DIRECT AND TWO-STEP PROCESSES IN SINGLE CHARGE EXCHANGE REACTIONS WITHIN A UNIFIED MODEL

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The NUMEN project: a new way to provide data-driven information on neutrino-less double-beta decay nuclear matrix element

20 May 2025, 16:50
25m
Ischia

Afternoon Session 2

Speaker

Alessandro Spatafora (INFN-LNS an

Probing two-body charge-exchange transition densities with heavy ion reactions

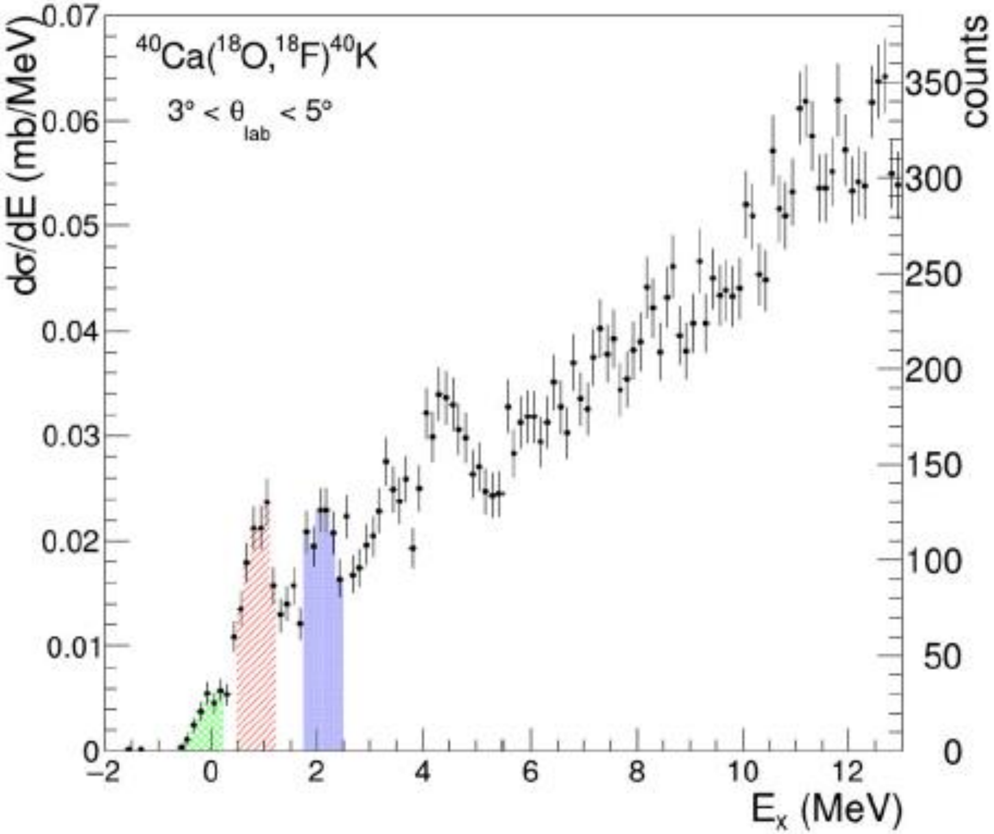
21 May 2025, 12:50
25m
Ischia

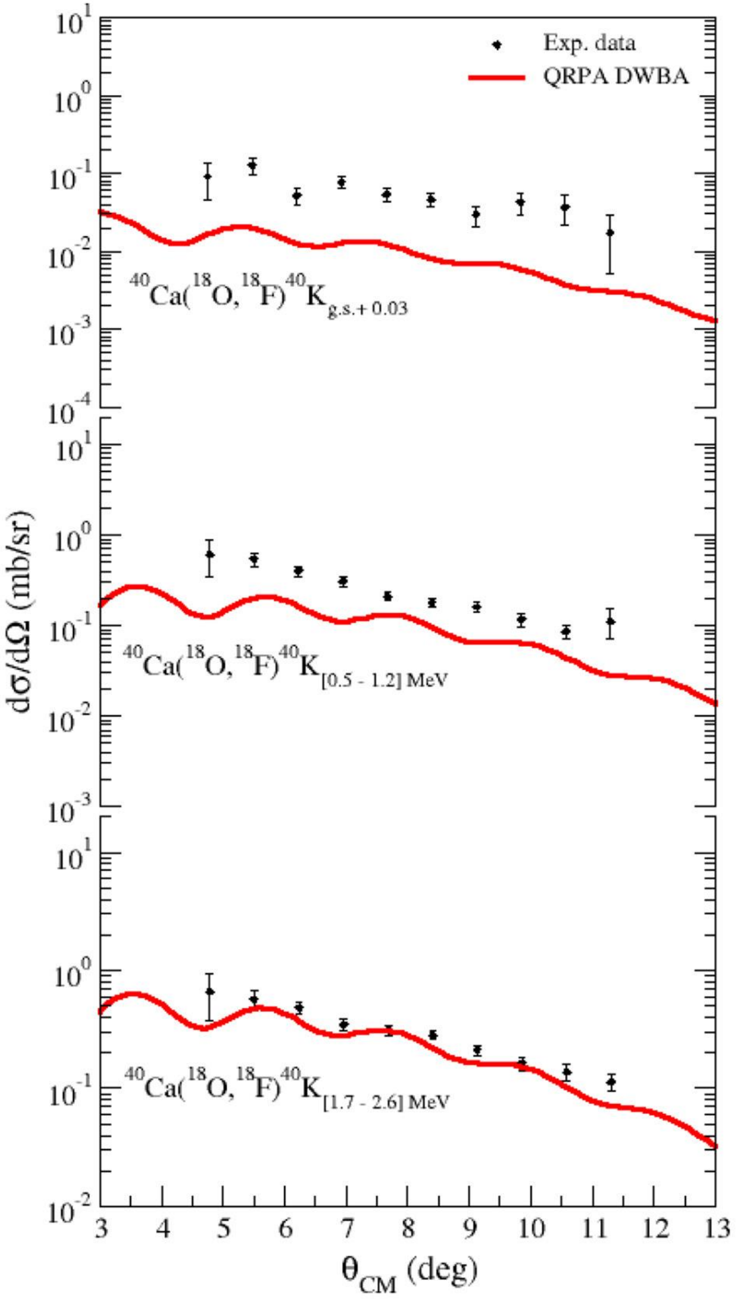
Morning Session 2

Speaker

Maria Colonna (Istituto Nazionale di Fisica Nucleare)

Cavallaro, Manuela, et al.
"A constrained analysis of the 40Ca (18O, 18F) 40K direct charge exchange reaction mechanism at 275 MeV."
Frontiers in Astronomy and Space Sciences 8 (2021): 659815.





DWBA with QRPA:

$$\frac{d^2\sigma}{dE_x d\Omega} = \frac{E_\alpha E_\beta}{(2\pi\hbar^2c^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_A+1)(2J_a+1)} \tag{3}$$

$$\sum_{M_A,\, M_a, M_B, M_b}\left|\int d^3 r \chi^*_{\beta,\mathbf{k}_\beta}(r)\, F^{(SCE)}_{\alpha\beta}(\mathbf{r})\, \chi_{\alpha,\mathbf{k}_\alpha}(\mathbf{r})\right|^2$$

