$ @ $p_T \approx 1$ TeV

- Still a preliminary analysis! A more detailed study of **NLO EW** corrections will follow

NLO QCD: [Bredenstein *et al*, 0807.1248; 2008]

NLO QCD to $t\bar{t}b\bar{b}j$: [Buccioni *et al*, 1907.13624; 2019]

Status: [CMS collaboration, 2309.144422; 2023]

Conclusions

- **EW** sector of **SM** rich of phenomenological non-trivial aspects:
  - Equivalence between longitudinal **gauge** bosons and **goldstones** at high energy
  - **Input**-schemes choice → *scheme uncertainties*
  - Decays of **gauge** bosons, mandatory in $\mathcal{O}(\alpha)$ computations → CMS
  - Enhanced higher order corrections:
    1. **ISR/FSR**
    2. Large mass ratios (corrections to ρ parameter & running of α)
    3. *Sudakov* logs at high energy
- Renovated interest: to match experimental accuracy, **NLO EW** corrections are mandatory. In view precision studies in high-energy experiments also **NNLO EW** will be required → highly non-trivial!
- My outlook: automation of **NNLO EW** corrections in *Sudakov approximation*

References

- This talk collects the following references:

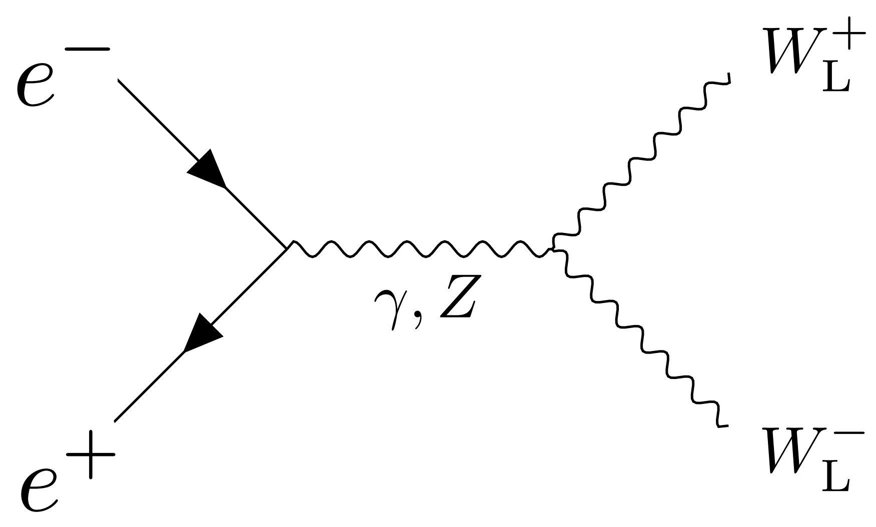
  - J. Lindert’s lectures @ MCnet Summer School 2022, Zakopane
  - S. Dittmaier’s lectures @ GGI workshop “Theory Challenges in the Precision Era of the LHC”, 2023, Florence
  - A. Denner & S. Dittmaier, “Electroweak Radiative Corrections for Collider Physics”; 1912.06823
  - Lectures on EW interactions and SM from from the Master degree “Physics of the fundamental interactions” at the University of Padova.
    The courses were respectively held by Pierpaolo Mastrolia and Paride Paradisi

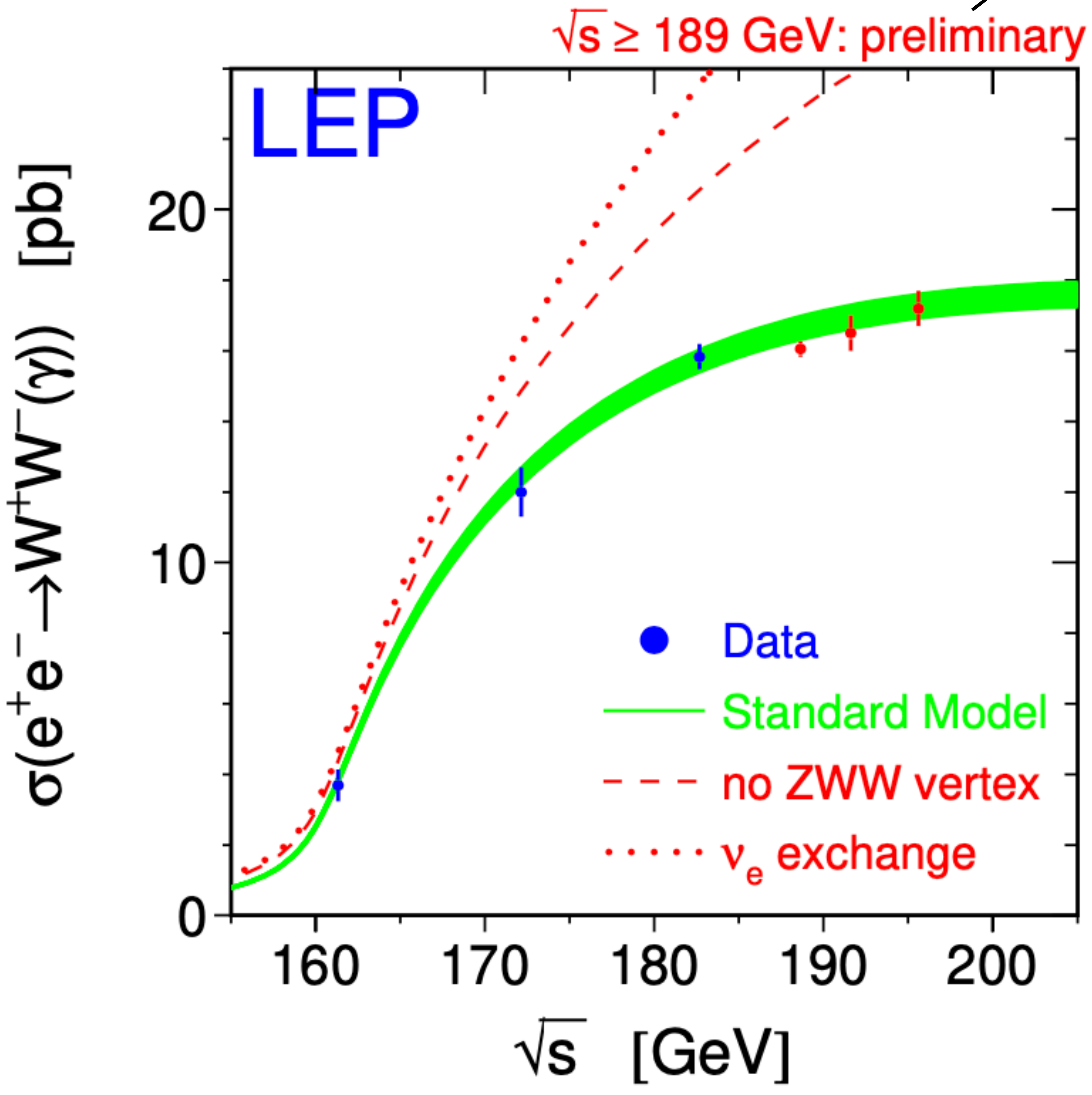
Backup

Goldstone Boson Equivalence Theorem (iv)

- Example 2: $e^+e^- \to W_\mathrm{L}^+ W_\mathrm{L}^-$

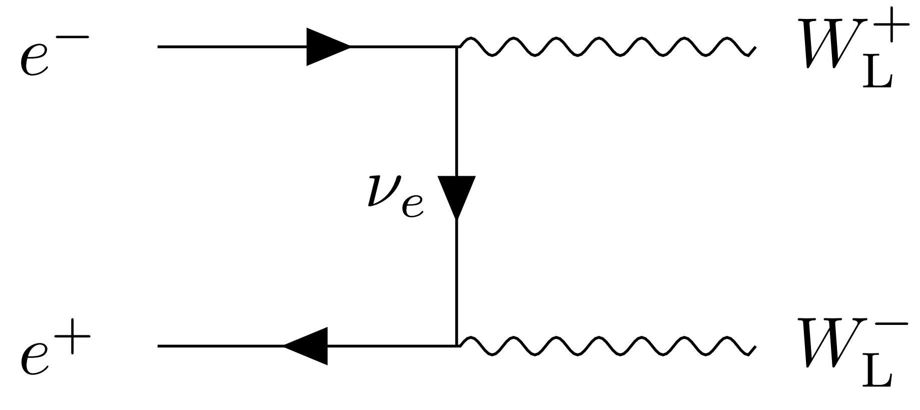
[Kunszt, 0004103; 2000]





$$\mathcal{M}(e_l^+ e_r^- \to W_\mathrm{L}^+ W_\mathrm{L}^-) \overset{s\gg m_W^2}{=} \bar{v}_\mathrm{R}\gamma^\mu u_\mathrm{R}\left[\frac{ie^2 m_Z^2}{2m_W^2 s}\right](k_+ - k_-)^\mu$$

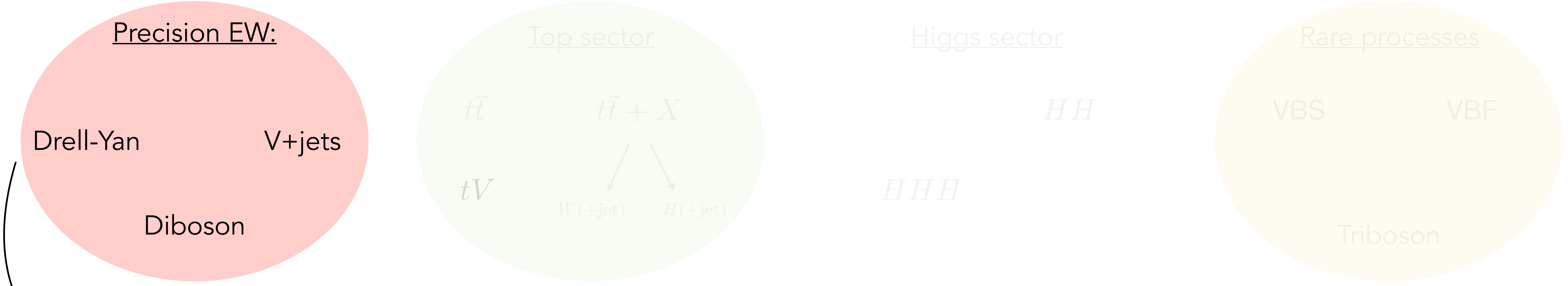
$$\overset{s\gg m_W^2}{=} \bar{v}_\mathrm{L}\gamma^\mu u_\mathrm{L}\frac{ie^2 s}{2m_W^2}\left[\frac{m_Z^2}{s(s-m_Z^2)} - \frac{1}{2s_\mathrm{w}^2(s-m_Z^2)}\right](k_+ - k_-)^\mu$$



$$\mathcal{M}_2(e_r^+ e_l^- \to W_\mathrm{L}^+ W_\mathrm{L}^-) \overset{s\gg m_W^2}{=} \bar{v}_\mathrm{L}\gamma^\mu u_\mathrm{L}\frac{ie^2}{4s_\mathrm{w}^2 m_W^2}(k_+ - k_-)^\mu$$

EW standard candles at LHC

EW standard candles at LHC



DY, a.k.a. production of a high-transverse-momentum lepton pair in hadronic collisions. It features large Xs and clean experimental signature.

CC DY distributions (E_{miss}, p_T, M_T) are fundamental for precise extraction of m_W

NC DY: large Xs and clean experimental signature. Used for different purposes:

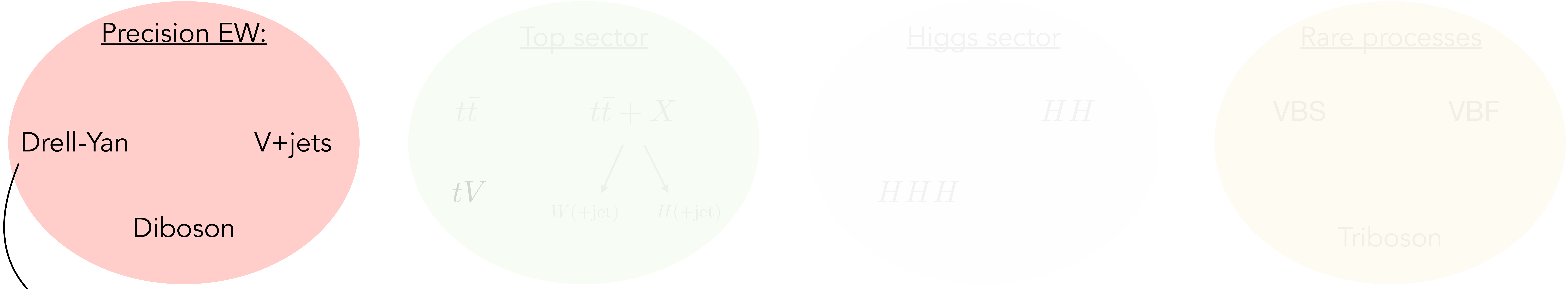
- Detector calibration
- PDFs constraining
- One of main irreducible backgrounds to NP searches at LHC in high-tail distributions

QCD: **N^{3}LO** [Camarda *et al*, 2103.04974; 2021], [Neumann *et al*, 2207.07056; 2022]

EW: **NLO** [Bauer *et al*, 0405191; 2004], **NNLO** Sudakov approx. [Jantzen *et al*, 0509157; 2008]

NNLO QCD x EW: [Armadillo *et al*, 2201.01754; 2022], [Armadillo *et al*, 2405.00612; 2024]

EW standard candles at LHC



DY, a.k.a. production of a high-transverse-momentum lepton pair in hadronic collisions. It features large Xs and clean experimental signature.

CC DY distributions (E_{miss}, p_T, M_T) are fundamental for precise extraction of W boson mass and width; studied for PDFs constraining as well

NC DY: used for different purposes:
- Detector calibration
- PDFs constraining
- One of main irreducible backgrounds to NP searches at LHC in high-tail distributions

QCD: **N^{3}LO** [Camarda *et al*, 2103.04974; 2021], [Neumann *et al*, 2207.07056; 2022]

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EW standard candles at LHC

<u>Precision EW:</u>

Drell-Yan

V+jets

Diboson

<u>Top sector</u>

$t\bar{t}$ $t\bar{t}+X$

tV $W(+\text{jet})$ $H(+\text{jet})$

<u>Higgs sector</u>

HH

HHH

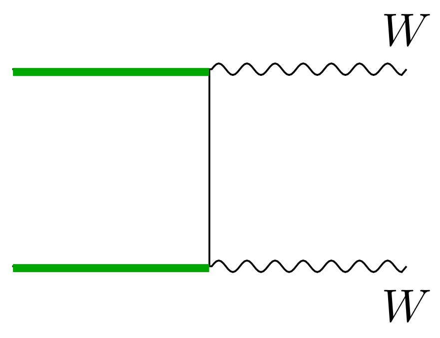
<u>Rare processes</u>

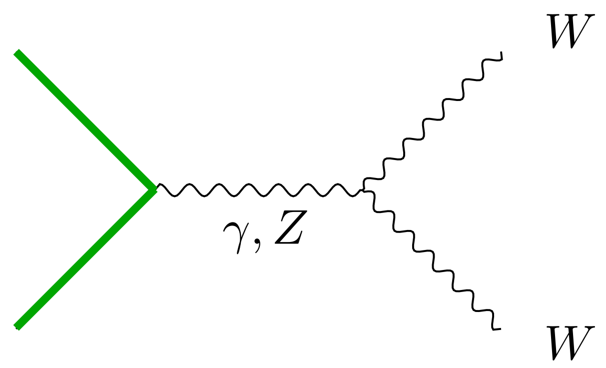
VBS VBF

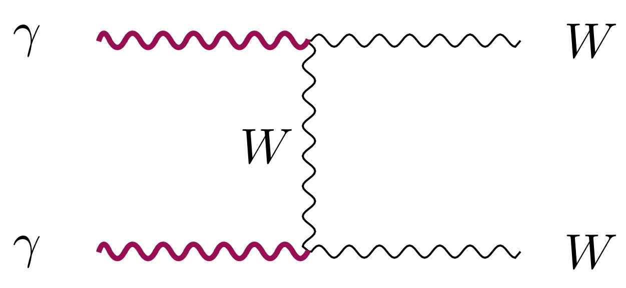
Triboson

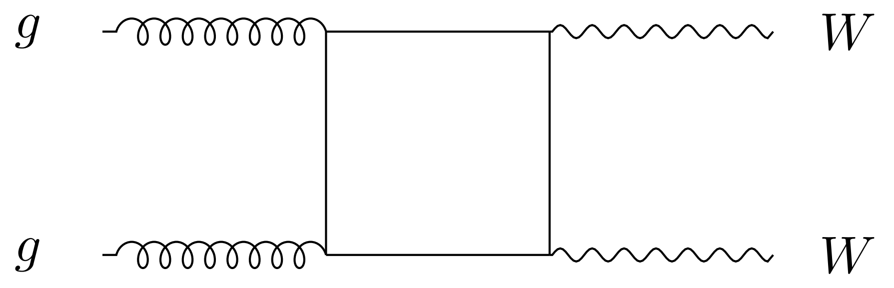
- Sensitivity to many PDFs:
  - gg in WW/WZ
  - $\gamma\gamma$ in WW
  - $\bar{u}d/u\bar{d}$ in WZ
  - $\bar{q}q$ in WW/ZZ

WW

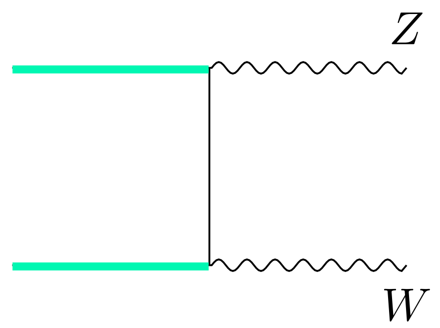


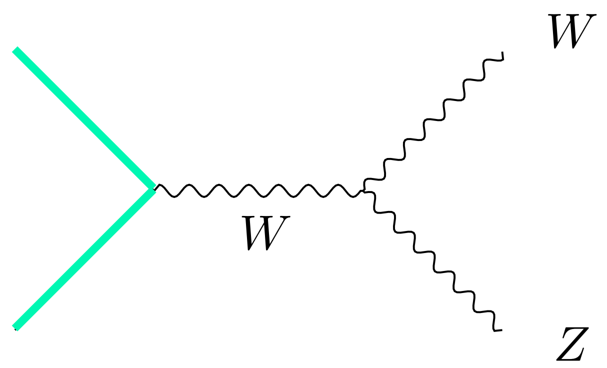




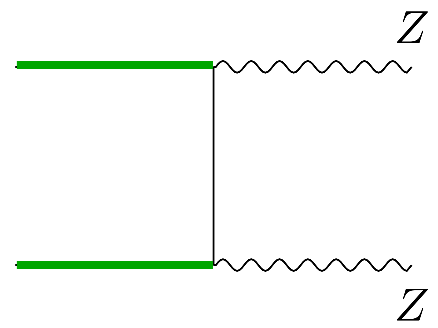


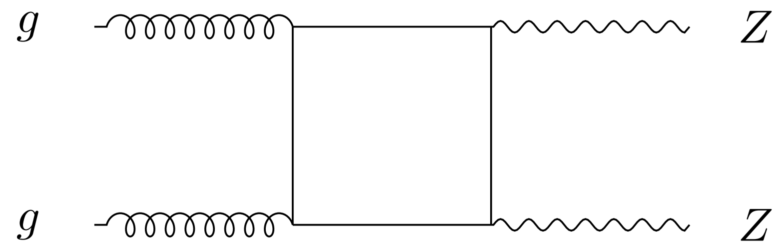
WZ





ZZ





- Sensible to trilinear **gauge** boson coupling
  - ZWW in WZ
  - $\gamma WW/ZWW$ in WW

Refs. in Sec. 4.2.2 from
[Lindert & **L.M.**, <u>2312.07927</u>; 2023]

EW standard candles at LHC

Precision EW:

Drell-Yan

V+jets

Diboson

Top sector

$t\bar{t}$

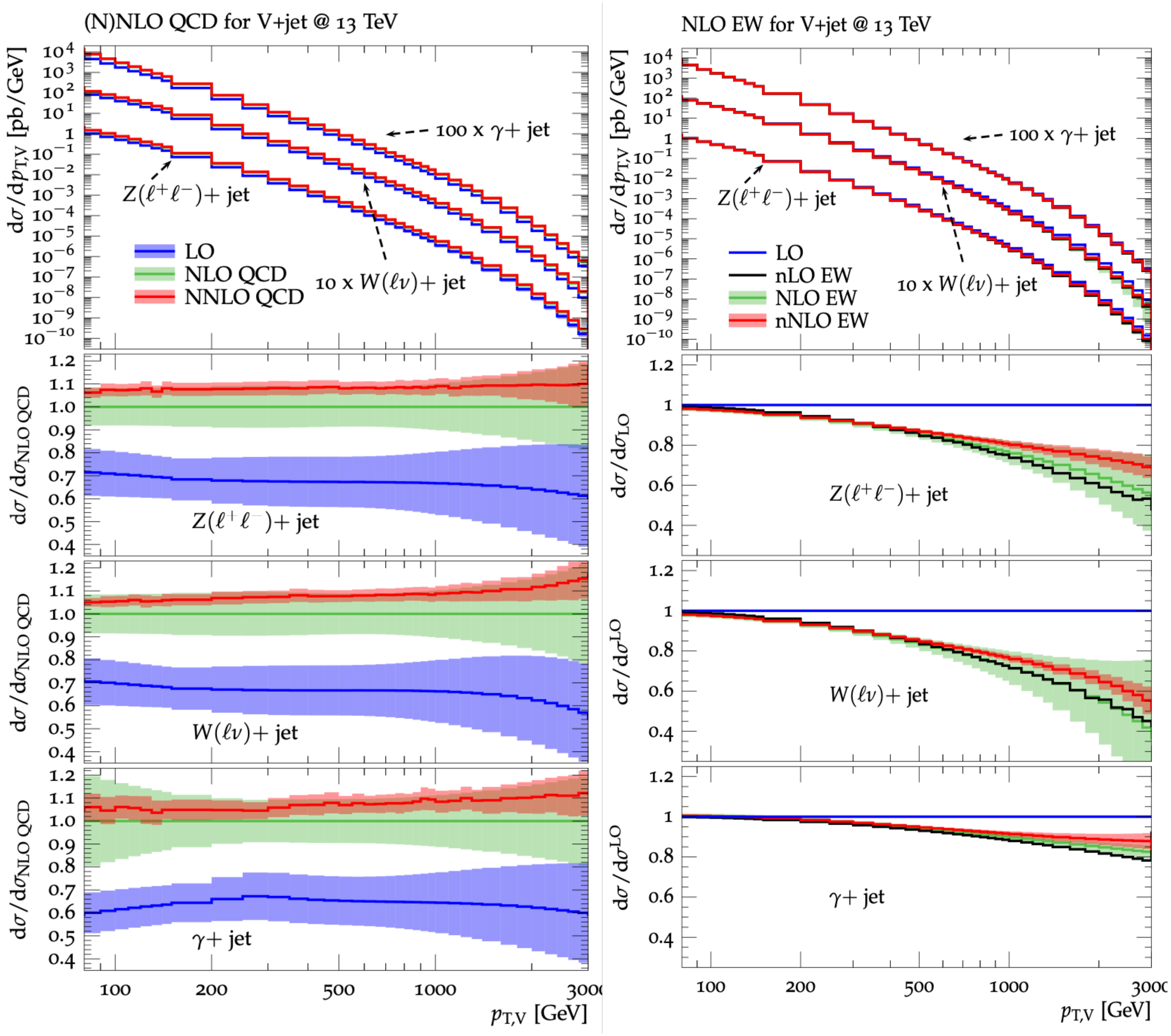
$t\bar{t}+X$

tV

$W(+\text{jet})$

$H(+\text{jet})$

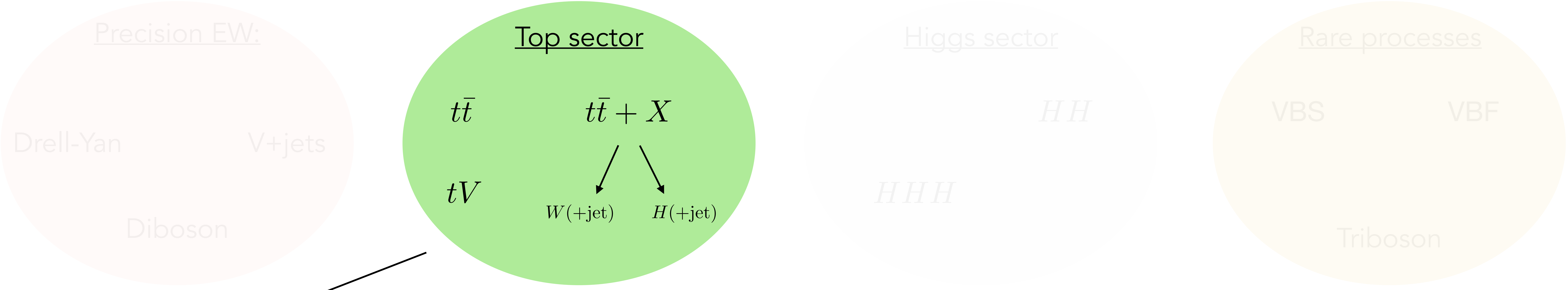
- Ubiquitous at the LHC
- Large production rate + clean signature from charged leptons pair
- Stringent tests of pQCD and constraints on PDFs
- Background for studies on Higgs and New Physics searches via $E_{T,miss}$



