How can we improve the p-values using muon tomography ?
A study using "fast"muon contribution to P

Per Grafstrom
University of Bologna
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We have seen that muon flux measurement will help in determining the depth and this will improve the max dating.

Now we are going to discuss how muon flux measurement will improve the p-values…

from my previous presentation

My consideration up to now have not addressed the important question how measurement of the muon flux at the surface and in the cave -possibly with momentum determination- can improve the p values which are important for the min/max method

Just one example…. Sample TM11 (30 m sample) gives a Max of 2.74Ma

If the P for fast muons would be 5x larger at 30 m we get Max 3.2 Ma

In order to understand to what extent measuring the muon flux in the cave helps we need to understand the relation between the muon flux and p values

Up to now we have calculated p-values using the approach described in Granger 2014.....

$$P_i = \sum_j A_{i,j} e^{-z/L_j}$$

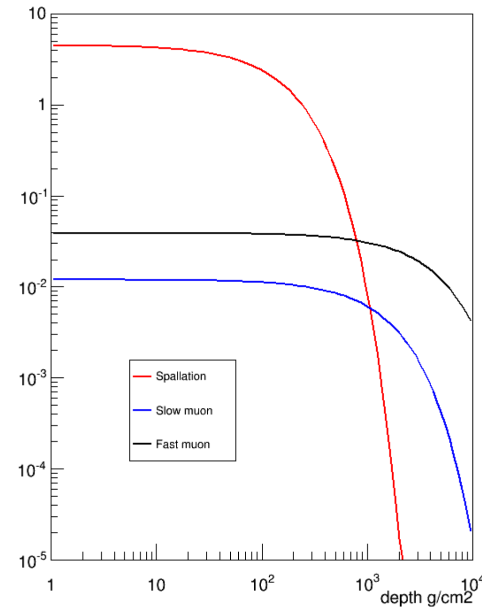
L_j are attenuation lengths for spallation, slow muons and fast muons

P_{10} Braucher 2011 (from Granger 2014)

A_{10}	L
4.5	160
0.012	1500
0.039	4320

This kind of parametrization is based upon depth profiles obtained from a long core drilled from the surface.

.....here I have used those values...
(which are normalized to sea level and high latitude(SLHL))



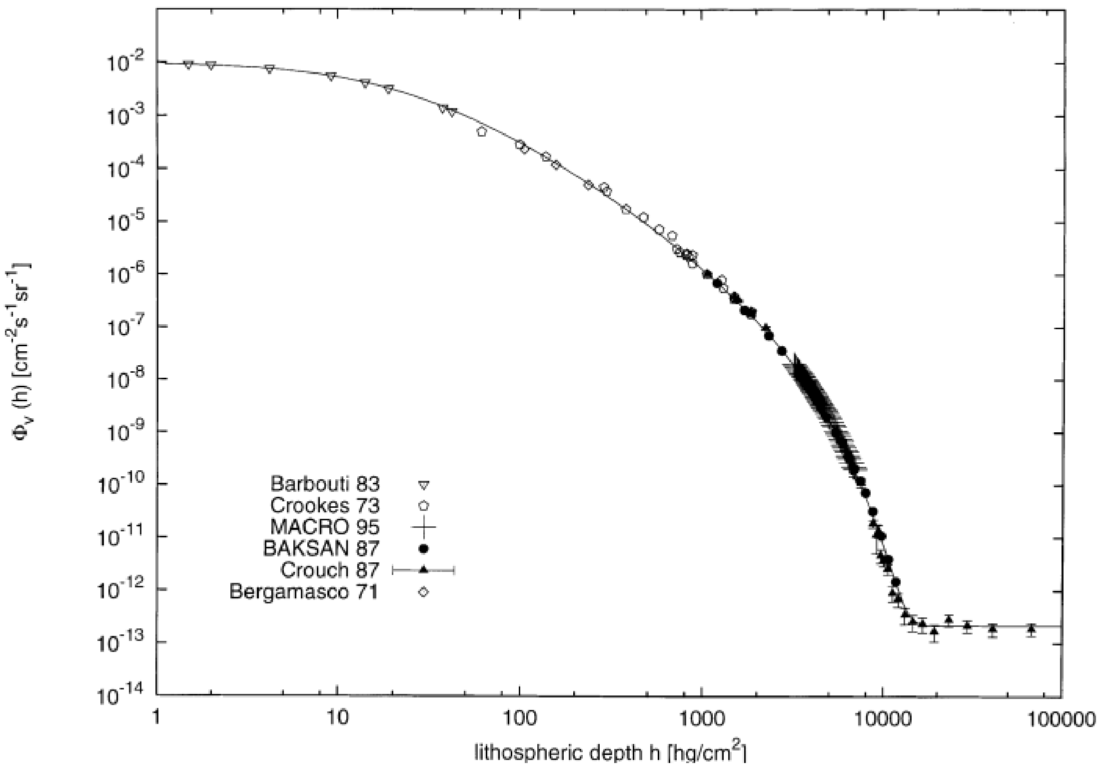
If we want to study how muon flux measurements can improve the p -value we can not use this approach because the muon flux and the cross sections are not separated!

