Dark Energy and key physical parameters of clusters of galaxies

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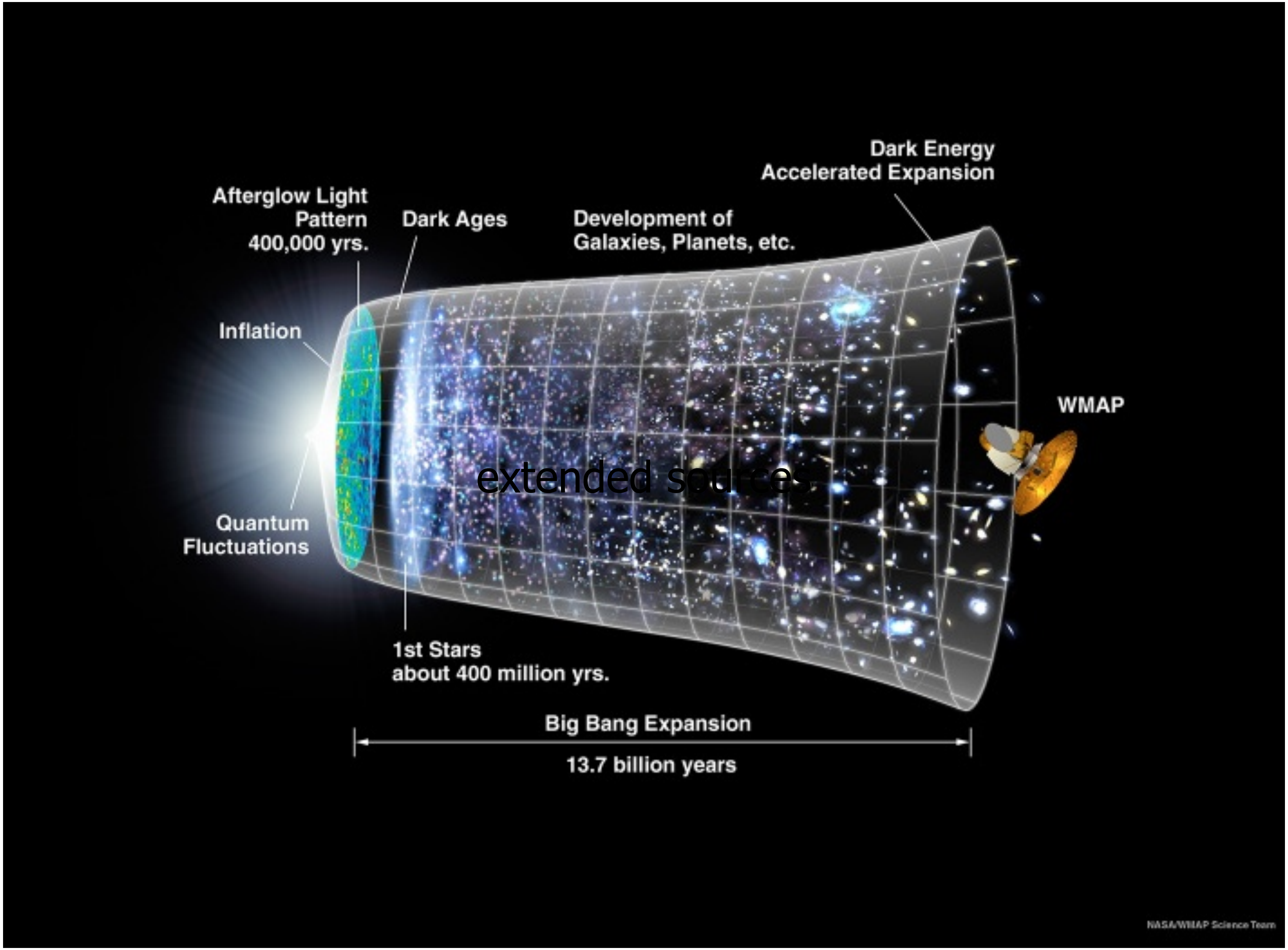


extended sources

Expanding Universe (Friedmann, Lemaitre, Hubble)

Uniform isotropic universe, Einstein equation

$$\frac{\dot{a}^2}{a^2} + \frac{k\,c^2}{a^2} = \frac{8\pi G}{3c^2}\varepsilon + \frac{\Lambda}{3}c^2.$$

Adiabatic expansion:

Для адиабатического расширения имеем

$$\frac{d\varepsilon}{\varepsilon + P} = -\frac{dV}{V} = -3\frac{da}{a}, \qquad \text{V - объем.}$$

Flat universe, *k*=0, unfinit universe, *a* – scale factor

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3c^2}\varepsilon + \frac{\Lambda}{3}c^2, \quad \varepsilon = \rho c^2 = \rho_* c^2 \frac{a_*^4}{a^4}. \qquad \frac{dx}{dt} = 2\sqrt{\frac{8\pi G\rho_*}{3}a_*^4 + \frac{\Lambda}{3}c^2x^2},$$

$$x = a^2 \qquad a^2 = a_*^2\sqrt{\frac{8\pi G\rho_*}{\Lambda c^2}}\,\mathrm{sh}\left(2\sqrt{\frac{\Lambda}{3}}ct\right).$$

Small **t** - beginning:

$$a^2 = a_*^2\sqrt{\frac{32\pi}{3}G\rho_* t}, \quad \rho = \frac{3}{32\pi G t^2}.$$

Large **t –** exponential expansion at non-zero cosmological constant:

$$a^2 = \frac{a_*^2}{2}\sqrt{\frac{8\pi G\rho_*}{\Lambda c^4}}\exp\left(2\sqrt{\frac{\Lambda}{3}}ct\right), \quad \rho = \frac{\Lambda c^4}{2\pi G}\exp\left(-4\sqrt{\frac{\Lambda}{3}}ct\right).$$

Hot Universe (Gamow, Penzias and Wilson)

Inflation

Early stages:

Exponential expansion (large Lambda term, “excited vacuum”, scalar field),

Inflation – decay of the “excited vacuum” or scalar field,

Stage of Friedmann expansion.

Non-zero Lambda term (much smaller), transition from Friedmann expansion to exponential expansion stage.

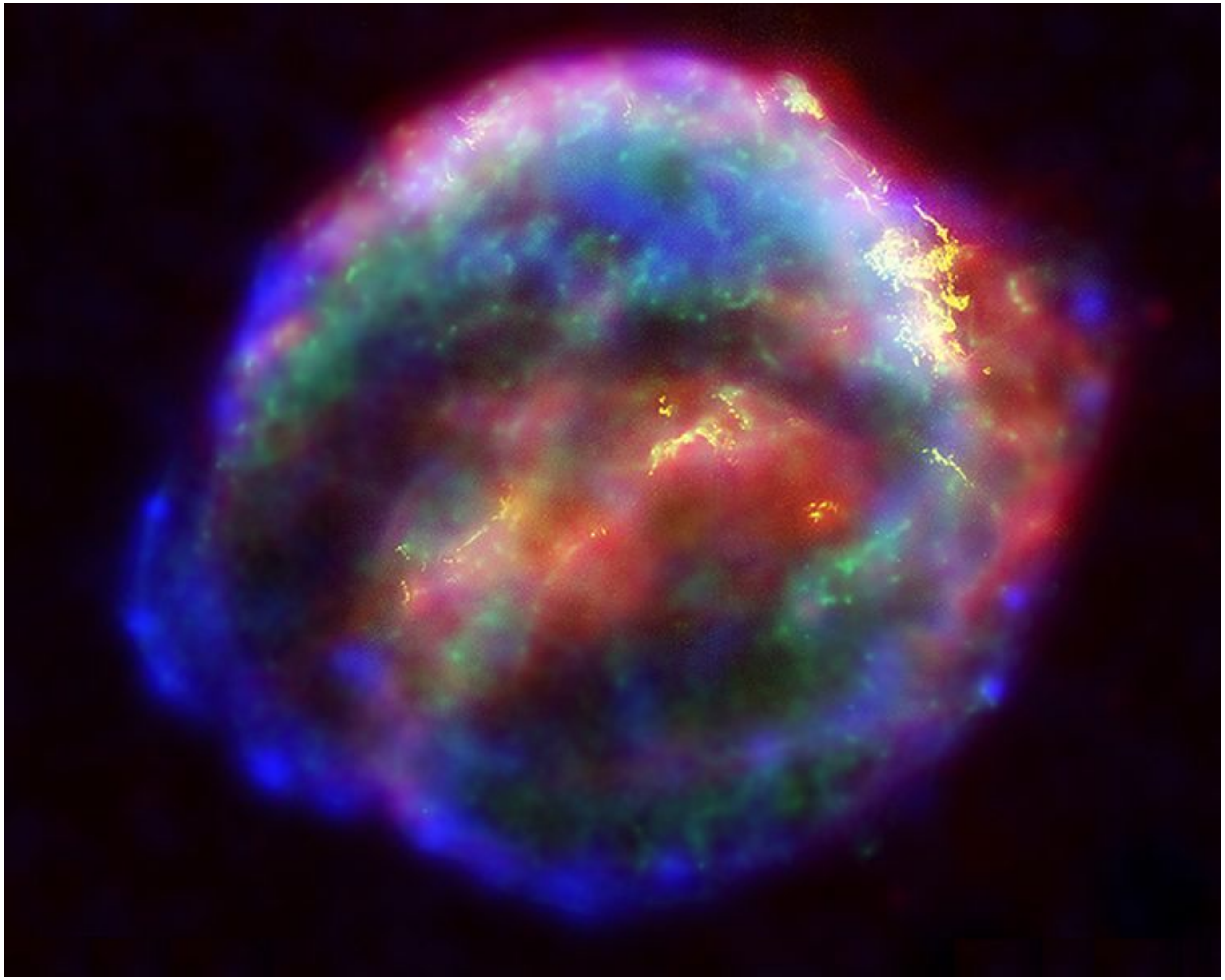
For discovery of the expansion law of the present universe we need independent measurements of the velocity and distance to very remote objects (galaxies, quasars, galaxy clusters)