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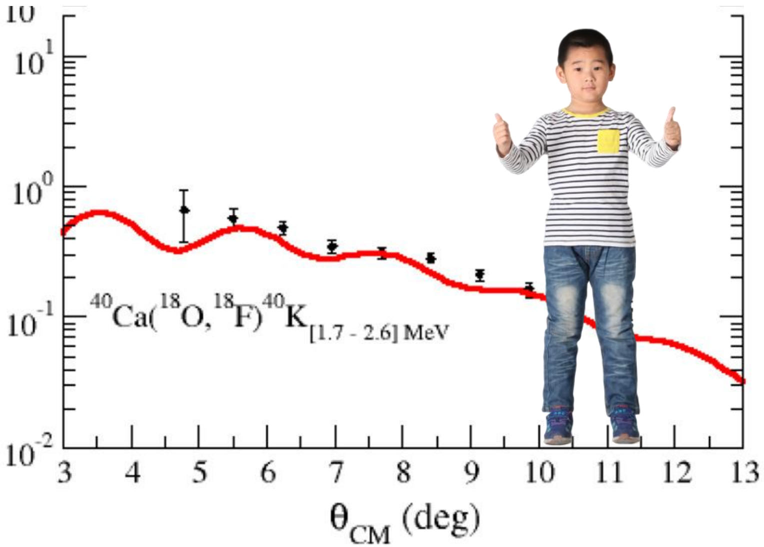
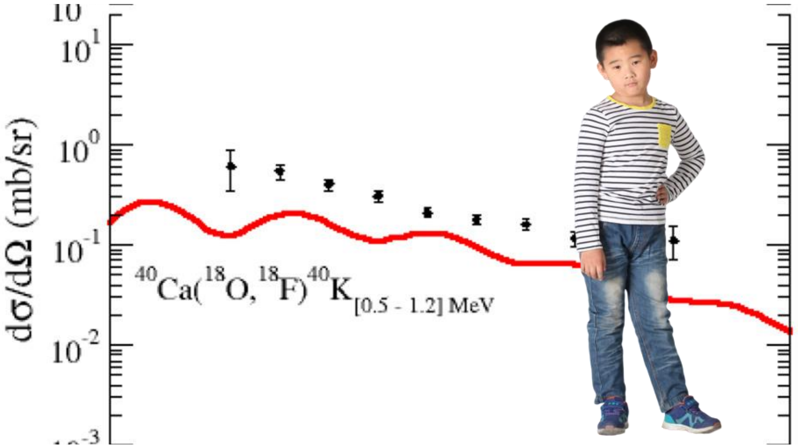
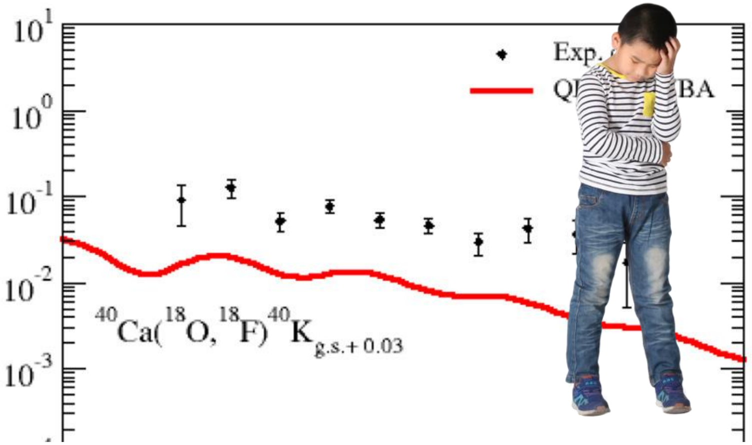






Possible effects for enhancing Xsection in *ground* and *low-lying excited states*:

1. The dynamics of polarization effects in nuclear structure
2. Two-step transfer mechanisms

Outline

Constraining optical potentials and spectroscopic information:

1. Elastic scattering
2. Inelastic scattering
3. One nucleon transfer reaction
4. Single charge exchange reaction
5. Two-step transfer reaction
6. Result

Conclusions and Outlook

1. Elastic scattering

$$U(R) = V^C(R) - V(R) - iW(R),$$

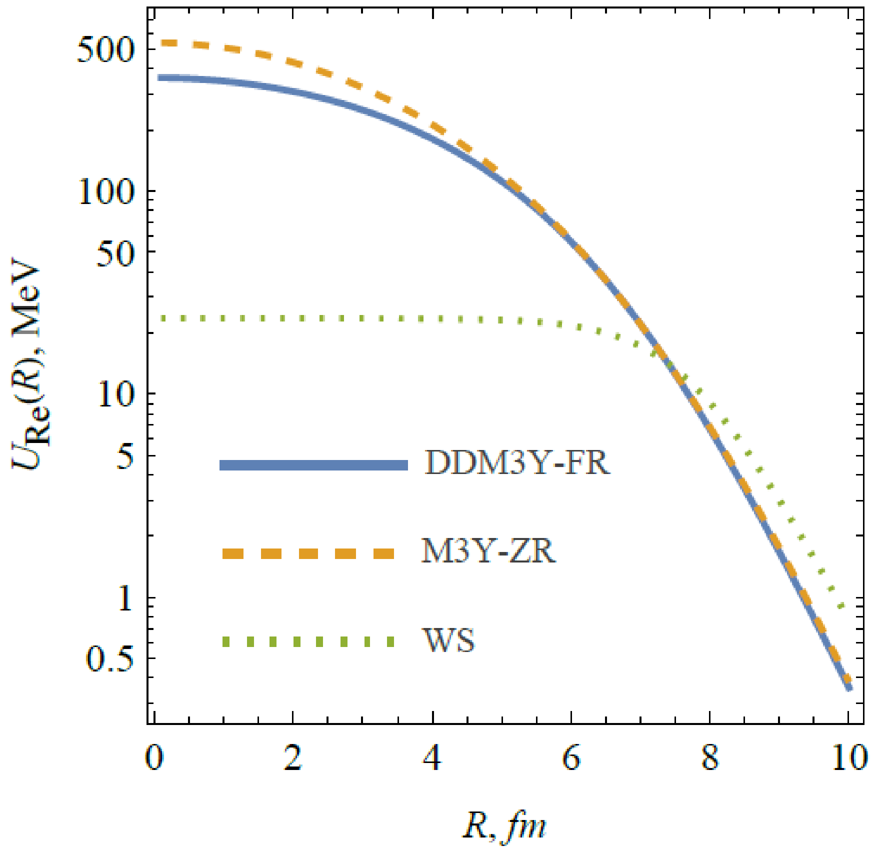
- Woods-Saxon (*WS*):

$$V(R) = V_0 \left(1 + \exp\left(\frac{R - R_v}{a_v}\right)\right)^{-1},$$

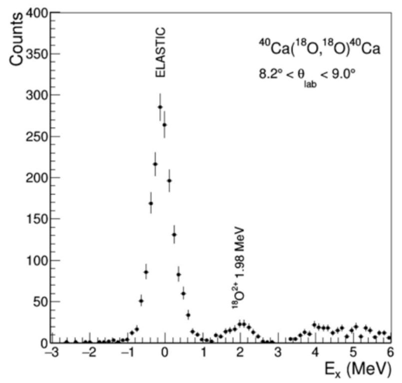
$$W(R) = W_0 \left(1 + \exp\left(\frac{R - R_w}{a_w}\right)\right)^{-1}.$$

- Double Folding potentials with Reid parametrizations:
  - Zero range Density independed M3Y (*M3Y-ZR*)
  - Finite range Density depended M3Y (*DDM3Y-FR*)

$$V(R) \equiv N_R V^{DF}(R) = \int dR\, \rho_1(r_1) V_{NN}(R_{NN}) \rho_2(r_2),$$



1. Elastic scattering



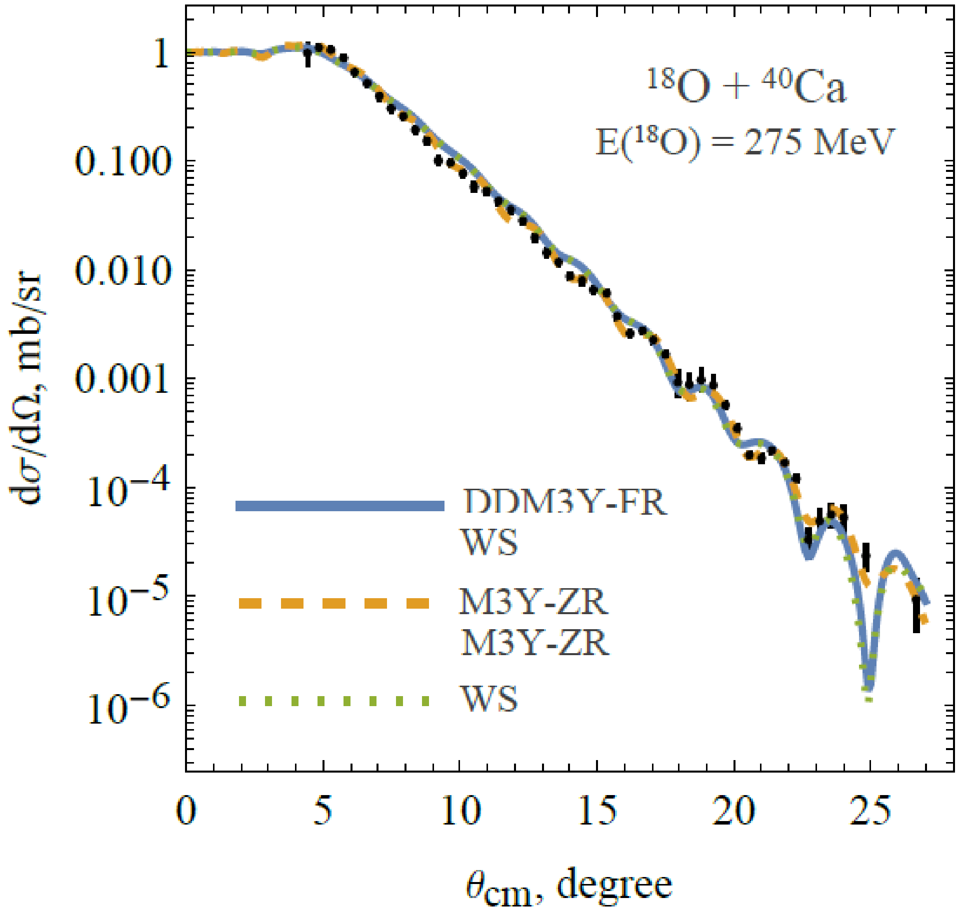
Chi-square test:

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{\sigma_i^{th} - \sigma_i^{exp}}{\sigma_i^{err}} \right)^2$$

Table 1. Optical potential parameters used in the OM calculations.

