L'ATTUALITÀ DELLA TEORIA DI FERMI

A 90 anni dalla Teoria di Fermi del Decadimento Beta

Guido Martinelli
INFN Sezione di Roma
Sapienza Università di Roma

DIPARTIMENTO DI FISICA

SAPIENZA
UNIVERSITÀ DI ROMA

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UTfit

Extended Twisted Mass Collaboration

PRACE

INFN Istituto Nazionale di Fisica Nucleare

Alcune Considerazioni di Carattere Generale:

Enrico Fermi, Tentativo di una teoria dell’emissione dei raggi “beta” (in Italian), Ric. Sci. 4, 491 (1933)

A 91 anni dalla Teoria di Fermi del Decadimento Beta

Enrico Fermi, Tentativo di una teoria dei raggi β (in Italian), Nuovo Cim. 11, 1 (1934)

Enrico Fermi, Versuch einer Theorie der β-Strahlen. I (in German), Z. Phys. 88, 161 (1934)

questo non è un seminario di Storia della Fisica

La Teoria di Fermi è un tentativo, molto ben riuscito
e dopo 90 anni tuttora attuale, di descrivere in maniera
quantitativa le disintegrazioni (i decadimenti) beta dei nuclei,
includendo il neutrino ipotizzato da Pauli. In una notazione moderna

$$(A, Z) \rightarrow (A, Z+1) + e^- + \nu$$

Ovvero nel nucleo iniziale un neutrone si trasforma in
protone con l'emissione (creazione) simultanea di
un elettrone e un neutrino

$${}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$$

Il Contesto Storico

Ho usato il materiale da *R. Barbieri* Il Nuovo Saggiatore Vol. 39 N. 3-4 (2023)
F. Vissani arXiv:2310.07834v1 [physics.hist-ph]

- 1920 Esistono solo protoni e elettroni. Atomi e Nuclei sono stati composti di questi costituenti elementari;
- Problemi con lo spin dei nuclei e con l'energia di legame nucleare;

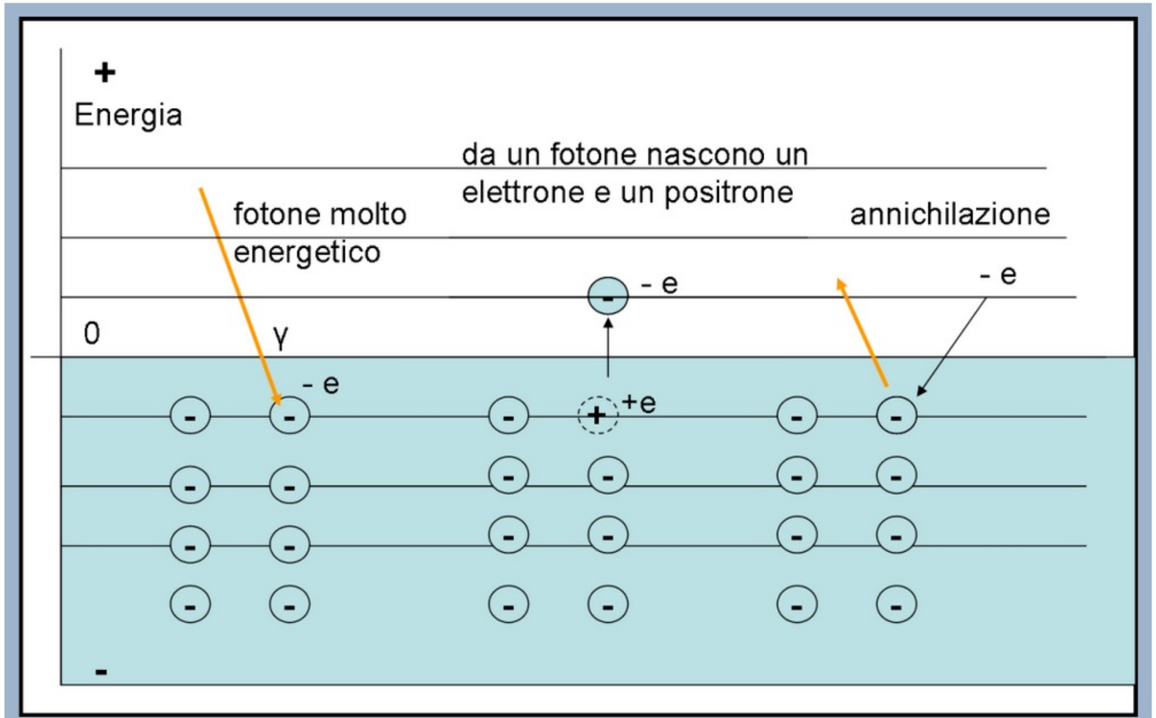
Composizione del Nucleo di Azoto e Relativa Statistica

model	year	^{14}N	statistics
pe	1920	$7\times(2p,e)$	fermion
$pe\nu$	1930	$7\times(2p,e,\nu)$	boson
pn	1932	$7\times(p,n)$	boson

Table 2: Particle content of the ^{14}N nucleus for different nuclear models and their prediction for the statistics. Its spin was measured and known to be integer, implying boson statistics.

Il Contesto Storico

- “Seconda quantizzazione”: valida per i fotoni (niente carica e massa nulla) mentre si pensa che protoni e elettroni sono ``conservati”, dunque le particelle emesse in una disintegrazione devono già esistere nel nucleo originale;



Il Contesto Storico

- 1930 Pauli propone l'esistenza del neutrino (chiamato inizialmente neutrone) per spiegare lo spettro dell'elettrone nel decadimento beta;
- 1932 Scoperta del neutrone da parte di J. Chadwick (e anche misura della massa molto vicina a quella del protone). Questo esclude che sia il neutrino;
- 1932 Scoperta del positrone da parte di C.D. Anderson;
- 1934 F. Joliot e I. Curie osservano decadimenti beta con emissione di positroni (radioattività artificiale).

CERTO UN PERIODO DENSO DI SVILUPPI FONDAMENTALI!

"Minimal Flavour Deconstruction" in 4d, explicit

B, Isidori, 2023

$$G = SU(3) \times SU(2) \times U(1)_Y^{[3]} \times U(1)_{B-L}^{[12]} \times U(1)_{T_{3R}}^{[2]} \times U(1)_{T_{3R}}^{[1]}$$

Scalars

Field	$U(1)_Y^{[3]}$	$U(1)_{B-L}^{[12]}$	$U(1)_{T_{3R}}^{[2]}$	$U(1)_{T_{3R}}^{[1]}$	$SU(3)\times SU(2)$
$H_{u,d}$	$-1/2$	0	0	0	$(\mathbf{1},\mathbf{2})$
χ^q	$-1/6$	$1/3$	0	0	$(\mathbf{1},\mathbf{1})$
χ^l	$1/2$	-1	0	0	$(\mathbf{1},\mathbf{1})$
ϕ	$1/2$	0	$-1/2$	0	$(\mathbf{1},\mathbf{1})$
σ	0	0	$1/2$	$-1/2$	$(\mathbf{1},\mathbf{1})$

VectorLike fermions

		$U(1)_Y^{[3]}$	$U(1)_{B-L}^{[12]}$	$U(1)_{T_{3R}}^{[2]}$	$U(1)_{T_{3R}}^{[1]}$	$SU(3)\times SU(2)$
light VL ($\alpha = 1,2$)	U_α	$1/2$	$1/3$	0	0	$(\mathbf{3},\mathbf{1})$
	D_α	$-1/2$	$1/3$	0	0	$(\mathbf{3},\mathbf{1})$
	E_α	$-1/2$	-1	0	0	$(\mathbf{1},\mathbf{1})$
heavy VL	U_3	0	$1/3$	$1/2$	0	$(\mathbf{3},\mathbf{1})$
	D_3	0	$1/3$	$-1/2$	0	$(\mathbf{3},\mathbf{1})$
	E_3	0	-1	$-1/2$	0	$(\mathbf{1},\mathbf{1})$

(a 16-plet/generation with neutrinos)

Most general $d \leq 4$

$$\mathcal{L}_Y^u = (y_3^u\,\bar{q}_3 u_3 H_u + y_{i\alpha}^u\,\bar{q}_i U_\alpha H_u + y_\alpha^{\chi_u}\,\bar{U}_\alpha u_3 \chi^q + y_{\alpha 2}^{\phi_u}\,\bar{U}_\alpha u_2 \phi + y_{\alpha 3}^{\phi_u}\,\bar{U}_{R\alpha} U_{L3}\phi + \hat{y}_{\alpha 3}^{\phi_u}\,\bar{U}_{L\alpha} U_{R3}\phi + y_1^{\sigma_u}\,\bar{U}_3 u_1 \sigma + \text{h.c.}) + M_{U_3}\,\bar{U}_3 U_3 + M_{U_\alpha}\,\bar{U}_\alpha U_\alpha$$

And similarly for $Y_{d,e}$ with $H_u \to H_d$

Chi riconosce in questa formula la Teoria di Fermi?

$$H_{Int} = g\left[\tau_{-}\bar{\psi}_e(x)\psi_\nu(x) + \tau_{+}\bar{\psi}_\nu(x)\psi_e(x)\right]$$

$\tau_{\pm}$ Sono le matrici di Isospin introdotte da Heisenberg
Non era chiaro che protoni e neutroni fossero particelle di Dirac

x Rappresenta le coordinate delle particelle pesanti ovvero il neutrone e il protone (nucleo iniziale o finale)

Le particelle leggere non esistono prima del decadimento nel nucleo ma vengono create come i fotoni, le particelle pesanti sono non-relativistiche e il loro numero è conservato

Evoluzione verso una forma più familiare

Generalizzando a tutte le possibili combinazioni di bilineari di Dirac

$$H_F = \frac{G_F}{\sqrt{2}} \sum_i \left[\bar{\psi}_p(x) O_i \psi_n(x)\, \bar{\psi}_e(x) O_i \psi_\nu(x) + h.c.\right]$$

$$O_i = S, P, V, A, T$$

In un linguaggio moderno questa teoria non è rinormalizzabile, contiene una scala di massa fondamentale, la scala di Fermi $v = (G_F\sqrt{2})^{-1/2} \simeq 246\,\mathrm{GeV}$, e la sezione d'urto protone-neutrino cresce con l'energia (M. Fierz).

