Lecture 2

Neutrino oscillations in vacuum and in matter

Neutrino oscillations

nobelprize.org

Neutrino oscillations

1957: Pontecorvo suggests oscillations between neutrinos & antineutrinos (only ν_e).

B. Pontecorvo, J. Exp. Theor. Phys. 33 (1957) 549.
B. Pontecorvo, J. Exp. Theor. Phys. 34 (1958) 247.

1962: Maki, Nakagawa and Sakata propose neutrino mixing between flavor eigenstates

$$\nu_1 = \nu_e \cos\delta + \nu_\mu \sin\delta,$$
$$\nu_2 = -\nu_e \sin\delta + \nu_\mu \cos\delta.$$

true neutrinos — weak neutrinos

2ν mixing

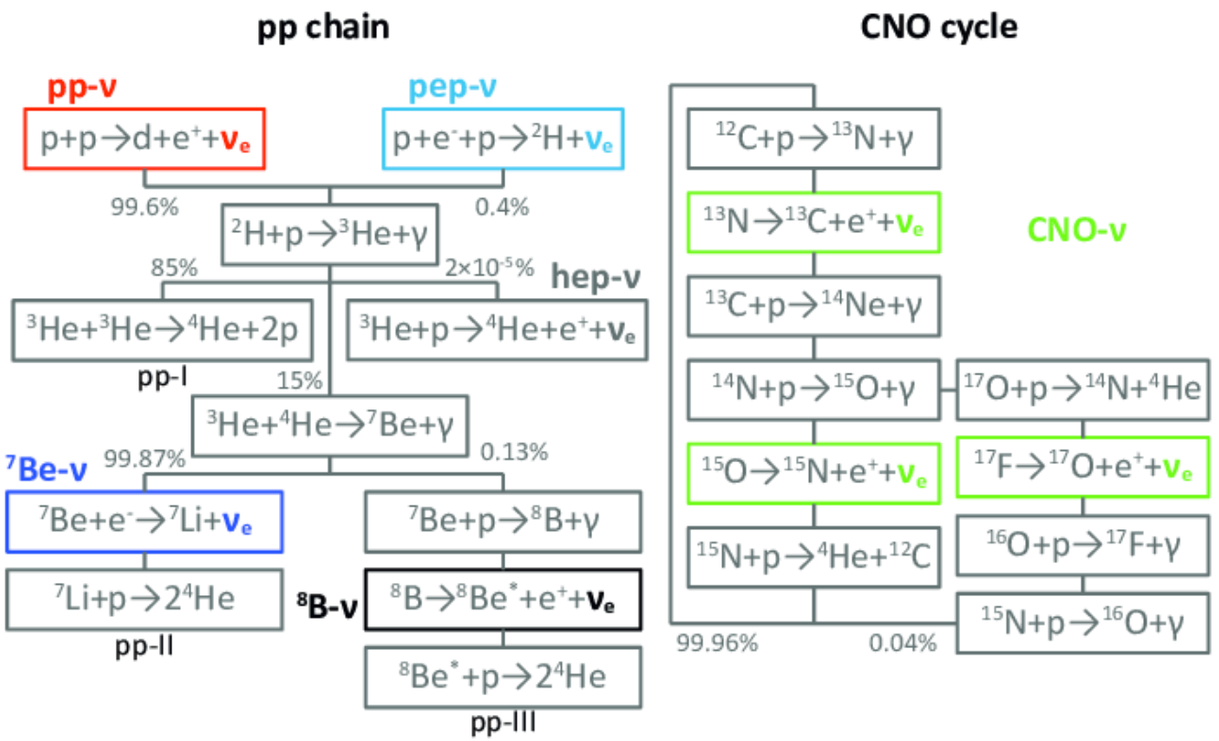
Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

1969: Gribov & Pontecorvo calculated the neutrino oscillation probability (in vacuum) for the first time

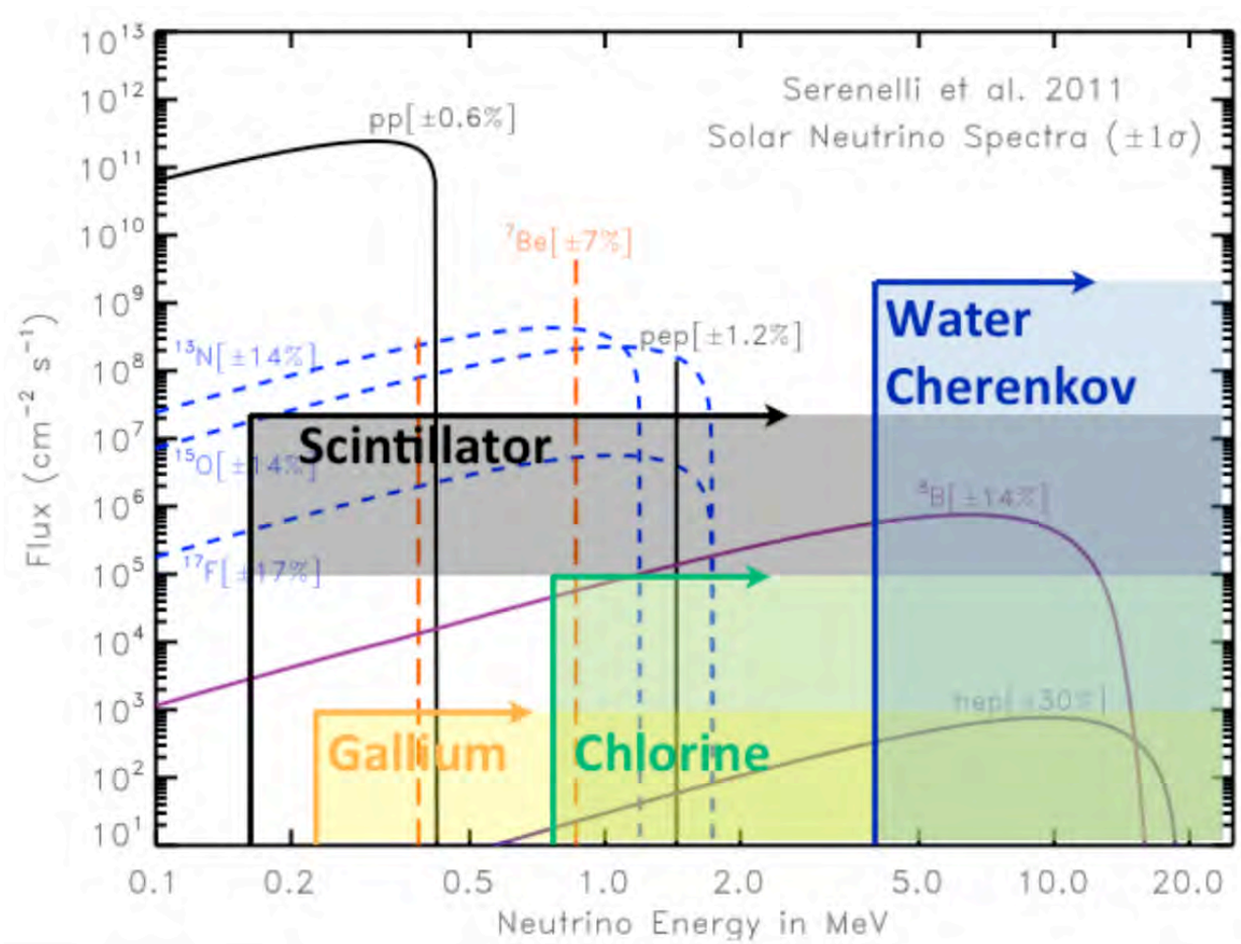
V. Gribov, B. Pontecorvo, Phys. Lett. B28 (1969) 493.

Solar neutrinos

$$4p \longrightarrow {}^4He + 2e^+ + 2\nu_e + \gamma$$



Standard Solar Model

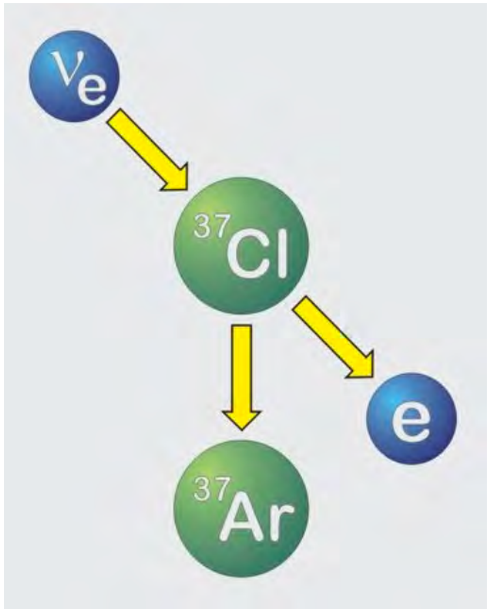


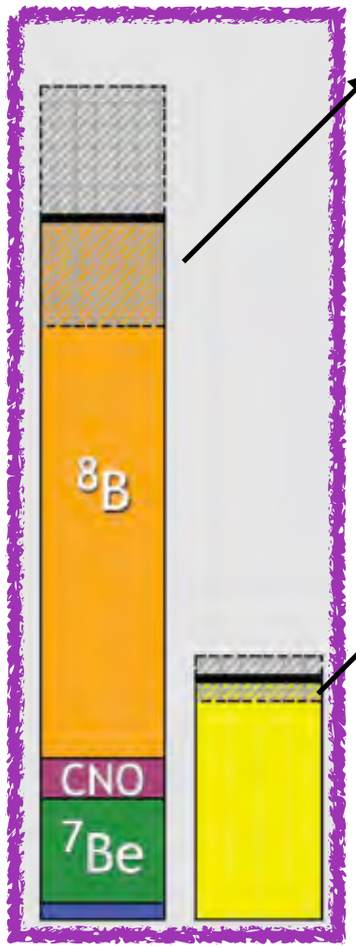
First indication of ν oscillations

Raymond Davis Jr.

1968: First observation of solar neutrinos by R. Davis in an underground experiment (Homestake gold mine, South Dakota) using 615 Tn of C_2Cl_4