Supernovae Ia – thermonuclear explosion SN – are used for these purposes, due to possibility to find its total luminosity by measurements of its light curve (type of a standard candle)

SN Ia

Composite X-ray and infrared image of the SN 1572 (Tycho's SN) remnant as seen by Chandra X-Ray Observatory, Spitzer Space Telescope, and Calar Alto Observatory

SN Ia



A false-color composite (HST/SIRTF) image of the supernova remnant nebula from SN 1604 (Kepler SN).

SN Ia,

Riess, A. G. + 19 authors

Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant (10 SNIa, 0.16 <= z <= 0.62)

The Astronomical Journal, Vol. 116, Issue 3, pp. 1009-1038 (1998)

Schmidt, B. P. + 23 authors

The Astrophysical Journal, Vol. 507: pp.46-63, 1998 November 1

The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type IA Supernovae

(>30 SNIa, 0.35 <= z <= 0.9)

Omega_M=0.4^+0.5_-0.4, Omega_Lambda=0.6^+0.4_-0.5

Unless supernovae are much different at high redshifts, the imperfection of SNe Ia as distance indicators will have a negligible impact on using SNe Ia as cosmological probes.

Perlmutter, S. + 23 authors (The Supernova Cosmology Project)

Measurements of the Cosmological Parameters Omega and Lambda from the First Seven Supernovae at Z >= 0.35

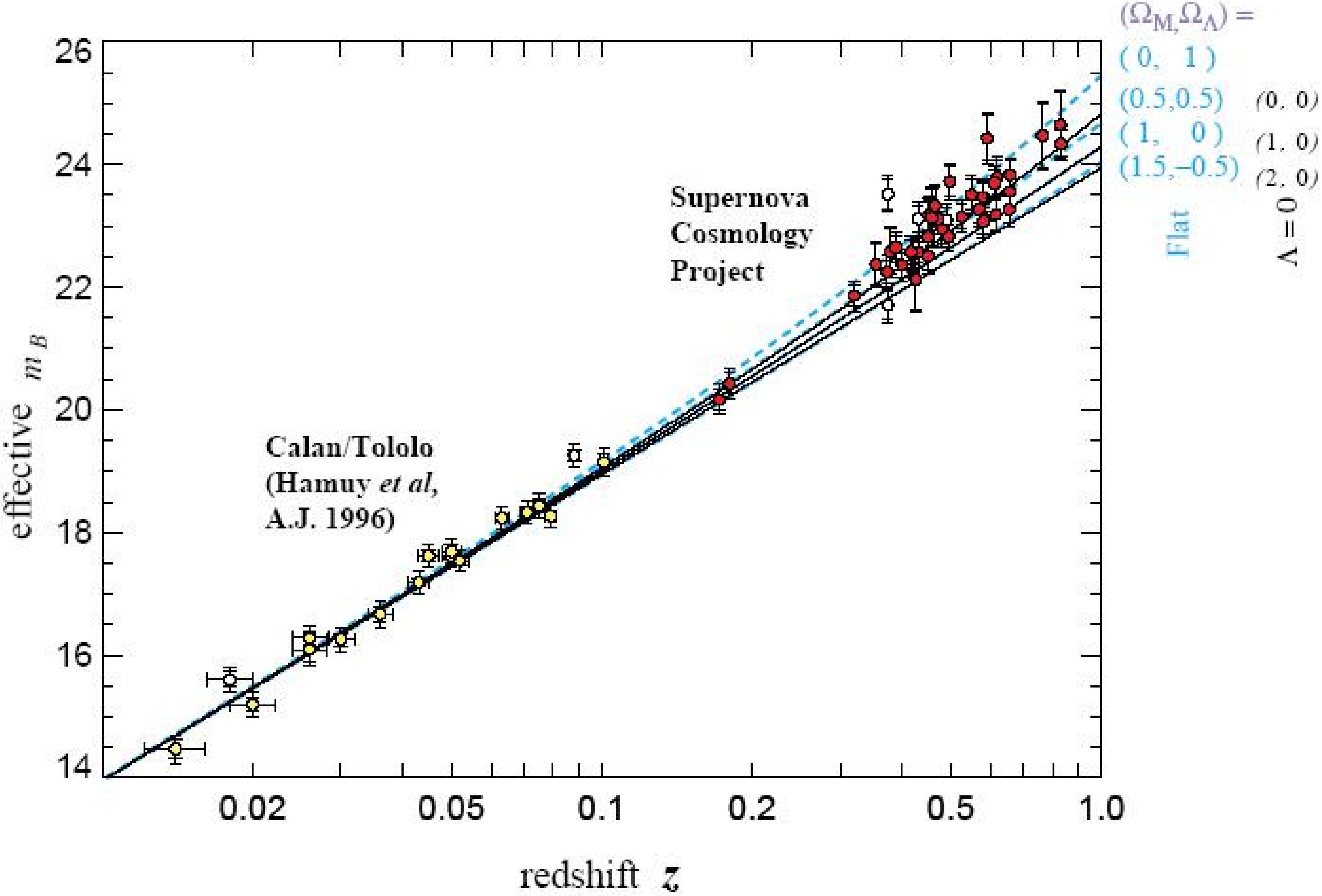
Astrophysical Journal v.483, pp.565-581 (1997)

For a spatially flat universe (Omega M + Omega Lamda = 1), we find Omega_{M} =0.94^{+0.34}_{-0.28} or, equivalently, a measurement of the cosmological constant, Omega _{ Lamda }=0.06^{+0.28}_{-0.34}

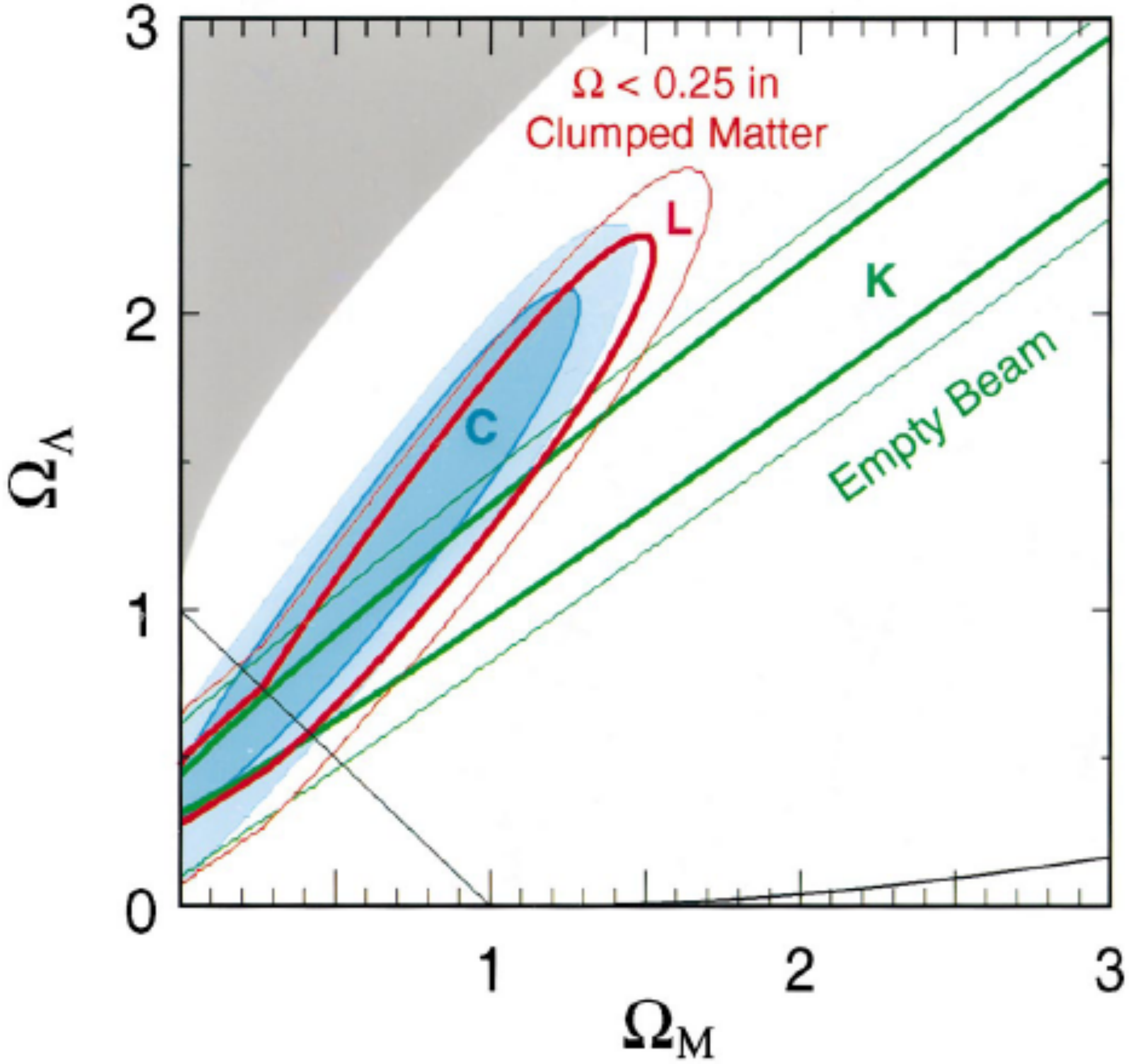
Perlmutter, S. + 32 authors (The Supernova Cosmology Project)