NNLO QCD + NLO EW: [Lindert *et al*, 1705.04664; 2017]

N^{3}LO QCD: [Chen *et al*, 2203.01565; 2024]

EW standard candles at LHC

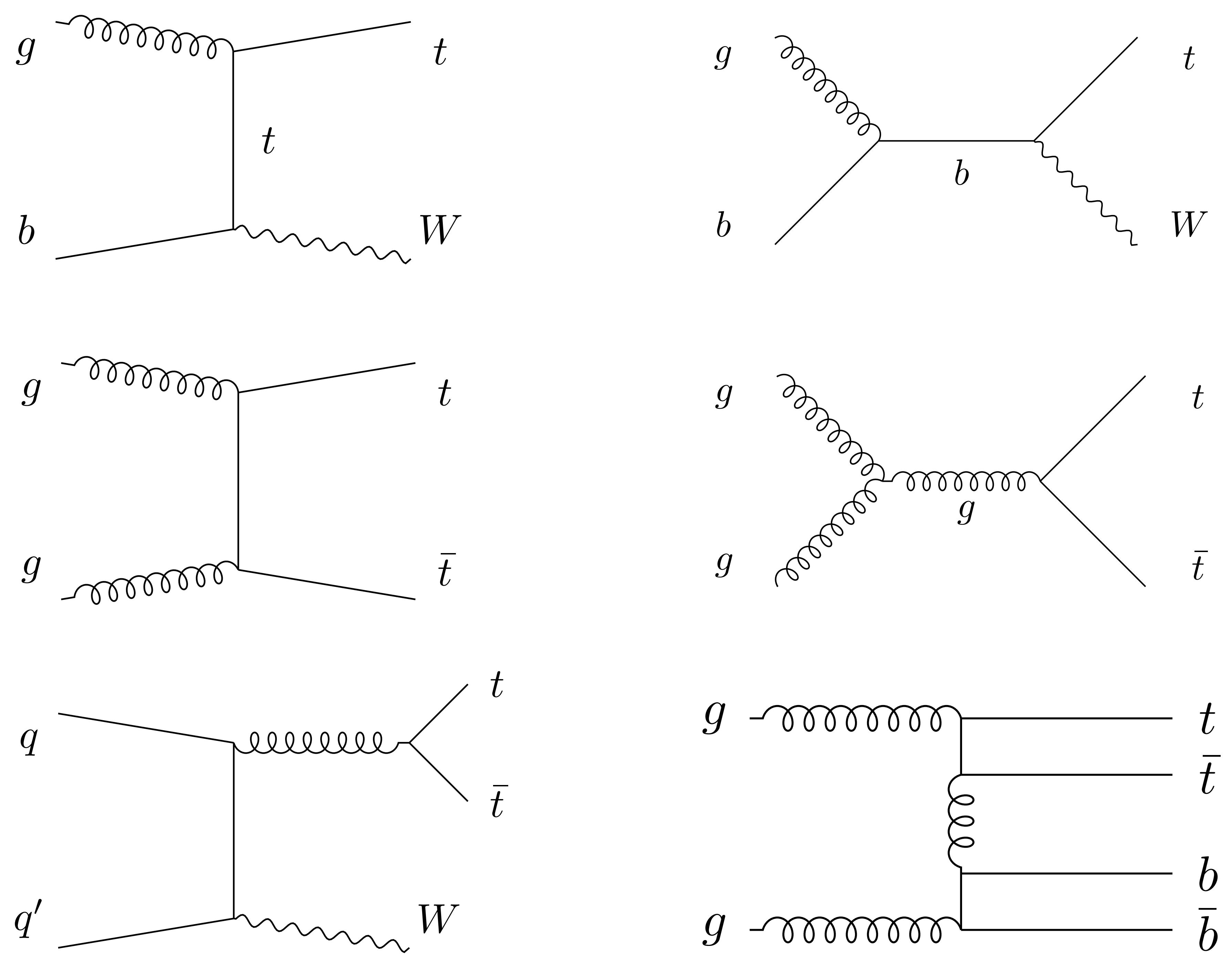


- **Top quark**:
  - only **quark** to decay before hadronisation → direct access to its properties
  - largely produced at LHC: mainly **top**-pair production in gluon fusion
- Backgrounds in many **Higgs** analyses and/or **BSM** searches
- Laboratory for testing **EWSB**

$tV:$

$t\bar{t}:$

$t\bar{t}+X:$



Th. status:

QCD: for tV, tt and ttV see Sec. 1 from [Broggio *et al*, 1907.04343; 2021]

EW: see Sec. 4.2.4 from [Lindert & **L.M.**, 2312.07927; 2023]

EW standard candles at LHC

Precision EW:

Drell-Yan

V+jets

Diboson

Top sector

$t\bar{t}$

$t\bar{t} + X$

tV

$W(+\text{jet})$

$H(+\text{jet})$

Higgs sector

HH

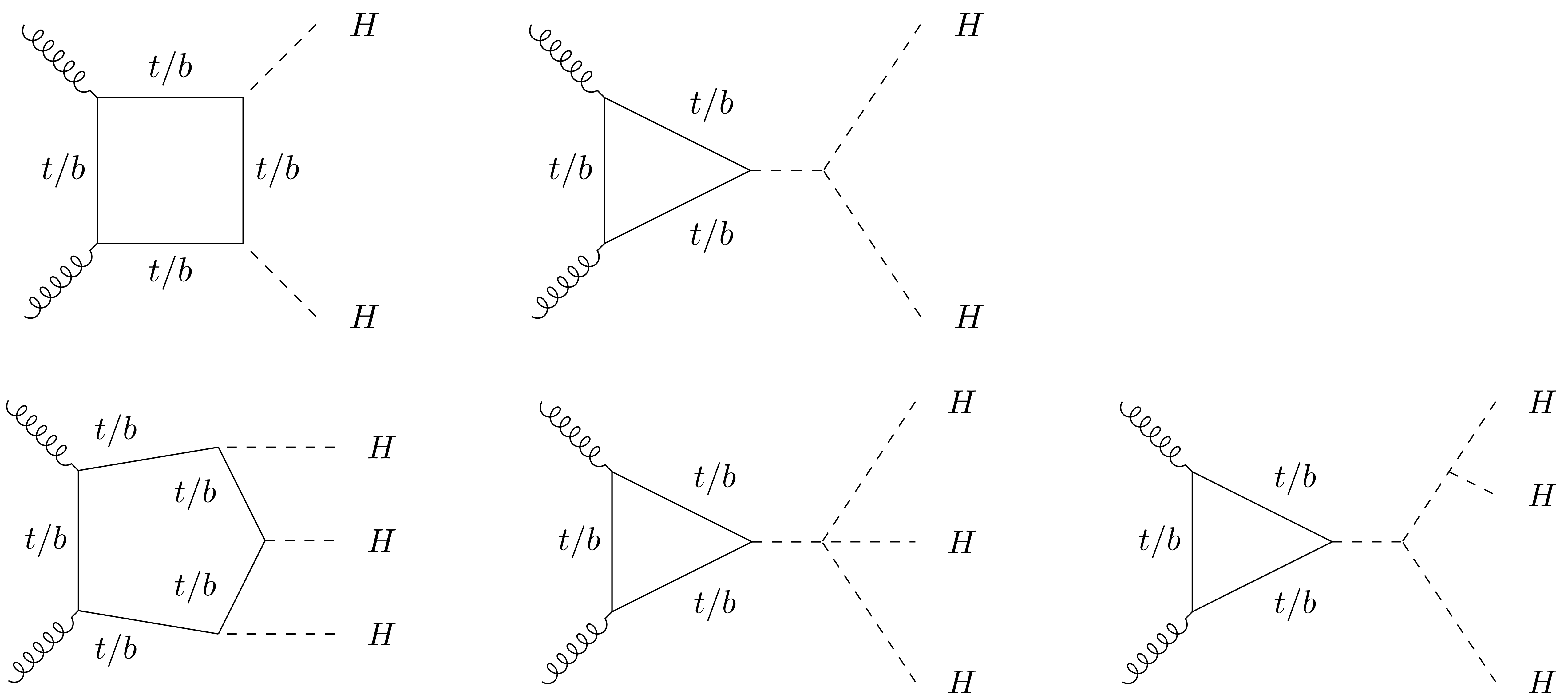
HHH

Rare processes

VBS

VBF

Triboson



- Sensitivity to **Higgs** triple and quartic self couplings λ and VEV

$$\lambda_3 = \lambda_4 = m_H^2/2v^2 = \lambda_{SM} \qquad HHH: \quad \lambda_3 v$$
$$HHHH: \quad \lambda_4/4$$

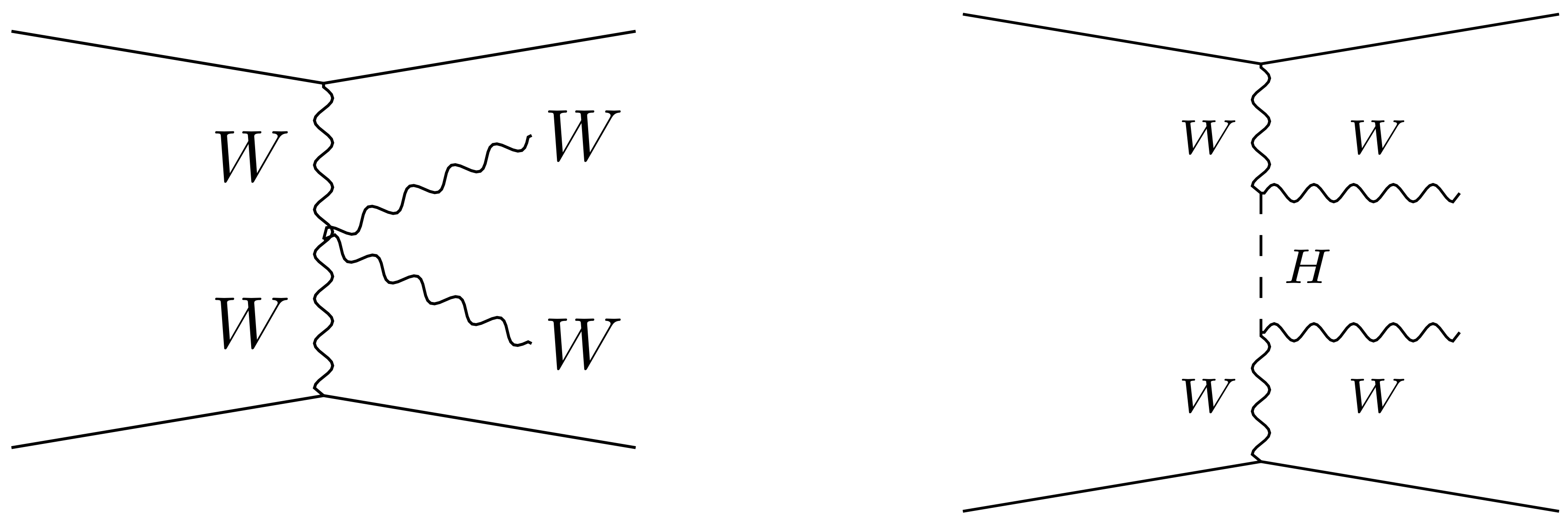
- HHH production rate very low at LHC but NP effects can enhance it
- Sensitivity to **top** and **bottom quark** couplings

HH: **NLO EW** [Bi *et al*, 2311.16963; 2024], **N**[3]**LO$_{\text{HTL}}$ QCD** [Bi *et al*, 1912.13001; 2020]

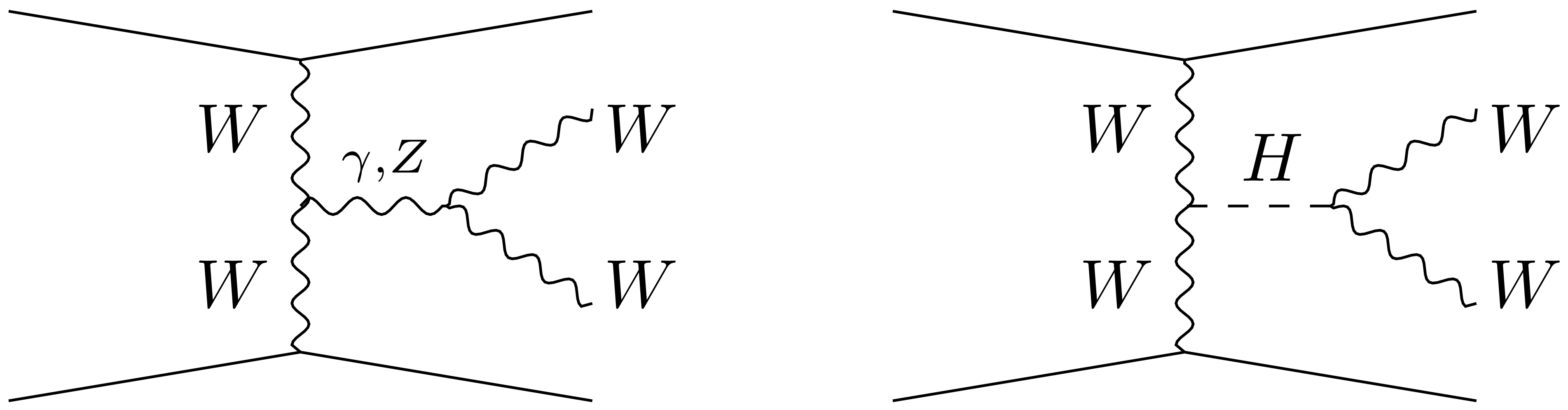
HHH: **NNLO$_{\text{HTL}}$ QCD** [de Florian *et al*, 1912.02760; 2020]

EW standard candles at LHC

VBS:



VBF:



- VBS: direct access to **EW** quartic **gauge** coupling + interplay with trilinear **gauge** couplings (e.g. in VBF)
- VBF and VBS: window to **SSB** through off-shell **Higgs**

VBS: **NLO QCD** and **NLO EW** see refs. in Sec. 3 in [Franzosi *et al*, 2106.01393; 2022]

VBF: **NLO QCD**, see refs. 49-55 in [Rauch, 1610.08420; 2016]

VBF-H: NLO EW [Ciccolini *et al*, 0707.0381; 2007]**, NNLO QCD** [Bolzoni *et al*, 1109.3717; 2012]

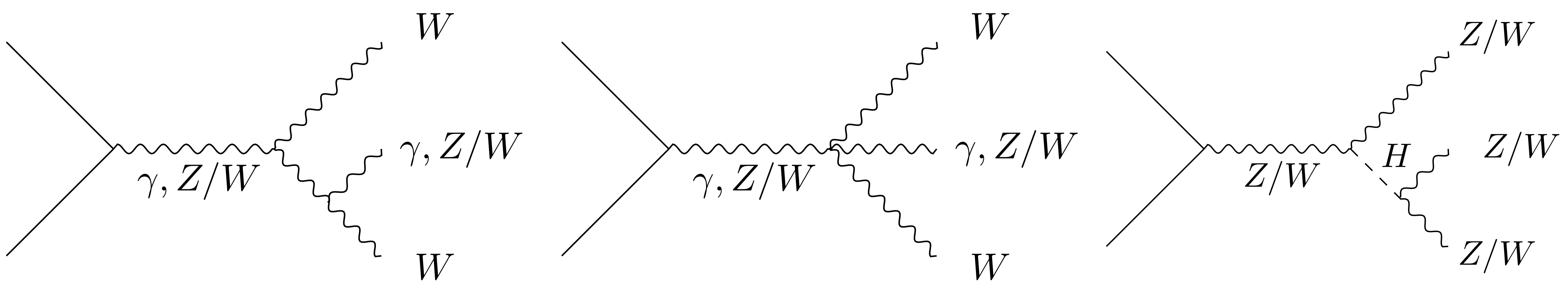
EW standard candles at LHC

process	experiment	submission date	reference
VVV	CMS	Jun. 2020	[1]
WWW	ATLAS	Jan. 2022	[2, 3]
$WW\gamma$	CMS	Oct. 2023	[4]
$WZ\gamma$	ATLAS	May 2023	[5]
$V\gamma\gamma$	CMS	May 2021	[6]
$Z\gamma\gamma$	ATLAS	Nov. 2022	[7]
$W\gamma\gamma$	ATLAS	Aug. 2023	[8]

Table from [Celada *et al*, 2407.09600; 2024].
Corresponding th. status: see refs. in Sec. 4.2.3 from [Lindert & **L.M.**, 2312.07927; 2023]

Also triboson processes grant

- direct access to **EW** quartic **gauge** coupling + interplay with trilinear **gauge** couplings (e.g. in VBF)
- VBF and VBS: window to **SSB** through off-shell **Higgs**



Higher order corrections (i)

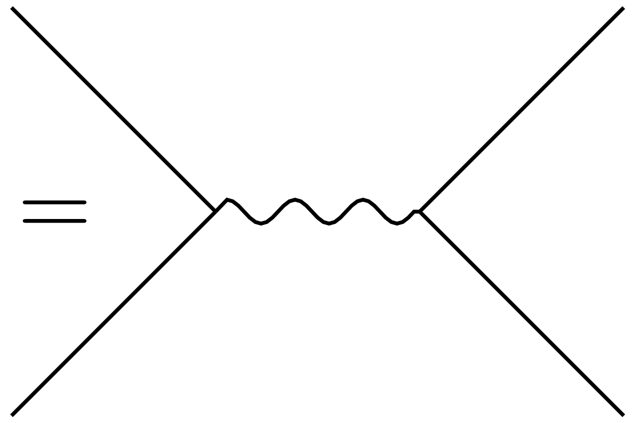
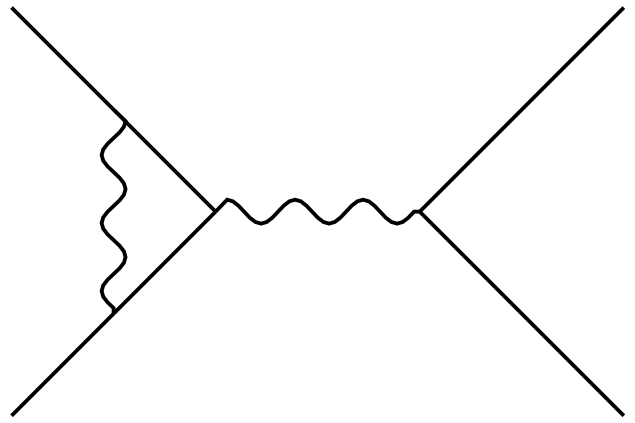
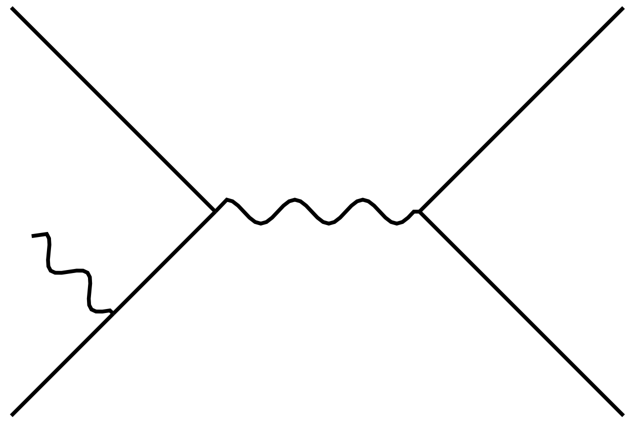
- Higher order corrections: perturbation theory with expansion parameters $\alpha \simeq 0.01$ and $\alpha_{\rm S} \simeq 0.1$:

$$\begin{aligned} d\sigma &= d\sigma_{\rm LO} + d\sigma_{\rm NLO} + d\sigma_{\rm NNLO} \\ &= d\sigma_{\rm LO} + \alpha_{\rm S}\, d\sigma_{\rm NLO\ QCD} + \alpha_{\rm EW} d\sigma_{\rm NLO\ EW} \\ &\quad + \alpha_{\rm S}^2\, d\sigma_{\rm NNLO\ QCD} + \alpha_{\rm EW}^2\, d\sigma_{\rm NNLO\ EW} + \alpha_S \alpha_{\rm EW} d\sigma_{\rm NNLO\ QCDxEW} + \ldots \end{aligned}$$

- At NLO:

$$\sigma|_{\rm NLO} = \frac{1}{2s}\int d\Phi_n \left[\underbrace{|\mathcal{M}_{\rm LO}|^2}_{B} + \underbrace{2\,{\rm Re}\left\{\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO,V}\right\}}_{V} \right] + \frac{1}{2s}\int d\Phi_{n+1} \underbrace{|\mathcal{M}_{\rm NLO,R}|^2}_{R}$$

where, e.g.