2002 Nobel Prize in Physics

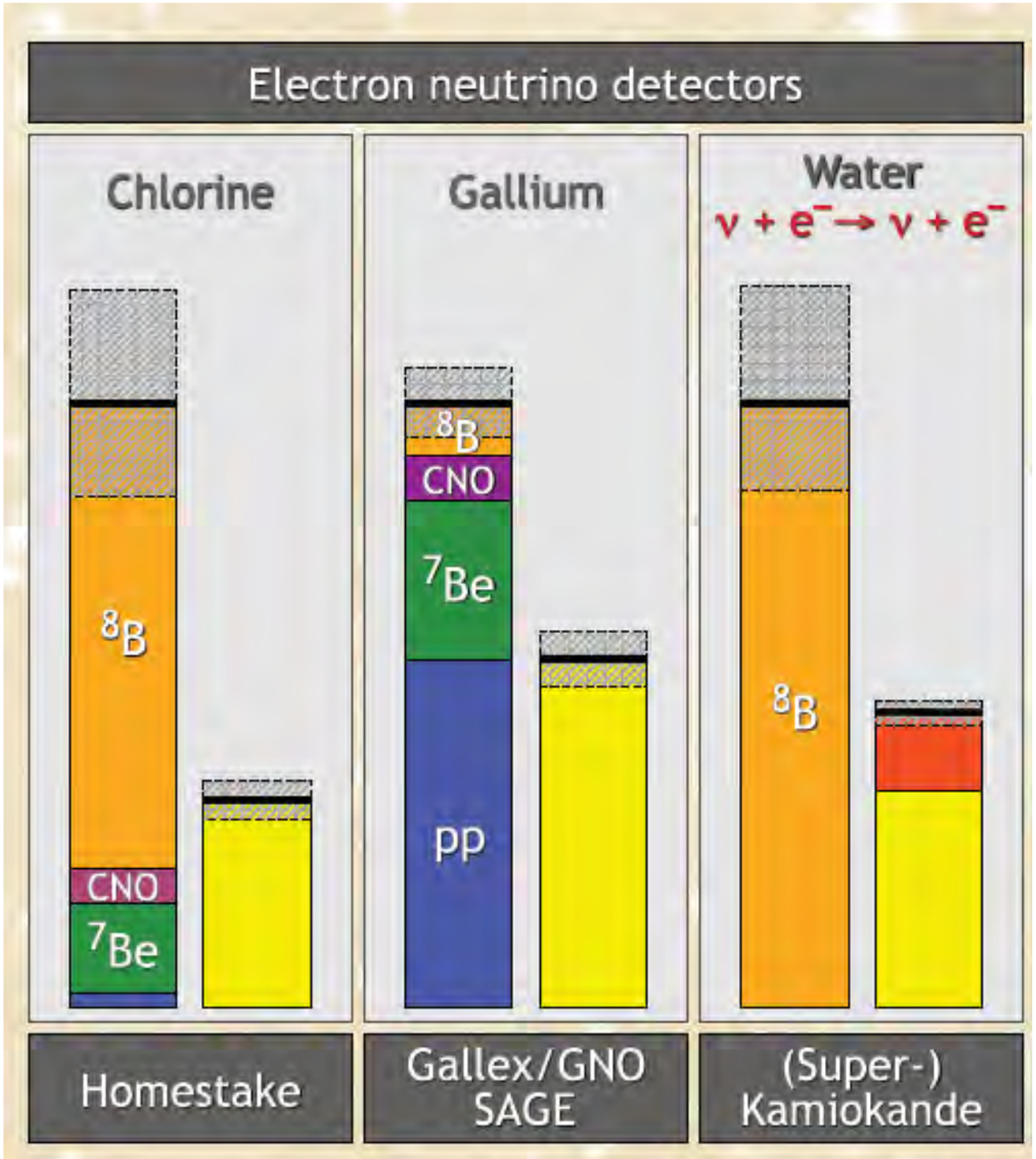




Expected neutrinos

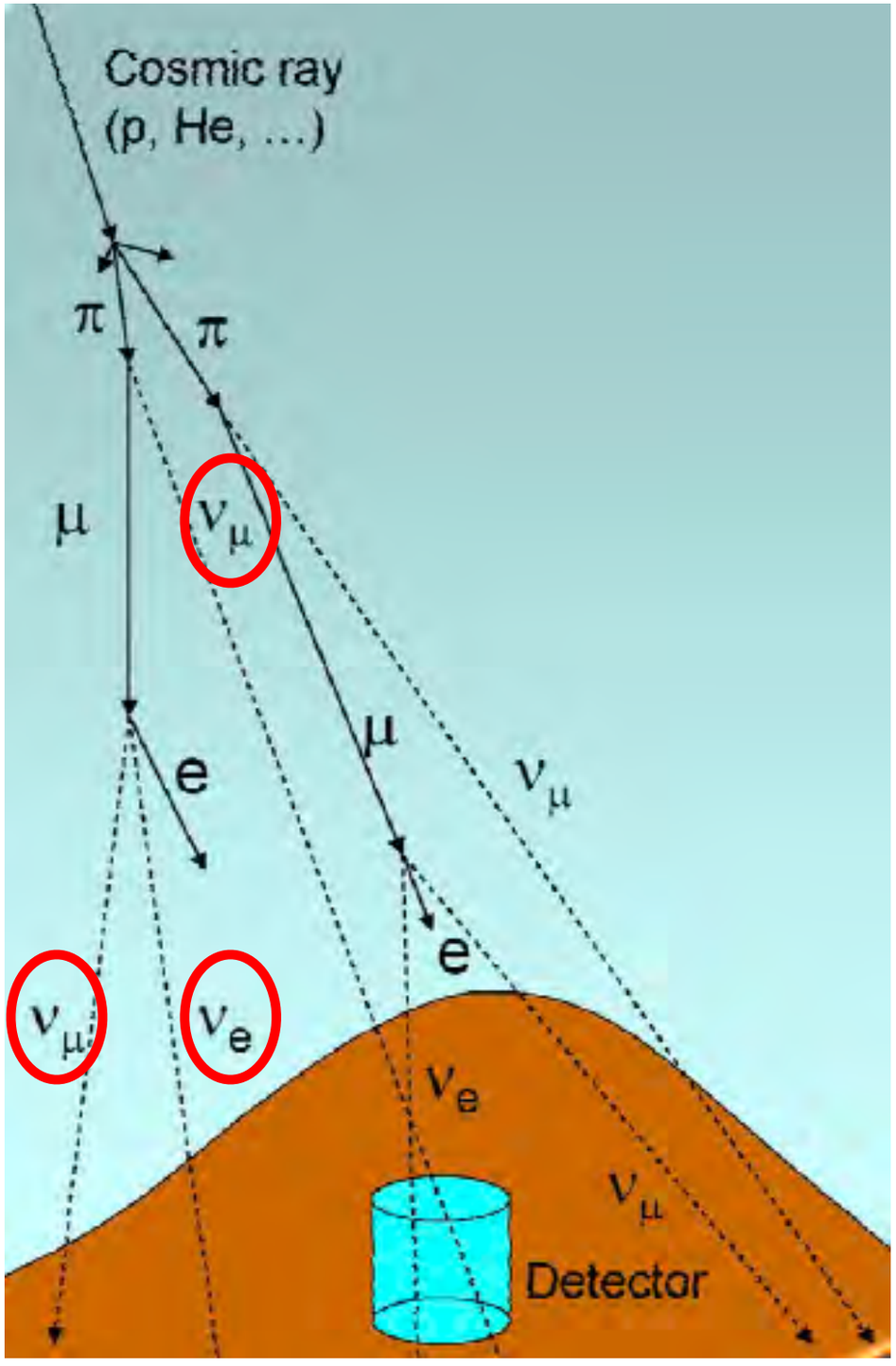
Observed neutrinos: 30% !!

The solar neutrino problem



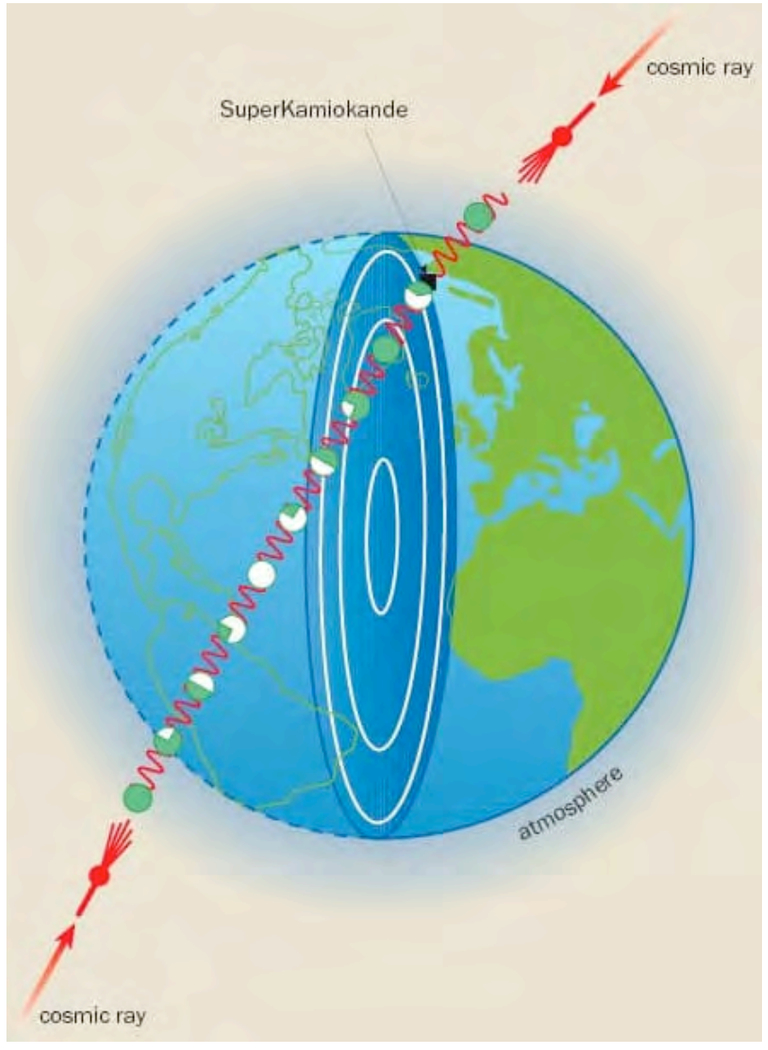
~30% ~50% ~40%

Atmospheric neutrinos



$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad \pi^+ \to \mu^+ + \nu_\mu$$

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \qquad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

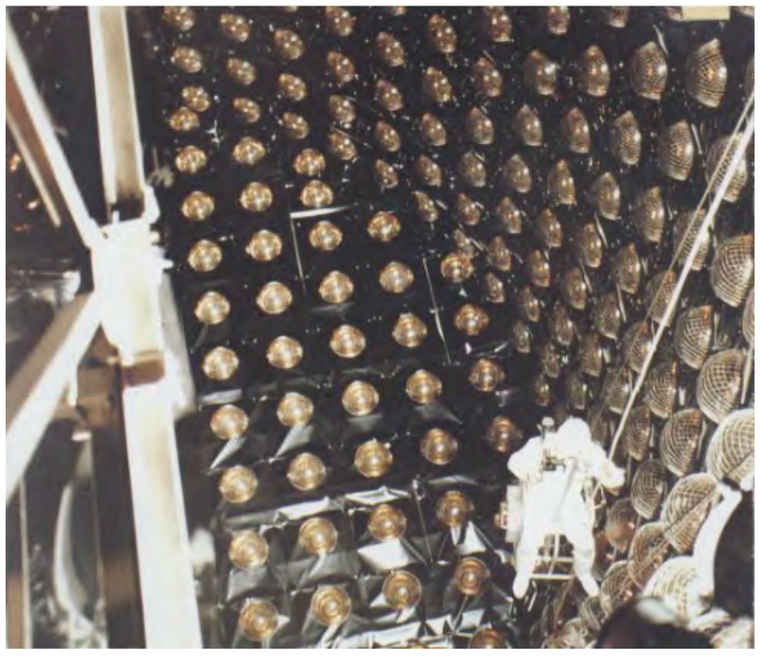


$$\nu_e + N \to e^- + N'$$
$$\nu_\mu + N \to \mu^- + N'$$

Atmospheric neutrinos

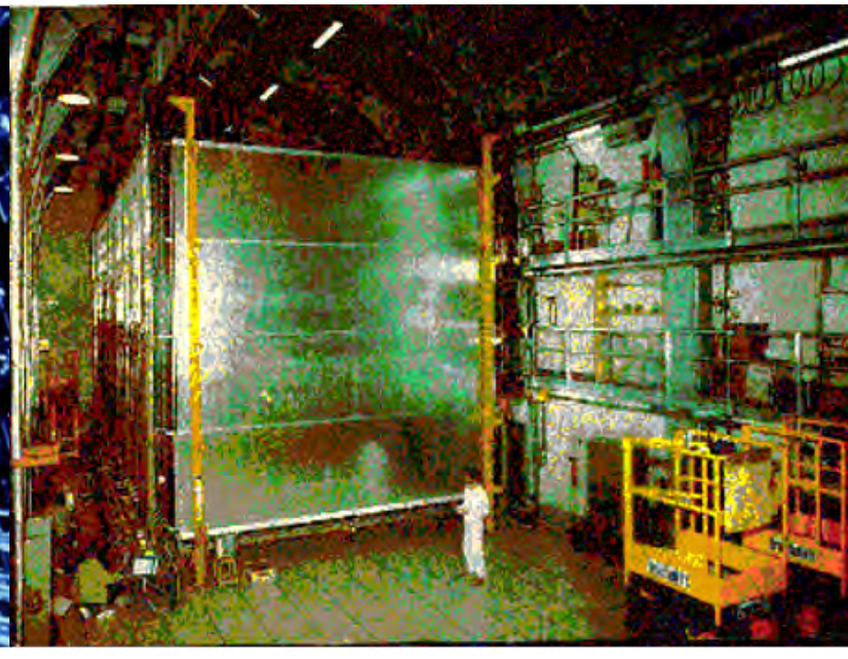
Background for proton decay searches

Detection process: $\nu_\mu + N \to \mu^- + N'$



Kamiokande, Japan
[1000 ton]

IMB, USA
[3300 ton]



Fréjus, France
[700 ton]

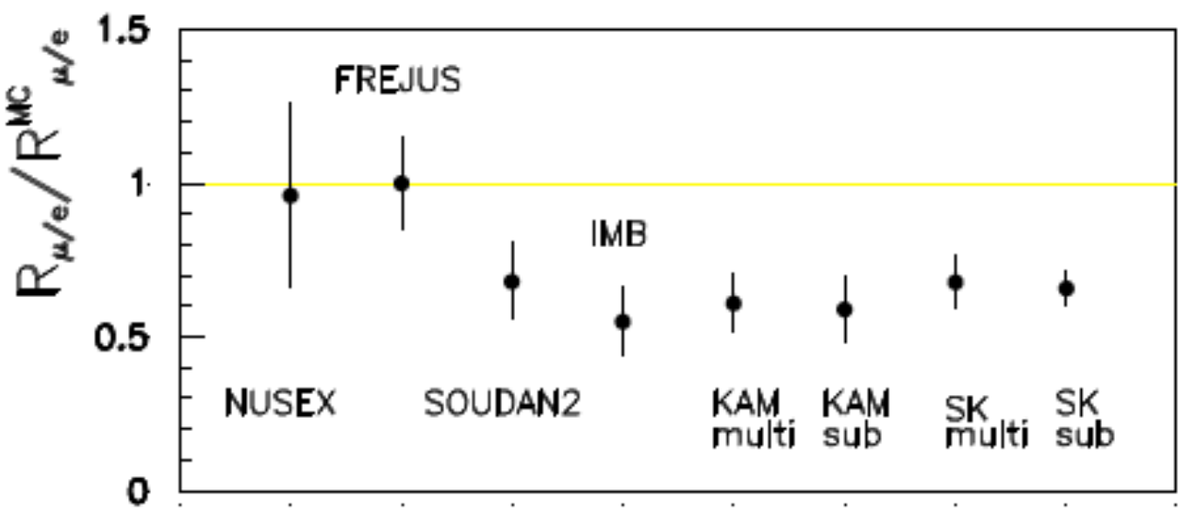
The atmospheric neutrino anomaly

1985: First indications of a deficit in the observed number of atmospheric ν_μ at IMB.

