Cryogenic Heat Transfer

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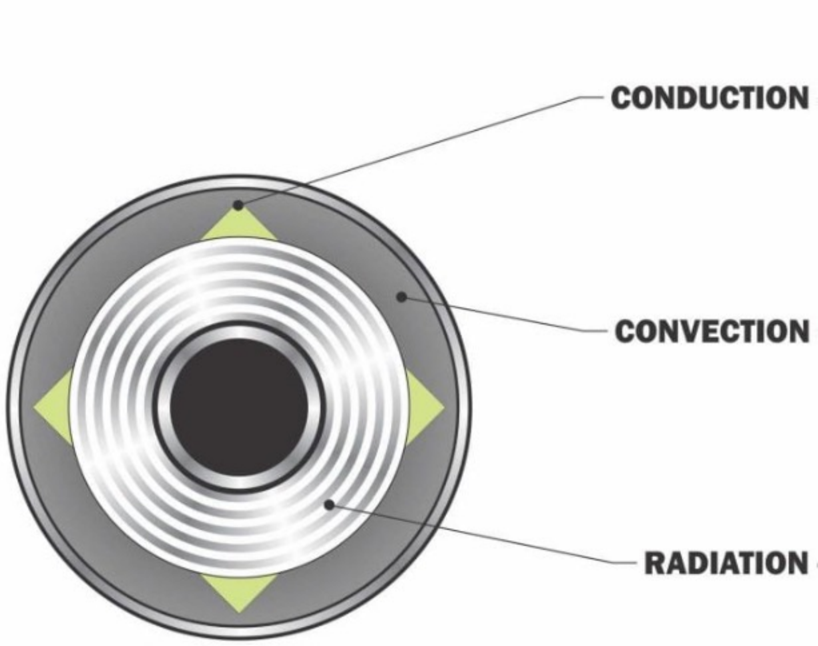


/Molecular conduction

Heat Transfer

In the following, often we refer as main figure qualifying the heat transmittion to the quantity

$$\Phi = (1/S)\,(1/\Delta T)\;dQ/dt$$

where

Q ➔ transmitted heat

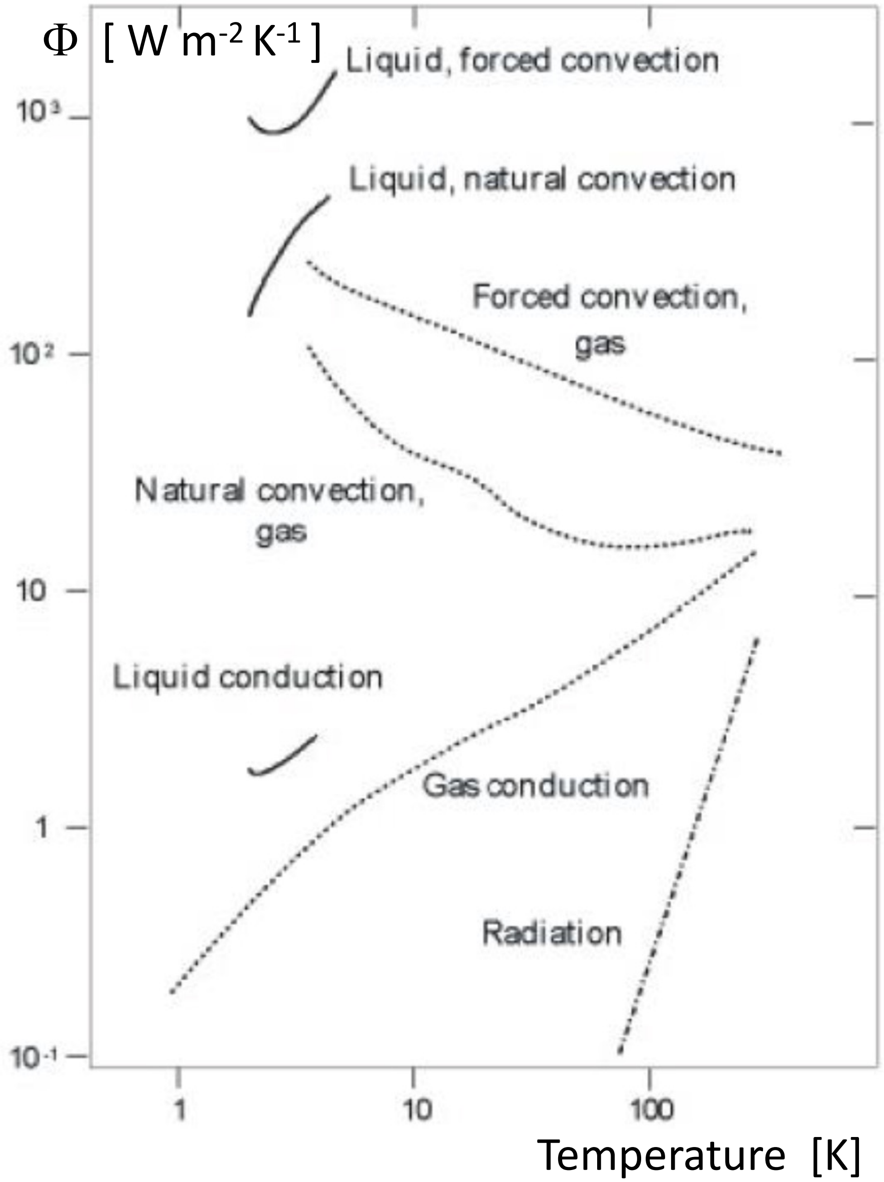
T ➔ temperature,

t ➔ time

ΔT ➔ the temperature difference between two unitary sections S

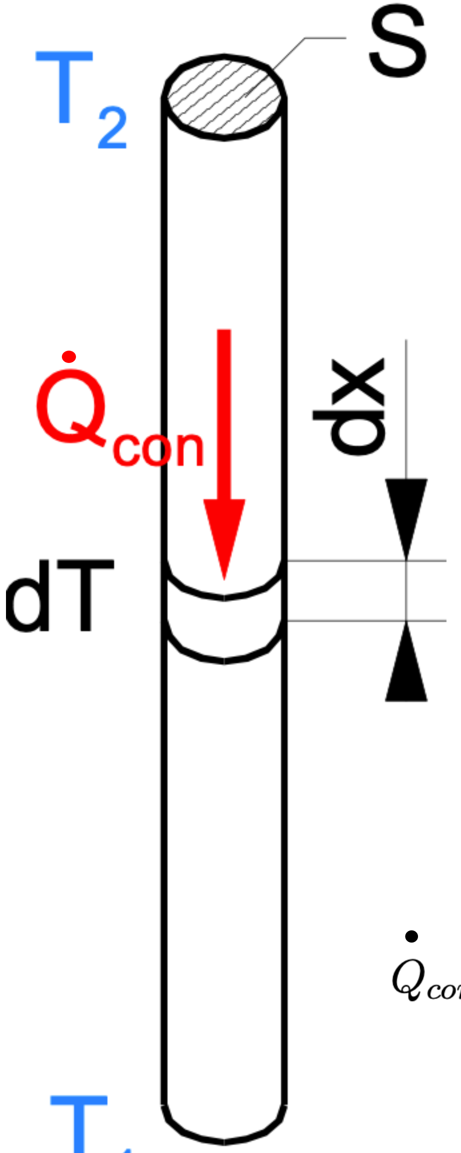
Main classification of the heat transfer mechanisms

- Conduction
- Convection/Molecular Conduction
- Radiation



Solid conduction - I

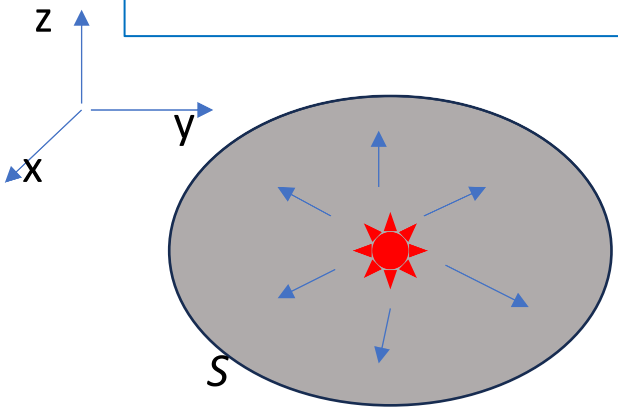
Heat power flow in the *x* direction depends on local derivative of temperature



$$\dot{Q}_x = -k(T)S\frac{\partial T}{\partial x}$$

Fourier law

$$\dot{Q} = \frac{\partial Q}{\partial t} \text{ [W]}$$



General case

$$\dot{Q}_{con} = -\int_S k(T)\ \vec{\nabla}T \cdot \vec{n}\ dS$$

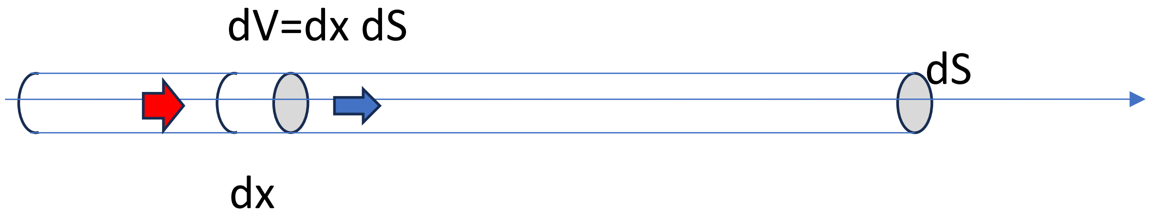
The case of thin bar of section *S* and length $\Delta L = L_2 - L_1$, ends at T_1 and T_2

$$\dot{Q}_{con} = -\frac{S}{\Delta L}\int_{L_1}^{L_2} k(T)\frac{\partial T}{\partial x}dx$$

if k(T)= k = constant

$$\dot{Q}_{con} = -\frac{S}{\Delta L}k(T_2 - T_1)$$

Solid Conduction – Heat propagation Theory **



Balance Equation of the Heat Exchange along the x direction

$$\underbrace{\rho(dS\ dx)c\ dT}_{\text{Heat absorbed by dV}} = -\Big[\underbrace{k\ dS\ dt\{\frac{\partial T}{\partial x}\}}_{\text{Heat flowing in}} - \underbrace{k\ dS\ dt\{[\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2}dx]\}}_{\text{Heat flowing out}}\Big]$$

and …after a bit of algebra….

$$\left(\frac{\rho\ c}{k}\right)\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$

Two lesson learned from this slide:

1) $(k/\rho\ c)$ ➔ {[W m^{-1} K^{-1}]/ [W s m^{-3} K^{-1}] }➔ [m^2 s^{-1}], called *thermal diffusivity*, tells us about the rate of the heat transfer inside the material.

2)The equation stands only in the case of an isolated body in the Universe, i.e. no other heat exchange mechanisms present (radiation , convection or molecular conduction).

