

Modeling Interaction of e-cloud & Microwaves

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Outline

- ▶ Objectives
- ▶ Radiation problem inside uniform waveguides
- ▶ Simulation setup
- ▶ Simulation results

Objectives

- ▶ Compute modes of LHC beam-pipe
- ▶ Implement microwave field using true beam pipe geometry in ecloud
- ▶ Excite LHC beam-pipe with microwave field
- ▶ Analyze em field radiated by the e^- -cloud

Radiation problem inside uniform waveguides

Observation:

- ▶ We deal with an arbitrary motion of charged particles (electrons)
- ▶ The em field associated with a particle is the Green function subject to the boundary conditions of the problem
- ▶ To take into account the images of the electron cloud, the modes of the waveguide are used. Especially useful when the cross-section is of non-canonical form.

⇒ use **time-domain** Greens function based on a modal expansion.

Radiation problem inside uniform waveguides

Applying the Lorenz Gauge:

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \phi = -\frac{\rho(\vec{r}, t)}{\epsilon_0}, \quad (1a)$$

$$\nabla^2 \mathcal{A} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathcal{A} = -\mu_0 \mathbf{j}(\vec{r}, t). \quad (1b)$$

By means of the Green functions, the solution can be expressed as

$$\phi(\vec{r}, t) = -\frac{1}{\epsilon_0} \int g(\vec{r}, t | \vec{r}', t') \rho(\vec{r}', t') dt' d^3 r', \quad (2a)$$

$$\mathcal{A}(\vec{r}, t) = -\mu_0 \int \overleftrightarrow{G}_A(\vec{r}, t | \vec{r}', t') \mathbf{j}(\vec{r}', t') dt' d^3 r'. \quad (2b)$$

Intermezzo - Green function of a parallel plate wg

image formulation

Point charge in parallel plate waveguide, e.g. e^-

infinite ground plane

...



...



$$G_{FS} = \frac{1}{4\pi} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$$

$$G_{BIS} = G_{FS}(x|x') - G_{FS}(x|-x')$$

a
↔

$$G_{PP} = \sum_{n=-\infty}^{+\infty} G_{BIS}(x|x' + 2na)$$

Intermezzo - Green function of a parallel plate wg

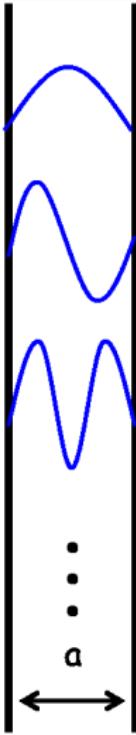
modal formulation

$$\phi_1(x) = \sin \frac{\pi x}{a}$$

$$\phi_2(x) = \sin \frac{2\pi x}{a}$$

$$\phi_3(x) = \sin \frac{3\pi x}{a}$$

⋮



$$G_{\text{PP}} = \sum_{n=-\infty}^{+\infty} g_n \phi_n(x) \cdot \phi_n(x')$$

Radiation problem inside uniform waveguides

The Green functions are defined by

$$\nabla^2 g - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} g = \mathbf{1} \cdot \delta(\vec{r} - \vec{r}') \delta(t - t'), \quad (3a)$$

$$\nabla^2 \overleftrightarrow{\mathbf{G}}_A - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \overleftrightarrow{\mathbf{G}}_A = \overleftrightarrow{\mathbf{I}} \cdot \delta(\vec{r} - \vec{r}') \delta(t - t'). \quad (3b)$$

+ boundary conditions of the em field.

Radiation problem inside uniform waveguides

The waveguide problem can be solved by separating

- ▶ the transverse problem → **boundary value problem**, and
- ▶ the longitudinal problem → **radiation/propagation problem**.