Instead we have to base our considerations on approaches that calculate the p-values from simulation of the muon flux and using measurements of the relevant the cross sections

Here below I will do this type of calculation step by step……
this will help us to see how the measurement of the muon flux in the cave will help in improving the p-values

(1) Start with the muon flux

Parametrization of the vertical muon flux….taken from Heisinger (2002)



$$\Phi_v(h) = \frac{258.5}{(h+210)\cdot([h+10]^{1.66}+75)} e^{-5.5\times 10^{-4}\cdot h} \text{ cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1}$$

for $h < 2000$ hg/cm^2 (1)

and

$$\Phi_v(h) = \left(1.82\times 10^{-6}\left(\frac{1211}{h}\right)^2 e^{-\frac{h}{1211}} + 2.84\times 10^{-13}\right) \text{cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1} \text{ for } h > 2000 \text{ hg/cm}^2$$

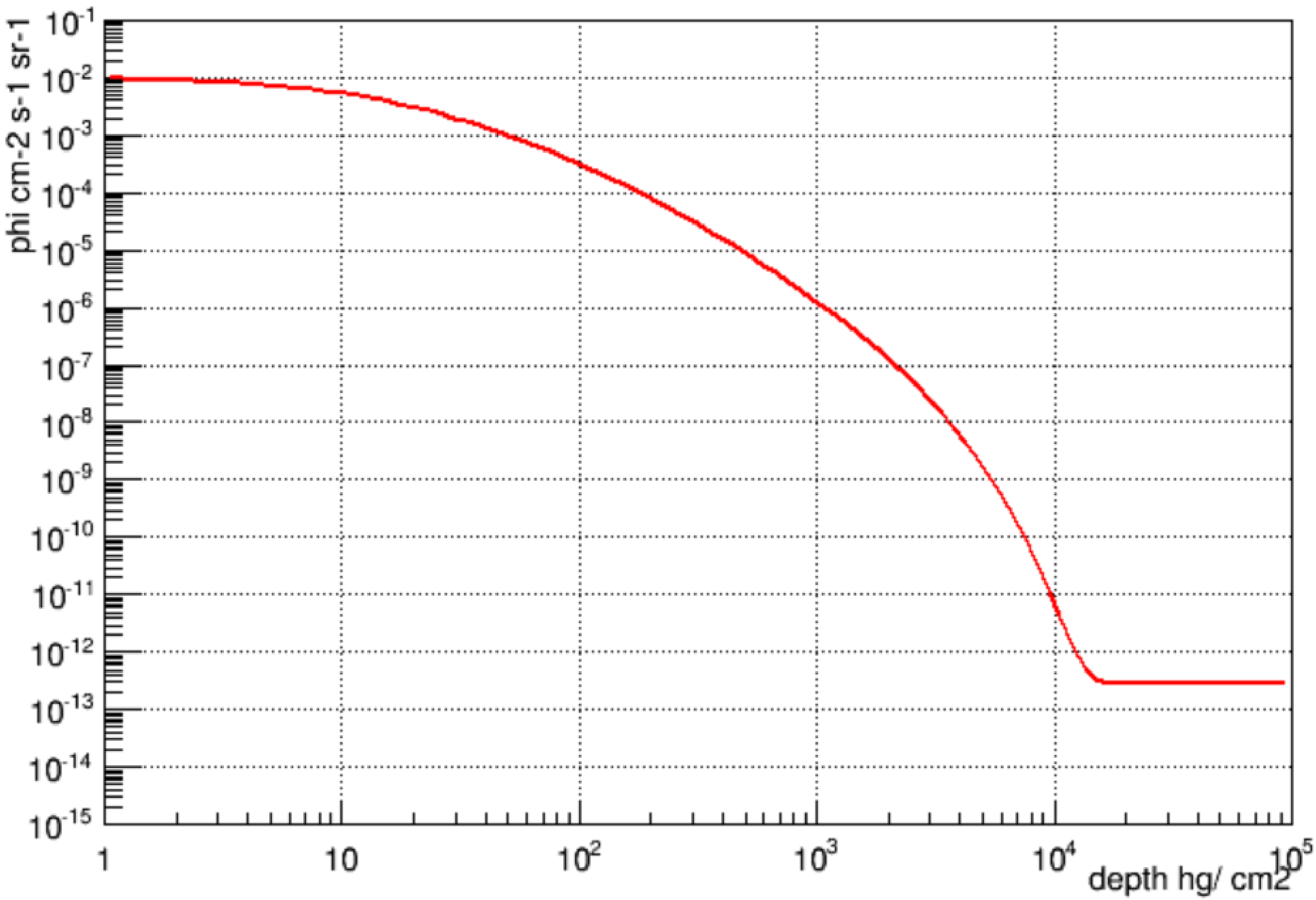
(2)