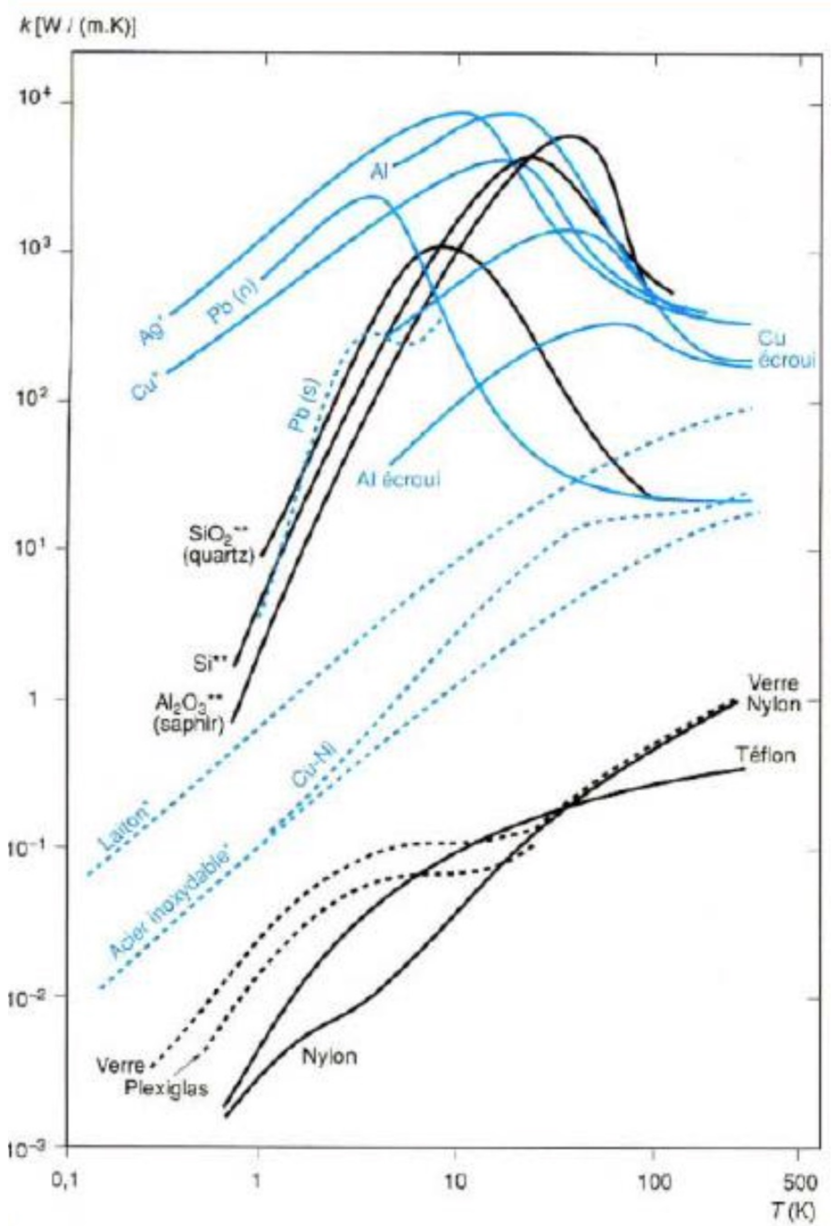
General Equation

$$\left(\frac{\rho\ c}{k}\right)\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T$$

Solid conduction - II

$$\int_0^T k(T')dT'$$ ➔ Integrated thermal conductivity [W m^{-1}]

	20 K	80 K	290 K
OFHC copper	11000	60600	152000
DHP copper	395	5890	46100
1100 aluminium	2740	23300	72100
2024 aluminium alloy	160	2420	22900
AISI 304 stainless steel	16.3	349	3060
G-10 glass-epoxy composite	2	18	153



k(T) ➔ thermal conductivity.

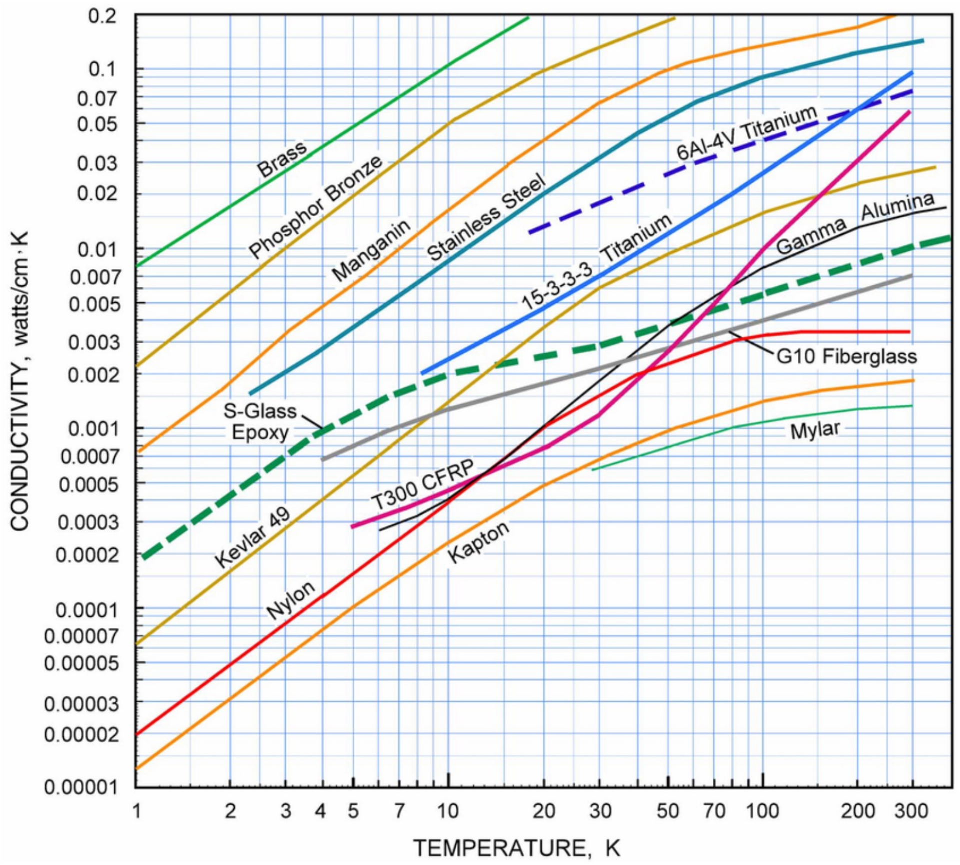
[*W K^{-1} m^{-1}*] SI units

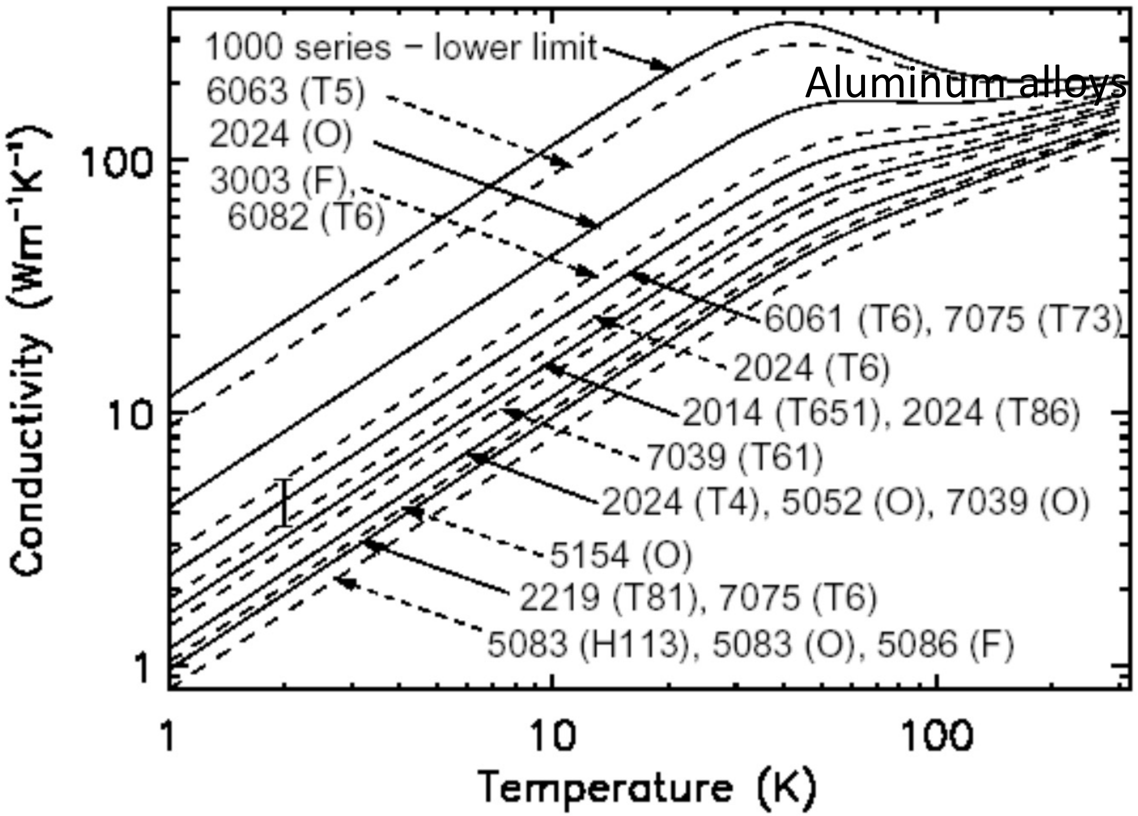
Solid Conduction - III

Some data to orient the choice in a cryo design….. then you must consult data books as for example

- *J.G. Weisend II et al., A reference guide for cryogenic properties of materials (SLAC-TN-03-023, Stanford, Calif., 2003).*
- *https://trc.nist.gov/cryogenics/materials/materialproperties.htm*
- …………………..

Common materials used in cryostat construction





Solid Conduction in Metals – IV

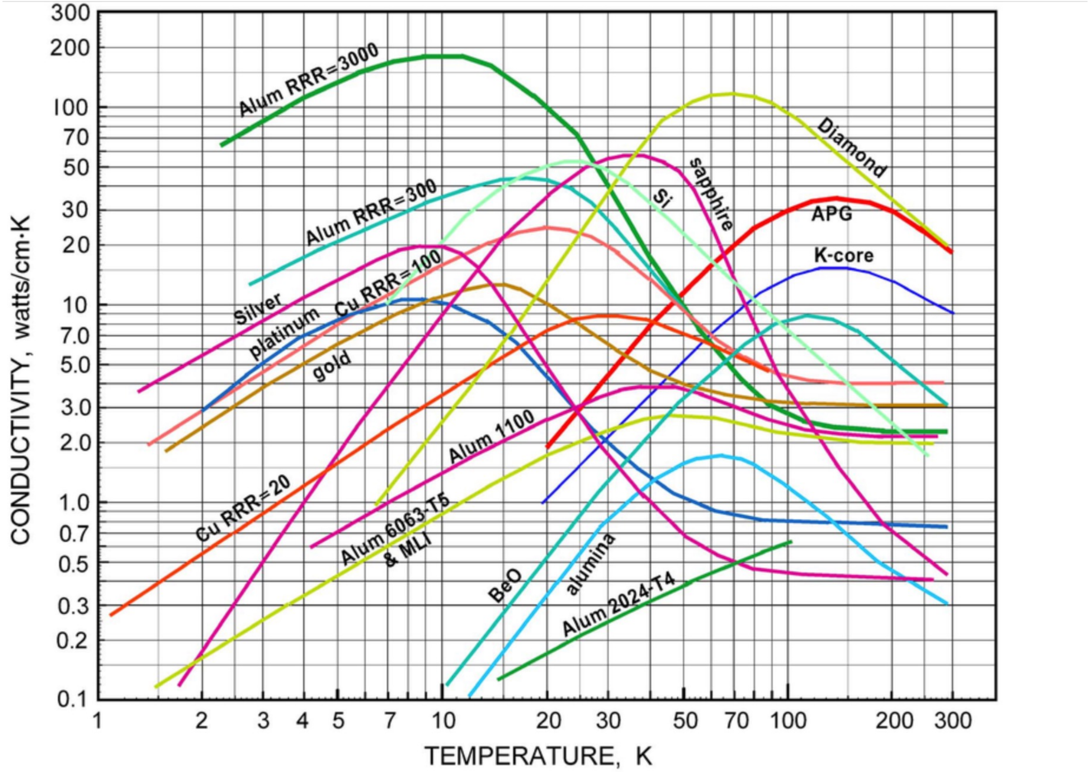
Wiedemann-Franz law ➔ the ratio of thermal conductivity k to the electrical conductivity σ of a metal is proportional to the temperature T via the coefficient L *(Lorentz factor)*

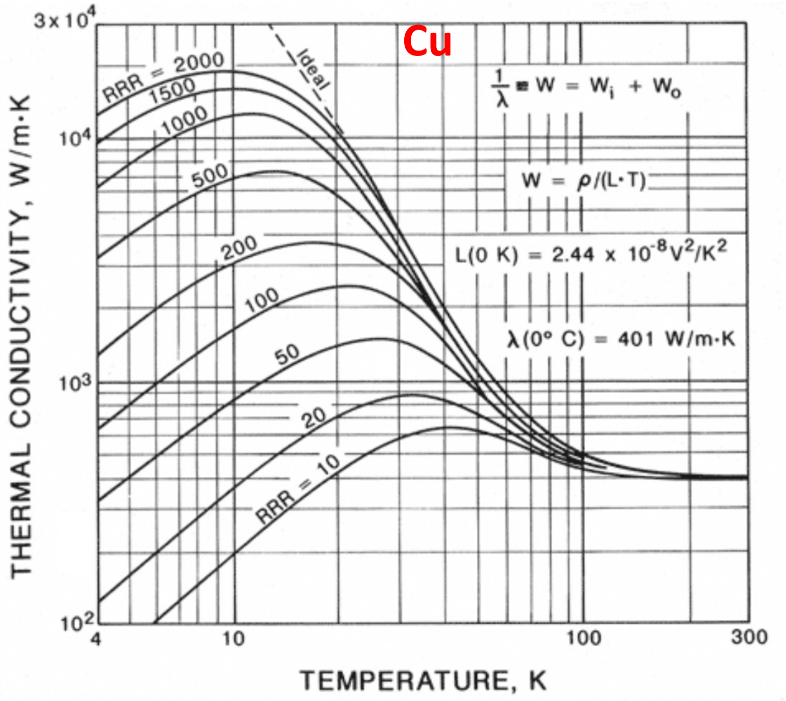
$$LT = \frac{\kappa}{\sigma} \quad \text{where} \quad L = \frac{\kappa}{\sigma T} = 2.45 \times 10^{-8} W\Omega K^{-2}$$

Residual Electric Resistivity Ratio ➔ $RRR = \frac{\rho_{300K}}{\rho_{0K}}$

Large RRR,high purity sample

Ideal case ρ_{0K} = 0 . However this is always false because of imputities and grain scattering at the boundary . *In the case of superconducting material the reference resitivity ρ_0 is the value just above the transition temperature.*





Suspensions of theInner thermal shields of a Cryostat

The Cryostat design follow the concept of a “ Mastrioska doll”.