Measurements of Omega and Lambda from 42 High-Redshift Suprnovae (redshifts between 0.18 and 0.83)

The Astrophysical Journal, Volume 517, Issue 2, pp. 565-586 (1999)



$(\Omega_M, \Omega_\Lambda) =$
(0, 1)
(0.5,0.5)
(1, 0)
(1.5,–0.5)
(0, 0)
(1, 0)
(2, 0)
Flat
$\Lambda = 0$
Supernova Cosmology Project
Calan/Tololo (Hamuy et al, A.J. 1996)
effective m_B
redshift z
26
24
22
20
18
16
14
0.02
0.05
0.1
0.2
0.5
1.0



PERLMUTTER ET AL. (1999)

Measurements of Cosmic Microwave Background fluctuations

Satellites: Relikt, COBE, WMAP (2001), Planck (2006)

Baloons: Boomegang, Maxima, CBI, ACBAR, …

Hot Universe, flat model, Ω =1

Dark energy (Λ – term) Ω =0.7, Dark matter (nonbarionic) Ω =0.26, Baryonic Ω =0.04

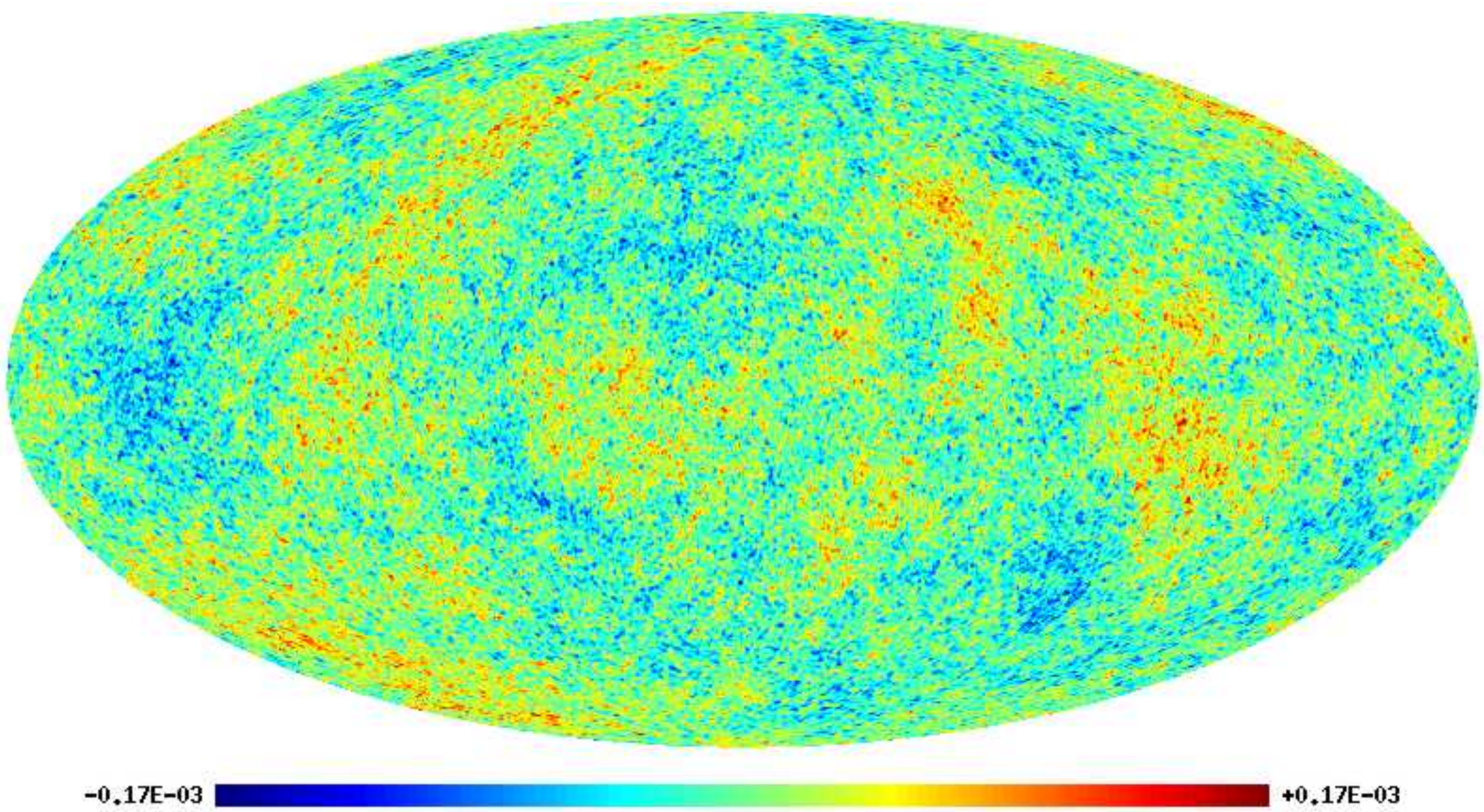
Equilibrium Planck radiation with temperature about 3 K was left as a result of expansion of the hot universe (n, gravitons).

Matter had separated from the radiation at redshift **Z** about 1000. Radiation preserves non-uniformities of that period.

Study of CMB fluctuations permitted to evaluate the global parameters of the universe:

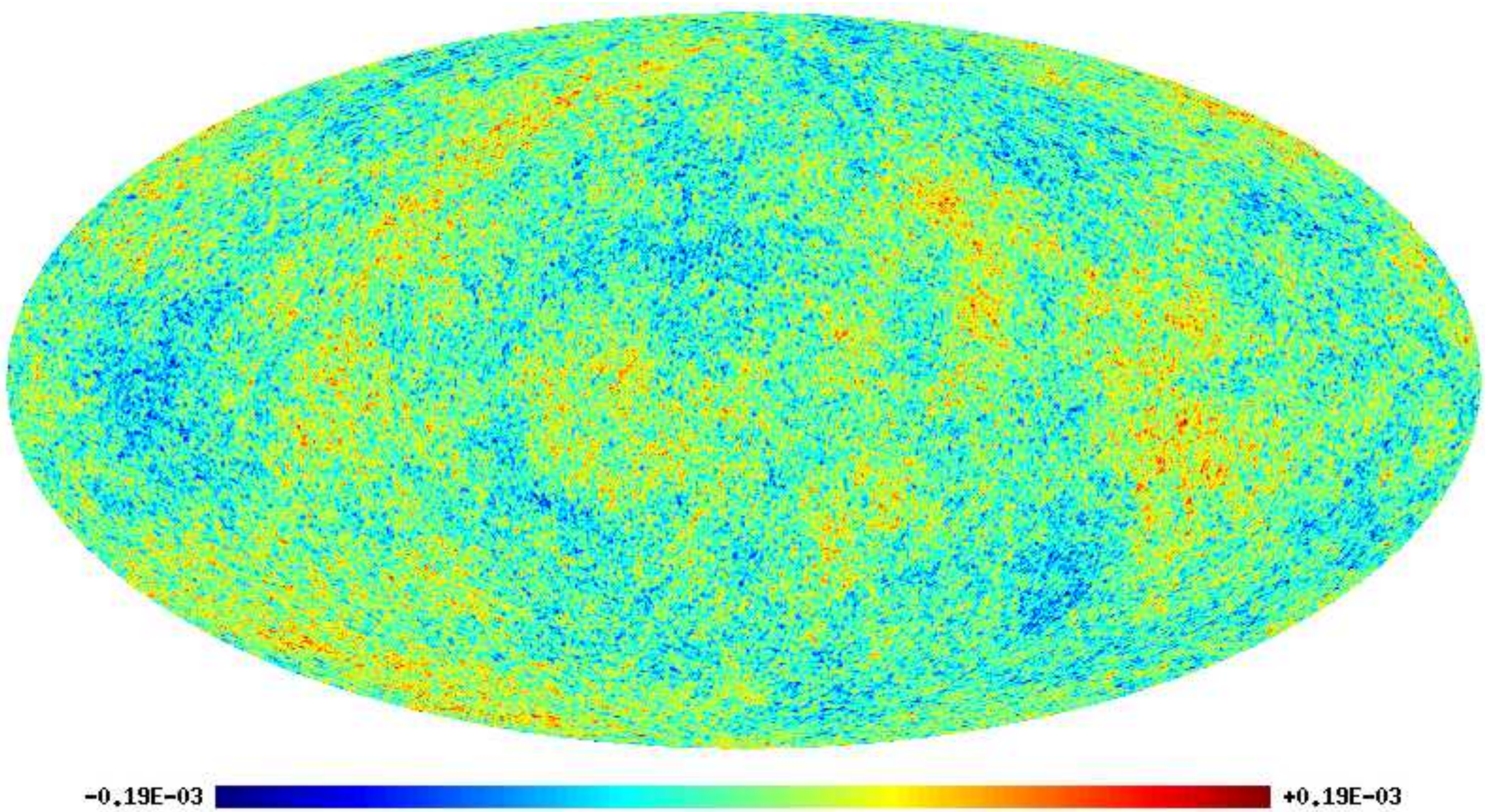
Ω and its ingradients, H – Hubble "constant", determining the rate of the universe expansion around us:

V=Hr, H~70 km/s/Mpc



WMAP

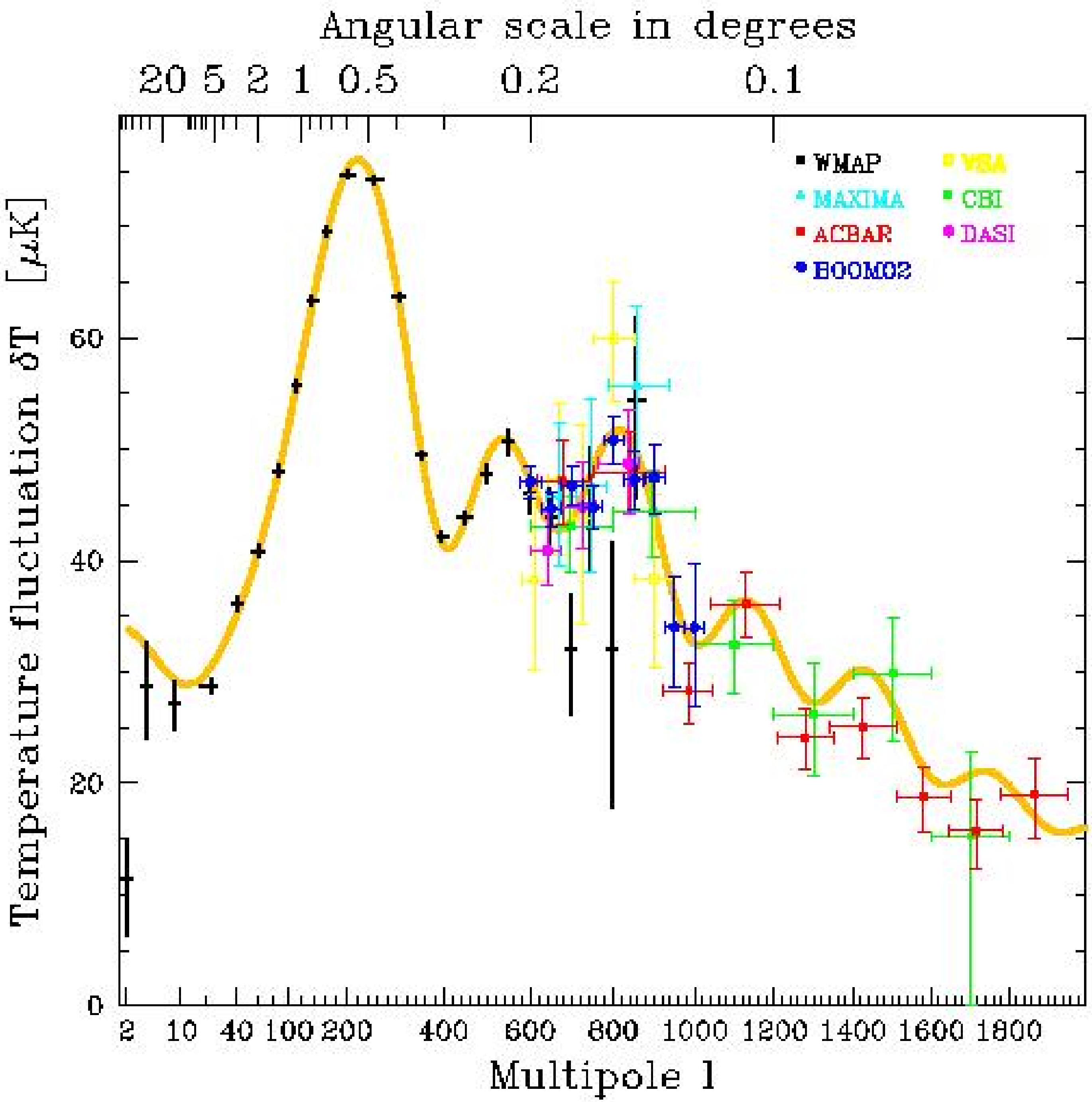
M. Liguori et al., 2003



Planck (simulation)

M. Liguori et al., 2003

All perturbations are correlated, so to the moment of recombination amplitudes of harmonics oscillale – Doppler peaks (**Sakharov oscillations**)



Angular scale in degrees
20 5 2 1 0.5 0.2 0.1
WMAP
VSA
MAXIMA
CBI
ACBAR
DASI
BOOM02
Temperature fluctuation δT [μK]
0
20
40
60
2 10 40 100 200 400 600 800 1000 1200 1400 1600 1800
Multipole l

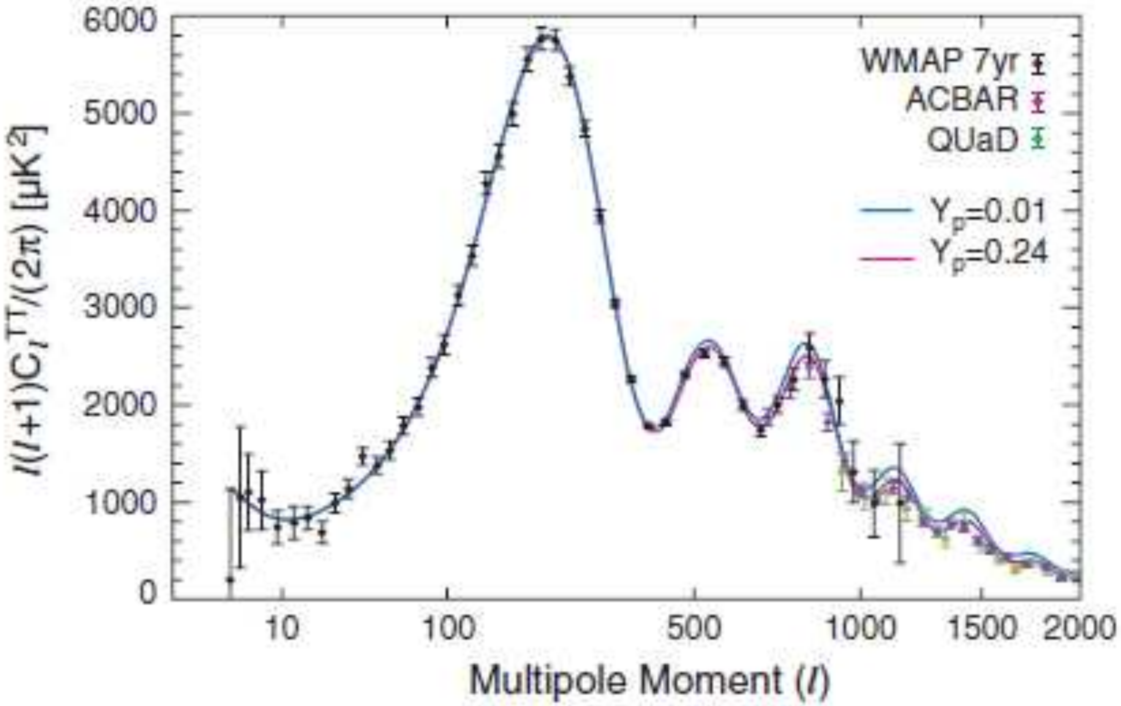


Figure 10. Primordial helium abundance and the temperature power spectrum. The data points are the same as those in Figure 7. The lower (pink) solid line (which is the same as the solid line in Figure 7) shows the power spectrum with the nominal helium abundance, $Y_p = 0.24$, while the upper (blue) solid line shows that with a tiny helium abundance, $Y_p = 0.01$. The larger the helium abundance is, the smaller the number density of electrons during recombination becomes, which enhances the Silk damping of the power spectrum on small angular scales, $l \gtrsim 500$.

