Shape coexistence of octupole shapes in the superheavy nucleus ^{286}No

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Peter Ring

Technical University Munich

Peking University

Collaborators:

Fangfang Xu, Li Bo, Pengwei Zhao

Content

- Why Covariant Description of Nuclei ?
- Applications for Exotic Deformations in 286No
- Problems with Tensor-Term
- Ab-Initio Derivation of Cov. DFT

Why covariant ?

1) Large fields $V \approx 350$ MeV, $S \approx -400$ MeV (from QCD)
2) Large spin-orbit splittings in nuclei (not fitted)
3) Success of relativistic Brueckner calculations
4) No three-body forces
5) Success of int. energy proton scattering (spin obs.)
6) Relativistic saturation mechanism
7) Consistent treatment of time-odd fields
8) Natural explanation of pseudo-spin symmetry
9) Connection to underlying theories ?
10) Use as many symmetries as possible !

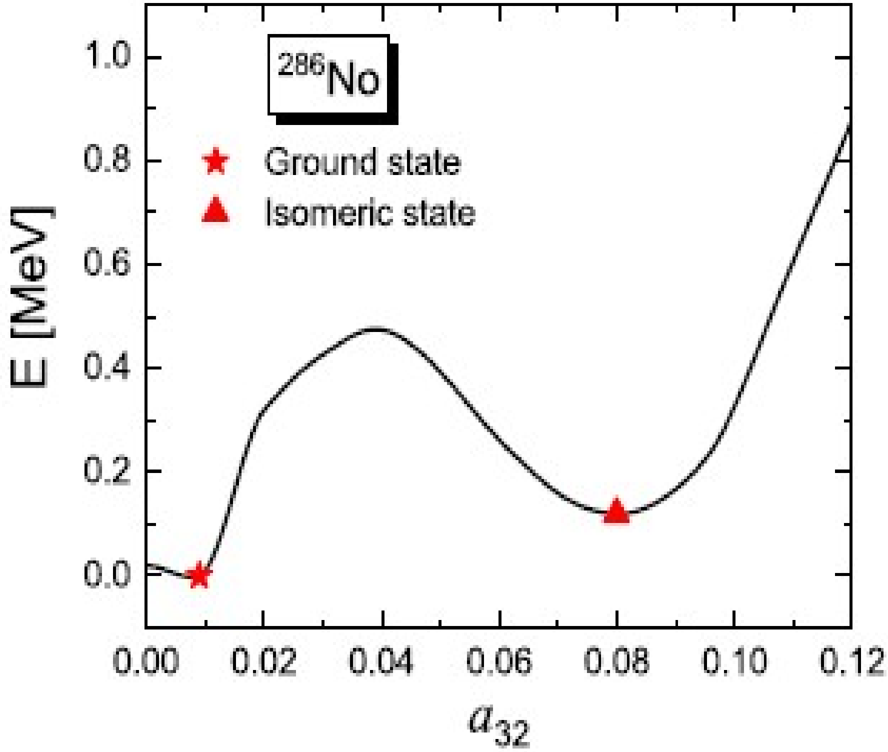
- many exotic deformations have been discussed in recent years
- we investigate the superheavy nucleus ^{286}No
- we use covariant density functional theory
- with the parameter set PC-PK1
- we solve the Dirac-equation on a 3D lattice space
- the intrinsic frame is defined by the octupol deformation
- the energy surface is obtained for

  5 quadrupole deformations $\alpha_{2\mu}$ (μ=-2,-1,0,1,2) and

  4 octupole deformations $\alpha_{3\mu}$ (μ=0,1,2,3)

Energies for groundstate and isomer

$a_{2\mu}$	a_{30}	a_{31}	a_{32}	a_{33}	E [MeV]
0.00	0.07	0.07	0.01	0.07	-2046.65
0.00	0.00	0.00	0.08	0.00	-2046.53