Observe units used

units on y-axis…..
/cm^2/sec/sr

units on x-axis
hg/cm^2 =1 meter water equivalent
100 cm water corresponds to 100g/cm2

Here my coding of the Heisinger formula to see that I am on the good track as I need the code for further calculations….
(The Heisinger parametrization of the flux is also used in the recent paper 2024 Sakurai et al)



(2) Now we have to go from muons/sec/cm^2/sr to muons/sec/cm^2
i.e we have to integrate of the horizontal angles

Use recipe of Heisinger 2002

$$\Phi(h,\theta) = \Phi_{\mathrm{v}}(h)\cdot\cos^{n(h)}\theta \qquad (3)$$

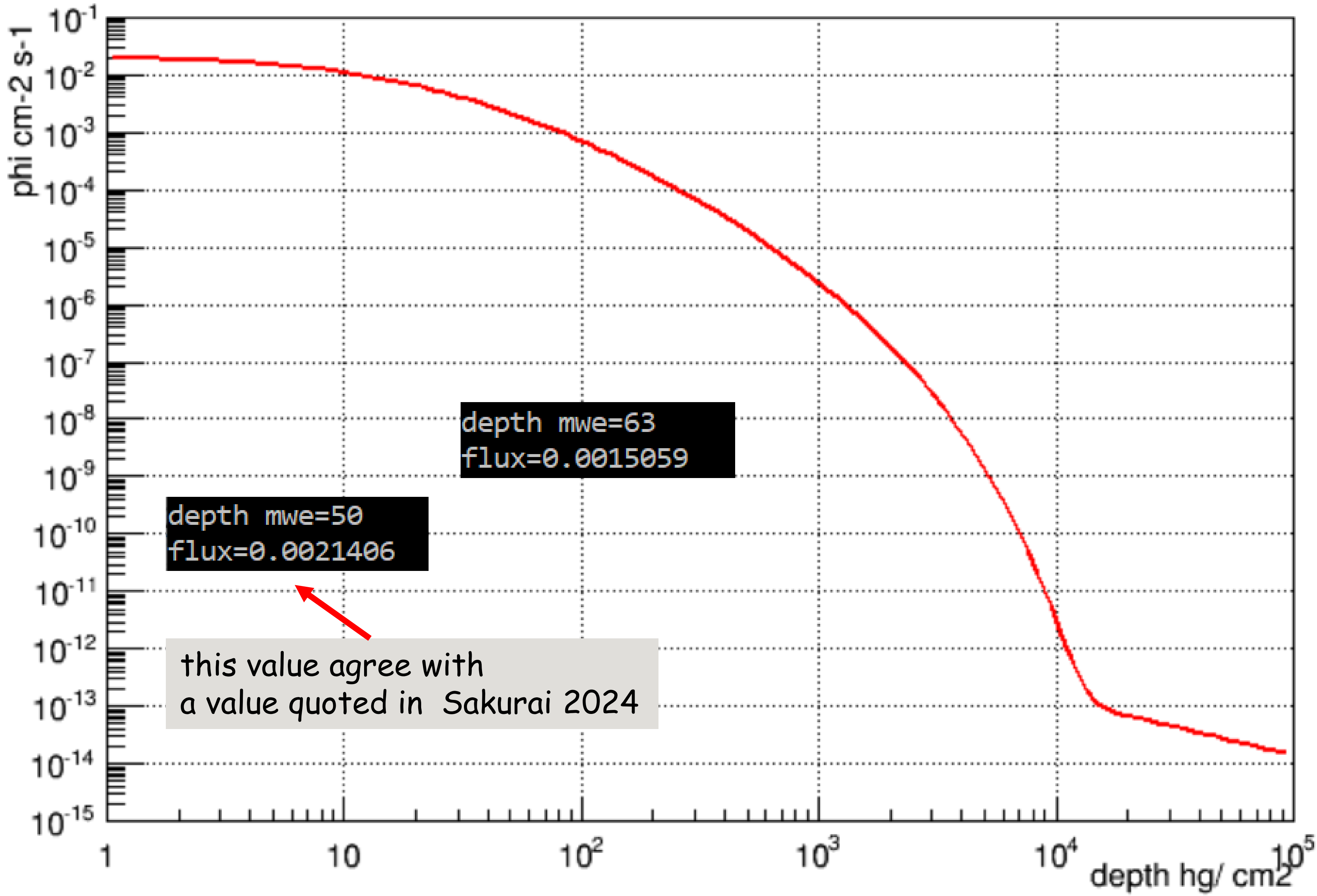
where $n(h)$ can be approximated by the function:

