Freedom in the chiral 3-nucleon force at N3LO

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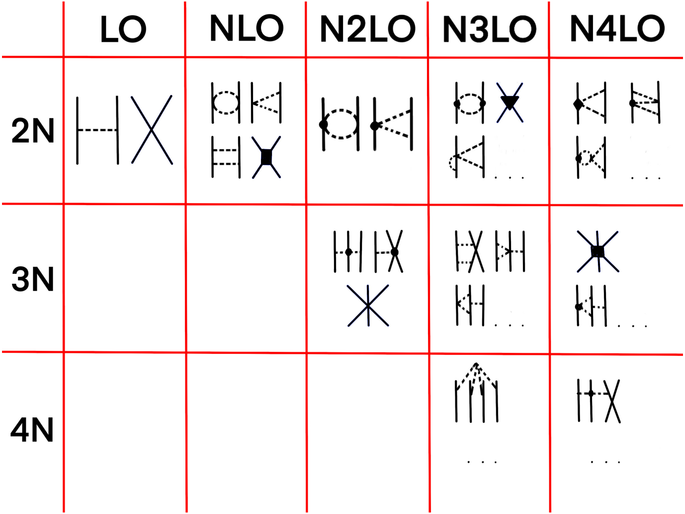
3N force is not parameter-free at N3LO



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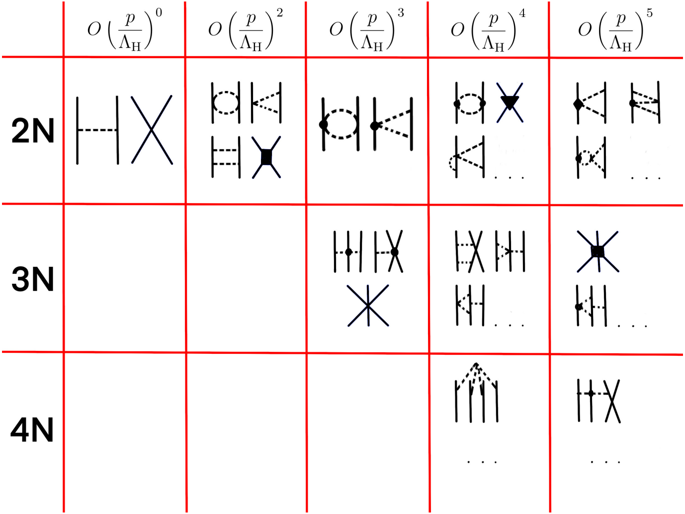

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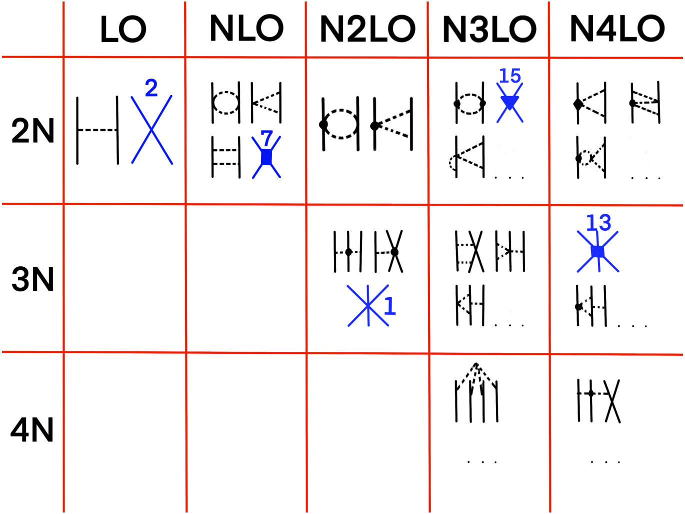
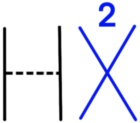


Freedom at LO



- OPE constrained by chiral symmetry (Goldberger-Treiman)
- unconstrained contact terms encode complicated short-distance dynamics
- minimal contact Lagrangian at LO

$$\mathcal{L} = -\frac{1}{2} C_S N^\dagger N N^\dagger N - \frac{1}{2} C_T N^\dagger \sigma_i N N^\dagger \sigma_i N$$

Freedom at NLO

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)$
O_1	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)^2 + \text{h.c.}$
O_2	$(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} N)$
O_3	$(N^\dagger N)(N^\dagger \overrightarrow{\boldsymbol{\nabla}}^2 N) + \text{h.c.}$
O_4	$i\,(N^\dagger \overrightarrow{\boldsymbol{\nabla}} N)\cdot(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \times \boldsymbol{\sigma} N) + \text{h.c.}$
O_5	$i\,(N^\dagger N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \times \overrightarrow{\boldsymbol{\nabla}} N)$
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O_8	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^k \overrightarrow{\nabla^j} N) + \text{h.c.}$
O_9	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \sigma^j \overrightarrow{\nabla^k} N) + \text{h.c.}$
O_{10}	$(N^\dagger \boldsymbol{\sigma} \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} N)$
O_{11}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^j} \sigma^k N)$
O_{12}	$(N^\dagger \sigma^j \overrightarrow{\nabla^k} N)(N^\dagger \overleftarrow{\nabla^k} \sigma^j N)$
O_{13}	$(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma}\, \overrightarrow{\nabla^j} N)(N^\dagger \sigma^j N) + \text{h.c.}$
O_{14}	$2\,(N^\dagger \overleftarrow{\boldsymbol{\nabla}} \sigma^j \cdot \overrightarrow{\boldsymbol{\nabla}} N)(N^\dagger \sigma^j N)$

[Ordoñez *et al.* PRC 53 (1996)]

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[Ordoñez *et al.* PRC 53 (1996)]

- only 12 independent operators since
  $O_7 + 2O_{10} = O_8 + 2O_{11}$ and $O_4 + O_5 = O_6$

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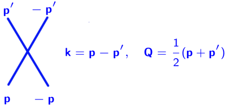

$$v(\mathbf{k},\mathbf{Q}) = C_1\,k^2 + C_2\,Q^2 + (C_3\,k^2 + C_4\,Q^2)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + i\,C_5\,\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{Q}\times\mathbf{k} + C_6\,\boldsymbol{\sigma}_1\cdot\mathbf{k}\,\boldsymbol{\sigma}_2\cdot\mathbf{k} + C_7\,\boldsymbol{\sigma}_1\cdot\mathbf{Q}\,\boldsymbol{\sigma}_2\cdot\mathbf{Q}$$

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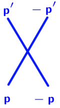

$\mathbf{k} = \mathbf{p} - \mathbf{p}', \quad \mathbf{Q} = \frac{1}{2}(\mathbf{p} + \mathbf{p}')$

$$v(\mathbf{k}, \mathbf{Q}) = C_1\, k^2 + C_2\, Q^2 + (C_3\, k^2 + C_4\, Q^2)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + i\, C_5\, \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot \mathbf{Q} \times \mathbf{k} + C_6\, \boldsymbol{\sigma}_1 \cdot \mathbf{k}\ \boldsymbol{\sigma}_2 \cdot \mathbf{k} + C_7\, \boldsymbol{\sigma}_1 \cdot \mathbf{Q}\ \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$$

- significance of the 5 remaining LECs?

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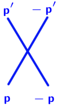

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- significance of the 5 remaining LECs? ($\vec{P}$-dependent *NN* interaction?)