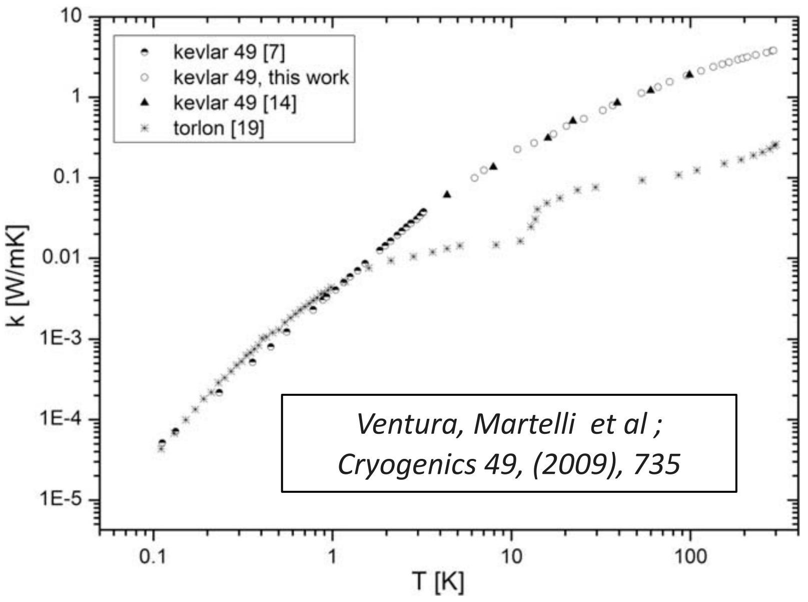
Lower is the temperature to be achieved in the innermost part, **higher** is the number of thermal shields.

A shields **must** be thermally insulated to the others.

Indipendent suspensions made of material with low thermal conductivty to stop the heat conduction via solid.

Composite material as **Kevlar® 49 cords**

(CUORE experiment and used to suspend the lowest detector stages of the X-IFU instrument on board of the Athena space telescope, which hosts superconducting sensor arrays operated at sub-Kelvin temperatures.)



Ventura, Martelli et al ; Cryogenics 49, (2009), 735

Tensile strength ~3000 MPa
Density 1.49 g/cm 3

When choosing a **metal**, *titanium* and *stainless steel* often emerge as top contenders.

Property	Titanium (Ti-6Al-4V)	Stainless Steel 304
k(T) [W k^{-1} m^{-1}]	~22	15-25 (alloy dep.)
Tensil strenght [Mpa]	1110	515-750 (alloy dep.)
Density [g/cm^3]	4.43	7.93
Strenght to weight ration [kN .m/kg]	280	70
Corrosion resistent	Yes	yes

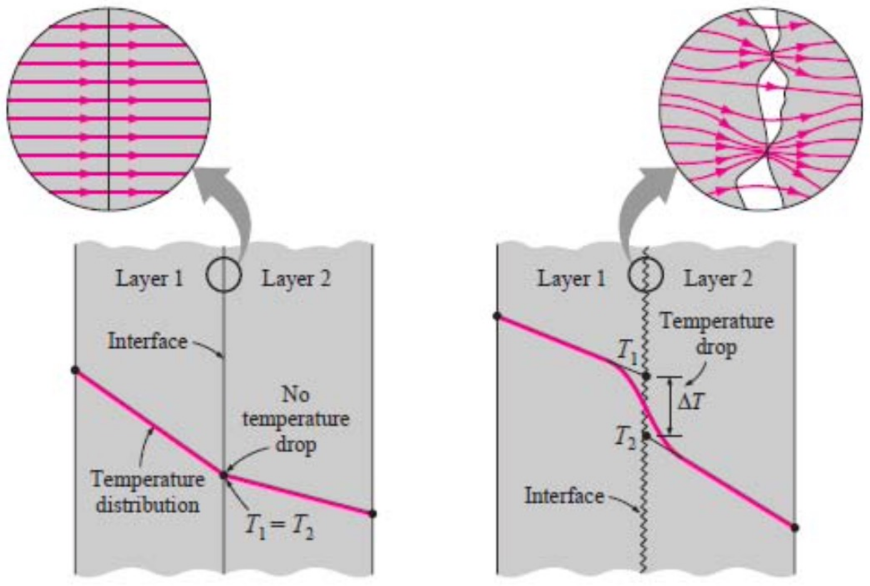
Thermal Contact Resistance

Two bodies in thermal contact may not be perfectly bonded and voids at their interface occur. We consider the case of power heat propagation $\dot{Q}_x$ through a surface contact S perpendicular to the *x* direction.

Imperfect contact at the interface perpendicular to *x*, causes a temperature drop ΔT

$\dot{Q}_x / (S\ \Delta T) = \alpha_x\ [W\ m^{-2}\ K^{-1}]$ ➔ thermal contact conductivity

$1/\alpha_x = R_x\ [K\ m^2\ W^{-1}]$ ➔ the thermal contact resistance



(a) Ideal (perfect) thermal contact

(b) Actual (imperfect) thermal contact

Be aware of its effect!

R_x values very difficult to predict

See for example the case of Cu-Cu coupling at 4.2 K (Dhuley R.C. Cryogenics 101 (2019) 111–124)

R_x *ranging in the interval* $2\ x10^{-2}$ - $2x20^{-4}$ *[K cm² W⁻¹]*

Generalization: make use of electric analogy

Thermal Contact Resistances in parallel :

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Gas Thermal Conductivity - I

The main mechanism in gases is the transfer of kinetic energy from the high-velocity (high-temperature) molecules to lower velocity (lower temperature) molecules.

A gas molecule will travel a (statistical) mean distance λ, (mean free path) before it collides with another molecule. In the collosion the energy change is

$$\Delta U = -\frac{dU}{dx}\lambda = -mc_v\frac{dT}{dx}\lambda$$

m ➔ molecular mass

$\frac{1}{3}\left(\frac{N}{V}\right)\bar{v}$ ➔ the number of molecules crossing a unit area per unit time

(*N/V*) ➔ number of molecules per unit volume

ρ = (Nm/V) ➔ density ➔ it depends on the pressure *P* and temperature *T* of the gas

$\bar{v}$ ➔average molecular velocity

Total energy flow per unit surface $\frac{\dot{Q}}{S} = \frac{1}{3}(\frac{N}{V})\bar{v}[-mc_v\frac{dT}{dx}\lambda]$

Thermal conductivity ➔ $k(T) = \frac{1}{3}(\frac{Nm}{V})c_v\bar{v}\lambda = \frac{1}{3}\rho c_v\bar{v}\lambda = \frac{1}{3}(\frac{P}{RT})c_v\bar{v}\lambda$

NOTE:
We assume the equation of the Ideal Gas

Gas Thermal Conductivity- II

Mean Free Path λ depends on gas viscosity μ

$$\lambda = \frac{\mu}{P}\left(\frac{\pi RT}{2}\right)^{1/2} = \frac{\mu}{\rho}\left(\frac{\pi}{2RT}\right)^{1/2}$$

Mean velocity of the gas molecules

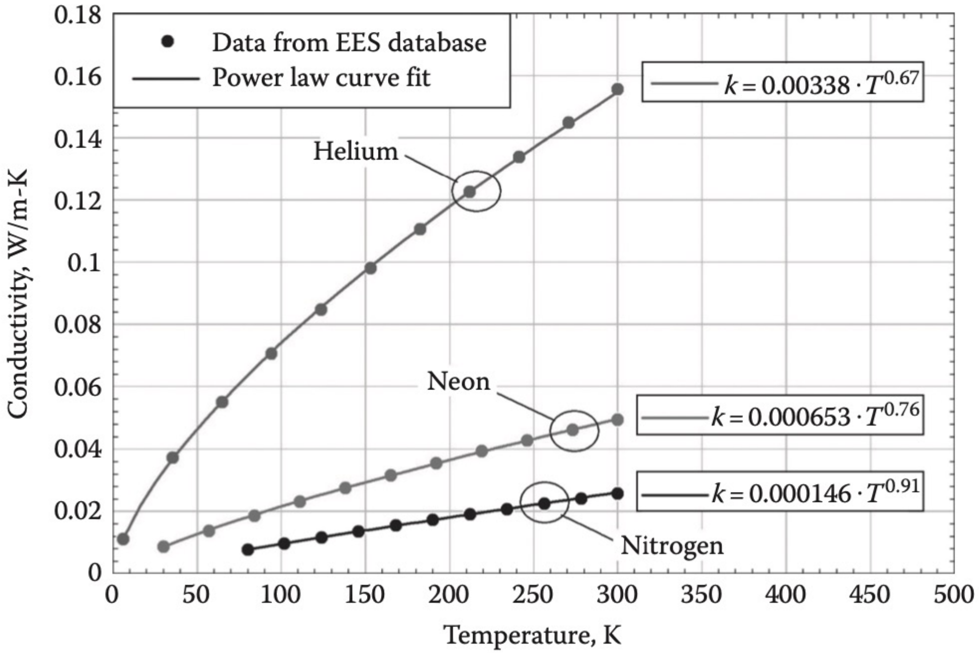
$$\bar{v} = \left(\frac{8RT}{\pi}\right)^{1/2}$$

and … after a bit of algebra … we realize that

k(t) = A T ½ with A= constant

This is the theory based on the assumption of an Ideal gas. However, looking at the experimental data we end up with a slight different conclusion

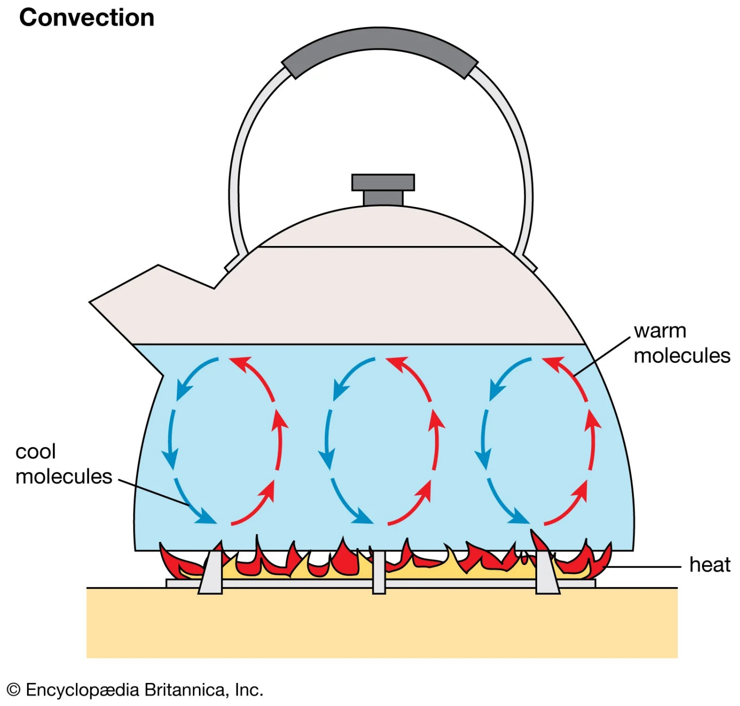
k(t) = A T^x with A= constant
with 06<x<0.9



Thermal Conductivity of Selected Gases at Atmospheric Pressure

Temperature		Thermal Conductivity, W/m-K				
°R	K	Helium	Hydrogen	Neon	Nitrogen	Oxygen
36	20	0.0262	—	—	—	—
54	30	0.0337	0.0252	0.00904	—	—
90	50	0.0467	0.0373	0.01270	—	—
135	75	0.0609	0.0524	0.01716	—	—
180	100	0.0737	0.0673	0.02144	0.00938	0.00909
225	125	0.0857	0.0820	0.02554	0.01174	0.01146
270	150	0.0969	0.0965	0.02946	0.01401	0.01378
360	200	0.1180	0.1246	0.03678	0.01828	0.01824
450	250	0.1375	0.1518	0.04346	0.02225	0.02246
540	300	0.1560	0.1779	0.04956	0.02597	0.02649

Gas Heat Transfer - Convection -I



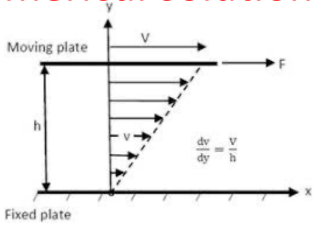
Convection : process by which heat is transferred by movement of a heated fluid. It can be single or multiphase fluid. Most commonly (Newtonian fluid) it is due to thermal expansion and gravity (buoyancy)

Theory rather complicated: *set of nonlinear partial differential equations* resulting from the Conservation of Momentum Principle. In the case a Newtonian fluid*, (shear stress proportional shear–strain rate**), with density constant and the buoyancy force written in terms of the thermal expansion coefficient β_t , we end up with *Navier–Stokes* equations + conservation of thermal energy equation.➔ **not straightforward solution of the coupled equations.**

Natural convection systems: the energy equation and the momentum equations are coupled because of the presence of the temperature in the buoyancy force term.