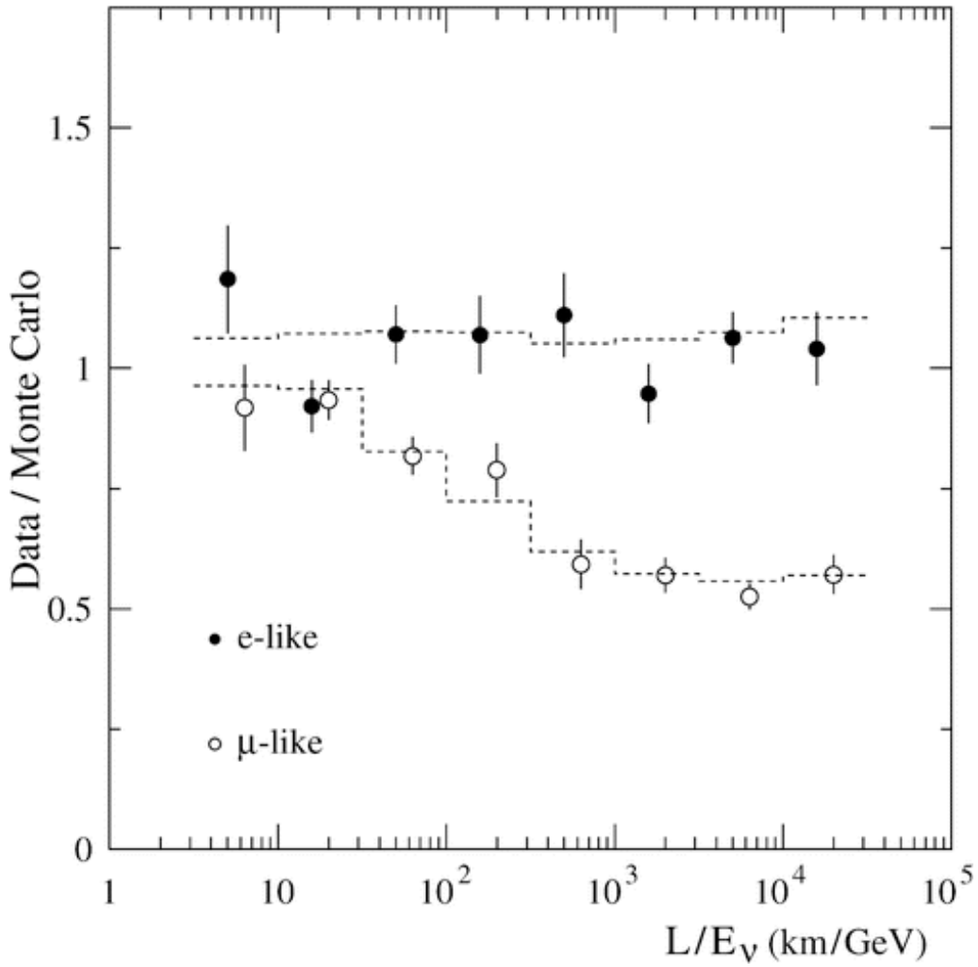
$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad \pi^+ \to \mu^+ + \nu_\mu$$

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \qquad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \simeq 2$$

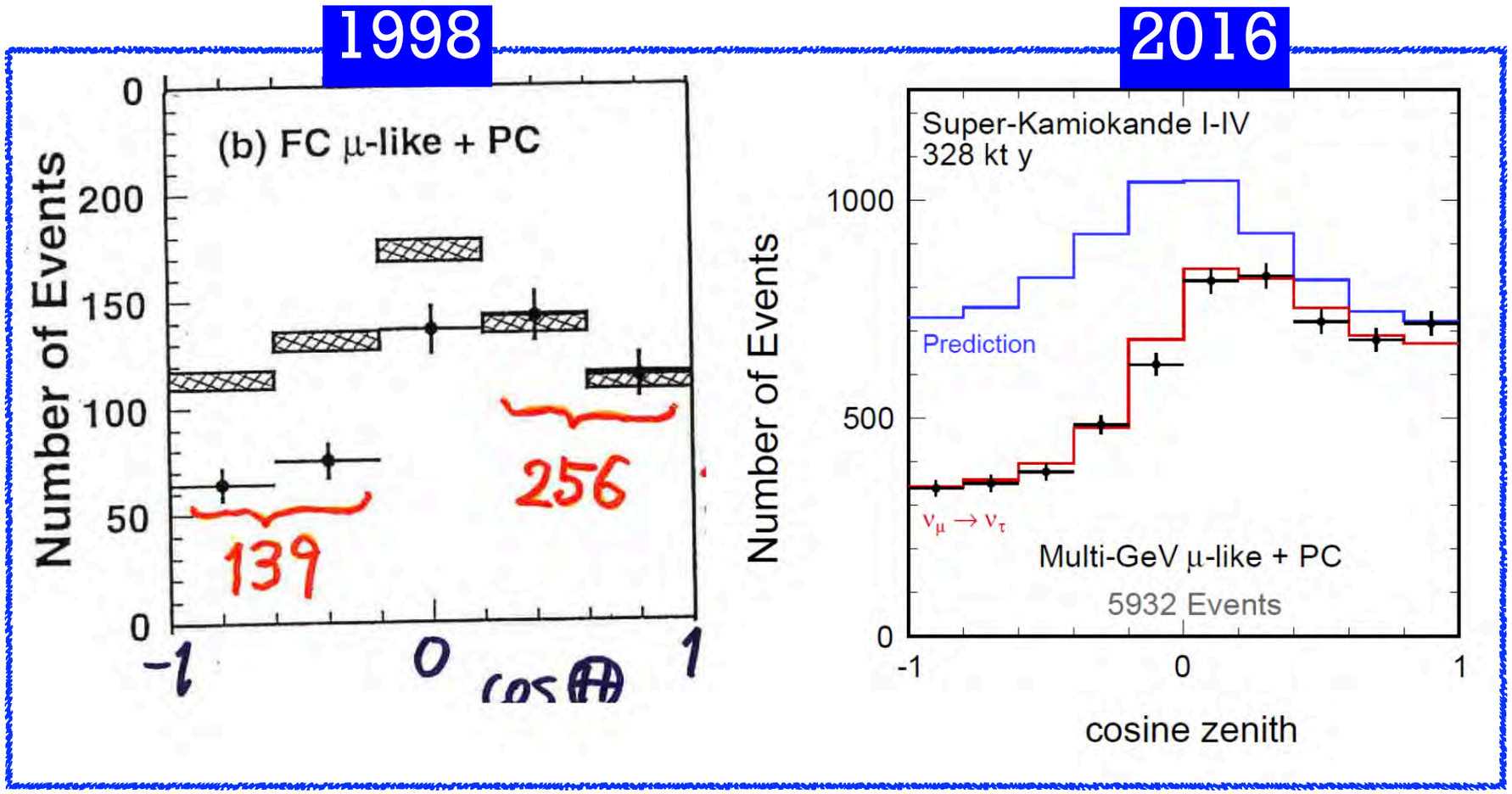


1994: **Kamiokande** finds the ν_μ deficit depends on the distance travelled by the neutrino and its energy.

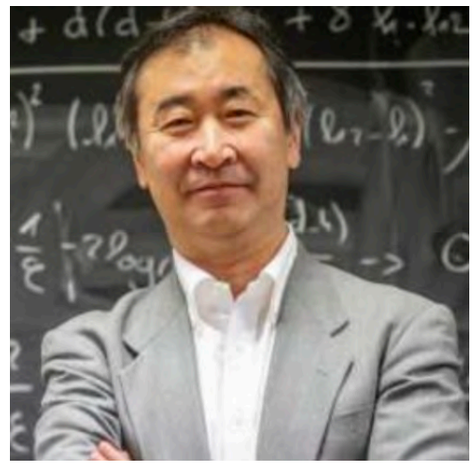


The atmospheric neutrino anomaly

1998: Discovery of atmospheric neutrino oscillations in **Super-Kamiokande**



From T. Kajita



2015 Nobel Prize in Physics

oscillation channel $\nu_\mu \to \nu_\tau$

➡ first evidence for non-zero **neutrino masses.**

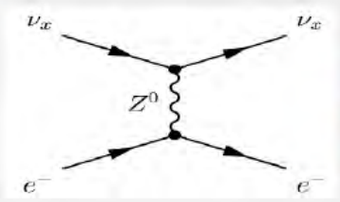
Sudbury Neutrino Observatory (SNO)

SNO Coll, PRL89 (2002) 011301

SNO interactions

Elastic-scattering (ES):

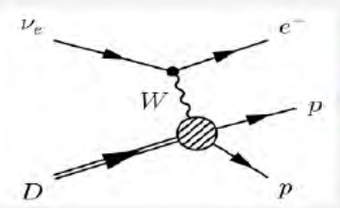
$$\nu_x + e^- \rightarrow \nu_x + e^-$$



ν_e mainly
strong directional
sensitivity

Charged-currents (CC):

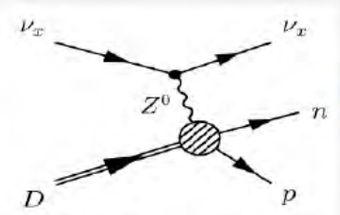
$$\nu_e + d \rightarrow p + p + e^-$$



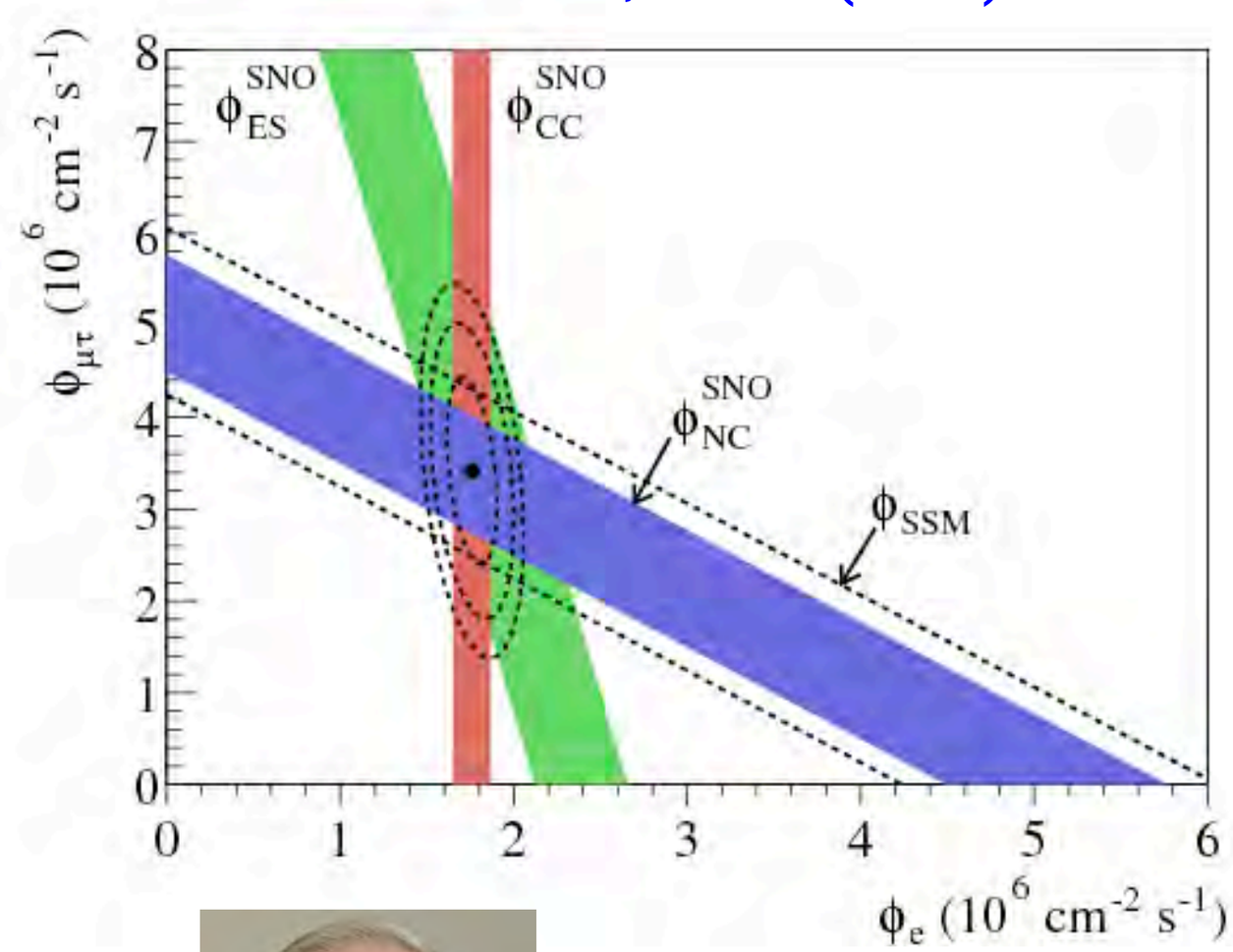
ν_e only
E_e well correlated
with E_ν

Neutral-currents (NC):

