Effective Field Theory approach for radiative corrections to superallowed β decays.

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with V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, S. Gandolfi, G. King, S. Novario

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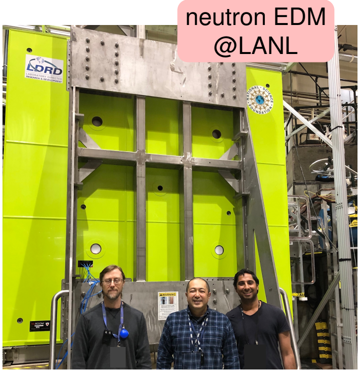
Finding BSM: energy vs. precision frontier



- SM is (likely) a low-energy EFT of a more complete theory

  matter-antimatter asymmetry, origin and nature of neutrino masses, dark matter . . .

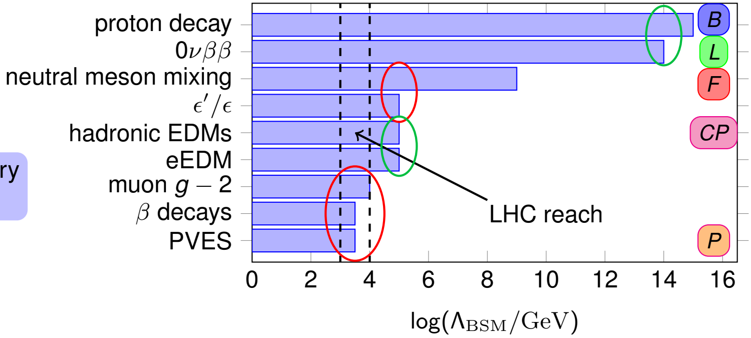
1. LHC to directly create new particles at 1-10 TeV
2. search for tiny indirect effects in low-energy precision experiments

   neutrinoless $\beta\beta$ decay, electric dipole moments, $\mu \to e$ conversion,
   muon and electron $g-2$, β decays, . . .

Finding BSM: energy vs. precision

$$O = (Q/\Lambda_{\mathrm{BSM}})^n$$

competitive/complementary to energy frontier

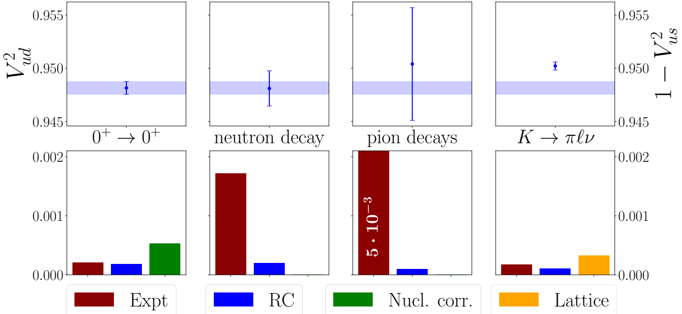


a. observables w/o (w. negligible) SM background: violation of SM symmetries, doors to new sectors?
b. observables w. SM background: need strong experiment/theory synergy to claim BSM discovery

$$\Lambda_{\mathrm{BSM}} \sim \frac{0.246\,\mathrm{TeV}}{(\delta^2_{\mathrm{exp}} + \delta^2_{\mathrm{th}})^{1/4}}$$

- $\lesssim 0.5 \cdot 10^{-3}$ experimental/theory uncertainties needed to probe $>$ 10 TeV scale

CKM unitarity tests



$V_{ud} = 0.97367 \pm 0.00032$
$V_{us}(K_{\ell 3}) = 0.2233 \pm 0.0005$

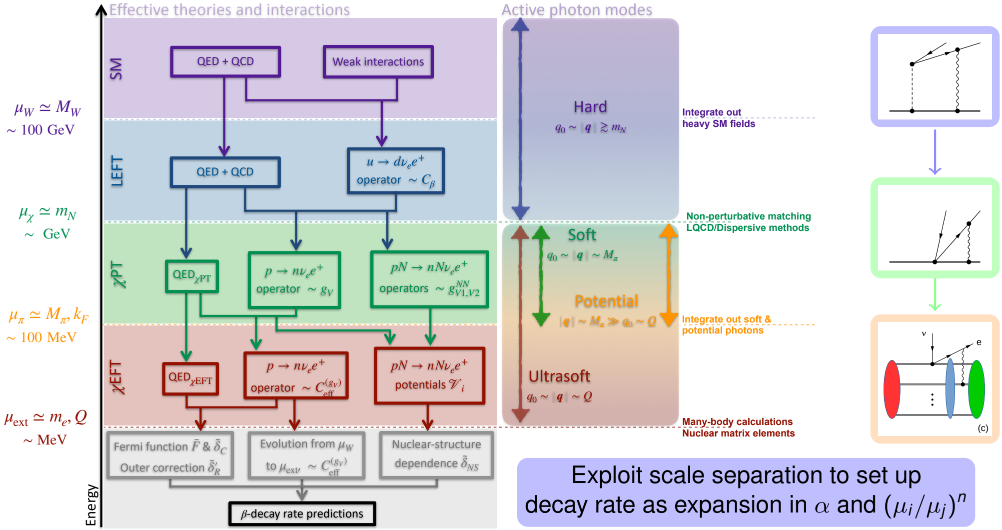
- the SM predicts $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- re-evaluation of "inner radiative correction" and precise lattice QCD calculation of $f_+(0)$