Forced convection systems (with negligible buoyancy force), the momentum equations may be solved first to obtain the velocity distribution, and then the energy equation may be solved to obtain the temperature distribution (assuming that density and viscosity are approximately constant).

In many complex situations, the only available solution technique is through a numerical solution of these differential equations.



*** newtonian fuids ➔ air, water,oil,... non newtonia: blood and in general polymeric fuids*

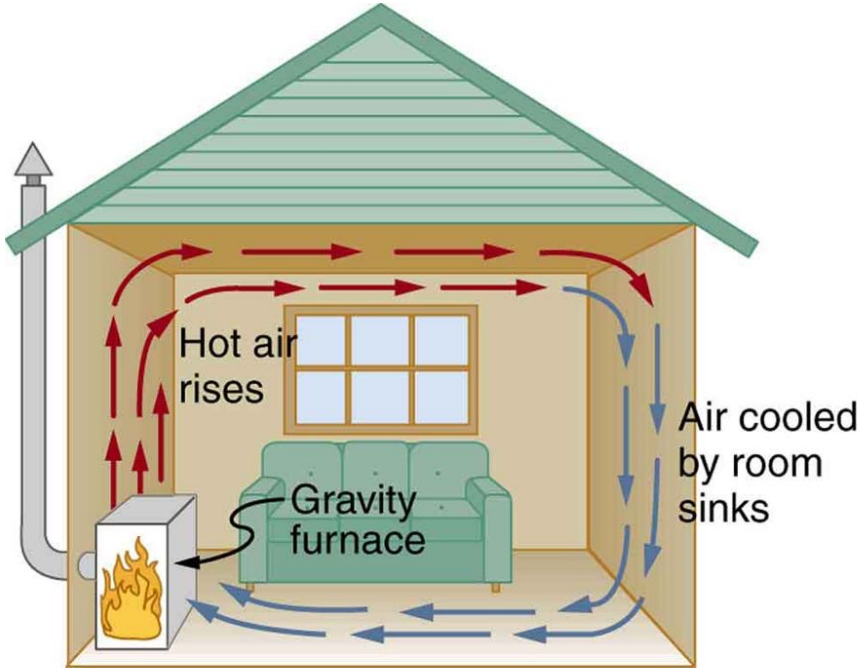
** Strain rate ➔ space gradient of the fluid velocity along the direction pertendicular to the shear stress*

Gas Heat Transfer - Convection -II

Simplified approach to evaluate the heat transfer via convection for a Newtonian fluid:

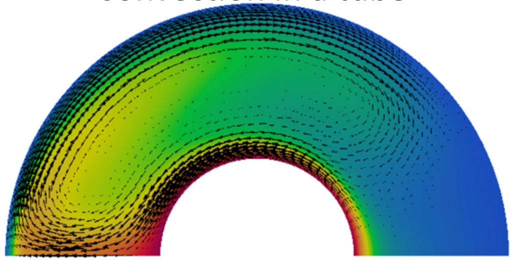
the Newtonian law

$$\frac{\dot{Q}}{S} = h\Delta T$$



- h ➔ convection heat transfer coefficient [W K^{-1} m^{-2}]
- S ➔ surface area exposed to fluid
- ΔT ➔ temperature difference between the surface and the fluid driving the heat

Convection in a tube

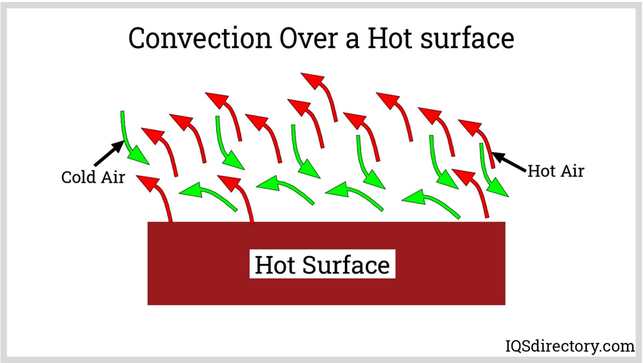


$$\Delta T = T_s - T_f$$

➔ The internal flow case (***fluid in a pipe***),
T_s temperature of the wall,
$T_f = T_{ave}$ mean temperature of the gas

$$\Delta T = T_s - T_\infty$$

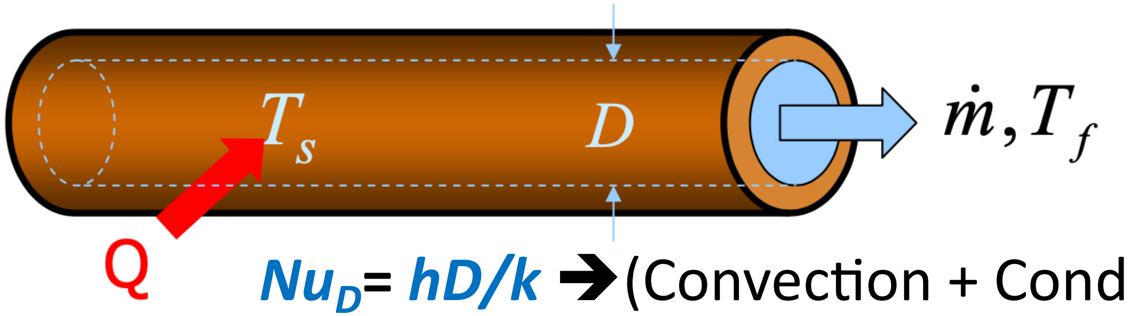
➔ The external flow case (***flow across a surface***),
T_s temperature of the wall,
T_∞ free stream temperature of the gas



Gas Heat Transfer - Convection - III

Convection is a relevant in several cryogenic applications:

- Heat exchangers
- Cryogenic fluid storage
- Cooling of superconducting magnets
- Low temperature instrumentation



Single phase convection of a classic fluid in a tube of diameter D:
The convection coefficient *h* can be extracted from the *Nusselt number* Nu_D (We have to include also the characteristic length of the duct *D*. For complex shapes, the length D may be defined as the volume of the fluid body divided by the surface area.) **$h = Nu_D (k/D)$**

$Nu_D = hD/k$ ➔(Convection + Conduction heat Transfer)/(Conduction heat transfer) with *k* ➔ *thermal conductivity of the fluid*

The Nusselt numbers depend on the fluidynamic regime and geometry considered (see heat exchange lesson) on which an entire course should be devoted!!

Laminar flow	Turbolent flow	Definitions
Laminar flow $Nu_D \underline{\sim} 4$	Turbolent flow $Nu_D = Const \cdot Re^n\, Pr^m$ (0.023 $Re^{4/5}$ $Pr^{2/5}$ 0.023 $Re^{4/5}$ $Pr^{2/5}$ Dittus-Boelter)	*Re* ➔ D Γ/ μ Reynolds number *Pr* = μ c/k➔Prandtl number

$\Gamma = \dot{m}/S$ ➔ specif mass flow rate, μ ➔ fluid viscosity, *c* ➔ fluid specif heat, ρ ➔ fluid density

Note : The relation $Nu_D = hD/k$ results from the assumption that heat transfer at S (expressed by the Newtonian law) is the same quantity given in terms of the thermal conductivity *k* of the fluid.

Gas Heat Transfer - Convection - IV

The heat transfer coefficient *h* is of the surface roughness *e* for laminar flow

Independent: Poiseuille law →

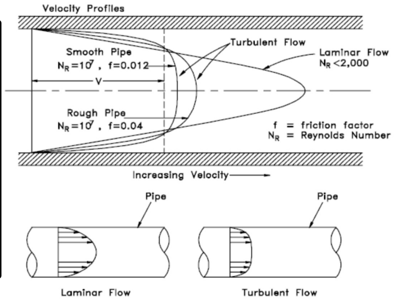
$$\Delta p = \frac{32\mu L}{\rho D^2}\left(\frac{\dot{m}}{S}\right)$$

R_e< 2000

The roughness *e* will affect the heat transfer coefficient in turbulent flow under the following condition: →

$$\left(\frac{e}{D}\right) Re > 5$$

R_e>3500



Turbolent case: the pressure drop *Δp* along the tube of length *L*, *depends on the* friction *f* , *which is a function of* *the* roughness *e*

$$\Delta p = \frac{fL}{2\rho D}\left(\frac{\dot{m}}{S}\right)^2$$

with $S= \pi D^2/4$

Friction factor *f* for a rough tube (turbolent case)

$$\frac{1}{\sqrt{f}} = 0.782\ln\left[\frac{1}{0.135\left(\frac{e}{D}\right)+\left(\frac{6.5}{Re}\right)}\right]$$

Typical Roughness for Pipes and Tubes

Material	Roughness, *e* µm
Drawn tubing	1.5
Brass	7.5
Glass	7.5
Steel or wrought iron	45
Galvanized iron	150
Cast iron	260

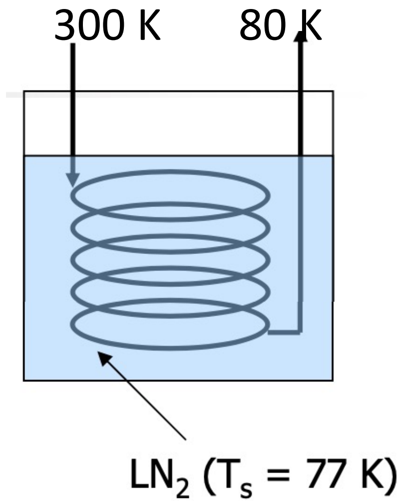
Gas heat Transfer – Convection V - A numerical example- I

The problem:

Helium flows in a helicoidal tube made of copper, which is immersed in a nitrogen bath at 77 K. The mass rate of the helium gas is 1g/s, the allowed pressure drop between inlet and outlet of the tube is ΔP=10 kPa and the temperature of the gas at the inlet and outlet should be 300 K and 80 K respectively. Determine the geometric parameters D and L of the tube.

Helium gas data: C_p = 5.2 kJ/kg K; μ = 15 x 10^{-6} Pa s; ρ = 0.3 kg/m^3, k = 0.1 W/m K (300 -80 K mean values)

Friction factor of helium f = 0.02 flowing in turbulent regime in a copper tube Copper



Solution:

We considere a small cylindric volume of gas moving in the tube in the unitary time: its mass rate is m which transfer heat via convection to the wall surface of the tube at temperature $T_{s.}$

The mass $\dot{m}$ of the gas loses heat power - $\dot{m}\, c_p\, dT$ via convection trough the lateral surface (π D dx) of the gas volume: h (π D dx) ($T - T_s$).

Assuming the stationary condition :

$$-\dot{m}\, c_p\, dT/dx = h\,(\pi D)\,(T - T_s)$$

The differenzial equation admits the following solution for T

$$\Delta T_f\,(x) = T_s - T(x) = (T_s - T)_{x=0}\ \exp[-(h\pi D/\dot{m} c_p)\,x]$$

being ΔT_f the temperature difference beetwen gas and tube at the interface.