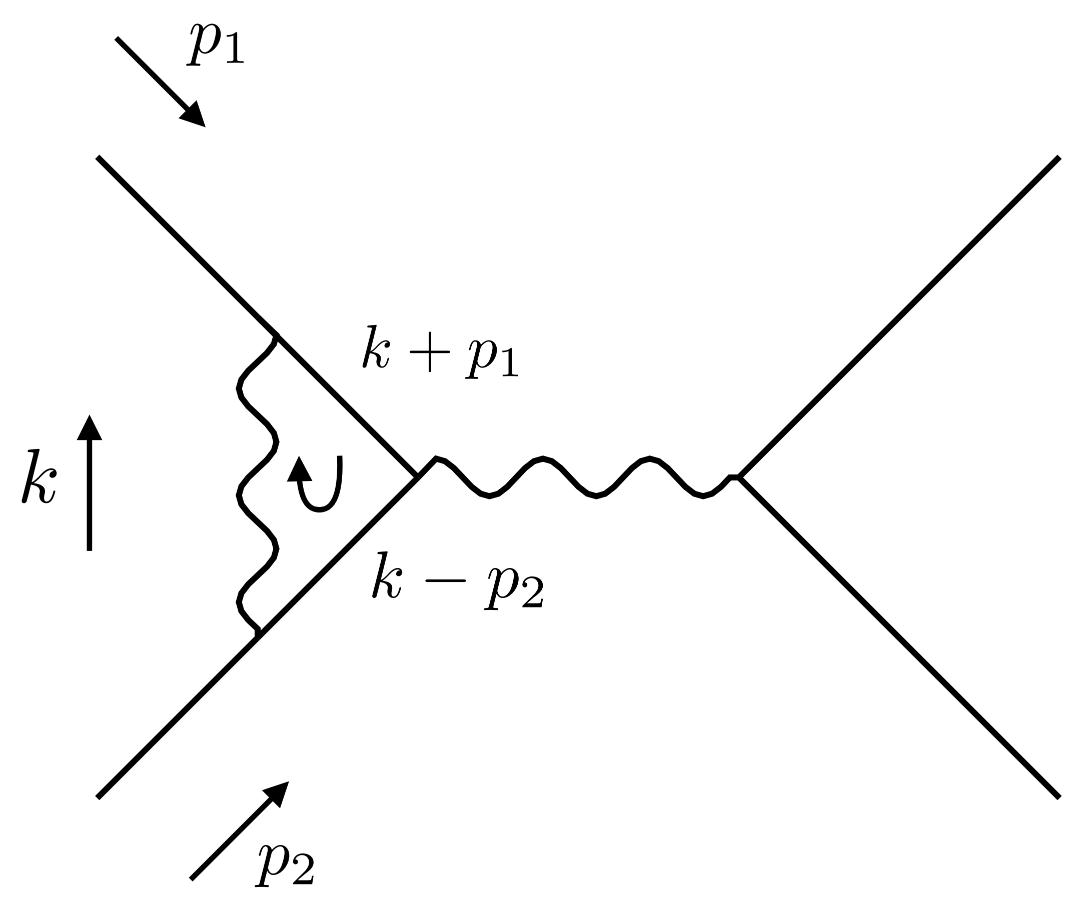
$\mathcal{M}_{\rm LO} =$  $\mathcal{M}_{\rm NLO,V} =$  $\mathcal{M}_{\rm NLO,R} =$ 

Higher order corrections (ii)

- **Virtual** (a.k.a. **loop**) **corrections**: emission and reabsorption of virtual particle ⇒ same initial and final states

- Integration over the momentum of the unobserved particle ⇒ in general **UV** and **IR** divergent

- E.g.: triangle diagram

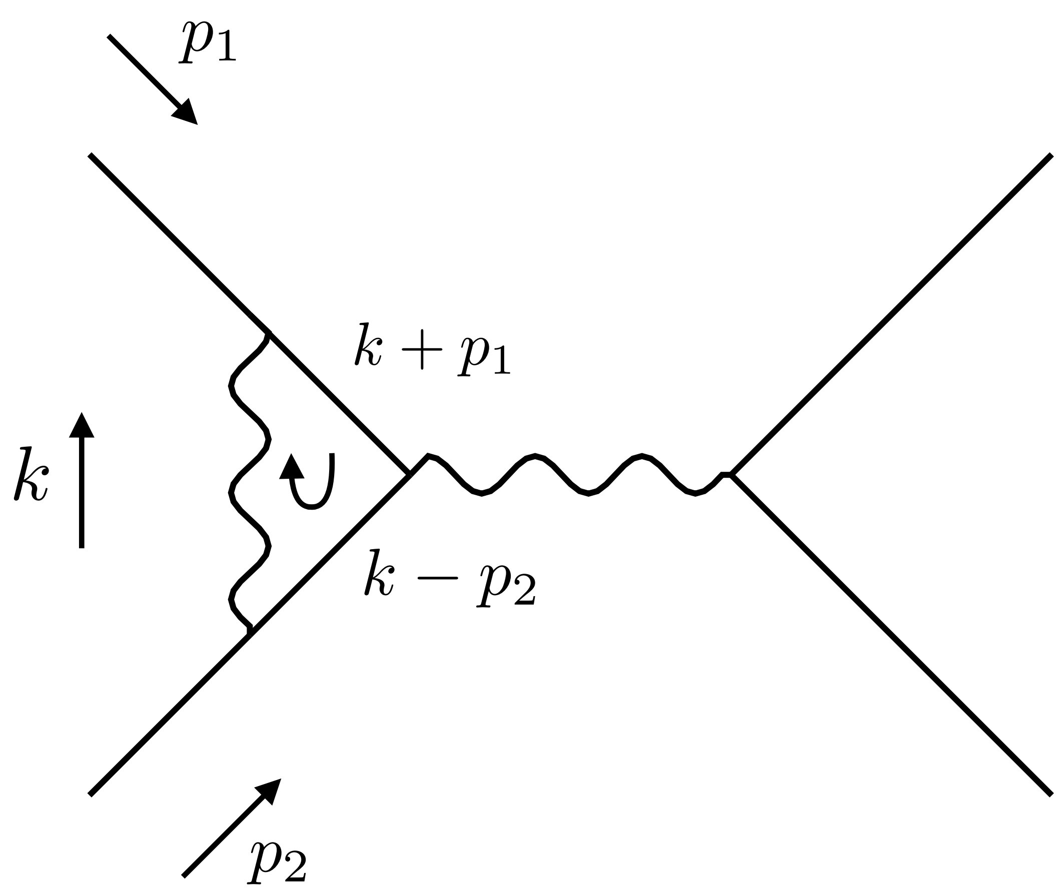


has the following **loop**-integral

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \gamma^\alpha \frac{i(\not{k} - \not{p}_2 + m)}{(k-p_2)^2 - m^2 - i0} \gamma^\mu \frac{i(\not{k} + \not{p}_1 + m)}{(k+p_1)^2 - m^2 - i0} \gamma_\alpha \frac{-i}{k^2 - i0}$$

Higher order corrections (iii)

- E.g.: triangle diagram



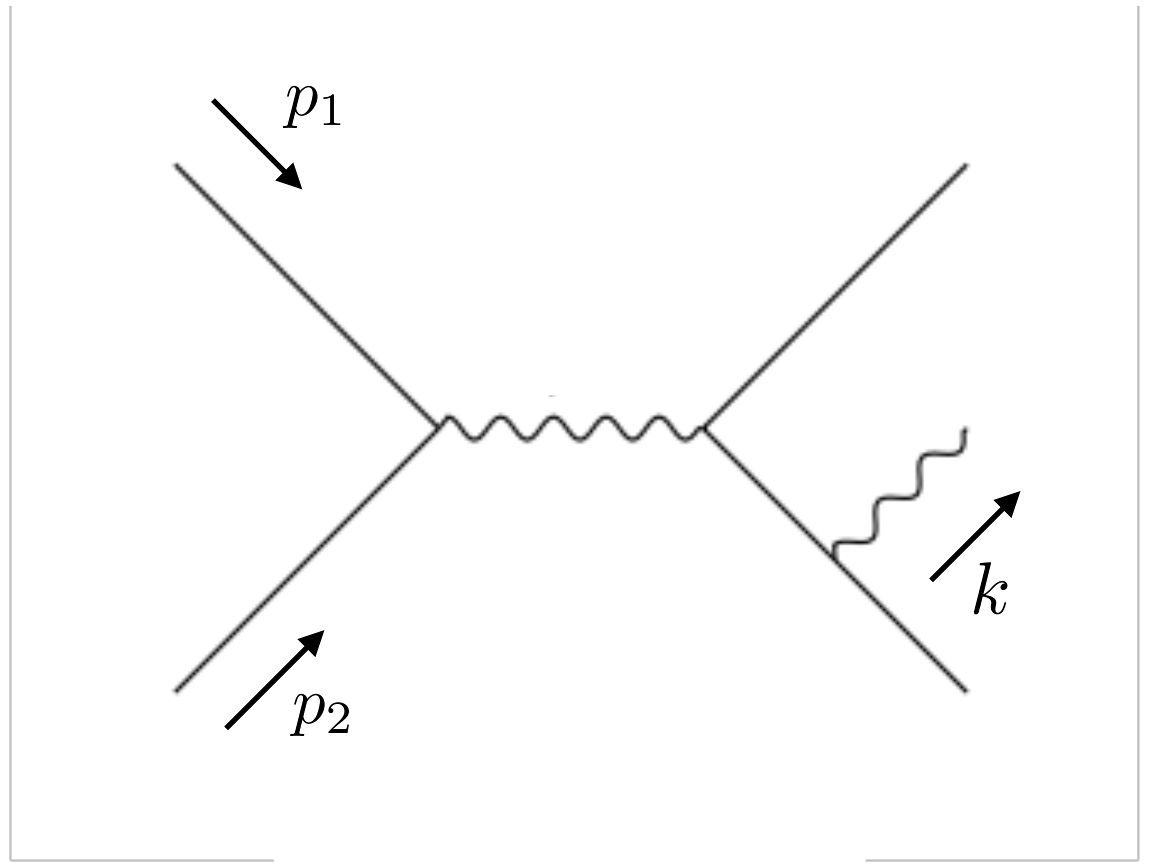
has the following **loop**-integral

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \gamma^\alpha \frac{i(\not{k} - \not{p}_2 + m)}{(k - p_2)^2 - m^2 - i0} \gamma^\mu \frac{i(\not{k} + \not{p}_1 + m)}{(k + p_1)^2 - m^2 - i0} \gamma_\alpha \frac{-i}{k^2 - i0}$$

- **UV** divergent: for $k \to \infty$ it behaves like $\log k$
- **IR** divergent: $\begin{cases} k \to 0 & \Rightarrow \text{soft divergence} \\ k \propto p_{1/2} \quad \text{if} \quad m = 0 & \Rightarrow \text{collinear divergence} \end{cases}$

Higher order corrections (iv)

- **IR** singularities appear also in **real corrections**



- In the soft limit $k \to 0$ the amplitude factories w.r.t. the LO one as

QED *Eikonal factor*

$$\mathcal{M}_{\mathrm{NLO,R}} = ie\,\underbrace{Q\left(\frac{p\cdot\epsilon^*}{p\cdot k}\right)}_{\mathcal{E}^{\mathrm{QED}}(p)}\mathcal{M}_{\mathrm{LO}}$$

and exposes the **IR** divergence

Higher order corrections (v)

- However, the **KLN theorem** assures that for any ***physical observable*** the **IR** singularities appearing in **virtual** and **real** corrections cancel against each other

- Note that this does not take place at the amplitude level but only after integration of the respective (and different) phase spaces

$$\sigma|_{\mathrm{NLO}} = \frac{1}{2s}\int \boxed{d\Phi_n} \left[\underbrace{|\mathcal{M}_{\mathrm{LO}}|^2}_{B} + \underbrace{2\,\mathrm{Re}\left\{\mathcal{M}_{\mathrm{LO}}\mathcal{M}^*_{\mathrm{NLO,V}}\right\}}_{V} \right] + \frac{1}{2s}\int \boxed{d\Phi_{n+1}} \underbrace{|\mathcal{M}_{\mathrm{NLO,R}}|^2}_{R}$$

NB: this is a general sketch, technicalities related to hadronic initial states are not discussed here

Higher order corrections (vi)

- **UV** singularities are removed through renormalisation

- This is a two-step procedure:
  - Regularisation: **UV** divergences are identified and expressed in terms of a *regulator*

    The *regulator* is an *ad-hoc* parameter, observables must not depend on it

  - Renormalisation: **UV** divergences are removed by a subtraction scheme, which redefines the parameters of the theory

- Typically

  Dimensional regularisation

  On-shell / MS / $\overline{\text{MS}}$ scheme

On-shell renormalisation (ii)

- Example: **OS** renormalisation for transverse W boson
  - Relations between bare and renormalised quantities through *counterterms* (CT)

$$m_{W,0}^2 = m_W^2 + \delta m_W^2 \qquad\qquad W_0^{\pm} = \left(1 + \frac{1}{2}\delta Z_W\right) W^{\pm}$$

from which it follows the relation

$$\hat{\Sigma}_T^W(k^2) = \Sigma_T^W(k^2) - \delta m_W^2 + (k^2 - m_W^2)\delta Z_W$$

  - *Counterterms* are fixed by imposing renormalisation conditions

On-shell renormalisation (ii)

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  - *Counterterms* are fixed by imposing renormalisation conditions

▲ Renormalised mass = physical mass, i.e. pole of the corresponding propagator/ zero of 2-point function projected into physical states (polarization vectors/spinors)

$$\mathrm{Re}\, \hat{G}_{\mu\nu}^W(k)\epsilon^\nu(k)\Big|_{k^2=m_W^2} = 0 \quad \Rightarrow \quad \mathrm{Re}\, \hat{\Sigma}_T^W(m_W^2) = 0 \quad \Rightarrow \quad \delta m_W^2 = \Sigma_T^W(m_W^2)$$

$$\hat{G}_{\mu\nu}^W(k) = -ig_{\mu\nu}(k^2 - m_W^2) - i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\hat{\Sigma}_T^W(k^2)$$

On-shell renormalisation (iii)

- Example: **OS** renormalisation for transverse W boson

  - Relations between bare and renormalised quantities through *counterterms* (CT)

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  - *Counterterms* are fixed by imposing renormalisation conditions

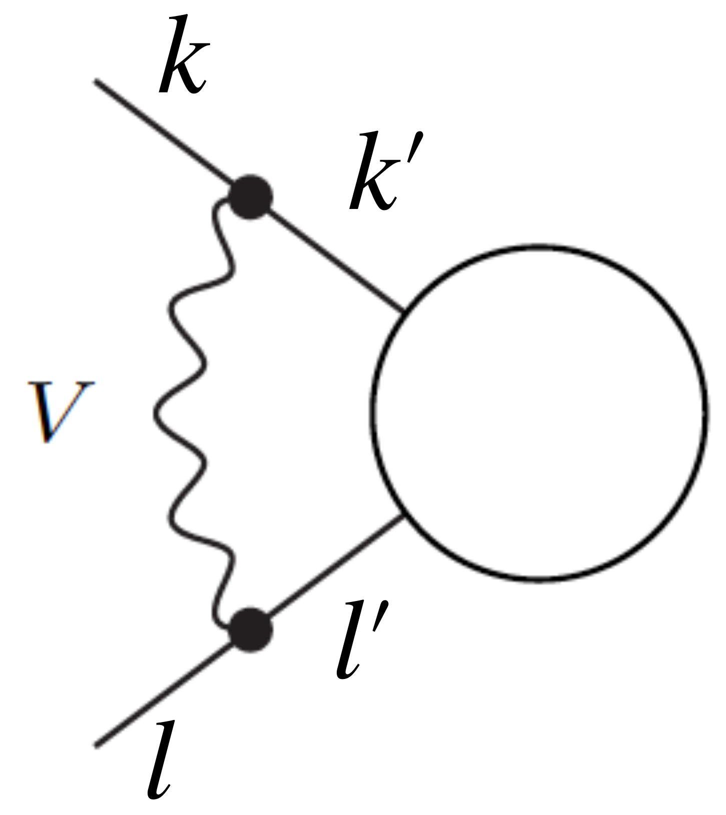
▲ Field *counterterms* (FRCs): close to its pole each propagator corresponds to its LO expression with $m_0 \to m$
With this condition **OS** particles do not mix and propagators have residue one:

$$\lim_{k^2 \to m_W^2} \frac{1}{k^2 - m_W^2} \mathrm{Re}\, \hat{G}_{\mu\nu}^W(k)\epsilon^\nu(k)\Big|_{k^2=m_W^2} = -i\epsilon_\mu(k) \quad \Rightarrow \quad \mathrm{Re}\, \frac{\partial \hat{\Sigma}_T^W(k^2)}{\partial k^2}\Big|_{k^2=m_W^2} = 0 \quad \Rightarrow \quad \delta Z_W = -\frac{\partial \Sigma_T^W(k^2)}{\partial k^2}\Big|_{k^2=m_W^2}$$

$$\hat{G}_{\mu\nu}^W(k) = -ig_{\mu\nu}(k^2 - m_W^2) - i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\hat{\Sigma}_T^W(k^2)$$

Double Logs

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V



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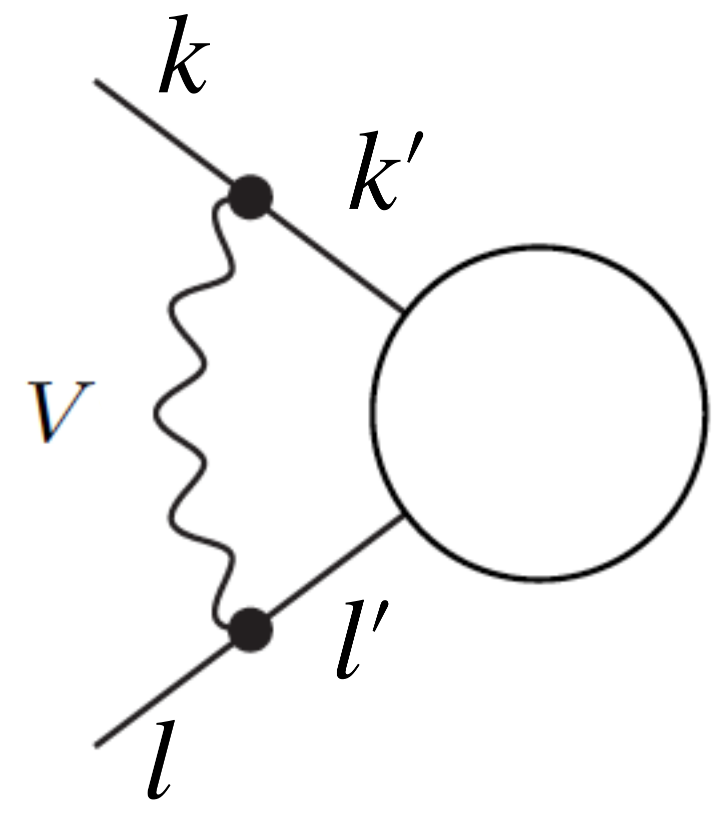


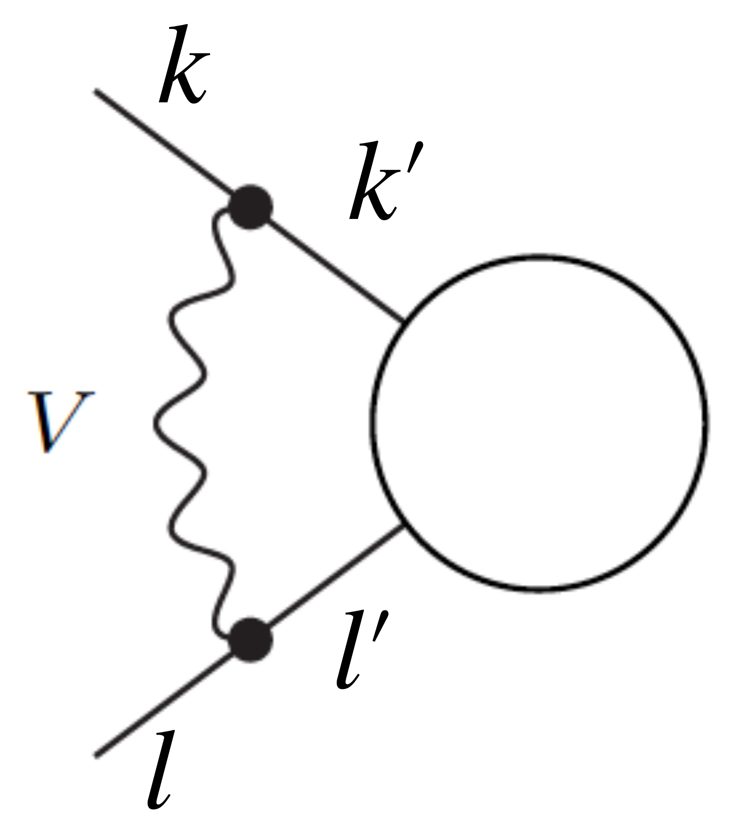
- In the *Eikonal approximation*[1], the loop integral reduces to the scalar three-point function C_0, which ***factorises***

$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_k \sum_{l<k} \sum_V \sum_{k',l'} \frac{\alpha}{4\pi} I^V_{kk'} I^{\bar{V}}_{ll'} \underbrace{\left[\log^2 \frac{|r_{kl}|}{m_V^2} - 2i\pi\Theta(r_{kl}) \log \frac{|r_{kl}|}{m_V^2}\right]}_{\propto\, C_0|_{\mathrm{LA}}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

with $r_{kl} = (p_k + p_l)^2$

[1]NB: external longitudinal gauge bosons require GBET

Double Logs



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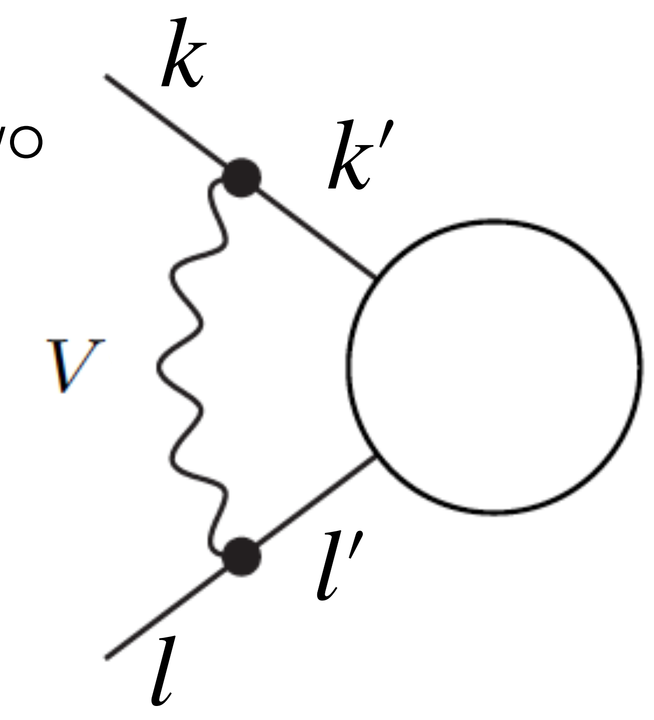
with $r_{kl} = (p_k + p_l)^2$