DF potential	V_0 MeV	R_v fm	a_v fm	J_{Re} MeV fm	W_0 MeV	R_w fm	a_w fm	J_{Im} MeV fm	R_C fm	χ^2/N
WS	23.60	1.27	0.687	67.9	5.75	1.44	0.69	1.82	7.85	2.59
M3Y-ZR	0.67*			266.9	67.3	6.05	1.05	112.3	7.85	7.17
DDM3Y-FR	0.55*			230.9	147.7	5.03	1.08	159.0	7.85	7.20

* stands for the normalization factor N_R



2. Inelastic scattering

Coupling potential:

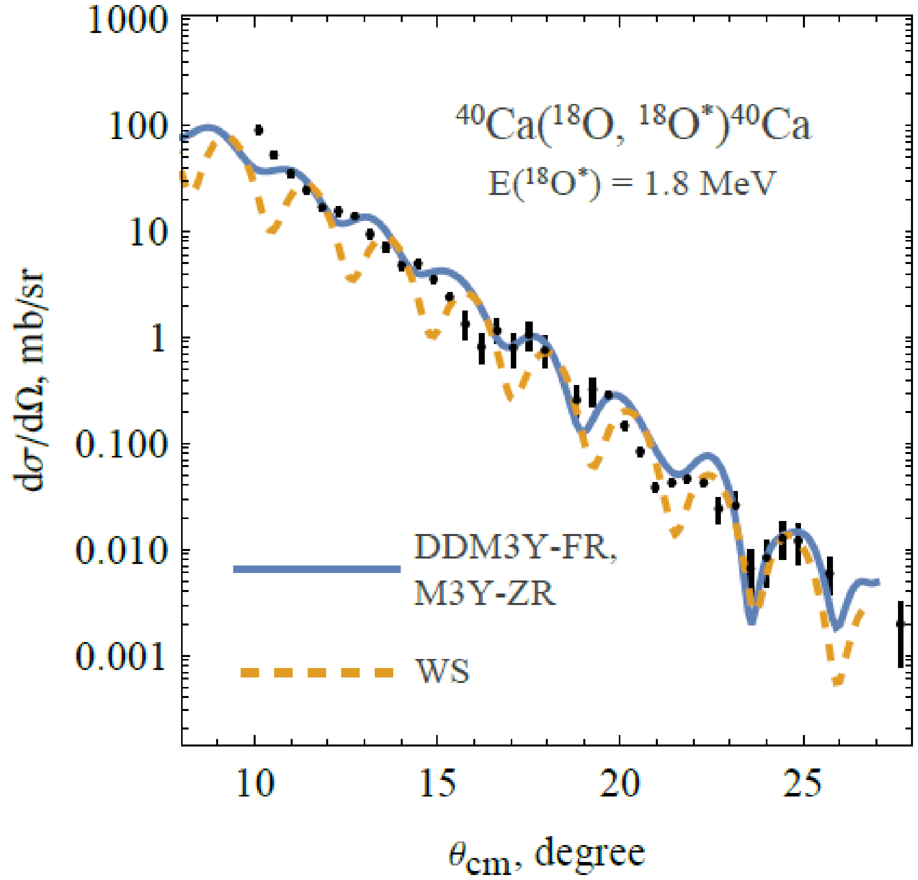
$$V_\lambda(R) = -\beta_\lambda R_V \left| \frac{dV}{dR} \right| - i\beta_\lambda R_W \left| \frac{dW}{dR} \right|$$

The deformation parameter:

$$\beta_\lambda = 0.289$$

DDM3Y-FR & M3Y-ZR

WS



3. One nucleon transfer reaction

$$[E_i - H_i]\psi_i(\boldsymbol{R}_i) = \sum_{j\neq i} \langle \phi_i | \mathscr{H} - E | \phi_j \rangle \psi_j(\boldsymbol{R}_j)$$

$$\mathscr{H} - E = H_i - E_i + V_i \quad \text{(the } \textit{post} \text{ form)}$$
$$\text{or} \quad = H_j - E_j + V_j \quad \text{(the } \textit{prior} \text{ form)}.$$

$$\langle \phi_i | \mathscr{H} - E | \phi_j \rangle = V_{ij}^{post} + [H_i - E_i] K_{ij} \quad (\textit{post})$$
$$\text{or} \quad = V_{ij}^{prior} + K_{ij}[H_j - E_j] \quad (\textit{prior})$$

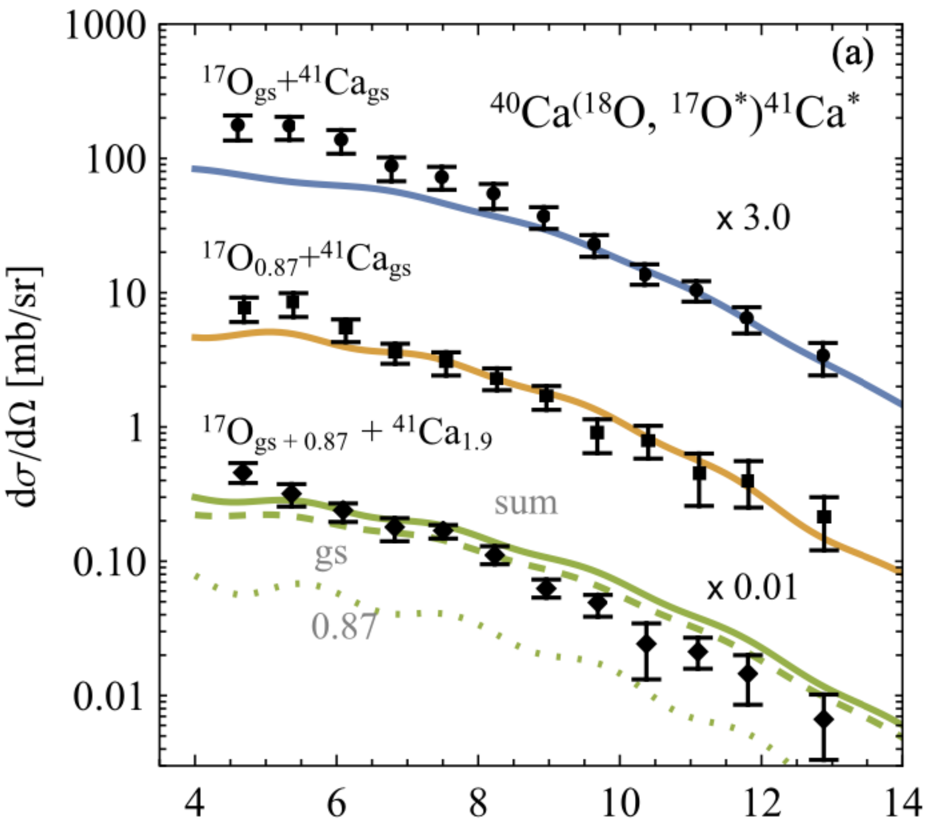
generated by fitting the depth of the WS potential