  C. Y. Seng, M. Gorchtein, H. Patel, M. Ramsey-Musolf, '18 A. Czarnecki, W. Marciano, A. Sirlin, '19; A. Bazavov *et al*, '18
- led to $\sim 2\sigma$-3σ tension in CKM unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -(1.65 \pm 0.73) \cdot 10^{-3}$$

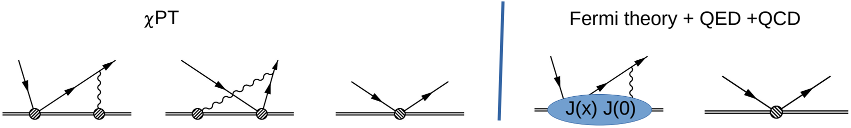
BSM physics in the 5-10 TeV range?

Why EFTs? Scales in β decays



Exploit scale separation to set up decay rate as expansion in α and $(\mu_i/\mu_j)^n$

Hard modes. One-body vector coupling g_V



$$\mathcal{L}_{\rm Fermi} = -2\sqrt{2} G_F V_{ud} C_\beta \bar{e}_L \gamma^\mu \nu_L \, \bar{q} \tau^+ \gamma_\mu q + \mathcal{L}_{\rm QED} \Longrightarrow -\sqrt{2} G_F V_{ud} \, \bar{e}_L \gamma^0 \nu_L \, \left\{ g_V \bar{N} \tau^+ N + g_{V1}^{NN} \bar{N} \tau^+ N \, \bar{N} N + \dots \right\}$$

- contributions from photons with $|\vec{q}| \sim \Lambda_{\rm QCD} \Longrightarrow$ photon modes are not in the low-energy EFT
- electromagnetism is hidden in EFT low-energy constants

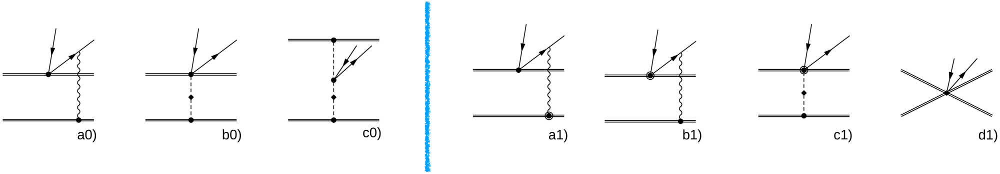
$$g_V = 1 + \sum_n \left(\frac{\alpha}{4\pi} \right)^n g_V^{(n)}, \qquad g_{\rm V1}^{NN} = \sum_n \left(\frac{\alpha}{4\pi} \right)^n g_{V1}^{(n)}$$

One body-coupling g_V

- perturbative contributions from matching & running from m_W
- non-perturbative contribs. from the "Wγ" box C. Y. Seng, M. Gorchtein, H. Patel, M. Ramsey-Musolf, '18

$$T^{\mu\nu}_{VA,\,0}(q,v) = \frac{\tau^a_{ij} \delta^{\sigma'\sigma}}{12} \frac{i}{6} \int \mathrm{d}^d x \, e^{iq\cdot x} \langle N(k,\sigma',j) | T \left[\bar{q} \gamma^\mu q(x) \, \bar{q} \gamma^\nu \gamma_5 \tau^a q(0) \right] | N(k,\sigma,i) \rangle.$$

Potential modes. $\bar{\delta}_{\rm NS}$ and two-body weak currents



- the second important regime is $q_0 \ll |\mathbf{q}| \sim k_F$ "potential"
- photon emission puts the nuclear intermediate state far off-shell
- can neglect nuclear excitation energy in energy denominators & sum over intermediate states

similar to "closure approximation" in $0\nu\beta\beta$

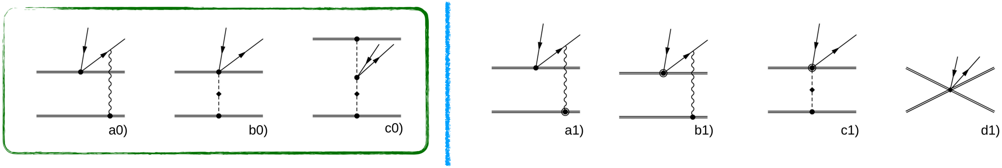
- $\bar{\delta}_{\rm NS}$ given by matrix element of a two- or higher-body current $\mathcal{V}^0$

$$\bar{\delta}_{\rm NS} = \sqrt{2}\langle f|\mathcal{V}^0|i\rangle$$

- $\mathcal{V}^0$ has an expansion in α and Q/Λ_χ as "standard" vector & axial 2- and 3-body currents

$$\mathcal{V}^0 = \alpha \sum_{n,m} \mathcal{V}_{n,m}\, \alpha^n\, \varepsilon_\chi^m \qquad \varepsilon_\chi = \frac{Q}{\Lambda_\chi}$$

