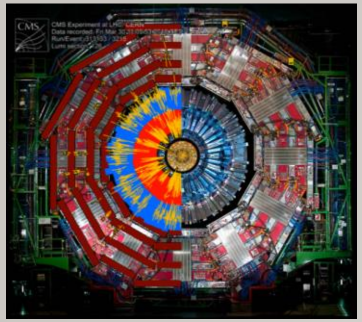

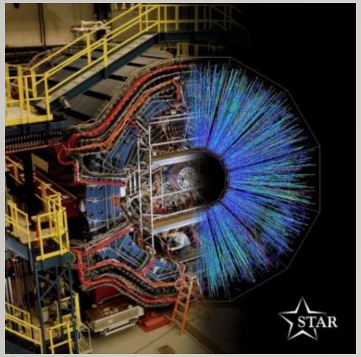

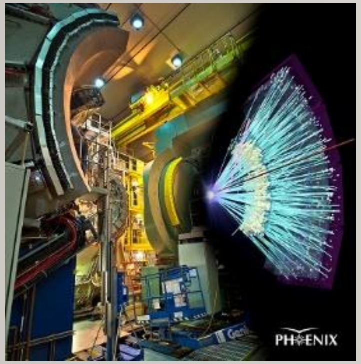
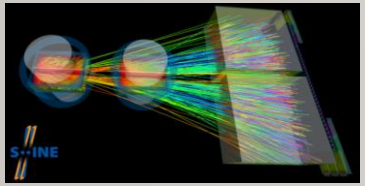

FEMTOSCOPY WITH LÉVY DISTRIBUTIONS FROM SPS TO LHC

MÁTÉ CSANÁD (EÖTVÖS UNIVERSITY)

INFN/BOLOGNA ALICE GROUP SEMINAR, SEPTEMBER 24, 2024

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
- Recent experimental results
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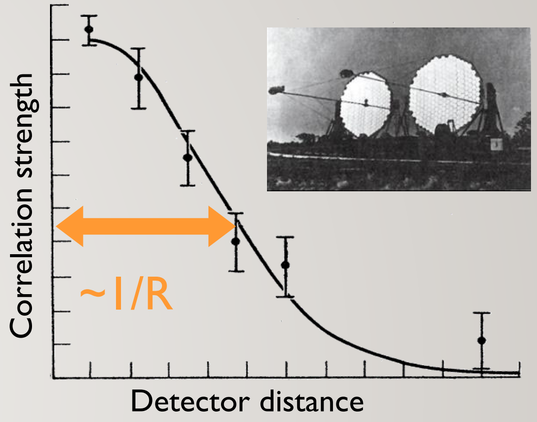
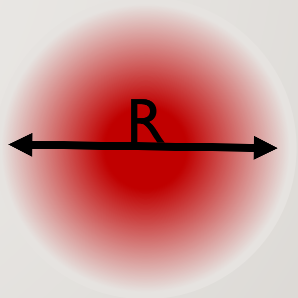
FEMTOSCOPY IN HIGH ENERGY PHYSICS

- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to source $S(r)$

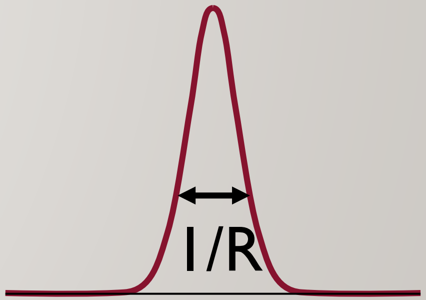
    $C(q) \cong 1 + \left|\int S(r) e^{iqr} dr\right|^2$ (under some assumptions)
  - Also with distance distribution $D(r)$:

    $C(q) \cong 1 + \int D(r) e^{iqr} dr$
  - Neglected: pair reconstruction, final state interactions, multi-particle correlations, coherence, …
- What is the source shape? Can be explored via femtoscopy



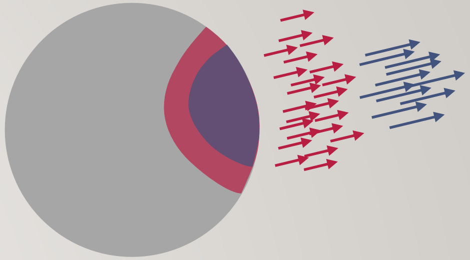


source function S(r)

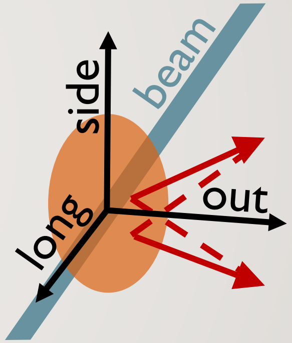


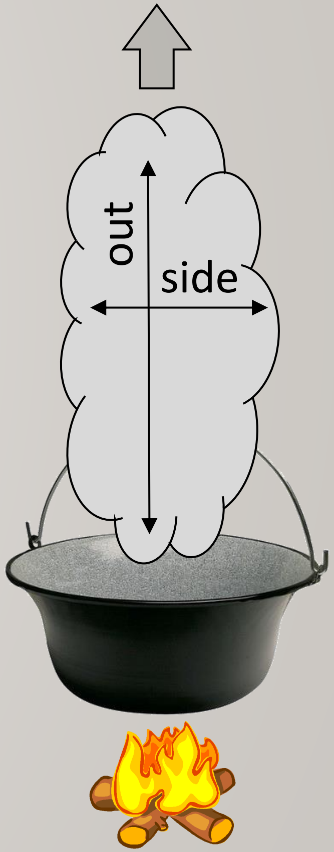
correlation funct. C(q)

WHY IS THE SOURCE SIZE IMPORTANT?



- Measured size related to expansion velocity and speed of sound
  - Testing Equation of State and phase transition (softest point)
  - Momentum dependence: collective flow (faster particles: smaller region)
- 3D measurements: connection to phase transition
  - Bertsch-Pratt coordinates:
    long (beam), *out* (pair momentum), *side* (⊥ to both)





- From a simple hydro calculation:

$$R^2_{\text{out}} = \frac{R^2}{1+u_T^2 m_T/T_0} + \beta_T^2 \Delta\tau^2, \qquad R^2_{\text{side}} = \frac{R^2}{1+u_T^2 m_T/T_0}$$

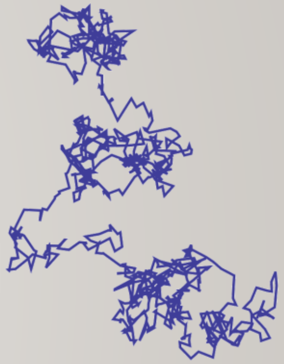
- LHC and RHIC top energy: $R_{\text{out}} \approx R_{\text{side}} \rightarrow$ no strong 1st order phase transition
- Plus lots of other details: pre-equilibrium flow, initial state, EoS, …

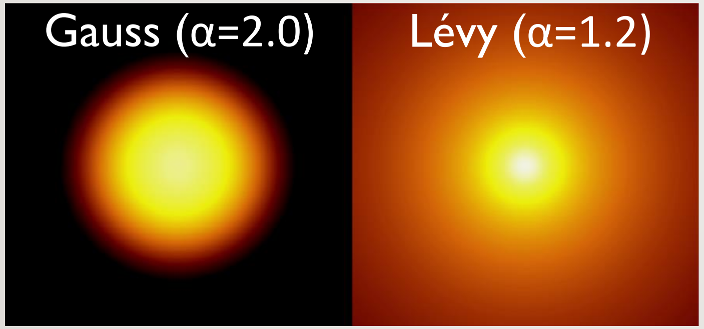
Chapman et al., PRL74(1995)4400, Csörgő and Lörstad, PRC54(1996)1390, Pratt, NPA830(2009)51C

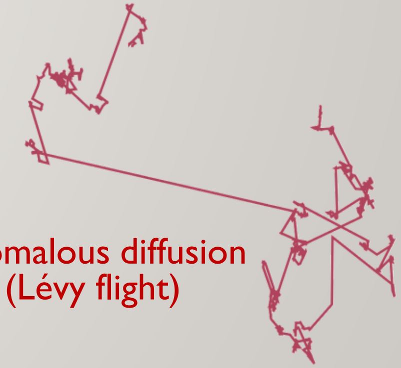
LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = (2\pi)^{-3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$, Gauss if $\alpha = 2$, Cauchy if $\alpha = 1$
  - Suggested by Csörgő, Hegyi and Zajc in Eur.Phys.J. C36 (2004) 67-78
  - First observed in L3 (Novák, Csörgő, Metzger, …) at LEP (e^+e^-) Eur.Phys.J.C 71 (2011) 1648
  - Special cases: α = 2 Gaussian, α = 1 Cauchy

Normal diffusion





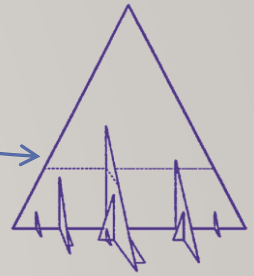


Anomalous diffusion
(Lévy flight)

- Shape of the correlation functions with Lévy source: $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$
- A possible reason for Lévy source: anomalous diffusion, many others

WHY DO LÉVY SHAPES APPEAR, WHY IS IT IMPORTANT?

- Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
  - Fractal nature of fragmented jets, creating power-law tails in spatial correlations
  - See also talk by Yacine Mehtar-Tani at ExploreQGP workshop in Belgrade

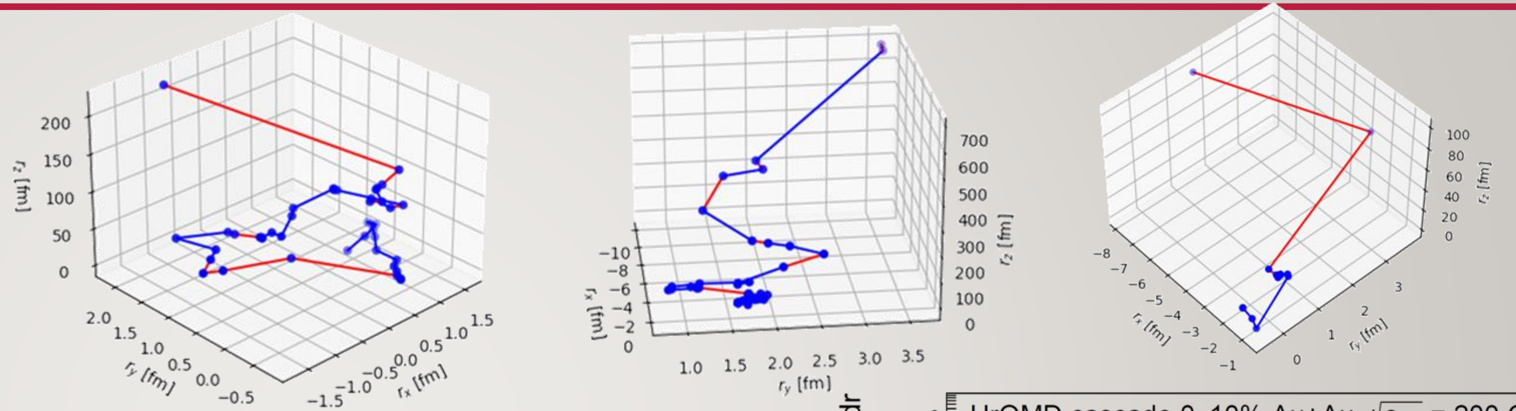


- Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
  - At the critical point: correlations at all scales ⇒ critical opalescence appears
  - Spatial correlations: power law with critical exponent η ⇔ Lévy-stability exponent α

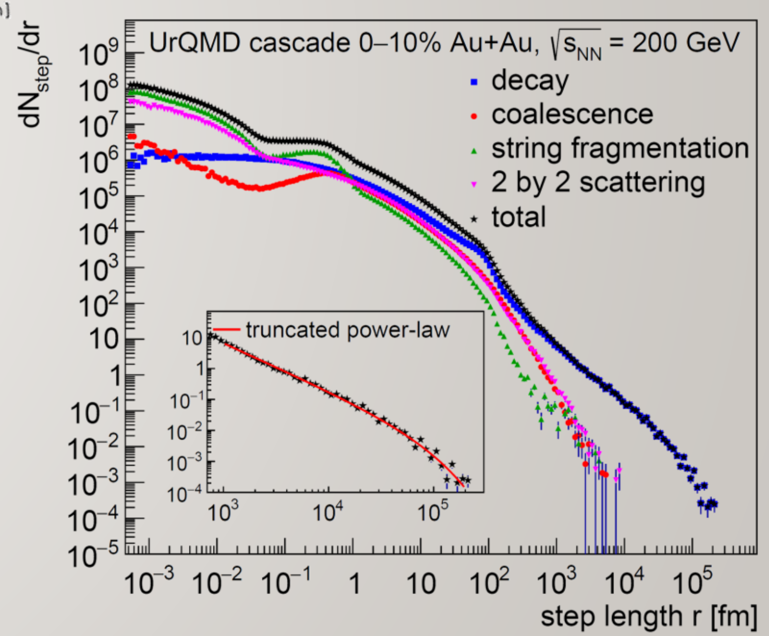
- Hadronic rescattering, Lévy flight (Braz.J.Phys. 37 (2007) 1002; Entropy 24 (2022) 308)
- Direction averaging and non-sphericality (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
- Event averaging (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
- Resonance decays (similarly to used fuel rods of power plants)
  (Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002; Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308)

LÉVY FLIGHT IN SIMULATIONS

- Increasing mean free path: stepsizes increase
- Examples from UrQMD:

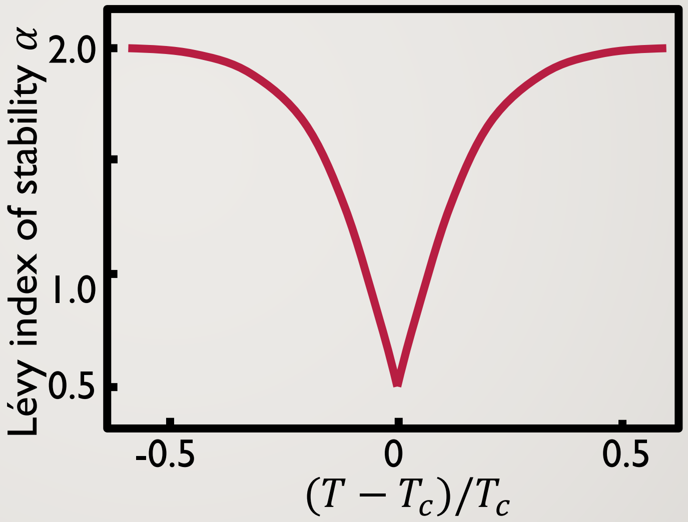


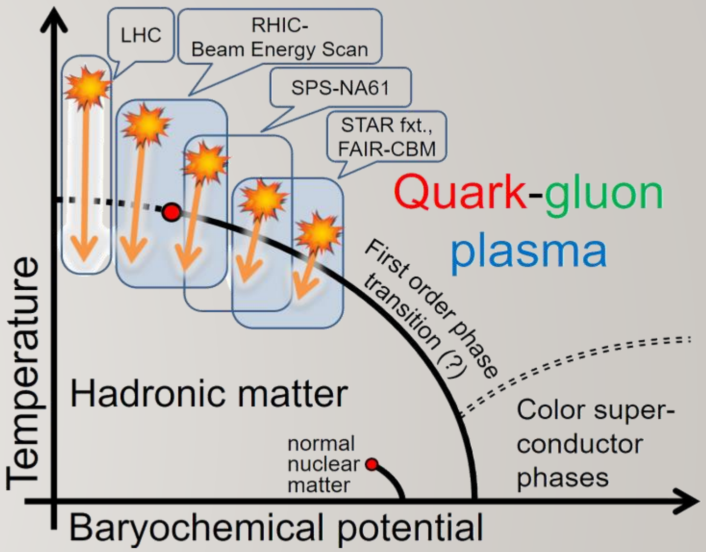
- If stepsize distribution has no finite width: generalized central limit theorem, Lévy-stable limiting distributions
- Indeed, observed in UrQMD
- Truncated power-law appearing
- More detailed simulations needed, see later in this presentation
- See details also in arXiv:2409.10373



LÉVY INDEX AS A CRITICAL EXPONENT?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$; Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: η = 0.50±0.05
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in α_{Levy} proximity of CEP?



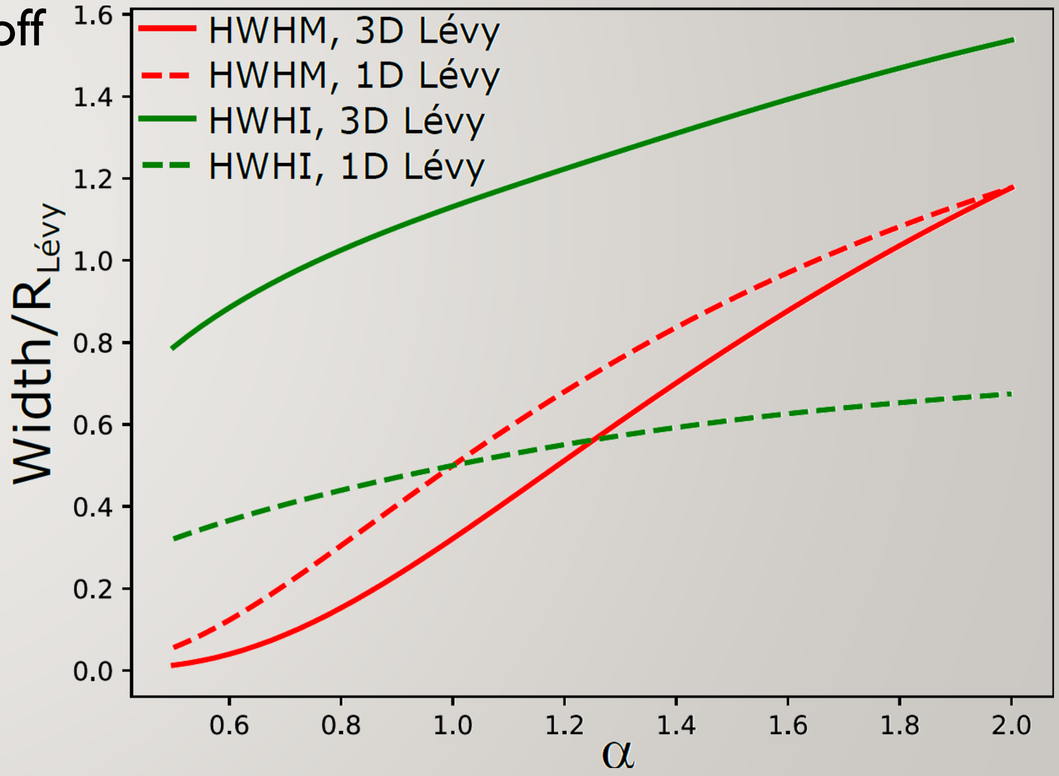


- Finite-size/time & non-equilibrium effects → what does power-law tail mean?
  - Finite-size effects not important? See e.g. Fytas et al, PRE108 (2023) 044146 Ballesteros et al., PLB387 (1996) 125

WHAT IS THE TRUE SIZE OF THE SOURCE?

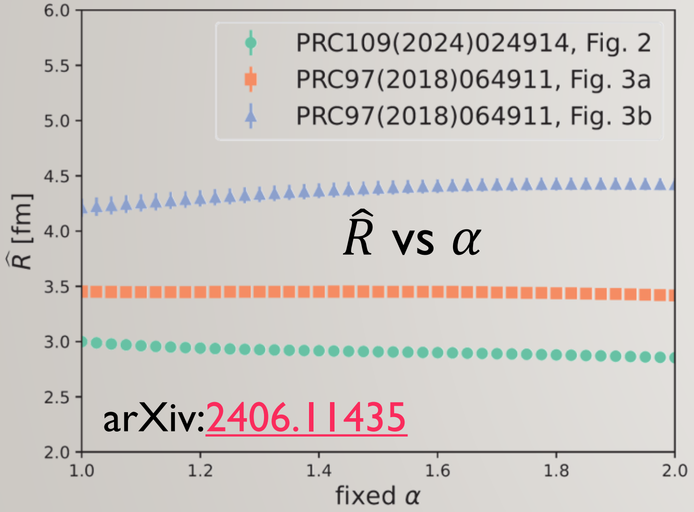
- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail depends on α
- If $S(r)$ Lévy, $D(r)$ Lévy with same α and $R \to 2^{1/\alpha}R$
- In principle, RMS $= \infty$ if $\alpha < 2$, practice: depends on cutoff
- What do Gaussian HBT radii mean?
  Important w.r.t. CEP search
- Alternative measures:
  - HWHI: (half) width at half integral
  - HWHM: (half) width at half max
  - Large difference between 1D and 3D relative width
  - Width (normalized by R) nontrivially depends on α
  - If $\alpha = 2$ or $\alpha = 1$ assumed: deviation from true scale

arXiv:2401.01249



HBT RADII WITH GAUSSIAN ASSUMPTION VS LÉVY FIT

- Significant peak in $R^2_{out} - R^2_{side}$ vs $\sqrt{s_{NN}}$ observed at STAR, PRC92(2015)014904, with Gaussian fits
- Signal of critical point, but what would happen for free-α fits?
- Let's utilize $\hat{R}$ scaling: $\hat{R} = \frac{\mathrm{R}}{\lambda\cdot(1+\alpha)}$ approx. independent of α

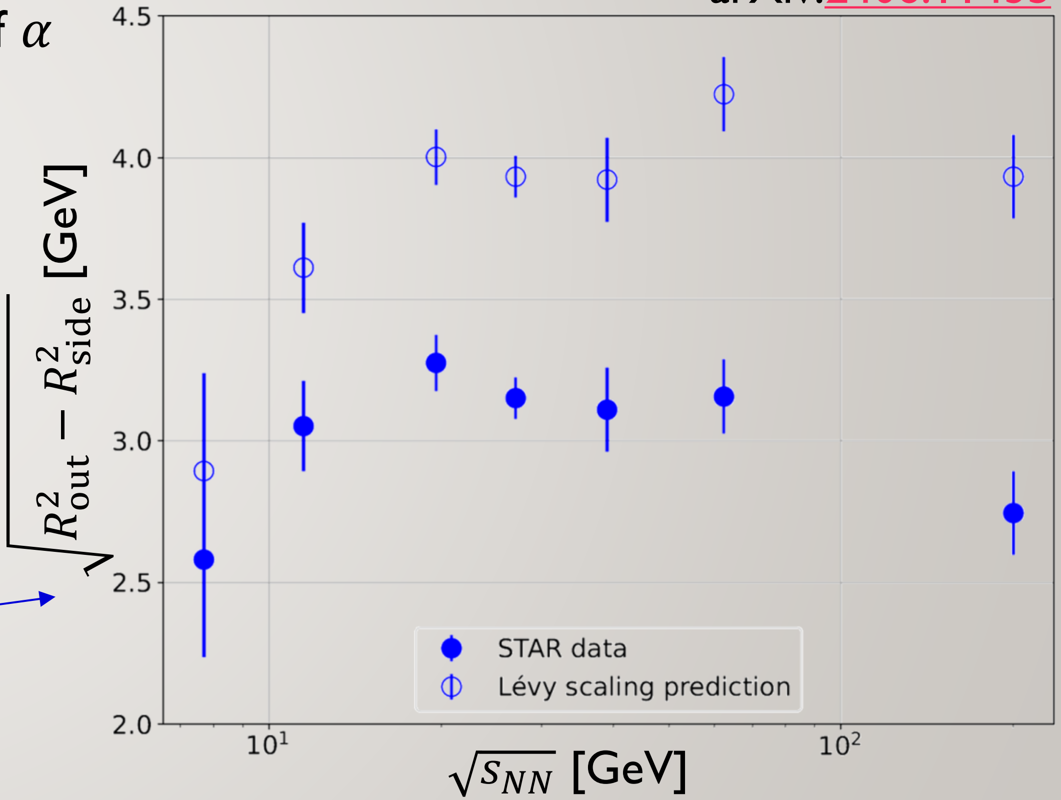


Based on STAR prelim. results:

$\alpha \to 1.85 \cdot \left(\frac{\sqrt{s_{NN}}}{1\ \mathrm{GeV}}\right)^{-0.06}$

$\lambda \to 0.60 \cdot \left(\frac{\sqrt{s_{NN}}}{1\ \mathrm{GeV}}\right)^{0.06}$

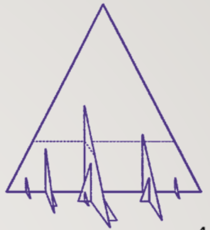
arXiv:2406.11435

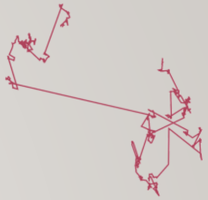


- Radii can be extrapolated from $\alpha = 2$ to free α
- Peak transforms into plateau!