By integrating from inlet *(x = 0, T = T_1)* to outlet *(x = L, T = T_2)* →

$$\Delta T_{lm} = \frac{\Delta T_f(x=0) - \Delta T_f(x=L)}{\ln\left(\Delta T_f(x=0) / \Delta T_f(x=L)\right)} = \frac{(300\text{-}77) - (80\text{-}77)\ \text{K}}{\ln\,[(300\text{-}77)/(80\text{-}77)]}$$

→ 51 K

Δt_{lm} → mean temperature difference between the surface and the fluid driving the heat

Gas heat Transfer – Convection V - A numerical example - II

Total Heat lost in the unit time by the helium gas

$$\dot{Q} = \dot{m} C_p (T_{in} - T_{out}) = 1 \text{ g/s} \times 5.2 \text{ J/g K} \times 220 \text{ K} = 1144 \text{ W}$$

Effective total heat power transferred via convection

$$\dot{Q} = h\,L\,D\pi\,\Delta T_{lm}$$

By imposing the equilibrium condition $\dot{m} C_p (T_{in} - T_{out}) = h\,L\,D\pi\,\Delta T_{lm}$ ➔ $h\,L\,D\pi = 1144/51$ W/K

- The heat transfer coefficient is a function of Re_D and $Pr = 0.67$
- Assuming the flow is turbulent and fully developed, use the Dittus Boelter correlation

$$Nu_D = \frac{hD}{k_f} = 0.023\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.3} \quad \text{and} \quad \mathrm{Re}_D = \frac{4\dot{m}}{\pi D \mu}$$

Substituting the Re_D and solving for h

$$h = 0.023 \frac{k_f}{D} \left(\frac{4\dot{m}}{\pi D \mu} \right)^{0.8} \mathrm{Pr}^{0.3} = \frac{0.0247 \times 0.1 \text{ W/m K} \times (10^{-3} \text{ kg/s})^{0.8}}{(15 \times 10^{-6} \text{ Pa s})^{0.8} \times D^{1.8}} = 0.07/D(\text{m})^{1.8}$$

$$h\pi DL = 0.224 \times (L/D^{0.8}) = 22.4 \text{ W/K}$$

⬅ 1 equation two unknown: D and L

Gas heat Transfer – Convection V - A numerical example - III

$$h\pi DL = 0.224 \times (L/D^{0.8}) = 22.4 \text{ W/K} \qquad (1)$$

We need a second equation. We will use the information about the pressure drop *ΔP =10000 Pa* and *f=0.02*

$$\Delta p = f\frac{L}{2\rho D}\left(\frac{\dot{m}}{A_{flow}}\right)^2 \quad \text{with} \quad A_{flow} = \pi D^2/4$$

$$\Delta p \approx 0.016\frac{\dot{m}^2}{\rho}\frac{L}{D^5} = \frac{0.016 \times (10^{-3}\text{ kg/s})^2}{0.3\text{ kg/m}^3}\left(\frac{L}{D^5}\right) = 5.33 \times 10^{-8}\left(\frac{L}{D^5}\right) = 10000\text{ Pa} \qquad (2)$$

Solving the systema of equations (1) and (2) we get:

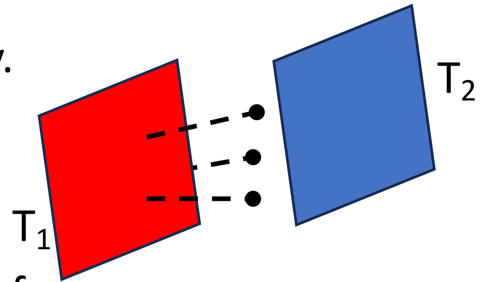
$$D = \left[5.33 \times 10^{-6}/\Delta p\right]^{1/4.2} = 6.2\text{ mm}$$

$$L = 100 \times (0.0062\text{ m})^{0.8} = 1.7\text{ m}$$

Molecular conduction - I

In the vacuum container surrounding the cold screens of the cryostat, the vacuum pressure that the mean path λ of the residual molecules achieves values of the order of 1 m. is sufficiently low ($<10^{-2}$ Pa) so that

- molecules travel from the warm to the cold walls without collision among the other molecules.
- the gas thermal conductivity is function of the number of molecules present as their mean velocity.



Heat power tansferred from the surface at temperature T_1 to the parallel surface at T_2:

$$\dot{Q} = \frac{a_0}{4}\frac{\gamma+1}{\gamma-1}\sqrt{\frac{2R}{\pi M}}\frac{T_2 - T_1}{\sqrt{T}}p$$

with

$$a_0 = \frac{a_1 a_2}{a_2 + (A_2/A_1)(1-a_2)a_1},$$

P ➔ Residual pressure between the two surfaces
M➔ molecular weight of the residual gas
γ ➔ ratio of the specif heats of the gas
R ➔ Ideal gas constant ➔ 8.314 J mol^{-1} K^{-1}
T➔ Gas temperature
A_1 and A_2 ➔ surface areas at T_1 and T_2
a_1 and a_2 ➔ accomodation cofficients of the gas to the two surfaces

Definition of the accomodation coefficient:
$a = (E_{in} – E_{refl}) / (E_{in} – E_{wall})$
E_{in} ➔ incident energy flux of molecule
E_{refl} ➔ reflected energy flux of molecule
E_{wall} ➔ energy flux of molecule if emitted in thermal equilibrium with the wall
a=1 ➔ complete accommodation, a = 1 ➔ diffuse reflection, a=0➔adiabatic or specular reflection

If $A_1 = A_2$ ➔ $a_0 = (a_1 a_2) / (a_1 + a_2 – a_1 a_2)$

…and

If ➔ $(a_1 = a_2 = a)$, $a_0 = a/(2-a)$

$a_0 = a/2$ for a ➔ 0
$a_0 = a$ for a ➔ 1

Molecular Conduction -II

$$\dot{Q} = \frac{a_0}{4}\frac{\gamma+1}{\gamma-1}\sqrt{\frac{2R}{\pi M}}\frac{T_2-T_1}{\sqrt{T}}p$$

Two problems are present in this theoretical formula that makes his use almost impossible:

1) identification of the gas temperature T
2) p is the **local** pressure of the residual gas (at the unknown temperature T)

In a typical cryogenic system n the molecular conductive regime, **the pressure gauge is at room temperature, T_{room}, connected to the cryogenic vacuum through a tube of length *l* >> *d*, the tube diameter**

In this condiction the gauge measure the pressure p_{gau} *which* behaves as $p/(T)^{1/2}$ = *constant* [Corrucci R. J. Vacuum 7-8 , 19 (1959)]

$$\dot{Q} = \frac{a_0}{4}\frac{\gamma+1}{\gamma-1}\sqrt{\frac{2R}{\pi M}}\frac{T_2-T_1}{\sqrt{T_{295}}}p_{gau}$$

Summary of measured gas accommodation coefficients with 304 stainless steel

Gas	Machined	Polished
Helium	0.36±0.02	0.40±0.02
Nitrogen	0.80±0.02	0.80±0.02
Argon	0.87±0.02	0.87±0.02

Trott W. M. et al , SAND 2006 – 6432C - American Institute of Aeronautics and Astronautics

Thermal Radiation - I

Radiation heat transfer is the heat transfer via the electromagnetic radiation emitted from one surface, transmitted through the intervening space and absorbed by another surface.

Energy transport of e.m. radiation:

- in vacuum velocity c = 2.99792 x 10^8 m/s ,
- in the material v=c/n with n refraction index
  - ✓ n=1 within less than 0.01% for almost all the gas
  - ✓ n=1.3 – 1.6 for liquids
  - ✓ $n_{He\text{-}gas}$ = 1.000035392 at 273 K , p=1 atm λ =0.5876 μm
  - ✓ $n_{He\text{-}liq}$ = 1.0206$\pm$0.0012 at 4.2 K, λ =0.546176 μm (above λ point , He I)
  - ✓ $n_{He\text{-}liq}$ = 1.0269$\pm$0.0004 at 2.26 K , λ =0.546176 μm (above λ point , He I)
  - ✓ $n_{He\text{-}liq}$ = 1.0269$\pm$0.0004 at 2.18 K, λ =0.546176 μm (below λ point , He II)

} [Burton E. F. , Nature (1937)]

The frequency *f* and corresponding wavelength λ of the electromagnetic waves depends on the temperature of the material emitting the radiant energy:

$$v = \lambda f$$

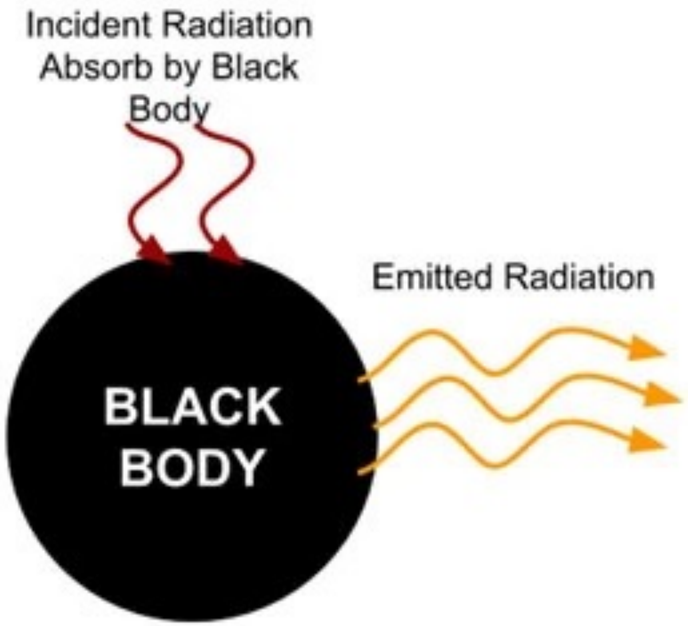
Thermal radiation generally involves λ between about 0.1 and 100 µm ($3<f<300$ THz), we can have contributions in all these three bandwidths

- UV (ultraviolet) → 0.1–0.4 µm
- Visible light wavelength → 0.4 - 0.76 µm
- IR (infrared) → 0.76–100 µm

Thermal Radiation - I

The Black body:

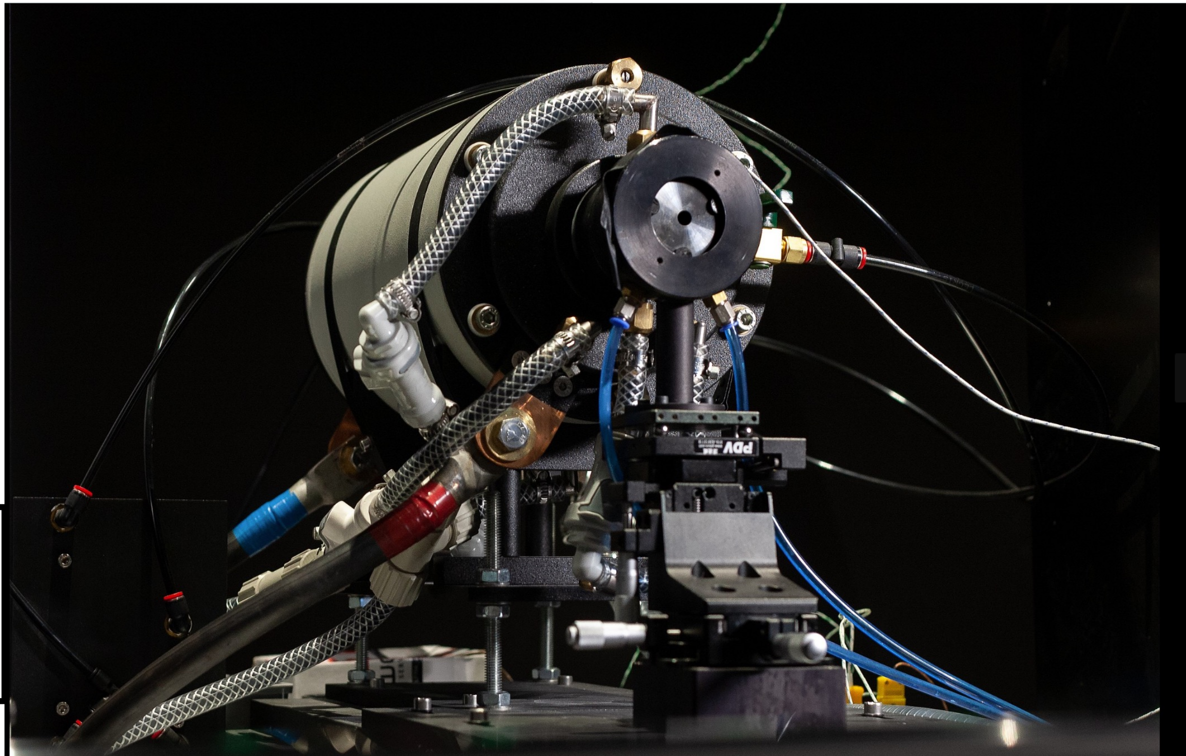
a surface that ***emits the maximum possible amount of radiation at any wavelength for a prescribed temperature***.