$\bar{\delta}_{\mathrm{NS}}$ at $\mathcal{O}(\alpha\epsilon_\chi)$

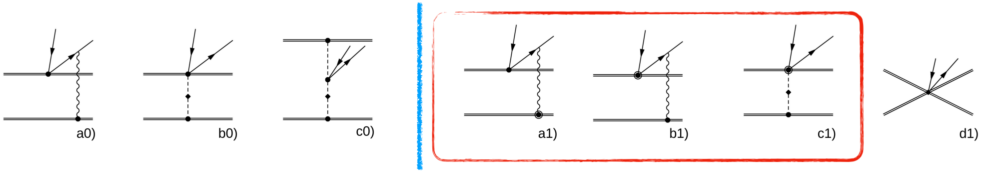


- diagrams with LO weak and EM vertices are proportional to external momenta

$$\mathcal{V}_E^0 = \frac{g_V}{3}\left(\frac{E_0}{2} + 4E_e\right)\sum_{j<k}\frac{e^2}{\mathbf{q}^4}\left(\tau^{+(j)}P_p^{(k)} + P_p^{(j)}\tau^{+(k)}\right)$$
$$\longrightarrow -g_V\alpha\,(R_A E_0)\left(\frac{1}{6} + \frac{4E_e}{3E_0}\right)\sum_{j<k}\frac{r_{jk}}{2R_A}\left(\tau^{+(j)}P_p^{(k)} + P_p^{(j)}\tau^{+(k)}\right)$$

- correction suppressed by E_e/k_F, still very important as it grows with Z
- replaces some terms linear in the nucleus charge radius R in standard approach

$\bar{\delta}_{\rm NS}$ at $\mathcal{O}(\alpha\epsilon_\chi)$



- diagrams with NLO EM vertices do not require external momenta $\mathcal{O}(\alpha\varepsilon_\chi)$
- long range corrections mediated by nucleon magnetic moments and recoil corrections

$$\mathcal{V}_0^{\rm mag}(\mathbf{q}) = \sum_{j<k} \frac{e^2}{3}\frac{g_A}{m_N}\frac{1}{\mathbf{q}^2}\left(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{\sigma}^{(k)} + \frac{1}{2}S^{(jk)}\right)\left[(1+\kappa_p)\tau^{+(j)}P_p^{(k)} + \kappa_n\tau^{+(j)}P_n^{(k)} + (j\leftrightarrow k)\right],$$

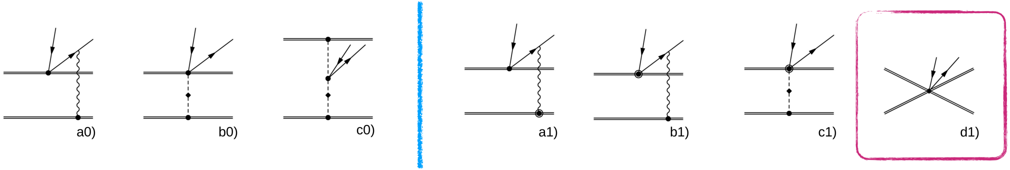
$$\mathcal{V}_0^{\rm rec}(\mathbf{q},\mathbf{P}) = \sum_{j<k}\left[-i\frac{e^2 g_A}{4m_N}\frac{\tau^{+(j)}P_p^{(k)}}{\mathbf{q}^4}((\mathbf{P}_j-\mathbf{P}_k)\times\mathbf{q})\cdot\boldsymbol{\sigma}^{(j)}\right],$$

very similar to I. S. Towner, '92

- $\mathcal{V}_0^{\rm mag}$ is a Coulomb-like potential $\Longrightarrow$ induces UV sensitivity when acting in ${}^1S_0$ channel

  same as for m_π dependence of nuclear force & $0\nu\beta\beta$ D. Kaplan, M. Savage, M. Wise, '96 V. Cirigliano, *et al*, '18

$\bar{\delta}_{\rm NS}$ at $\mathcal{O}(\alpha\epsilon_\chi)$



- to absorb cut-off dependence, need

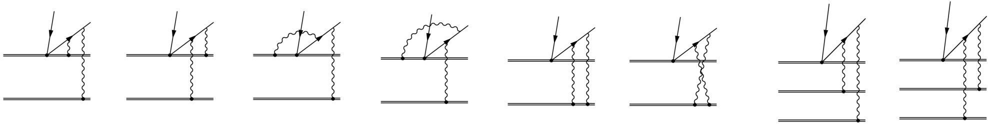
$$\mathcal{V}_0^{\rm CT} = e^2\left(g_{V1}^{NN} O_1 + g_{V2}^{NN} O_2\right), \qquad O_1 = \sum_{j\neq k} \tau^{+(j)} \mathbb{1}_k, \quad O_2 = \sum_{j<k} \left[\tau^{+(j)}\tau_3^{(k)} + (j \leftrightarrow k)\right].$$