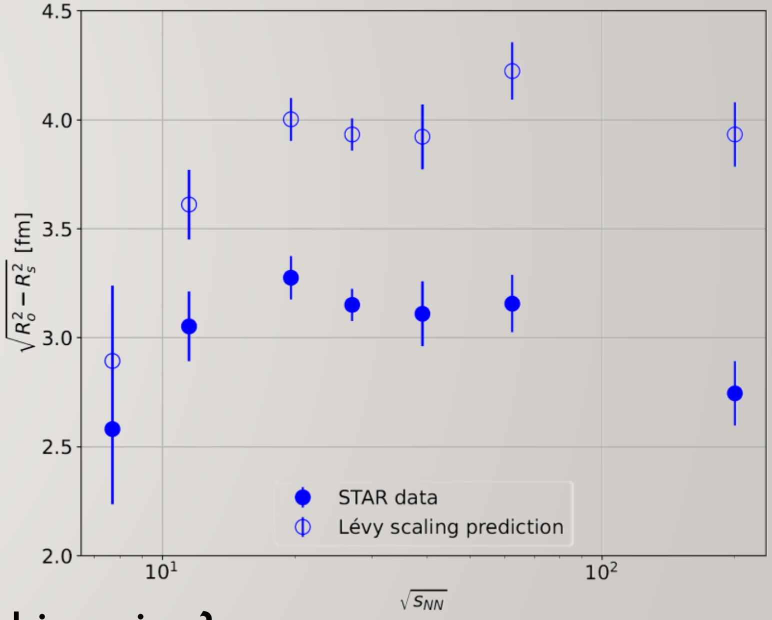
SO WHY SHOULD ANYONE ASSUME LÉVY SOURCES?

- Extra parameter (α) has physical meaning in each of the physical reasons of its appearance
  - Jet fragmentation, critical phenomena, anomalous diffusion





- When measuring source size (R) or strength (λ) with Gaussian assumption, actual size and shape information are entangled
- Why not try it?
  - One more parameter (with its ups and downs, e.g., interparam. corr.)
  - Coulomb more complicated (but by now not too complicated)
  - Note: radius means something else than Gaussian radius



- Why assume anything? Can't we just use spherical decomposition and imaging?
  - Not if we want to quantify details of the source, such as size or strength (e.g., to measure R_{out}, R_{side} vs $\sqrt{s_{NN}}$)

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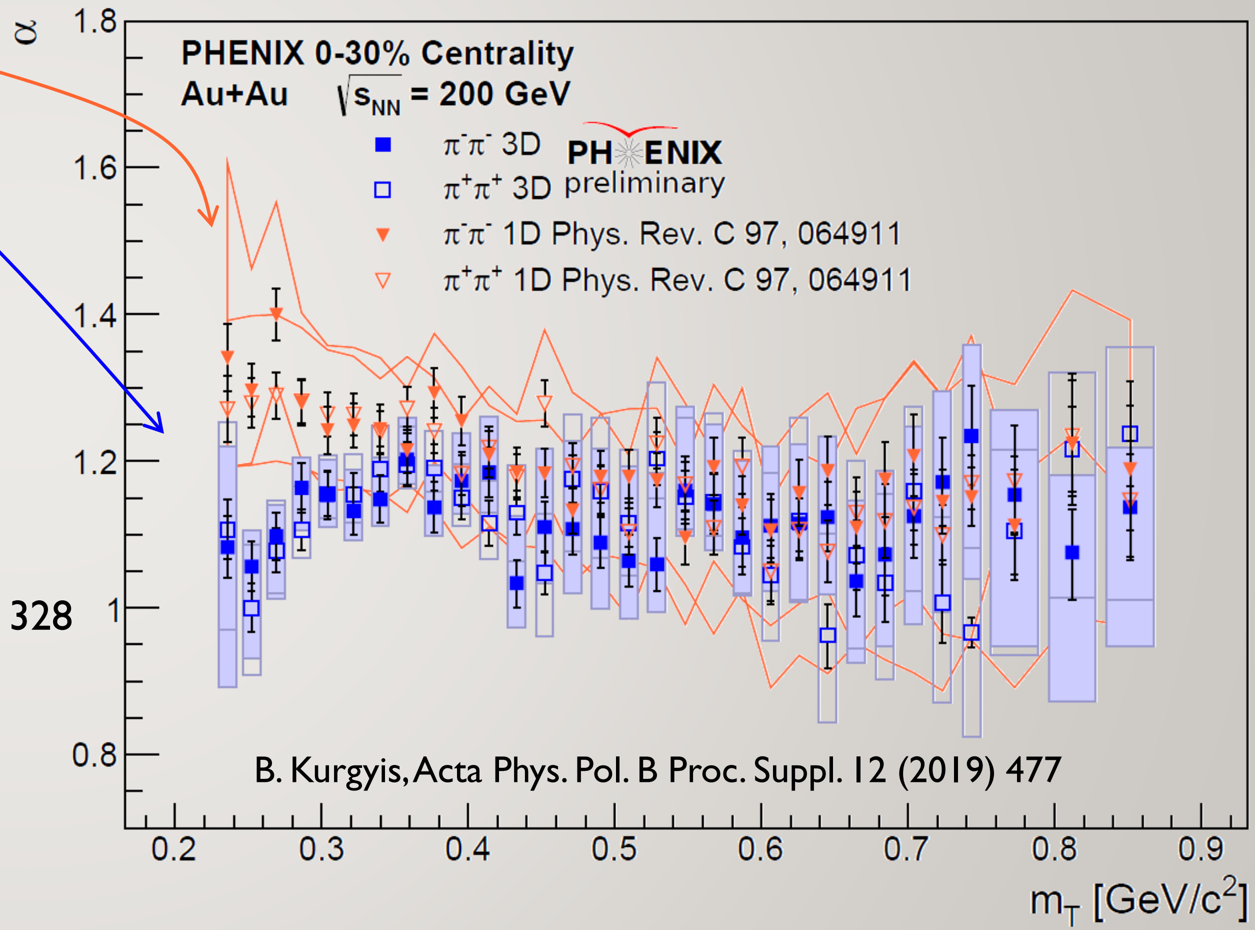
LÉVY EXPONENT VERSUS TRANSVERSE MASS, 1D AND 3D

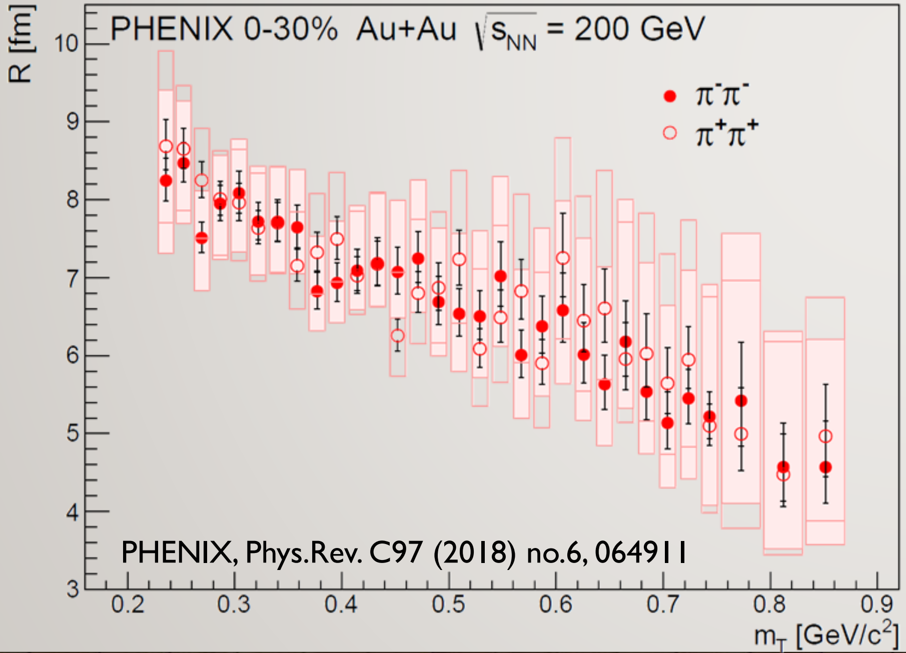
- Lévy exponent α in 3D close to 1D result
- On average still far from 2
- Observable differences at low m_T
  - Maybe due to lack of spherical symmetry?
- Coulomb effect for non-spherical sources?
  - Approximation possible
    Kurgyis, Kincses, Csanád, Nagy, Universe 9 (2023) 7, 328
  - If spherical in LCMS, radius in PCMS:

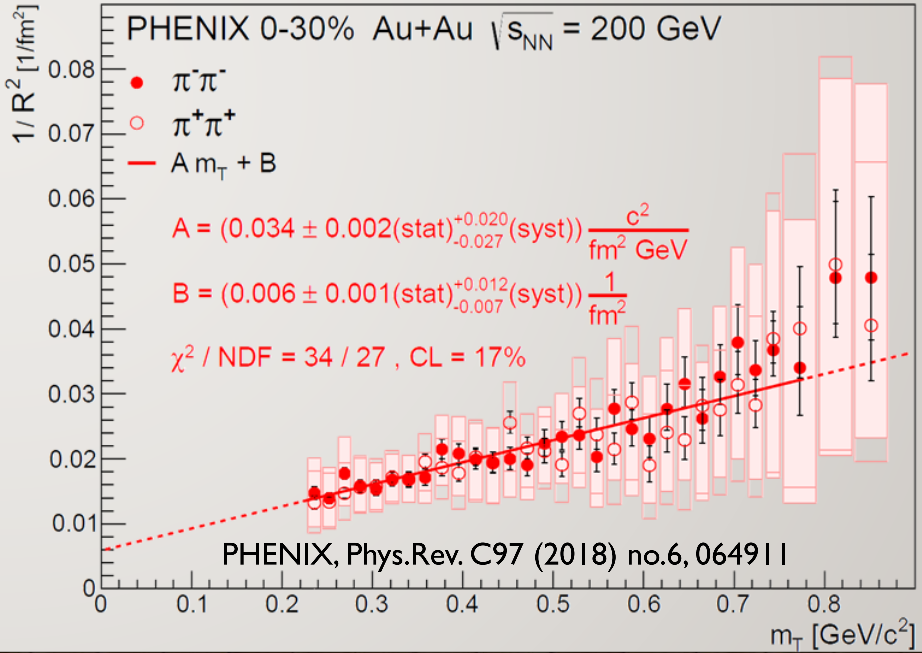
$$R_{PCMS} = \sqrt{\frac{1-2\beta_T^2/3}{1-\beta_T^2}} \cdot R_{LCMS}$$



LÉVY SCALE PARAMETER R AT RHIC

- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
  - RMS of a Lévy source: in principle infinity, obtained value depends on cutoff
- What do model calculations, simulations say about this?
- Hydro behavior ($1/R^2 \sim m_T$) not invalid; but: predicted for Gaussian case only!



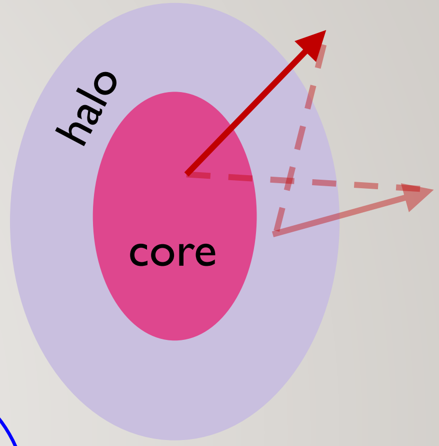


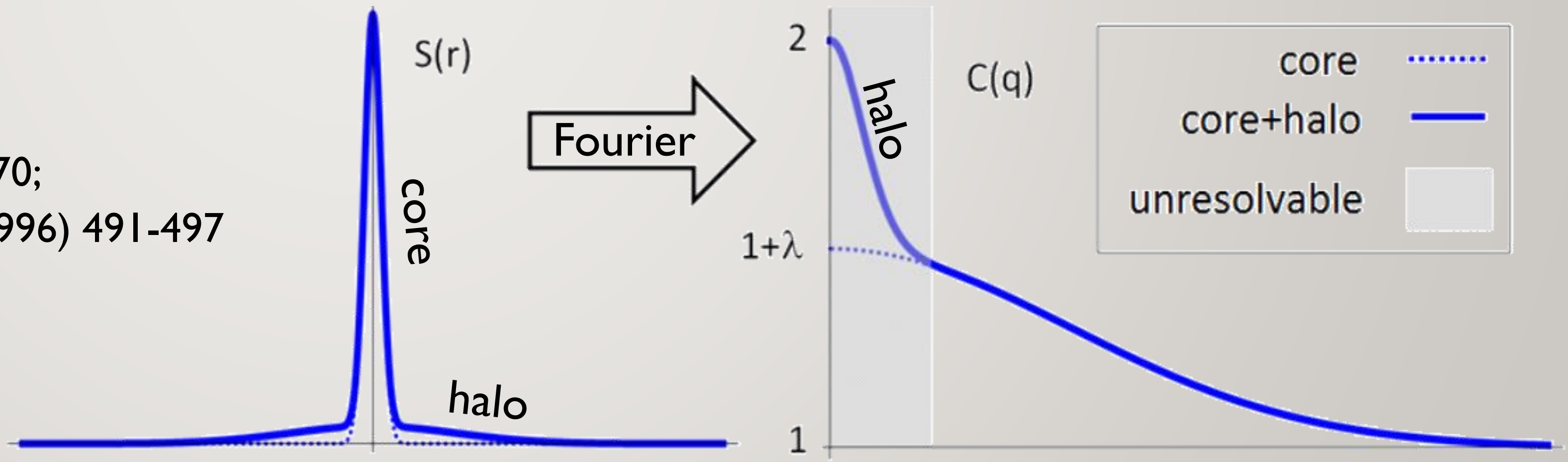
CORRELATION STRENGTH λ: CORE/HALO PICTURE

- Two-component source: core and halo
  - Core: hydrodynamically expanding, thermal medium
  - Halo: long lived resonances ($\gtrsim$ 10 fm/c, ω,η,η',K_0^S,...)
  - Halo unresolvable experimentally
  - Define $f_C = N_{\rm core}/N_{\rm total}$
- True $q \to 0$ limit: $C(0) = 2$
- Apparently $C(q \to 0) \to 1 + \lambda$
- $\lambda(m_{\rm T}) = f_C^2(m_{\rm T})$

Bolz et al, Phys.Rev. D47 (1993) 3860-3870;
Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497

- Can be cross-checked with three-particle correlations (see backup)

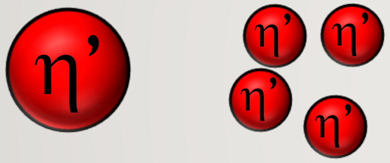




CORRELATION STRENGTH λ: IN-MEDIUM MASS?

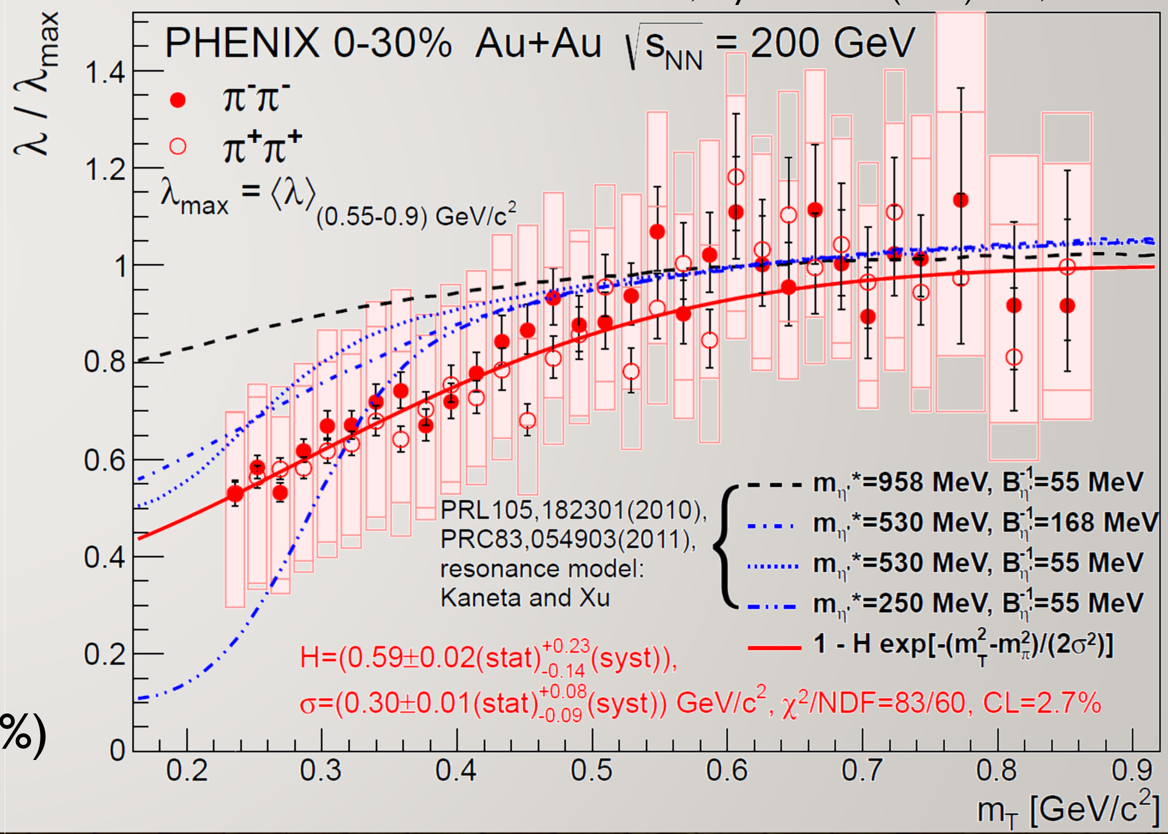
- Connection to chiral restoration
  - Decreased η' mass → more η' produced → more decay pions → λ decreases
  - Kinematics: η' → ππππ with low m_T → decreased $\lambda(m_T)$ specifically at low m_T
  - Dependence on in-medium η' mass?

Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301



$$T < T^*$$
$$m_{\eta'} > m^*_{\eta'}$$
$$N_{\eta'} < N^*_{\eta'}$$
$$\lambda > \lambda^*$$

PHENIX, Phys.Rev. C97 (2018) no.6, 064911

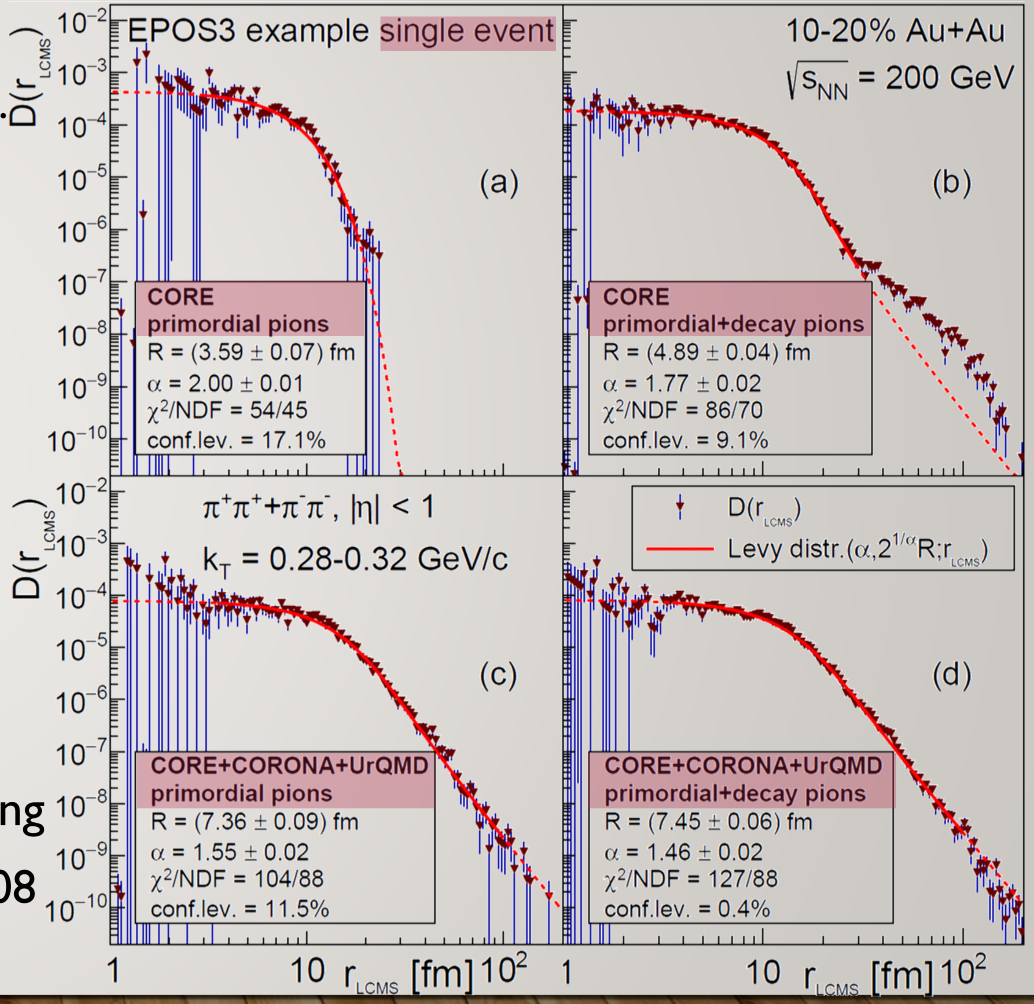


- Results not incompatible with this
- Recall: 3D results similar to 1D
- Would need direct check with photons ($\eta' \to \gamma\gamma$, 2.3%)
- Centrality dependent analysis in collaboration review

CONTENTS OF THIS TALK

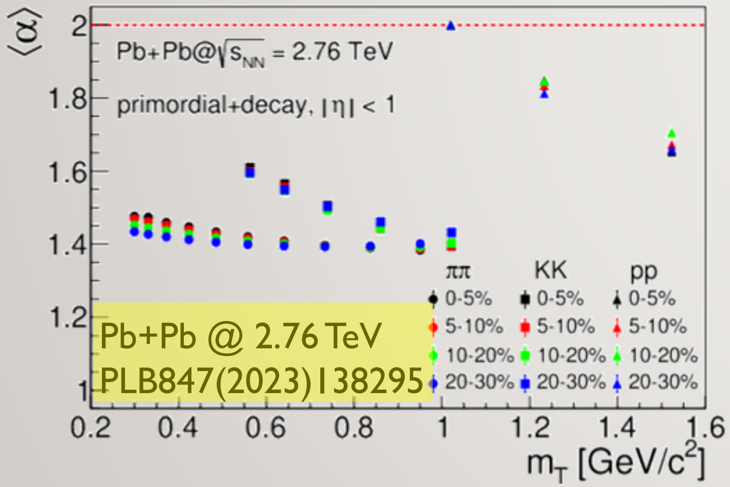
EVENT BY EVENT SHAPE ANALYSIS WITH EPOS IN AU+AU

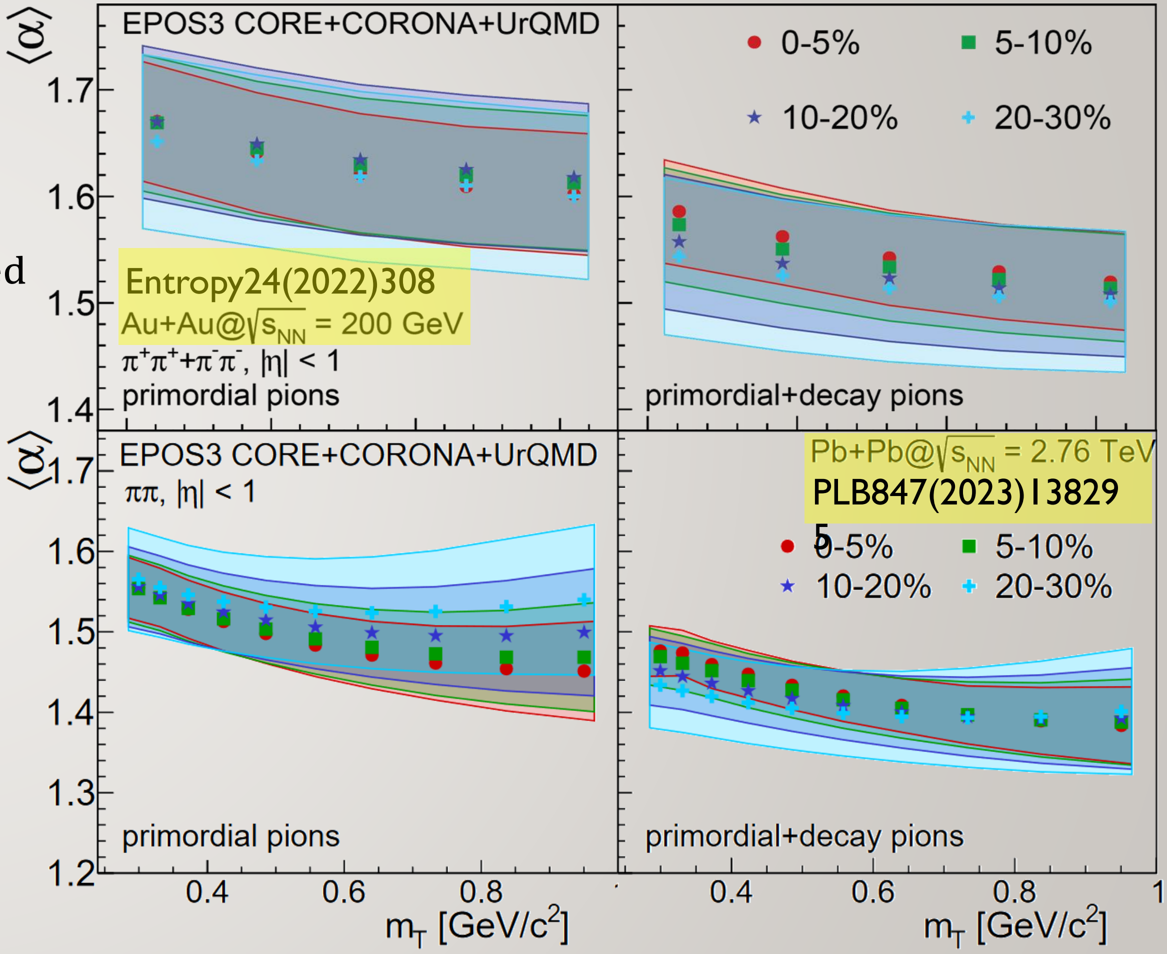
- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
  - Core-Corona, viscous hydro (vHLLE), cascades, UrQMD
- Pair distribution calculated: $D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$
  - Angle-averaged radial source distribution of like-sign pion pairs
- Investigated cases:
  a) CORE, primordial pions: close to Gaussian
  b) CORE, with decay products: power-law structures
  c) CORE+CORONA+UrQMD, primordial pions: Lévy shape
  d) CORE+CORONA+UrQMD, with decay products: Lévy shape
- Lévy shape in single events; source size versus m_T: hydro scaling
  - 200 GeV AuAu: Kincses, Stefaniak, Cs., Entropy 24 (2022) 308
  - 2.76 TeV PbPb: Kórodi, Kincses, Cs., PLB 847 (2023) 138295



