Shape coexistence in medium-mass nuclei

Dorian Frycz
Mail: dorianfrycz@fqa.ub.edu
Universitat de Barcelona & Institut de Ciències del Cosmos

Collaborators: **J. Menéndez** (UB), A. Rios (UB), **A. Poves** (UAM), B. Bally (CEA, Paris-Saclay), T. R. Rodríguez (UCM) & A. M. Romero (Fujitsu)

Deformation is everywhere:

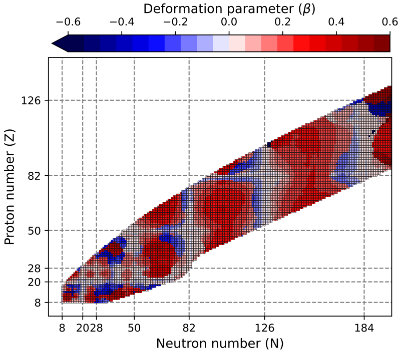
- Magical nuclei are spherical
- Deformation is enhanced mid-shell

Quadrupole deformation:

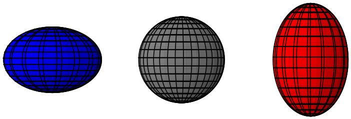
- Most important shape
- Prolate: elongated sphere
- Oblate: flattened sphere

Shape coexistence:

- Different shapes coexist within the **same nucleus** at different energies
- Present in most nuclei



P. Möller, et al. Atom. Data Nucl. Data Tabl. 109-110 (2016)



Oblate Spherical Prolate

Intrinsic vs laboratory frame

Deformation is characterized in the intrinsic frame: (β, γ)

However, measurements are done in lab frame: $(B(E2), Q_s)$

Relation between lab and intrinsic frame:

$$Q_s(J) = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_{0,s},$$

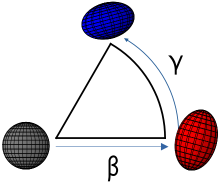
$$B(E2, J_i \to J_f) = \frac{5 {Q_{0,t}}^2}{16\pi} \langle J_i K 20 | J_f K \rangle$$

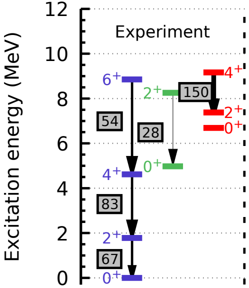
- If $Q_{0,s} \simeq Q_{0,t}$: well deformed rotor

1. For even-even nuclei (J=0) $\to$ $Q_s = 0$
2. Triaxial shapes diminish Q_s
3. $B(E2)$: scattered across out-band states

$\to$ Perfect axial rotor assumption!

Shape invariants $\langle Q^n \rangle$: model independent measure of deformation





Shape invariants: Intermediate-state expansion

Couple Q_2 (tensor) to obtain a scalar:

- $\langle Q^2 \rangle = \langle [Q_2 \times Q_2]_0 \rangle \rightarrow \beta$
- $\langle Q^3 \rangle = \langle [[Q_2 \times Q_2]_2 \times Q_2]_0 \rangle \rightarrow \gamma$
- All $\langle Q^n \rangle$ up to $\langle Q^6 \rangle$ are needed for fluctuations of (β, γ)

  K. Kumar, Phys. Rev. Lett. 28, 249 (1972)

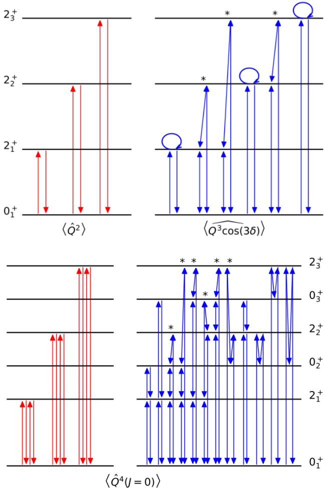
  D. Cline, Nuclear Structure 313–326 (1985)

Intermediate-state expansion:

- Expansion in terms of $M_{if} = \langle i || Q || f \rangle \leftarrow Q_s$ or $B(E2)$
- Needs **all the nuclear states**!
- Harder convergence for higher $\langle Q^n \rangle$

$$\langle Q^2 \rangle_i \sim \sum_t M_{it} M_{ti}$$

$$\langle Q^3 \rangle_i \sim \sum_{u,t} M_{iu} M_{ut} M_{ti}$$



Henderson, J. Phys Rev C 102, 054306 (2020)