Applying the Fourier Transform to the longitudinal variable $\textcolor{red}{z}$ and the time $\textcolor{green}{t}$ yields 2D Helmholtz equations for the transverse problem in the spectral-frequency (β - ω) domain

$$(\nabla_t^2 + k_c^2) \hat{g}(\vec{r}_t | \vec{r}'_t) = 1 \cdot \delta(\vec{r}_t - \vec{r}'_t), \quad (4a)$$

$$(\nabla_t^2 + k_c^2) \overleftrightarrow{\hat{G}}_{\text{A}}(\vec{r}_t | \vec{r}'_t) = \overleftrightarrow{l}_t \cdot \delta(\vec{r}_t - \vec{r}'_t). \quad (4b)$$

$$k_c^2 = k_0^2 - \beta^2 \quad k_0 = \frac{\omega}{c_0}$$

Radiation problem inside uniform waveguides

The transverse part can be expressed as infinite series of eigen solutions

$$\hat{g}(\vec{r}_t | \vec{r}'_t) = \sum_l \frac{1}{k_c^2 - k_l^2} \hat{g}_l(\vec{r}_t) \hat{g}_l(\vec{r}'_t), \quad (5a)$$

$$\overleftrightarrow{\hat{G}}_{\text{A}}(\vec{r}_t | \vec{r}'_t) = \sum_l \frac{1}{k_c^2 - k_l^2} \hat{A}_l(\vec{r}_t) \hat{A}_l^{\text{tr}}(\vec{r}'_t). \quad (5b)$$

Note, $k_c = k_c(\beta, \omega)$, thus the (β, ω)-dependent part is isolated in the coefficients of the infinite series.

Radiation problem inside uniform waveguides

The longitudinal- (z) and time- (t) -dependency is obtained by applying the inverse Fourier transform

$$\Lambda(k_l, z-z', t-t') := \mathcal{F}_{2D}^{-1} \left\{ \frac{1}{k_c^2 - k_l^2} \right\} = \frac{c_0}{2} \begin{cases} J_0 \left(k_l \sqrt{c_0(t-t') - |z-z'|} \right), & c_0(t-t') > |z-z'| \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

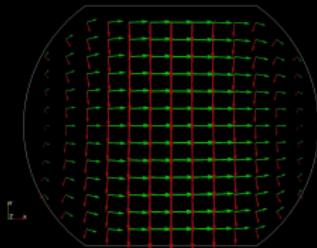
$$g(\vec{r}, t | \vec{r}', t') = \sum_l \Lambda(k_l, z-z', t-t') \hat{g}_l(\vec{r}_t) \hat{g}_l^{\text{tr}}(\vec{r}'_t), \quad (7a)$$

$$\overleftrightarrow{G}_A(\vec{r}, t | \vec{r}', t') = \sum_l \Lambda(k_l, z-z', t-t') \hat{A}_l(\vec{r}_t) \hat{A}_l^{\text{tr}}(\vec{r}'_t). \quad (7b)$$

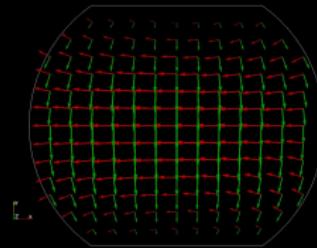
Modes of LHC beam pipe

The eigen solutions \hat{g} and \hat{G}_A of the LHC beam pipe are computed using the BI-RME algorithm¹

