



ELTE  
EÖTVÖS LORÁND  
TUDOMÁNYEGYETEM

# Pion femtoscopy at NA61/SHINE

Barnabás Pórfy

Eötvös Loránd University, HUN-REN Wigner RCP



University of Bologna, CosmicAntiNuclei Seminar  
2024.09.24

# The experiment

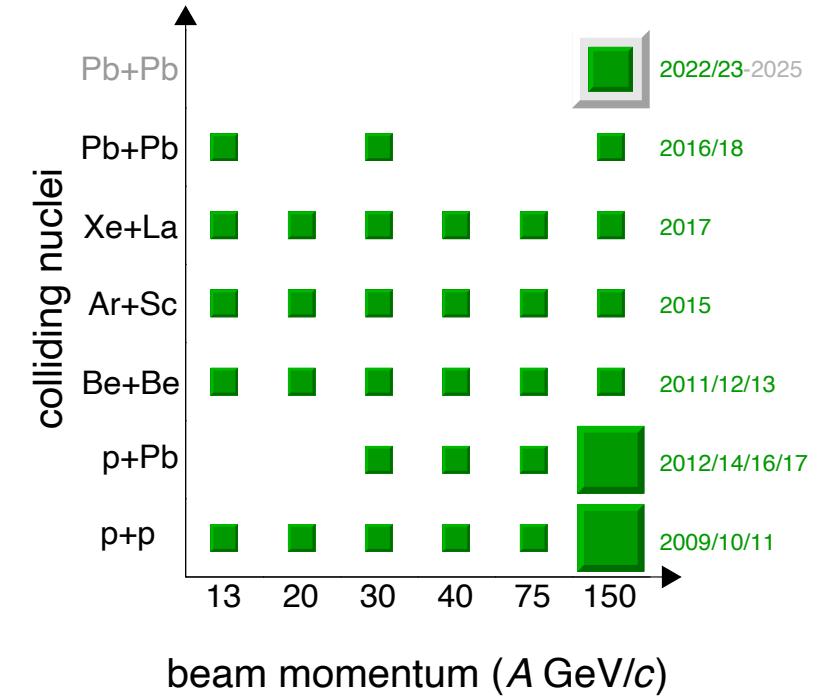
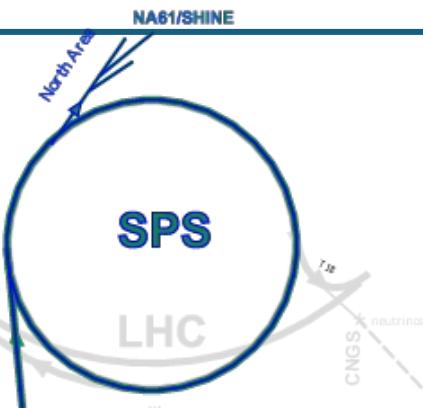
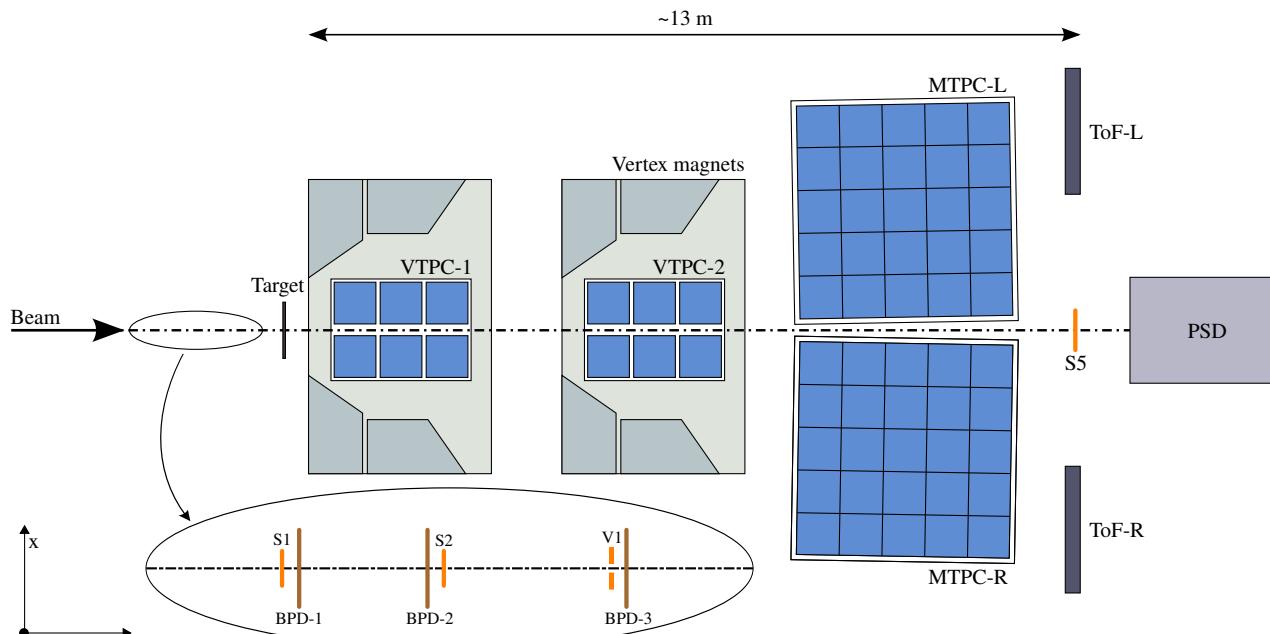
## NA61/SHINE, CERN, SPS

- Strong interaction
- Neutrino measurements
- Cosmic ray measurements

## Fixed target

## Various systems, various energies

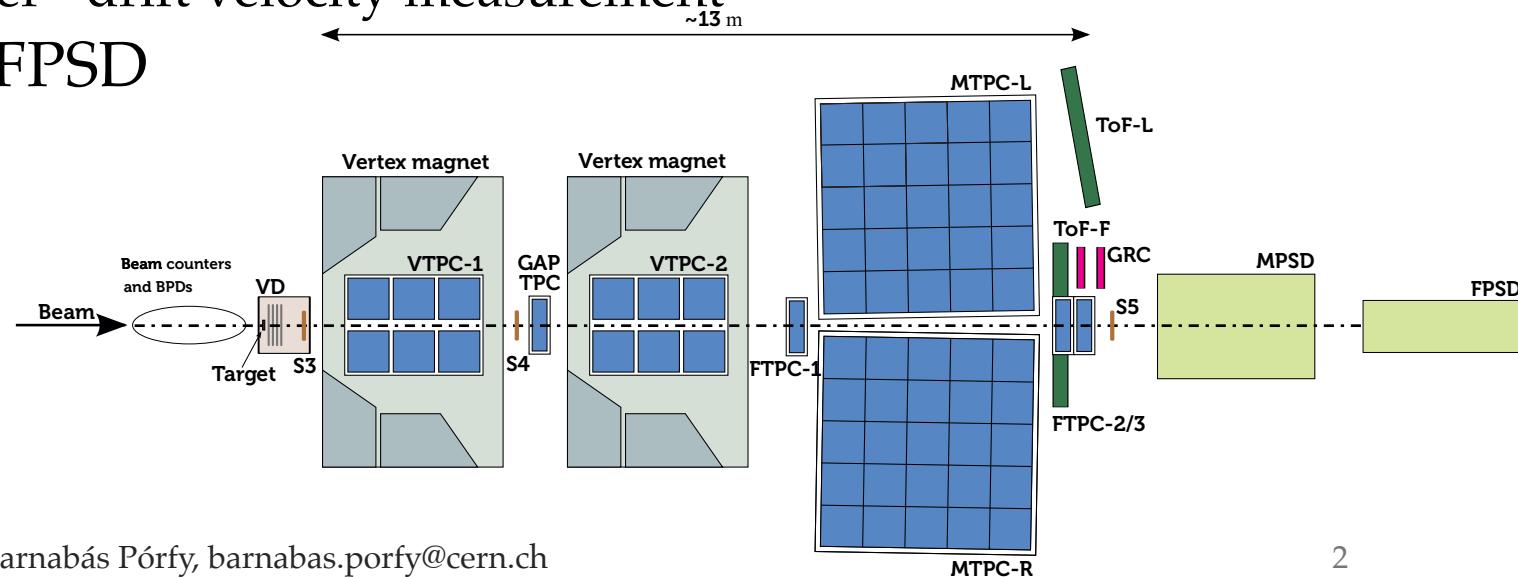
## After LS2 ~ 20x faster data taking rate (readout from ALICE ☺)



$$\sqrt{s_{\text{NN}}} = 5.3, 6.3, 7.7, 8.8, 11.9, 16.8$$

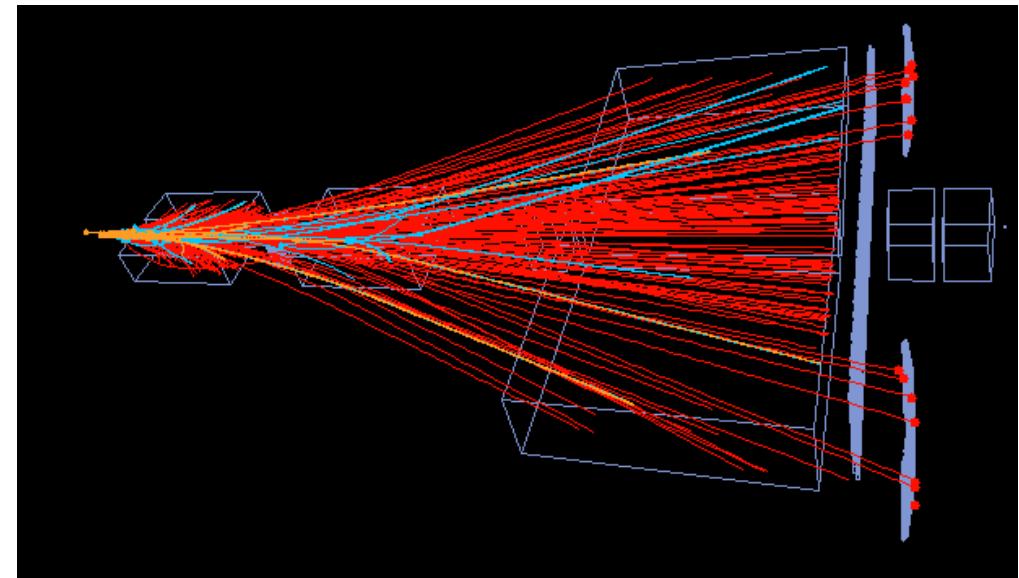
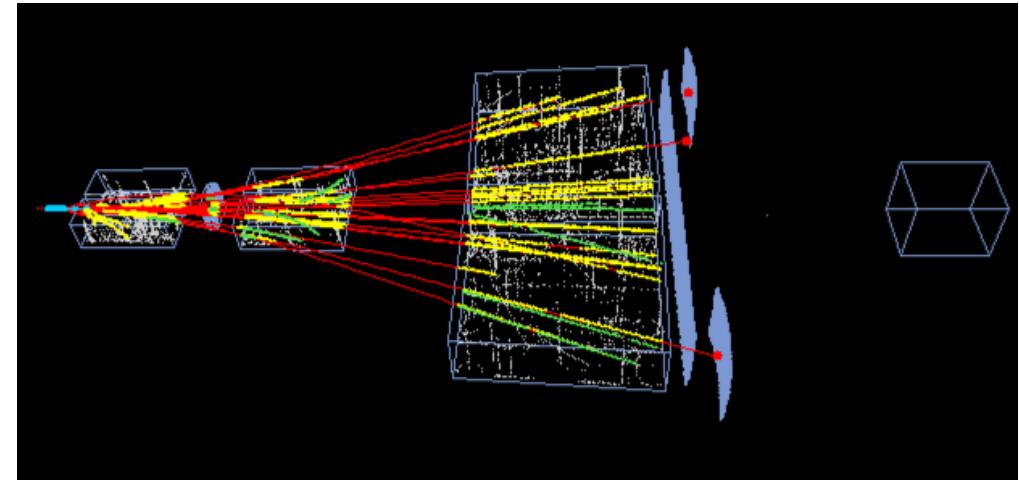
# NA61 after the LS2

- Upgrade of DAQ + new trigger system (TDAQ)
  - Detector readouts replaced → data taking rate increase 20x
  - TPCs - ALICE (frontends, readout control units); other detectors - DRS4
- Construction of:
  - Vertex Detector - open-charm measurements
  - Long Target Tracker – neutrino program
  - ToF-F wall
  - Multi-gap Resistive Plate Chamber based ToF-L (ToF-R under construction)
  - Beam Position Detector
  - Geometry Reference Chamber - drift velocity measurement
- Upgrade of PSD: MPSD + FPSD



# Example event

- Event sample in
    - Be+Be at  $150A$  GeV/c – 2011
    - Ar+Sc at  $150A$  GeV/c – 2015
- $\rightarrow \sqrt{s_{NN}} \approx 16.8$  GeV  
 $\sim 3M$  events



# Lévy approach – a summary

Based on experimental results: Gaussian not describe data well

Generalized central limit theorem  
Lévy distribution (stable, symmetric):

$$\mathcal{L}(\alpha, R; r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

- Generalization of Gauss, power-law  $\sim r^{-(d-2+\alpha)}$
- $\alpha = 2$  Gauss,  $\alpha = 1$  Cauchy
- Momentum correlation related to source  $S(x)$ :

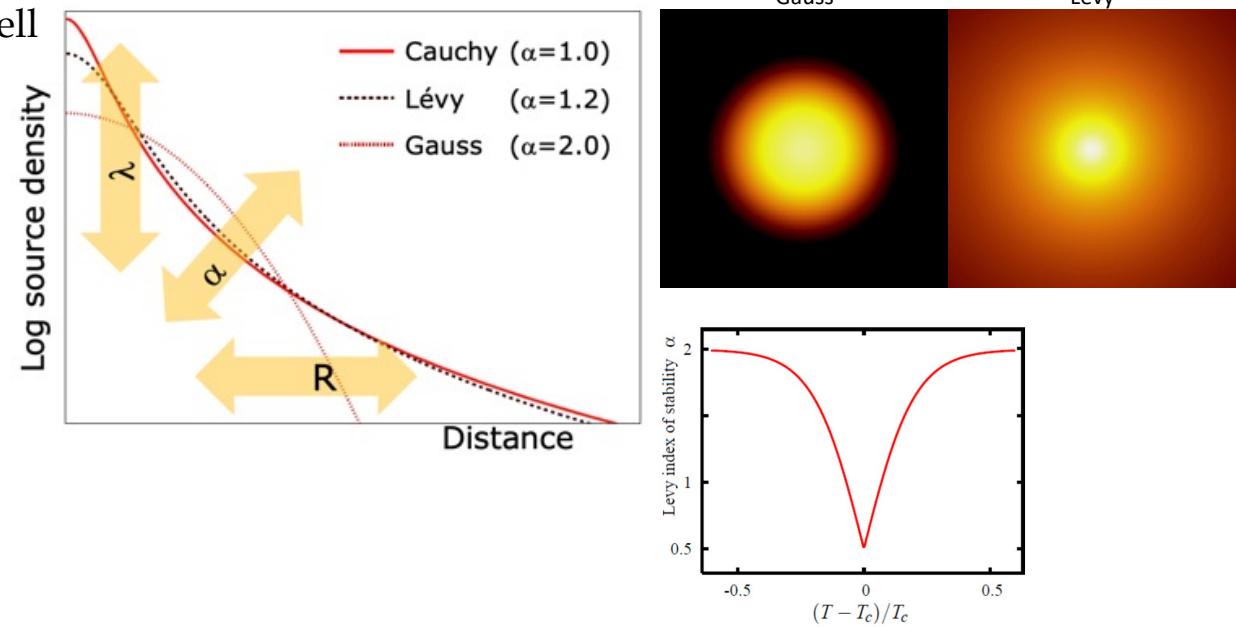
$$C(q) \cong 1 + \lambda \cdot |\tilde{S}(q)|^2$$