Shape invariants: Sum rule method

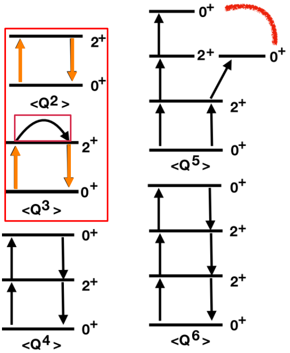
1). Apply the $\hat{Q}_{20}$ operator to a state:
$\hat{Q}_{20}|0^+\rangle = \text{SR1}(2^+)^{1/2}\overline{|2^+(1)\rangle}$
$\rightarrow \langle Q^2 \rangle = 5\text{SR1}(2^+)$

- $\overline{|2^+(1)\rangle}$ not an eigenstate but contains the whole quadrupole strenght

2). Evaluate $\langle\langle Q \rangle\rangle = \langle 2^+(1)|Q|2^+(1)\rangle/\sqrt{5}$
$\rightarrow \langle Q^3 \rangle = \langle Q^2 \rangle \langle\langle Q \rangle\rangle$

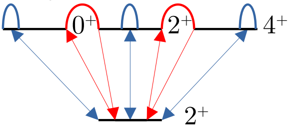
3). Apply $\hat{Q}_{20}$ a second and third time to obtain $\langle Q^4 \rangle$, $\langle Q^5 \rangle$, and $\langle Q^6 \rangle$

- **Exact** calculation (up to numeric)
- Computationally cheap
- Currently only for $J = 0$
- **New!:** extension for any J up to $\langle Q^4 \rangle$



Poves, A., Nowacki, F. & Alhassid, Y. Phys. Rev. C 101, 054307 (2020)

To be published:



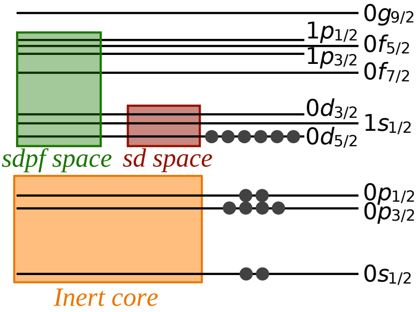
Shell model valence space

Schrödinger equation

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

- **Interacting shell model**:
  $\mathcal{H}_{\text{eff}} = \mathcal{H}_0 + \mathcal{H}_{\text{res}}$
  $\mathcal{H}_{\text{res}}$: valence space
- Slater determinant basis $\{\Phi_i\}$
- Phenomenological interactions: USDB and SDPF-NR

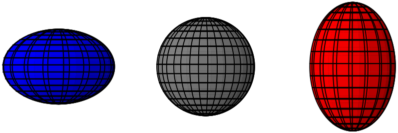
Caurier E., et al. Rev. Mod. Phys. **77**, 427 (2005)



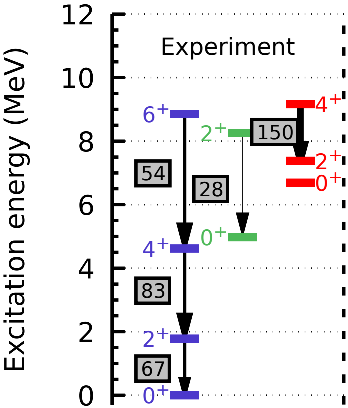
Valence space: $Z_v = N_v = 6$ (^{28}Si)
Inert core: ^{16}O nucleus

- **Coexistence** of different collective structures:
  1. 0_1^+ (0.0 MeV): Oblate bandhead of a rotational band
  2. 0_2^+ (5.0 MeV): Vibration of the ground state
  3. 0_3^+ (6.7 MeV): Prolate bandhead of a rotational band
  4. Superdeformed rotational band? ($E \gtrsim$10 MeV)

Taniguchi, Y., et al. Phys. Rev. C **80**, 044316 (2009)