$$n(h) = 3.21 - 0.297\cdot\ln(h+42) + 1.21\times10^{-3}\,h \qquad (4)$$

Fig. 4 shows experimental results for $n(h)$ [21–32] together with the approximation given by Eq. 4. At sea level, we obtain $n(0)=2.1$. Using Eqs. 3 and 4, we obtain for the total muon flux, $\Phi(h)$:

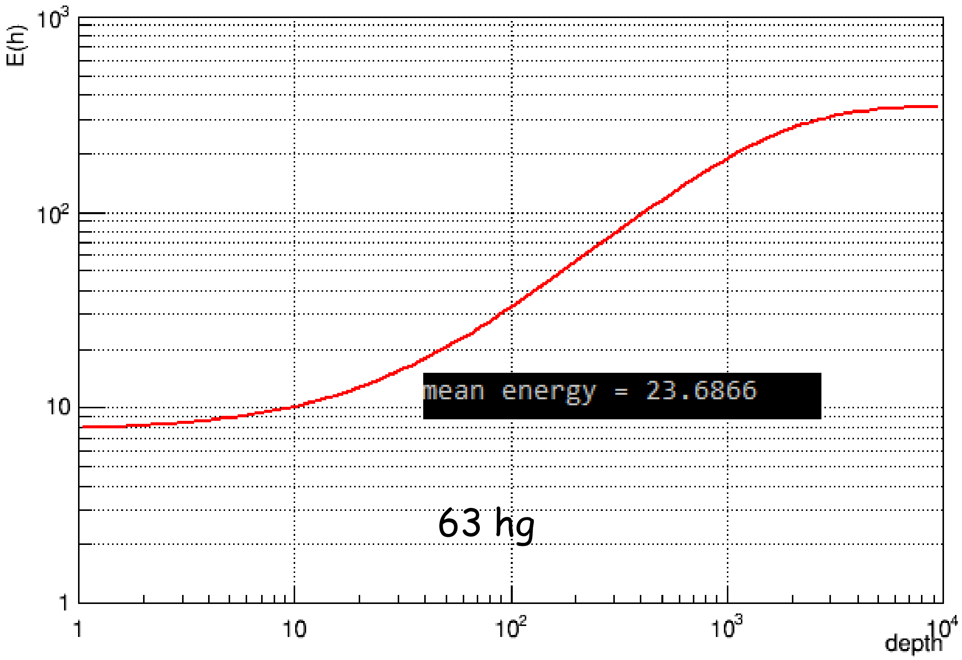
$$\Phi(h) = \int_{2\pi}\Phi(h,\theta)\mathrm{d}\Omega = \frac{2\pi}{n(h)+1}\cdot\Phi_{\mathrm{v}}(h) \qquad (5)$$

