High-Precision Determination of Radiative Corrections to Superallowed Nuclear Beta Decays

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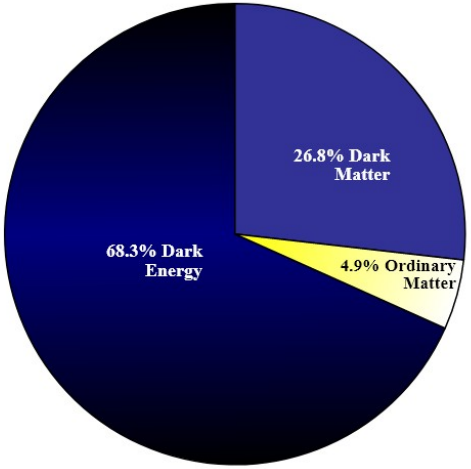
Outline

1. Beta decays and radiative corrections (RC)
2. Nucleus-independent RC
3. Nucleus-dependent RC
4. Summary and outlook

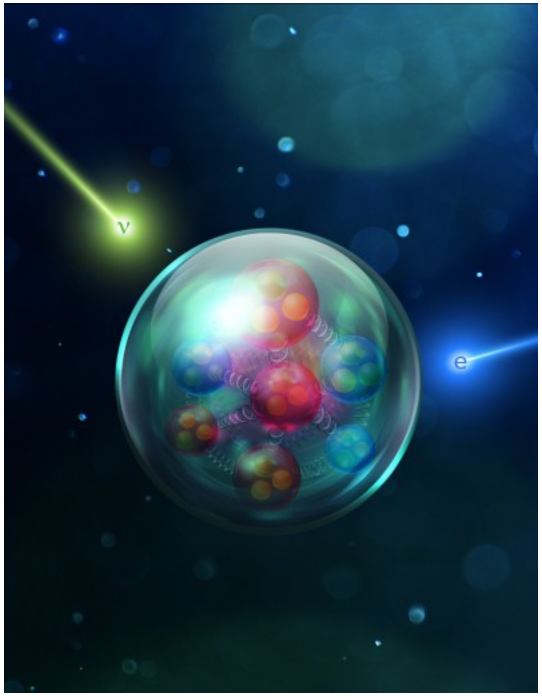
Many unresolved problems call for physics beyond the Standard Model (BSM) !

Image credit: Wikipedia

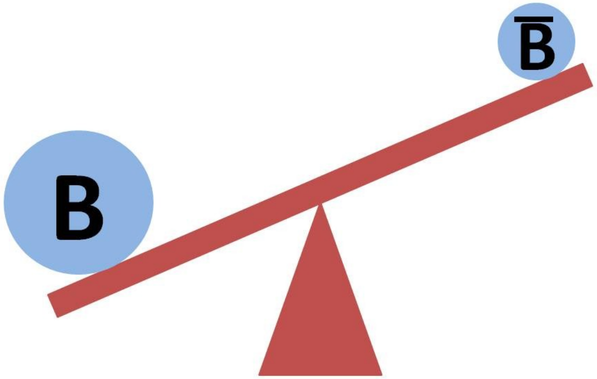


What is the origin of dark energy and dark matter?

Image credit: Jefferson Lab

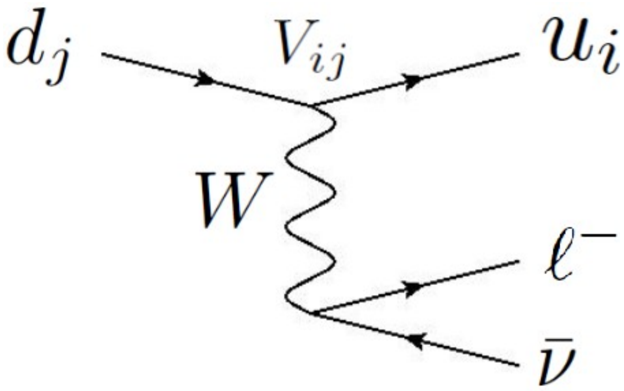


What is the nature of the neutrino mass?



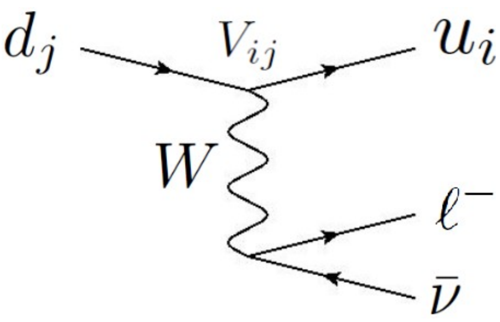
Why is there much more matter than antimatter in the observed universe?

Beta decay: Decay of **strong interaction bound states** through the emission of a **W-boson**



Testing the SM with beta decays

$$\psi_{d,f} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_f = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$



CKM matrix

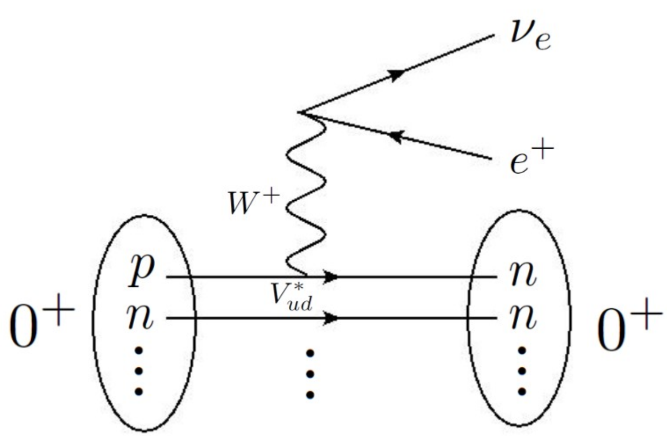
"First-row CKM unitarity"