Oblate Spherical Prolate



- $E \sim J(J+1)$; $J = 0^+, 2^+, 4^+ \ldots$
- Strong $B(E2)$ strengths

Shell model calculation of ^{28}Si

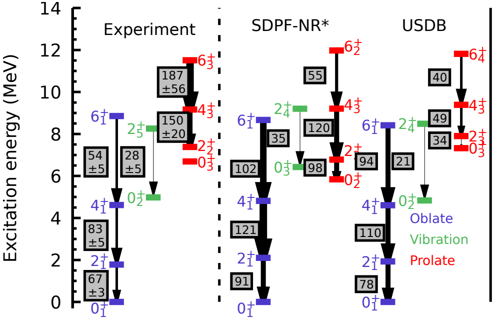
USDB: fails in B(E2) transitions of prolate band

SDPF*: Adjusted SDPF-NR† interaction to reproduce ^{28}Si shell gap

Additional deformation from *pf*-shell particles:

- Slight gain for oblate and vibration
- Significant gain in prolate deformation
- 1 particle in *pf*-shell (38% of *sdpf* 2p-2h)

†S. Nummela Phys. Rev. C **63**, 044316 (2001)



Experiment vs Theory (SDPF-NR*):

$$B(E2, 2^+_{\text{obl}} \to 0^+_{\text{obl}}) = 67 \pm 3 \text{ vs } 91 \; e^2\text{fm}^4$$

$$B(E2, 2^+_{\text{vib}} \to 0^+_{\text{vib}}) = 28 \pm 5 \text{ vs } 35 \; e^2\text{fm}^4$$

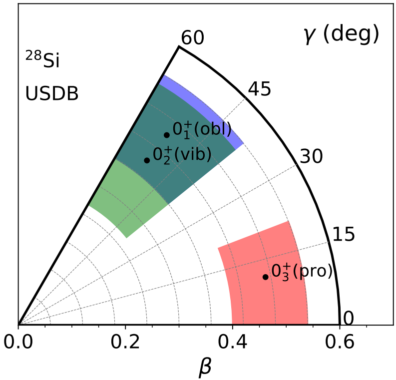
$$B(E2, 4^+_{\text{pro}} \to 2^+_{\text{pro}}) = 150 \pm 20 \text{ vs } 120 \; e^2\text{fm}^4$$

D. Frycz, et al. Phys. Rev. C 110, 054326 (2024)

Shape invariants for ^{28}Si

^{28}Si shape coexistence (USDB):

- Oblate: $\beta = 0.45 \pm 0.09$
  $\gamma = 53\ (39 - 60)$
- Vibration: $\beta = 0.39 \pm 0.13$
  $\gamma = 53\ (39 - 60)$
- Prolate: $\beta = 0.47 \pm 0.07$
  $\gamma = 11\ (0 - 21)$



- Shapes are γ **and** β **soft**
- Vibration is **similar** in deformation to oblate GS
- Prolate shape has similar deformation as GS

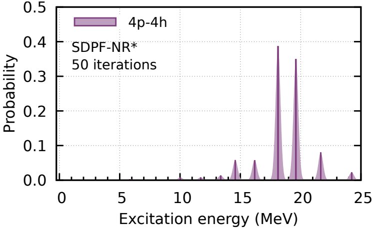
Superdeformed (SD) band

predicted with:

- Deformation: $\beta_{theo} \approx 1$
- **4p-4h** into *pf* shell
- $\sim$ 13 MeV bandhead

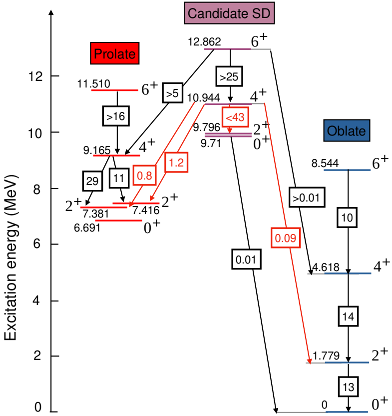
Taniguchi, Y., et al. Physical Review C, 2009. **80**, 044316

Our prediction lies at 18 MeV:



D.