In papers of **A. Chernin:, Physics-Uspekhi, 44, 1099 (2001), and Physics-Uspekhi, 51, 267 (2008),** the question was raised about a possible influence of the existence of the cosmological constant on the properties of the Hubble flow in the local galaxy cluster – close vicinity of our Galaxy. Basing on the observations of Karachentsev et al. (2006), he concluded that the presence of the the dark energy (DE) is responsible for the formation of this Hubble flow.

The importance of the DE for the structure of the local galaxy cluster (LC) depends on the level of the influence of DE on the dynamic properties. In particular, it is necessary to check, if the LC may exist in the equilibrium state, at present values of DE density, and the LC densities of matter, consisting of the baryonic, and dark matter (BM and DM).

A&A 541, A84 (2012)

A brief analysis of self-gravitating polytropic models with a non-zero cosmological constant (Research Note)

M. Merafina, G. S. Bisnovatyi-Kogan, and S. O. Tarasov

We investigate the equilibrium and stability of polytropic spheres in the presence of a non-zero cosmological constant.
We solve the Newtonian gravitational equilibrium equation for a system with a polytropic equation of state of the matter $P = K\rho$^γ introducing a non-zero cosmological constant Λ.

Consider spherically symmetric equilibrium configuration in newtonian gravity, in presence of DE, represented by the cosmological constant \Lambda. In this case the gravitational force F_g in a spherically symmetric body is given by the formula

$$F_g = -\frac{Gm}{r^2} + \frac{\Lambda r}{3},$$

$$\frac{dm}{dr} = 4\pi\rho r^2. \qquad \rho_v = \frac{\Lambda}{8\pi G}. \qquad \frac{1}{\rho}\frac{dP}{dr} = -\frac{Gm}{r^2} + \frac{\Lambda r}{3}.$$

$$P = K\rho^{\gamma}, \quad \gamma = 1 + \frac{1}{n}.$$

The virial theorem

$$\varepsilon_g = -G\int_0^M \frac{mdm}{r}, \quad m = 4\pi\int_0^r \rho r^2 dr = -\frac{r^2}{G}\left(\frac{1}{\rho}\frac{dP}{dr} - \frac{\Lambda r}{3}\right), \quad M = m(r_{out}), \; r_{out} \equiv R.$$

Here *m* is written using the equilibrium equation (4). For the adiabatic star with the polytropic equation of state we have relations

$$\rho E = nP, \quad P = \frac{\rho E}{n}, \quad I = E + PV = \frac{n+1}{n}E, \quad V = \frac{1}{\rho}.$$

Here *E* and I are thermal energy and enthalpy per gram, and V is a specific volume. Aftersome transformations, we obtain the following relations between the energies

$$\varepsilon_g = -\frac{3}{5-n}\frac{GM^2}{R} - \frac{\Lambda}{2(5-n)}MR^2 - \frac{2n-5}{5-n}\varepsilon_\Lambda,$$

$$\varepsilon_{th} = \frac{n}{5-n}\frac{GM^2}{R} + \frac{n\Lambda}{6(5-n)}MR^2 + \frac{5n}{5-n}\varepsilon_\Lambda.$$

Here $\varepsilon_\Lambda = -\frac{4\pi G\rho_v}{3}\int_0^M r^2 dm = -\frac{\Lambda}{6}\int_0^M r^2 dm$, is defined below in (39), where the additive constant in the energy definition of ε_Λ is chosen so that $\varepsilon_\Lambda = 0$ at $\Lambda = 0$, $M = 0$, $\varepsilon_{th} = \int_0^M E\, dm$. Let us make an additional transformation of the expression for ε_g. By partial integration we obtain

$$\varepsilon_{tot} = \frac{n-3}{5-n}\frac{GM^2}{R} + \frac{(n-3)\Lambda}{6(5-n)}MR^2 + \frac{2n}{5-n}\varepsilon_\Lambda.$$

Introduce non-dimensional polytropic variables

$$(\xi,\ \theta_n) \quad \text{so that } r = \alpha\xi,\ \rho = \rho_0\theta_n^n,$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta_n}{d\xi}\right) = -\theta_n^n + \beta. \quad \alpha^2 = \frac{(n+1)K}{4\pi G}\rho_0^{\frac{1}{n}-1}, \qquad \beta = \frac{\Lambda}{4\pi G\rho_0} = \frac{2\rho_v}{\rho_0}$$

Equilibrium solutions

At $n = 1$ the equation (7) is linear, and has an analytic solution. Following Chandrasekhar (1939), let us introduce a new variable $\chi = \xi\theta_1$. After the substitution the equation (7) is transforms into

At $\beta = 0$ we have a well known analytic polytropic solution (Chandrasekhar, 1939)

$$\theta_1 = \frac{\sin\xi}{\xi},$$

satisfying the boundary conditions in the center $\theta_1(0) = 1$, $\theta_1'(0) = 0$. At nonzero β, the solution satisfying the boundary conditions in the center is written as

$$\theta_1 = (1 - \beta)\frac{\sin\xi}{\xi} + \beta.$$

The radius of the configuration is defined by the first zero of the solution (21), or (22). For the Emden solution (21) the outer radius corresponds to $\xi_{out} = \pi$, and for the solution of (22) with DE, the radius of the configuration is determined by the transcendental equation

$$(1 - \beta)\frac{\sin\xi_{out}}{\xi_{out}} + \beta = 0. \tag{23}$$

This equation has real solutions only at $\beta < \beta_c$, so that at the outer boundary not only θ_1, but also $\theta_1' = 0$ at $\beta = \beta_c$. It follows from (22)

$$\theta_1' = (1 - \beta)\left(\frac{\cos\xi}{\xi} - \frac{\sin\xi}{\xi^2}\right). \tag{24}$$

therefore the parameters β_c and $\xi_{out,c}$ for the limiting equilibrium solution, for the polytropic configuration with $n = 1$ in presence of DE is determined by the algebraic equations

$$(1-\beta_c)\frac{\sin\xi_{out,c}}{\xi_{out,c}} + \beta_c = 0, \quad \tan\xi_{out,c} = \xi_{out,c}, \quad \pi < \xi_{out,c} < \frac{3\pi}{2}. \tag{25}$$

Some general relations

Equilibrium mass:

$$M_n = 4\pi\int_0^{r_{out}} \rho r^2 dr = 4\pi\left[\frac{(n+1)K}{4\pi G}\right]^{3/2} \rho_0^{\frac{3}{2n}-\frac{1}{2}} \int_0^{\xi_{out}} \theta_n^n \xi^2 d\xi.$$

$$M_n = 4\pi\left[\frac{(n+1)K}{4\pi G}\right]^{3/2} \rho_0^{\frac{3}{2n}-\frac{1}{2}} \left[-\xi_{out}^2\left(\frac{d\theta_n}{d\xi}\right)_{out} + \frac{\beta\xi_{out}^3}{3}\right].$$

$$M_n = 4\pi\rho_0\alpha^3\left[-\xi_{out}^2\left(\frac{d\theta_n}{d\xi}\right)_{out} + \frac{\beta\xi_{out}^3}{3}\right]$$

For the limiting configuration with \beta=\beta_c

$$\theta_n(\xi_{out}) = 0, \quad \frac{d\theta_n}{d\xi}|_{\xi_{out}} = 0.$$

$$M_{n,lim} = 4\pi\rho_0\alpha^3\frac{\beta_c\xi_{out}^3}{3} = \frac{4\pi}{3}r_{out}{}^3\beta_c\rho_{0c} = \frac{4\pi}{3}r_{out}{}^3\bar{\rho}_{mc},$$

so that the limiting value β_c is exactly equal to the ratio of the average matter density $\bar{\rho}_c$ of the limiting configuration, to its central density ρ_{0c}

$$\beta_c = \frac{\bar{\rho}_c}{\rho_{0c}}. \tag{31}$$

For the Emden solution at $\beta = 0$ we have $\frac{\rho_0}{\bar{\rho}_0} = 3.290,\ 5.99,\ 54.18$ for $n = 1,\ 1.5,\ 3$

The equilibrium equation for this case is

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta_3}{d\xi}\right) = -\theta_3^3 + \beta. \tag{35}$$

The mass of the configuration is written as

$$M_3 = 4\pi\left[\frac{K}{\pi G}\right]^{3/2}\hat{M}_3, \quad \hat{M}_3 = -\xi_{out}^2\left(\frac{d\theta_3}{d\xi}\right)_{out} + \frac{\beta\xi_{out}^3}{3}. \tag{36}$$

The Emden model at $\beta = 0$ has only one value of the mass, independent on the density (neutral equilibrium configuration). At $\beta \neq 0$ the dependence on the density appears,

because the function θ_3 is different for different β, and along the curve $\hat{M}_3(\hat{\rho}_0)|_\Lambda$ the value of β is inversely proportional to $\hat{\rho}_0$.

