Electron Cloud Effects in Heavy Ion and Proton Machines

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Outline

**FAIR**
- Introduction
- Electron cloud effects due to the residual gas ionization in coasting beam
- Simulation results for FAIR coasting beams
- Simulation results for FAIR bunched beams
- FAIR Conclusions and Outlook

**Short Relativistic Proton Bunches**
- Simulation of electron cloud wake fields and stopping powers for short LHC like bunches.
- Conclusions and Outlook
FAIR
Facility for Antiproton and Ion Research

Existing facility
UNILAC/SIS-18 GSI facility:
provides ion-beam source
and injector for FAIR

Particles:
• heavy ions
• protons

Energies:
• up to 2.7 GeV/u for ions
• 29 GeV/u for protons

Beams:
• bunched(fast extraction, acceleration)
• coasting(at slow extraction)

Intensities:
• $4 \cdot 10^{13}$ protons
• $5 \cdot 10^{11}$ U$^{28+}$
How Many Electrons Produces One Ion Per Second due to Ionization?

Pressure in SIS18 vacuum chamber is 
\[ P = 10^{-11} \text{ Torr} = 353000 \text{ cm}^{-3} \]

The higher is the charge of ion the faster it can be neutralized by the residual gas ionization.

Neutralization time scale \( \sim 1 \text{ s} \)

Comparable or smaller than slow extraction time

\[ V_i = \sigma \beta c \rho g \]

The same pressure is used in simulations for SIS100.

Cross sections are calculated using Kaganovich, NJP 2006
Electron Interaction with Coasting Beam

Electron-beam coupled motion for fixed neutralization is described by

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + \omega_0 \frac{\partial}{\partial \theta} \right)^2 y_i + Q_b^2 \omega_0^2 y_i &= -Q_i^2 \omega_0^2 (y_i - \bar{y}_e) \\
\frac{d^2 y_e}{dt^2} &= -Q_e^2 \omega_0^2 (y_e - \bar{y}_i)
\end{align*}
\]

- Driving term
- \( Q_e \) – electron trapping tune
- \( Q_b \) – betatron tune

\( Q_{i,sc} \) is neglected it play role at injection in SIS18 at 11.4 MeV/u

\( Q_{e,sc} \) is also neglected at this moment
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\]

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\( Q_e,sc \) is also neglected at this moment

Algebraic equation for eigenfrequencies. \( \text{Im}(Q) \) – gives the instability growth rate

\[
(Q^2 - Q_e^2)[(n-Q)^2 - Q_b^2 - Q_i^2] - Q_e^2 Q_i^2 = 0
\]

If \( dp/p \) is present and chromaticity is corrected the threshold growth rate is given by

\[
\gamma_{\text{thresh}} = \sqrt{\frac{2}{\pi}} \omega_0 \eta n \frac{dp}{p}
\]

*Intensity dependent beam instabilities, \( N_g \)
Coasting Beam Operation in FAIR. Simple Analysis of Parameters.

If intensity is limited by the space charge limit at injection in SIS18

Total number of particles

\[ N_i \propto \frac{m_i}{Z^2} \]

Electron trapping frequency

\[ \omega_e = \sqrt{\frac{ZN e^2}{2\pi \epsilon_0 a^2 m_e}} \propto \sqrt{\frac{m_i}{Z}} \]

Threshold

\[ \gamma_{damp} \approx \sqrt{\frac{2}{\pi \omega_e}} \frac{dp}{p} \eta \propto \sqrt{\frac{m_i}{Z}} \]
Coasting Beam Operation in FAIR. Simple Analysis of Parameters.

If intensity is limited by the space charge limit at injection in SIS18

Total number of particles
\[ N_i \propto \frac{m_i}{Z^2} \]

Ionization rate
\[ V_i = N_i \rho_{\text{gas}} Z^2 s(\beta, \gamma) \propto m_i \]

Electron trapping frequency
\[ \omega_e = \sqrt{\frac{Z N e^2}{2 \pi \epsilon_0 a^2 m_e}} \propto \sqrt{\frac{m_i}{Z}} \]

Number of electrons
\[ n_e = V_i T \propto m_i \]

Threshold
\[ \gamma_{\text{damp}} \approx \sqrt{\frac{2}{\pi} \omega_e \frac{dp}{p} \eta} \propto \sqrt{\frac{m_i}{Z}} \]