TE_1

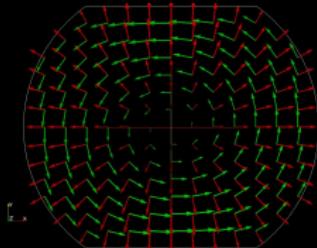


TE_2

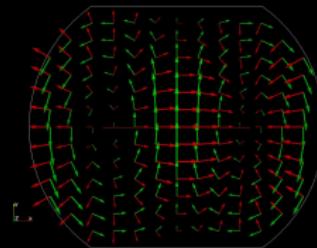


E

TM_1



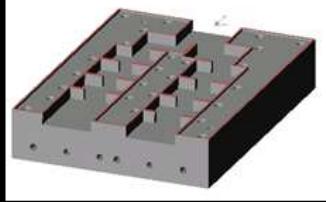
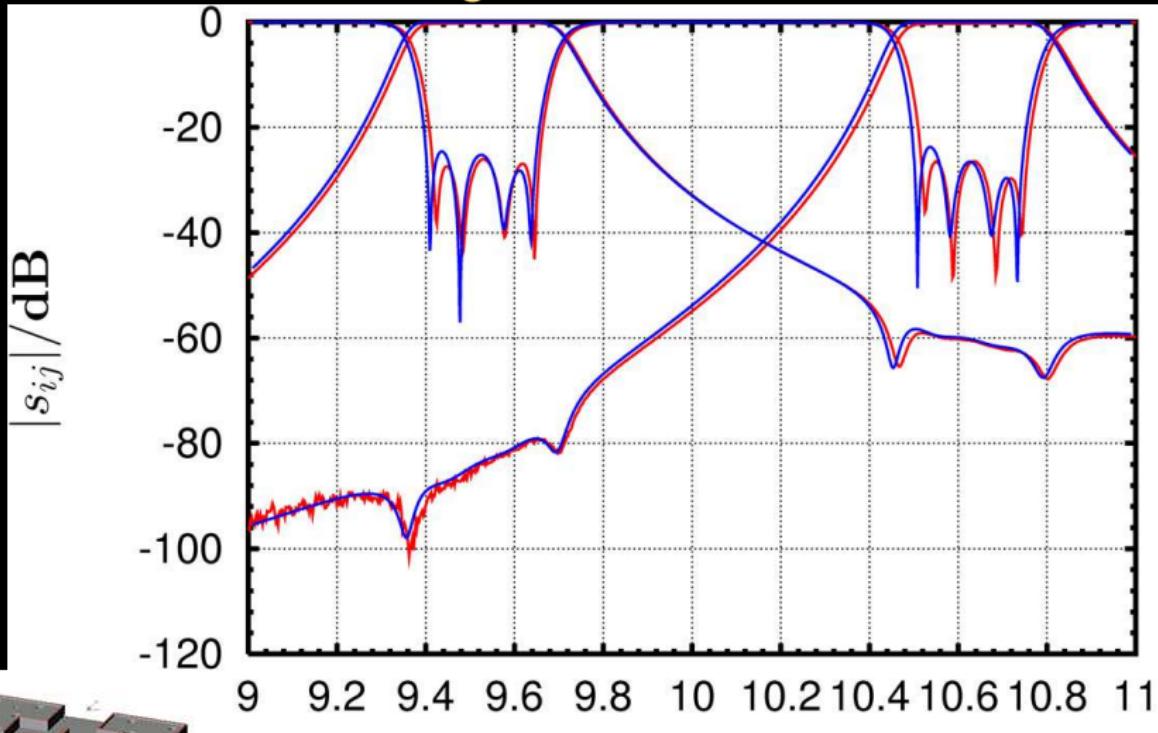
TM_2



H

¹Conciauro, Guglielmi, Sorrentino, *Advanced Modal Analysis*, Wiley, 1999.

Intermezzo - CEM: waveguides



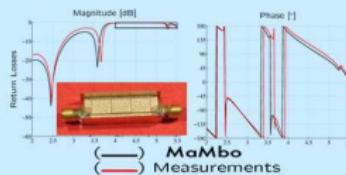
f/GHz

Intermezzo - CEM: planar structures

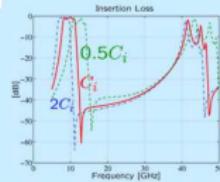
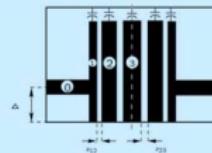
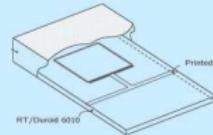
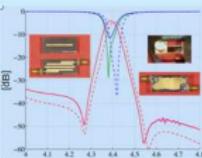
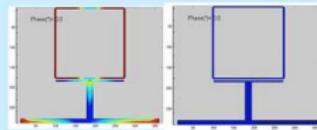
Multilayered Boxed Printed Circuits

MaMbo

- Hybrid mesh (triangles/rectangles)
- Stable & efficient model for multilayered media
- Complete losses model