AVERAGE LÉVY EXPONENT VS TRANSVERSE MASS

- $\langle\alpha\rangle$ versus m_T and centrality: small dependence
  - 200 GeV Au+Au: Entropy 24 (2022) 308
  - 2.76 TeV Pb+Pb: Phys. Lett. B 847 (2023) 138295
- With or without decays at RHIC: $\alpha_{\mathrm{EPOS}} > \alpha_{\mathrm{measured}}$
- Particle type dependence analyzed as well



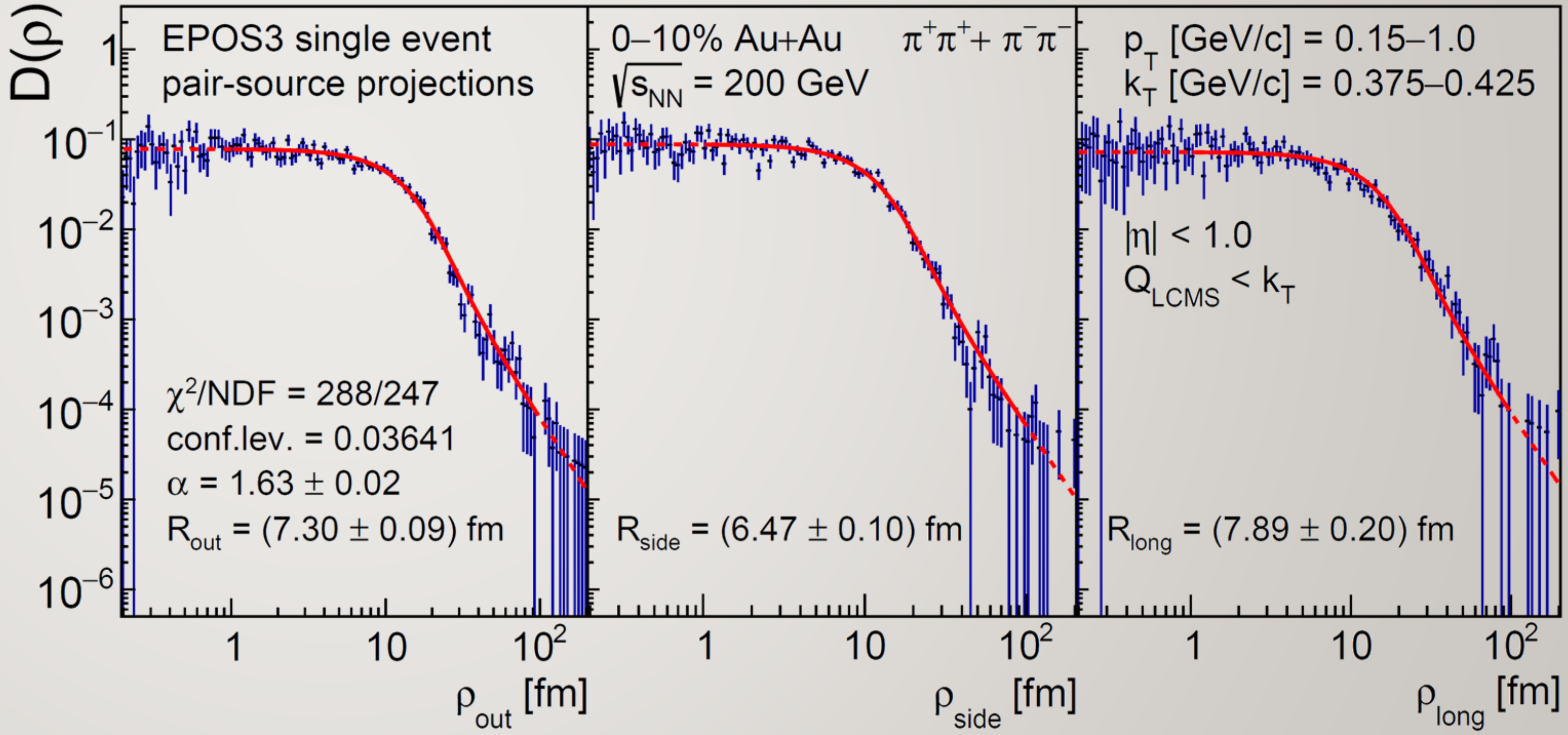


- 3D analysis as well

3D EVENT-BY-EVENT EPOS ANALYSIS

- Similar analysis performed with 3D pair source as well: Lévy-fits describe EPOS results
- Not angular averaging causes event-by-event Lévy shapes
- 1D projections shown: →
- Note: spatial variable
  $\vec{\rho} = \vec{r} - \vec{\beta} t$, where $\vec{\beta} = \vec{K}/K_0$
- In LCMS:
  $\vec{K} = (K_{out}, 0,0)$ thus
  $q_0 = q_{out}\beta_{out}$
- Need to boost ρ to LCMS

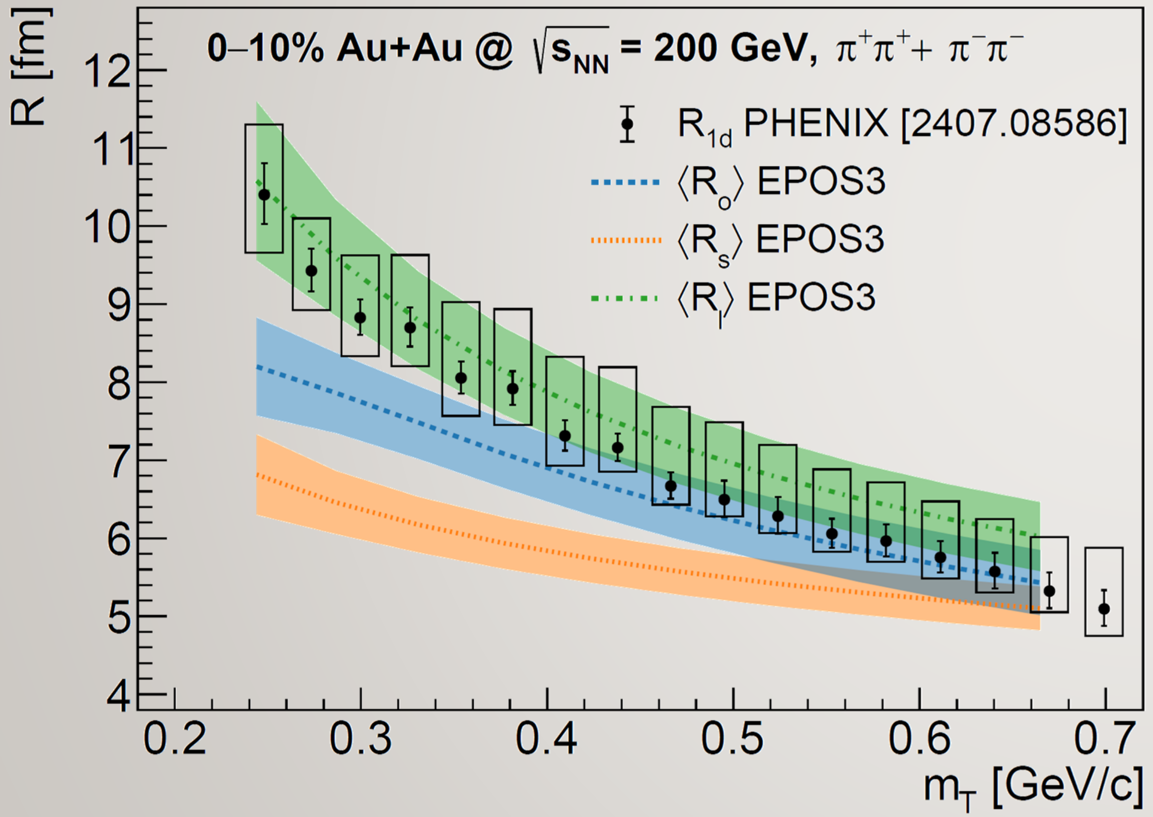
arXiv:2409.10373

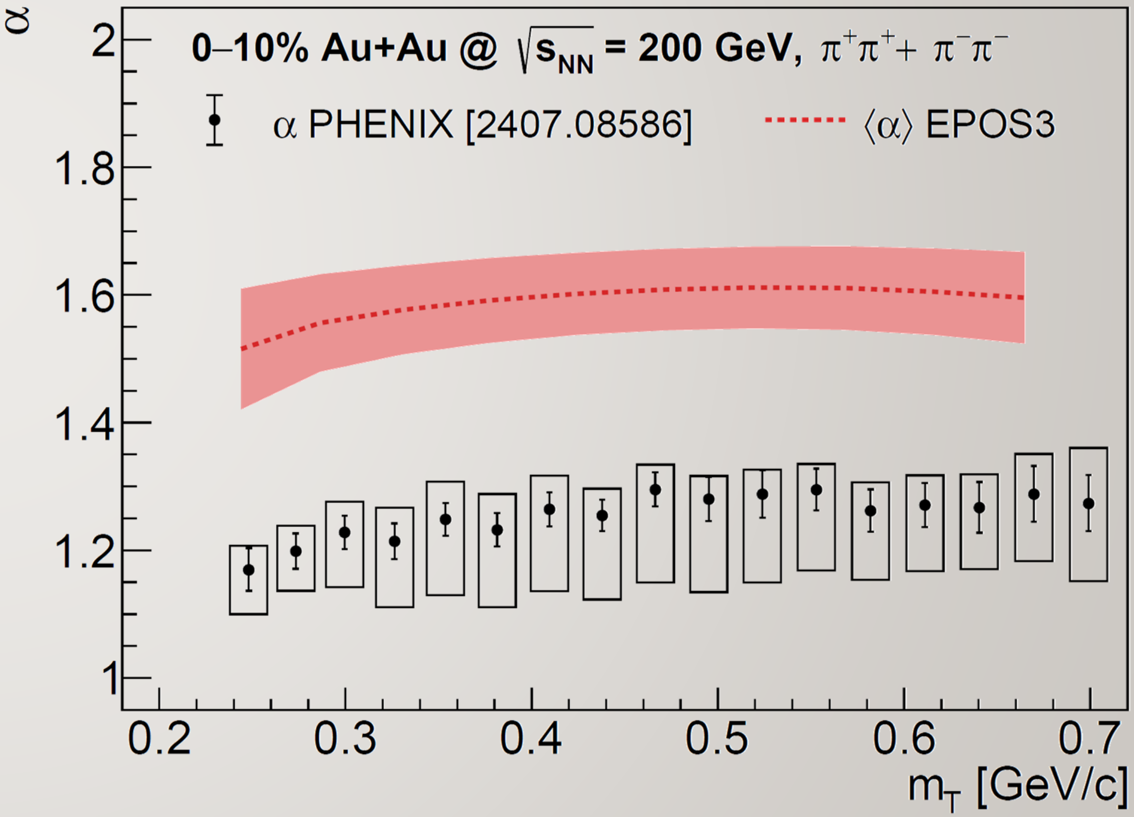


3D EVENT-BY-EVENT EPOS ANALYSIS: LÉVY PARAMETERS

- Within EPOS, 3D and 1D results fully compatible
- Compare to 1D PHENIX measurement: similar radius but much smaller α

arXiv:2409.10373





ROLE OF INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $\left|\Psi_2^{(0)}(r)\right|^2 = 1 + e^{iqr}$:

$$C_2(q,K) \cong \int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r,K)e^{iqr}\,dr$$

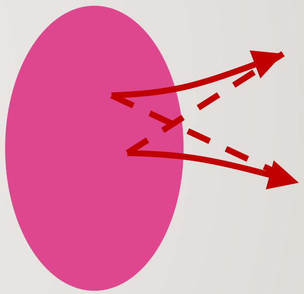
- If there is interaction: $\Psi_2^{(0)}(r) \to \Psi_2^{(\mathrm{int})}(r_1, r_2)$
- For Coulomb:

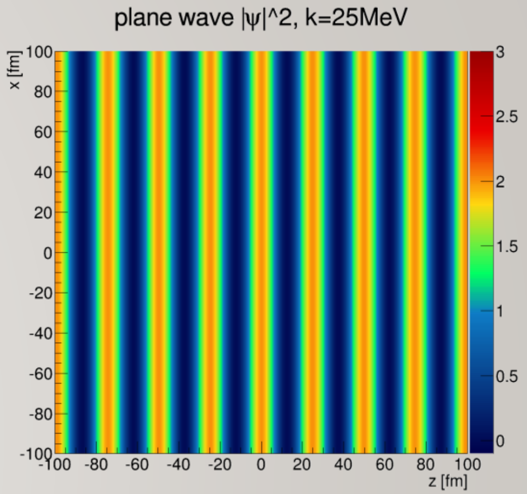
$$\left|\Psi_2^{(C)}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot \left(\text{complicated hypergeometric expression}\right)$$

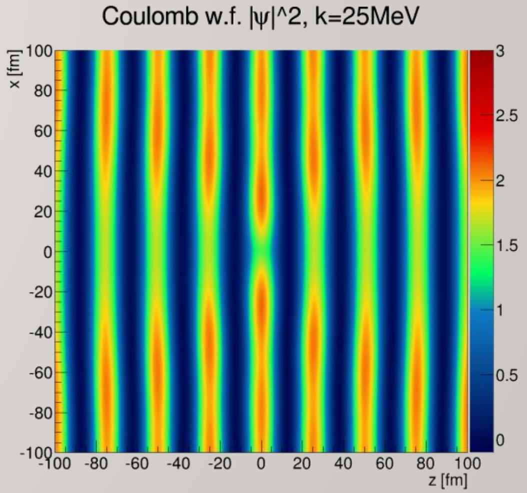
- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\mathrm{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r,K)\left|\Psi_2^{(C)}(r)\right|^2 dr}{\int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr}$$

- Complication: need for integrating power-law tails

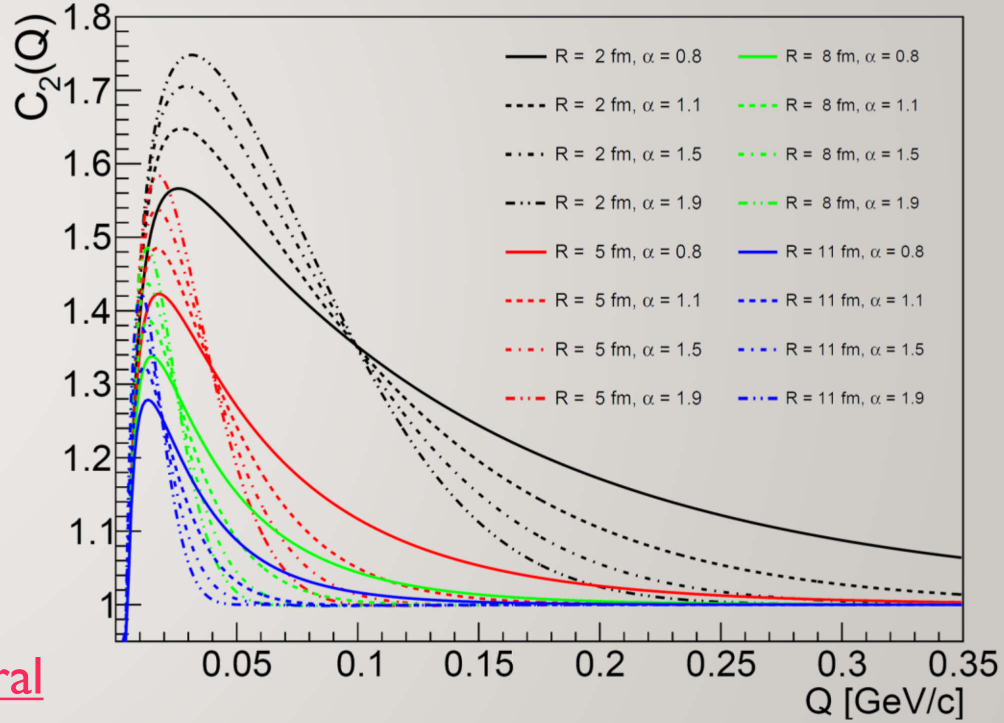






HOW TO CALCULATE THE COULOMB EFFECT

- Calculating correlation functions with the Coulomb effect included: time consuming in the past
- Method used in early analyses: Coulomb correction calculated for fixed radius and shape
  - For example, fixing $R = 5$ fm and $\alpha = 2$
- More consistent method: correlation function with Coulomb FSI precalculated in a tabular form
  - Iterative fitting, see e.g., PHENIX, PRC97 (2018) 6, 064911
- Convenient, but somewhat restricted method: interpolating functional form, in a limited R, α range
  - See Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238, used in arXiv:2306.11574 [CMS], EPJC83(2023)919 [NA61]



- A novel method: github.com/csanadm/CoulCorrLevyIntegral
  Nagy, Purzsa, Csanád, Kincses Eur. Phys. J. C 83, 1015 (2023), code at

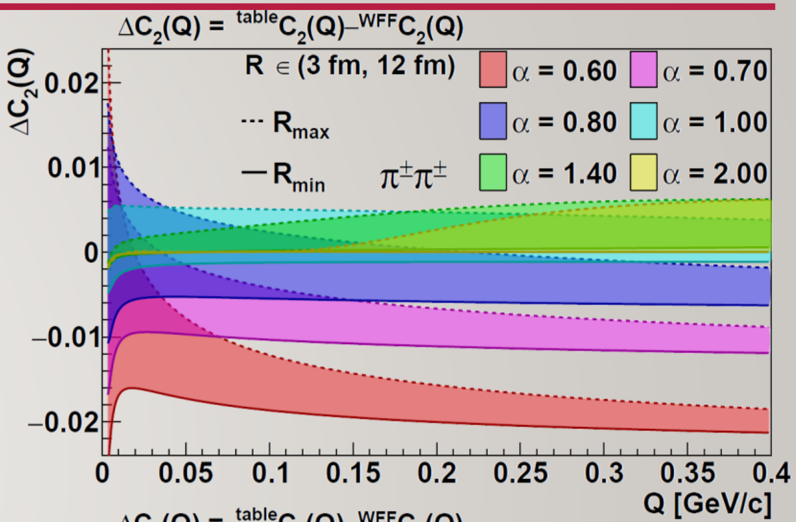
A NOVEL METHOD FOR LÉVY SHAPES WITH COULOMB FSI

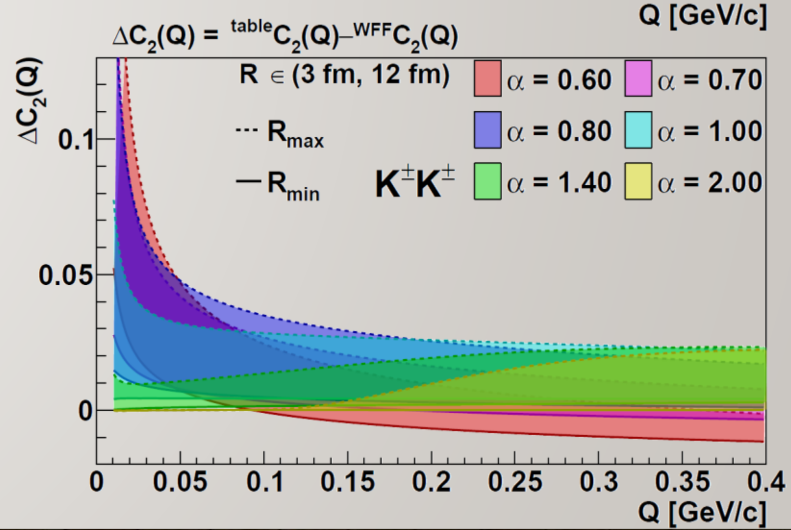
- Recent mathematical development:

  Coulomb integral $C_2(Q) = \int d^3r \left|\psi_Q(r)\right|^2 D(r)$ can be performed
  - $D(r)$ is expressible as a Fourier transform:
    $D(r) = \int d^3q e^{iqr} f(q)$, for example $D(r)$ Lévy: $f(q) = e^{-|qR|^\alpha}$
  - Integrals $\int d^3r$ and $\int d^3q$ unfortunately cannot be exchanged
  - Calculation can still be performed via Lebesgue and Fubini theorems

- Result: $C_2(Q) = |\mathcal{N}|^2 \left(1 + f(Q) + \frac{\eta}{\pi}[A_{1s}[f](Q) + A_{2s}[f](Q)]\right)$,

  where $|\mathcal{N}|^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$ (Gamow), $\eta = \frac{mc^2\alpha}{\hbar c Q}$, A_{ns} functionals
  - Few percent difference to numerical (tabularized) values used earlier
  - Details in Nagy, Purzsa, Csanád, Kincses Eur. Phys. J. C 83, 1015 (2023), code at github.com/csanadm/CoulCorrLevyIntegral