- Consequence of C_0 ***factorisation***: DL are ***universal***, i.e. process independent

[1]NB: external longitudinal gauge bosons require GBET

Double Logs: LSC, SSC, S-SSC

DL originate when two external legs exchange a ***soft and collinear* (SC)** gauge boson V



- DL can be split into

→ **Leading *Soft-Collinear* (LSC)**: angular independent, single sum over external legs

$$\delta^{\mathrm{LSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{k'} \sum_V \delta^{\mathrm{LSC,V}}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\mathrm{LSC,\ V}}_{kk'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{s}{m_V^2}\right)}$$

→ **Subleading *Soft-Collinear* (SSC)** and **Sub-SSC (SSSC)**: angular dependent, double sum over external legs

$$\delta^{\mathrm{(S-)SSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{l<k} \sum_{k',l'} \sum_V \delta^{\mathrm{(S-)SSC,V}}_{kk'll'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

$$\delta^{\mathrm{SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log\left(\frac{s}{m_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)} \qquad \delta^{\text{S-SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{|r_{kl}|}{s}\right)}$$

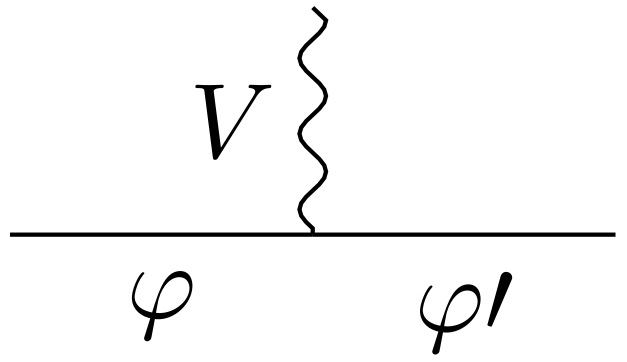
Formally not part of LA and omitted in original DP, but needed for reliable estimates as firstly pointed out in [Pagani, Zaro 2110.03714; *2021*]

$\boxed{\text{LA:} \quad s \sim r_{kl} \equiv (p_k + p_l)^2 \gg m^2_{\mathrm{Z,W}} \qquad \forall k, l}$

[Denner and Pozzorini 0010201; *2001*]

DL sketch

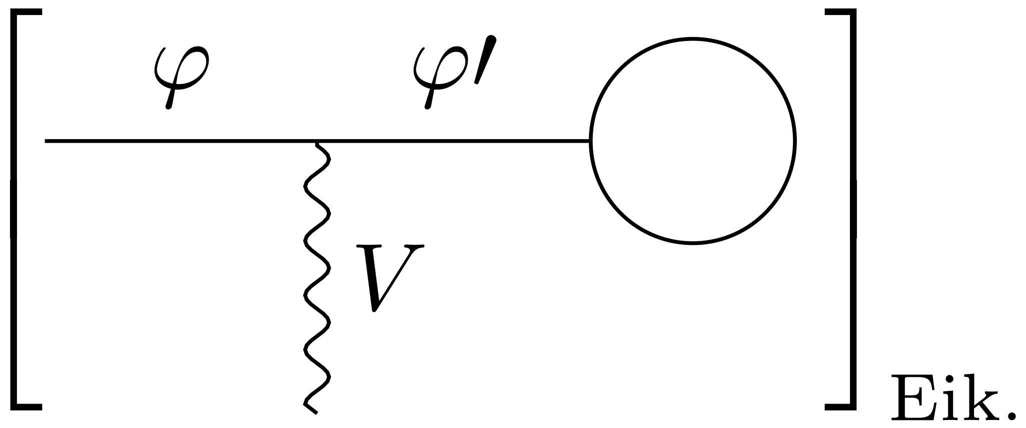
- General form of **EW** Feynman rules as in [Denner 0709.1075; *2007*]



$$= ieI^V_{\varphi\varphi\prime}T$$

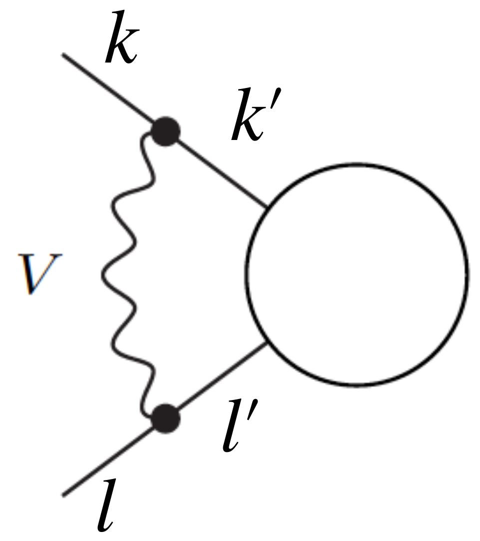
where T contains the Lorentz structure of the vertex (e.g., in $\gamma\bar{f}f$: $I^V_{\varphi\varphi\prime} = -Q_f, \quad T = \gamma^\mu$)

- When contracted with external w.f.s. in computing one-loop amplitudes, it follows



$$\propto 2ieI^V_{\varphi\varphi\prime}p^\mu_\varphi \mathcal{M}_0^{\dots\varphi'\dots}$$

so that



$$\sim \sum_V \sum_{k',l'} \frac{\alpha}{4\pi} I^V_{kk'} I^{\bar{V}}_{ll'}\, r_{\varphi\varphi'} C_0|_{\text{Eik.}} \mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i'_l}\dots\varphi_{i_n}}, \qquad C_0|_{\text{Eik.}} \propto \frac{1}{r_{\varphi\varphi'}}\left[\log^2\frac{r_{\varphi\varphi'}}{m_V^2}\right]$$

Single Logs

- SL have a triple origin

Single Logs: PR

- SL have a triple origin

→ **PR**: UV renormalisation of **EW** dimensionless parameters

$$\boxed{g_{i,0} = g_i + \delta g_i = \left(1 + \delta Z_{g_i}\right) g_i}$$

$$\mu_{i,0}^2 = \mu_i^2 + \delta\mu_i^2$$

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right)\varphi_j$$

$$\boxed{g_{i,0} = g_i + \delta g_i = \left(1 + \delta Z_{g_i}\right) g_i}$$

Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

Single Logs: PR & WFR

- SL have a triple origin

→ **PR**: UV renormalisation of **EW** dimensionless parameters

$$\boxed{g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i}$$

→ **WF**: wave-function renormalisation of external fields

$$\boxed{\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right) \varphi_j}$$

yields to the ***factorised*** correction

$$\delta^{\mathrm{WF}} \mathcal{M}^{\varphi_{i_1} \dots \varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_k \sum_{k'} \delta^{\mathrm{WF}}_{kk'} \mathcal{M}_0^{\varphi_{i_1} \dots \varphi_{i'_k} \dots \varphi_{i_n}}, \qquad \delta^{\mathrm{WF}}_{kk'} = \frac{1}{2}\delta Z_{kk'}$$

$$\mu^2_{i,0} = \mu^2_i + \delta\mu^2_i$$

$$\boxed{\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right) \varphi_j}$$

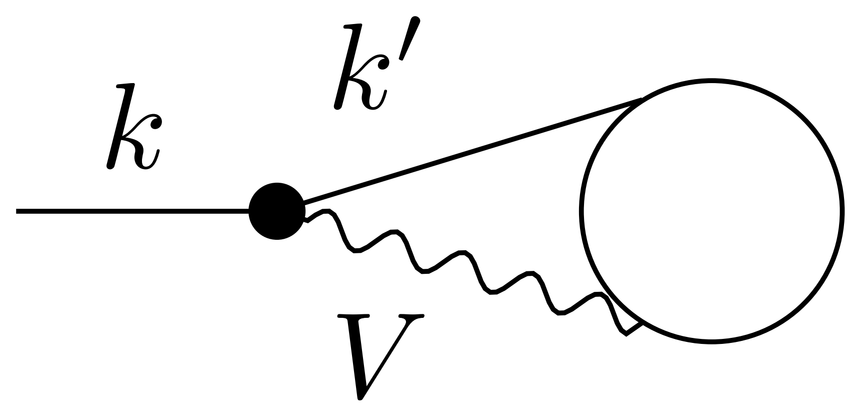
$$\boxed{g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i}$$

Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

Single Logs: Coll

- SL have a triple origin

→ **Coll**: external leg emission of a collinear gauge boson



Single Logs: Coll

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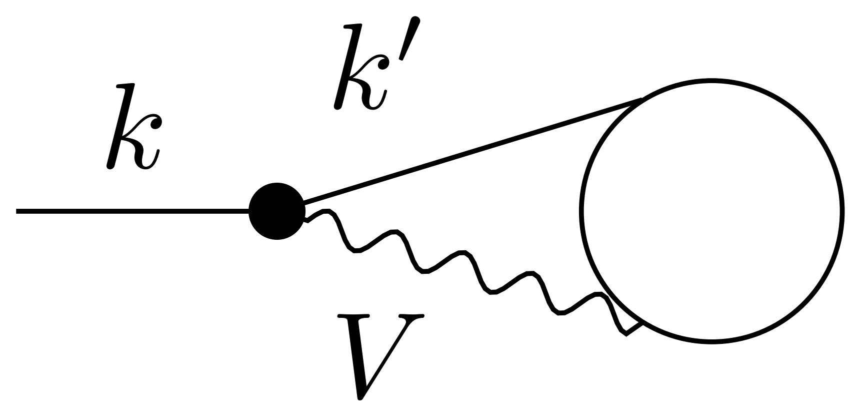

Its evaluation in *Collinear approximation* leads to the ***factorised*** contribution

$$\delta^{\rm coll}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \stackrel{\rm LA}{=} \sum_k \sum_{k'} \delta^{\rm coll}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}\,, \qquad \delta^{\rm coll}_{kk'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar V}\log\left(\frac{s}{m_V^2}\right)$$

Single Logs: Coll & C

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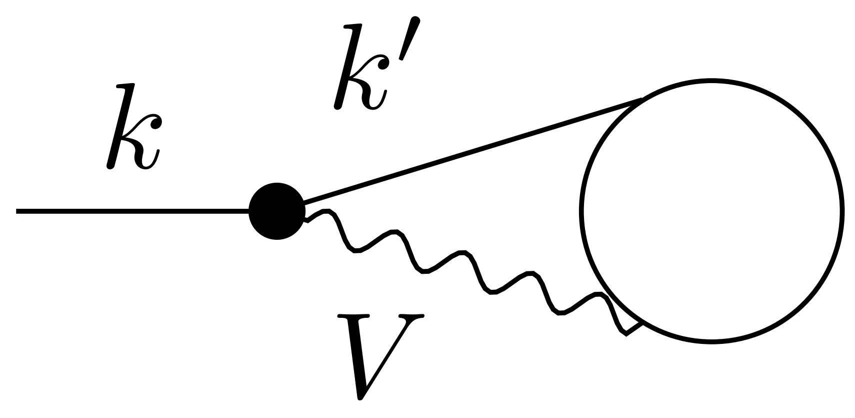


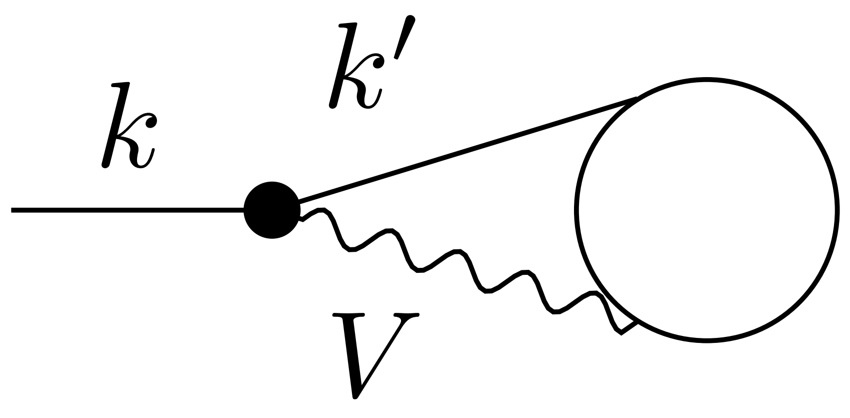
Its evaluation in *Collinear approximation* leads to the ***factorised*** contribution

$$\delta^{\mathrm{coll}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k}\sum_{k'}\delta^{\mathrm{coll}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\mathrm{coll}}_{kk'} \sim \frac{\alpha}{4\pi}\sum_{V} I^{V} I^{\bar{V}}\log\left(\frac{s}{m_V^2}\right)$$

→ **C**: Full gauge-invariant SL correction associated to external fields:

$$\delta^{\mathrm{C}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k}\sum_{k'}\delta^{\mathrm{C}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\mathrm{C}}_{kk'} = \left(\delta^{\mathrm{coll}}_{kk'} + \delta^{\mathrm{WF}}_{kk'}\right)\Big|_{\mu^2=s}$$

Coll sketch



- Original integral

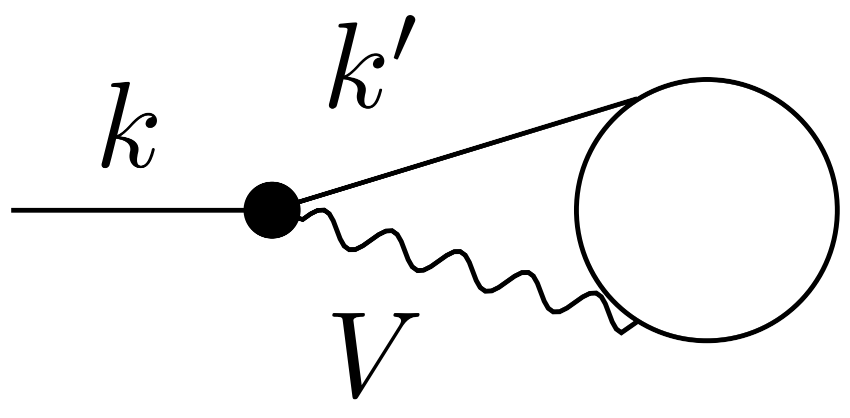
$$\sim \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{N(q)}{(q^2 - m_V^2 + i0)[(p_k - q)^2 - m_k^2 + i0]}$$

- In Sudakov parametrisation $q^\mu = x p_k^\mu + y l^\mu + q_\mathrm{T}^\mu$, after y integration the integral reduces to

$$\sim \mu^{4-D} \int_0^1 dx \int \frac{d^{D-2} q_\mathrm{T}}{(2\pi)^{D-2}} \frac{N(x, y_i, q_\mathrm{T})}{|\vec{q}_\mathrm{T}|^2 + \Delta(x)}$$

with $\Delta(x) = (1-x) m_V^2 + x m_{k'}^2 - x(1-x) p_k^2$ regulating the logarithmic singularity

Coll sketch



- In Sudakov parametrisation

$$\sim \mu^{4-D} \int_0^1 dx \int \frac{d^{D-2} q_{\mathrm{T}}}{(2\pi)^{D-2}} \frac{N(x, y_i, q_{\mathrm{T}})}{|\vec{q}_{\mathrm{T}}|^2 + \Delta(x)}$$

with $\Delta(x) = (1-x)m_V^2 + x m_{k'}^2 - x(1-x)p_k^2$ regulating the logarithmic singularity

- Since we restrict to logarithmic mass-singular contributions, all terms of order

$$|\vec{q}_{\mathrm{T}}|^2,\ p_k^2,\ m_V,\ m_k,\ y_i\,(\propto |\vec{q}_{\mathrm{T}}|^2/p_k l)$$

can be neglected in $N(q)$

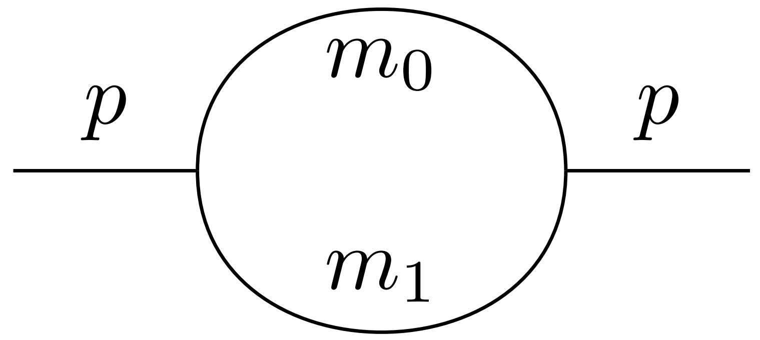
- *Collinear approximation*

    ▲ Substitute $N(x,y,q_{\mathrm{T}}) \to N(x,0,0)$, i.e. $q^\mu \to x p_k^\mu$

    ▲ Neglect all mass terms in $N(x,0,0)$

Single Logs: PR

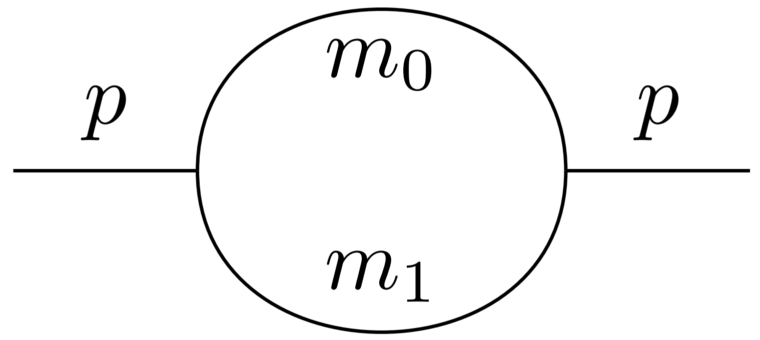
- Generic two-point function



$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}\left(p, m_0, m_1\right) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{\left(q^2 - m_0^2 + \mathrm{i}\varepsilon\right)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

Single Logs: PR

- Generic two-point function

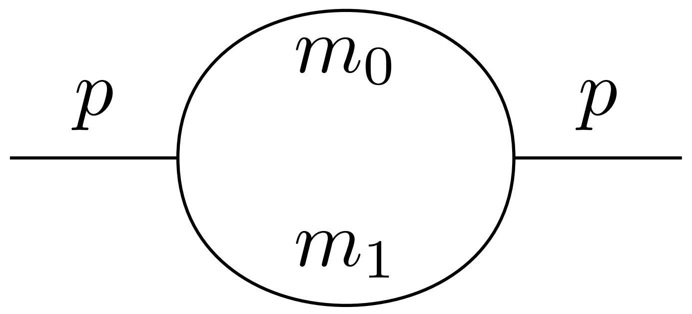


$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}(p, m_0, m_1) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{(q^2 - m_0^2 + \mathrm{i}\varepsilon)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

- In LA $\mu^2 = s \gg p^2, m_0^2, m_1^2 \Rightarrow$ four possible hierarchy of masses

$$\begin{aligned}
&(a) \quad m_i^2 \ll p^2 \text{ and } p^2 - m_{1-i}^2 \ll p^2 \quad \text{for } i = 0 \text{ or } \quad i = 1, \\
&(b) \quad \text{not} \quad (a) \text{ and } m_i^2 \not\gg p^2 \text{ for } i = 0, 1, \\
&(c) \quad m_0^2 = m_1^2 \gg p^2 \\
&(d) \quad m_i^2 \gg p^2 \not\gg m_{1-i}^2 \text{ for } i = 0 \text{ or } i = 1
\end{aligned}$$

Single Logs: PR



- Generic two-point function

$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}\left(p, m_0, m_1\right) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{\left(q^2 - m_0^2 + \mathrm{i}\varepsilon\right)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

- Results for two point functions and their derivatives

$$B_0\left(p, m_0, m_1\right) \overset{\mathrm{LA}}{=} \log \frac{\mu^2}{M^2},$$

$$B_1\left(p^2, m_0, m_1\right) \overset{\mathrm{LA}}{=} -\frac{1}{2} \log \frac{\mu^2}{M^2},$$

$$\frac{1}{p^2} B_{00}\left(p^2, m_0, m_1\right) \overset{\mathrm{LA}}{=} \frac{3m_0^2 + 3m_1^2 - p^2}{12p^2} \log \frac{\mu^2}{M^2},$$