- General approach: Gauss  $\rightarrow$  Lévy shaped source
$$C(q) = 1 + \lambda \cdot e^{-(|qR|)^{2 \rightarrow \alpha}}$$
- Actual and mixed pair distribution  $\rightarrow C(q) = \frac{A(q)}{B(q)} \cdot N$
- QCD universality class  $\leftrightarrow$  3D Ising

Halasz et al., Phys.Rev.D58 (1998) 096007,  
Stephanov et al., Phys.Rev.Lett.81 (1998) 4816

- Spatial correlation:  $\sim r^{-(d-2+\eta)}$   
Lévy source:  $\sim r^{-(d-2+\alpha)}$ ;  $\alpha \Leftrightarrow \eta$ ?  
Csörgő, Hegyi, Novák, Zajc, AIP Conf. Proc. 828 (2006) 525-532
- At CEP:

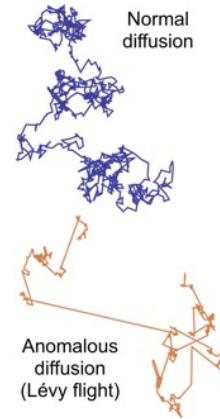
- Random field 3D Ising:  $\eta = 0.50 \pm 0.05$   
Rieger, Phys.Rev.B52 (1995) 6659
- 3D Ising:  $\eta = 0.03631(3)$   
El-Showk et al., J.Stat.Phys.157 (4-5): 86



Possible reasons for the appearance of Lévy:

QCD jets; Anomalous diffusion; Critical behavior; ...

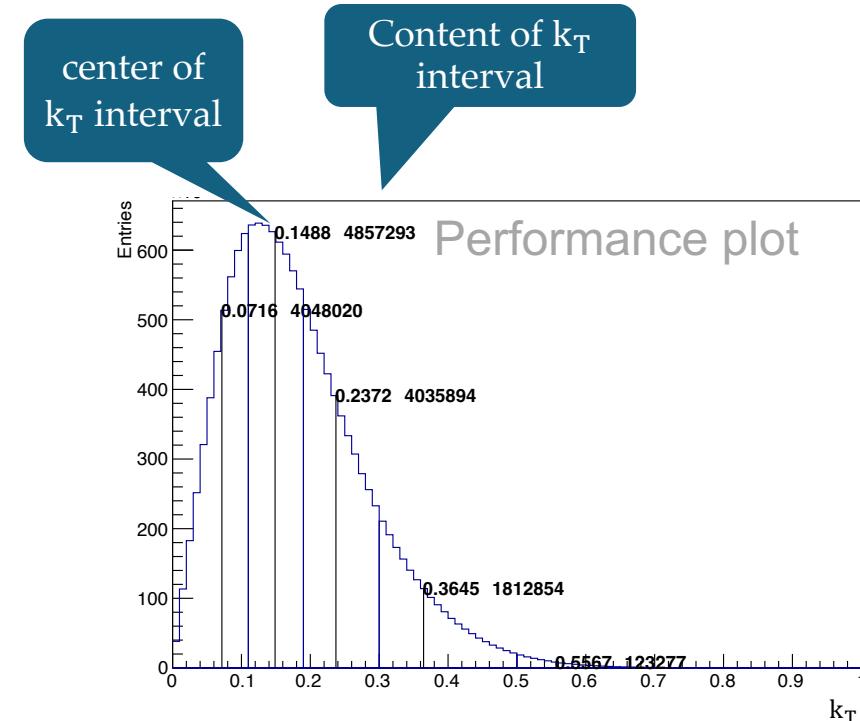
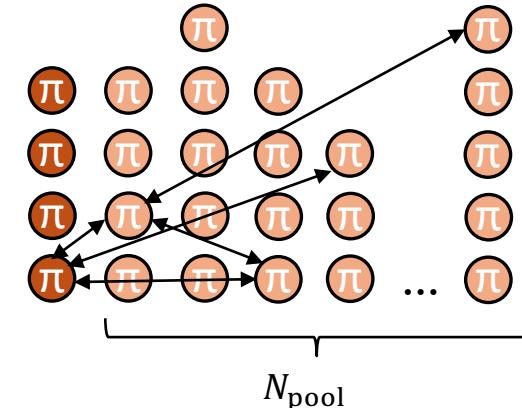
Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon.B36 (2005) 329-337  
Csanád, Csörgő, Nagy, Braz.J.Phys.37 (2007) 1002  
Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc.828 (2006) 525-532  
Metzler, Klafter, Physics Reports 339 (2000) 1-77  
Kincses, Stefaniak, Csanád, Entropy 24 3 (2022) 308  
Kórodi, Kincses, Csanád, Phys.Lett.B 847 (2023)



# Usual problems, questions

## Usual check-list and questions

- 1D or 3D? (Usually decided before...)
- Maximum multiplicity in an event
  - Needed for background mixing
  - Determines size of  $N_{\text{pool}}$
- Pair average transverse momenta
  - How large intervals?
  - How many?
  - How should they be distributed?
- Momentum difference variable
  - $q_{\text{inv}}$  or  $q_{\text{LCMS}}$



# Usual questions – What is the correct variable?

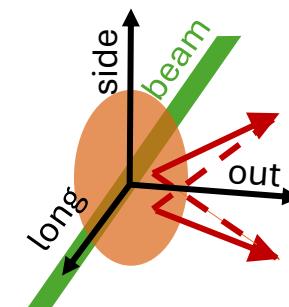
- Correlation function is measured as a function of momentum difference  $\mathbf{q}$
- Useful to use Longitudinally CoMoving System (Bertsch-Pratt) S. Pratt, Phys. Rev. D33, 72 (1986)  
 ● average momentum orthogonal to beam  
 ● source is approx. spherically symmetric in nucleus-nucleus collision
- G. Bertsch, M. Gong, M. Tohyama, Phys. Rev. C37, 1896 (1988)
- In cases where statistics is not enough: use 1D momentum difference
- Need to check two options:  $q_{\text{LCMS}}$  or  $q_{\text{inv}}$  (=PCMS)

$$q_{\text{inv}} = \sqrt{-(E_1 - E_2)^2 + (p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + (p_{1z} - p_{2z})^2}$$

**BUT!**  $q_{\text{inv}} = q_{\text{out}}^2(1 - \beta_T)^2 + q_{\text{side}}^2 + q_{\text{long}}^2 \rightarrow$  even if  $q_{\text{out}}$  is large and  $\beta_T \approx 0 \rightarrow q_{\text{inv}}$  can be small!  $\beta_T = \frac{K_T}{K_0}$

This makes  $q$  observer ( $q_{\text{LCMS}}$  or  $q_{\text{inv}}$ ) dependent

- Can we decide which option is best...?

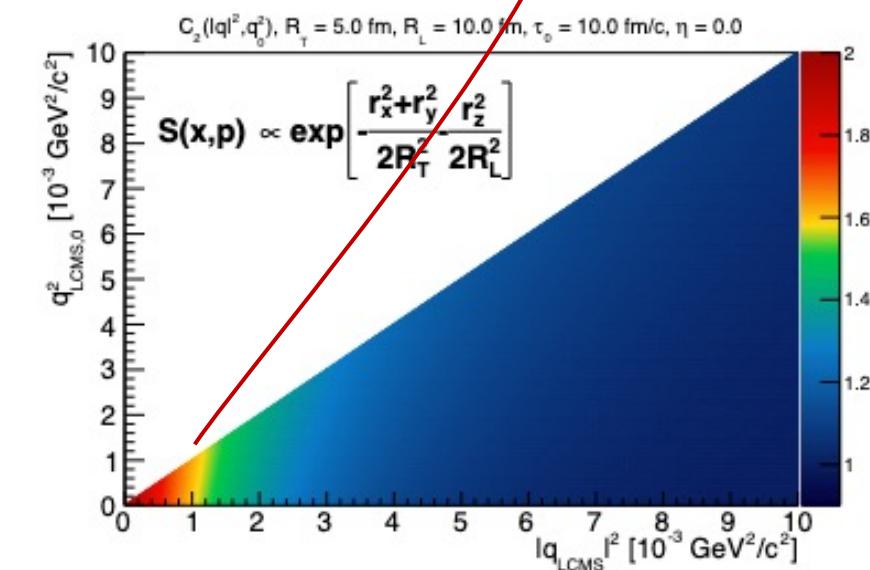
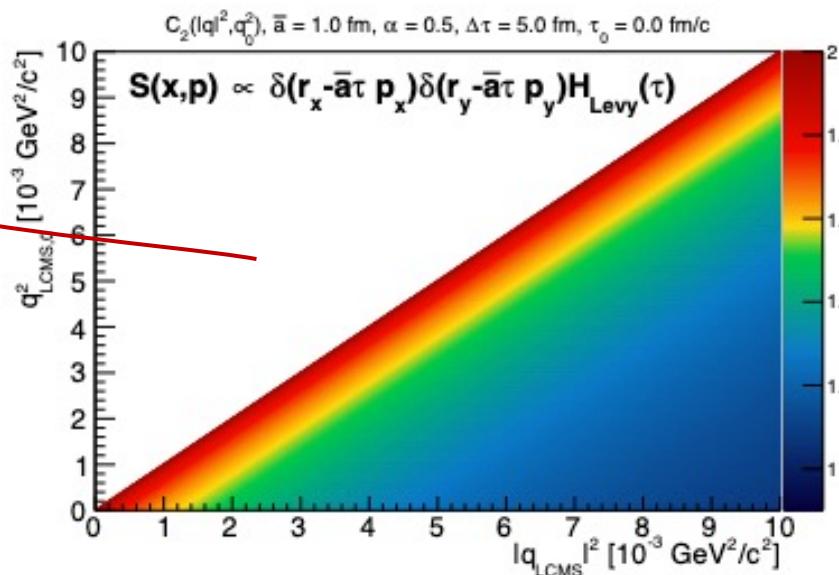


# Usual questions – What is the correct variable?

- Yes! We can measure  $q_0 = |E_1 - E_2|$  vs.  $|\mathbf{q}|$
- In case  $q_{\text{inv}}$  is the correct variable, we expect maximum diagonally
- In case  $q_{\text{LCMS}}$  is the correct variable, we expect maximum near 0

$$|\mathbf{q}_{\text{LCMS}}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{z,\text{LCMS}}^2}$$

$$q_{z,\text{LCMS}}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}$$



# Cuts and usual problems

## Usual cuts

- ➊ Centrality selection
  - ➌ statistics, compatibility with other data
- ➋ How to mix the events
- ➌ Rapidity cut (around midrapidity)
- ➍ PID
  - ➌ Bethe Bloch and / or ToF?
  - ➌ Pions, kaons, protons?
- ➎ Track splitting
  - ➌ visible spike in actual pair distribution
- ➏ Track merging
  - ➌ visible in pair distance (geometrical, momentum space,...)
- ➐ Fitting range
  - ➌ where to start and where to finish

# (Two-particle) femtoscopy measurement steps

After the good tracks are selected... create:

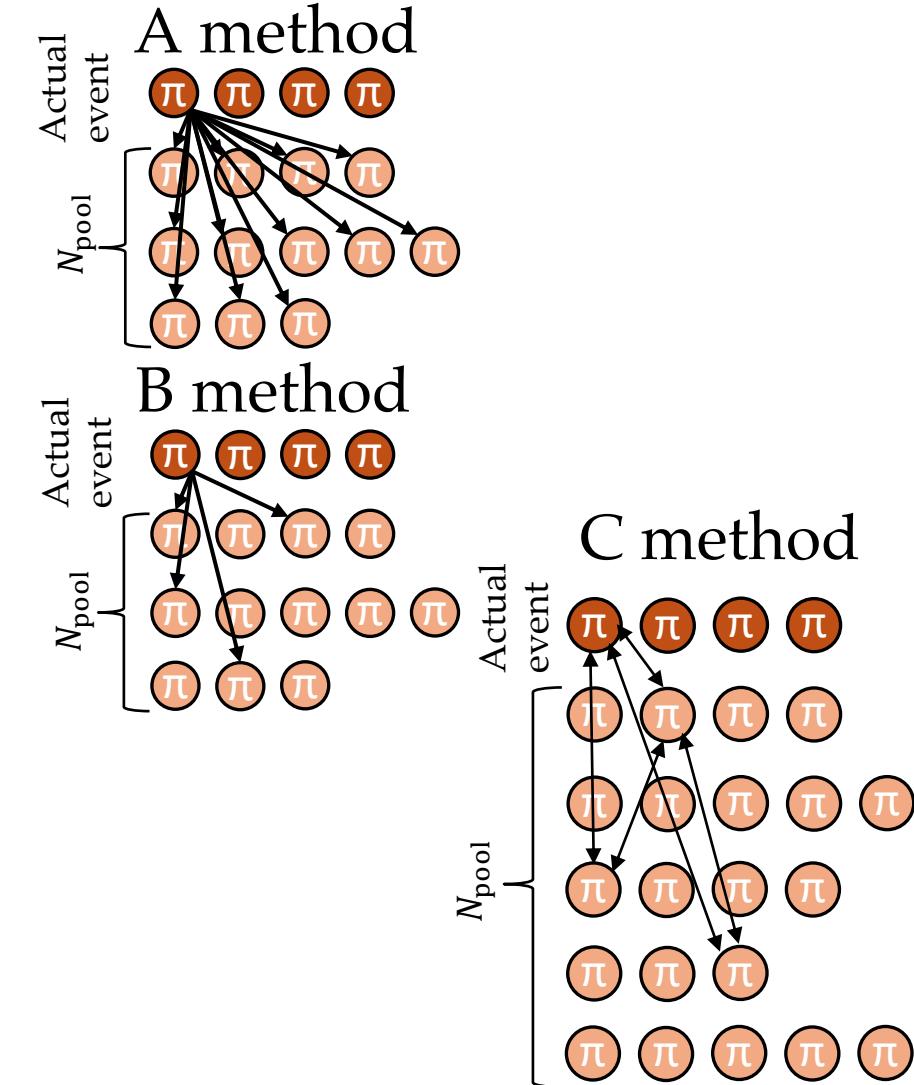
- ➊  $A(q)$ : Actual event relative momentum distribution
  - ➌ pairs from same event
- ➋  $B(q)$ : Background event relative momentum distribution
  - ➌ pairs from mixed event
- ➌  $C(q) = A(q)/B(q) \cdot \mathcal{N}$
- ➍ Measure  $C(q)$  for various  $m_T$  bins
  - ➌  $m_T$  = average transverse mass of the pairs
- ➎ Fit  $C(q)$  with suitable function
- ➏ Determine  $m_T$  dependence of fit parameters