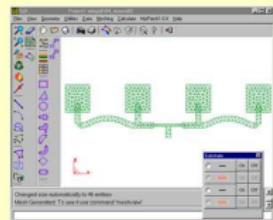
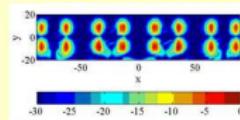


- Lumped elements
- Coax-type and waveguide ports

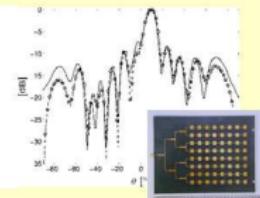
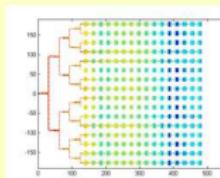


Multilayered Planar Antennas

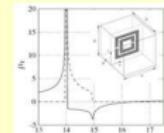
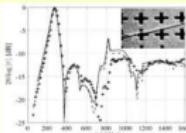
POLARIS



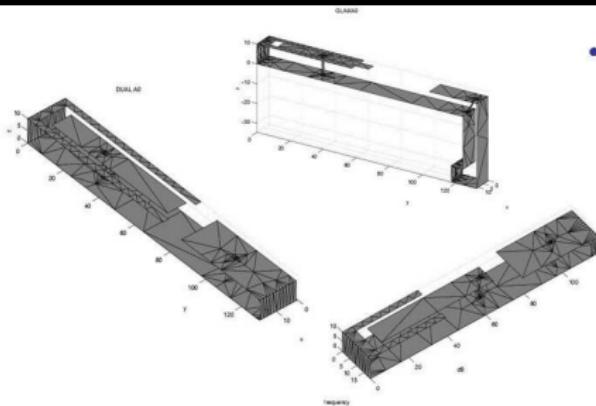
- Efficient treatment of large structures
- Computation of Input impedance and Near & Far field



