Extended Random Phase Approximation with modern NN+NNN interactions

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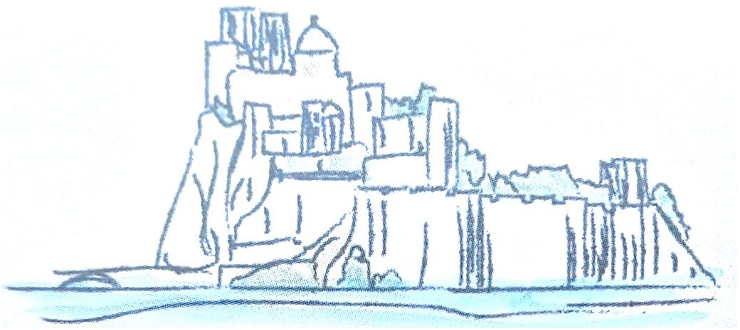

14th International Spring Seminar on Nuclear Physics, Ischia

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Outline

- Introduction: HF+RPA

- newly developed framework for HF+MPTP(2) + **(E)RPA** calculations
  master thesis of Radek Folprecht

- comparison with available to HF+MPBT and IMSRG(2) calculations

- calculations of selected closed shell nuclei with chiral NN+NNN potentials

  (correlation energies, low-lying spectra, dipole response)

Nuclear many-body problem

nuclear structure $\rightarrow$ many-body Schrödinger equation $\boxed{\rightarrow \textbf{approximations}}$

$$i\hbar \frac{\partial \Psi(1,2 \ldots A)}{\partial t} = \boldsymbol{H}\Psi(1,2 \ldots A)$$

$$\boldsymbol{H} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A} \boldsymbol{V}(i,j) + \sum_{i<j<k}^{A} \boldsymbol{V}(i,j,k)$$

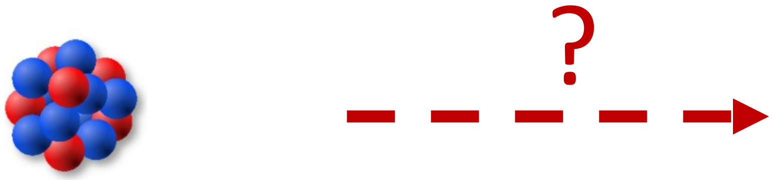
interaction models

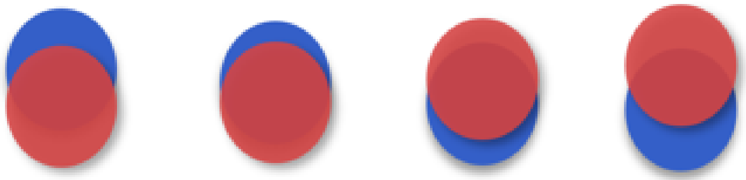
NN and NNN interactions from chiral effective field theory

ab initio methods

phenomenological interactions

DFT, SM





Chiral EFT interactions

Chiral perturbation theory (ChpT)

- hierarchy of nuclear forces
- systematic improvements
- many-body forces

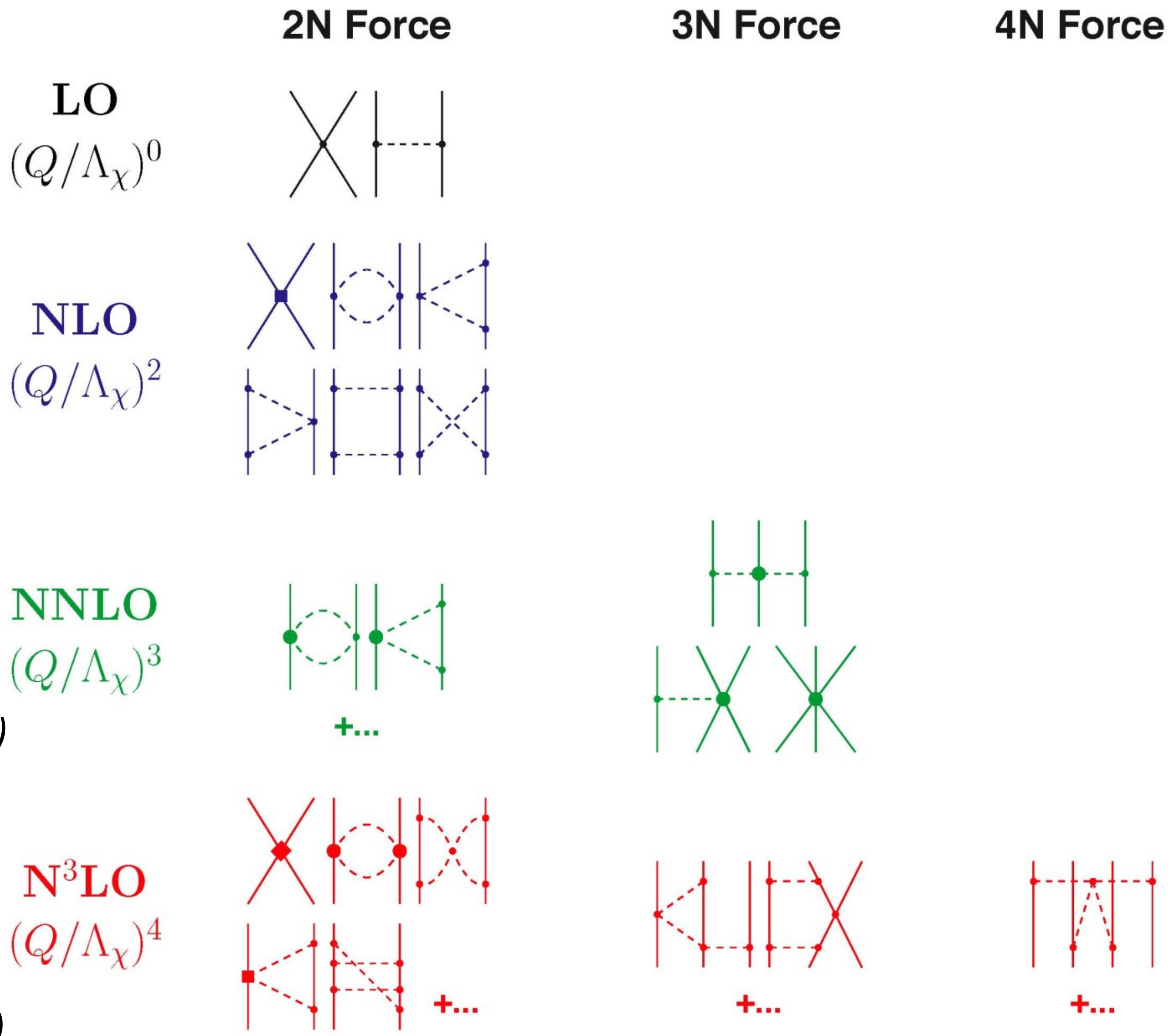
several fitting strategies

→ different predictions for many-body systems

N^2LO_{sat}
Ekström A. et al., Phys. Rev C 91, 051301 (2015)

Δ -full nuclear interactions
(Gothenburg–Oak Ridge)

$\Delta N^2LOGO(394)$, $\Delta N^2LOGO(450)$
Jiang W. G. et al., Phys. Rev C102,054301(2020)



LEC's have to be fitted to selected set of experimental data

Mean-field and beyond …

- intrinsic Hamiltonian with NN+NNN interactions

$$H = \sum_{a,d} t_{ad} a_a^\dagger a_d + \frac{1}{4} \sum_{\substack{a,b,\\d,e}} {}^{\mathrm{NN}}V_{abde} a_a^\dagger a_b^\dagger a_e a_d + \frac{1}{36} \sum_{\substack{a,b,c,\\d,e,f}} \boxed{{}^{\mathrm{NNN}}V_{abcdef} a_a^\dagger a_b^\dagger a_c^\dagger a_f a_e a_d}$$

- reference state → Hartree-Fock approximation (HF)