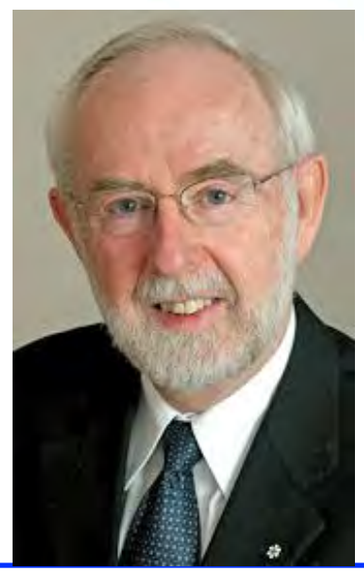
$$\nu_x + d \rightarrow p + n + \nu_x$$



All flavors equally
Total neutrino flux

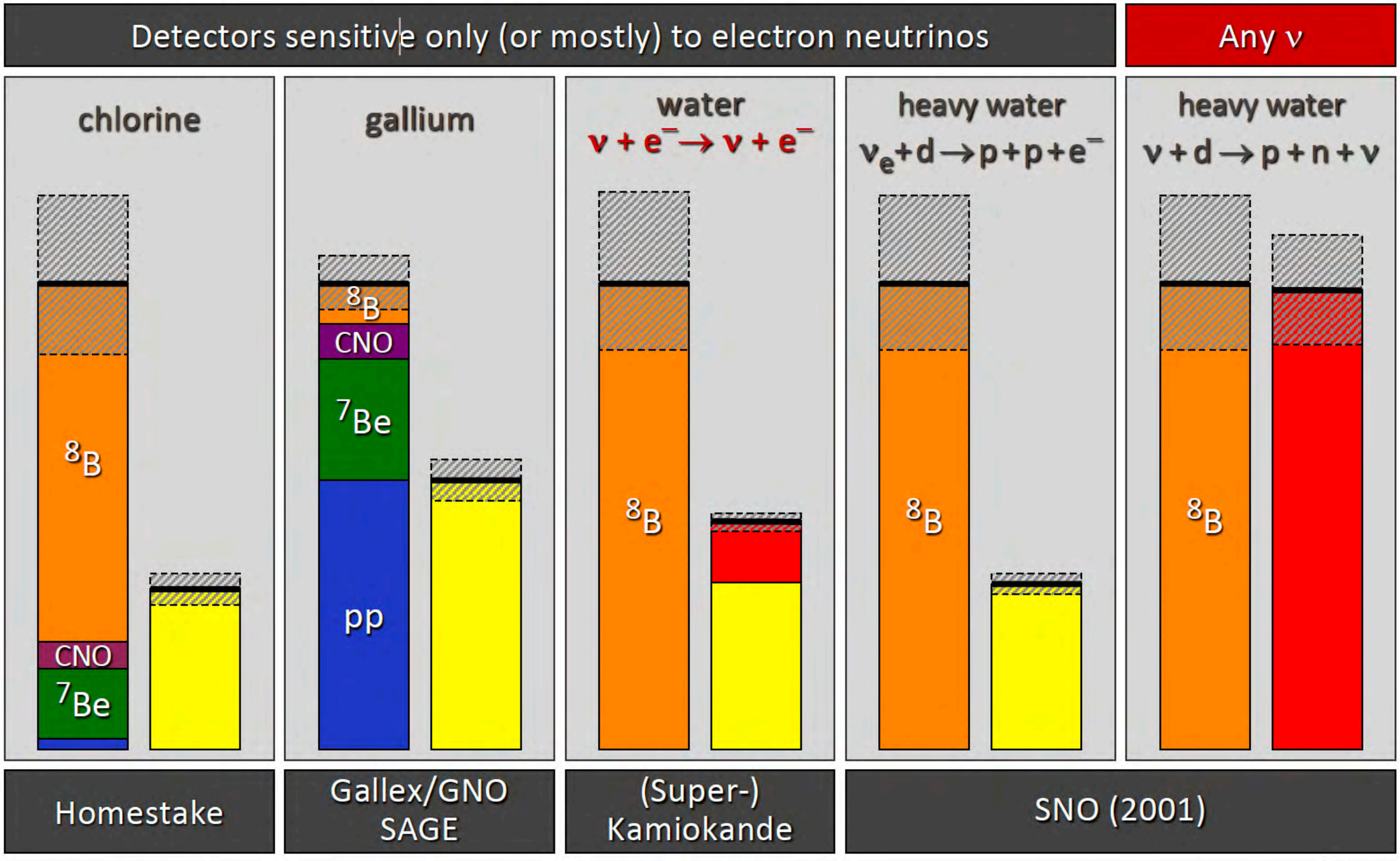


conversion $\nu_e \rightarrow \nu_\mu, \nu_\tau$



2015 Nobel Prize in Physics

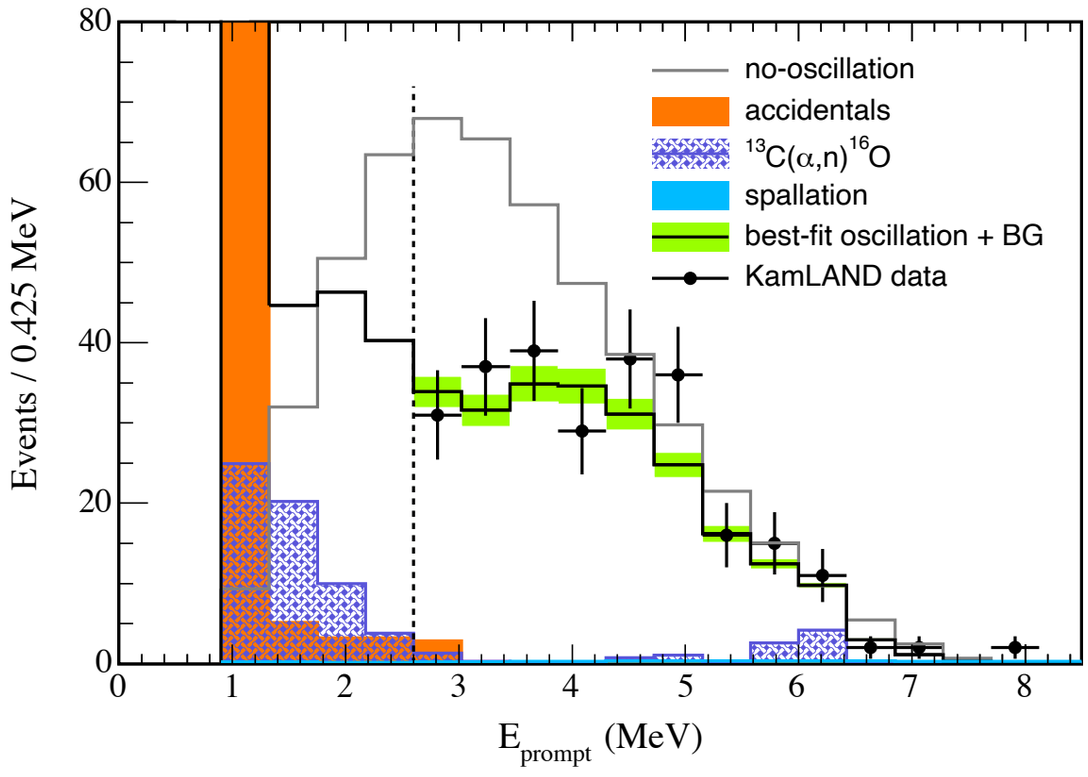
The solar neutrino ~~problem~~



conversion $\nu_e \rightarrow \nu_\mu, \nu_\tau$

Other important results

2002: The reactor experiment KamLAND observed neutrino oscillations consistent with the solar anomaly.



KamLAND Coll, PRL 90 (2003) 021802

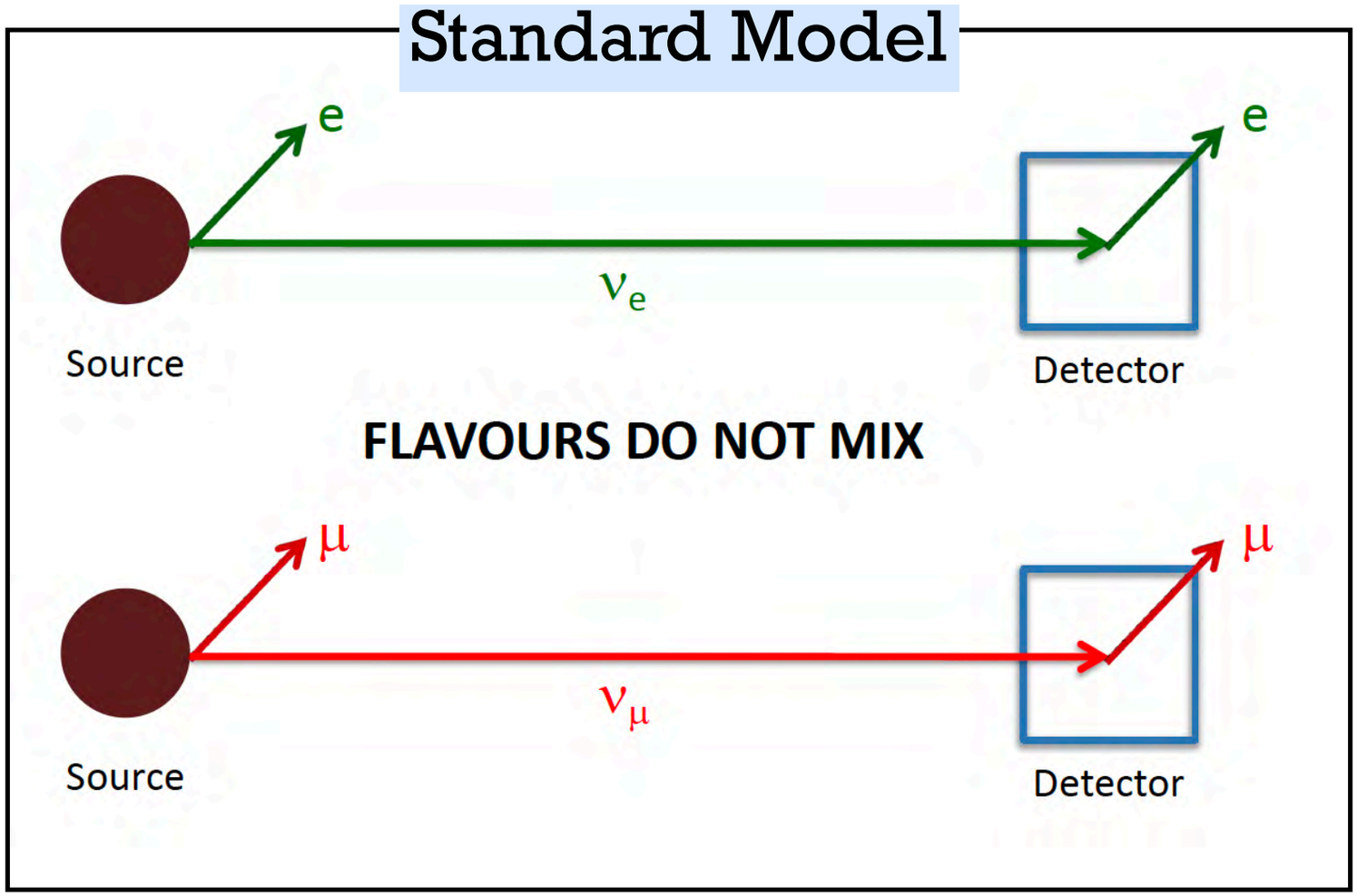
2002: Results of the accelerator experiment K2K consistent with ν_μ oscillations as in the atmospheric anomaly (MINOS, T2K, NOvA).

2011: $\nu_\mu \rightarrow \nu_e$ oscillations observed in long-baseline accelerator experiments.

2011: Double Chooz confirmed reactor antineutrino oscillations in a baseline of ~1 km (Daya Bay, RENO).