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Precision level: ~0.01%

"Superallowed" beta decays of T=1, J^p=0$^+$ nuclei

$$\boxed{i(0^+) \rightarrow f(0^+) + e^+ + \nu_e}$$

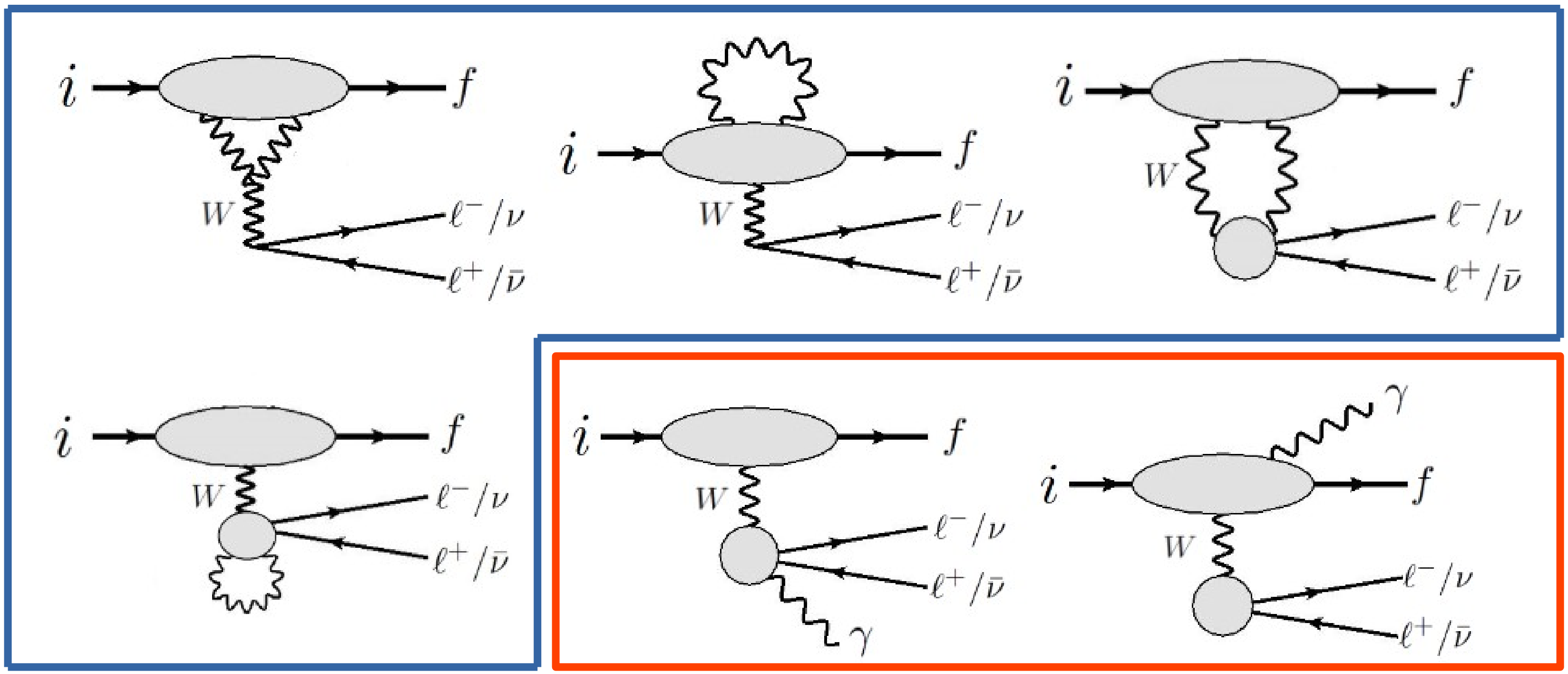


Provides the **best measurement of V_{ud} :**

- Pure **Fermi** transition
- **23** measured transitions
- **15** with lifetime precision better than **0.23%**

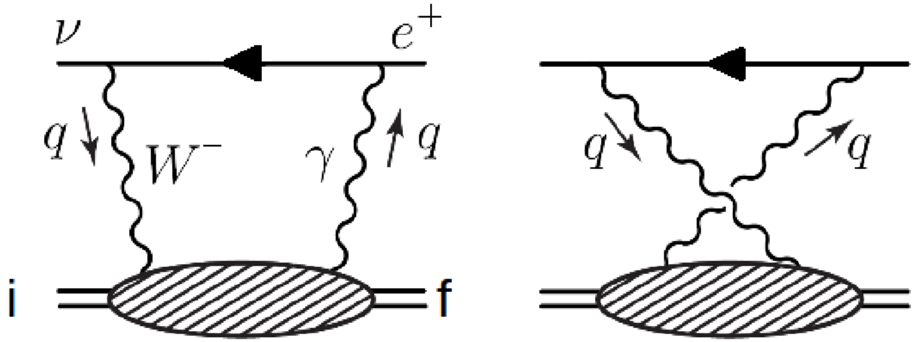
Hardy and Towner, 2020 PRC

Radiative corrections (RC): Perturbations induced by virtual and real gauge bosons



Typical size: $\gtrsim \frac{\alpha}{\pi} \sim 10^{-3}$

Needed to high precision for V_{ud} extraction!