The dependence M(\rho_c), showing decreasing of mass with increase of the central density, corresponds, for the adiabatic power equal to the polytropic one, to a dynamically unstable configuration, according to the static criterium of stability (Zeldovich, 1963). When the vacuum infuence is small, it is possible to investigate the stability of the adiabatic configuration by the approximate energetic method

$$\varepsilon = \varepsilon_{th} + \varepsilon_g + \varepsilon_\Lambda + \varepsilon_{GR}$$
$$= \int_0^M E\, dm - 0.639 G M^{5/3} \rho_0^{1/3} - 0.104 \Lambda M^{5/3} \rho_0^{-2/3} - 0.918 \frac{G^2 M^{7/3}}{c^2} \rho_0^{2/3}.$$

$$\xi_{out} = 6.897, \quad \frac{\rho_0}{\bar{\rho}} = 54.18, \quad r_{out} = \alpha \xi_{out} = \frac{M^{1/3} \rho_0^{-1/3}}{0.426}, \quad r = \alpha \xi = r_{out} \frac{\xi}{\xi_{out}}.$$

The equilibrium configuration is determined by the zero of the first derivative of the energy over \rho_c, and stable configuration has a positive second derivative. It is more convenient to take derivatives over \rho_c^{1/3}*, so we have*

$$3\rho_0^{-4/3} \int_0^M P \frac{dm}{\phi(m/M)} - 0.639 G M^{5/3} + 0.208 \Lambda M^{5/3} \rho_0^{-1} - 1.84 \frac{G^2 M^{7/3}}{c^2} \rho_0^{1/3} = 0,$$

$$9\rho_0^{-5/3}\int_0^M \left(\gamma-\frac{4}{3}\right) P\frac{dm}{\phi(m/M)} - 0.624\Lambda M^{5/3}\rho_0^{-4/3} - 1.84\frac{G^2M^{7/3}}{c^2} > 0.$$

$$\gamma = \left(\frac{\rho}{P}\frac{\partial P}{\partial \rho}\right)|_S, \quad \rho = \rho_0\phi\left(\frac{m}{M}\right),$$

DE input in the stability of the configuration is negative, and similar to the fof the general relativistic correction. Contrary, the presence of the dark matter, as a background, has a positive influence, increasing the dynamic stability of the configuration (McLaughlin and Fuller, 1996; Bisnovatyi-Kogan, 1998). **The adiabatic star with a polytropic power 4/3, which is in the state of a neutral stability in the pure newtonian gravity, becomes unstable in presence of DE.**

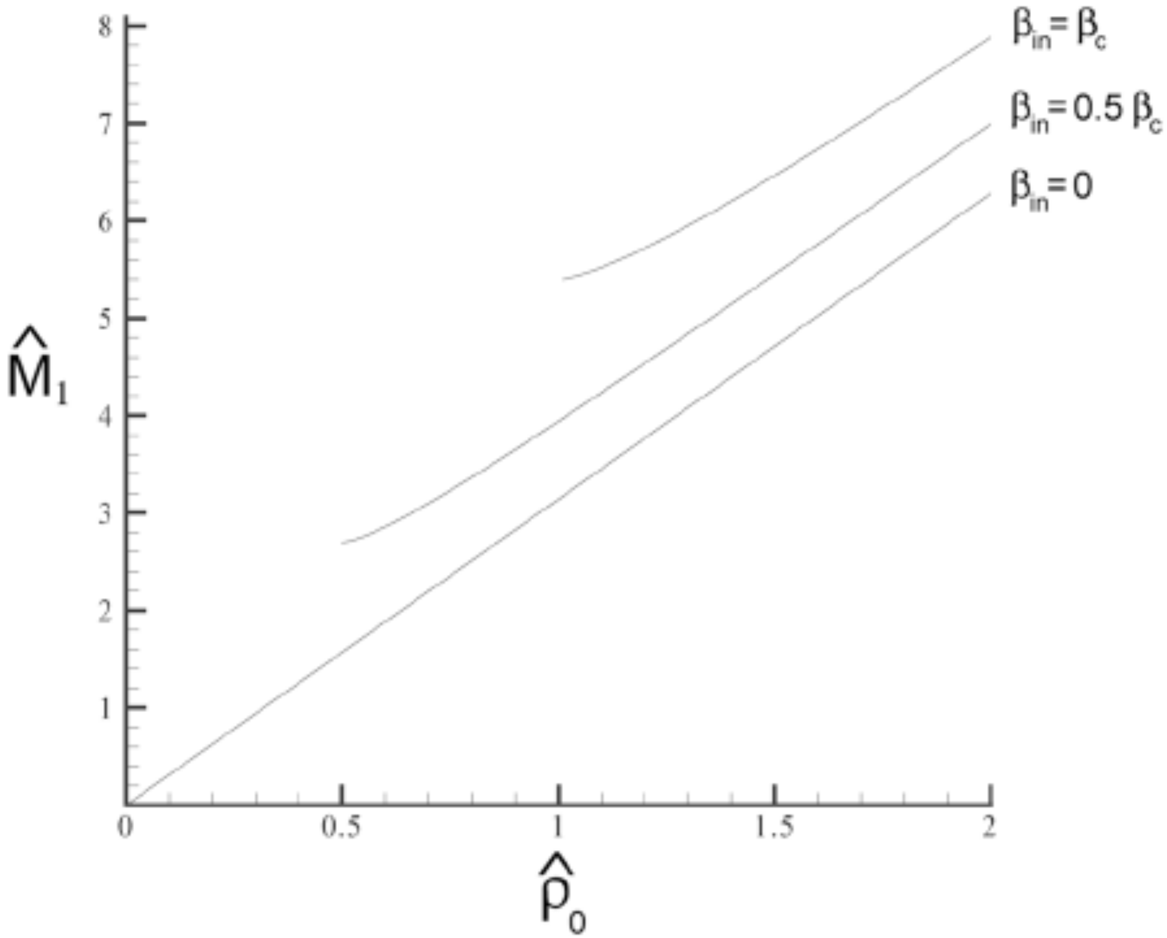


Fig. 1. Non-dimensional mass $\hat{M}_1$ of the equilibrium polytropic configurations at $n = 1$ as a function of the non-dimensional central density $\hat{\rho}_0$, for different values of $\beta_{\rm in}$. The cosmological constant Λ is the same along each curve. The curves at $\beta_{\rm in} \neq 0$ are limited by the configuration with $\beta = \beta_{\rm c}$.

