Charged lepton flavour violation: (nearly) theory overview

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Brief overview

(I) The Standard Model and beyond

A new lepton sector & further SM problems
EFT approach to NP (and the probing power of flavours...)

(II) Lepton observables: BSM arena

Going beyond minimal New Physics
Lepton observables - cLFV muon transitions, cLFV tau decays
(Brief EFT parenthesis)
cLFV at higher energies - meson decays to collider searches
Toolboxes

(III) New Physics paths to cLFV

General NP models & peculiar patterns
Models of neutrino mass generation - vanilla seesaws, low-scale seesaws
The singlet seesaw
The SUSY type I seesaw (and RPV example)

(IV) The power of cLFV - hints on models of New Physics

The SM and its lepton sector

Lepton sector: charged and neutral fermions

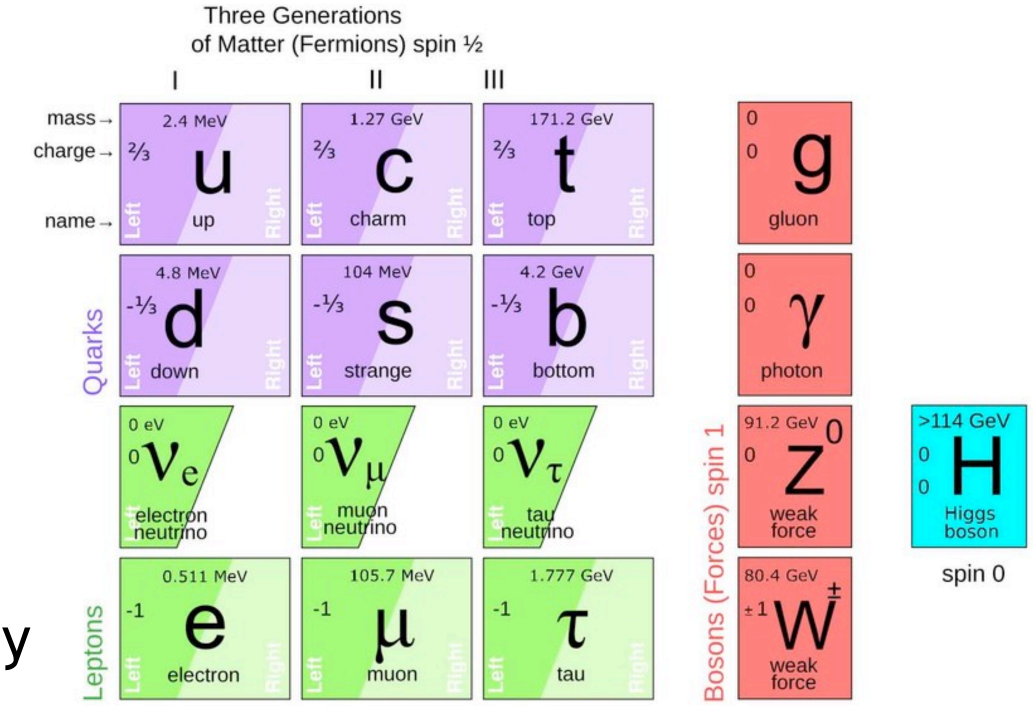
- In the **SM**, three families of quarks and leptons

  Lepton sector: colourless states, "charged" under $SU(2)_L \otimes U(1)$

$\Rightarrow$ 3 massive $\ell^\pm$ $\quad (m_\ell \propto v Y^\ell)$

$\Rightarrow$ massless ν $\rightsquigarrow$ no leptonic mixing...

Minimalistic description based on available data at the birth of the SM



However: **massless neutrino** hypothesis challenged by solar and atmospheric neutrino data!

Explanation of the "**solar ν**" and **atmospheric "ν"** problems $\Rightarrow$ **neutrino oscillations!**

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_{k,j=1}^{3} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

$$\nu_\alpha = \boldsymbol{U}^{\text{PMNS}}_{\boldsymbol{\alpha i}} \nu_i$$

$$\Delta m^2_{kj} = m^2_{\nu_j} - m^2_{\nu_k}$$

Neutrino oscillations $\Rightarrow$ **massive neutrinos** and **non-trivial leptonic mixing!**

1st "laboratory" discovery of *physics beyond the SM (BSM)*

The need for new physics

- Neutrino oscillations ⇒ massive neutrinos and non-trivial leptonic mixing!

  1st "laboratory" discovery of *physics beyond the SM (BSM)*

- **Matter** **dominated Universe:** explaining **the baryon asymmetry of the Universe (BAU)**

  (i) **initial** asymmetric composition ✘ (incompatible with inflation)

  (ii) **statistical fluctuations** during evolution ✘ (negligible effects)

  (iii) large scale **spatial separation** ✘ (incompatible with evolution of primordial Universe)

  ... **Dynamical generation! "Baryon-genesis"** ✔

Sakharov's conditions for a (successful) BAU

a priori, all are present in the SM! (electroweak baryogenesis)

- If originally symmetric Universe, **baryon number violation**

  Sphaleron production ⇒ B & L number violation

- Differentiate matter from antimatter, **CP violation**

  CPV from CKM mechanism highly suppressed...

- Suppress inverse processes, **out of (thermal) equilibrium**

  Strong 1st order EW phase transition ? soft crossover for a "heavy Higgs" (125 GeV)

Explain the BAU ⇒ BSM physics is also required!

The need for new physics

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- **Dark Matter in the Universe**

  PLANCK, WMAP, ... & Galactic dynamics ⇒ most **matter is "dark"** $\Omega_{\text{CDM}} = 0.259 \pm 0.006$

  "ordinary (SM) matter" - a tiny fraction of mass-energy density $\Omega_b = 0.049 \pm 0.001$

  Dark matter candidate: massive, non-luminous, no strong interactions...

  (at best) weakly interacting, stable!

  No such candidate in the Standard Model!

The need for new physics

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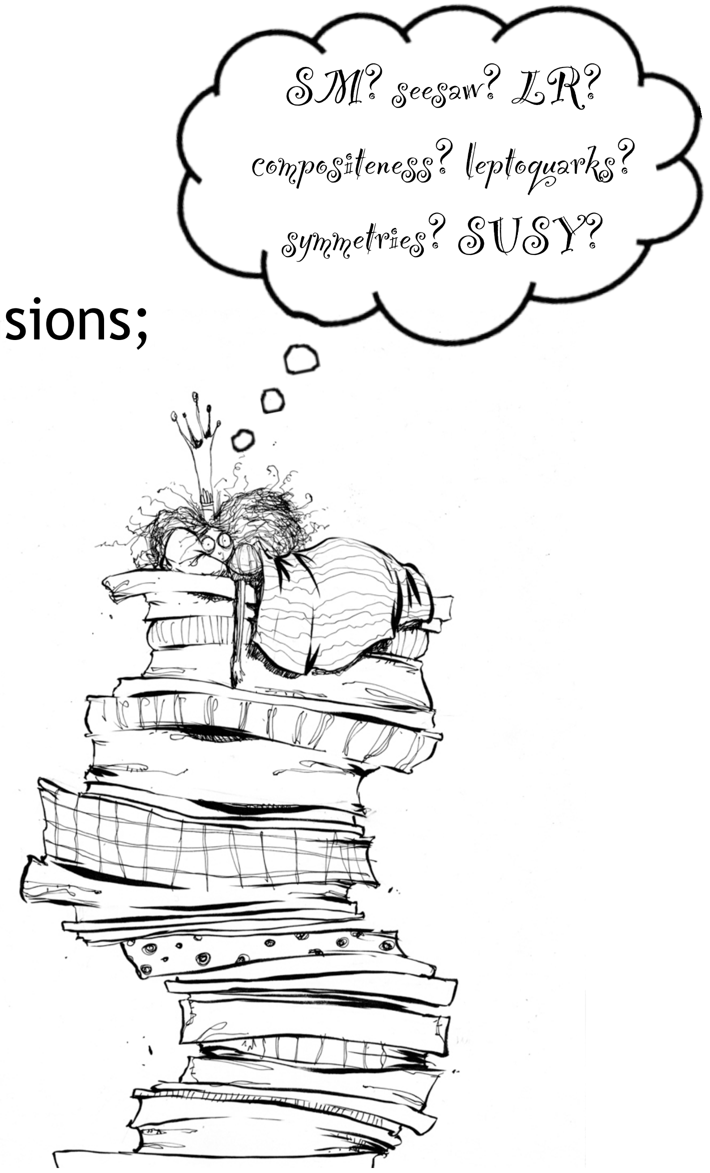
- And a number of ***"theoretical caveats & puzzles"***:

  Unification of interactions, desert of fundamental scales...
  Hierarchy problem, metastability of vacuum...
  Strong CP problem, flavour puzzle, ...
  Accidental symmetries, and many "just-so" constructions...

Beyond the SM

NP model in a nutshell

- New Physics is indeed needed - but which new physics model?
  - New interactions? Additional states? At which **scale?**
- "Golden rule" - extensions of the SM should **ideally:**
  - (i) address (at least one) its observational problems & tensions;
  - (ii) ease/render less severe its **theoretical caveats;**
  - (iii) **be falsifiable;**
  - (iv) avoid creating further issues and/or tensions!



- Extensive ensemble of models...
  - From **minimal extensions**
    - (additional states, in general to address *one* problem),
      - to **comprehensive constructions...**

In all cases, **SM** interpreted as a low-energy limit of a (complete, yet unknown) NP model

Two approaches to identify the model (necessarily) at work

⇒ Study **various classes** of well-motivated **models**

⇒ **Model-independent, effective approach (EFT)**

EFT approach to New Physics

SM interpreted as a low-energy limit of a (complete, yet unknown) NP model

⇒ **Model-independent**, **effective field theory approach (EFT)**

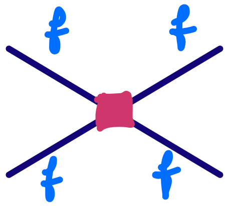
$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \sum_{n\geq5} \frac{1}{\Lambda^{n-4}}\, \mathscr{C}^n(g,Y,\dots)\,\mathscr{O}^n(\ell,q,H,\gamma,\dots)$$

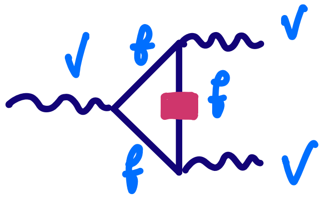
effective **operators**

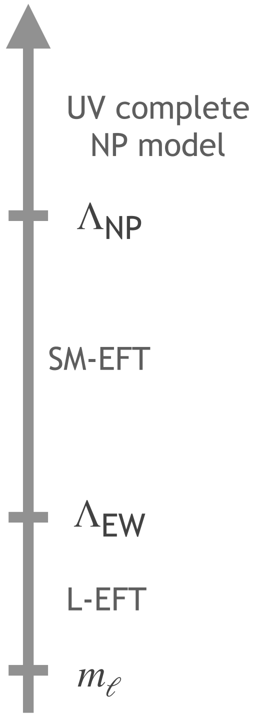
(unknown) NP **scale**

effective coefficients

$\mathscr{O}^5 \rightsquigarrow$ Weinberg operator (m_ν)
$\mathscr{O}^6 \rightsquigarrow$ flavoured contributions
(among many others!)







Derive the new "effective" interactions (vertices, ...), and compute contributions to observables

Agnostic approach, allowing to generically parametrise NP effects

on observables forbidden in SM and/or observables suggesting deviations from SM

$$\mathscr{A} \sim \mathscr{A}\left(\frac{\mathscr{C}^6}{\Lambda^2_{\text{NP}}}\right) + \dots$$

$$\mathscr{A} \sim \mathscr{A}^{\text{SM}} + \mathscr{A}\left(\frac{\mathscr{C}^6}{\Lambda^2_{\text{NP}}}\right) + \dots$$

⇒ master SM prediction!

EFT approach to New Physics

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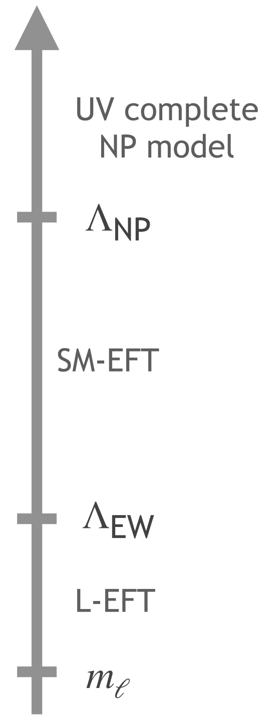

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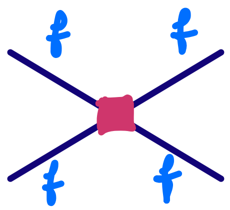
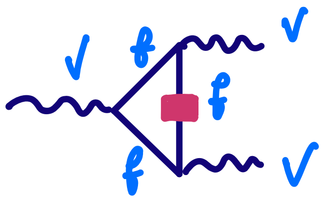




Cast **current data** (limits, ...) in terms of $\mathscr{C}^6_{ij}$ and Λ^2_{NP}

and attempt at **inferring info** on the **dominant operator**, and **scale of NP**

⇒ Beyond $(V-A)$ structure? New vector/axial, (pseudo)scalar or tensor currents?
Flavour violation beyond SM flavour paradigm?

⇒ But **many unknowns:** minimal assumptions must be made, e.g.

"natural" Λ_{NP} → constrain $\mathscr{C}^6_{ij}$
"natural" $\mathscr{C}^6_{ij} \approx 1$ → hint on Λ_{NP}

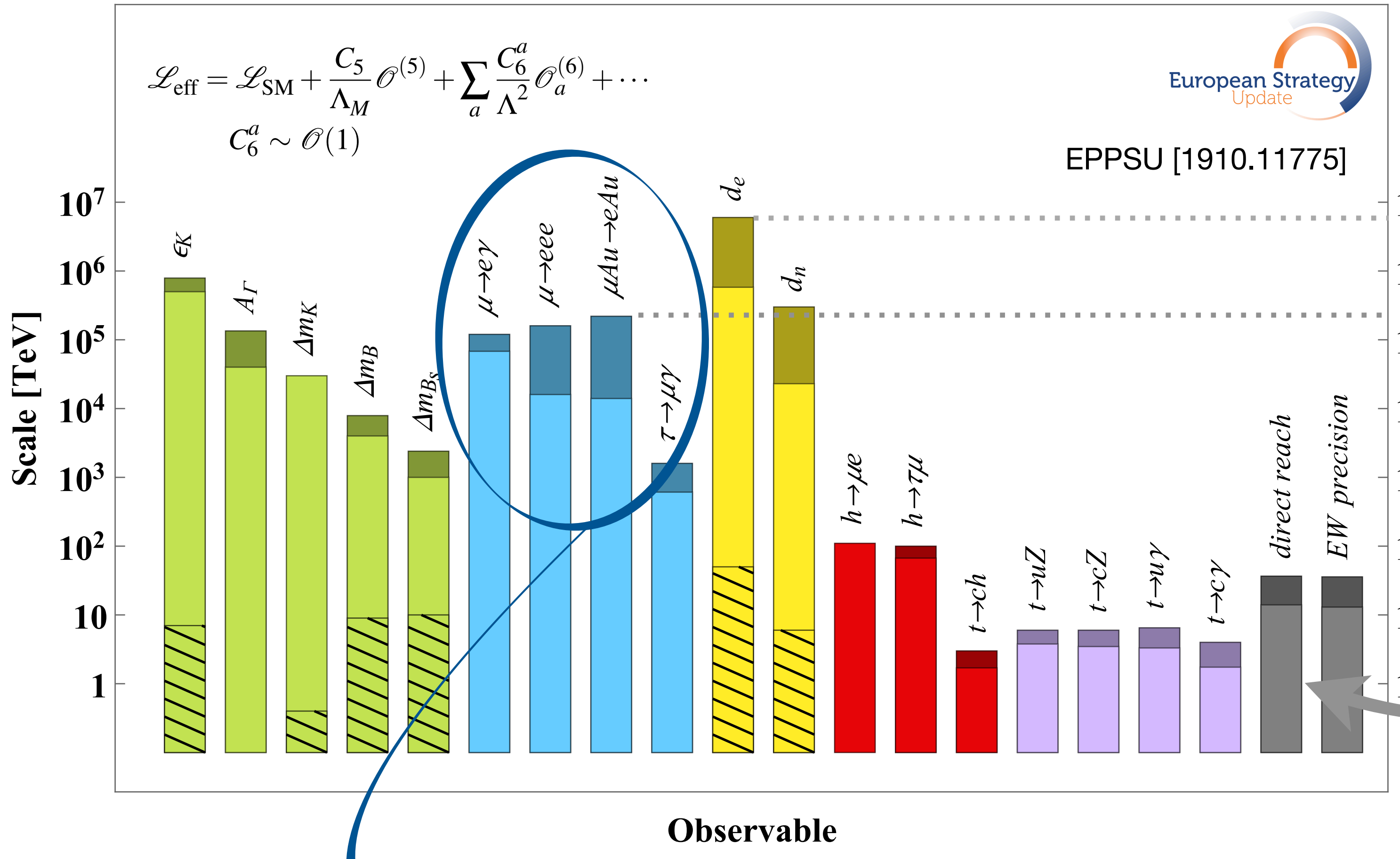
The probing power of flavour & CPV

SM interpreted as a low-energy limit of a (complete, yet unknown) NP model

⇒ **Model-independent**, **effective approach (EFT)**

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \sum_{n\geq 5} \frac{1}{\Lambda^{n-4}} \mathscr{C}^n(g, Y, \ldots)\, \mathscr{O}^n(\ell, q, H, \gamma, \ldots)$$

Cast **current data** in terms of $\mathscr{C}^6_{ij}$ and Λ_{NP} : $\mathscr{C}^6_{ij} \approx 1 \Rightarrow$ **bounds on** Λ_{NP}



Flavour observables:
probes sensitive to very high NP **scales**
$\Lambda_{\text{NP}} \sim \mathscr{O}(10^5$ TeV)
well beyond collider's **reach!**

charged lepton flavour violating observables!

Lepton observables: BSM arena

Lepton flavours: from ν oscillations...

SM lepton sector: (strictly) **massless neutrinos**

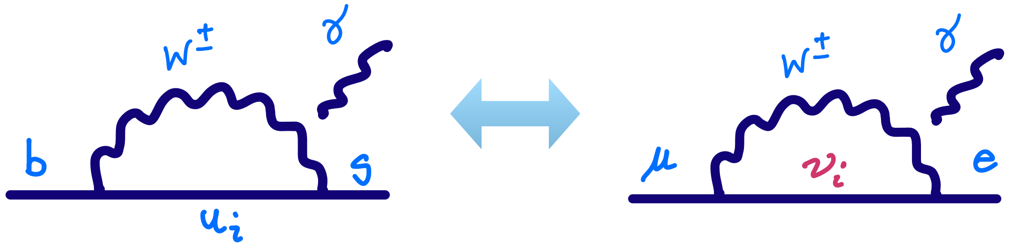
conservation of total lepton number and lepton flavours

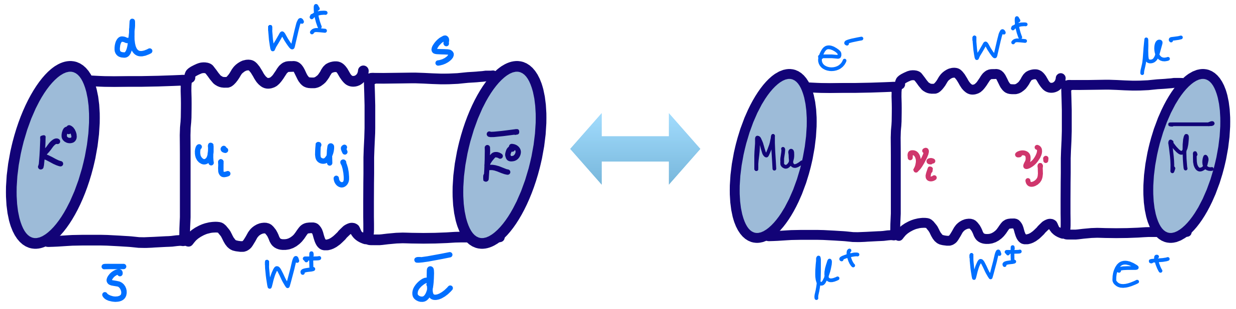
tiny leptonic **EDMs** (4-loop... $d_e^{\text{CKM}} \leq 10^{-38} e$ cm)

Neutrino oscillations: SM description **insufficient!** Added complexity to the flavour problem...

Violation of lepton flavour in neutral lepton sector opens a wide door

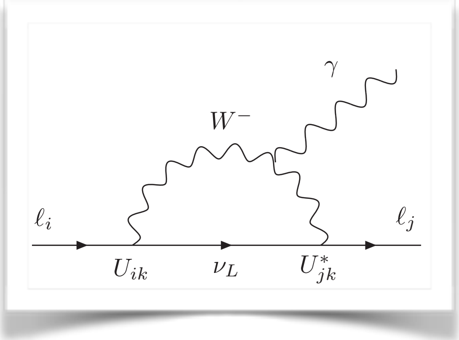
to **flavour violation in the charged lepton sector!**





How general is this once we extend the SM to accommodate $\nu_\alpha \leftrightsquigarrow \nu_\beta$?

In the most **minimal extension** **SM$_{m_\nu}$** [SM$_{m_\nu}$ = "ad-hoc" m_ν (Dirac), U_{PMNS}]



$$\boldsymbol{\Gamma(\mu \to e\gamma)} = \frac{m_\mu^5}{16\pi}(|A_L|^2 + |A_R|^2)$$

$$\sum_i \frac{U^*_{ei}U_{\mu i}}{(k^2 - m_i^2)} = \sum_i \frac{U^*_{ei}U_{\mu i}}{k^2} + \sum_i \frac{U^*_{ei}U_{\mu i}}{k^2}\left(\frac{m_i^2}{k^2}\right) + \mathcal{O}\left(\frac{m_i^4}{k^4}\right) \ \& \ \sum_i U^*_{ei}U_{\mu i}\frac{m_i^2}{M_W^2} = U^*_{e2}U^{e2}\frac{\Delta m^2_{21}}{M_W^2} + U^*_{e3}U^{e3}\frac{\Delta m^2_{31}}{M_W^2}$$

$$\textbf{BR}(\boldsymbol{\mu \to e\gamma}) = \frac{3\alpha_e}{32\pi}\left|\sum_i U^*_{ei}U_{\mu i}\frac{m_i^2}{M_W^2}\right|^2 \qquad \Rightarrow \textbf{BR}(\boldsymbol{\mu \to e\gamma}) \approx \mathbf{10^{-54\div-55}}$$

Lepton flavours: from ν oscillations...

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conservation of total lepton number and lepton flavours

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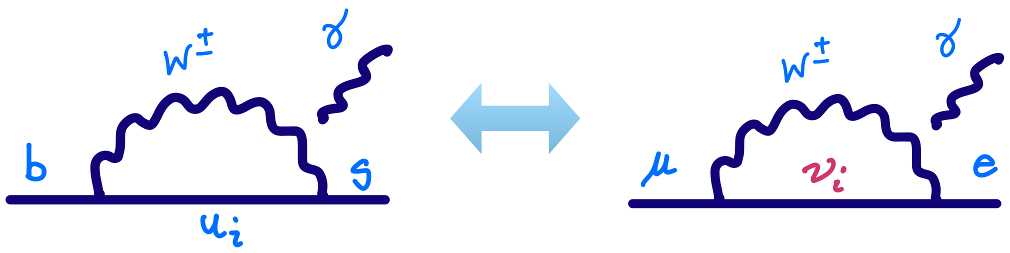

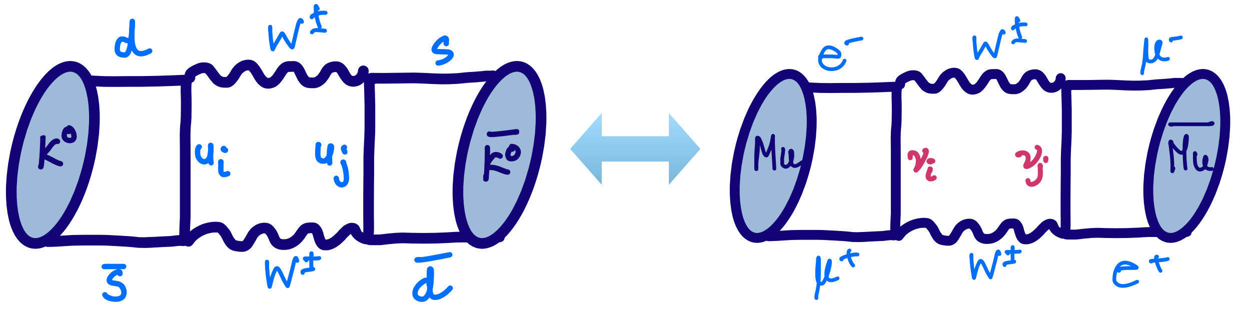

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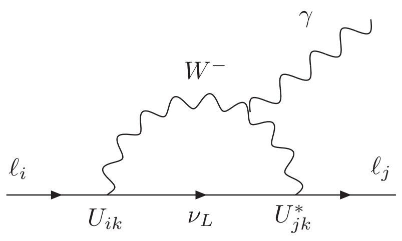


total **lepton number** still **conserved** (LNC)

lepton EDMs still beyond observation (2-loop contributions from δ_{CP})

cLFV possible... but **not observable!!** **BR($\mu \to e\gamma$)** $\sim 10^{-54}$

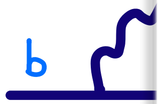
cLFV, LNV, lepton EDMs, ...: observation of **SM-forbidden leptonic modes**

$\Rightarrow$ **Discovery of New Physics!** (possibly before direct signal @ LHC)

Lepton flavours: from ν oscillations...

SM lepton sector: (strictly) **massless neutrinos**

Neu



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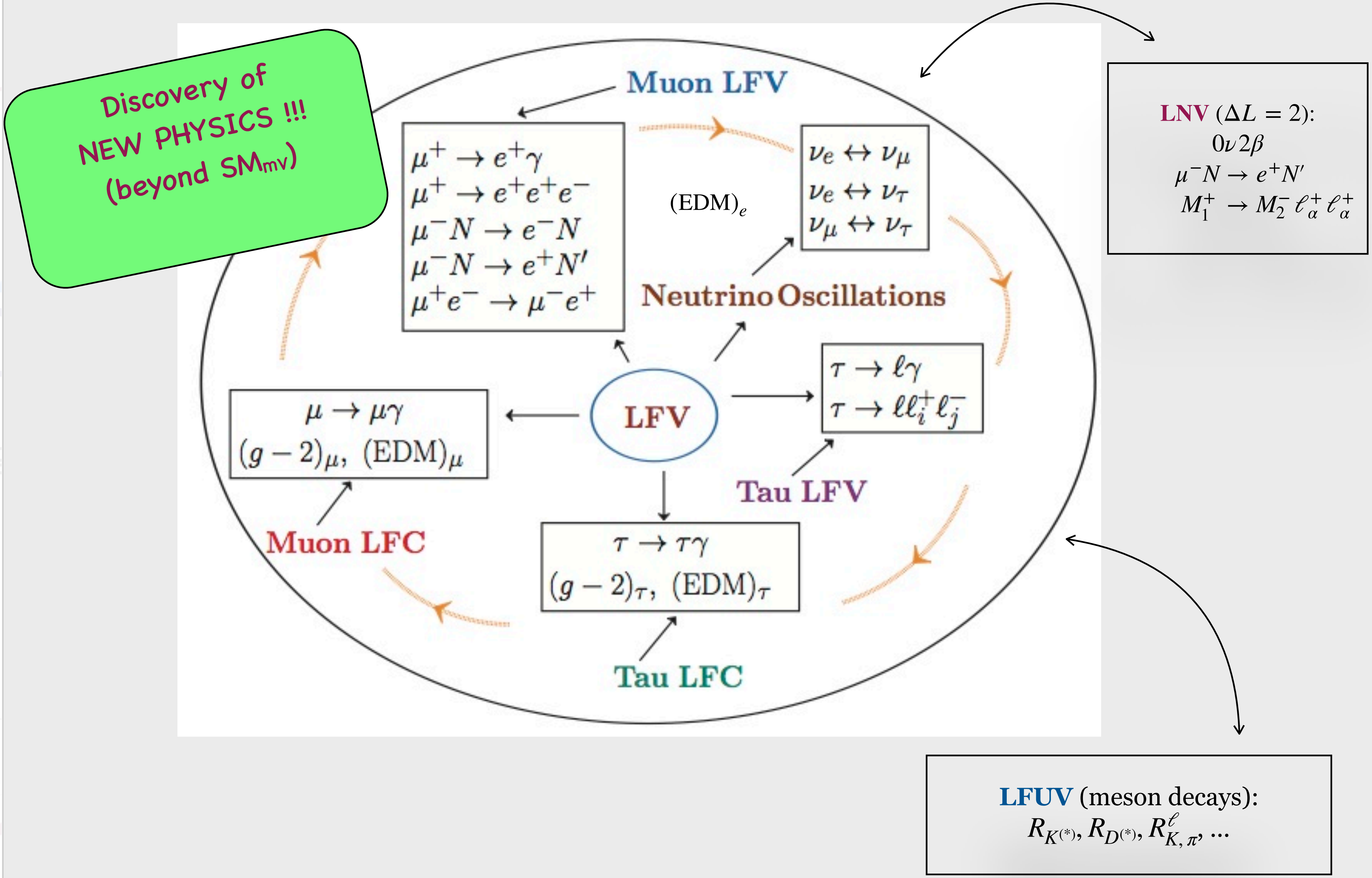

Discovery of NEW PHYSICS !!! (beyond $\text{SM}_{m\nu}$)

Muon LFV

$\mu^+ \to e^+\gamma$
$\mu^+ \to e^+e^+e^-$
$\mu^-N \to e^-N$
$\mu^-N \to e^+N'$
$\mu^+e^- \to \mu^-e^+$

$(\text{EDM})_e$

$\nu_e \leftrightarrow \nu_\mu$
$\nu_e \leftrightarrow \nu_\tau$
$\nu_\mu \leftrightarrow \nu_\tau$

Neutrino Oscillations

LFV

$\tau \to \ell\gamma$
$\tau \to \ell\ell_i^+\ell_j^-$

Tau LFV

$\mu \to \mu\gamma$
$(g-2)_\mu,\ (\text{EDM})_\mu$

Muon LFC

$\tau \to \tau\gamma$
$(g-2)_\tau,\ (\text{EDM})_\tau$

Tau LFC

LNV $(\Delta L = 2)$:
$0\nu 2\beta$
$\mu^-N \to e^+N'$
$M_1^+ \to M_2^- \ell_\alpha^+ \ell_\alpha^+$

LFUV (meson decays):
$R_{K^{(*)}}, R_{D^{(*)}}, R^{\ell}_{K,\pi}, \ldots$

Lepton observables - a very vast array

- **Generic New Physics observables** in the lepton sector:
  - Lepton number violation (e.g. neutrino masses, $0\nu 2\beta$ decays, ...)
  - Electric and (anomalous) magnetic moments - d_ℓ, $(g-2)_\ell$
  - **charged lepton flavour violation**

Back to $\mathscr{L}^{\text{eff}}$: cast observables in terms of $\mathscr{C}_{ij}$ and Λ_{NP}

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathcal{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6 \mathcal{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \ldots + \frac{\mathscr{C}_7 \mathcal{O}^7}{\Lambda^3_{\text{LNV}}}(0\nu 2\beta) + \ldots$$

Majorana ν masses

Kinetic corrections, ...
EW precision, top physics, ...
Electric dipole & anomalous magnetic moments, ...
cLFV (dipole, 3 body, matter assisted, ...)

Lepton number violation,
cLFV & LNV,
...

Deceptively simple task... different new physics scales, numerous operators!

Technically very involved, *even if no "SM background"...*

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- **General description:** processes (sector by sector approach)
  experimental setup; current sensitivity & future prospects
  phenomenological implications (EFT approach)

Lepton observables: cLFV muon processes

cLFV: muon observables

Muons - ideal **probe for NP**: from lepton **flavour universality** tests,
to **anomalous magnetic moments**, ... to cLFV!

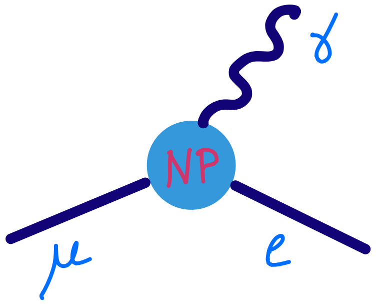
Muon cLFV - extensive opportunities, numerous observables, relying on very intense beams

- **Leptonic decays:** radiative $\mu \to e\gamma$ and three-body $\mu \to 3e$
  muonic atoms $\mu^-(A,Z) \to e^-(A,Z)$ & LNV $\mu^-(A,Z) \to e^+(A,Z-2)^*$
  nuclear assisted Coulomb decays $\mu^- e^- \to e^- e^-$
  Muonium oscillations $\mathrm{Mu}(\mu^+ e^-) - \overline{\mathrm{Mu}}(\mu^- e^+)$ and decays $\mathrm{Mu}(\mu^+ e^-) \to e^+ e^-$
  Light "invisible" searches (e.g. $\mu \to e\phi$, ...)

- And further! **Semi-leptonic decays:** $M \to (M')\mu\ell$
  And at colliders: $Z \to \mu\tau$, $H \to \mu\tau$ (e.g. FCC-ee, CEPC, ...);
  high p_T dilepton tails in $pp \to \mu\ell$ ...
  Numerous channels at a future muon collider!

Muons: *lightest "unstables"* - clean objects, ideal & versatile probes for new physics searches
At the centre of a world-wide comprehensive programme - experiments *and* theory

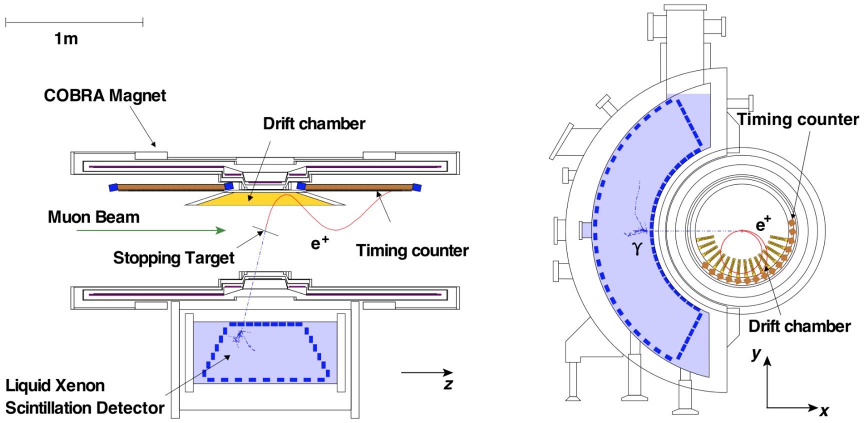
cLFV muon channels: radiative decays



- cLFV decay: $\mu^+ \to e^+ \gamma$
- **Event signature:** $E_e = E_\gamma = m_\mu/2 \quad (\sim 52.8$ MeV)
  Back-to-back $e^+ - \gamma \quad (\theta \sim 180^\circ)$; Time coincidence

- **Backgrounds** ⇒ prompt physics & accidental
  Prompt: radiative μ decays ($\mu \to e\,\bar{\nu}_e \nu_\mu \gamma$, very low E_ν) $[\propto R_\mu]$
  Accidental: coincidence of γ with positron from Michel decays $\mu \to e\,\bar{\nu}_e \nu_\mu$:
  photon from $\mu \to e\,\bar{\nu}_e \nu_\mu \gamma$; γ from in-flight $e^+ e^-$ annihilation $[\propto R_\mu^2]$

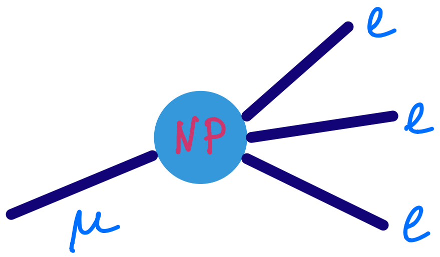
- **Experimental status:**
  First searches (!) in 1940's [MEG Coll., 1605.05081]
  Advent of intense muon beams in 2000's MEG @ PSI
  BR($\mu^+ \to e^+ \gamma$) $\leq 4.2 \times 10^{-13}$ (90% CL)



- **Future prospects:** [MEG II Coll., 2201.008200]
  MEG II (@ PSI): BR($\mu^+ \to e^+ \gamma$) $\leq 6 \times 10^{-14}$

very hard to go beyond 10^{-15} without conceptually different approach

cLFV muon channels: 3-body decays



- cLFV decay: $\boldsymbol{\mu^+ \to e^+ e^- e^+}$
- **Event signature:** $\Sigma E_e = m_\mu$; $\Sigma \vec{P}_e = \vec{0}$
  common vertex; Time coincidence

- **Backgrounds** ⇒ physics & accidental

  Physics: multi-body μ decays ($\mu \to e\,\bar{\nu}_e \nu_\mu\, e^+ e^-$, very low E_ν)

  Accidental: Bhabha scattering of Michel e^+ from $\mu \to e\,\bar{\nu}_e \nu_\mu$ decays with atomic $e^+ e^-$
  Michel positrons with $e^+ e^-$ from γ conversion

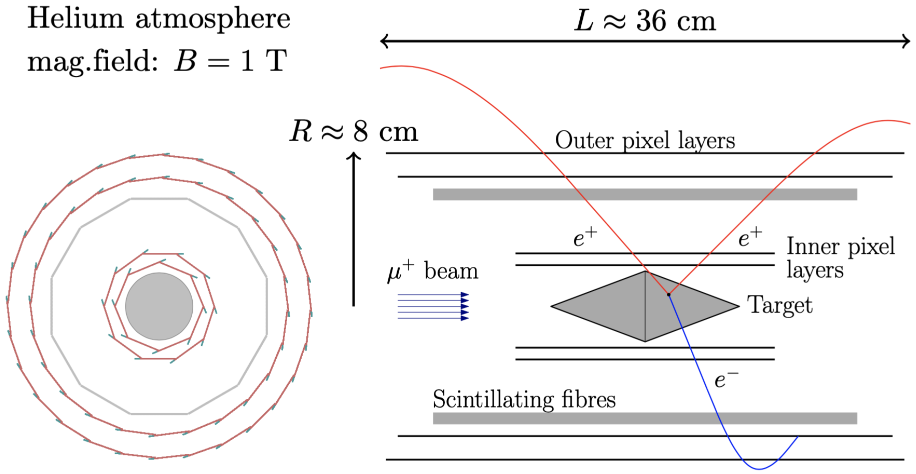
- **Experimental status:**

  SINDRUM @ PSI [SINDRUM Coll., '88]

  BR($\boldsymbol{\mu^+ \to e^+ e^- e^+}$) $\leq 1.0 \times 10^{-12}$ (90% CL)

- **Future prospects:** [Mu3e Coll., 2009.11690]

  Mu3e (@ PSI): expected sensitivity $\mathcal{O}(10^{-15})$ for Phase I
  with HIMB, $\mathcal{O}(10^{-16})$ for Phase II [Aiba et al, 2111.05788]



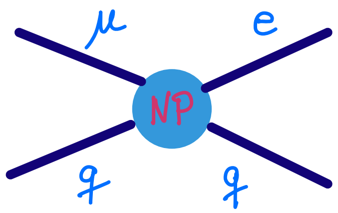
cLFV in muonic atoms: $\mu - e$ conversion

- **Muonic atoms:** 1s bound state formed when μ^- stopped in target

  SM allowed processes: decay in orbit (DIO) $\mu^- \to e^- \nu_\mu \bar{\nu}_e$

  nuclear capture $\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)$

- In the presence of **New Physics** - cLFV neutrinoless $\mu^- - e^-$ conversion



$$\boldsymbol{\mu^- + (A,Z) \to e^- + (A,Z)}$$

- **Event signature:** single mono-energetic electron

  $$E_{\mu e} = m_\mu - E_B(A,Z) - E_R(A,Z)$$

  For Aluminium, Lead, Titanium $\leadsto E_{\mu e} \approx \mathcal{O}(100 \text{ MeV})$