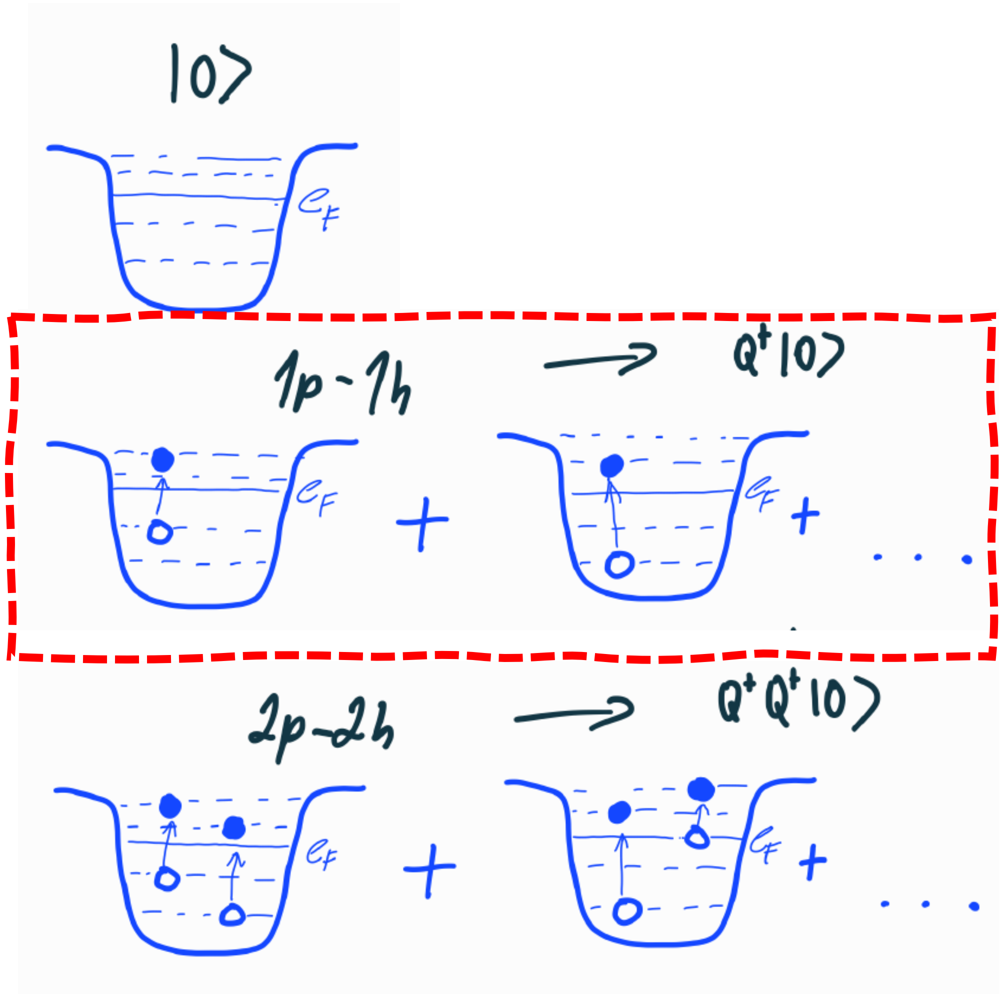
$$H = E_0^{\mathrm{HF}} + \sum_a {}^{\mathrm{N}}\varepsilon_a : c_a^\dagger c_a : + \boxed{V^{\mathrm{Res}}}$$

- **residual interaction** → correlations

  ground state properties → many-body Perturbation Theory (MBTP), Coupled cluster, Greens-function…
  In Medium Similarity Renormalisation Group (IMSRG)

  excited states and response functions, GRs → Greens function methods, LIT-Coupled cluster
  (Extended) Random Phase Approximation (ERPA), Second RPA, Equation of Motion Phonon Method (EMPM) …

Microscopic picture of collective vibrations



mean-field

bulk properties of ground states (binding energies, radii)

HF, HFB

harmonic approximation

gross features of GRs energy centroids, integrated strength

(Q)RPA 1-phonon

"complex" configurations

fragmentation, spreading widths, anharmonic effects, fine structures of GRs

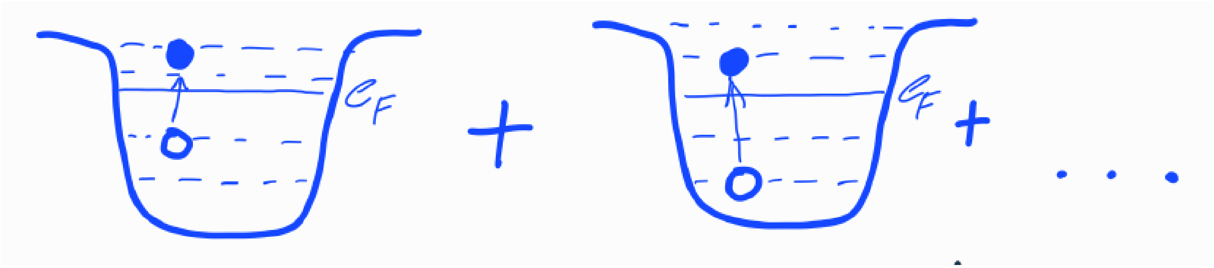
SRPA (*RPA + 2p2h***)**
PVC (*1p1h x RPA phonon***)**
QPM (*1+2+3 QRPA*)
RQTBA(*2qp x QRPA phonon***)**
EOM (*2qp x 2-phonon***)**
EMPM(*1+2+3 phonon***)**

Random Phase Approximation (RPA)

Excited states → phonons

$$\left[H,\, Q_\nu^\dagger\right] |\mathrm{RPA}\rangle = (E_\nu - E_0)|\nu\rangle$$

$$|\nu\rangle \equiv Q_\nu^\dagger|0\rangle \equiv \sum_{ph} \left[X^\nu_{ph} B^\dagger_{ph} - Y^\nu_{ph} B_{ph}\right]|0\rangle$$



$$B^\dagger_{ph} \equiv a^\dagger_p a_h, \quad B_{ph} \equiv a^\dagger_h a_p$$

Quasiboson approximation : valid if correlations are small

$$\langle \mathrm{RPA}| \left[a^\dagger_h a_p,\, a^\dagger_q a_g\right] |\mathrm{RPA}\rangle \simeq \cancel{\langle \mathrm{HF}| \left[a^\dagger_h a_p,\, a^\dagger_q a_g\right] |\mathrm{HF}\rangle = \delta_{p,q}\delta_{h,g}}$$

What if correlations are strong? → trully correlated g.s. → fully self-consistent RPA

D. J. Rowe, Rev. Mod. Phys 40, 153 (1968), Schuck P. et al., Physics Reports 929 (2021) 1–84

Extended Random Phase Approximation (ERPA)

different names in literature: Improved RPA (IRPA), or Extended RPA (ERPA)

- metallic clusters

Catara F., Piccito G., Sambataro M. and Van Giai N., Phys. Rev. B 54, 17536 (1996)
*Catara F. et al. , Phys. Rev. B **58**, 16070 (1998)*
*Voronov V. et al, Phys. Part. Nucl. **31**, 904 (2000)*

- application with modern NN potential - UCOM

P. Papakonstantinou, R. Roth, and N. Paar, Phys. Rev. C 75, 014310 (2007)

- **application with chiral NN+NNN potentials**

Extended Random Phase Approximation (ERPA)

- towards selfconsistent (first order) RPA → beyond Quasiboson Approximation (QBA)

$$|\nu\rangle \equiv Q^{\dagger}_{\nu}|0\rangle \equiv \sum_{ph} [X^{\nu}_{ph} B^{\dagger}_{ph} - Y^{\nu}_{ph} B_{ph}]|0\rangle$$

- renormalization of RPA equations in terms of one-body density matrix (OBDM) occupation probabilities

$$B^{\dagger}_{ph} \equiv \sum_{q>\mathrm{F}} \sum_{g<\mathrm{F}} \boxed{N_{phqg}}\, a^{\dagger}_{q} a_{g}$$

- **diagonal approximation of OBMD** $\langle \mathrm{RPA}| a^{\dagger}_{a} a_{b} |\mathrm{RPA}\rangle = {}^{\mathrm{N}}\rho_{ab}\, \delta_{a,b}$

$$N_{phqg} \equiv \delta_{p,q}\, \delta_{h,g} \boxed{\left({}^{\mathrm{N}}\rho_{hh} - {}^{\mathrm{N}}\rho_{pp}\right)^{-1/2}}$$

Extended Random Phase Approximation (ERPA)