- Planar Periodic Structures, FSS and metamaterials

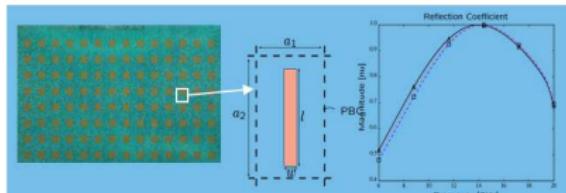


Intermezzo - CEM: 3D and periodic structures

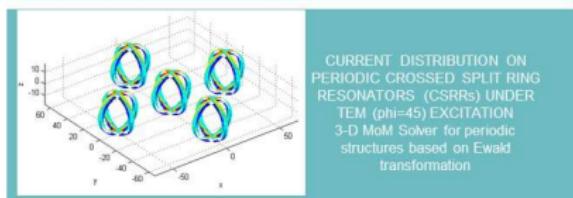


- Modelling and optimizing 3D antennas for mobile applications like GSM

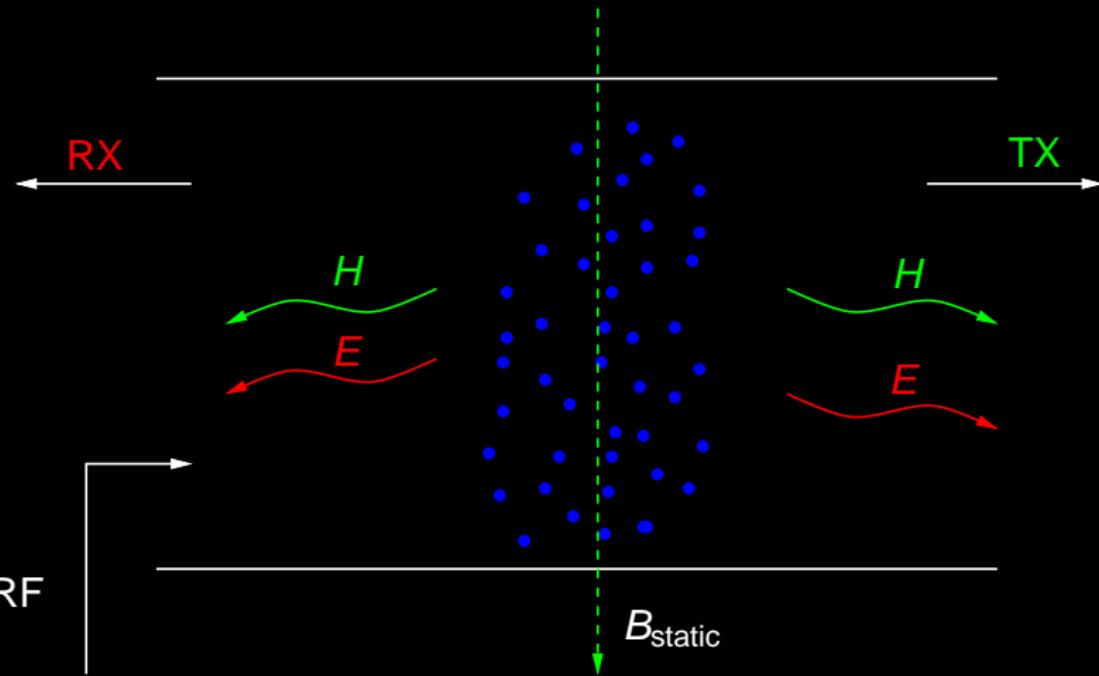
... in periodic structures



- Planar periodic structures (FSS, antenna arrays, ...)
- 3D objects in a 2D/3D periodic lattice SIE-MoM procedure
- An improved retrieval procedure of artificial materials under oblique plane-wave incidence

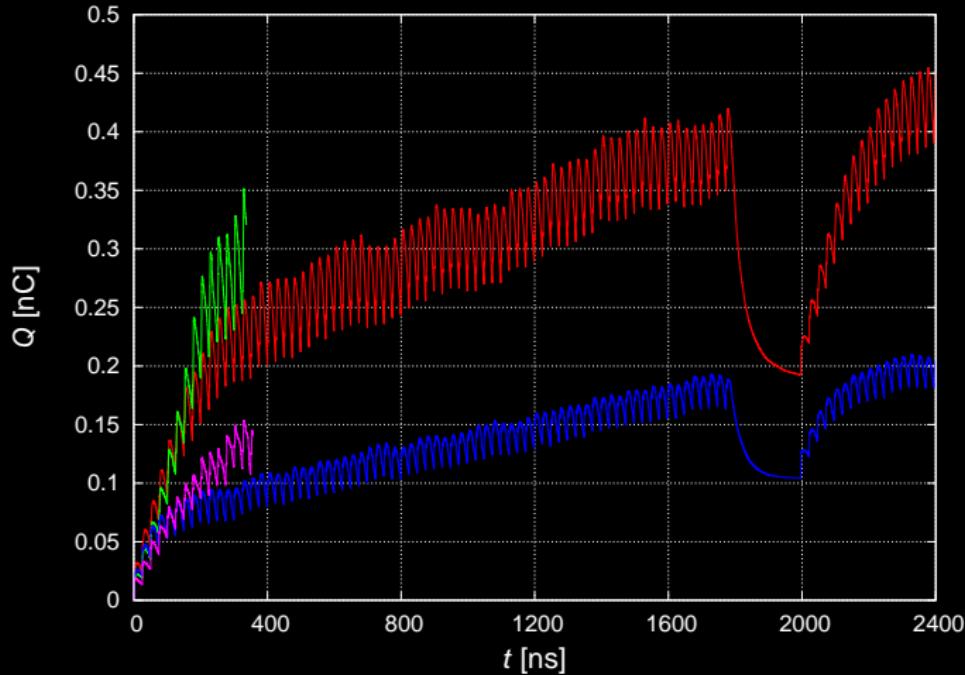


Simulation setup



RF: fundamental mode of beam-pipe is used to excite microwave field and to measure the radiated field of the e^- -cloud.