- in Towner's approach, UV divergence regulated by axial and magnetic form factors
- g_{V1}^{NN} and g_{V2}^{NN} could be extracted from lattice QCD or calculating/modeling $W\gamma$ box at large Q^2

  for dispersive approach to $\delta_{\rm NS}$ see C. Y. Seng's talk on Friday
- or fit them to superallowed transitions with V_{ud}
- for illustration, will assume "improved NDA"

$$g_{V1}^{\rm NN} \pm g_{V2}^{\rm NN} = \pm \frac{1}{m_N}\frac{1}{4F_\pi^2}$$

$\mathcal{O}(\alpha^2)$ corrections to $\delta_{\rm NS}$

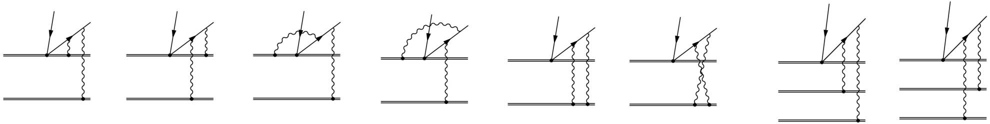


- at $\mathcal{O}(\alpha^2)$, the Fermi function and the nuclear-structure-independent correction δ_2 depend logarithmically on R A. Sirlin and R. Zucchini, '86, W. Jaus and G. Rasche, '87

$$f = 1 + \alpha Z + \alpha^2 Z^2 \log E_e R \ldots, \qquad \delta_2 = \alpha^2 Z \log E_e R$$

can we understand large logs in terms of scale separation?

$\mathcal{O}(\alpha^2)$ corrections to $\delta_{\rm NS}$



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can we understand large logs in terms of scale separation?

- "soft" photons induce two more 2- and 3-body operators

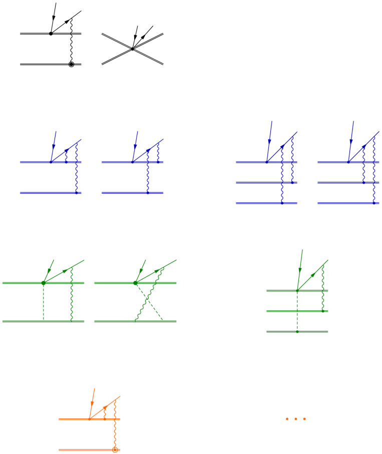
$$\mathcal{V}^0 = C_\delta(\mu, \Lambda)\, \mathcal{V}_\delta + C_\delta^{\rm 3b}(\mu, \Lambda)\, \mathcal{V}_\delta^{\rm 3b} + C_+ \mathcal{V}_+(\Lambda) + C_+^{\rm 3b} \mathcal{V}_+^{\rm 3b}(\Lambda)$$

- matching coefficients $C_\delta^{\rm (3b)}$ have right μ-dependence to reproduce $\log R$ in Fermi function and δ_2
- logarithmic 2- and 3-body potentials provide an operator definition of "R"

$$C_+ \mathcal{V}_+(\mathbf{r}, \Lambda) = -g_V \frac{\alpha^2}{2} \sum_{j<k} \log(r_{jk}^2 \Lambda^2) \left(\tau^{+(j)} P_p^{(k)} + \tau^{+(k)} P_p^{(j)} \right).$$

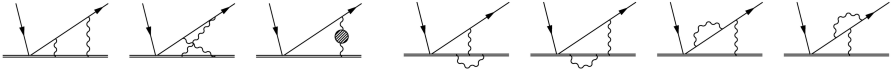
Extension of the δ_{NS} operator to higher orders

	2N current	3N current	4N current
NLO $\mathcal{O}(G_F \alpha \epsilon_\chi)$			
NLO$_\alpha$ $\mathcal{O}(G_F \alpha^2)$			
N^{2}LO $\mathcal{O}(G_F \alpha \epsilon_\chi^2)$			
N^{2}LO$_\alpha$, N^{3}LO $\mathcal{O}(G_F \alpha^3, G_F \alpha^2 \epsilon_\chi, G_F \alpha \epsilon_\chi^2)$		...	...



derive $\mathcal{V}^0$ at higher orders
test convergence and theory errors

Usoft modes. The Fermi and Sirlin functions



- last step of the calculation involves photon momentum modes $|q_0| \sim |\vec{q}| \sim E_e \ll R^{-1}$

  Fermi & Sirlin functions and "outer corrections"