Here my result of the muon flux after horizontal integration



Now we have the muon flux at 6300 g/cm2

(3)Now we need the mean energy of the muons as the cross section depends on the muon energy



Units hg

My coding of Heisinger 2002

$$\overline{E'(h)} = 7.6\ \text{GeV} + 321.7\ \text{GeV}(1-\mathrm{e}^{-8.059\times10^{-4}\cdot h}) + 50.7\ \text{GeV}(1-\mathrm{e}^{-5.05\times10^{-5}\cdot h}) \qquad (11)$$

(4) Now we need the cross section and its energy dependence

(4a) For the energy dependence I use again Heisinger 2002

the cross-section depends on the muon energy E[GeV] as:

$$\sigma(E) = \sigma_0 \cdot E^{\alpha} \tag{13}$$

with $\sigma_0 = \sigma(1\ \mathrm{GeV})$. The exponent α is about $\alpha \approx 0.75$. For the measured nuclides σ_0 can be

(4b) Now the cross section

A table from the 2024 Sakurai paper (measurements at 160 GeV using the CERN muon beam)

TABLE V. Cross section data for ^{10}Be and ^{26}Al productions. The cross sections of the boxed silica plate and the three portions of granite core quartz were obtained in our muon exposure experiment. The uncertainties in this work are propagated from the errors in measured production rates in Table IV without any systematic error.

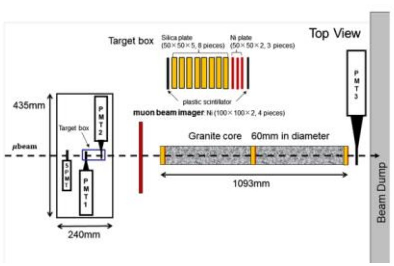
	Synthetic silica plate	Front quartz	Middle quartz	Back quartz	Heisinger *et al.* (190 GeV) [13]	Balco (190 GeV) [11]
^{10}Be (μb)	9.2 ± 0.6	12.0 ± 0.7	20.6 ± 1.1	27.2 ± 1.9	94 ± 13	$37.8 - 53.2$
^{26}Al (μb)	132 ± 8	154 ± 20	402 ± 32	486 ± 44	1410 ± 170	$521 - 739$

Sakurai et al 160 GeV 2024

Heisinger et al 190 Gev 2002

Balco 2017 Benchmarking of geological calibration data

Two mechanisms



Furthermore, the PHITS and FLUKA analysis of the particle contribution to the ^{10}Be and ^{26}Al production rates indicated that positive muons produce those nuclides at a constant rate and secondary particles produce them at a rate that increases with respect to granite core location. This suggests direct muon-induced spallation and secondary particle-induced spallation. The analysis also indicated that

to start with I use the back quartz….

(5)remains number atoms per gram SiO_2

Number of Si atoms per gram SiO_2 = N_a / (28+2*16=60)

Number of O atoms per gram SiO_2 = 2*N_a / (28+2*16=60)

N_a = Avogadros number

Now we have the 5 ingredients and can calculate the p-value at any depth

(1) muon flux /cm2/s/sr
(2) integrate over horizontal angles
flux/cm2/s
(3) mean energy of muons at a given depth
(4) Cross section at this energy
(5) number of atoms per gram SiO2

