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Constraining spacetime torsion with the Moon and Mercury

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[Phys. Rev. D **83**, 2011], [Gen. Rel. Gravitation **38**, 2011]

Plan of the talk

- short introduction to the problem and to torsion
- constraining torsion with the Moon (geodetic precession, three-body computation)
- constraining torsion with Mercury (perihelion computation)
- constraining torsion with LAGEOS (Lense-Thirring effect).

• In Einstein General Relativity (GR) spacetime is a manifold endowed with a connection ${\Gamma^{\nu}}_{\lambda\mu}$ coming from a locally lorentzian metric $g_{\lambda\mu}$. In particular ${\Gamma^{\nu}}_{\lambda\mu}$ is symmetric, i.e.

$$ {\Gamma^{\nu}}_{\lambda\mu} = {\Gamma^{\nu}}_{\mu\lambda} \tag{1} $$

and gives origin to a nonvanishing curvature tensor. Because of (1), in GR torsion is not present.

• From the point of view of differential geometry, it seems to be natural to consider also a nonvanishing torsion tensor, and *remove* assumption (1).

Mao, Tegmark, Guth and Cabi (MTGC) [Phys. Rev. D **76**, 2007] claim that if there are theories giving rise to detectable torsion in the Solar system, they should be tested experimentally. They

- develop a parametrized theory imposing symmetry principles
- compute the precessions of gyroscopes
- put constraints on torsion with Gravity Probe B experiment.

In the teleparallel theory of Hayashi and Shirafuji [Phys. Rev. D **19**, 1979] a massive body generates a torsion field. Gravitational forces are due entirely to spacetime torsion and not to curvature.

Our plan: to follow the nonstandard parametrized torsion model of MTGC, in the case of orbits of a test body.
We compute the torsion corrections to:

- the precession of the pericenter of a body (either the Moon or LAGEOS) orbiting around a central mass,
- the orbital geodetic precession,
- the orbital Lense-Thirring effect.

We then use the measured Moon geodetic precession, Mercury's perihelion advance and the data from the measurements of LAGEOS satellites, to put contraints on torsion.

Torsion of $\Gamma^{\nu}{}_{\lambda\mu}$ (E. Cartan, 1922)

Model of spacetime (Riemann-Cartan spacetime):

- a metric $g_{\lambda\mu}$, which yields the length $ds^2 = g_{\lambda\mu} dx^{\lambda} dx^{\mu}$
- a nonsymmetric connection $\Gamma^{\nu}{}_{\lambda\mu} \neq \Gamma^{\nu}{}_{\mu\lambda}$

not independent, because they are required to be compatible one each other: the Γ-parallel transport conserves the g scalar product. The torsion tensor T reads as:

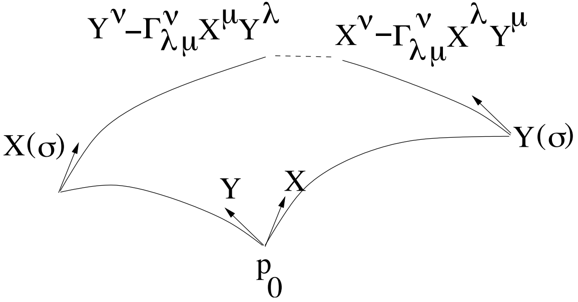
$$T_{\lambda\mu}{}^{\nu} := \frac{1}{2}\left(\Gamma^{\nu}{}_{\lambda\mu} - \Gamma^{\nu}{}_{\mu\lambda}\right)$$

$\Gamma^{\nu}{}_{\mu\nu}$ is determined uniquely by $g_{\mu\nu}$ and by the torsion tensor as follows:

$$\Gamma^{\nu}{}_{\lambda\mu} = \left\{ {\nu \atop \lambda\mu} \right\} + T_{\lambda\mu}{}^{\nu} + T^{\nu}{}_{\mu\lambda} + T^{\nu}{}_{\lambda\mu}$$

where $\{\cdot\}$ is the Levi-Civita connection.

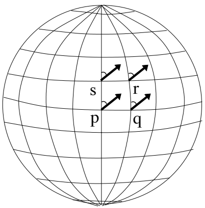
Geometric meaning of torsion tensor and asymmetry of a connection: an attempt to construct an infinitesimal parallelogram with sides $d\sigma$ spanned by two vectors X^λ, Y^μ, leads to a pentagon whose closing side is $\left(\Gamma^\nu{}_{\lambda\mu} - \Gamma^\nu{}_{\mu\lambda}\right)(d\sigma)^2 + O((d\sigma)^3)$

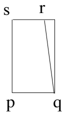


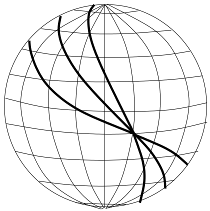
The hessian of a function defined on M is not symmetric.
Examples: density of dislocations in a crystal (Hehl et al, 1973).