- for Fermi function, use $\overline{\mathrm{MS}}$ calculation of R. Hill and R. Plestid, '23

$$\bar{F}(\beta,\mu) = \frac{4\eta}{(1+\eta)^2}\frac{2(1+\eta)}{\Gamma(2\eta+1)^2}|\Gamma(\eta+iy)|^2 e^{\pi y}\left(\frac{2|\mathbf{p}_e|}{\mu}e^{1/2-\gamma_E}\right)^{2(\eta-1)}, \qquad \eta = \sqrt{1-\alpha^2 Z^2} \quad y = \mp Z\alpha/\beta$$

- recover the standard Fermi function by $\mu \to R^{-1} e^{1/2-\gamma_E}$
- find standard Sirlin function at $\mathcal{O}(\alpha)$
  + with consistent NLL resummation of $\log \frac{m_e}{k_F}$ by getting $\mathcal{O}(\alpha^2)$ anomalous dimension from HQET

  X. D. Ji and M. Ramsey-Musolf, '91; V. Gimenez, '92; D. J. Broadhurst and A. G. Grozin, '99, V. Cirigliano, W. Dekens, EM, O. Tomalak, '23

(Partial) Summary: factorization of the lifetime

$$\frac{1}{t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} \left[C_{\rm eff}^{(g_V)}(\mu)\right]^2 \bar{f}(\mu) \times [1 + \bar{\delta}'_R(\mu)] \, (1 + \bar{\delta}_{\rm NS}) \, (1 - \bar{\delta}_C)$$

$\left[C_{\rm eff}^{(g_V)}(\mu)\right]^2$

- matching at electroweak scale, RG evolution from m_W to $\mu \sim m_N$ [pQCD]
- 1-body matching Fermi theory to χPT at $\mu \sim m_N$ [dispersive + LQCD methods]
- matching and running from m_N to $\mu \sim k_F$ [χPT, pQED]

$\bar{\delta}_{\rm NS}$

- two nucleon LECs [dispersive + LQCD methods], [data driven?]
- nuclear matrix elements [QMC, ab initio methods, shell model . . .]

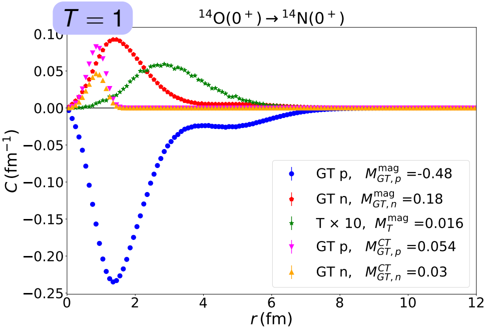
$\bar{\delta}_C$

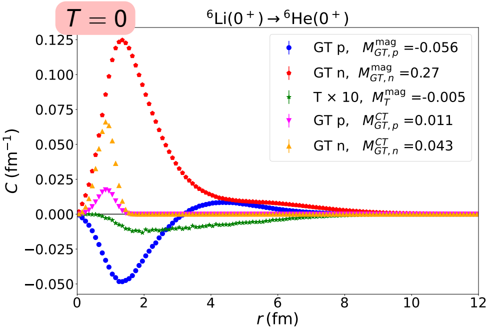
- isospin breaking in nuclear wavefunction [QMC, ab initio methods, shell model . . .]

$\bar{\delta}'_R, \bar{f}$

- long wavelength modes [pQED]

Nuclear matrix elements in ^{14}O





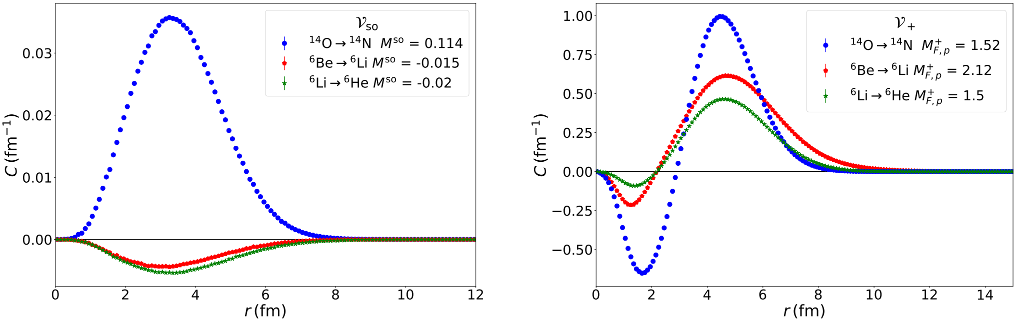
V. Cirigliano, W. Dekens, J. de Vries. **S. Gandolfi**, M. Hoferichter, EM, '24

- nuclear matrix elements in $A = 6$ (for benchmarking) and $A = 14$

$$M^{\mathcal{O}}_{\mathrm{GT,N}} = \langle f | \sum_{j<k} h^{\mathcal{O}}_{\mathrm{GT}}(r_{jk})\, \boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\sigma}^{(k)} \left[\tau^{+(j)} P^{(k)}_N + (j \leftrightarrow k) \right] | i \rangle \qquad h^{\mathrm{mag}}_{\mathrm{GT,p}} = \frac{g_A}{3 m_N} \frac{1 + \kappa_p}{r}$$

- VMC calculation with N^{2}LO local chiral potential at $R_0 = 1$ fm — A. Gezerlis, I. Tews *et al*, '14
- for local terms, Gamow-Teller component dominates, tensor small.
- contact can be sizable (depending on LECs)

NME in ^{14}O: energy independent components.