The relativistic answer

$1\otimes 1$	$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$
	$\widetilde{O}_2$	$\frac{1}{4m^2}(\bar\psi i\overleftrightarrow{\partial}^\mu\psi)(\bar\psi i\overleftrightarrow{\partial}_\mu\psi)$
	$\widetilde{O}_3$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$
	$\widetilde{O}_4$	$\frac{1}{16m^4}(\bar\psi\overleftrightarrow{\partial}^\mu\overleftrightarrow{\partial}^\nu\psi)(\bar\psi\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_\nu\psi)$
$1\otimes\gamma$	$\widetilde{O}_5$	$\frac{1}{2m}(\bar\psi i\overleftrightarrow{\partial}^\mu\psi)(\bar\psi\gamma_\mu\psi)$
	$\widetilde{O}_6$	$\frac{1}{8m^3}(\bar\psi i\overleftrightarrow{\partial}^\mu i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_\nu\psi)$
$1\otimes\gamma\gamma_5$	$\widetilde{O}_7$	$\frac{1}{8m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi i\overleftrightarrow{\partial}^\mu\psi)\partial^\nu(\bar\psi i\overleftrightarrow{\partial}^\alpha\gamma^\beta\gamma_5\psi)$
$\gamma_5\otimes\gamma_5$	$\widetilde{O}_8$	$(\bar\psi i\gamma_5\psi)(\bar\psi i\gamma_5\psi)$
$\gamma_5\otimes\sigma$	$\widetilde{O}_9$	$\frac{1}{4m^2}\epsilon_{\mu\nu\alpha\beta}(\bar\psi i\gamma_5\psi)\partial^\mu(\bar\psi i\overleftrightarrow{\partial}^\nu\sigma^{\alpha\beta}\psi)$
$\gamma\otimes 1$	$\widetilde{O}_{10}$	$\frac{1}{2m}(\bar\psi\gamma^\mu\psi)(\bar\psi i\overleftrightarrow{\partial}_\mu\psi)$
	$\widetilde{O}_{11}$	$\frac{1}{8m^3}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi i\overleftrightarrow{\partial}_\mu i\overleftrightarrow{\partial}_\nu\psi)$
	$\widetilde{O}_{12}$	$\frac{1}{8m3}(\bar\psi\gamma^\mu\psi)\partial^2(\bar\psi i\overleftrightarrow{\partial}_\mu\psi)$
	$\widetilde{O}_{13}$	$\frac{1}{32m^5}(\bar\psi\gamma^\mu\overleftrightarrow{\partial}^\nu\overleftrightarrow{\partial}^\alpha\psi)(\bar\psi i\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_\nu\overleftrightarrow{\partial}_\alpha\psi)$
$\gamma\otimes\gamma$	$\widetilde{O}_{14}$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$
	$\widetilde{O}_{15}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_\nu\psi)$
	$\widetilde{O}_{16}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\psi)\partial^2(\bar\psi\gamma_\mu\psi)$
	$\widetilde{O}_{17}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu\overleftrightarrow{\partial}^\nu\overleftrightarrow{\partial}^\alpha\psi)(\bar\psi\gamma_\mu\overleftrightarrow{\partial}_\nu\overleftrightarrow{\partial}_\alpha\psi)$
	$\widetilde{O}_{18}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi\gamma_\nu i\overleftrightarrow{\partial}_\mu\psi)$
	$\widetilde{O}_{19}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\nu i\overleftrightarrow{\partial}^\alpha\psi)(\bar\psi\gamma_\nu i\overleftrightarrow{\partial}_\mu i\overleftrightarrow{\partial}_\alpha\psi)$
$\gamma\otimes\gamma\gamma_5$	$\widetilde{O}_{20}$	$\frac{1}{4m^2}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\psi)\partial^\nu(\bar\psi i\overleftrightarrow{\partial}^\alpha\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{21}$	$\frac{1}{16m^4}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\overleftrightarrow{\partial}^\gamma\psi)\partial_\nu(\bar\psi i\overleftrightarrow{\partial}^\alpha\overleftrightarrow{\partial}_\gamma\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{22}$	$\frac{1}{16m^4}\epsilon_{\nu\alpha\beta\gamma}(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\nu\psi)\partial^\alpha(\bar\psi\gamma^\beta\gamma_5 i\overleftrightarrow{\partial}_\mu i\overleftrightarrow{\partial}^\gamma\psi)$
$\gamma\gamma_5\otimes 1$	$\widetilde{O}_{23}$	$\frac{1}{8m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5 i\overleftrightarrow{\partial}^\nu\psi)\partial^\alpha(\bar\psi i\overleftrightarrow{\partial}^\beta\psi)$
$\gamma\gamma_5\otimes\gamma$	$\widetilde{O}_{24}$	$\frac{1}{4m^2}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5\psi)\partial^\nu(\bar\psi\gamma^\alpha i\overleftrightarrow{\partial}^\beta\psi)$
	$\widetilde{O}_{25}$	$\frac{1}{16m^4}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5\overleftrightarrow{\partial}^\gamma\psi)\partial^\nu(\bar\psi\gamma^\alpha i\overleftrightarrow{\partial}^\beta\overleftrightarrow{\partial}_\gamma\psi)$
$\gamma\gamma_5\otimes\gamma\gamma_5$	$\widetilde{O}_{26}$	$(\bar\psi\gamma^\mu\gamma_5\psi)(\bar\psi\gamma_\mu\gamma_5\psi)$
	$\widetilde{O}_{27}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\gamma_5 i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi\gamma_\mu\gamma_5 i\overleftrightarrow{\partial}_\nu\psi)$
	$\widetilde{O}_{28}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\gamma_5\psi)\partial^2(\bar\psi\gamma_\mu\gamma_5\psi)$
	$\widetilde{O}_{29}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu\gamma_5\overleftrightarrow{\partial}^\nu\overleftrightarrow{\partial}^\alpha\psi)(\bar\psi\gamma_\mu\gamma_5\overleftrightarrow{\partial}_\nu\overleftrightarrow{\partial}_\alpha\psi)$
	$\widetilde{O}_{30}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\gamma_5 i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi\gamma_\nu\gamma_5 i\overleftrightarrow{\partial}_\mu\psi)$
$\gamma\gamma_5\otimes\sigma$	$\widetilde{O}_{31}$	$\frac{1}{4m}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5\psi)(\bar\psi i\overleftrightarrow{\partial}^\nu\sigma^{\alpha\beta}\psi)$
	$\widetilde{O}_{32}$	$\frac{1}{4m}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5 i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi\sigma^{\alpha\beta}\psi)$
	$\widetilde{O}_{33}$	$\frac{1}{16m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5 i\overleftrightarrow{\partial}^\gamma\psi)(\bar\psi i\overleftrightarrow{\partial}^\nu i\overleftrightarrow{\partial}_\gamma\sigma^{\alpha\beta}\psi)$
	$\widetilde{O}_{34}$	$\frac{1}{16m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5 i\overleftrightarrow{\partial}^\nu i\overleftrightarrow{\partial}^\gamma\psi)(\bar\psi\sigma^{\alpha\beta} i\overleftrightarrow{\partial}_\gamma\psi)$
	$\widetilde{O}_{35}$	$\frac{1}{16m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5\psi)\partial^2(\bar\psi i\overleftrightarrow{\partial}^\nu\sigma^{\alpha\beta}\psi)$
	$\widetilde{O}_{36}$	$\frac{1}{64m^5}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5\overleftrightarrow{\partial}^\gamma\overleftrightarrow{\partial}^\delta\psi)(\bar\psi i\overleftrightarrow{\partial}^\nu\overleftrightarrow{\partial}_\gamma\overleftrightarrow{\partial}_\delta\sigma^{\alpha\beta}\psi)$
	$\widetilde{O}_{37}$	$\frac{1}{16m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\mu\gamma_5 i\overleftrightarrow{\partial}^\nu i\overleftrightarrow{\partial}_\gamma\psi)(\bar\psi i\overleftrightarrow{\partial}^\alpha\sigma^{\beta\gamma}\psi)$
	$\widetilde{O}_{38}$	$\frac{1}{16m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\gamma^\gamma\gamma_5 i\overleftrightarrow{\partial}^\mu\psi)(\bar\psi i\overleftrightarrow{\partial}_\gamma i\overleftrightarrow{\partial}^\nu\sigma^{\alpha\beta}\psi)$
$\sigma\otimes\gamma_5$	$\widetilde{O}_{39}$	$\frac{1}{4m^2}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu}\psi)\partial^\alpha(\bar\psi i\overleftrightarrow{\partial}^\beta i\gamma_5\psi)$
$\sigma\otimes\gamma\gamma_5$	$\widetilde{O}_{40}$	$\frac{1}{4m}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi i\overleftrightarrow{\partial}^\alpha\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{41}$	$\frac{1}{4m}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi)(\bar\psi\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{42}$	$\frac{1}{16m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu}\overleftrightarrow{\partial}^\gamma\psi)(\bar\psi i\overleftrightarrow{\partial}^\alpha\overleftrightarrow{\partial}_\gamma\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{43}$	$\frac{1}{8m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2(\bar\psi i\overleftrightarrow{\partial}^\alpha\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{44}$	$\frac{1}{16m^3}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\overleftrightarrow{\partial}^\gamma\psi)(\bar\psi\overleftrightarrow{\partial}_\gamma\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{45}$	$\frac{1}{64m^5}\epsilon_{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu}\overleftrightarrow{\partial}^\gamma\overleftrightarrow{\partial}^\delta\psi)(\bar\psi i\overleftrightarrow{\partial}^\alpha\overleftrightarrow{\partial}_\gamma\overleftrightarrow{\partial}_\delta\gamma^\beta\gamma_5\psi)$
	$\widetilde{O}_{46}$	$\frac{1}{8m^3}\epsilon^{\mu\nu\alpha\beta}(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha i\overleftrightarrow{\partial}^\gamma\psi)(\bar\psi i\overleftrightarrow{\partial}^\beta\gamma_\gamma\gamma_5\psi)$
$\sigma\otimes\sigma$	$\widetilde{O}_{47}$	$\frac{1}{2}(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$
	$\widetilde{O}_{48}$	$\frac{1}{8m^2}(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi)(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_\alpha\psi)$
	$\widetilde{O}_{49}$	$\frac{1}{32m^4}(\bar\psi\sigma^{\mu\nu}\overleftrightarrow{\partial}^\alpha\overleftrightarrow{\partial}^\beta\psi)(\bar\psi\sigma_{\mu\nu}\overleftrightarrow{\partial}_\alpha\overleftrightarrow{\partial}_\beta\psi)$
	$\widetilde{O}_{50}$	$\frac{1}{8m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2(\bar\psi\sigma_{\mu\nu}\psi)$
	$\widetilde{O}_{51}$	$\frac{1}{8m^2}(\bar\psi\sigma^{\mu\alpha} i\overleftrightarrow{\partial}^\nu\psi)(\bar\psi i\overleftrightarrow{\partial}_\alpha\sigma_{\mu\nu}\psi)$
	$\widetilde{O}_{52}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\alpha} i\overleftrightarrow{\partial}^\beta\overleftrightarrow{\partial}^\gamma\psi)(\bar\psi i\overleftrightarrow{\partial}_\alpha\overleftrightarrow{\partial}_\gamma\sigma_{\mu\beta}\psi)$
	$\widetilde{O}_{53}$	$\frac{1}{32m^4}\epsilon_{\mu\nu\gamma\delta}\epsilon_{\alpha\beta\rho\sigma}(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\gamma i\overleftrightarrow{\partial}^\rho\psi)(\bar\psi\sigma^{\alpha\beta} i\overleftrightarrow{\partial}^\delta i\overleftrightarrow{\partial}^\sigma\psi)$
	$\widetilde{O}_{54}$	$\frac{1}{32m^4}\epsilon_{\mu\nu\gamma\delta}\epsilon_{\alpha\beta\rho\sigma}(\bar\psi\sigma^{\mu\nu}\psi)\partial^\delta\partial^\sigma(\bar\psi\sigma^{\alpha\beta} i\overleftrightarrow{\partial}^\gamma i\overleftrightarrow{\partial}^\rho\psi)$