Results - ecloud build up



no external field

$f_0 = 4100 \text{ MHz}$

$V_{\text{RF}} = 10 \text{ kV}$, $E_{\text{RF}} = 355 \text{ kV/m}$

$P_{\text{rms}} = 148 \text{ kW}$

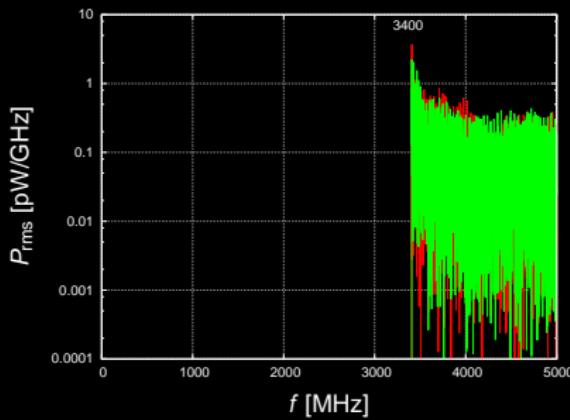
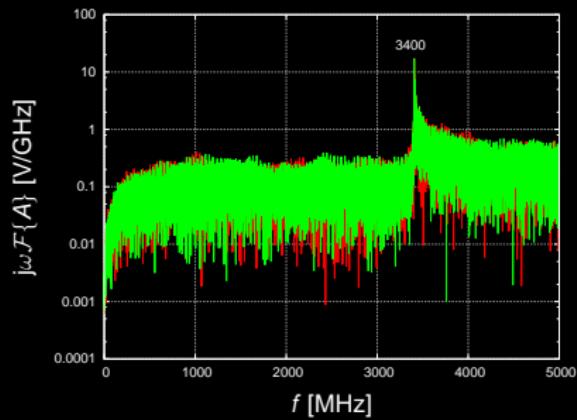
$B_{\text{static}} = 8.33 \text{ T}$

$B_{\text{static}} = 8.33 \text{ T}$, $f_0 = 4100 \text{ MHz}$

$V_{\text{RF}} = 10 \text{ kV}$, $E_{\text{RF}} = 355 \text{ kV/m}$

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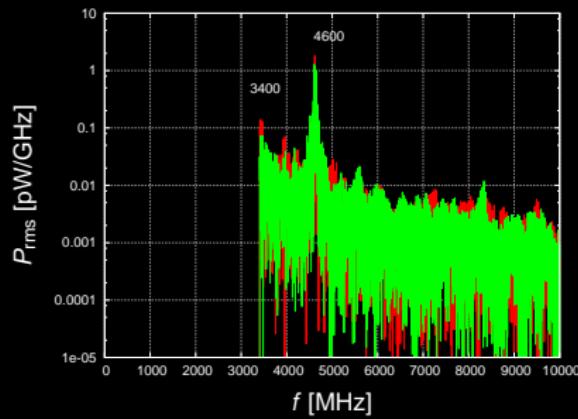
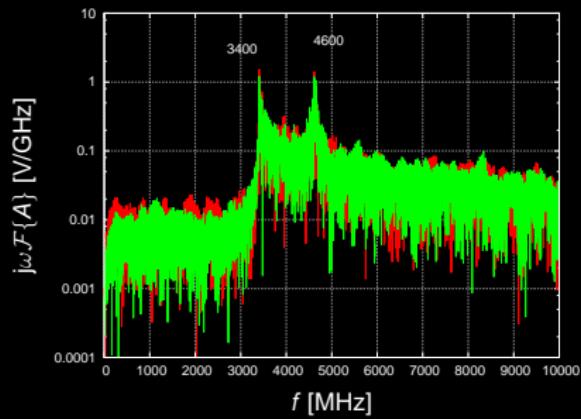
Results - no external field