- **correlated RPA ground state** is determined from a system of coupled equations analogous to RPA
  specific choice of renormalization
  equations are expanded in Y ($|Y|^2 \ll 1$, exact up to $O(Y^4)$)
  linearization of 2-body density matrix
- nonlinear problem → iterative solution

$$\begin{pmatrix} \mathbb{A} & \mathbb{B} \\ -\mathbb{B}^* & -\mathbb{A}^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = E_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

$$\mathbb{A}_{phqg} = \frac{1}{2} \left(\frac{D_{ph} + D_{qg}}{\sqrt{D_{ph} D_{qg}}} \right) \left[\delta_{h,g} \, {}^{\mathrm{N}}\varepsilon_{pq} - \delta_{p,q} \, {}^{\mathrm{N}}\varepsilon_{hg} \right] + D_{ph}^{1/2} D_{qg}^{1/2} \, {}^{\mathrm{NN}}\mathcal{V}_{hqpg}$$

$$\mathbb{B}_{phqg} = D_{ph}^{1/2} D_{qg}^{1/2} \, {}^{\mathrm{NN}}\mathcal{V}_{hgpq}$$

$$\boxed{D_{ph} \equiv \left({}^{\mathrm{N}}\rho_{hh} - {}^{\mathrm{N}}\rho_{pp} \right)}$$

Hamiltonian

- ***NuHamil*** **: A numerical code to generate nuclear two- and three-body matrix elements from chiral effective field theory,** ***Miyagi T., Eur. Phys. J. A (2023) 59:150***

N^2LO_{sat}, $\Delta N^3LO_{GO}(394)$, $\Delta N^3LO_{GO}(450)$ …

- spherical HO basis – model space given by the number of major shells, typical number e_{max}=14

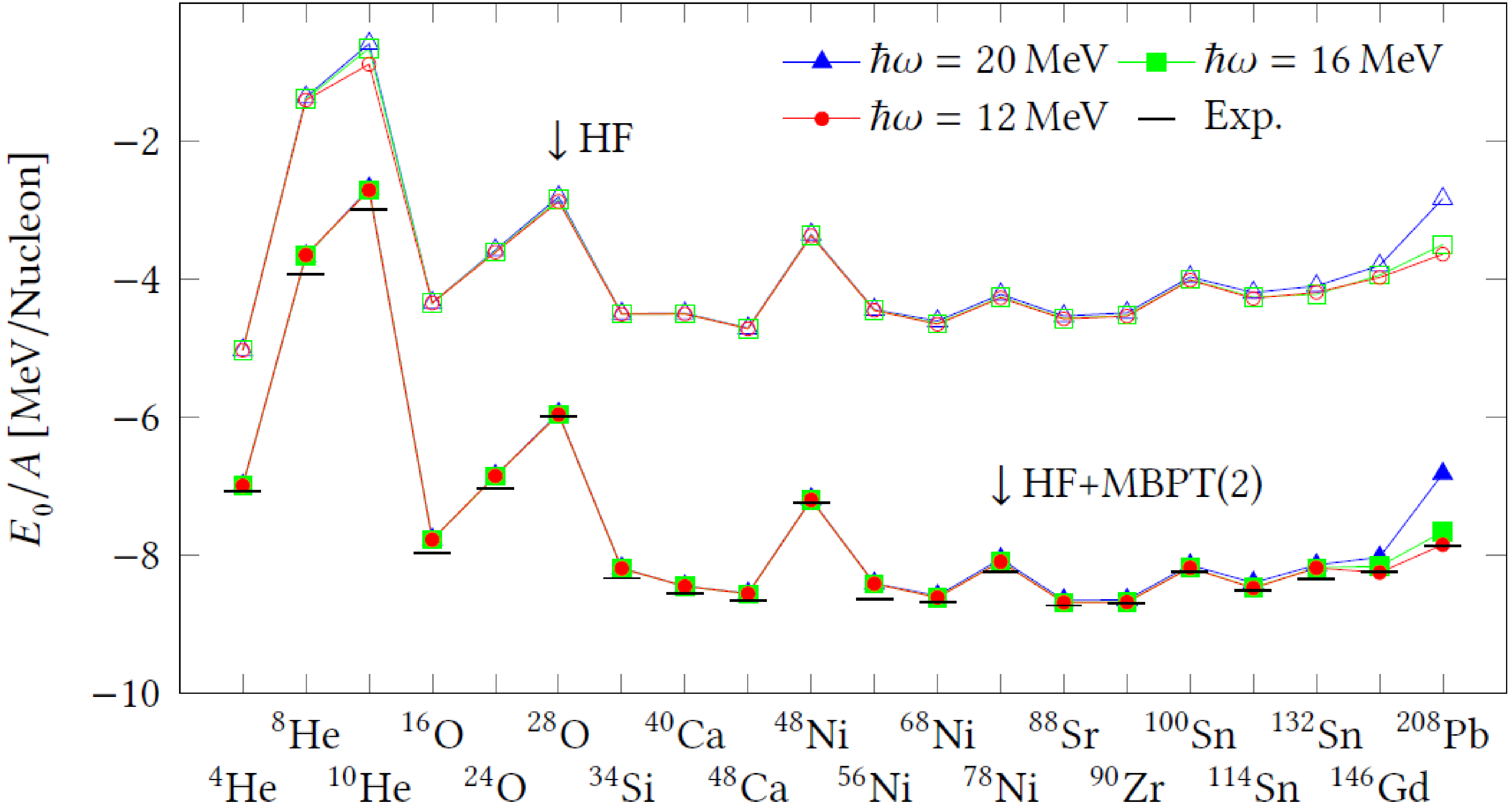
- **NNN matrix elements must be truncated** → $e_{3max} \leq 16$

- **Normal-ordered 2-body Approximation** → clever storage of NNN matrix elements up to e_{3max}=28

Miyagi T. , Stroberg S. R., Navrátil P., Hebeler K., and Holt J. D., Phys. Rev. C 105, 014302 (2022).

$$V^{\mathrm{Res}} = \frac{1}{4} \sum_{a,b,d,e} {}^{\mathrm{NN}}V_{abde} :c^\dagger_a c^\dagger_b c_e c_d: + \frac{1}{4} \sum_{a,b,c,d,e,f} {}^{\mathrm{NNN}}V_{abcdef} \, {}^{\mathrm{N}}\rho_{cf} :c^\dagger_a c^\dagger_b c_e c_d: + \cancel{\frac{1}{36} \sum_{a,b,c,d,e,f} {}^{\mathrm{NNN}}V_{abcdef} :c^\dagger_a c^\dagger_b c^\dagger_c c_f c_e c_d:} \, .$$

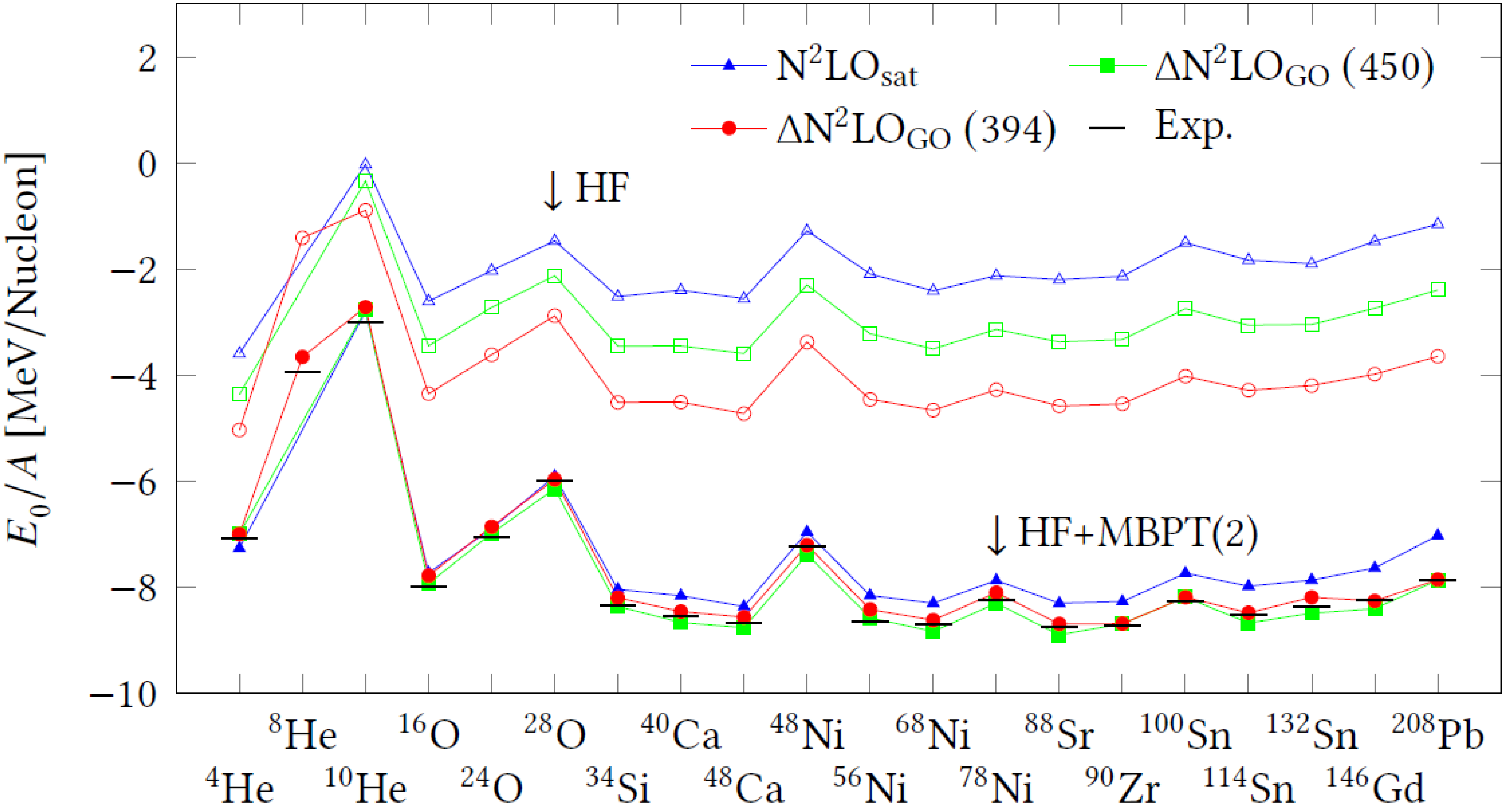
Binding energies: HF+MBPT(2)