[LG + M. Viviani, FBS 49 (2011)]

- build the *NN* contact Lagrangian in terms of relativistic nucleon fields
- many more operators due to the large nucleon mass
- non-relativistic reduction

$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m}\begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} \end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix} \boldsymbol{\nabla}^2 \\ 0 \end{pmatrix}\right] N(x)$$

- only 9 combinations of non-relativistic operators up to $O(p^2)$

$$O_S + \frac{1}{4m^2}(O_1 + O_3 + O_5 + O_6)$$

$$O_T - \frac{1}{4m^2}(O_5 + O_6 - O_7 + O_8 + 2\,O_{12} + O_{14})$$

$+$ 7 “c.o.m operators”

The algebraic answer

$$\begin{aligned}
&[J^i, J^j] = i\epsilon^{ijk} J^k, \quad &&[K^i, K^j] = -i\epsilon^{ijk} J^k, \quad &&[J^i, K^j] = i\epsilon^{ijk} K^k, \quad &&[P^\mu, P^\nu] = 0, \\
&[K^i, P^j] = -i\delta^{ij} H, \quad &&[J^i, P^j] = i\epsilon^{ijk} P^k, \quad &&[K^i, H] = -iP^i, \quad &&[J^i, H] = 0.
\end{aligned}$$

The algebraic answer

$$[J^i, J^j] = i\epsilon^{ijk} J^k, \quad [K^i, K^j] = -i\epsilon^{ijk} J^k, \quad [J^i, K^j] = i\epsilon^{ijk} K^k, \quad [P^\mu, P^\nu] = 0,$$
$$[K^i, P^j] = -i\delta^{ij} H, \quad [J^i, P^j] = i\epsilon^{ijk} P^k, \quad [K^i, H] = -iP^i, \quad [J^i, H] = 0.$$

in the instant-form $H = H_0 + H_I$, $\vec{K} = \vec{K}_0 + \vec{K}_I$

The algebraic answer

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in the instant-form $H = H_0 + H_I$, $\vec{K} = \vec{K}_0 + \vec{K}_I$

$$\vec{K}_I = \int d^3\boldsymbol{x}\vec{x}\mathcal{H}_I(x) + \delta\vec{W}$$

The algebraic answer

$$\begin{array}{llll} [J^i, J^j] = i\epsilon^{ijk} J^k, & [K^i, K^j] = -i\epsilon^{ijk} J^k, & [J^i, K^j] = i\epsilon^{ijk} K^k, & [P^\mu, P^\nu] = 0, \\ [K^i, P^j] = -i\delta^{ij} H, & [J^i, P^j] = i\epsilon^{ijk} P^k, & [K^i, H] = -iP^i, & [J^i, H] = 0. \end{array}$$

in the instant-form $H = H_0 + H_I$, $\vec{K} = \vec{K}_0 + \vec{K}_I$

$$\vec{K}_I = \int d^3\boldsymbol{x} \vec{x} \mathcal{H}_I(x) + \delta \vec{W}$$

expand in soft momenta

$$H_0 = H_0^{(0)} + H_0^{(2)} + H_0^{(4)} + \ldots, \quad \boldsymbol{K}_0 = \boldsymbol{K}_0^{(-1)} + \boldsymbol{K}_0^{(1)} + \boldsymbol{K}_0^{(3)} + \ldots,$$

and solve order by order for $H_I = H_{NN} + H_{3N} + \ldots$, and $\delta \vec{W}$

The algebraic answer

$$[J^i, J^j] = i\epsilon^{ijk} J^k, \quad [K^i, K^j] = -i\epsilon^{ijk} J^k, \quad [J^i, K^j] = i\epsilon^{ijk} K^k, \quad [P^\mu, P^\nu] = 0,$$
$$[K^i, P^j] = -i\delta^{ij} H, \quad [J^i, P^j] = i\epsilon^{ijk} P^k, \quad [K^i, H] = -iP^i, \quad [J^i, H] = 0.$$

in the instant-form $H = H_0 + H_I$, $\vec{K} = \vec{K}_0 + \vec{K}_I$

$$\vec{K}_I = \int d^3\boldsymbol{x} \vec{x} \mathcal{H}_I(x) + \delta\vec{W}$$

expand in soft momenta

$$H_0 = H_0^{(0)} + H_0^{(2)} + H_0^{(4)} + ..., \quad \boldsymbol{K}_0 = \boldsymbol{K}_0^{(-1)} + \boldsymbol{K}_0^{(1)} + \boldsymbol{K}_0^{(3)} + ...,$$

and solve order by order for $H_I = H_{NN} + H_{3N} + ...$, and $\delta\vec{W}$

$$H_{NN} = H_{NN}^{(3)} + H_{NN}^{(5)} + H_{NN}^{(7)} + ...$$
$$H_{3N} = H_{3N}^{(6)} + H_{3N}^{(8)} + ...$$
$$\delta\vec{W} = \delta\vec{W}^{(4)} + ...$$

Resulting *drift interaction* at NLO

genuine *NN* interaction

$$
\begin{aligned}
v(\mathbf{k},\mathbf{Q}) &= C_1\,k^2 + C_2\,Q^2 + (C_3\,k^2 + C_4\,Q^2)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + i\,C_5\,\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{Q}\times\mathbf{k} \\
&+ C_6\,\boldsymbol{\sigma}_1\cdot\mathbf{k}\;\boldsymbol{\sigma}_2\cdot\mathbf{k} + C_7\,\boldsymbol{\sigma}_1\cdot\mathbf{Q}\;\boldsymbol{\sigma}_2\cdot\mathbf{Q}\;,
\end{aligned}
$$

Resulting *drift interaction* at NLO

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\begin{aligned}
v(\mathbf{k},\mathbf{Q}) &= C_1\,k^2 + C_2\,Q^2 + (C_3\,k^2 + C_4\,Q^2)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + i\,C_5\,\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{Q}\times\mathbf{k} \\
&+ C_6\,\boldsymbol{\sigma}_1\cdot\mathbf{k}\;\boldsymbol{\sigma}_2\cdot\mathbf{k} + C_7\,\boldsymbol{\sigma}_1\cdot\mathbf{Q}\;\boldsymbol{\sigma}_2\cdot\mathbf{Q}\;,
\end{aligned}
$$

$\vec{P}$-dependent interaction

$$
\begin{aligned}
v_{\mathbf{P}}(\mathbf{k},\mathbf{Q}) &= i\,C_1^*\,\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\cdot\mathbf{P}\times\mathbf{k} + C_2^*\,(\boldsymbol{\sigma}_1\cdot\mathbf{P}\;\boldsymbol{\sigma}_2\cdot\mathbf{Q} - \boldsymbol{\sigma}_1\cdot\mathbf{Q}\;\boldsymbol{\sigma}_2\cdot\mathbf{P}) \\
&+ (C_3^* + C_4^*\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,P^2 + C_5^*\,\boldsymbol{\sigma}_1\cdot\mathbf{P}\;\boldsymbol{\sigma}_2\cdot\mathbf{P}\;,
\end{aligned}
$$

with *fixed* LECs as $1/m^2$ boost corrections to the LO interaction

$$
C_1^* = \frac{C_S - C_T}{4m^2}\;,\quad C_2^* = \frac{C_T}{2m^2}\;,\quad C_3^* = -\frac{C_S}{4m^2}\;,\quad C_4^* = -\frac{C_T}{4m^2}\;,\quad C_5^* = 0\;,
$$

[LG *et al.* PRC81 034005 (2010)]

Drift effects

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Variational Monte Carlo calculations of ^{3}H and ^{4}He with a relativistic Hamiltonian

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In relativistic Hamilton
action in the $\mathbf{P}_{ij} = 0$ res
total momentum $\mathbf{P}_{ij}$ and
terms: $\delta v_{\rm RE}$, $\delta v_{\rm LC}$, $\delta v_{\rm TP}$, a
relation, Lorentz contractio
butions of $\delta v_{\rm RE}$ and $\delta v_{\rm LC}$ h
for ^{3}H and ^{4}He. In this pa
$\delta v_{\rm TP}$ and $\delta v_{\rm QM}$. These are

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Nucleon-nucleon potential: Drift effects

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(Received 4 May 2006; published 17 October 2006; publisher error corrected 31 October 2006)

In the rest frame of a many-body system, used in the calculation of its static and scattering properties, the center of mass of a two-body subsystem is allowed to drift. We show, in a model-independent way, that drift corrections to the nucleon-nucleon potential are relatively large and arise from both one- and two-pion exchange processes. As far as chiral symmetry is concerned, corrections to these processes begin, respectively, at $\mathcal{O}(q^2)$ and $\mathcal{O}(q^4)$. The two-pion exchange interaction also yields a new spin structure, which promotes the presence of P waves in trinuclei and is associated with profile functions that do not coincide with either central or spin-orbit ones. In principle, the new spin terms should be smaller than the $\mathcal{O}(q^3)$ spin-orbit components. However, in the isospin-even channel, a large contribution defies this expectation and gives rise to the prediction of important drift effects.