Start with depth =6300 g/cm2

Results and comparison with the exponential approach

p10 at depth of 6300 g/cm2

Units =atoms/gr/year

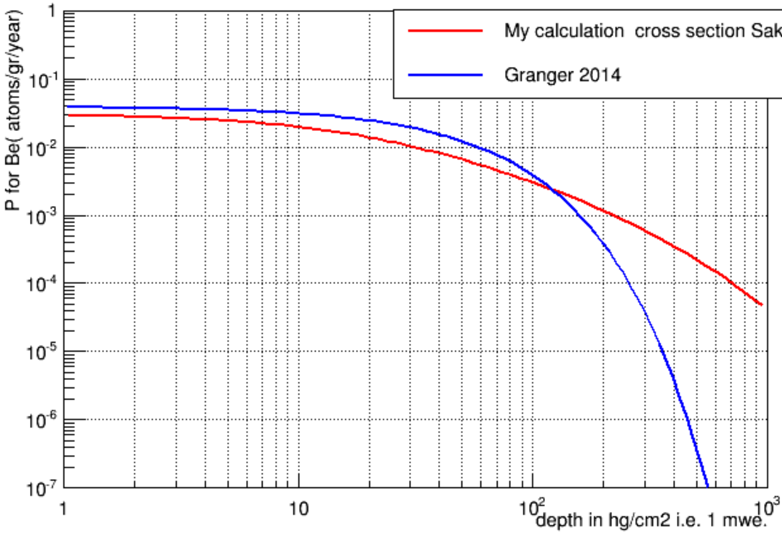
My calculation	Granger 2014	Braucher directly			
0.0052	0.0091	0.0098			

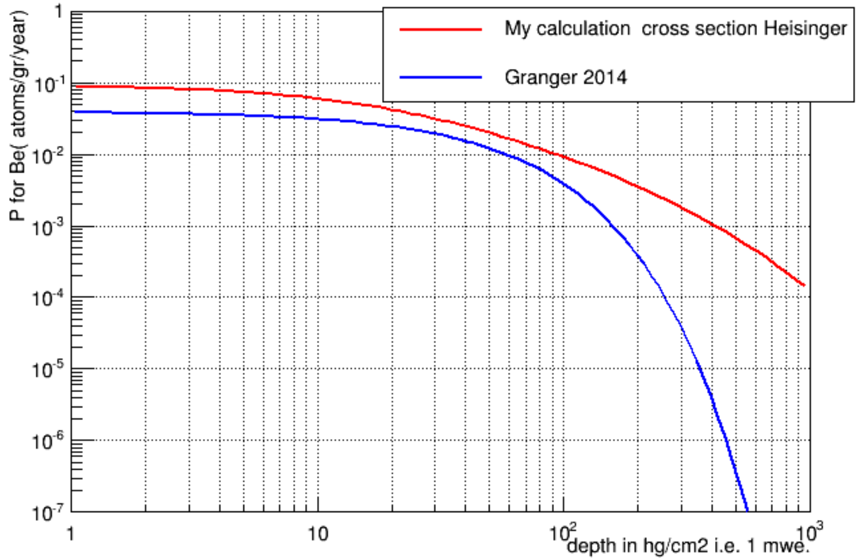
p26 at depth of 6300 g/cm2

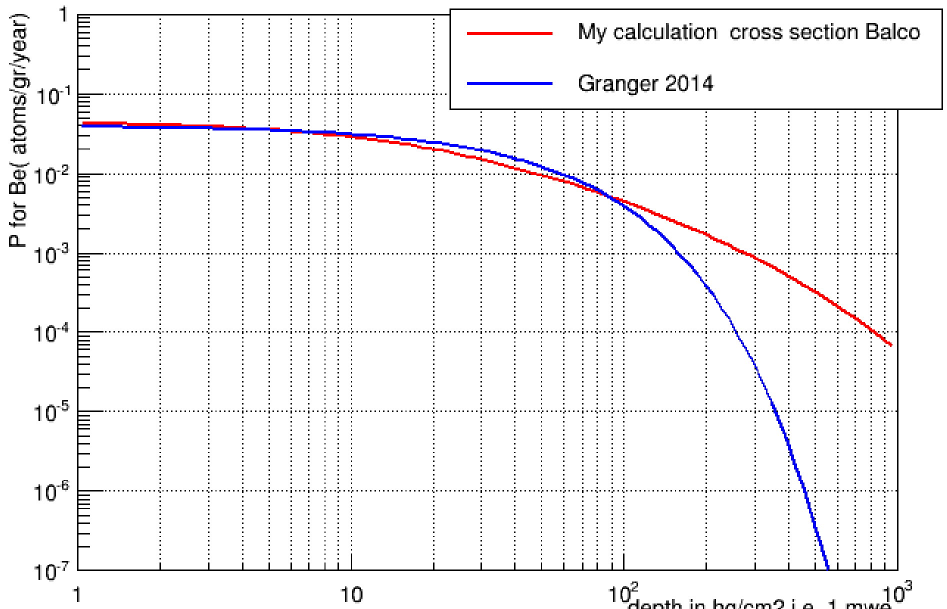
My calculation	Granger 2014	Braucher directly			
0.046	0.061	0.017			