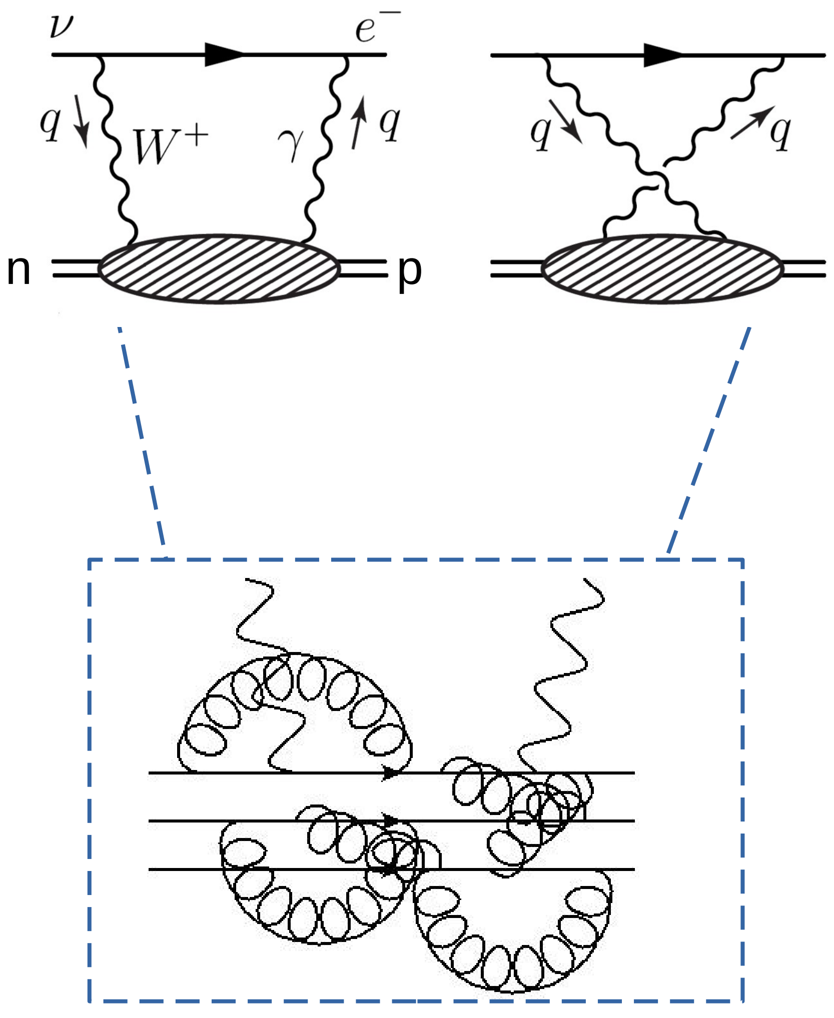


RC depends on hadron physics + **nuclear physics**

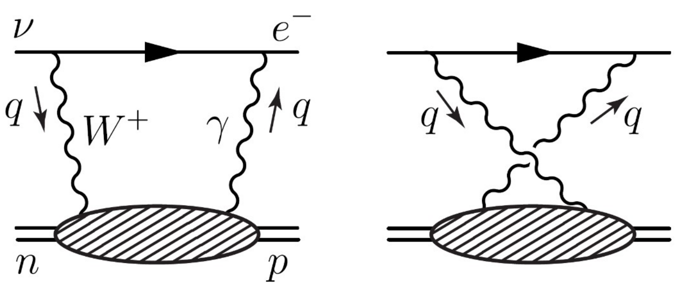
$$\text{Nucleus RC} = \underbrace{(\text{Nucleon RC})}_{\Delta_R^V} + \underbrace{(\text{Nucleus RC} - \text{Nucleon RC})}_{\delta_{\text{NS}}}$$

Nuclear structure (NS)-dependent RC

Single-nucleon RC



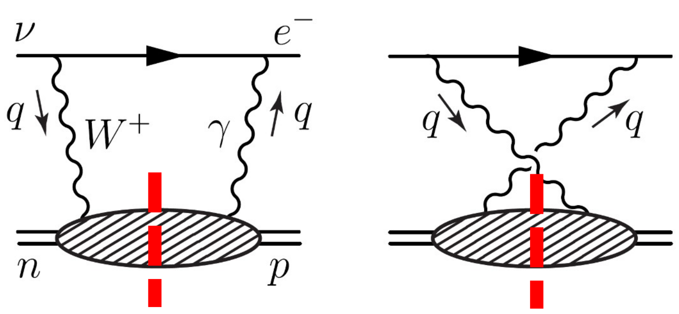
Virtual γ probes QCD at **ALL scales**!