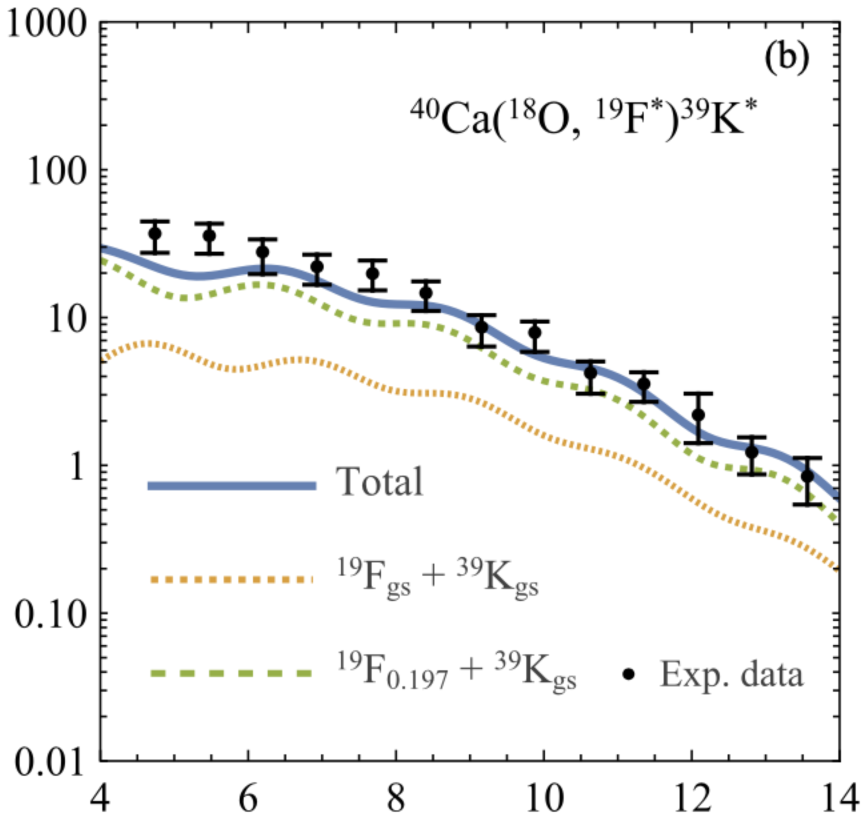
calculated through ***KSHELL*** code using *YSOX* for p-sd, *sdpf-mu* for sd-pf interactions

$$\phi_i(\xi_i) \equiv \phi_{JM}(\xi_c, \mathbf{r}),$$
$$\phi_{JM}(\xi_c, \mathbf{r}) = \sum_{ljI} A_{lsj}^{jIJ} \left[\phi_I(\xi_c) \otimes \phi_{lsj}(\mathbf{r})\right]_{JM}$$

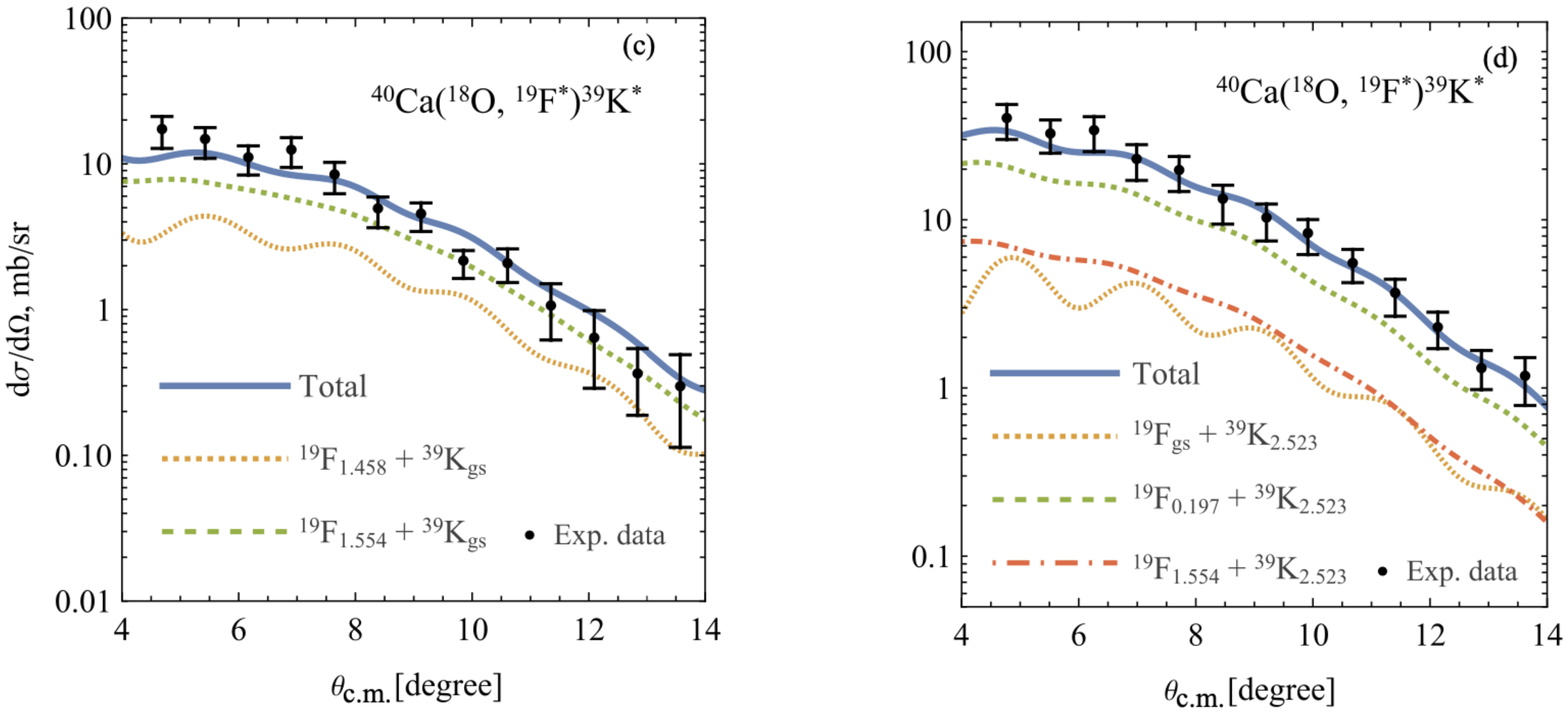
Thompson, Ian J. "Coupled reaction channels calculations in nuclear physics." Computer Physics Reports 7.4 (1988): 167-212.

3. One-nucleon transfer reaction





3. One-nucleon transfer reaction



4. Single charge exchange reaction: formalism

$$F(\mathbf{R})=\langle I_B M_B T_B M_{TB};s_b m_b t_b m_{tb} \mid \sum_{t,p} V_{t,p} \mid s_a m_a t_a m_{ta};I_A M_A T_A M_{TA}\rangle ,$$

$$V_{t,p}=V(\mathbf{r}_{t,p})=\sum_{\substack{s_0,t_0\\K=0,2}} V^K_{s_0 t_0}(r_{t,p})C^K_{s_0}\sum_{\substack{\nu,\nu'\\M,n_0}}(-)^{M+n_0}\langle s_0 s_0 \nu\nu' \mid KM\rangle \sigma^{s_0}_{\nu}(p)\sigma^{s_0}_{\nu'}(t)\tau^{t_0}_{n_0}(p)\tau^{t_0}_{-n_0}(t)Y_{K,-M}(\hat{r}_{t,p})$$

$$F(\mathbf{R})=\sqrt{4\pi}\sum_{\substack{JSL_{\mathrm{tr}}\\M_{\mathrm{tr}}}} i^{-L_{\mathrm{tr}}}h(ABab,JSL_{\mathrm{tr}})F^{JSL_{\mathrm{tr}}}_{M_{\mathrm{tr}}}(\mathbf{R})$$

$$h(ABab,JSL_{\mathrm{tr}})=(-)^{I_A-M_A+s_b-m_b+S+M_S}\langle I_A M_A I_B - M_B \mid JM_J\rangle \langle s_a m_a s_b - m_b \mid SM_s\rangle \langle JM_J SM_S \mid L_{\mathrm{tr}} - M_{\mathrm{tr}}\rangle$$

$$F^{JSL_{\mathrm{tr}}}_{M_{\mathrm{tr}}}(\mathbf{R})=\sum_{\substack{l_1,l_2,L_{12}\\Ks_0t_0n_0}} d^{JSL_{\mathrm{tr}}}_{l_1l_2L_{12};Ks_0t_0n_0} f^{JSL_{\mathrm{tr}},M_{\mathrm{tr}}}_{l_1l_2L_{12};Ks_0t_0}(\mathbf{R}) ,$$