Driving term
\[ \omega_i = \sqrt{\frac{Z n_e e^2}{2 \pi \epsilon_0 a^2 m_i \gamma}} \propto \sqrt{Z} \]
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Driving term

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Threshold

\[ \gamma_{damp} \approx \sqrt{\frac{2}{\pi}} \omega_e \frac{dp}{p} \eta \propto \sqrt{m_i} \]

For beams with equal energies with intensities defined by the space charge limit U\(^{73+}\) is less stable than U\(^{28+}\) and Ar\(^{18+}\)

<table>
<thead>
<tr>
<th>Ion</th>
<th>(\sqrt{\frac{A}{Z}}) damp</th>
<th>(\sqrt{Z}) drive</th>
</tr>
</thead>
<tbody>
<tr>
<td>U(^{73+})</td>
<td>1.80</td>
<td>8.5</td>
</tr>
<tr>
<td>U(^{28+})</td>
<td>2.91</td>
<td>5.3</td>
</tr>
<tr>
<td>Ar(^{18+})</td>
<td>1.49</td>
<td>4.2</td>
</tr>
</tbody>
</table>

p | 1 | 1

If the vacuum is good then threshold is not reached. Why?
Estimation of the Coulomb Heating Effect

Electrons collide with beam ions and gain energy

**Heating rate**

\[
V_{\text{heat}} = \frac{dW_e}{dt} = E_0 \frac{4\pi c \rho_i r_e^2 Z_i^2}{\beta L_{\text{Coul}}}
\]

**Lifetime**

\[
t_{\text{life}} = \frac{U_{\text{wall}}}{V_{\text{heat}}}
\]

**Neutralization degree**

\[
\chi_e = \frac{V_i t_{\text{life}}}{N_i Z}
\]

*Z cancels

N_i – ion intensity

V_i – ionization rate

U_{\text{wall}} – wall potential

E_0 – rest energy

*Zenkevich, AIP Proceedings 480, 1998
Estimation of the Coulomb Heating Effect

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Lifetime

\[
t_{\text{life}} = \frac{U_{\text{wall}}}{V_{\text{heat}}}
\]

Neutralization degree

\[
\chi_e = \frac{V_i t_{\text{life}}}{N_i Z}
\]

At 1 GeV/u assuming radius of the pipe 5 cm

<table>
<thead>
<tr>
<th>Ion</th>
<th>(N_i)</th>
<th>(V_{\text{heat}})</th>
<th>(U_{\text{wall}})</th>
<th>(t_{\text{life}})</th>
<th>(\chi_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar(^{18+})</td>
<td>(2 \cdot 10^{11})</td>
<td>143 eV/s</td>
<td>26 eV</td>
<td>0.18 s</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>U(^{28+})</td>
<td>(5 \cdot 10^{11})</td>
<td>868 eV/s</td>
<td>104 eV</td>
<td>0.12 s</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>U(^{73+})</td>
<td>(7 \cdot 10^{10})</td>
<td>826 eV/s</td>
<td>38 eV</td>
<td>0.046 s</td>
<td>&lt;1%</td>
</tr>
</tbody>
</table>

\(\sigma = 9\ mm\)

\(\sigma_r = 5\ mm\)

\(N_i\) – ion intensity
\(V_i\) – ionization rate
\(U_{\text{wall}}\) – wall potential
\(E_0\) – rest energy

*Zenkevich, AIP Proceedings 480, 1998
Thresholds in Linear Theory with Landau Damping due to $dp/p$

Intensities of particles are scaled according to $Z^2/m_i$ space charge limit.
Neutralization is fixed. $dp/p=0.5 \cdot 10^{-3}$ $\beta=0.86$

<table>
<thead>
<tr>
<th>Neutralization / %</th>
<th>Neutralization / %</th>
<th>Neutralization / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$530 \text{ ms}$</td>
<td>$75 \text{ ms}$</td>
<td>$54 \text{ ms}$</td>
</tr>
</tbody>
</table>

$U^{28+}$ has the biggest threshold. Protons has the smallest threshold.

\[
\omega_e^2 = \frac{Z N e^2}{2 \pi \varepsilon_0 a^2 m_e} = Q_e^2 \omega_0^2
\]

\[
\omega_i^2 = \frac{Z n_e e^2}{2 \pi \varepsilon_0 a^2 m_e \gamma} = Q_i^2 \omega_0^2 = Q_e^2 \omega_0^2 \chi \frac{m_e}{m_i \gamma}
\]

Taking into account speed of neutralization the threshold will be first reached by $U^{73+}$
Width of Electron Oscillation Spectrum

Spectrum of electrons produced by residual gas ionization.