Calculations for ^{286}No

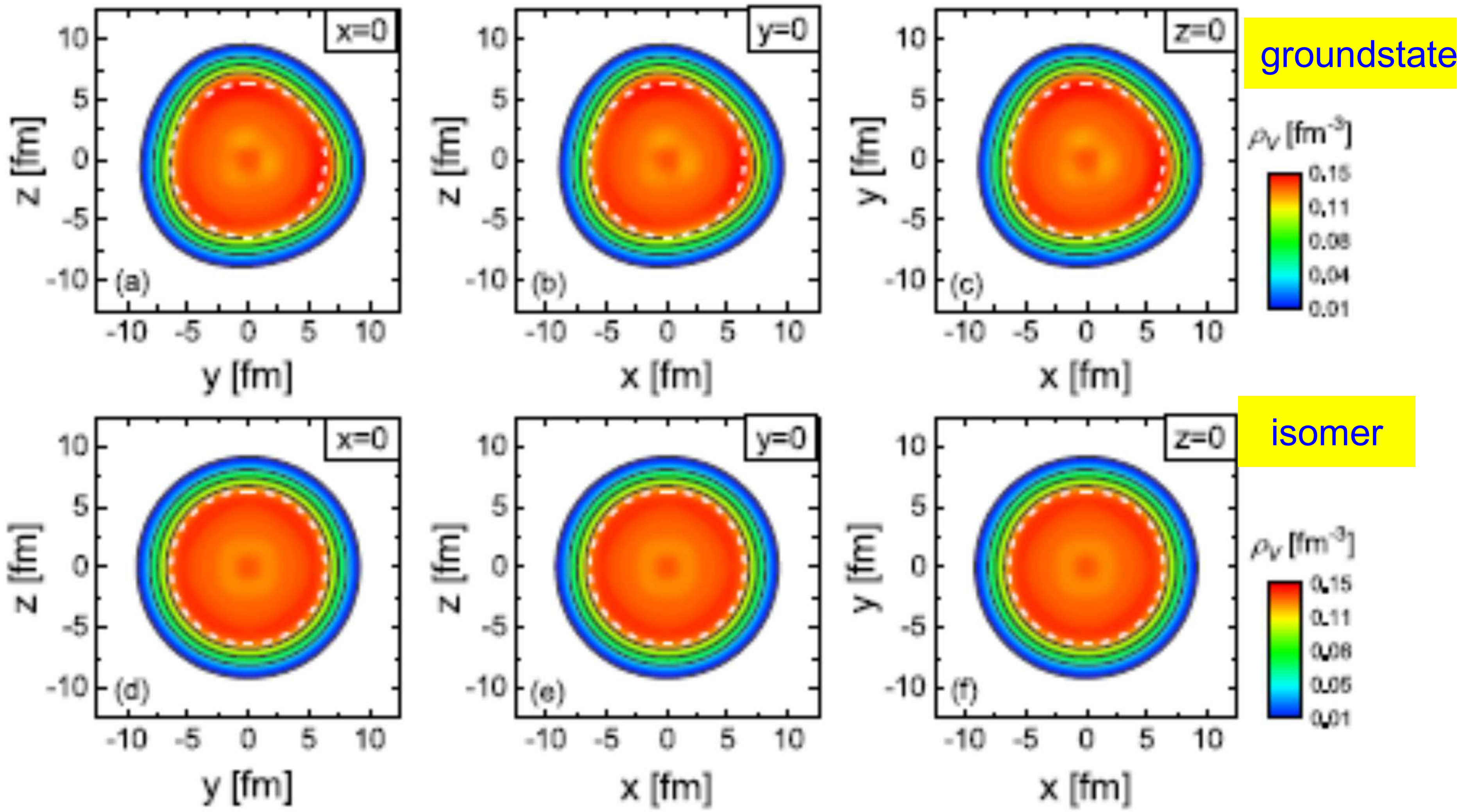
g.s: pure non-axial octupole

isomer: tetrahedral

isomer 120 keV

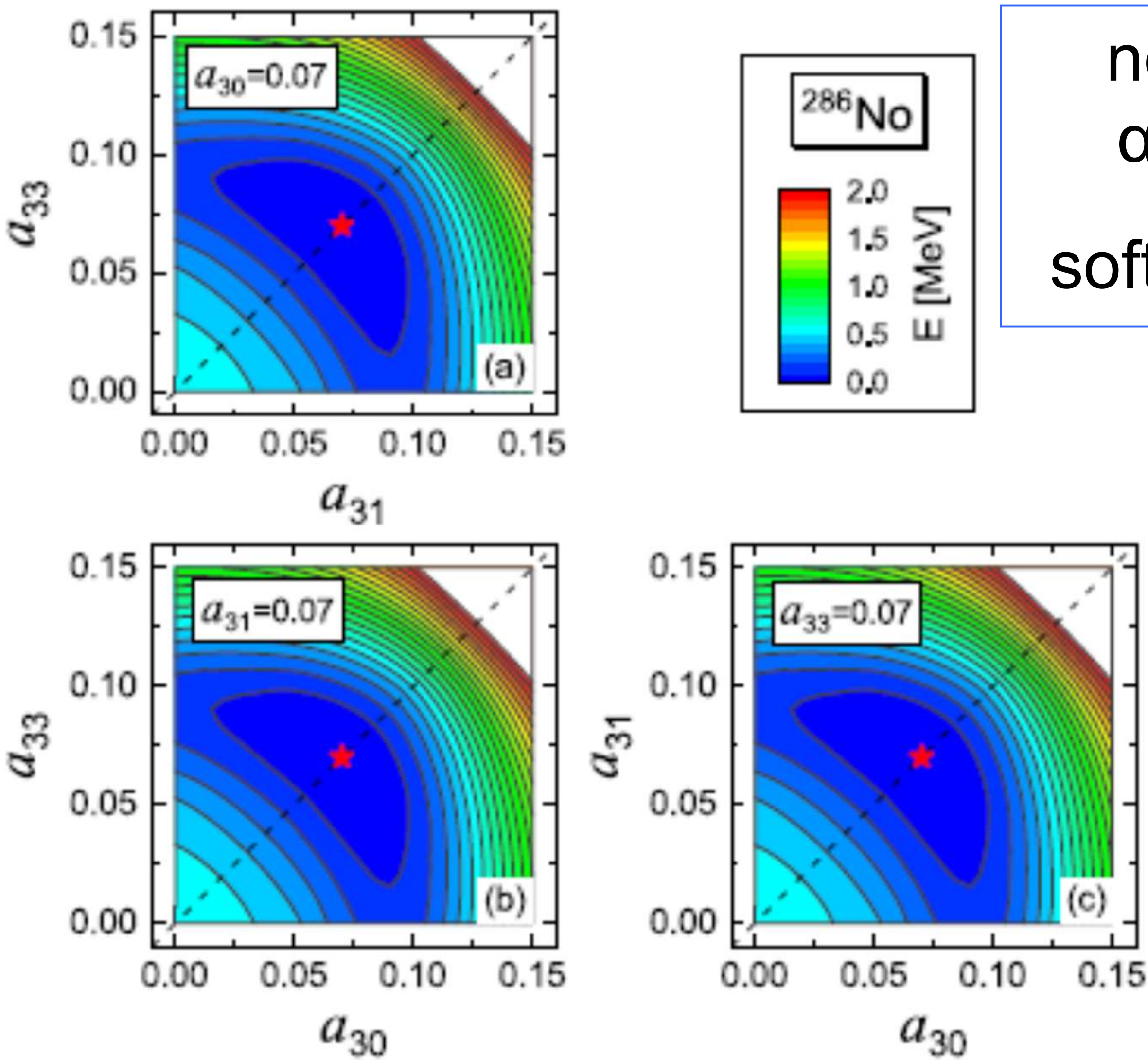
barrier 500 keV

Density distributions



FU Fangfang, LI Bo, P.R., ZHAO Pengwey, Phys. Lett. B 856 (2024) 138893

Softeness around the groundstate

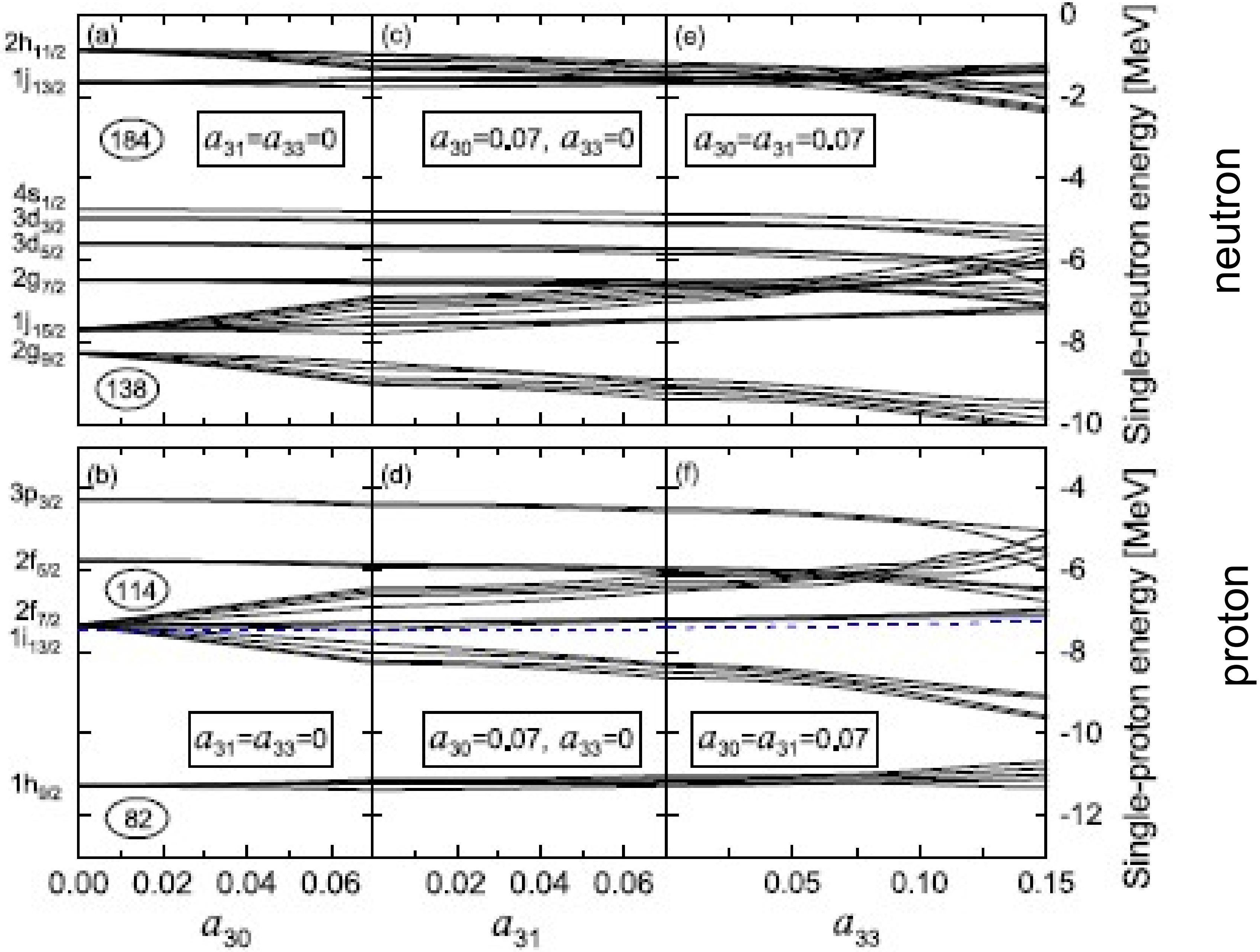


non-axial groundstate

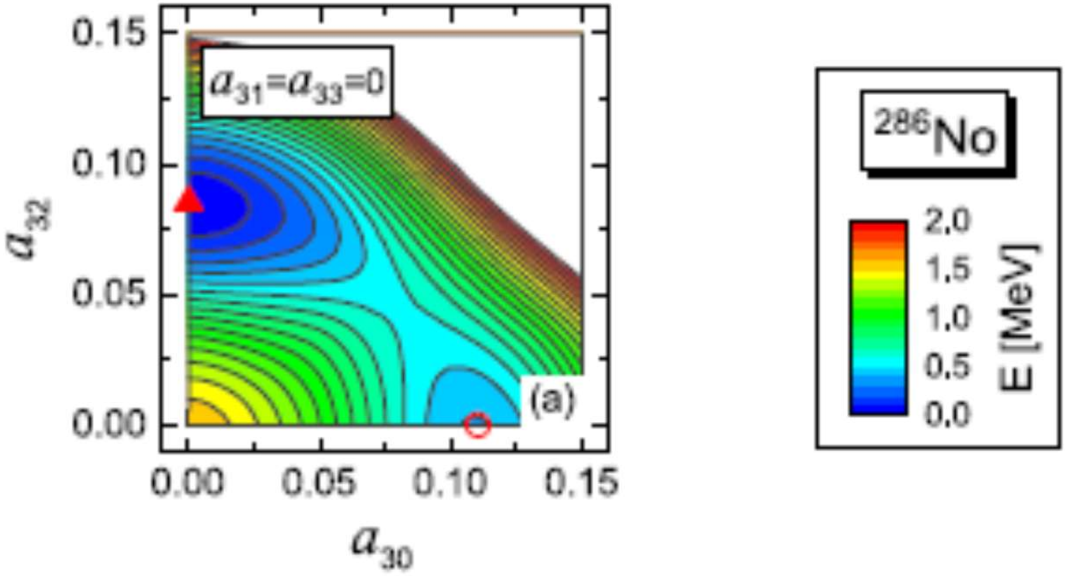
$\alpha_{30} = \alpha_{31} = \alpha_{33} = 0.07$