$$d^{JSL_{\mathrm{tr}}}_{l_1l_2L_{12};Ks_0t_0n_0}=(4\pi)^{-1/2}i^{L_{\mathrm{tr}}-(l_1+l_2)}(-)^{s_a-s_b+S+n_0}C^K_{s_0} \times \hat{s}_b\hat{I}_B\hat{L}_{12}\hat{K}\begin{Bmatrix} S & J & L_{\mathrm{tr}}\\ l_2 & l_1 & L_{12}\\ s_0 & s_0 & K\end{Bmatrix}\langle T_B T_{Bz} \mid T_A T_{Az} t_0 - n_0\rangle \langle t_b t_{bz} \mid t_a t_{az} t_0 n_0\rangle ,$$

$$f^{JSL_{\mathrm{tr}},M_{\mathrm{tr}}}_{l_1l_2L_{12};Ks_0t_0}(\mathbf{R})=\int\int V^K_{s_0t_0}(r_{t,p})g^{l_1s_0Jt_0}_{AB}(r_1)g^{l_2s_0St_0}_{ab}(r_2)\{[Y^*_{l_2}(\hat{r}_2)Y^*_{l_1}(\hat{r}_1)]^{L_{12}}Y^*_K(\hat{r}_{t,p})\}^{L_{\mathrm{tr}}}d\mathbf{r}_1 d\mathbf{r}_2 ,$$

4. Single charge exchange reaction: formalism

$$f^{JSL_{\rm tr},M_{\rm tr}}_{l_1 l_2 L_{12};Ks_0 t_0}(\mathbf{R}) = \int\int V^K_{s_0 t_0}(r_{t,p})\, g^{l_1 s_0 J t_0}_{AB}(r_1)\, g^{l_2 s_0 S t_0}_{ab}(r_2) \{[Y^*_{l_2}(\hat{r}_2) Y^*_{l_1}(\hat{r}_1)]^{L_{12}} Y^*_K(\hat{r}_{t,p})\}^{L_{\rm tr}} d\mathbf{r}_1 d\mathbf{r}_2 ,$$

$$g^{l_1 s_0 J t_0}_{AB}(r_1) = \langle I_B T_B \| \sum_t \frac{\delta(r_1 - r_t)}{r_t^2} T^{l_1 s_0 J}(t) \tau^{t_0}(t) \| I_A T_A \rangle ,$$

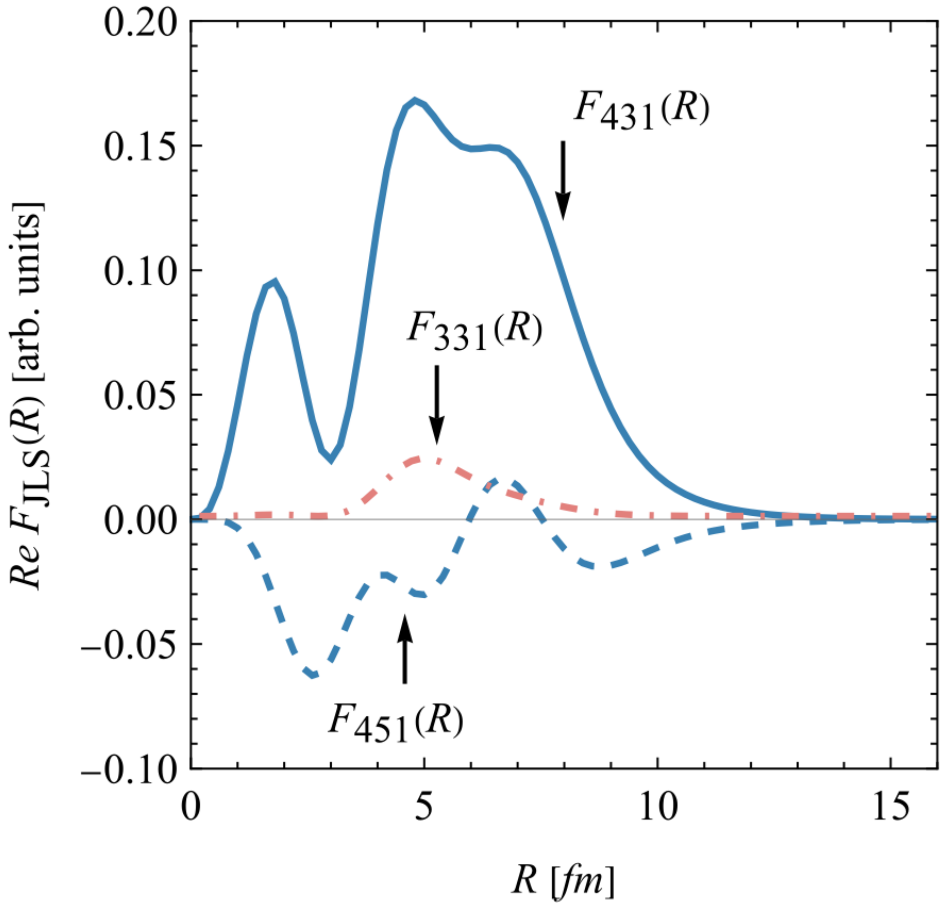
$$g^{l_2 s_0 S t_0}_{ab}(r_2) = \langle s_b t_b \| \sum_p \frac{\delta(r_2 - r_p)}{r_p^2} T^{l_2 s_0 S}(p) \tau^{t_0}(p) \| s_a t_a \rangle ,$$

$$T^{LSJ}_{M_J} = \sum_{MM_s} i^L Y_{LM} \sigma^S_{M_S} \langle LMSM_S \mid JM_J \rangle .$$

$$g^{l_1 s_0 J t_0}_{AB}(r_1) = \sum_{\rho_i \rho_j} C^{l_1 s_0 J}_{\rho_i \rho_j}(ABt_0) \Phi^B_{n_i l_i J_i}(r_1) \Phi^A_{n_j l_j J_j}(r_1) ,$$

$$\mathrm{OBTD}(AB\rho_i\rho_j; Jt_0) = \frac{\langle I_B T_B \| \{a^\dagger_{\rho_i} \tilde{a}_{\rho_j}\}^{Jt_0} \| I_A T_A \rangle}{\hat{J}\hat{t}_0} .$$

4. Single charge exchange reaction: formfactor

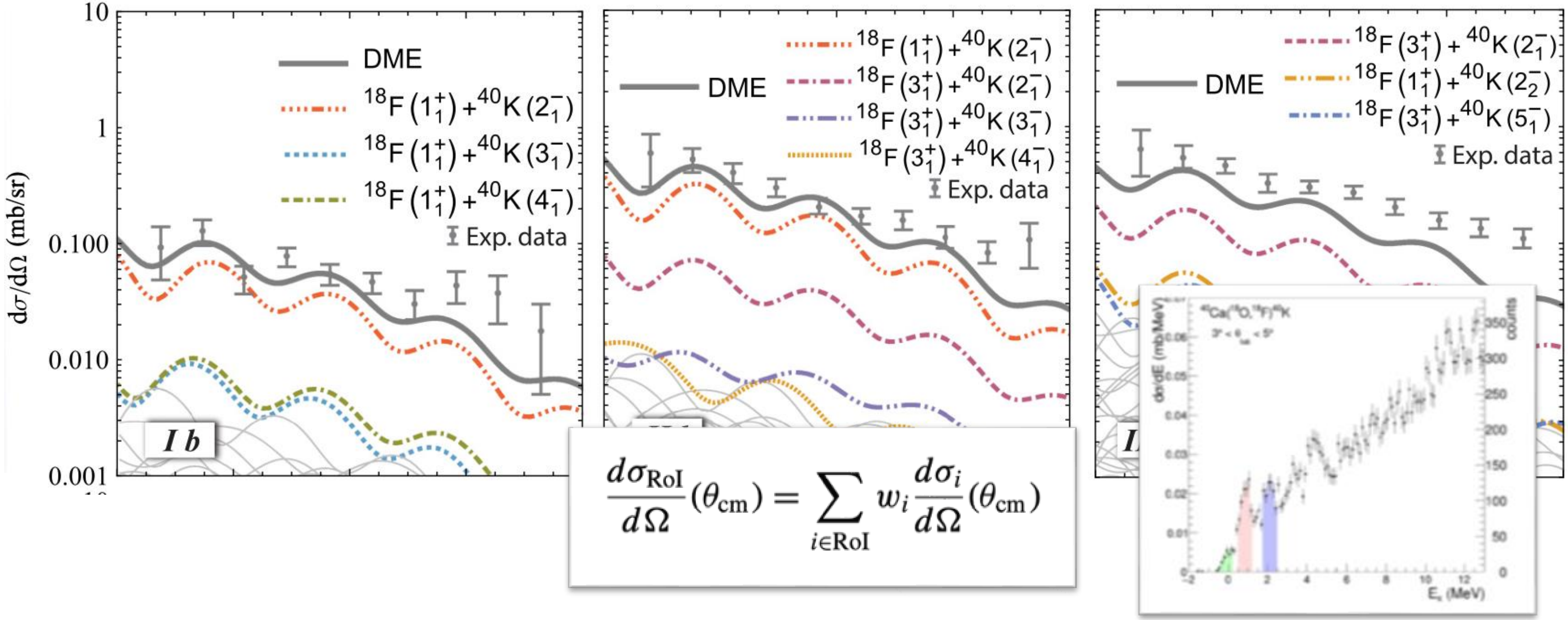


Typical form factor $F_{JLS}(R)$ calculated in this work with the transferred momenta JLS:

$F_{431}(R)$ (continuous blue line) and $F_{451}(R)$ (dashed blue line) are for the exit channels $^{18}F(1^+)$+ $^{40}K(4^-)$ and $^{18}F(1^+_1)$ + $^{40}K(3^-_1)$

$F_{331}(R)$ (dotted-dashed red line) is for the channel $^{18}F(1^+_1)$ + $^{40}K(3^-_1)$.

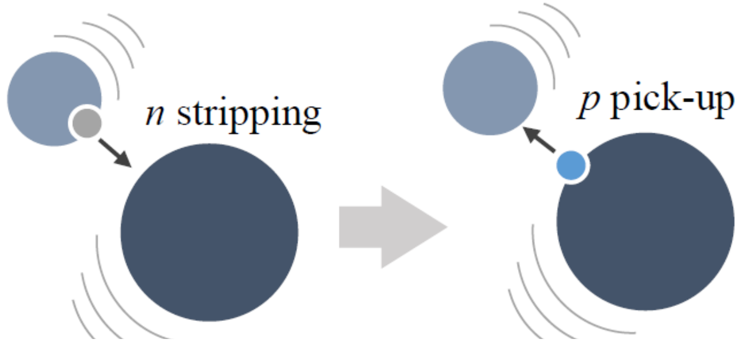
4. Single charge exchange reaction: cross sections



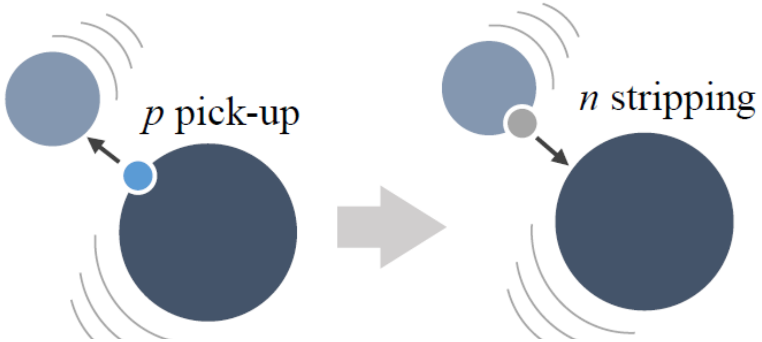
$$\frac{d\sigma_{\mathrm{RoI}}}{d\Omega}(\theta_{\mathrm{cm}}) = \sum_{i \in \mathrm{RoI}} w_i \frac{d\sigma_i}{d\Omega}(\theta_{\mathrm{cm}})$$

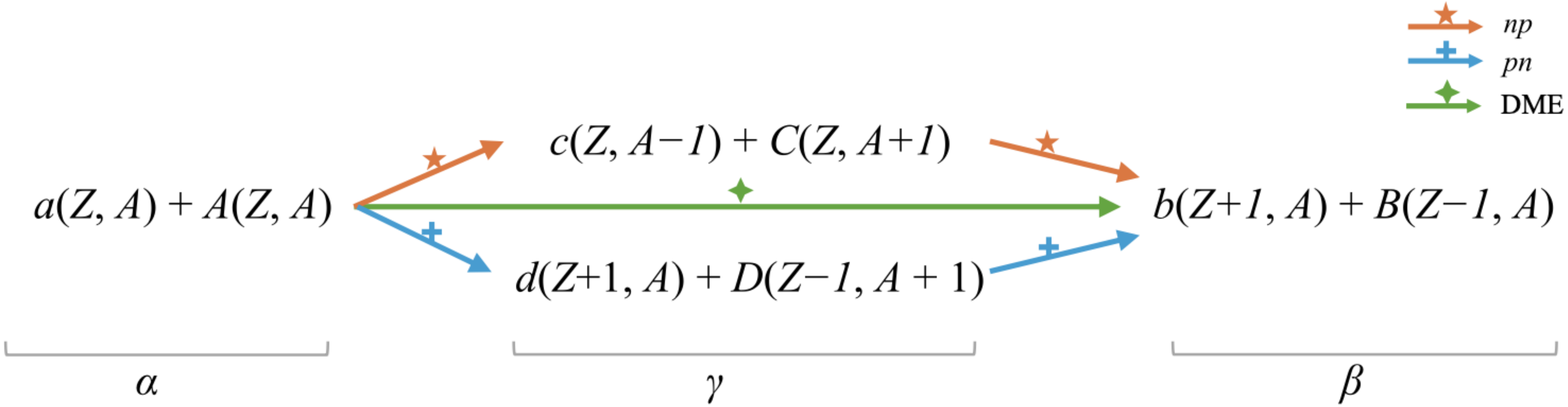
5. Two-step transfer: Coupling scheme

np – two-step transfer mechanism:



pn – two-step transfer mechanism:





5. Two-step transfer formalism

$$[(E-\varepsilon_\alpha)-K_\alpha-(\alpha|V_\alpha|\alpha)]u_\alpha(\mathbf{r}_\alpha)\approx 0$$

$$[(E-\varepsilon_\gamma)-K_\gamma-(\gamma|V_\gamma|\gamma)]u_\gamma(\mathbf{r}_\gamma)\approx \int d\mathbf{r}_\alpha K_{\gamma\alpha}(\mathbf{r}_\gamma,\mathbf{r}_\alpha)u_\alpha(\mathbf{r}_\alpha)$$

$$[(E-\varepsilon_\beta)-K_\beta-(\beta|V_\beta|\beta)]u_\beta(\mathbf{r}_\beta)\approx \int d\mathbf{r}_\alpha K_{\beta\alpha}(\mathbf{r}_\beta,\mathbf{r}_\alpha)u_\alpha(\mathbf{r}_\alpha)$$

$$+\int d\mathbf{r}_\gamma K_{\beta\gamma}(\mathbf{r}_\beta,\mathbf{r}_\gamma)u_\gamma(\mathbf{r}_\gamma).$$

$$T_{\beta\alpha}=T^{(1)}_{\beta\alpha}+T^{(2)}_{\beta\alpha}$$

$$T^{(1)}_{\beta\alpha}=\langle\tilde{u}^{(-)}_\beta|K_{\beta\alpha}|\tilde{u}^{(+)}_\alpha\rangle$$

$$T^{(2)}_{\beta\alpha}=\langle\tilde{u}^{(-)}_\beta|K_{\beta\gamma}\tilde{G}^{(+)}_\gamma K_{\gamma\alpha}|\tilde{u}^{(+)}_\alpha\rangle$$

$$T^{(2)}_{\beta\alpha}(p,p')=\hat{T}^{(2)}_{\beta\alpha}(p,p')+T^{(NO)}_{\beta\alpha}(p,p')$$

5. Two-step transfer: formalism

$$T^{(2)}_{\beta\alpha}(p,p') = \hat{T}^{(2)}_{\beta\alpha}(p,p') + T^{(NO)}_{\beta\alpha}(p,p')$$

$$\hat{T}^{(2)}_{\beta\alpha}(\text{prior-post}) = \langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|W_{\beta}|\psi_{\gamma})\tilde{G}^{(+)}_{\gamma}(\psi_{\gamma}|W_{\alpha}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle$$

$$\hat{T}^{(2)}_{\beta\alpha}(\text{post-post}) = \langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|W_{\beta}|\psi_{\gamma})\tilde{G}^{(+)}_{\gamma}(\psi_{\gamma}|W_{\gamma}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle$$

$$\hat{T}^{(2)}_{\beta\alpha}(\text{post-prior}) = \langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|W_{\gamma}|\psi_{\gamma})\tilde{G}^{(+)}_{\gamma}(\psi_{\gamma}|W_{\gamma}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle .$$

$$\hat{T}^{(2)}_{\beta\alpha}(\text{prior-prior}) = \langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|W_{\gamma}|\psi_{\gamma})\tilde{G}^{(+)}_{\gamma}(\psi_{\gamma}|W_{\alpha}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle$$