Number of electrons produced outside $\sigma_r$ is over 60%.

Spectrum width is about 20% of linear trapping tune.

Strongly shifted to lower frequencies.

This causes significant change in thresholds.
General Principle of Simulation in Rigid Slice Code

This is the same principle as in HEADTAIL and ECLoud codes.

In our case one and the same code is used for the build-up studies and for the instabilities simulations.
Coasting Beam Rigid Slice Model with Landau Damping

Build-up simulations in bunches were performed using ECLOUD and our own code.

Transversely excited coasting beam

For different harmonics $n$ there are different instability thresholds.

\[ v_n = \sqrt{\frac{2}{\pi}} \sigma_{\omega_0, n} \]

one turn amplitude decrease

\[ \alpha_n = e^{-\sqrt{\frac{2}{\pi}} \sigma_{\omega_0, n} T} \]
Coasting Beam Rigid Slice Model with Landau Damping

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For different harmonics n there are different instability thresholds*

\[ \nu_n = \sqrt{\frac{2}{\pi}} \sigma_{\omega_0, n} \]

one turn amplitude decrease

\[ \alpha_n = e^{-\sqrt{\frac{2}{\pi}} \sigma_{\omega_0, n} T} \]

\( (x_0, x_1, \ldots, x_N) \xrightarrow{\text{FFT}} (k_0, k_1, \ldots, k_N) \xrightarrow{} (\alpha_0 k_0, \alpha_1 k_1, \ldots, \alpha_N k_N) \)

*A. Hofmann,, Landau Damping, CERN school*
Coasting Beam Rigid Slice Model with Landau Damping

Build-up simulations in bunches were performed using ECLoud and our own code Transversely excited coasting beam

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\[ (x_0, x_1, ..., x_N) \xrightarrow{\text{FFT}} (k_0, k_1, ..., k_N) \xrightarrow{\alpha_0 k_0, \alpha_1 k_1, ..., \alpha_N k_N} \]

Inverse FFT gives us a new damped set of beam coordinates which is used in the next iteration.
This way we reproduce the instability threshold in a rigid slice simulation.

The production of electrons happens every timestep. Interaction is HEADTAIL like.

*A. Hofmann,, Landau Damping, CERN school
Comparing Rigid Model with Full Particle-in-Cell Model

Electron cloud effects have similarities with impedances.

To check the correctness of our model we apply the broad band impedance to our rigid beam and compare with the full Particle-in-Cell model as well as with analytical theory.

**Broad band impedance**

\[
Z_{Tr}(\omega) = \frac{\omega_r}{\omega} \frac{Z_0}{1 + i Q \left( \frac{\omega}{\omega_r} - \frac{\omega}{\omega_r} \right)}
\]

**Stability Border**

\[
\sqrt{\frac{2}{\pi}} \omega_r \frac{dp}{p} \eta = \frac{N q^2 \beta}{8 \pi^2 m_i \gamma R Q_x} R e(Z_{Tr})
\]
Build-up and Instability in SIS100 Without Coulomb Heating

To reduce the simulation time ionization rate was increased by factor 100.

Exponential instability.

Linear growth of amplitude

Scan over parameters revealed that among $U^{73+}$, $U^{28+}$, $Ar^{18+}$ and protons only $U^{73+}$ reaches the threshold in several seconds. Threshold is significantly higher then density due to the heating.
Electron Cloud Build-up During Bunched Beam Operation

Scan over bunch parameters in SIS100 to reveal dangerous conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>1080 m</td>
</tr>
<tr>
<td>Design bunch length, $\sigma_z$</td>
<td>4 m</td>
</tr>
<tr>
<td>Ion type</td>
<td>$U^{28+}$</td>
</tr>
<tr>
<td>Intensity</td>
<td>$5 \cdot 10^{11}$</td>
</tr>
<tr>
<td>Energy</td>
<td>1 GeV/u</td>
</tr>
</tbody>
</table>

Possible danger:
SIS100 will not be covered by any coatings.