soft in octupole directions

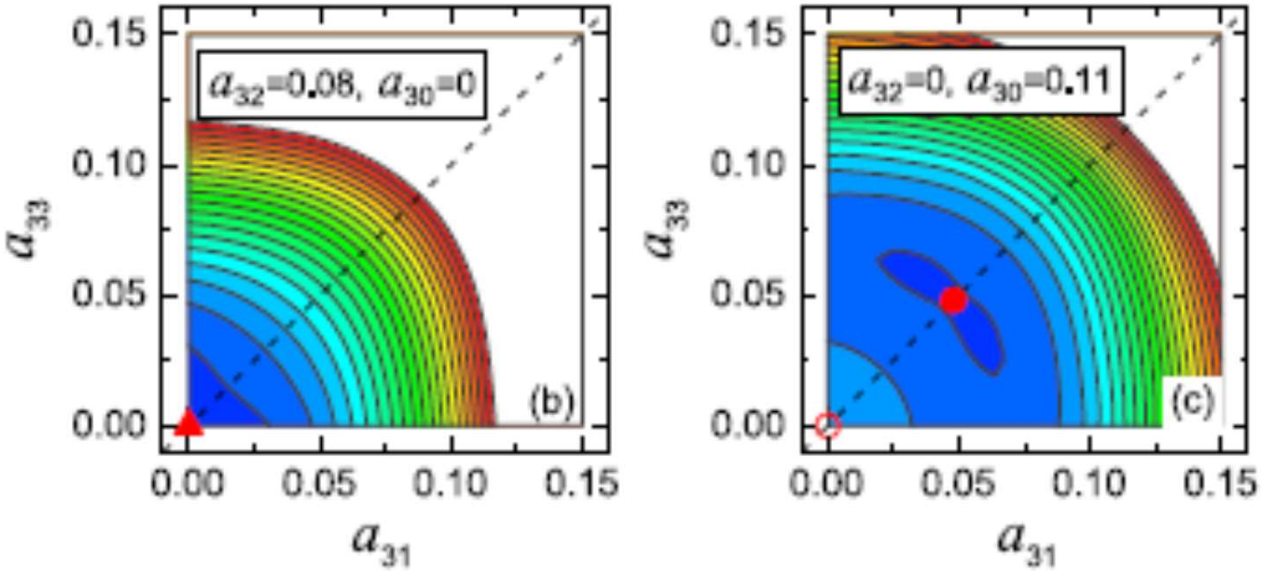
Single particle energies



Potential energy surfaces in:



In the α_{30},α_{32} plane



in α_{31},α_{33} plane

	α_{30}	α_{31}	α_{32}	α_{33}
non-axial oct. g.state	0.7	0.7	0.01	0.7
tetrahedrial isom.st.	0	0	0.08	0
prear-like isom.state	0.11	0.5	0	0.5

Conclusions:

- we study the shape of ^{286}No
- we use covariant density functional theory
- on a three-dimensional lattice space

Results:

- we find:
- a non-axial ground state with octupole shape
- a tetrahedral isomeric state with 0.12 MeV excitation energy
- In addition a pear-like isomeric state

Future: calculations with coll. Hamiltonian
to determine vibrational states

Problem

single particle energies $\Leftrightarrow$ tensor forces

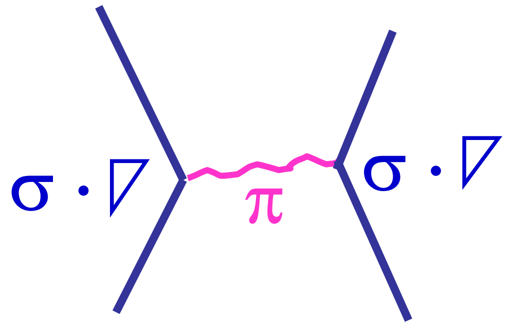
Tensor interaction by pion exchange:

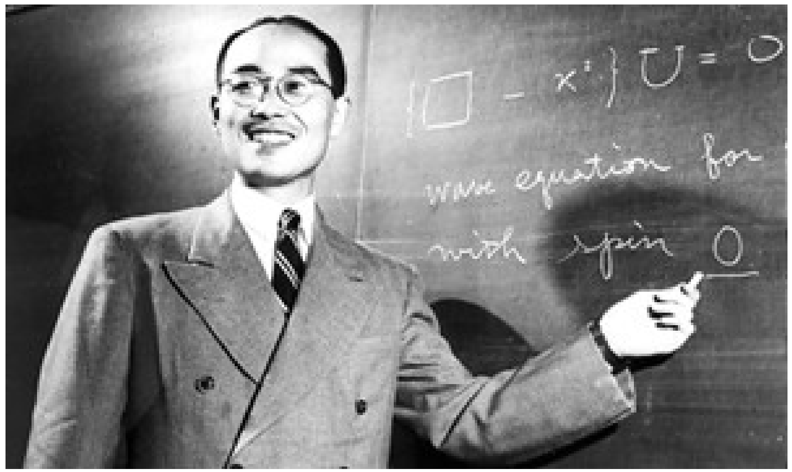
$$V_T = (\tau_1\tau_2)\left([\sigma_1\sigma_2]^{(2)}\, Y^{(2)}(\Omega) \right) Z(r)$$

contributes only to **S=1** states

relative motion

π meson : primary source

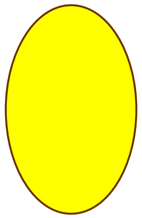




Yukawa

thanks to Takaharu Otsuka

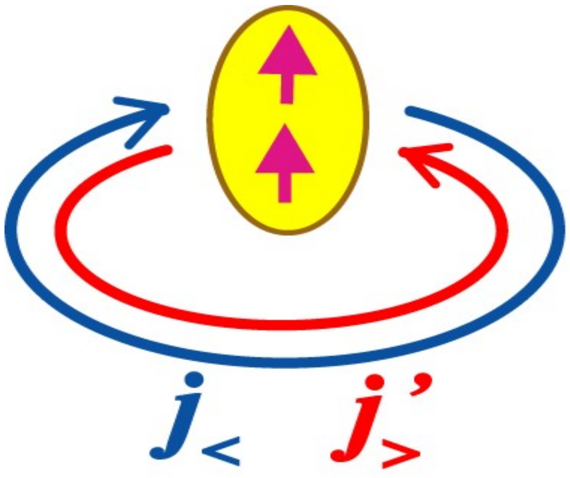
Monopole effects due to the tensor force:

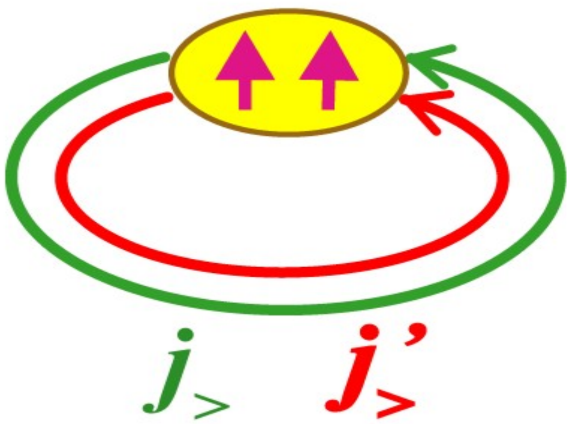


wave function of relative motion

↑↑ spin of the nucleon

large relative momentum **small relative momentum**





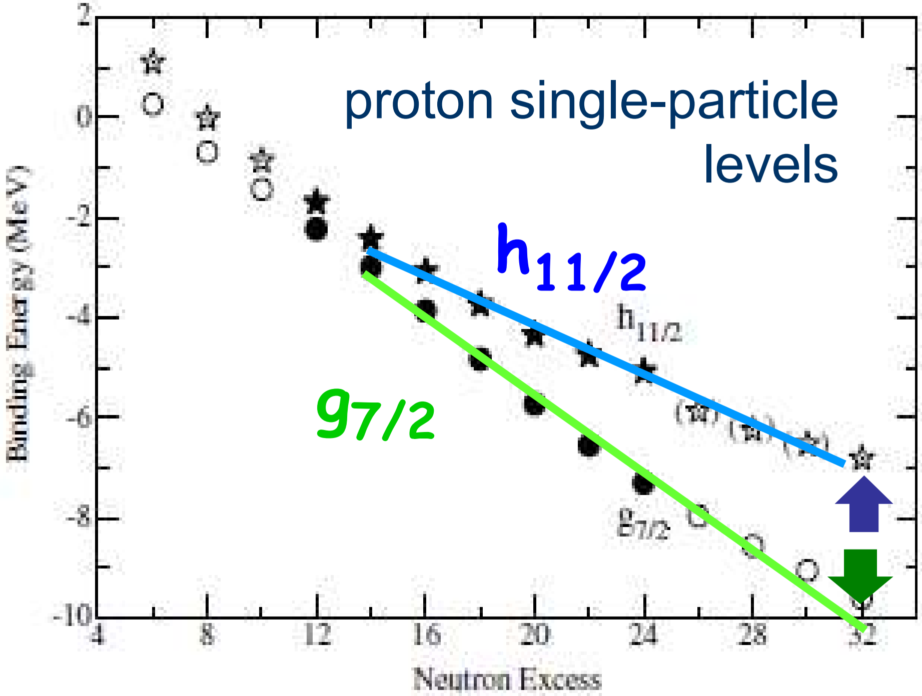
attractive **repulsive**

$$j_> = \ell + \tfrac{1}{2}, \quad j_< = \ell - \tfrac{1}{2}$$

Otsuka et al, PRL 95, 232502 (2005)

An example are the Sb isotopes (Z=51):

Experiment: J. P. Schiffer et al., Phys. Rev. Lett, 92 162501, (2004)



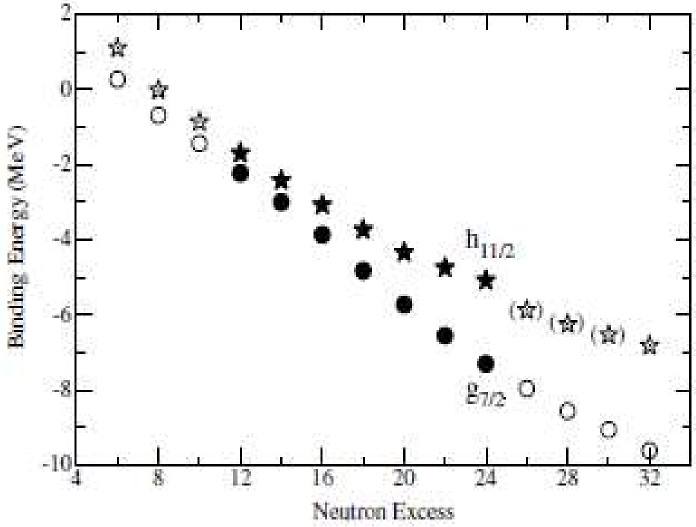
change is driven by neutrons in $1h_{11/2}$

$h_{11/2}$ - $h_{11/2}$ repulsive

$h_{11/2}$ - $g_{7/2}$ attractive

tensor force increases splitting by ~ 2 MeV

Without tensor, no EDF (Skyrme, Gogny, RMF) explains these facts.

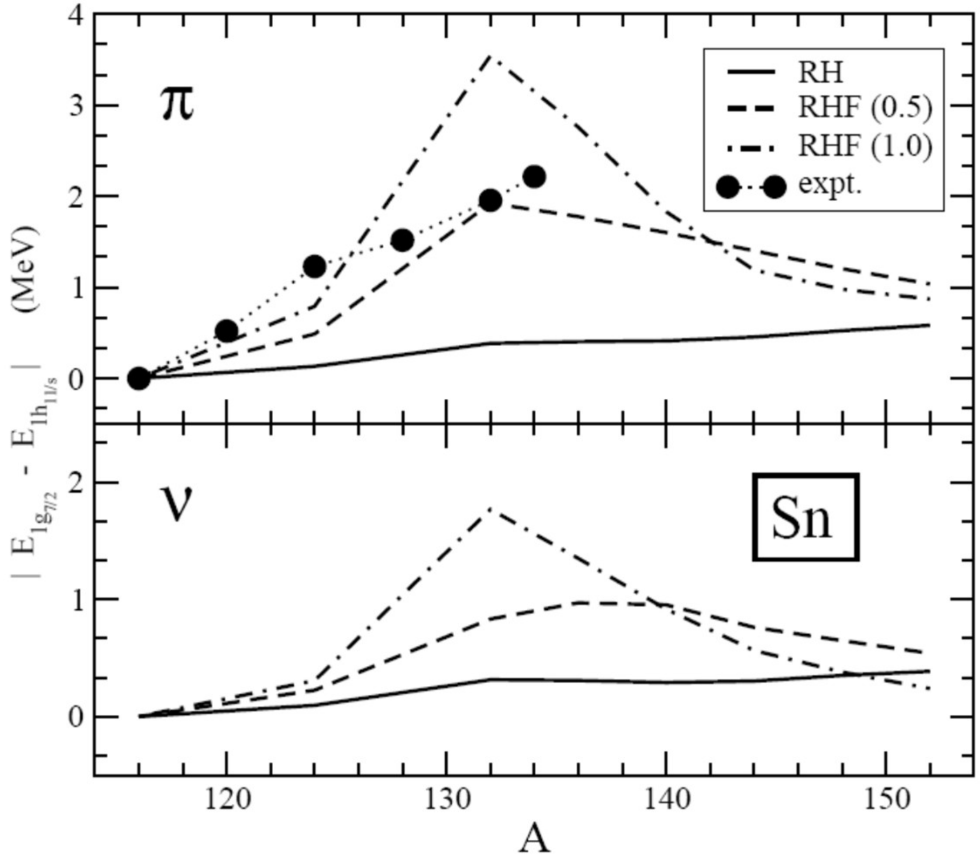


Experiment:
J. P. Schiffer et al., Phys. Rev. Lett, 92 162501, (2004)

$E_{1h_{11/2}} - E_{1g_{7/2}}$

Effect of a tensor force in Relativistic Hartree-Fock

RH → RHF



Theory: Lalazissis, Karatzikos, Serra, Otsuka, P.R., Phys. Rev. C89, 041301 (2009)

Particle-vibrational coupling (PVC) energy dependent self-energy

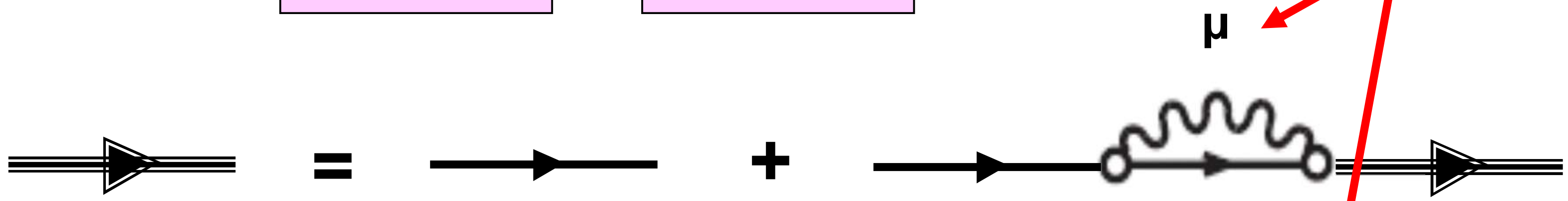
eff. Potential v_{eff}
$\rightarrow$ self-energy Σ