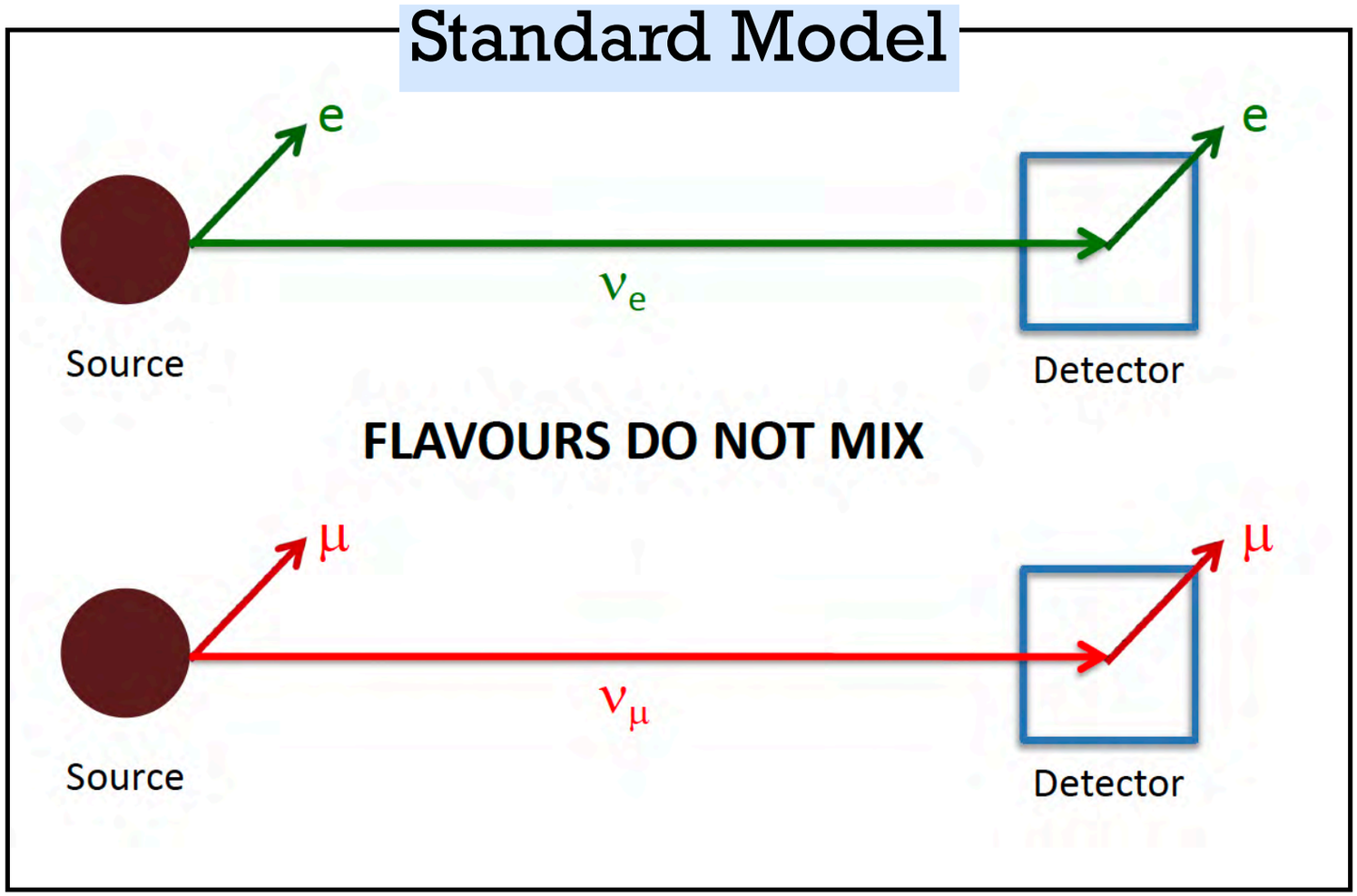
neutrino oscillations have been observed in solar, atmospheric, reactor and accelerator neutrino experiments.

Neutrino oscillations

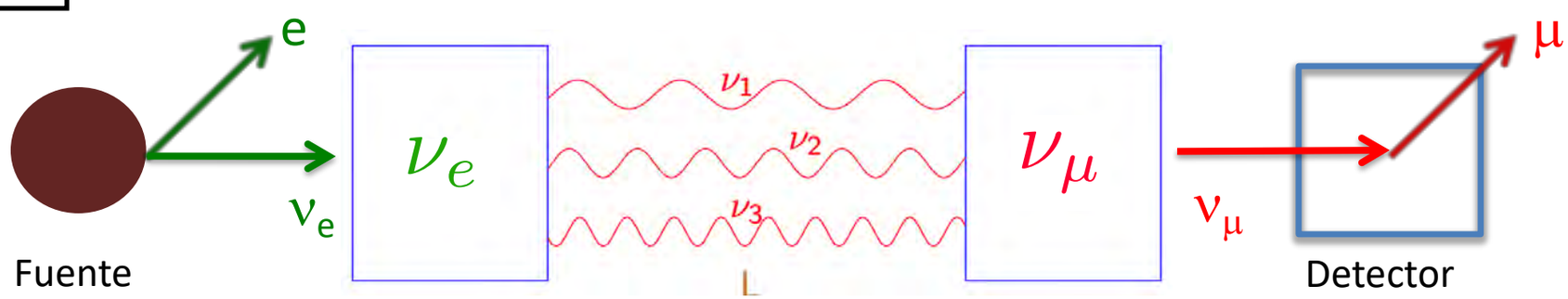


However

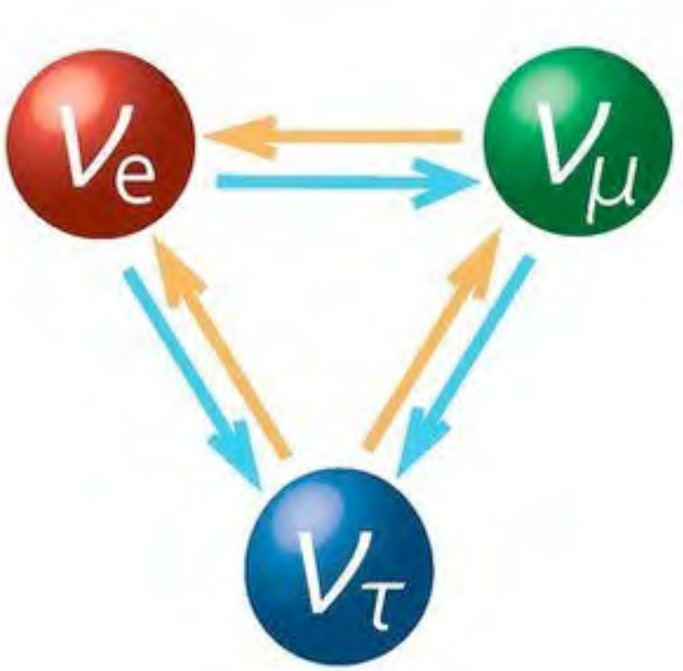
Neutrino oscillations



$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$$



Neutrino oscillations: formalism



Neutrino mixing

✦ Mixing described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$$

✦ NxN unitary matrix: NxN real parameters

→ N(N-1)/2 mixing angles + N(N+1)/2 phases (not all observables!)

✦ Leptonic weak charged current:

N(N+1)/2 phases

$$J^\rho = 2 \sum_\alpha \overline{l_{\alpha L}} \gamma^\rho \nu_{\alpha L} = \sum_\alpha \sum_k \overline{l_{\alpha L}} \gamma^\rho U_{\alpha k} \nu_{\alpha L}$$

✦ Lagrangian invariant under global phase transformations of Dirac fields:

$$l_\alpha \to e^{i\theta_\alpha} l_\alpha \,, \quad \nu_k \to e^{i\phi_k} \nu_k$$

$$J^\rho \to 2 \sum_{\alpha,k} \overline{l_{\alpha L}} e^{-i(\theta_\alpha - \phi_1)} \gamma^\rho U_{\alpha k} e^{i(\phi_k - \phi_1)} \nu_{kL}$$

N N-1

(2N-1) phases from U reabsorbed in the fields

(N-1)(N-2)/2 physical phases

Neutrino mixing

✦ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \to e^{i\phi_k}\nu_k \qquad \Rightarrow \qquad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \to e^{2i\phi_k}\nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

→ only N phases can be eliminated by rephasing charged lepton fields (neutrino fields can not be rephased!!):

$$J^\rho \to 2\sum_{\alpha,k} \overline{l_{\alpha L}} e^{-i\theta_\alpha} \gamma^\rho U_{\alpha k} \nu_{kL}$$

N

N(N+1)/2 - N = N(N-1)/2 physical phases for Majorana neutrinos

→ N(N-1)/2 **physical phases:** (N-1)(N-2)/2 Dirac phases → effect in ν oscil.

(N-1) Majorana phases → relevant for 0νββ

Neutrino mixing

- 2-neutrino mixing depends on 1 angle only (+1 Majorana phase)

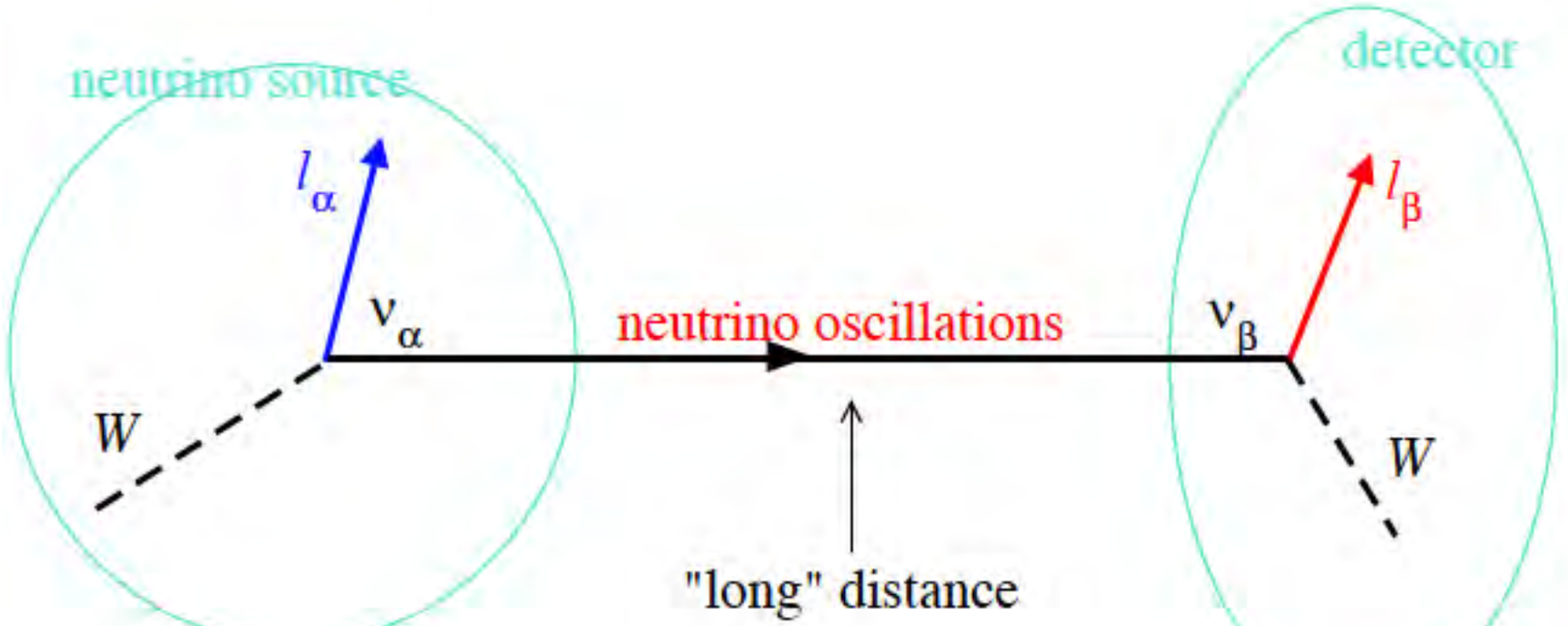
$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL reactor + LBL solar + KamLAND

Neutrino oscillations picture



Production

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$$

coherent superposition of massive states

Propagation

$$\nu_j : e^{-i\frac{m_j^2 L}{2E}}$$

different propagation phases change ν_j composition

Detection

$$\langle\nu_\beta| = \sum_j \langle\nu_j| U_{\beta j}$$

projection over flavour eigenstates

Neutrino oscillation probability

Neutrino oscillation amplitude:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta(t) | \nu_\alpha(0) \rangle = \sum_j \underbrace{\langle \nu_\beta | \nu_j(t) \rangle}_{\text{detection}} \underbrace{\langle \nu_j(t) | \nu_j(0) \rangle}_{\text{propagation}} \underbrace{\langle \nu_j(0) | \nu_\alpha \rangle}_{\text{production}}$$

Neutrino oscillation probability: $P_{\nu_\alpha \to \nu_\beta} = \left| \sum_j U_{\beta j} e^{-i \frac{m_j^2 L}{2E}} U^*_{\alpha j} \right|^2$