ΔN^2LO_{GO} (394)

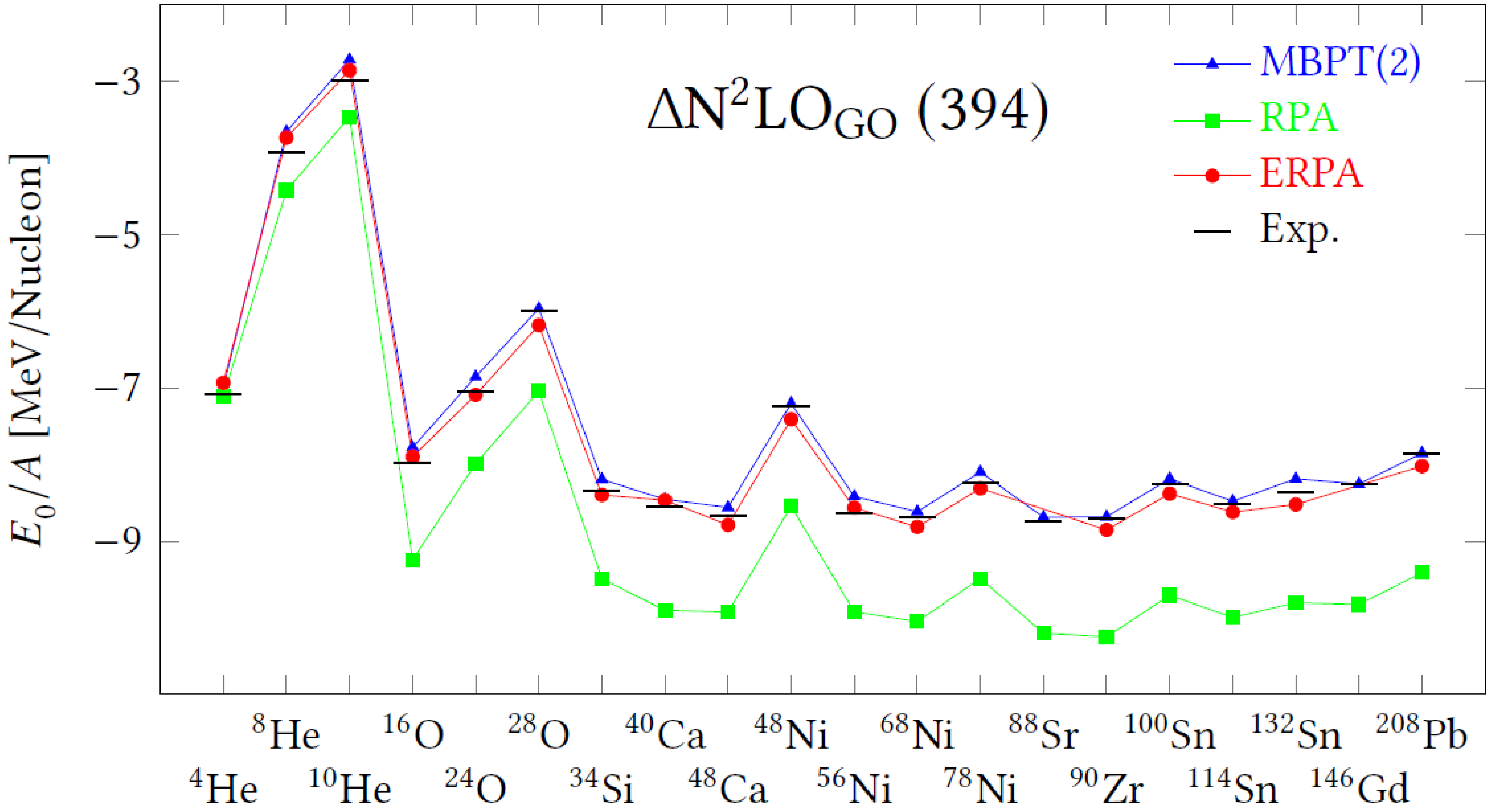
$e_{max}=14$, $e_{3max}=28$

Binding energies: HF+MBPT(2)



