Inflation and the Theory of Cosmological Perturbations

Exercises

1. **Friedmann equations**

   Using the FLRW metric and Einstein equations, find the Friedmann equations.

2. **Matter-Radiation Transition**

   Calculate the epoch of matter-radiation equality as a function of redshift.

3. **Flatness Problem**

   Consider an FLRW model dominated by a perfect fluid with equation of state $P = w\rho$, for $w = \text{const}$. Show that

   $$\frac{\mathrm{d}\Omega}{\mathrm{d}N} = (1 + 3w)\Omega(\Omega - 1)$$

   and discuss the stability of the solution $\Omega = 1$.

4. **Horizon Problem**

   The (comoving) particle horizon at the time t equals the amount of conformal time τ between $t_i = 0$ and t. In a flat universe filled with matter, radiation and dark energy, we have

   $$H^2 = H_0^2 \left[\Omega_r a^{-4} + \Omega_m a^{-3}\right] \tag{0.1}$$

   where H_0 is the Hubble constant and Ω_i are the densities today (in units of the critical density) ignoring the contribution from dark energy.

   Show that

   $$\tau = \frac{2}{\sqrt{\Omega_m H_0^2}} \left[\sqrt{a + a_{\mathrm{eq}}} - \sqrt{a_{\mathrm{eq}}}\right] \tag{0.2}$$

   What is the conformal time today? What is it at recombination? You may leave your answer in terms of the Hubble constant.

   Calculate the angle subtended by the horizon at recombination.

5. **E-folds and reheating temperature**

   Using entropy conservation and assuming instantaneous reheating at temperature T_{rh},

show that we need a number of e-folds of

$$N = 53 + \log\left[\frac{T_{\rm rh}}{10^{10}{\rm GeV}}\right] + \frac{1}{2}\log|\Omega_i - 1| \tag{0.3}$$

to solve the flatness problem.

6. **Photons in deSitter**

   Show that photons are not produced during inflation.

References

1. Inflation and Cosmological Perturbations, Antonio Riotto, https://arxiv.org/pdf/hep-ph/0210162
2. Cosmology Master's Degree course by Daniel Baumann at University of Amsterdam