Due aspetti rivoluzionari: le particelle sono create dal vuoto e non sono preesistenti e questo è il primo esempio di una Teoria Effettiva

Nel frattempo molte scoperte e sviluppi nella fisica delle interazioni deboli, seguendo R. Barbieri

i) The discovery of parity violation in the β decay of ^{60}Co by Chien-Shiung Wu [3] in 1956.

ii) The identification of the vector component of the hadronic current with the charged component of the SU(2) isospin current (Conserved Vector Current).

iii) The universality between the β decay and the μ decay.

iv) The extension by Nicola Cabibbo [4] of the CVC hypothesis to include the current with change of *strangeness* – by the name given to a number of relatively long-lived particles discovered since 1947 – thus making the hadronic part of the weak current a member of an SU(3) octet.

v) The Partial Conservation of the Axial-vector Current (PCAC): Unlike the case of isospin SU(2), or of SU(3), no approximate axial symmetry seems present in the hadron spectrum. No parity doublet is observed, *e.g.*, in the nucleons. Yoshiro Nambu [5] pointed out in 1960 that the strong interactions are nevertheless invariant under chiral transformations, but the symmetry is spontaneously broken, with pions – or the full octet of pseudo-scalar mesons in the case of SU(3) – being the corresponding Nambu-Goldstone bosons.

Effective Hamiltonians for Neutron and Muon Weak decays

$$H_F = -\frac{G_F}{\sqrt{2}}\left(\bar{p}(x)\gamma^{\mu}\left(1-\frac{g_A}{g_V}\gamma^5\right)n\right)(\bar{e}\,\gamma_{\mu}(1-\gamma_5)\nu_e)$$

$$H_F = \frac{-G_F}{\sqrt{2}}\left(\bar{\nu}_{\mu}\gamma^{\mu}(1-\gamma_5)\mu\right)\left(\bar{e}\gamma_{\mu}(1-\gamma_5)\ \nu_e\right)$$

CKM matrix elements, Strong interactions effects and Radiative Corrections neglected

Which are the limits of validity of the Fermi theory?

PRESENT: the Standard Model and beyond

Vacuum Energy

Hierarchy

Vacuum Stability

$$\mathcal{L} = \Lambda^4 + \Lambda^2 H^2 + \lambda H^4 +$$

Higgs meson 2012

$$(D_\mu H)^2 + \bar{\psi} \not{D} \psi + F^2_{\mu\nu} + F_{\mu\nu}\tilde{F}_{\mu\nu}$$

Strong $\not{CP}$

charm quark 1974

neutral currents 1973

$$Y H \bar{\psi}\psi + \frac{1}{\Lambda}(\bar{L}H)^2 + \frac{1}{\Lambda^2}\sum_i C_i O_i + \ldots$$

Buchmuller&Wyler '88

Flavor puzzle

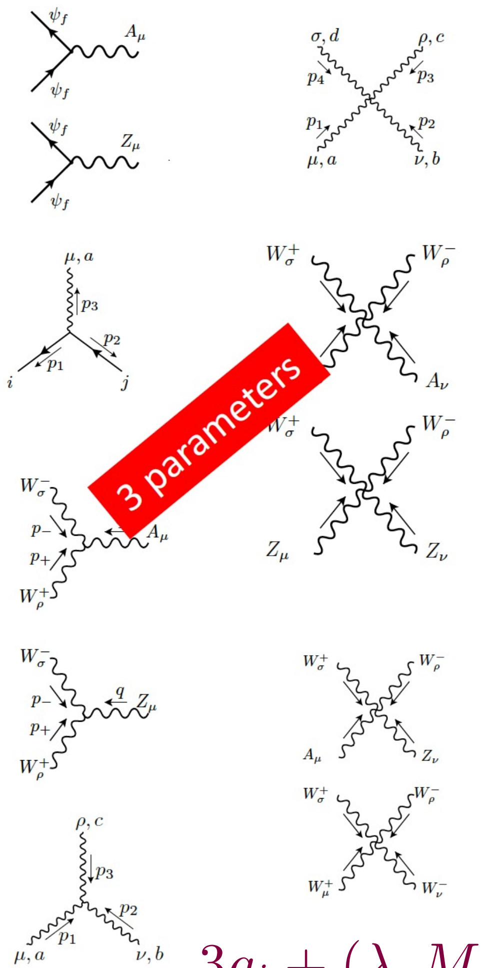
Neutrino Masses

New Physics

Possible breaking of accidental symmetries

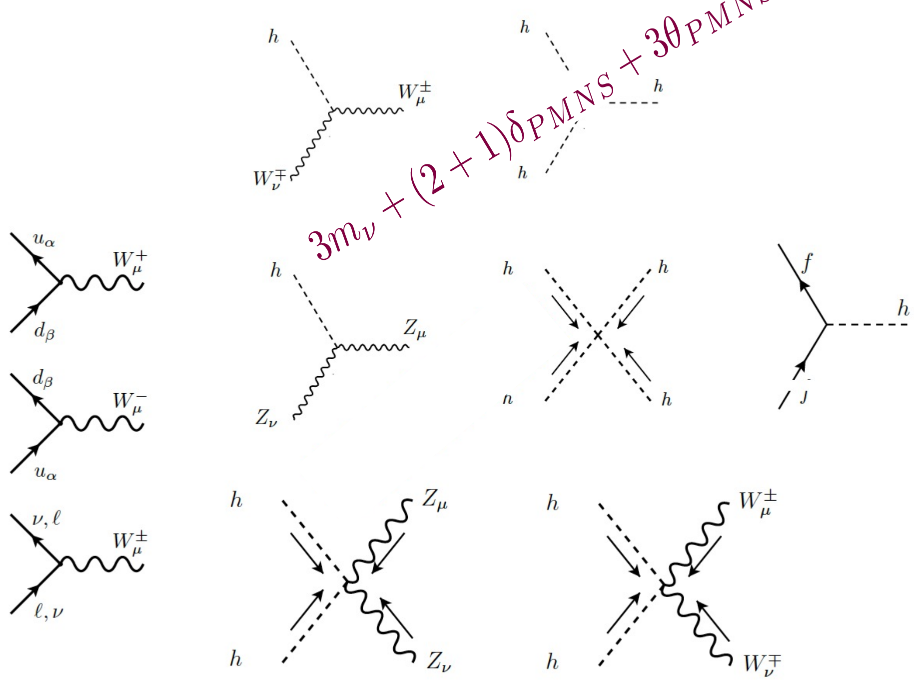
The Standard Model

$SU(3) \times SU(2) \times U(1)_Y$



Higgs + gauge principle

$3m_\nu + (2+1)\delta_{PMNS} + 3\theta_{PMNS} = 9$



from elegance to caos !!

If we are looking for the suspect that could be hiding some secret obviously the higgs is the one!

$$3g_i + (\lambda, M_H) + 6m_q + 3m_\ell + \delta + 3\theta_{CKM} + \theta_{QCD} = 19$$

The Weirdness of the Standard Model

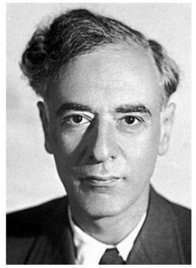
$$3m_\nu + (2+1)\delta_{PMNS} + 3\theta_{PMNS} = 9$$

- Three families

"who ordered that ?" I. Rabi

- Fundamental breaking of Parity

"space cannot be asymmetric!" L. Landau



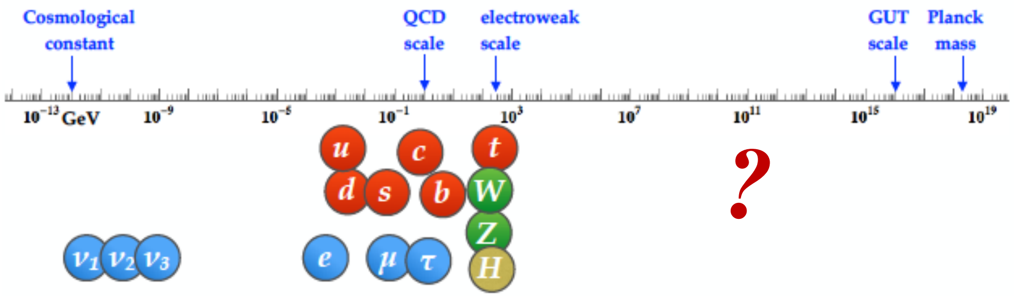
- Predictivity: 3 gauge couplings+ 16 higgs couplings (+ 7 higgs-neutrino) !
  + the coupling θ of strong CP violation

"has too many arbitrary features for [its] predictions
to be taken very seriously" S. Weinberg '67

$$3g_i + (\lambda, M_H) + 6m_q + 3m_\ell + \delta + 3\theta_{CKM} + \theta_{QCD} = 19$$

Zupan

J. ZUPAN



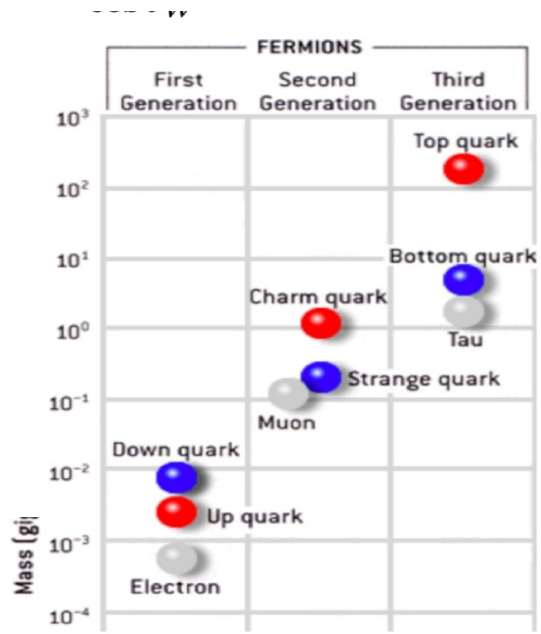


Illustration from a G. Isidori talk

$m_\nu \leq 1\, eV$

Quark Masses from Lattice QCD

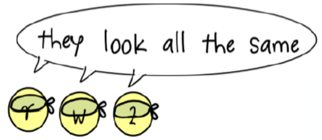
Input	Lattice/Exp
$m_u^{\overline{\rm MS}}(2\,{\rm GeV})$	2.20(9) MeV
$m_d^{\overline{\rm MS}}(2\,{\rm GeV})$	4.69(2) MeV
$m_s^{\overline{\rm MS}}(2\,{\rm GeV})$	93.14(58) MeV
$m_c^{\overline{\rm MS}}(3\,{\rm GeV})$	993(4) MeV
$m_c^{\overline{\rm MS}}(m_c^{\overline{\rm MS}})$	1277(5) MeV
$m_b^{\overline{\rm MS}}(m_b^{\overline{\rm MS}})$	4196(19) MeV
$m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}})$ (GeV) to be updated	163.44(43)