Freedom at N3LO

The "c.o.m. interaction" (depending on 15 LECs)

Freedom at N3LO

The "c.o.m. interaction" (depending on 15 LECs)

$$v(\mathbf{k},\mathbf{Q}) = D_1\,k^4 + D_2\,Q^4 + D_3\,k^2Q^2 + D_4\,(\mathbf{k}\times\mathbf{Q})^2\,\left(D_5\,k^4 + D_6\,Q^4 + D_7\,k^2Q^2 + D_8\,(\mathbf{k}\times\mathbf{Q})^2\right)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$
$$+\left(D_9\,k^2 + D_{10}\,Q^2\right)(-i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{Q}))\left(D_{11}\,k^2 + D_{12}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})$$
$$+\left(D_{13}\,k^2 + D_{14}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{Q})\,(\boldsymbol{\sigma}_2\cdot\mathbf{Q})D_{15}\,(\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\;\boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}))\,.$$

Freedom at N3LO

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$$+\left(D_9\,k^2 + D_{10}\,Q^2\right)(-i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{Q}))\left(D_{11}\,k^2 + D_{12}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})$$
$$+\left(D_{13}\,k^2 + D_{14}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{Q})\,(\boldsymbol{\sigma}_2\cdot\mathbf{Q})D_{15}\,(\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\;\boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}))\,.$$

what about the $\vec{P}$-dependent interaction?

Freedom at N3LO

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$$v(\mathbf{k},\mathbf{Q}) = D_1\,k^4 + D_2\,Q^4 + D_3\,k^2 Q^2 + D_4\,(\mathbf{k}\times\mathbf{Q})^2 \left(D_5\,k^4 + D_6\,Q^4 + D_7\,k^2 Q^2 + D_8\,(\mathbf{k}\times\mathbf{Q})^2\right) \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$
$$+ \left(D_9\,k^2 + D_{10}\,Q^2\right)(-i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{Q})) \left(D_{11}\,k^2 + D_{12}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})$$
$$+ \left(D_{13}\,k^2 + D_{14}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{Q})(\boldsymbol{\sigma}_2\cdot\mathbf{Q}) D_{15}\,(\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\ \boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}))\,.$$

what about the $\vec{P}$-dependent interaction?

- identify a basis of non relativistic operators at $O(p^4)$

Freedom at N3LO

The "c.o.m. interaction" (depending on 15 LECs)

$$
\begin{aligned}
v(\mathbf{k},\mathbf{Q}) &= D_1\, k^4 + D_2\, Q^4 + D_3\, k^2 Q^2 + D_4\, (\mathbf{k}\times\mathbf{Q})^2 \left(D_5\, k^4 + D_6\, Q^4 + D_7\, k^2 Q^2 + D_8\, (\mathbf{k}\times\mathbf{Q})^2\right) \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
&+ \left(D_9\, k^2 + D_{10}\, Q^2\right) (-i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{Q})) \left(D_{11}\, k^2 + D_{12}\, Q^2\right) (\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\sigma}_2\cdot\mathbf{k}) \\
&+ \left(D_{13}\, k^2 + D_{14}\, Q^2\right) (\boldsymbol{\sigma}_1\cdot\mathbf{Q})\,(\boldsymbol{\sigma}_2\cdot\mathbf{Q}) D_{15}\, (\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\ \boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}))\,.
\end{aligned}
$$

what about the $\vec{P}$-dependent interaction?

- identify a basis of non relativistic operators at $O(p^4)$
- write down an overcomplete set of relativistic operators

Freedom at N3LO

The "c.o.m. interaction" (depending on 15 LECs)

$$v(\mathbf{k},\mathbf{Q}) = D_1\,k^4 + D_2\,Q^4 + D_3\,k^2Q^2 + D_4\,(\mathbf{k}\times\mathbf{Q})^2\,\left(D_5\,k^4 + D_6\,Q^4 + D_7\,k^2Q^2 + D_8\,(\mathbf{k}\times\mathbf{Q})^2\right)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$
$$+\left(D_9\,k^2 + D_{10}\,Q^2\right)(-i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{Q}))\left(D_{11}\,k^2 + D_{12}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})$$
$$+\left(D_{13}\,k^2 + D_{14}\,Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{Q})(\boldsymbol{\sigma}_2\cdot\mathbf{Q})D_{15}\,(\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\;\boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}))\,.$$

what about the $\vec{P}$-dependent interaction?

- identify a basis of non relativistic operators at $O(p^4)$
- write down an overcomplete set of relativistic operators
- express the latter in the chosen basis using the non-relativistic expansion

$$\psi(x) = [\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m}\begin{pmatrix} 0 \\ \sigma\cdot\nabla \end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix} \nabla^2 \\ 0 \end{pmatrix} - \frac{3i}{16m^3}\begin{pmatrix} 0 \\ \sigma\cdot\nabla\nabla^2 \end{pmatrix} + \frac{11}{128m^4}\begin{pmatrix} \nabla^4 \\ 0 \end{pmatrix}]N(x) + O\left(p^5\right)\,.$$

Freedom at N3LO

The "c.o.m. interaction" (depending on 15 LECs)

$$
\begin{aligned}
v(\mathbf{k},\mathbf{Q}) = {} & D_1\, k^4 + D_2\, Q^4 + D_3\, k^2 Q^2 + D_4\, (\mathbf{k}\times\mathbf{Q})^2 \left(D_5\, k^4 + D_6\, Q^4 + D_7\, k^2 Q^2 + D_8\, (\mathbf{k}\times\mathbf{Q})^2\right) \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
& + \left(D_9\, k^2 + D_{10}\, Q^2\right)(-i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{Q}))\left(D_{11}\, k^2 + D_{12}\, Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}) \\
& + \left(D_{13}\, k^2 + D_{14}\, Q^2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{Q})(\boldsymbol{\sigma}_2\cdot\mathbf{Q}) D_{15}\, (\boldsymbol{\sigma}_1\cdot(\mathbf{k}\times\mathbf{Q})\ \boldsymbol{\sigma}_2\cdot(\mathbf{k}\times\mathbf{Q}))\,.
\end{aligned}
$$

what about the $\vec{P}$-dependent interaction?

- identify a basis of non relativistic operators at $O(p^4)$
- write down an overcomplete set of relativistic operators
- express the latter in the chosen basis using the non-relativistic expansion

$$
\psi(x) = [\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m}\begin{pmatrix} 0 \\ \sigma\cdot\nabla \end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix} \nabla^2 \\ 0 \end{pmatrix} - \frac{3i}{16m^3}\begin{pmatrix} 0 \\ \sigma\cdot\nabla\nabla^2 \end{pmatrix} + \frac{11}{128m^4}\begin{pmatrix} \nabla^4 \\ 0 \end{pmatrix}]N(x) + O\left(p^5\right)\,.
$$