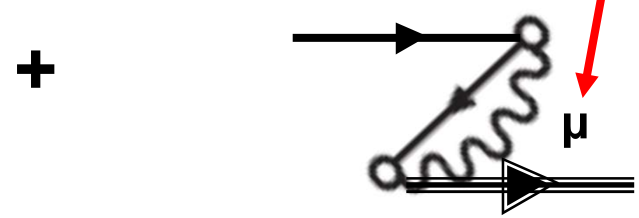
$$\Sigma = S + V + \Sigma(\omega)$$

mean field

pole part

RPA-modes





Dyson equation

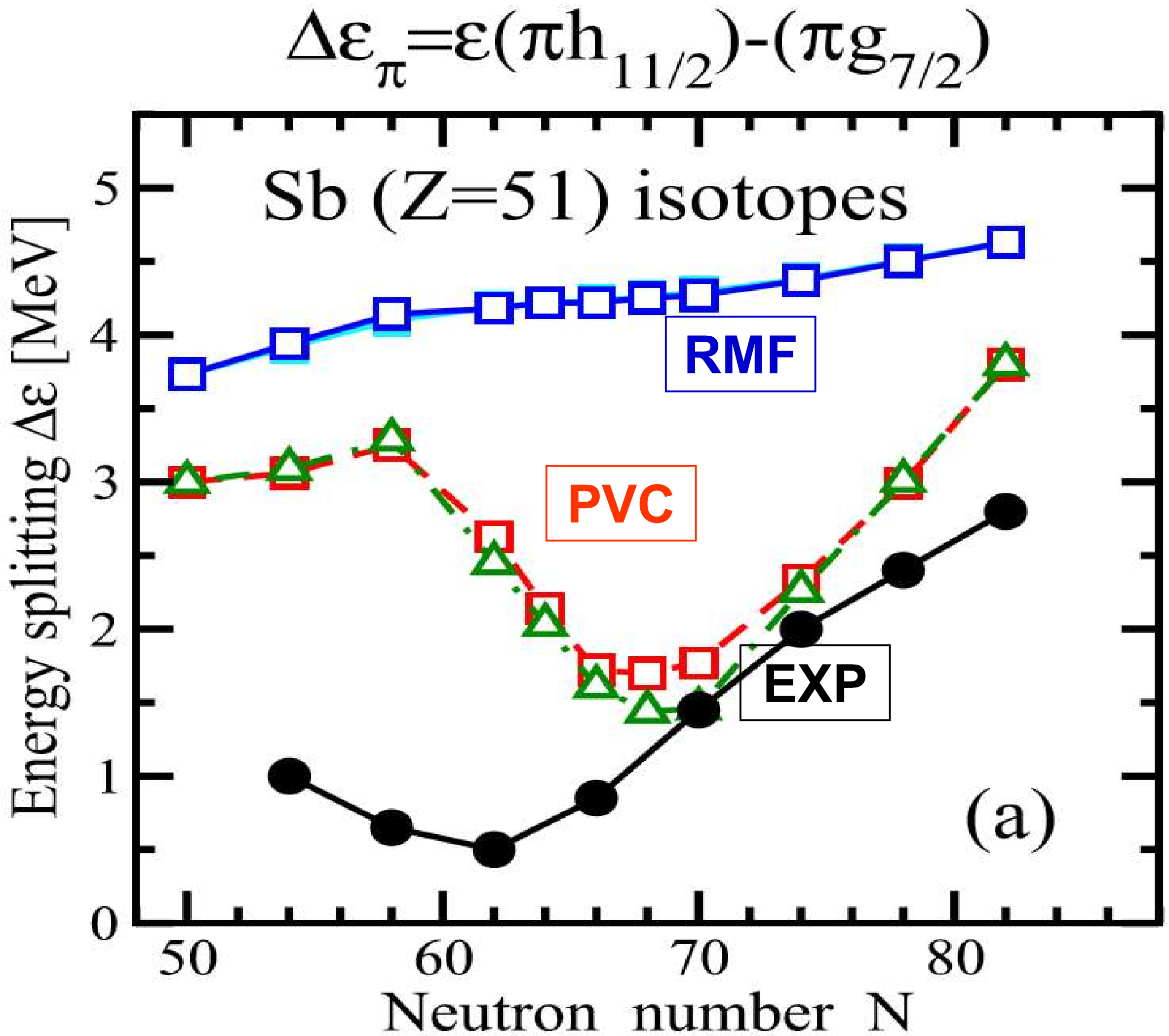
Dyson-equation

single particle strength:

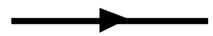
$$S_\nu = \left[1 - \frac{d\Sigma_{\nu\nu}}{d\omega}\Big|_{\omega=\epsilon_\nu}\right]^{-1}$$

non-relativistic investigations:
Ring, Werner (1973)
Hamamoto, Siemens (1976)
Perazzo, Reich, Sofia (1980)
Bortignon et al (1980)
Bernard, Giai (1980)
Platonov (1981)
Kamerdzhiev, Tselyaev (1986)

Influence of Particle-Vibrational Coupling

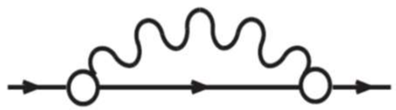


mean field



+

pole part



Experiment: J. P. Schiffer et al., Phys. Rev. Lett, 92 162501, (2004)
Theory: A.V. Afanasjev, E. Litvinova, Phys. Rev. C92, 044307 (2015)

- Ab-initio derivation of density functional theory

    first attempts of ab-initio go back to the fifties:

    Brueckner theory:

        based on the mean-field concept

        effective density-dep. interaction: G[ρ]

        mother of nuclear density functional theory

- Non-relativistic BHF fails: Three-body forces
- 1980: Relativistic BHF: no NNN necessary

  problems:

  a) no exact solution of RBHF in nuclear matter

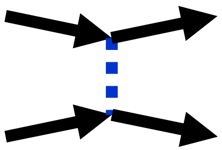
      many different approximations

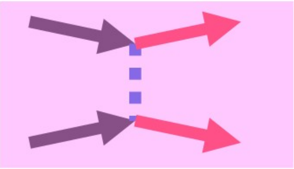
  b) we need RBHF in finite nuclei (tensor?)

Brueckner theory (1958):

Brueckner,Gammel, Phys. Rev. 109, 1023 (1958)

- The nucleons in the interior of the nuclear medium do not feel the same bare force V, as the nucleons feel in free space.



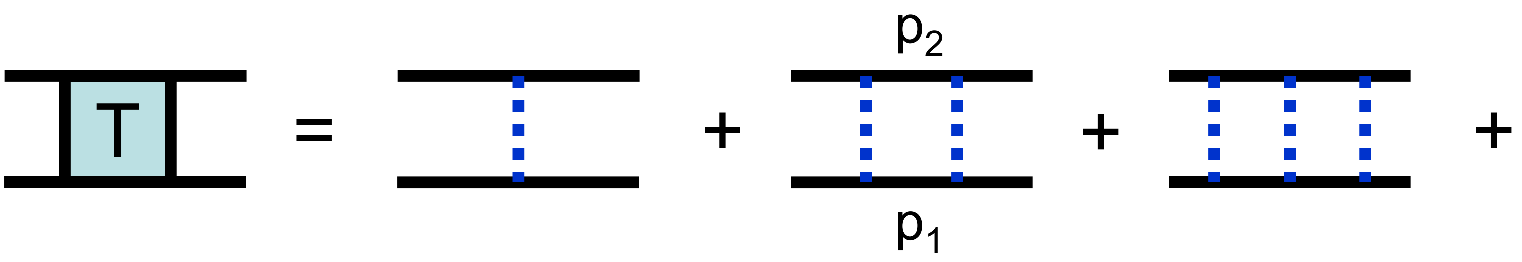


- They feel an effective force G.
- The Pauli principle prohibits the scattering into states, which are already occupied in the medium.
- Therefore this force $G(\rho)$ depends on the density
- This force G is much weaker than the bare force V.
- Nucleons move nearly free in the nuclear medium and feel only a strong attraction at the surface (shell model)

Free nucleon-nucleon scattering:

Lippmann-Schwinger-Eq.