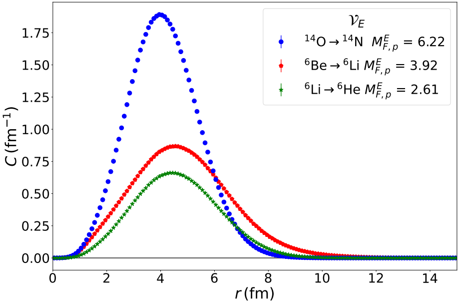
- spin-orbit term is very small for $A = 6$, significant for $A = 14$
- the α^2 $\mathcal{V}_+$ potential vanishes for $\Lambda = (1.02R)^{-1}$, with $R^2 = 5/3\langle r^2 \rangle$

  standard choice $\mu = R$ works pretty well for ^{14}O!
- putting everything together

$$\bar{\delta}^{(0)}_{\rm NS} = -\left[3.1|^{\rm mag}_{\rm GT} - 0.2|_{\rm T} - 1.2|_{\rm SO} \pm 0.9|_{\rm LEC} + 0.1|_{\alpha^2} \right] \cdot 10^{-3} = -(1.9 \pm 0.9) \cdot 10^{-3}$$

- compares well with $\delta^{(B)}_{NS} = -1.96(50) \cdot 10^{-3}$ J. Hardy and I. Towner, ‘20

NME for ^{14}O. Energy dependent pieces



$$\mathcal{V}_E \propto \alpha \sum_{j<k} \frac{r_{jk}}{2R_A} \left(\tau^{+(j)} P_p^{(k)} + \tau^{+(k)} P_p^j\right)$$

- the energy dependent operator leads to (neglecting small π-exchange corrections)

$$\bar{\delta}^E_{\rm NS} = \mp \frac{2}{g_V M_{\rm F}^{(0)}} \langle f^{(0)} | \mathcal{V}_E | i^{(0)} \rangle \left(\frac{E_0 + 8E_e}{6} + \frac{m_e^2}{2E_e} \right)$$

- corrections matches closely $\mathcal{O}(\alpha ZRE)$ “finite size” corrections if

$$M^E_{F,p} = M_F^{(0)} Z \frac{R}{2R_A} \sim 5.6 \Longrightarrow 10\% \text{ too small}$$

can we replace all “R” in phase space corrections with NMEs?

Extraction of V_{ud} from $^{14}\mathrm{O} \Longrightarrow {}^{14}\mathrm{N}$

- attempt a V_{ud} extraction using **only** ^{14}O

$$V_{ud} = 0.97364(10)_{\rm exp}(12)_{g_V}(22)_{\mu}(12)_{\delta_C}(43)_{g_V^{NN}}(20)_{\delta^E_{\rm NS}} = 0.97364(56)_{\rm total},$$

g_V one-body error

dominated by $W\gamma$ box, Lattice QCD calculations in progress

μ scale variation $\mu_{\rm low} = \{E_0, 4E_0\}$ in usoft EFT,

missing $\mathcal{O}(\alpha^2 Z)$ corrections, calculations in progress

$g_V^{\rm NN}$ LEC uncertainty

"fake" error, will go away with calculation & fit to multiple isotopes

$\delta^E_{\rm NS}$ rough estimate of higher orders in energy-dep. part

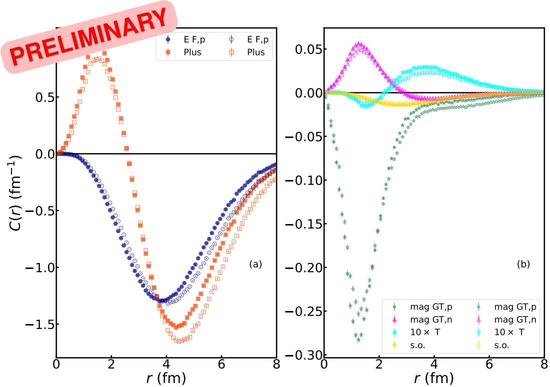
$g_V^{\rm NN}$, $\delta^E_{\rm NS}$ to be replaced with careful assessment of missing orders in EFT expansion

- compares well with what we would get from Towner and Hardy

$$V_{ud}\left[{}^{14}\mathrm{O}\right] = 0.97405(13)_{\rm exp}(9)_{\Delta_R^V}(12)_{\delta_C}(31)_{\delta_{\rm NS}}[37]_{\rm total},$$

Warning: we have said nothing of δ_C . . .