Example (E. Cartan, 1923). Consider the unit sphere in $\mathbb{R}^3$ without the poles, with $g_{\lambda\mu}$ induced (riemannian) metric. We call two vectors parallel if their angles wih the meridian through that point coincide. Then

- $\Gamma^{\nu}{}_{\lambda\mu}$ is compatible with the metric
- $\Gamma^{\nu}{}_{\lambda\mu}$ has vanishing Riemann tensor (parallel transport a vector along a quadrilateral made up of latitudes and longitudes)
- $\Gamma^{\nu}{}_{\lambda\mu}$ has nonvanishing torsion, $T^{\phi}_{\theta\phi}$
- the autoparallels of $\Gamma^{\nu}{}_{\lambda\mu}$ differ from geodesics, and intersect the meridians at a constant angle.







Testing spacetime torsion by secular perturbations of orbits:

- geodetic precession (de Sitter 1916) of the Moon perigee: 19.2 mas/yr in GR;
- correction to the secular precession of the perihelion of Mercury: $43''$/century in GR;
- Lense-Thirring effect (1918) on the longitude of the nodes of the LAGEOS satellites: 31 mas/yr in GR.

Torsion modifies such effects. Measurements of orbital elements (by Laser Ranging, Moon and LAGEOS satellites) and Radar Ranging (Mercury) can be used to constrain torsion.

Main working assumptions:

- weak field approximation and slow motion
- spherical (resp. axial, with Earth spherically symmetric and uniformly rotating) symmetry
- bodies move along autoparallel trajectories, and not along geodesics. The system expressing autoparallel trajectories is

$$\frac{d^2x^\nu}{d\tau^2} + \Gamma^\nu{}_{\lambda\mu}\frac{dx^\lambda}{d\tau}\frac{dx^\mu}{d\tau} = 0,$$

  the velocity vector is transported parallel to itself. Only autoparallel trajectories depend on torsion parameters. The antisymmetric part of $T^\nu{}_{\lambda\mu}$ cannot be measured.
- in the computation of the geodetic precession (a three-body problem, Sun and Earth supposed nonrotating) we assume that the we can superimpose linearly the metric and torsion fields of Sun and Earth to obtain the global fields:

$$g_{\lambda\mu} = (g_{\lambda\mu})_{\mathrm{Sun}} + (g_{\lambda\mu})_{\mathrm{Earth}} \qquad T_{\lambda\mu}{}^{\nu} = (T_{\lambda\mu}{}^{\nu})_{\mathrm{Sun}} + (T_{\lambda\mu}{}^{\nu})_{\mathrm{Earth}}$$

Metric and torsion tensors around a spherically symmetric body (Sun/Earth) in spherical coordinates (t, r, θ, ϕ). If m is the mass of the body, and γ and β are PPN parameters, to second order in $m/r << 1$ we have

$$ds^2 = -h(r)dt^2 + f(r)dr^2 + r^2\left[d\theta^2 + \sin^2\theta d\phi^2\right]$$

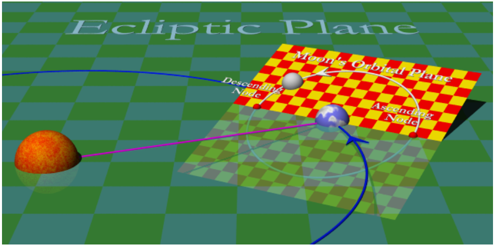
$$h(r) = 1 - 2\frac{m}{r} + 2(\beta - \gamma)\frac{m^2}{r^2}, \qquad f(r) = 1 + 2\gamma\frac{m}{r}$$

where all other PPN parameters are supposed to be negligible. Symmetry arguments imply that the general expression of the torsion tensor is

$$T_{tr}{}^{t} = t_1\frac{m}{2r^2} + t_3\frac{m^2}{r^3}$$

$$T_{r\theta}{}^{\theta} = S_{r\phi}{}^{\phi} = t_2\frac{m}{2r^2} + t_4\frac{m^2}{r^3}$$

t_1, t_2, t_3, t_4 are torsion parameters (all other components vanish. In addition t_4 will not enter in our computations).
$t_1 = 0$ will be fixed by the imposing the Newtonian limit.