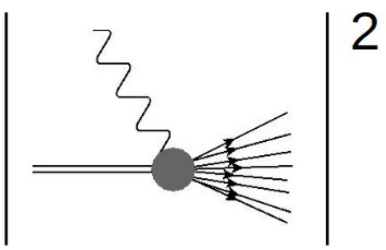
Dispersion relation (DR) analysis

CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL

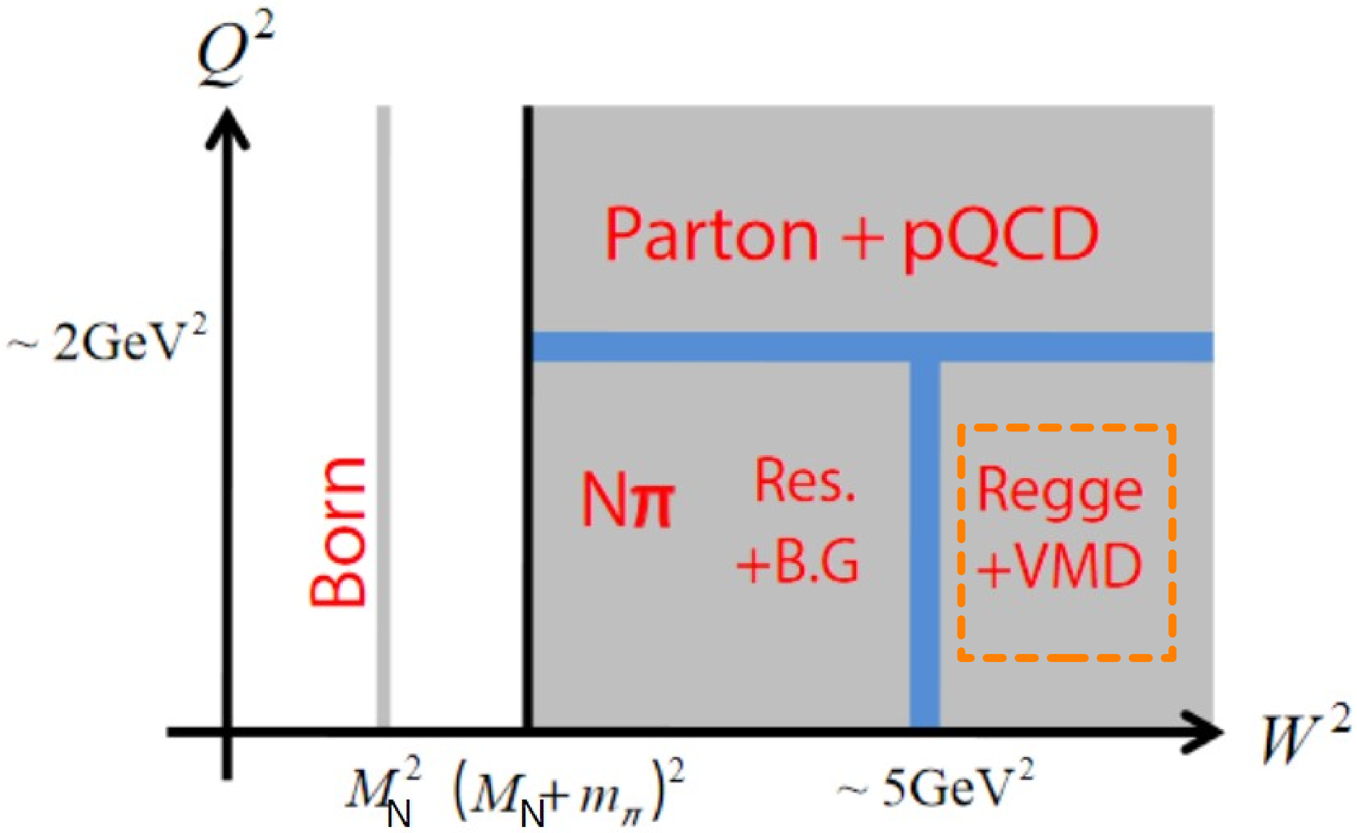
Cauchy's theorem



Optical theorem



Single-nucleon structure function

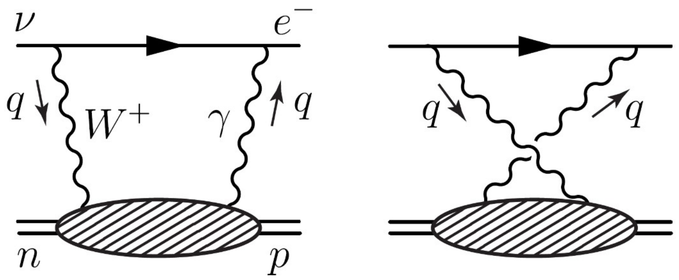


Data Input:

Structure functions from **inclusive neutrino-nucleon scattering**

Independent assessment with **lattice QCD**

Ma, Feng, Gorchtein, Jin, Ma, Liu, CYS and Wang, 2023 PRL



Direct computation of **four-point correlation functions**

$$\langle p | T\left\{ J^{\mu}_{\rm em}(x) J^{\nu}_{W}(y) \right\} | n \rangle$$

1 2 3 4

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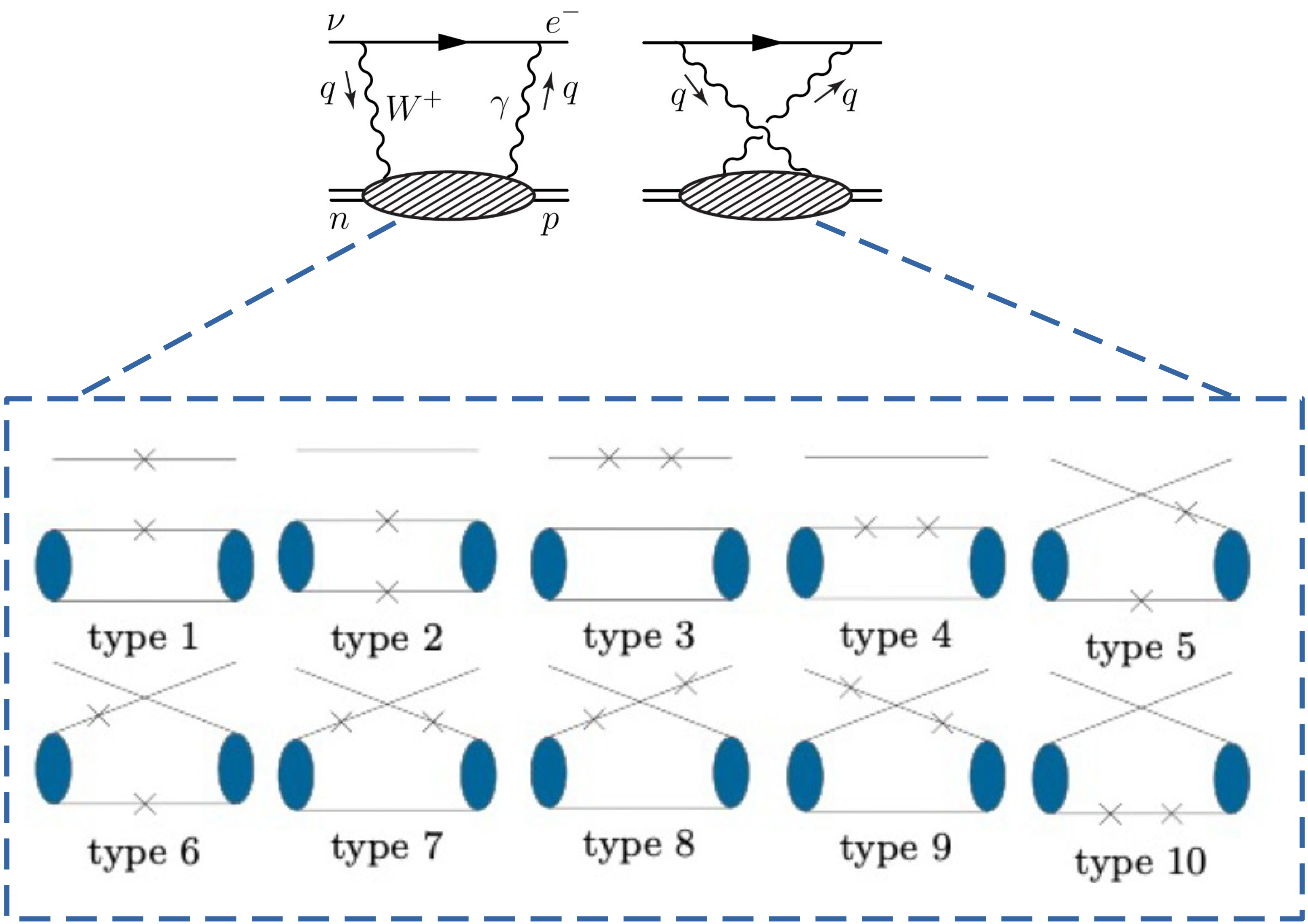