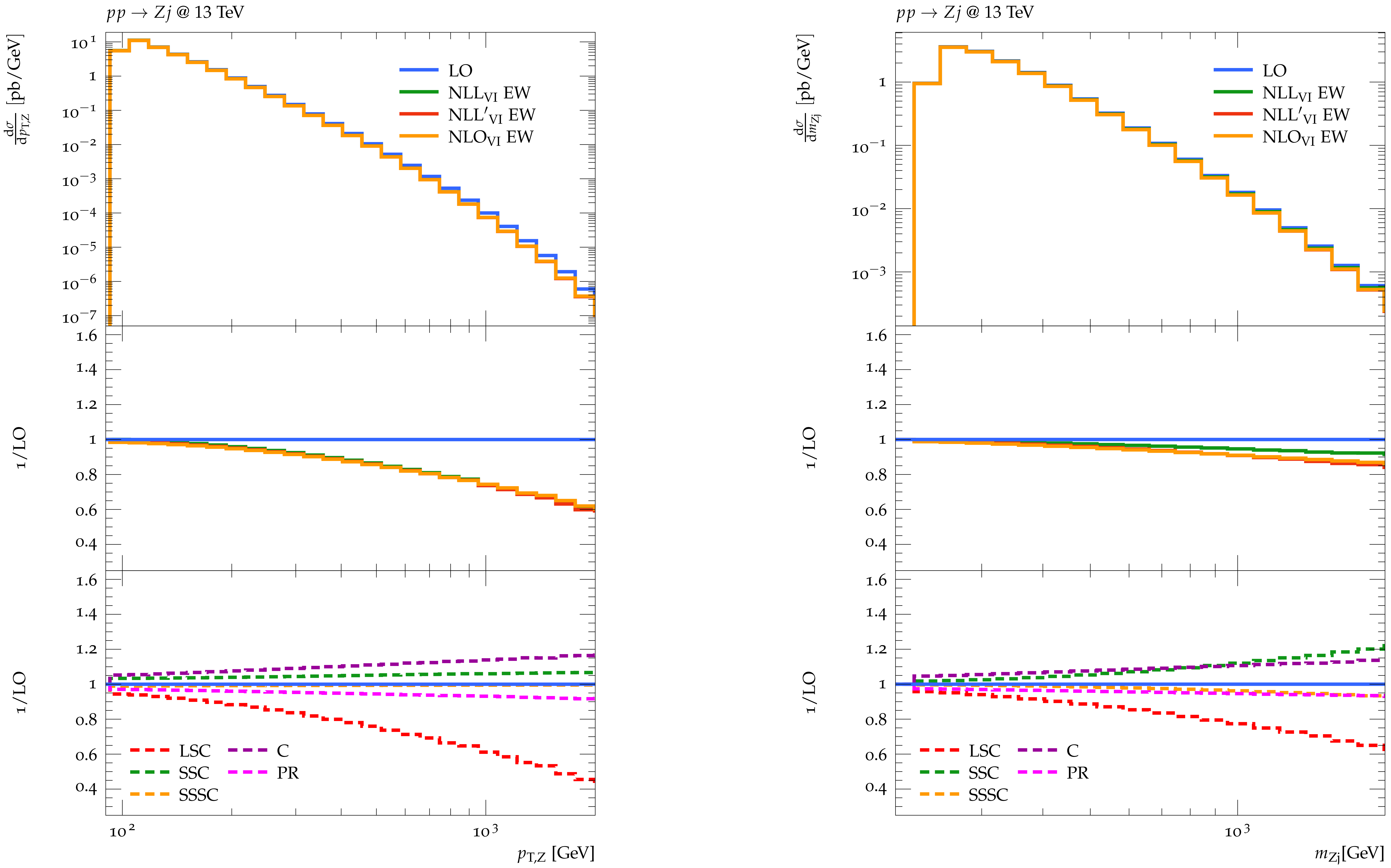
$$\frac{1}{p^2} g^{\mu\nu} B_{\mu\nu}\left(p^2, m_0, m_1\right) \overset{\mathrm{LA}}{=} \frac{m_0^2 + m_1^2}{p^2} \log \frac{\mu^2}{M^2}$$

$$p^2 B_0'\left(p, m_0, m_1\right) \overset{\mathrm{LA}}{=} \frac{1}{2} \log \frac{m_{1-i}^2}{m_i^2} = \frac{1}{2} \log \frac{p^2}{\lambda^2},$$

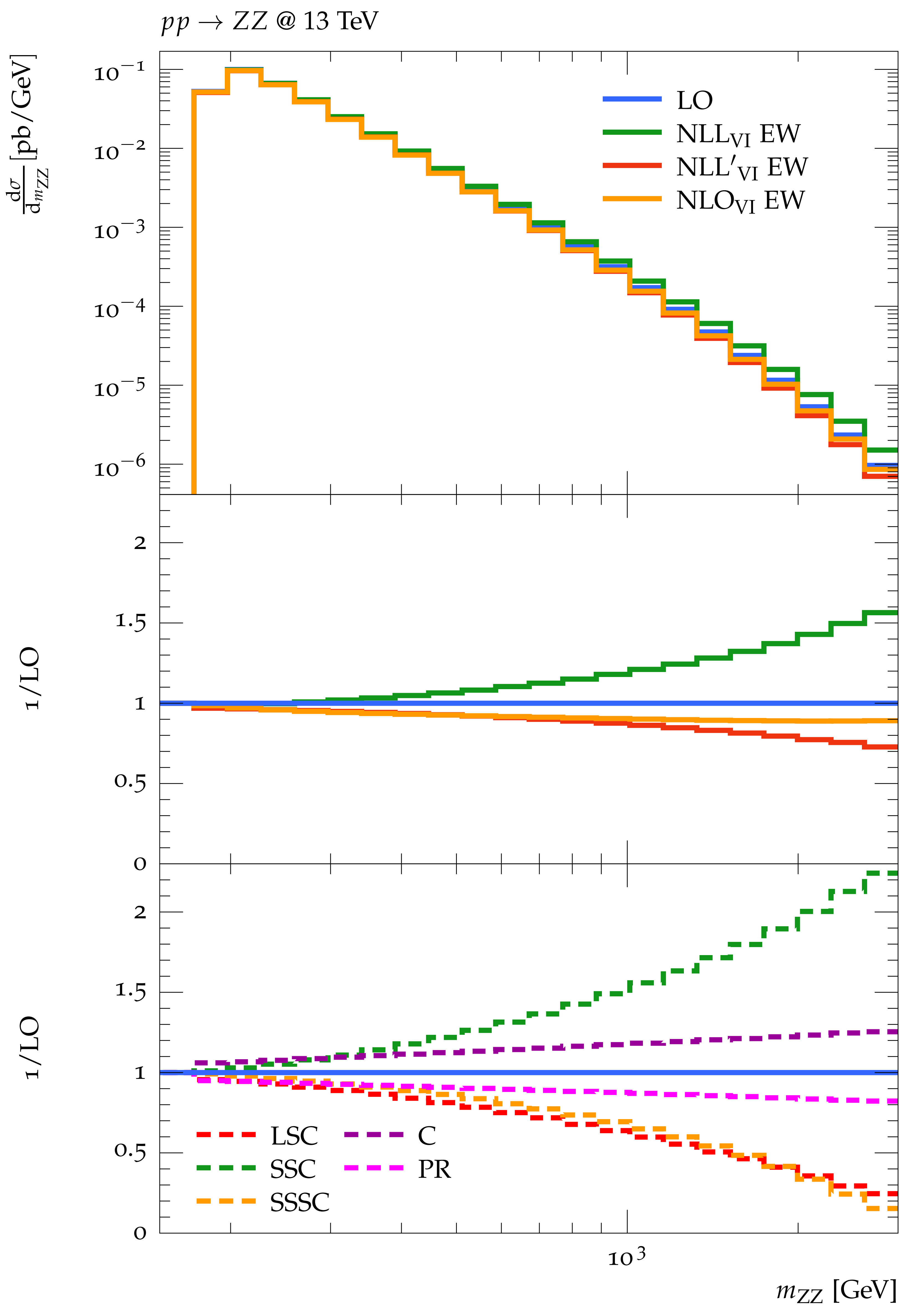
$$p^2 B_1'\left(p^2, m_0, m_1\right) + \frac{1}{2} p^2 B_0'\left(p, m_0, m_1\right) \overset{\mathrm{LA}}{=} -\frac{1}{4} \log \frac{m_0^2}{m_1^2}$$

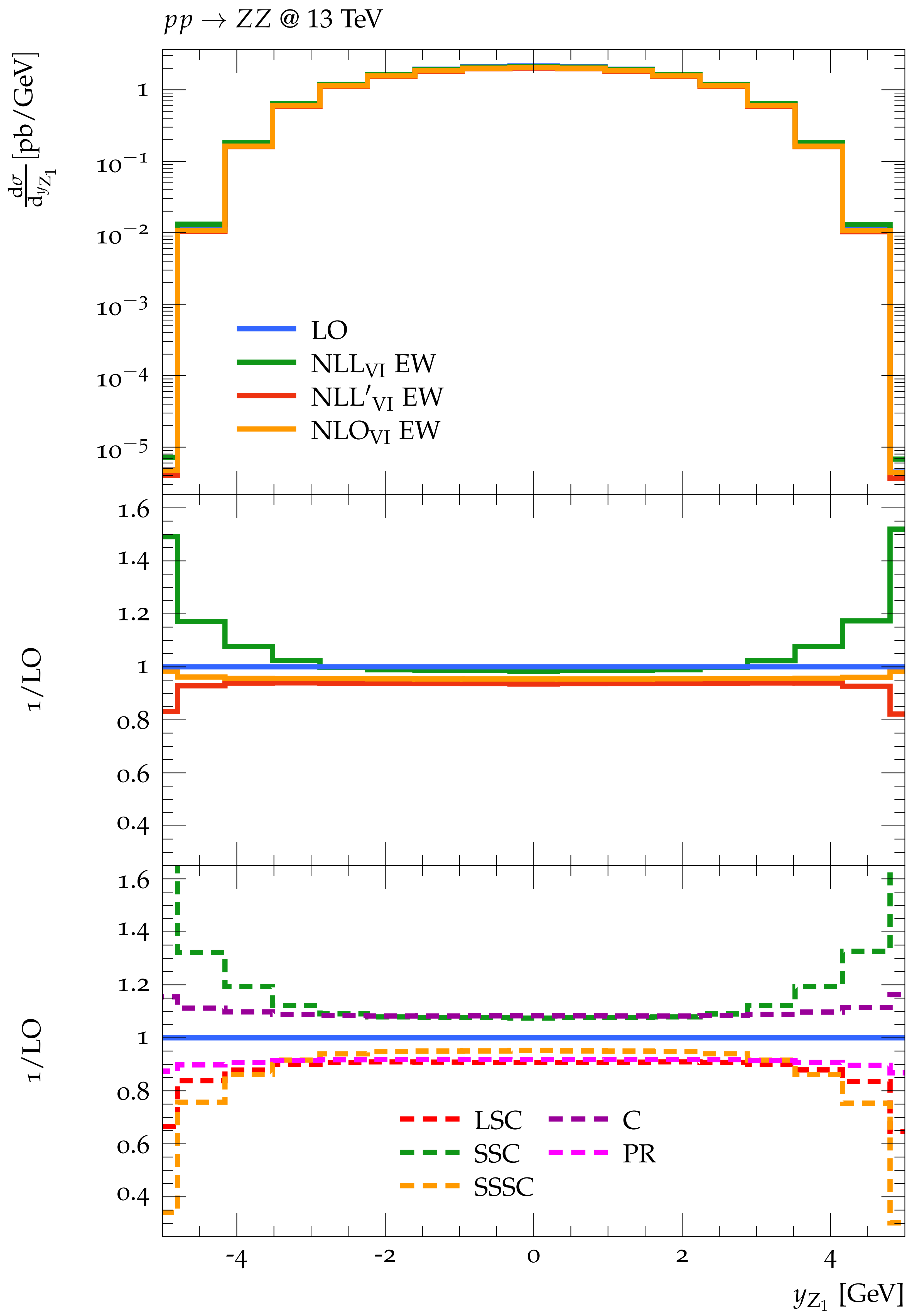
Additional results

Results: $pp \to Zj$



Results: $pp \to ZZ$





Two considerations from the rapidity distribution:

- The inclusion of **SSSC** allows for a better *Sudakov* approximation, in particular for $|y_Z| < 3$
- For very forward configurations, i.e. outside the central region $|y_Z| < 3$, **SSC** and **SSSC** rapidly grow

Results: $pp \to W^+W^-$

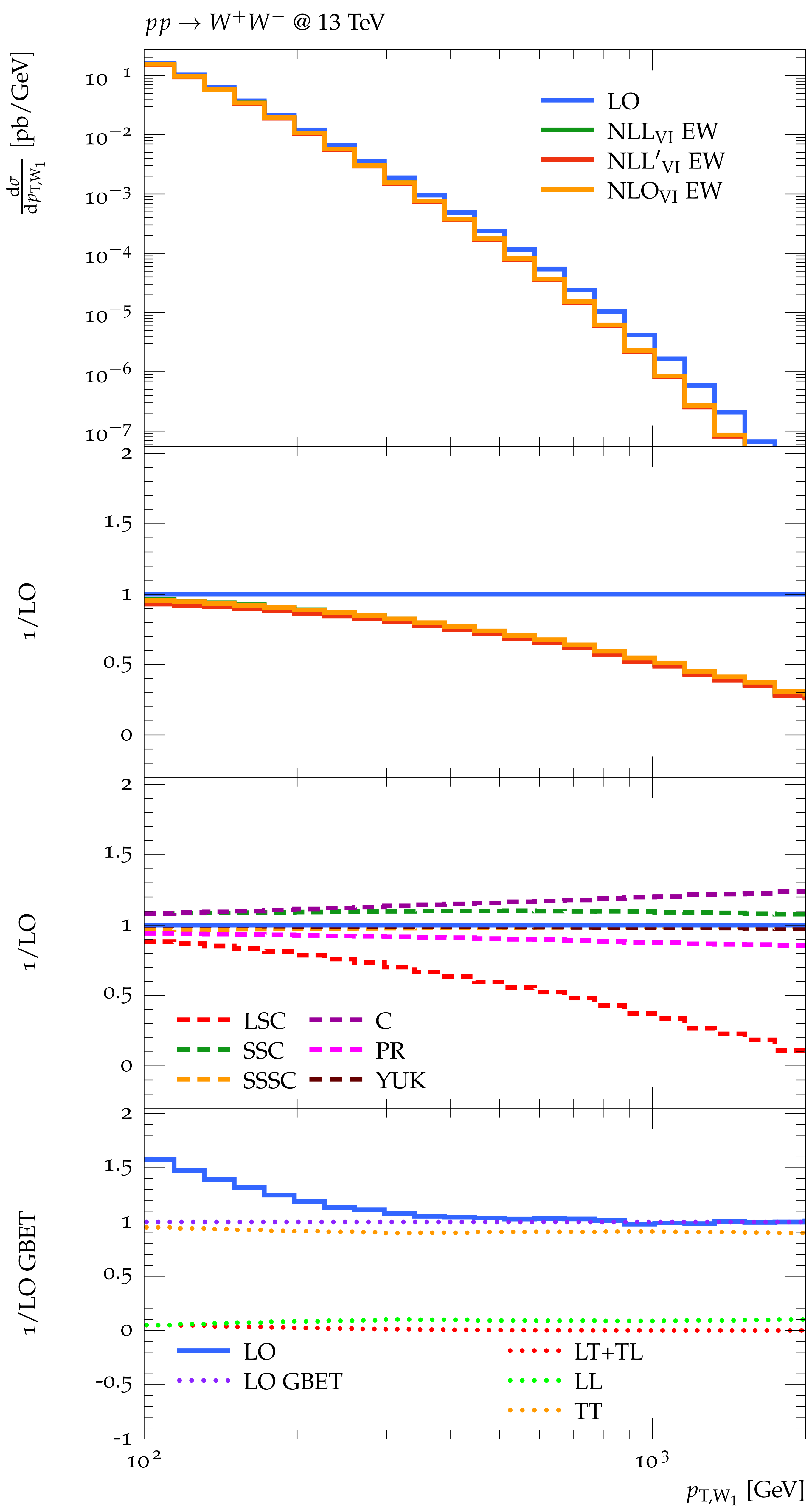
NLL EW: [Accomando et al, 0409247; 2004]

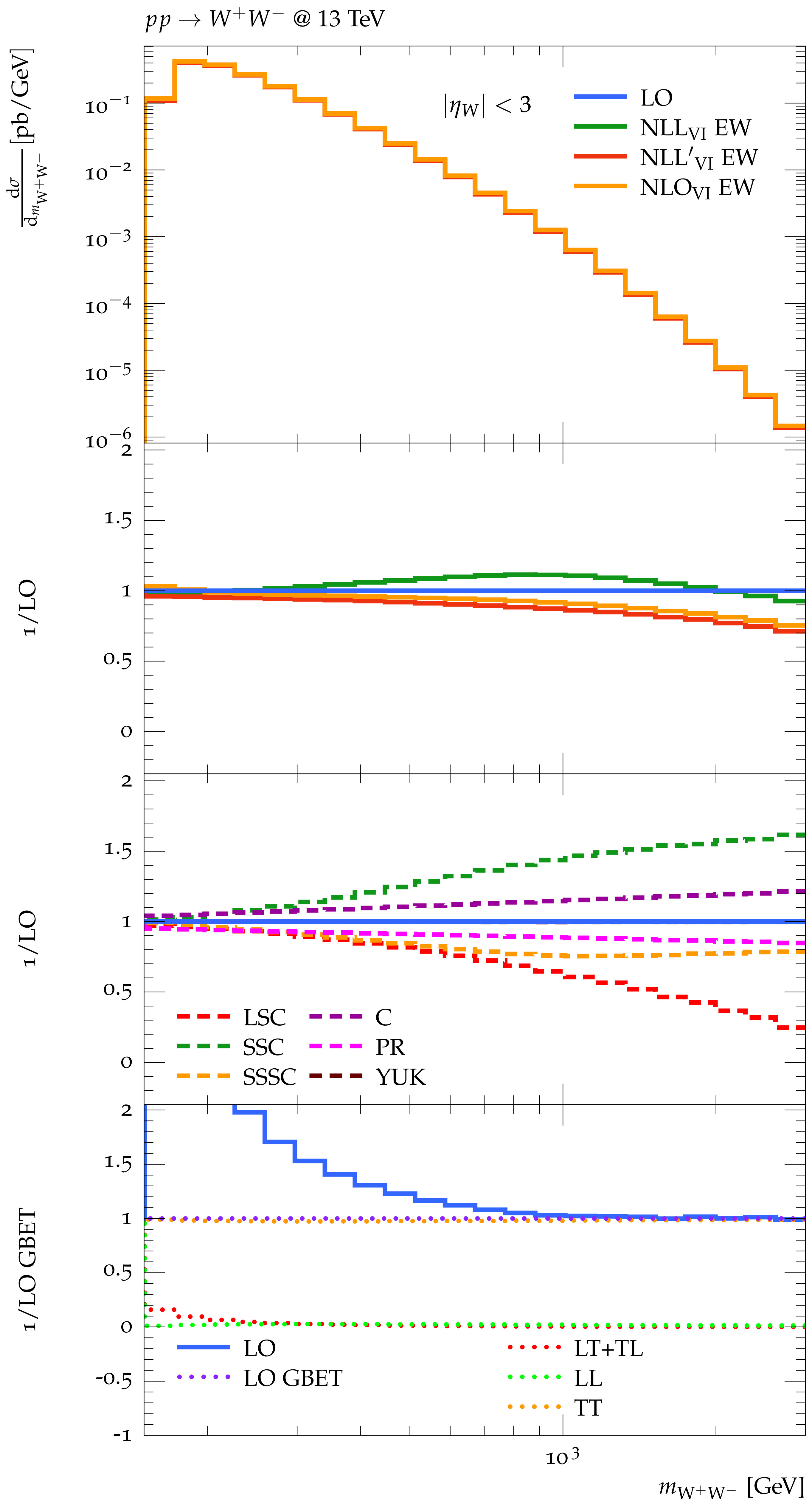
Full **NLO EW**: [Bierweile *et al*, 1208.3147; 2012]

Full **NLO**: [Baglio *et al*, 1307.4331; 2016]

Mixed NLO QCD - EW: [Bräuer *et al*, 2005.12128; 2020]

NNLO QCD+NLO EW: [Grazzini *et al*, 1912.00068; 2020]





Here **LT** and **TL** polarisation configurations are mass-suppressed while mixed **TT** and **LL** are not.

However, **LT** and **TL** are several orders of magnitude smaller than both **TT** and **LL**.

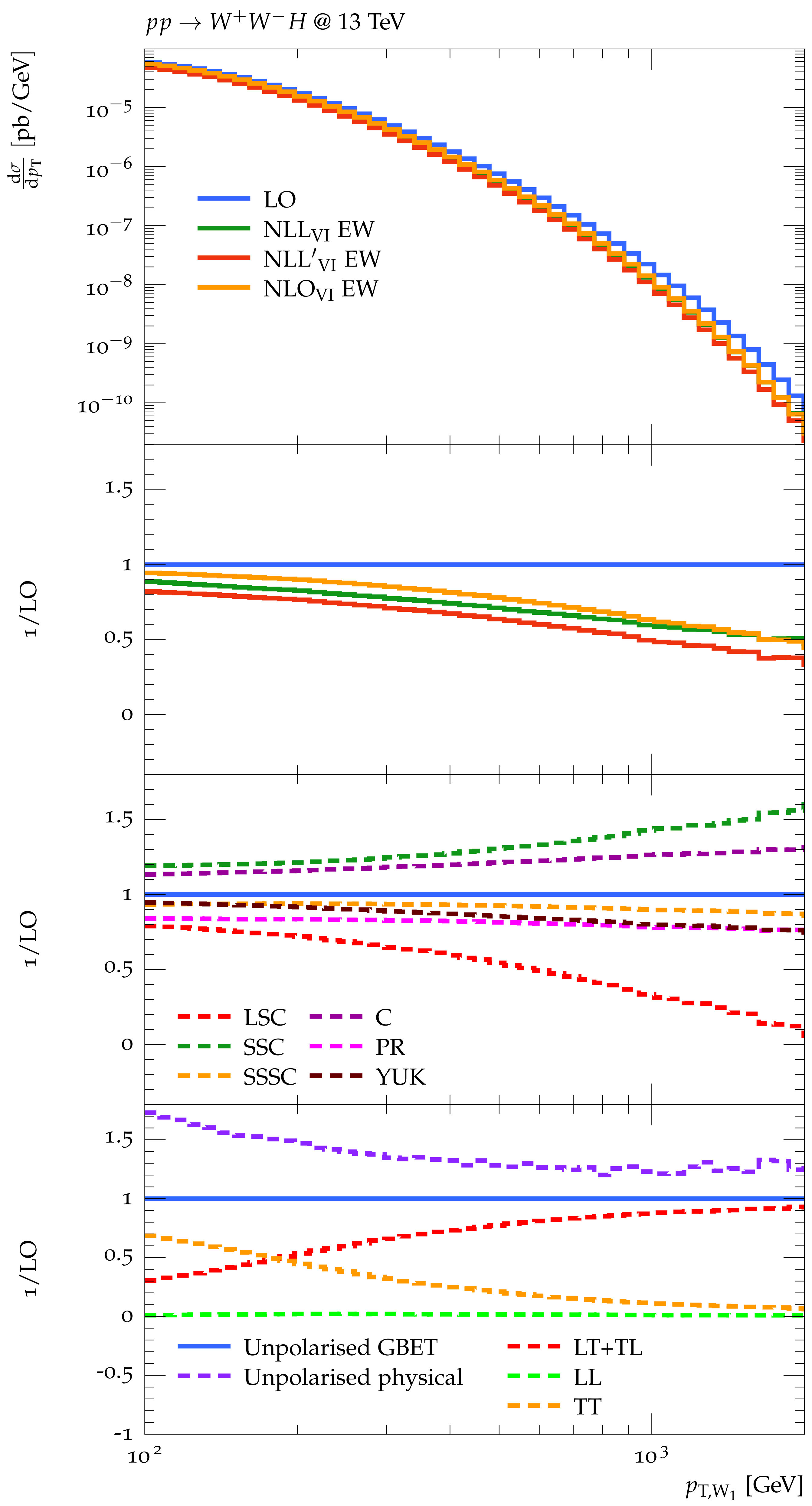
Within this setup, *Sudakov* approximation can be directly employed for these observables