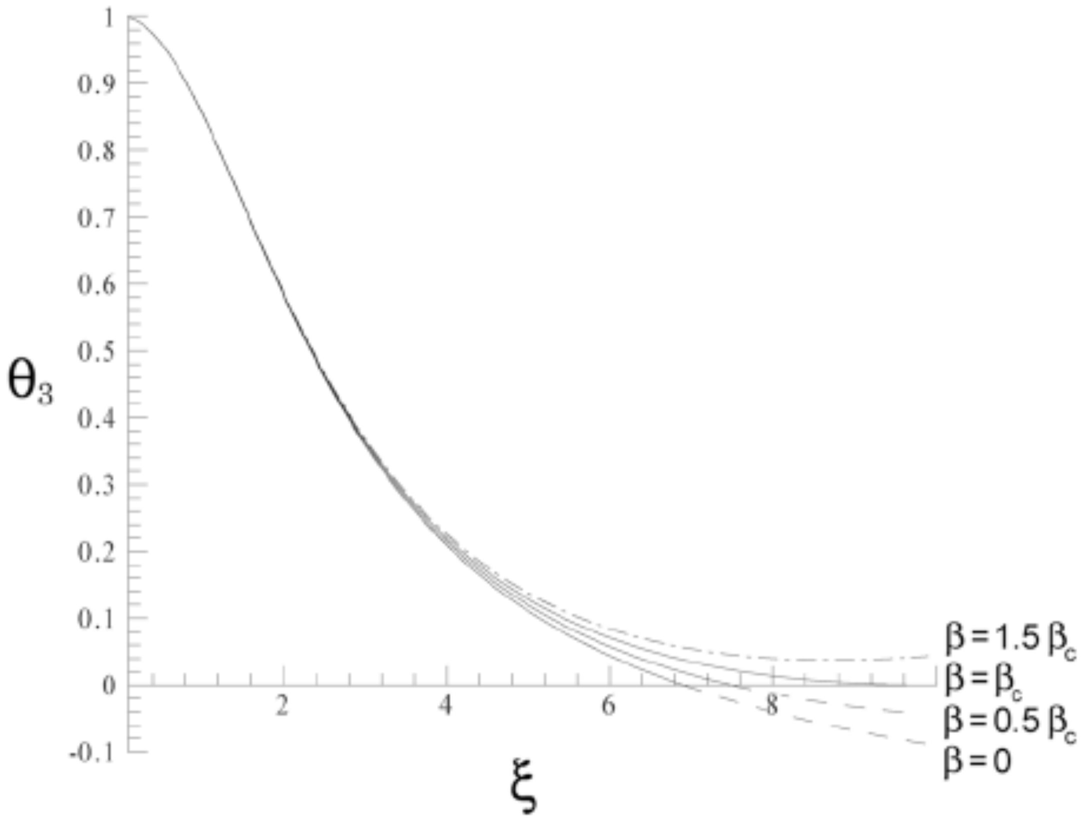


Fig. 2. The density distribution for configurations at $n = 3$ with $\beta = 0$, $\beta = 0.5\beta_c$, and $\beta = \beta_c$. The curves are marked with the values of β. The non-physical solution at $\beta = 1.5\beta_c$, which does not have an outer boundary, is given by the dash-dot line. The non-physical parts of the solutions at $\beta \leq \beta_c$, behind the outer boundary, are given by the dash lines. The solutions asymptotically approach, at large ξ, the horizontal line $\theta_3 = \beta^{1/3}$.

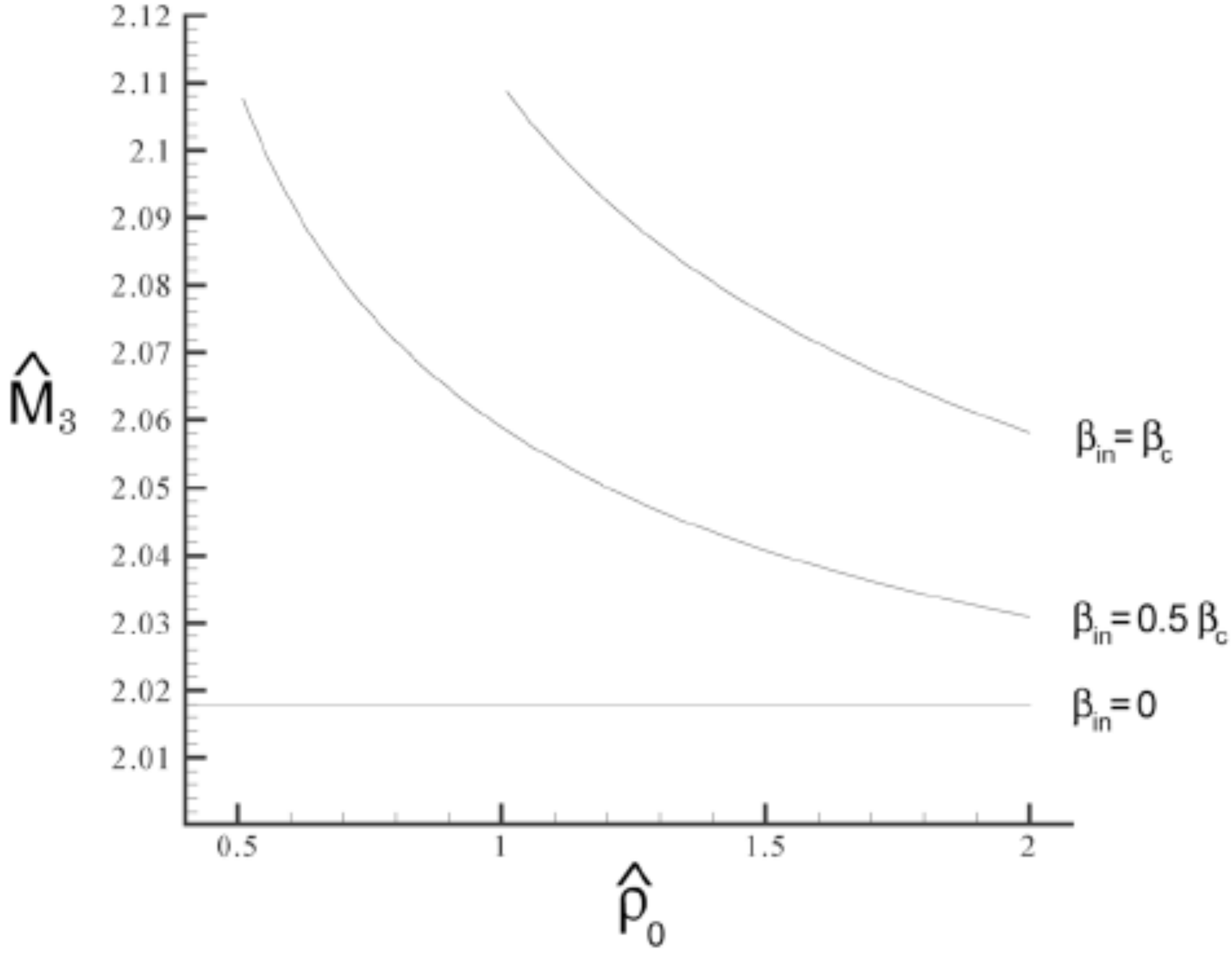


Fig. 3. Same as in Fig. 1, for $n = 3$.

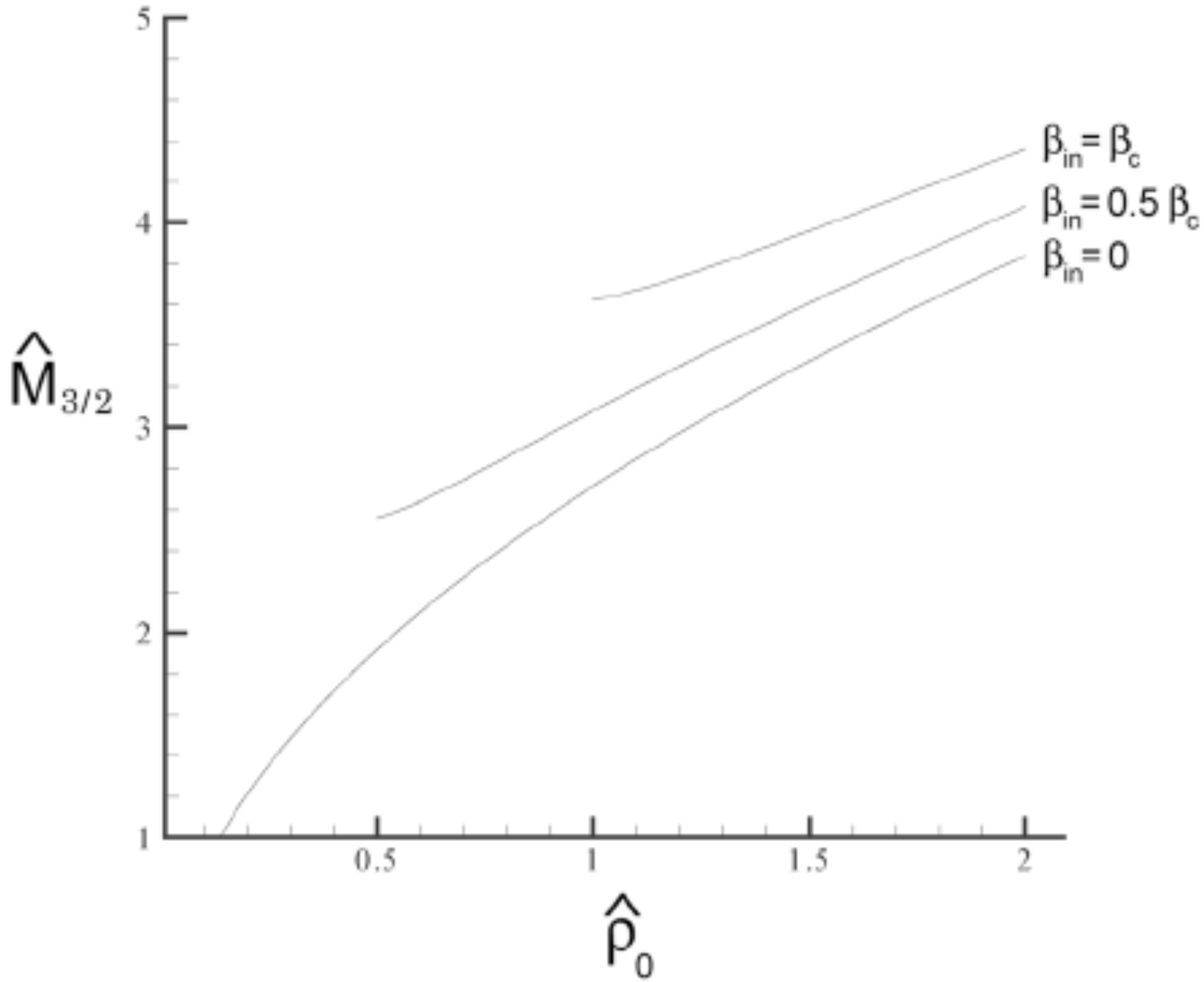


Fig. 4. Same as in Fig. 1, for $n = 3/2$.