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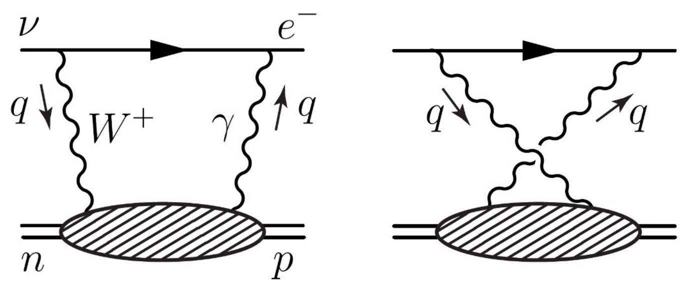
Ma, Feng, Gorchtein, Jin, Ma, Liu, CYS and Wang, 2023 PRL



$$(\Delta_R^V)_{\mathrm{lat}} = 0.02439(15)_{\mathrm{lat}}(10)_{\mathrm{HO}}$$

$$(\Delta_R^V)_{\mathrm{DR}} = 0.02467(20)_{\mathrm{data}}(10)_{\mathrm{HO}}$$

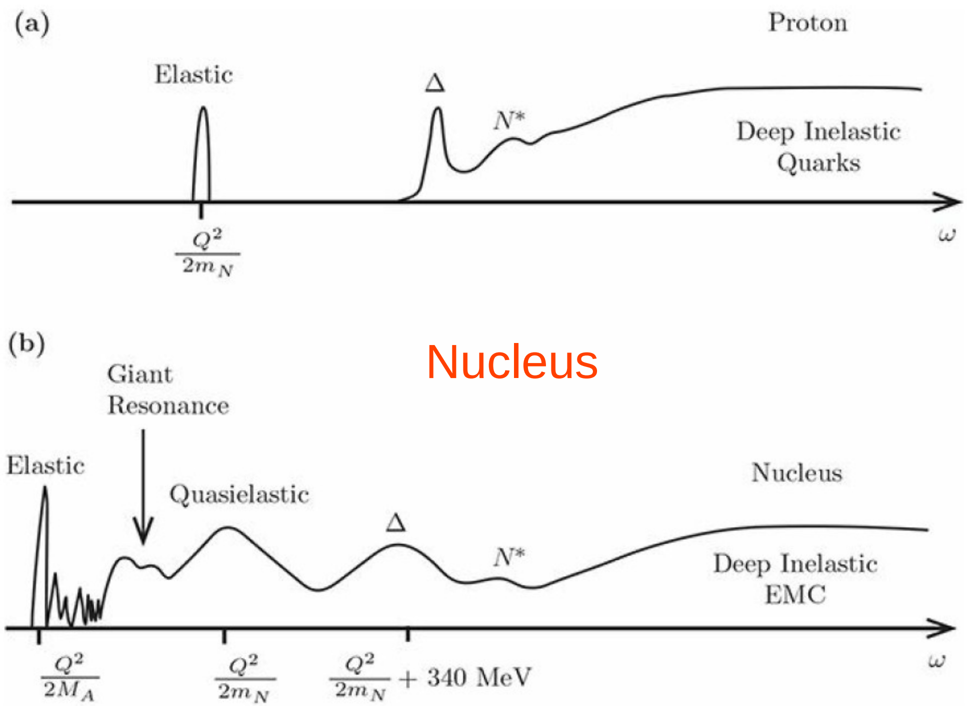
Lattice result in excellent agreement with DR + data at 1σ level

Nuclear structure effects in RC, δ_{NS}

Originates from the **difference between the nucleon and nuclear absorption spectrum**

CYS, Gorchtein and Ramsey-Musolf, 2019 PRD

Nucleon

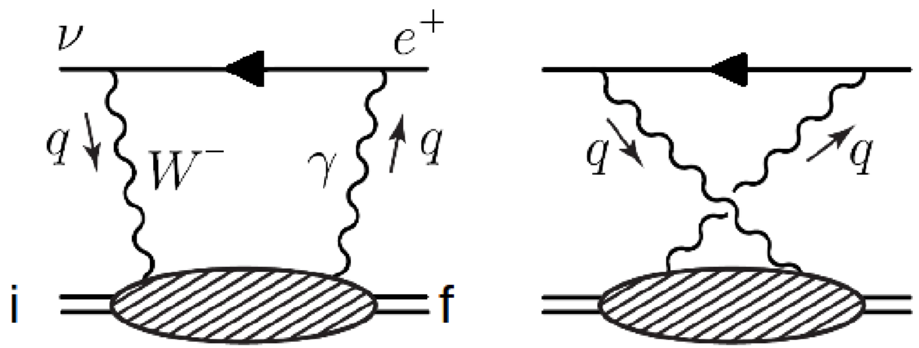


Nucleus

Donnelly, Formaggio, Holstein, Milner and Surrow, "Foundations of Nuclear and Particle Physics"