A blackbody is

- a **perfect** radiation **absorber**→ it absorbs all of the radiation incident on its surface)
- a **perfect diffuse radiator** → the blackbody radiation emission is independent of direction (no preferred direction of radiation emission)

Black Body radiator used in CARLO laboratory in Poland. It is an approximation of a model described by Planck's law utilized as a spectral irradiance standard.



Thermal Radiation – II -

Spectrum of the black body radiation $W_{\lambda s}$: thermal power per unit surface emitted at a given λ per unitary interval of λ.
It tells us what are the wavlenghts that contribute more to the energy emission

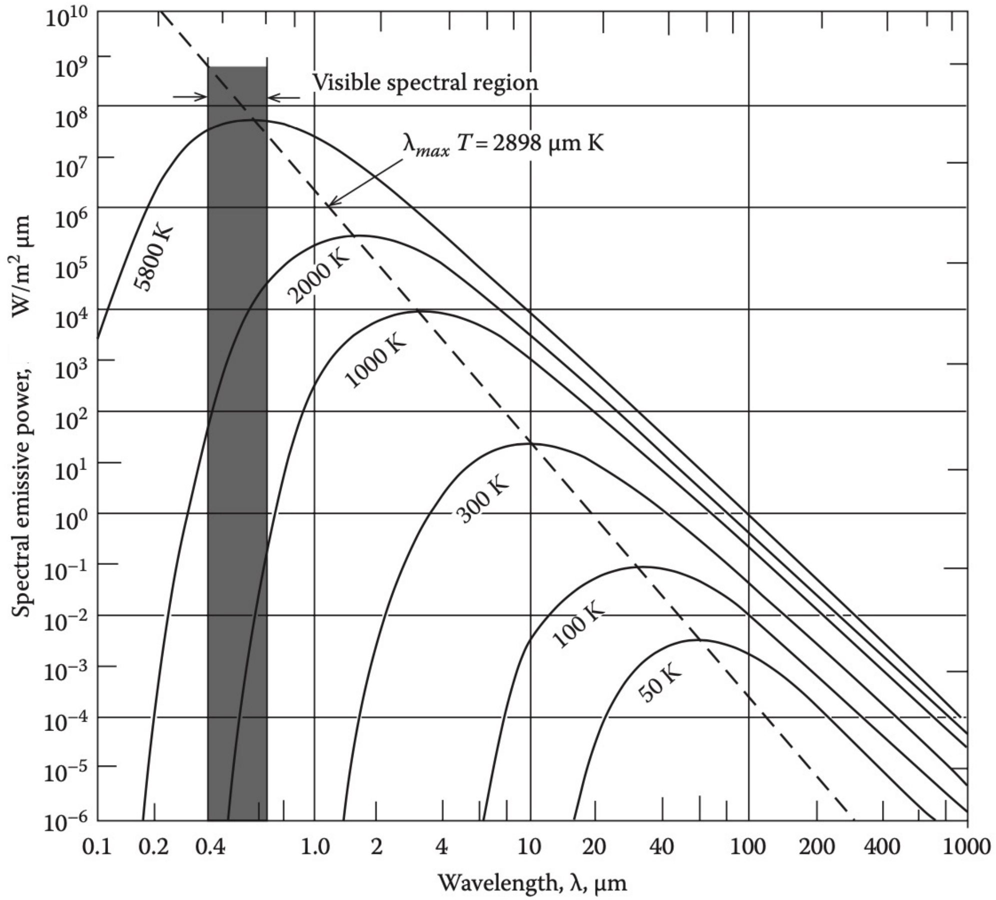
$$W_{\lambda bb}(\lambda, T) = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$$

- $C_1 = 2\pi h_P c^2$ = 3.7415´10^{-16} W m^2
- $C_2 = h_P c/k_B$ =0.014388 m K
- h_P = 6.6256 × 10^{-34} J s = Planck's constant
- k_B = 1.3805 × 10^{-23} J/K = Boltzmann constant

$$W_{bb}(T) = \int_0^\infty W_{\lambda bb}(\lambda, T)\, d\lambda = \sigma T^4$$

Stefan–Boltzmann equation

$$\sigma = \frac{2\pi^5 k_B^4}{15 c_0^2 h_P^3} = \frac{C_1}{15}\left(\frac{\pi}{C_2}\right)^4$$ =56.60x10^{-9} W/(m^2 K^4)



Wein displacement *law* ➔ $\lambda_{max} T = C_3$ = 2897.8 μm K

Examples:

300 K ➔ λ_{max}= 9.66 μm

4.2 K ➔ λ_{max}= 690 μm

Thermal Radiation - III

Nature is more complicated: **most materials that we use, do not behave as blackbodies!**
Additional material properties should be defined!

Emissivity e_λ ➔ ratio of the actual surface density power $W_e(\lambda, T)$ emitted from a surface to the surface density power $W_{\lambda\, bb}(\lambda, T)$ emitted by a blackbody at the same temperature T and **at wavelenght λ** ➔ $e_\lambda = W_e(\lambda, T) / W_{\lambda\, bb}(\lambda, T)$

Absorptivity α_λ ➔ ratio of the power absorbed by a surface to the power incident on the surface at wavelenght λ .

Reflectivity r_λ ➔ ratio of the power reflected from the surface to the power incident on the surface at wavelenght λ .

Transmissivity t_λ ➔ ratio of the power transmitted through a material to the power incident on the surface at wavelenght λ

Kirchhoff relation ➔ $e_\lambda = \alpha_\lambda$ for any material at a given temperature!

Total Power per unit surface W(T) emitted by the body at a given temperature T $$W(T) = \int_o^\infty e_\lambda W_{\lambda bb}(\lambda, T) d\lambda$$

Total emissivity $$e = \frac{W(T)}{\sigma T^4}$$

Thermal Radiation - IV

Total absorptivity

$$\alpha = \frac{G_{abs}}{G} = \frac{\int_0^\infty \alpha_\lambda G_\lambda(T_{inc})d\lambda}{\int_0^\infty G_\lambda(T_{inc})d\lambda}$$

G_λ ➔ incident power per unit surface, a function of the temperature T_{inc} of the source of the incident radiation

In general, the radiation properties are also functions of the direction of the emitted or incident radiation.
For angles of incidence < 60° (measured from the normal to the surface), radiation properties relatively independent of direction for both electrical conductors and electrical insulators

Grey surfaces ➔ **their radiation properties are independent of wavelength and direction (diffusion).**

absorptivity = emissivity

(at the temperature of the radiation incident on the surface)

From an energy balance applied to the surface:

$$\alpha + r + t = 1$$

For ***opaque bodies (t = 0)***

$$\alpha + r = 1$$

Thermal Radiation - V

e ➔ function of the electrical resistance of the material.

Parker and Abbot (1964) suggested the following correlation between ***e*** and the electrical resistivity r_e in [Ω cm] and T [K]

$$e = 0.766\sqrt{r_e T} - 0.0175(r_e T)^{3/2} - [0.309 - 0.0889\ln(r_e T)](r_e T)$$

[Parker and Abbot *Theoretical and experimental studies of the total emittance of metals. Symposium on Thermal Radiation to Solids*, pp. 11–28. NASA SP-55. Washington, DC: Government Printing Office 1964]

Example 304 stanless steel

T= 80 K , $r_e = 52 \times 10^{-6}\ \Omega\,\mathrm{cm}$

$e = 0.049406 - 4.70 \times 10^{-6} - 0.003313 = 0.0461$

40% of the measured value!

Lesson learned: the actual surface condition is extremely important in determining the emissivity of a surface!

Total Emissivity for Selected Materials

Material	Emissivity	Temp, K
3M black paint (80 μm thick) on copper substrate	0.892	4.2
	0.896	20
	0.910	77
	0.935	300
Aluminum, polished (2024-T6) (33 μin. roughness)	0.018	4.2
	0.018	20
	0.023	77
	0.050	300
Copper, polished (41 μin. roughness)	0.054	4.2
	0.055	20
	0.070	77
	0.100	300
304 Stainless steel, polished (27 μin. roughness)	0.078	4.2
	0.087	20
	0.13	77
	0.17	300
Aluminum, mechanical polish	0.058	4.2
	0.10	77
Aluminum, oxide layer	0.074	4.2
	0.49	77
Copper, as received	0.062	4.2
	0.12	77
304 Stainless steel, mech. polish	0.074	4.2
	0.12	77
	0.16	300
304 Stainless steel, as received	0.12	4.2
	0.34	77
Aluminum coating (79 nm thick) on 6 μm polyester film	0.021	307
Aluminum coating (38 nm thick) on both sides of Mylar film	0.025	307
Copper coating (68 nm thick) on 6 μm polyester film	0.013	307
Silver coating (76 nm thick) on 6 μm polyester film	0.0133	307

Thermal Radiation Transport - I

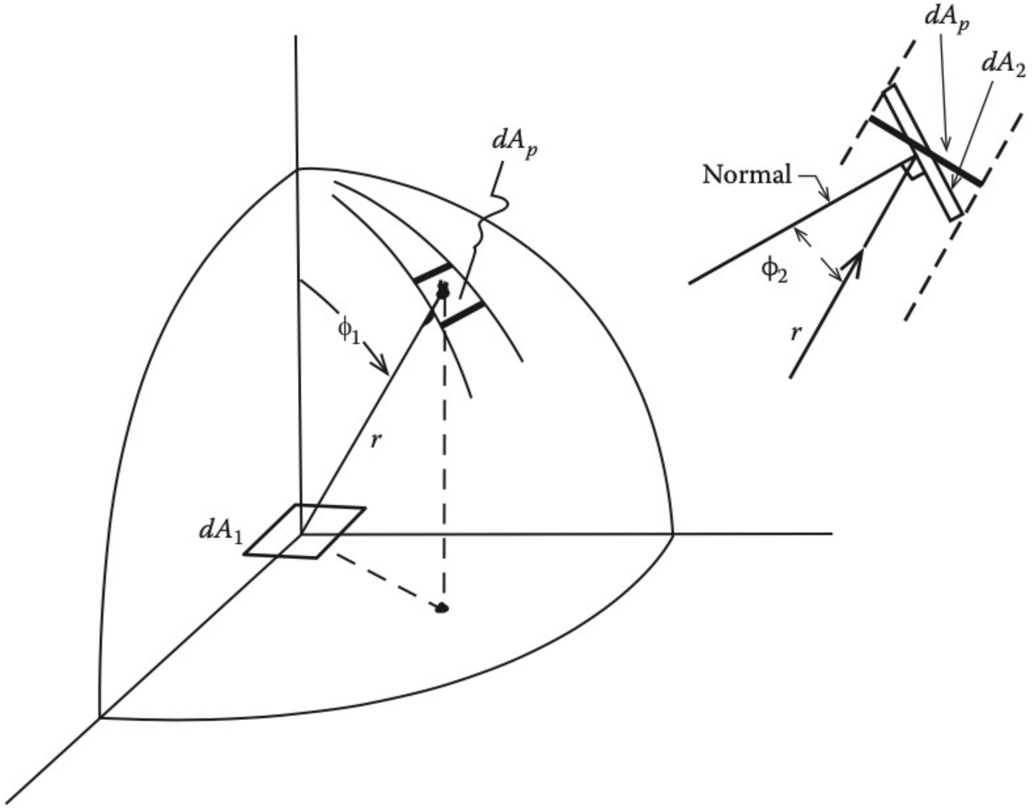
When two surfaces exchange radiant energy, some of the energy radiated from one surface may not be intercepted by the second surface.

To take into account this problem, we define a geometrical paramenter, which dependes on the specif configuration considered

The configuration factor $F_{ij} = \dfrac{\textit{Radiation leaving surface i that goes directly to surface j}}{\textit{Total radiation leaving surface i}}$

surface dA_1 ➔emitter ; surface 2 ➔ receiver

$$F_{d1\rightarrow 2} = \int_{A_2} \frac{\cos\phi_1 \cos\phi_2}{\pi r^2} dA_2$$



Thermal Radiation Transport - II

Two properties of the fconfiguration factor:

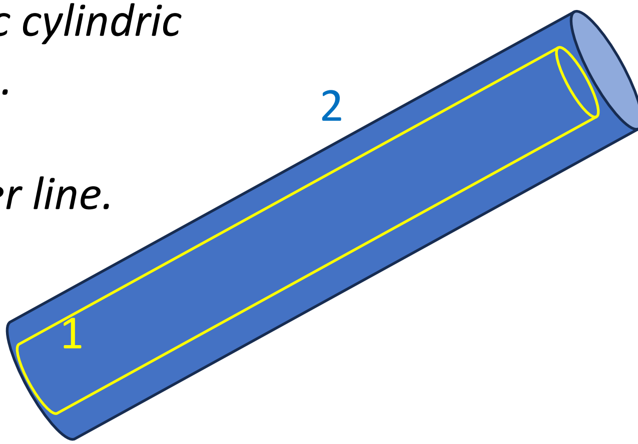
- the reciprocity rule ➔ $F_{i,j} A_i = F_{j,i} A_j$

Example: *a long vacuum-jacketed transfer line (L is the length) is composed by two concentric cylindric tubes. The surface n.1 is the inner one of diameter D_1 , the outer of diameter D_2) is surface 2.*

In this case we have $F_{1,2} = 1$ because all of the radiation leaving the inner line strikes the outer line.