$\xi^\alpha, \rho_\oplus$ heliocentric rect. coordinates of the Earth
X^α, Δ heliocentric rect. coordinates of the Moon
x^α, r geocentric rect. coordinates of the Moon
Autoparallel trajectories ($m_\odot$ mass of the Sun):

$$\frac{d^2x^\alpha}{dt^2} + m_\odot\left(\frac{X^\alpha}{\Delta^3} - \frac{\xi^\alpha}{\rho_\oplus^3}\right) = 2(\beta - t_3)m_\odot^2\left(\frac{X^\alpha}{\Delta^4} - \frac{\xi^\alpha}{\rho_\oplus^4}\right)$$
$$+ (t_2 + 2)m_\odot\left(\frac{\dot\Delta\dot\xi^\alpha}{\Delta^2} - \frac{\dot\rho_\oplus\dot\xi^\alpha}{\rho_\oplus^2}\right) + 3\gamma m_\odot\left(\frac{X^\alpha\dot\Delta^2}{\Delta^3} - \frac{\xi^\alpha\dot\rho_\oplus^2}{\rho_\oplus^3}\right)$$
$$- (2\gamma + t_2)m_\odot\left(\frac{X^\alpha\sum_\sigma(\dot X^\sigma)^2}{\Delta^3} - \frac{\xi^\alpha\sum_\sigma(\dot\xi^\sigma)^2}{\rho_\oplus^3}\right)$$

Using perturbative methods in Celestial Mechanics, we find the secular precession of the node Ω of the satellite orbiting around Earth:

$$(\delta\Omega)_{\rm sec} = \frac{1}{2}\frac{m_\odot \nu_0}{\rho_\oplus}\left(1 + 2\gamma + \frac{3}{2}t_2\right)$$

where ν_0 is the angular velocity of the Earth. $(\delta\Omega)_{\rm sec}$ is independent of the details of the satellite.

The same result is obtained for the secular precession of the lunar perigee

$$(\delta\widetilde{\omega})_{\rm sec} = \frac{1}{2}\frac{m_\odot \nu_0}{\rho_\oplus}\left(1 + 2\gamma + \frac{3}{2}t_2\right)$$

When torsion is zero the usual PPN formulas are found (in particular, when $\gamma = \beta = 1$ de Sitter's formulas).

Constraining torsion with the Moon

$$b \equiv \frac{\text{geodetic precession with torsion}}{\text{geodetic precession in GR}} = \frac{1}{3}(1+2\gamma) + \frac{t_2}{2}$$

Using the Lunar Laser Ranging data giving the relative deviation from GR (Williams et al., 2004), we find

$$|b-1| < 0.0064$$

Using the Cassini measurement $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$ (Bertotti et al., 2003) gives

$$|t_2| < 0.0128$$

Constraining torsion with Mercury

We use the autoparallel trajectories. If $\widetilde{\omega}$ is the longitude of the pericenter, the secular contribution $(\delta\widetilde{\omega})_{\rm sec}$ reads as

$$(\delta\widetilde{\omega})_{\rm sec} = (2 + 2\gamma - \beta + 2t_2 + t_3)\,\frac{m_\odot}{a(1-e^2)}v$$

a is the semimajor axis of the orbit, e is the eccentricity and v the true anomaly. Then

$$B \equiv \frac{\text{perihelion precession with torsion}}{\text{perihelion precession in GR}} = \frac{1}{3}(2+2\gamma-\beta+2t_2+t_3)$$

Using the planetary radar ranging data giving the relative deviation from GR of 10^{-3} (Shapiro et al., 1989), we find

$$|B-1| < 0.001.$$

Using the Cassini measurement one gets $|1-\beta+t_3| < 0.0286$. If in addition we assume $\beta = 1 + (1.2 \pm 1.1) \times 10^{-4}$ (Williams et al, 2004), then

$$|t_3| < 0.0286$$

Constraining torsion with LAGEOS

Lense-Thirring effect. Metric around a uniformly rotating spherical body (Earth):

$$ds^2 = -\left(1 - 2\frac{m}{r}\right) dt^2 + \left(1 + 2\gamma\frac{m}{r}\right) dr^2 + r^2\left(d\theta^2 + \sin^2\theta\ d\phi^2\right)$$
$$- 2\left(1 + \gamma + \frac{\alpha_1}{4}\right)\frac{J}{r}\sin^2\theta\ dt d\phi$$

m mass and J angular momentum of the body, α_1 PPN parameter.
The torsion tensor depends on t_1, t_2 but also on another set $w_1, \ldots, w_5$ of torsion parameters.
Using the corresponding autoparallel trajectories of the satellites (LAGEOS), the precession of the node Ω reads as

$$(\delta\Omega)_{\mathrm{sec}} = \frac{J}{a^3(1-e^2)^{3/2}}\left(1 + \gamma + \frac{\alpha_1}{4} - \frac{w_2 - w_4}{2}\right)$$

a the semimajor axis of the satellite, e the eccentricity of the orbit.

$$b_\Omega \equiv \frac{\text{precession of the node with torsion}}{\text{precession of the node in GR}} = \frac{1}{2}(1+\gamma+\frac{\alpha_1}{4}-\frac{w_2-w_4}{4})$$

Using $|\alpha_1| < 10^{-4}$ and the measurements of Ciufolini, Pavlis (2004), we get

$$-0.36 < w_2 - w_4 < 0.44$$

Reasoning similarly for the perigee, we eventually have

$$-0.22 < 0.11w_1 - 0.20w_2 - 0.06w_3 + 0.20w_4 + 0.06w_5 < 0.42$$