$\bar{\delta}_{\rm NS}$ in ^{10}C

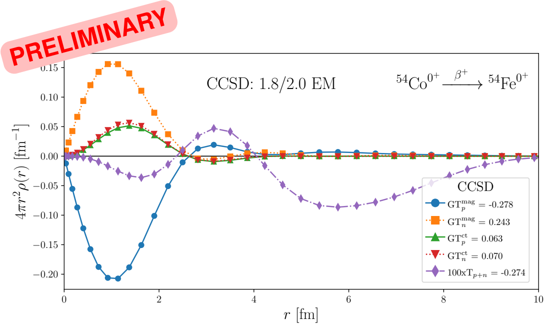


Model	$\bar{\delta}_{\rm NS}^{(0)}$
NV2+3-Ia	$-(4.23 + 0.28 \pm 0.61) \cdot 10^{-3}$
NV2+3-Ia$*$	$-(4.41 + 0.30 \pm 0.55) \cdot 10^{-3}$
AV18+UX	$-(4.55 + 0.39 \pm 0.48) \cdot 10^{-3}$
AV18+IL7	$-(4.87 + 0.29 \pm 0.61) \cdot 10^{-3}$

NV-Ia vs. NV-Ia* interactions;
thanks to **G. King**

- first step towards uncertainty quantification
- evaluation of $\bar{\delta}_{\rm NS}$ in ^{10}C with 4 different interactions and 2 QMC methods
- about 10% variation, but significantly larger than shell model

Coupled cluster calculations in medium mass nuclei



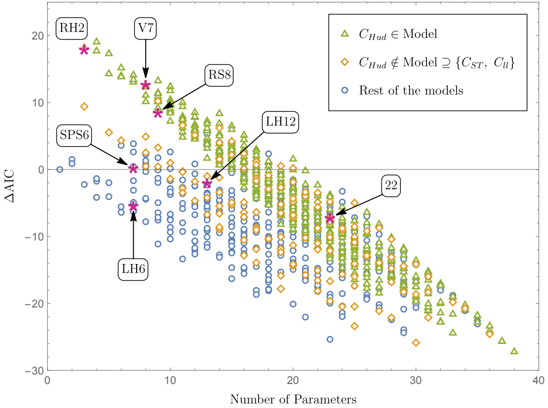
Transition	$M^{\mathrm{mag}}_{\mathrm{GT+T,p}}$	$M^{\mathrm{mag}}_{\mathrm{GT+T,n}}$	$M^{\mathrm{CT}}_{\mathrm{GT,p}}$	$M^{\mathrm{CT}}_{\mathrm{GT,n}}$
^{10}C → ^{10}B	-0.327	0.082	0.044	0.027
^{14}O → ^{14}N	-0.486	0.239	0.050	0.031
^{26}Al → ^{26}Mg	-0.221	0.229	0.048	0.060
^{34}Cl → ^{34}S	-0.368	0.305	0.047	0.056
^{38}K → ^{38}Ar	-0.407	0.330	0.048	0.059
^{42}Sc → ^{42}Ca	-0.229	0.218	0.053	0.062
^{46}V → ^{46}Ti	-0.268	0.225	0.058	0.066
^{50}Mn → ^{50}Cr	-0.270	0.231	0.060	0.067
^{54}Co → ^{54}Fe	-0.273	0.235	0.062	0.070

thanks to **S. Novario** (WashU)

- ongoing coupled cluster calculations 9 nuclei, including some of the most accurate
- should allow to check if joint fit to V_{ud} + two-body LECs does impact V_{ud} accuracy

need δ_C with same interactions!

Implications of the Cabibbo anomaly



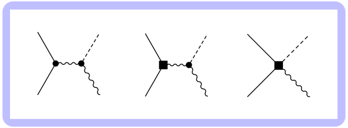
$$\Delta\text{AIC} = \text{AIC}_{\text{SM}} - \text{AIC}$$

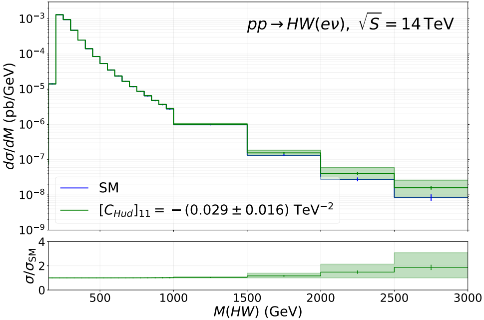
$$[C_{Hud}]_{1j}\, \bar{u}_R \gamma^\mu d_R^j W_\mu (1 + h/v)$$

V. Cirigliano, W. Dekens, J. de Vries, EM, T. Tong, '24

- new physics explanation of the Cabibbo anomaly are **not** ruled out
- analyzed β decays, electroweak precision obs. and Drell-Yan data
- with 37 SMEFT operators organized in 10 categories $\Longrightarrow$ leading to 1024 "models"
- right-handed charged-current couplings of quarks to W bosons significantly improve the fit