Table 3 *Full lattice inputs. The values of the different quantities have been obtained by taking the weighted average of the $N_f = 2+1$ and $N_f = 2+1+1$ FLAG numbers.*

Hints of NP structure: Flavor symmetries of the SM

- Standard Model (SM) gauge sector is flavor blind and CP conserving

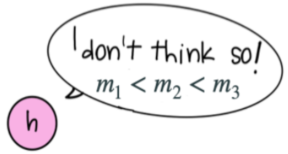
$$\mathcal{G}_F(\mathrm{SM}) = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$$



The Higgs introduces the only known non-gauge couplings

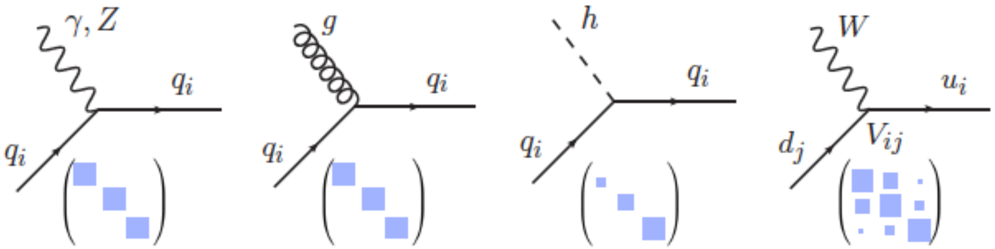
Turn on Yukawas

$$Y_{ij} \bar{\Psi}^i_L H \Psi^j_R$$



$$\mathcal{G}_F(\mathrm{SM}) = U(1)_B \times U(1)_L$$

Higgs couplings are not flavor blind



courtesy of B.A. Stefanek

electromagnetic **neutral currents** **charged currents**

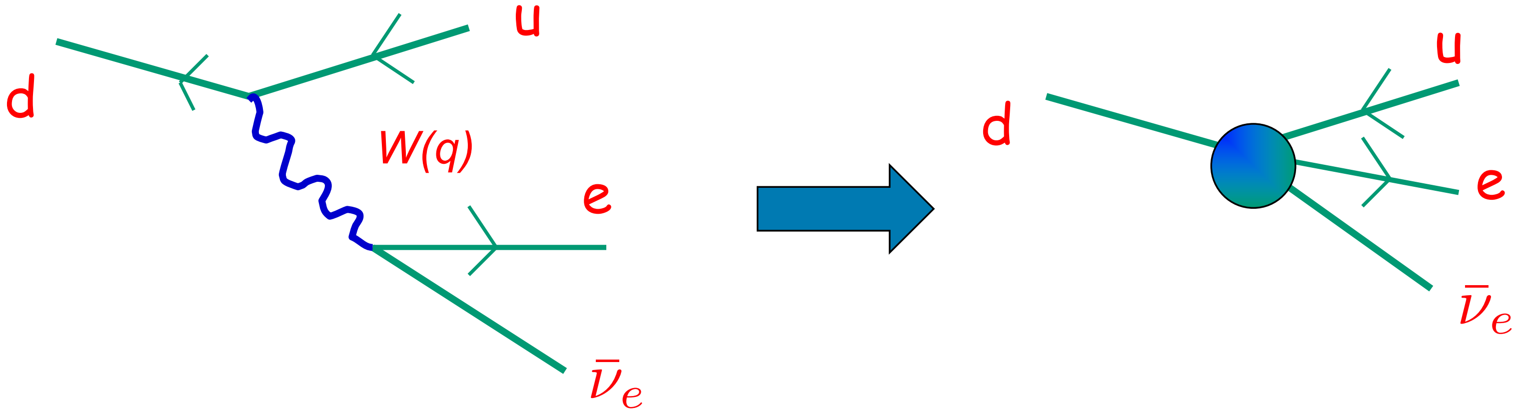
$$\mathcal{L}_{int} = -eA^{\mu}J_{\mu}^{em} - \frac{g_W}{2\cos\theta_W}Z^{\mu}J_{\mu}^{Z} - \frac{g_W}{2\sqrt{2}}[W^{\mu}(J^{W})_{\mu}^{\dagger} + h.c.]$$

$$J_{\mu}^{Z} = 2J_{\mu}^{3} - 2\sin^2\theta_W J_{\mu}^{em}$$

$$L_{CC}^{weak\,int} = \frac{g_W}{\sqrt{2}}(J_{\mu}^{-}W_{\mu}^{+} + h.c.)$$

$$\rightarrow \quad \frac{g_W}{\sqrt{2}}(\bar{u}_L \mathbf{V}^{CKM}\gamma_{\mu}d_L W_{\mu}^{+} + \ldots)$$

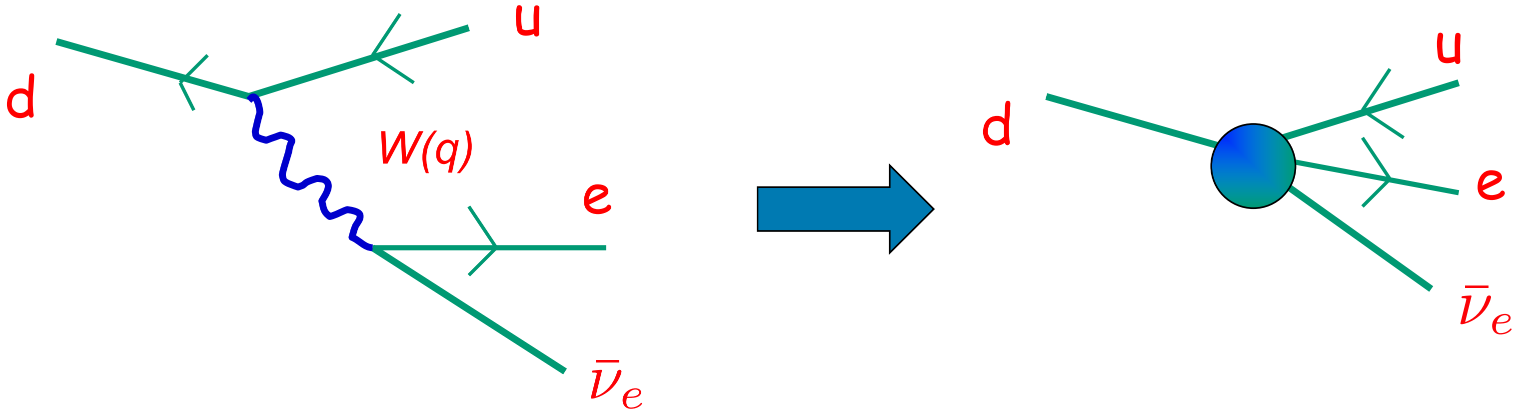
The Effective Hamiltonian, Wilson OPE and QCD Corrections



$$\frac{(-ig_W)^2}{8}\bar{u}\gamma^\mu(1-\gamma_5)d\, i\frac{-g_{\mu\nu}+q_\mu q_\nu/M_W^2}{q^2-M_W^2}\,\bar{e}\gamma^\nu(1-\gamma_5)\nu_e$$

$$A \sim \langle p|\bar{u}\gamma^\mu(1-\gamma_5)d|n\rangle\, i\frac{-g_{\mu\nu}+q_\mu q_\nu/M_W^2}{q^2-M_W^2}\,\bar{e}\gamma^\nu(1-\gamma_5)\nu_e$$

The Effective Hamiltonian, Wilson OPE and QCD Corrections



G_F ha dimensione di M^{-2}

Operatore di dimensione 6

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}}\,[\bar{u}\gamma_\mu(1-\gamma_5)d][\bar{e}\gamma^\mu(1-\gamma_5)\nu_e]\;\; V_{ud}$$

$$i\frac{-g_{\mu\nu}+q_\mu q_\nu/M_W^2}{q^2-M_W^2+i\varepsilon}\sim i\frac{g_{\mu\nu}}{M_W^2} \qquad q^2 << M_W^2 \qquad \frac{G_F}{\sqrt{2}}=\frac{g_W^2}{8M_W^2}$$

GENERAL FRAMEWORK: THE OPE

$A_{\mathbf{FI}}\ (2\pi^4)\ \delta^4\ (p_F - p_I) = \int d^4x\ d^4y\ D_{\mu\nu}(x, M_W)$