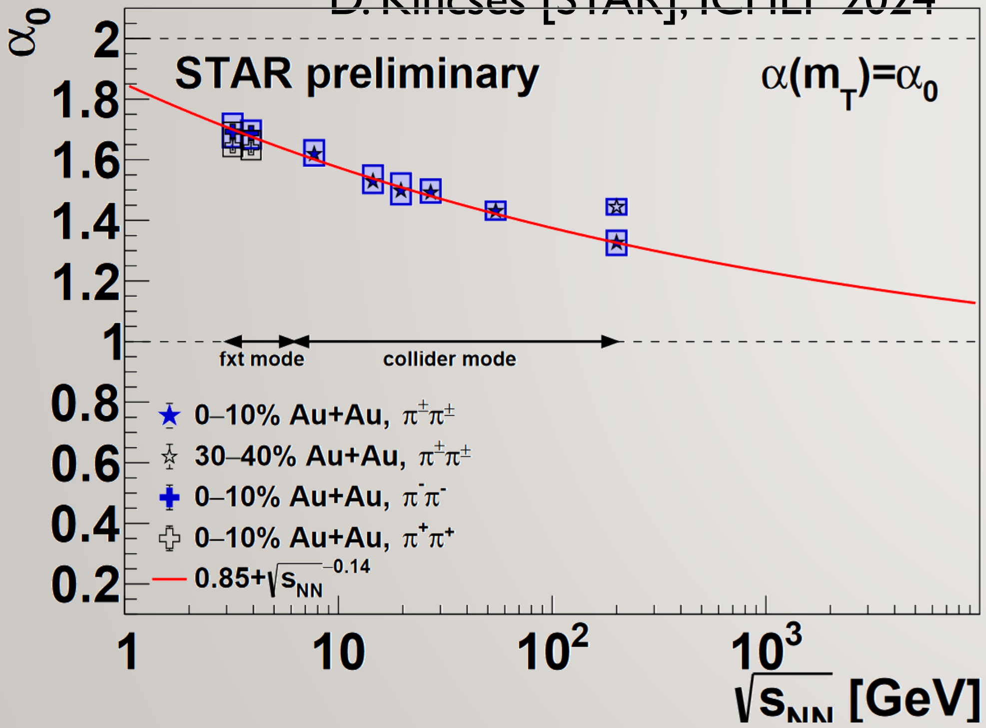


CONTENTS OF THIS TALK

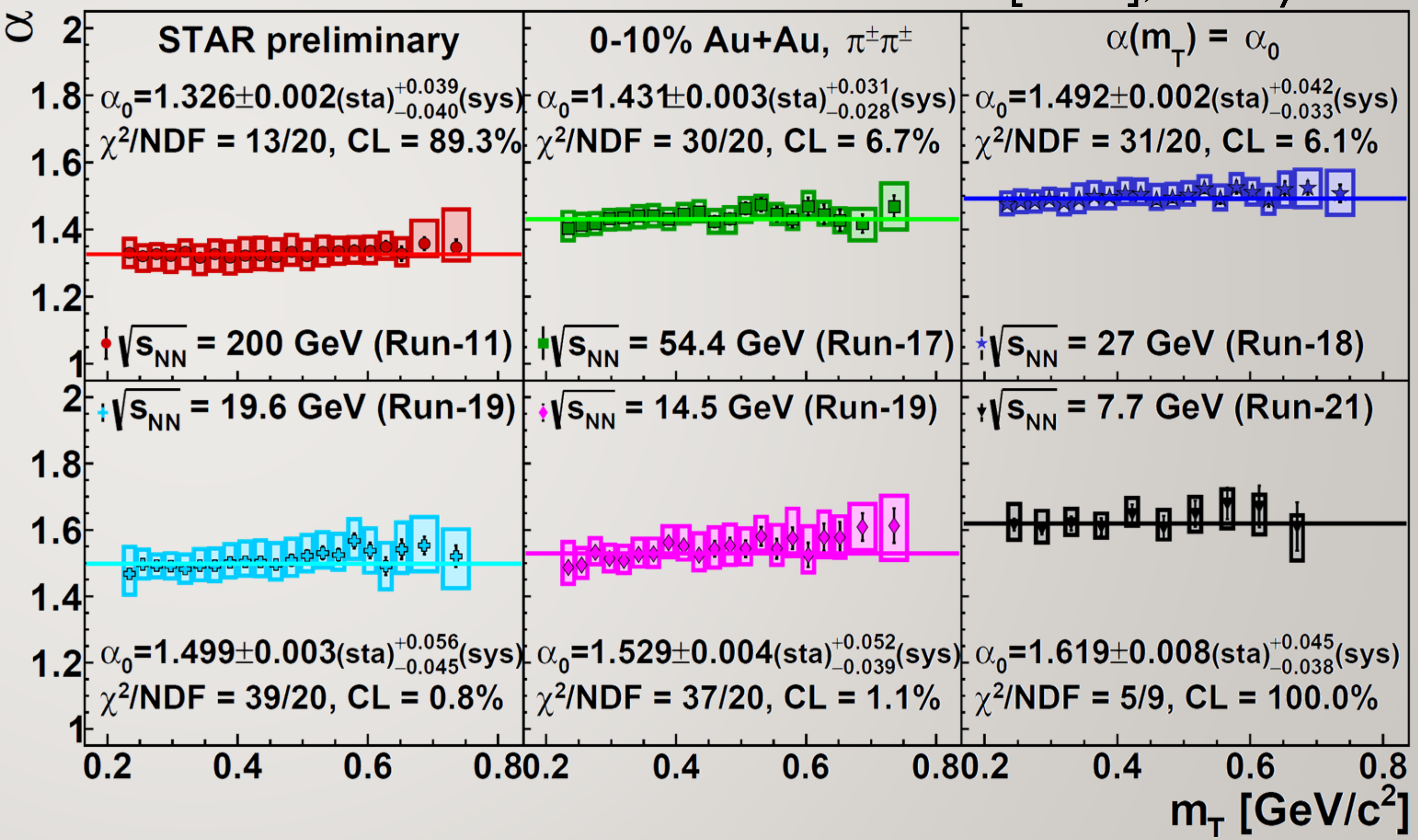
STAR RESULTS: COLLISION ENERGY DEPENDENCE OF α

- Lévy-stability parameter α approximately constant as a function of m_T
- Clear collision energy dependence

D. Kincses [STAR], ICHEP 2024



D. Kincses [STAR], Zimányi 2023

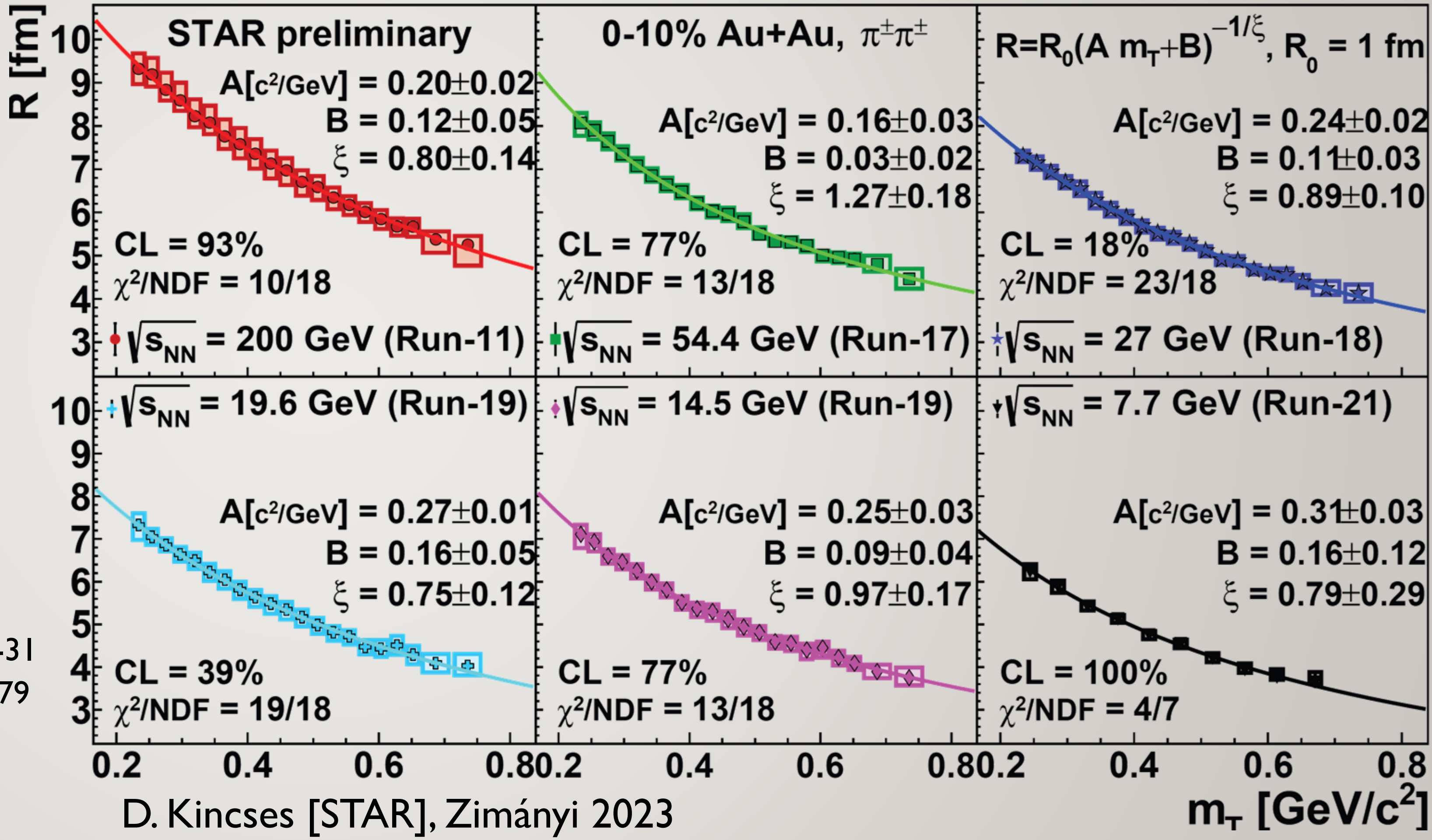


STAR RESULTS: COLLISION ENERGY DEPENDENCE OF R

- Decrease with $\sqrt{s_{NN}}$
- Decreasing trend with m_T
  - Connected to flow?
  - Fits with functional form of $R = R_0(Am_T + B)^{-1/\xi}$ describe data
- Hydro calculations (R_{Gauss}) obtain usually $\xi = 2$

Makhlin, Sinyukov, Z. Phys. C 39, 69 (1988)
Csörgő, Lörstad, Phys. Rev. C 54, 1390 (1996)
Chapman, Scotto, Heinz, Acta P.H.A 1 (1995) 1-31
Csanád, Csörgő, Lörstad, Ster, JPG30(2004)S1079

- Here: $\xi \approx 1$



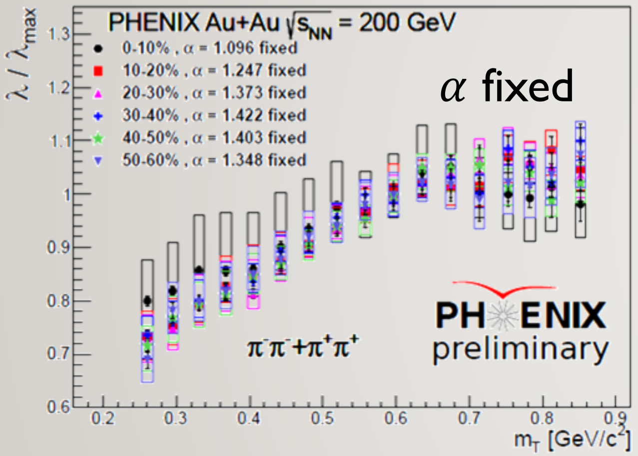
D. Kincses [STAR], Zimányi 2023

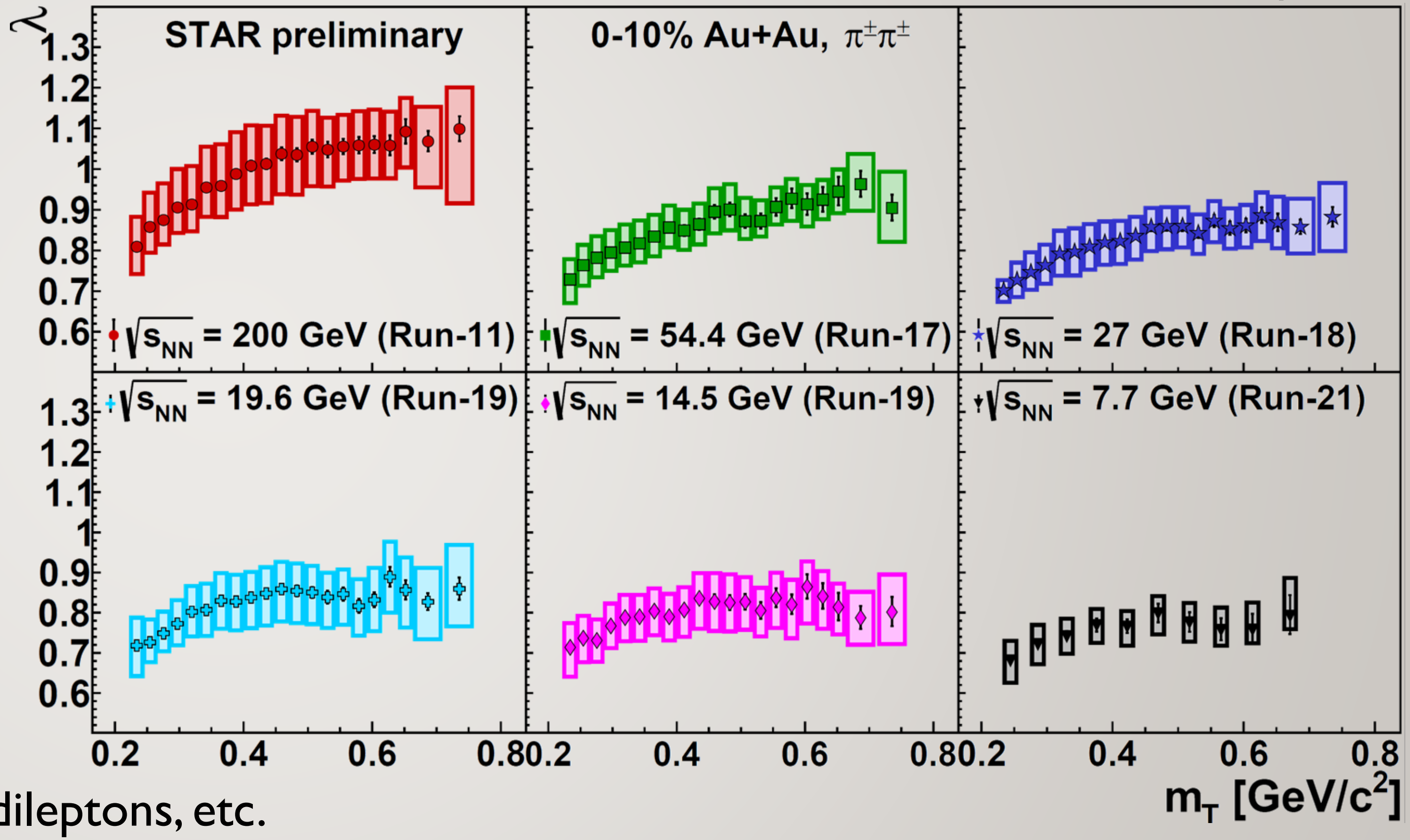
HOLE IN $\lambda(m_T)$: ALL MEASUREMENTS AT RHIC

D. Kincses [STAR], Zimányi 2023

- Hole apparent for all energies, somewhat dependently of centrality

S. Lökös, Universe 4 (2018) 31

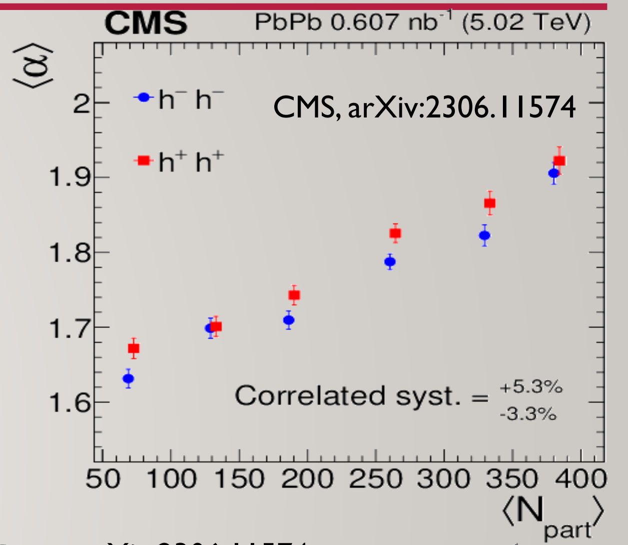


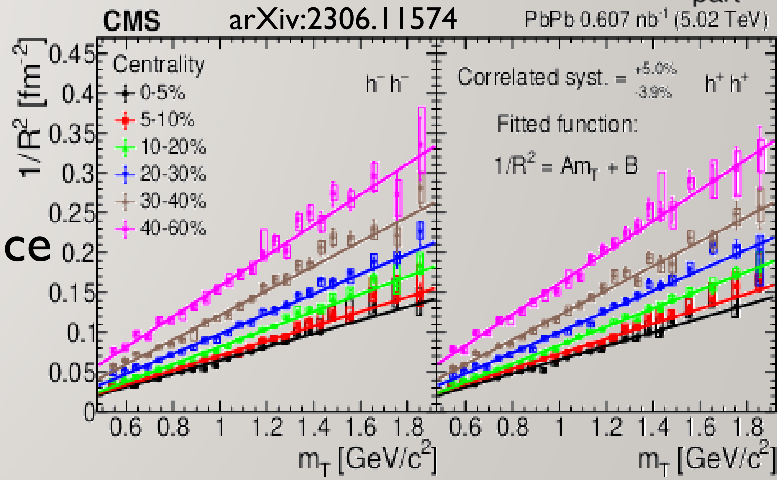


- Due to reduced η' mass?
- Sign for chiral restoration?
- To be cross-checked with photons, dileptons, etc.

CHARGED HADRON ANALYSIS IN 5 TEV PB+PB WITH CMS

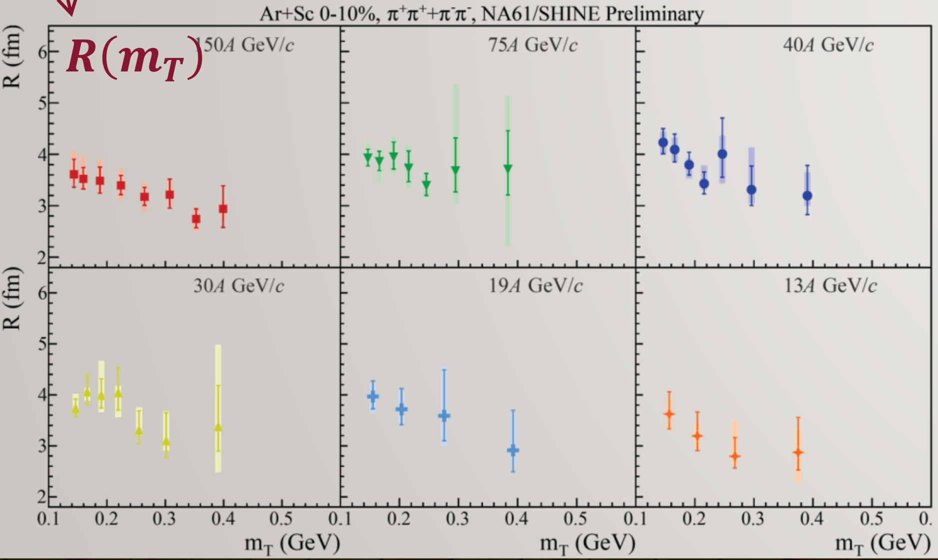
- Lévy index α measured:
  - Far from Cauchy
  - Clearly not Gaussian
  - Closer to Gaussian for large N_{part}, unlike RHIC
- Lévy scale R: hydro scaling confirmed
  - In every centrality class, despite non-Gaussianity
  - Hubble coefficient can be extracted: 0.12-0.18 c/fm, larger than at RHIC
- Correlation strength λ also analyzed
- Low-Q deviation cross-checked with Monte-Carlo: two-track acceptance
- Final CMS result: Phys.Rev.C109(2024)024914, arXiv:2306.11574
  - Preliminary results in proceedings: B. Kórodi, Universe 9 (2023) 7, 318

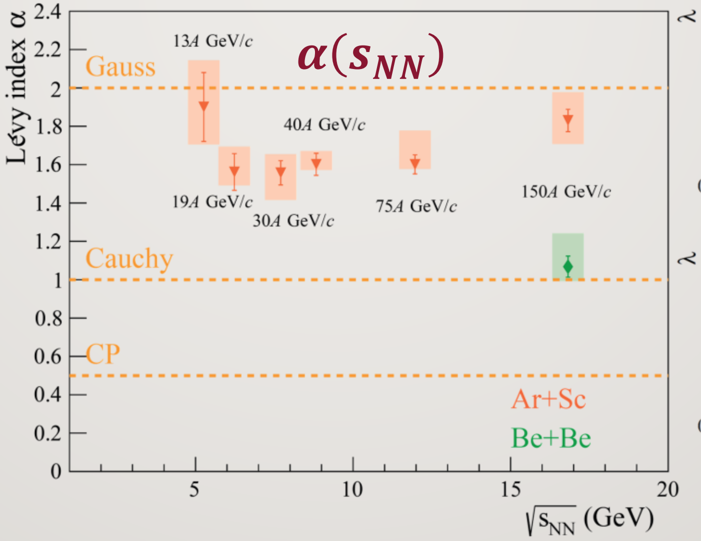


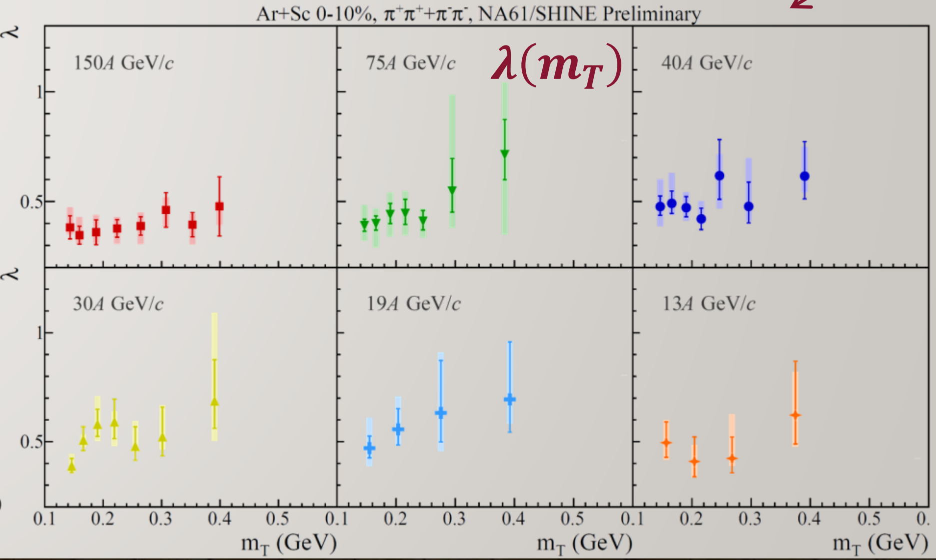


NA61/SHINE RESULTS

- Lévy scale R of Ar+Sc [arXiv:2406.02242 & ICHEP 2024] and Be+Be [Eur.Phys.J.C 83 (2023) 10, 919]:
  - Compatible with initial geometry factor 1.6 between Ar+Sc and Be+Be; small energy dependence
  - Decrease with m_T due to transverse flow?
- No m_T dependence in λ, in contrast to RHIC result – can be turned off?
- Lévy index α [see ICHEP talk]: significant difference between Ar+Sc and Be+Be, small energy dependence



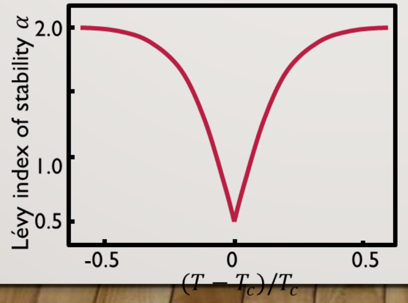


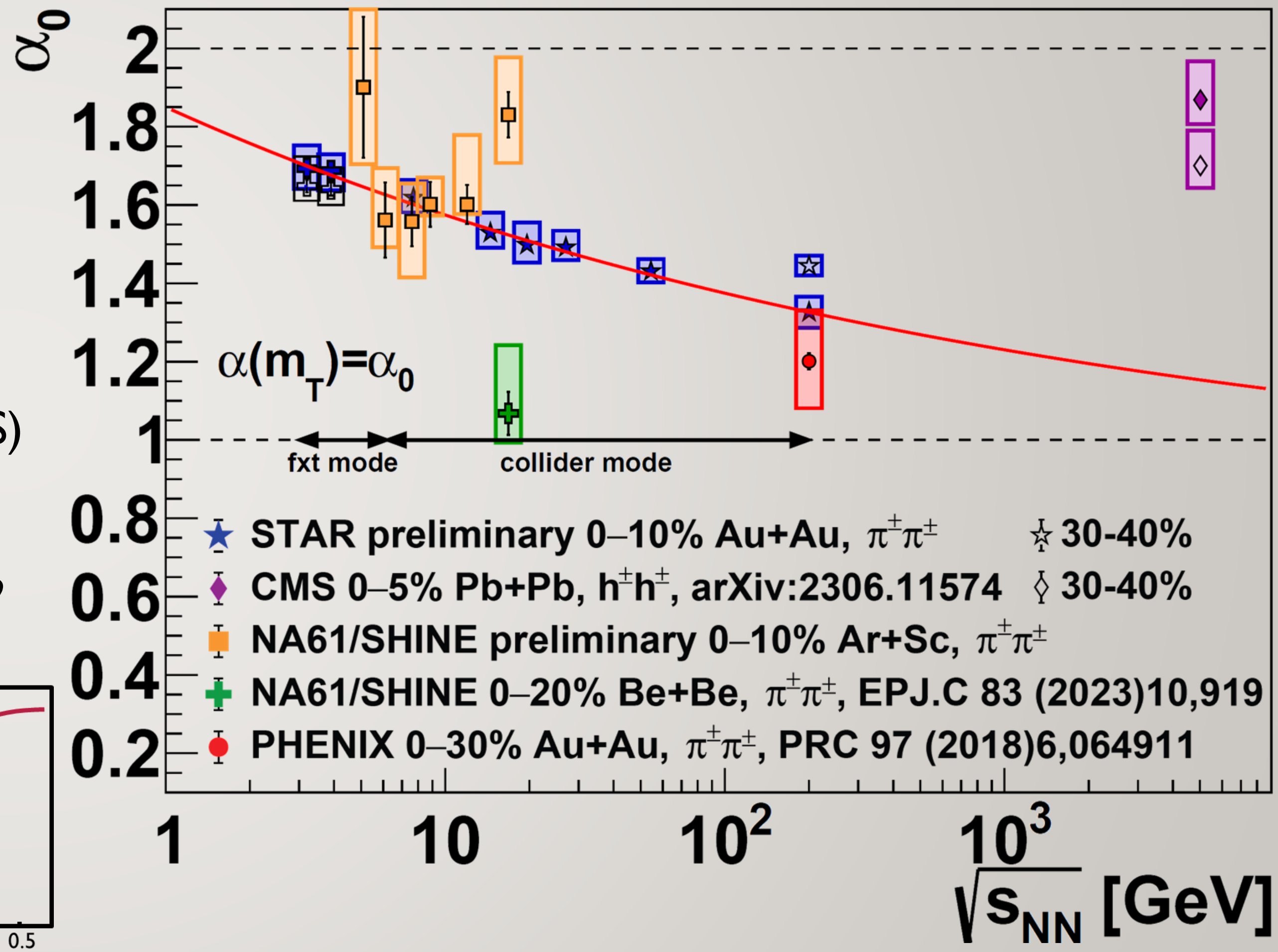


COLLISION ENERGY AND SYSTEM SIZE DEPENCENCE

- Pb+Pb at LHC and Ar+Sc at SPS: largest α
- Difference between small and large systems?
  - Pb+Pb and Au+Au are large
  - Ar+Sc and Be+Be (and p+p) small
- Very different momentum range (low-p_T cut)?
  - 0 GeV (NA61), 0.2 GeV (STAR), 0.5 GeV (CMS)
  - Checked: this does not change α
- Reason behind interesting energy dependence?
- Centrality dependence?
  - In pp, $\alpha = 1$ used often
- Compare to CEP expectation:

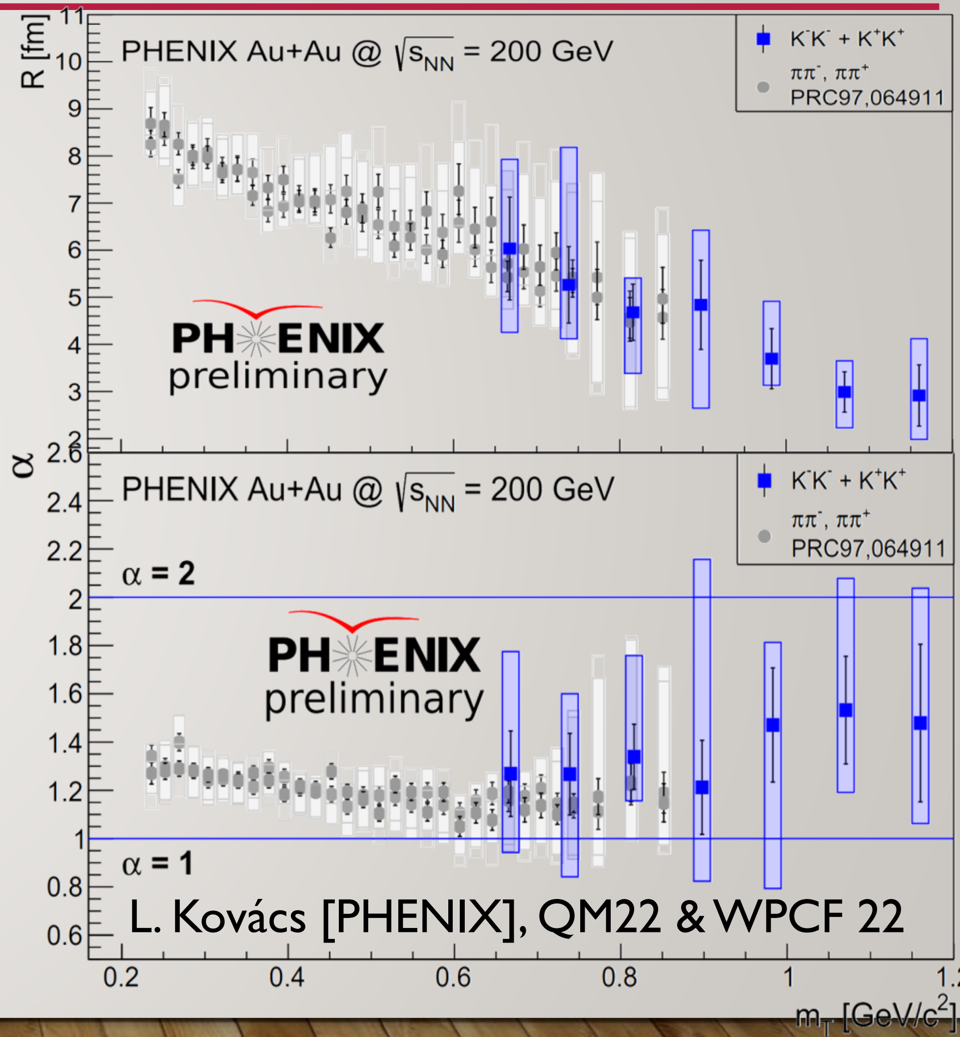
Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67





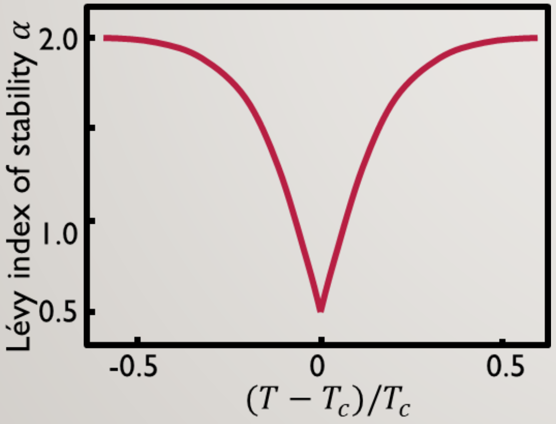
KAONS AT PHENIX AND STAR

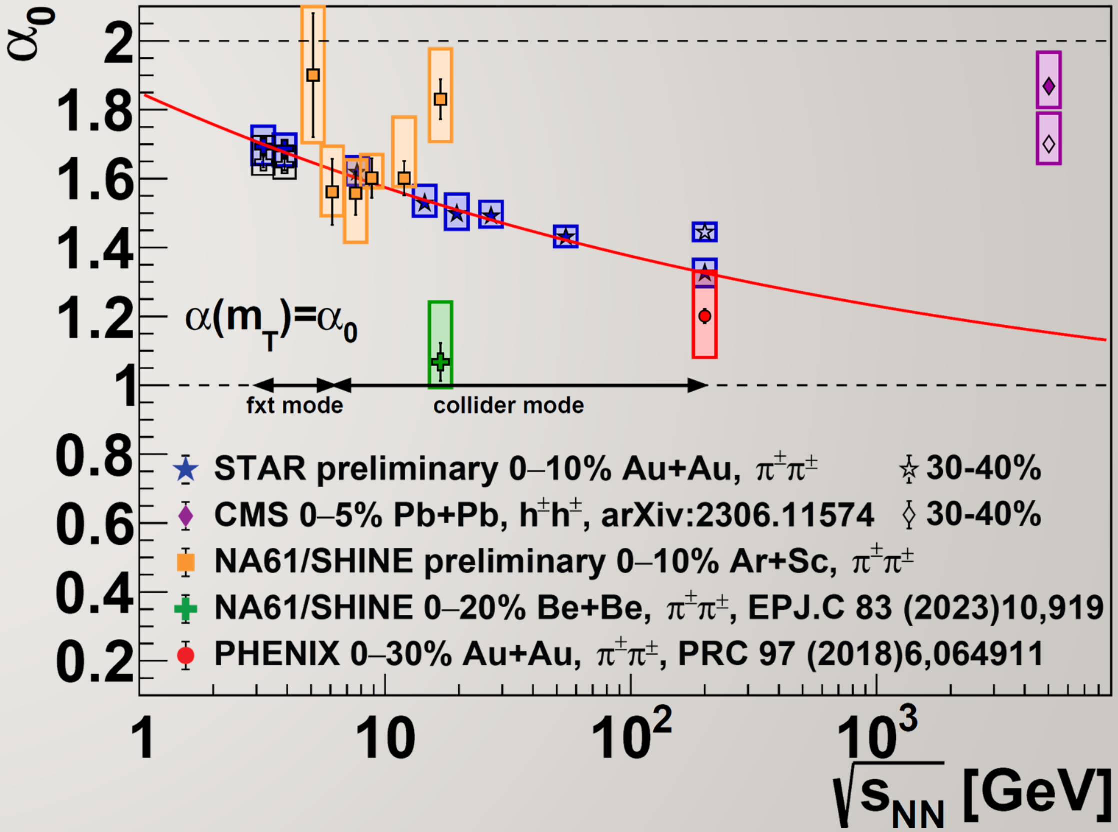
- Preliminary analysis performed at PHENIX and STAR
- Kaon and pion data seem compatible at the same m_T
- Lévy scale R shows hydro type of scaling with m_T
  - R depending on m_T but not on particle type separately
- $\alpha(K) \geq \alpha(\pi)$, but anomalous diffusion suggests opposite
  Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]
- Dominant mechanism creating Lévy source?
  - Not only rescattering?
  - Anomalous hydro at the sQGP stage?
- PHENIX prelim. results: L. Kovács, Universe 9 (2023) 7, 336
- STAR prelim. results: A. Mukherjee, Universe 9 (2023) 7, 300



STABILITY PARAMETER α FROM SPS TO LHC

- Different values for small (Be+Be) & medium (Ar+Sc) systems at SPS
  - Also true for Pb+Pb and p+p at LHC? (pp: $\alpha = 1$ assumed so far)
- Medium and large systems: non-monotonic trend
  - Centrality dependence different at RHIC and LHC
  - More regions to be explored
- Compare to expectation cartoon based on Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67

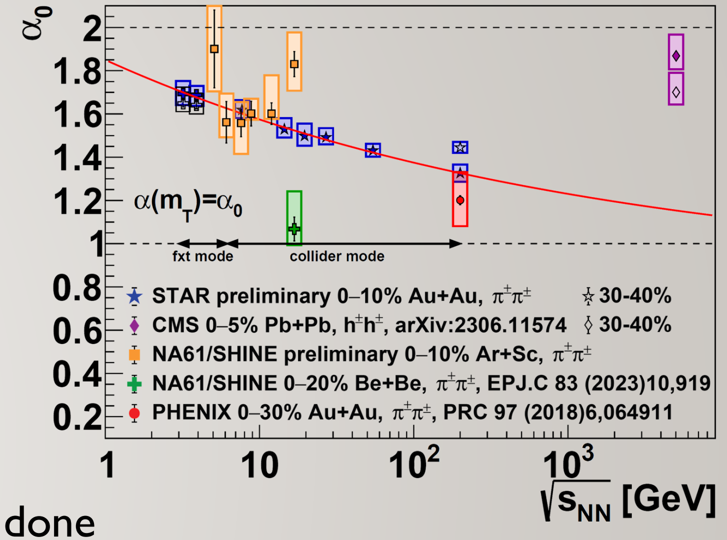




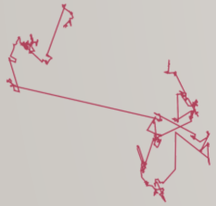
CONTENTS OF THIS TALK

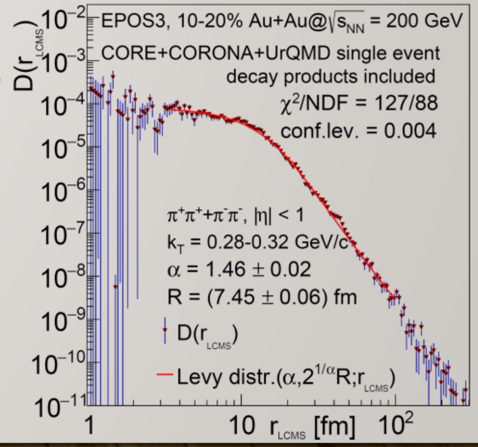
CONCLUSIONS AND OUTLOOK

- Lévy sources from SPS to RHIC and LHC
  - **Lévy α**: between 1 and 2, increases with $\sqrt{s_{NN}}$?
    - Contrary to expectations, $\alpha(K) \geq \alpha(\pi)$
  - **Lévy R**: hydro scaling, despite not Gaussian
  - **Lévy λ**: signs of η' in-medium mass modification

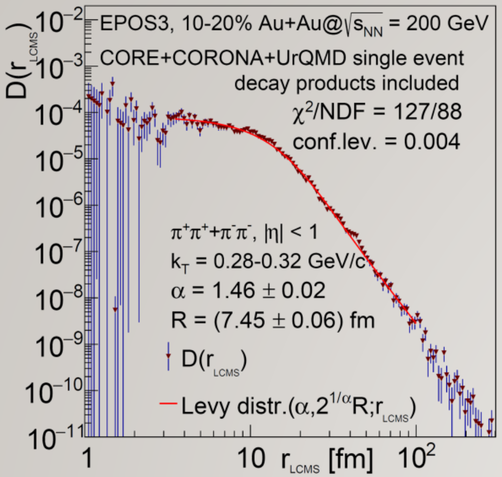


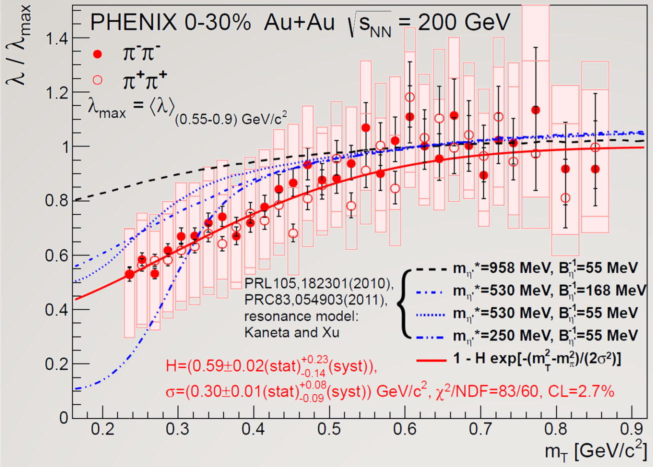
- Possible reasons:
  - Jet fragmentation → not dominant in AA collisions
  - **Critical phenomena** → maybe at lowest RHIC energies and SPS
  - Directional averaging → source <u>is</u> (approx.) spherical in LCMS, 3D cross-check done
  - Event averaging → event-by-event simulations show: Lévy not due to averaging
  - **Resonance decays** → part of the reason, not enough alone (Lévy without it as well)
  - **Hadronic rescattering, Lévy flight** → $\alpha(K) \geq \alpha(\pi)$ puzzling

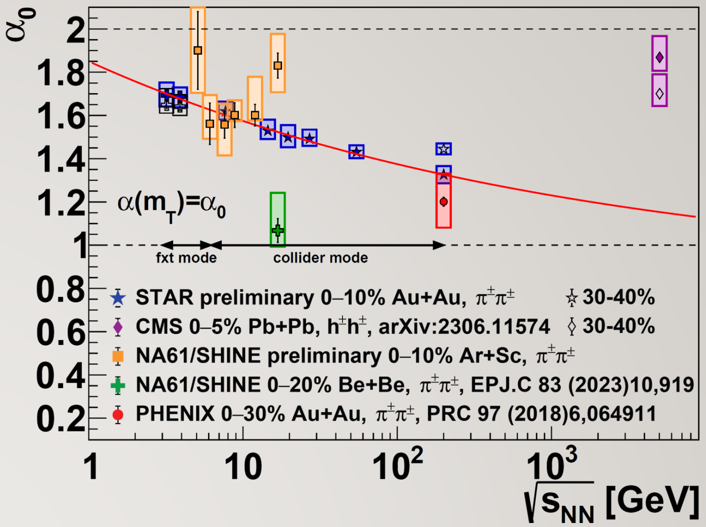




- Questions to be answered:
  - When measuring α, what effects need to be considered?
  - Particle type dependence? Anomalous diffusion in the quark stage?
  - Why α nonmonotonic versus $\sqrt{s_{NN}}$? System size dependence?







THANK YOU FOR YOUR ATTENTION

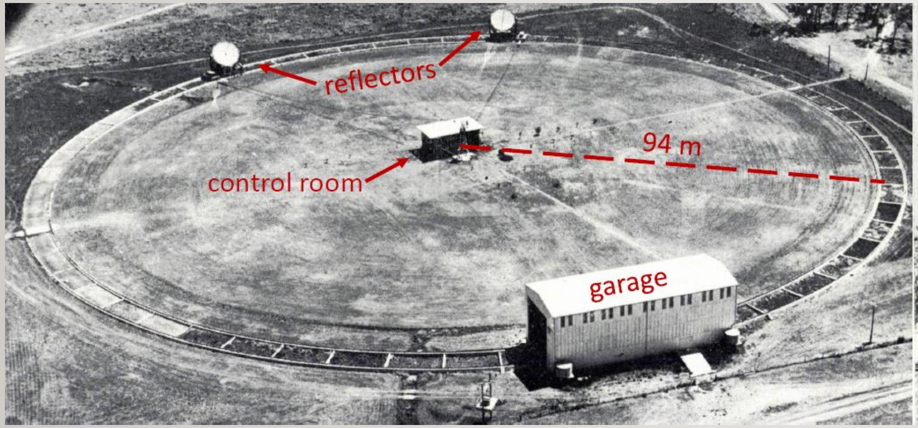
And if you are interested in these topics:

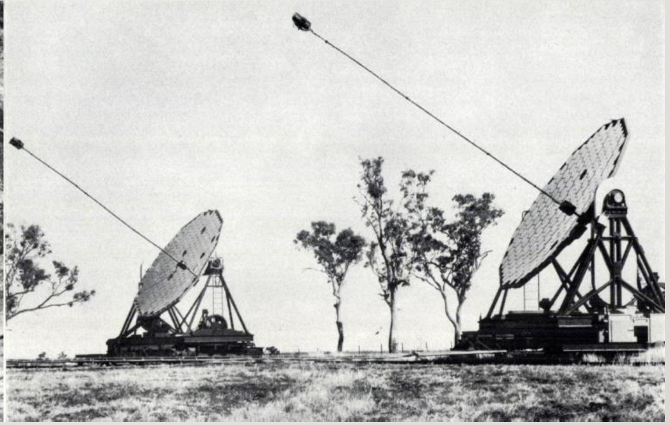
https://zimanyischool.kfki.hu/24/

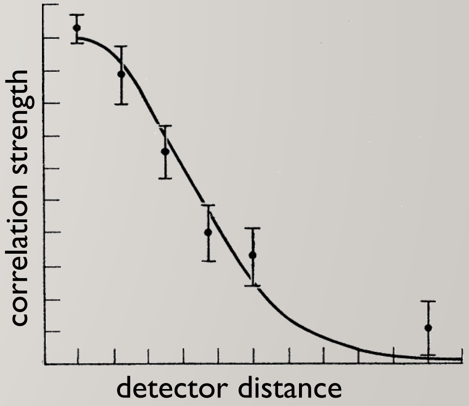
BACKUP

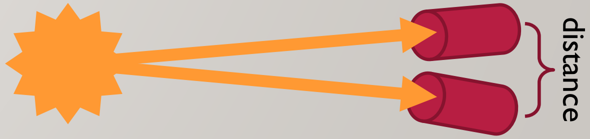
A SURPRISING DISCOVERY: HBT CORRELATIONS

- Radio astronomy: Jansky, 1933, weird 24h oscillation; stars also emit EM radiation in the radio domain!
- R. H. Brown: investigated radio waves from stars
  - Jordell Bank (optical and radio telescopes), tabletop experiment (optics), Narrabri (stellar interferometer)
- R. Q. Twiss helped to understand results mathematically
- Weird correlation in all experiments: joint intensity ”too frequent”, interference?
- „Interference between two different photons never occurs”
  P.A. M. Dirac, Quantum Mechanics (Oxford UP, London, 1958)







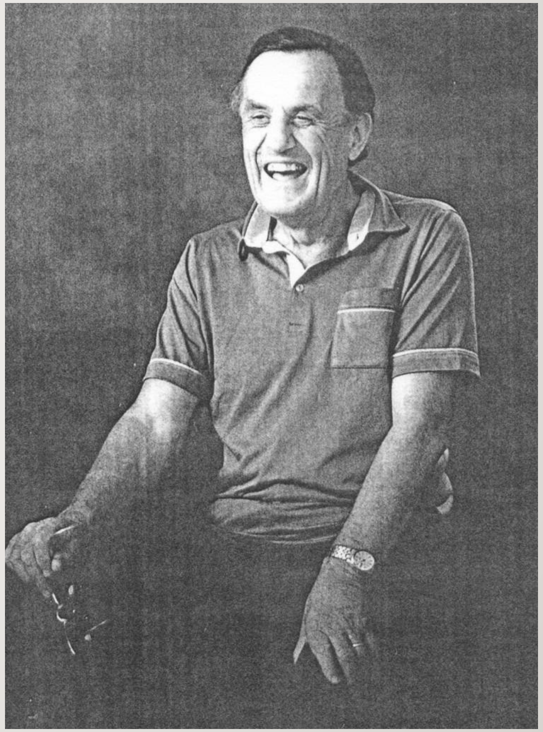


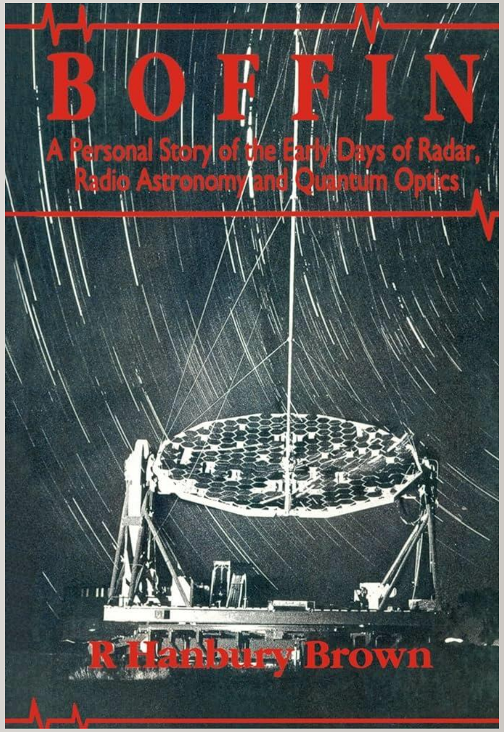
IGNORANCE IS SOMETIMES A BLISS IN SCIENCE

„In fact, to a surprising number of people the idea that the arrival of photons at two separated detectors can ever be correlated was not only heretical but patently absurd, and they told us so in no uncertain terms, in person, by letter, in print, and by publishing the results of laboratory experiments, which claimed to show that *we were wrong*."

„I was a long way from being able to calculate, whether it would be sensitive enough to measure a star.
... my education in physics had stopped far short of the quantum theory. Perhaps just as well, *ignorance is sometimes a bliss in science*."

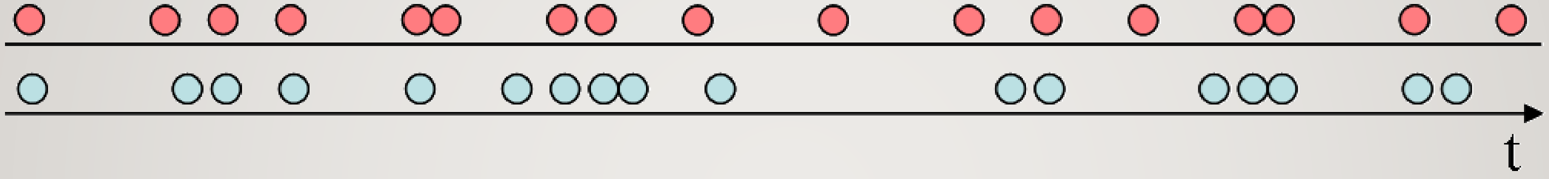
R. H. Brown, Boffin: a personal story of radar, radio astronomy and quantum optics
(Taylor & Francis, 1991)



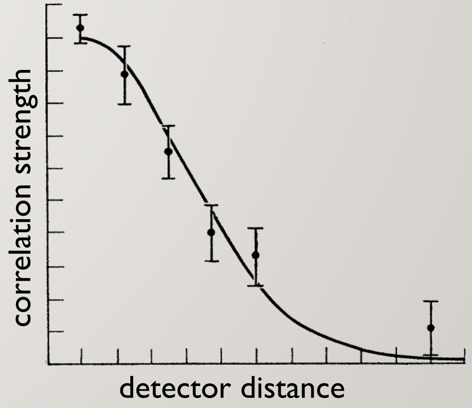


THE HBT CORRELATION

- Observation by R. H. Brown: decreasing detector distance increases correlations in detector signals
- Joint intensity ”too frequent”: $I(A,B) > I(A)I(B)$

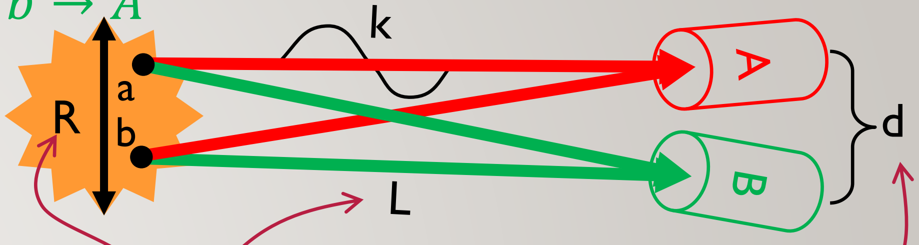


- Reason for correlations? Interference?
- „Interference between two different photons never occurs”
  P.A. M. Dirac, Quantum Mechanics (Oxford UP, London, 1958)
- Why does the correlation reduce with distance?