  *Which target?** For coherent conversion, maximal rates for* $30 \leq Z \leq 60$

- **Backgrounds** ⇒ Only physics! μ decay in orbit, beam purity, cosmic rays, …

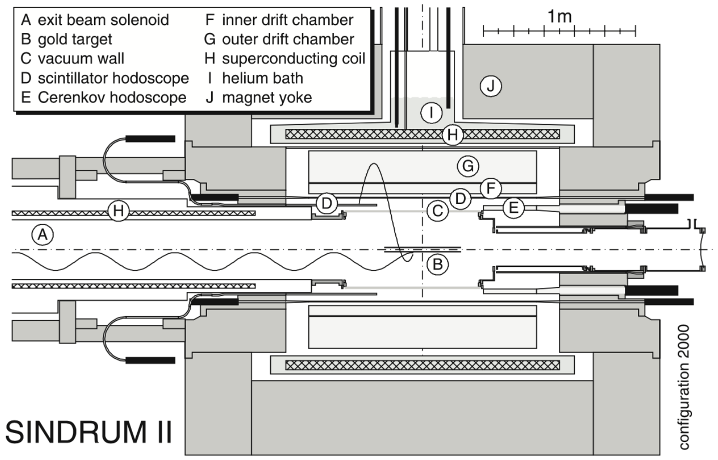
- **Experimental status:** [SINDRUM II Coll., '06]

  SINDRUM @ PSI: CR($\mu^- - e^-$, Au) $\leq 7.1 \times 10^{-13}$ (90% CL)

- **Future prospects:**

  Mu2e (@ FNAL) - $\mathcal{O}(10^{-17})$, [Bartoszek et al, 1501.05241]

  [Abramishvili et al, '20] COMET (@ JPARC) - $\mathcal{O}(10^{-15} - 10^{-17})$, …



SINDRUM II

cLFV in muonic atoms: $\mu - e$ conversion

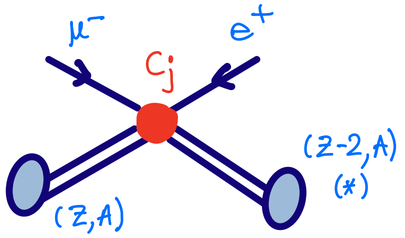
- **Muonic atoms:** 1s bound state formed when $\boldsymbol{\mu^-}$ stopped in target

  SM allowed processes: decay in orbit (DIO) $\mu^- \to e^- \nu_\mu \bar{\nu}_e$

  nuclear capture $\mu^- + (A,Z) \to \nu_\mu + (A, Z-1)$

- In the presence of **New Physics** - cLFV & LNV **($\Delta L = 2$)** neutrinoless $\boldsymbol{\mu^- - e^+}$ conversion

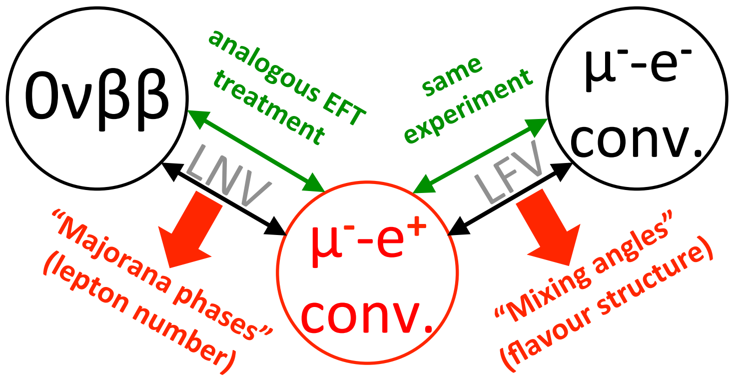
$$\boldsymbol{\mu^- + (A,Z) \to e^+ + (A, Z-2)^*}$$



$\mu^- - e^-$ conversion: coherent process, single nucleon, nuclear ground states

$\mu^- - e^+$ conversion: 2 nucleons ($\Delta Q = 2$), possibly excited final state

A **unique connection** between LNV (in association with Majorana nature and possibly, neutrino mass generation) and cLFV


0νββ
analogous EFT treatment
same experiment
μ^--e^- conv.
LNV
LFV
μ^--e^+ conv.
“Majorana phases” (lepton number)
“Mixing angles” (flavour structure)

LNV-Alternatives:
μ^--μ^+ conversion
$K^+ \longrightarrow \pi^+\mu^-\mu^-$

LFV-Alternatives:
$\mu \longrightarrow e + \gamma$
$\mu \longrightarrow 3e$

[see e.g. Geib et al, 1609.09088]

cLFV in muonic atoms: $\mu - e$ conversion

- **Muonic atoms:** 1s bound state formed when μ^- stopped in target

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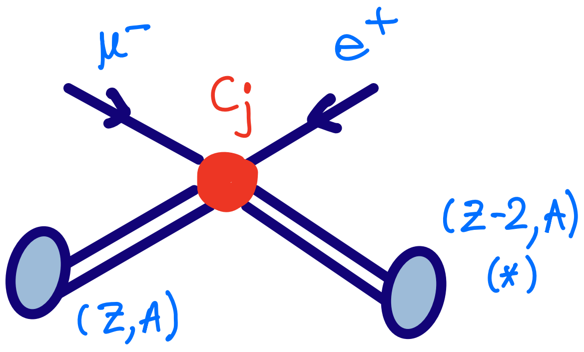


- **Event signature:** single positron - but complex energy spectrum

$$E_{\mu e}^{N^*} = m_\mu - E_B(A,Z) - E_R(A,Z) - \Delta_{Z-2^{(*)}}$$

For Aluminium (giant dipole resonance) $\rightsquigarrow E_{\mu^- e^+}^{\text{Al, GDR}} \approx \mathcal{O}(83.9\text{ MeV})$

- **Experimental status:**

Collaboration	year	Process	Bound
PSI/SINDRUM	1998	μ^-+Ti$\to e^+$+Ca*	3.6×10^{-11}
PSI/SINDRUM	1998	μ^-+Ti$\to e^+$+Ca	1.7×10^{-12}

- **Future prospects:**

  Best sensitivity expected for Ca, S and Ti targets (possibly $\sim \mathcal{O}(\text{few} \times 10^{-15})$)

[Yeo et al,'17]

cLFV in muonic atoms: Coulomb enhanced decay

- **Muonic atoms:** 1s bound state formed when $\boldsymbol{\mu^-}$ stopped in target

- In the presence of **New Physics** - cLFV muonic atom decay $\boldsymbol{\mu^- e^- \to e^- e^-}$

  Initial μ^-, e^-: 1s states bound in Coulomb field of muonic atom's nucleus

  Coulomb interaction increases wave function overlap

  rate strongly enhanced in large Z atoms, $\Gamma \gtrsim (Z-1)^3$

  Larger phase space (compared with $\mu \to 3e$)

- **Event signature:** back-to-back electrons, $E_{e^-} \approx m_\mu/2$

- **Backgrounds** ⇒ similar to neutrinoless conversion

- Experimental status - new observable!

  possibly included in future physics runs (e.g. COMET)

cLFV muonium decays

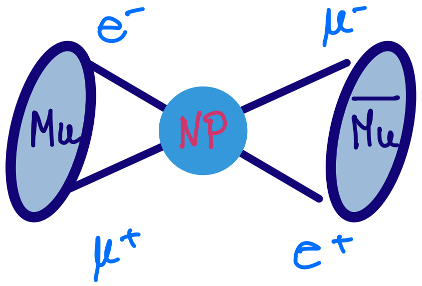
- **Muonium:** $\mu^+ e^-$

  Hydrogen-like Coulomb bound state, free of hadronic interactions!

  Powerful laboratory for **EW** tests and cLFV

- In the presence of **New Physics** - Muonium oscillations and Muonium decays

  - Mu-$\overline{\text{Mu}}$ oscillation



Spontaneous conversion $\mu^+ e^- \leftrightsquigarrow \mu^- e^+$

Reflects a double (individual) lepton number violation $|\Delta L_e| = |\Delta L_\mu| = 2$

Rate (typically) suppressed by external magnetic fields

Detection: reconstruct Michel electron from μ^- decays and shell positron

Experimental status: MACS - $P(\text{Mu} - \overline{\text{Mu}}) < 8.3 \times 10^{-11}$ [Willmann et al, 1999]

Future prospects: MACE, AMF (@FNAL)

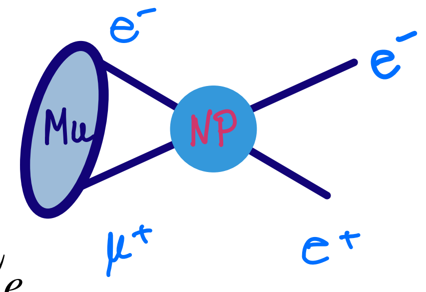
[Bai et al, 2203.11406]

- Mu decays

  $\mu^+ e^- \to e^+ e^-$

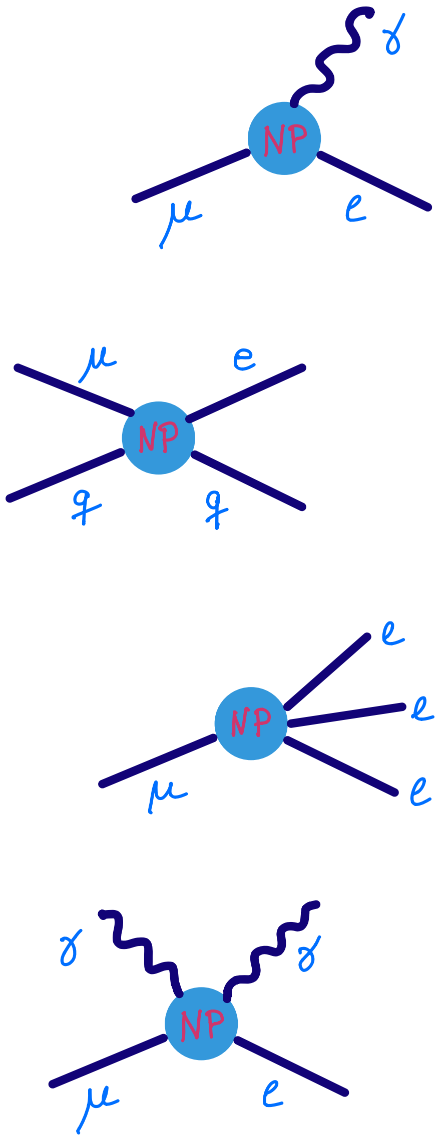
  Clear signal compared to SM-allowed muonium decay, Mu $\to e^+ e^- \bar{\nu}_\mu \nu_e$

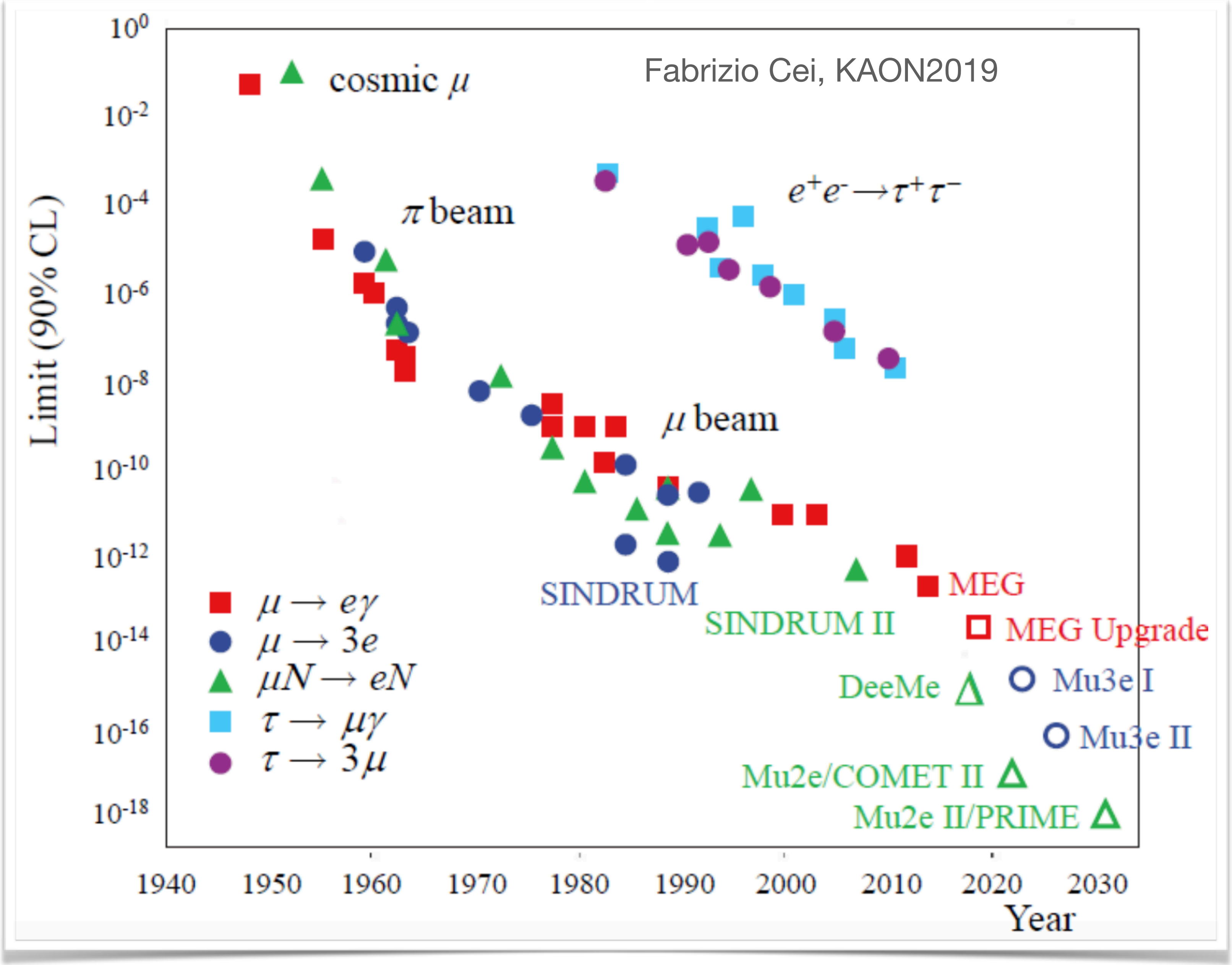
  No available bounds, no clear roadmap...



cLFV muon observables: experimental status

Searching for tiny cLFV effects ⇒ high-intensity sources for excellent sensitivities

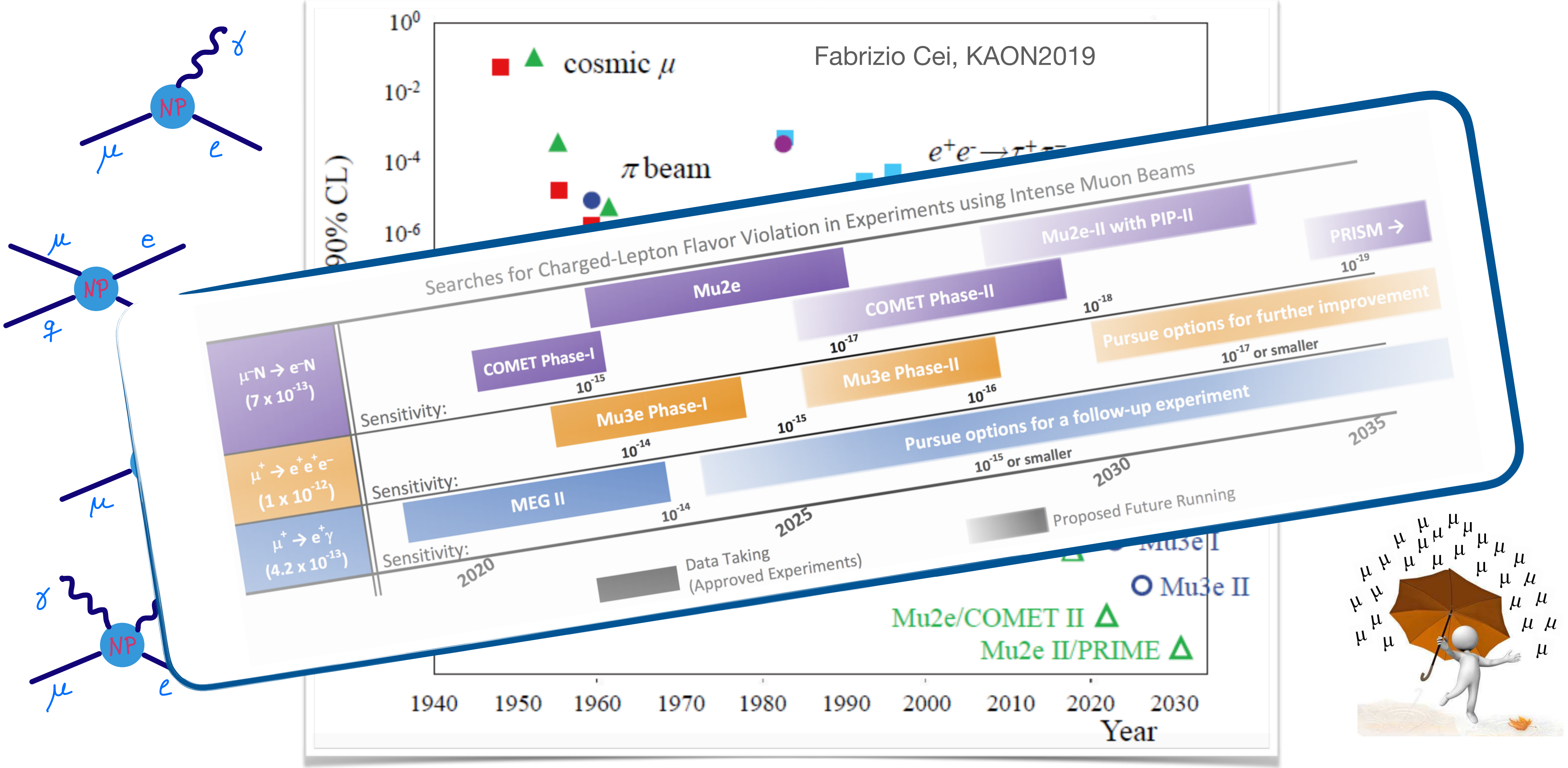




⇒ **Need many many (really many!) muons:** excellent sensitivity with **current sources,**
Amazing prospects with advent of **high-intensity beams** (PSI, FNAL, J-PARC)
and beyond?... ***Muon facility? Muon collider?***

cLFV muon observables: experimental status

Searching for tiny cLFV effects ⇒ high-intensity sources for excellent sensitivities



⇒ **Need many many (really many!) muons:** excellent sensitivity with **current sources,**
Amazing prospects with advent of **high-intensity beams** (PSI, FNAL, J-PARC)
and beyond?... ***Muon facility? Muon collider?***

Lepton observables: cLFV tau processes

cLFV tau decays: leptonic and more

Tau leptons - **heaviest of all charged leptons!** *Cannot have "intense tau beams"* 😄

Copious production at B-factories (BaBar, Belle, LHCb, Belle II, ...)

Production and decay: $e^+ e^- \to \tau^+ \tau^-$

signal "hemisphere"

tagging "hemisphere" (e.g. $\tau^+ \to \bar{\nu}_\tau \nu_e e^+$)

- **cLFV tau decays:** abundant modes! Pure leptonic, semileptonic (2- and 3-body), ...

- **Radiative decay: $\tau^\pm \to \ell^\pm \gamma$**
  - **Event signature:** $E_{\text{final}} - \sqrt{s}/2 = \Delta E \sim 0$; $M_{\text{final}} = M_{\ell\gamma} \sim m_\tau$
  - **Backgrounds** ⇒ coincidence of isolated leptons with γ (ISR, FSR); mistagging

- **3-body leptonic decay: $\tau^\pm \to \ell_i^\pm \ell_j^\mp \ell_k^\pm$**
  - **Event signature:** $E_{3\ell} - \sqrt{s}/2 \sim 0$; $M_{3\ell} \sim m_\tau$
  - **Backgrounds** ⇒ No irreducible backgrounds!

    Small background from $q\bar{q}$ and Bhabha pairs, ...

cLFV tau decays: leptonic and more

Tau leptons - **heaviest of all charged leptons!** *Cannot have "intense tau beams"* :)

Copious production at B-factories (BaBar, Belle, LHCb, Belle II, ...)

Production and decay: $e^+ e^- \to \tau^+ \tau^-$

signal "hemisphere"

tagging "hemisphere" (e.g. $\tau^+ \to \bar{\nu}_\tau \nu_e e^+$)

- **cLFV tau decays:** abundant modes! Pure leptonic, semileptonic (2- and 3-body), ...

- Semi-leptonic cLFV tau decays

  2-body final state: $\boldsymbol{\tau \to \ell h^0}$ (pseudoscalar, scalar or vector neutral meson)

  3-body final state: $\boldsymbol{\tau \to \ell h_i h_j}$ $(h \leftrightsquigarrow \pi^\pm, K^\pm, K_s^0)$

- cLFV exotic modes (also lepton & baryon number violating)

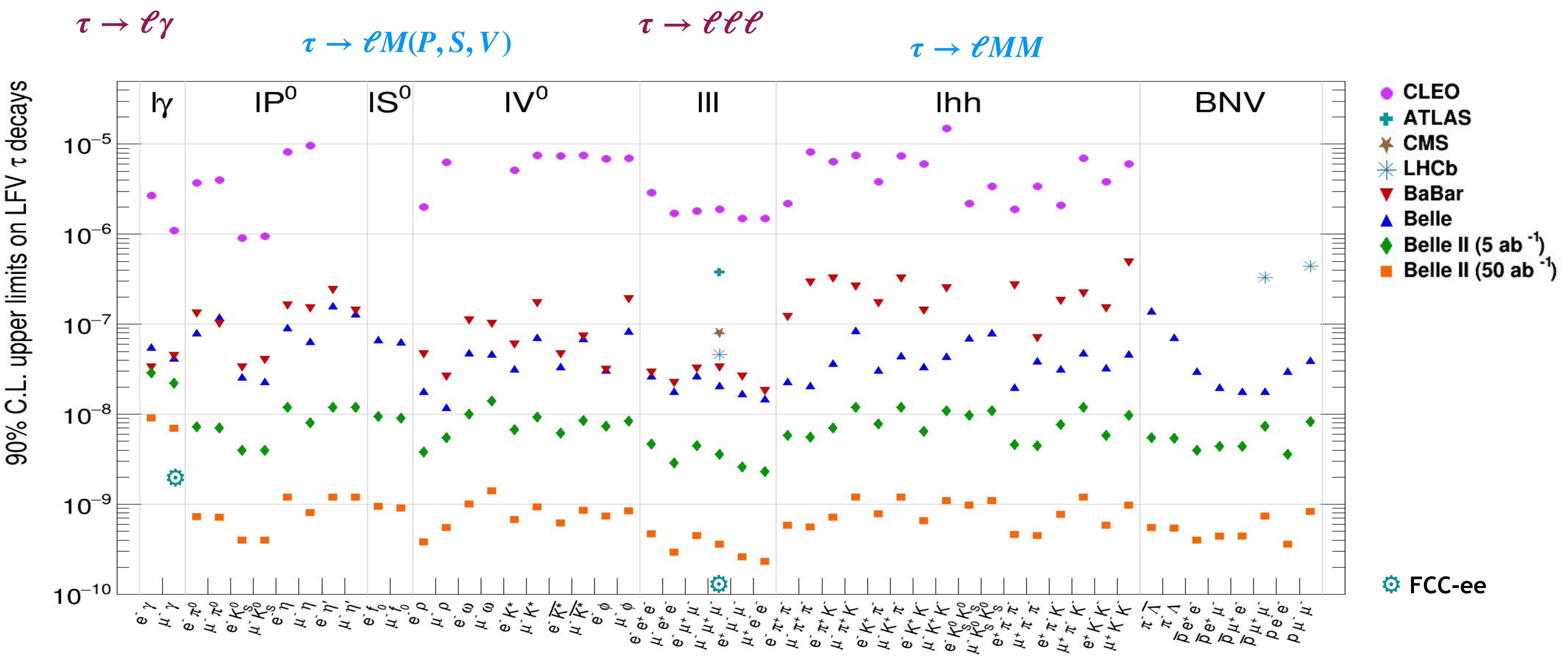
  $\boldsymbol{\tau^- \to \ell^+ h_i^\pm h_j^\pm}$ $(h \leftrightsquigarrow \pi^\pm, K^\pm)$ $\Rightarrow$ LNV

  $\boldsymbol{\tau^- \to \Lambda h^-}$ $(h \leftrightsquigarrow \pi^\pm, K^\pm)$ $\Rightarrow$ LNV & BLV

  $\boldsymbol{\tau \to p \ell_i \ell_j}$ $\Rightarrow$ LNV & BLV

cLFV tau decays: experimental status

Tau cLFV - extensive array of modes!



⇒ **Tau cLFV:** increasingly good prospects!

At **B-factories** (LHCb, **Belle II**)

and also at **FCC-ee**

Decay	Present bound	FCC-ee sensitivity
$Z \to \mu e$	0.75×10^{-6}	10^{-10}–10^{-8}
$Z \to \tau\mu$	12×10^{-6}	10^{-9}
$Z \to \tau e$	9.8×10^{-6}	10^{-9}
$\tau \to \mu\gamma$	4.4×10^{-8}	2×10^{-9}
$\tau \to 3\mu$	2.1×10^{-8}	10^{-10}

cLFV lepton observables: (effective approach)

cLFV: EFT approach to New Physics *(again)*

SM interpreted as a low-energy limit of a (complete, yet unknown) NP model

$\Rightarrow$ **model-independent**, **effective approach (EFT)**

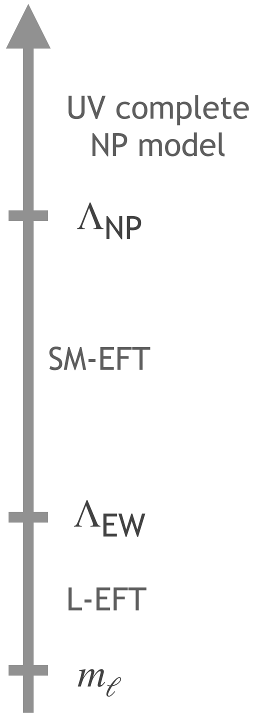
$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \sum_{n\geq 5} \frac{1}{\boldsymbol{\Lambda}^{n-4}} \mathscr{C}^n(g, Y, \dots)\, \mathscr{O}^n(\ell, q, H, \gamma, \dots)$$

effective **operators**

(unknown) NP **scale**

effective coefficients

$\mathscr{O}^5 \rightsquigarrow$ Weinberg operator (m_ν)
$\mathscr{O}^6 \rightsquigarrow$ flavoured contributions
(among many others!)



Cast observables in terms of $\mathscr{C}_{ij}$ and Λ_{NP}; Apply **current data** (limits, ...)

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5 \mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \boxed{\frac{\mathscr{C}_6 \mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta)} + \dots + \frac{\mathscr{C}_7 \mathscr{O}^7}{\Lambda'^3_{\text{LNV}}}(0\nu 2\beta) + \dots$$

$\Rightarrow$ **cLFV data to constrain** $\mathscr{C}_{ij}$ and/or **infer sensitivity** of process to large sets of $\mathscr{C}_{ij}$

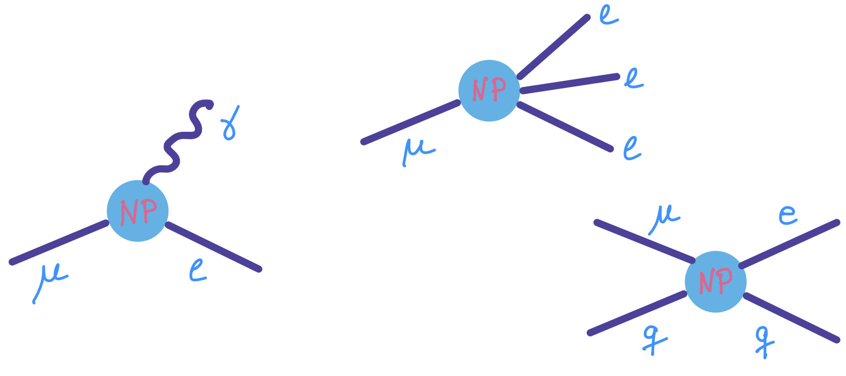
$\Rightarrow$ **Hints on** Λ_{NP} (and on **properties** of new states & **nature** of couplings)

Deceptively simple task... different new physics scales, numerous operators!
Technically very involved, *even if no "SM background"...*

Muon cLFV: EFT approach to New Physics

Cast current data (limits, ...) in terms of $\mathscr{C}_{ij}$ and Λ_{NP}: cLFV operators ($\mathscr{O}^6$)

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5\mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \boxed{\frac{\mathscr{C}_6\mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta)} + \dots$$



- **QED & QCD & NP** effective Lagrangian, many involved operators!

$$\boxed{\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}}\\ &+ \frac{1}{\Lambda^2}\Bigg\{C_L^D O_L^D + \sum_{f=q,\ell}\left(C_{ff}^{V\,LL}O_{ff}^{V\,LL} + C_{ff}^{V\,LR}O_{ff}^{V\,LR} + C_{ff}^{S\,LL}O_{ff}^{S\,LL}\right)\\ &\quad + \sum_{h=q,\tau}\left(C_{hh}^{T\,LL}O_{hh}^{T\,LL} + C_{hh}^{S\,LR}O_{hh}^{S\,LR}\right) + C_{gg}^L O_{gg}^L + L \leftrightarrow R\Bigg\} + \text{h.c.},\end{aligned}}$$

$$\begin{aligned}O_{ff}^{S\,LL} &= (\bar{e}P_L\mu)\left(\bar{f}P_Lf\right),\\ O_{hh}^{S\,LR} &= (\bar{e}P_L\mu)\left(\bar{h}P_Rh\right),\\ O_{hh}^{T\,LL} &= (\bar{e}\sigma_{\mu\nu}P_L\mu)\left(\bar{h}\sigma^{\mu\nu}P_Lh\right),\\ O_{gg}^L &= \alpha_s\, m_\mu G_F\,(\bar{e}P_L\mu)\,G^a_{\mu\nu}G_a^{\mu\nu}\end{aligned}$$

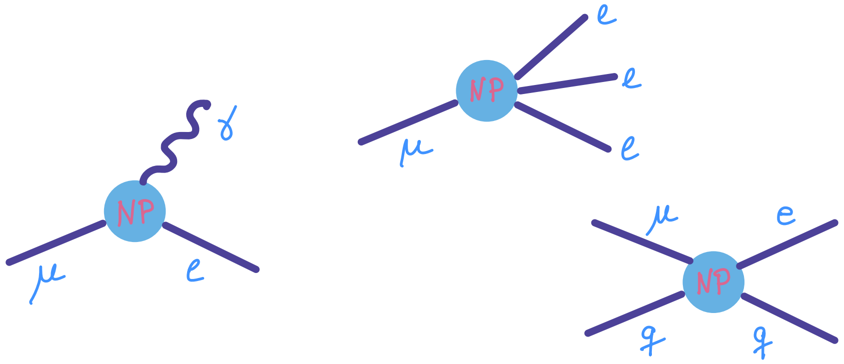
$$\begin{aligned}O_L^D &= e\, m_\mu\,(\bar{e}\sigma^{\mu\nu}P_L\mu)\,F_{\mu\nu},\\ O_{ff}^{V\,LL} &= (\bar{e}\gamma^\mu P_L\mu)\left(\bar{f}\gamma_\mu P_Lf\right),\\ O_{ff}^{V\,LR} &= (\bar{e}\gamma^\mu P_L\mu)\left(\bar{f}\gamma_\mu P_Rf\right),\end{aligned}$$

... and further "mixing" effects, from RGE running (including loop effects) ...

Muon cLFV: EFT approach to New Physics

Cast current data (limits, ...) in terms of $\mathscr{C}_{ij}$ and Λ_{NP}: cLFV operators ($\mathscr{O}^6$)

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5\mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \boxed{\frac{\mathscr{C}_6\mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta)} + \dots$$



- Simple examples: at leading order one has

BR($\mu \to e\gamma$) $\simeq 384\pi^2 \frac{v^4}{\Lambda^4}\left(|C_{D,L}|^2 + |C_{D,R}|^2\right)$

BR($\mu \to eee$) $\simeq \frac{v^4}{\Lambda^4}\left[\frac{1}{8}|C_{S,LL}|^2 + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)\,e|C_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2\right] + (L \leftrightarrow R)$

CR($\mu - e$, N): *far more involved* (nuclear target effects, spin (in)-dependent contributions, ...)

$$\approx \frac{1}{\Gamma_{\text{cap}}}\frac{m_\mu^5}{\Lambda^4}\left[\left|eC_L^D\,D_N + 4\left(G_F m_\mu m_p\,\tilde{C}^{SL}_{(p)}\,S_N^{(p)} + \tilde{C}^{VR}_{(p)}\,V_N^{(p)} + p \to n\right)\right|^2 + (L \leftrightarrow R)\right]$$

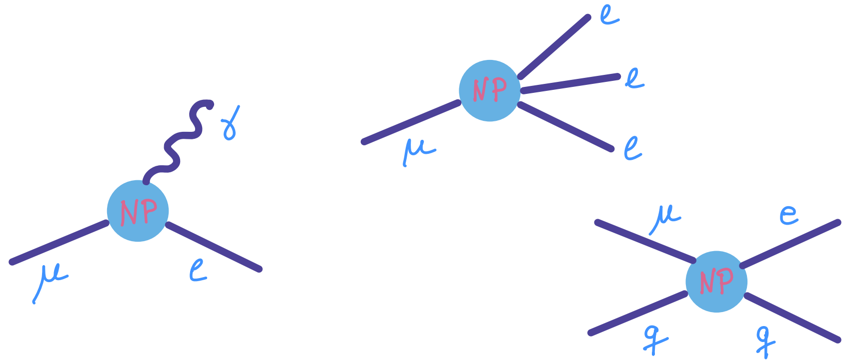
$D_N, S_N^{(p/n)}, V_N^{(p/n)}$: *nuclear "overlap integrals" between lepton wave functions and nucleon densities (target-dependent)*

- What can we learn? Use data to **constrain** (combinations) $\mathscr{C}_6$ and hint on Λ_{cLFV}...

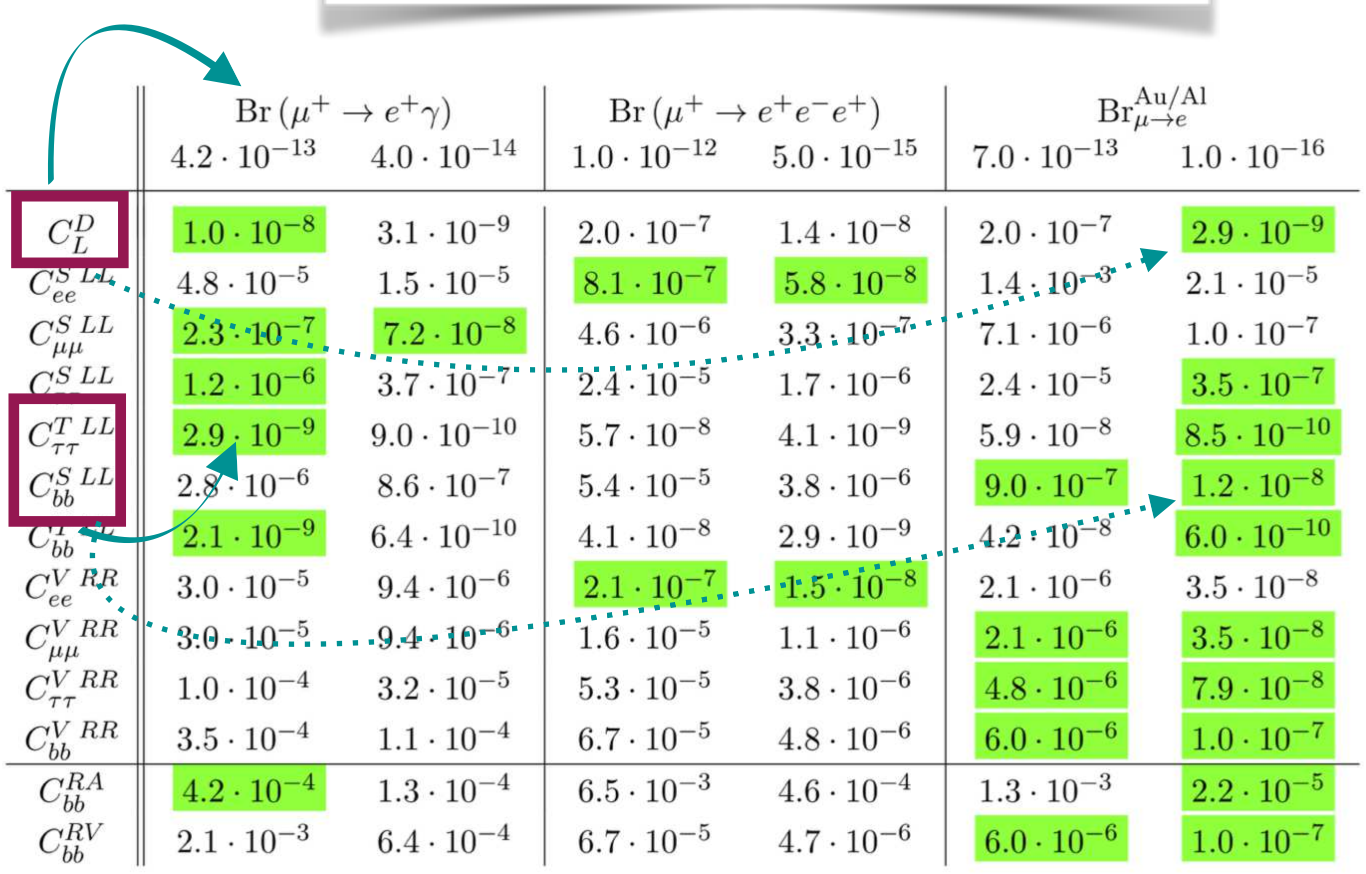
Muon cLFV: EFT approach to New Physics

Cast current data (limits, ...) in terms of $\mathscr{C}_{ij}$ and Λ_{NP}: cLFV operators ($\mathscr{O}^6$)

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \frac{\mathscr{C}_5\mathscr{O}^5}{\Lambda_{\text{LNV}}}(m_\nu) + \frac{\mathscr{C}_6\mathscr{O}^6}{\Lambda^2_{\text{cLFV}}}(\ell_\alpha \leftrightarrow \ell_\beta) + \dots$$



Simple *"one-at-a-time"* limits:



	$\text{Br}(\mu^+ \to e^+\gamma)$		$\text{Br}(\mu^+ \to e^+e^-e^+)$		$\text{Br}^{\text{Au/Al}}_{\mu\to e}$	
	$4.2 \cdot 10^{-13}$	$4.0 \cdot 10^{-14}$	$1.0 \cdot 10^{-12}$	$5.0 \cdot 10^{-15}$	$7.0 \cdot 10^{-13}$	$1.0 \cdot 10^{-16}$
C_L^D	$1.0 \cdot 10^{-8}$	$3.1 \cdot 10^{-9}$	$2.0 \cdot 10^{-7}$	$1.4 \cdot 10^{-8}$	$2.0 \cdot 10^{-7}$	$2.9 \cdot 10^{-9}$
$C_{ee}^{S\,LL}$	$4.8 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$	$8.1 \cdot 10^{-7}$	$5.8 \cdot 10^{-8}$	$1.4 \cdot 10^{-3}$	$2.1 \cdot 10^{-5}$
$C_{\mu\mu}^{S\,LL}$	$2.3 \cdot 10^{-7}$	$7.2 \cdot 10^{-8}$	$4.6 \cdot 10^{-6}$	$3.3 \cdot 10^{-7}$	$7.1 \cdot 10^{-6}$	$1.0 \cdot 10^{-7}$
$C^{S\,LL}$	$1.2 \cdot 10^{-6}$	$3.7 \cdot 10^{-7}$	$2.4 \cdot 10^{-5}$	$1.7 \cdot 10^{-6}$	$2.4 \cdot 10^{-5}$	$3.5 \cdot 10^{-7}$
$C_{\tau\tau}^{T\,LL}$	$2.9 \cdot 10^{-9}$	$9.0 \cdot 10^{-10}$	$5.7 \cdot 10^{-8}$	$4.1 \cdot 10^{-9}$	$5.9 \cdot 10^{-8}$	$8.5 \cdot 10^{-10}$
$C_{bb}^{S\,LL}$	$2.8 \cdot 10^{-6}$	$8.6 \cdot 10^{-7}$	$5.4 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$	$9.0 \cdot 10^{-7}$	$1.2 \cdot 10^{-8}$
$C_{bb}^{T\,LL}$	$2.1 \cdot 10^{-9}$	$6.4 \cdot 10^{-10}$	$4.1 \cdot 10^{-8}$	$2.9 \cdot 10^{-9}$	$4.2 \cdot 10^{-8}$	$6.0 \cdot 10^{-10}$
$C_{ee}^{V\,RR}$	$3.0 \cdot 10^{-5}$	$9.4 \cdot 10^{-6}$	$2.1 \cdot 10^{-7}$	$1.5 \cdot 10^{-8}$	$2.1 \cdot 10^{-6}$	$3.5 \cdot 10^{-8}$
$C_{\mu\mu}^{V\,RR}$	$3.0 \cdot 10^{-5}$	$9.4 \cdot 10^{-6}$	$1.6 \cdot 10^{-5}$	$1.1 \cdot 10^{-6}$	$2.1 \cdot 10^{-6}$	$3.5 \cdot 10^{-8}$
$C_{\tau\tau}^{V\,RR}$	$1.0 \cdot 10^{-4}$	$3.2 \cdot 10^{-5}$	$5.3 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$	$4.8 \cdot 10^{-6}$	$7.9 \cdot 10^{-8}$
$C_{bb}^{V\,RR}$	$3.5 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$	$6.7 \cdot 10^{-5}$	$4.8 \cdot 10^{-6}$	$6.0 \cdot 10^{-6}$	$1.0 \cdot 10^{-7}$
C_{bb}^{RA}	$4.2 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$6.5 \cdot 10^{-3}$	$4.6 \cdot 10^{-4}$	$1.3 \cdot 10^{-3}$	$2.2 \cdot 10^{-5}$
C_{bb}^{RV}	$2.1 \cdot 10^{-3}$	$6.4 \cdot 10^{-4}$	$6.7 \cdot 10^{-5}$	$4.7 \cdot 10^{-6}$	$6.0 \cdot 10^{-6}$	$1.0 \cdot 10^{-7}$

[Crivellin et al, 2017 (courtesy of M. Pruna)]

$\Rightarrow$ BR($\mu \to e\gamma$) depends on dipole C_D

(but mixing effects from RGE running and loop contributions render it **also sensitive** to scalar/tensor/vector contributions, even for $q\bar{q}$ operators)

Unexpected findings!