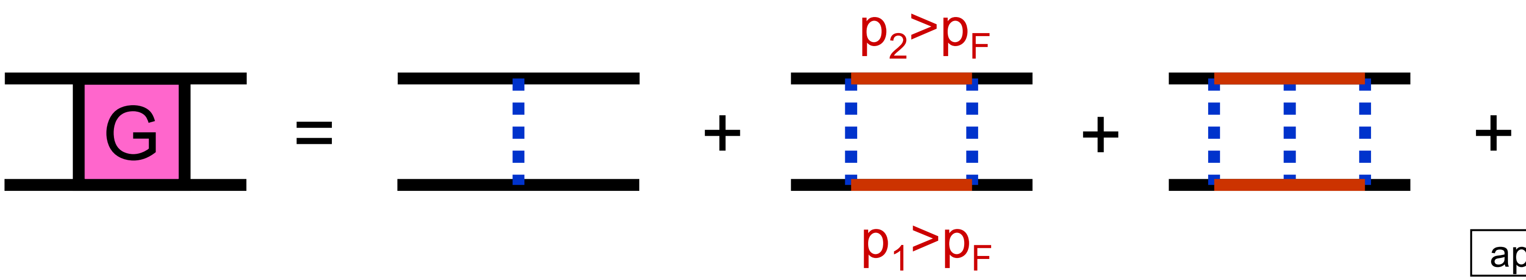
$$\langle \mathbf{k}_1\mathbf{k}_2|T|\mathbf{k}_1'\mathbf{k}_2'\rangle = \langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{k}_1'\mathbf{k}_2'\rangle + \sum_{\mathbf{p}_1\mathbf{p}_2}\langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{p}_1\mathbf{p}_2\rangle \frac{1}{E-\frac{\mathbf{p}_1^2}{2m}-\frac{\mathbf{p}_2^2}{2m}+i\eta}\langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{k}_1'\mathbf{k}_2'\rangle$$



exact

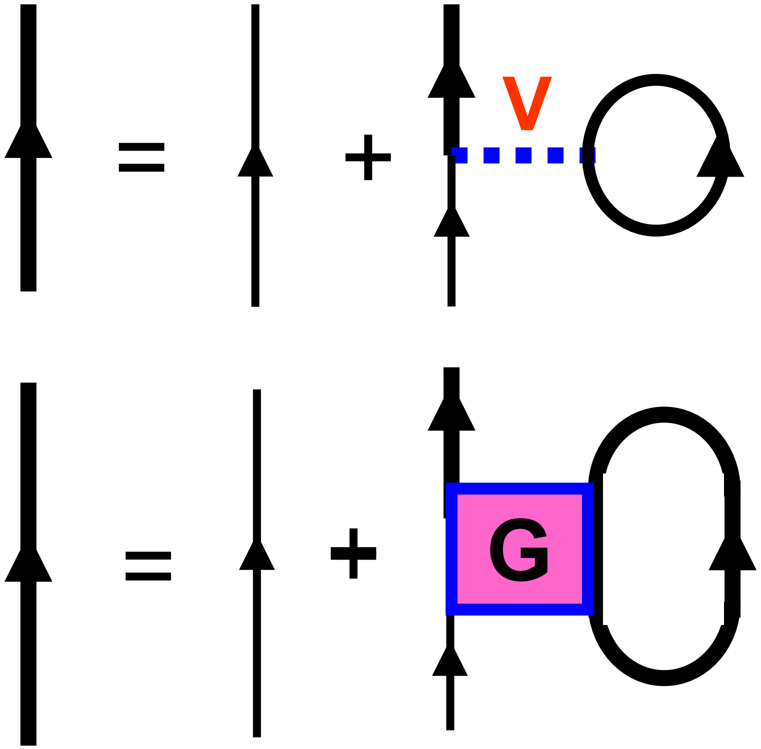
Scattering in the nuclear medium:

Bethe-Goldstone-Eq.



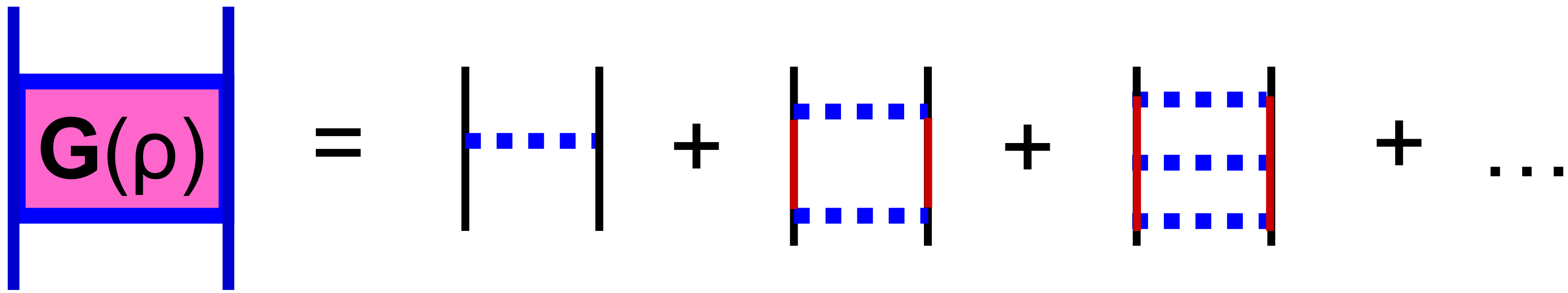
approximation

Ab-initio: Relativistic Brueckner Hartree-Fock:



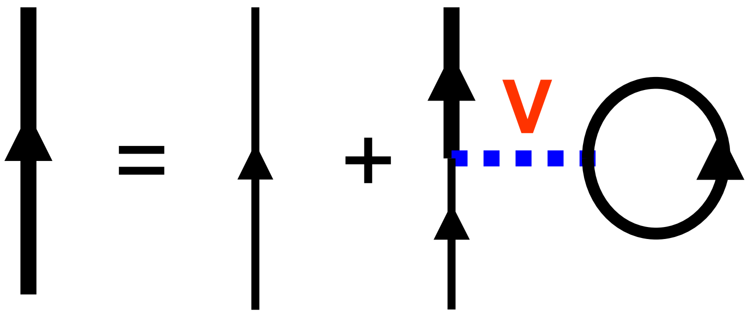
Hartree-Fock

Brueckner Hartree-Fock

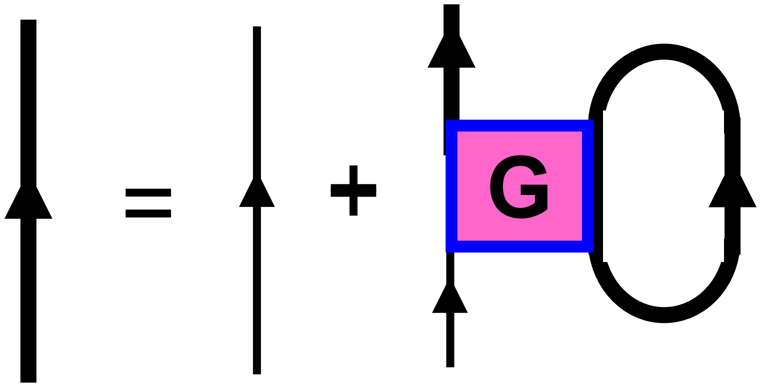


Summing up all ladder diagramms

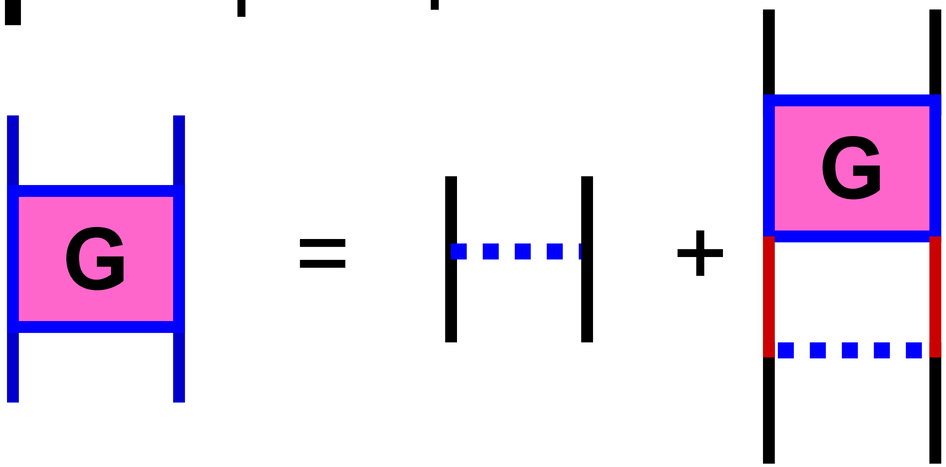
Ab-initio: Relativistic Brueckner Hartree-Fock:



Hartree-Fock



Brueckner Hartree-Fock

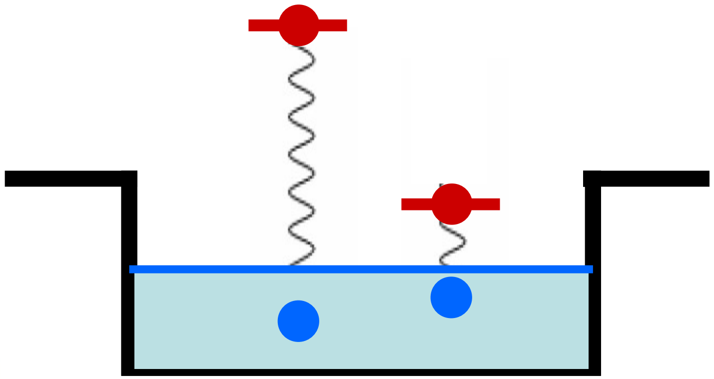


Bethe-Goldstone

Bethe-Goldstone equation:

- ω is the starting energy
- V is realistic interaction
- Q_F is the Pauli operator

$$G(\omega) = V + VQ_F \frac{1}{\omega - H_{HF}} Q_F G(\omega)$$



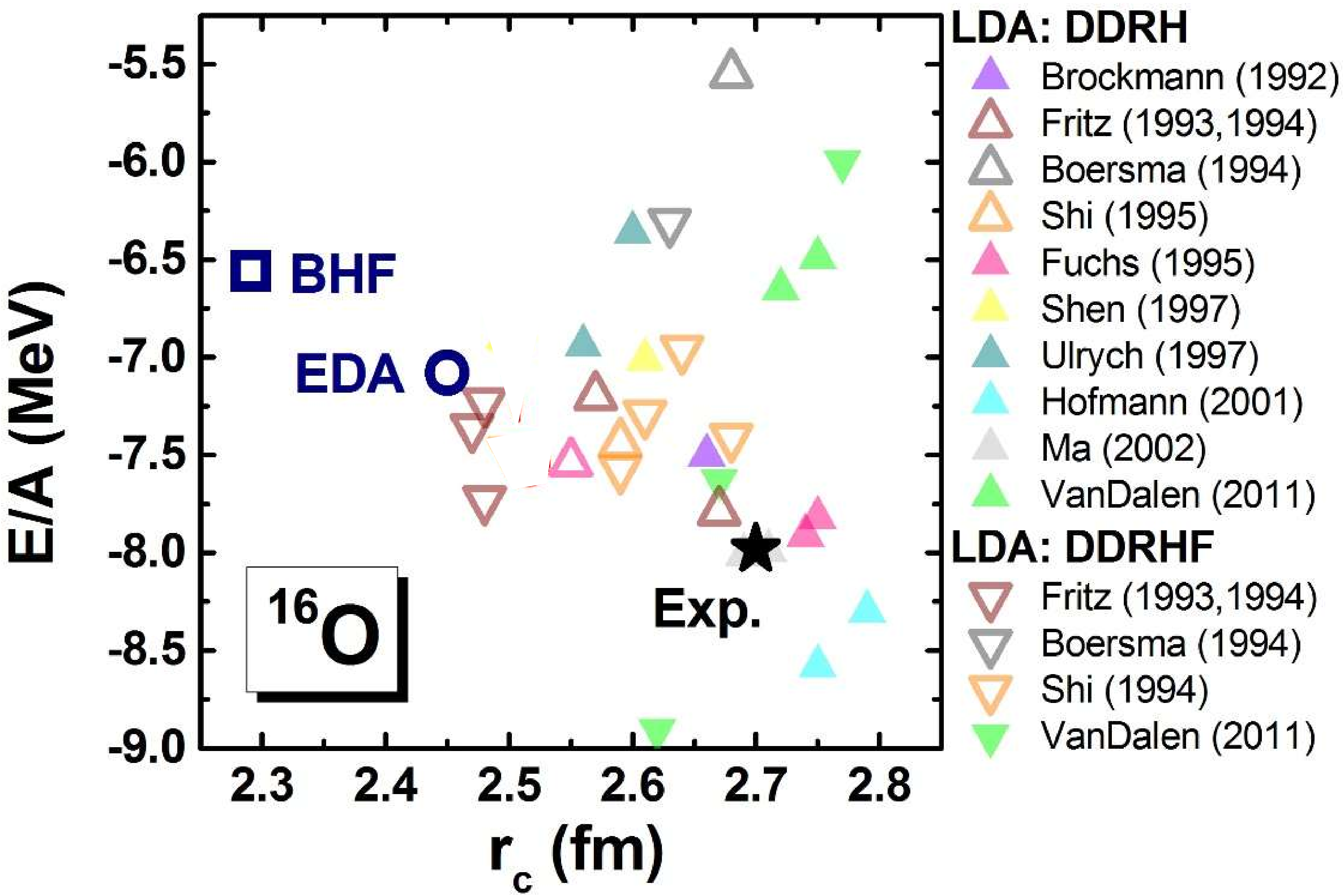
$$G(\omega) = V + VP_F(\omega)G(\omega)$$

$$P_F(\omega) = \sum_{m_1 m_2 > \varepsilon_F} |m_1 m_2\rangle \frac{1}{\omega - \varepsilon_{m_1} - \varepsilon_{m_2}} \langle m_1 m_2|$$

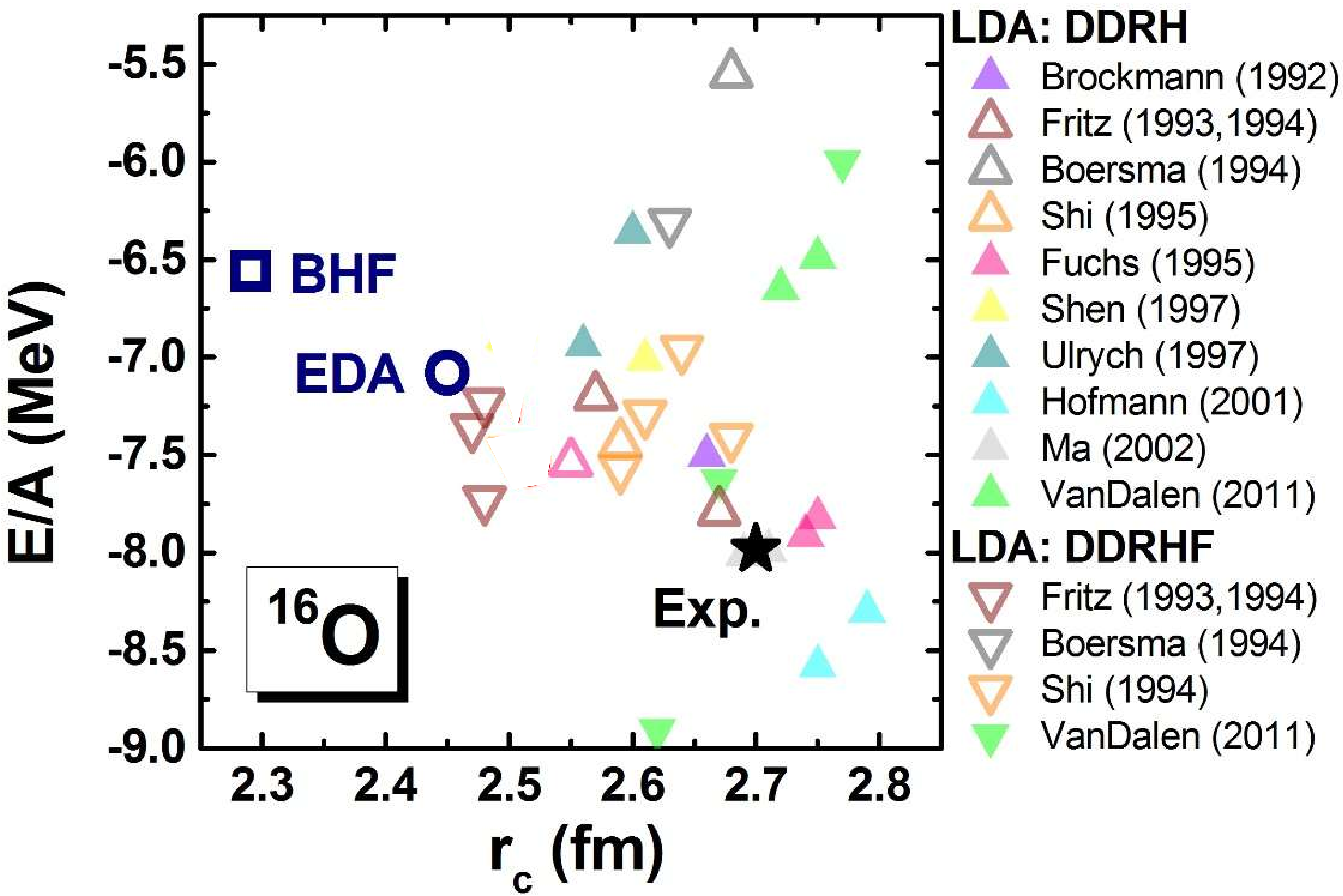
$$G(\omega) = \frac{1}{1 - VP_F(\omega)} V$$