Implications of the Cabibbo anomaly





- Higgs properties: $pp \to hW$

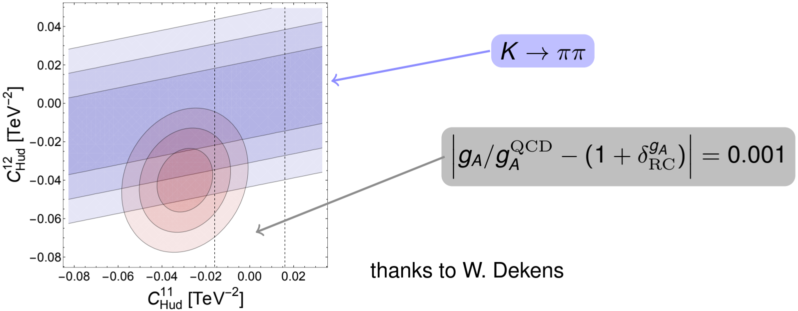
$$\mu_{WH}(13\mathrm{TeV}) = \frac{\sigma^{W^+H} + \sigma^{W^-H}}{\sigma^{W^+H}_{\mathrm{SM}} + \sigma^{W^-H}_{\mathrm{SM}}} - 1 = 160\left[v^2 C_{Hud}\right]^2_{11} + 90\left[v^2 C_{Hud}\right]^2_{12} = 5.2 \cdot 10^{-4}$$

too small to be seen even at HL-LHC

- but the effect grows at large H-W invariant mass
- RH explanation of the Cabibbo anomaly $\Longrightarrow$ look for high-invariant mass resonances in HW

β decays indirectly probe Higgs properties!

Implications of the Cabibbo anomaly



thanks to W. Dekens

- RH give large contributions to $I = 2$ amplitude in $K \to \pi\pi$
- and would shift the value of g_A

$$\frac{g_A}{g_V} = g_A^{\rm QCD}\left(1 + \delta_{\rm RC}^{g_A} - v^2 \frac{[C_{Hud}]_{11}}{V_{ud}}\right),$$

- best fit to the Cabibbo anomaly shifts g_A by 0.2%, more than current UCNA error
- test currently limited by theory errors on $\delta g_A^{\rm QCD}/g_A = 0.8\%$ and $\delta_{\rm RC}^{g_A} \approx 1\%$

FLAG '24; V. Cirigliano *et al*, '22; C. Y. Seng, 24; C. Y. Seng, 24; V. Cirigliano, W. Dekens, EM, O. Tomalak, '24;

Conclusion

- precision β decays are important probes of BSM physics

  ...but they require precise theoretical input!
- EFTs provide a systematic organizational tool for assessment/re-evalution of th. uncertainties

- "refactored" lifetime formula according to photon virtuality
- derived lowest-order two-body operators for $\bar{\delta}_{\rm NS}$ in chiral EFT
- clarified some nuclear-structure dependence of nuclear-structure-independent corrections
- resummed large logs of $E_e R$
- performed first preliminary QMC and CC calculations in ^{10}C, ^{14}O and medium mass nuclei

TO DO:

- higher order corrections to $\delta_{\rm NS}$ and "outer corrections"

  does the χEFT expansion converges?
- more rigorous assessment of nuclear theory error on the NMEs

  multiple chiral Hamiltonians, different cut-offs, multiple methods
- calculate δ_C with the same method

$\mathcal{O}(\alpha^2)$ corrections to $\delta_{\rm NS}$

- the matching coefficients have right μ dependence to reproduce $\log R$ in Fermi function and δ_2

$$C_\delta = -g_V(\mu)\frac{\alpha^2}{2}\left(\log\frac{\mu^2}{\Lambda^2} - \frac{13}{8} + 2\gamma_E\right), \qquad C_\delta^{\rm 3b} = -g_V(\mu)\alpha^2\left(\frac{1}{4}\log\frac{\mu^2}{\Lambda^2} + \frac{\gamma_E}{2} - \frac{3}{8}\right).$$

- the arbitrary scale Λ is canceled by $\mathcal{V}_+$ and $\mathcal{V}_+^{\rm 3b}$ operators, e.g.

$$C_+\mathcal{V}_+(\mathbf{r},\Lambda) = -g_V\frac{\alpha^2}{2}\sum_{j<k}\log(r_{jk}^2\Lambda^2)\left(\tau^{+(j)}P_p^{(k)} + \tau^{+(k)}P_p^{(j)}\right),$$

nuclear matrix element provides an operator definition of "R" in large logs

- upon integrating out soft and potential modes

$$g_V \to C_{\rm eff}^{g_V}, \qquad C_{\rm eff}^{g_V}(\mu_\pi) = g_V\left[1 + Z(C_\delta - C_\delta^{\rm 3b}) + Z^2 C_\delta^{\rm 3b}\right]$$

and RGE for $C_{\rm eff}^{g_V}$ allows to resum large logs