Important bottom line here;
If we would measure the muon rate in the cave than we could immediately compare with the simulated rate used here and correct the p-value according to the measured muon rate and thus improve the precision in the dating

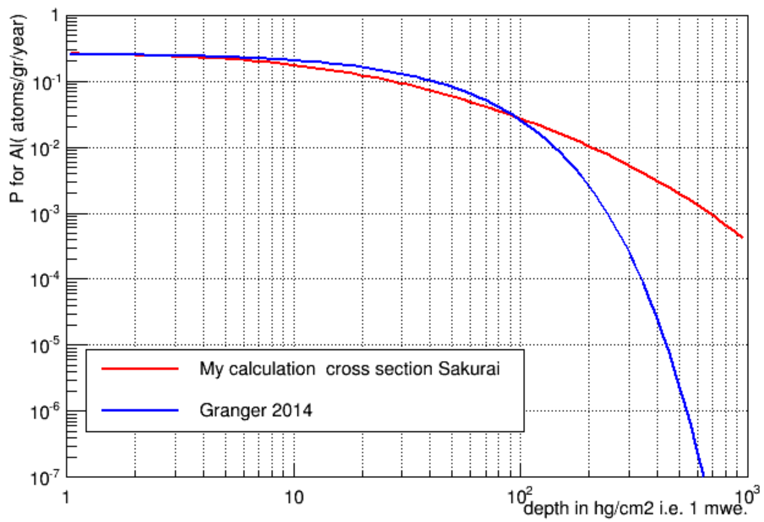
Now look at depth dependence
First Be
Compare different cross sections