$$m_T \equiv \sqrt{m_\pi^2 + k_T^2}$$

# Event mixing

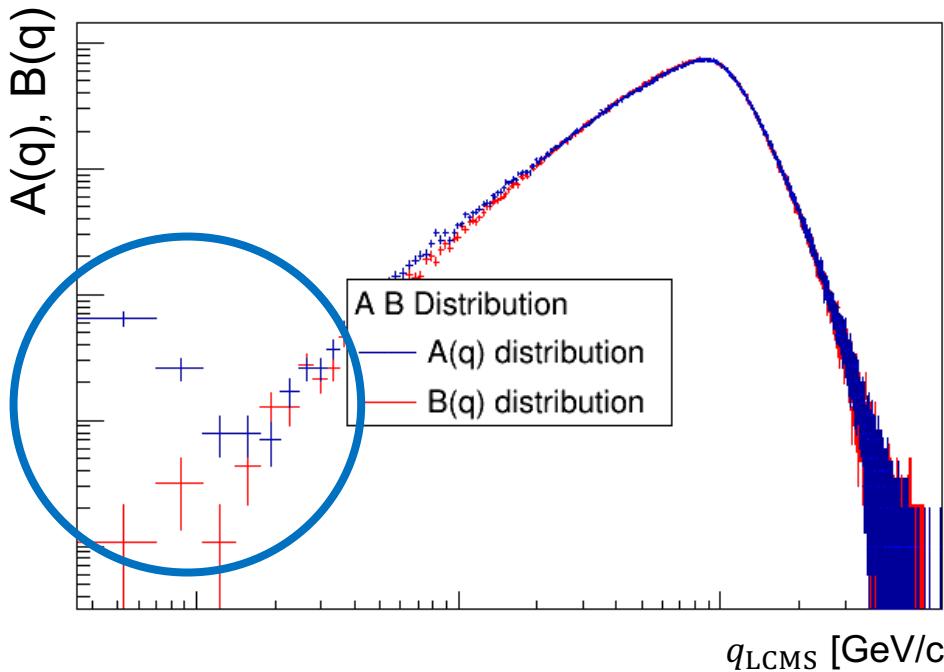
General idea: fill up  $N_{\text{pool}}$  events from previous actual events, in different classes – centrality, multiplicity,...

- Actual / current event with  $n$  pions
- A method**: make pairs with all pions of the pool to all actual pions
- B method**: make pairs with all actual pions to a random selection of  $n$  background pions
- C method**: select randomly one event and select randomly one pion, make pairs
- In our (and most) case, C method is preferred and used

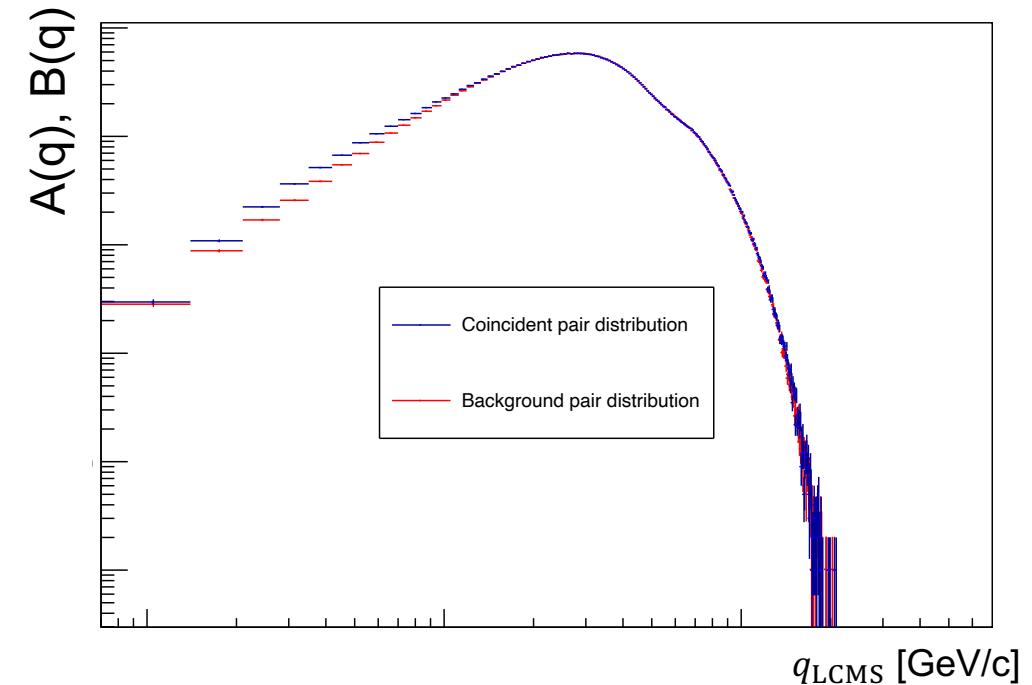
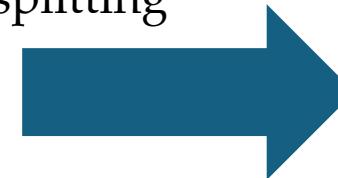


# A-B and track splitting

Track splitting spike in low-q is a telling sign in  $A(q)$ ,  $B(q)$

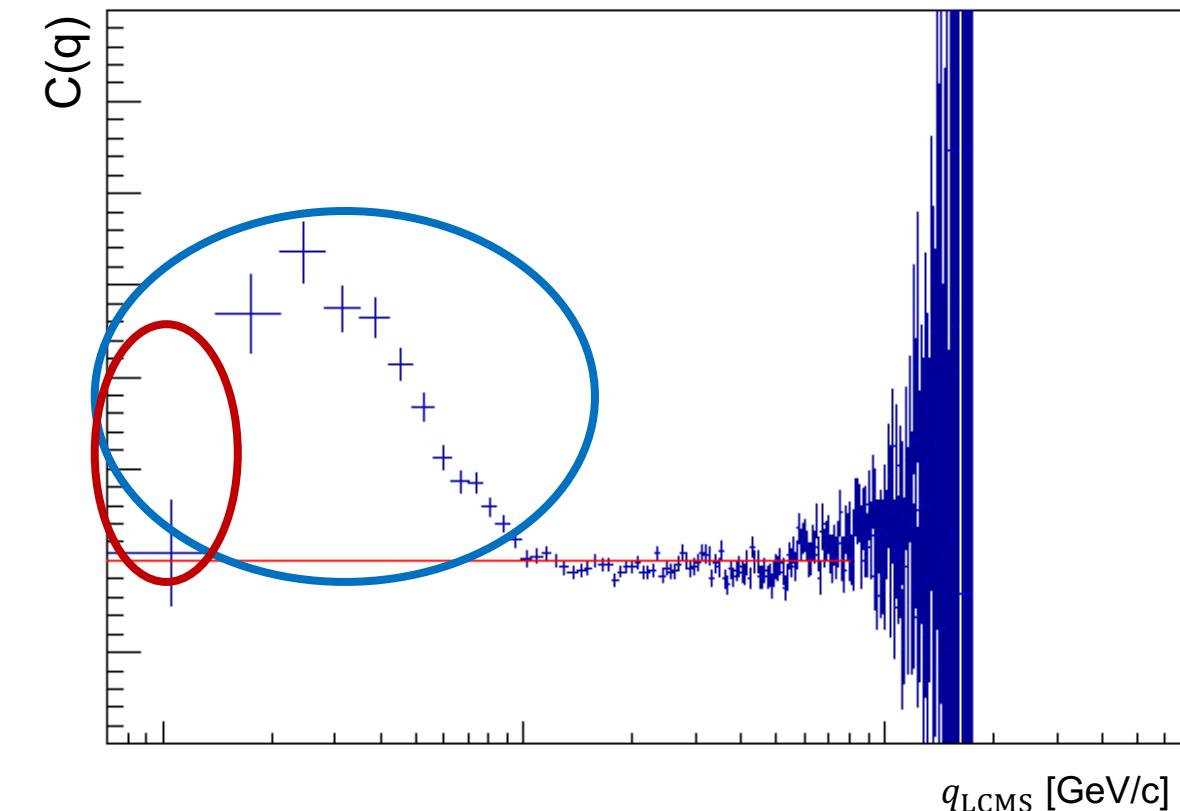
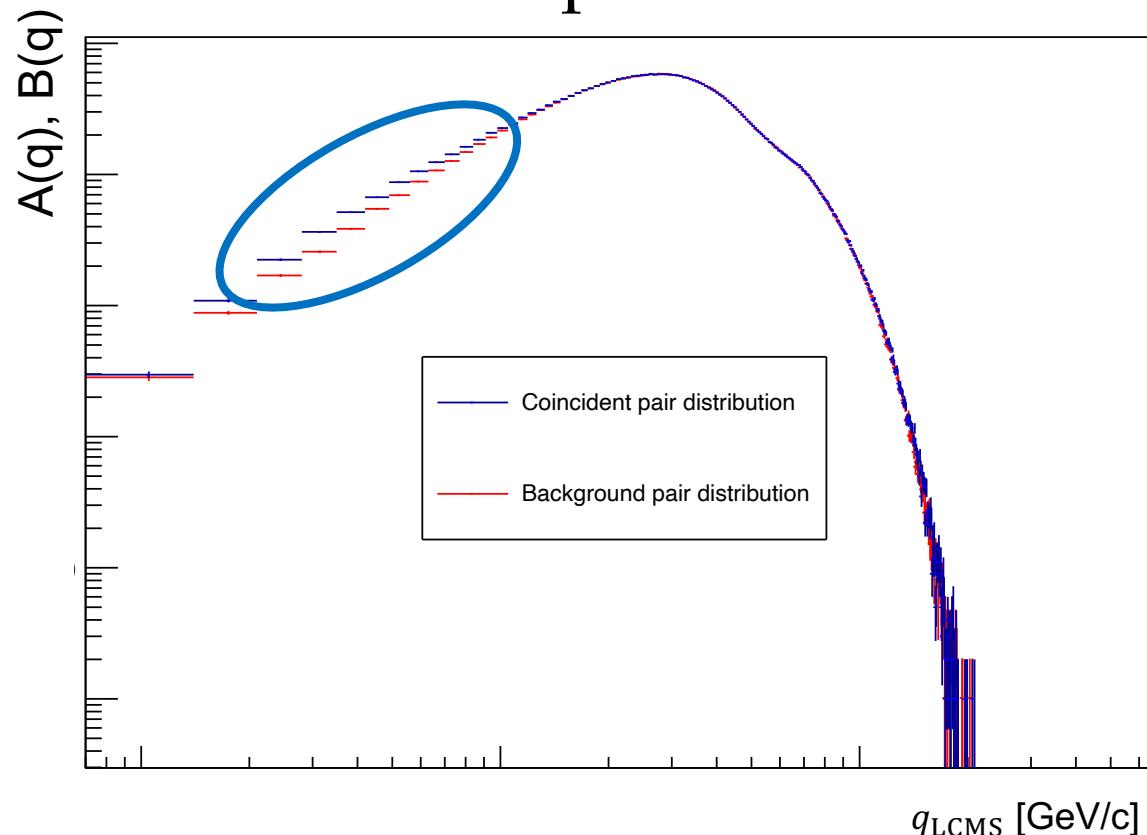


After applying  
selection for track  
splitting



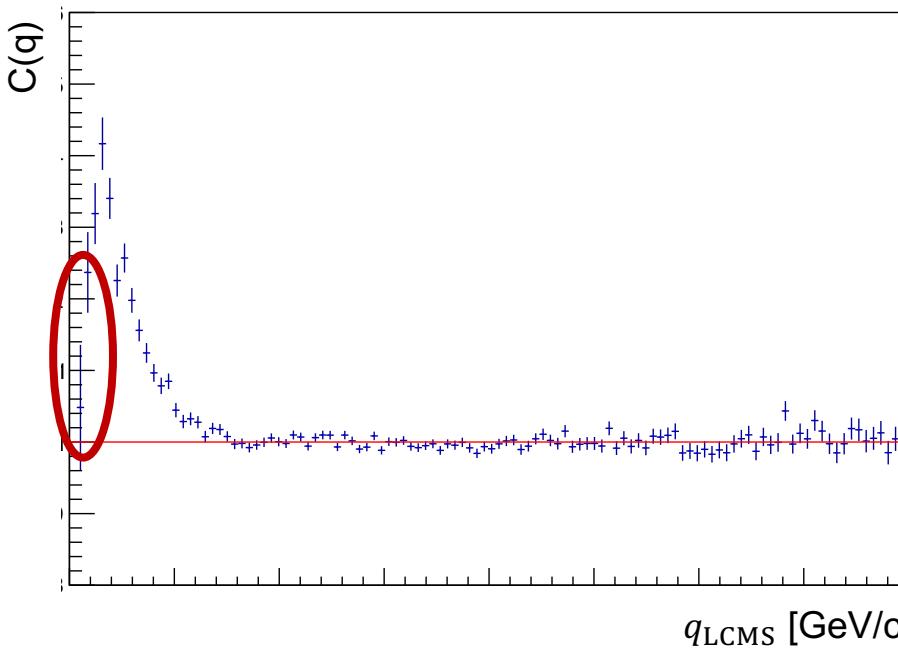
# A healthy A-B-C

- Example plots of  $A(q)$ ,  $B(q)$ ,  $C(q)$
- B-E effect visible in blue circle
- Coulomb repulsion in red circle, in case of like charged pairs