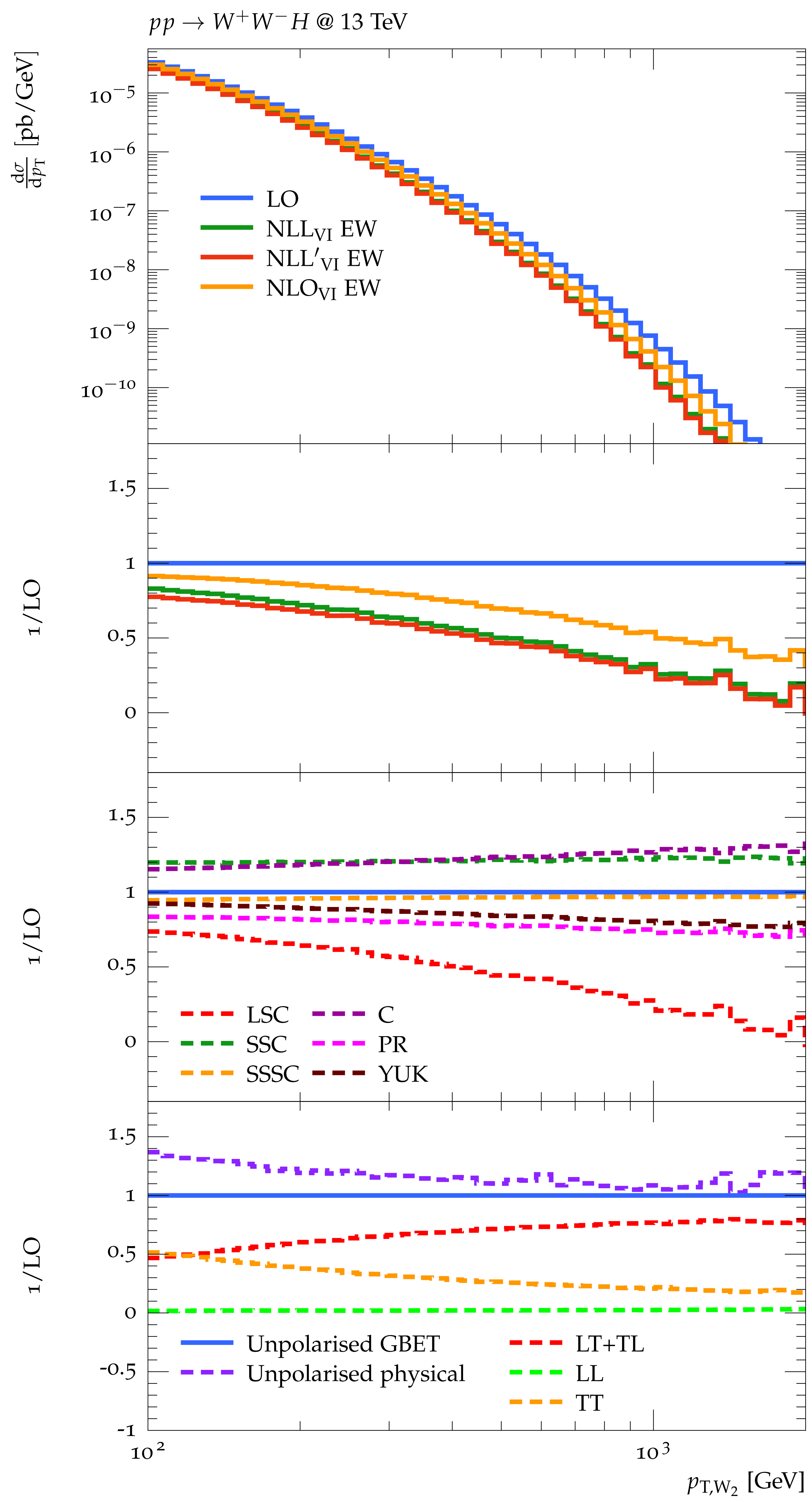
NLO QCD: [Mao et al, 0903.2885; 2009]

Full **NLO**: [Alwall *et al*, 1405.0301; 2014]

NLO QCD PS: [Baglio, 1609.05907; 2016]

Results: $pp \to W^+W^-H$





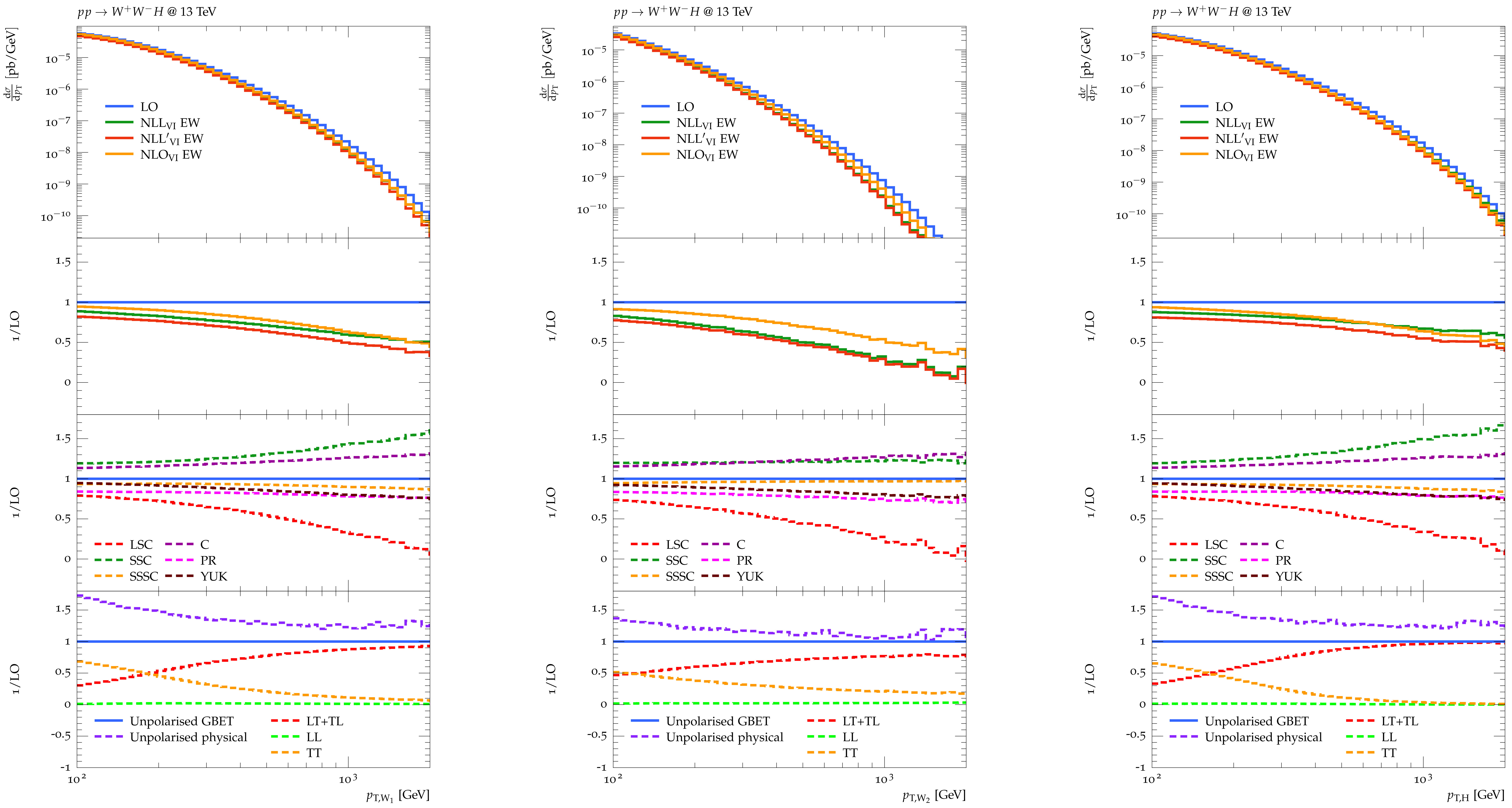
Here **TT** and **LL** polarisation configurations are mass-suppressed while mixed **LT** and **TL** are not

Small but sizeable contribution to the LO coming from **TT**. In the tail

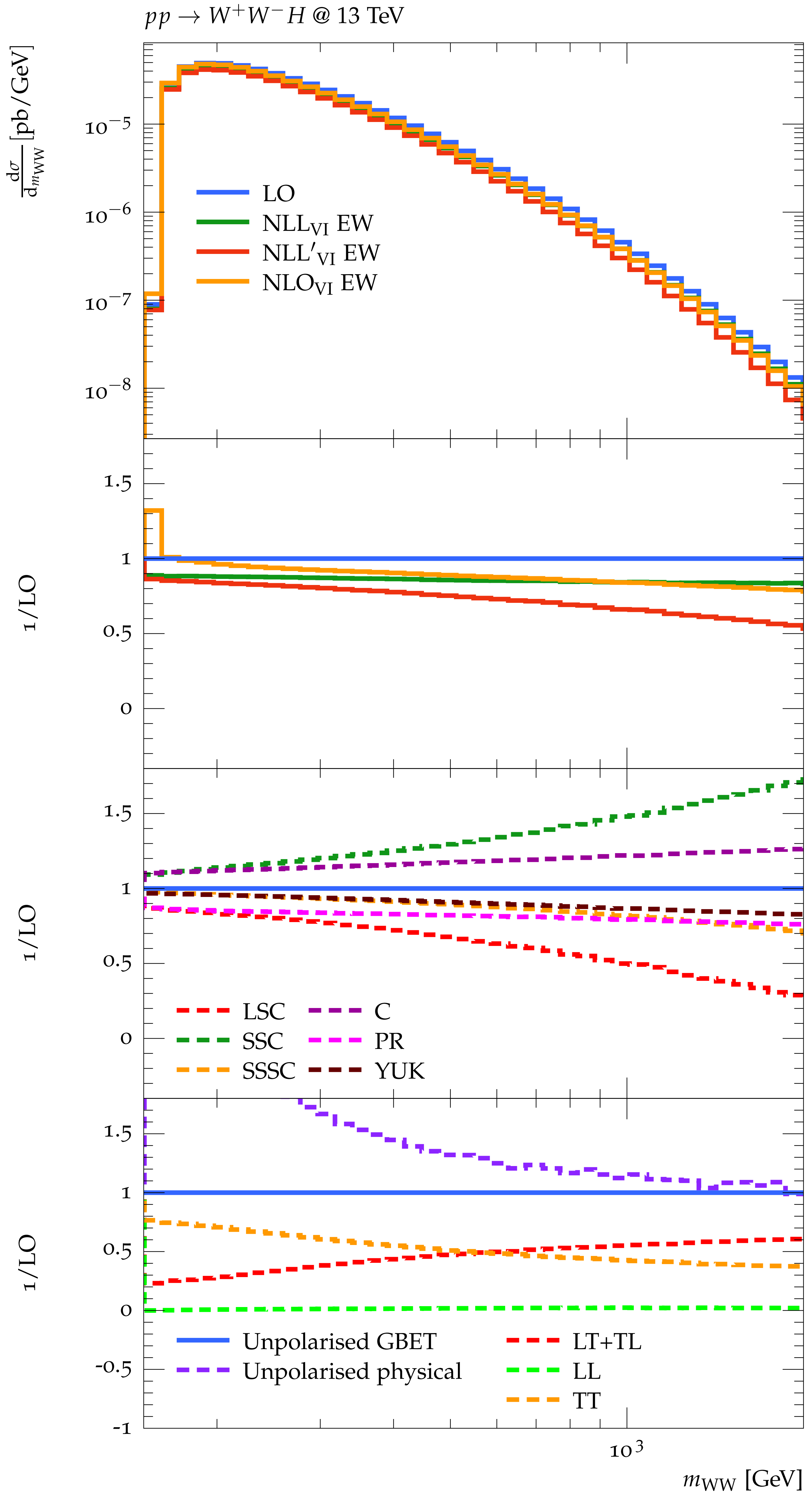
- $p_{\mathrm{T,W_1}}$: $\sim 5\,\%$
- $p_{\mathrm{T,W_2}}$: $\sim 15\,\%$

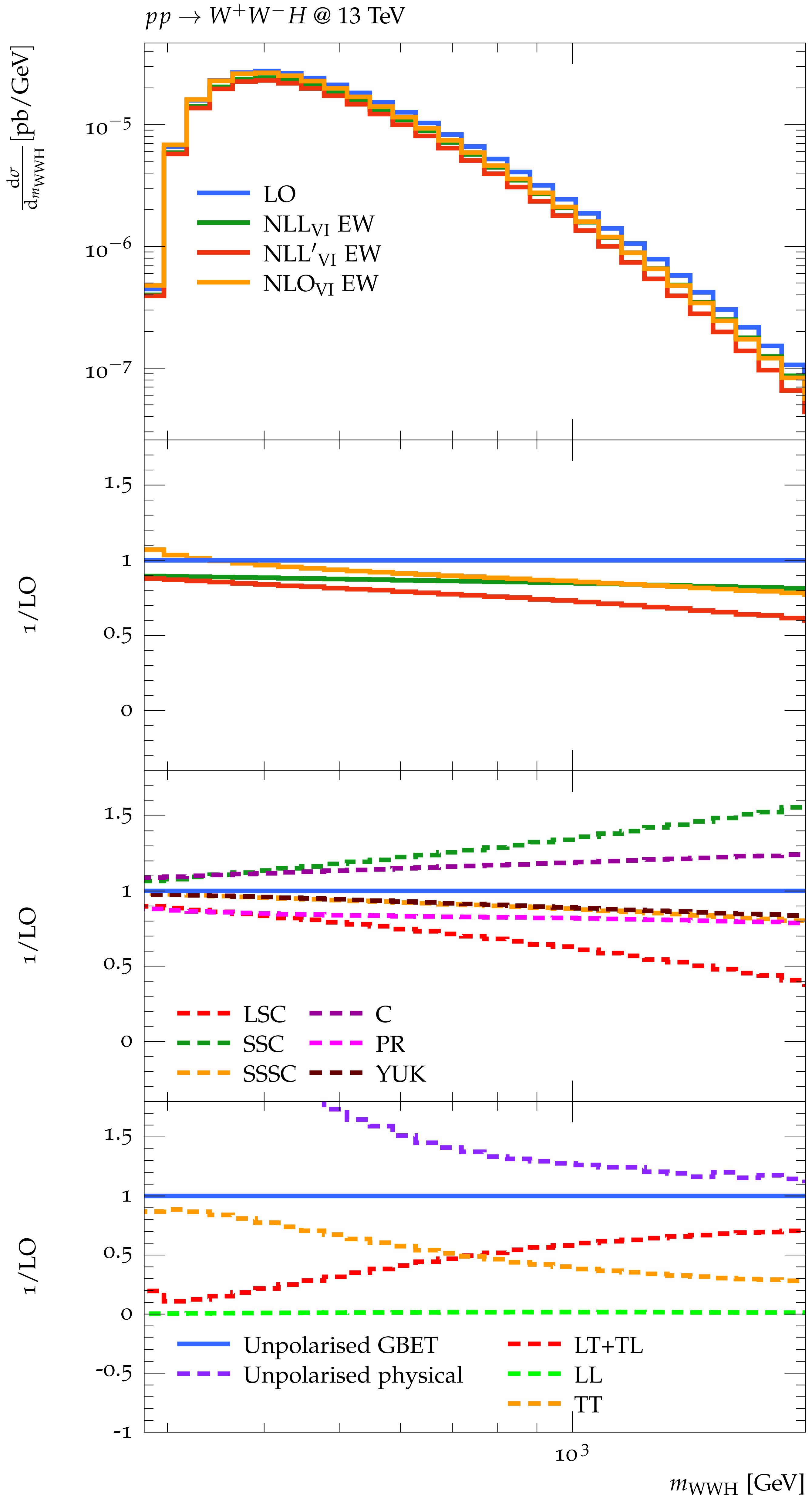
Within this setup, *Sudakov* approximation cannot be directly employed for these observables

Results: $pp \to W^+W^-H$



Results: $pp \to W^+W^-H$



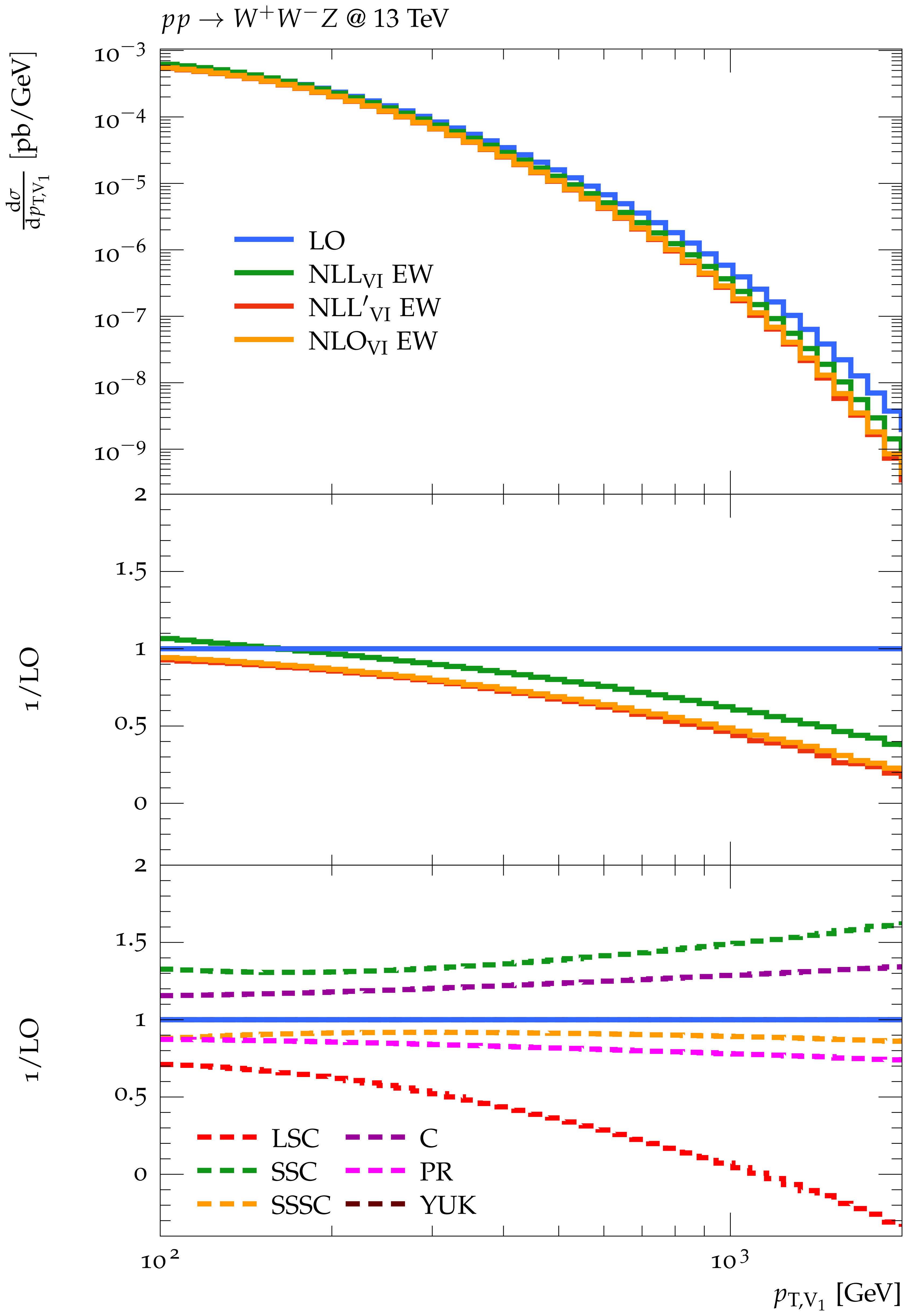


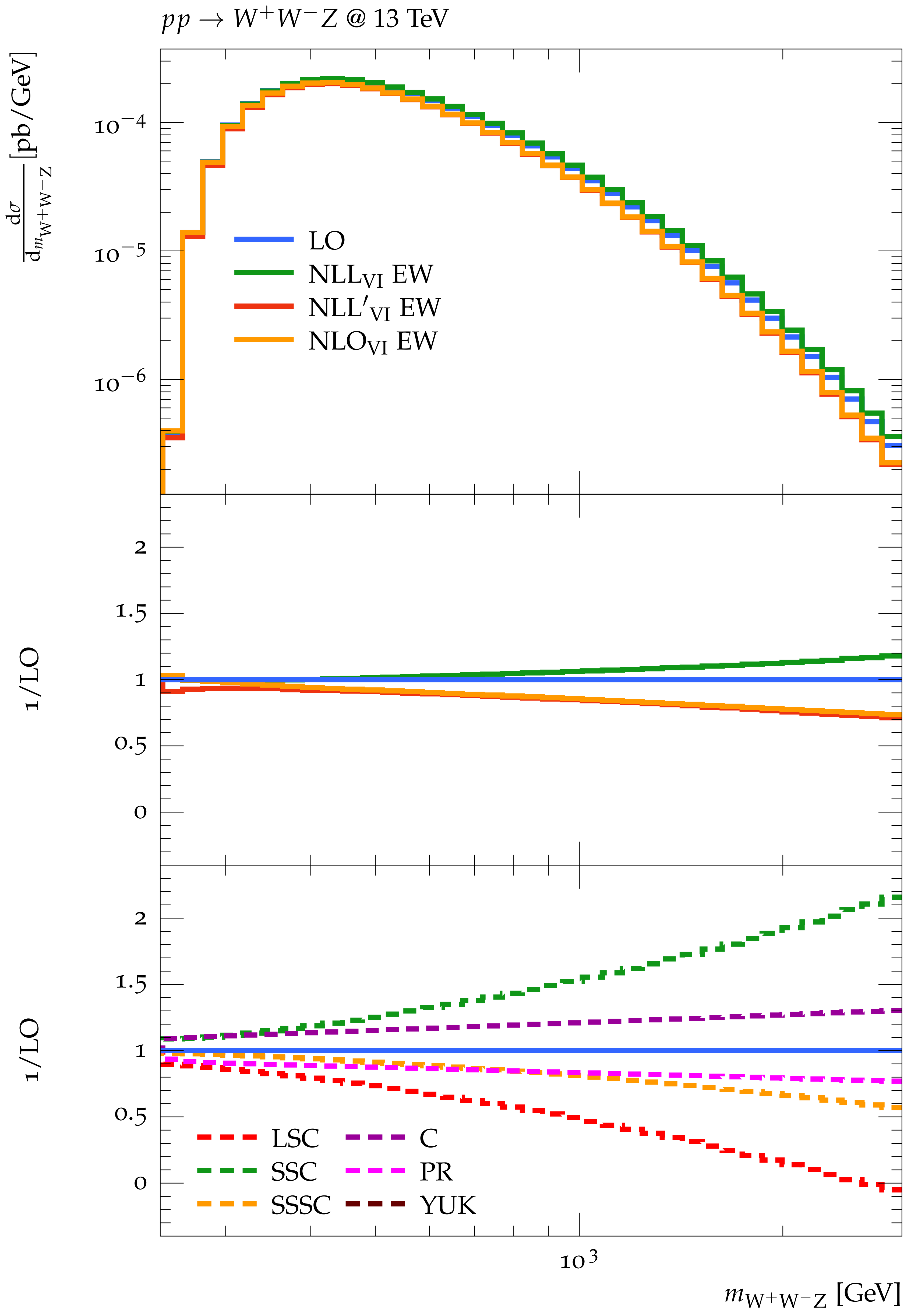
Too big contribution to the LO coming from the mass-suppressed **TT** fraction, around $30 - 40\,\%$