Frycz, et al. Phys. Rev. C **110**, 054326 (2024)



Experimental attempts

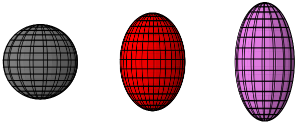
- $B(E2, 4^+ \rightarrow 2^+) \leq 217 efm^2$
- Not found: $\beta_{exp} \leq 0.6$

Morris, L. et al. Phys. Rev. C **104**, 054323 (2021)

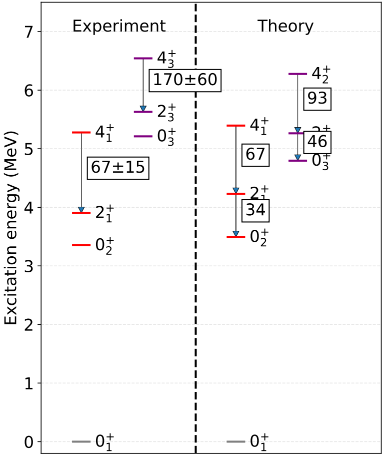
Doubly magical nucleus
$(Z = N = 20)$

Shape coexistence in ^{40}Ca:

1. 0_1^+ (0.0 MeV):
   Spherical 0p-0h
2. 0_2^+ (3.4 MeV):
   Deformed 4p-4h
3. 0_3^+ (5.2 MeV):
   Superdeformed 8p-8h



Spherical Normal Superdeformed



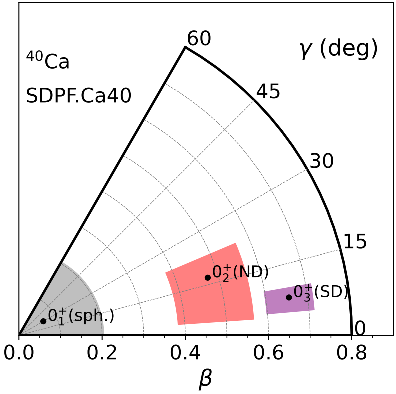
Accurate description of ^{40}Ca

E. Caurier, J. Menéndez, F. Nowacki, and A. Poves
Phys. Rev. C **75**, 054317 (2007)

Shape invariants for ^{40}Ca

^{40}Ca shape coexistence (SDPF):

- "Spherical":
  $\beta = 0.07 \pm 0.14$
  $\gamma = -\ (0 - 60)$
  Compatible with $\beta = 0$ but reaches up to $\beta = 0.2$
- Normal deformed:
  $\beta = 0.47 \pm 0.09$
  $\gamma = 17\ (4 - 23)$
  Considerable fluctuations
- Superdeformed:
  $\beta = 0.66 \pm 0.06$
  $\gamma = 8\ (5 - 10)$
  Lower fluctuations for SD

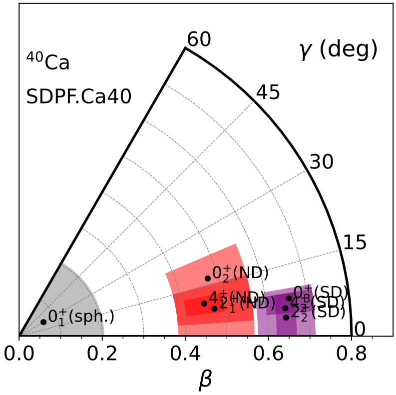


Band evolution in ^{40}Ca

Band evolution of (β, γ):

^{40}Ca	$\beta \pm \Delta\beta$	γ ($\Delta\gamma$)
0^+_{sph}	0.07±0.14	- (0-60)
0^+_{ND}	0.47±0.09	17 (4-23)
2^+_{ND}	0.47±0.09	8 (0-15)
4^+_{ND}	0.45±0.05	10 (7-12)
0^+_{SD}	0.66±0.06	8 (5-10)
2^+_{SD}	0.64±0.07	4 (0-9)
4^+_{SD}	0.64±0.02	6 (0-9)

- Band states overlap
- (β, γ) are consistent
- Robust method to identify states with the same intrinsic shape



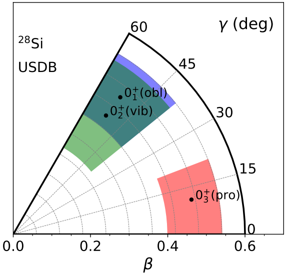
Conclusions

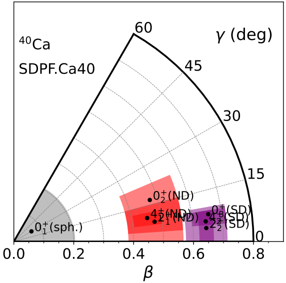
Conclusions:

- Shape coexistence is challenging to describe
- **Shape invariants provide a method to identify shapes**
- (β, γ) fluctuations are large
- **Extension of sum rule method for any** J (up to $\langle Q^4 \rangle$)
- **Deformation parameters are constant across the band**

Outlook:

- Shape invariants for odd nuclei
- Study of $^{48\pm1}$Cr
- Octupole shape invariants?