- extract the linearly independent combinations

Non relativistic basis

Operator	Operator
$O_S = 1$	$O_1^{*\prime\prime} = ik^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_T = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_2^{*\prime\prime} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2 - \mathbf{k} \cdot \boldsymbol{\sigma}_2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_1' = k^2$	$O_3^{*\prime\prime} = k^2(\mathbf{Q} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2 - \mathbf{Q} \cdot \boldsymbol{\sigma}_2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_2' = Q^2$	$O_4^{*\prime\prime} = k^2 P^2$
$O_3' = k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_5^{*\prime\prime} = k^2 P^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_4' = Q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_6^{*\prime\prime} = (\mathbf{k} \cdot \mathbf{P})^2$
$O_5' = i \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot \mathbf{Q} \times \mathbf{k}$	$O_7^{*\prime\prime} = (\mathbf{k} \cdot \mathbf{P})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_6' = \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_8^{*\prime\prime} = P^2 \, \mathbf{k} \cdot \boldsymbol{\sigma}_1 \, \mathbf{k} \cdot \boldsymbol{\sigma}_2$
$O_7' = \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_9^{*\prime\prime} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2 + \mathbf{k} \cdot \boldsymbol{\sigma}_2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_1^{*\prime} = i \frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2}{2} \cdot \mathbf{P} \times \mathbf{k}$	$O_{10}^{*\prime\prime} = k^2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_2^{*\prime} = \boldsymbol{\sigma}_1 \cdot \mathbf{P} \boldsymbol{\sigma}_2 \cdot \mathbf{Q} - \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \mathbf{P}$	$O_{11}^{*\prime\prime} = iQ^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_3^{*\prime} = P^2$	$O_{12}^{*\prime\prime} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{Q} \times \mathbf{k} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_4^{*\prime} = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 P^2$	$O_{13}^{*\prime\prime} = iP^2(\mathbf{k} \times \mathbf{Q} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_5^{*\prime} = \boldsymbol{\sigma}_1 \cdot \mathbf{P} \boldsymbol{\sigma}_2 \cdot \mathbf{P}$	$O_{14}^{*\prime\prime} = i\mathbf{P} \cdot \mathbf{k}(\mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_1'' = k^4$	$O_{15}^{*\prime\prime} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{P} \times \mathbf{k} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_2'' = Q^4$	$O_{16}^{*\prime\prime} = iP^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_3'' = k^2 Q^2$	$O_{17}^{*\prime\prime} = Q^2(\mathbf{Q} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2 - \mathbf{Q} \cdot \boldsymbol{\sigma}_2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_4'' = (\mathbf{k} \times \mathbf{Q})^2$	$O_{18}^{*\prime\prime} = P^2 Q^2$
$O_5'' = k^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{19}^{*\prime\prime} = (\mathbf{P} \cdot \mathbf{Q})^2$
$O_6'' = Q^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{20}^{*\prime\prime} = P^2 Q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_7'' = k^2 Q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{21}^{*\prime\prime} = (\mathbf{P} \cdot \mathbf{Q})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_8'' = (\mathbf{k} \times \mathbf{Q})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{22}^{*\prime\prime} = Q^2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_9'' = \frac{ik^2}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{23}^{*\prime\prime} = \mathbf{P} \cdot \mathbf{Q}(\mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2 + \mathbf{P} \cdot \boldsymbol{\sigma}_2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_{10}'' = \frac{iQ^2}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{24}^{*\prime\prime} = P^2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2$
$O_{11}'' = k^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_{25}^{*\prime\prime} = P^2(\mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2 - \mathbf{P} \cdot \boldsymbol{\sigma}_2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_{12}'' = Q^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_{26}^{*\prime\prime} = P^4$
$O_{13}'' = k^2 \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_{27}^{*\prime\prime} = P^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_{14}'' = Q^2 \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_{28}^{*\prime\prime} = P^2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_{15}'' = \boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \, \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})$	
$O_{16}'' = i\mathbf{k} \cdot \mathbf{Q} \, \mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$	
$O_{17}'' = \mathbf{k} \cdot \mathbf{Q} \, (\mathbf{k} \times \mathbf{P}) \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)$	

Redundant set of relativistic operators

$\Gamma_A \otimes \Gamma_B$	Operators	Gradient structures
$1 \otimes 1$	$\tilde{O}_{1-6} = \bar{\psi}\psi\bar{\psi}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
$1 \otimes \gamma$	$\tilde{O}_{7-9} = \frac{i}{2m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\psi\bar{\psi}\gamma_\mu\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$1 \otimes \gamma\gamma_5$	$\tilde{O}_{10-12} = \frac{-1}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\overleftrightarrow{\partial}_{\mu}\psi\partial_\nu\bar{\psi}\gamma_\alpha\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma_5 \otimes \gamma_5$	$\tilde{O}_{13-15} = -\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma_5 \otimes \sigma$	$\tilde{O}_{16} = \frac{1}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_5\overleftrightarrow{\partial}^{\gamma}\overleftrightarrow{\partial}_{\mu}\psi\partial_\nu\bar{\psi}\sigma_{\alpha\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	1
$\gamma \otimes \gamma$	$\tilde{O}_{17-19} = \bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{20-22} = \frac{-1}{4m^2}\bar{\psi}\gamma_\mu\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\gamma^\nu\overleftrightarrow{\partial}^{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma \otimes \gamma\gamma_5$	$\tilde{O}_{23} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\overleftrightarrow{\partial}^{\gamma}\overleftrightarrow{\partial}_{\nu}\psi\partial_\alpha\bar{\psi}\gamma_\gamma\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{24} = \frac{-i}{16m^4}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma^\gamma\overleftrightarrow{\partial}_{\mu}\psi\partial_\nu\bar{\psi}\gamma_\alpha\gamma_5\overleftrightarrow{\partial}_{\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	1
	$\tilde{O}_{25-27} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\overleftrightarrow{\partial}_{\nu}\psi\partial_\alpha\bar{\psi}\gamma_\beta\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{28-30} = \frac{i}{4m^2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\psi\partial_\nu\bar{\psi}\gamma_\alpha\gamma_5\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \gamma\gamma_5$	$\tilde{O}_{31-36} = \bar{\psi}\gamma^\mu\gamma_5\psi\bar{\psi}\gamma_\mu\gamma_5\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)^2, (\overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B)d^2, d^4\}$
	$\tilde{O}_{37-39} = \frac{-1}{4m^2}\bar{\psi}\gamma^\mu\gamma_5\overleftrightarrow{\partial}^{\nu}\psi\bar{\psi}\gamma_\nu\gamma_5\overleftrightarrow{\partial}_{\mu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\gamma\gamma_5 \otimes \sigma$	$\tilde{O}_{40-42} = \frac{-i}{8m^3}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\gamma_5\overleftrightarrow{\partial}_{\nu}\overleftrightarrow{\partial}^{\gamma}\psi\bar{\psi}\sigma_{\alpha\gamma}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{43-45} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\sigma_{\nu\alpha}\overleftrightarrow{\partial}_{\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{46-48} = \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}\gamma_\mu\gamma_5\overleftrightarrow{\partial}_{\nu}\psi\bar{\psi}\sigma_{\alpha\beta}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
$\sigma \otimes \sigma$	$\tilde{O}_{49-51} = \bar{\psi}\sigma_{\mu\nu}\psi\bar{\psi}\sigma^{\mu\nu}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$
	$\tilde{O}_{52-54} = \frac{-1}{4m^2}\bar{\psi}\sigma^{\mu\alpha}\overleftrightarrow{\partial}^{\beta}\psi\bar{\psi}\sigma_{\mu\beta}\overleftrightarrow{\partial}_{\alpha}\psi$	$\{1, \overleftrightarrow{d}_A \cdot \overleftrightarrow{d}_B, d^2\}$

to be expanded non-relativistically

[E. Filandri + LG, PLB 841 137957 (2023)]

26 independent operators

Operator	Operator
$O_S = 1$	$O_1^{*\prime\prime} = ik^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_T = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_2^{*\prime\prime} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2 - \mathbf{k} \cdot \boldsymbol{\sigma}_2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_1' = k^2$	$O_3^{*\prime\prime} = k^2(\mathbf{Q} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2 - \mathbf{Q} \cdot \boldsymbol{\sigma}_2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_2' = Q^2$	$O_4^{*\prime\prime} = k^2 P^2$
$O_3' = k^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_5^{*\prime\prime} = k^2 P^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_4' = Q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_6^{*\prime\prime} = (\mathbf{k} \cdot \mathbf{P})^2$
$O_5' = i \frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2} \cdot \mathbf{Q} \times \mathbf{k}$	$O_7^{*\prime\prime} = (\mathbf{k} \cdot \mathbf{P})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_6' = \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_8^{*\prime\prime} = P^2 \, \mathbf{k} \cdot \boldsymbol{\sigma}_1 \, \mathbf{k} \cdot \boldsymbol{\sigma}_2$
$O_7' = \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_9^{*\prime\prime} = \mathbf{k} \cdot \mathbf{P}(\mathbf{k} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2 + \mathbf{k} \cdot \boldsymbol{\sigma}_2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_1^{*\prime} = i \frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2}{2} \cdot \mathbf{P} \times \mathbf{k}$	$O_{10}^{*\prime\prime} = k^2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_2^{*\prime} = \boldsymbol{\sigma}_1 \cdot \mathbf{P} \boldsymbol{\sigma}_2 \cdot \mathbf{Q} - \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \mathbf{P}$	$O_{11}^{*\prime\prime} = iQ^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_3^{*\prime} = P^2$	$O_{12}^{*\prime\prime} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{Q} \times \mathbf{k} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_4^{*\prime} = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 P^2$	$O_{13}^{*\prime\prime} = iP^2(\mathbf{k} \times \mathbf{Q} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_5^{*\prime} = \boldsymbol{\sigma}_1 \cdot \mathbf{P} \boldsymbol{\sigma}_2 \cdot \mathbf{P}$	$O_{14}^{*\prime\prime} = i\mathbf{P} \cdot \mathbf{k}(\mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_1'' = k^4$	$O_{15}^{*\prime\prime} = i\mathbf{Q} \cdot \mathbf{P}(\mathbf{P} \times \mathbf{k} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2))$
$O_2'' = Q^4$	$O_{16}^{*\prime\prime} = iP^2(\mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2))$
$O_3'' = k^2 Q^2$	$O_{17}^{*\prime\prime} = Q^2(\mathbf{Q} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2 - \mathbf{Q} \cdot \boldsymbol{\sigma}_2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1)$
$O_4'' = (\mathbf{k} \times \mathbf{Q})^2$	$O_{18}^{*\prime\prime} = P^2 Q^2$
$O_5'' = k^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{19}^{*\prime\prime} = (\mathbf{P} \cdot \mathbf{Q})^2$
$O_6'' = Q^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{20}^{*\prime\prime} = P^2 Q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_7'' = k^2 Q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{21}^{*\prime\prime} = (\mathbf{P} \cdot \mathbf{Q})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_8'' = (\mathbf{k} \times \mathbf{Q})^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	$O_{22}^{*\prime\prime} = Q^2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_9'' = \frac{ik^2}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{23}^{*\prime\prime} = \mathbf{P} \cdot \mathbf{Q}(\mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2 + \mathbf{P} \cdot \boldsymbol{\sigma}_2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_{10}'' = \frac{iQ^2}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{Q} \times \mathbf{k})$	$O_{24}^{*\prime\prime} = P^2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2$
$O_{11}'' = k^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_{25}^{*\prime\prime} = P^2(\mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_2 - \mathbf{P} \cdot \boldsymbol{\sigma}_2 \, \mathbf{Q} \cdot \boldsymbol{\sigma}_1)$
$O_{12}'' = Q^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k}$	$O_{26}^{*\prime\prime} = P^4$
$O_{13}'' = k^2 \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_{27}^{*\prime\prime} = P^4 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
$O_{14}'' = Q^2 \boldsymbol{\sigma}_1 \cdot \mathbf{Q} \boldsymbol{\sigma}_2 \cdot \mathbf{Q}$	$O_{28}^{*\prime\prime} = P^2 \, \mathbf{P} \cdot \boldsymbol{\sigma}_1 \, \mathbf{P} \cdot \boldsymbol{\sigma}_2$
$O_{15}'' = \boldsymbol{\sigma}_1 \cdot (\mathbf{k} \times \mathbf{Q}) \, \boldsymbol{\sigma}_2 \cdot (\mathbf{k} \times \mathbf{Q})$	