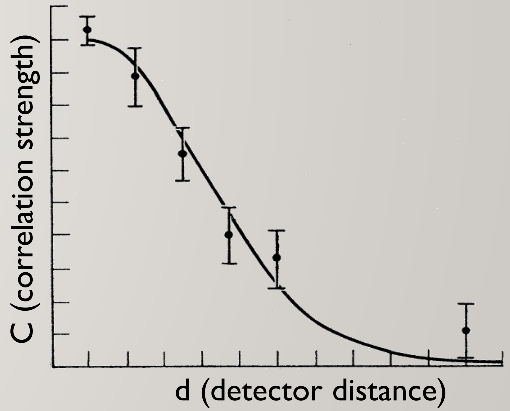
HBT EFFECT: QUANTUM EXPLANATION

- „Symmetrized wavefunction”: $a \to A$ and $b \to B$ or $a \to B$ and $b \to A$
  - Cannot distinguish which photon is happening in which detector
  - Important: photon is the detection event
  - Detection possibilities are symmetrized



- With a plane wave (with k wavenumber) from two point-like sources, the normalized joint probability, called correlation function, is:

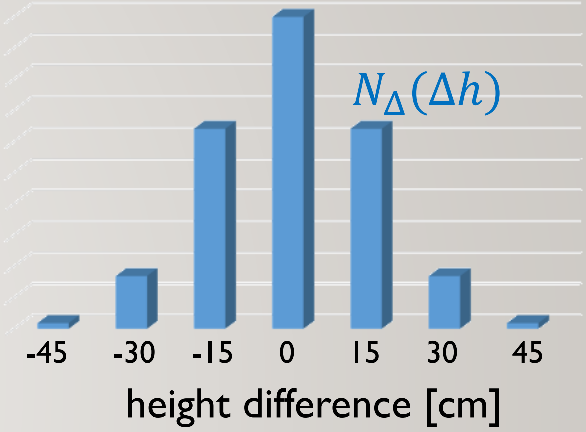
$$C(A,B) = \frac{P(A,B)}{P(A)P(B)} = 1 + \cos\left(\frac{kR}{L}d\right)$$



- Correlation width inversely proportional to source size
- Bose-Einstein correlation!
- Similar explanation can be given based on classical waves (as Mike Lisa often says: there is no h in HBT)
  - Deep reason: classical scalar or vector waves behave as bosons, statistically

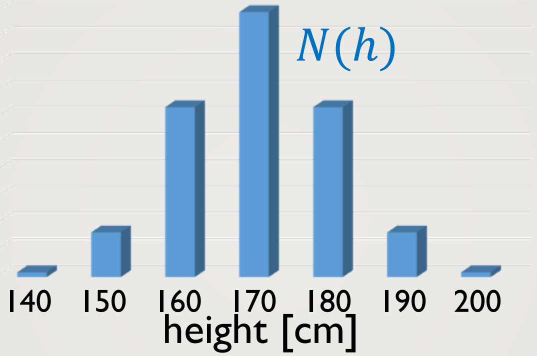
WHY ARE CORRELATIONS USEFUL?

- Frequency of a given height in a population/sample: $N(h)$
- Height densities: $N_\Delta(\Delta h) = \langle N(h)N(h+\Delta h)\rangle$, averaging on h



- Simplest case:

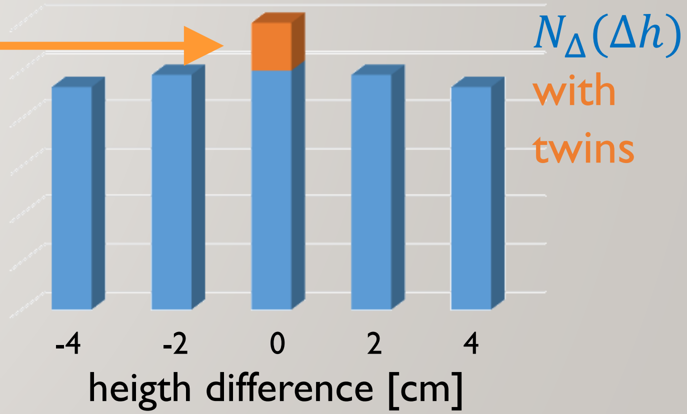
  both $N(h)$ and $N_\Delta(\Delta h)$ Gaussian



- Except if lot of identical twins:

  unexpected number of same height pairs –

  increase at zero!

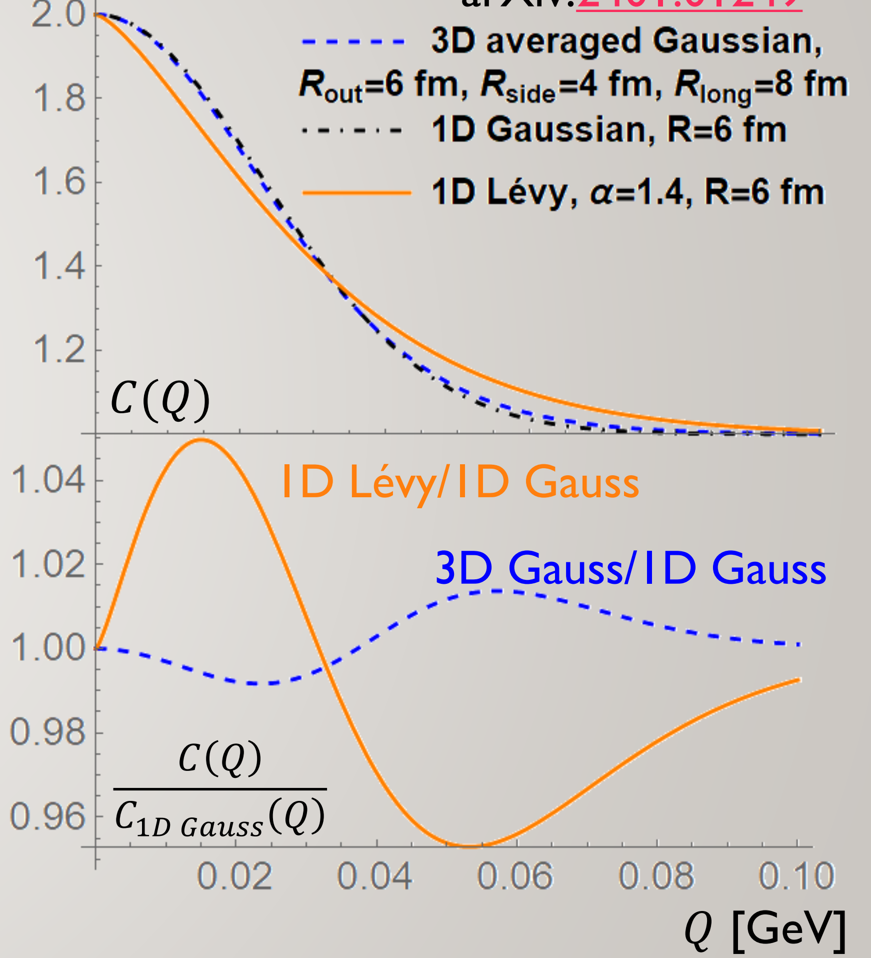


- Then $N_\Delta(0) > \langle N(h)N(h)\rangle$
- Correlation: tells the number of identical twins!

ROLE OF ANGULAR AVERAGING USING AN 1D ANALYSIS

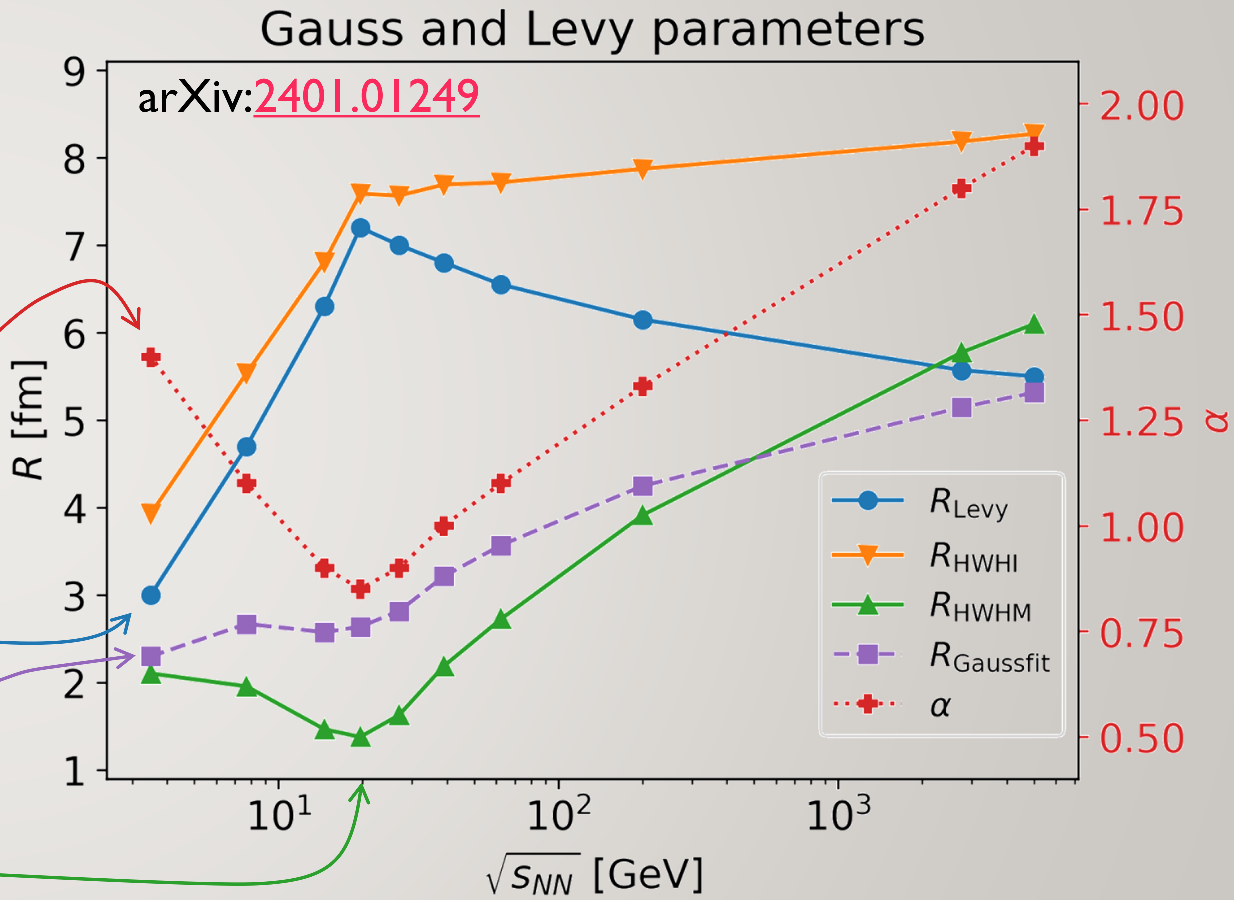
- Approximate spherical symmetry observed with Gaussian radii
  - ALICE: PRL118 (2017) 1702.01612, STAR: PRC92 (2015), 1403.4972
- Preliminary 3D Lévy analysis by PHENIX showed $\alpha_{1\mathrm{D}} \approx \alpha_{3\mathrm{D}}$
  - APPolB Proc. S. 12 (2019) arXiv:1809.09392
- Further check usually done: geometry explored in $(q_0, |q|)$ and (q_L, q_T) → sphericality valid
- 3D Gaussian does *not* result in 1D Lévy
  - Difference: several percent
  - Available experimental precision: much better than this difference
  - Experimental uncertainties in this analysis: few permille (‰)

arXiv:2401.01249



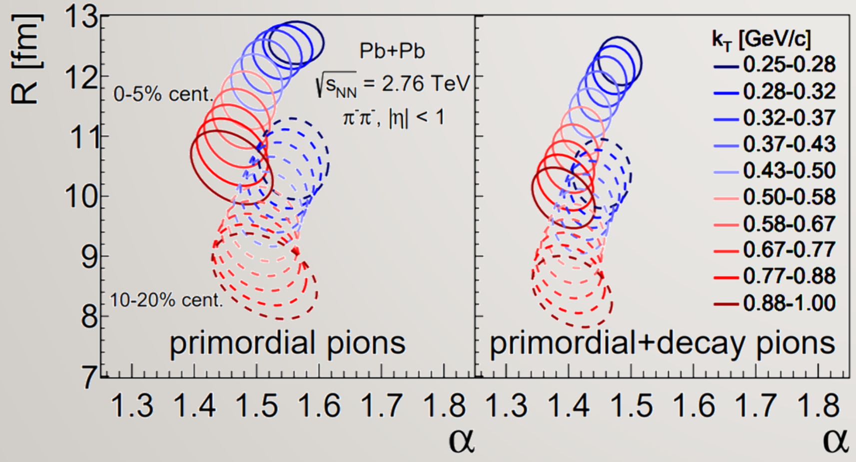
SOURCE SIZE MEASURES AROUND THE CRITICAL POINT?

- Main Lévy source parameters: R_{Levy}, α
- Other source size measures:
  - R_{Gaussfit}: $C(Q; R_{\text{Levy}}, \alpha)$ fitted with α = 2 fixed
  - R_{HWHM}: half width at half maximum
  - R_{HWHI}: half width at half integral
- **Simulated scenario:**
  - minimum in α vs. s_{NN}
  - maximum in R_{Levy} vs. s_{NN}
- **Observation:**
  - R_{Gaussfit}: approximately monotonic increase
  - Minimum in R_{HWHM}!
  - Trend change in R_{HWHI}!

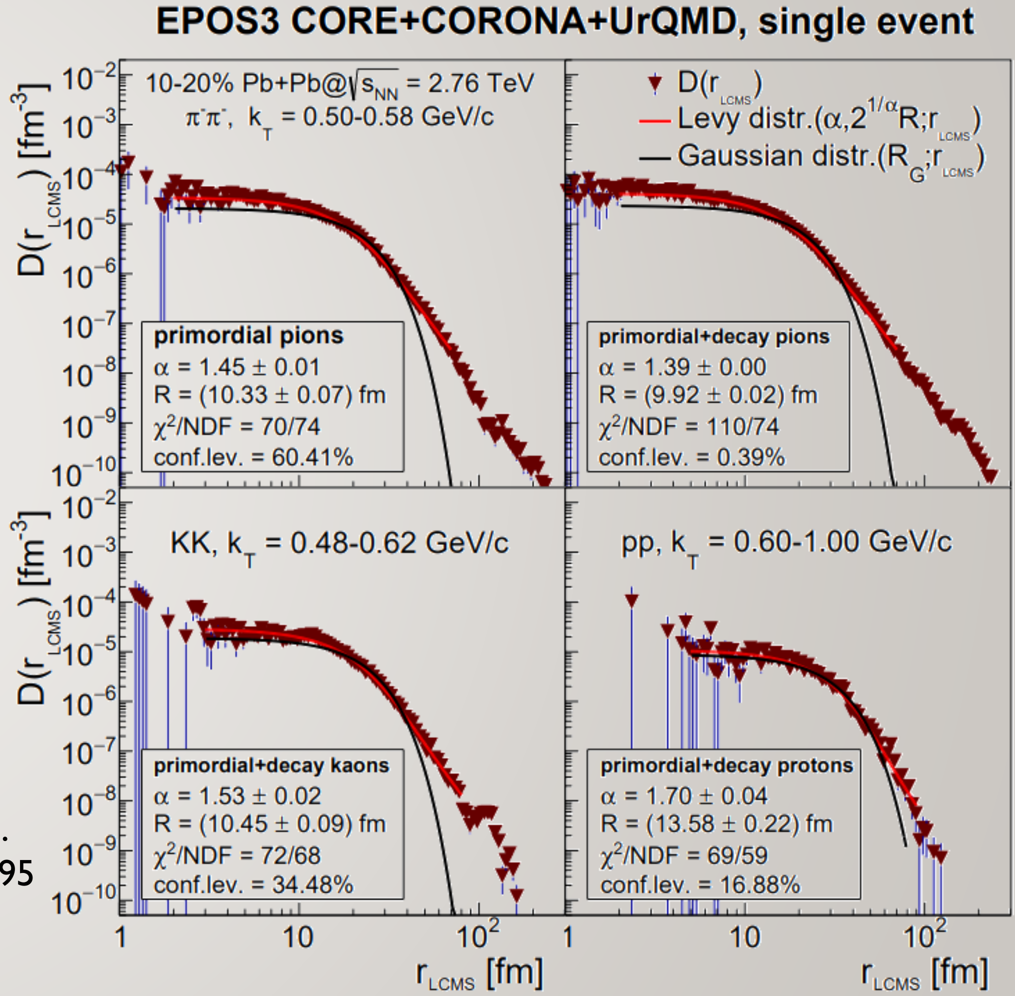


EPOS IN 2.76 TEV PBPB, EVENT-BY-EVENT

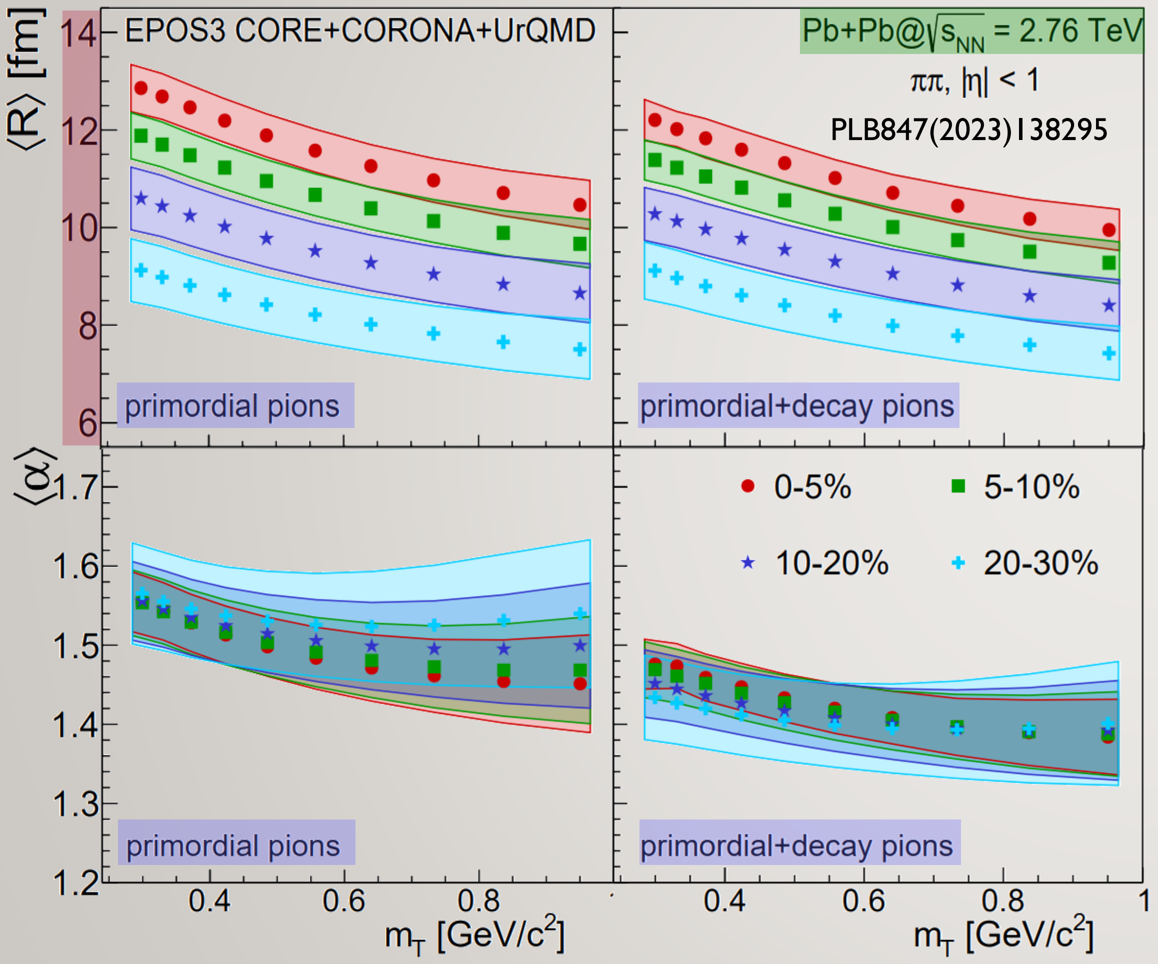
- Pion and kaon pair distributions calculated in individual EPOS events
$$D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$$
- Lévy source parameters determined for 800k events separately
  - Fit limits: from 2-5 fm to 70-100 fm
  - Criterion for acceptance: confidence level > 0.1%
  - Strongly non-Gaussian shapes observed
- Separately for various centrality and k_T classes, primordial or decays