$\langle F\ |T[\ J_\mu(y+x/2)\ J^\dagger_\nu(y-x/2)]\ |\ I\ \rangle$ ➡

$\langle F\ |\ H^{\Delta S=1}\ |\ I\ \rangle =$

$$G_F/\sqrt{2}\ V_{ud}\ V_{us}^* \quad \Sigma_i\ C_i(\mu)\ \frac{\langle F\ |\ Q_i(\mu)\ |\ I\ \rangle}{(M_W)^{di-6}}$$

di= dimension of the operator $Q_i(\mu)$

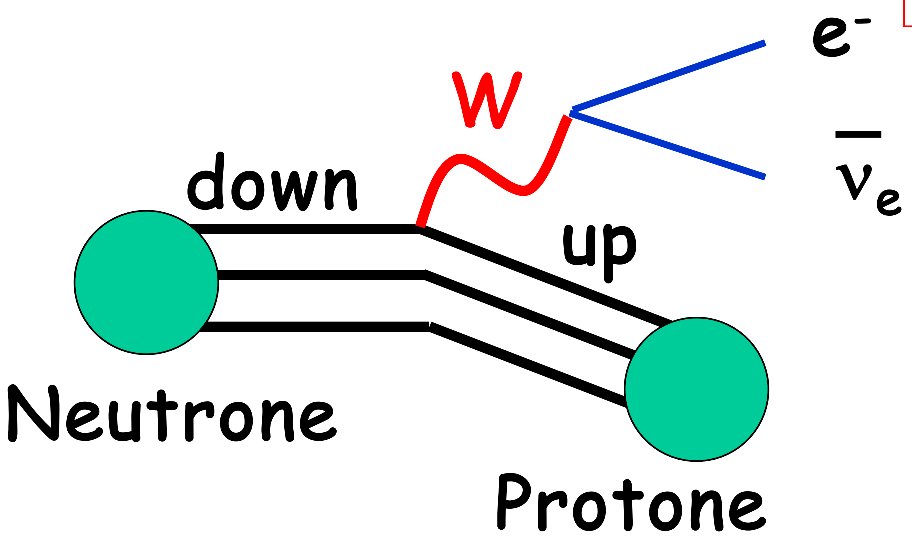
$C_i(\mu)$ Wilson coefficient: it depends on M_W/μ and $\alpha_W(\mu)$

$Q_i(\mu)$ local operator renormalized at the scale μ

Weak Interactions

β-Decays

$$n \rightarrow p + e^- + \bar{\nu}_e$$



$$^{60}_{27}\mathrm{Co} \rightarrow ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} [\bar{u}\gamma_\mu(1-\gamma_5)d][\bar{e}\gamma^\mu(1-\gamma_5)\nu_e] \quad V_{ud}$$

$$i\frac{-g_{\mu\nu} + q_\mu q_\nu / M_W^2}{q^2 - M_W^2 + i\varepsilon} \sim i\frac{g_{\mu\nu}}{M_W^2} \qquad q^2 << M_W^2 \qquad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

$$H_F = \frac{-G_F}{\sqrt{2}} \left(\bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu\right) \left(\bar{e}\gamma_\mu (1-\gamma_5)\ \nu_e\right)$$

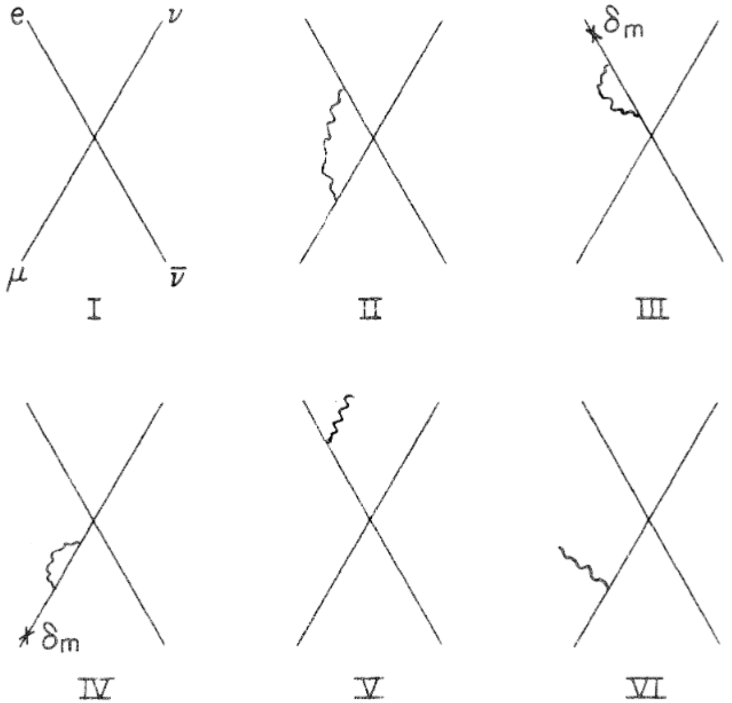
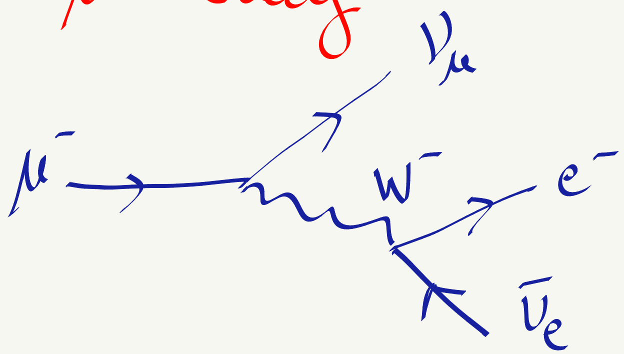


FIGURE I. FEYNMAN DIAGRAMS FOR MUON DECAY.

μ-decay

[e13]



$$H_{eff} = -\frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu \times \bar{e}\gamma_\mu(1-\gamma_5)\nu_e$$

$$\frac{d\Gamma}{dx} = \frac{G_F^2}{96\pi^3}\sqrt{x^2-4\rho^2}\; x(3-2x)$$

$x = 2E_e/m_\mu$ $\qquad \rho = m_e/m_\mu$

$$2\rho \leq x \leq 1+\rho^2$$

$\rho = 0$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Very recently, in an important theoretical development, van Ritbergen and Stuart completed the evaluation of $\mathcal{O}(\alpha^2)$ corrections to τ_μ in the local V–A theory, in the limit $m_e^2/m_\mu^2 \to 0$ [7,8]. Their final answer can be expressed succinctly as

$$C(m_\mu) = \frac{\alpha(m_\mu)}{\pi} c_1 + \left(\frac{\alpha(m_\mu)}{\pi}\right)^2 c_2, \tag{1}$$

$$c_1 = \frac{1}{2}\left(\frac{25}{4} - \pi^2\right); \qquad c_2 = 6.700. \tag{2}$$

In Eq. (1), $\alpha(m_\mu)c_1/\pi$ is the one-loop result [1] and $\alpha(m_\mu)$ is a running coupling defined by

$$\alpha(m_\mu) = \frac{\alpha}{1 - \left(\frac{2\alpha}{3\pi} + \frac{\alpha^2}{2\pi^2}\right)\ln\frac{m_\mu}{m_e}}. \tag{3}$$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}[1 + C(m_\mu)]f(\rho^2) + O(\frac{m_\mu^2}{M_W^2})$$

$$f(z) = 1 - 8z - 12z^2\log z + 8z^3 - z^4$$

L'HAMILTONIANA DI FERMI E` OVUNQUE
Classification of the processes in the SM

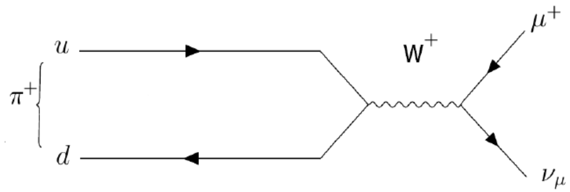
$$q^2 \ll M_W^2$$

Leptonic Decays

the prototype of these decays is given by

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu \tag{35}$$

which at the fundamental level is given by



$$-\frac{G_F}{\sqrt{2}} V_{ud}^* \left(\bar{d}(x)\gamma_\mu(1-\gamma_5)u(x)\right)\right) \left(\bar{\nu}_\mu(x)\gamma_\mu(1-\gamma_5)\mu(x)\right)\right)$$

Other possible leptonic decays are given by

$$K^+ \longrightarrow \mu^+ + \nu_\mu$$
$$D^+ \longrightarrow \mu^+ + \nu_\mu$$
$$B^+ \longrightarrow \tau^+ + \nu_\tau$$
$$\pi^+ \longrightarrow e^+ + \nu_e$$

the latter process is suppressed by chirality

Decadimenti semi leptonici

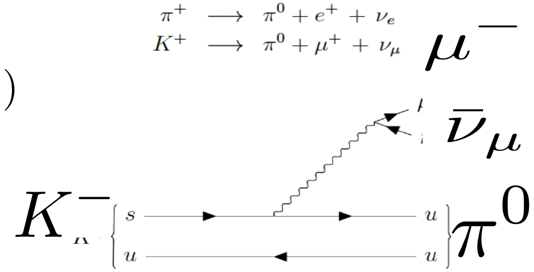
Semi-leptonic Decays

these are the better sources to measure the absolute values of the CKM matrix elements |V ij|

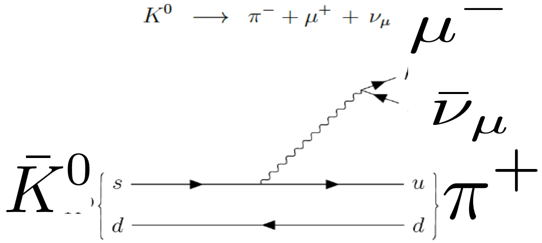
other possible semi-leptonic decays are the following

$$-\frac{G_F}{\sqrt{2}} V_{us} \left(\bar{u}(x)\gamma_\mu(1-\gamma_5)s(x)\right)\right) \left(\bar{\mu}(x)\gamma_\mu(1-\gamma_5)\nu_\mu(x)\right)\right)$$