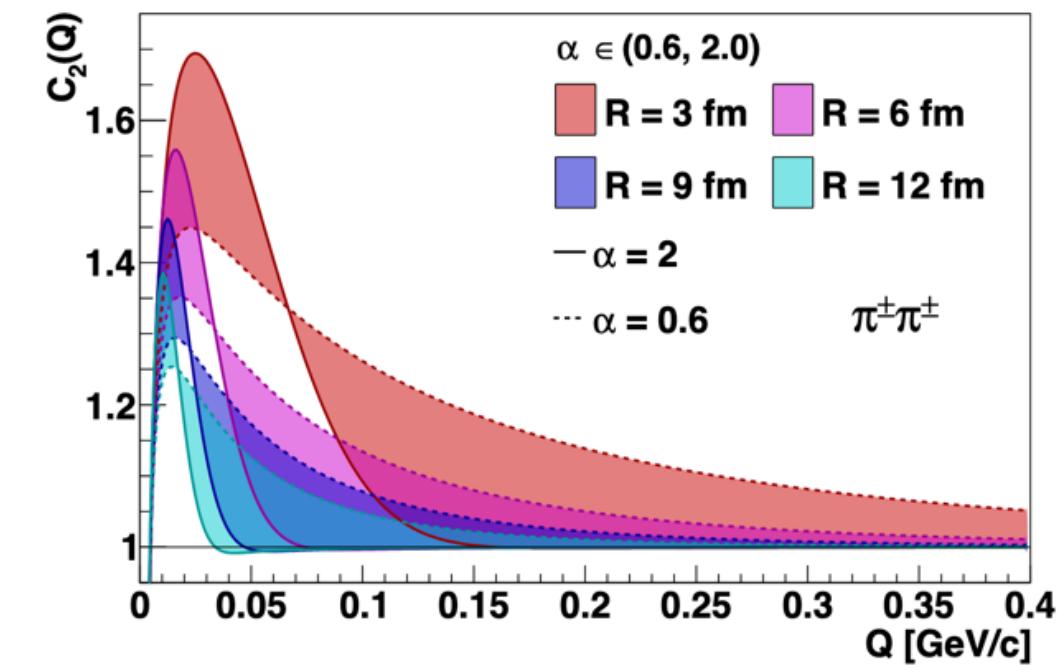


# Coulomb correction

- Dealing with like charged pairs → Coulomb repulsion
- Coulomb correction is needed (CC)
- Calculation is possible via a novel method of numerical integration
- Plug-n-play tool available:  
<https://github.com/csanadm/CoulCorrLevyIntegral>



Nagy, M., Purzsa, A., Csanad, M. and Kincses, Daniel,  
*Eur.Phys.J.C* 83 (2023) 11, 1015



# Coulomb correction „correction”

- Measuring in LCMS, but Coulomb correction is in PCMS
- This is negligible, but 1D spherical source in LCMS  
NOT spherical in PCMS

- Boost R to PCMS:  $R_{\text{PCMS}} = \sqrt{\frac{1-\frac{2}{3}\beta_T^2}{1-\beta_T^2}} \cdot R$

where  $\beta_T = \frac{K_T}{m_T}$

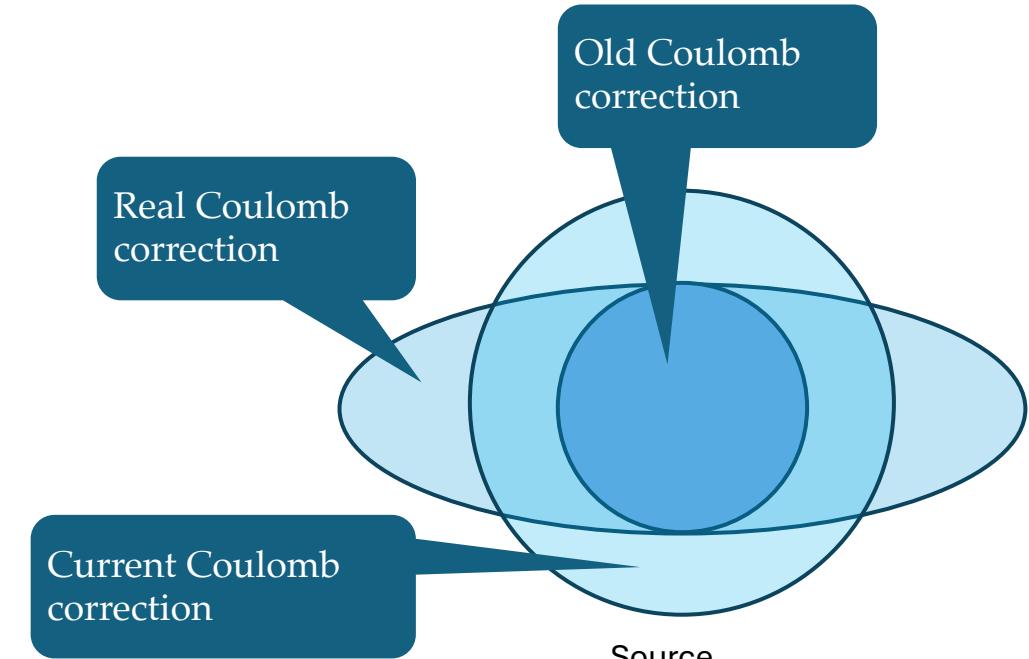
$$q_{\text{inv}} \approx \sqrt{1 - \beta_T^3} \cdot q_{\text{LCMS}}$$

$$K_{\text{Coul}}(q, R) = K_{\text{Coul}} \left( \sqrt{1 - \beta_T^3} \cdot q, \sqrt{\frac{1-\frac{2}{3}\beta_T^2}{1-\beta_T^2}} \cdot R \right)$$

Use this source for Coulomb correction

- Old Coulomb correction: fits into the source
  - smaller in size
- Current Coulomb correction: same size
  - smaller in longitudinal
  - larger in transversal

B. Kurygis, D. Kincses, M. Nagy, and M. Csanad, Universe 2023, 9(7), 328



# Fitting with Lévy parametrization

- Formula without final state interactions:

$$C^0(q) = 1 + \lambda \cdot e^{-(|qR|)^{\alpha}}$$

LCMS momentum difference

$$q = |\mathbf{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$

- Fitting formula, Bowler- Sinyukov:

Yu. Sinyukov et al., Phys. Lett. B432 (1998) 248,  
M.G. Bowler, Phys. Lett. B270 (1991) 69

$$C(q) = N \cdot \left( 1 - \lambda + \left( 1 + e^{-(qR)^{\alpha}} \right) \cdot \lambda \cdot K_{\text{Coul}}(q) \right) \cdot (1 + \varepsilon q)$$

N – normalization,  $\varepsilon$  – background linearity

- Physics parameters

$\lambda$ : correlation strength (core/halo model)

Core-halo model:  
Csörgő, Lörstad, Zimányi, Z.Phys.C71 (1996)  
Bolz et al, Phys.Rev. D47 (1993) 3860-3870

$R$ : Lévy scale parameter (similar to HBT size, ~length of homogeneity)

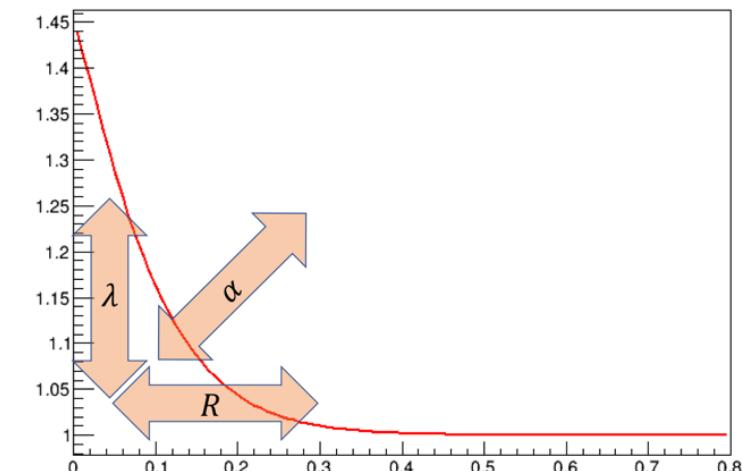
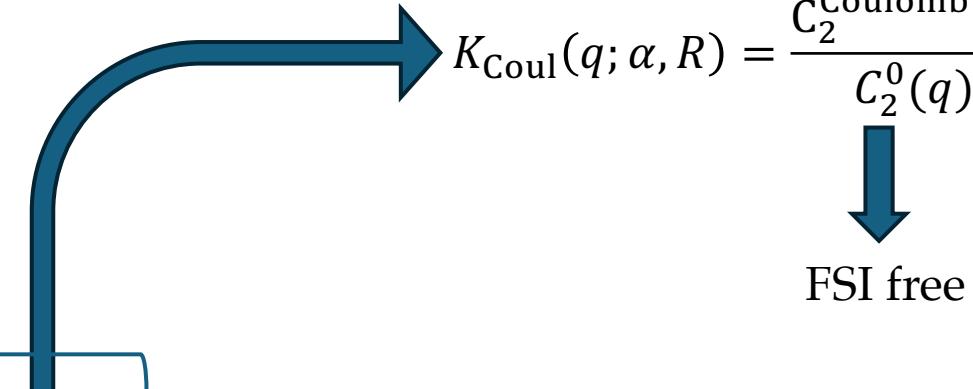
$\alpha$ : Lévy index (CEP proximity, shape of the source):

M. Nagy, A. Purzsa, M. Csanád, D. Kincses  
Eur. Phys. J. C 83, 1015 (2023)

Coulomb correction:

$$K_{\text{Coul}}(q; \alpha, R) = \frac{C_2^{\text{Coulomb}}(q)}{C_2^0(q)}$$

FSI free



# Core-halo model

- Hydrodynamically increasing core → pion emission

- Results in two-component core + halo source:

$$S(x) = S_{\text{core}}(x) + S_{\text{halo}}(x)$$

- Core: thermal hydro medium, size  $\lesssim 10$  fm

- Halo: long lived resonances ( $\omega, \eta, K_0^s, \dots$ ), size  $\gtrsim 50$  fm

- Halo is not seen due to detector resolution

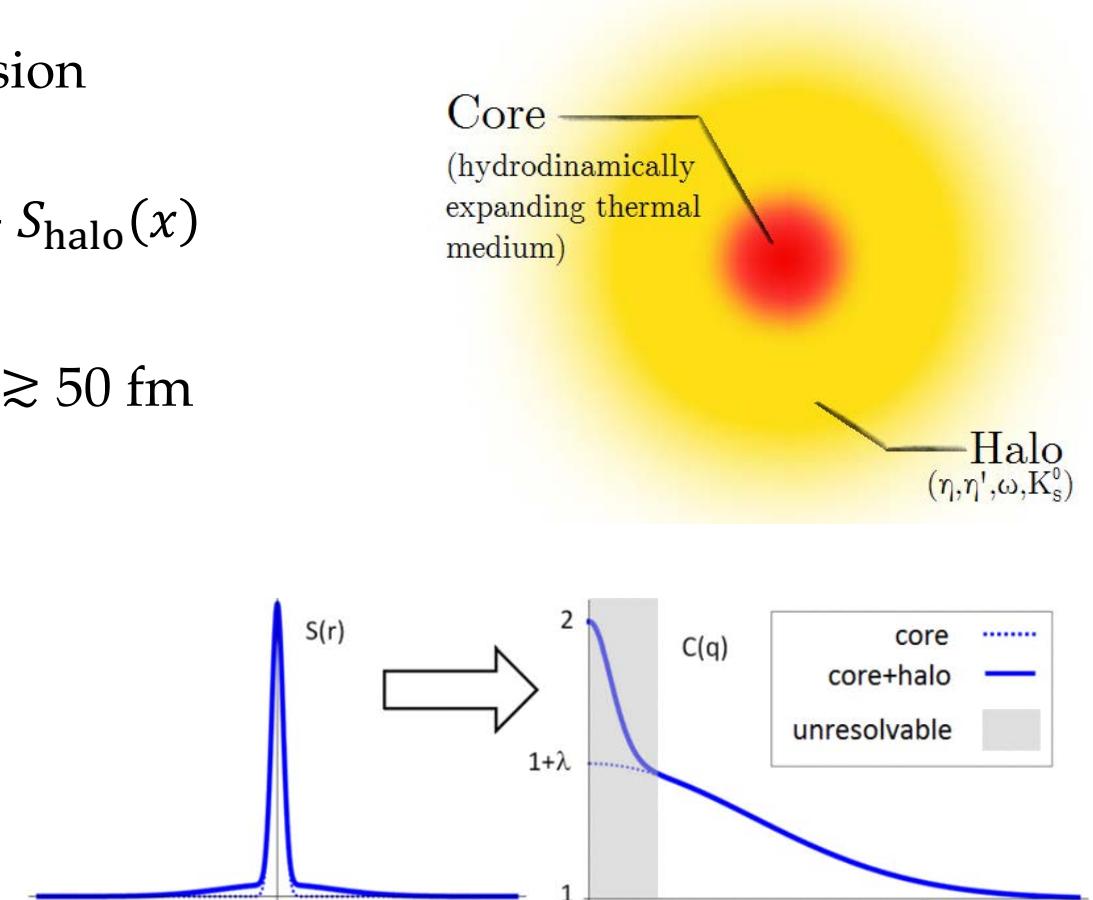
- Real  $q \rightarrow 0$ , at  $C(q=0) = 2$

- Experimentally:  $C(q \rightarrow 0) \rightarrow 1 + \lambda$

- Experimentally  $q \lesssim 5 - 10$  MeV/ $c$  not accessible

- Halo size “corresponds” to small momenta

- Define  $\lambda = \left( \frac{N_{\text{core}}}{N_{\text{core}} + N_{\text{halo}}} \right)^2$



# Fitting and fitting ranges

- Goodness-of-fit:  $\chi^2$ , NDF  $\rightarrow$  confidence level

- #1 rule: good fit should have  $CL > 0.01\%$

- Below that, it shouldn't be accepted

- Fit ranges start and end of fit ( $q_{\min}$ ,  $q_{\max}$ ) must be optimized

- Select by eye  $q_{\max}$  where  $C(q)$  is "flat" already

- Performing fit with  $q_{\min}$  steps bin-by-bin

- starting point of the scan can be around B-E peak

- ending point of the scan can be around middle to end of the B-E slope

- Display all obtained parameters with values, error bars + highlight if fit failed

- How to select fit ranges? All below should be considered simultaneously

- Fit converged

- Parameter values are believable ( $\alpha \geq 2$ ,  $\lambda \geq 1$ )

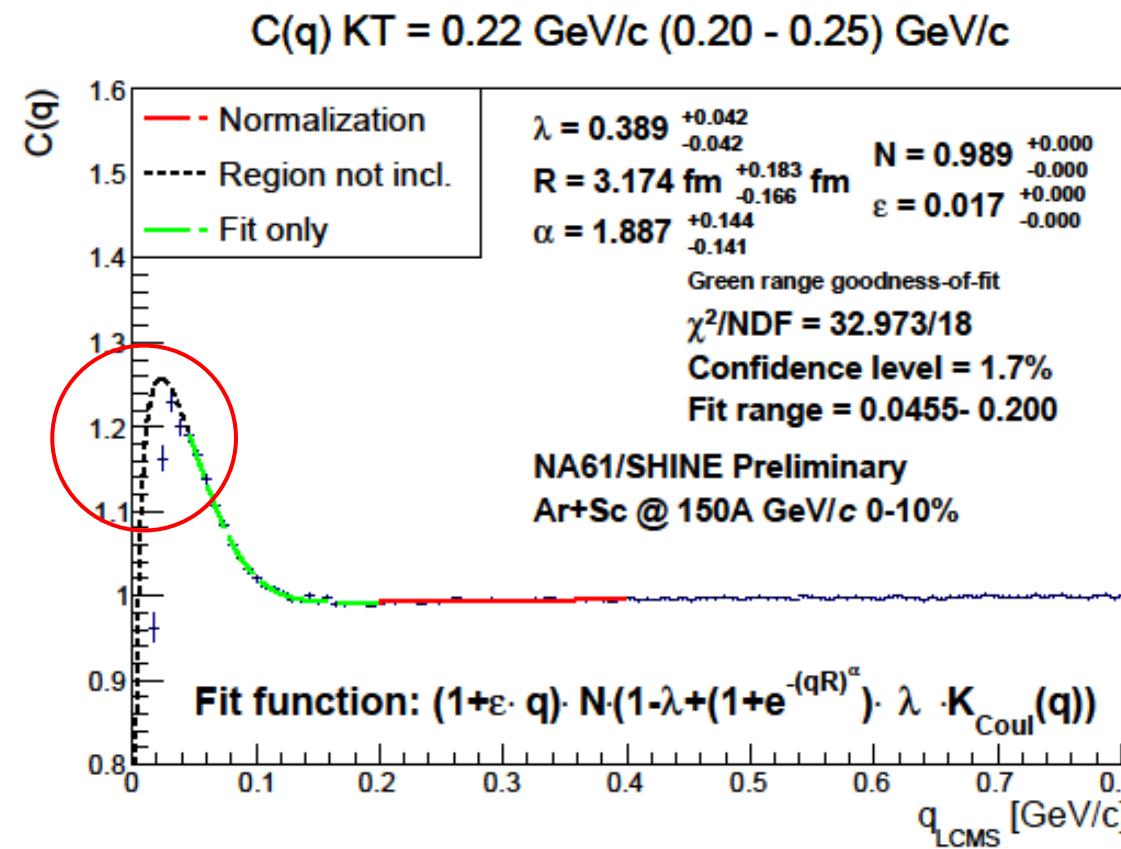
- Trend of parameters

- Generally, fit  $q_{\min}$  should increase with increasing  $K_T$  bins



# Low- $q$ range issue – MC correction?

- Low- $q$  range – fits overestimate the data
  - Experimental artefact?
- Maybe use Monte Carlo to correct this effect?