- Include as many observables & operators **as possible!**

  (e.g. $\mu e\gamma\gamma$ contact interactions, angular observables in polarised $\mu \to 3e$ decays, ...)

[Davidson et al, 2007.09612]

[Bolton, Petcov, 2204.03468]

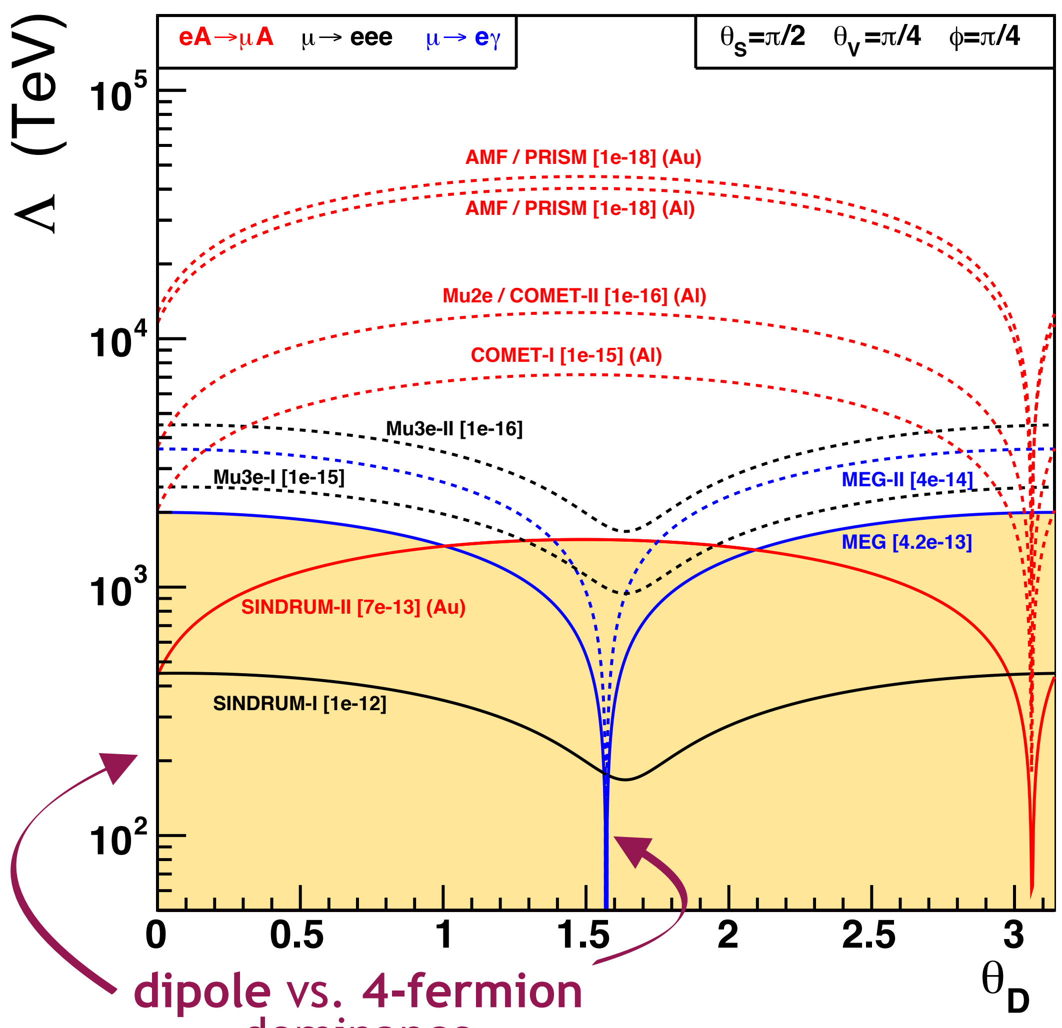
Muon cLFV: EFT approach to New Physics

Results of a recent **EFT** approach to **muon transitions:**

$$\mathscr{L}^{\text{eff}}_{\text{NP, cLFV}} = \frac{1}{\Lambda^2}\Big[C_D(\bar{e}\sigma^{\nu\rho}P_R\mu)F_{\nu\rho} + C_S(\bar{e}P_R\mu)(\bar{e}P_Re) + C_{VR}(\bar{e}\gamma^\nu P_L\mu)(\bar{e}\gamma_\nu P_Re) + C_{VL}(\bar{e}\gamma^\nu P_L\mu)(\bar{e}\gamma_\nu P_Le) +$$

$$+ C_{\text{N-light}}\mathscr{O}_{\text{N-light}} + C_{\text{N-heavy}\perp}\mathscr{O}_{\text{N-heavy}\perp}\Big]$$

$$\vec{C} = \{C_D, C_S, C_{VR}, C_{VR}, C_{VL}, C_{\text{N-light}}, C_{\text{N-heavy}\perp}\}$$



dipole vs. 4-fermion dominance

$$2\sqrt{2}\,C_D \approx \frac{\cos\theta_D}{\Lambda^2}$$

[Davidson & Echenard, 2204.00564]

$$\Rightarrow \text{BR}(\mu \to e\gamma) \simeq 384\pi^2 \frac{v^4}{\Lambda^4}|\vec{C}\,.\,\hat{e}_{DR}|^2 \;\rightsquigarrow\; \lesssim \text{BR}^{\text{exp (future)}}$$

and likewise for other observables

BR($\mu \to 3e$), CR($\mu - e$, N), Muonium oscillations…

Sensitivity to NP scales (current & future):

MEG ($\mu \to e\gamma$) $\leftrightsquigarrow$ $\Lambda_{\text{cLFV}} \sim \mathscr{O}(10^3\ \text{TeV})$ [dipole]

steadily improved by **Mu3e** $\sim \mathscr{O}(5 \times 10^3\ \text{TeV})$

SINDRUM II ($\mu - e$, Au) $\leftrightsquigarrow$ $\Lambda_{\text{cLFV}} \sim \mathscr{O}(10^3\ \text{TeV})$ [4f]

Mu2e/COMET II ($\mu - e$, Al) $\leftrightsquigarrow$ $\Lambda_{\text{cLFV}} \lesssim \mathscr{O}(10^4\ \text{TeV})$
[either dipole or 4f]

Muon cLFV: EFT approach & conversion in nuclei

⇒ **cLFV data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to operator** $\mathscr{O}^6$)

- Fully exploring the potential of atomic (elastic) **muon-electron conversion,** CR($\mu - e$,N):

  Comparatively more involved theoretical approach!

  Explore target-nucleus dependence to distinguish dominant operator (hint on **NP** model!)

  [extensive contributions since Kitano et al, 0203110! see Davidson et al, 1810.01884; Heeck et al, 2203.00702, ...]

  In the advent of an observation (@ Mu2e, COMET ⤳ using Aluminium targets)

  prepare **choice of future targets**

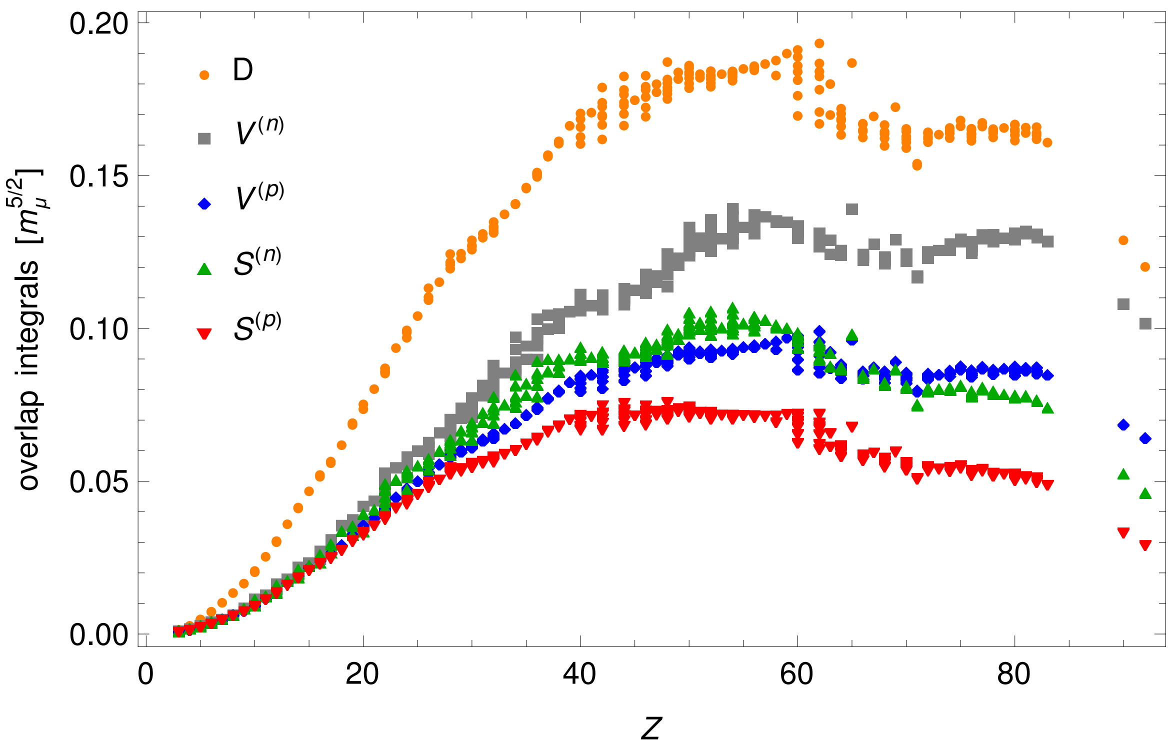
  Which offer the **largest complementarity** with respect to Al?

$$\mathrm{BR_{SI}}(\mu A \to eA) = \frac{32 G_F^2}{\Gamma_{\mathrm{capture}}} \left[\left| C_{V,R}^{pp} V^{(p)} + C_{S,L}^{pp'} S^{(p)} + C_{V,R}^{nn} V^{(n)} + C_{S,L}^{nn'} S^{(n)} + C_{D,L} \frac{D}{4} \right|^2 + \{L \leftrightarrow R\} \right] .$$

Overlap integrals:

more distinguishable at **large Z** !

Better disentangle dominant NP contributions...



[Heeck et al, 2203.00702]

Muon cLFV: EFT approach & conversion in nuclei

⇒ **cLFV data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to operator** $\mathscr{O}^6$)

- Fully exploring the potential of atomic (elastic) **muon-electron conversion,** CR($\mu - e$,N):

  Comparatively more involved theoretical approach!

  Explore target-nucleus dependence to distinguish dominant operator (hint on **NP** model!)

  [extensive contributions since Kitano et al, 0203110! see Davidson et al, 1810.01884; Heeck et al, 2203.00702, ...]

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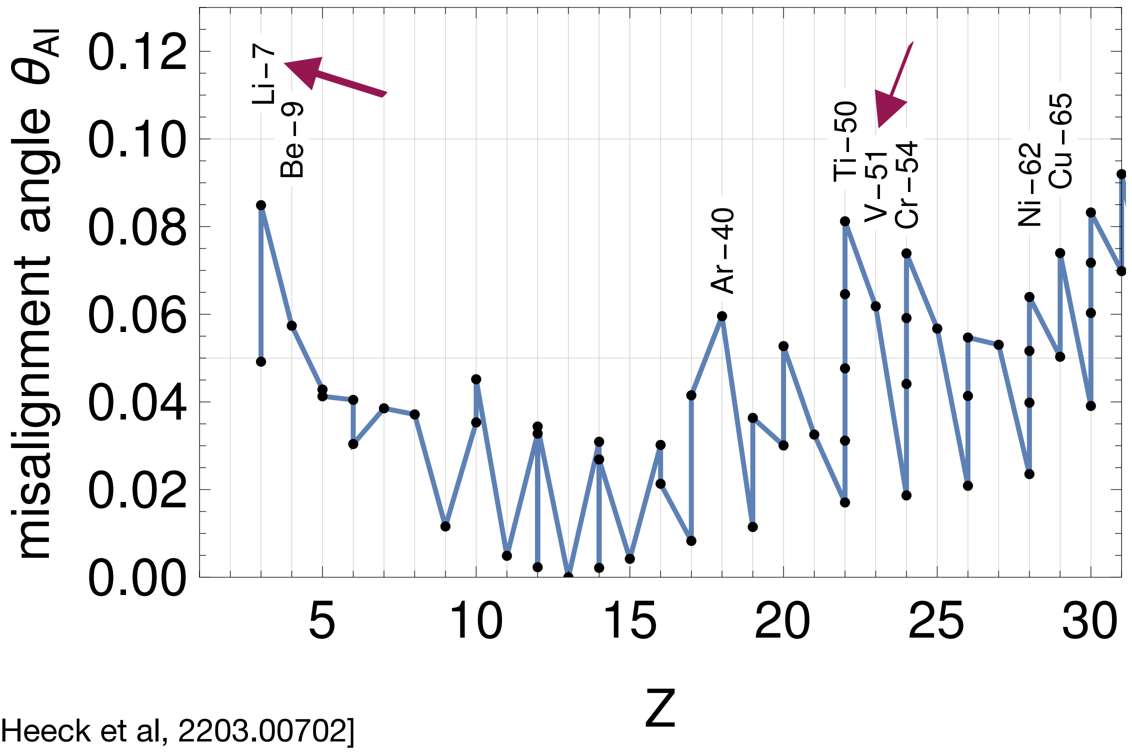
Which offer the **largest complementarity** with respect to Al? $\boldsymbol{\theta_{\mathrm{Al}}}$



[Heeck et al, 2203.00702]

- **Heavier nuclei** (Au, Pb)! ... not feasible...
  (pulsed beams)
- Among experimental-friendly $Z \leq 25$ targets

  several (theoretically good) candidates

  Li-7, Ti-50, Ti-49, Cr-54, .., V-51

⇒ **Li-7 and/or V-51 :** preferable "**second**" targets
post CR($\mu - e$,Al) observation

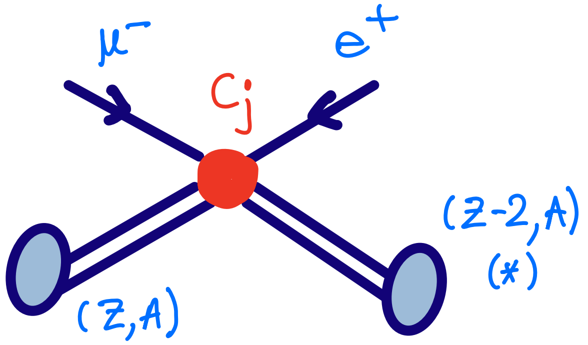
Muon cLFV: EFT approach & conversion in nuclei

⇒ **cLFV data to constrain** $\mathscr{C}^6$ (and **infer sensitivity** of a process **to operator** $\mathscr{O}^6$)

- Fully exploring the potential of atomic (elastic) **muon-electron conversion,** CR($\mu - e$,N):

  And of its lepton number violating counterpart, $\boldsymbol{\mu^- + (A,Z) \to e^+ + (A,Z-2)^{(*)}}$

  A **unique connection** between **LNV** (in association with Majorana nature and possibly, neutrino mass generation) and **cLFV**



From a theoretical point of view, not straightforward!

- Higher-dimension operators in $\mathscr{L}^{\text{eff}}$ (dim 6, 10, 14...)
- Nuclear matrix elements extremely hard to compute!

$$\Gamma^{\text{LNV}}_{\mu e} \approx \frac{G_F^4\, g_A^4}{32\pi^2}\,|\epsilon^2_{C_j}|\,\frac{m_e^2 m_\mu^2}{R^2}\,|F(Z-2,E_e)|\ <\phi_\mu>^2\ |\mathscr{M}^{(\mu^-,e^+)}|^2$$

(only two $\mathscr{M}^{(\mu^-,e^+)}$ known, for Ti-48...)

[Domin et al, 0409033; Simkovic et al, 0103029]

⇒ Very hard to draw implications... **Must tackle NME!**

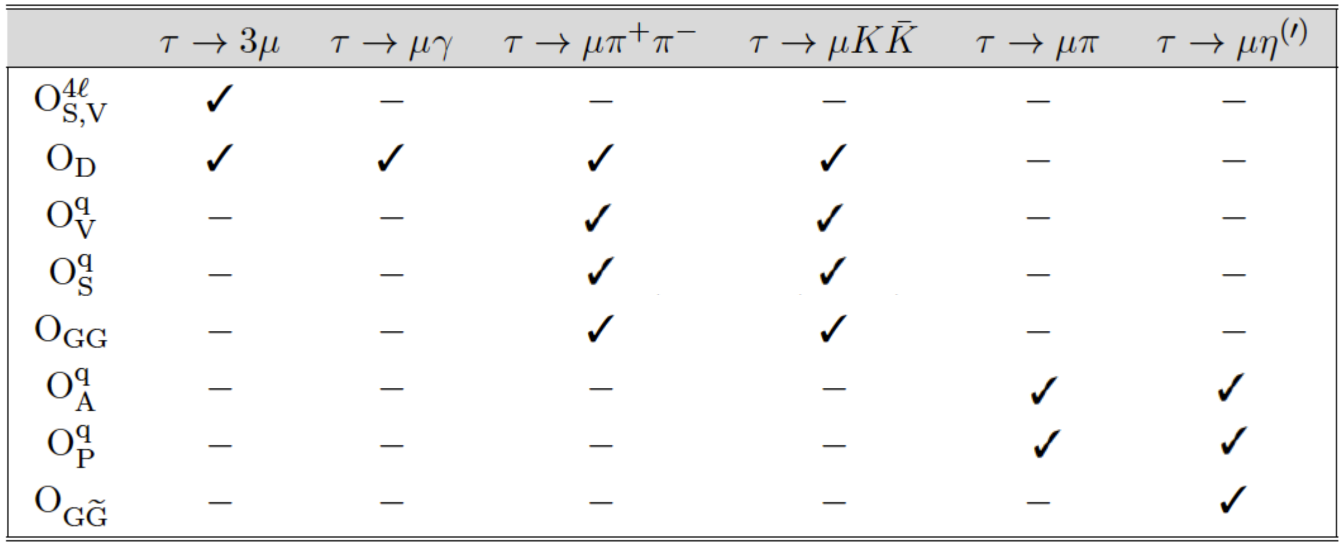
Tau cLFV: (semi-) leptonic modes

Flavour violating tau **decays: large number** of modes

Leptonic (radiative, three-body) as well as semi-leptonic (light mesons, 2- and 3-body)

⇒ theoretically much more involved (scales, hadronisation, ...)

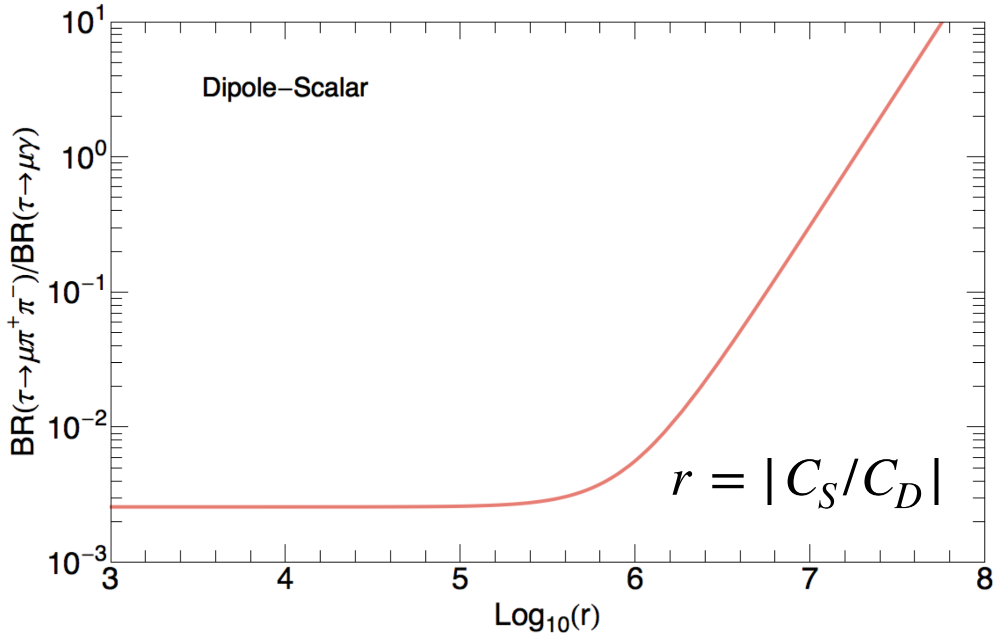
⇒ larger set of (tree-level) contributing operators (e.g. numerous $qq\ell\ell$, gluon, ...)!



	$\tau\to 3\mu$	$\tau\to\mu\gamma$	$\tau\to\mu\pi^+\pi^-$	$\tau\to\mu K\bar{K}$	$\tau\to\mu\pi$	$\tau\to\mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓	✓	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓	✓
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓	✓
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

More challenging to **disentangle** operator dominance... *(even @ tree level!)*

[Celis et al, 1403.5781]



Example: for only $C_D, C_S \neq 0$

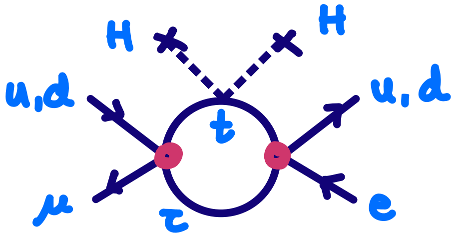
study **ratios of cLFV observables**

Less trivial interpretation

τ-FV can also lead to muon cLFV:

$(\mu\to\tau)\times(\tau\to e)$

complementary sensitivity to $C_{\tau\ell Qq}$



[Ardu et al, 2202.09246]

Tau cLFV: (semi-) leptonic modes

Flavour violating tau decays: comparatively **large number** of modes

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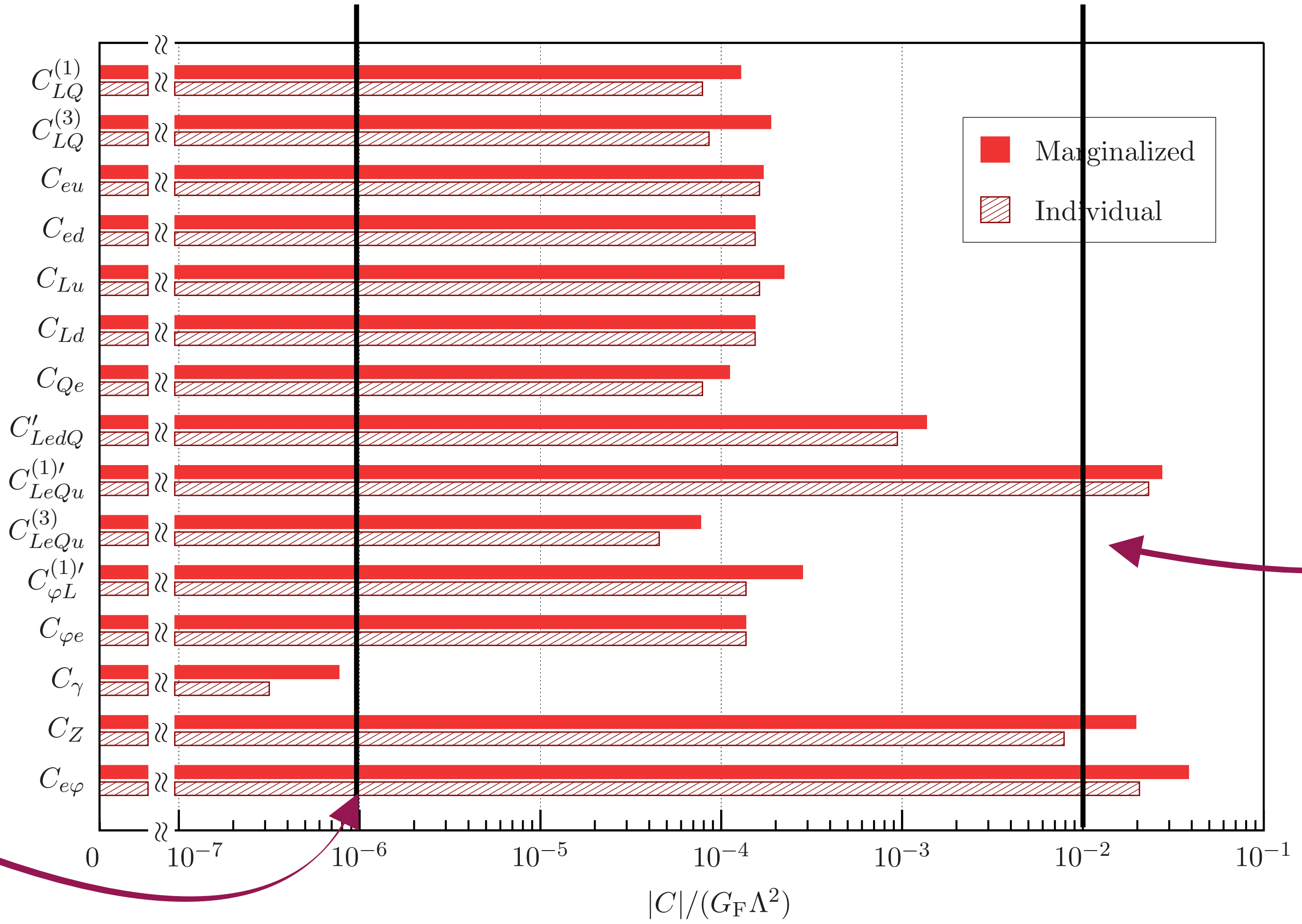


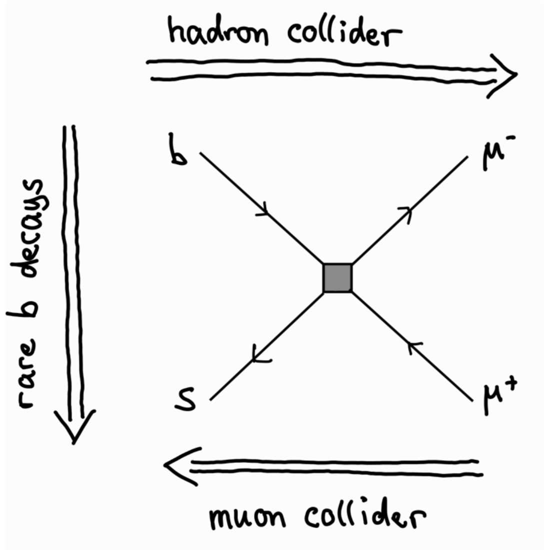
For $C \approx 1$,
$\Lambda_{NP} \sim 3$ TeV

For $C \approx 1$,
$\Lambda_{NP} \sim 300$ TeV

[Banerjee et al, 2203.14919]

Overview of Belle II limits on relevant coefficients (and NP scales) for **cLFV tau decays**

cLFV at higher energies: from meson decays to colliders (EFT studies...)



[From Altmannshofer, 2023]

Meson decays: LFUV, cLFV and more

- **Meson decays:** excellent hunting grounds for **"leptophilic"** New Physics

  ⇒ **deviations from SM** (lepton flavour universality violation, angular distributions, ...)

  ⇒ **new phenomena** (cLFV, LNV, ...)

- **cLFV semileptonic meson** decays!

"Old" experimental results (around 2018)

But *insight into plethora of modes*

Comparative illustration of **sensitivity**

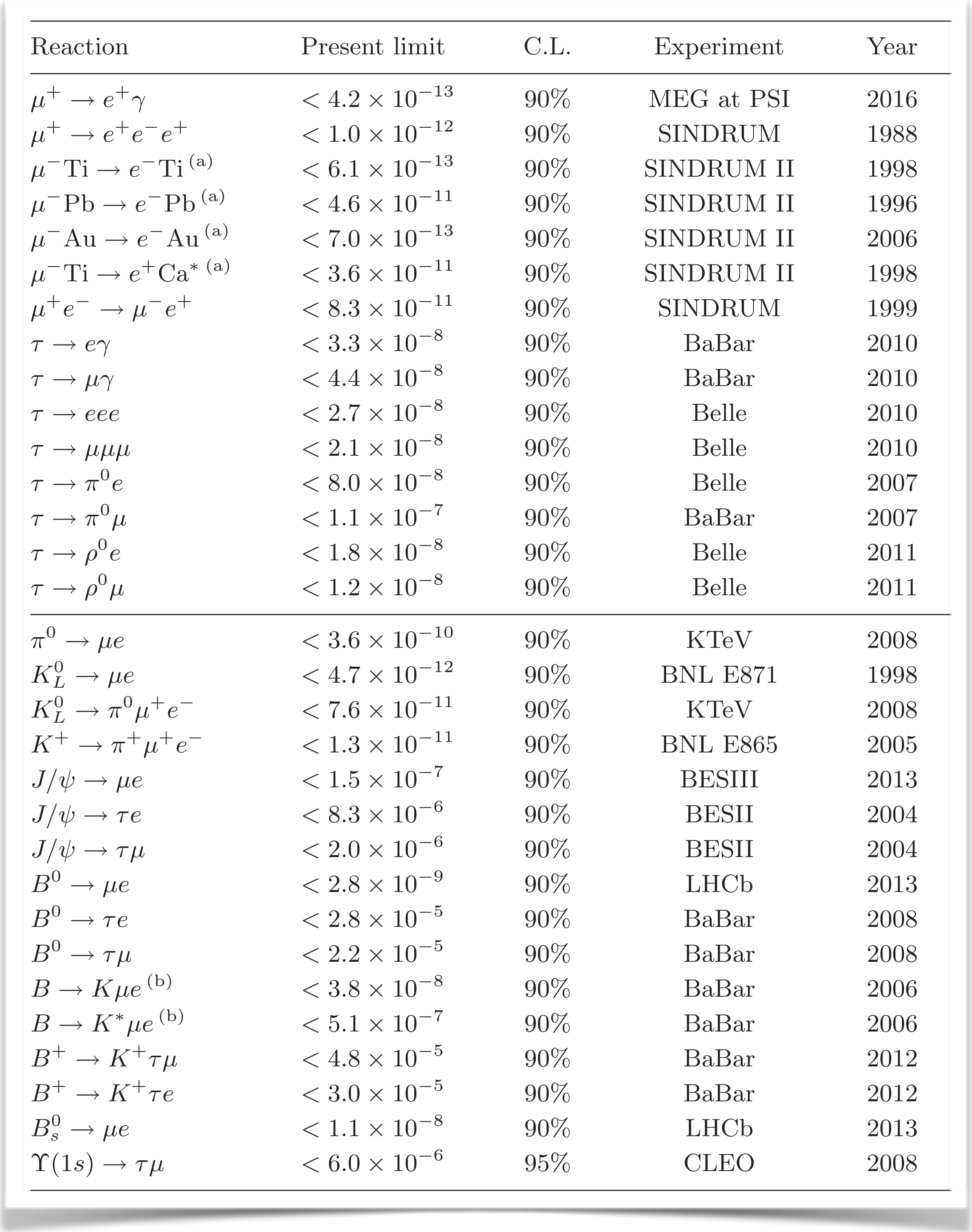
in different systems: muons, taus, mesons...

Although **heavy meson cLFV** decays offer

less powerful probes,

impressive bounds from **rare kaon decays!**

$$\text{BR}(K_L^0 \to \mu e) \; < 4.7 \times 10^{-12}$$



Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$ (a)	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$ (a)	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$ (a)	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006
$\mu^-\mathrm{Ti} \to e^+\mathrm{Ca}^*$ (a)	$< 3.6 \times 10^{-11}$	90%	SINDRUM II	1998
$\mu^+e^- \to \mu^-e^+$	$< 8.3 \times 10^{-11}$	90%	SINDRUM	1999
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	90%	BaBar	2010
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	90%	BaBar	2010
$\tau \to eee$	$< 2.7 \times 10^{-8}$	90%	Belle	2010
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	90%	Belle	2010
$\tau \to \pi^0 e$	$< 8.0 \times 10^{-8}$	90%	Belle	2007
$\tau \to \pi^0 \mu$	$< 1.1 \times 10^{-7}$	90%	BaBar	2007
$\tau \to \rho^0 e$	$< 1.8 \times 10^{-8}$	90%	Belle	2011
$\tau \to \rho^0 \mu$	$< 1.2 \times 10^{-8}$	90%	Belle	2011
$\pi^0 \to \mu e$	$< 3.6 \times 10^{-10}$	90%	KTeV	2008
$K_L^0 \to \mu e$	$< 4.7 \times 10^{-12}$	90%	BNL E871	1998
$K_L^0 \to \pi^0\mu^+e^-$	$< 7.6 \times 10^{-11}$	90%	KTeV	2008
$K^+ \to \pi^+\mu^+e^-$	$< 1.3 \times 10^{-11}$	90%	BNL E865	2005
$J/\psi \to \mu e$	$< 1.5 \times 10^{-7}$	90%	BESIII	2013
$J/\psi \to \tau e$	$< 8.3 \times 10^{-6}$	90%	BESII	2004
$J/\psi \to \tau\mu$	$< 2.0 \times 10^{-6}$	90%	BESII	2004
$B^0 \to \mu e$	$< 2.8 \times 10^{-9}$	90%	LHCb	2013
$B^0 \to \tau e$	$< 2.8 \times 10^{-5}$	90%	BaBar	2008
$B^0 \to \tau\mu$	$< 2.2 \times 10^{-5}$	90%	BaBar	2008
$B \to K\mu e$ (b)	$< 3.8 \times 10^{-8}$	90%	BaBar	2006
$B \to K^*\mu e$ (b)	$< 5.1 \times 10^{-7}$	90%	BaBar	2006
$B^+ \to K^+\tau\mu$	$< 4.8 \times 10^{-5}$	90%	BaBar	2012
$B^+ \to K^+\tau e$	$< 3.0 \times 10^{-5}$	90%	BaBar	2012
$B_s^0 \to \mu e$	$< 1.1 \times 10^{-8}$	90%	LHCb	2013
$\Upsilon(1s) \to \tau\mu$	$< 6.0 \times 10^{-6}$	95%	CLEO	2008