Is solved in each step of the iteration

Relativistic BHf for finite nuclei:

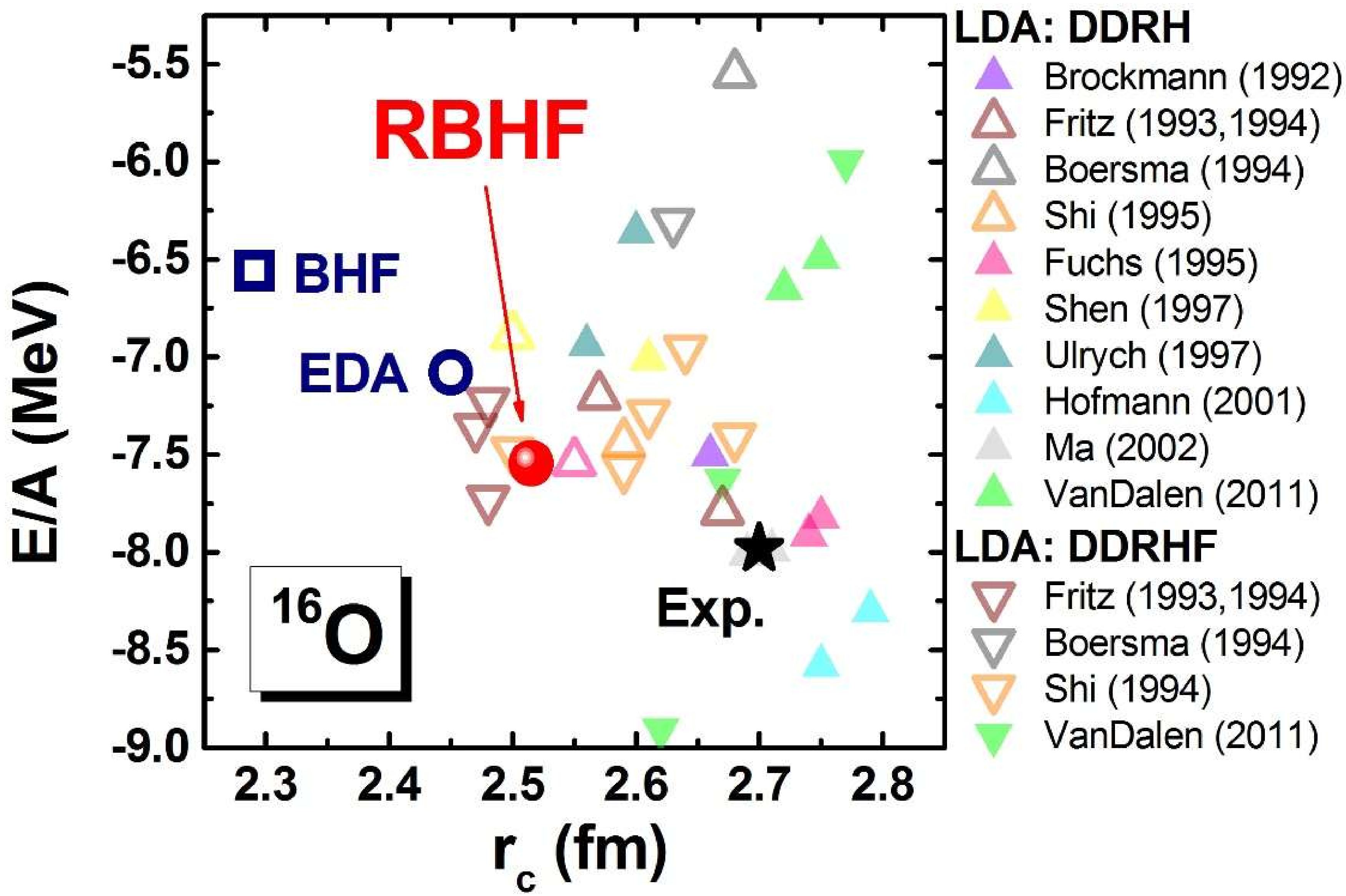


Relativistic BHf for finite nuclei:

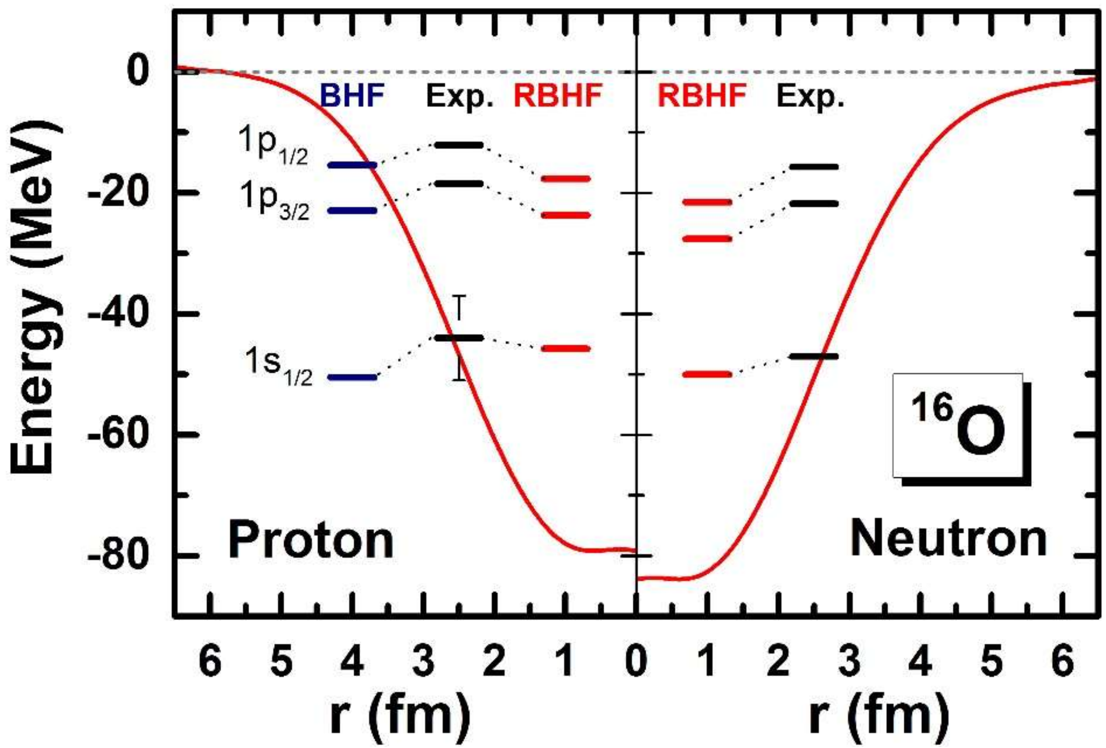


Relativistic BHF for finite nuclei:

S.H. Shen et al (2017).



Single particle spectrum:



Shen, Hu, HZL, Meng, Ring, Zhang, *Chin. Phys. Lett.* **33**, 102103 (2016)

- a first *ab initio* calculations for finite nuclei in the **relativistic** scheme
- **Spin-orbit** splitting is reproduced well from the bare interaction
- **benchmark** for various LDA calculations

Outlook for the future:

- simplify the calculations:

  Brueckner theory with renormalized forces ($V_{low\ k}$) …

  Local density approximation under control
- heavy nuclei and the tensor force
- open shell nuclei: pairing, deformation
- optical potential
- short range correlations

Thanks to my collaborators:

Shihang Shen (Jülich)
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Haozhao Liang (RIKEN)
Sibo Wang (Beijing)
H. Tong (Beijing)

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Thank you