Then

$F_{2,1} = F_{1,2} (A_1/A_2) = (\pi L D_1/\pi L D_2) = (D_1/D_2)$

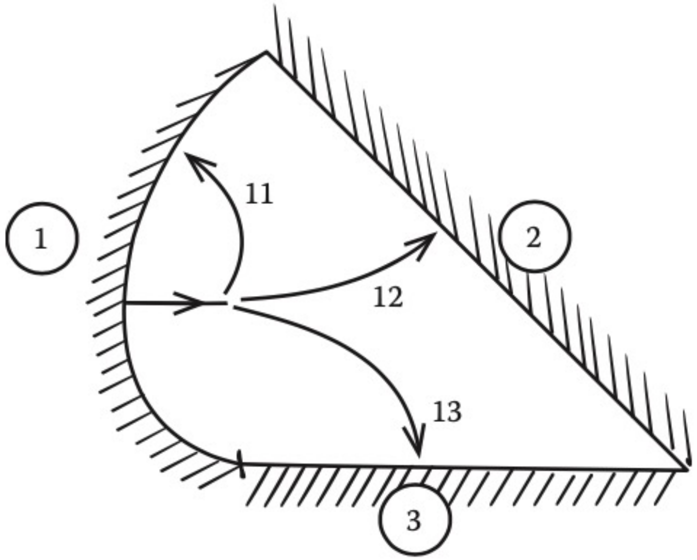


- the enclosure rule (or summation rule)

$$\sum_{j=1}^{N} F_{i,j} = 1 \quad \text{for } i = 1, 2, \ldots, N$$

The radiant energy leaving surface 1 must either strike itself (surface 1) or the other surfaces (surface 2 and 3).

$$F_{1,1} + F_{1,2} + F_{1,3} = 1$$



Thermal Radiation Transport - III

The configuration factor for radiant heat transfer between two finite-size areas

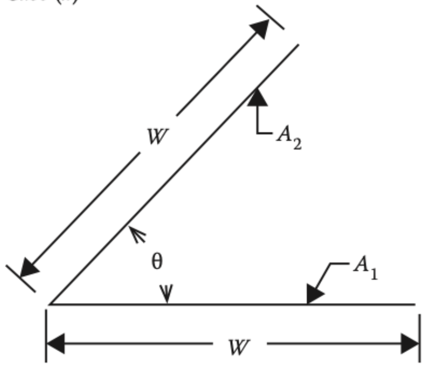
$$A_1 F_{12} = \int_{A1} F_{d1-2} dA_1 = \int_{A1}\int_{A2} \frac{\cos\phi_1 \cos\phi_2 dA_1 dA_2}{\pi r^2}$$

$$F_{12} = \frac{1}{A_1}\int_{A1}\int_{A2} \frac{\cos\phi_1 \cos\phi_2 dA_1 dA_2}{\pi r^2}$$

The evaluation of the integrals often requires numerical integration !

Simplest case: two infinite parallel surfaces ➔ ***F_{ij}=1***

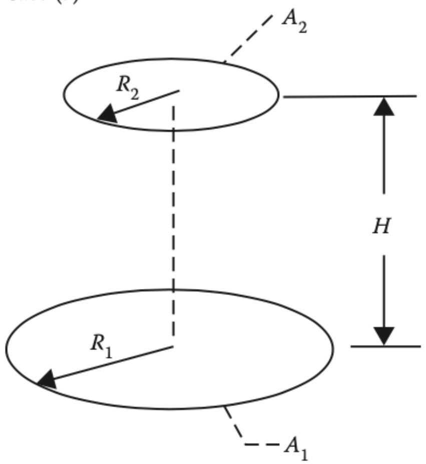
Case (a)



Two infinitely long plates of equal width W, having an included angle θ

$$F_{1,2} = 1 - \sin\left(\tfrac{1}{2}\theta\right)$$

Case (b)

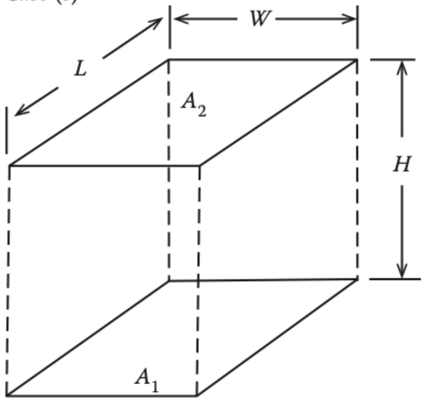


Parallel circular disks with centers along the same normal, spaced a distance H apart.

$$X = 1 + \frac{H^2 + R_2^2}{R_1^2}$$

$$F_{1,2} = \tfrac{1}{2}\left[X - \sqrt{X^2 - 4\left(\frac{R_2}{R_1}\right)^2}\right]$$

Case (c)



Identical parallel directly opposed rectangles spaced a distance H apart

$$X = \frac{L}{H}; \quad Y = \frac{W}{H}$$

$$Z_1 = \sqrt{\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}}$$

$$Z_2 = X\sqrt{1+Y^2}\tan^{-1}\left[\frac{X}{\sqrt{1+Y^2}}\right] + Y\sqrt{1+X^2}\tan^{-1}\left[\frac{Y}{\sqrt{1+X^2}}\right]$$

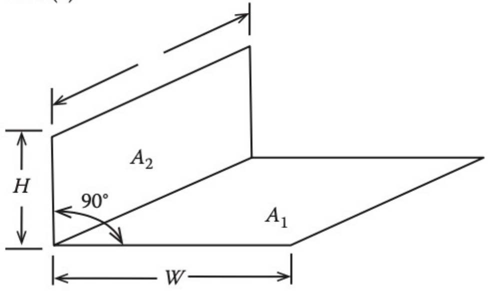
$$Z_3 = X\tan^{-1}(X) + Y\tan^{-1}(Y)$$

$$F_{1,2} = \left(\frac{2}{\pi XY}\right)\left[\ln(Z_1) + Z_2 - Z_3\right]$$

(Continued)

Configuration Factor Expressions

Case (d)



Two rectangles having a common side L with an included angle of 90°

$$X = \frac{H}{L}; \quad Y = \frac{W}{L}$$

$$Z_1 = Y\tan^{-1}\left(\frac{1}{Y}\right) + X\tan^{-1}\left(\frac{1}{X}\right) - \sqrt{X^2+Y^2}\tan^{-1}\left(\frac{1}{\sqrt{X^2+Y^2}}\right)$$

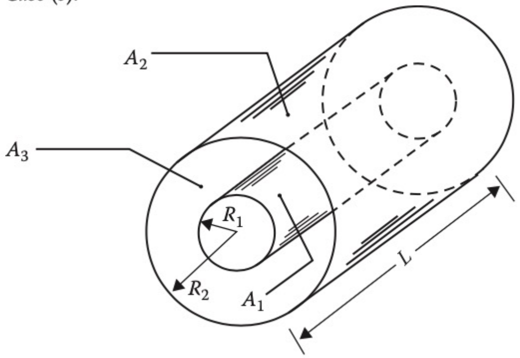
$$Z_2 = \frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}$$

$$Z_3 = \frac{Y^2(1+X^2+Y^2)}{(1+Y^2)(X^2+Y^2)}$$

$$Z_4 = \frac{X^2(1+X^2+Y^2)}{(1+X^2)(X^2+Y^2)}$$

$$F_{1,2} = \left(\frac{1}{4\pi Y}\right)\left[4Z_1 + \ln(Z_2) + Y^2\ln(Z_3) + X^2\ln(Z_4)\right]$$

Case (e):



Two concentric cylinders of length L. The inner cylinder is surface 1; the outer cylinder is surface 2; and one end annular disk is surface 3.

$$X = \frac{L}{R_1}; \quad Y = \frac{R_2}{R_1}$$

$$Z_1 = X^2 + Y^2 - 1$$

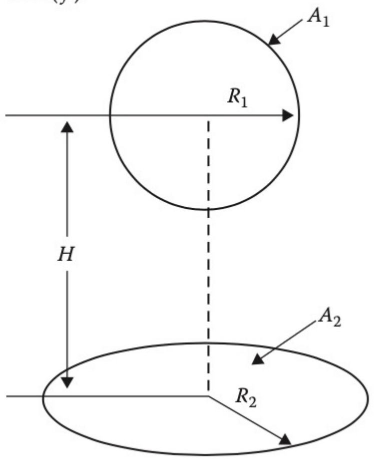
$$Z_2 = X^2 - Y^2 + 1$$

$$Z_3 = \sqrt{(Z_1+2)^2 - 4Y^2}\cos^{-1}\left(\frac{Z_2}{Z_1 Y}\right) + Z_2\sin^{-1}\left(\frac{1}{Y}\right) - \tfrac{1}{2}\pi Z_1$$

$$F_{1,3} = \left(\frac{1}{2\pi}\right)\left[\cos^{-1}\left(\frac{Z_2}{Z_1}\right) - \left(\frac{Z_3}{2X}\right)\right]$$

$$F_{1,2} = 1 - F_{1,3}$$

Case (f)



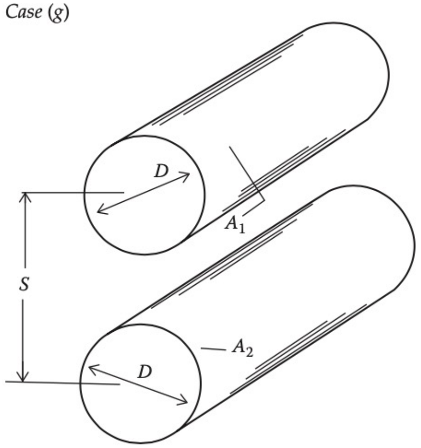
Sphere of radius R_1 to disk of radius R_2 with the normal to the disk passing through the center of the sphere. Distance between the sphere center and the disk is H.

$$X = \frac{R_2}{H}$$

$$F_{1,2} = \tfrac{1}{2}\left(1 - \frac{1}{\sqrt{1+X^2}}\right)$$

Configuration Factor Expressions

Case (g)



Two very long parallel cylinders with centers spaced a distance S apart.

$$X = \frac{S}{D}$$

$$F_{1,2} = F_{2,1} = \frac{1}{\pi}\left[\sqrt{X^2 - 1} + \sin^{-1}\left(\frac{1}{X}\right) - X\right]$$

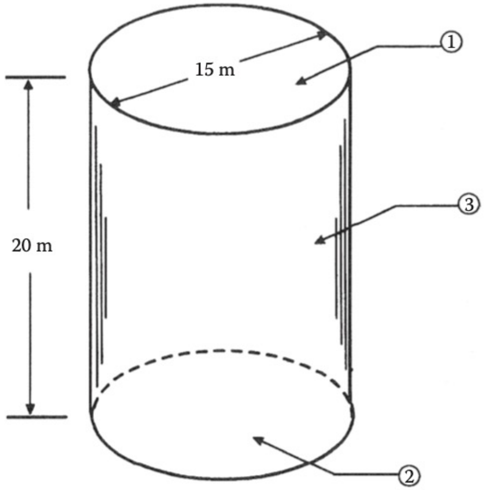


FIGURE 8.6
Space environmental chamber for Example 8.5.