e_{max}=14, e_{3max}=28

Binding energies

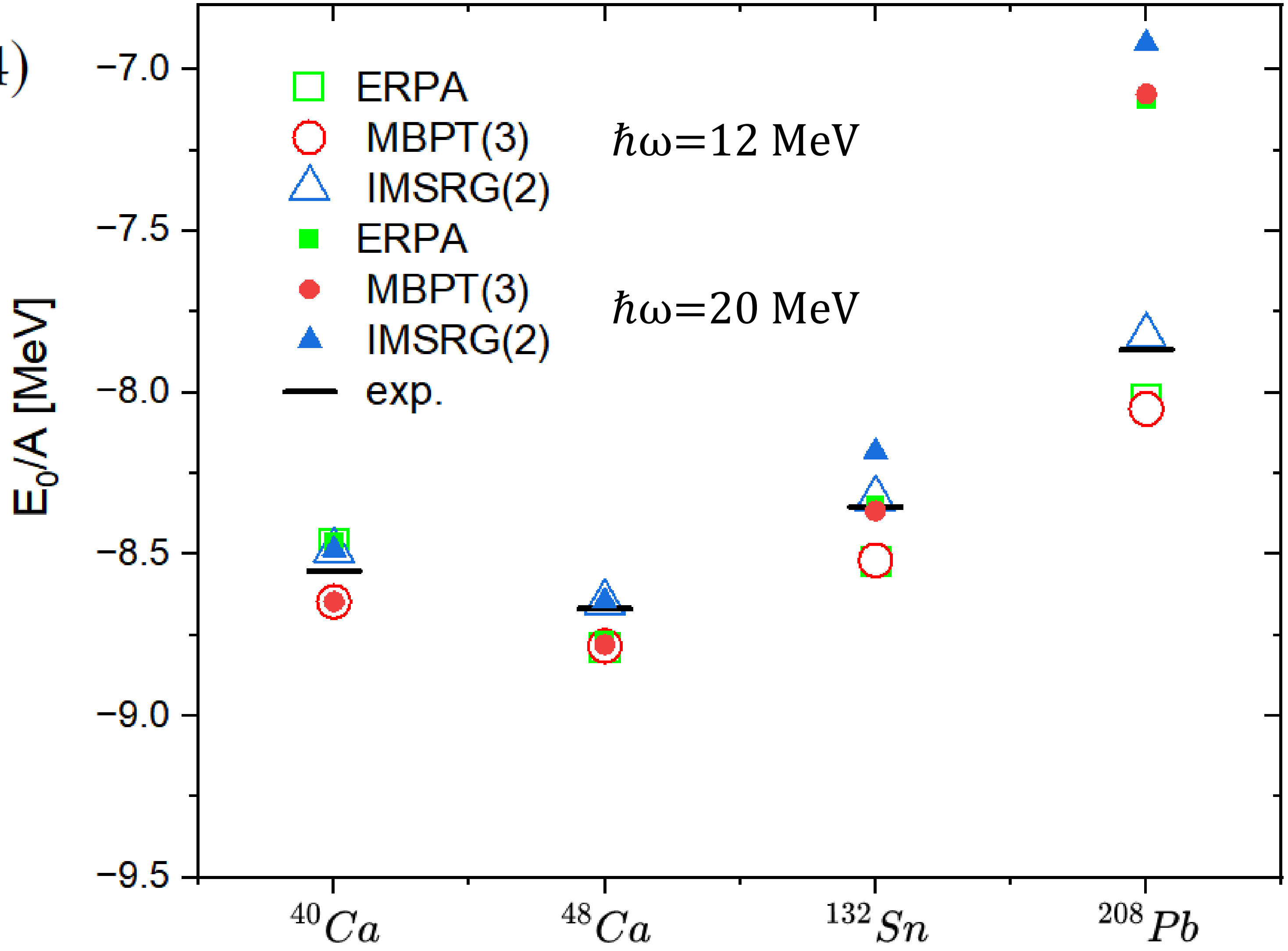


$\hbar\omega$=12 MeV e_{max}=14, e_{3max}=28

$$\boxed{E_0^{\mathrm{RPA}} = E_0^{\mathrm{HF}} - \sum_{\nu} E_\nu \sum_{p>\mathrm{F}} \sum_{h<\mathrm{F}} |Y_{ph}^{\nu}|^2}$$

Binding energies

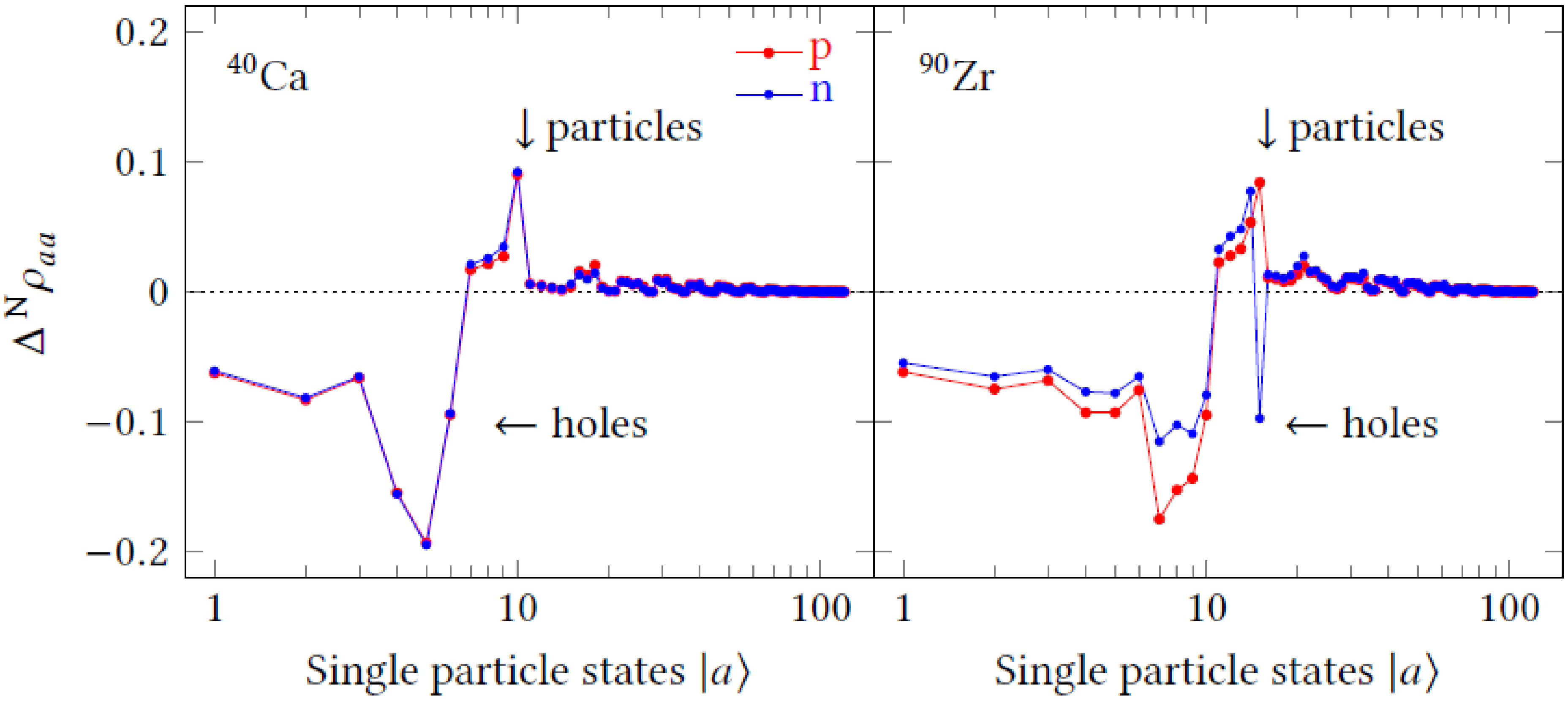
ΔN^2LO_{GO} (394)



e_{max}=14, e_{3max}=28

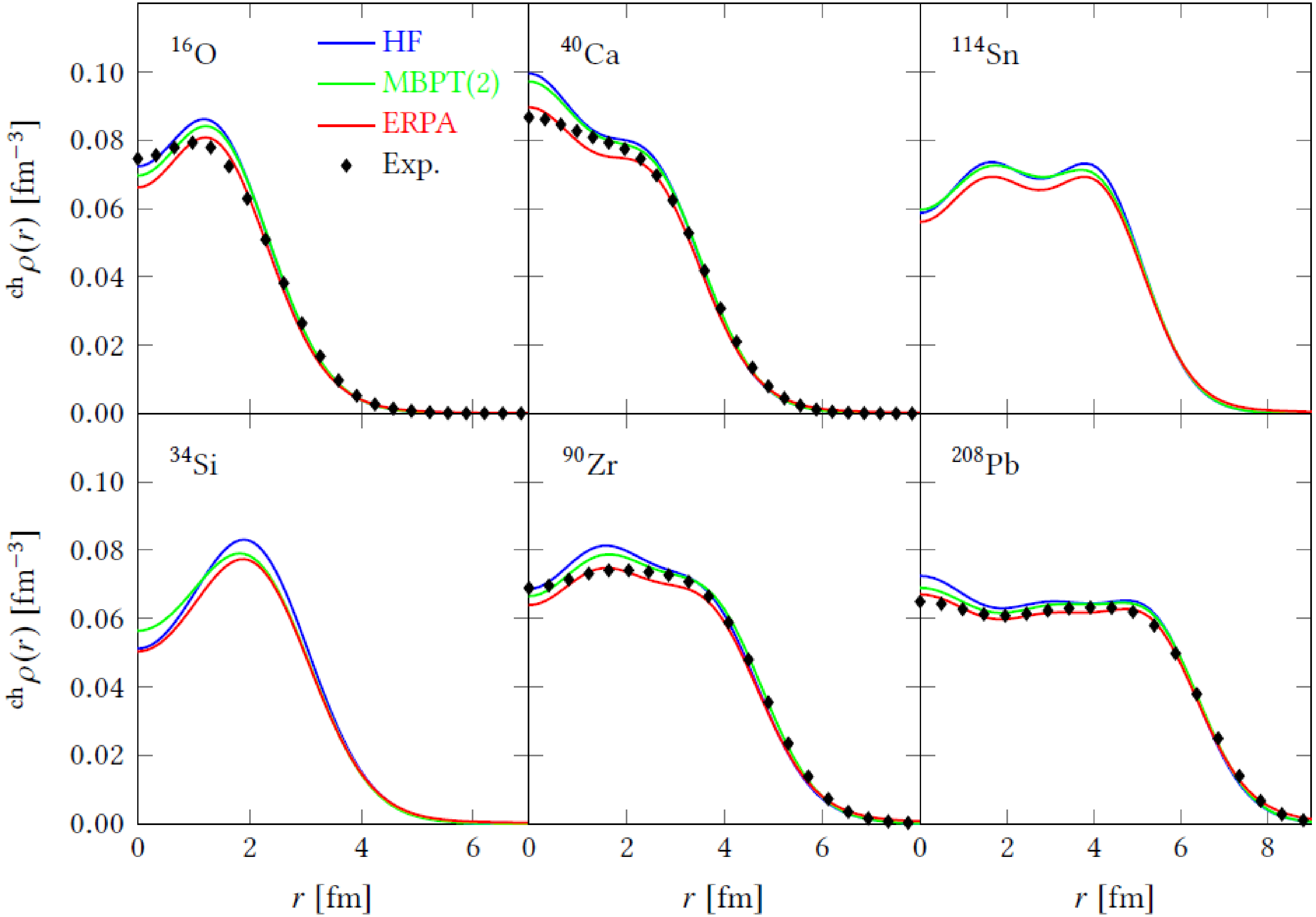
Single-particle occupation depletion in ERPA

$$\Delta\, {}^{\mathrm{N}}\rho_{aa} \equiv {}^{\mathrm{N}}\rho_{aa}^{\mathrm{ERPA}} - {}^{\mathrm{N}}\rho_{aa}^{\mathrm{HF}}$$