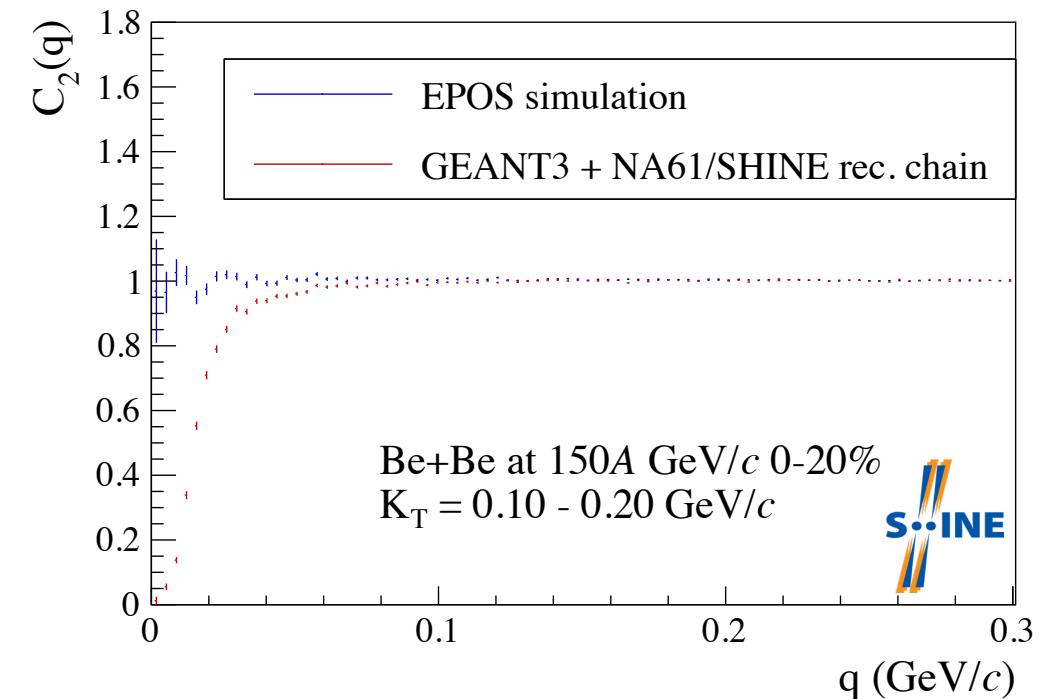
# Monte Carlo correction idea

- Correction is possible via:

$$R_{\text{paireff}} = \frac{C_{\text{rec}}}{C_{\text{sim}}} \rightarrow C(q)/R_{\text{paireff}}$$

- Correction is unstable

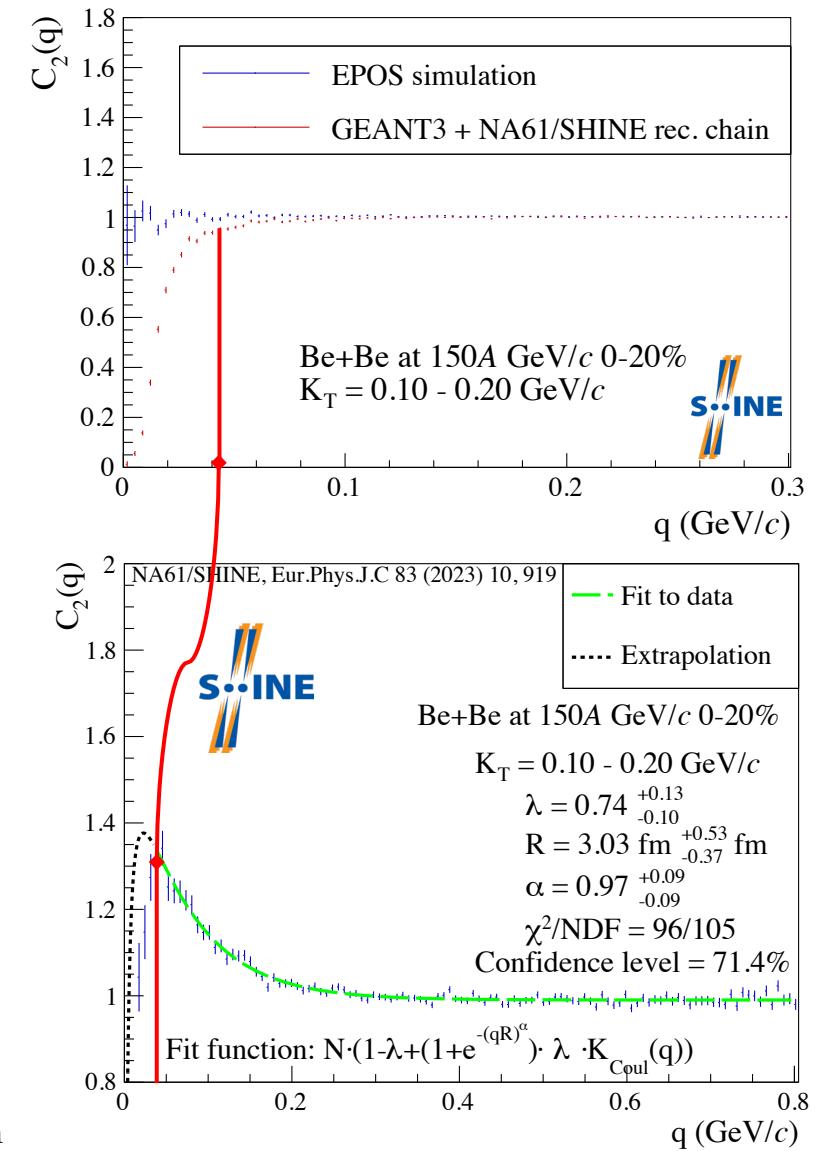
OK, then what can we use it for?



# Monte Carlo estimation

We can estimate the start of this strong cutoff

- B-E and Coul. effect not present in EPOS simulation
  - $C_2(q) \approx \text{const.}$
- Low-q range behavior in data:
  - Fits overestimate data
  - Theoretical correlation function cannot describe
  - Observed in Be+Be, Ar+Sc
- Strong cutoff observable
  - Several possibilities...
  - Might be experimental artefact?
- Visible deviation from generated (simulated)
  - Effects such as track merging present
- Low-q region (until reco.  $\approx 1$ ) can be excluded
  - Two Track Distance (track merging) cut not used

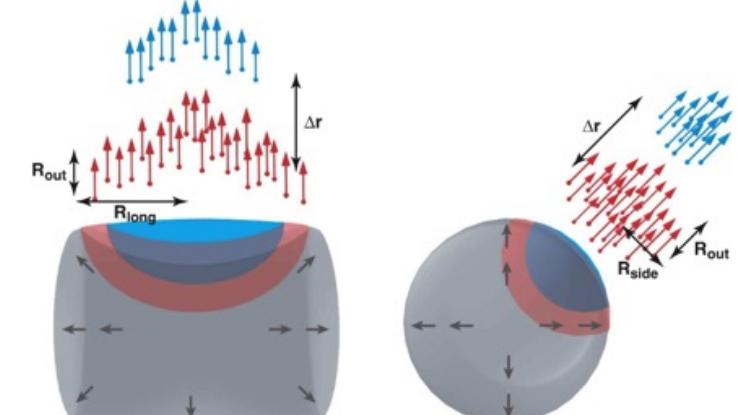
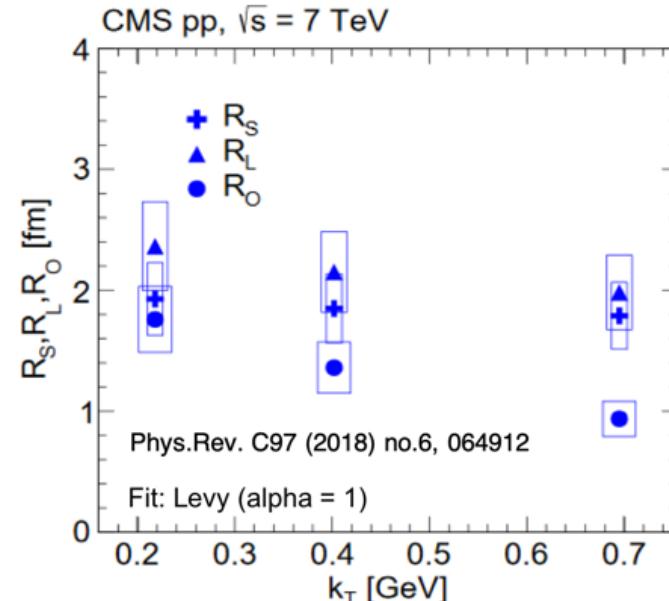
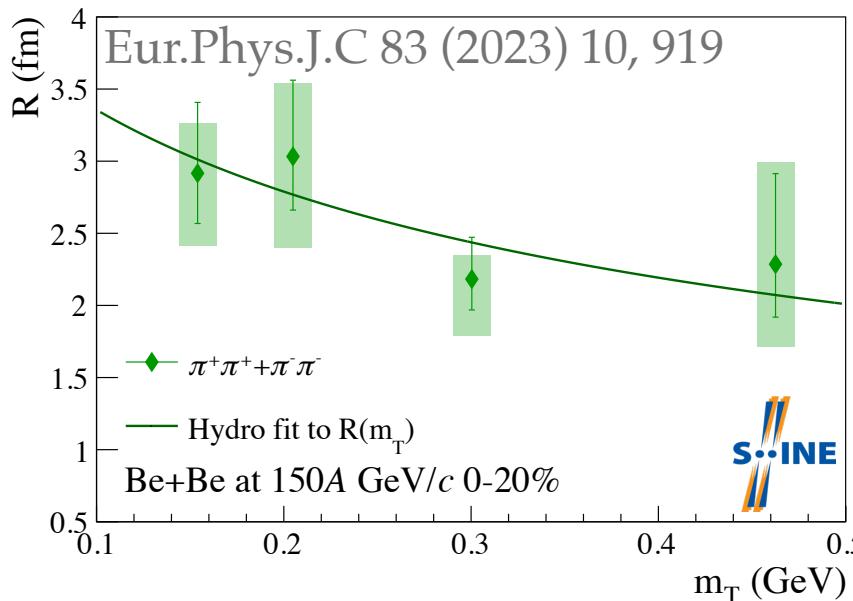


# $^{7}\text{Be} + ^{9}\text{Be}$ at highest energy

- Lévy scale parameter  $R$  – related to length of homogeneity
  - Region in space
  - Emission of particles are coherent, correlated
- System size comparable to p+p
- $R$  decreasing trend,  $\frac{1}{R^2} \propto m_T$  (calculated for Gaussian shape!)

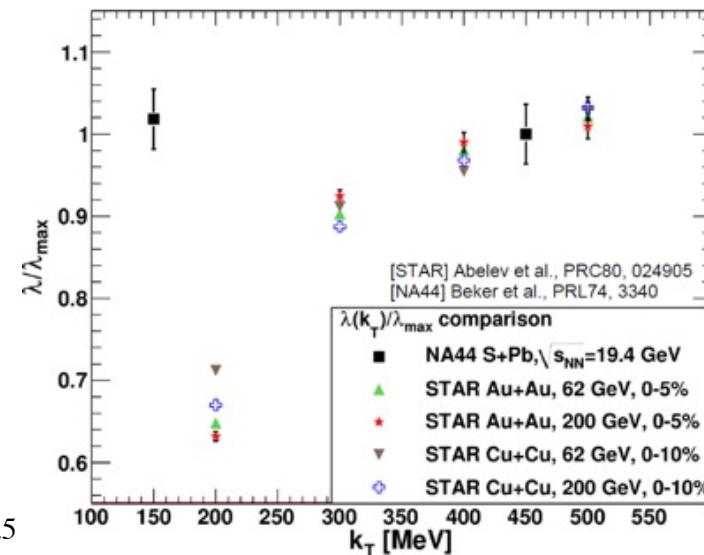
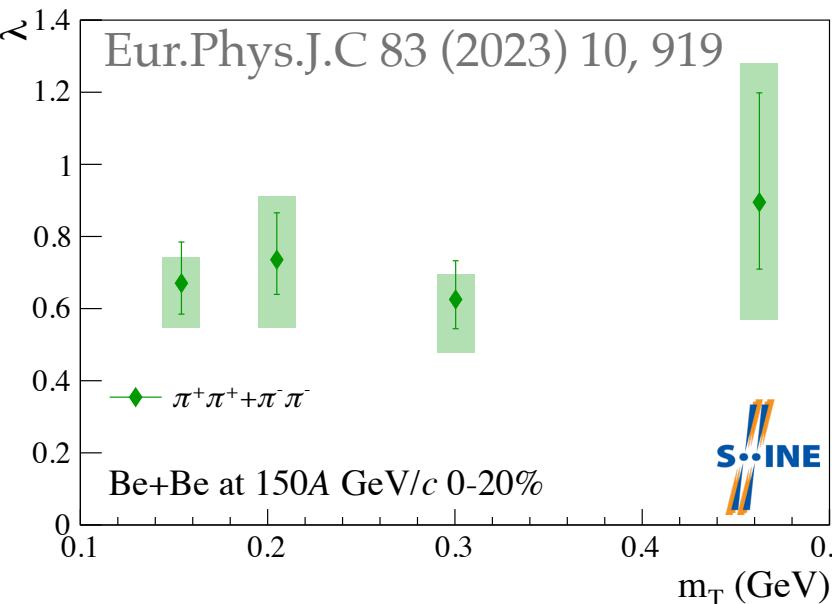
Csörgő, Lörstad, Phys.Rev.C54 (1996) 1390

$$m_T \equiv \sqrt{m_\pi^2 + k_T^2}, k_T \equiv 0.5 \sqrt{k_X^2 + k_y^2}$$



# $^{7}\text{Be} + ^{9}\text{Be}$ at highest energy

- Correlation strength / intercept parameter  $\lambda$ 
  - Related to the core-halo model
- Small size  $\sim p+p$
- $\lambda \sim \text{constant}$ , no decrease with low  $m_T$
- Compared to STAR, PHENIX: lack of decrease at low- $m_T$