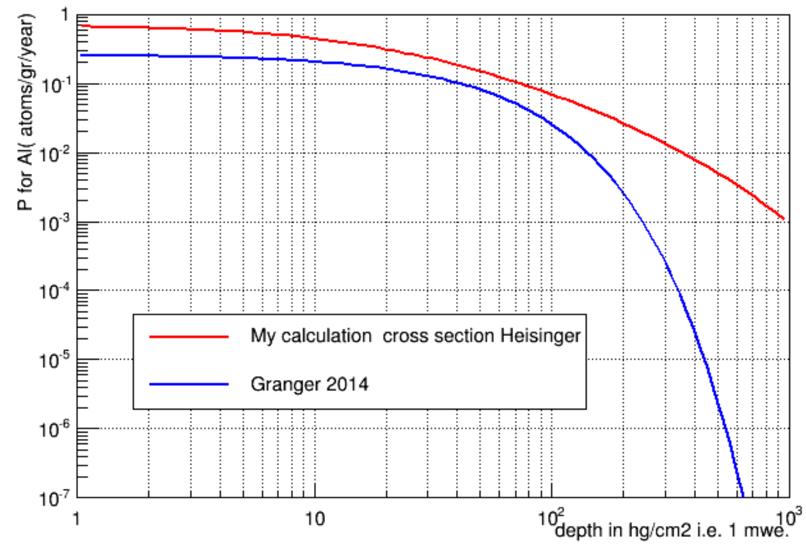


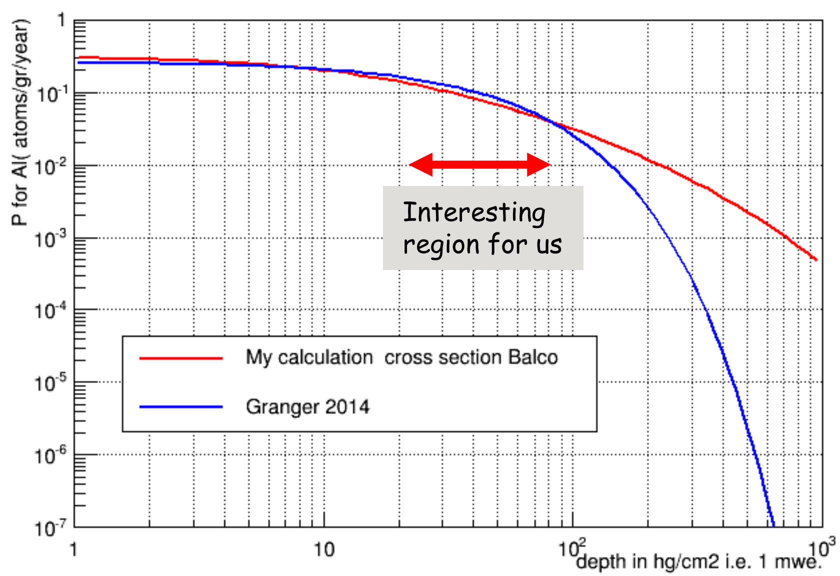




Now look at depth dependence
Now Al
Compare different cross sections







Amazing agreement between my flux driven calculation and the Granger calculation based upon drilled long cores…

A concrete example....before concluding

14.7.3.3.1 Example: Rietputs Formation, lower Vaal River

The Vaal River is a major tributary to the Orange River in South Africa. It drains a large part of South Africa, heading near Johannesburg and joining the Orange near the center of the country. The lower part of the Vaal River contains the best known and most studied terrace sequence in southern Africa (see Gibbon et al., 2009, and references therein). The terrace

I have reproduced with my calculation both values below

Table 4 Cosmogenic nuclide data and minimum burial age for Windsorton Pit 5

Sample	*[^{10}Be]^a ($\times 10^6$ at g^{-1})*	*[^{26}Al] ($\times 10^6$ at g^{-1})*	N_{10}^* ($\times 10^{-3}$)	N_{26}^* ($\times 10^{-3}$)	*Minimum age (Ma)*
Pit 5	0.364 ± 0.015	1.26 ± 0.11	21.5 ± 0.9	21.3 ± 1.9	1.35 ± 0.21

Adjusted for local production rates of 8.45 and 57.4 at g^{-1} year^{-1}.
[a]Normalized against standard 07KNSTD.

Table 5 Postburial production and maximum burial age for Windsorton Pit 5

Sample	*Depth (m)*	$P_{26,\,pb}$ *(at g^{-1} year^{-1})*	$P_{10,\,pb}$ *(at g^{-1} year^{-1})*	$N_{26,\,pb}$ *($\times 10^6$ at g^{-1})*	$N_{10,\,pb}$ *($\times 10^6$ at g^{-1})*	*Maximum age (Ma)*
Pit 5	15	0.150	0.022	0.117	0.017	1.46 ± 0.21

Assuming overburden density 1.9 g cm^{-3}.

Assuming overburden density 1.9 g cm^{-3}.

In this example they (and I) used the Heisinger depth profile
If we would measure the muon flux and would fine x2 or x1/2 of the one used in the Heisinger profile the max time would change accordingly:

μ flux
x 2 Maximum age 1.55 (Ma)
x 1/2 Maximum age 1.42 (Ma)

Conclusion

We have calculated the p-values from "first principles"

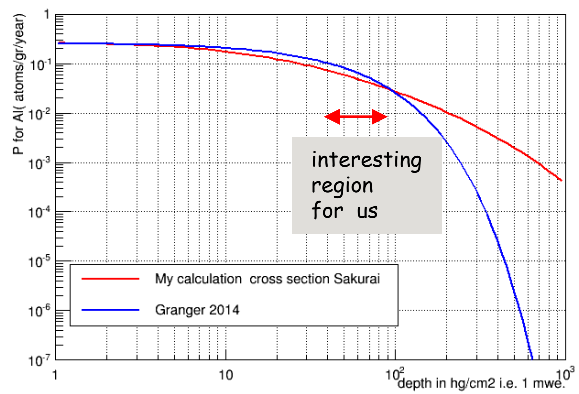
Amazing agreement between my calculation and Granger !

Completely different input

- long core drilled from the surface

versus

- calculation based on muon flux and σ



We have a tool !!!

A measurement of the muon flux in the cavity can immediately be translated to a new p-value by comparing the measurement with the flux used for the calculation

the corresponding change in burial time will be big/small depending on how much the flux measurement deviates from the calculated flux

Back up