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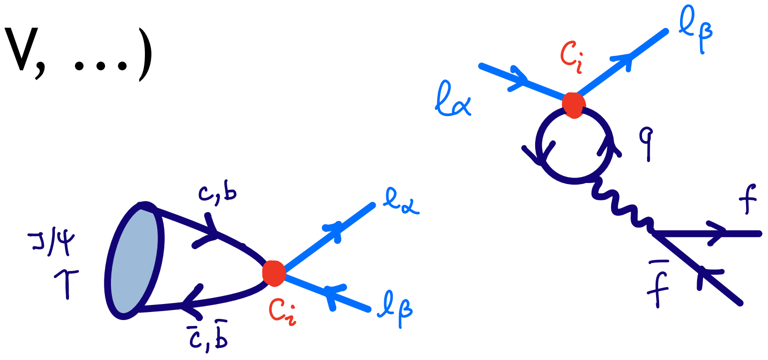
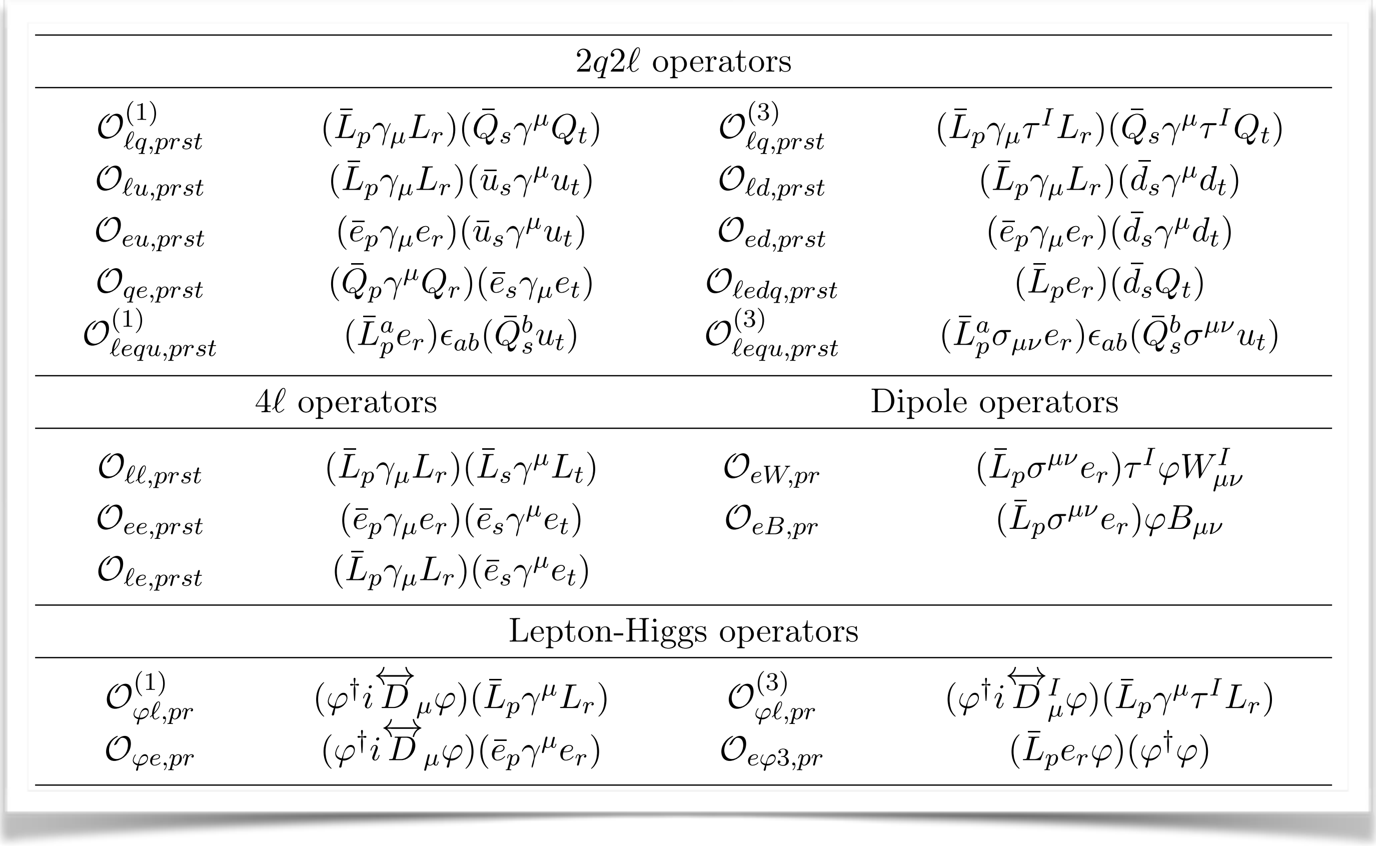
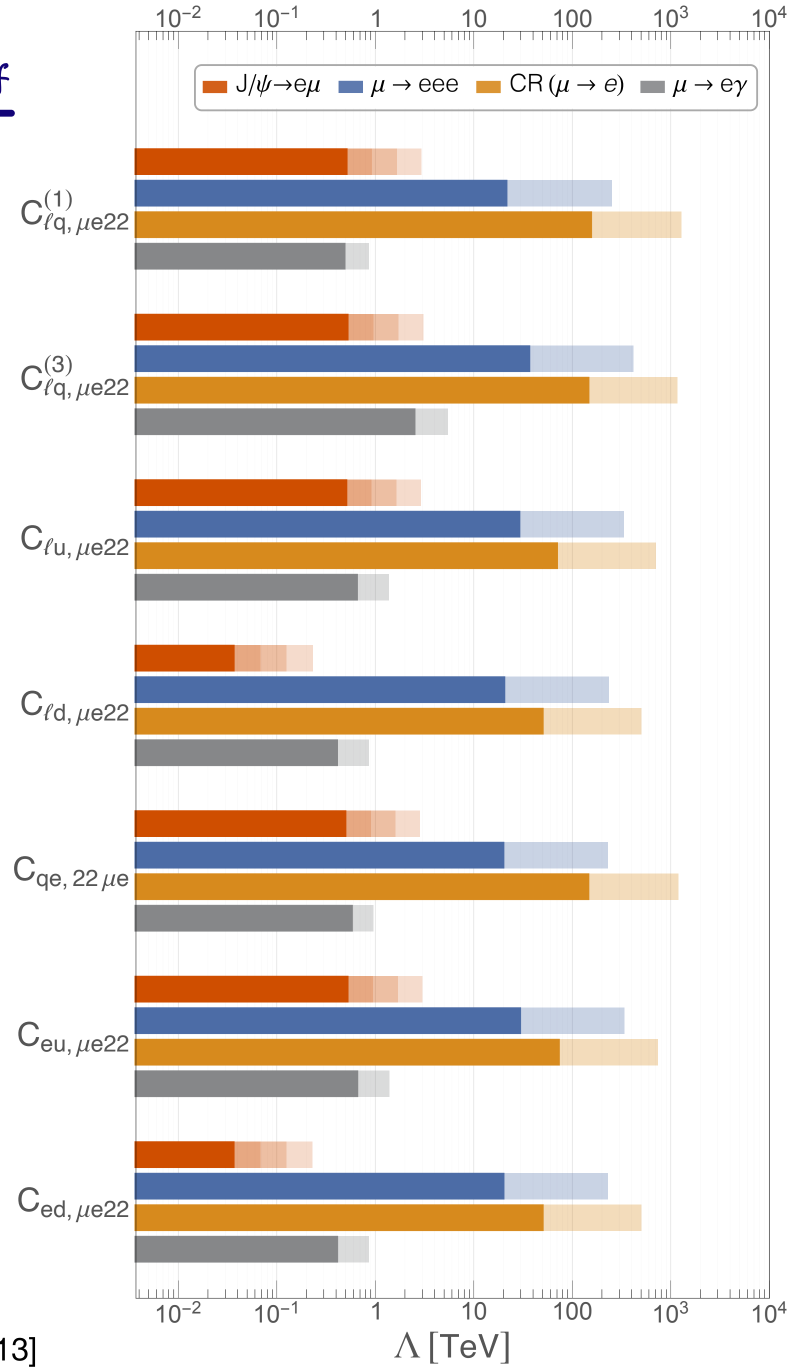




2q2ℓ operators			
$\mathcal{O}^{(1)}_{\ell q,prst}$	$(\bar{L}_p\gamma_\mu L_r)(\bar{Q}_s\gamma^\mu Q_t)$	$\mathcal{O}^{(3)}_{\ell q,prst}$	$(\bar{L}_p\gamma_\mu\tau^I L_r)(\bar{Q}_s\gamma^\mu\tau^I Q_t)$
$\mathcal{O}_{\ell u,prst}$	$(\bar{L}_p\gamma_\mu L_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{\ell d,prst}$	$(\bar{L}_p\gamma_\mu L_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{eu,prst}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{ed,prst}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{qe,prst}$	$(\bar{Q}_p\gamma^\mu Q_r)(\bar{e}_s\gamma_\mu e_t)$	$\mathcal{O}_{\ell edq,prst}$	$(\bar{L}_p e_r)(\bar{d}_s Q_t)$
$\mathcal{O}^{(1)}_{\ell equ,prst}$	$(\bar{L}^a_p e_r)\epsilon_{ab}(\bar{Q}^b_s u_t)$	$\mathcal{O}^{(3)}_{\ell equ,prst}$	$(\bar{L}^a_p\sigma_{\mu\nu}e_r)\epsilon_{ab}(\bar{Q}^b_s\sigma^{\mu\nu}u_t)$
4ℓ operators		Dipole operators	
$\mathcal{O}_{\ell\ell,prst}$	$(\bar{L}_p\gamma_\mu L_r)(\bar{L}_s\gamma^\mu L_t)$	$\mathcal{O}_{eW,pr}$	$(\bar{L}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$
$\mathcal{O}_{ee,prst}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{eB,pr}$	$(\bar{L}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$
$\mathcal{O}_{\ell e,prst}$	$(\bar{L}_p\gamma_\mu L_r)(\bar{e}_s\gamma^\mu e_t)$		
Lepton-Higgs operators			
$\mathcal{O}^{(1)}_{\varphi\ell,pr}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{L}_p\gamma^\mu L_r)$	$\mathcal{O}^{(3)}_{\varphi\ell,pr}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{L}_p\gamma^\mu\tau^I L_r)$
$\mathcal{O}_{\varphi e,pr}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$	$\mathcal{O}_{e\varphi 3,pr}$	$(\bar{L}_p e_r\varphi)(\varphi^\dagger\varphi)$



Comparative study of the **probing power of quarkonium** (charmonium) $\mu - e$ **cLFV decays** for relevant $\mathscr{C}_{\mu e} = 1$

[recent study - Calibbi et al, 2207.10913]

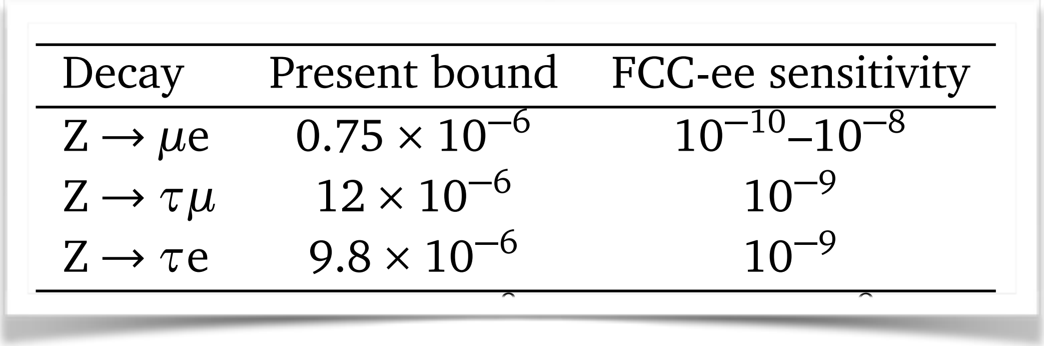
cLFV collider searches: "heavy" decays

- **SM "heavy" states** - **Higgs, Z-bosons, top** quarks, all abundantly **produced at the LHC!**

cLFV top-quark decays @ LHC: BR($t \to u\, e\, \mu$) $\lesssim 10^{-7}$; BR($t \to c\, e\, \mu$) $\lesssim 10^{-6}$

cLFV Z^0 decays ⇒ extensively searched for at **LEP & LHC**

excellent prospects for **FCC-ee in its Z-pole runs!**



Decay	Present bound	FCC-ee sensitivity
$Z \to \mu e$	0.75×10^{-6}	10^{-10}–10^{-8}
$Z \to \tau\mu$	12×10^{-6}	10^{-9}
$Z \to \tau e$	9.8×10^{-6}	10^{-9}

cLFV Higgs decays ⇒ profit from a **Higgs factory (LHC)!**

BR($H \to \tau\mu$) $\lesssim 2.8 \times 10^{-3}$, BR($H \to \tau e$) $\lesssim 4.7 \times 10^{-3}$ (ATLAS)

BR($H \to \tau\mu$) $\lesssim 1.5 \times 10^{-3}$, BR($H \to \tau e$) $\lesssim 2.2 \times 10^{-3}$ (CMS)

In the future, lepton colliders (circular or linear) offer striking advances (even $e\mu$):

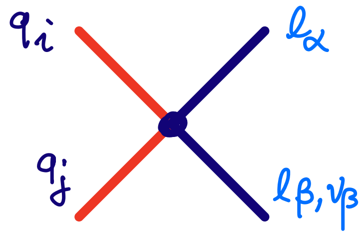
BR($H \to e\mu$) $\lesssim 1.2 \times 10^{-5}$ @ FCC-ee/CEPC, BR($H \to e\mu$) $\lesssim 2.1 \times 10^{-5}$ @ ILC

BR($H \to \tau\ell$) $\lesssim 1.5 \times 10^{-4}$ @ FCC-ee/CEPC, BR($H \to \tau\ell$) $\lesssim 2.4 \times 10^{-4}$ @ ILC

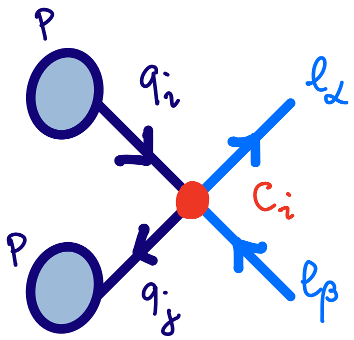
From low to high energies: spinning operators

- Albeit leading to **formally different transitions**, the same leptonic and semi-leptonic operators can be at the origin of flavour violating transitions in very distinct contexts

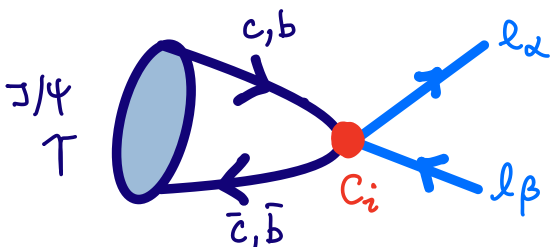
Consider a 4-fermion quark-lepton operator ($q_i q_j \ell_\alpha \ell_\beta$), with $i = j, \alpha \neq \beta$



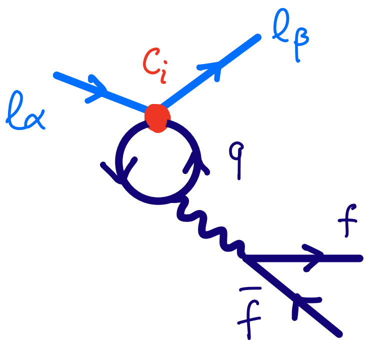
One operator can source rare LHC cLFV decays (rich "flavour" content!), cLFV semileptonic decays, muon-electron conversion, ...



$pp \to \ell_\alpha \ell_\beta$



$J/\Psi \to \ell_\alpha \ell_\beta$



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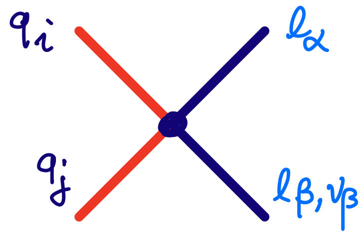

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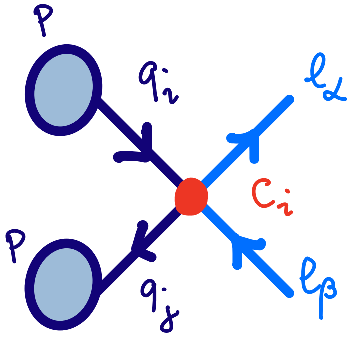

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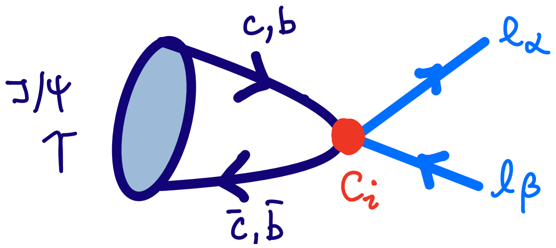

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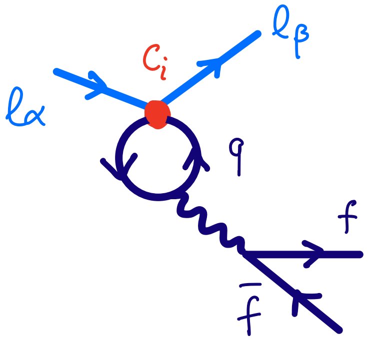

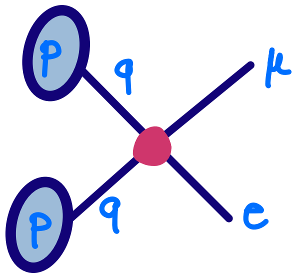

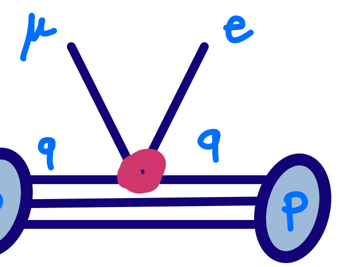

$\mu \to e$, N

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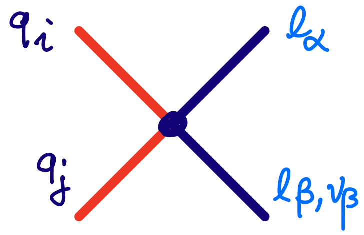
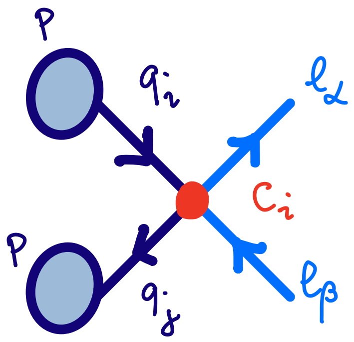
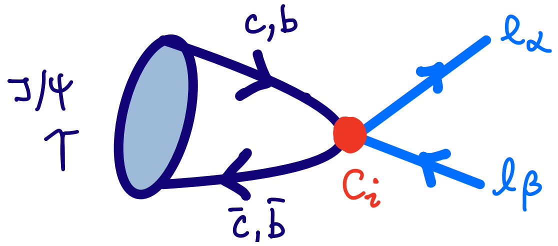




$pp \to \ell_\alpha \ell_\beta$



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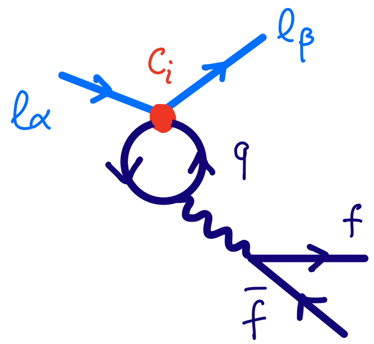
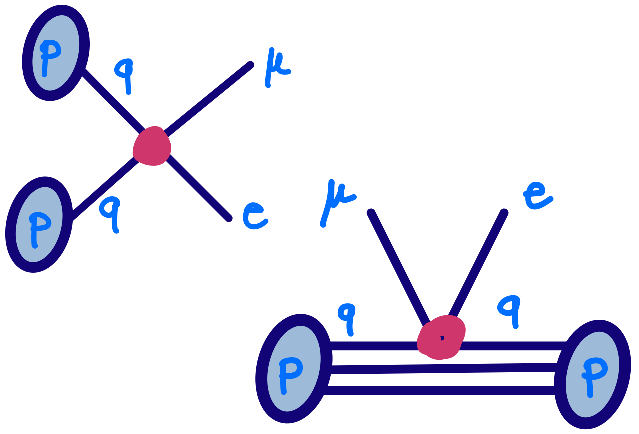
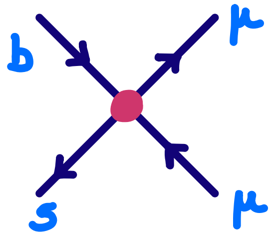
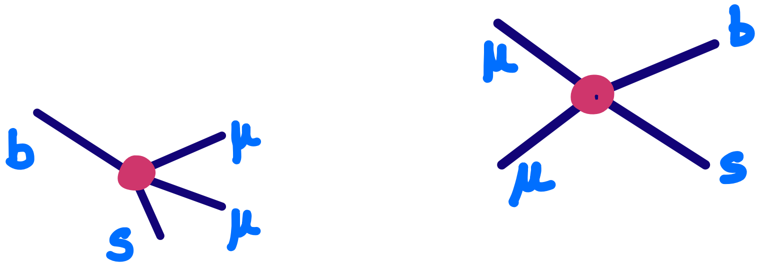




$\mu \to e, \mathrm{N}$



$\Rightarrow b \to s\ell\ell$ at a $\mu^+\mu^-$ collider



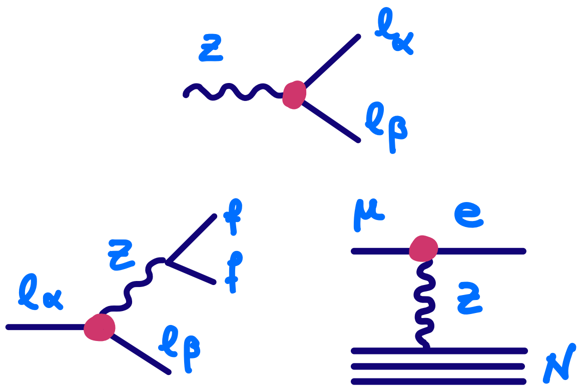
Lepton flavours @ high Tera-Z

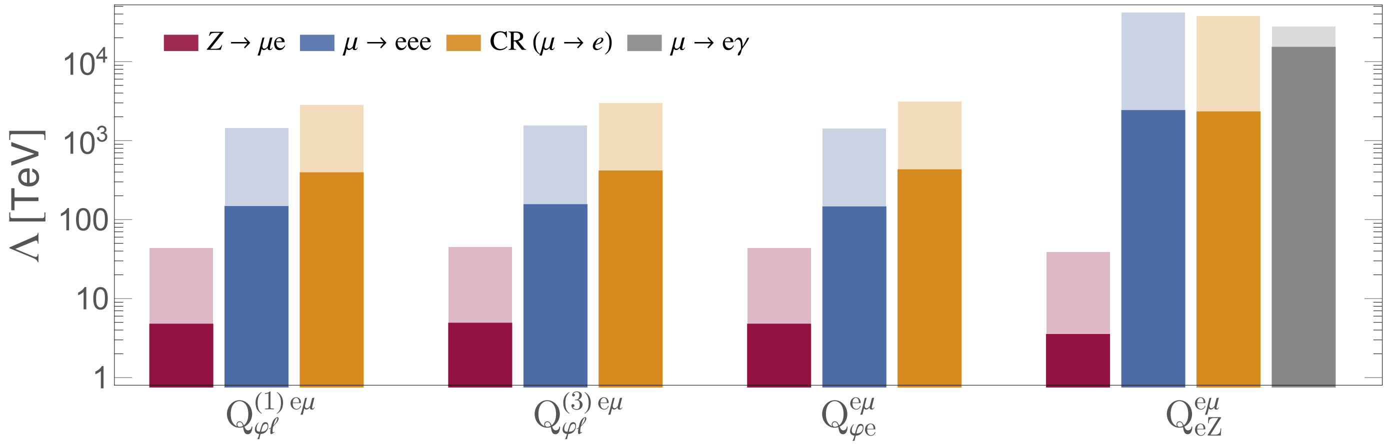
High-energy colliders: also **high-intensity frontier** (amazing luminosities!)

LHC ⇝ abundant sources of flavour in pp collisions

(and also a Higgs-factory...)

TeraZ factory (FCC-ee, CEPC) ⇝ **EW precision & flavour violation**





TeraZ factory ⇝ **cLFV Z decays**

[Calibbi et al, 2107.10273]

For $Z \to \mu e$ better sensitivity of

dedicated (low-energy) cLFV searches

$\mu \to eee$, $\mu - e$ conversion

Lepton flavours @ high Tera-Z

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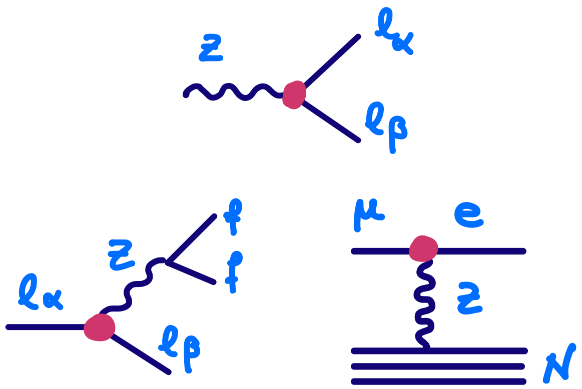

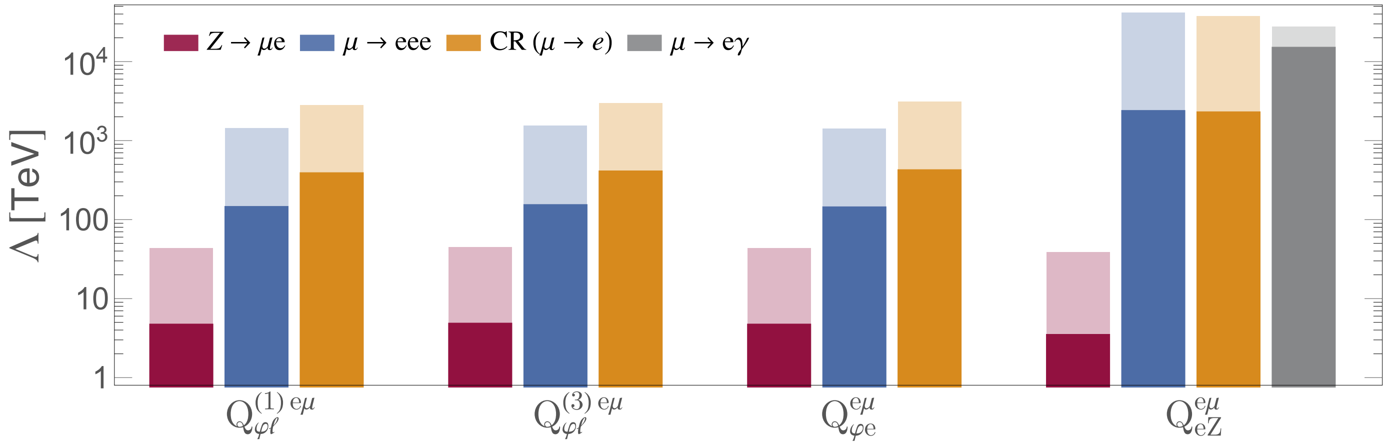

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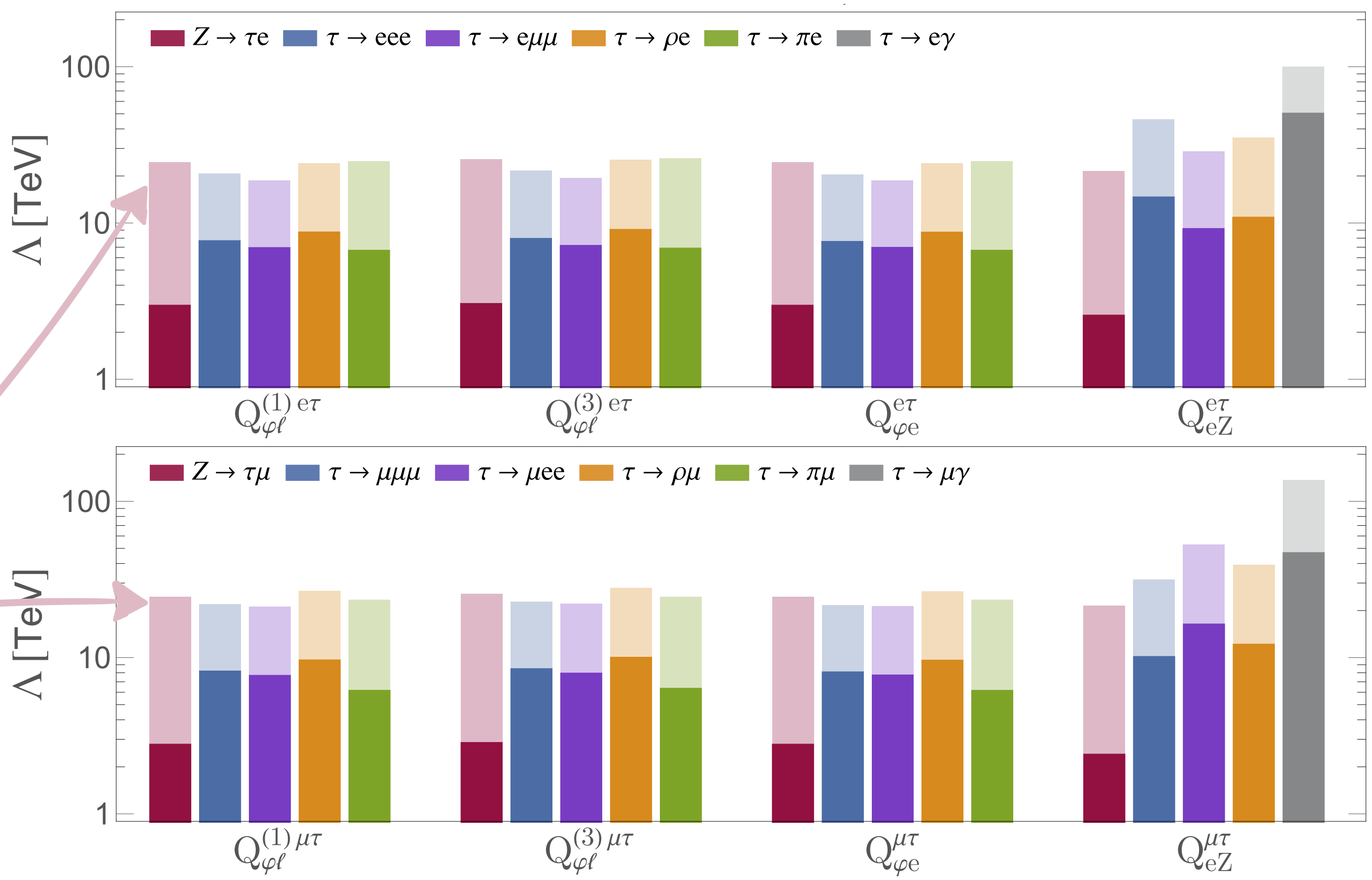

Promising potential of **TeraZ factory** to probe **NP** at the origin of $Z \to \tau\ell$ decays

(competitive with low-energy cLFV)

cLFV collider searches: new resonances

- At high-energy colliders, **on-shell production** of "heavy" New Physics states
  - ⇒ new propagators (scalars, vectors, fermions)
  - ⇒ new interactions (possibly violating lepton flavour!)

Multiplicity, composition, ... properties of the final state(s) are strongly model-dependent

Striking NP signatures (with little SM background), additional contribution to E_{miss}, ...

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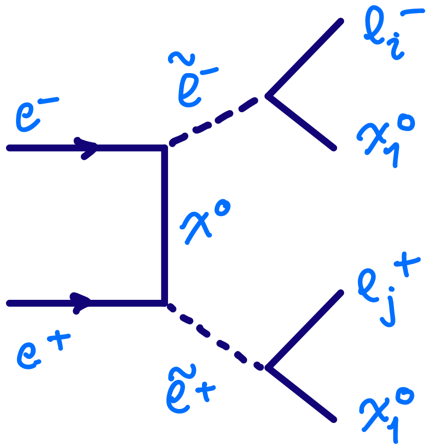
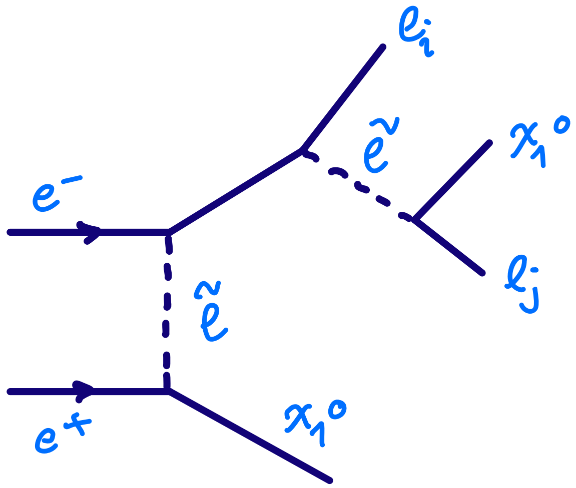
Striking NP signatures (with little SM background), additional contribution to E_miss, ...

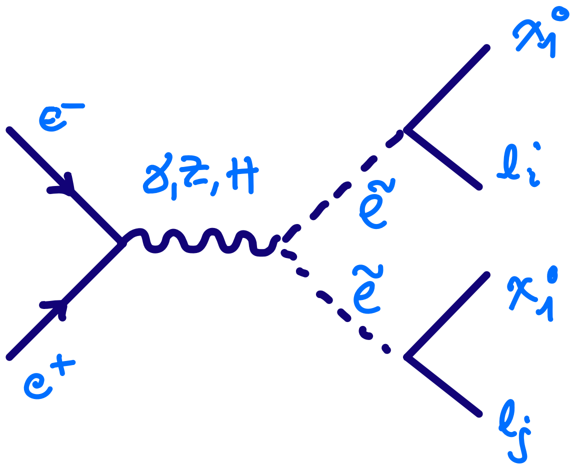
- Example: SUSY cLFV interactions (in neutral current interactions) $\chi^0 \tilde{\ell}_\alpha \ell_\beta$

$$p\,p \to \ldots \to \chi_2^0 \to \ell_\alpha^\pm \ell_\beta^\mp + E^T_\text{miss} \qquad \text{(LHC)}$$

$$e^+ e^- \to \mu^+ e^- + E_\text{miss} \qquad \text{(FCC-ee, LC, ...)}$$







@ FCC-ee: $e^+ e^- \to \ell_i \ell_j + E_\text{miss}$

Toolboxes: neutrinos, lepton flavours and more

And now?

- SM lepton flavours are strictly conserved...
  - ⇒ but neutrino oscillations imply that neutrinos are massive and that there is flavour mixing in the neutral lepton sector
  - ⇒ a **neutrino mass generation mechanism must be included!**

- *If lepton flavours are not conserved, if neutrinos are massive - New Physics is there*

- Many cLFV observables (and not only!) at low- and high-energies, searched for in a **word-wide effort**

- How does NP generate contributions to cLFV? Are all contributions "observable"?

- What can we **expect** from (well-motivated) **models of neutrino mass generation?**

- Can cLFV observables hint at the NP model? Or contribute to falsify it?

- And... How does one compute all this???

Some hints from a young colleague

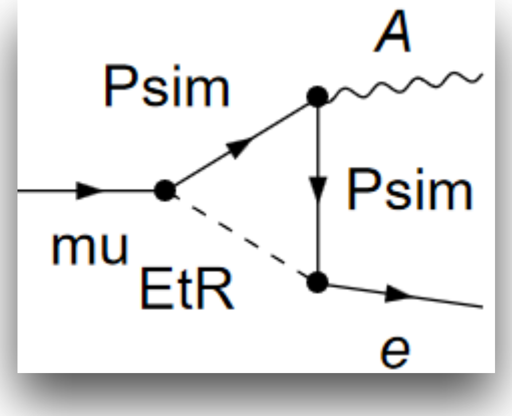
Semi-Automatic Amplitude calculation

Feynrules

.mod and .pars files

- Tool used to implement Lagrangians to be read by a vast landscape of cutting edge analysis tools (Madgraph, Omega, FeynArts…).

FeynCalc + FeynArts and FeynHelpers



- FeynArts to draw topologies and extract the corresponding bare Amplitude.
- FeynCalc to do reduce said Amplitudes (D-dimensionnal Integration, Dirac, Lorentz and Spinor algebra…).
- FeynHelpers (especially the PackageX commands) to analyse Loop functions (UV divergences, PaVe function analytic expressions…)

Thanks to Adrian Darricau (LPC)!

Useful links: FeynCalc: https://feyncalc.github.io/ , FeynArts: https://feynarts.de/ , FeynRules: https://arxiv.org/abs/1310.1921 , FeynHelpers: https://arxiv.org/abs/1611.06793

- Many **dedicated tools:** ***SARAH***, ***SPheno***, ***Flavio***, ...

  Compute observables and confront with available bounds

https://sarah.hepforge.org/
https://spheno.hepforge.org/
https://flav-io.github.io/

- **Develop your own code!** (from experience gathered in analysing a given set of models...)

New Physics paths to cLFV: models of neutrino mass generation

Lepton flavours: from ν oscillations...

SM lepton sector: (strictly) **massless neutrinos**

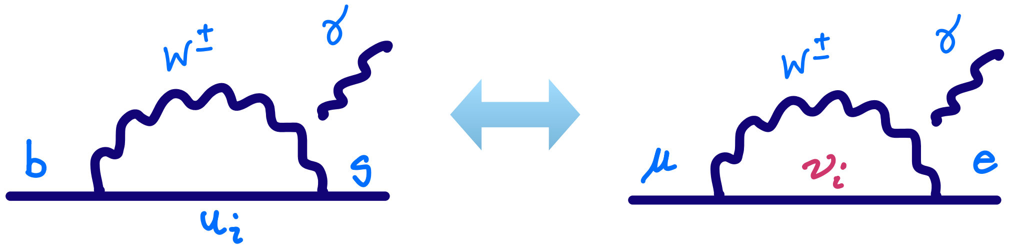
conservation of total lepton number and lepton flavours

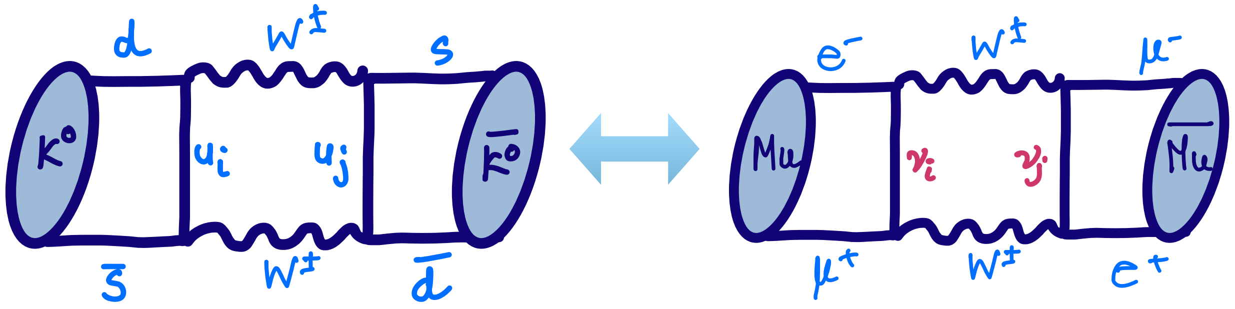
tiny leptonic **EDMs** (4-loop... $d_e^{\text{CKM}} \leq 10^{-38} e$ cm)

Neutrino oscillations: SM description **insufficient!** Added complexity to the flavour problem...

Violation of lepton flavour in neutral lepton sector opens a wide door

to **flavour violation in the charged lepton sector!**





How general is this once we extend the SM to accommodate $\nu_\alpha \leftrightsquigarrow \nu_\beta$ *?*

- Several paths to charged lepton flavour violation (and other New Physics effects):

  For Dirac neutrinos, analogous to rare quark transitions and decays

  ("CKM-sourced" → "PMNS-sourced") $\rightsquigarrow$ **tiny contributions to cLFV** observables...

  More generically, in association **to lepton mixing "PMNS-sourced" and**

  to new heavy mediators present in mechanisms of neutrino mass generation

  Due to the presence of new lepton flavour violating interactions (unrelated to m_ν)

Lepton flavours: from ν oscillations...

SM lepton sector: (strictly) **massless neutrinos**
conservation of total lepton number and lepton flavours
tiny leptonic **EDMs** (4-loop... $d_e^{\text{CKM}} \lesssim 10^{-38} e$ cm)

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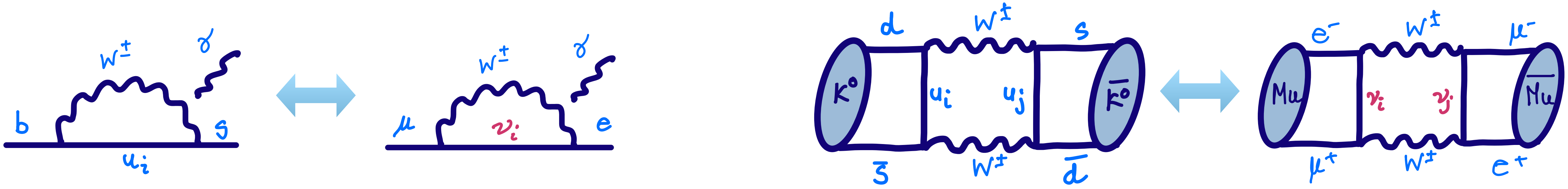

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- Several paths to charged lepton flavour violation (an … s effects):
  For Dirac neutrinos, analogous to rar…
  ("CKM-sourced" … o **cLFV** observables...
  More gen… sourced" and
  … mechanisms of neutrino mass generation
  Due to th… lepton flavour violating interactions (unrelated to m_ν)

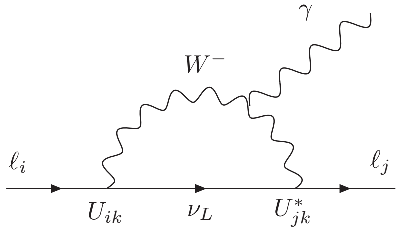
An overview of contributions to cLFV transitions in distinct realisations of the **Seesaw mechanism of neutrino mass generation!**

New Physics and (Majorana) m_ν: cLFV

Neutrino masses (brief "how to"...)