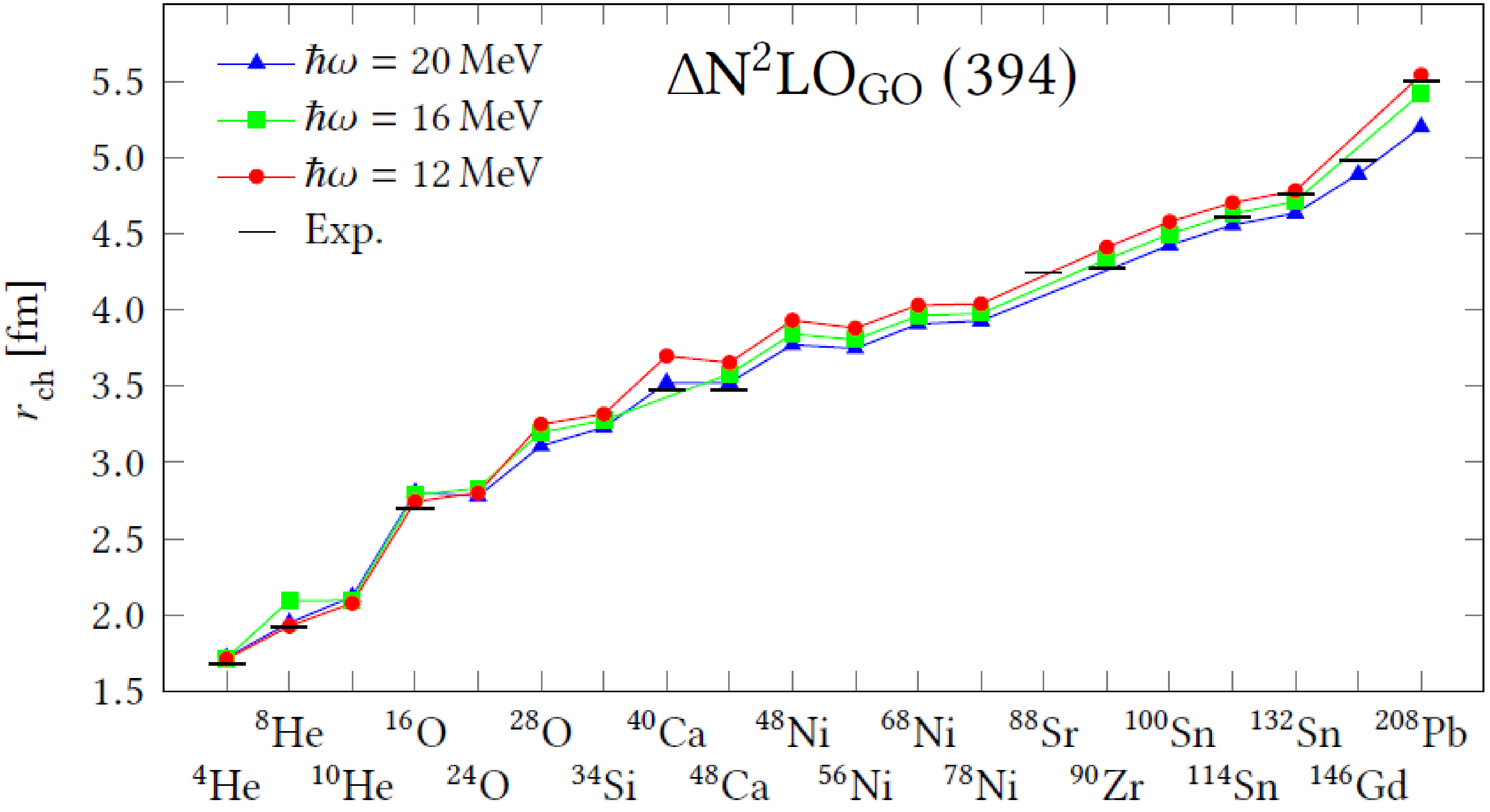
Charged densities

ΔN^2LO_{GO} (394)



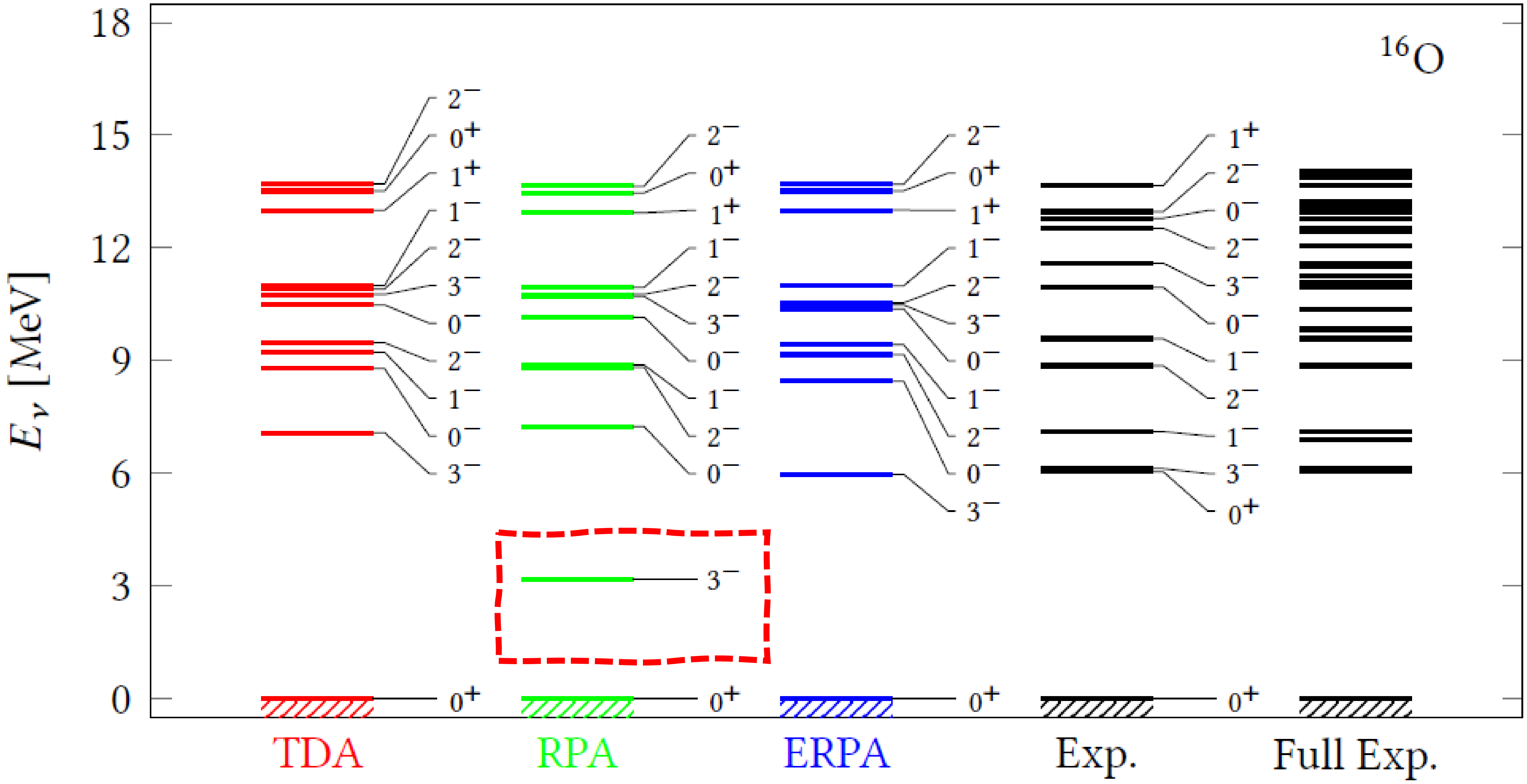
$\hbar\omega$=12 MeV e_{max}=14, e_{3max}=28