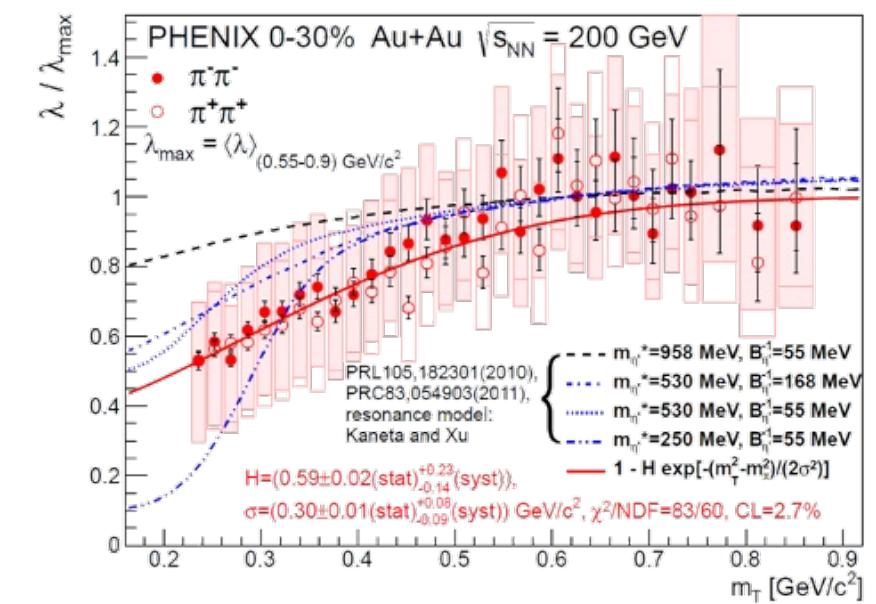
- Possible explanation for increasing  $N_{\text{halo}}$ :
- In-medium-mass modification of  $\eta'$ 
    - Large mass due to chiral symmetry breaking
    - Potential chiral symmetry restoration  $\rightarrow$  lower mass
    - Lower mass  $\rightarrow$  more produced  $\rightarrow$  larger halo

Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028

Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205

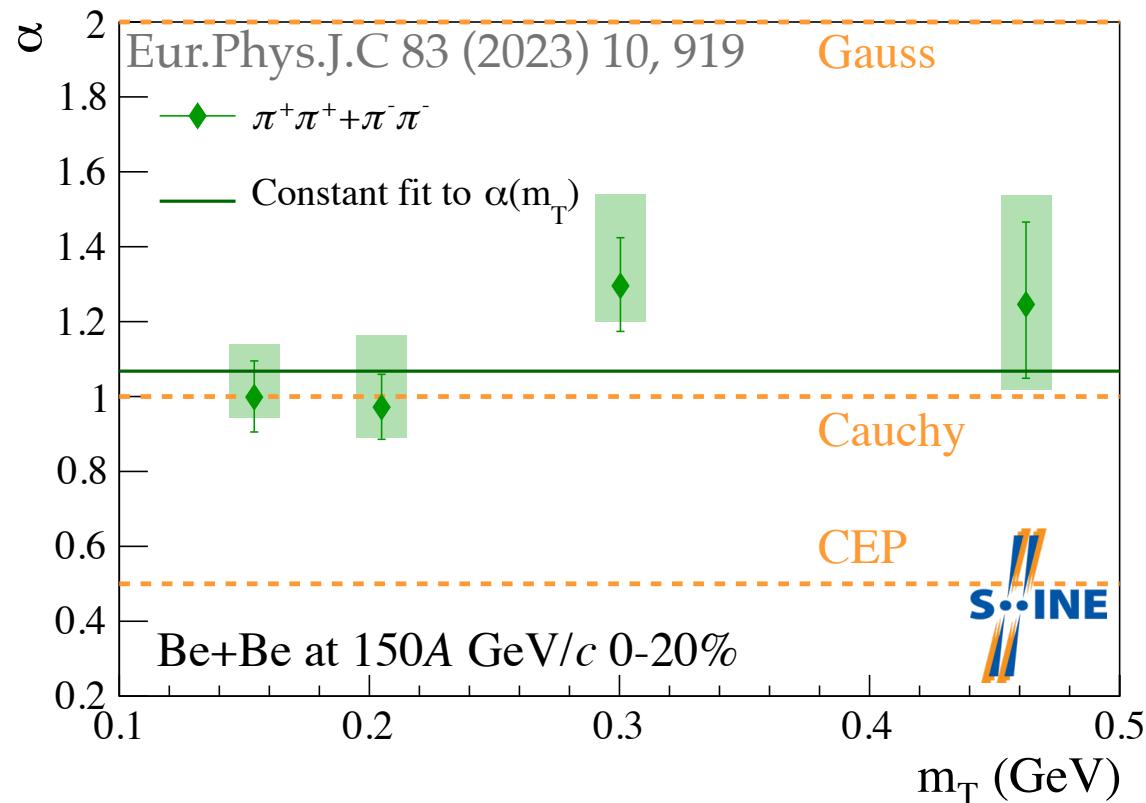
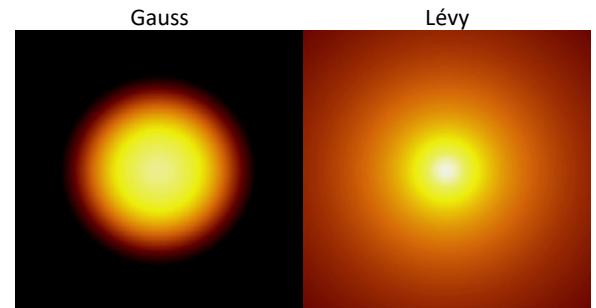
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301

PHENIX, Phys.Rev. C97 (2018) no.6, 064911



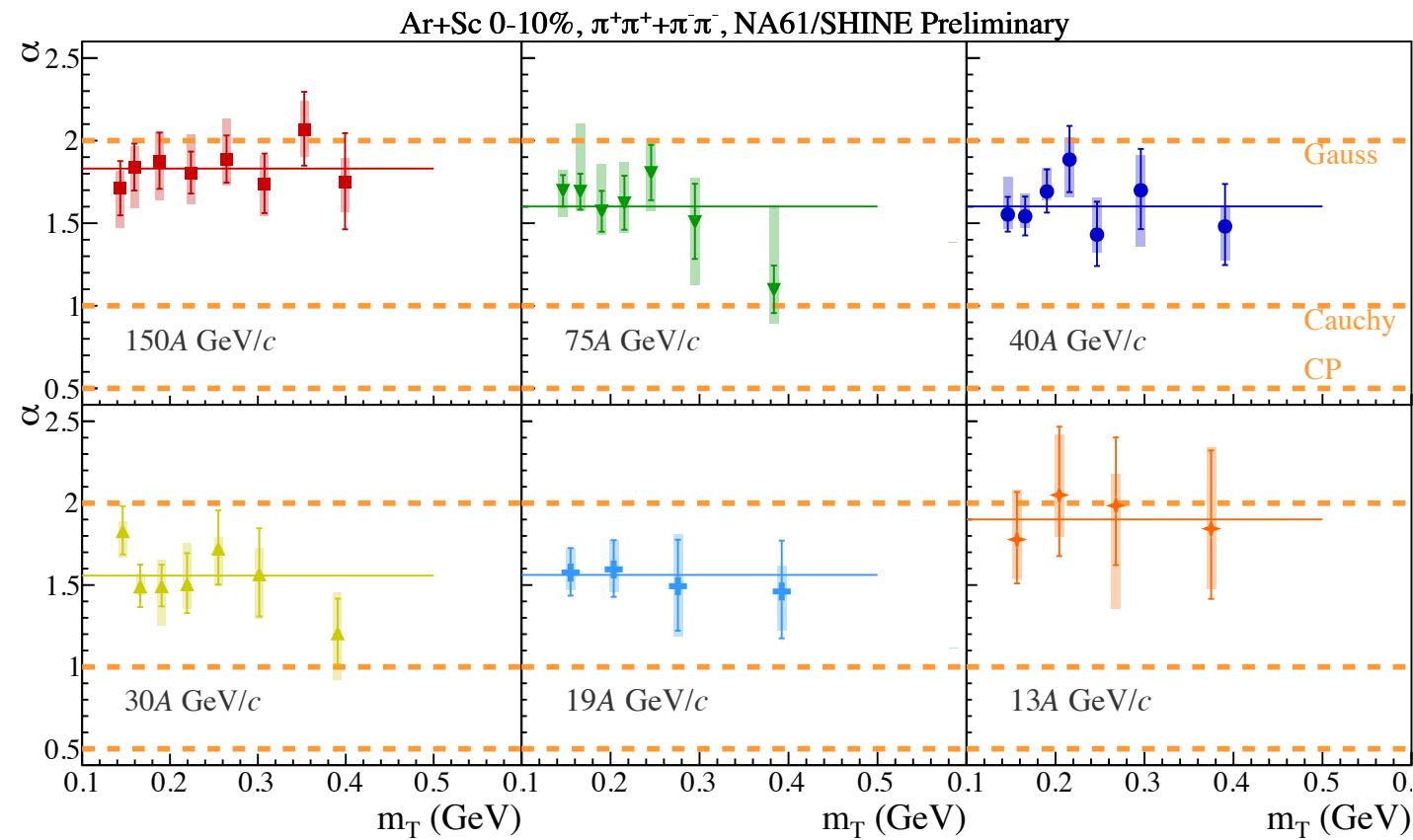
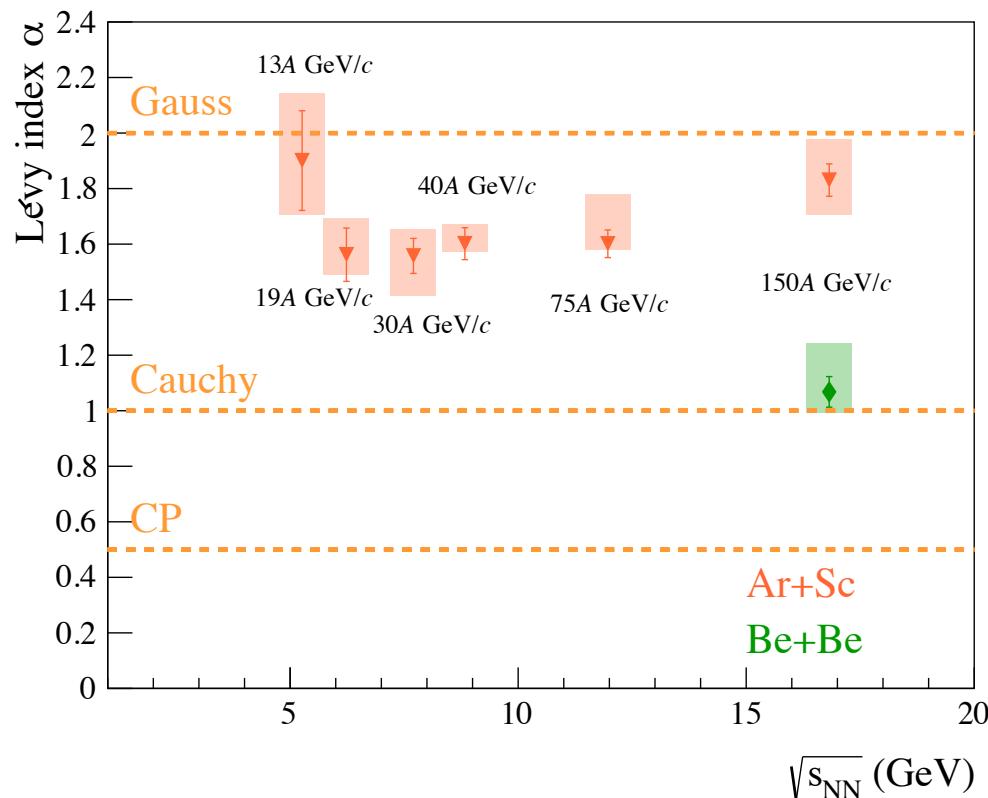
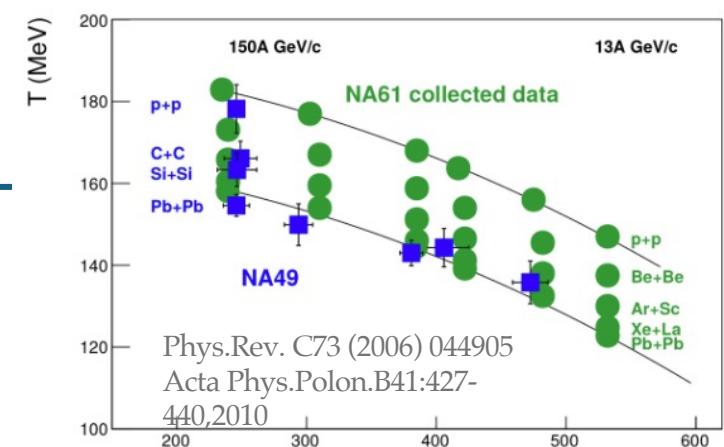
# $^{7}\text{Be} + ^{9}\text{Be}$ at highest energy

- Lévy stability index  $\alpha$ : shape of spatial correlation
  - Small system, comparable to p+p
  - $\alpha \approx 1 \rightarrow$  Cauchy shape,  $\neq$  Gauss,  $\neq$  CEP
  - Compatible with Lévy assumption



# Energy dependence in $^{40}\text{Ar} + ^{45}\text{Sc}$

- Different energies, different  $T, \mu_B$
- Interesting trend:  $\sim \sqrt{s_{\text{NN}}} = 6 - 8 \text{ GeV}$  minimum?