Most minimal possibility: **SM extended** by Dirac RH neutrinos (impose L conservation)

$\Rightarrow \mathscr{L}_{m_\nu} \sim -Y^\nu L \tilde{H} \nu_R$ but tiny Yukawa couplings, $\mathcal{O}(10^{-13})$

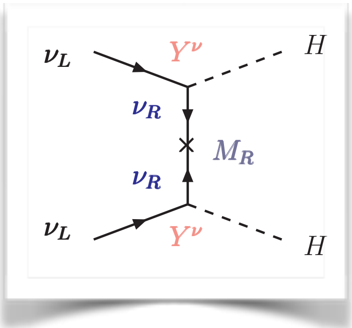


No impact for cLFV; GIM-like suppression due to smallness of m_{ν_i}

BR($\mu \to e\gamma$) $\sim 10^{-54}$ and similarly for **other observables...**

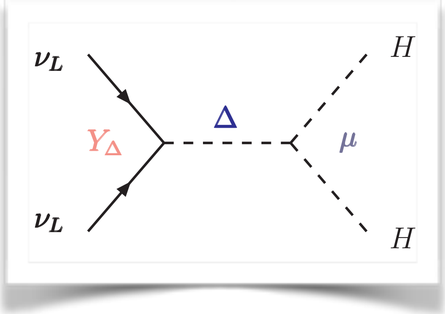
Allow for L violation: realisations of **Weinberg** operator! $\mathscr{L}^5_{m_\nu} \sim \frac{\mathscr{C}^5}{\Lambda_{\text{NP}}} (\bar{L}^c H H L)$

Tree-level seesaw realisations



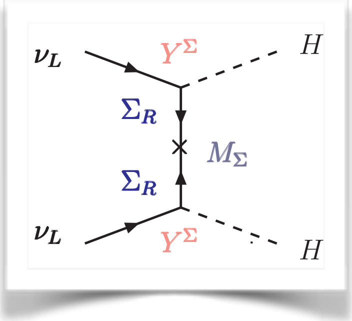
Type I (fermion singlet)

$$\boxed{m_\nu \sim (Y^\nu v)^T \frac{1}{M_R} (Y^\nu v)}$$



Type II (scalar triplet)

$$\boxed{m_\nu \sim \frac{Y_\Delta \mu}{2} \frac{v^2}{M^2_\Delta}}$$



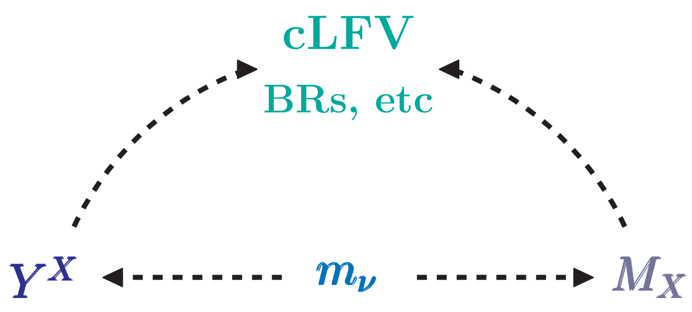
Type III (fermion triplet)

$$\boxed{m_\nu \sim (Y_\Sigma v)^T \frac{1}{M_\Sigma} (Y_\Sigma v)}$$

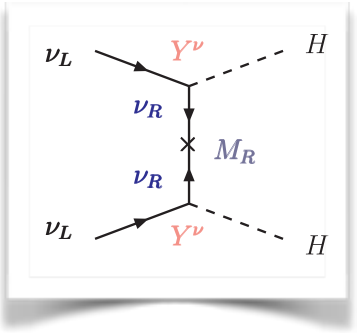
All successfully accounting for oscillation data... so far, *no hint from experimental searches!*

cLFV from "standard" Seesaws

- **Mechanisms for neutrino mass generation:** delicate "balance" between
  sources of flavour violation (new couplings, e.g. Y^ν) and
  masses of new propagators
  $\Rightarrow$ account for oscillation data (**observation!**)


cLFV
BRs, etc
Y^X
m_ν
M_X

- **Type I Seesaw:** extend the SM via (Majorana) right-handed sterile fermions



$\rightsquigarrow$ an enlarged spectrum

$\rightsquigarrow$ extended mixings

$$\boxed{m_\nu \sim (Y^\nu v)^T \frac{1}{M_R} (Y^\nu v)}$$

If **light neutrino masses** generated by
"natural" new physics $\Rightarrow$ very high energy NP scale

$Y^\nu \sim \mathcal{O}(1)$ $\quad$ $M_R \sim 10^{14-16}$ GeV

$U^T \mathscr{M}_\nu^{6\times 6} U$ = diag(m_{ν_i}) $\quad$ $U = \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix}$

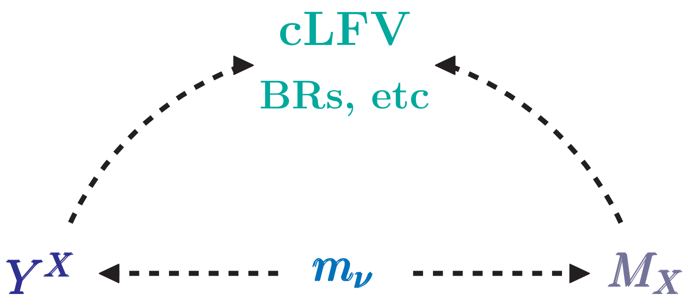
leptonic mixing $\approx U_{\text{PMNS}}$
(unitary to very good approximation)

negligible active-sterile mixings ($\theta \propto m_D^\dagger M_R^{-1}$)

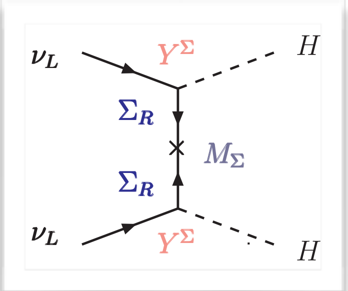
$\Rightarrow$ Decoupled new physics! No contributions for cLFV observables,
no resonance within collider reach...

cLFV from "standard" Seesaws

- **Mechanisms for neutrino mass generation:** delicate "balance" between
  sources of flavour violation (new couplings, e.g. Y^ν) and
  masses of new propagators
  ⇒ account for oscillation data (**observation!**)



- **Type III Seesaw:** extend the SM via SU(2) **triplet fermions**



$$\boxed{m_\nu \sim (Y_\Sigma v)^T \frac{1}{M_\Sigma} (Y_\Sigma v)}$$

⤳ an enlarged spectrum

⤳ extended mixings

If **light neutrino masses** generated by

"natural" new physics ⇒ very high energy NP scale

$Y_\Sigma \sim \mathcal{O}(1)$ $M_\Sigma \sim 10^{14-16}$ GeV

negligible mixings between active neutrinos and NP states ($\theta \propto m_\Sigma^\dagger M_\Sigma^{-1}$)

⇒ Decoupled new physics! No contributions for cLFV observables,
no resonance within collider reach...

cLFV from "standard" Seesaws

- **Mechanisms for neutrino mass generation:** delicate "balance" between
  sources of flavour violation (new couplings, e.g. Y^ν) and
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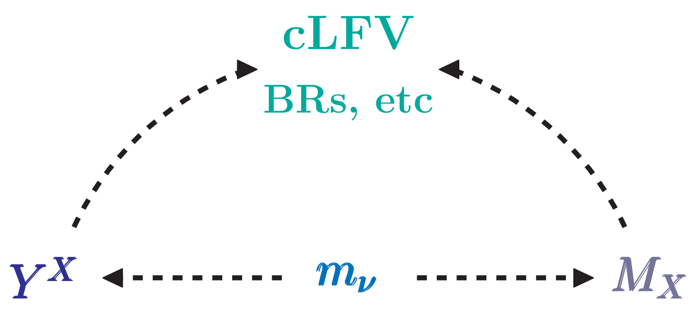
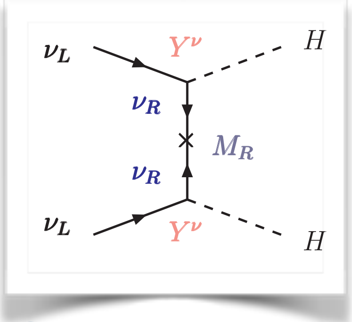
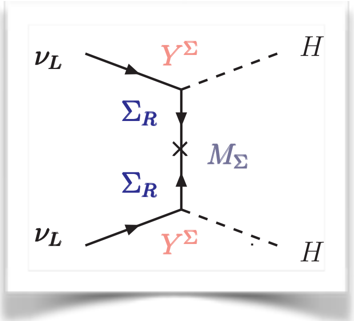
  $\Rightarrow$ account for oscillation data (**observation!**)



- **Type I & III Seesaw:** a quick **EFT detour** - *integrate out the heavy mediators* (N_R, Σ)





Dimension 5
(Weinberg operator)

$$Y_\nu^T M_R^{-1} Y_\nu (\overline{L^c_L}\tilde{\phi}^*)\,(\tilde{\phi}^\dagger L_L)$$

$$\boxed{m_\nu \sim (Y^\nu v)^T \frac{1}{M_R}(Y^\nu v)}$$

[see Broncano et al, 0210271]

$$Y_\Sigma^T M_\Sigma^{-1} Y_\Sigma (\overline{L^c_L}\tilde{\phi}^*)\,(\tilde{\phi}^\dagger L_L)$$

$$\boxed{m_\nu \sim (Y_\Sigma v)^T \frac{1}{M_\Sigma}(Y_\Sigma v)}$$

[see Abada et al, 0707.4058]

Dimension 6

$$Y_\nu^\dagger M_R^{-2} Y_\nu (\overline{L_L}\tilde{\phi}^*)\, i\not{\partial}(\tilde{\phi}^\dagger L_L)$$

$$Y_\Sigma^\dagger M_\Sigma^{-2} Y_\Sigma (\overline{L_L}\vec{\tau}\tilde{\phi})\, \not{D}(\tilde{\phi}^\dagger \vec{\tau} L_L)$$

$\Rightarrow$ **suppression** of "light neutrino masses" entails strong **suppression** of NP effects!

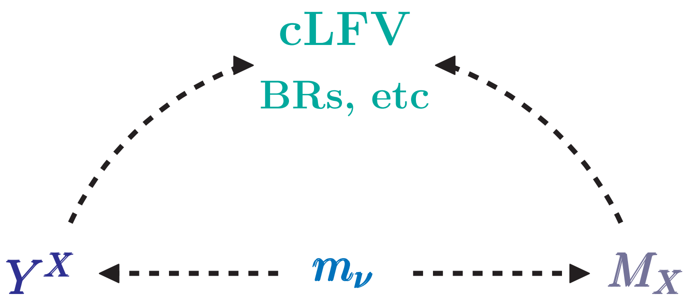
cLFV from "standard" Seesaws

- **Mechanisms for neutrino mass generation:** delicate **"balance"** between

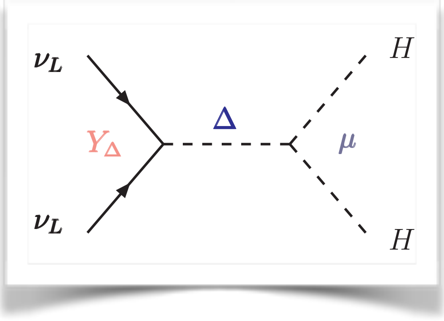
  sources of flavour violation (new couplings, e.g. Y^ν) and

  masses of new propagators

  $\Rightarrow$ account for **oscillation data** (**observation!**)



- **Type II Seesaw:** extend the SM via SU(2) **triplet scalars**



⇝ an enlarged spectrum

⇝ extended mixings

$$\boxed{m_\nu \sim \frac{Y_\Delta\, \mu}{2}\,\frac{v^2}{M_\Delta^2}}$$

A different scenario: additional ingredient!

"natural" new physics $\not\Rightarrow$ very **high energy NP scale**

Smallness of m_ν **also** from (tiny) μ coupling

for "natural" Y_Δ and not "too heavy" M_Δ

[see Abada et al, 0707.4058]

Dimension 5 $\quad 4\, Y_\Delta\, \mu\, M_\Delta^{-2}\, (\overline{L^c_L}\tilde{\phi}^*)\, (\tilde{\phi}^\dagger L_L)$

Dimension 6 $\quad Y_\Delta\, Y_\Delta^\dagger\, M_\Delta^{-2}\, (\overline{L_L}\gamma_\mu L_L)\, (\overline{L_L}\gamma^\mu L_L)$

$\Rightarrow$ **suppression** of **"light neutrino masses"** decorrelated from **contribution to NP effects!**

Seesaw scales and phenomenology

- **Light neutrino masses** generated by **"natural"** new physics at a very **high energy scale**
  (*"vanilla"* or standard high-scale seesaw)

  If $Y \sim \mathcal{O}(1) \Rightarrow M_{\text{NP}} \sim 10^{14-16}$ GeV $\rightsquigarrow$ hierarchy problem (if $M_R \geq 10^7$ GeV)

  $\Rightarrow$ Decoupled new physics! No contributions at high energy, or high intensity...

  $\Rightarrow$ Only hypothesise a viable BAU from "vanilla" thermal leptogenesis $(M_R \geq 10^9 \ldots)$

- **Light neutrino masses** generated via **tiny couplings** of (very) **light sector**

  M_R even below the MeV! Also "natural" since $M_R \to 0$ restores L conservation

  $\Rightarrow$ Contributions to phenomena suppressed by very light fermion masses

  $\Rightarrow$ Potential conflict with active neutrino data (extremely large mixings θ_{as})

- Accept a certain **tuning** between **couplings** and **scale,** sacrificing "naturality" of couplings

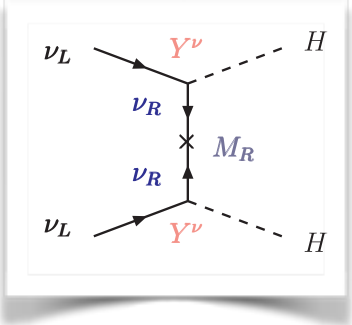
  $\Rightarrow$ Explore regimes of comparatively lower M_{NP}

- **Rely on symmetry-protection!** Smallness of neutrino masses from
  approximate symmetry conservation (e.g. lepton number conservation)

"Low-scale seesaw realisations"

Low-scale models of m_ν generation: type I seesaw

- Addition of **3 "heavy" Majorana** right-handed neutrinos ν_R to the SM
  but explore considerably lighter range for M_R **MeV** $\leq M_R \leq 10^{\text{few}}$ **TeV**



Type I (fermion singlet)

$$\boxed{m_\nu \sim (Y^\nu v)^T \frac{1}{M_R} (Y^\nu v)}$$

After EW symmetry breaking, **6 states** in the neutral lepton spectrum

$$\mathcal{M}_\nu^{6\times6} = \begin{pmatrix} 0 & Y^\nu\, v \\ (Y^\nu)^T\, v & M_R \end{pmatrix}$$

3 light neutrinos $m_\nu \approx -\,v^2 Y_\nu^T M_R^{-1} Y_\nu$

3 heavy states $m_N \approx M_R$

Enlarged 6×6 **mixing matrix** $U^T \mathcal{M}_\nu^{6\times6} U$ = diag(m_{ν_i})

$$U = \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix}$$

Non-negligible active-sterile mixings! ($\theta \propto m_D^\dagger M_R^{-1}$)

Non-unitary leptonic mixing $\tilde{U}_{\text{PMNS}}$

Low-scale realisations of the Type I seesaw open door to a very rich phenomenology
from cLFV signals, to collider searches

Similar implications for **low-scale Type III**
(but important direct/indirect constraints due to the *non-singlet nature of new states...*)

Low-scale models for m_ν: Inverse Seesaw

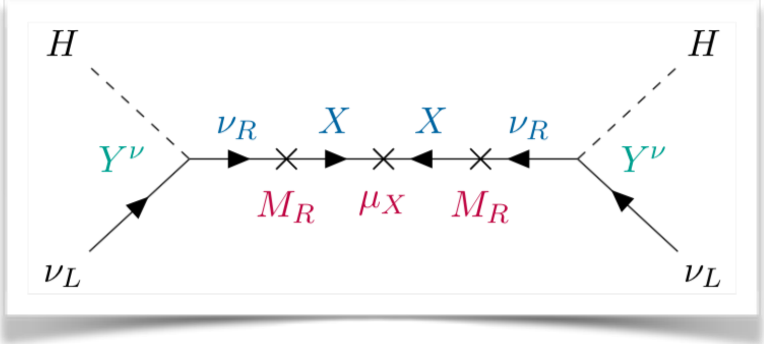
- **Variants** of type I seesaw aiming at a natural realisation of a low-scale m_ν mechanism
- Addition of **two new species** of **fermionic gauge singlets**

n_R right-handed neutrinos ν_R ($L_{\nu_R} = 1$) and n_X extra sterile states X ($L_X = -1$)

$$\mathscr{L}^{(3,3)}_{\text{ISS}} = -\, Y^\nu \bar{L}\, \tilde{H}\, \nu_R - M_R\, \bar{\nu}^c_R\, X - \frac{1}{2}\mu_X \bar{X}^c X$$

lepton number violating!

[Mohapatra and Valle, '86]



ISS(3,3)

$$\mathcal{M}^{9\times 9}_{\text{ISS}} = \begin{pmatrix} 0 & Y_\nu v & 0 \\ Y_\nu^T v & 0 & M_R \\ 0 & M_R & \mu_X \end{pmatrix} \Rightarrow \begin{cases} \text{3 light } \nu : m_\nu \approx \frac{(Y_\nu v)^2}{(Y_\nu v)^2 + M_R^2}\mu_X \\ \text{3 pseudo-Dirac pairs} : m_{N^\pm} \approx M_R \pm \mu_X \end{cases}$$

$$\boxed{m_\nu \sim (Y^\nu v)^T \frac{\mu_X}{M_R^2} (Y^\nu v)}$$

For **natural values** of $Y^\nu \sim \mathcal{O}(1)$

interplay of **two scales** driving smallness of m_ν: M_R and μ_X

Comparatively "light" heavy spectrum ($\Lambda_{\text{EW}} \leftrightarrow$ TeV) for

small values of μ_X (around eV - keV)

Natural ('t Hooft criterium) since B-L conservation restored when $\mu_X \to 0$!

Symmetry protected "smallness" of m_ν - approximate **LNC**

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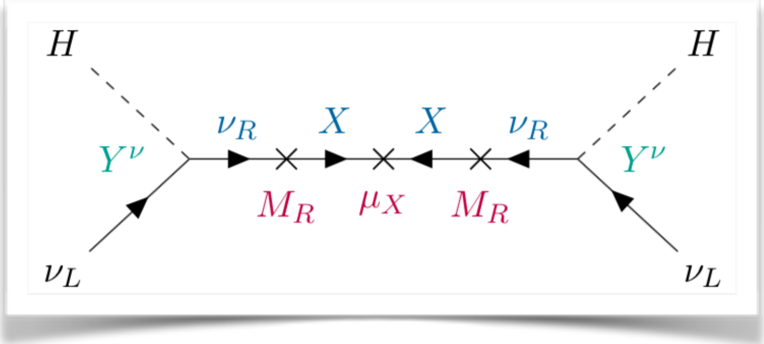
[Mohapatra and Valle, '86]

lepton number violating!



ISS(3,3)

$$\mathcal{M}^{9\times9}_{\text{ISS}} = \begin{pmatrix} 0 & Y_\nu v & 0 \\ Y_\nu^T v & 0 & M_R \\ 0 & M_R & \mu_X \end{pmatrix} \Rightarrow \begin{cases} \textbf{3 light } \boldsymbol{\nu} : m_\nu \approx \frac{(Y_\nu v)^2}{(Y_\nu v)^2 + M_R^2}\, \mu_X \\ \textbf{3 pseudo-Dirac pairs} : m_{N^\pm} \approx M_R \pm \mu_X \end{cases}$$

$$\boxed{m_\nu \sim (Y^\nu v)^T \frac{\mu_X}{M_R^2} (Y^\nu v)}$$

For **natural values** of $Y^\nu \sim \mathcal{O}(1)$

interplay of **two scales** driving smallness of m_ν: M_R and μ_X

Comparatively "light" heavy spectrum ($\Lambda_{\text{EW}} \leftrightarrow$ TeV) for

small values of μ_X (around eV - keV)

⇒ Despite **small** $m_\nu \sim \mu_X \frac{m_D^2}{M_R^2}$, a **"low" NP** scale $\sim M_R$, and **sizeable mixings** ($\theta \propto m_D^\dagger M_R^{-1}$) !

Low-scale models for m_ν: Inverse Seesaw

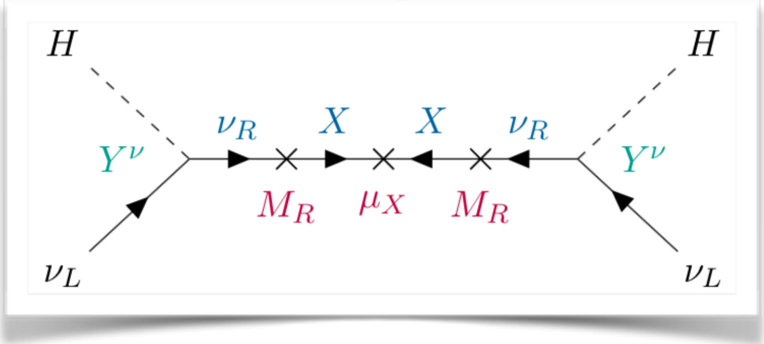
- **Variants** of type I seesaw aiming at a natural realisation of a low-scale m_ν mechanism
- Addition of **two new species** of **fermionic gauge singlets**

n_R right-handed neutrinos ν_R ($L_{\nu_R} = 1$) and n_X extra sterile states X ($L_X = -1$)

$$\mathscr{L}^{(3,3)}_{\text{ISS}} = -\, Y^\nu \bar{L}\, \tilde{H}\, \nu_R - M_R\, \bar{\nu}^c_R\, X - \frac{1}{2}\mu_X \bar{X}^c X$$

[Mohapatra and Valle, '86]

lepton number violating!



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Similar frameworks: $\mathscr{L}_{\text{SM}} + m_D\, \bar{N}_R\, \nu_L + M_N\, \bar{N}_R\, N_L$ (addition of new species, $N_{L,R}$)

sizeable mixings ($\theta \propto m_D^\dagger M_N^{-1}$) even for vanishing m_ν

[Branco et al, '88; Kersten and Smirnov, 0705.3221]

Low-scale seesaws: phenomenological impact

- **New Physics states** at a comparatively **low scale;**
  non-negligible mixings between new states and SM leptons...
  How does this **actually lead** to abundant **phenomenological implications?**

- Presence of **non-negligible mixings** between **active neutrinos and NP states** has **non-negligible consequences** for the **leptonic mixing matrix**, i.e. U_{PMNS}

  Recall (for Type I and variants): $U^T \mathscr{M}_\nu^{(3+n)\times(3+n)} U = \text{diag}(m_{\nu_i})$

$U(3+n,3+n)$: unitary matrix

$$\boldsymbol{U} = \begin{pmatrix} \boldsymbol{U_{\nu\nu}} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix}$$

leptonic mixing $\approx U_{\text{PMNS}}$ (3×3)

active-sterile mixings: $\boldsymbol{U_{\nu N} = U_{\nu N}(\theta)}$

$(\theta \propto m_D^\dagger M_R^{-1})$

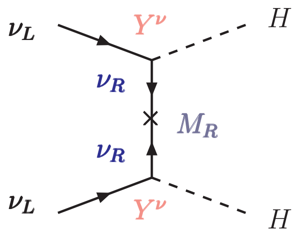
For **non-negligible** active-sterile mixings, $U_{\nu N} \neq 0 \quad \Rightarrow$ **non-unitary** $\tilde{U}_{\text{PMNS}}$

Modified leptonic currents!

Extensive implications for EW precision observables, flavour conservating transitions, and **cLFV!**

cLFV and the seesaw: distinctive features

- **Seesaw realisations:** distinctive expectations for numerous cLFV observables
  topology strongly depends on nature of new mediators



Type I (fermion singlet)

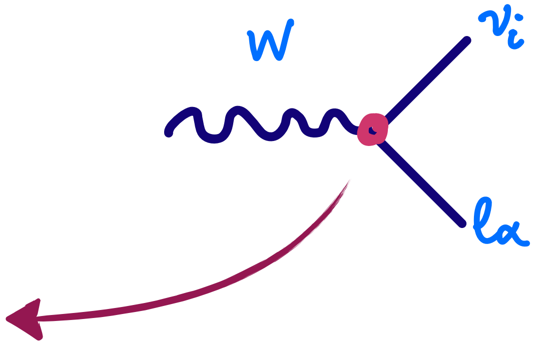
SM + (sterile) ν_R

$$\mathscr{L}_{\text{Type I}} \supset v\, Y_\nu \bar{\nu}_L \nu_R + \frac{1}{2} M_R \bar{\nu}^c_R \nu_R$$

active-sterile mixings: $\nu_L - \nu_R$

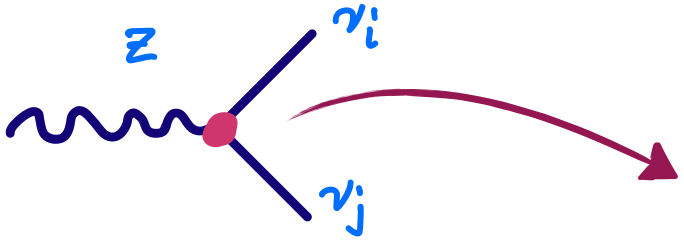
$$\Rightarrow \theta \approx \mathcal{O}(m_D^\dagger M_R^{-1})$$

- **Type I seesaw - sterile fermions:**
  No interactions with gauge bosons; only neutral leptons and Higgs, $\boldsymbol{H^0}$
  Possible searches at colliders (displaced vertices, LNV); constraints from EWPO, cLFV, ...



$$\propto \sum_{j=1}^{3+n_s} U_{\alpha j}$$

$$\propto (1-\eta) U_{\text{PMNS}}$$



$$\propto \sum_{i,j=1}^{3+n_s} \left(\sum_{\rho} U^\dagger_{i\rho} U_{\rho j} \right)$$

$$\propto (1-2\eta) U_{\text{PMNS}}$$

Deviations from unitarity: $\tilde{U}_{\text{PMNS}} = (1-\eta) U_{\text{PMNS}}$; $\eta = \frac{1}{2}\theta\theta^\dagger$

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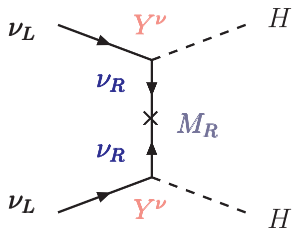

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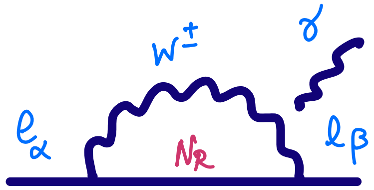
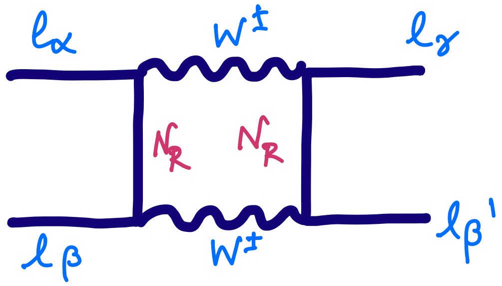
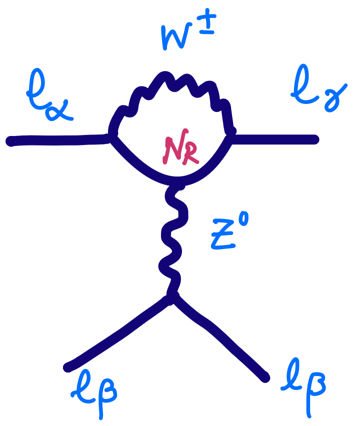
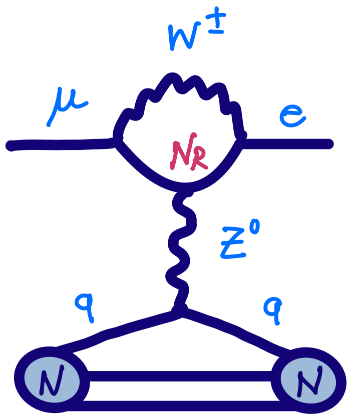








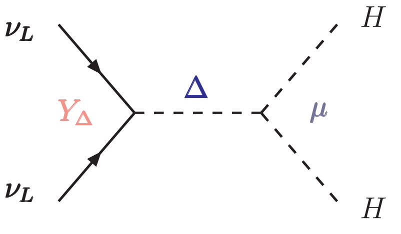
All @ loop-level...

cLFV and the seesaw: distinctive features

- **Seesaw realisations:** distinctive expectations for numerous cLFV observables
  topology strongly depends on nature of new mediators

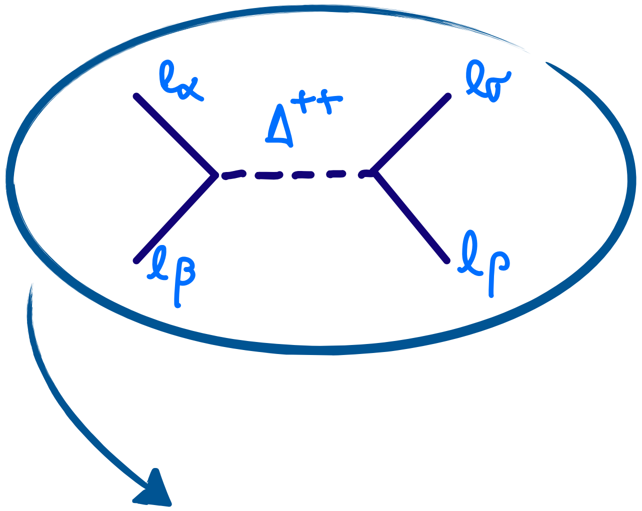
SM + triplet Δ $\qquad \Delta = \Delta^0, \Delta^+, \Delta^{++}$

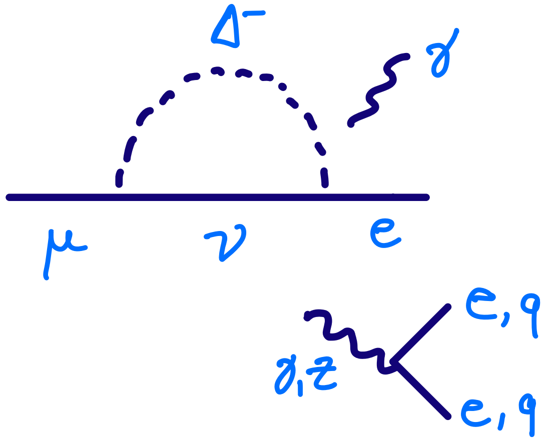
$$\mathscr{L}_{\text{Type II}} \supset Y_\Delta \bar{L}_L \vec{\tau} L_L^c \vec{\Delta} + \mu_\Delta \phi^\dagger \vec{\tau} \tilde{\phi} \vec{\Delta}$$



Type II (scalar triplet)

- **Type II seesaw - triplet scalars:**
  Interactions with gauge bosons; direct couplings with matter
  Important bounds from direct collider searches and precision physics (and cLFV!)





Tree-body decays @ tree-level...

cLFV and the seesaw: distinctive features

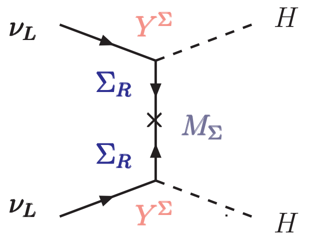
- **Seesaw realisations:** distinctive expectations for numerous **cLFV observables**
  topology strongly depends on nature of new mediators

SM + (hypercharge-less) Σ $\qquad \Sigma = \Sigma^+, \Sigma^0, \Sigma^-$

$$\mathscr{L}_{\text{Type II}} \supset v\, Y_\Sigma\, \Sigma^+ \ell^- + v\, Y_\Sigma\, \Sigma^0 \nu + M_\Sigma \bar{\Sigma}\, \Sigma$$

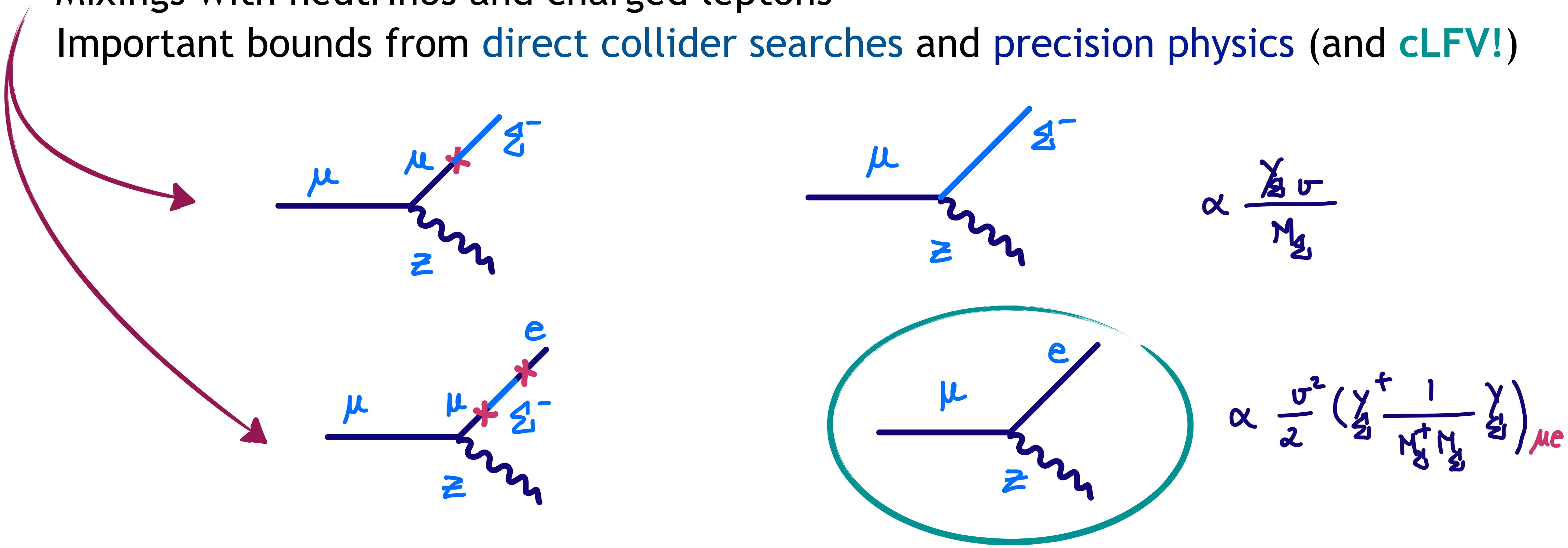
Fermion-triplet mixings: $\Sigma^0 - \nu$ and $\Sigma^{+c} - \ell^-$

$$\Rightarrow \theta \approx \mathcal{O}(v Y_\Sigma M_\Sigma^{-1})$$



Type III (fermion triplet)

- **Type III seesaw - triplet fermions:**
  Interactions with gauge bosons $(\bar{\Sigma}^- \Sigma^- Z, \bar{\Sigma}^+ \Sigma^+ Z, \bar{\Sigma}^0 \Sigma^+ W^-, \bar{\Sigma}^0 \Sigma^- W^+)$
  Mixings with neutrinos and charged leptons
  Important bounds from direct collider searches and precision physics (and **cLFV!**)



$\propto \frac{Y_\Sigma v}{M_\Sigma}$

$\propto \frac{v^2}{2} \left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger M_\Sigma} Y_\Sigma \right)_{\mu e}$

cLFV and the seesaw: distinctive features

- **Seesaw realisations:** distinctive expectations for numerous **cLFV observables**
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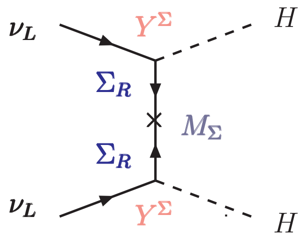

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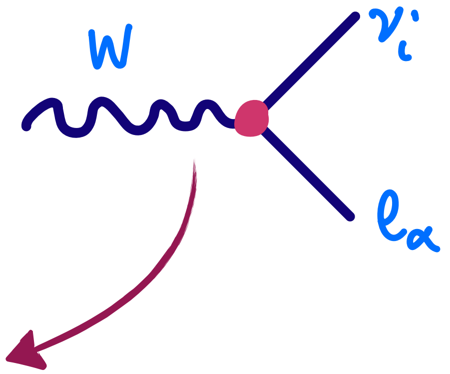
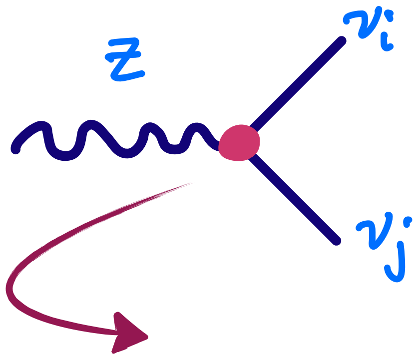
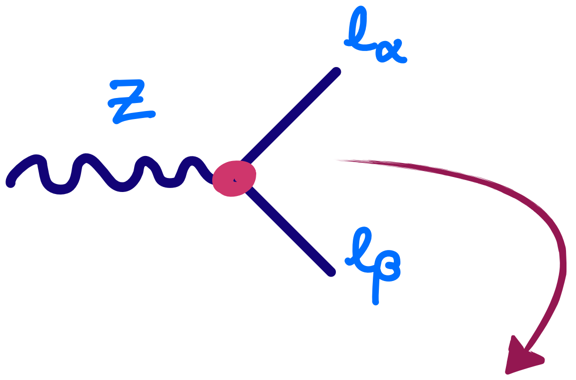


$\propto (1+\varepsilon) U_{\text{PMNS}}$



$\propto (1-2\varepsilon) U_{\text{PMNS}}$



$\propto (1+4\varepsilon) U_{\text{PMNS}}$

Deviations from unitarity: $\varepsilon = \frac{1}{2} m_\Sigma^\dagger M_\Sigma^{-2} m_\Sigma$

cLFV and the seesaw: distinctive features

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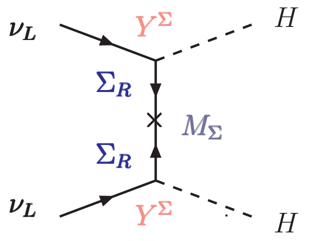

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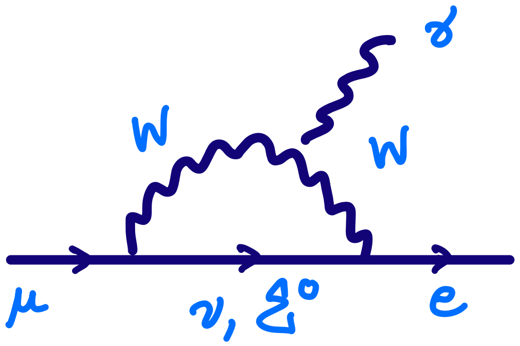
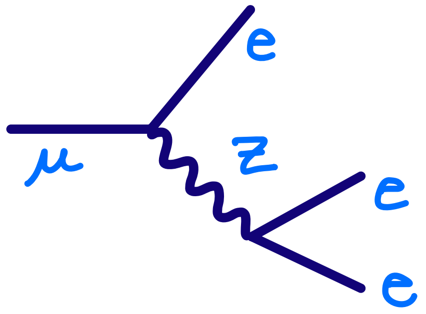
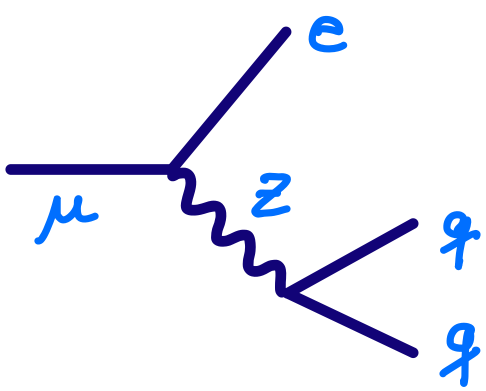






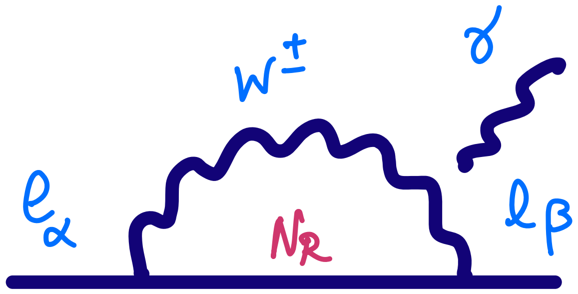
Tree-body decays and conversion in nuclei @ tree-level...
due to the modified $Z\ell_\alpha\ell_\beta$ vertex!