Significantly higher energies are required to further suppress **TT** and apply the _Sudakov_ approximation

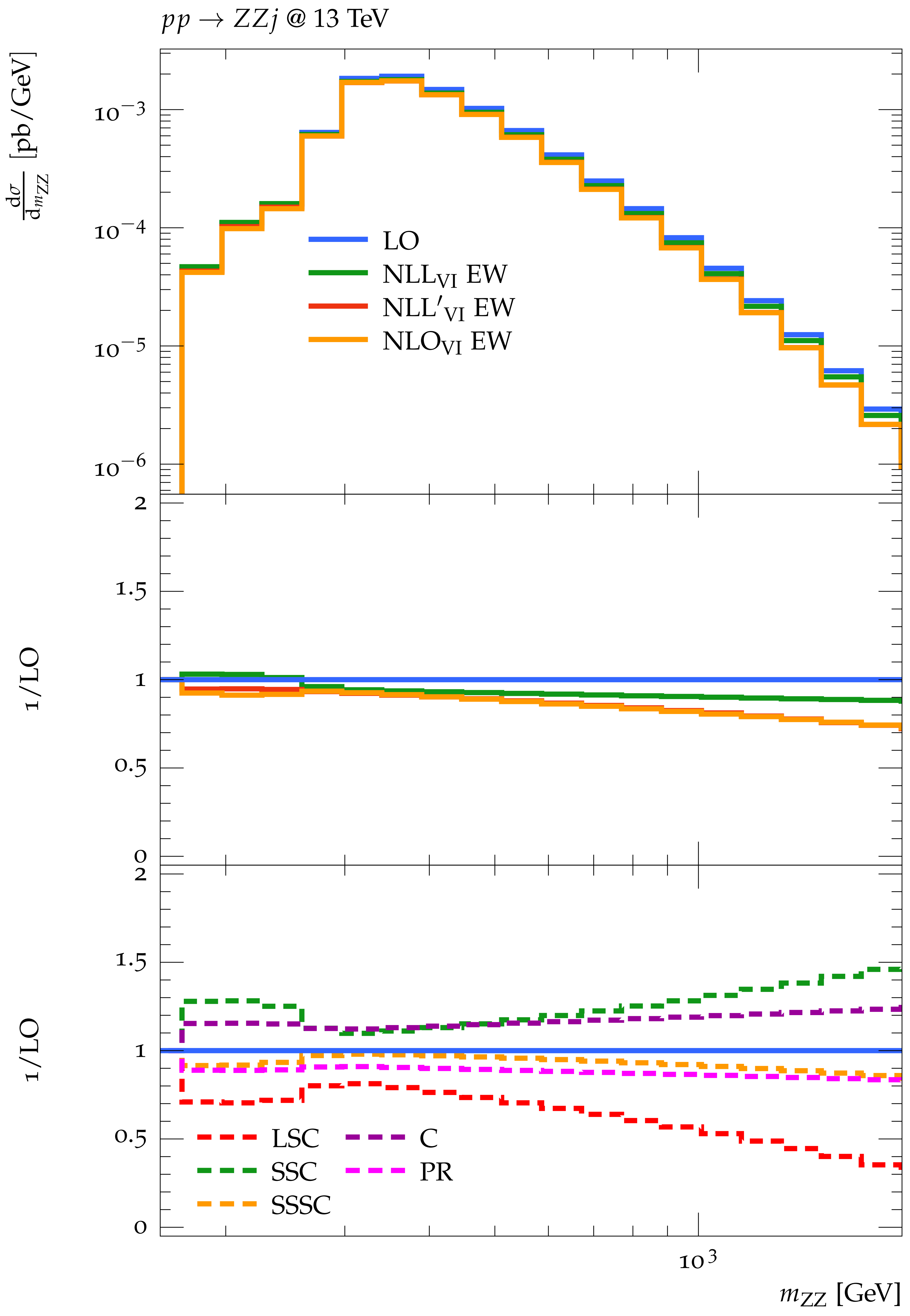
Less appealing solution: systematically derive and implement all mass-suppressed corrections