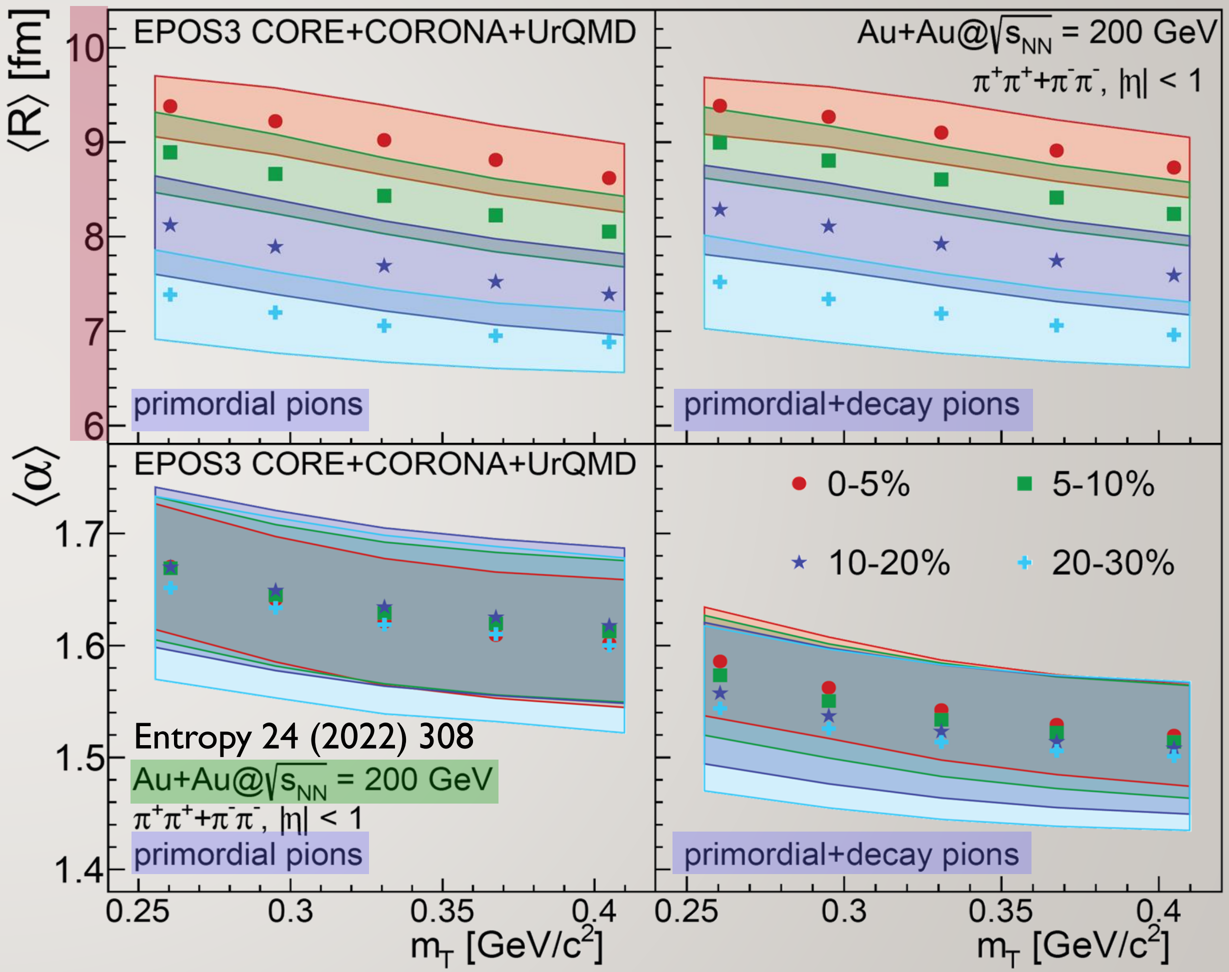


Kórodi, Kincses, Cs.
PLB847(2023)138295
arXiv:2212.02980



EPOS @ LHC VS RHIC

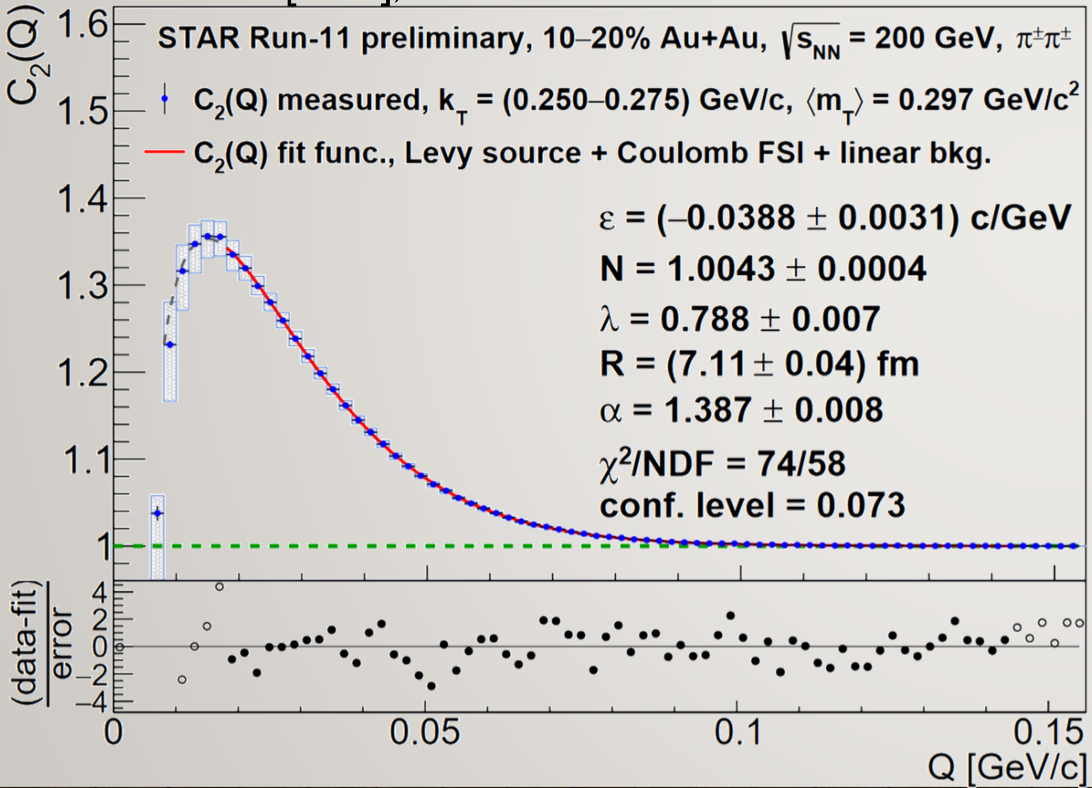




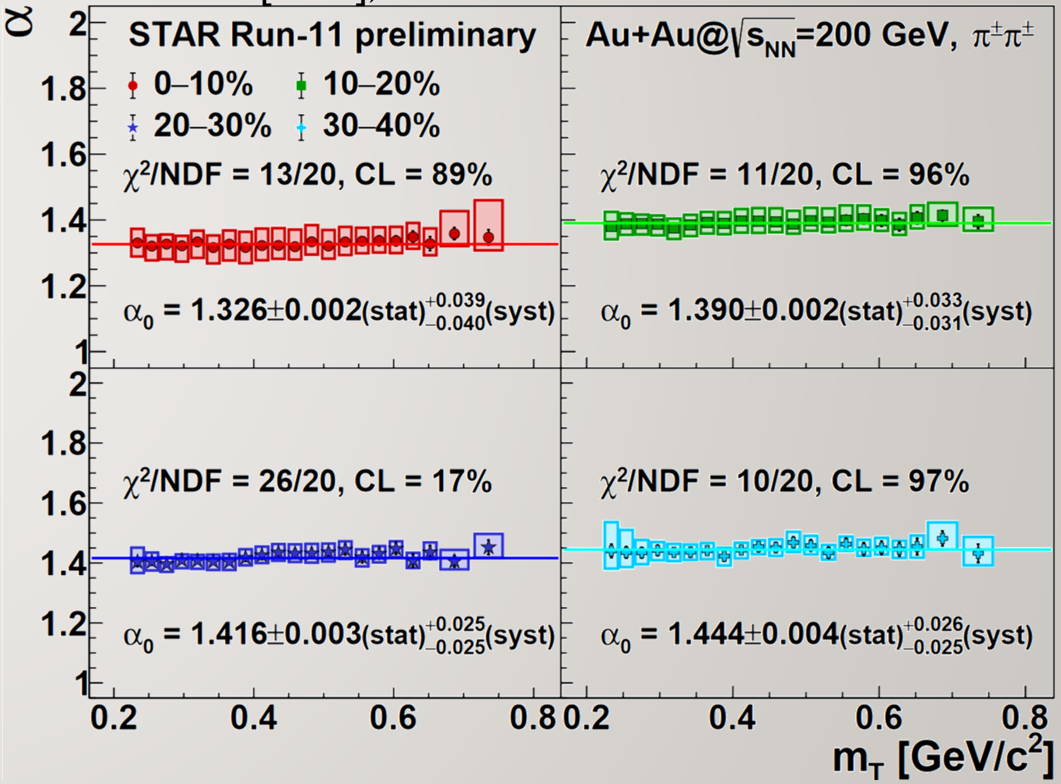
STAR RESULTS: CENTRALITY DEPENDENCE AT 200 GEV

- Run-11 Au+Au at 200 GeV, ~ 550 M events, PID by TPC+TOF, 21 m_T and 4 centrality bins
- Source far from Gaussian, constant in m_T, slight increase of α for peripheral collisions ($1.326 \rightarrow 1.444$)

D. Kincses [STAR], WPCF 2023



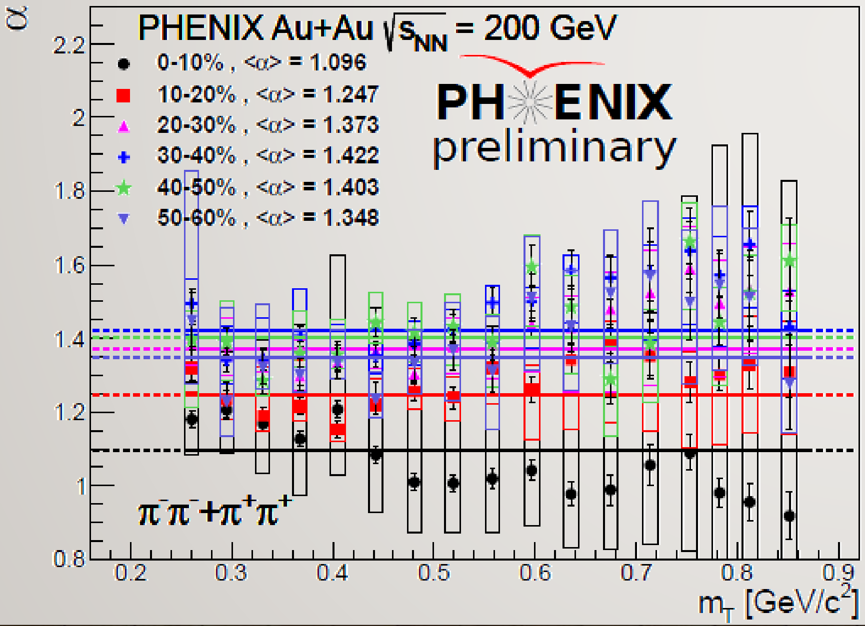
D. Kincses [STAR], WPCF 2023

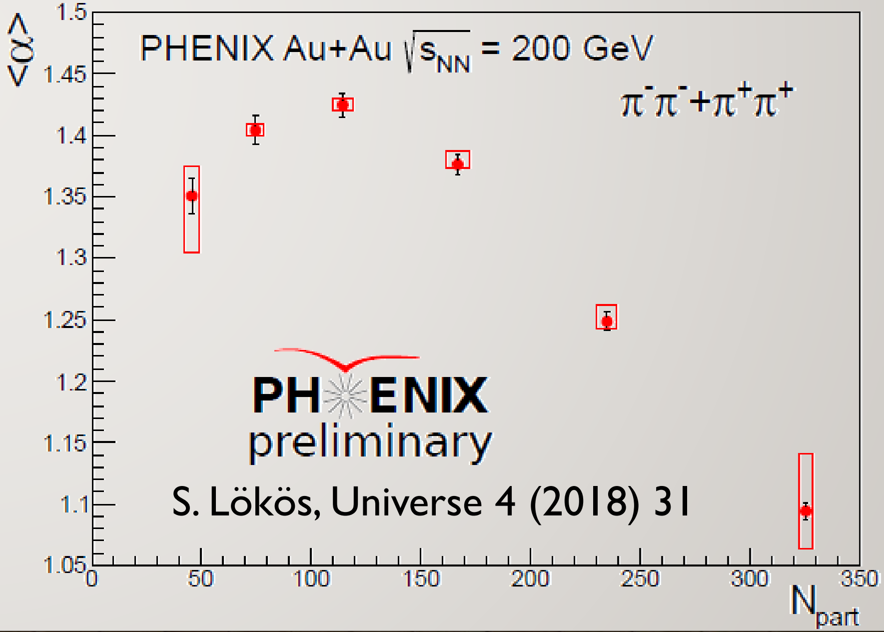


LEVY HBT EXPERIMENT PHENOMENOLOGY NEW DATA

ANALYZING THE CENTRALITY DEPENDENCE

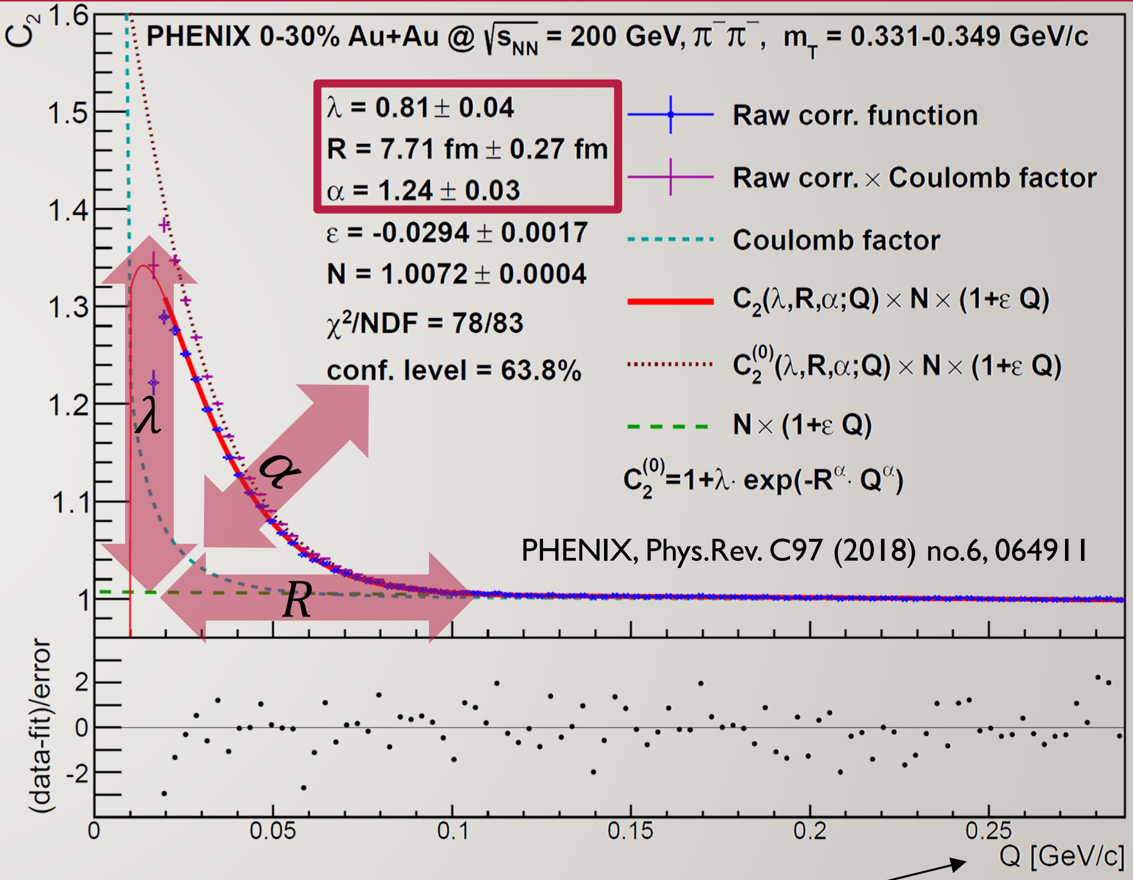
- α vs m_T: Slightly non-monotonic, averaging still possible
- $\langle\alpha\rangle$ vs N_{part}: Slightly non-monotonic, strong decreasing for large N_{part}
- No clear interpretation or understanding of this trend, need theory comparison
- Final data and publication in under collaboration review in PHENIX





EXAMPLE $C_2(Q_{LCMS})$ CORRELATION FUNCTION

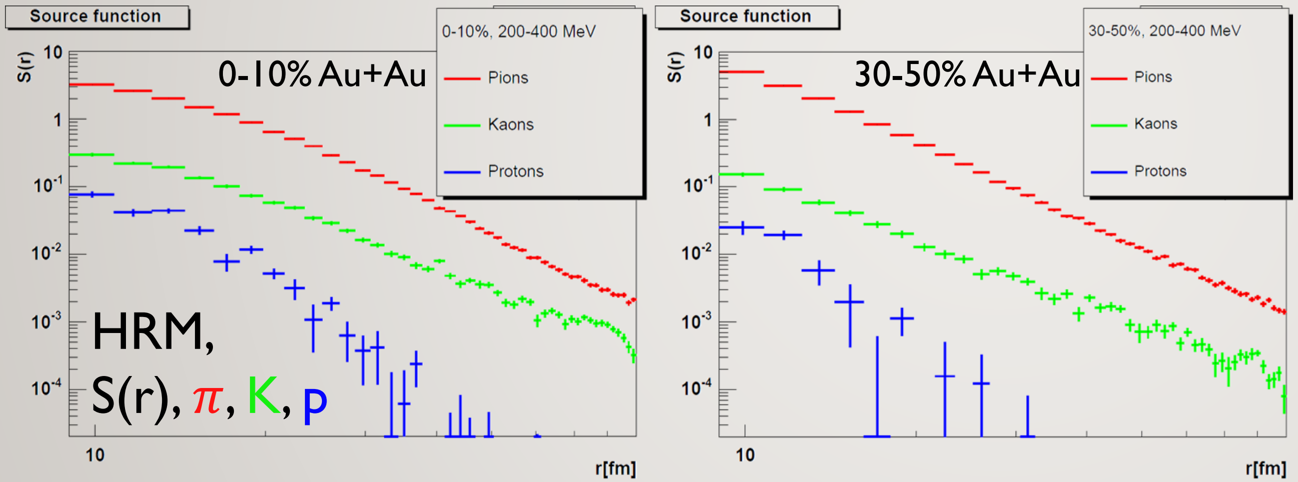
- Correlation function: spherical in LCMS
  - 1D measurement possible
  - Done in several m_T bins
- Fit with calculation based on Lévy distribution
- Only converging fits with good confidence level should be accepted
- Physical parameters:
  R, λ, α measured versus pair m_T



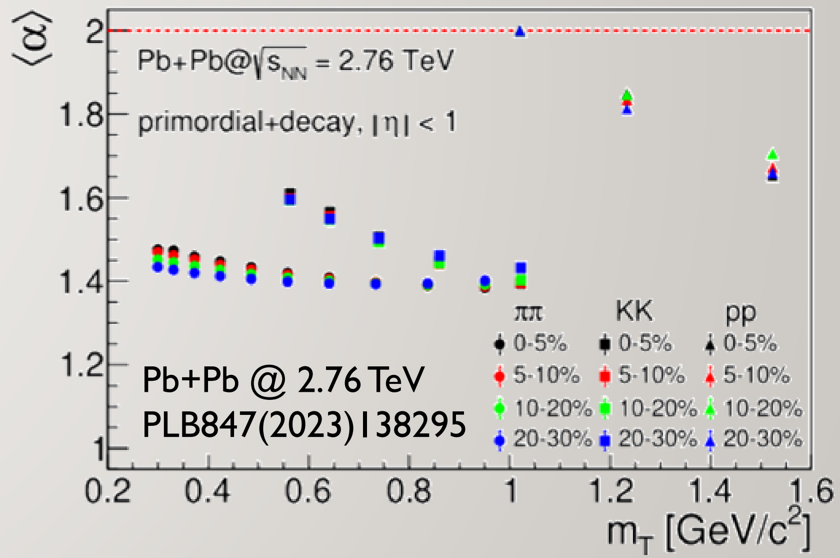
Q: 1D momentum difference in Longitudinally CoMoving System (LCMS)

THE IMPORTANCE OF A KAON ANALYSIS

- Kaons: smaller cross-section, larger mean free path
- Mean free path increases more during a time-step → heavier power-law tail?
- Prediction for π, K, p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering
  Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]
  Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



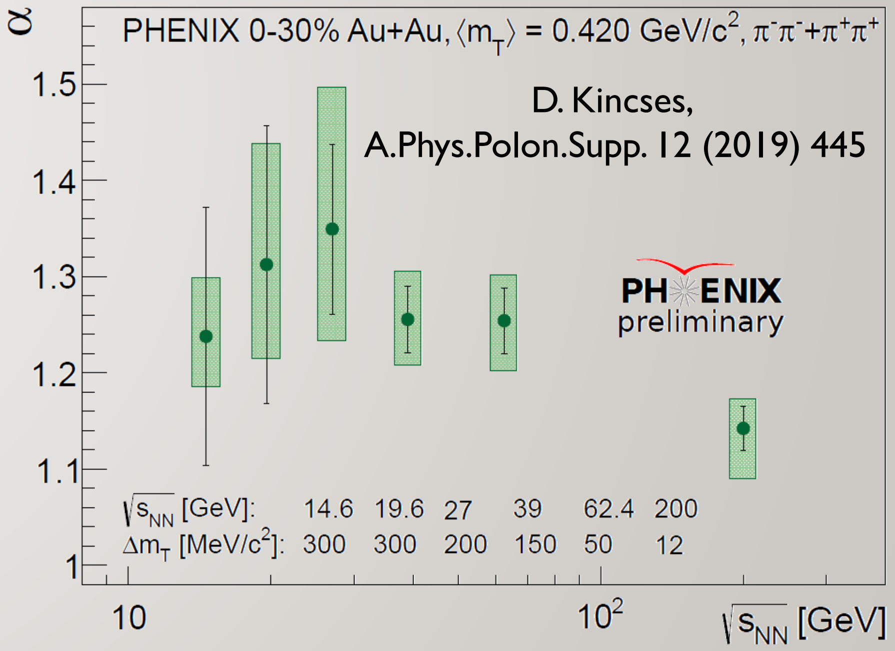
Particle type dependence also analyzed in EPOS:



- Kaon HBT radii: m_T scaling or its violation for Lévy scale R?
- Prediction validated in HRM: $\alpha(p) > \alpha(\pi) > \alpha(K)$; not valid in EPOS: $\alpha(p) > \alpha(K) > \alpha(\pi)$

COLLISION ENERGY DEPENDENCE AT PHENIX

- $\langle\alpha\rangle$ approximately monotonic versus $\sqrt{s_{NN}}$
  - No clear interpretation or understanding of this trend
  - With widely varying m_T interval (due to statistics) → may influence outcome
  - Important w.r.t. shape averaging interpretation of $\alpha \neq 2$
- Lévy exponent α still far from conjectured CEP limit of 0.5
  - Very much dependent on m_T bin width
  - Working on final results…



A CROSS-CHECK: THREE-PION LÉVY HBT

- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

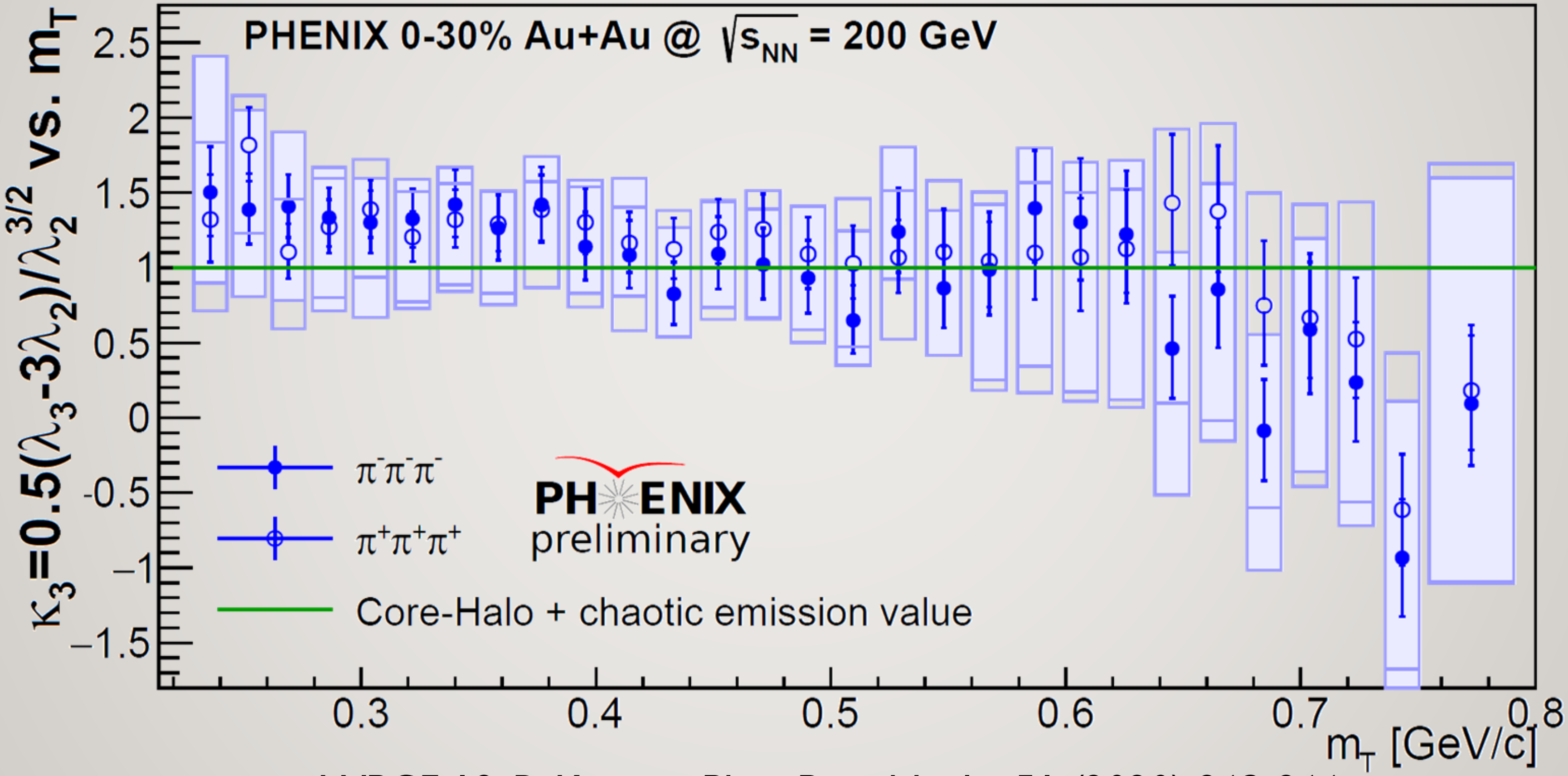
$$\lambda_2 = f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

$$\lambda_3 = 2f_C^3[(1-p_C)^3 + 3p_C(1-p_C)^2] + 3f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
  - does not depend on f_C
  - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?
  - Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
  - Random fields create random phase shift, on average distorts Bose-Einstein correlations
    Csanád et al., Gribov-90 (2021) 261-273 [arXiv:2007.07167]

TEST OF CORE-HALO MODEL / COHERENCE

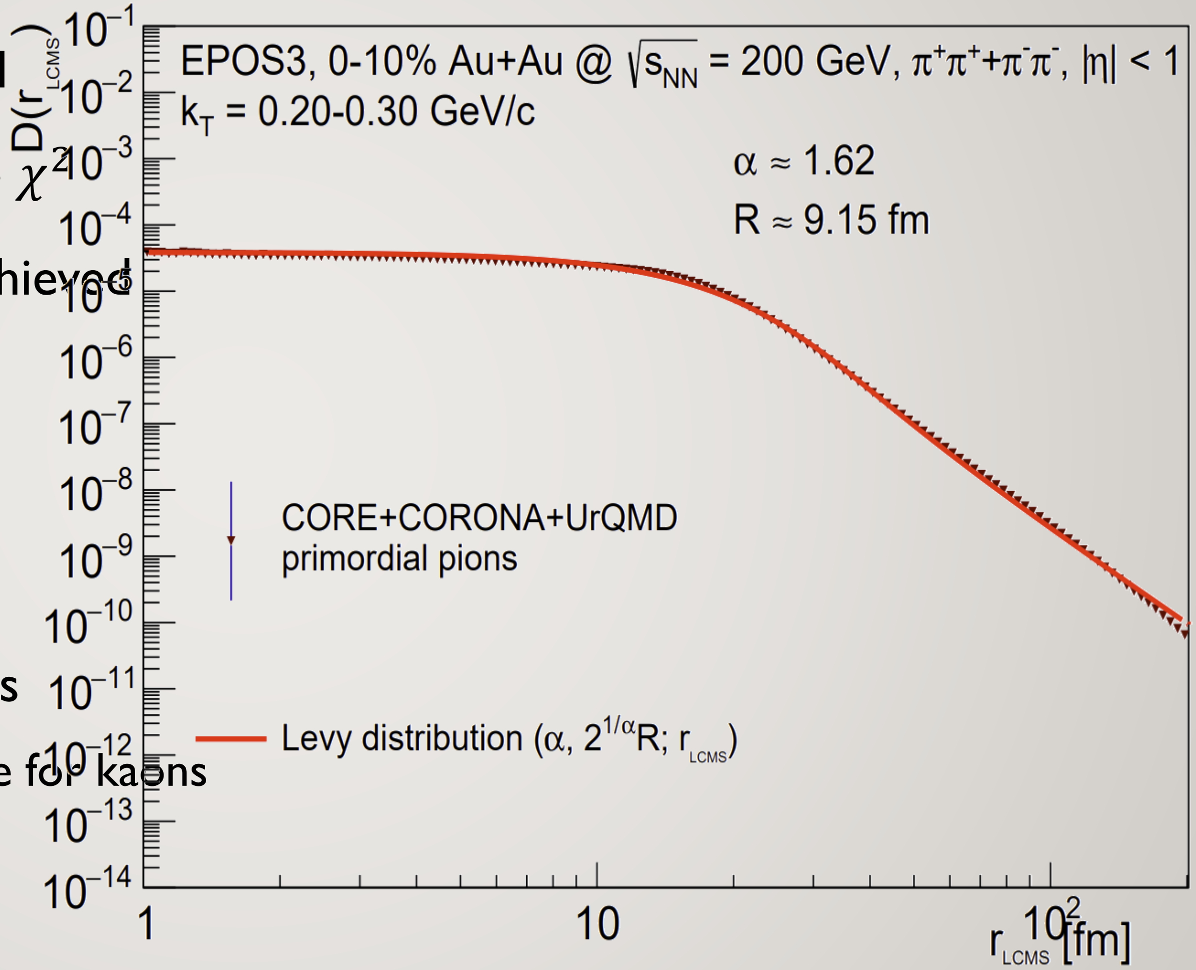
- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence



WPCF 19, B. Kurgyis, Phys. Part. Nuclei 51 (2020) 263-266

ROLE OF EVENT AVERAGING?