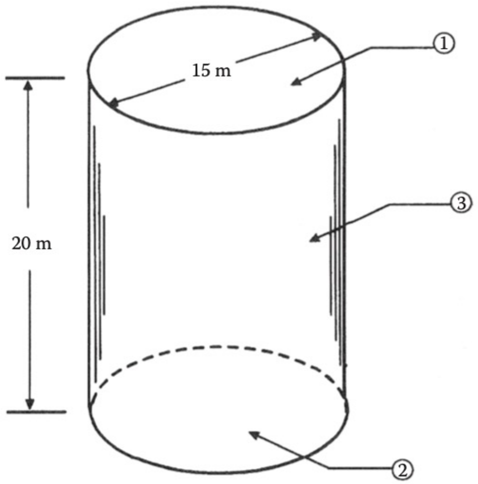
We may use the reciprocity rule to determine the configuration factor $F_{3,1}$ (the fraction of the energy leaving the side surface that strikes the top surface):

$$F_{3,1} = \left(\frac{A_1}{A_3}\right)F_{1,3} = \left(\frac{\frac{1}{4}\pi D^2}{\pi DL}\right)F_{1,3} = \left(\frac{D}{4L}\right)F_{1,3}$$

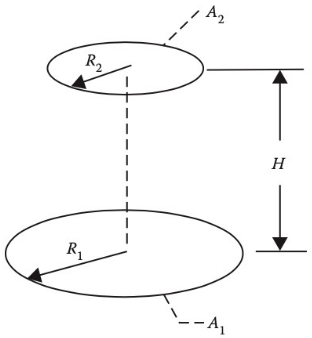
$$F_{3,1} = \left[\frac{(15)}{(4)(20)}\right](0.8889) = 0.1667 = \frac{1}{6}$$

Example:

In this cylindric enclosure we need to compute the configuration factor F_{13}



- The top surface (1) cannot "see" any part of itself ➔ F_{11}=0
- Configuration factor $F_{1,2}$ from the top (surface 1) to the bottom (surface 2) –> *case (b) of our table*



$$X = 1 + \frac{H^2 + R_2^2}{R_1^2} = 1 + \frac{(20)^2 + (7.50)^2}{(7.50)^2} = 9.111$$

$$F_{1,2} = \frac{1}{2}\left[9.111 - \sqrt{(9.111)^2 - (4)\left(\frac{7.50}{7.50}\right)^2} = 0.1111 = \frac{1}{9}\right]$$

- We apply the summation rule to the enclosure

$$F_{1,1} + F_{1,2} + F_{1,3} = 1 = 0 + 0.1111 + F_{1,3}$$

$$F_{1,3} = 1 - 0.1111 = 0.8889 = \frac{8}{9}$$

Radiation Heat Exchange between Grey surfaces

The black bodies absorb the entire incident radiation and this aspect makes the calculation procedure of heat exchange between black bodies simpler.

- One has to only determine the view factor.

- Real surfaces do not absorb the whole of the incident radiation: a part is reflected back to the radiating surface.

For non-grey-body the computation is even more complex: the absorptivity and emissivity are not uniform in all directions and for all wavelengths.

Radiation heat flux between Grey surfaces – parallel surface case - I

Surface 1 emits E_1

Surface 2 absorbs $E_1\alpha_2$

Surface 2 reflects $E_1(1-\alpha_2)$

Surface 1 absorbs $E_1(1-\alpha_2)\alpha_1$

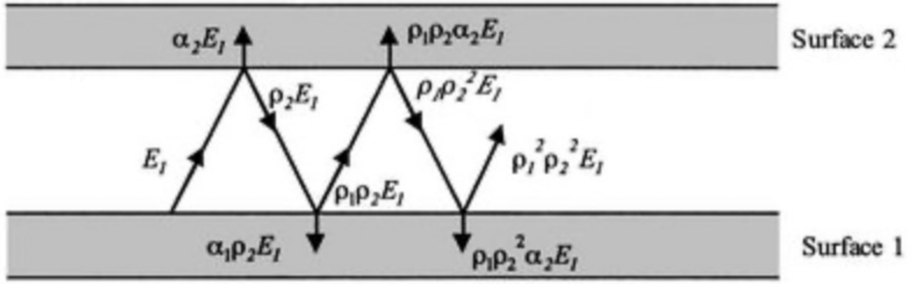
Surface 1 reflects $E_1(1-\alpha_2)(1-\alpha_1)$

Surface 2 absorbs $E_1(1-\alpha_2)(1-\alpha_1)\alpha_2$

Surface 2 reflects $E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2)$

Surface 1 absorbs $E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2)\alpha_1$

...



Surface 2 emits E_2

Surface 1 absorbs $E_2\alpha_1$

Surface 1 reflects $E_2(1-\alpha_1)$

Surface 2 absorbs $E_2(1-\alpha_1)\alpha_2$

Surface 2 reflects $E_2(1-\alpha_1)(1-\alpha_2)$

...

Adding all the contributions absorbed in 1

$$E_1(1-\alpha_2)\alpha_1 + E_1(1-\alpha_2)\alpha_1(1-\alpha_2)(1-\alpha_1) + \ldots \xrightarrow{\beta=(1-\alpha_1)(1-\alpha_2)} E_1(1-\alpha_2)\alpha_1(1+\beta+\beta^2+\ldots). \Rightarrow \frac{E_1(1-\alpha_2)\alpha_1}{1-\beta}$$

and those generated and absorbed in 2 ➡

$$E_2 - \left(\frac{E_2(1-\alpha_1)\alpha_2}{1-\beta}\right) = \frac{E_2\alpha_1}{1-\beta}$$

The net heat flux from 1 to 2

$$E_1 - \frac{E_1(1-\alpha_2)\alpha_1}{1-\beta} - \frac{E_2\alpha_1}{1-\beta} \Rightarrow \frac{E_1\alpha_2 - E_2\alpha_1}{\alpha_1+\alpha_2-\alpha_1\alpha_2}.$$

Radiation heat flux between Grey surfaces – parallel surface case - II

Being

Kirchoff's law
$e_1 = \alpha_1$ and $e_2 = \alpha_2$

$E_1 = e_1 \, \sigma T_1^4$

$E_2 = e_2 \, \sigma T_2^4$

The final expression for heat transfer between grey, planar, surfaces is

$$\dot{Q}_{12}^{rad} = \frac{e_1 \sigma T_1^4 - e_2 \sigma T_2^4}{e_1 + e_2 - e_1 e_2} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

Radiant Exchange between two generic grey surfaces- I

Grey surface ➔ e_λ independent of wavelength + diffused emission (e_λ independent of direction of the radiation).

The radiant heat exchange is derived from the Stefan–Boltzmann equation, modified by few factors:

- the **configuration factor**, F_{12}, which accounts for the fact that not all of the radiation leaving one surface is intercepted by the second surface,

- a **total emissivity factor** *Fe* accounting that the surface is not a blackbody (total emissivity of the surface less than 1), and the radiation heat transfer involves reflections and absorptions between the two surfaces (see the parrallel case)

The heat power $\dot{Q}$ exchanged **by surface A_1 at T_1** with surface A_2 at T_2 can be written as

$$\dot{Q} = F_e F_{1,2} \sigma A_1 (T_2^4 - T_1^4)$$

Radiant Exchange between two grey surfaces- III

Emissivity Factor Expressions F_e

Case (*a*): Small body (1) enclosed by a much larger surface (2).	$F_e = e_1$
Case (*b*): Infinite parallel flat plates	$\dfrac{1}{F_e} = \dfrac{1}{e_1} + \dfrac{1}{e_2} - 1$
Case (*c*): Long concentric cylinders or concentric spheres. Subscript (1) denotes the enclosed surface.	$\dfrac{1}{F_e} = \dfrac{1}{e_1} + \left(\dfrac{A_1}{A_2}\right)\left(\dfrac{1}{e_2} - 1\right)$
Case (*d*): Two gray surfaces connected by a reradiating non-conducting surface (Based on the area A_1).	$\dfrac{1}{F_e F_{1,2}} = \dfrac{1}{e_1} - 1 + \left(\dfrac{A_1}{A_2}\right)\left(\dfrac{1}{e_2}\right) + \dfrac{1}{F_0}$ where $F_0 = \dfrac{1 - \left(\dfrac{A_1}{A_2}\right) F_{12}^2}{1 + \left(\dfrac{A_1}{A_2}\right)(1 - 2F_{12})}$

Radiant Exchange between two Gray Surfaces- IV

Example:

Long transfer line

D_1= 114 mm , e_1 =0.12 , T_1 =100 K

D_2 = 164 mm , e_2 =0.16 , T_2 = 320 K

Determine the radiant power transferred per unit length !

- Configuration factor $F_{1,2} = 1$: *the radiation from the inner surface (1) to the outer one (2) is unity because all of the radiation leaving the inner line strikes the outer line surface.*

- For the emissivity factor we use formula (c) of the table reported in previous slide

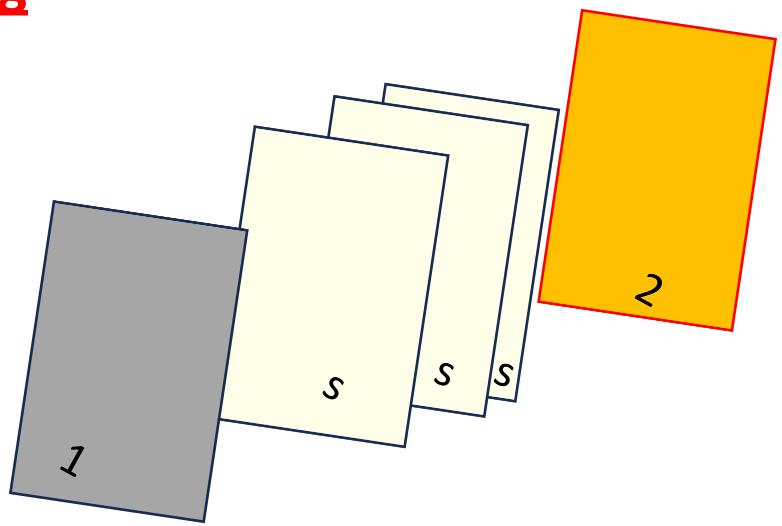
$$\frac{1}{F_e} = \frac{1}{e_1} + \left(\frac{A_1}{A_2}\right)\left(\frac{1}{e_2} - 1\right) \longrightarrow \frac{1}{F_e} = \frac{1}{0.12} + \left(\frac{114}{161}\right)\left(\frac{1}{0.16} - 1\right) = 12.051 \longrightarrow \boxed{F_e = 0.0830}$$

- The radiation heat transfer rate per unit area of the inner line is $\dot{Q} = F_e F_{1,2} \sigma A_1 (T_2^4 - T_1^4)$

$$\frac{\dot{Q}}{A_1} = (1)(0.0830)(56.69\times10^{-9})(320^4 - 100^4) = 48.86\ \mathrm{W/m^2} \longrightarrow \frac{\dot{Q}}{L} = \left(\frac{\dot{Q}}{A_1}\right)\left(\frac{A_1}{L}\right) = (48.86)(\pi)(0.114) = \boxed{7.50\ \mathrm{W/m}}$$

How to Reduce the Radiant Exchange between two Gray Surfaces- I

Radiation heat transfer may be reduced by interposing floating shields of highly reflective material between the hot and cold surfaces *(Assumption: No contact among all the surfaces!!)*.



$$\frac{\dot{Q}}{A} = F_{e1}\sigma(T_{s1}^4 - T_1^4)$$

$$\frac{\dot{Q}}{A} = F_{e2}\sigma(T_{s2}^4 - T_{s1}^4)$$

$$\cdots$$

$$\frac{\dot{Q}}{A} = F_{e(N+1)}\sigma\left(T_2^4 - T_{sN}^4\right)$$

e_s emissivity of the shields

e_1 and e_2 boundary emissivities

$$T_2^4 - T_1^4 = \frac{\dot{Q}}{\sigma A}\left(\frac{1}{F_{e1}} + \frac{1}{F_{e2}} + \cdots + \frac{1}{F_{e(N+1)}}\right) = \frac{\dot{Q}}{F_e \sigma A}$$

$$\frac{1}{F_e} = \left(\frac{1}{e_1} + \frac{1}{e_S} - 1\right) + (N-1)\left(\frac{2}{e_S} - 1\right) + \left(\frac{1}{e_S} + \frac{1}{e_2} - 1\right)$$

How to Reduce the Radiant Exchange between two Gray Surfaces- II

Example:

Boundary surfaces with emissivities $e_1 = e_2 = 0.80$

10 shields ($N = 10$) having emissivities of $e_S = 0.05$

$$\frac{1}{F_e(\text{no shields})} = \frac{1}{0.80} + \frac{1}{0.80} - 1 = 1.500$$

$$F_e(\text{no shields}) = 0.6667 = \frac{2}{3}$$

$$\frac{1}{F_e(10\text{ shields})} = (2)\left(\frac{1}{0.80} + \frac{1}{0.05} - 1\right) + (10-1)\left(\frac{2}{0.05} - 1\right)$$

$$F_e(10\text{ shields}) = \frac{1}{391.5} = 0.002554$$

Radiant heat transfer rate reduced by a factor of (0.002554/0.6667) = 0.00383 = 1/261 !!!

M.L.I. physics principle !