Formulation of δ_{NS} theory framework:

CYS and Gorchtein, 2023 PRC;
Gorchtein and CYS, 2024 Ann.Rev.Nucl.Part.Sci



Nuclear Green's function

$$\text{RC Integrand} \sim \boxed{\langle f|J^x_{\text{em}}(\vec{q})G(M+q_0+i\varepsilon)J^y_{W,A}(-\vec{q}) + \ldots |i\rangle}$$

$$\overset{\text{DR}}{\sim} \boxed{\sum_X \delta(q_0+M-E_X)\langle f|J^x_{\text{em}}(\vec{q})|X\rangle\langle X|J^y_{W,A}(-\vec{q})|i\rangle}$$

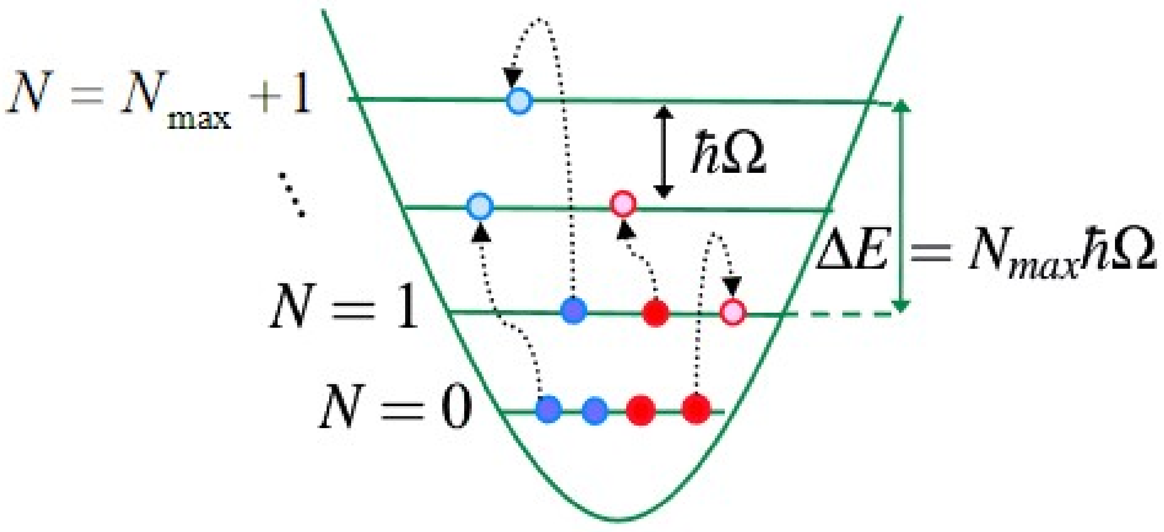
Nuclear response function

Serve as foundations for **nuclear *ab initio* calculations**

First calculation with **No Core Shell Model (NCSM)**

Gennari, Drissi, Gorchtein, Navratil and CYS, 2025 PRL

- Utilizes discrete harmonic oscillator (HO) basis up to $N=N_{max}+N_{Pauli}$
- Nuclear interactions from Chiral EFT



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$$\langle f | J^x_{\rm em}(\vec{q}) G(M+q_0+i\varepsilon) J^y_{W,A}(-\vec{q}) + \dots \; | i \rangle$$

$$\uparrow$$

$$\frac{1}{z-H}$$

Lanczos algorithm

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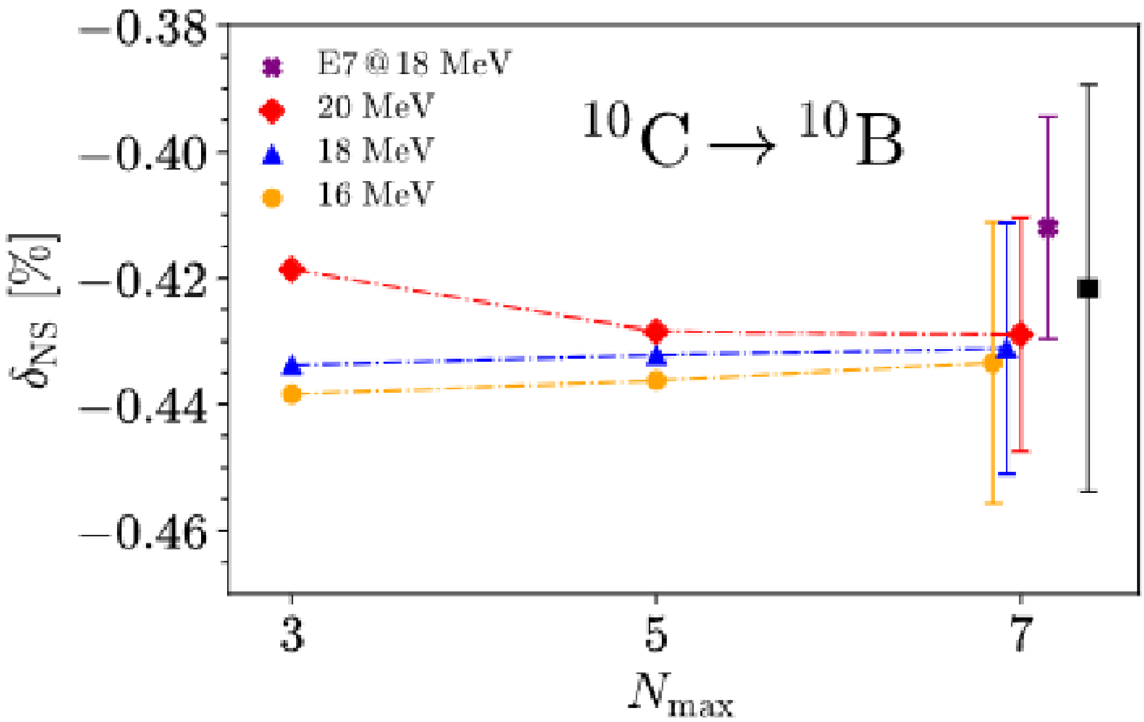
$$\langle f | J^x_{\rm em}(\vec{q}) G(M + q_0 + i\varepsilon) J^y_{W,A}(-\vec{q}) \; + \ldots \; | i \rangle$$