Advantage:
1. Beam pipe size is significantly smaller and potential $U_{\text{wall}}$ should be also smaller
2. 2 empty buckets = 216 m of free space where electrons decay

Electron cloud density is negligible for SEY < 2.2
Conclusion and Outlook for FAIR

Coasting beams
- Realistic Landau damping in rigid slice coasting beam model
- Faster instability for highly charged heavy ions
- Proton beam has a low threshold.
- Time for electrons to accumulate in proton beam is very long
- Coulomb heating is very powerful electron loss mechanism under the designed FAIR conditions
- Most probably the Instability thresholds for heavy ions will not be reached in FAIR
- Pressure deteriorated to $10^{-9}$-$10^{-8}$ Torr (beam loss) can make the threshold reachable.

Bunched beams
- No multipacting happens in SIS18 and in SIS100 for the designed bunch length and intensities for SEY $< 2.0$-$2.2$

Outlook: To analyse the effect of other sources(losses on the wall)
Outline

• Introduction
• Electron cloud production
• Electron cloud effects
• Simulation models
• Simulation results for FAIR bunched beams
• Simulation results for FAIR coasting beams
• FAIR Conclusions and Outlook

• Simulation of electron cloud wake fields and stopping powers for short LHC like bunches.
• LHC Conclusions and Outlook
Energy Loss of Relativistic Short Proton Bunches in Electron Clouds

Density profile of electron cloud pinched in the field of the bunch

If there is already an electron cloud when the bunch passes it is attracted towards the center of the bunch.

The non-uniformity of the cloud results into the longitudinal electric field which tries to stop the bunch.

This is seen in measurements of rf phase shift

Energy Loss and RF Phase Shift

Bunch line density:

$$\lambda_z = \frac{N_i}{\sqrt{2 \pi \sigma_z^2}} \exp \left(-\frac{z^2}{2 \sigma_z^2} \right)$$

Stopping power:

$$\frac{dW}{ds} = -\int \rho_i(r) E_z(r) \, dr = -q \int \lambda(z) E_z(z) \, dz$$

Energy loss per turn and particle:

$$\Delta W_z = \frac{L}{N_i} \frac{dW}{ds}$$

rf phase shift:

$$\sin (\Delta \varphi_s) = \frac{\Delta W_p}{q V_{rf}}$$

$$N_e = 10^{12}-10^{13} \text{m}^{-3}$$
Obtaining Longitudinal Wake Fields in Code with 2D Poisson Solver

Transverse electric field of the bunch:

\[ E_r^i(r, z) = \frac{q \lambda(z, t)}{2 \pi \epsilon_0 r} \left[ 1 - \exp \left( -\frac{r^2}{2 \sigma^2_r} \right) \right] \]

Electron’s equation of motion:

\[ r'' + k^2(r, z)r = \frac{e E_r^i(r, z)}{m_e c^2} \]

where

\[ \omega_{pe} = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}} \quad k = \omega_{pe} / c \]

Electron cloud’s space charge field:

\[ \epsilon_0 \nabla \cdot E_e(r, z) = \rho_e(r, z) \]

The same model used for FAIR simulations

2D cloud distribution is saved each step

At the end of simulations one has a 3D density profile which is used to find \( E_z \)

Need to crosscheck with 3D code
Comparison of Longitudinal Wakes in 2D code and VORPALAL

VORPAL is a commercial program for 3D electro-magnetic Particle-in-Cell simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch rms length</td>
<td>0.11 m</td>
</tr>
<tr>
<td>Bunch intensity</td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>Bunch radius</td>
<td>2 mm</td>
</tr>
</tbody>
</table>

Plasma oscillations

VORPAL results agree very well with the simplified 2D ES simulations.
Energy Loss of Short Bunches

Transverse field of the Gaussian bunch

\[ E^i_r(r, z) = \frac{q \lambda(z, t)}{2\pi \epsilon_0 r} \left[ 1 - \exp \left( -\frac{r^2}{2 \sigma^2_r} \right) \right] \]

If the bunch is short most of the electrons see a short transverse kick

\[ \Delta_\perp p(b) = \frac{1}{c} \int_{-\infty}^{\infty} F_\perp(b, s) ds \quad F_\perp = -e E^i_\perp(b, s) \]

The energy used to kick electrons comes to stopping power

\[ \frac{dW}{ds} = \frac{n_e}{2m_e} \int_0^R 2\pi \Delta p^2_\perp(b) b \, db \]

\[ S = \frac{dW}{ds} \approx \frac{4\pi}{\epsilon_0} Q^2_i n_e r_e \ln \left( \frac{R}{a} \right) \]

\[ \frac{d \Delta \varphi}{ds} \approx \frac{4\pi Q_i n_e r_e}{\epsilon_0 V_{rf}} \ln \left( \frac{R}{a} \right) \]