cLFV - the singlet seesaw case

cLFV in low-scale seesaw (type I and variants)

- Consider a low-scale realisation of a type I seesaw and variants: SM $+\nu_R + \ldots$
  leading to **non-negligible mixing** between active (3) and sterile states (n_S)

- **Contributions to cLFV transitions:** simple examples 😀
  (all transitions at loop-level!)



$$\mathrm{BR}(\ell_\beta \to \ell_\alpha \gamma) = \frac{\alpha_w^3\, s_w^2}{256\,\pi^2}\,\frac{m_\beta^4}{M_W^4}\,\frac{m_\beta}{\Gamma_\beta}\left|G_\gamma^{\beta\alpha}\right|^2$$

$$\boxed{G_\gamma^{\beta\alpha} = \sum_{i=1}^{3+n_s} \mathcal{U}_{\alpha i}\,\mathcal{U}^*_{\beta i}\, G_\gamma(x_i)}$$

Form factor

$$\begin{aligned} G_\gamma(x) &= -\frac{x(2x^2+5x-1)}{4(1-x)^3} - \frac{3x^3}{2(1-x)^4}\log x\,, \\ G_\gamma(x) &\xrightarrow[x\gg 1]{} \frac{1}{2}\,, \\ G_\gamma(0) &= 0\,. \end{aligned}$$

Loop function
(and relevant $x_i = m_i^2/M_W^2$ limits)

cLFV in low-scale seesaw (type I and variants)

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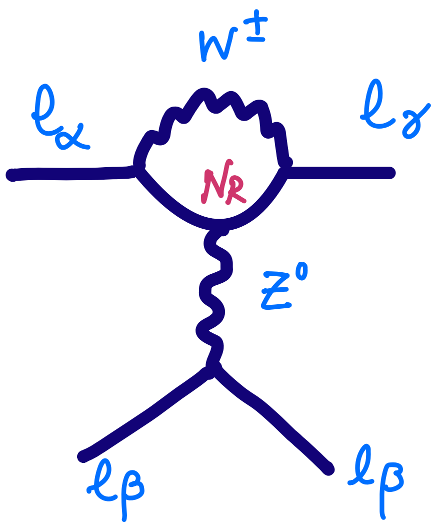

$$\begin{aligned}\mathrm{BR}(\ell_\beta \to 3\ell_\alpha) \;=\; & \frac{\alpha_w^4}{24576\,\pi^3}\,\frac{m_\beta^4}{M_W^4}\,\frac{m_\beta}{\Gamma_\beta}\times\Bigg\{2\left|\frac{1}{2}F_{\text{box}}^{\beta 3\alpha}+F_Z^{\beta\alpha}-2s_w^2\,(F_Z^{\beta\alpha}-F_\gamma^{\beta\alpha})\right|^2 \\ & + 4s_w^4\,|F_Z^{\beta\alpha}-F_\gamma^{\beta\alpha}|^2 + 16 s_w^2\,\mathrm{Re}\left[(F_Z^{\beta\alpha}-\frac{1}{2}F_{\text{box}}^{\beta 3\alpha})\,G_\gamma^{\beta\alpha *}\right] \\ & - 48 s_w^4\,\mathrm{Re}\left[(F_Z^{\beta\alpha}-F_\gamma^{\beta\alpha})\,G_\gamma^{\beta\alpha *}\right] + 32 s_w^4\,|G_\gamma^{\beta\alpha}|^2\left[\log\frac{m_\beta^2}{m_\alpha^2}-\frac{11}{4}\right]\Bigg\}\end{aligned}$$

Form factors

$$\begin{aligned}F_\gamma^{\beta\alpha} &= \sum_{i=1}^{3+n_s}\mathcal{U}_{\alpha i}\mathcal{U}_{\beta i}^*\,F_\gamma(x_i)\,,\\ F_Z^{\beta\alpha} &= \sum_{i,j=1}^{3+n_s}\mathcal{U}_{\alpha i}\mathcal{U}_{\beta j}^*\left[\delta_{ij}\,F_Z(x_j)+C_{ij}\,G_Z(x_i,x_j)+C_{ij}^*\,H_Z(x_i,x_j)\right]\,,\\ F_{\text{box}}^{\beta 3\alpha} &= \sum_{i,j=1}^{3+n_s}\mathcal{U}_{\alpha i}\mathcal{U}_{\beta j}^*\left[\mathcal{U}_{\alpha i}\mathcal{U}_{\alpha j}^*\,G_{\text{box}}(x_i,x_j)-2\mathcal{U}_{\alpha i}^*\mathcal{U}_{\alpha j}\,F_{\text{Xbox}}(x_i,x_j)\right]\,,\end{aligned}$$

$$C_{ij} = \sum_{\rho=1}^{3}\mathcal{U}_{i\rho}^\dagger\,\mathcal{U}_{\rho j}$$

Loop functions:
$F_\gamma, F_Z, G_Z, H_Z, G_{box}, F_{Xbox}$

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- Consider a **low-scale realisation** of a **type I seesaw** and variants: SM $+\nu_R + \ldots$
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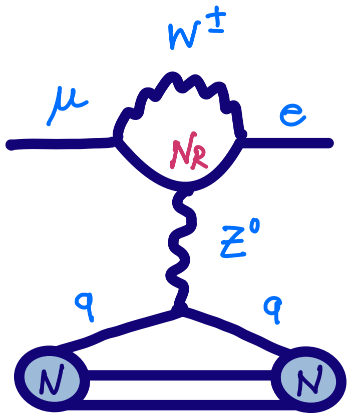


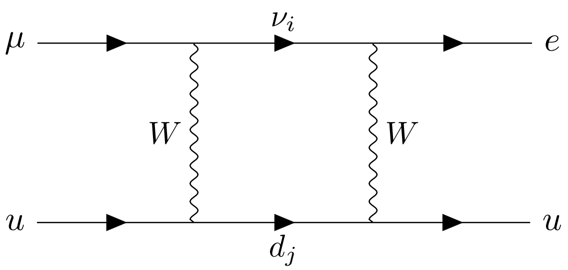
$$\mathrm{CR}(\mu - e,\, \mathrm{N}) = \frac{2G_F^2\, \alpha_w^2\, m_\mu^5}{(4\pi)^2\, \Gamma_{\mathrm{capt.}}} \left| 4V^{(p)}\left(2\widetilde{F}_u^{\mu e} + \widetilde{F}_d^{\mu e}\right) + 4V^{(n)}\left(\widetilde{F}_u^{\mu e} + 2\widetilde{F}_d^{\mu e}\right) + s_w^2\, \frac{G_\gamma^{\mu e} D}{2e} \right|^2$$

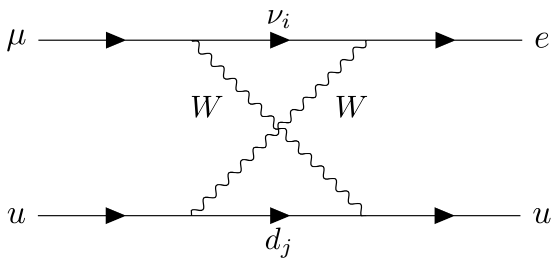
Form factors

$$\begin{aligned}
\widetilde{F}_d^{\mu e} &= -\frac{1}{3}s_w^2 F_\gamma^{\mu e} - F_Z^{\mu e}\left(\frac{1}{4} - \frac{1}{3}s_w^2\right) + \frac{1}{4}F_{\mathrm{box}}^{\mu edd}\,,\\
\widetilde{F}_u^{\mu e} &= \frac{2}{3}s_w^2 F_\gamma^{\mu e} + F_Z^{\mu e}\left(\frac{1}{4} - \frac{2}{3}s_w^2\right) + \frac{1}{4}F_{\mathrm{box}}^{\mu euu}\,.\\
F_{\mathrm{box}}^{\mu euu} &= \sum_{i=1}^{3+n_s}\ \sum_{q_d=d,s,b} \mathcal{U}_{ei}\mathcal{U}_{\mu i}^*\, V_{uq_d}\, V_{uq_d}^*\ F_{\mathrm{box}}(x_i, x_{q_d})\,,\\
F_{\mathrm{box}}^{\mu edd} &= \sum_{i=1}^{3+n_s}\ \sum_{q_u=u,c,t} \mathcal{U}_{ei}\mathcal{U}_{\mu i}^*\, V_{q_u d}\, V_{q_u d}^*\ F_{\mathrm{Xbox}}(x_i, x_{q_u})\,,
\end{aligned}$$

Nuclear form factors (overlap integrals)

$$\boxed{D,\, V^{(p)} \text{ and } V^{(n)}}$$





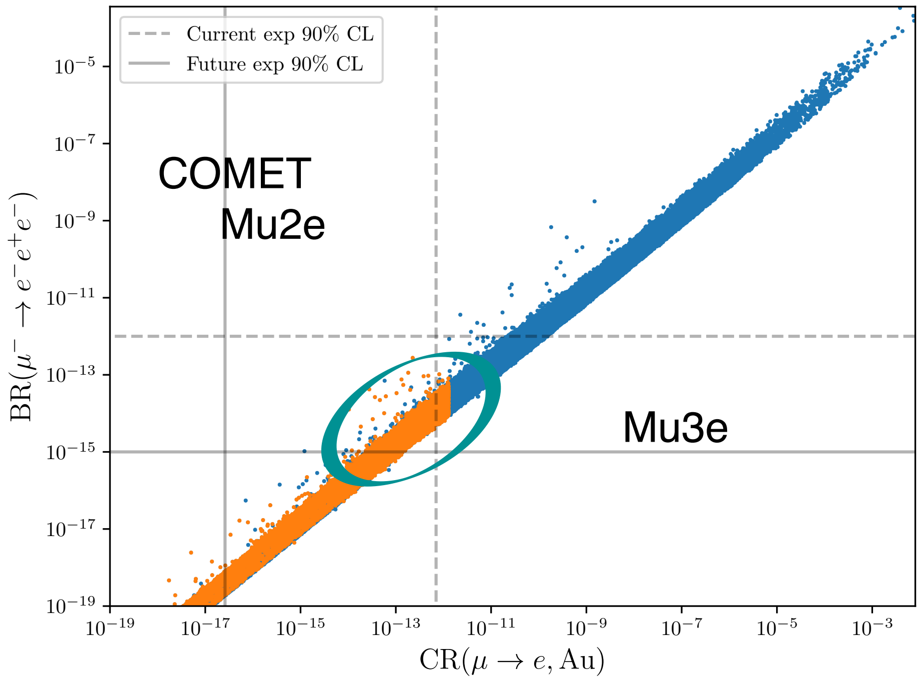
Loop functions:

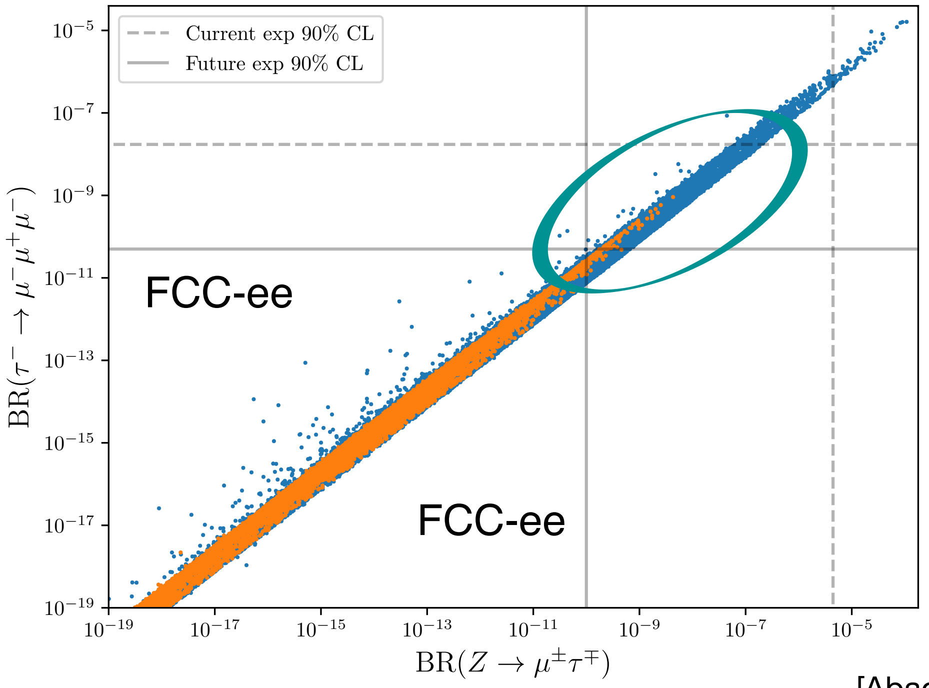
$F_\gamma, F_Z, G_Z, H_Z, G_{box}, F_{box}, F_{Xbox}$

Low-scale models for m_ν: Inverse Seesaw

Inverse seesaw: well-motivated **low-scale mechanism of neutrino mass generation**

ISS(3,3) $\Rightarrow$ SM + 3 ν_R + 3 X *(rich phenomenology $\Rightarrow$ **testability!**)*





[Abada, Kriewald, Pinsard, Rosauro, AMT, '23]

$\Rightarrow$ **Abundant "flavour" signals: cLFV** transitions (at low and high energies)

Regimes *already disfavoured* from current bounds!

cLFV actively constrains parameter space of **ISS**

$\Rightarrow$ **Opportunities** to observe cLFV in (near-)future facilities:

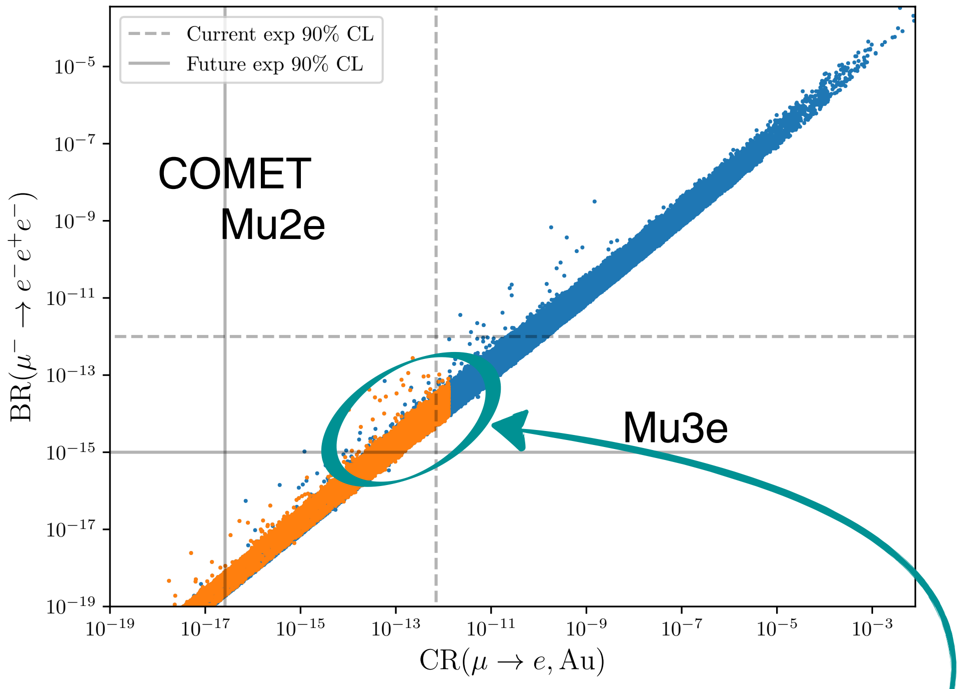
$\mu - e$ sector @ Mu3e, COMET & Mu2e

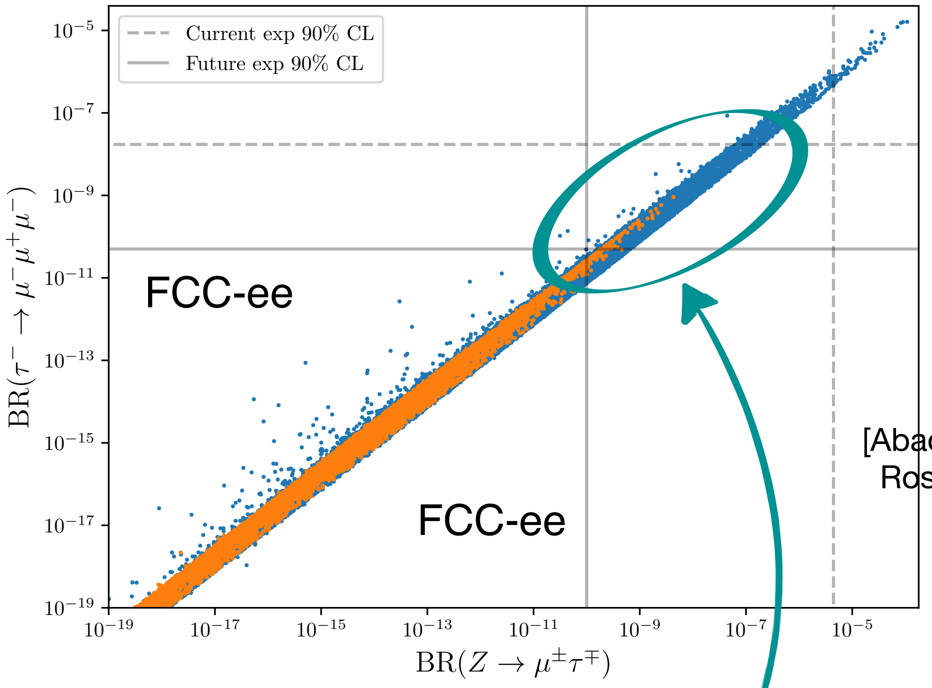
$\tau - \mu$ sector @ Belle II, FCC-ee, ...

Low-scale models for m_ν: Inverse Seesaw

Inverse seesaw: well-motivated **low-scale mechanism of neutrino mass generation**

ISS(3,3) ⇒ SM + 3 ν_R + 3 X *(rich phenomenology ⇒* ***testability!****)*





[Abada, Kriewald, Pinsard, Rosauro, AMT, '23]

⇒ **Correlated observables!** $\mu \to 3e$ vs. $\mu - e$ conversion

and $Z \to \mu\tau$ vs. $\tau \to 3\mu$

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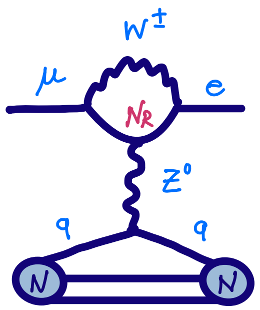

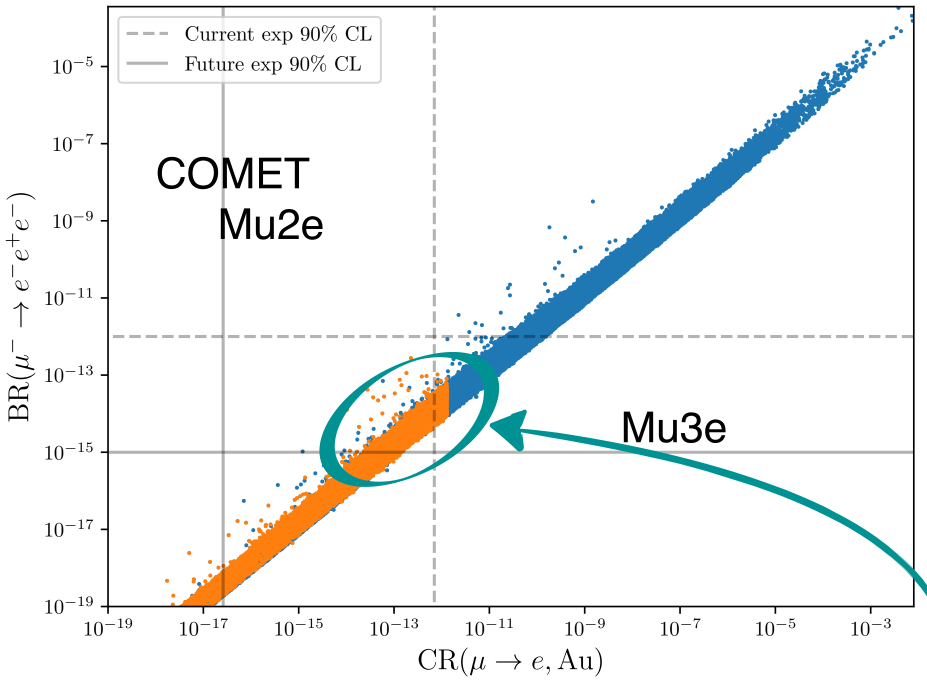

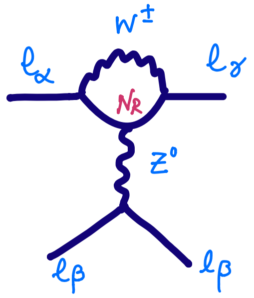

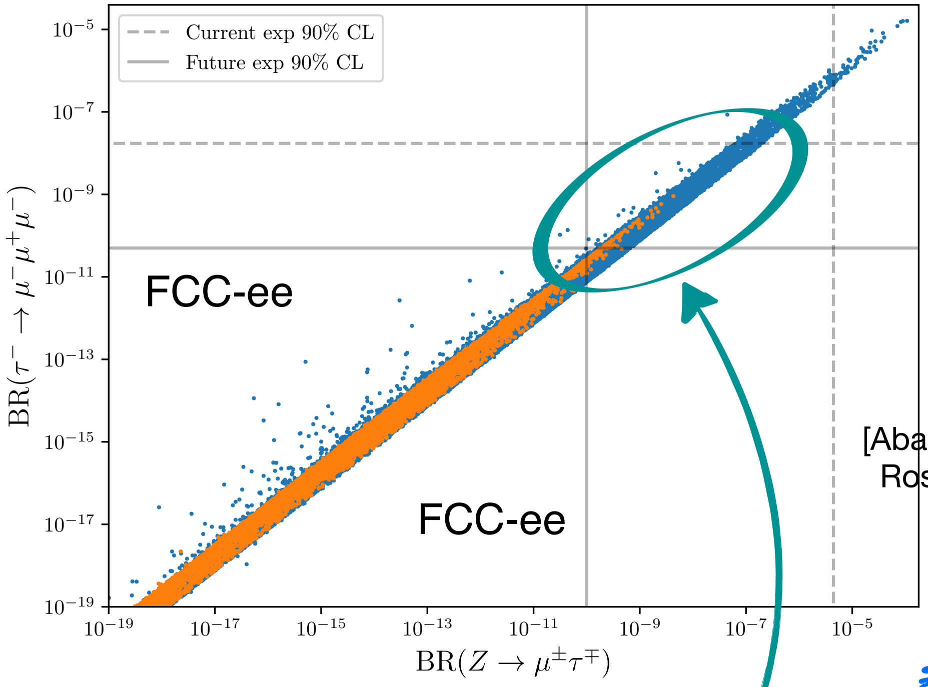

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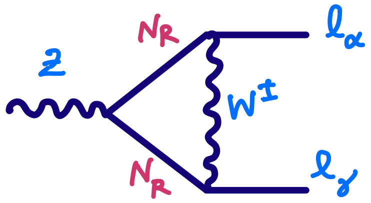

$\Rightarrow$ **Correlated observables!** $\mu \to 3e$ vs. $\mu - e$ conversion

and $Z \to \mu\tau$ vs. $\tau \to 3\mu$

A consequence of the **dominant contribution of Z-penguins** in the 3-body decays and in neutrinoless conversion in nuclei (for the most "observable" regimes...)

Observation of $\mu \to 3e$ $\Rightarrow$ *observation* of $\mu - e$ conversion
$\tau \to 3\mu$ $\Rightarrow$ *observation* of $Z \to \mu\tau$

testability!?

cLFV and the seesaw: peculiar patterns

- **Seesaw realisations:** distinctive expectations for numerous cLFV observables
  ratios of observables to **identify seesaw mediators** & constrain their **masses??**

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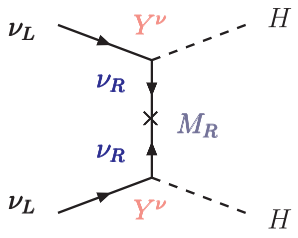
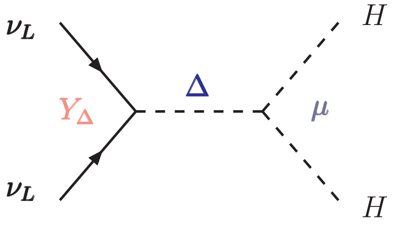
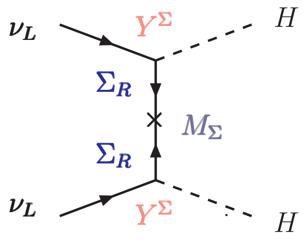


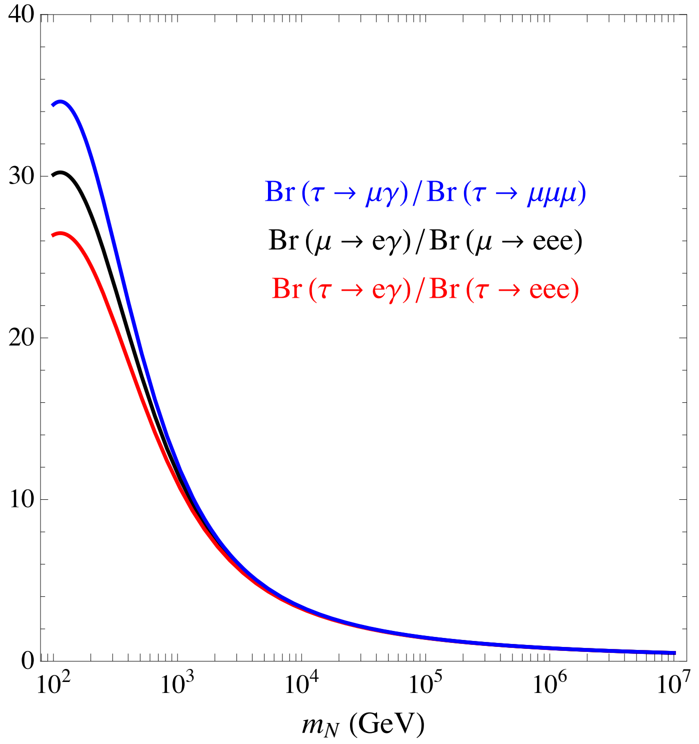
Type I (fermion singlet)



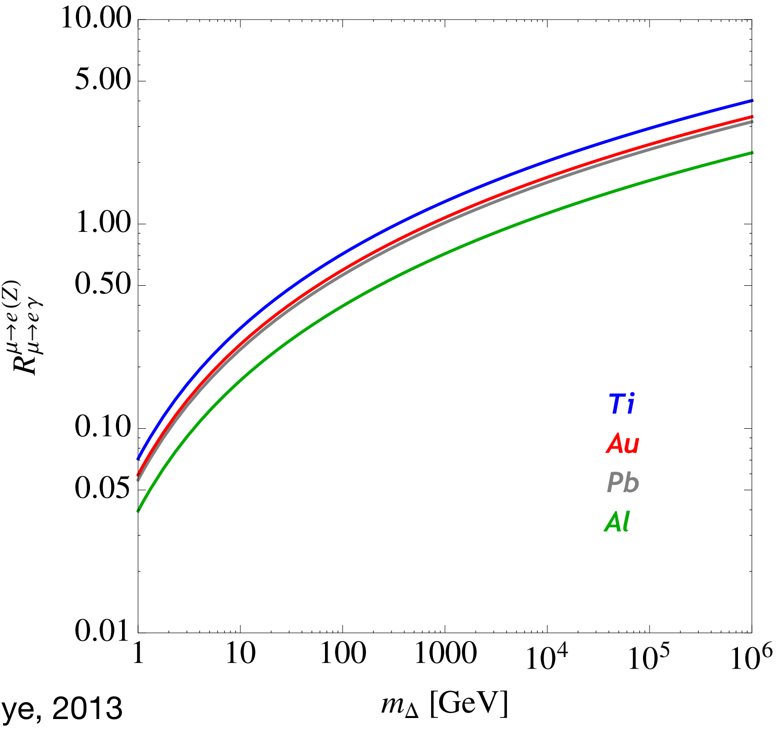
Type II (scalar triplet)



Type III (fermion triplet)



Hambye, 2013



$$\frac{\text{BR}(\mu\to e\gamma)}{\text{BR}(\mu\to 3e)} = 1.3\times10^{-3}$$

$$\frac{\text{BR}(\tau\to\mu\gamma)}{\text{BR}(\tau\to 3\mu)} = 1.3\times10^{-3}$$

$$\frac{\text{BR}(\mu\to e\gamma)}{\text{CR}(e-\mu,\text{Ti})} = 3.1\times10^{-4}$$

[adapted from Calibbi et al, 1709.00294]

cLFV patterns reflect the **topology** of contributions associated with the **new mediators** (dipole or Z-dominated, tree vs. loop, ...)

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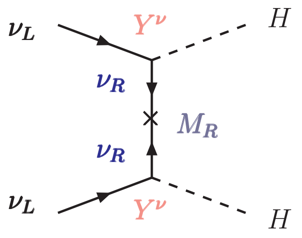

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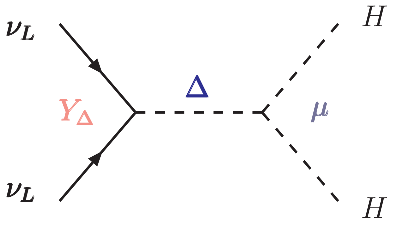

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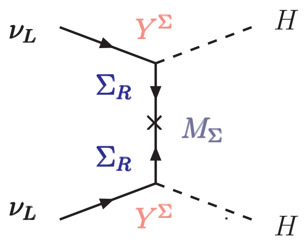

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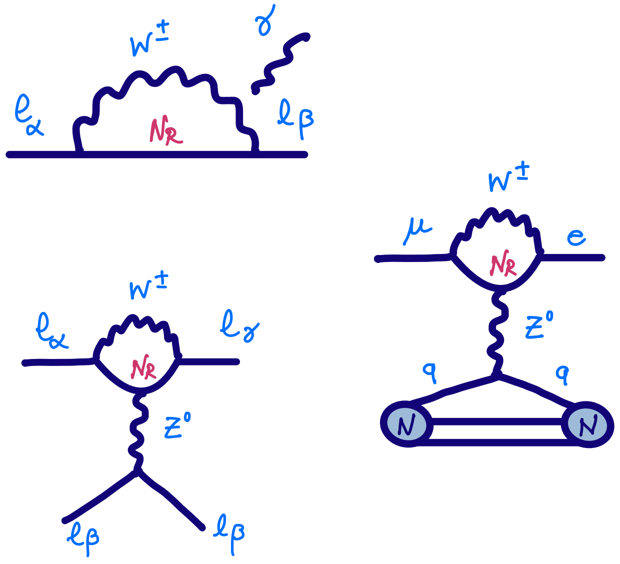

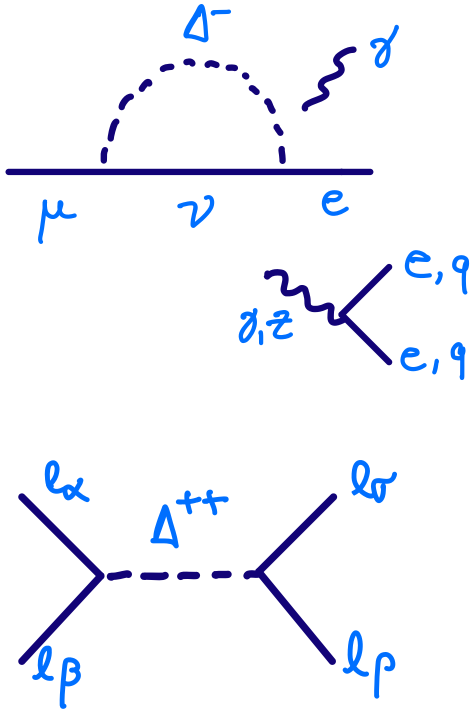

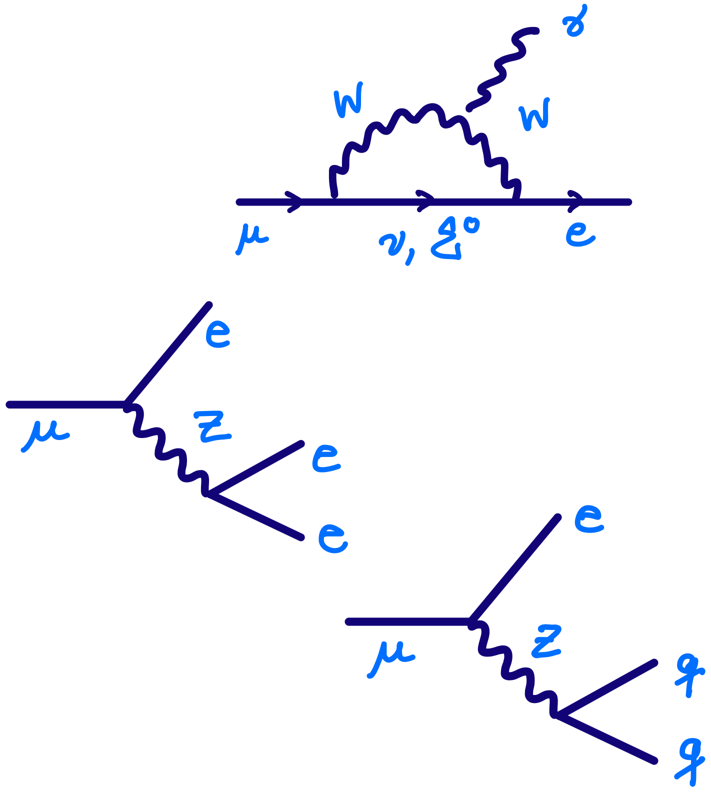

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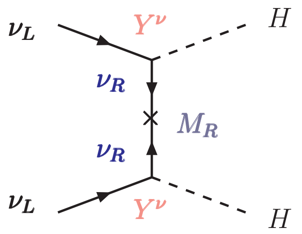

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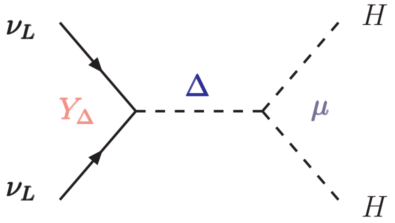

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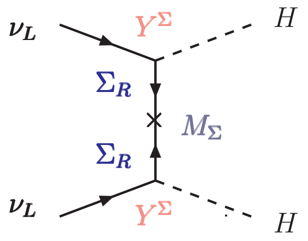

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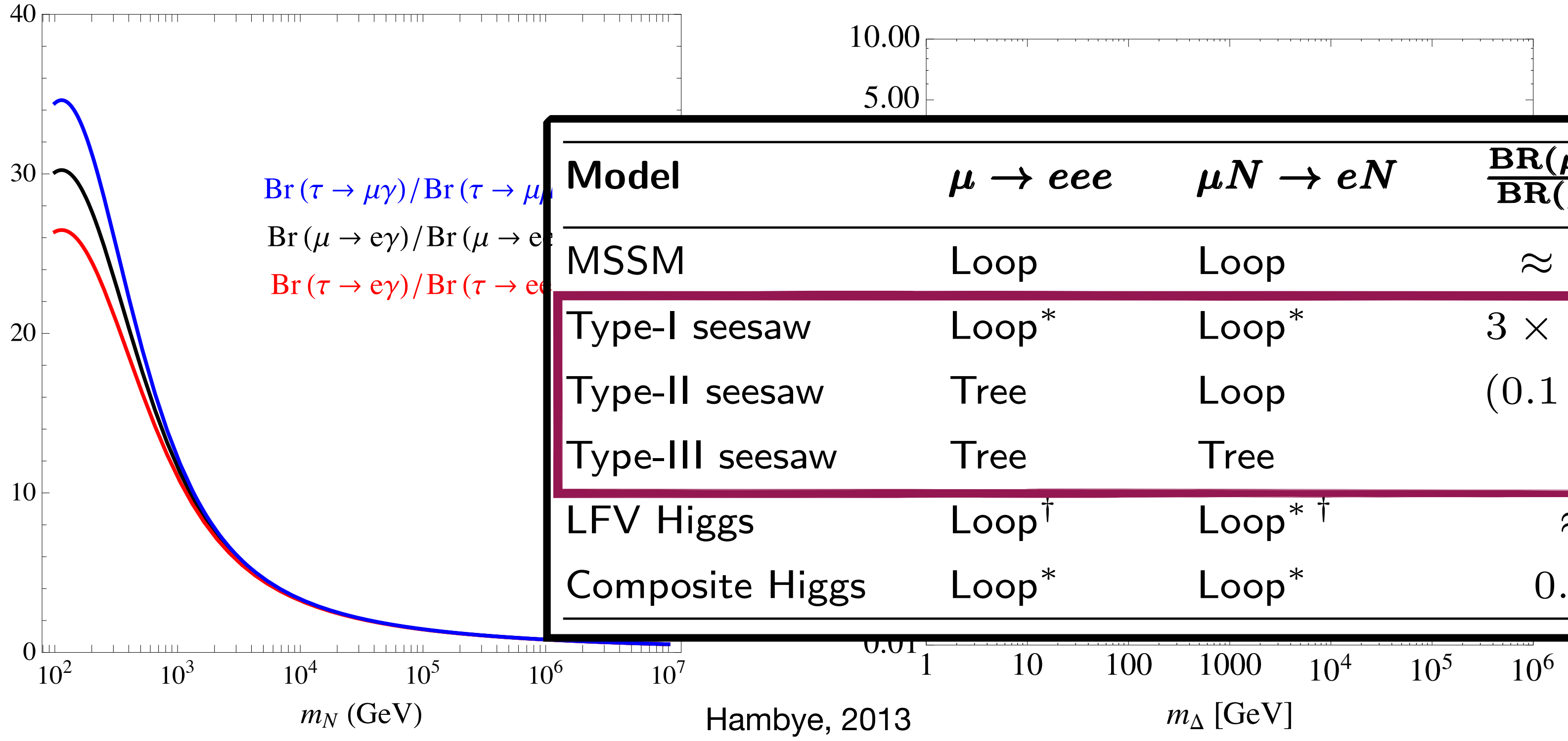

Hambye, 2013

Model	$\mu \to eee$	$\mu N \to eN$	$\frac{\mathrm{BR}(\mu\to eee)}{\mathrm{BR}(\mu\to e\gamma)}$	$\frac{\mathrm{CR}(\mu N\to eN)}{\mathrm{BR}(\mu\to e\gamma)}$
MSSM	Loop	Loop	$\approx 6 \times 10^{-3}$	$10^{-3} - 10^{-2}$
Type-I seesaw	Loop*	Loop*	$3 \times 10^{-3} - 0.3$	$0.1 - 10$
Type-II seesaw	Tree	Loop	$(0.1 - 3) \times 10^3$	$\mathcal{O}(10^{-2})$
Type-III seesaw	Tree	Tree	$\approx 10^3$	$\mathcal{O}(10^3)$
LFV Higgs	Loop†	Loop* †	$\approx 10^{-2}$	$\mathcal{O}(0.1)$
Composite Higgs	Loop*	Loop*	$0.05 - 0.5$	$2 - 20$

[adapted from Calibbi et al, 1709.00294]

cLFV patterns reflect the **topology** of contributions associated with the **new mediators**
(dipole or Z-dominated, tree vs. loop, ...)