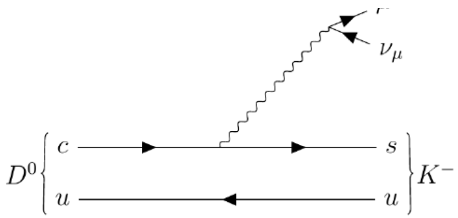
$$\pi^+ \longrightarrow \pi^0 + e^+ + \nu_e$$
$$K^+ \longrightarrow \pi^0 + \mu^+ + \nu_\mu$$



$$K^0 \longrightarrow \pi^- + \mu^+ + \nu_\mu$$



$$D^0 \longrightarrow K^+ + \mu^+ + \nu_\mu$$

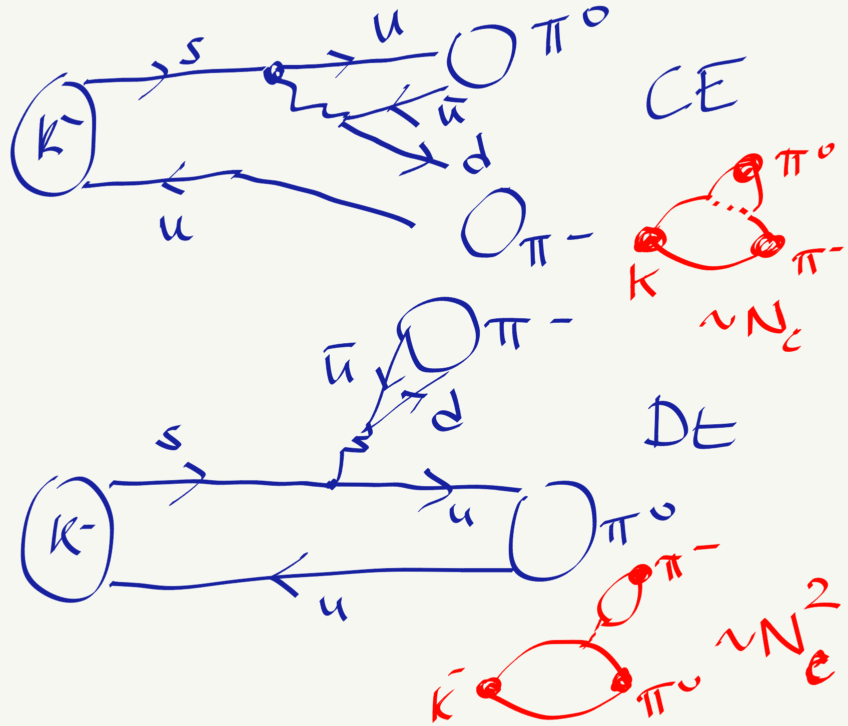


Non-leptonic Decays

Penguins contractions and all that

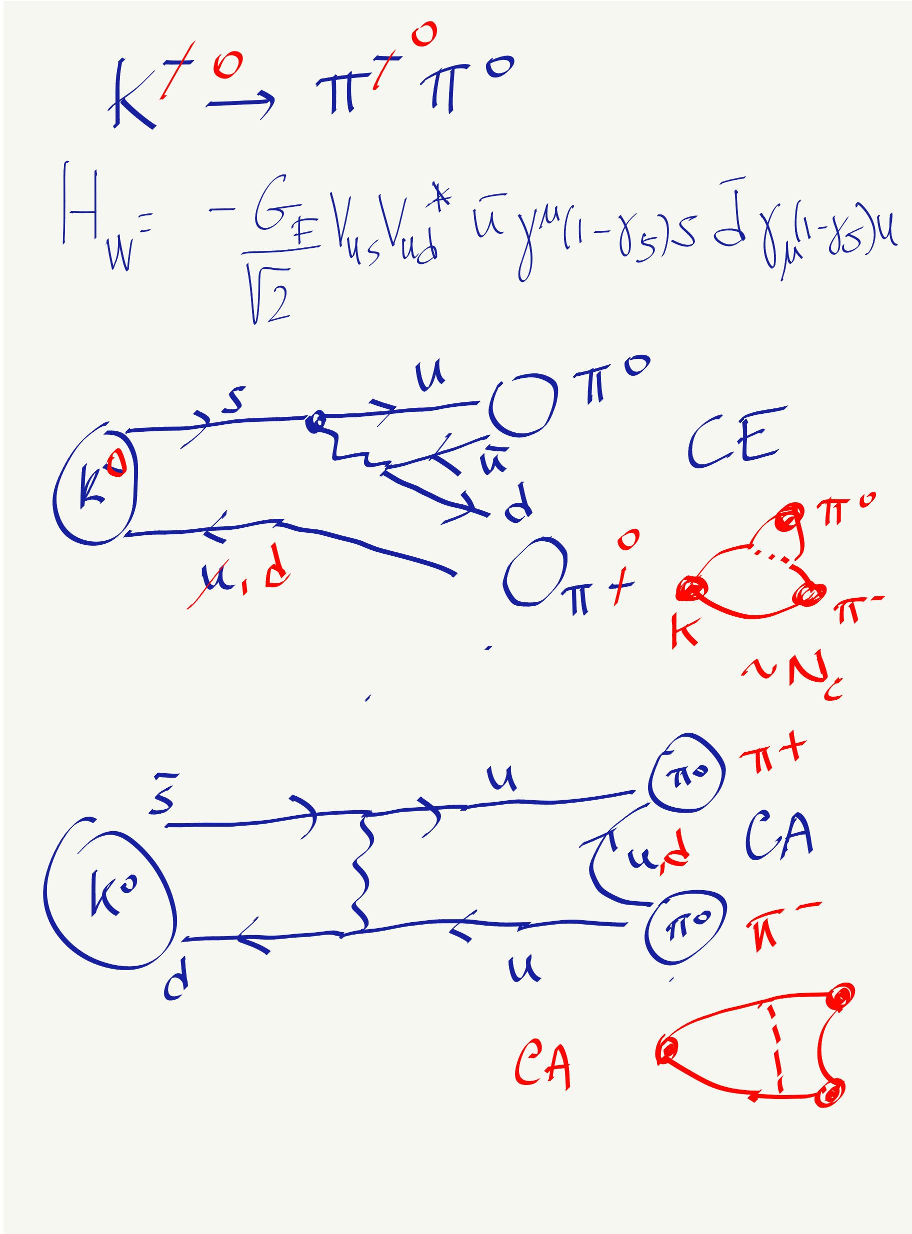
$$K^- \to \pi^- \pi^0$$

$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \, \bar{u} \gamma^\mu (1-\gamma_5) s \, \bar{d} \gamma_\mu (1-\gamma_5) u$$



Non-leptonic Decays

Penguins contractions and all that

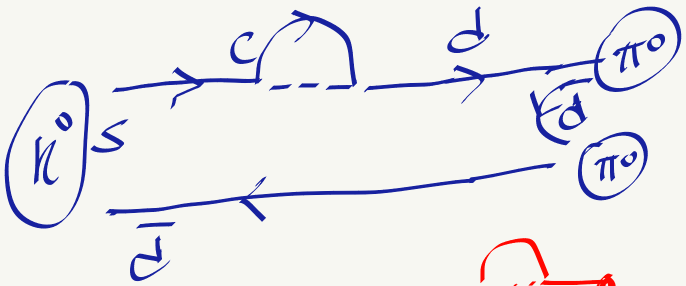


Non-leptonic Decays

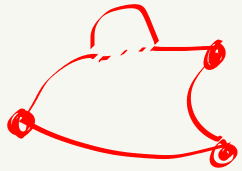
Penguins contractions and all that

Penguins diagrams

$$H = -\frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^* \, \bar{c}\gamma_\mu(1-\gamma_5)s \, \bar{d}\gamma^\mu(1-\gamma_5)c$$



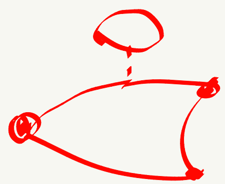
CP



CP

other ops

$$\sim V_{cs} V_{cd}^* \, \bar{d}\gamma_\mu(1-\gamma_5)s \, \bar{c}\gamma^\mu(1-\gamma_5)c$$



DP

All Topologies

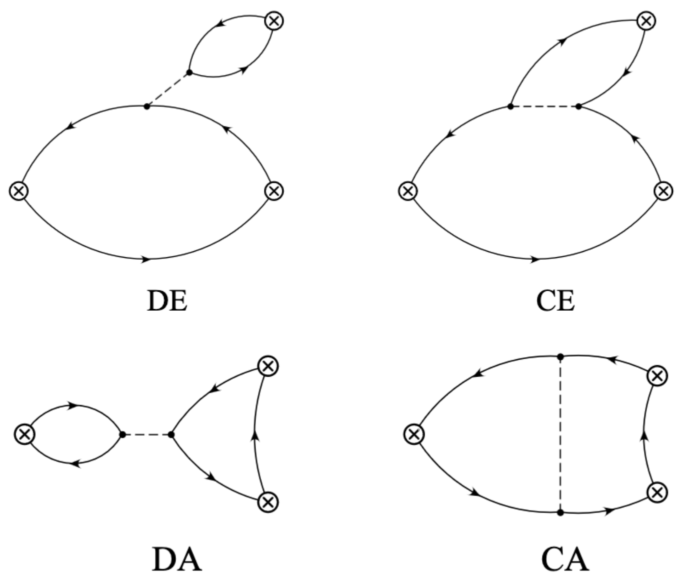
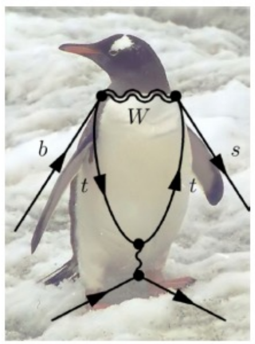


Figure 1: *Non-penguin diagrams. The dashed line represents the four-fermion operator.*

All Topologies



D. Kovalskyi's seminar @ CERN (26/7/22)

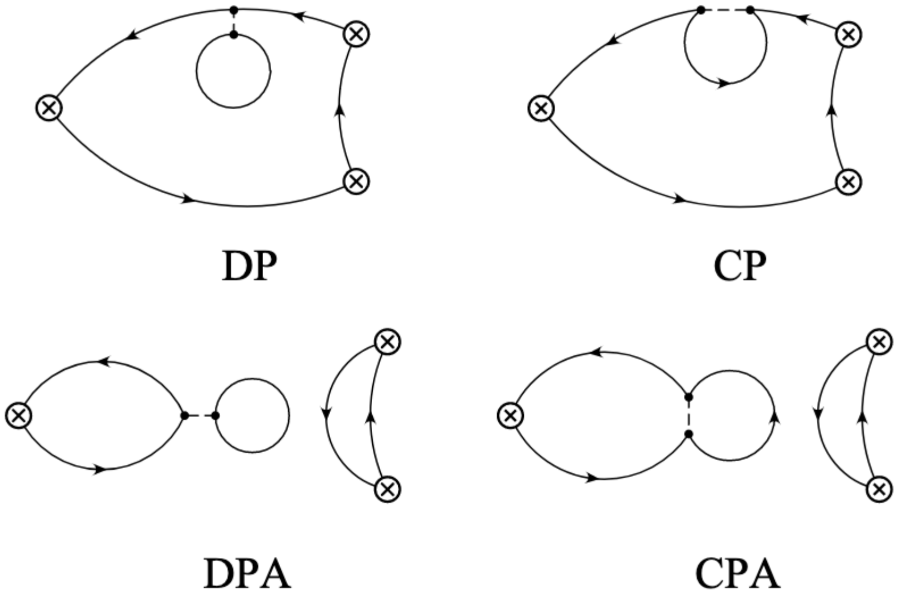
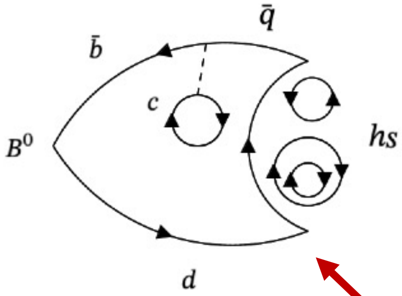