High electron densities shield the bunch field and the stopping power goes down.
Energy Loss at High Electron Densities

Equation for electron offset
\[ \delta'' + \frac{\omega^2}{c^2} \delta = k^2 (b, z) b \]

Oscillator amplitude at s=∞
\[ \hat{\delta}(b) = \frac{b}{k_e} \int_{-\infty}^{\infty} k(b, s)^2 \cos(k_e s) \, ds \]

Stopping power is proportional to energy transferred to oscillations
\[ \frac{dW_e}{ds} = \frac{1}{2} m_e n_e \omega_{pe}^2 \int_0^R 2\pi \hat{\delta}^2 b \, db \]
\[ \frac{dW_e}{ds} \approx \frac{Q^2 k^2}{4\pi \epsilon} \ln \left( \frac{R}{a} \right) \exp \left( -k_e^2 \sigma_z^2 \right) \]
Longitudinal Wakes with Multi-Bunch Effects

Saturated e-cloud density:

\[ n_{es} \approx \frac{E_s}{\pi m_e c^2 R^2 r_e} \approx 10^{12} \text{m}^{-3} \]

Ring-like density distribution

Realistic cloud acts weaker on the bunch → weaker stopping power
Transverse Wake Fields for the $k=0$ Head-Tail Mode (offset)

In comparison with longitudinal case transverse wakes are obtained using 2D solver directly in the code

$$\Delta r = 0.004 \text{ mm}$$

Pinching of the cloud around the bunch with and offset

Transverse wake fields obtained with 2D PIC and VORPAL

The only disagreement is seen at the end of the bunch. However, the fraction of beam particles affected is very small.
Transverse Wake Fields for the k=1 Head-Tail mode (tilt)

In this simulation bunch is traveling along the pipe with an angle between the pipe and the bunch axis

\[ \tan(\varphi) = 0.01 \]

Pinching of the cloud around the tilted bunch

The agreement is again very good.
Conclusions and Outlook

- Analytical theory connecting rf phase shift and electron cloud density
- Realistic cloud shape reduces significantly the stopping power if the electron number is preserved
- Electron cloud wake field obtained in 2D electrostatic simulations and 3D electromagnetic VORPAL simulations agree very well
- 2D Poisson solver can be used for short relativistic bunches

Future work:
- fast e-cloud solver on GPUs
- Parametrization of the wake fields (Impedances ?)

German government accepted proposal for 3 years on electron cloud studies in LHC and FAIR.
Thank you for your attention!

Thank you for your attention and questions.
Simplified Model of Emittance Growth of an Oscillating Beam

What happens with the emittance when beam oscillates?

Initial emittance

\[ \epsilon_{x,0} = N_i \frac{\sigma_r^2}{\beta_x} \]

Oscillations with coherent energy

\[ \epsilon_{coh} = N_i \frac{\sigma_{coh}^2}{\beta_x} \]

If there is a damping due to \( \frac{dp}{p} \) and restoring energy source

\[ \gamma_{damp} = \sqrt{\frac{2}{\pi}} \omega_0 \frac{dp}{p} n \eta \]

Emittance growth is linear

\[ \epsilon_x(t) = \epsilon_{x,0} + 2\gamma_{damp} \epsilon_{coh} t \]

Table 1. How fast does emittance double if oscillation amplitude is \( 10^{-4} \) m, \( \beta=0.86 \)

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>( \frac{dp}{p} )</th>
<th>10(^{-4})</th>
<th>2 ( 10^{-4} )</th>
<th>5 ( 10^{-4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=20</td>
<td></td>
<td>15.4 s</td>
<td>7.7 s</td>
<td>3.1 s</td>
</tr>
<tr>
<td>n=30</td>
<td></td>
<td>10.3 s</td>
<td>5.1 s</td>
<td>2.1 s</td>
</tr>
<tr>
<td>n=100</td>
<td></td>
<td>3.2 s</td>
<td>1.5 s</td>
<td>0.6 s</td>
</tr>
</tbody>
</table>
Scan Over Intensities for U$^{73+}$ with Lowest Momentum Spreads

Threshold electron densities given by the simulation are much bigger than the limiting densities due to the Coulomb heating.