$O_{16}'' = i\mathbf{k} \cdot \mathbf{Q} \, \mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$

$O_{17}'' = \mathbf{k} \cdot \mathbf{Q} \, (\mathbf{k} \times \mathbf{P}) \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)$

26 independent operators

Operator	Operator
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$O_T = \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_2^{*\prime\prime} = \mathbf{k}\cdot\mathbf{P}(\mathbf{k}\cdot\boldsymbol{\sigma}_1\,\mathbf{Q}\cdot\boldsymbol{\sigma}_2-\mathbf{k}\cdot\boldsymbol{\sigma}_2\,\mathbf{Q}\cdot\boldsymbol{\sigma}_1)$
$O_1' = k^2$	$O_3^{*\prime\prime} = k^2(\mathbf{Q}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2-\mathbf{Q}\cdot\boldsymbol{\sigma}_2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1)$
$O_2' = Q^2$	$O_4^{*\prime\prime} = k^2P^2$
$O_3' = k^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_5^{*\prime\prime} = k^2P^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_4' = Q^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$O_6^{*\prime\prime} = (\mathbf{k}\cdot\mathbf{P})^2$
$O_5' = i\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{Q}\times\mathbf{k}$	$O_7^{*\prime\prime} = (\mathbf{k}\cdot\mathbf{P})^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_6' = \boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}$	$O_8^{*\prime\prime} = P^2\,\mathbf{k}\cdot\boldsymbol{\sigma}_1\,\mathbf{k}\cdot\boldsymbol{\sigma}_2$
$O_7' = \boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}$	$O_9^{*\prime\prime} = \mathbf{k}\cdot\mathbf{P}(\mathbf{k}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2+\mathbf{k}\cdot\boldsymbol{\sigma}_2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1)$
$O_1^{*\prime} = i\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\cdot\mathbf{P}\times\mathbf{k}$	$O_{10}^{*\prime\prime} = k^2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2$
$O_2^{*\prime} = \boldsymbol{\sigma}_1\cdot\mathbf{P}\boldsymbol{\sigma}_2\cdot\mathbf{Q}-\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\mathbf{P}$	$O_{11}^{*\prime\prime} = iQ^2(\mathbf{k}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2))$
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$O_4^{*\prime} = \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2P^2$	$O_{13}^{*\prime\prime} = iP^2(\mathbf{k}\times\mathbf{Q}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2))$
$O_5^{*\prime} = \boldsymbol{\sigma}_1\cdot\mathbf{P}\boldsymbol{\sigma}_2\cdot\mathbf{P}$	$O_{14}^{*\prime\prime} = i\mathbf{P}\cdot\mathbf{k}(\mathbf{Q}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2))$
$O_1'' = k^4$	$O_{15}^{*\prime\prime} = i\mathbf{Q}\cdot\mathbf{P}(\mathbf{P}\times\mathbf{k}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2))$
$O_2'' = Q^4$	$O_{16}^{*\prime\prime} = iP^2(\mathbf{k}\times\mathbf{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2))$
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$O_4'' = (\mathbf{k}\times\mathbf{Q})^2$	$O_{18}^{*\prime\prime} = P^2Q^2$
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$O_9'' = \frac{ik^2}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{Q}\times\mathbf{k})$	$O_{23}^{*\prime\prime} = \mathbf{P}\cdot\mathbf{Q}(\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{Q}\cdot\boldsymbol{\sigma}_2+\mathbf{P}\cdot\boldsymbol{\sigma}_2\,\mathbf{Q}\cdot\boldsymbol{\sigma}_1)$
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$O_{13}'' = k^2\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}$	$O_{27}^{*\prime\prime} = P^4\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$O_{14}'' = Q^2\boldsymbol{\sigma}_1\cdot\mathbf{Q}\boldsymbol{\sigma}_2\cdot\mathbf{Q}$	$O_{28}^{*\prime\prime} = P^2\,\mathbf{P}\cdot\boldsymbol{\sigma}_1\,\mathbf{P}\cdot\boldsymbol{\sigma}_2$
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$O_{17}'' = \mathbf{k}\cdot\mathbf{Q}\,(\mathbf{k}\times\mathbf{P})\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)$	

$$v_P(\mathbf{k},\mathbf{Q}) = \text{drift terms} + iD_{16}\boldsymbol{k}\cdot\boldsymbol{Q}\,\boldsymbol{Q}\times\boldsymbol{P}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)+D_{17}\boldsymbol{k}\cdot\boldsymbol{Q}\,(\boldsymbol{k}\times\boldsymbol{P})\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)$$

$\rightarrow$ 2 "forgotten" LECs @N3LO contributing in larger systems than 2N

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It can be absorbed with a suitable unitary transformation of the 1-body kinetic energy H_0

$$U^\dagger H_0 U \longrightarrow H_0 + [H_0, \alpha_4 T_4 + \alpha_5 T_5] = H_0 - V_{NN}(\boldsymbol{P})$$

with

$$U = \mathrm{e}^{\alpha_4 T_4 + \alpha_5 T_5}, \quad \alpha_4 = -\frac{m}{8} D_{16}, \quad \alpha_5 = -\frac{m}{4} D_{17}$$

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and

$$T_4 = i\epsilon^{ijk} \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N \sim i\boldsymbol{P} \times \boldsymbol{Q} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$$

$$T_5 = \int d^3\boldsymbol{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right] \sim (\boldsymbol{P} \cdot \boldsymbol{\sigma}_1 \boldsymbol{k} \cdot \boldsymbol{\sigma}_2 - \boldsymbol{P} \cdot \boldsymbol{\sigma}_2 \boldsymbol{k} \cdot \boldsymbol{\sigma}_1)/2$$

[LG *et al.* PRC 102 064003 (2020)]

Further unitary transformations

$$U = \mathrm{e}^{\alpha_i T_i} \longrightarrow \quad U^\dagger H_0 U = \sum_i \alpha_i$$

[P. Reinert *et al.* Eur. Phys. J. A 54 (2018) 86]

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[P. Reinert *et al.* Eur. Phys. J. A 54 (2018) 86]

$$\begin{aligned}
T_1 &= \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i (N^\dagger N) \\
T_2 &= \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^i (N^\dagger \sigma^j N) \\
T_3 &= \int d^3\boldsymbol{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right]
\end{aligned}$$

Further unitary transformations

$$U = \mathrm{e}^{\alpha_i T_i} \longrightarrow \quad U^\dagger H_0 U = \sum_i \alpha_i$$

17

[P. Reinert *et al.* Eur. Phys. J. A 54 (2018) 86]

$$
\begin{aligned}
T_1 &= \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i (N^\dagger N) \\
T_2 &= \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^i (N^\dagger \sigma^j N) \\
T_3 &= \int d^3\mathbf{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right] \\
T_4 &= i\epsilon^{ijk} \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N \\
T_5 &= \int d^3\mathbf{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right]
\end{aligned}
$$

Further unitary transformations

$$U = \mathrm{e}^{\alpha_i T_i} \longrightarrow \quad U^\dagger H_0 U = \sum_i \alpha_i$$