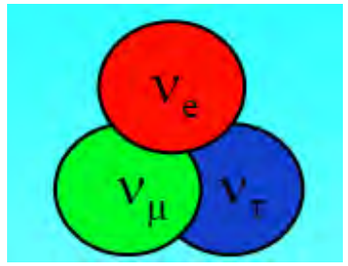
$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i>j} Re \left(U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j} \right) \sin^2 \left(\frac{\Delta m^2_{ij} L}{4E} \right) +$$

$$+ 2 \sum_{i>j} Im \left(U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j} \right) \sin \left(\frac{\Delta m^2_{ij} L}{2E} \right)$$

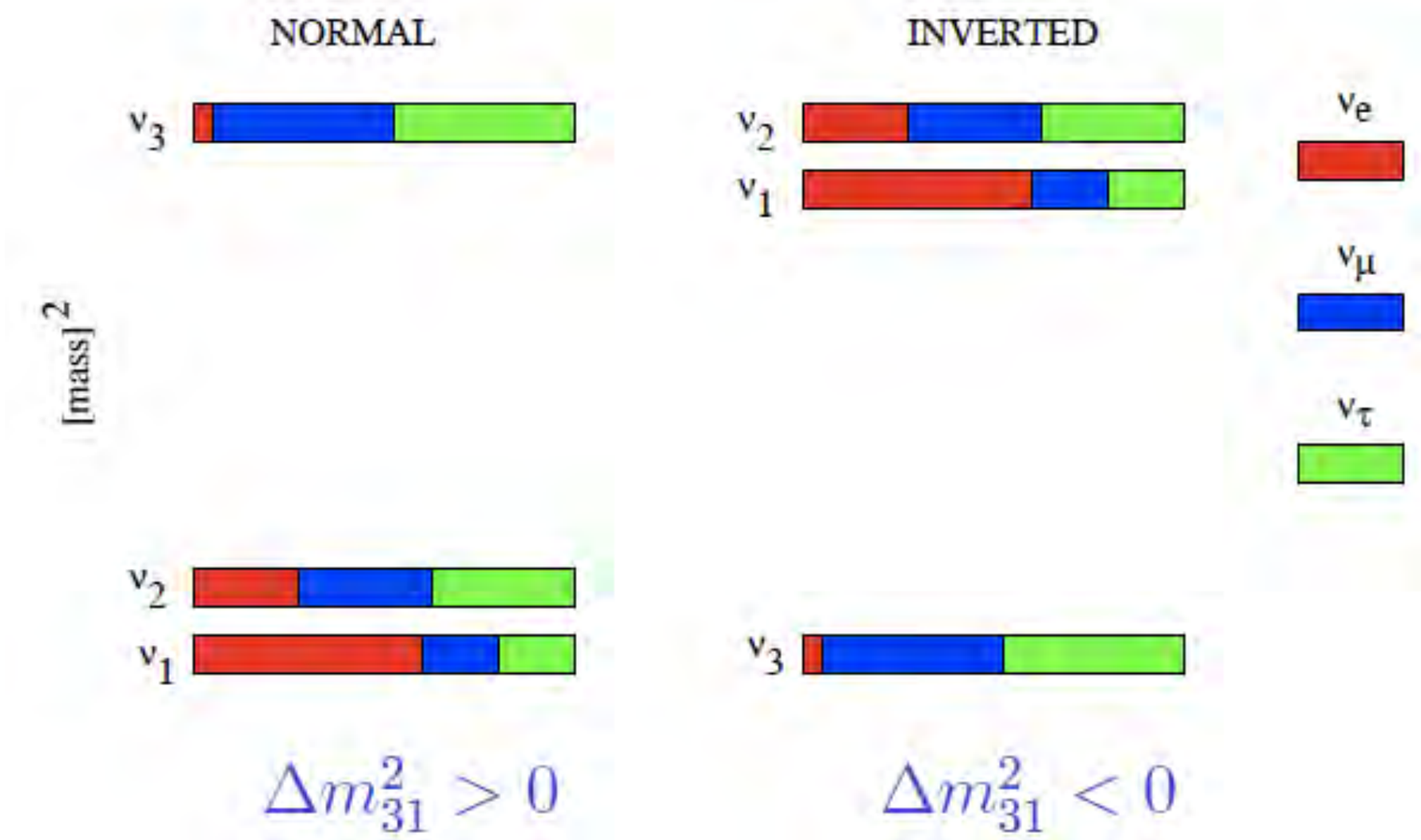
❋ **For a realistic derivation considering uncertainties in E and L and wave packet treatment see:**

Giunti & Kim, Fundamentals of Neutrino Physics and Astrophysics. Oxford University Press, 2007.

Two possible mass orderings



- Δm^2_{21} : solar + KamLAND (positive)
- Δm^2_{31} : atmospheric + LBL accelerator + SBL reactor (sign?)



Two-neutrino oscillations

- Two-neutrino mixing matrix: $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

- Two-neutrino oscillation probability ($\alpha \neq \beta$):

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m^2_{21} L}{2E}} \right|^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2_{21} L}{4E} \right)$$

- The **oscillation phase**:

$$\phi = \frac{\Delta m^2_{21} L}{4E} = 1.27 \frac{\Delta m^2_{21}[\mathrm{eV}^2] L[\mathrm{km}]}{E[GeV]}$$

→ short distances, ϕ << 1: oscillations do not develop, $P_{\alpha\beta} = 0$

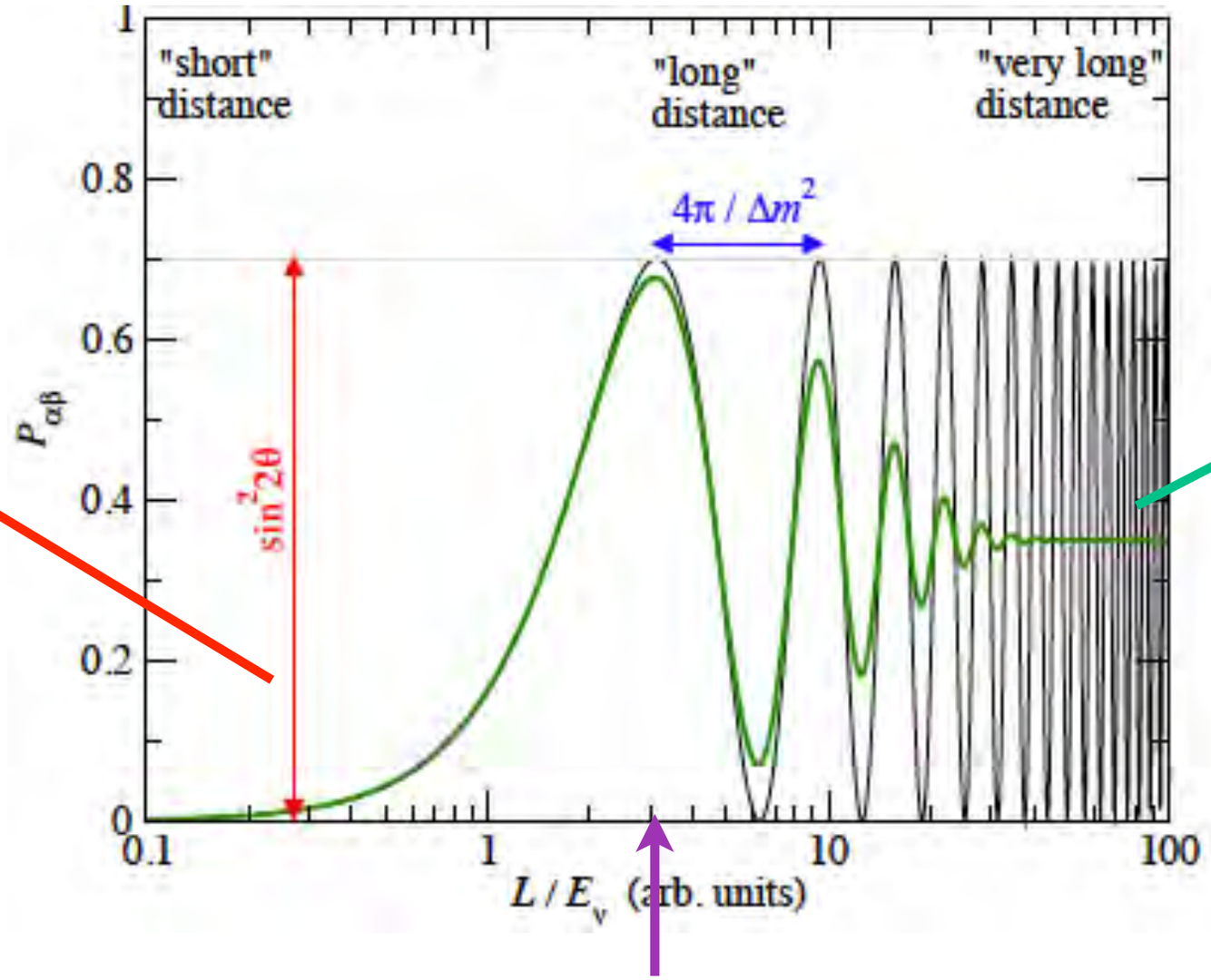
→ long distance, $\phi \sim 1$: oscillations are observable

→ very long distances, ϕ >> 1: oscillations are averaged out:

$$P_{\alpha\beta} \simeq \frac{1}{2} \sin^2 2\theta$$

2-neutrino oscillation probability

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$



oscillation amplitude

averaged oscillations

oscillation length:

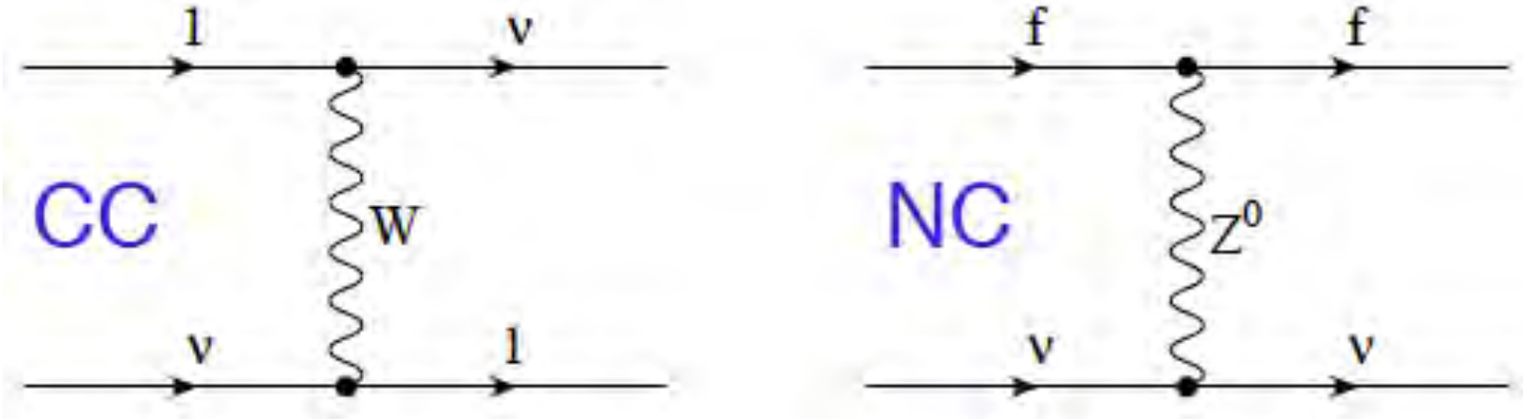
$$L_{\rm osc} = \frac{4\pi E}{\Delta m^2}$$

first oscillation maximum:

Matter effects on neutrino oscillations

✦When neutrinos pass trough matter, the interactions with the particles in the medium induce an **effective potential** for neutrinos.

[→ the coherent forward scattering amplitude leads to an index of refraction for neutrinos. **L. Wolfenstein, 1978**]



→ modifies the **mixing between flavor states and mass eigenstates** as well as the eigenvalues of the Hamiltonian, leading to a different oscillation probability with respect to vacuum oscillations.