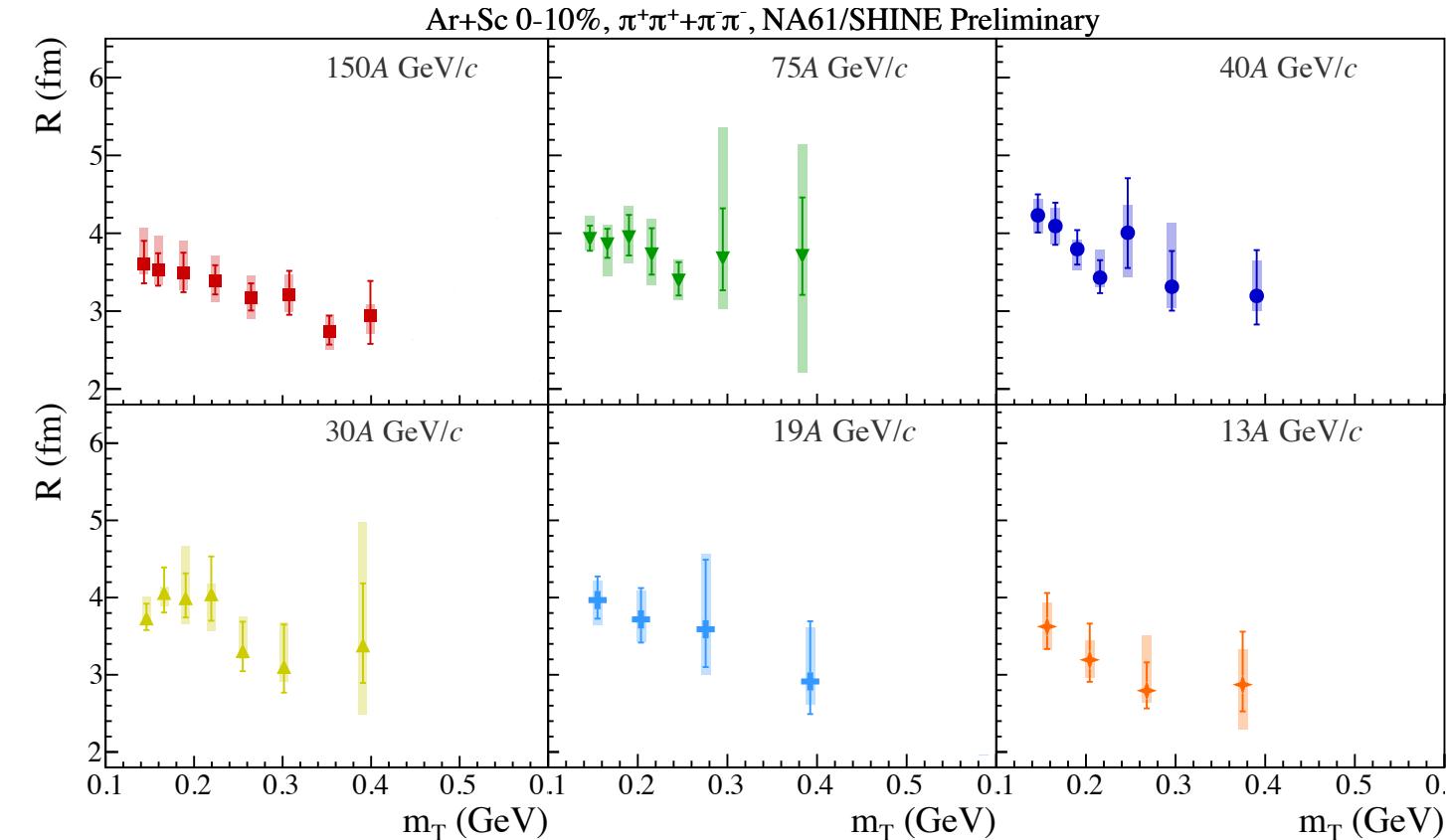
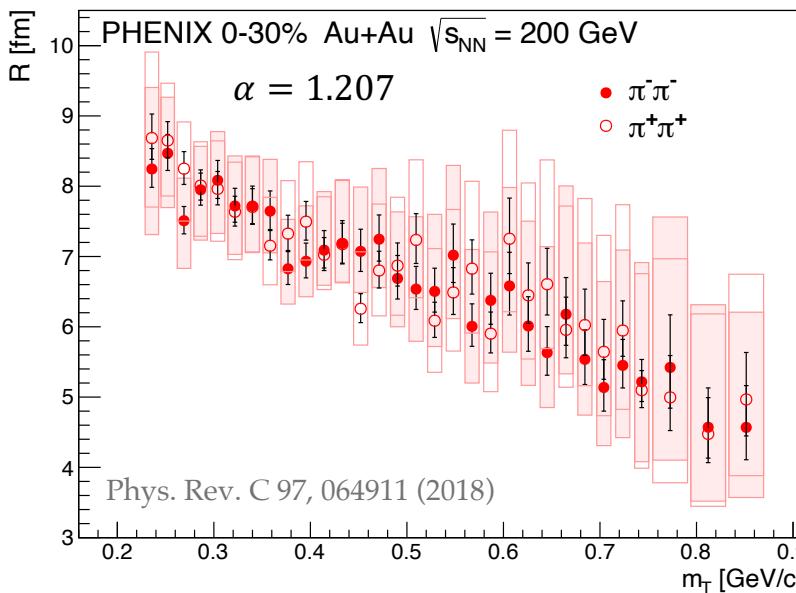


# Energy dependence in $^{40}\text{Ar} + ^{45}\text{Sc}$

- For Gaussian sources  $R \sim 1/\sqrt{m_T}$

Csörgő, Lörstad, Phys.Rev.C54 (1996) 1390

- Decreasing trend but  $\alpha < 2!$ 
  - Present in other measurements as well

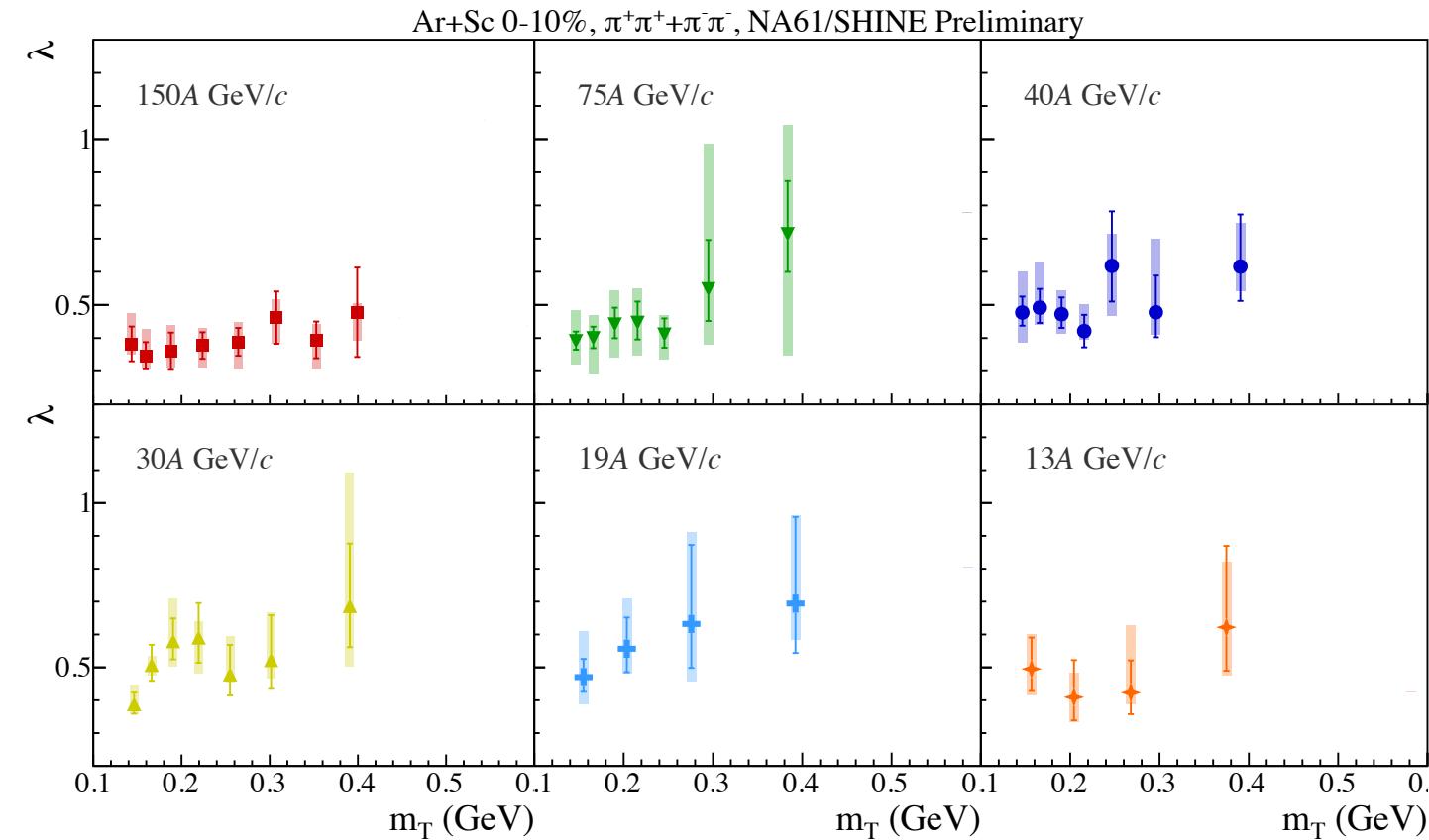
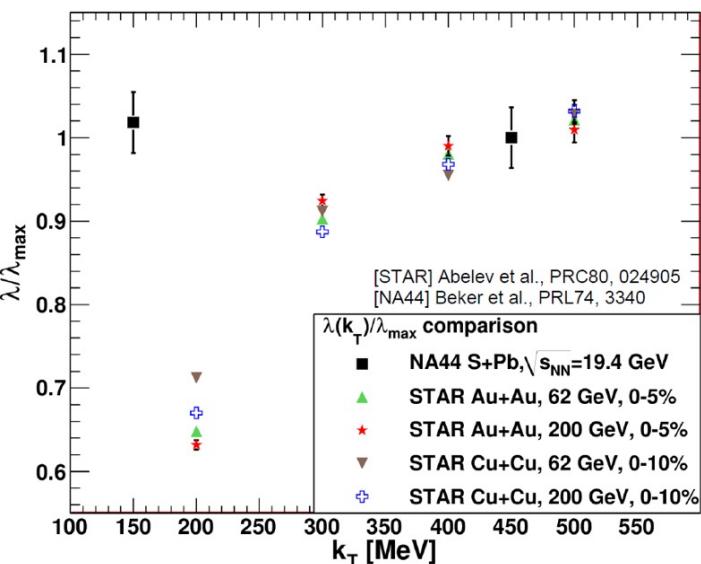


# Energy dependence in $^{40}\text{Ar} + ^{45}\text{Sc}$

- Almost constant (except maybe 30A GeV/c)
- Halo component increases at RHIC;  $\lambda = \left( \frac{N_{\text{core}}}{N_{\text{core}} + N_{\text{halo}}} \right)^2$

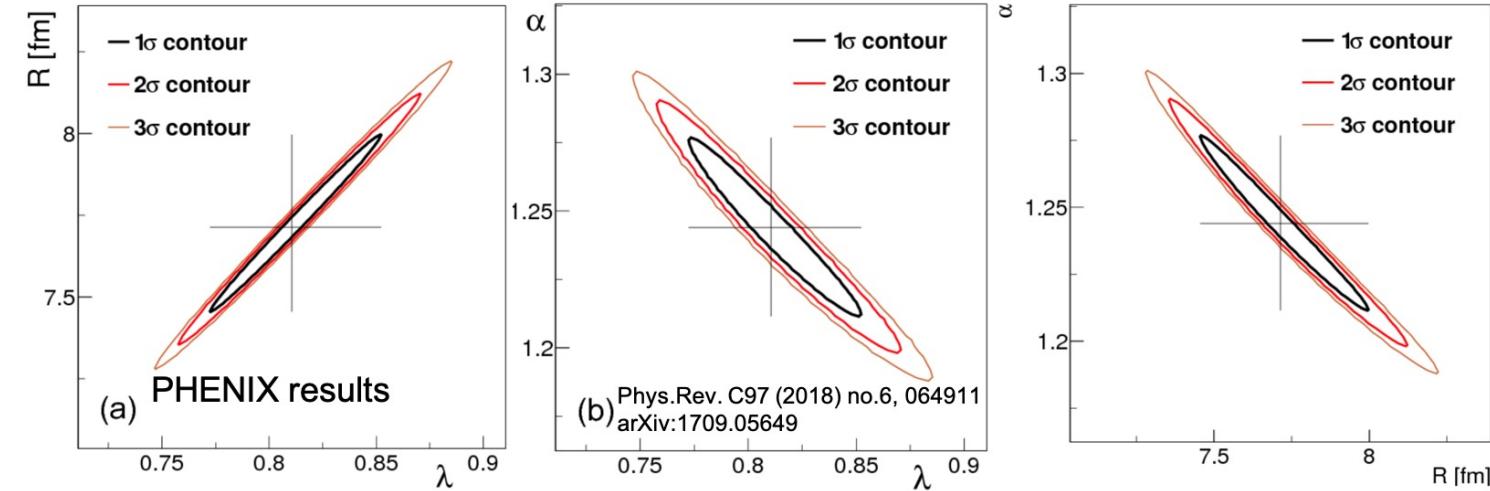
S. E. Vance et al, Phys.Rev.Lett. 81 (1998) 2205-2208  
T. Csörgő et al, Phys.Rev.Lett. 105 (2010) 182301  
A. Adare for PHENIX Collaboration, Phys.Rev. C97 (2018) no.6, 064911