[P. Reinert *et al.* Eur. Phys. J. A 54 (2018) 86]

$$\begin{aligned}
T_1 &= \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i (N^\dagger N) \\
T_2 &= \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^i (N^\dagger \sigma^j N) \\
T_3 &= \int d^3\boldsymbol{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right] \\
T_4 &= i\epsilon^{ijk} \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N \sim i\boldsymbol{P} \times \boldsymbol{Q} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \\
T_5 &= \int d^3\boldsymbol{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right] \sim (\boldsymbol{P} \cdot \boldsymbol{\sigma}_1 \boldsymbol{k} \cdot \boldsymbol{\sigma}_2 - \boldsymbol{P} \cdot \boldsymbol{\sigma}_2 \boldsymbol{k} \cdot \boldsymbol{\sigma}_1)/2
\end{aligned}$$

Further unitary transformations

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[P. Reinert *et al.* Eur. Phys. J. A 54 (2018) 86]

$$\begin{aligned}
T_1 &= \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i (N^\dagger N) \\
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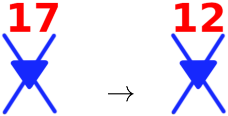
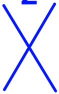
T_3 &= \int d^3\boldsymbol{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right] \\
T_4 &= i\epsilon^{ijk} \int d^3\boldsymbol{x} N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N \sim i\boldsymbol{P} \times \boldsymbol{Q} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \\
T_5 &= \int d^3\boldsymbol{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right] \sim (\boldsymbol{P} \cdot \boldsymbol{\sigma}_1 \boldsymbol{k} \cdot \boldsymbol{\sigma}_2 - \boldsymbol{P} \cdot \boldsymbol{\sigma}_2 \boldsymbol{k} \cdot \boldsymbol{\sigma}_1)/2
\end{aligned}$$



Induced 3N forces

The same unitary transformation induces 3N interactions

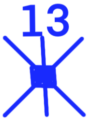
$U^\dagger \Big[C_{S/T}$ 2  $\Big] U = \sum_i mD_i C_{S/T}$ 13 

Induced 3N forces

The same unitary transformation induces 3N interactions

$$U^\dagger \left[C_{S/T} \right.$$

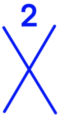
$$\left. \right] U = \sum_i m D_i C_{S/T}$$

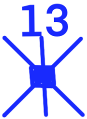


$$\begin{aligned}
V \;=\; & \sum_{i\neq j\neq k} (E_1 + E_2 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)(\boldsymbol{L}\cdot\boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k) \boldsymbol{\sigma}_j\cdot\hat{\boldsymbol{r}}_{ij} \boldsymbol{\sigma}_k\cdot\hat{\boldsymbol{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\
& + (E_{11} + E_{12} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) \boldsymbol{\sigma}_k\cdot\hat{\boldsymbol{r}}_{ij} \boldsymbol{\sigma}_j\cdot\hat{\boldsymbol{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
\end{aligned}$$

Induced 3N forces

The same unitary transformation induces 3N interactions

$$U^\dagger \left[C_{S/T} \right.$$ 

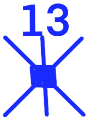
$$\left. \right] U = \sum_i m D_i C_{S/T}$$ 

$$
\begin{aligned}
V \;=\; & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)(\boldsymbol{L} \cdot \boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\
& + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
\end{aligned}
$$

Induced 3N forces

The same unitary transformation induces 3N interactions

$U^\dagger \Big[C_{S/T}$ 

$\Big] U = \sum_i m D_i C_{S/T}$ 

$$
\begin{aligned}
V \; = \; & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\boldsymbol{L} \cdot \boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
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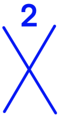
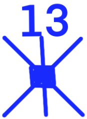
& + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
\end{aligned}
$$

Induced 3N forces

The same unitary transformation induces 3N interactions

$$U^\dagger \left[C_{S/T}$$ 

$$\right] U = \sum_i m D_i C_{S/T}$$ 

$$\begin{aligned}
V \; = \; & \sum_{i\neq j\neq k} (E_1 + E_2 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k) (\boldsymbol{L}\cdot\boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k) \boldsymbol{\sigma}_j\cdot\hat{\boldsymbol{r}}_{ij} \boldsymbol{\sigma}_k\cdot\hat{\boldsymbol{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\
& + (E_{11} + E_{12} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) \boldsymbol{\sigma}_k\cdot\hat{\boldsymbol{r}}_{ij} \boldsymbol{\sigma}_j\cdot\hat{\boldsymbol{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
\end{aligned}$$

$$
\begin{aligned}
E_1 &= C_S(\alpha_1+\alpha_2)+C_T(\alpha_1-2\alpha_2),\\
E_2 &= C_T(3\alpha_2+2\alpha_3-8\alpha_4+2\alpha_5),\\
E_3 &= 2C_S(\alpha_2+2\alpha_3/3)+C_T(2\alpha_1-\alpha_2-2\alpha_3/3+8\alpha_4-2\alpha_5),\\
E_4 &= 2C_S\alpha_2/3+C_T(2\alpha_1-7\alpha_2-2\alpha_3+8\alpha_4-2\alpha_5)/3,\\
E_5 &= 2C_S(\alpha_2+2\alpha_3/3)+2C_T(\alpha_1-2\alpha_2-\alpha_3/3+4\alpha_4-\alpha_5),\\
E_6 &= 2C_S\alpha_2/3+2C_T(\alpha_1-2\alpha_2-\alpha_3+4\alpha_4-\alpha_5)/3,\\
E_7 &= 24C_T\alpha_4,\\
E_8 &= 8C_T\alpha_4,\\
E_9 &= C_S\,(3\alpha_2+2\alpha_3-\alpha_4+2\alpha_5)+C_T\,(3\alpha_1-6\alpha_2-4\alpha_3+11\alpha_4-4\alpha_5)\,,\\
E_{10} &= C_S\,(\alpha_2-\alpha_4)+C_T(\alpha_1-2\alpha_2+5\alpha_4),\\
E_{11} &= C_S(3\alpha_2+2\alpha_3+\alpha_4-2\alpha_5)+C_T(3\alpha_1-6\alpha_2-4\alpha_3-11\alpha_4+4\alpha_5),\\
E_{12} &= C_S(\alpha_2+\alpha_4)+C_T(\alpha_1-2\alpha_2-5\alpha_4),\\
E_{13} &= -4C_T(4\alpha_4-\alpha_5).
\end{aligned}
$$

with $\alpha_i \sim O(m) \sim O(p^{-1})$ enhanced (Weinberg counting)

$$
\alpha_1 = \frac{m}{16}\,(16D_1+D_2+4D_3)\,,\quad \alpha_2 = \frac{m}{16}\,(16D_5+D_6+4D_7)\,,\quad \alpha_3 = \frac{m}{32}\,(D_{14}+16D_{11}+4D_{12}+4D_{13})\,,
$$

$$
\alpha_4 = \frac{m}{2}D_{16},\quad \alpha_5 = \frac{m}{16}\,(8D_{17}-D_{14}-16D_{11}-4D_{12}-4D_{13})\,,
$$

[Skibinski et al. FBS65(2024)]

Emulator for Nd scattering – fit to the true data at 10,70, and 135 MeV (786 data points)

TABLE III. The values of strengths c_i found in the least squares fit to the data from Table II at the three energies $E = 10$, 70, and 135 MeV.

c_D	-1.49 ± 0.06
c_E	-1.27 ± 0.06
c_{E_1}	6.40 ± 0.33
c_{E_2}	7.80 ± 0.36
c_{E_3}	6.97 ± 0.34
c_{E_4}	-2.06 ± 0.13
c_{E_5}	-0.36 ± 0.05
c_{E_6}	0.52 ± 0.03
c_{E_7}	-7.40 ± 0.14
c_{E_8}	-2.61 ± 0.05
c_{E_9}	-4.59 ± 0.22
$c_{E_{10}}$	-0.98 ± 0.05
$c_{E_{13}}$	-1.14 ± 0.05

c_D	c_E	c_{E_1}	c_{E_2}	c_{E_3}	c_{E_4}	c_{E_5}	c_{E_6}	c_{E_7}	c_{E_8}	c_{E_9}	$c_{E_{10}}$	$c_{E_{13}}$
1.000	-0.122	0.068	0.202	0.040	0.107	-0.237	-0.566	0.124	0.094	-0.052	0.038	-0.066
-0.122	1.000	0.048	-0.166	0.067	-0.084	-0.059	-0.144	-0.284	-0.298	-0.127	0.128	0.191
0.068	0.048	1.000	0.945	0.982	-0.843	0.087	-0.291	0.551	0.500	0.176	0.228	0.027
0.202	-0.166	0.945	1.000	0.934	-0.770	-0.112	-0.356	0.629	0.581	-0.007	0.041	-0.184
0.040	0.067	0.982	0.934	1.000	-0.917	0.098	-0.214	0.574	0.528	0.111	0.093	-0.012
0.107	-0.084	-0.843	-0.770	-0.917	1.000	-0.317	-0.114	-0.583	-0.556	-0.172	0.053	-0.035
-0.237	-0.059	0.087	-0.112	0.098	-0.317	1.000	0.540	0.113	0.126	0.898	0.521	0.719
-0.566	-0.144	-0.291	-0.356	-0.214	-0.114	0.540	1.000	-0.243	-0.197	0.253	-0.175	0.167
0.124	-0.284	0.551	0.629	0.574	-0.583	0.113	-0.243	1.000	0.995	0.108	-0.044	-0.205
0.094	-0.298	0.500	0.581	0.528	-0.556	0.126	-0.197	0.995	1.000	0.102	-0.068	-0.203
-0.052	-0.127	0.176	-0.007	0.111	-0.172	0.898	0.253	0.108	0.102	1.000	0.795	0.787
0.038	0.128	0.228	0.041	0.093	0.053	0.521	-0.175	-0.044	-0.068	0.795	1.000	0.746
-0.066	0.191	0.027	-0.184	-0.012	-0.035	0.719	0.167	-0.205	-0.203	0.787	0.746	1.000