Charged radii

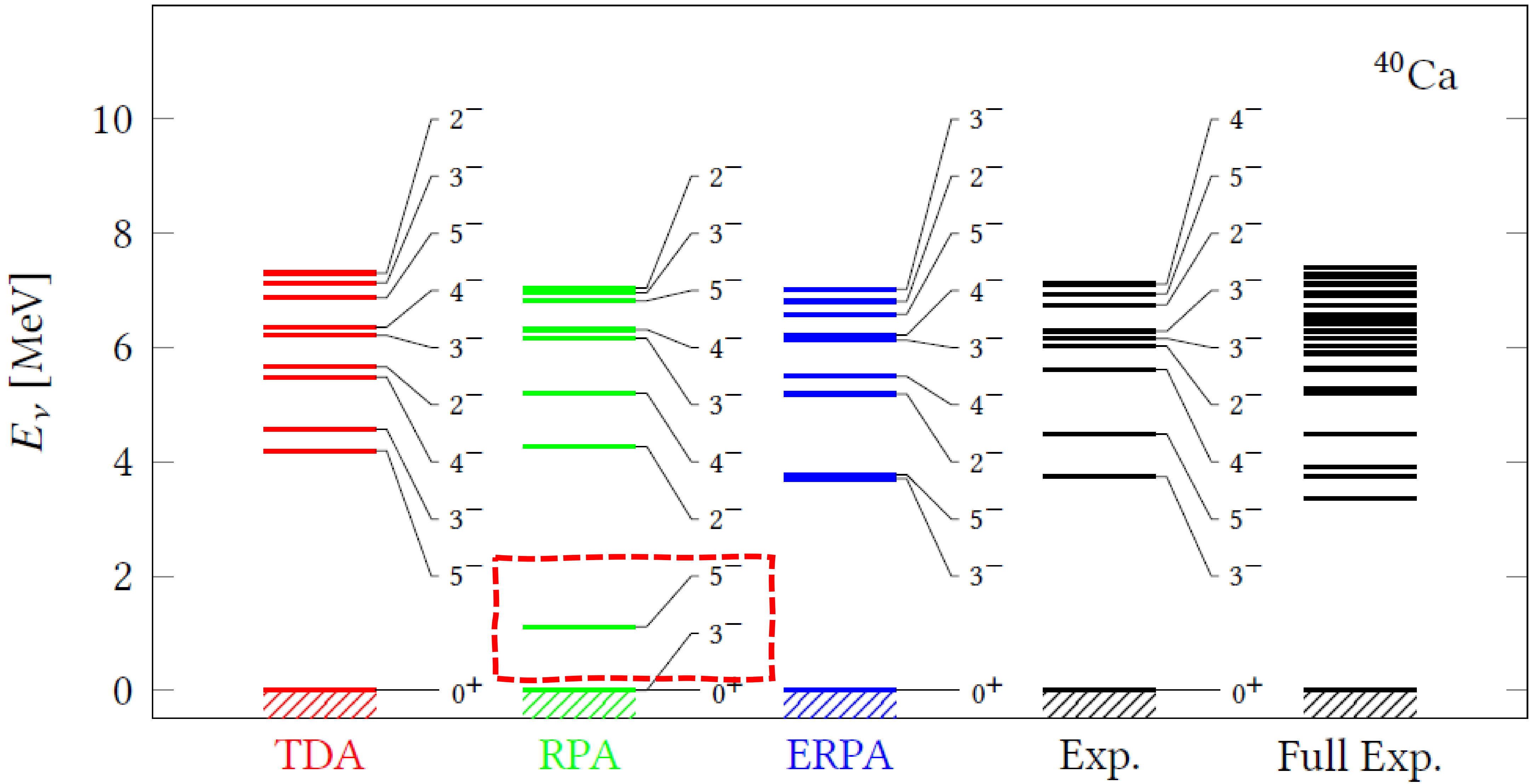


e_{max}=14, e_{3max}=28

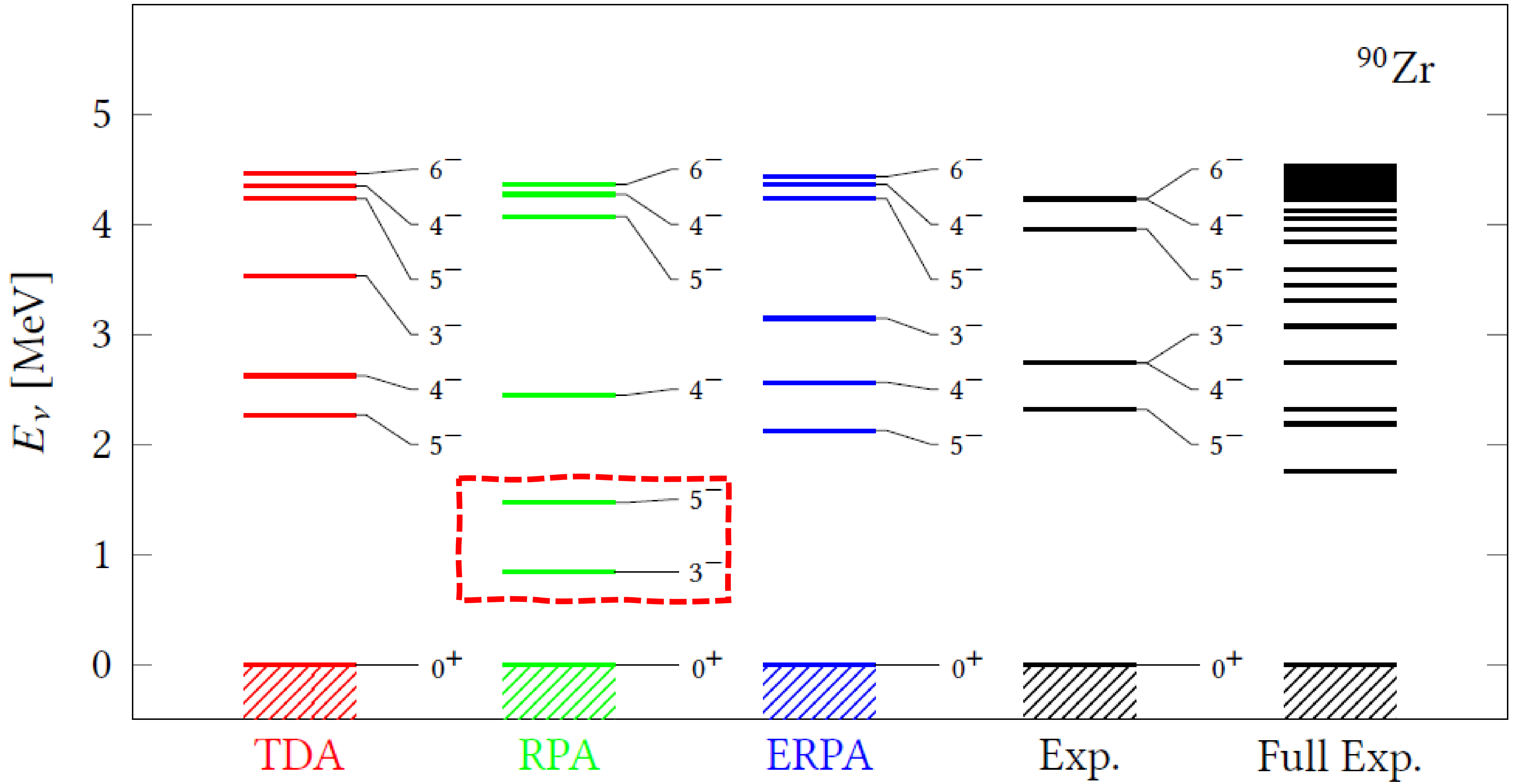
Low-energy spectra



Low-energy spectra



Low-energy spectra



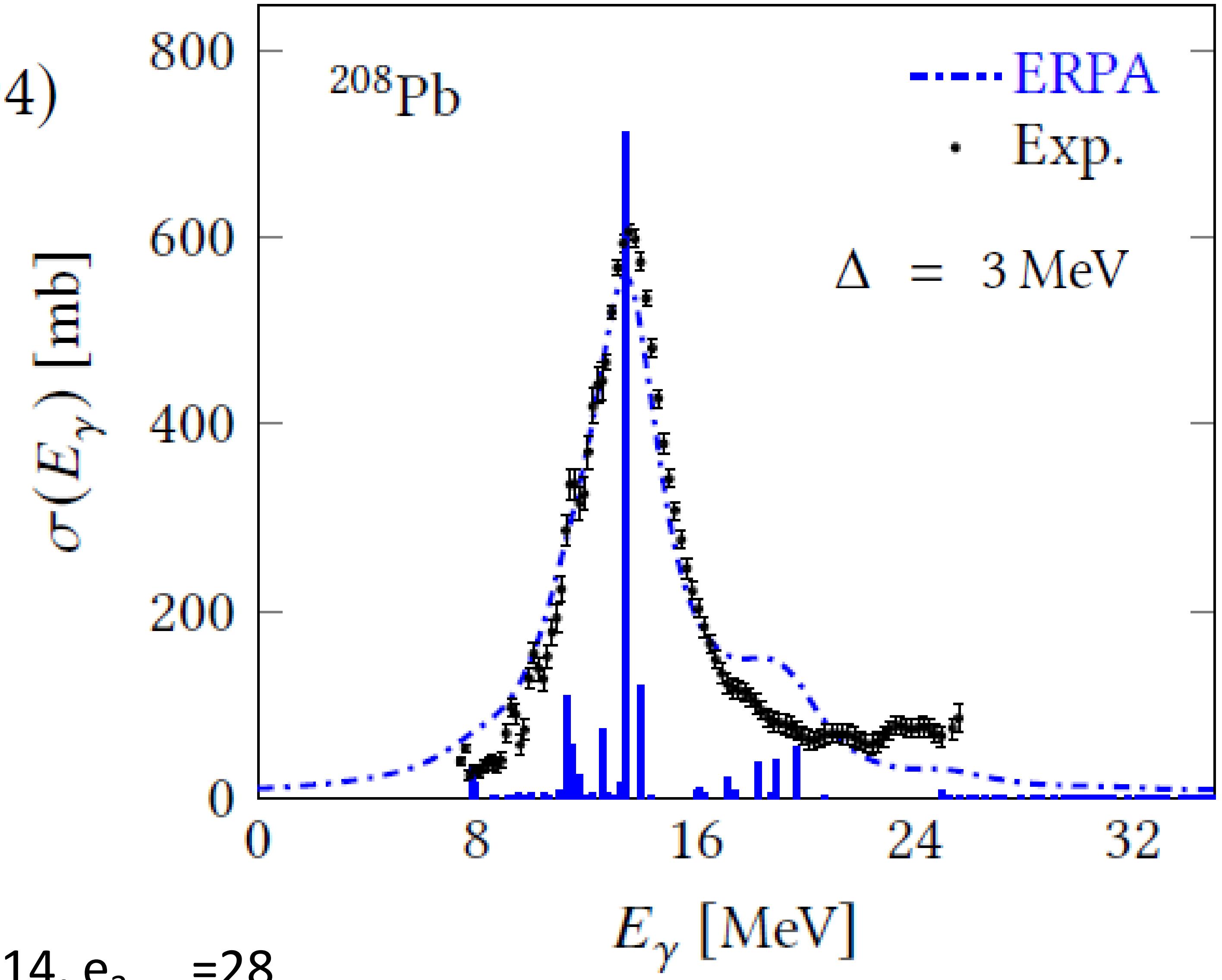
E3 transitions

$B(E3; 0_1^+ \rightarrow 3_1^-)$

Nuclide	TDA $[10^3 \cdot e^2 \cdot fm^6]$	RPA $[10^3 \cdot e^2 \cdot fm^6]$	ERPA $[10^3 \cdot e^2 \cdot fm^6]$	Exp. $[10^3 \cdot e^2 \cdot fm^6]$
^{16}O	0.7	2.9	1.2	1.5 ± 0.1
^{40}Ca	5.4	14.8	12.0	20.4 ± 1.7
^{48}Ca	5.7	17.2	8.6	8.3 ± 0.2
^{90}Zr	27.1	396.6	51.6	108.0 ± 9.0
^{114}Sn	7.0	87.6	29.1	100.0 ± 12.0
^{132}Sn	41.1	133.1	58.3	–
^{208}Pb	123.6	773.8	209.7	611.0 ± 9.0

Giant dipole resonance

ΔN^2LO_{GO} (394)