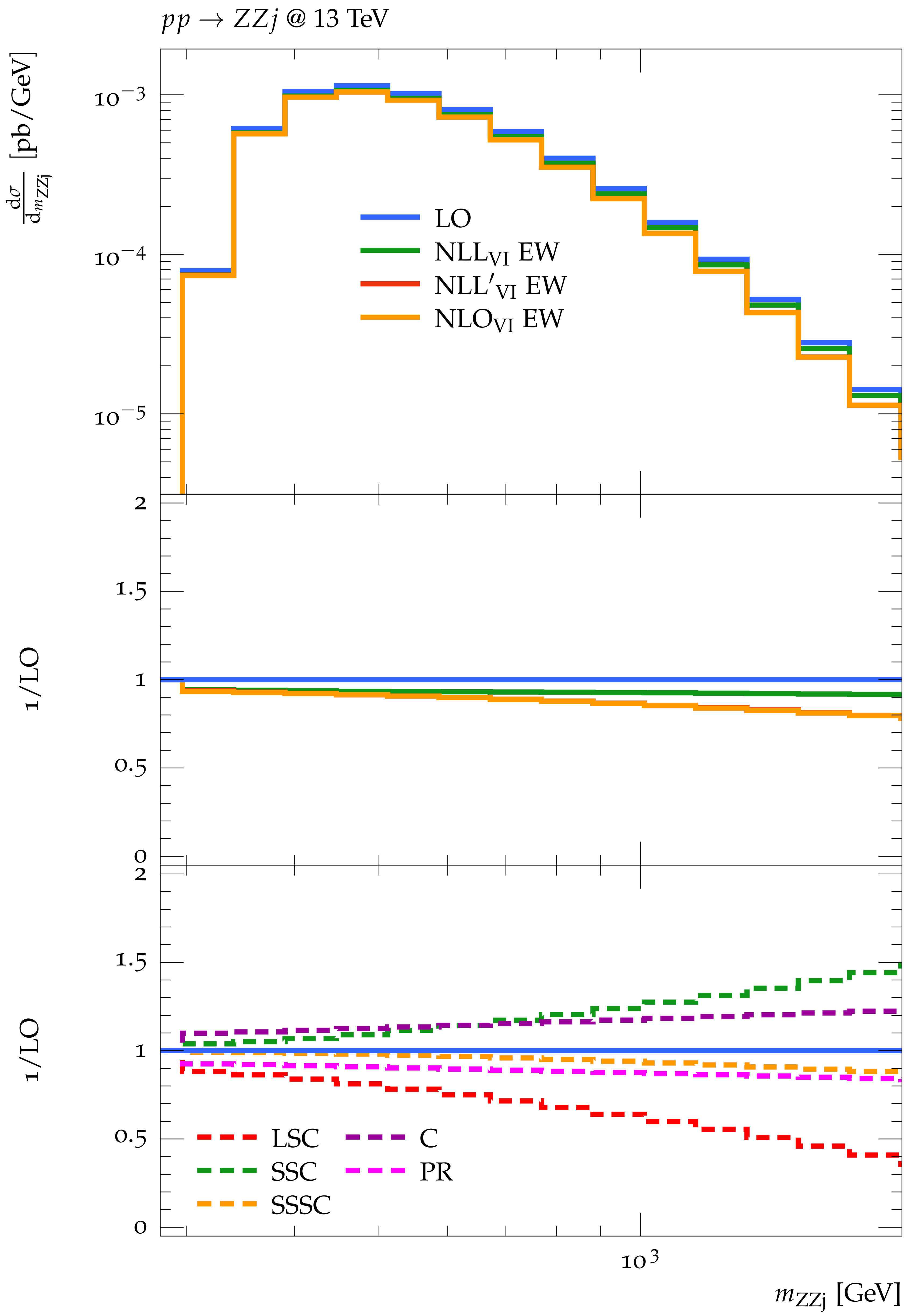
Results: $pp \to W^+W^-Z$



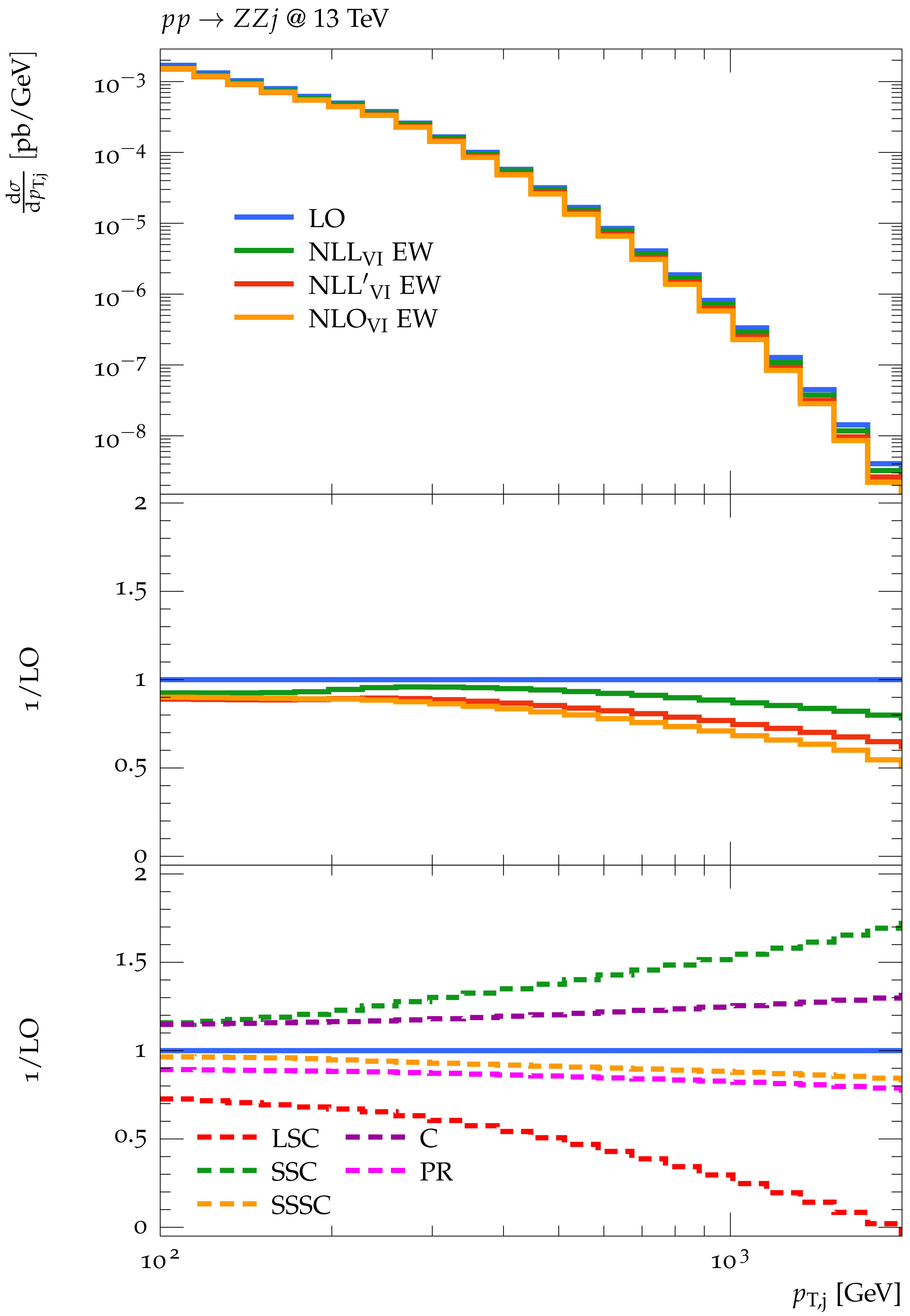


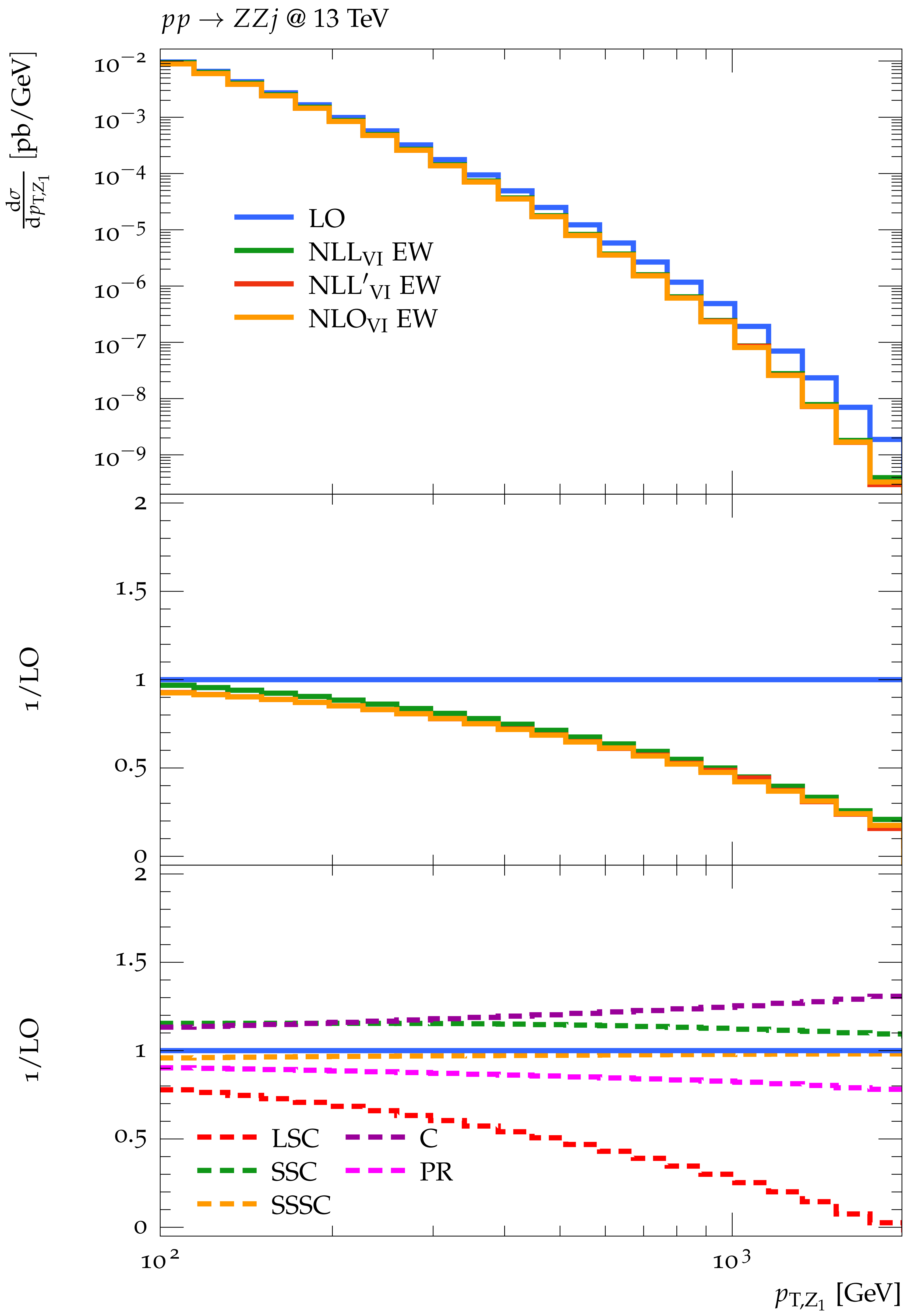
Results: $pp \to ZZj$



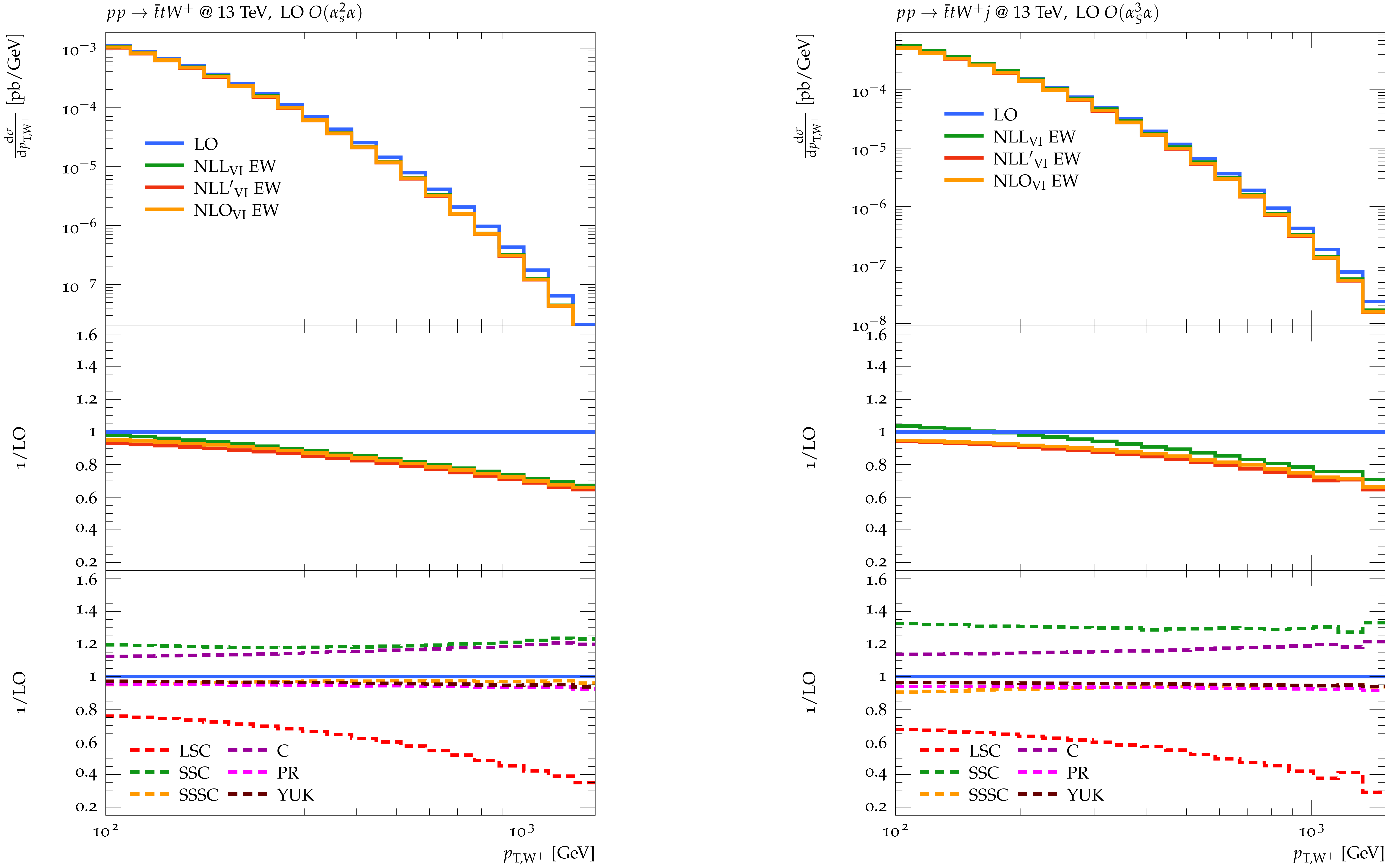


Results: $pp \to ZZj$

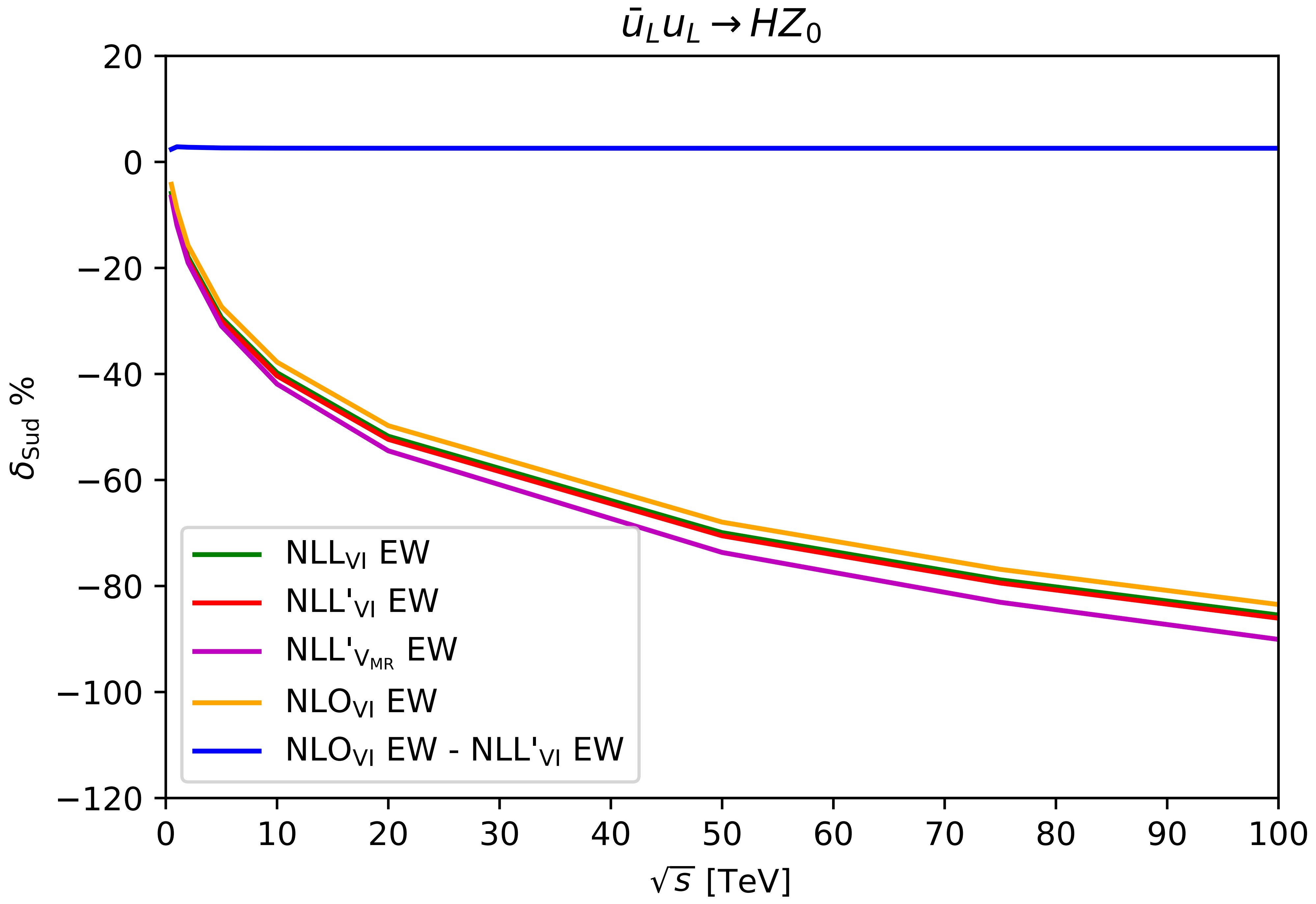


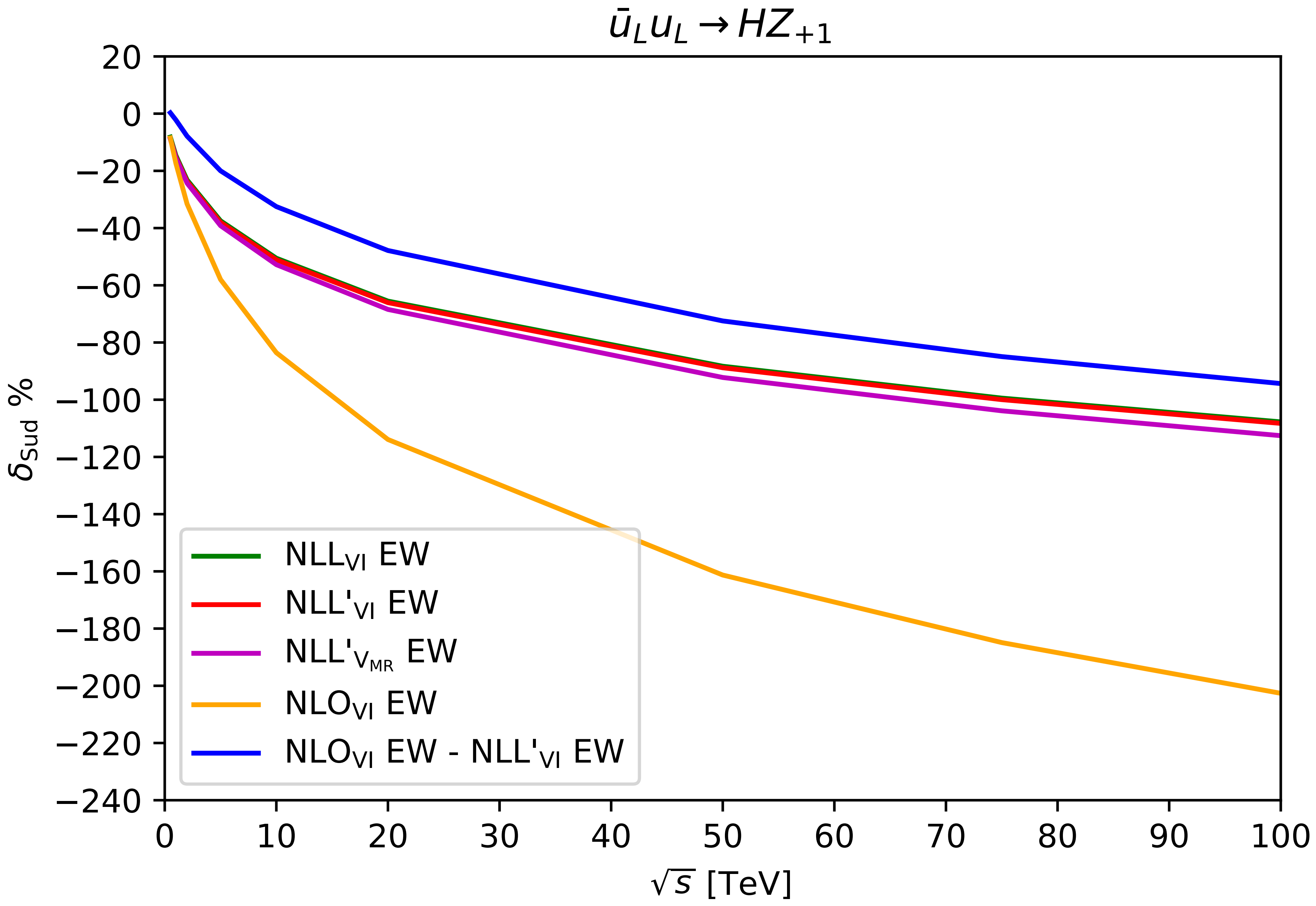


Results: $pp \to ttW^+$ & $pp \to ttW^+j$



Amplitude-level validation: $\sqrt{s}$ scan





- In *Sudakov* approximation: keep only double and singular logarithmic corrections

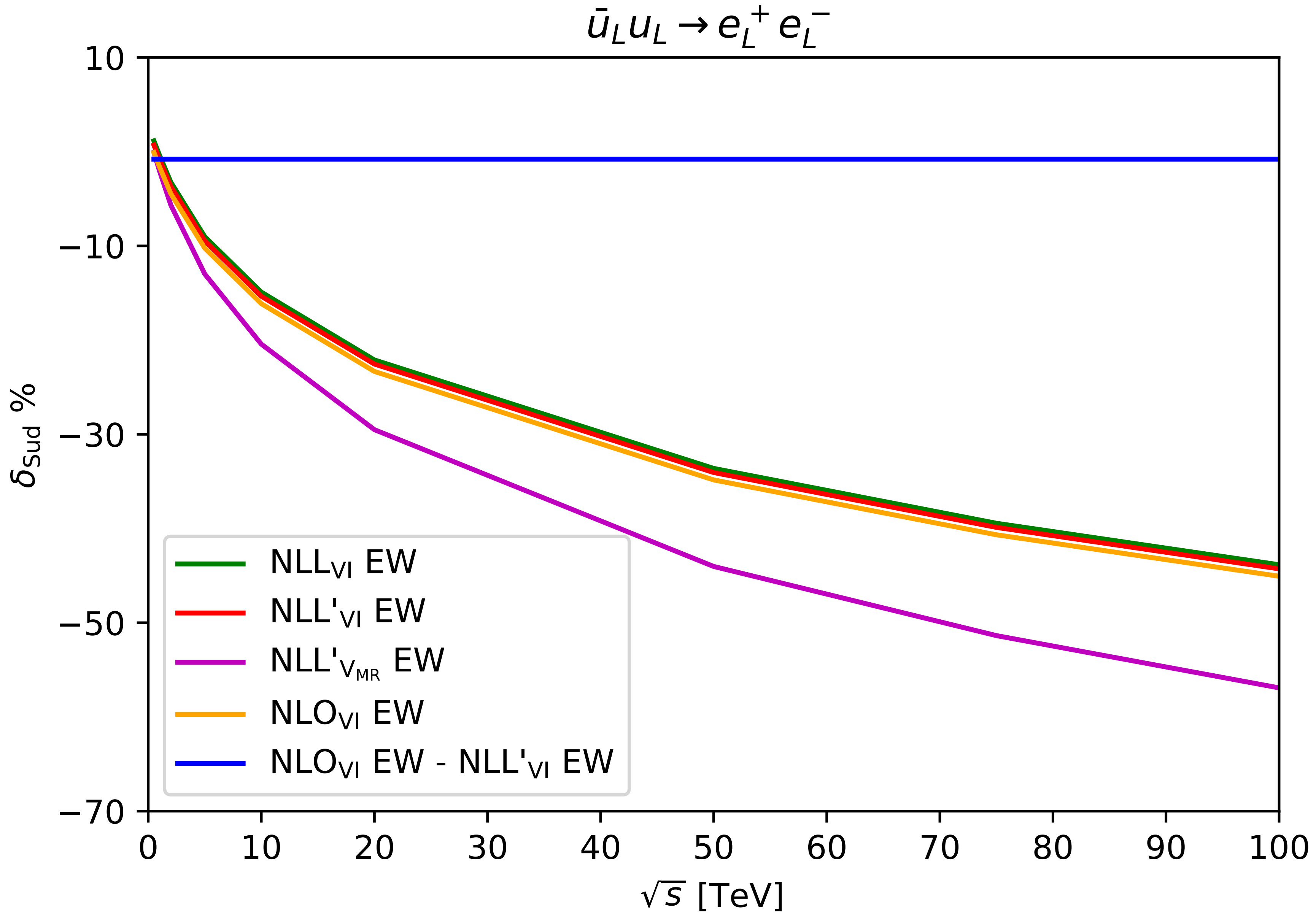
$$\delta^{\rm DL}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \sim E^d L \qquad \delta^{\rm SL}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \sim E^d l$$

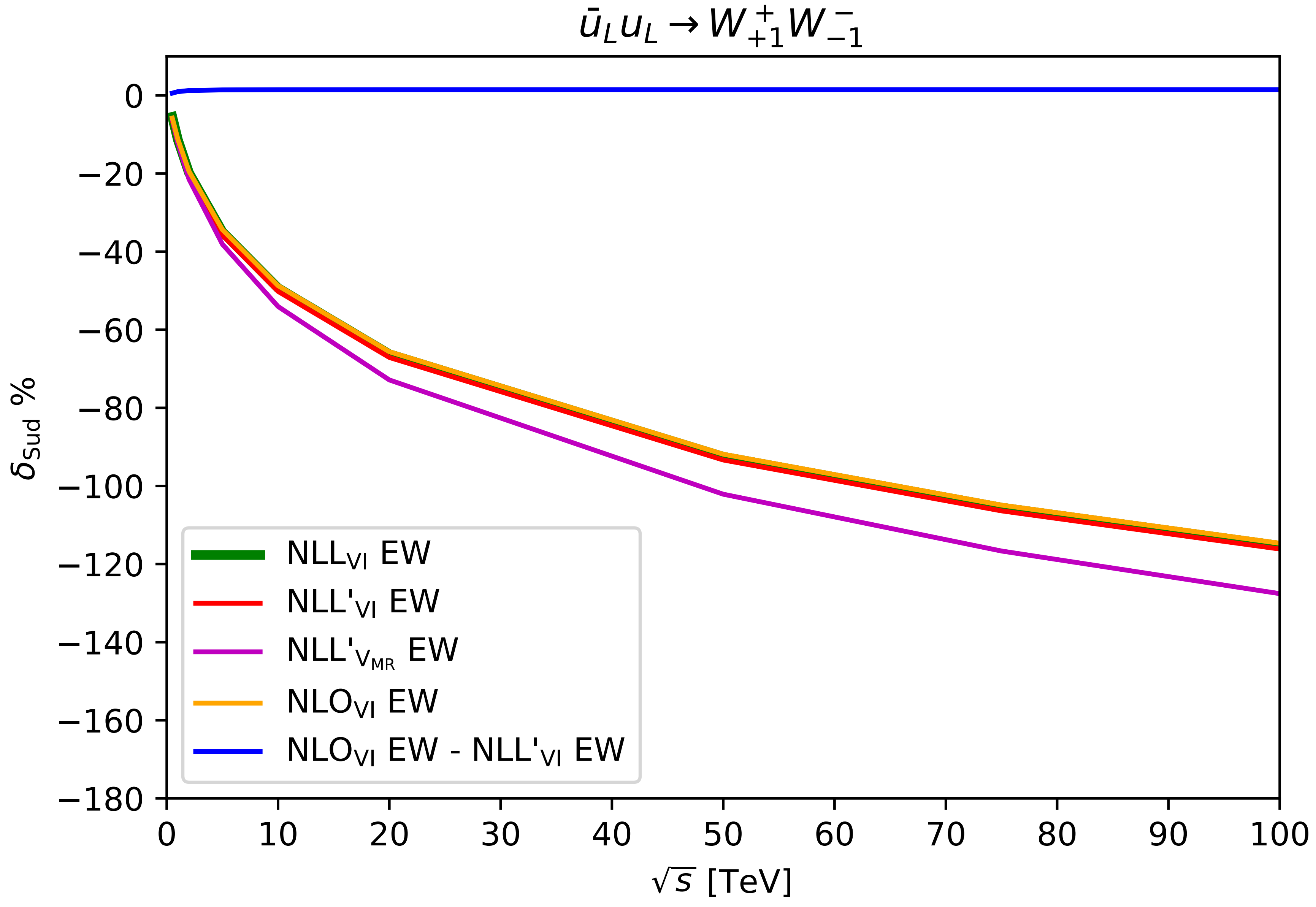
neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim M^n E^{d-n} L)$ terms

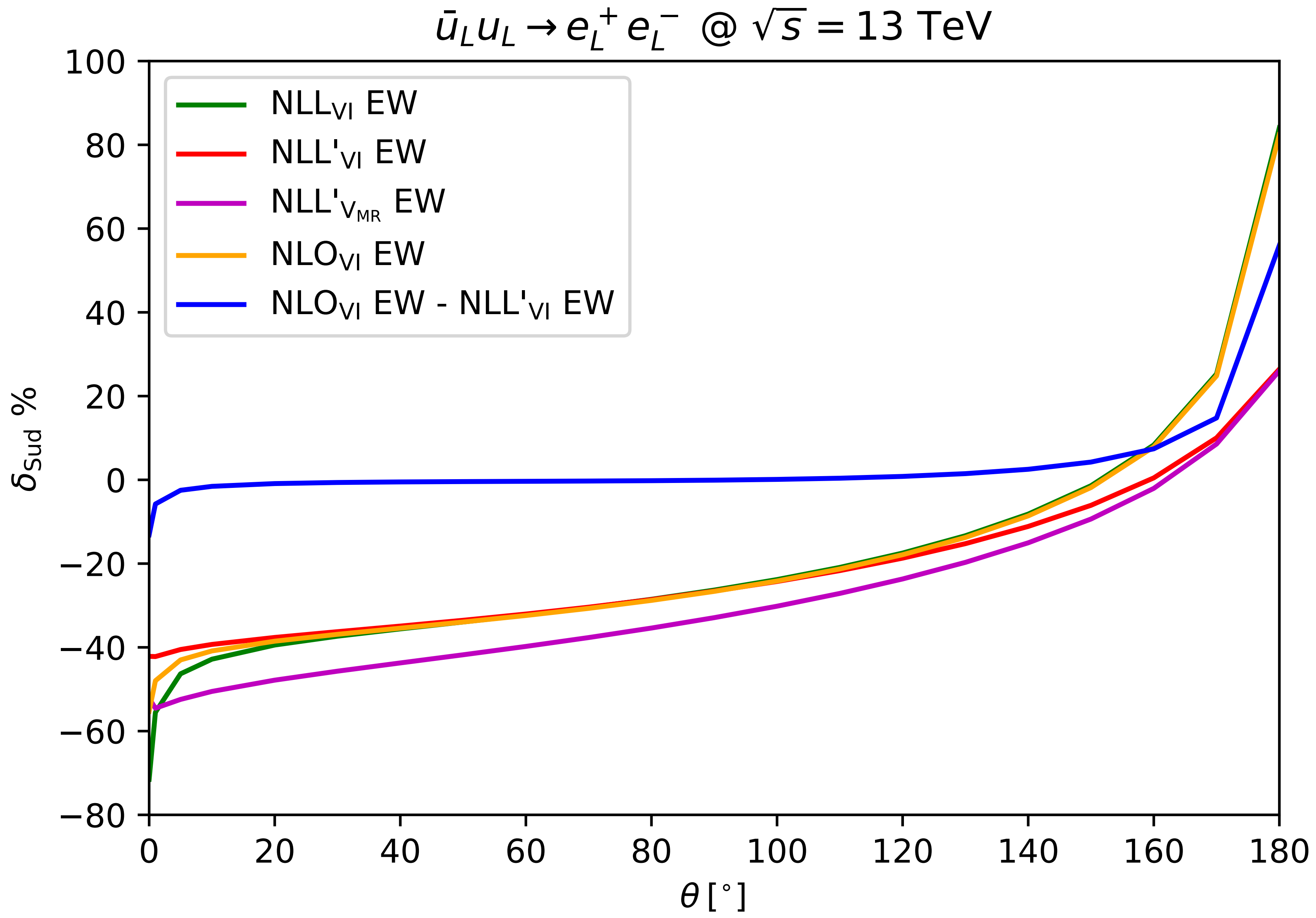
- In the high energy limit and for non mass-suppressed [2] matrix elements we expect $\mathrm{NLO_{VI}\ EW - NLL'_{VI}\ EW} \propto \ const$

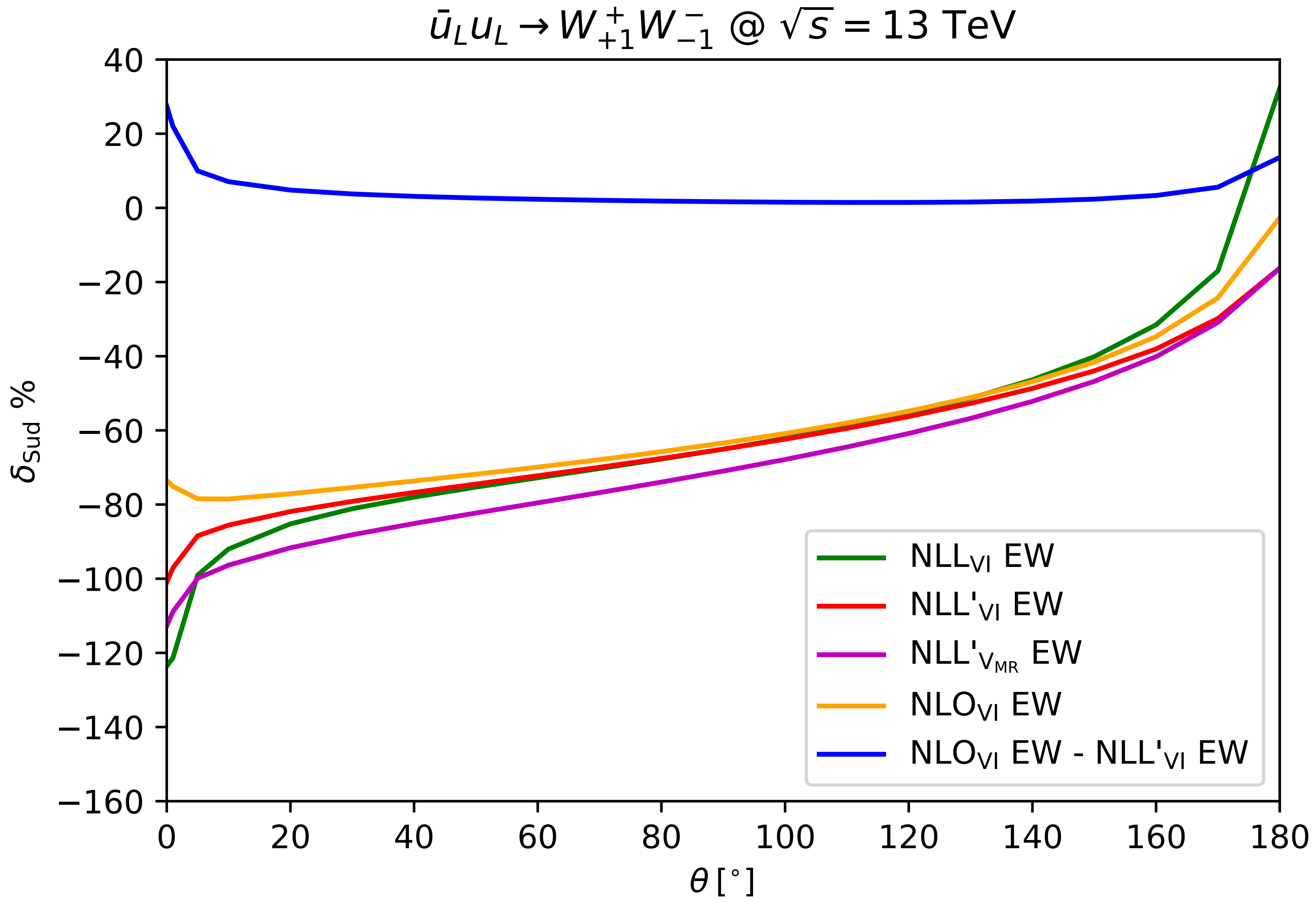
[2]NB: non mass-suppressed configurations scale like $\sim \sqrt{s}^{4-n}$

Amplitude-level validation: $\sqrt{s}$ and θ scans

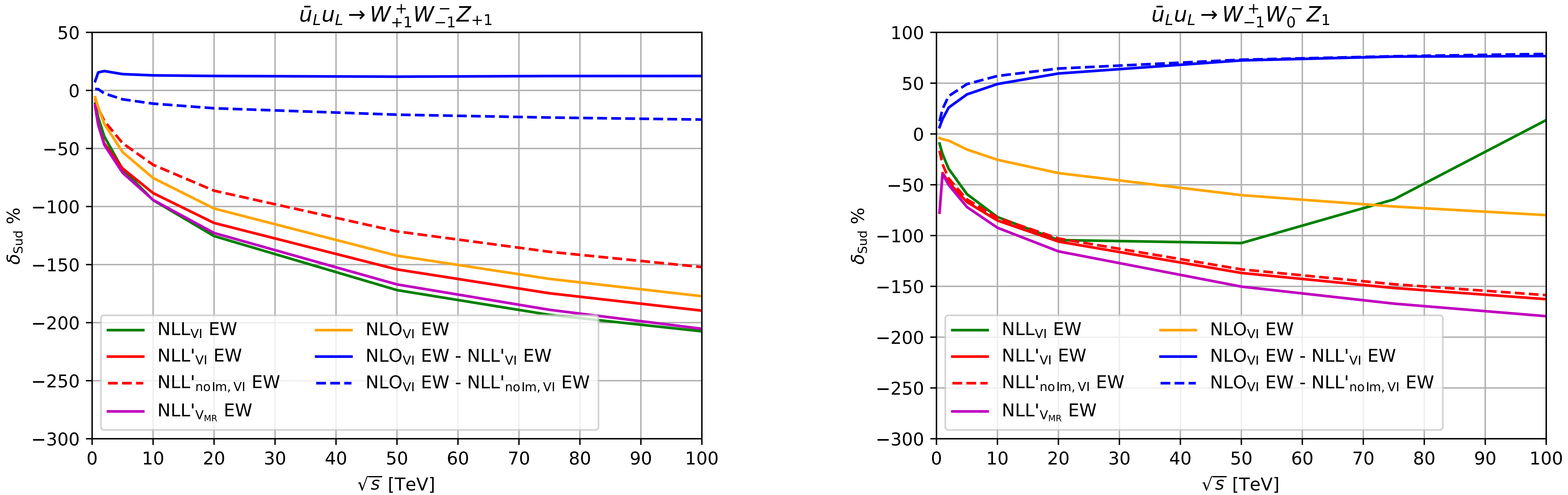








Amplitude-level validation: $\sqrt{s}$ scan



- In the high energy limit and for non mass-suppressed matrix elements we expect $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW} \propto \mathit{const}$

- Inclusion of the phase in DL from the LA of C_0, i.e.

$$C_0|_{\rm LA} \propto \left[\log^2\frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl})\log\frac{|r_{kl}|}{M_V^2}\right]$$

is crucial in $2 \to n$ processes with $n \geq 3$: without phase $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW}$ shows a logarithmic dependence. This has been firstly noticed in [Pagani, Zaro 2110.03714; 2021]