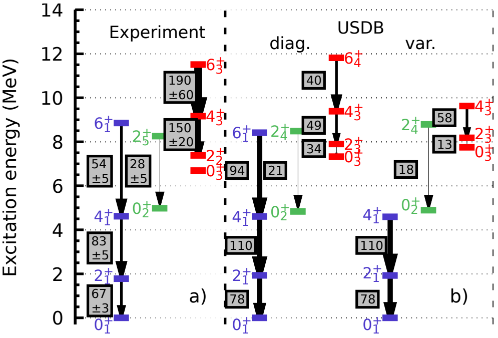


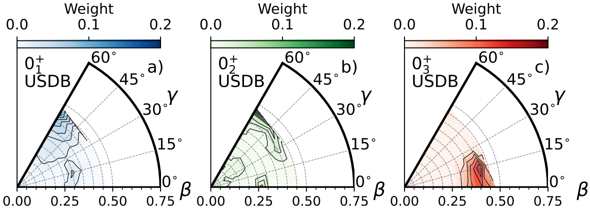
Spectrum of ^{28}Si (USDB)

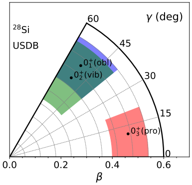
Oblate rotational band: **well described**, slightly more deformed

Vibrational band based on the ground state is also **well described**

Prolate rotational band has too weak $B(E2)$



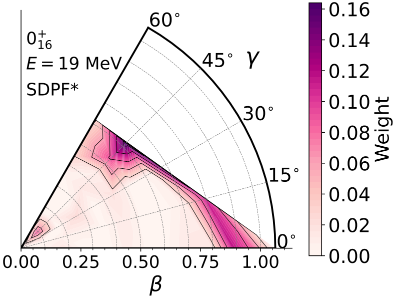




Full *sdpf* space calculation

- Full *sdpf* space
- Superdeformed state ($\beta \geq 0.6$)
- $\sim$ **3 particles** into the *pf* shell
- Energy: $E \approx 19$ MeV
- In agreement with the shell model calculation

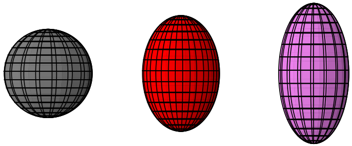
PGCM calculation in *sdpf* space with SDPF-NR* interaction:



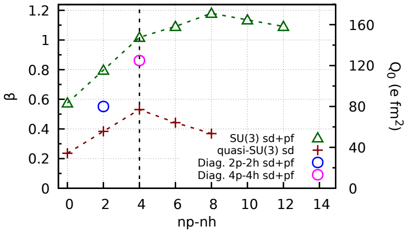
Fixed *np*-*nh* configurations

Analytical SU(3) models:

- *sd*-shell ($\beta \leq 0.5$)
- *sdpf* space ($\beta \geq 0.5$)
  SD for $\geq$4p-4h ($\beta \approx 0.8$)



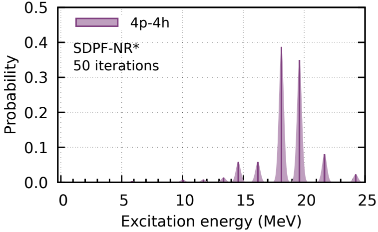
Spherical   Normal   Superdeformed



Lanczos strength function:
Decomposition of a fixed 4p-4h configuration into the fully mixed states of the Hamiltonian:

$$|0^+_{np\text{-}nh}\rangle = \frac{1}{N}\sum_\sigma S(\sigma)|0^+_\sigma\rangle$$