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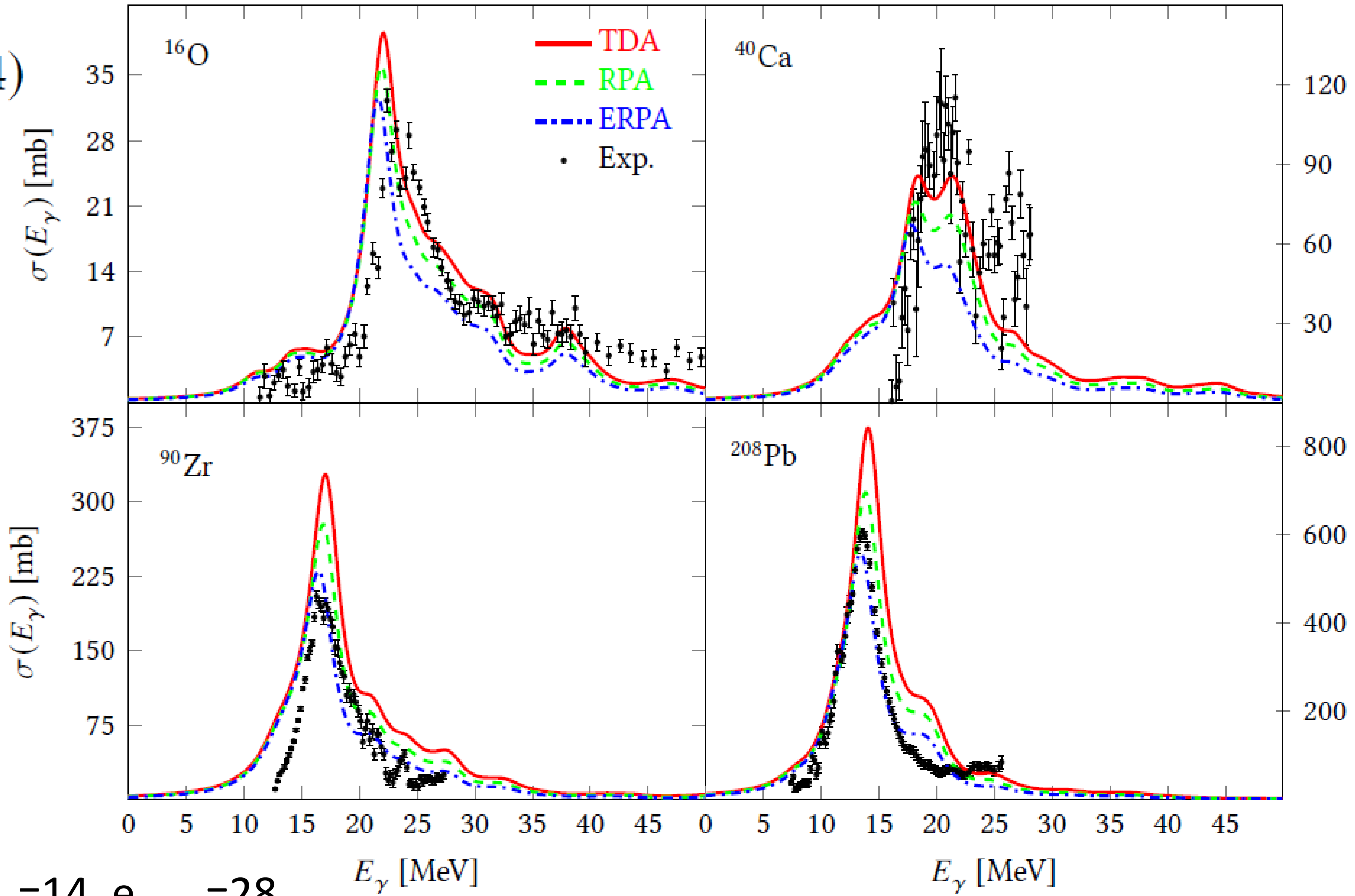
Giant dipole resonance

ΔN^2LO_{GO} (394)



$\hbar\omega$=12 MeV e_{max}=14, e_{3max}=28

Summary

- first implementation of ERPA with NN+NNN chiral potentials, in particular $\Delta N^2LOGO(394)$
- significant improvement with respect to RPA, reasonable description of **correlation energies** and some **low-lying levels** and gross properties of **GDRs** in spherical closed subshell nuclei

Prospects:

- more systematic calculations with different NN+NNN potentials
- quasiparticle extension
- complex configurations

Collaborators

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Thank you for you attention!