Figure 2: *Penguin diagrams.*



for heavy mesons many particles in the final state

GENERAL FRAMEWORK: THE OPE

$A_{FI}\ (2\pi^4)\ \delta^4\ (p_F - p_I) = \int d^4x\ d^4y\ D_{\mu\nu}(x, M_W)$

$\langle F | T[\ J_\mu (y+x/2)\ J^\dagger_\nu (y-x/2)] | I \rangle$

$\langle F | H^{\Delta S=1} | I \rangle =$

$$G_F/\sqrt{2}\ V_{ud}\ V_{us}^* \qquad \Sigma_i\, C_i(\mu)\ \frac{\langle F | Q_i(\mu) | I \rangle}{(M_W)^{di-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_W/μ and $\alpha_W(\mu)$

$Q_i(\mu)$ local operator renormalized at the scale μ

$$H_F = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \, [\bar{u}\gamma_\mu (1-\gamma_5) d] \, [\bar{s}\gamma^\mu (1-\gamma_5) u]$$

GENERAL FRAMEWORK

$$H^{\Delta S=1} = G_F/\sqrt{2}\, V_{ud}\, V_{us}^{*} \left[(1-\tau) \Sigma_{i=1,2}\, z_i (Q_i - Q^c_i) + \tau\, \Sigma_{i=1,10} (z_i + y_i)\, Q_i \right]$$

Where y_i and z_i are short distance coefficients, which are known in perturbation theory at the NLO (Buras et al. + Ciuchini et al.)

$$\tau = -V_{ts}^{*} V_{td} / V_{us}^{*} V_{ud}$$

We have to compute $A^{I=0,2}_i = \langle (\pi\, \pi)_{I=0,2} \,|\, Q_i \,|\, K \rangle$ with a non perturbative technique (lattice, QCD sum rules, 1/N expansion etc.)

New local four-fermion operators are generated

$Q_1 = (\bar{s}_L{}^A \gamma_\mu u_L{}^B)(\bar{u}_L{}^B \gamma_\mu d_L{}^A)$ Current-Current
$Q_2 = (\bar{s}_L{}^A \gamma_\mu u_L{}^A)(\bar{u}_L{}^B \gamma_\mu d_L{}^B)$

$Q_{3,5} = (\bar{s}_R{}^A \gamma_\mu d_L{}^A)\sum_q (\bar{q}_{L,R}{}^B \gamma_\mu q_{L,R}{}^B)$ Gluon
$Q_{4,6} = (\bar{s}_R{}^A \gamma_\mu d_L{}^B)\sum_q (\bar{q}_{L,R}{}^B \gamma_\mu q_{L,R}{}^A)$ Penguins

$Q_{7,9} = 3/2(\bar{s}_R{}^A \gamma_\mu d_L{}^A)\sum_q e_q (\bar{q}_{R,L}{}^B \gamma_\mu q_{R,L}{}^B)$ Electroweak
$Q_{8,10} = 3/2(\bar{s}_R{}^A \gamma_\mu d_L{}^B)\sum_q e_q (\bar{q}_{R,L}{}^B \gamma_\mu q_{R,L}{}^A)$ Penguins

+ Chromomagnetic end electromagnetic operators

$$\mathcal{A}(K \to \pi\pi) = \sum_i C_W^i(\mu)\langle\pi\pi|O_i(\mu)|K\rangle$$

GENERAL FRAMEWORK

$$\langle H^{\Delta S=1} \rangle = G_F/\sqrt{2}\, V_{ud}\, V_{us}^{*} \ldots \Sigma_i\, C_i(\mathbf{a}) \langle Q_i(\mathbf{a}) \rangle$$

$M_W = 100$ GeV

perturbative regime

FERMI EFFECTIVE THEORY

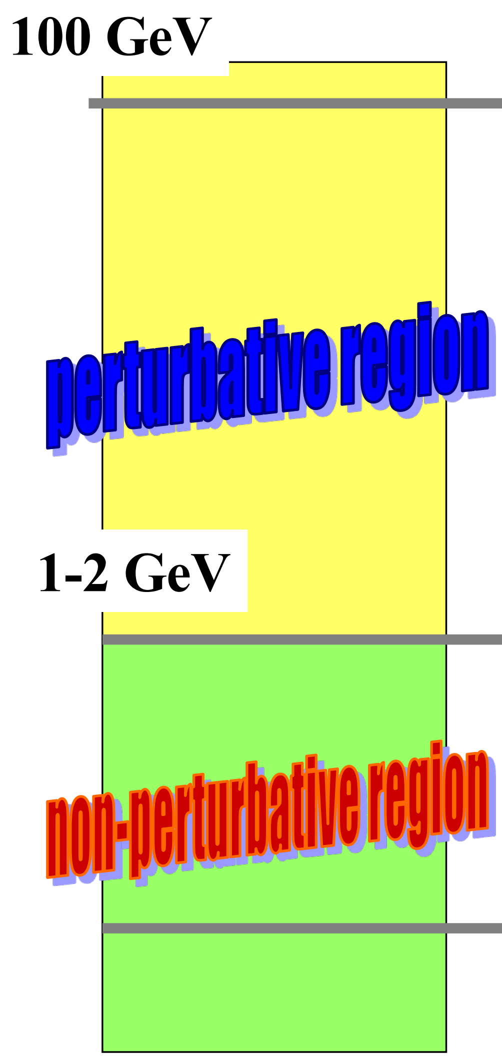
Effective Theory - quark & gluons

$a^{-1} = 2\text{-}5$ GeV

Hadronic non-perturbative region

Chiral regime

$\Lambda_{QCD}, M_K = 0.2\text{-}0.5$ GeV



Large mass scale: heavy degrees of freedom (m_t , M_W, M_s) are removed and their effect included in the Wilson coefficients

renormalizazion scale μ (inverse lattice spacing 1/a); this is the scale where the quark theory is matched to the effective hadronic theory

Scale of the low energy process $\Lambda \sim M_W$

<u>THE SCALE PROBLEM:</u> Effective theories prefer low scales, Perturbation Theory prefers large scales

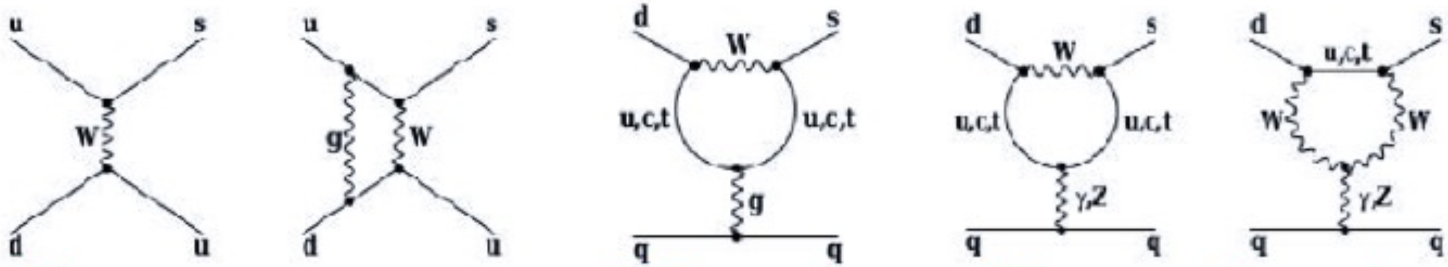
Weak Hamiltonian for $K \to \pi\pi$

Weak Hamiltonian is given by local four-quark operator

Courtesy by Xu Feng

$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)]\, Q_i \right\}, \quad \tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = 1.543 + 0.635i$
- $z_i(\mu)$ and $y_i(\mu)$ are perturbative Wilson coefficients
- Q_i are local four-quark operator

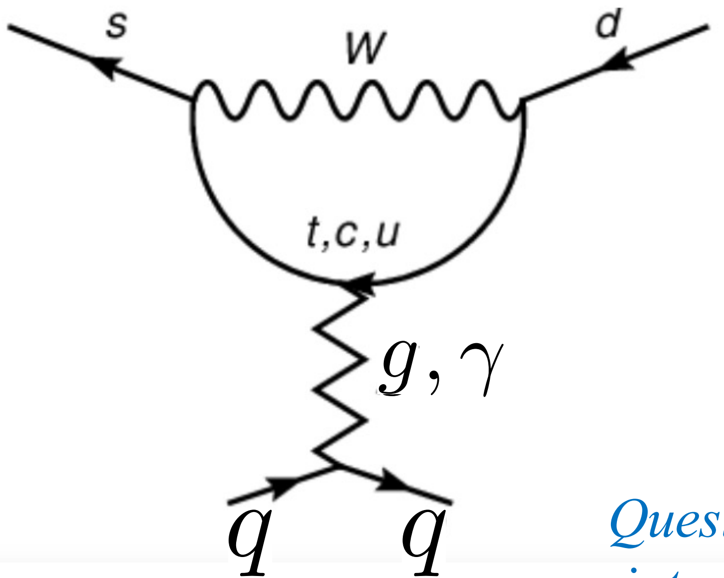


Current-current operator Q_1, Q_2 dominate $\mathrm{Re}[A_0]$, $\mathrm{Re}[A_2]$

QCD penguin $Q_3 - Q_6$ Q_6 dominate $\mathrm{Im}[A_0]$

Electro-weak penguin $Q_7 - Q_{10}$ Q_7, Q_8 dominate $\mathrm{Im}[A_2]$

Anche i Diagrammi a Pinguino generano l'Hamiltoniana di Fermi



$$A \sim C(q^2)\left(-g_{\mu\nu}q^2 + q_\mu q_\nu\right)\bar{s}\gamma_\mu(1-\gamma_5)d$$

$$i\frac{-g_{\mu\nu} + (1-\alpha)q_\mu q_\nu/q^2}{q^2}\bar{q}\gamma_\nu q$$

$$= iC(q^2)\ [\bar{s}\gamma_\mu(1-\gamma_5)d]\ [\bar{q}\gamma_\mu q]$$

Questo si può generalizzare a tutti gli ordini nelle interazioni forti e elettromagnetiche

New local four-fermion operators are generated

$Q_1 = (\bar{s}_L{}^A \gamma_\mu u_L{}^B)(\bar{u}_L{}^B \gamma_\mu d_L{}^A)$ Current-Current
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+ Chromomagnetic end electromagnetic operators

$$\mathcal{A}(K \rightarrow \pi\pi) = \sum_i C_W^i(\mu) \langle \pi\pi | O_i(\mu) | K \rangle$$

Rare Penguin Radiative Decays

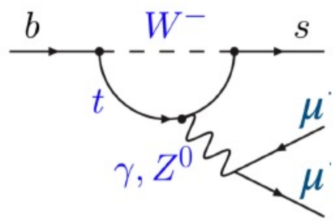
The main issue in this Group Meeting will be represented by b → s quark transitions: the best example is offered by the neutral-current semileptonic B → K(*) l+ l- transitions!