Energies: 4p-4h at **18-20 MeV**



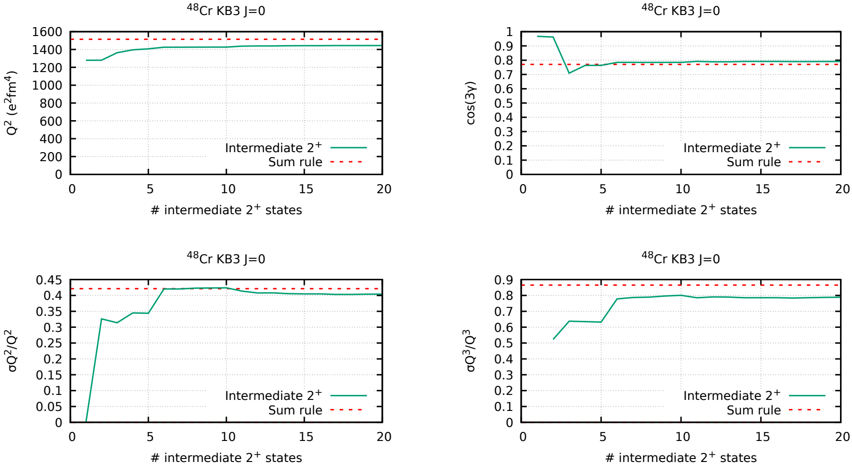
Exact diagonalization (ISM)

- **Most accurate solution** of $\mathcal{H}_{\text{eff}}|\Psi\rangle = E|\Psi\rangle$
- Large set of simple Slater determinants $|\Phi_i\rangle = c^\dagger_{i1} c^\dagger_{i2} \dots c^\dagger_{iA} |0\rangle$
- Best suited for smaller valence spaces (*sd*)
- Cannot explore a single degree of freedom ($Q_{\lambda\mu}$)

Beyond-mean-field (PGCM)

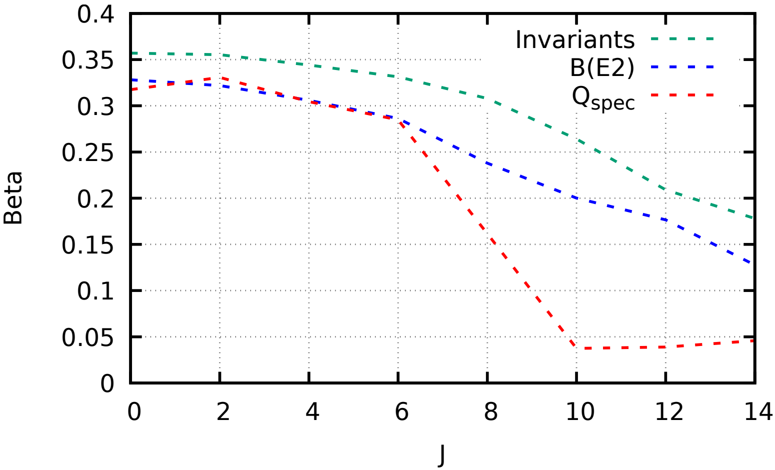
- **Approximate solution** to $\mathcal{H}_{\text{eff}}|\Psi\rangle = E|\Psi\rangle$
- Smaller set of more complex wavefunctions (HFB) $\beta^\dagger_k = \sum_l (U_{lk} c^\dagger_l + V_{lk} c_l)$
- Alternative for large valence spaces (*sdpf*)
- Exploration of relevant degrees of freedom ($Q_{\lambda\mu}$)

A word of caution:

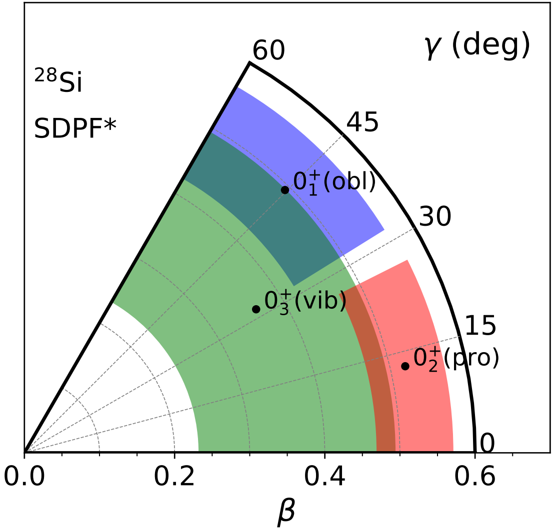


The convergence is worse for more complex nuclei and other J

Backbending ^{48}Cr



SDPF Si-28 invariants



Shape invariants