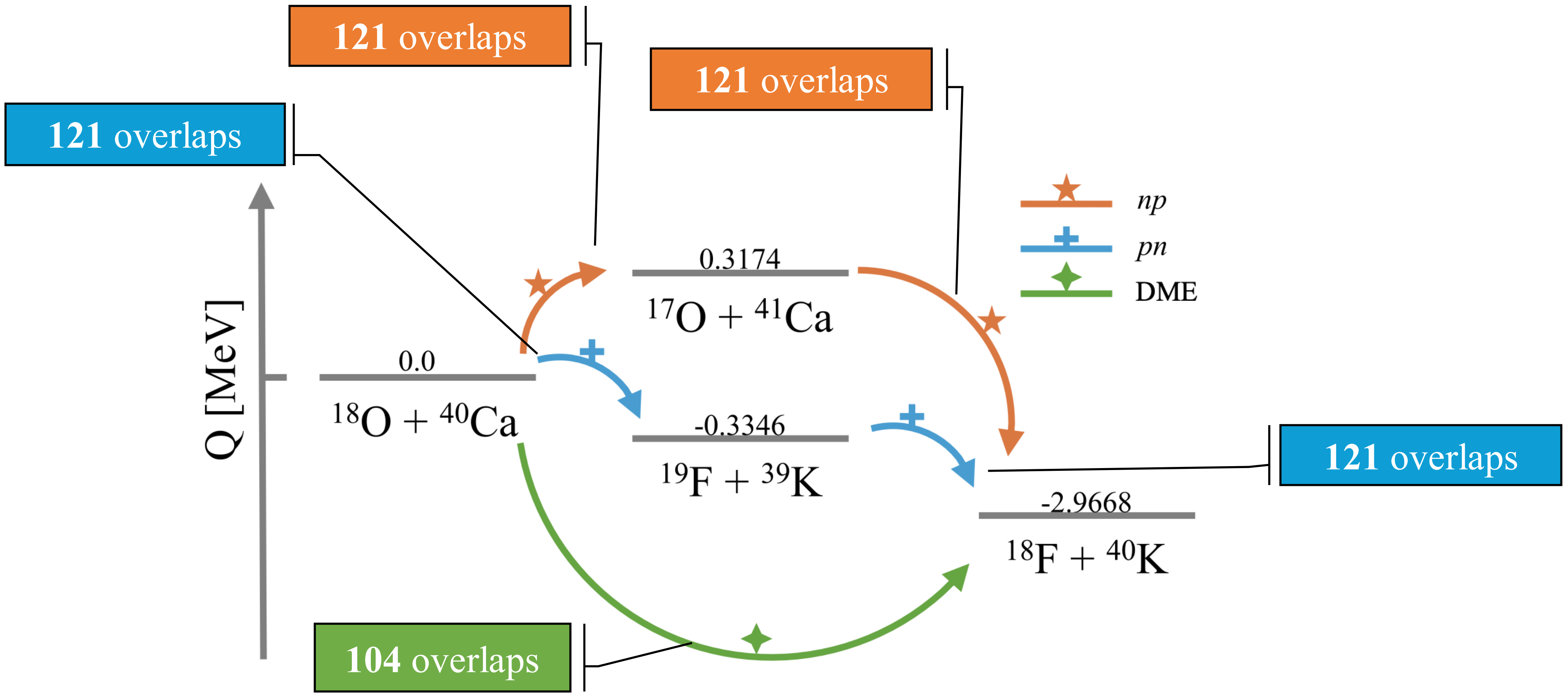
$$T^{(NO)}_{\beta\alpha}(\text{prior-post}) = 0$$

$$T^{(NO)}_{\beta\alpha}(\text{post-post}) = -\langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|W_{\beta}|\psi_{\gamma})(\psi_{\gamma}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle$$

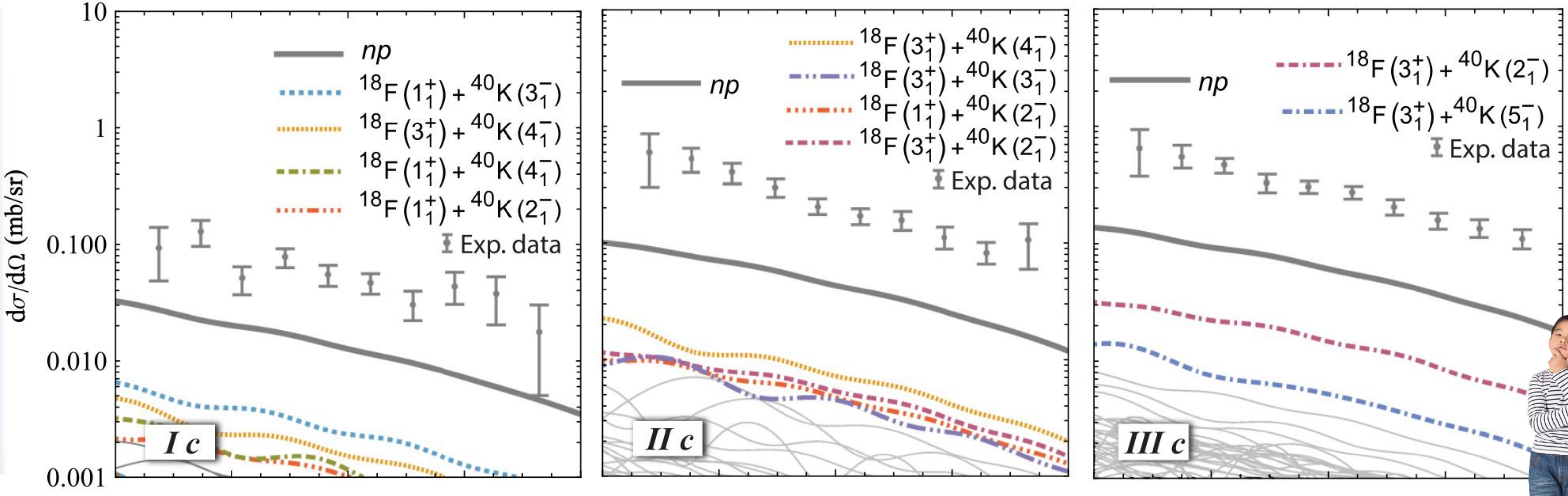
$$\begin{aligned} T^{(NO)}_{\beta\alpha}(\text{post-prior}) &= -\langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|W_{\beta}|\psi_{\gamma})(\psi_{\gamma}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle \\ &\quad - \langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|\psi_{\gamma})(\psi_{\gamma}|W_{\gamma}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle \\ &= -\langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|W_{\gamma}|\psi_{\gamma})(\psi_{\gamma}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle - \langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|\psi_{\gamma})(\psi_{\gamma}|W_{\alpha}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle \end{aligned}$$

$$T^{(NO)}_{\beta\alpha}(\text{prior-prior}) = -\langle \tilde{u}^{(-)}_{\beta}\psi_{\beta}|\psi_{\gamma})(\psi_{\gamma}|W_{\alpha}|\tilde{u}^{(+)}_{\alpha}\psi_{\alpha}\rangle .$$