Application to Local cluster (LC)

The question about the importance of DE to the dynamics of the Local Cluster (LC) was raised by Chernin (2008). Presently accepted values of the DE density $\rho_v = (0.72\pm0.03) \times 10^{-29}$ g/cm^3, the mass of the Local Group, including its dark matter input, is between $MLC \sim 3.5 \times 10^{12} M_{solar}$, and $\sim 1.3 \times 10^{12} M_{solar}$. The radius *RLC* was estimated by measuring the velocity dispersion v_t of galaxies in the LC, and by the application of the virial theorem. The estimated velocity dispersion of galaxies in the LC, which has been found to equal $v_t = 63$ km s^{-1}, is very close to the value of the local Hubble constant $H = 68$ *km* s^{-1} *Mpc*$^{-1}$ (Karachentsev et al. 2006). The similarity between these values indicates the great difficulties in dividing the measured velocities between regular and chaotic components. The radius of the LC have values between 1.5 Mpc and 4 Mpc. Chernin et al. (2009) identifies the radius *RLC with the* radius R_v of the zero-gravity force, which is identical to the one corresponding to our critical model with $\beta = \beta_c$, in which the average matter density is equal to $2\rho_v$: $1.2 < M_{LC} < 3.7 \times 10^{12} M_{solar}$, and $1.1 < R_v < 1.6 Mpc$. All these estimations show the importance of the presently accepted value of DE density on the structure and dynamics of the outer parts of LC, and its vicinity.

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Dark energy and key physical parameters of clusters of galaxies

G.S. Bisnovatyi-Kogan · A.D. Chernin

We study physics of clusters of galaxies embedded in the cosmic dark energy background. Under the assumption that dark energy is described by the cosmological constant, we show that the dynamical effects of dark energy are strong in clusters like the Virgo cluster. Specifically, the key physical parameters of the dark mater halos in clusters are determined by dark energy:

(1) the halo cut-off radius is practically, if not exactly, equal to the zero-gravity radius at which the dark matter gravity is balanced by the dark energy antigravity;

(2) the halo averaged density is equal to two densities of dark energy;

(3) the halo edge (cut-off) density is the dark energy density with a numerical factor of the unity order slightly depending on the halo profile.

$$F(r) = F_N(r) + F_E(r) = -G\frac{M}{r^2} + \frac{8\pi G}{3}\rho_\Lambda r.$$

The total force *F* and the acceleration are both zero at the distance

$$r = R_\Lambda = \left[\frac{M}{\frac{8\pi}{3}\rho_\Lambda}\right]^{1/3}.$$

Here R_*lambda* is the zero-gravity radius

This radius is an absolute upper limit for the radial size *R* of a static cluster:

Taking for an estimate the total mass of the Virgo cluster (dark matter and baryons) M = (0.6–1.2) × 10^15 M_solar and the cosmological dark energy density *ρ_v,* one finds the zero-gravity radius of the Virgo cluster: R _*lambda*= (9–11) Mpc.

Estimation of Lambda from the galaxy cluster parameters, with zero gravity radius

The gravitational potential $\Phi(r)$ inside the cluster comes from the Poisson equation:

$$\Delta\Phi = 4\pi G(\rho - 2\rho_\Lambda). \tag{31}$$

Restricting ourselves for simplicity by the model of the isothermal halo, we find from (31) together with (14), (16):

$$\frac{d\Phi}{dr} = -\frac{8\pi G}{3}\rho_\Lambda r\left[1 - \left(\frac{R_\Lambda}{r}\right)^2\right], \quad 0 \le r \le R_\Lambda. \tag{32}$$

In accordance with what was said above, the acceleration $-\frac{d\Phi}{dr} = 0$ at $r = R_\Lambda$. The extremum (maximum) of the potential Φ is located at the same distance $r = R_\Lambda$.

The potential comes from integration of (32):

$$\Phi(r) = -\frac{4\pi G}{3}\rho_\Lambda r^2\left[1 - 2\left(\frac{R_\Lambda}{r}\right)^2 \ln\frac{r}{R_\Lambda}\right] + C. \tag{33}$$

The constant C may be found from the boundary condition at $r = R_\Lambda$:

$$\Phi(R_\Lambda) = -\frac{4\pi G}{3}\rho_\Lambda R_{ZG}^2 + C = -\frac{4\pi G}{3}\rho_\Lambda R_\Lambda^2 - \frac{GM}{R_\Lambda}. \tag{34}$$

It implies that $C = -\frac{GM}{R_\Lambda}$. Then the maximum of the potential

$$\Phi_{max} = -\frac{3}{2}\frac{GM}{R_\Lambda} = -\frac{3}{2}G\left(\frac{8\pi}{3}\rho_\Lambda\right)^{1/3} M^{2/3}. \tag{35}$$

The value of Φ_{max} depends on the cluster matter mass M and the universal dark energy density. Its value is the same for any halo profile. It gives a quantitative measure to the deepness of the cluster potential well and determines the characteristic isothermal velocity of the gravitationally bound particles (galaxies, intracluster plasma particles and dark matter particles) in the cluster:

$$V_{iso} = |\Phi_{max}|^{1/2} = G^{1/2}\left(\frac{3}{2}\right)^{1/2}\left(\frac{8\pi}{3}\rho_\Lambda\right)^{1/6} M^{1/3}$$

$$= 780\left[\frac{M}{10^{15} M_\odot}\right]^{1/3}.$$

As we see, this velocity is rather close to the mean velocity dispersion, $V \simeq 700$ km/s, of the galaxies in the Virgo cluster

The plasma isothermal temperature

$$T_{iso} = \frac{Gm}{3k} V_{iso}^2 = \frac{m}{3k}\left(\frac{8\pi}{3}\rho_\Lambda\right)^{1/3} M^{2/3}$$

$$= 3 \times 10^7 \left[\frac{M}{10^{15} M_\odot}\right]^{2/3} K,$$

where k, m are the Boltzmann constant and the hydrogen atom mass. The temperature is roughly equal to the temperature of the hot X-ray emitting plasma in clusters like the Virgo. Identifying theoretical value V_iso with the observed value V for typical clusters, we see that the matter mass of a cluster can be estimated, if the velocity dispersion of its galaxies is measured:

$$M = \left(\frac{2}{3G}\right)^{3/2} \left(\frac{8\pi}{3}\rho_\Lambda\right)^{-1/2} V_{iso}^3 = 10^{15} M_\odot \left(\frac{V}{780\ \mathrm{km/s}}\right)^3$$

In a similar way, the mass may be found, if the theoretical value of the temperature T_*iso* is identified with the measured temperature of the intracluster plasma:

$$M = \left(\frac{3k}{Gm}\right)^{3/2} \left(\frac{8\pi}{3}\rho_\Lambda\right)^{-1/2} T_{iso}^{3/2} = \left(\frac{T}{2 \times 10^7\ K}\right)^{3/2} \times 10^{15} M_\odot$$

Finally, if the matter mass of a cluster and its velocity dispersion or its plasma temperature are measured independently, enable one to estimate the density of the dark energy:

$$\rho_{loc} = \rho_\Lambda \left(\frac{M}{10^{15} M_\odot} \right)^{-2} \left(\frac{V}{780 \text{ km/s}} \right)^6,$$

$$\rho_{loc} = \rho_\Lambda \left(\frac{M}{10^{15} M_\odot} \right)^{-2} \left(\frac{T}{3 \times 10^7 \ K} \right)^3.$$

The empirical data on clusters like the Virgo are consistent with assumption that the radius of the cluster coincides with the zero gravity radius, and the dark energy density is universal in the universe.

Conclusions

The density of DE, measured from SN Ia distributions, and spectra of fluctuations CMB perturbations, imply the necessity to take it into account in calculations of the structure of galaxy clusters. We have done these calculations in the simple polytropic model, in which it is possible to see how the model changes with increasing of the influence of DE.

The existing observational indefiniteness in the parameters of LC indicate to the dynamic importance of DE in the scale of the galaxy clusters.

Observational data on the Virgo cluster suggest its radius R is roughly, if not exactly, equal to system's zero-gravity radius R_Λ. For the Virgo cluster $R \simeq R_\Lambda \simeq 10$ Mpc.