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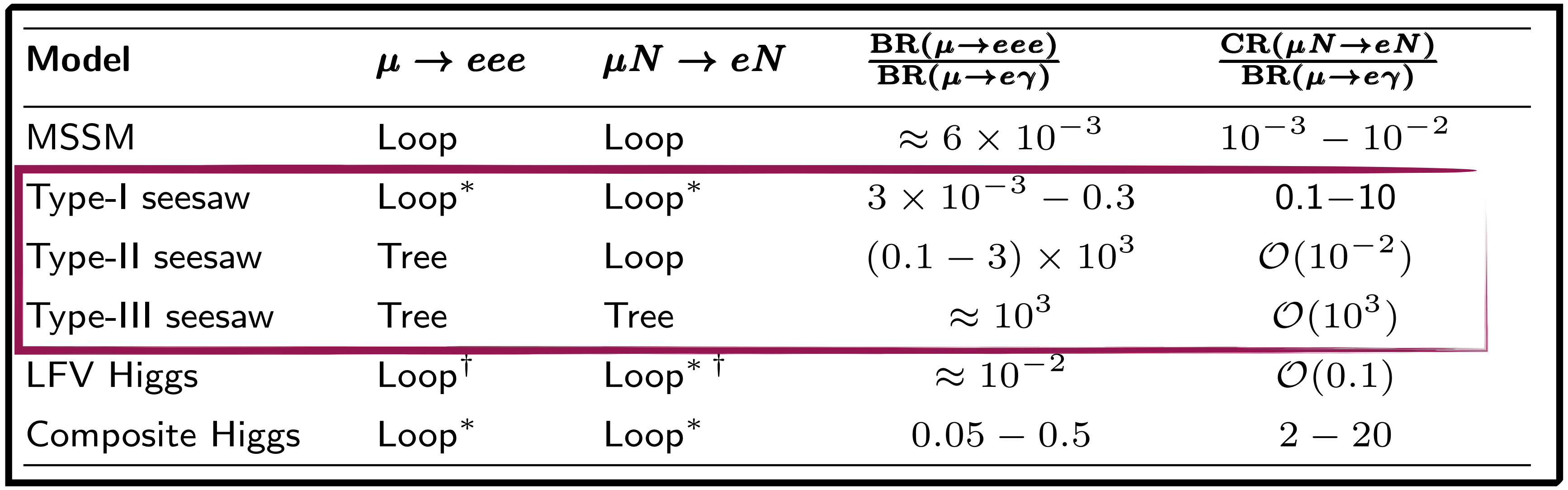

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Composite Higgs	Loop*	Loop*	$0.05 - 0.5$	$2 - 20$

[adapted from Calibbi et al, 1709.00294]

Upon experimental determination of rates for cLFV transitions:

comparison of $\left.\frac{\mathrm{BR}(\mu\to 3e)}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\mathrm{exp}}$ **with** $\left.\frac{\mathrm{BR}(\mu\to 3e)}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\mathrm{NP\text{-}th}}$

and of $\left.\frac{\mathrm{CR}(\mu-e,\mathrm{N})}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\mathrm{exp}}$ **with** $\left.\frac{\mathrm{CR}(\mu-e,\mathrm{N})}{\mathrm{BR}(\mu\to e\gamma)}\right|_{\mathrm{NP\text{-}th}}$

⇨ Probe NP model
at the source
of cLFV

cLFV from neutrino masses in extended frameworks: the SUSY seesaw

(Type I) SUSY seesaw

- **Minimal SUSY** constructions do **not include** a mechanism for ν mass generation

  R-parity conserved, no ν_R, no Higgs triplets, ...

- **Embed a seesaw** (type I, II or III) in the context of (otherwise) **flavour conserving** SUSY

  Minimal Supersymmetric Standard Model - cMSSM

  universal soft breaking terms at GUT scale: $M^{\tilde f}_{L,R} = M_0, A_{\tilde f} = A_0$

- Example: **cMSSM + type I seesaw**

$$W = \hat N^c\, Y_\nu\, \hat L\ \hat H_2 + \hat E^c\, Y_\ell\, \hat L\ \hat H_1 + \frac{1}{2}\, \hat N^c\, M_R\, \hat N^c$$

$$\mathscr{L}_{\text{soft}} : (m_{\tilde L})^2_{ij} = (m_{\tilde E})^2_{ij} = (m_{\tilde N})^2_{ij} = (M_{\tilde L})^2_{ij} = M_0^2\, \delta_{ij}\,;\ (A_\ell)_{ij} = A_0\, (Y_\ell)_{ij}, (A_\nu)_{ij} = A_0\, (Y_\nu)_{ij}$$

"Standard high-scale" seesaw: $M_R \sim \mathcal{O}(10^{10-16}$ GeV$)$, $Y^\nu \sim \mathcal{O}(1)$

$$\boxed{m_\nu \sim (Y^\nu v)^T \frac{1}{M_R} (Y^\nu v)}$$

- Origin of **contributions to lepton flavour violating** processes?

  How to generate sizeable $\delta^{LL}_{\alpha\beta} = \dfrac{(m^2_{\tilde L})_{\alpha\beta}}{\bar m^2_{\tilde L}}, \delta^{LR}_{\alpha\beta}, \delta^{RR}_{\alpha\beta}, \ldots$? **Flavour-blind** soft-breaking terms!

- RGE-induced lepton flavour mixing from non-trivial structure of Y^ν

  (required to account for oscillation data!)

(Type I) SUSY seesaw

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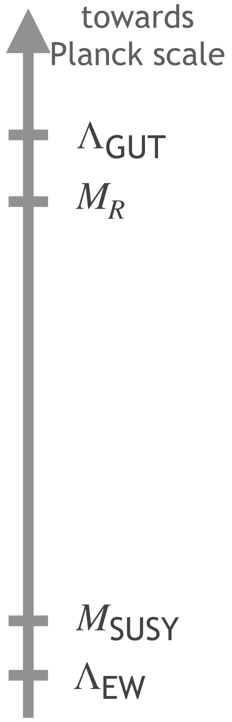
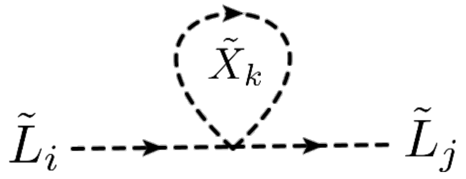


Before decoupling at M_R, contributions from right-handed (s)neutrinos to RGE-running of slepton soft-breaking terms



$\sim (\mathbf{Y}_k^\dagger \mathbf{Y}_k)_{ij}$

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$$(\Delta m^2_{\tilde{E}})_{ij} = 0\ ;\ L_{kl} \equiv \log\left(\frac{M_X}{m_{M_k}}\right)\,\delta_{kl}\,,$$

Soft-masses scale (almost trivially) with Y^ν (*Leading Log approximation*)

Approximately, one has $\text{BR}(\ell_\alpha \to \ell_\beta \gamma) \approx \frac{\alpha^3}{G_F^2}\frac{\tan^2\beta}{\tilde{m}^4_{\text{SUSY}}}(\delta^{LL}_{\alpha\beta})^2\,\text{BR}(\ell_\alpha \to \ell_\beta \nu_\alpha \bar{\nu}_\beta)$,

$\Rightarrow$ If Y^ν is large, sizeable RGE-induced $\delta^{LL}_{\alpha\beta}, \delta^{LR}_{\alpha\beta}, \delta^{RR}_{\alpha\beta}$

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- Contributions to low-energy cLFV observables in a type I "standard" SUSY seesaw

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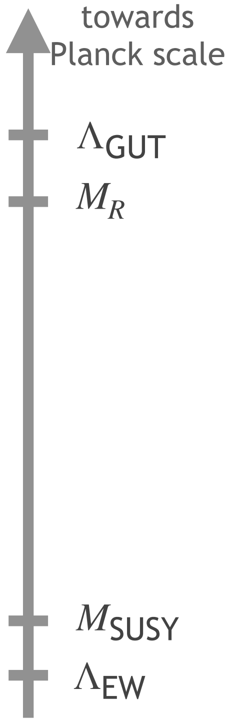

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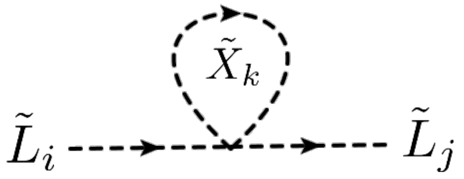

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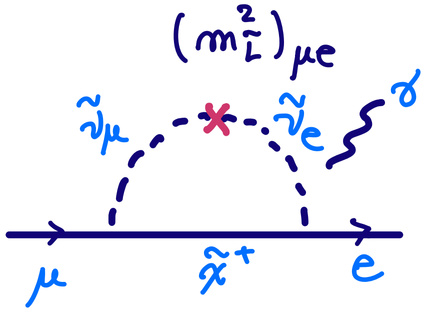


$\Rightarrow$ If Y^ν is large, sizeable RGE-induced $\delta^{LL}_{\alpha\beta}, \delta^{LR}_{\alpha\beta}, \delta^{RR}_{\alpha\beta}$

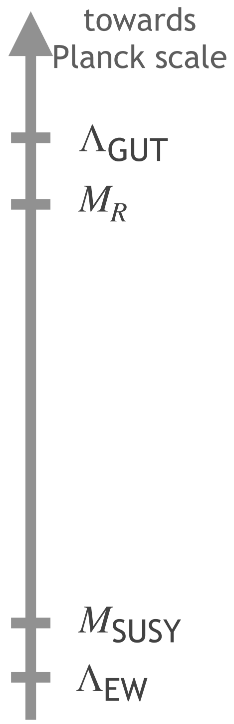
$\Rightarrow$ If new mediators not too heavy, $M_{\text{SUSY}} \sim \mathcal{O}(\text{TeV})$

$\}$ Sizeable cLFV !

(Type I) SUSY seesaw

- **Contributions to low-energy cLFV observables in a type I "standard" SUSY seesaw**

- **RGE-induced lepton flavour mixing** from **non-trivial structure of Y^ν**



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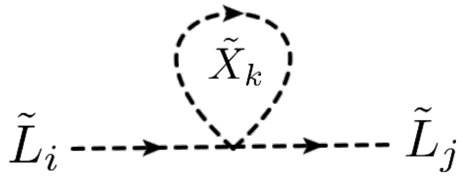

$$\sim (\mathbf{Y}_k^\dagger \mathbf{Y}_k)_{ij}$$

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$\Rightarrow$ ratios independent of **non-flavoured SUSY parameters!**

e.g. $$\frac{\text{BR}(\tau \to \mu\gamma)}{\text{BR}(\mu \to e\gamma)} \approx \left|\frac{\delta^{LL}_{\tau\mu}}{\delta^{LL}_{\mu e}}\right|^2 \frac{\text{BR}(\tau \to \mu\nu_\tau\bar{\nu}_\mu)}{\text{BR}(\mu \to e\nu_\mu\bar{\nu}_e)}$$

(Type I) SUSY seesaw

- Contributions to low-energy cLFV observables in a type I "standard" SUSY seesaw

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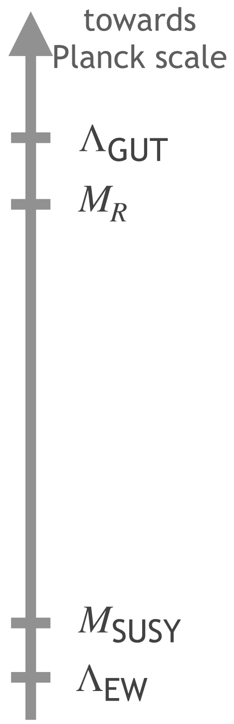

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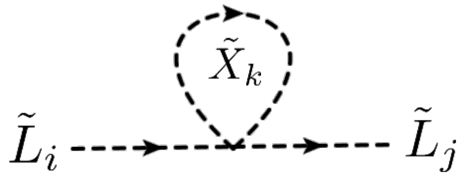

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- N.B.: effects from general SUSY LFV hard to disentangle from seesaw induced cLFV...

(Type I) SUSY seesaw: leptonic cLFV

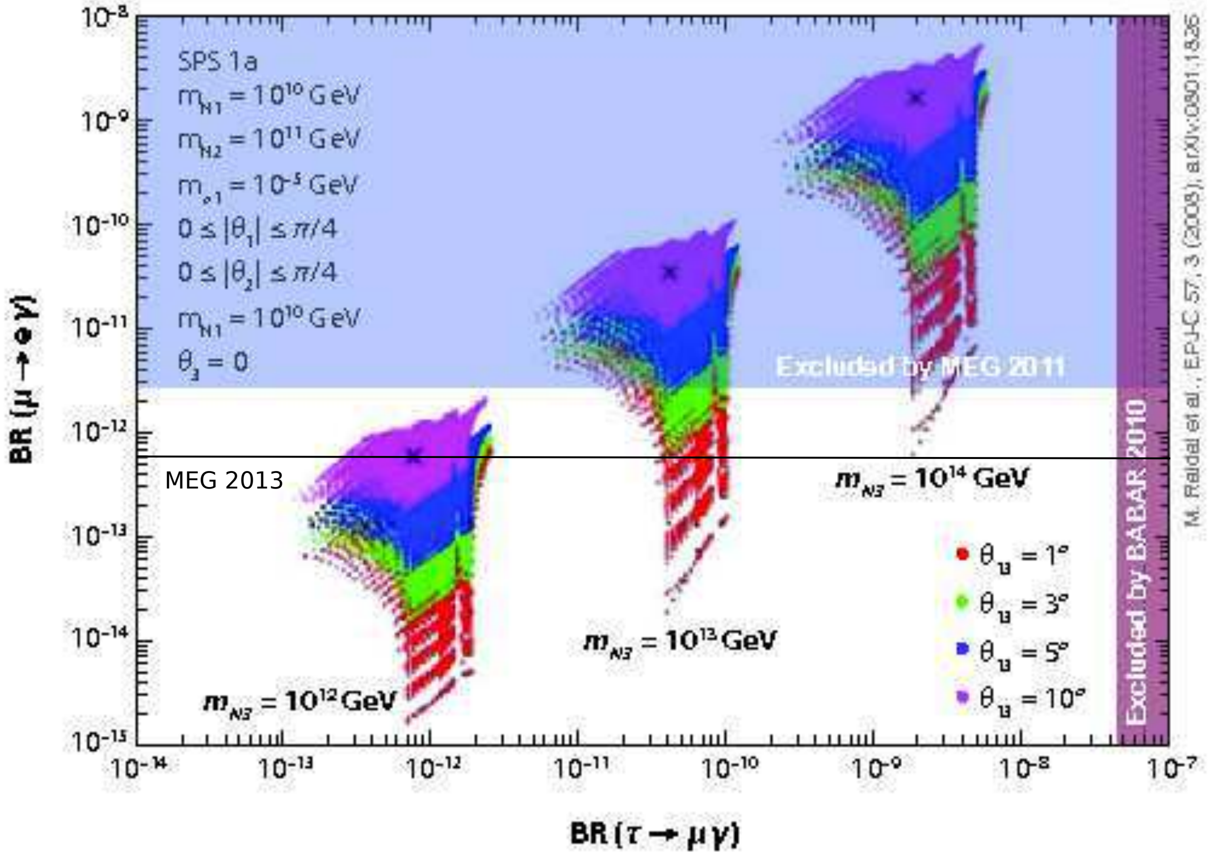
- Embed the **seesaw** mechanism in the framework of **flavour-conserving SUSY models**

  Flavour-blind SUSY breaking (cMSSM-like): lepton flavour mixing from Y^ν

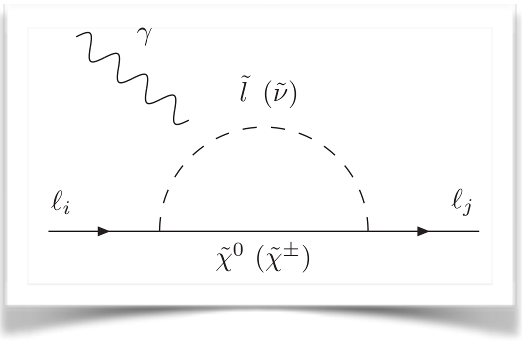
  At GUT scale, **large** $Y^\nu \sim \mathcal{O}(1)$ (type I "vanilla seesaw")

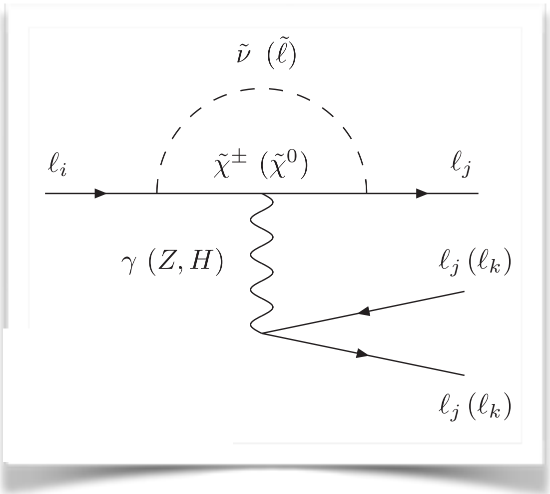
  Flavour violation in (s)lepton sector $(M_{\tilde{\ell},\tilde{\nu}}, A^{\tilde{\ell},\tilde{\nu}})$: **RGE-running** $(Y^\nu, M_{\text{GUT}} \to M_R)$

  $\Rightarrow$ cLFV transitions driven by exchange of **virtual SUSY states** (masses around TeV scale)



[Antusch, Arganda, Herrero, AMT, '06-'08]





Unique source of cLFV (Y^ν)

$\Rightarrow$ all observables strongly related

Synergy of low-energy cLFV observables

$\Rightarrow$ hints on the **seesaw scale** M_R

(Type I) SUSY seesaw: cLFV @ LHC

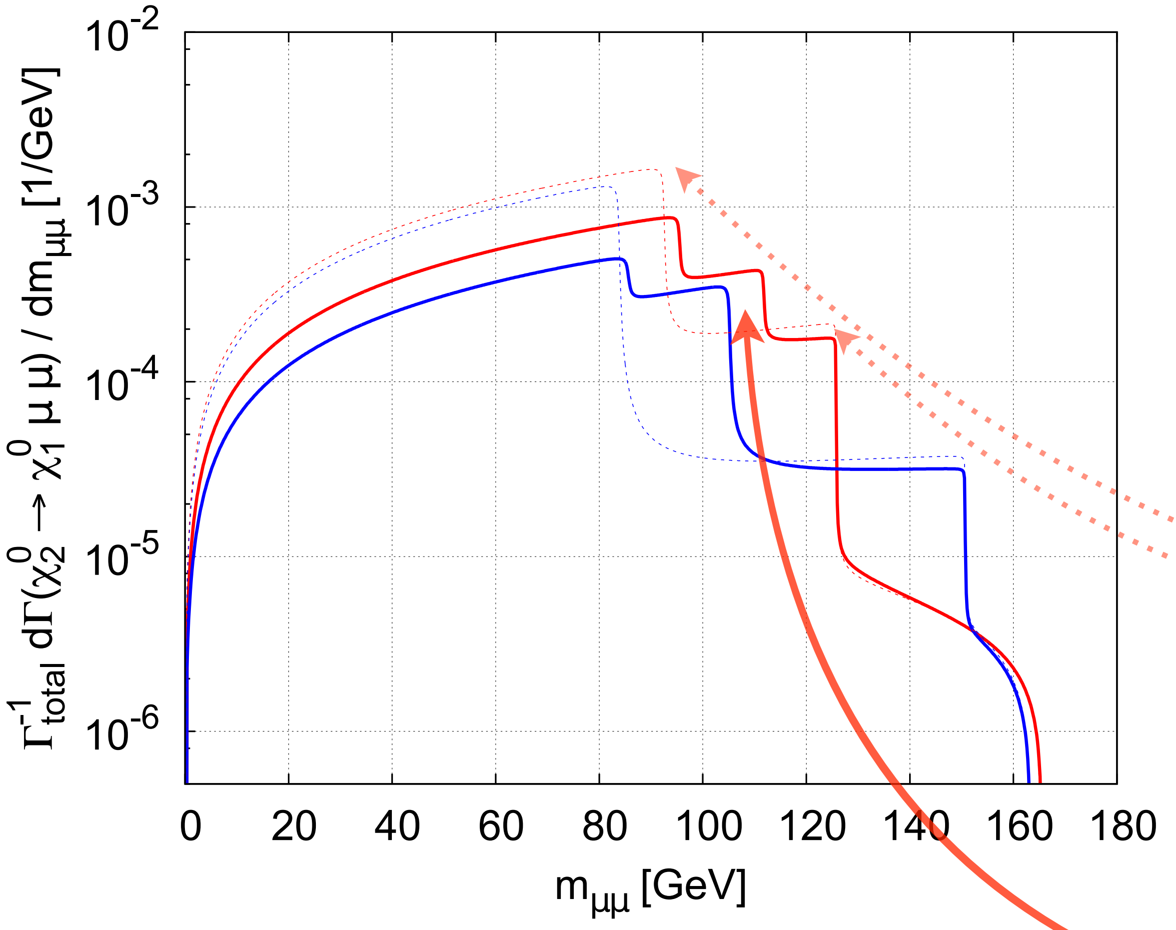
- Embed the seesaw mechanism in the framework of flavour-conserving SUSY models

  Flavour-blind SUSY breaking (cMSSM-like): lepton flavour mixing from Y^ν

  At GUT scale, **large $Y^\nu \sim \mathcal{O}(1)$** (type I "vanilla sesesaw")

  Flavour violation in (s)lepton sector $(M_{\tilde{\ell},\tilde{\nu}}, A^{\tilde{\ell},\tilde{\nu}})$: **RGE-running $(Y^\nu, M_{\text{GUT}} \to M_R)$**

  $\Rightarrow$ **At colliders:** on-shell sparticle production $\tilde{\ell}$ (masses around TeV scale)



[Abada, Figueiredo, Romao, AMT, '10]

LHC: cLFV in neutral current interactions $\chi^0\, \tilde{\ell}_i\, \ell_j$

$$\chi_2^0 \to \ell^\pm \ell^\mp + E^T_{miss}$$

$\Rightarrow$ **new edges** in **dilepton mass distributions**

Flavour conserving case: double triangular $m_{\mu\mu}$

$$\chi_2^0 \to \tilde{\mu}_{L,R}\mu \to \chi_1^0 \mu\mu$$

cLFV from type I SUSY seesaw:

new edge in $m_{\mu\mu}$ from intermediate light $\tilde{\tau}$

$$\chi_2^0 \to \tilde{\tau}\mu \to \chi_1^0 \mu\mu$$

cLFV @ LHC!

Rp-violating supersymmetry

- The **MSSM** (and variants, constrained cases) $\rightsquigarrow$ **not the most generic SUSY realisation!**
- Discrete symmetry (i.e. **R-parity**) assumed: $(-1)^R \equiv (-1)^{3(B-L)+2s}$

  $\Rightarrow$ **SM** particles are **R-even**; **SUSY** partners are **R-odd**

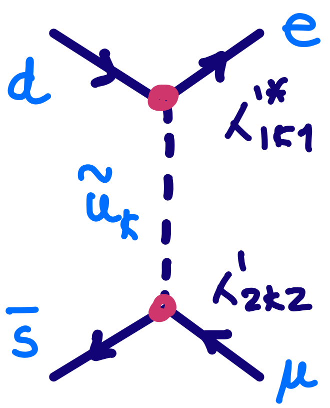
  $\Rightarrow$ **sparticles** are **pair-produced; LSP** is **stable** (DM candidate!)

- Most general case, $W_{RpV} = \frac{\lambda_{ijk}}{2} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k + \mu_i L_i H_u$

cLFV transitions
(even @ tree level!)

baryon number violating
$\Rightarrow$ proton decay...

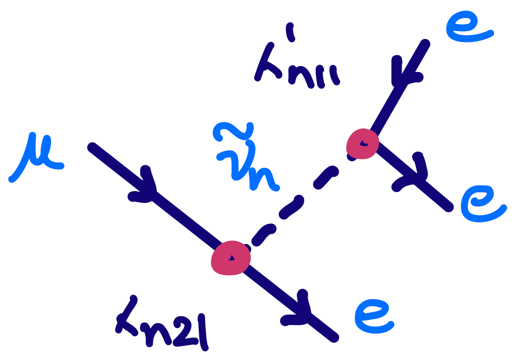
- Strong **constraints** on $\lambda^{(\prime)}_{ijk}$ from cLFV leptonic and (semi) leptonic **decays!**



$\text{BR}(K^0_L \to \mu e) < 4.7 \times 10^{-12}$

$\Rightarrow |\lambda'^*_{1k1}\lambda'^*_{2k2}| \lesssim 10^{-4}$ (for $M_{\text{SUSY}} \sim \mathcal{O}$(TeV))

RpV SUSY - leptoquarks!



$\text{BR}(\mu \to 3e) < 10^{-12}$

$\Rightarrow |\lambda_{n21}\lambda^*_{n11}| \lesssim 7 \times 10^{-9}$ $(n \neq 1)$

$\lambda\,\lambda'$: **tree-level** contributions to $\mu - e$ conversion ; $\mu \to e\gamma$ @ 1-loop from $\lambda\,\lambda, \lambda'\,\lambda'$

The power of cLFV:
hints on models of New Physics

NP models of flavour: so many possibilities!

Extensive contributions in recent years - driven by **NP hints** (m_ν, AMMs, B-anomalies...)

⇒ exploring *flavoured signatures of BSM* realisations

Neutrino mass generation and the **BAU**

Higgs portal to flavours

DM flavoured portals

Symmetry-enforced

sterile fermions

vector-like quarks & leptons

additional Higgs

Z'

leptoquarks

Majorons

...

axion-like particles

extra scalars

flavons

UV-complete models: GUTs, Supersymmetry, extra dimensions, ...

Ultimately addressing all (several) SM problems, and **testable** (via flavours?!) **??**

Can **lepton flavours** help us disentangle the **NP model at work?**

Or **falsify** candidates?

NP models of flavour: so many possibilities!

Extensive contributions in recent years - driven by **NP hints** (m_ν, AMMs, B-anomalies...)

⇒ exploring *flavoured signatures of BSM* realisations

	AC	RVV2	AKM	δLL	FBMSSM	LHT	RS
$D^0-\bar{D}^0$	★★★	★	★	★	★	★★★	?
ϵ_K	★	★★★	★★★	★	★	★★	★★★
$S_{\psi\phi}$	★★★	★★★	★★★	★	★	★★★	★★★
$S_{\phi K_S}$	★★★	★★	★	★★★	★★★	★	?
$A_{\rm CP}(B\to X_s\gamma)$	★	★	★	★★★	★★★	★	?
$A_{7,8}(B\to K^*\mu^+\mu^-)$	★	★	★	★★★	★★★	★★	?
$A_9(B\to K^*\mu^+\mu^-)$	★	★	★	★	★	★	?
$B\to K^{(*)}\nu\bar{\nu}$	★	★	★	★	★	★	★
$B_s\to\mu^+\mu^-$	★★★	★★★	★★★	★★★	★★★	★	★
$K^+\to\pi^+\nu\bar{\nu}$	★	★	★	★	★	★★★	★★★
$K_L\to\pi^0\nu\bar{\nu}$	★	★	★	★	★	★★★	★★★
$\mu\to e\gamma$	★★★	★★★	★★★	★★★	★★★	★★★	★★★
$\tau\to\mu\gamma$	★★★	★★★	★	★★★	★★★	★★★	★★★
$\mu+N\to e+N$	★★★	★★★	★★★	★★★	★★★	★★★	★★★
d_n	★★★	★★★	★★★	★★	★★★	★	★★★
d_e	★★★	★★★	★★	★	★★★	★	★★★
$(g-2)_\mu$	★★★	★★★	★★	★★★	★★★	★	?

[Altmannshofer et al, '10]

AC: RH currents & U(1) flavour sym

RVV2: SU(3)-flavoured MSSM

AKM: RH currents & SU(3) family sym

δLL: CKM-like currents

FBMSSM: flavour-blind MSSM

LHT: Little Higgs (T-parity)

RS: Warped extra dimensions

Expected **impact** for **observables:**

★★★	large effects
★★	small, but visible
★	unobservable

Densely populated sector!

cLFV transitions amongst the most **sensitive observables** to numerous **NP models!**

cLFV in extended frameworks: illustrative examples

Low-scale models for m_ν: DM connection (?)

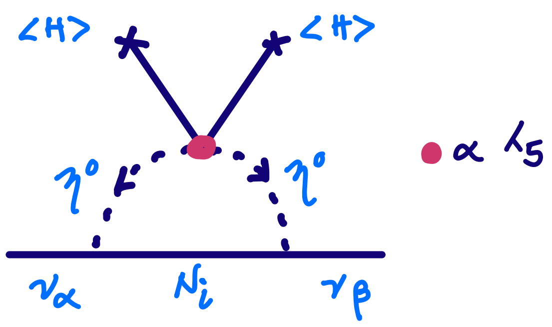
Scotogenic models: a link between neutrino mass generation and **dark matter!**

[Ma, 2006] Additional Z_2 symmetry: stabilises dark matter candidate ... but

$\Rightarrow$ neutrino masses @ 1-loop

[Review on phenomenology of generalised scotogenic models: Hagedorn et al, 1804.04117]

A minimal realisation: extend SM by **inert scalar doublet η** and **RH neutrinos N_R**

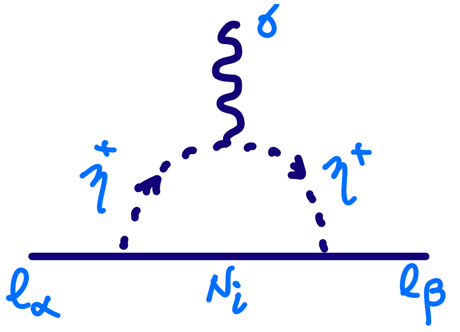


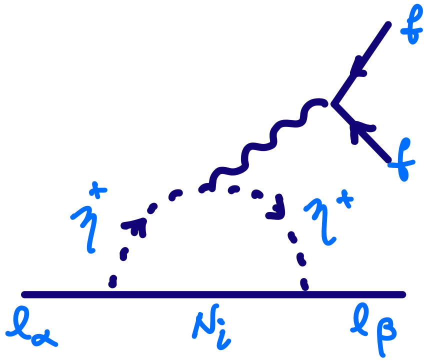
$$\mathcal{M}^\nu_{ij} \simeq \boxed{\frac{\lambda_5}{16\pi^2}}\,\frac{2\,Y^\eta_{ik}\,Y^\eta_{jk}\,v^2}{M^2_{N_i}}\left[\frac{M^2_{N_k}}{m_0^2-M^2_{N_k}}+\frac{M^4_{N_k}}{(m_0^2-M^2_{N_k})^2}\log\left(\frac{M^2_{N_k}}{m_0^2}\right)\right]$$

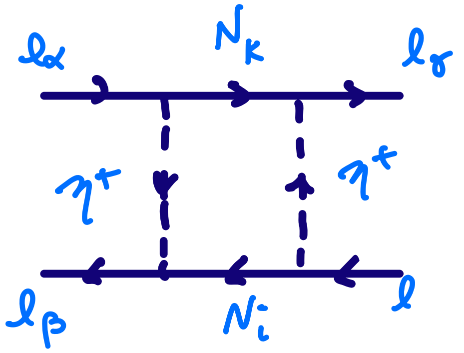
(for $\lambda_5 \ll 1$, with $m_0 = m_{\eta_R} \sim m_{\eta_I}$)

Suppression of neutrino masses:
*smallness of λ_5 and **loop** factors!*

cLFV observables: numerous contributions from η and/or N_R







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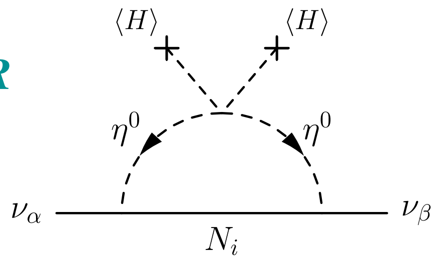
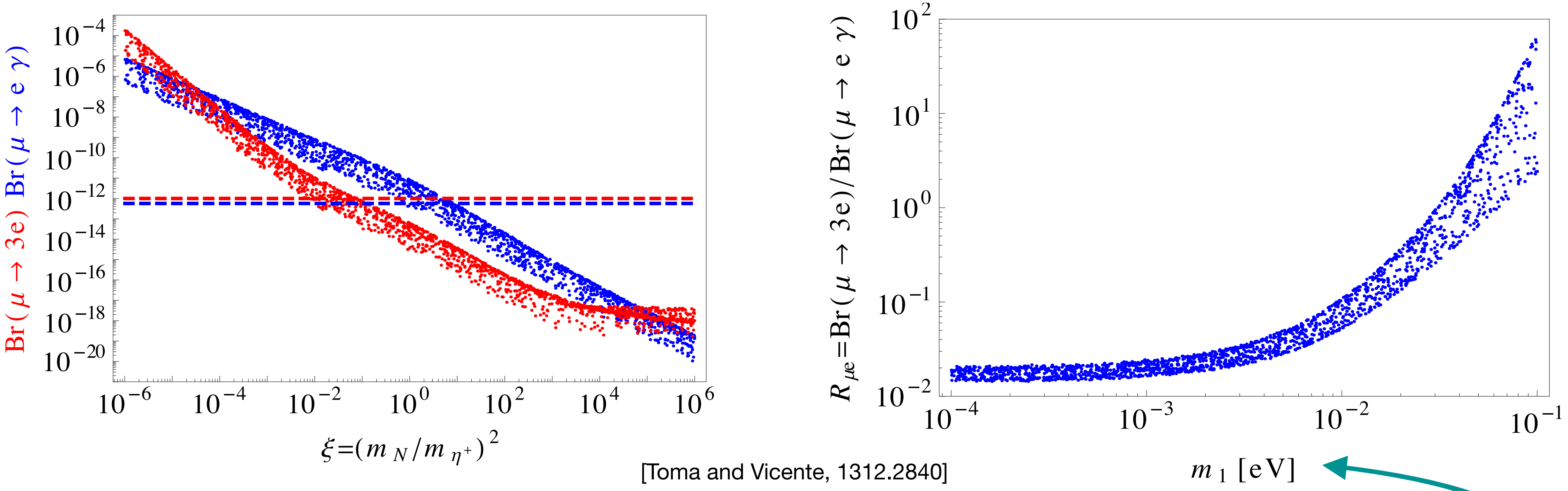


cLFV observables: hints on the **nature of the DM candidate** (η or N_R) and ν **mass scale**



[Toma and Vicente, 1312.2840]

Determination of $R_{\mu e}$ = BR($\mu \to 3e$)/BR($\mu \to e\gamma$) $\Rightarrow$ hints on **lightest neutrino mass** m_{ν_1}

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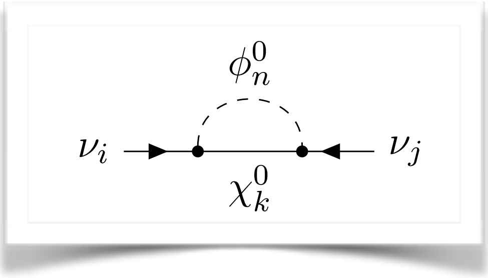
Recent example: SM extended by $SU(2)_L$ **Weyl fermions, Majorana fermion singlets & scalars**

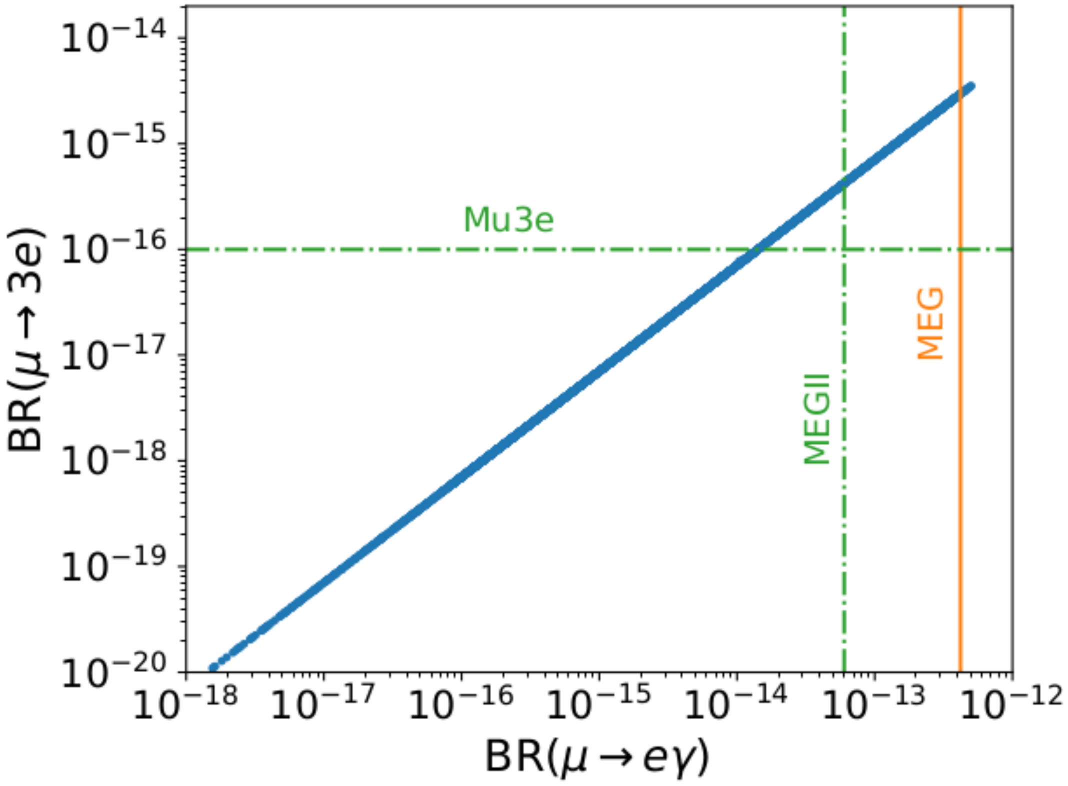
$\Rightarrow$ **ν mass generation, DM candidates, $(g-2)_\mu$** and **BAU** via leptogenesis

cLFV observables: strict correlation between BR$(\mu \to e\gamma)$, BR$(\mu \to 3e)$ [dipole dominated]

muon cLFV decays $\Rightarrow$ falsify model @ MEG II and **Mu3e !**



[Alvarez et al, 2301.08485]



Enhanced gauge symmetries: Left-Right models

- **Minimal** Left-Right extensions of the SM (non SUSY)

  Extensions of the **SM gauge group** (restore parity!)

  $$SU(2)_L \otimes U(1) \longrightarrow SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

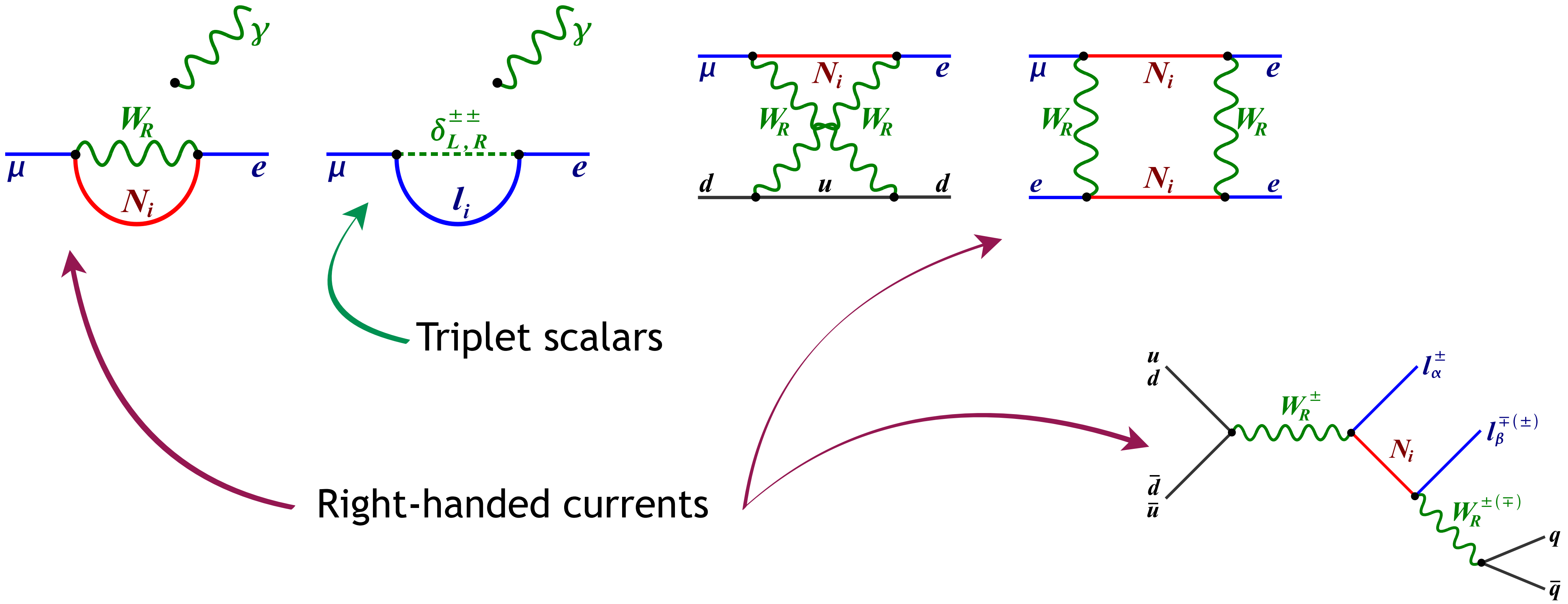
  ν_R automatically included, Z_R & $W_R^\pm$ currents, extended Higgs sector (bidoublet & triplet)

  $$M_\nu \approx \begin{pmatrix} y_M v_L & y_D m_{\text{EW}} \\ y_D^T m_{\text{EW}} & y_M v_R \end{pmatrix}$$

  Dirac masses ($\propto \Lambda_{\text{EW}}$)

  Majorana masses (dynamical generation)

  Abundant contributions to cLFV observables at **low and high-energies** (colliders)!