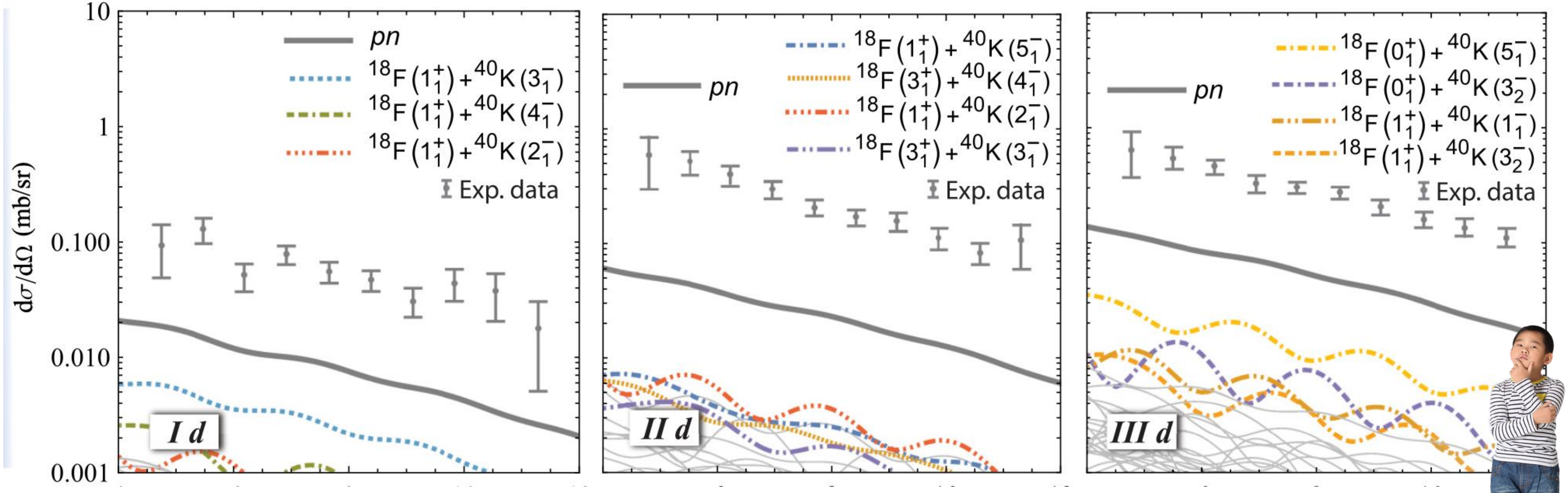
5. Two-step transfer: Coupling scheme



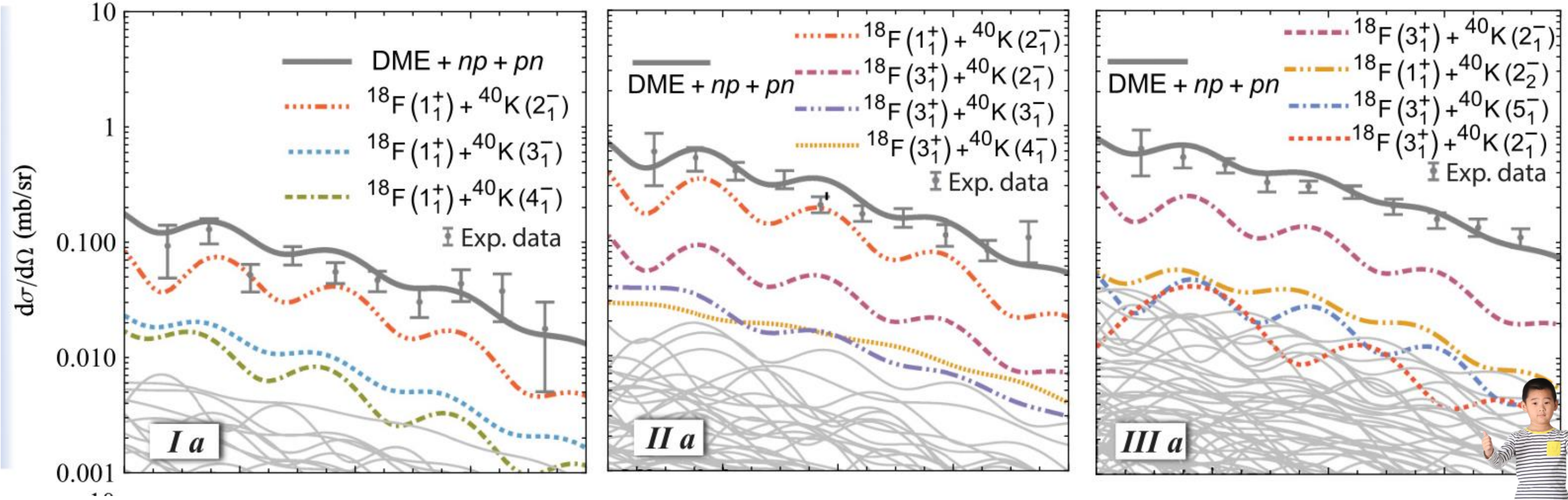
5. Two-step transfer: *np* mechanism



5. Two-step transfer: ***pn*** mechanism

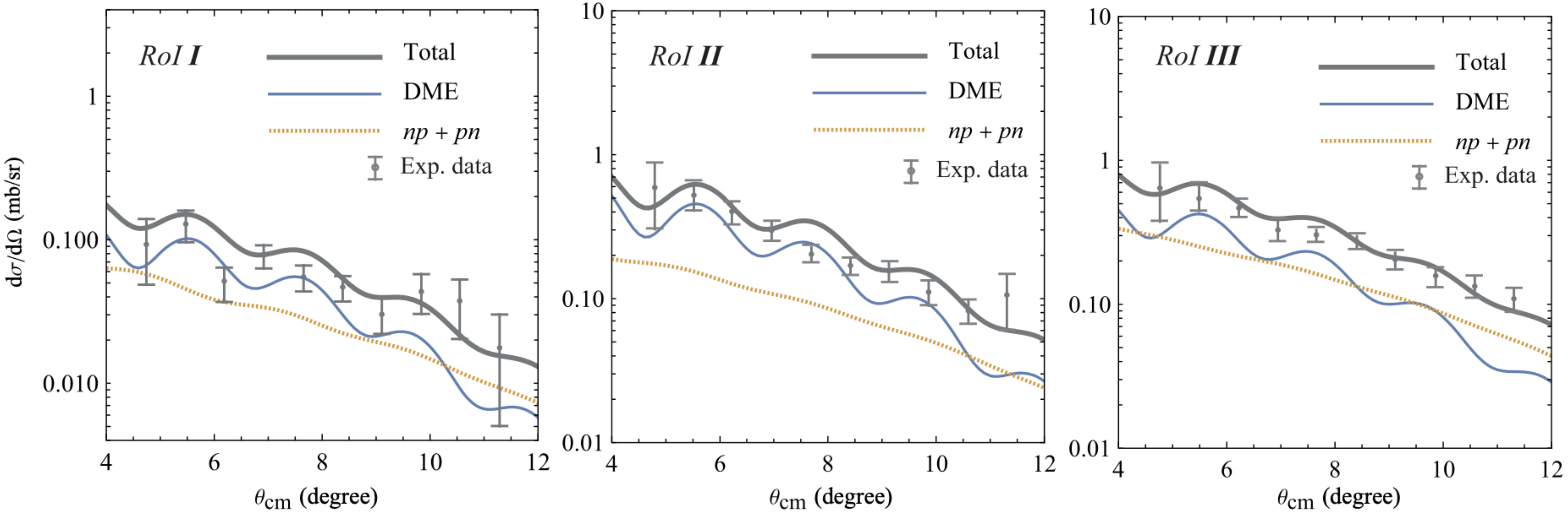


6. Result



10
1
0.100
0.010
0.001
dσ/dΩ (mb/sr)
DME + np + pn
18F (1+1) + 40K (2−1)
18F (1+1) + 40K (3−1)
18F (1+1) + 40K (4−1)
Exp. data
I a
DME + np + pn
18F (1+1) + 40K (2−1)
18F (3+1) + 40K (2−1)
18F (3+1) + 40K (3−1)
18F (3+1) + 40K (4−1)
Exp. data
II a
DME + np + pn
18F (3+1) + 40K (2−1)
18F (1+1) + 40K (2−2)
18F (3+1) + 40K (5−1)
18F (3+1) + 40K (2−1)
Exp. data
III a

6. Result



RoI I
RoI II
RoI III
Total
DME
np + pn
Exp. data
dσ/dΩ (mb/sr)
θcm (degree)
10
1
0.100
0.010
0.10
0.01
4
6
8
10
12

Conclusions and Outlook

- the DME mechanism exhibits the most significant dominance
- the ***pn*** mechanism was found to generally contribute more significantly to the reaction, especially at higher excited states, but the ***np*** mechanism also plays an essential role in lower excited states
- the shell model is effective in describing SCE reactions involving medium-mass nuclei, providing an alternative to the QRPA approach
- other nuclear models based on the mean-field approach may be required to describe SCE reactions involving heavy nuclei
- To explore other systems of interest studied within the NUMEN project

Thank you for your attention!