Effective matter potential

- Effective four-fermion interaction Hamiltonian (CC+NC)

$$H_{\rm int}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}} \overline{\nu_\alpha} \gamma_\mu (1-\gamma_5) \nu_\alpha \sum_j \overline{f} \gamma_\mu (g_V^{\alpha,f} - g_A^{\alpha,f} \gamma_5) f$$

in ordinary matter: f=e^-,p,n

To obtain the **matter-induced potential** we integrate over f-variables,
For a non-relativistic unpolarised neutral medium

$$V_{\rm matt} = \sqrt{2} G_F \, {\rm diag}(N_e - \frac{1}{2} N_n, -\frac{1}{2} N_n, -\frac{1}{2} N_n)$$

- only ν_e are sensitive to CC (no μ,τ in ordinary matter)
- **NC** has the same effect for all flavours → it has **no effect on evolution**

  (however it can be important in presence of sterile neutrinos)
- for **antineutrinos** the potential has opposite sign

2-neutrino oscillations in matter

- Hamiltonian in **vacuum** in the flavour basis:

$$H_f^{\rm vac} = U H_m U^\dagger = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- Effective hamiltonian in **matter**

$$H_f^{\rm matt} = H_f^{\rm vac} + V_{\rm eff} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V_{CC} & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

$$V_{CC} = \sqrt{2} G_F N_e$$

Diagonalizing the Hamiltonian, we identify the mixing angle and mass splitting in matter:

$$H_f^{\rm matt} = \frac{\Delta M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

In general: $N_e = N_e(x)$, so θ_M and ΔM^2 will be function of x as well

→ however, in some cases analytical solutions can be obtained

2-ν oscillations in constant matter

- If N_e is constant (good approximation for oscillations in the Earth crust):

  → θ_M and ΔM^2 are constant as well

  → we can use vacuum expression for oscillation probability, replacing "vacuum" parameters by "matter" parameters:

$$P_{\alpha\beta} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta M^2 L}{4E}\right)$$

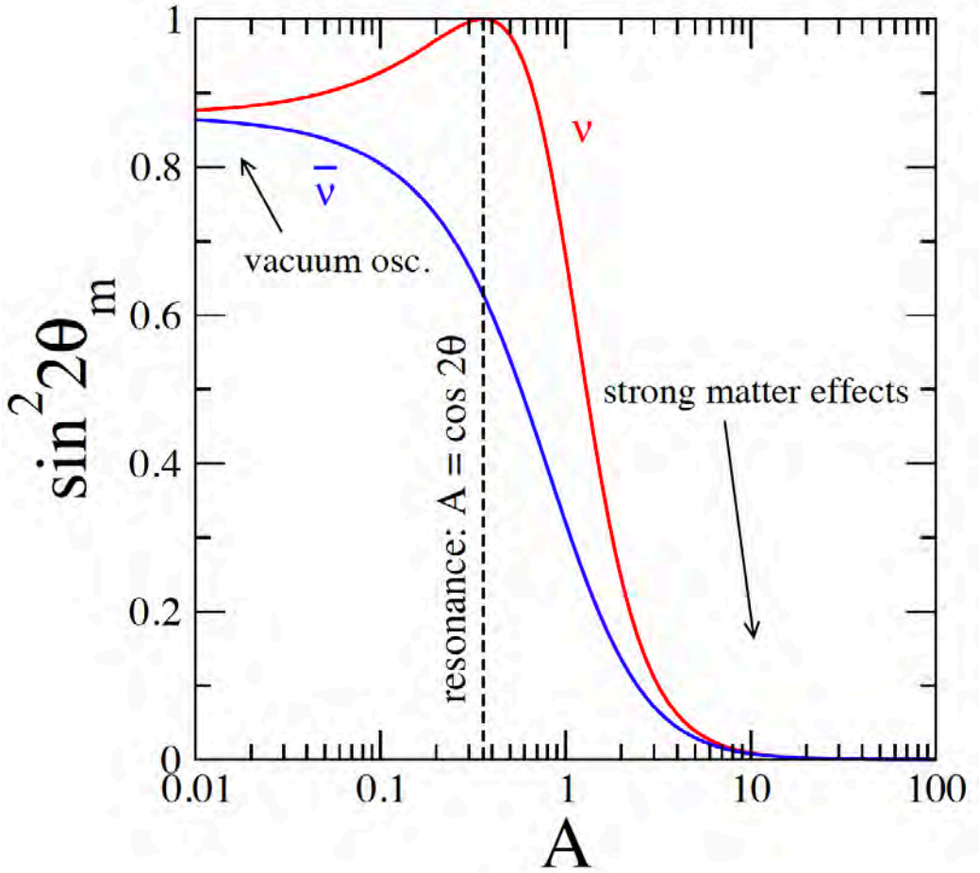
$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \qquad A = \frac{2EV}{\Delta m^2}$$

$$\Delta M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

There is a **resonance** effect for A = cos2θ → **MSW effect**

Wolfenstein, 1978. Mikheyev & Smirnov, 1986

2-ν oscillations in constant matter



mixing angle in matter:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$A = \frac{2EV}{\Delta m^2}$$

- A << cos2θ, small matter effect → vacuum oscillations: $\theta_M = \theta$
- A >> cos2θ, matter effects dominate → oscillations suppressed: $\theta_M \approx \pi/2$
- A = cos2θ, resonance takes place → maximal mixing $\theta_M \approx \pi/4$

→ **resonance condition** is satisfied for neutrinos for $\Delta m^2 > 0$

for antineutrinos for $\Delta m^2 < 0$

2-ν oscillations in varying matter

- If N_e varies with time (nu beam propagating through the Earth or the Sun)

→ diagonalization of H_{matt} at every instant to obtain $\theta_M(t)$ and $\Delta M^2(t)$

→ evolution of the **instantaneous eigenstates in matter ν_i^m :**

$$i\frac{d}{dt}\nu_\alpha = i\frac{d}{dt}\left[U(\theta_M)\nu_i^m\right] = i\frac{d}{dt}U(\theta_M)\nu_i^m + U(\theta_M)i\frac{d}{dt}\nu_i^m$$

On the other hand:

$$i\frac{d}{dt}\nu_\alpha = H_f\nu_\alpha = U(\theta_M)H_{\rm diag}\left(\frac{\Delta M^2}{4E}\right)U(\theta_M)^\dagger\nu_\alpha = U(\theta_M)H_{\rm diag}\left(\frac{\Delta M^2}{4E}\right)\nu_i^m$$

$$\Rightarrow \quad i\frac{d}{dt}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix} = \begin{pmatrix}-\Delta M^2/4E & -i\dot\theta_M\\ i\dot\theta_M & \Delta M^2/4E\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$

→ the presence of off-diagonal terms induce the **mixing of ν_i^m states**

2-ν oscillations in varying matter

$$i\frac{d}{dt}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}=\begin{pmatrix}-\Delta M^2/4E & -i\dot\theta_M\\ i\dot\theta_M & \Delta M^2/4E\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$

adiabatic approximation $\quad |\dot\theta_M| \ll \Delta M^2/4E$

- Adiabaticity condition:

$$\gamma^{-1} \equiv \frac{2\dot\theta_M}{\Delta m^2/2E} = \frac{\sin 2\theta \Delta m^2/2E}{(\Delta M^2/2E)^3}|\dot V_{CC}| \ll 1$$

adiabaticity parameter — from the instantaneous expression of θ_M

the typical value in the Sun: $\quad \gamma^{-1} \sim \frac{\Delta m^2}{10^{-9}\mathrm{eV}^2}\frac{\mathrm{MeV}}{E}$

→ adiabaticity applies up to 10 GeV

Solar neutrinos: the MSW effect

✦ neutrino oscillations in matter were first discussed by Wolfenstein, Mikheyev and Smirnov (MSW effect)

✦ 2nu approx: electron neutrino is born at the center of the Sun as:

$$|\nu_e\rangle = \cos\theta_M |\nu_1^m\rangle + \sin\theta_M |\nu_2^m\rangle$$

→ ν_1^m and ν_2^m evolve adiabatically until the solar surface and propagate in vacuum from the Sun to the Earth:

$$P(\nu_e \to \nu_e) = P_{e1}^{\rm prod} P_{1e}^{\rm det} + P_{e2}^{\rm prod} P_{2e}^{\rm det}$$

$$P_{e1}^{\rm prod} = \cos^2\theta_M\,, \quad P_{1e}^{\rm det} = \cos^2\theta$$
$$P_{e2}^{\rm prod} = \sin^2\theta_M\,, \quad P_{2e}^{\rm det} = \sin^2\theta$$

$$\boxed{P_{ee} = \cos^2\theta_M \cos^2\theta + \sin^2\theta_M \sin^2\theta}$$

Solar neutrinos: the MSW effect

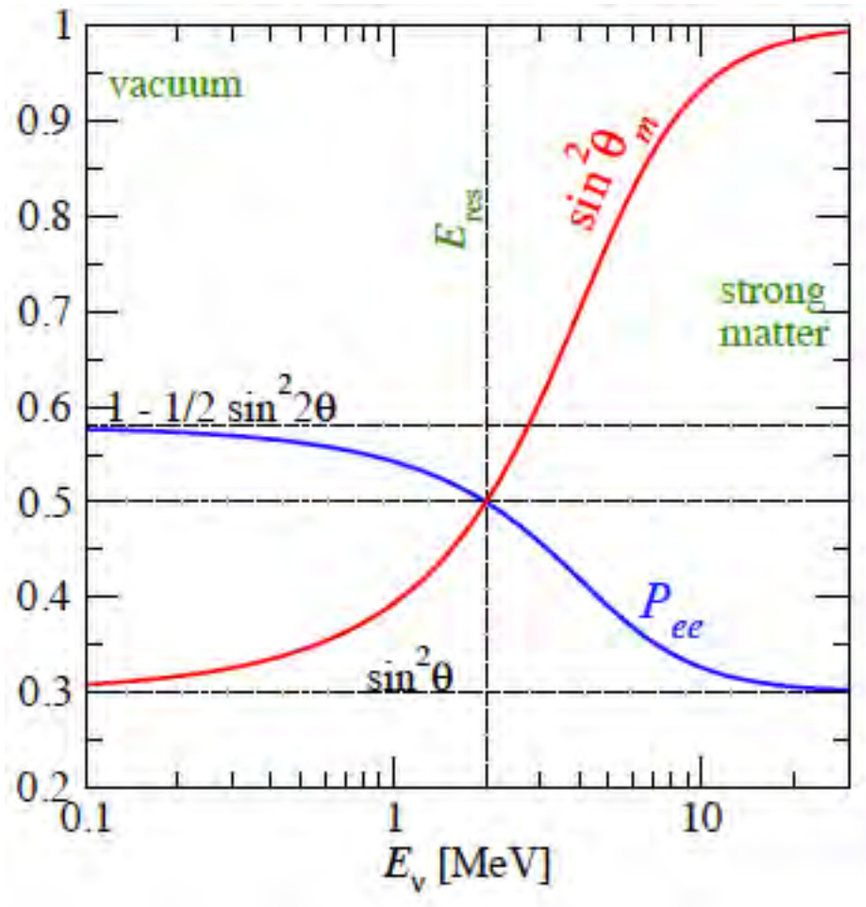
$$P_{ee} = \cos^2\theta_M \cos^2\theta + \sin^2\theta_M \sin^2\theta$$

- In the center of the Sun:

$$A = \frac{2EV}{\Delta m^2} \simeq 0.2 \left(\frac{E}{\text{MeV}}\right) \left(\frac{8\times 10^{-5}\text{eV}^2}{\Delta m^2}\right)$$

and resonance occurs for A = cos(2θ) = 0.4

→ $E_{res} \approx 2$ MeV



- For E < 2 MeV → vacuum osc: $\theta_M = \theta$:

$$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta$$

- For E > 2 MeV → strong matter effect: $\theta_M = \pi/2$: $P_{ee} = \sin^2\theta$

→ P_{ee} (E) will be crucial to understand solar neutrino data

Mass hierarchy in solar neutrinos

- Mixing angle in matter:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

→ **resonance condition** $A = \cos 2\theta$ is satisfied for neutrinos for $\Delta m^2 > 0$ and for antineutrinos for $\Delta m^2 < 0$ (change of sign in Vcc)

- Matter effects observed in solar neutrino data are in agreement with the presence of a resonance as predicted above:

→ since solar neutrinos are ν_e: $\Delta m^2_{21} > 0 \rightarrow m^2_2 > m^2_1$

Earth regeneration effect

- Neutrinos observed at night are also affected by Earth matter effects

- If neutrinos cross only the Earth mantle, P_{2e}^{det} is well approximated by the evolution of a constant potential:

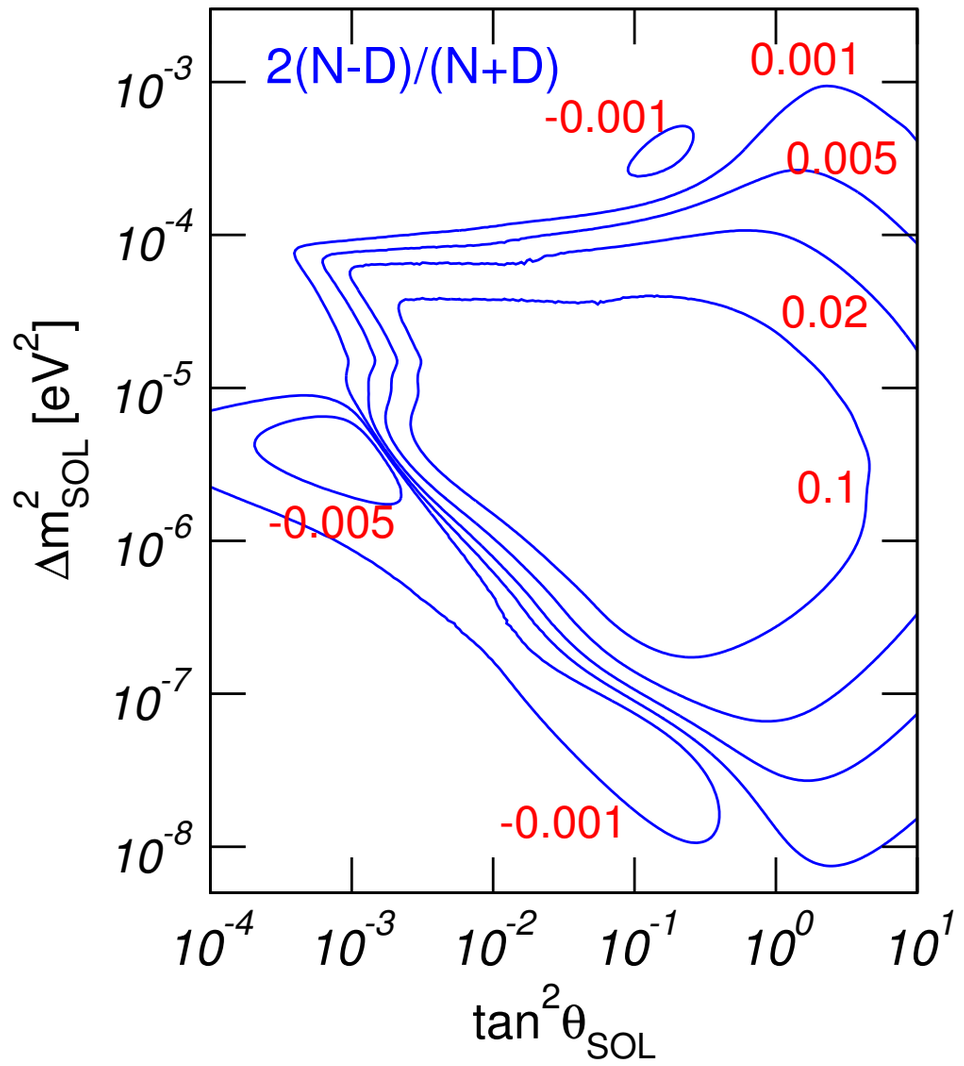
$$P_{2e}^{\rm det} = \sin^2\theta + f_{\rm reg}$$

prob. during day

regeneration term

$$f_{\rm reg} = \frac{4EV_{CC}}{\Delta m^2}\sin^2 2\theta_E \sin^2\frac{\pi L}{L_{\rm osc}}$$

$$P_{ee}^{\rm night} = P_{ee}^{\rm day} - \cos 2\theta_M f_{\rm reg}$$

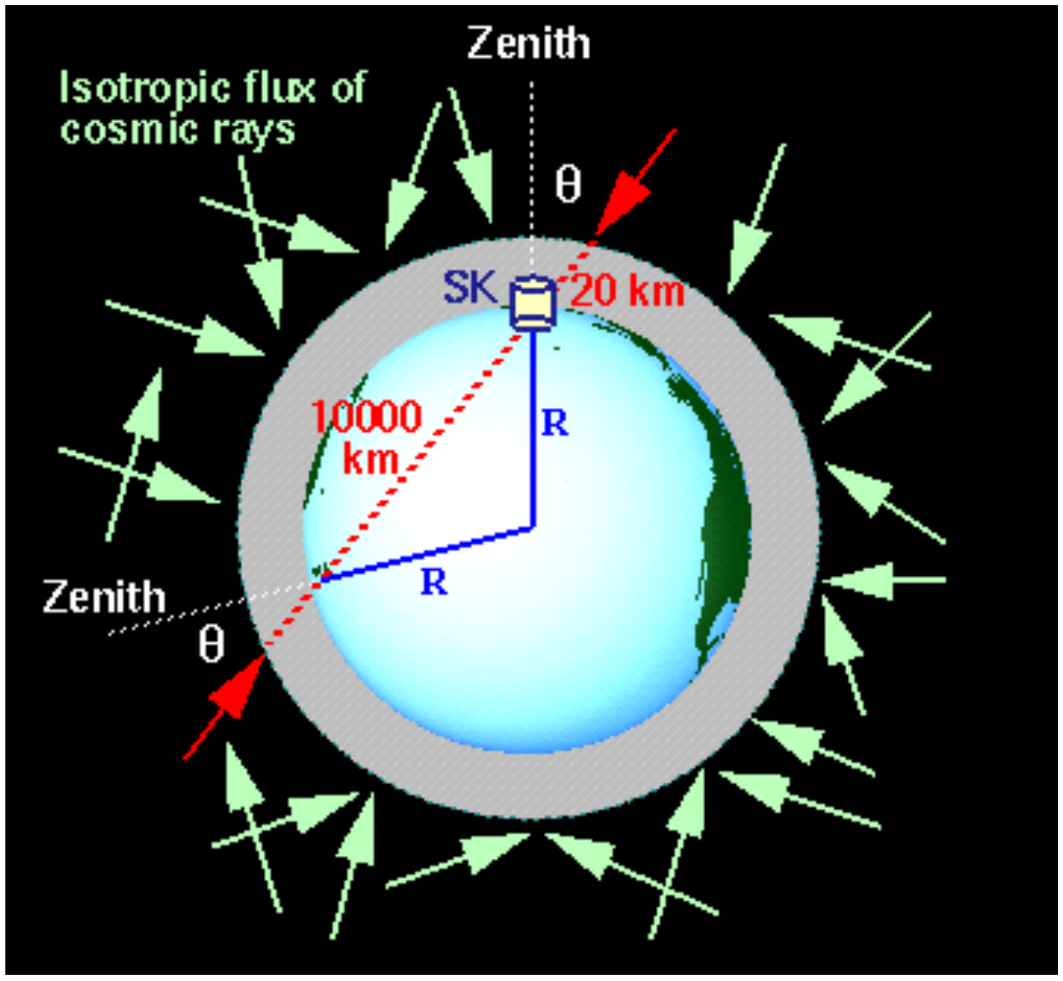


→ **day-night asymmetry:**

$$A_{\rm DN} \equiv 2\frac{(P_N - P_D)}{P_N + P_D}$$

- For the measured solar neutrino parameters $f_{reg} \sim +1\%$

Matter effects in atmospheric ν's



✦ Atmospheric neutrinos interact with the Earth mantle and core

✓ no matter effects in $\nu_\mu \rightarrow \nu_\tau$ channel

✓ MSW resonance in $\nu_\mu \rightarrow \nu_e$ channel

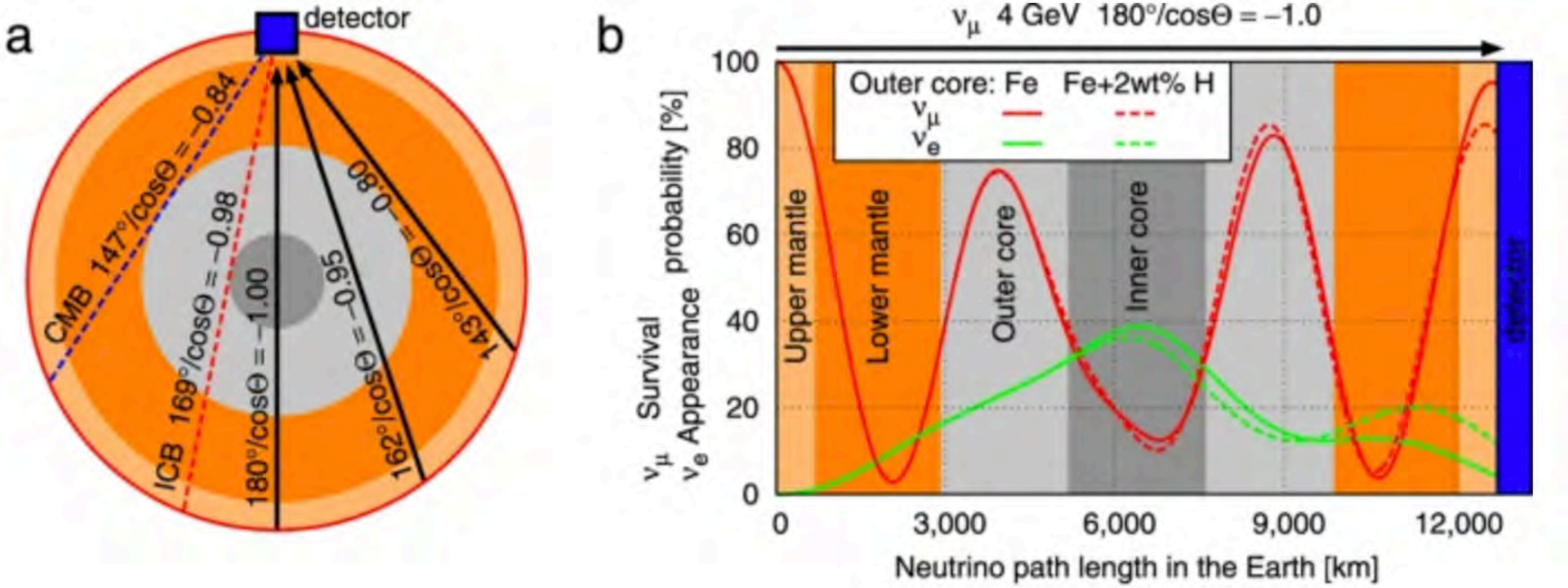
$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{4E}\sin 2\theta}{\frac{\Delta m^2}{4E}\cos 2\theta \mp \sqrt{2}G_F N_e}$$

(-) neutrinos (+)antineutrinos

➙ Matter effects on the atmospheric neutrino flux are sensitive to the mass ordering.

➙ they are harder to observe since $P_{\mu e} \propto \theta_{13}$

Matter effects in atmospheric ν's

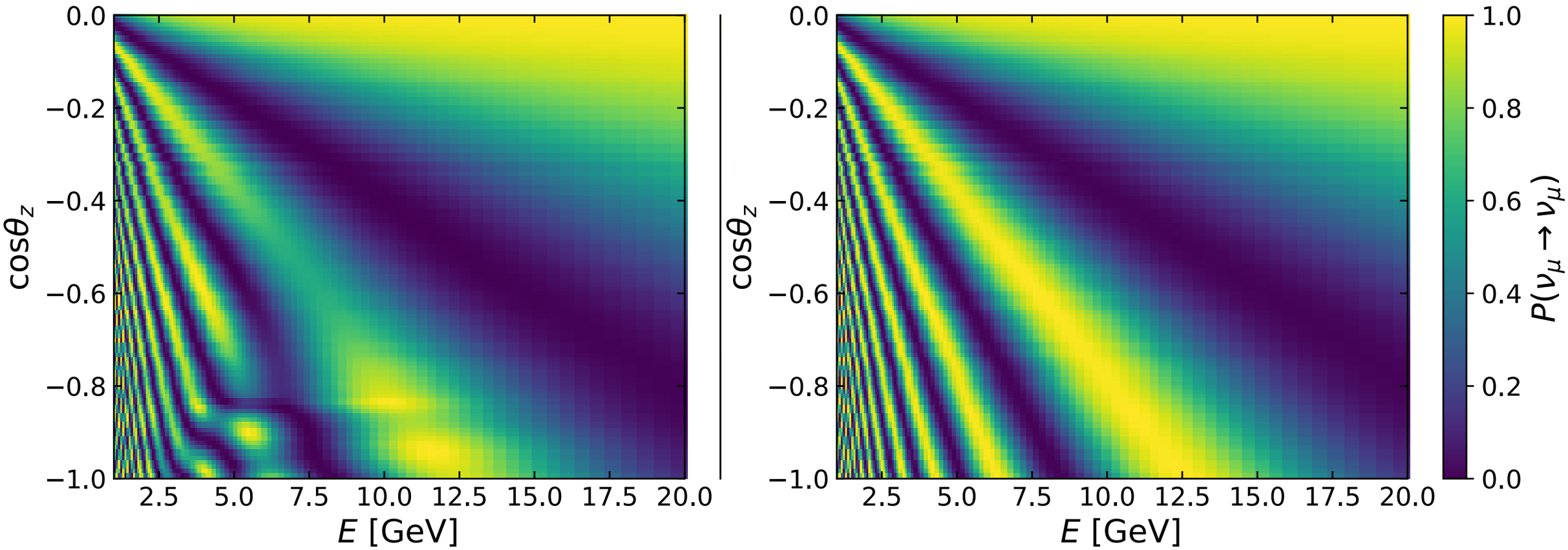


Rott, Taketa & Bose, Scientific Reports **5** (2015) 15225

Matter effects in atmospheric ν's

NO

IO



de Salas et al, arXiv:1806.11051

At E~ 3-8 GeV: MSW resonance for neutrinos and NO mass spectrum.

For antineutrinos ⟹ the resonance appears in IO