Many interesting properties:

1. **Loop-level processes (FCNCs are forbidden at tree level in the Standard Model)**
2. **CKM-suppressed decays, where**

$$J^{\mu}_{\text{charged}} = \bar{u}^i_L \boxed{V^{ij}_{CKM}} \gamma^{\mu} d^j_L + \bar{\nu}^i_L \gamma^{\mu} \ell^i_L$$

➡ **Rare transitions!**



L. Vittorio (LAPTh & CNRS, Annecy)

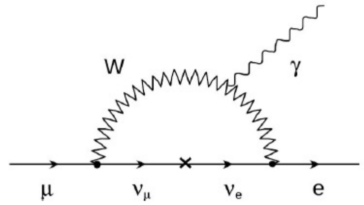
$$B^+ \to K^{(*)+} \gamma \qquad B^+ \to K^{(*)+} \mu^+ \mu^-$$

since different neutrinos have a mass and they can mix,

$\mu \to e\gamma$ is a possible decay which satisfies

all the symmetry

constraints



$$\mathcal{B}(\mu \to e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

note that the photon is emitted by

the W boson, analogy radiative B decays

FLAVOR CHANGING NEUTRAL CURRENTS

FCNC

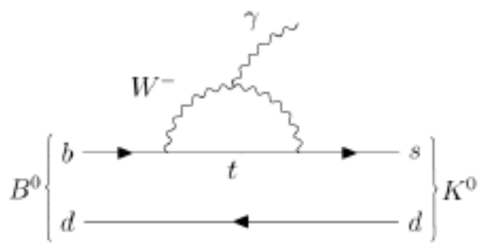


Figura 4: quark process

Radiative Penguins

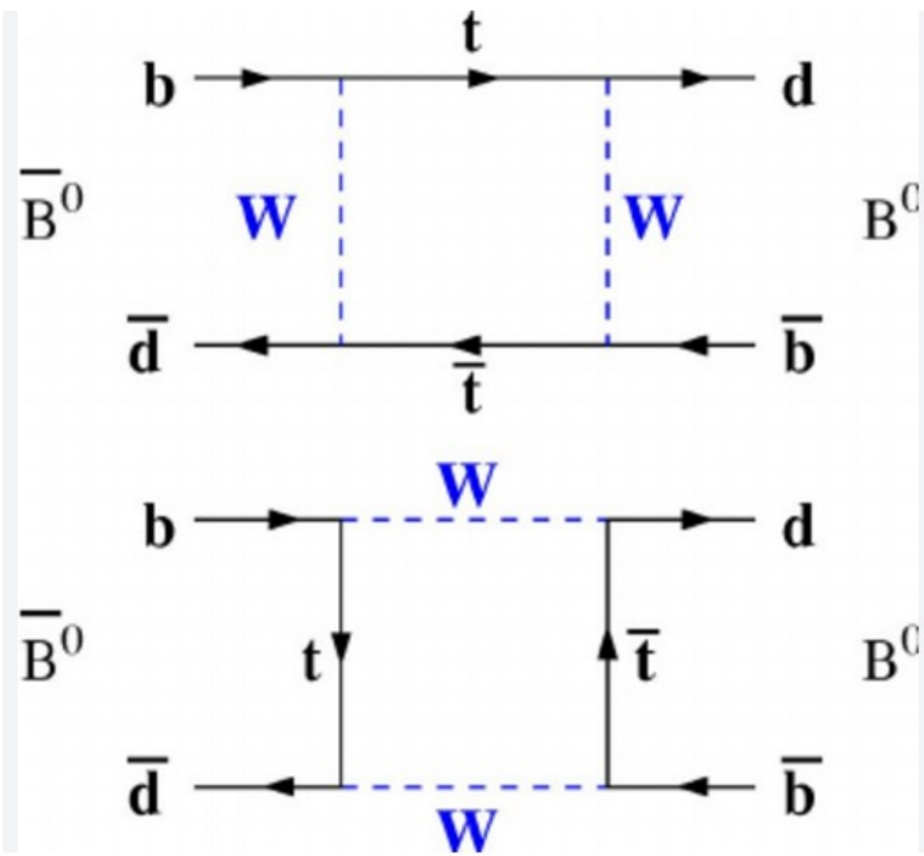
BOXES

Mixing of Neutral Mesons

$$K^0 \leftrightarrow \bar{K}^0$$

$$D^0 \leftrightarrow \bar{D}^0$$

$$B^0 \leftrightarrow \bar{B}^0$$



$$\Delta F = 2 \qquad H_{eff} = C(\mu)\left(\bar{d}\gamma_\mu(1-\gamma_5)b\right)\left(\bar{d}\gamma_\mu(1-\gamma_5)b\right)$$

$$\boxed{A \sim C(\mu)\langle B^0|\left(\bar{d}\gamma_\mu(1-\gamma_5)b\right)\left(\bar{d}\gamma_\mu(1-\gamma_5)b\right)|\bar{B}^0\rangle}$$

BOXES

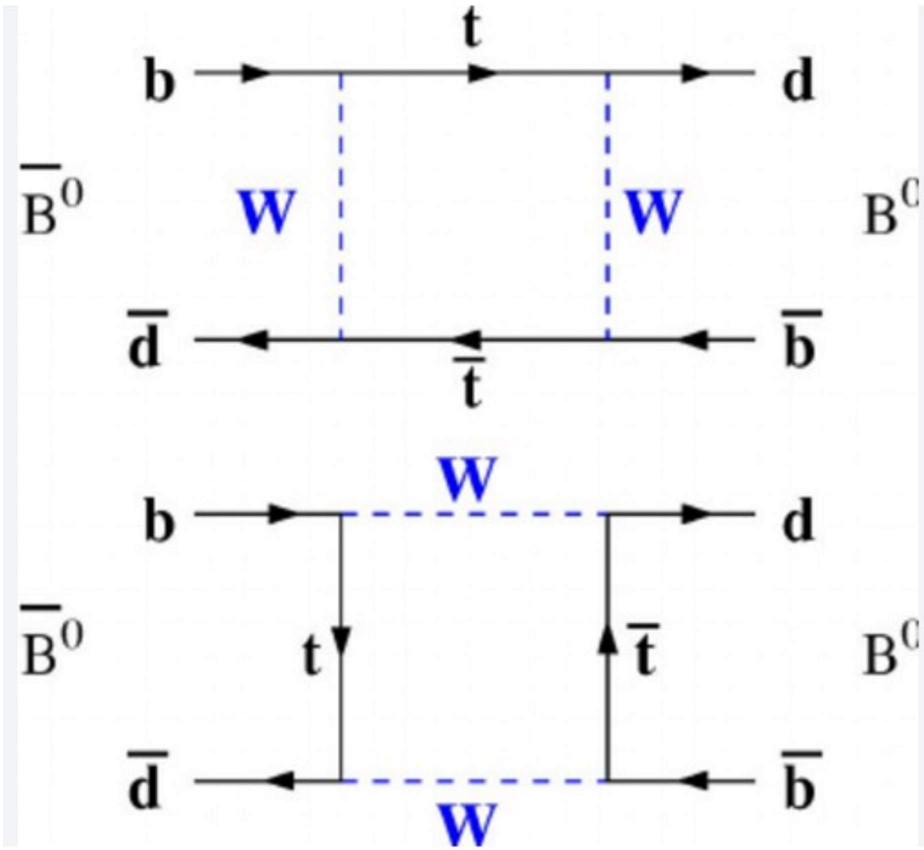
Mixing of Neutral Mesons

$$K^0 \leftrightarrow \bar{K}^0$$

$$D^0 \leftrightarrow \bar{D}^0$$

$$B^0 \leftrightarrow \bar{B}^0$$

in the case of kaons
also charm and up quarks
contribute
for D and K meson mixing
there are important long
distance contributions



$$\mathcal{H}_{\rm eff}^{\Delta B=2} = \frac{G_{\rm F}^2}{16\pi^2} M_W^2 \left(V_{tb}^* V_{tq}\right)^2 \eta_B S_0(x_t) \times$$
$$\times \left[\alpha_s^{(5)}(\mu_b)\right]^{-6/23} \left[1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5\right] Q^q(\Delta B = 2) + h.c.$$

QCD corrections

PENGUINS AND BOXES

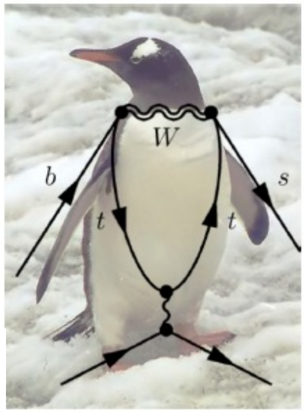
Pure leptonic Bs decays

$$Br(B_s \to l^+l^-) = \tau(B_s)\frac{G_F^2}{\pi}\left(\frac{\alpha}{4\pi\sin^2\Theta_W}\right)^2 \boxed{F_{B_s}^2} m_l^2 m_{B_s}\sqrt{1-4\frac{m_l^2}{m_{B_s}^2}}|V_{tb}^* V_{ts}|^2 Y^2(x_t)$$

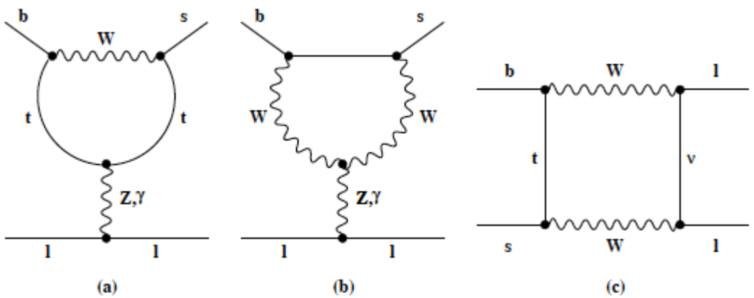
G. Buchalla and A.J. Buras, Nucl. Phys. B 400 (1993) 225.