Multipole expansion

$$\rho(\vec{q}) \;=\; \sqrt{4\pi} \sum_{J=0}^{\infty} (-i)^J \sqrt{2J+1} M_{J0}(\mathbf{q})$$

$$J(\vec{q},\lambda) \;=\; \begin{cases} \sqrt{4\pi} \sum_{J=0}^{\infty} (-i)^J \sqrt{2J+1} L_{J0}(\mathbf{q}) & , \quad \lambda = 0 \\ -\sqrt{2\pi} \sum_{J=1}^{\infty} (-i)^J \sqrt{2J+1} \left(\lambda T^{\rm mag}_{J\lambda}(\mathbf{q}) - T^{\rm el}_{J\lambda}(\mathbf{q}) \right) & , \quad \lambda = \pm 1 \end{cases}$$

δ_{NS} in $^{10}C \rightarrow {}^{10}B$ with NCSM:



Rapid convergence with increasing N_{max}

δ_{NS} in $^{10}C \rightarrow {}^{10}B$ with NCSM:

$$\delta_{\rm NS}(\%) \quad : \quad -0.345(35)$$ **Shell model**

Hardy and Towner, 2015 PRC

$$\rightarrow \quad -0.400(50)$$ **DR + Fermi gas model**

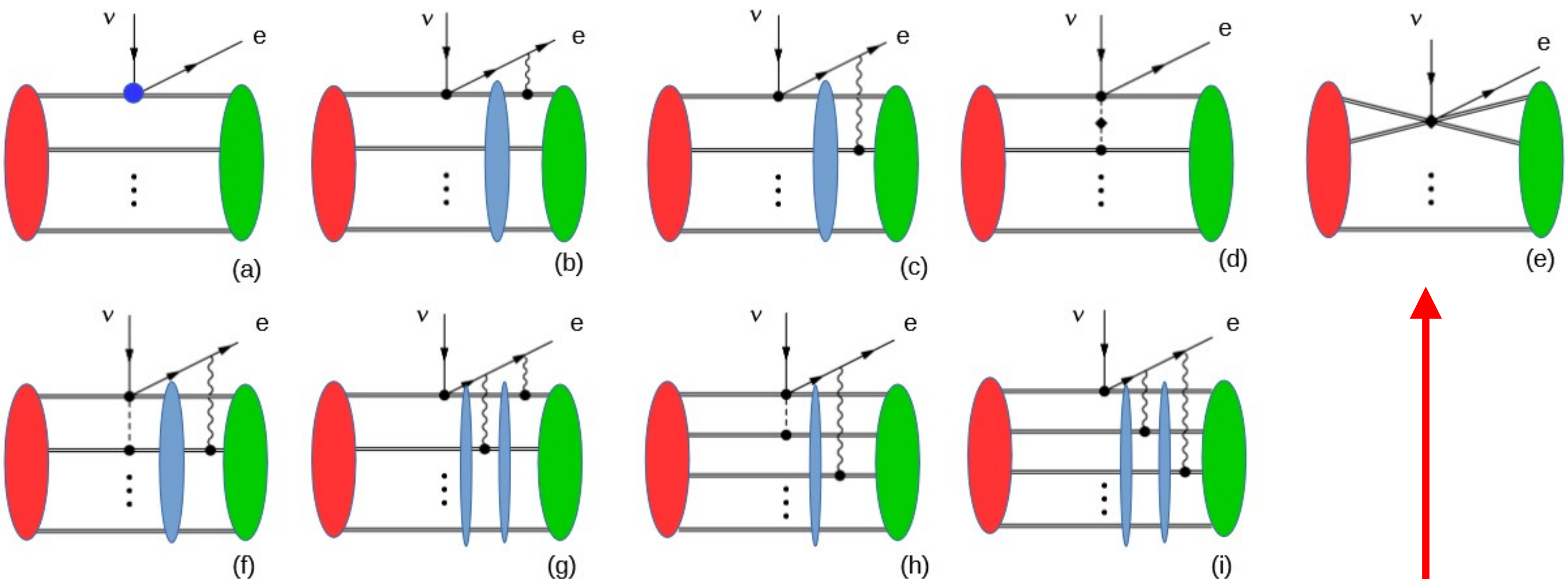
CYS, Gorchtein and Ramsey-Musolf, 2018 PRD;
Hardy and Towner, 2020 PRC