- Big values of $c_{E1}, c_{E2}, c_{E3}, c_{E7}, c_{E9}$
- Correlation coefficients close to ±1: $\rho(E_1,E_2)$, $\rho(E_2,E_3)$, $\rho(E_1,E_3)$, $\rho(E_3,E_4)$, $\rho(E_7,E_8)$
- Correlation coefficients close to 0: (c_D,c_E),$(c_D,c_{Ei,\text{ beside }E6})$,$(c_E,c_{Ei})$
- χ^2/data≈35

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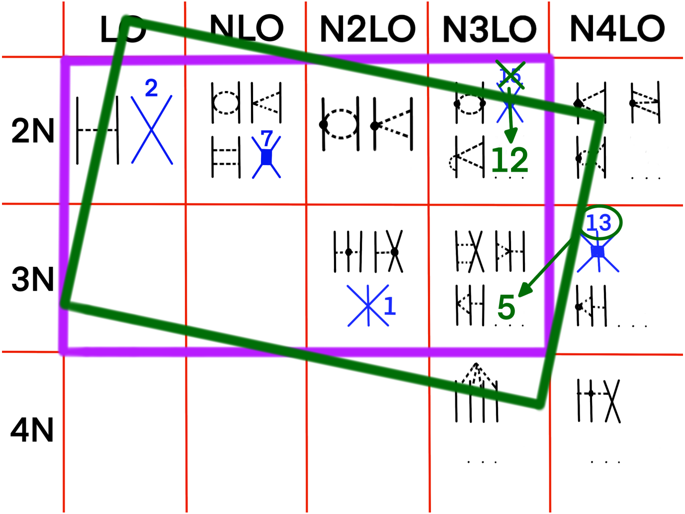
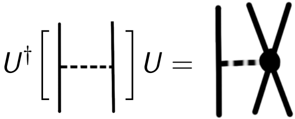
10

The new matrix of orders



Further topologies

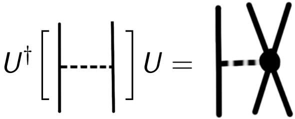
induced from one-pion-exchange



$$V_{3NF} = -\frac{g_A}{F_\pi} \sum_{i \neq j \neq k} \frac{\boldsymbol{k}_k \cdot \boldsymbol{\sigma}_k \, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k}{k_k^2 + m_\pi^2} \{\alpha_1 \boldsymbol{k}_j \cdot \boldsymbol{k}_k \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_i$$
$$+ \alpha_2 \left[\boldsymbol{k}_j \cdot \boldsymbol{k}_k \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_j + 2i \boldsymbol{k}_j \cdot (\boldsymbol{Q}_i - \boldsymbol{Q}_j) \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right]$$
$$+ (\alpha_3 + \alpha_5) \left[k_k^2 \boldsymbol{k}_j \cdot \boldsymbol{\sigma}_j - 2i \boldsymbol{k}_j \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{Q}_i \times \boldsymbol{\sigma}_i + 2i \boldsymbol{Q}_j \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{k}_j \times \boldsymbol{\sigma}_i\right]$$
$$+ (\alpha_3 - \alpha_5) \left[\boldsymbol{k}_j \cdot \boldsymbol{k}_k \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_j + 2i \boldsymbol{k}_j \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{Q}_j \times \boldsymbol{\sigma}_i - 2i \boldsymbol{Q}_i \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{k}_j \times \boldsymbol{\sigma}_i\right]$$
$$-2\alpha_4 \left[\boldsymbol{k}_k \cdot \boldsymbol{\sigma}_i \, \boldsymbol{k}_k \cdot \boldsymbol{Q}_j \times \boldsymbol{\sigma}_j - 2i \boldsymbol{k}_k \cdot \boldsymbol{Q}_i \, (\boldsymbol{k}_k \cdot \boldsymbol{Q}_i \, \boldsymbol{Q}_j \cdot \boldsymbol{\sigma}_i - \boldsymbol{k}_k \cdot \boldsymbol{Q}_j \, \boldsymbol{Q}_i \cdot \boldsymbol{\sigma}_i)\right]\}$$

Further topologies

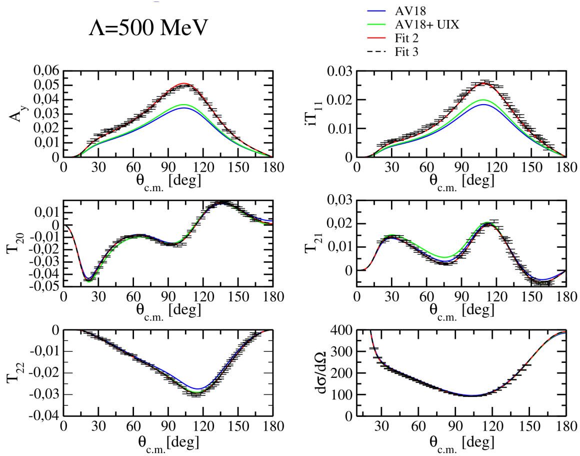
induced from one-pion-exchange



$$
\begin{aligned}
V_{3NF} = -\frac{g_A}{F_\pi} \sum_{i \neq j \neq k} \frac{\boldsymbol{k}_k \cdot \boldsymbol{\sigma}_k \, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k}{k_k^2 + m_\pi^2} \{ & \alpha_1 \boldsymbol{k}_j \cdot \boldsymbol{k}_k \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_i \\
& + \alpha_2 \left[\boldsymbol{k}_j \cdot \boldsymbol{k}_k \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_j + 2i \boldsymbol{k}_j \cdot (\boldsymbol{Q}_i - \boldsymbol{Q}_j) \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j \right] \\
& + (\alpha_3 + \alpha_5) \left[k_k^2 \boldsymbol{k}_j \cdot \boldsymbol{\sigma}_j - 2i \boldsymbol{k}_j \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{Q}_i \times \boldsymbol{\sigma}_i + 2i \boldsymbol{Q}_j \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{k}_j \times \boldsymbol{\sigma}_i \right] \\
& + (\alpha_3 - \alpha_5) \left[\boldsymbol{k}_j \cdot \boldsymbol{k}_k \, \boldsymbol{k}_k \cdot \boldsymbol{\sigma}_j + 2i \boldsymbol{k}_j \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{Q}_j \times \boldsymbol{\sigma}_i - 2i \boldsymbol{Q}_i \cdot \boldsymbol{\sigma}_j \, \boldsymbol{k}_k \cdot \boldsymbol{k}_j \times \boldsymbol{\sigma}_i \right] \\
& - 2\alpha_4 \left[\boldsymbol{k}_k \cdot \boldsymbol{\sigma}_i \, \boldsymbol{k}_k \cdot \boldsymbol{Q}_j \times \boldsymbol{\sigma}_j - 2i \boldsymbol{k}_k \cdot \boldsymbol{Q}_i \, (\boldsymbol{k}_k \cdot \boldsymbol{Q}_i \, \boldsymbol{Q}_j \cdot \boldsymbol{\sigma}_i - \boldsymbol{k}_k \cdot \boldsymbol{Q}_j \, \boldsymbol{Q}_i \cdot \boldsymbol{\sigma}_i) \right] \}
\end{aligned}
$$

$$
O(p^{-1}) \times O(p^5) \sim O(p^4) = \text{N3LO}
$$

Use of the freedom: resolution of the $p-d$ A_y-puzzle



Fitting procedure	2-param.	3-param.
χ^2/d.o.f.	2.1	1.9
e_0	-	0.459
$\tilde{\alpha}_4 C_S$	1.751	1.894
$\tilde{\alpha}_5 C_S$	-0.495	-1.175
${}^2a_{nd}$ (fm)	0.573	0.599

Summary and outlook

- There are 5 free LECs in the 3N force at N3LO to improve the description of scattering data
- $N - d\ A_y$ puzzle can be solved in this way

Summary and outlook

- There are 5 free LECs in the 3N force at N3LO to improve the description of scattering data
- $N-d$ A_y puzzle can be solved in this way
- current work towards adding the pion-range induced component with consistent chiral interactions
- explore the impact in $A>3$ systems

Summary and outlook

- There are 5 free LECs in the 3N force at N3LO to improve the description of scattering data
- $N - d\ A_y$ puzzle can be solved in this way
- current work towards adding the pion-range induced component with consistent chiral interactions
- explore the impact in $A > 3$ systems

thank you