RX TX

Results - static magnetic field

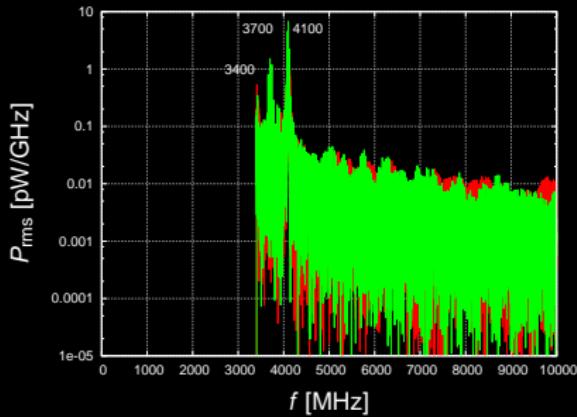
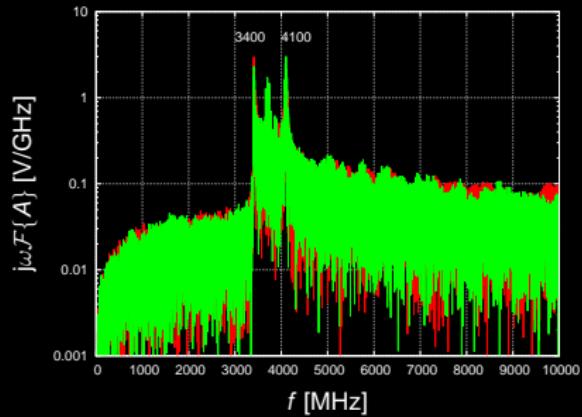
$$B_{\text{static}} = 8.33 \text{ T}$$



RX TX

Results - microwave field

$$f_0 = 4100 \text{ MHz}, V_{\text{RF}} = 10 \text{ kV}, E_{\text{RF}} = 355 \text{ kV/m}, P_{\text{rms}} = 148 \text{ kW}$$

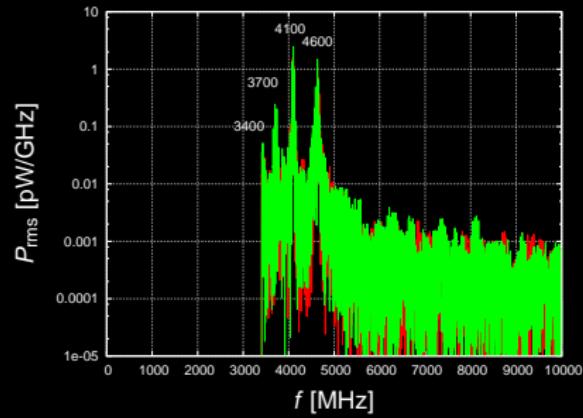
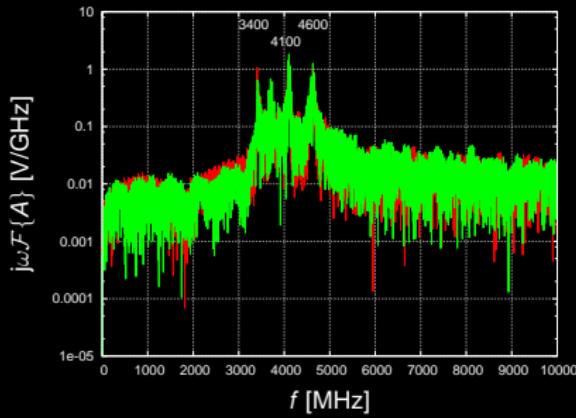


RX TX

Results - microwave & static magnetic field

$f_0 = 4100 \text{ MHz}$, $V_{\text{RF}} = 10 \text{ kV}$, $E_{\text{RF}} = 355 \text{ kV/m}$, $P_{\text{rms}} = 148 \text{ kW}$

$$B_{\text{static}} = 8.33 \text{ T}$$



RX TX

Summary & Future Work

- ▶ ecloud includes microwave field excitation using beam pipe geometry
- ▶ em field radiated by e^- -cloud can be analyzed
- ▶ The contributions in the spectrum at 3700 MHz and 4600 MHz remain unclear for the moment. Further analysis is necessary:
Dependency on
 - ▶ field strength of microwave and static magnetic field
 - ▶ microwave frequency
 - ▶ polarization of microwave field
- ▶ Excite beam pipe with a micorwave field having horizontal polarization (perpendicular to B_{static}) to check for Synchrotron effect.

Acknowledgements

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