$$\rightarrow \quad -0.422(31)$$ ***Ab initio NCSM***

Gennari, Drissi, Gorchtein, Navratil and CYS, 2025 PRL

Parallel effort: **Effective Field Theory (EFT) approach**

Cirigliano et al, 2024 PRL; 2024 PRC



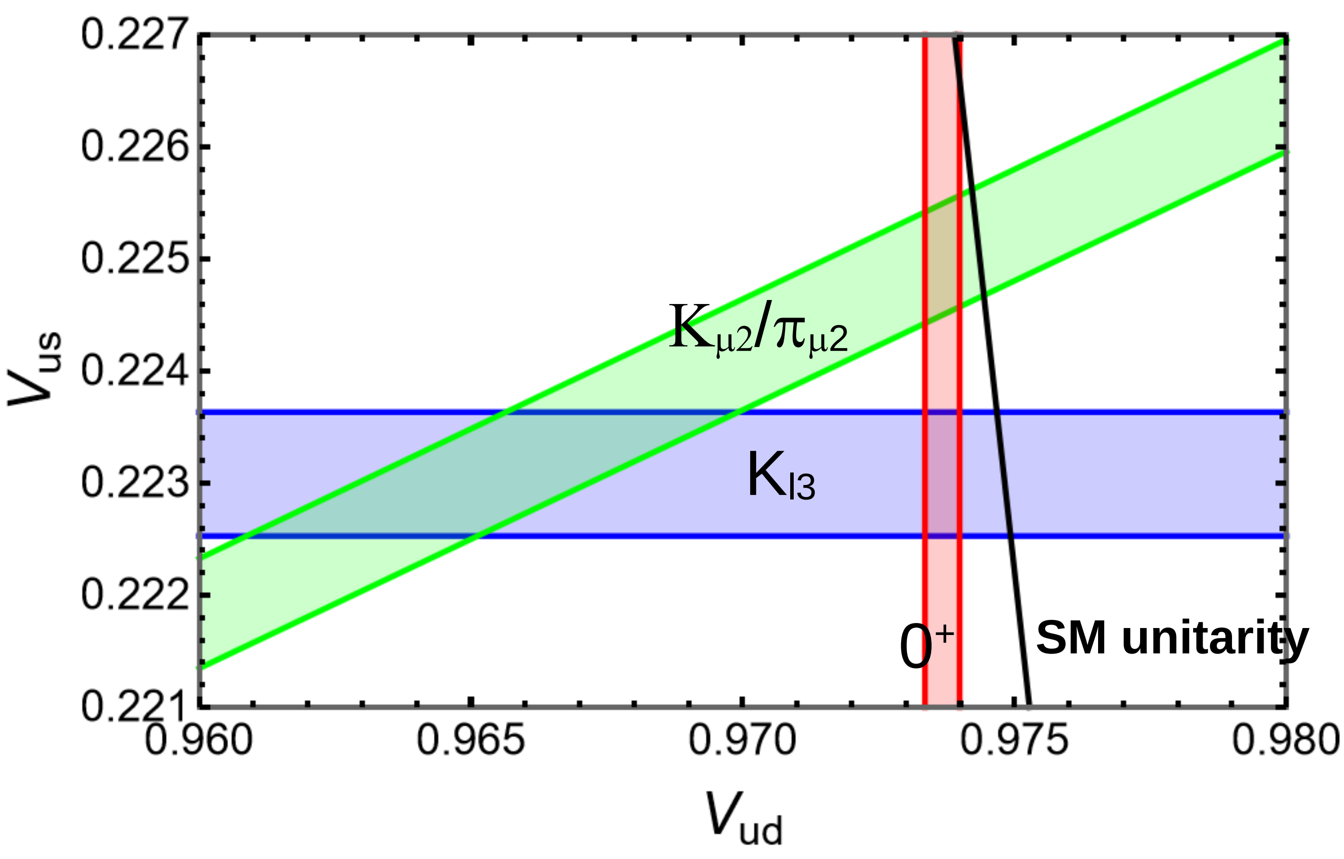
Numerical calculation is **less demanding**!

Limitation: Unknown **counterterms**

Talk by Emanuele Mereghetti

Similar problem in **0νββ**

Talks by Lotta Jokiniemi, Elina Kauppinen, Jason Holt...



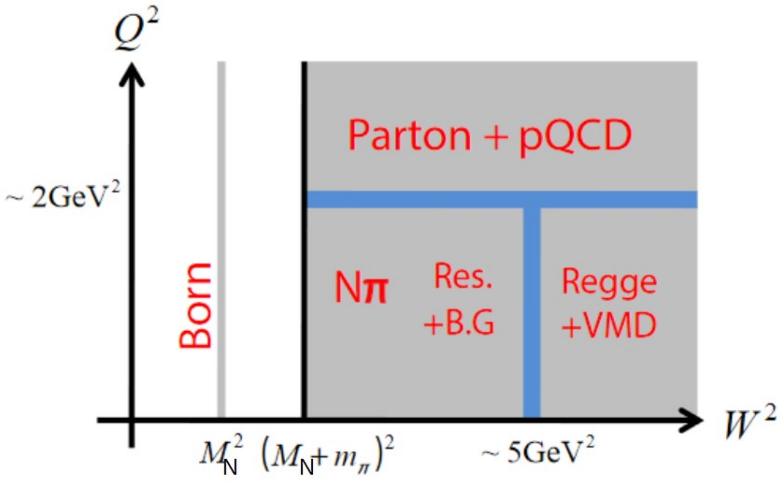
Unitarity deficit ~3σ, “Cabibbo angle anomaly”

Summary and outlook

- High-precision RC is needed to test SM in beta decays
- We pin down non-perturbative QCD effects by:
  - Hadron physics: DR + data / lattice
  - Nuclear physics: Nuclear *ab initio* calculations
- New studies of RC unveiled CKM unitarity violation

Summary and outlook

- Neutron decay RC: Towards a community consensus



	[60, 61]	[92]	[67]	[93]	[87]	Our value
Born	1.06(6)	1.06(6)	1.05(4)	0.99(10)	1.06(6)	1.06(6)
πN+Res.	0.05(1)	0.05(1)	0.04(1)	-	-	0.05(1)
Regge	0.51(8)	0.56(9)	0.52(7)	0.38(3)	0.53(7)	0.54(6)
DIS	2.17	2.16	2.20(3)	2.16	2.16	2.20(3)

Gorchtein and CYS, 2023 Universe

Summary and outlook

- Interplay between different approaches to δ_{NS}

CYS et al, in progress

$$\delta_{\rm NS} = \frac{2}{M_F}\langle \underbrace{\mathcal{V}_0^{\rm mag}(\vec{r}) + \mathcal{V}_0^{\rm rec,1}(\vec{r}) + \mathcal{V}_0^{\rm CT}(\vec{r})\rangle_{fi}}_{\text{Nuclear matrix element of EFT potentials}}$$

Nuclear Green's function/ response function

Simultaneous analysis of **two superallowed transitions** will determine the two unknown counterterms in EFT

Same game can be played for **0νββ**