Not really visible at SPS



# Interparameter correlation

- Known characteristic of Lévy fit
- Observed in PHENIX, in NA61/SHINE
  - Statistical uncertainty: via Minos for  $1\sigma$  contours
  - Statistical uncertainty of a given parameter: side length of "bounding box" of N-dimensional contour surface
  - 2D projections of N-dim. contour shown below
  - "Cross" shows parameter uncertainty, equal to contour projection
- To reduce the correlation – one parameter
  - $\alpha$  – approx. constant in  $m_T$
  - $R - m_T$  dependent fit, based on hydro
 
$$A/\sqrt{1 + m_T/B}$$



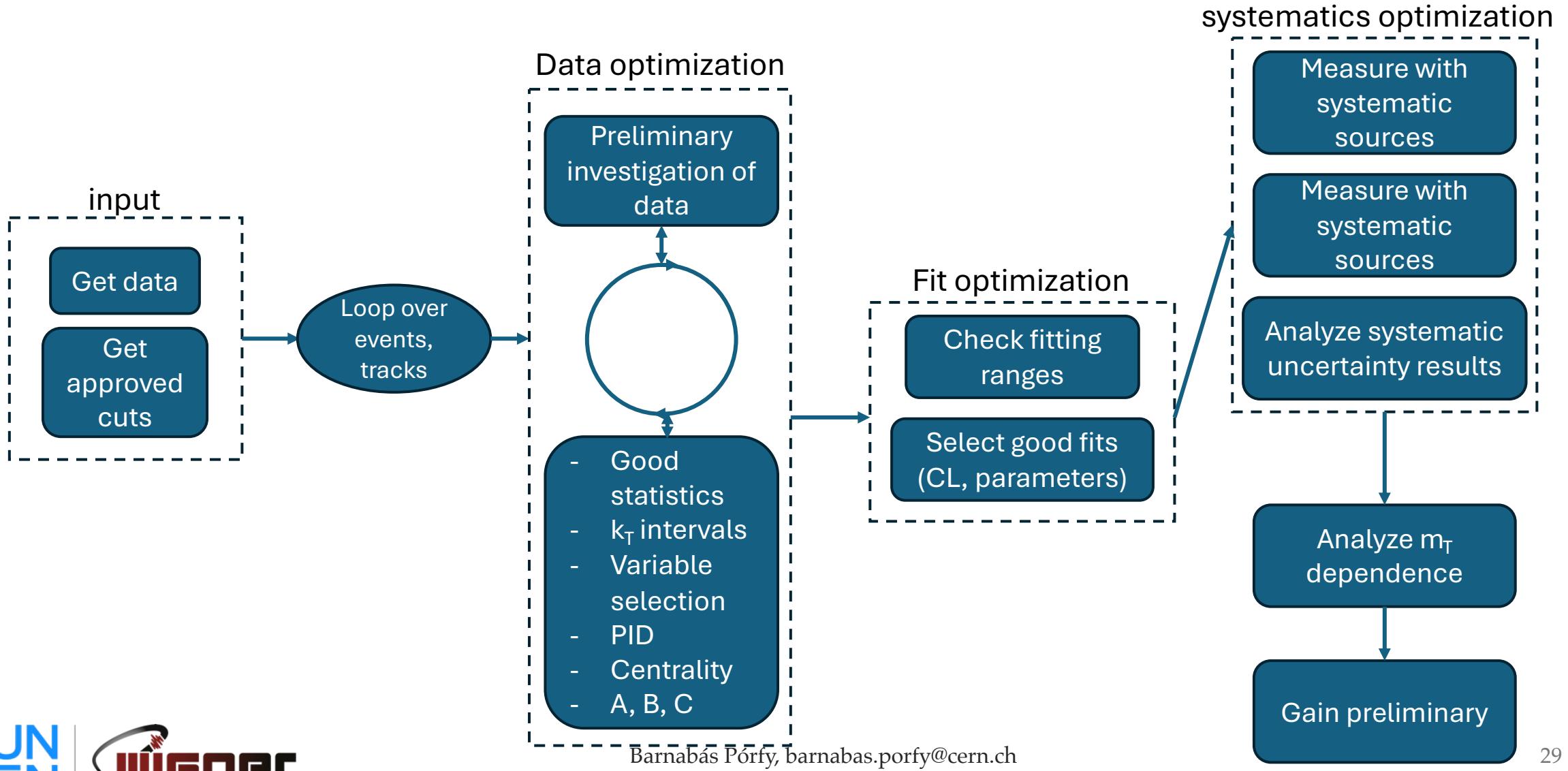
# Calculation of Systematic Uncertainties

$$\bullet \delta P^\pm(i) = \sqrt{\sum_n \frac{1}{N_n^{j^\pm}} \sum_{j \in J_n^\pm} \left( P_n^j(i) - P^0(i) \right)^2}$$

 Where:

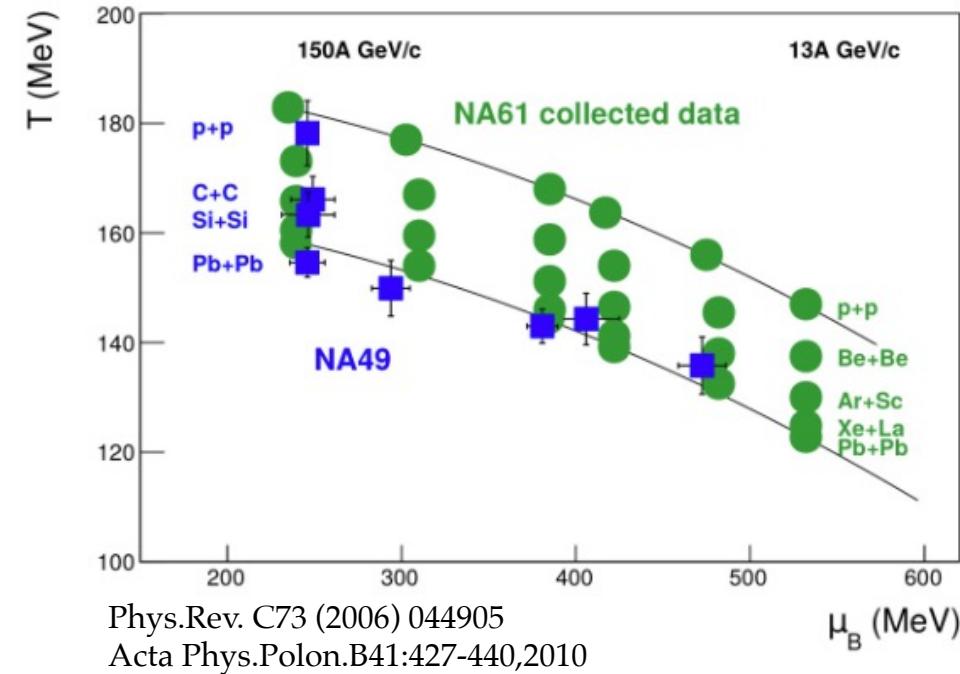
-  i bin (0,1,2,3,...), in our case:  $K_T$  bins
-   $P^0(i)$  value of parameter in i
-  n # of systematic source
-  j option used: standard, loose or strict (0,1,2)

# Analysis workflow



# Further plans

- Finalization and article on  $^{40}\text{Ar}+^{45}\text{Sc}$  energy scan results
- High multiplicity  $^{129}\text{Xe}+^{139}\text{La}$  energy scan measurement
  - 3D, if possible



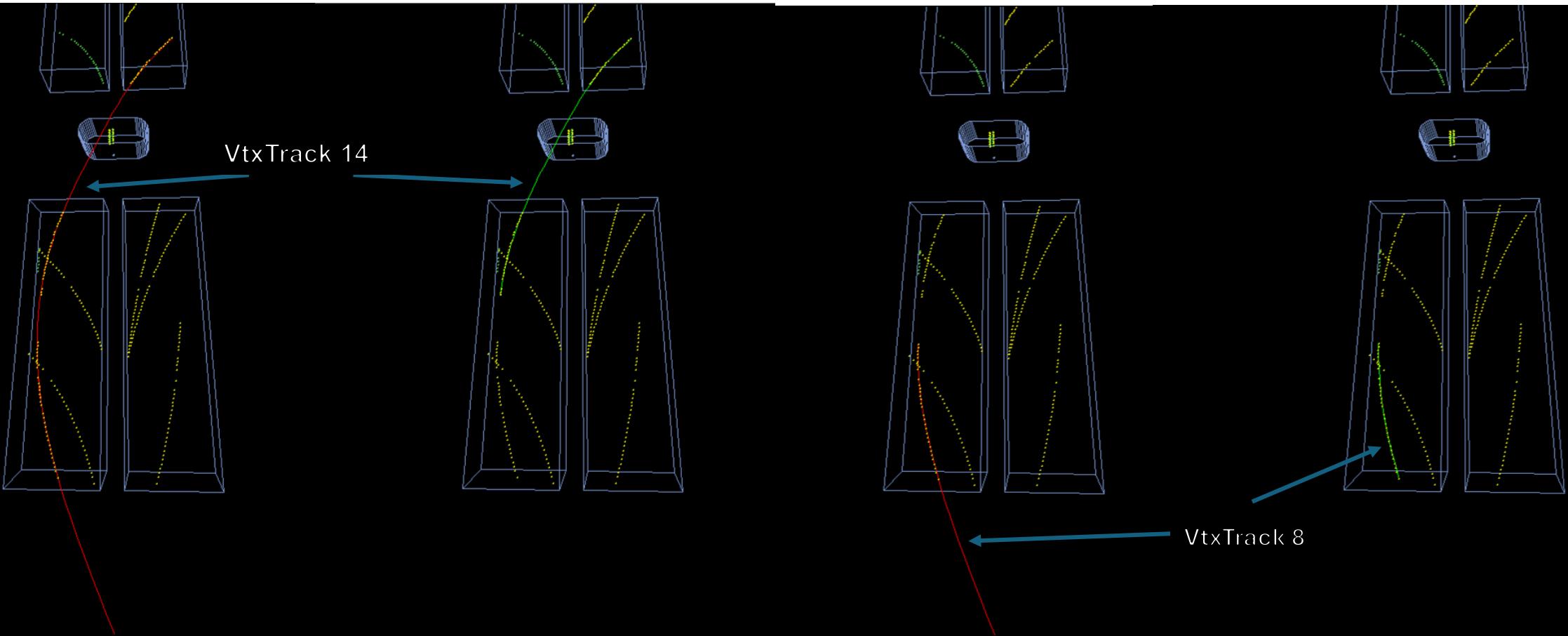


# Thank you for your attention!



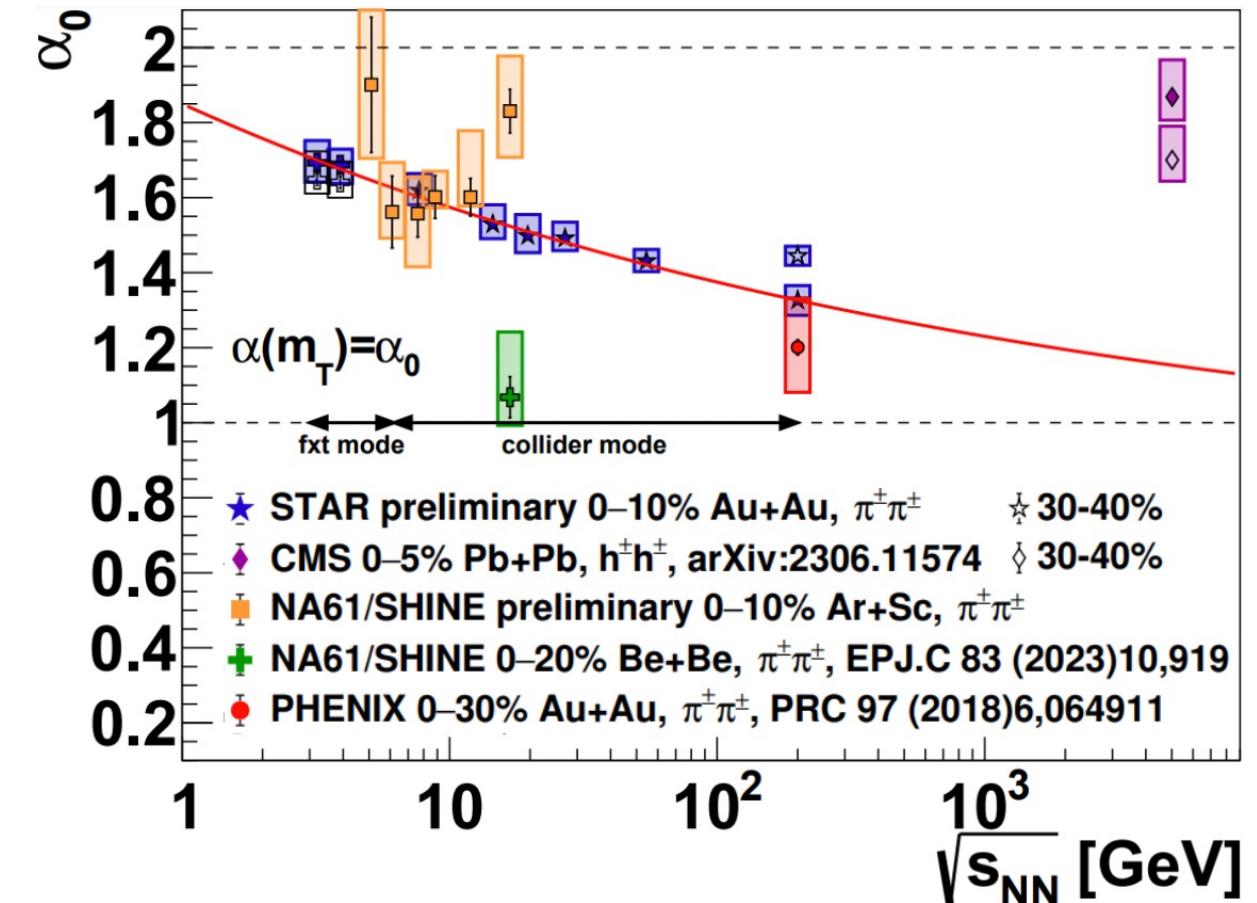
# Backup

# Split track



# Femtoscopy group results

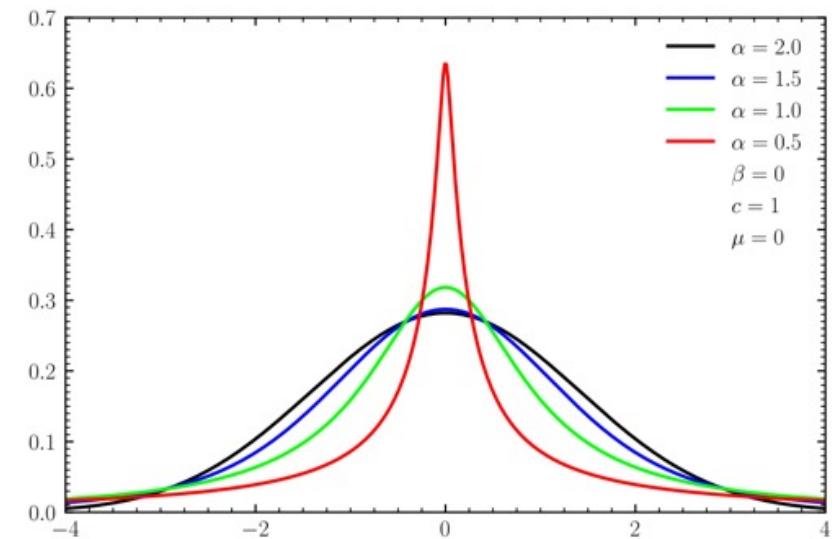
- ➊ Different experiments, different acceptances
- ➋ System size dependence?
- ➌ Centrality dependence?
- ➍ Energy dependence?



# Stable distribution

- If a lin. comb. of two indep. random variables with this distribution has the same distribution
- $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(q) e^{-ixq} dq$ , where the characteristic function:
- $\varphi(q; \alpha, \beta, R, \mu) = \exp(iq\mu - |Rq|^\alpha(1 - \beta \operatorname{sgn}(q)\Phi))$
- $\alpha$ : stability index
- $\beta$ : skewness, distribution is symmetric if  $= 0$
- $R$ : scale parameter
- $\mu$ : location parameter, if  $\beta = 0 \rightarrow$  median

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), & \alpha \neq 1 \\ -\frac{2}{\pi} \log|t|, & \alpha = 1 \end{cases}$$



- Sum of two random variables from a stable distribution gives something with the same values of  $\alpha$  and  $\beta$  (with possibly different values of  $\mu, R \dots$ )
- Ha  $\beta = 0 \Rightarrow$  Lévy symmetric alpha-stable distribution