- Event-averaged source also analyzed
- Not perfectly Lévy shape, very large χ^2
- Nevertheless: similar parameters achieved
  - Event averaged:
    $\alpha \approx 1.62, R \approx 9.15$ fm
  - Event-by-event:
    $\alpha \approx 1.66, R \approx 8.96$ fm
- More reasonable approach for kaons
  - No event-by-event analysis possible for kaons



SOURCE OR PAIR DISTRIBUTION?

- Under some circumstances (thermal emission, no interactions, …):

$$C_2(q,K) = \int S\left(r_1, K+\frac{q}{2}\right) S\left(r_2, K-\frac{q}{2}\right) |\Psi_2(r_1,r_2)|^2 dr_1 dr_2$$
$$\cong 1 + \left|\int S(r,K)e^{iqr}dr\right|^2$$

- Let us introduce the spatial pair distribution:

$$D(r,K) = \int S\left(\rho+\frac{r}{2},K\right) S\left(\rho-\frac{r}{2},K\right) d\rho$$

- Then the Bose-Einstein correlation function becomes:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr = 1 + \int D(r,K)e^{iqr}dr$$

- **Bose-Einstein correlations measure spatial pair distributions!**
- Coulomb and strong Final State Interactions? Under control for Lévy sources

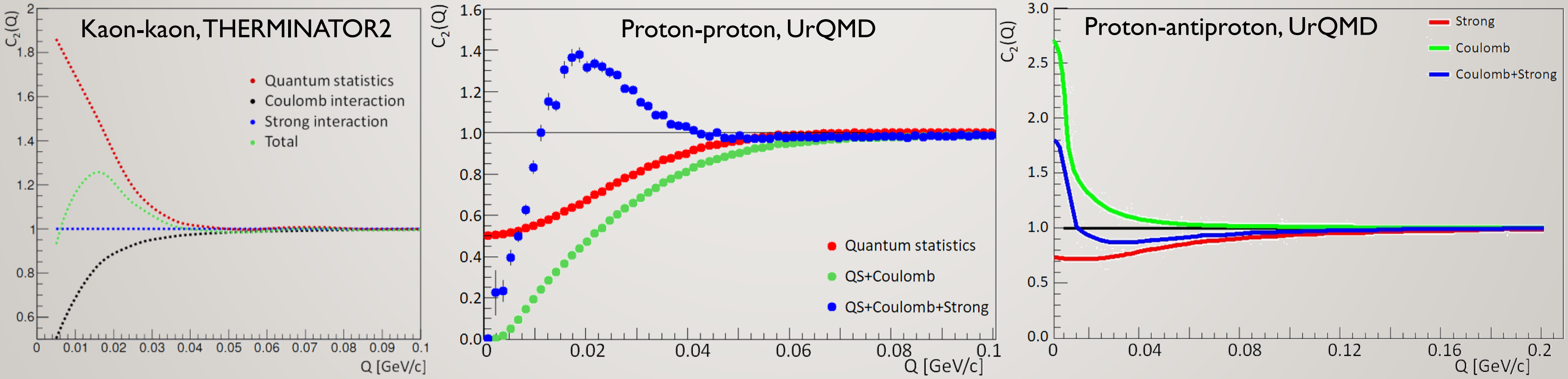
Csanad, Lökös, Nagy, Phys. Part. Nuclei 51 (2020) 238 [arXiv:1910.02231]
Kincses, Nagy, Csanad Phys. Rev. C102, 064912 (2020) [arXiv:1912.01381]

ROLE OF THE STRONG INTERACTION

- In case of other interactions or not identical bosons, the formula still works:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr$$

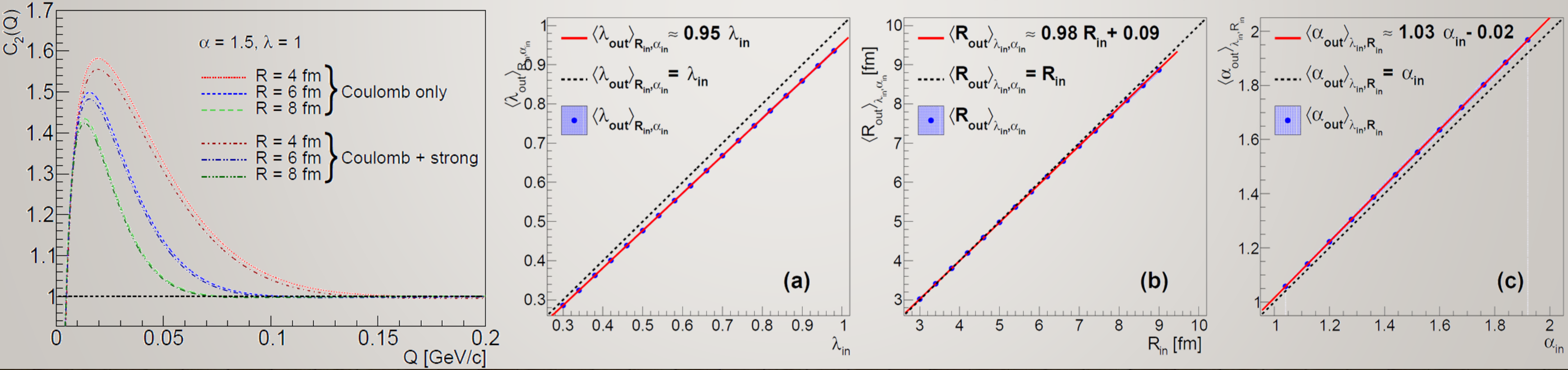
- Pair wave function determines $D \leftrightarrow C_2$ connection
- Mesons, baryons: strong interaction; fermions: anticorrelation
- Non-identical pairs: interaction modifies wave function



From e.g. H. Zbroszczyk's talk at Zimányi School 2019

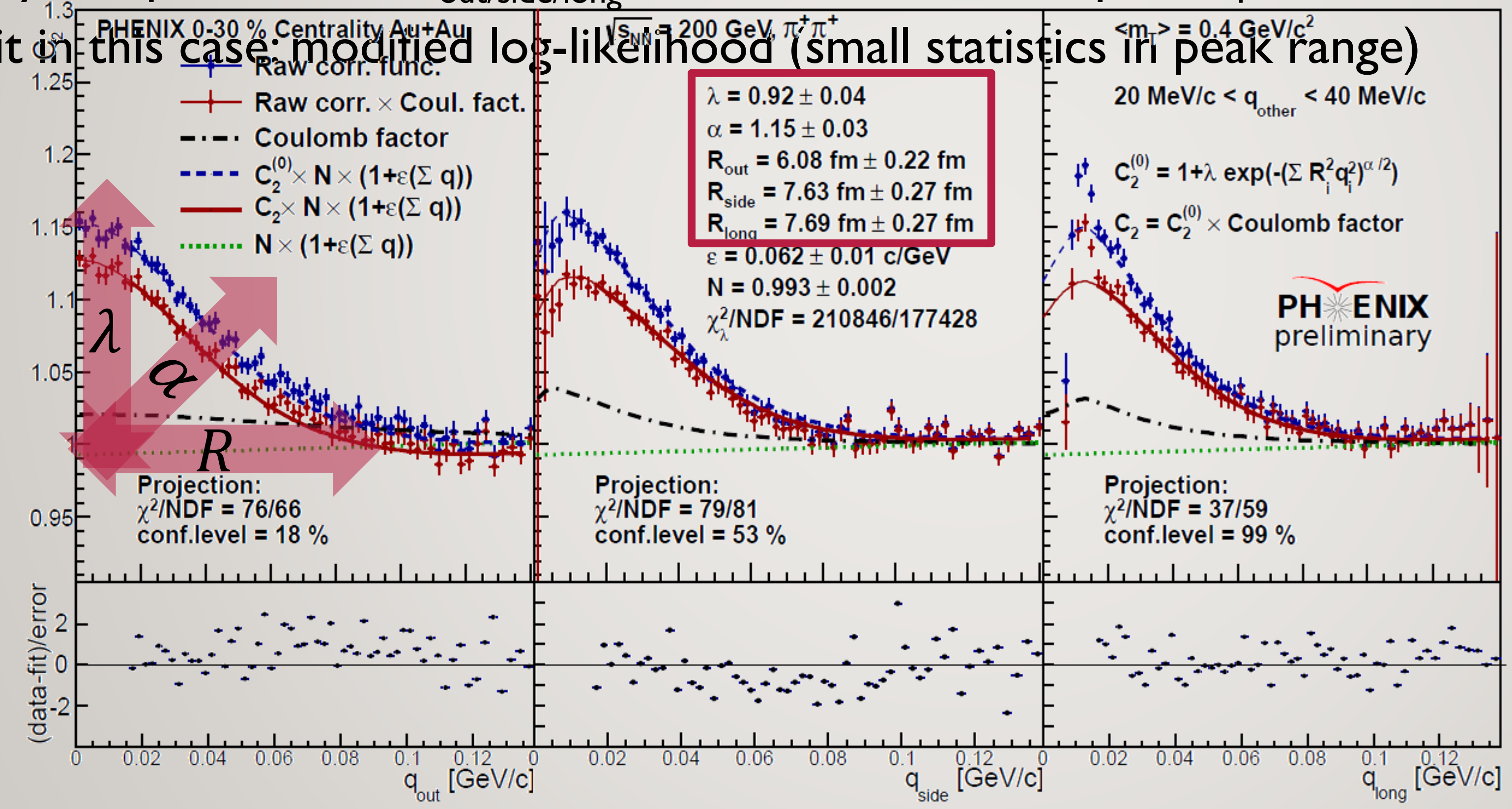
STRONG INTERACTION FOR PION PAIRS

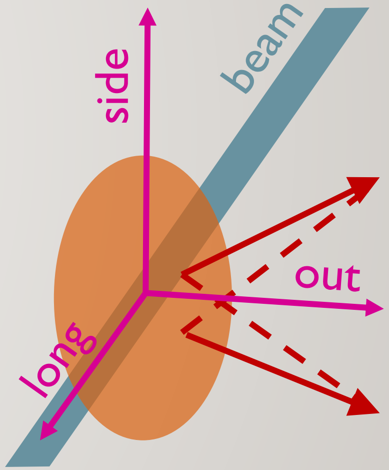
- Additional potential appearing
- Possible handling: strong phase shift, Modify s-wave component in wave func.
  R. Lednicky, Phys. Part. Nucl.40, 307 (2009)
- Small difference in case of pions
- Few percent modification in λ, α, see Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 064912



A CROSS-CHECK: 3D LÉVY FEMTOSCOPY

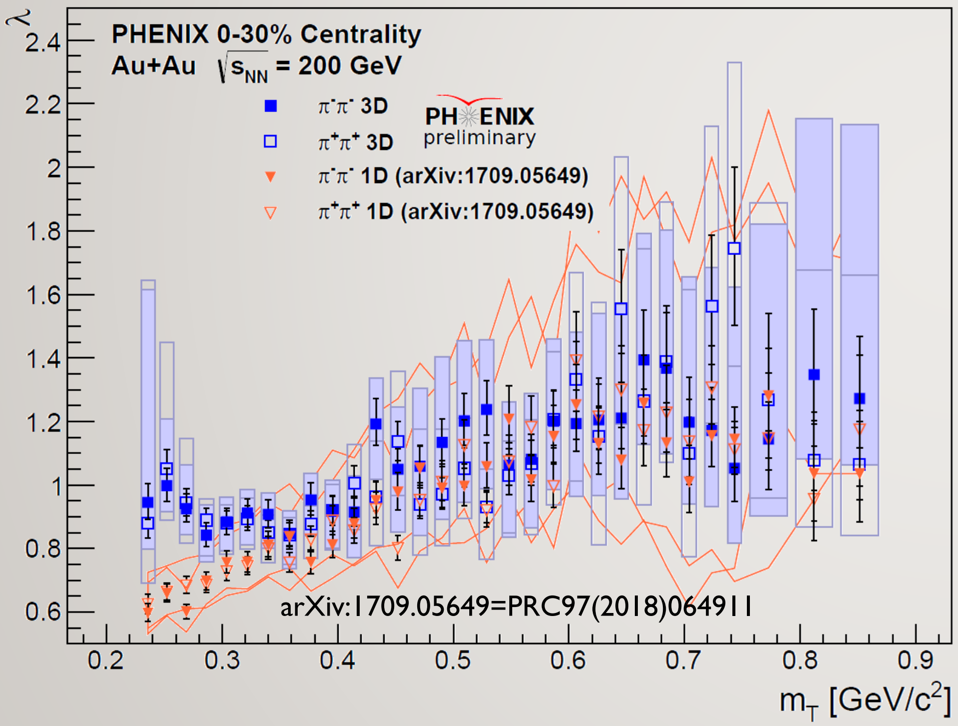
- Femtoscopy done in 3D: Bertsch-Pratt pair frame (out/side/long coordinates)
- Physical parameters: $R_{out/side/long}$ λ, α measured versus pair m_T
- Fit in this case: modified log-likelihood (small statistics in peak range)

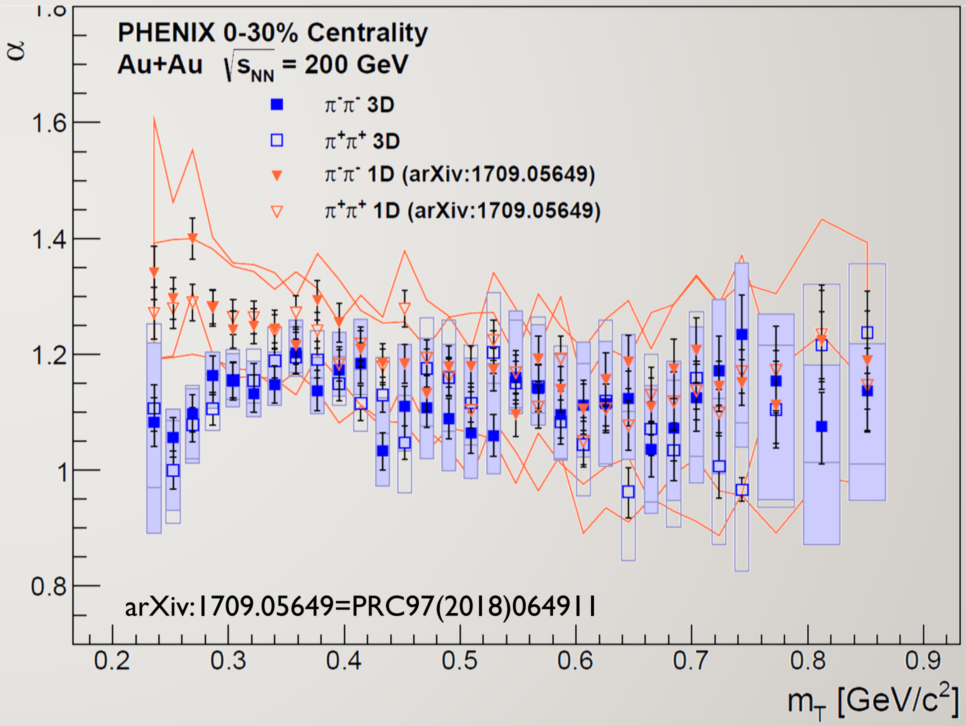




3D VERSUS 1D: STRENGTH λ AND SHAPE α

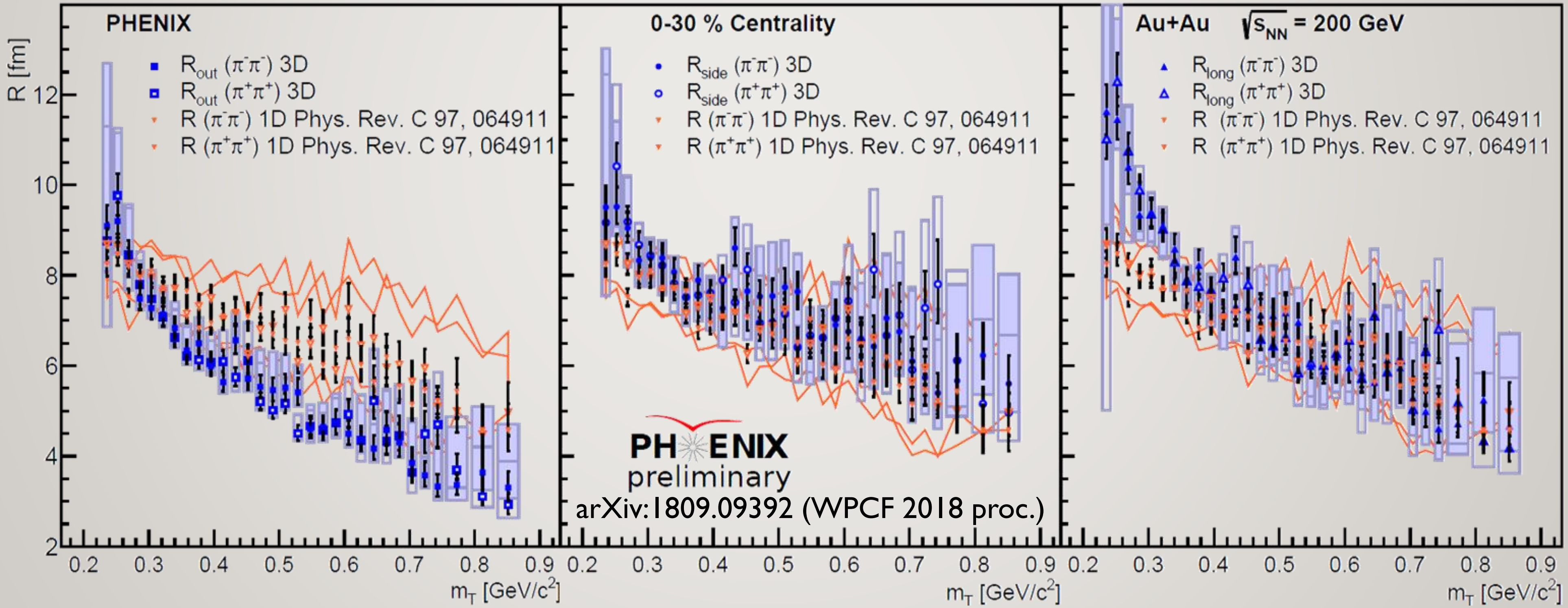
- Compatible with 1D (Q_{LCMS}) measurement of PRC97(2018)064911
- Small discrepancy at small mT: due to large Rlong at small mT?





LÉVY SCALES IN 3D

- Compatibility with 1D Lévy analysis
- Similar decreasing trend as Gaussian HBT radii, but hydro prediction based on Gaussian source
- Asymmetric source for small m_T, validity of Coulomb-approximation?



TWO-PARTICLE SPATIAL CORRELATIONS

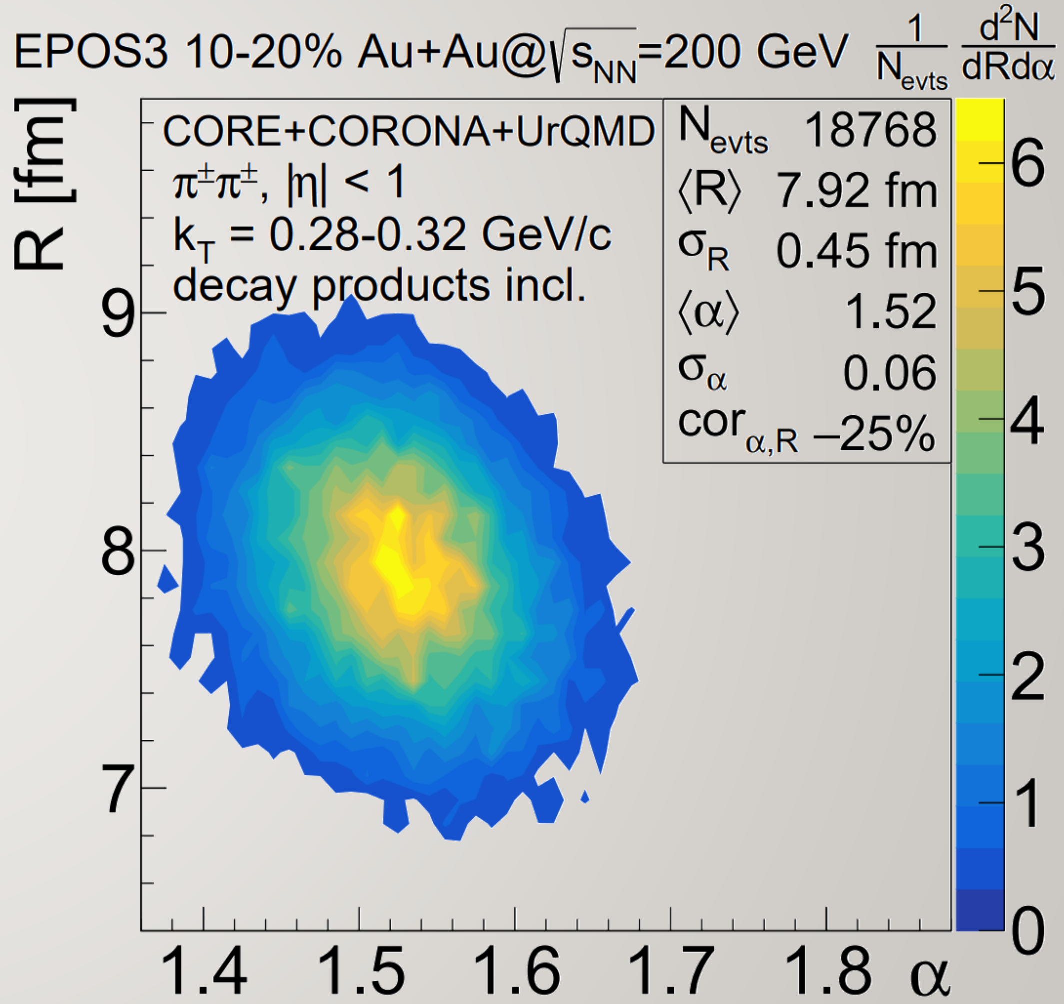
- Object to be investigated: two-particle source

$$D(r,K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

- Experimental results measure power-law tails, Lévy shapes
  - Measure momentum-space correlations, reconstruct $D(r)$ or fit its parameters
- Why do these Lévy shapes appear?
  - What physics does contribute to it? Rescattering, decays?
  - What role does event averaging have in it?
    Cimerman, Plumberg, Tomasik, Phys.Part.Nucl. 51 (2020) 282, PoS ICHEP2020 538
  - What do specific α values mean?
- Event generator models (like EPOS) – direct access to pair-source!
  - Phenomenological investigations of $D(r)$ possible
  - Effects can be turned off or on, investigated separately

DISTRIBUTION OF α, R PARAMETERS

- Normal distribution of α, R for given centrality & k_T
- Extract mean and std.dev,
- Investigate centrality & k_T dependence
- kT dependence investigated around the peak of the pair-kT distr. to have adequate stat.



EPOS SUMMARY

- D(r) calculated in EPOS evt-by-evt
- Lévy fits done evt-by-evt
- Non-Gaussianity in single events
- Extracting mean, & std.dev. of R, α
- m_T & centrality dependence