Many interesting properties:

1. **Helicity suppressed**
2. **Non-perturbative hadronic contributions enter via Bs decay constant**



valskyi's seminar @ CERN (26/7/22)



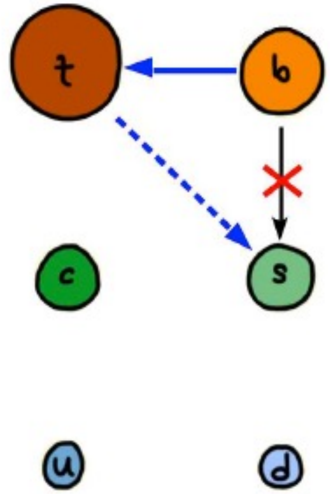
Lowest order diagrams
QCD corrections at NLO or NNLO

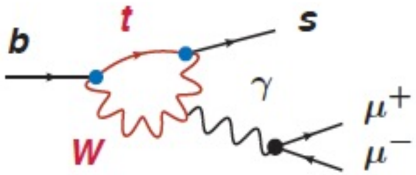
Flavor Changing Neutral Currents in the SM

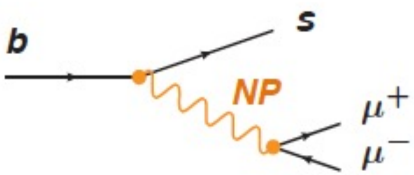
LA TEORIA DI FERMI OLTRE IL MODELLO STANDARD

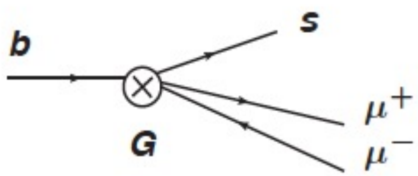
In the SM, flavor changing neutral currents (FCNCs) are absent at the tree level

FCNCs can arise at the loop level
they are suppressed by loop factors
and small CKM elements







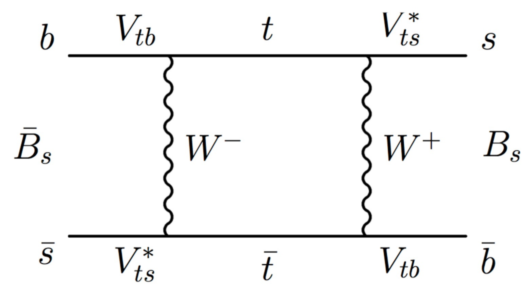


$$G \sim \frac{1}{16\pi^2} \frac{g^4}{m_W^2} \frac{m_t^2}{m_W^2} V_{tb} V_{ts}^* + \frac{C_{NP}}{\Lambda_{NP}^2}$$

→ measuring low energy flavor observables gives information on new physics flavor couplings and the new physics mass scale

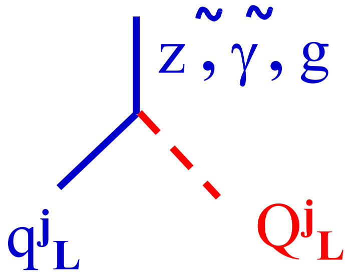
B^0 - $\overline{B^0}$ mixing

Standard Model CKM



$$\Delta m_{B_s} = \frac{G_F^2 M_W^2}{16\pi^2} A^2 \lambda^6 F_{tt} \left(\frac{m_t^2}{M_W^2}\right) \langle B_s | \left(\bar{s}\gamma_\mu(1-\gamma_5)b\right)^2 | \bar{B}_s \rangle$$

Hadronic matrix element



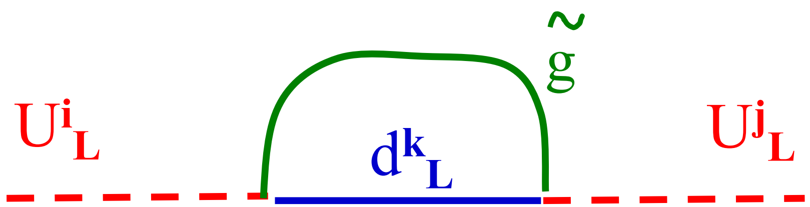
In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case

We may either

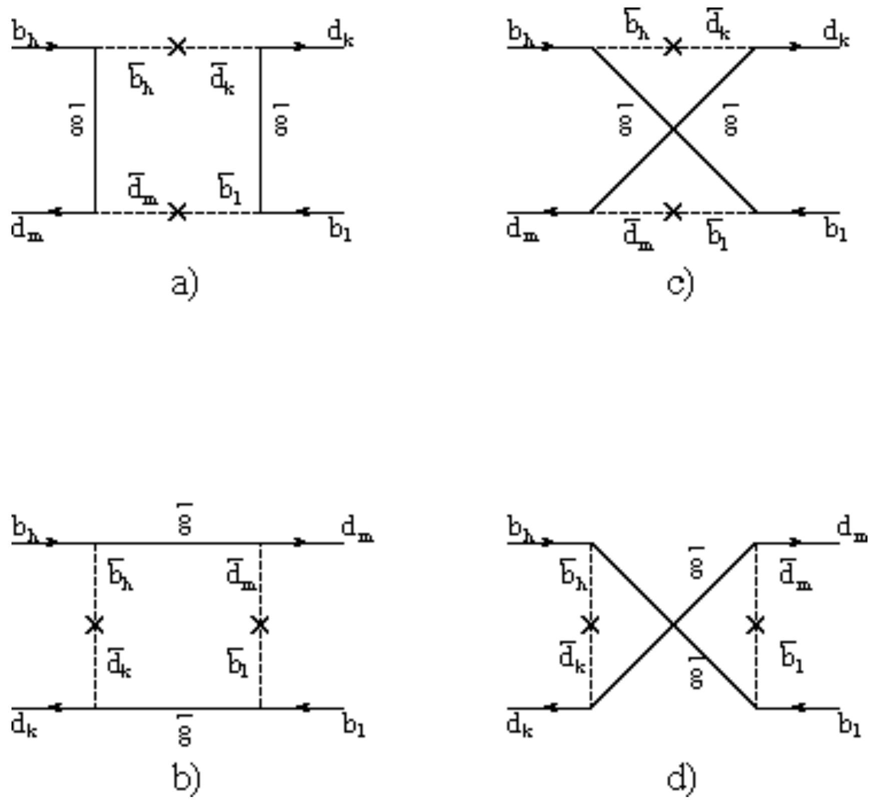
Diagonalize the SMM

or Rotate by the same Matrices the SUSY partners of the u- and d- like quarks

$(Q^j_L)' = U^{ij}_L Q^j_L$



In the latter case the Squark Mass Matrix is not diagonal



$$(m^2_Q)_{ij} = m^2_{average} 1_{ij} + \Delta m_{ij}^2 \qquad \delta_{ij} = \Delta m_{ij}^2 / m^2_{average}$$

Hamiltoniana di Fermi

$$H_F = \frac{-G_F}{\sqrt{2}} \left(\bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu\right) \left(\bar{e}\gamma_\mu (1-\gamma_5)\ \nu_e\right)$$

$$H_F = -\frac{G_F}{\sqrt{2}} J^\mu J_\mu^\dagger$$

1) Seconda quantizzazione per tutte le particelle elementari
2) Primo esempio di teoria effettiva di cui facciamo oggi larghissimo uso nella ricerca di nuova fisica
3) Come prodotto di correnti ha indicato la strada per la costruzione del Modello Standard (SM)
4) Strumento ancora oggi essenziale per la ricerca di fenomeni/segnali di fisica oltre lo SM

BACKUP SLIDES

$$A_0 = \sum_i C_i(\mu) \; \langle (\pi\,\pi) | Q_i(\mu) | K \rangle_{I=0} \; (1 - \Omega_{IB})$$

μ = renormalization scale
μ-dependence cancels if operator matrix elements are consistently computed

ISOSPIN BREAKING

$$A_2 = \sum_i C_i(\mu) \; \langle (\pi\,\pi) | Q_i(\mu) | K \rangle_{I=2}$$

Ω_{IB} = 0.25± 0.08 (Munich from Buras & Gerard)

0.25± 0.15 (Rome Group) 0.16± 0.03 (Ecker et al.)

0.10± 0.20 Gardner & Valencia, Maltman & Wolf, Cirigliano & al.

$$A^{I=0,2}_{i}(\mu) = \langle (\pi\,\pi)_{I=0,2}\, |Q_{i}(\mu)|\, K \rangle$$
$$= Z_{ik}(\mu\, a)\, \langle (\pi\,\pi)_{I=0,2}\, |Q_{k}(a)|\, K \rangle$$

Where $Q_i(a)$ is the bare lattice operator
And a the lattice spacing.

The effective Hamiltonian can then be read as:

$$\langle F \,|\, H^{\Delta S=1} \,|\, I \rangle = G_F/\sqrt{2} V_{ud} V_{us}^{*} \Sigma_i\, C_i\,(1/a)\, \langle F \,|\, Q_i\,(a) \,|\, I \rangle$$

In practice the renormalization scale (or 1/a) are the scales which separate short and long distance dynamics