Triplet scalars

Right-handed currents

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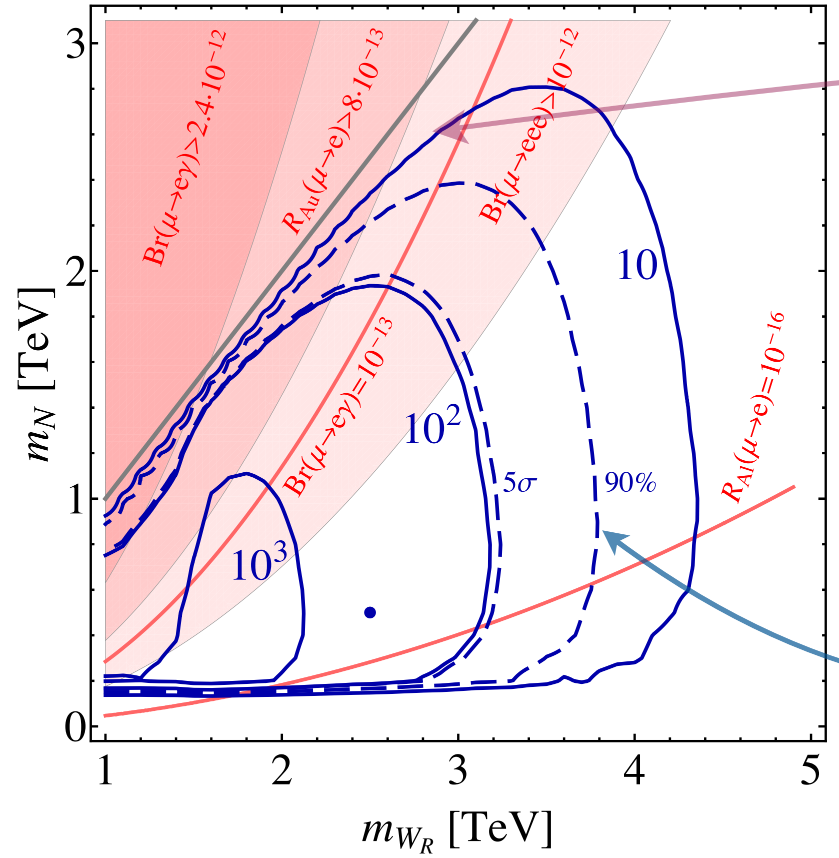
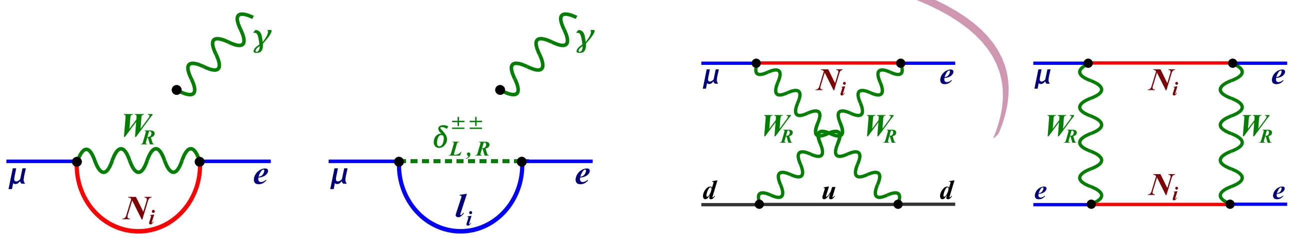
Dirac masses ($\propto \Lambda_{\mathrm{EW}}$)

Majorana masses (dynamical generation)

Extensive work in recent years (here just a "classic" example)



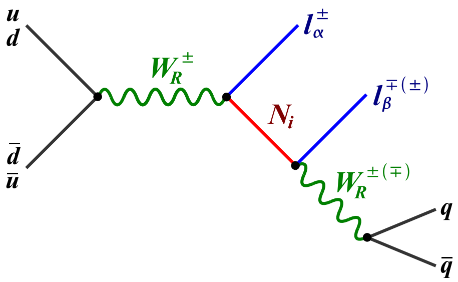
[Das et al, 1206.0656]



If **LHC** $\sqrt{s}$ above threshold for ν_R **production:**

dilepton cLFV signatures

$$pp \to W_R \to e^\pm \mu^\mp + 2 \text{ jets}$$



Complementary studies of Left-Right cLFV

⇒ **LHC signatures** and **low-energy rare decays**

GUT models & cLFV

- **Grand unified models:** several possibilities explored, from (SUSY) $SU(5)$ to $SO(10)$, ...

  Many realisations include mechanisms of ν **mass generation,** and open the door to flavour violation (at all levels)

  Realised at **very high scales** (M_{GUT}) - how to probe and test them?

GUT models: type II seesaw (and more)

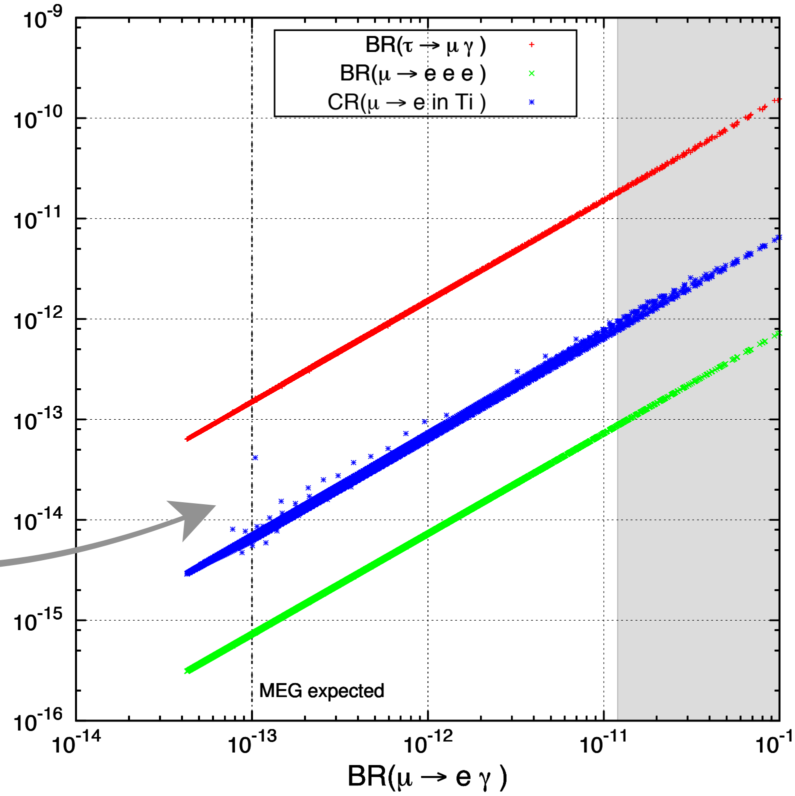
- **Grand unified models:** several possibilities explored, from (SUSY) $SU(5)$ to $SO(10)$, …
  - Many realisations include mechanisms of ν **mass generation,** and open the door to
    - flavour violation (at all levels)
  - Realised at **very high scales** ($M_{\rm GUT}$) - how to probe and test them?

Illustrative example: SUSY GUTs

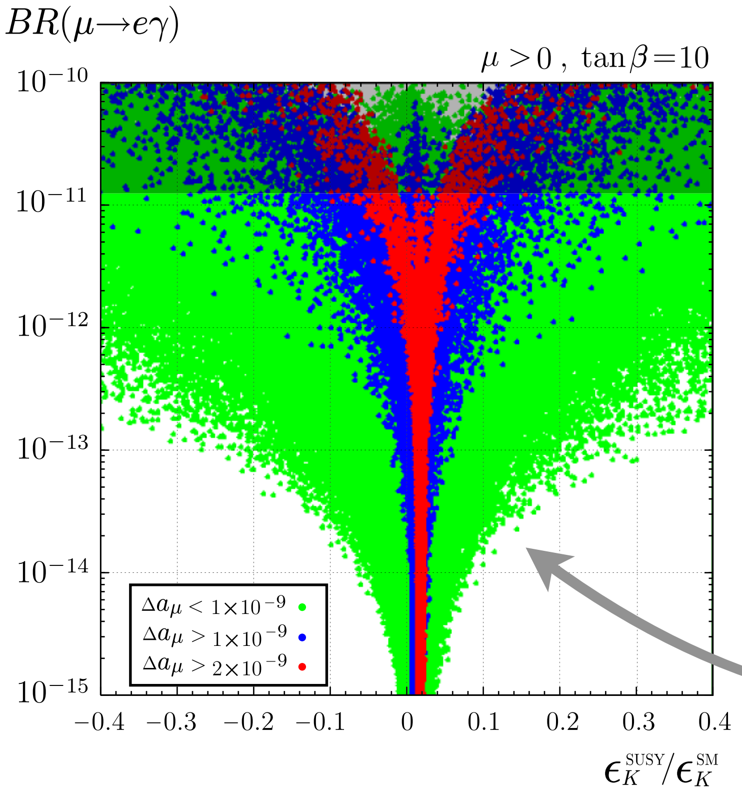
SO(10) **type II SUSY seesaw** (leptogenesis motivated)

reduce arbitrariness of Y^ν [CKM- and PMNS inspired textures]

⇒ highly correlated cLFV observables! **Falsifiable!**



[Calibbi et al, 0910.0377]



SU(5) **SUSY GUT** (& RH neutrinos)

strong dependency of CPV and flavour observables

in lepton & quark sectors!

[Buras et al, 1011.4853]

cLFV in general NP models

Extended Higgs sectors: 2 Higgs doublet model

- **Two Higgs Doublet:** an additional scalar doublet (H_2) - motivated by larger frameworks

  In general cases (e.g. non-SUSY), *doublets coupling to all species* lead to significant **flavour-changing-neutral currents** ⇒ conflict with data!

  Impose symmetries to avoid tree-level FCNC!

Type I: SM fermions couple only to one Higgs

Type II: up quarks couple to H_2; down quarks and charged leptons to H_1 (SUSY-like)

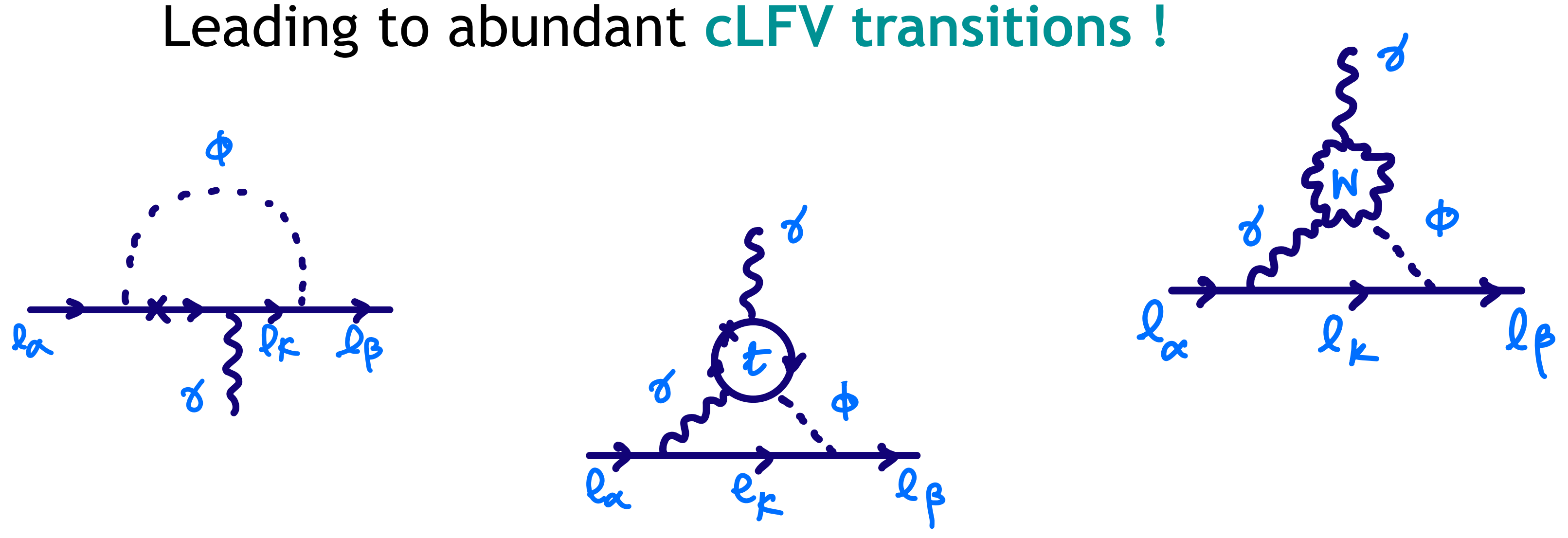
Generic Lagrangian:

"standard" Yukawas

$$-\mathscr{L}_{\text{2HDM}} = Y^{\ell}_{ij}\,\bar{\ell}_i\,H_1\,e_j + Y^{u}_{ij}\,\bar{q}_i\,\tilde{H}_1\,u_j + Y^{d}_{ij}\,\bar{q}_i\,H_1\,d_j + K^{\ell}_{ij}\,\bar{\ell}_i\,H_2\,e_j + K^{u}_{ij}\,\bar{q}_i\,\tilde{H}_2\,u_j + K^{d}_{ij}\,\bar{q}_i\,H_1\,d_j + \text{H.c.} + V(H_1,H_2)$$

"exotic" Yukawas

Leading to abundant **cLFV transitions !**

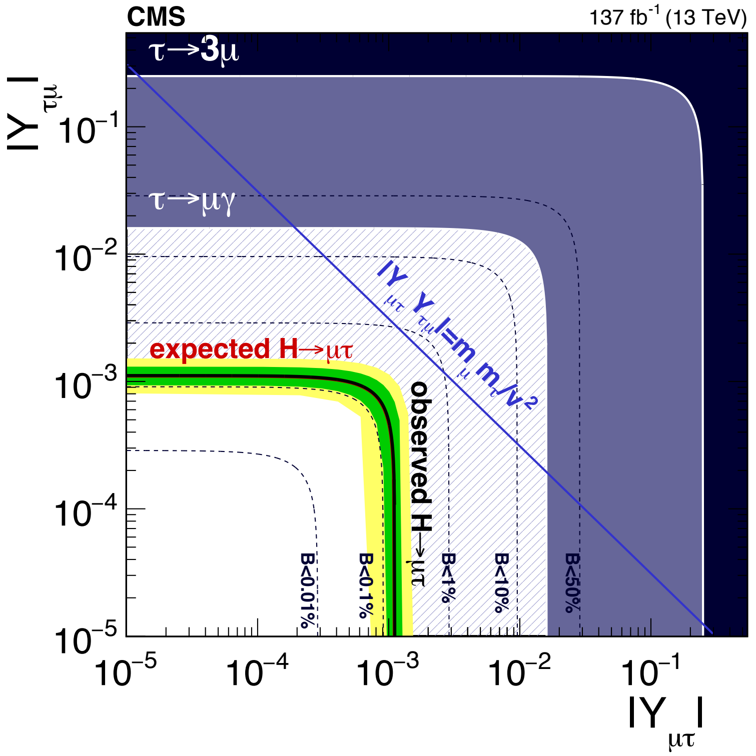


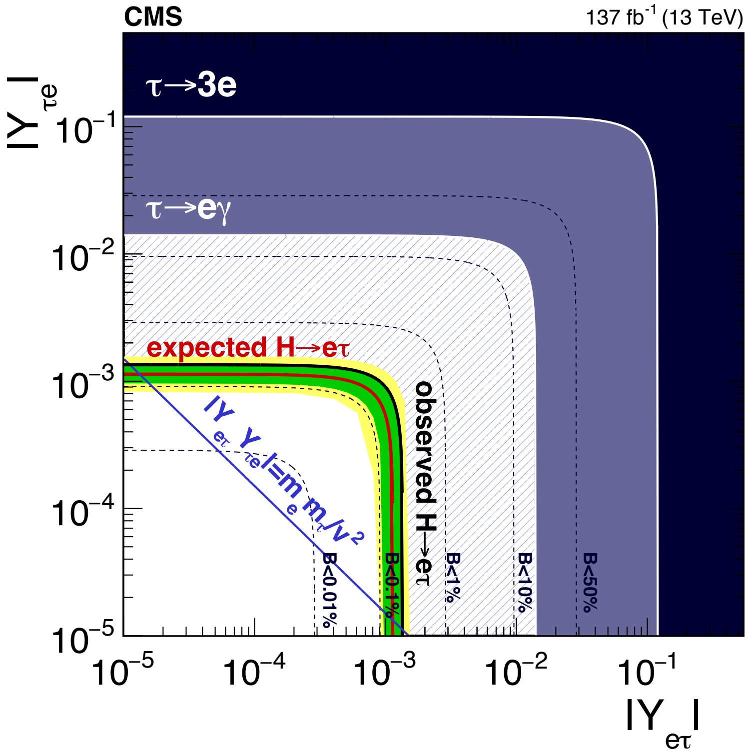
And to **cLFV Higgs decays:**

$$\Gamma(h \to \ell_i \ell_j) \propto m_h \,\frac{|K^{\ell}|^2_{ij} + |K^{\ell}|^2_{ji}}{16\pi}$$

Flavour violating Higgs: cLFV

- **Limits on "effective" Yukawa couplings (@ LHC)**





cLFV Higgs decays

$$\begin{aligned} \mathrm{BR}(H \to \tau\mu) &\lesssim 2.8 \times 10^{-3} \,, \\ \mathrm{BR}(H \to \tau e) &\lesssim 4.7 \times 10^{-3} \quad \text{(ATLAS)} \end{aligned}$$

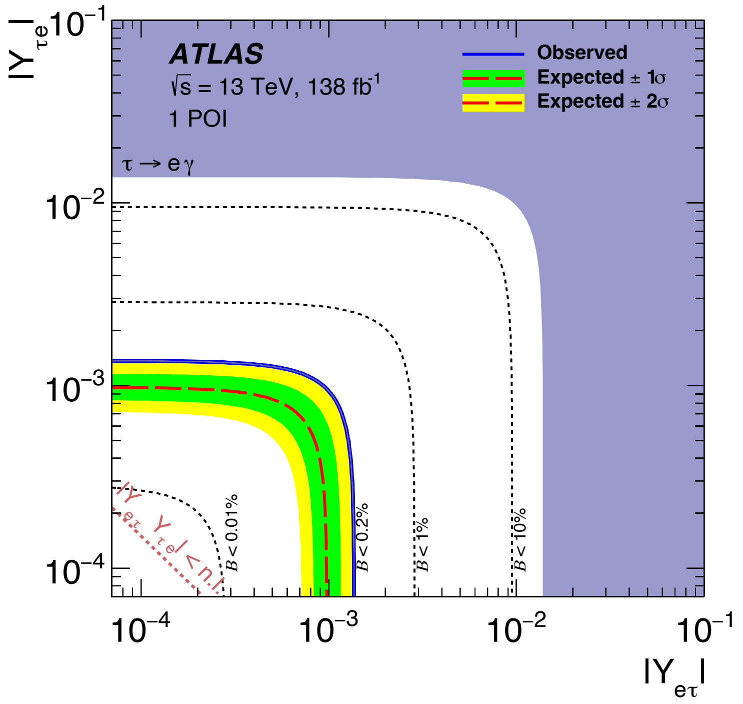
$$\begin{aligned} \mathrm{BR}(H \to \tau\mu) &\lesssim 1.5 \times 10^{-3} \,, \\ \mathrm{BR}(H \to \tau e) &\lesssim 2.2 \times 10^{-3} \quad \text{(CMS)} \end{aligned}$$

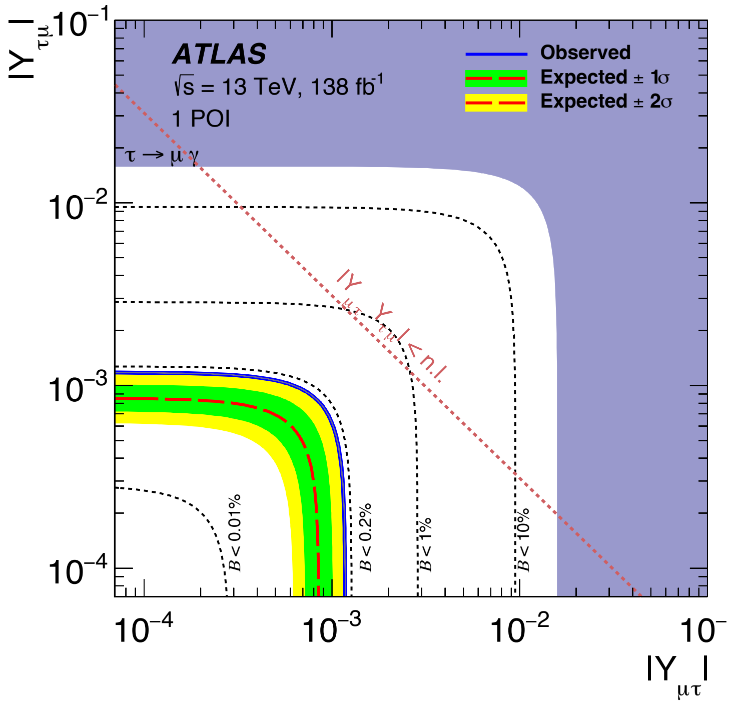
$$\Gamma(\mathrm{H} \to \ell^\alpha \ell^\beta) = \frac{m_\mathrm{H}}{8\pi}\left(|Y_{\ell^\alpha \ell^\beta}|^2 + |Y_{\ell^\beta \ell^\alpha}|^2\right)$$

[CMS Coll, 2105.03007]

$$|Y_{\ell\tau}|^2 + |Y_{\tau\ell}|^2 = \frac{8\pi}{m_H} \frac{\mathcal{B}(H \to \ell\tau)}{1 - \mathcal{B}(H \to \ell\tau)} \Gamma_H(\mathrm{SM})$$

[ATLAS Coll, 2302.05225]





Vector-like fermions: impact for cLFV

- **Massive vector-like fermions** (leptons) present in well-motivated SM extensions:

  composite Higgs models, warped extra dimensions, ad-hoc extensions, ...

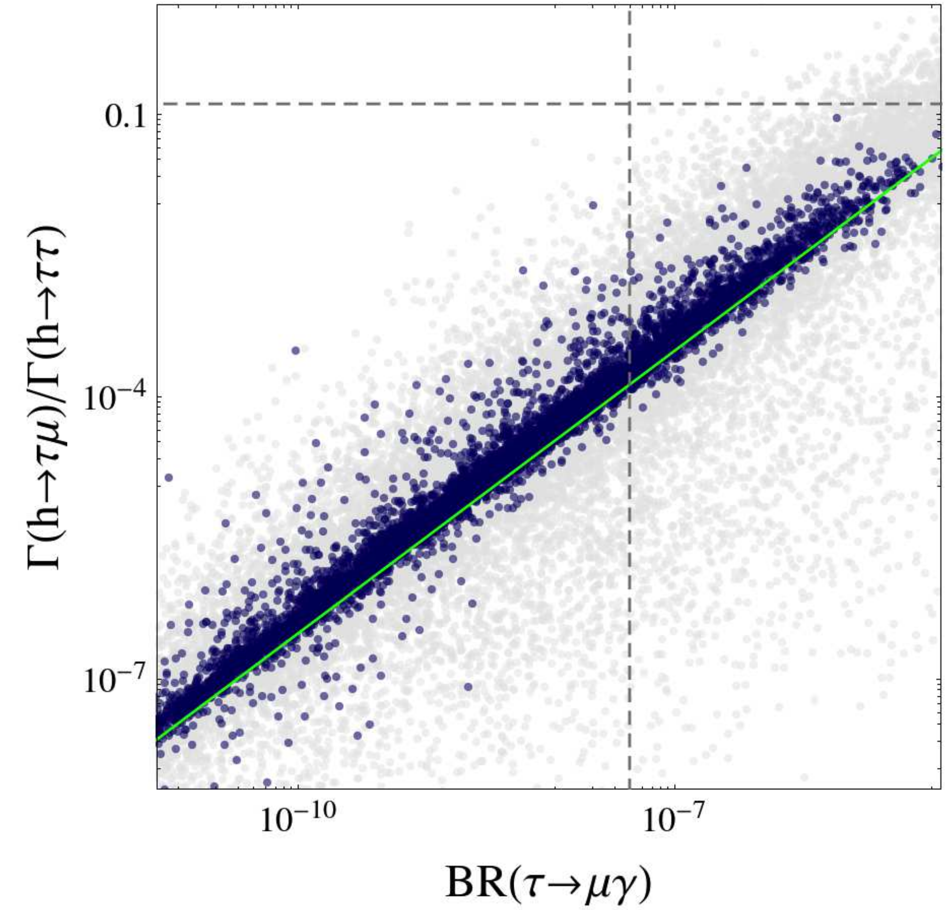
  Offer **new possibilities for neutrino mass generation:**

  dynamical Majorana mass generation, higher order contributions, ...

Illustrative view: generic set-up (composite Higgs inspired), SM + 3 generations of L_i^V and E_i^V

Neutrino masses from ν_R and interactions with vector-like partners

cLFV observables: contributions from reduced set of couplings ⇒ correlated rates!



[Falkowski et al, '14]

An example:

$$\frac{\text{BR}(h \to \ell_i \ell_j)}{\text{BR}(\ell_i \to \ell_j \gamma)} \approx \frac{4\pi}{3\alpha} \frac{\text{BR}(h \to \ell_i \ell_j)|_{\text{SM}}}{\text{BR}(\ell_i \to \ell_j \nu_i \bar{\nu}_j)}$$

⇒ Synergy between cLFV Higgs and cLFV radiative decays!

Geometric flavour: RS warped extra dimensions

- Embed 4dim space-time into 5dim AdS space (extra dim compactified on orbifold)

  Two branes (UV, IR) and **bulk** between them

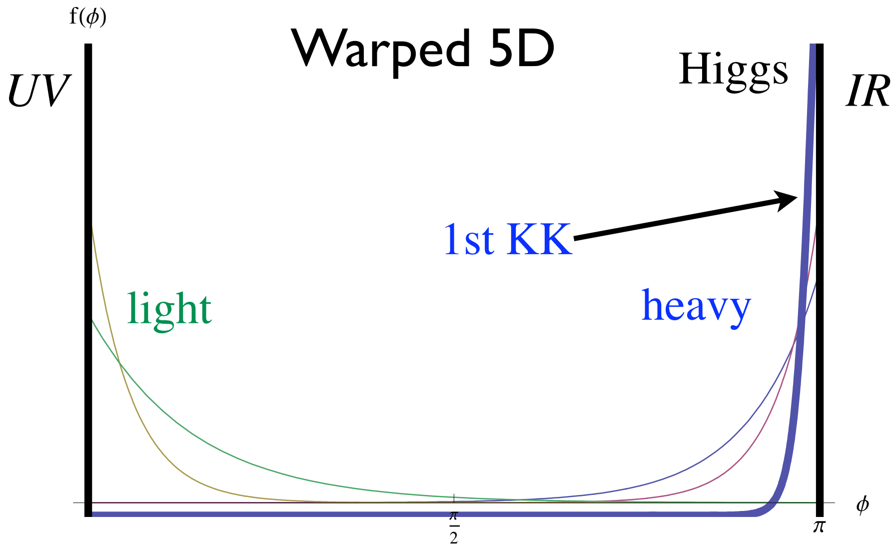
  Localise fields: Higgs close to *IR brane*

  SM fermions & gauge bosons on *bulk*

  KK excitations close to *IR brane*

  Interactions ↭ overlap of wave functions

  (L)FV from couplings of light fermions to KK excitations



$$M_{\text{TeV}} \simeq M_{\text{Planck}}\, e^{-\pi k L_5}$$

Geometrical distribution of fermions in bulk:

reproduce **hierarchy** in 4dim Yukawas from **"anarchic" $\mathcal{O}(1)$ dim5 couplings!**

Non-negligible phenomenological issues:

enlarge bulk symmetry to prevent violation of custodial SU(2) symmetry

additional "rescue" strategies to avoid excessive FCNCs,

to protect EW precision observables, ..., among other issues

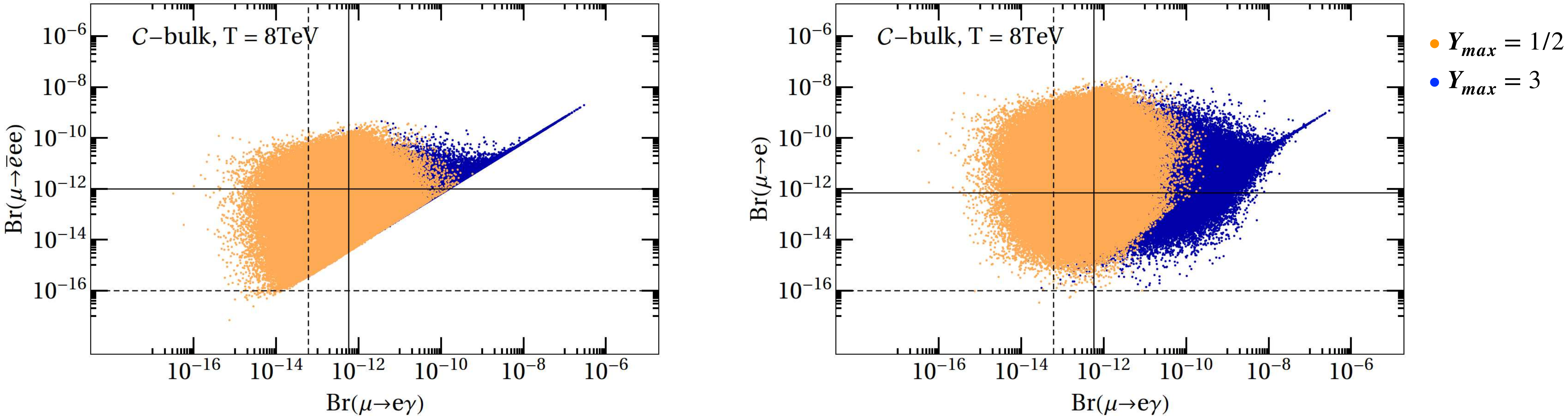
[Burdman, ’02; Agashe et al, ’04; Csaki et al, ’08; Blanke et al & Buras et al, ’08-’09;
Bauer et at, ’10; Vempati et al, ’12; Beneke et al, ’12-’15; and many others...]

Geometric cLFV: RS warped extra dimensions

- *Example*: **custodially protected** model, full inclusion of **all dim-6 cLFV** operators

  generical anarchic Yukawa couplings

  new gauge fields & KK-excitations of lepton fields ⇒ **cLFV transitions**



[Beneke et al, 1508.01705]

Most stringent constraints from $\mu \to e\gamma$ and $\mu - e$ **conversion**

τ decays comparatively less restrictive

Current $\mu - e$ **cLFV bounds** constrain **NP scale to be very heavy**, **beyond LHC reach**

$T_{KK} \gtrsim$ **4 TeV** (corresponding to $m^1_{KK} \gtrsim 10$ TeV)

Future $\mu - e$ sensitivities: exclude anarchic RS models (without additional symmetries)

up to **8 TeV** (corresponding to KK gluon masses around 20 TeV)

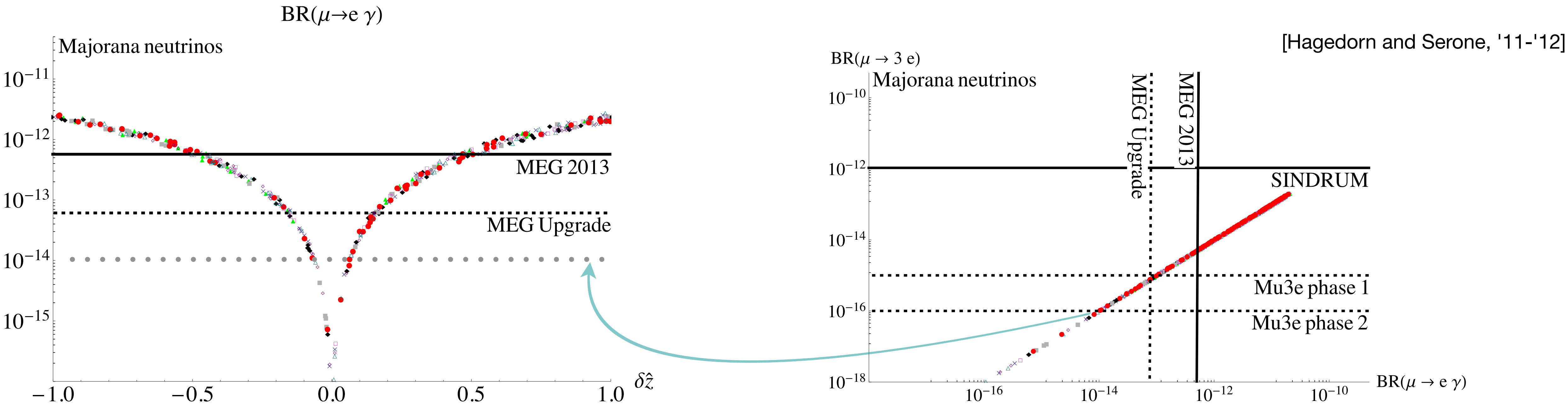
Composite Higgs and warped extra dimensions

- **Holographic composite Higgs** model based on enlarged symmetry, $\mathcal{G}_{SM} \times G_f$

  $G_f = X \times Z_N$, with $X = S_4, A_4, \Delta(96{,}384)$

  (Discrete) symmetries - predict the lepton mixing pattern (masses unconstrained)

  Applied to **5dim** model in warped space; both cases of Dirac and Majorana neutrinos



[Hagedorn and Serone, '11-'12]

cLFV observables (as well as EDMs) typically **below experimental bounds** ($m^1_{KK} \sim 3-4$ TeV)

MEG (I & II) bounds on $\mu \to e\gamma$ $\rightsquigarrow$ constrain the size of boundary kinetic terms!

Important role played in the future by Mu3e data

$\Rightarrow$ cLFV allows to infer relevant information on fundamental parameters

Concluding remarks

Outlook

Confirmed observations and several **"tensions"** suggest the need to go beyond the SM

In the lepton sector, ν**-masses** provided the 1st laboratory evidence of NP

Many experimental **"tensions"** nested in lepton-related observables

Lepton physics might offer valuable hints in **constructing and probing NP models**

New Physics can be **manifest via** cLFV, LNV, **...** even before any direct discovery!

(Synergy of) lepton observables can provide **information on the underlying NP model**

New Physics is there! Lepton physics might be a perfect **portal** to address SM problems

⇒ First hints on **preferred paths** to NP from EFT approach

⇒ Attempt at **identifying the** underlying model capable of accounting for *all* SM problems (m_ν, **DM** and **BAU**) and further "tensions" with observation!

cLFV emerges as extremely powerful probe to test and falsify NP in the lepton sector

Explore different paths, and **profit from amazing** experimental prospects **in the near future!**

Outlook

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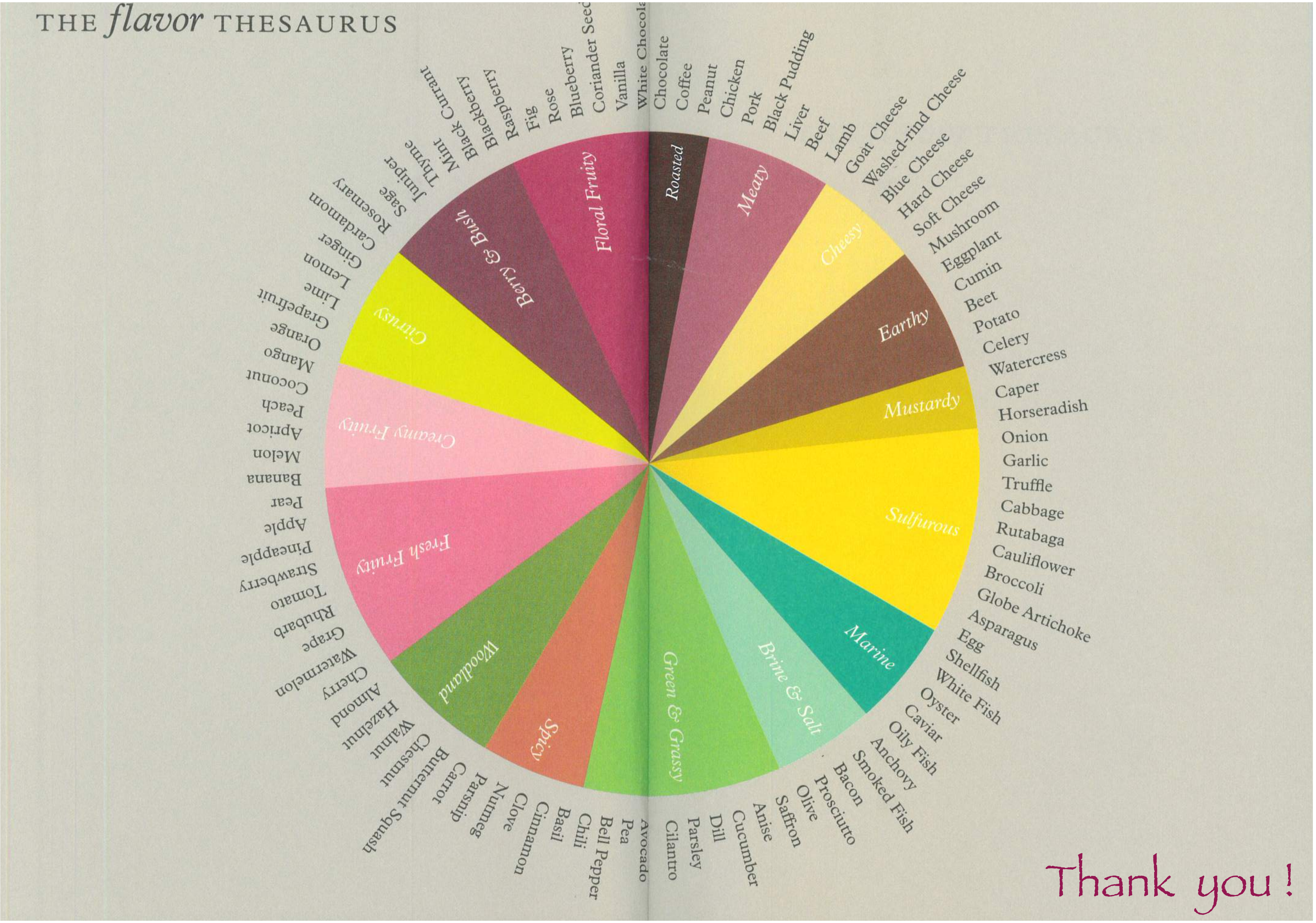
"Leave no (flavoured) stone unturned" –
leave no single grain of sand **unobserved**,
or **flavour unte(a)sted!** 😀

Happy flavours :)



Thank you !