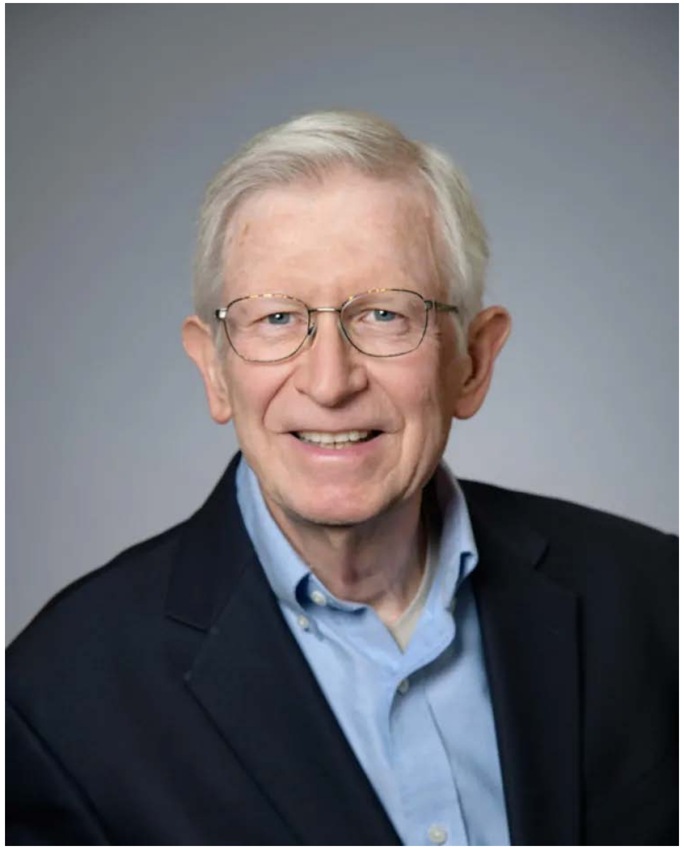
IMPROVED ACTIONS FOR NUCLEAR EFFECTIVE FIELD THEORY

Bira van Kolck

IN MEMORIAM

University of Arizona



Bruce Richard Barrett

1939-2025

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Properties of ^{12}C in the *Ab Initio* Nuclear Shell Model

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(Received 1 December 1999)

We obtain properties of ^{12}C in the *ab initio* no-core nuclear shell model. The effective Hamiltonians are derived microscopically from the realistic CD Bonn and the Argonne V8′ nucleon-nucleon (*NN*) potentials as a function of the finite harmonic oscillator basis space. Binding energies, excitation spectra, and electromagnetic properties are presented for model spaces up to $5\hbar\Omega$. The favorable comparison with available data is a consequence of the underlying *NN* interaction rather than a phenomenological fit.

PACS numbers: 21.60.Cs, 21.30.Fe, 27.20.+n

A deep but unpretentious thinker

A noble friend and colleague

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No-core shell model in an effective-field-theory framework

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Abstract

We present a new approach to the construction of effective interactions suitable for many-body calculations by means of the no-core shell model (NCSM). We consider an effective field theory (EFT) with only nucleon fields directly in the NCSM model spaces. In leading order, we obtain the strengths of the three contact interactions from the condition that in each model space the experimental ground-state energies of ^{2}H, ^{3}H and ^{4}He be exactly reproduced. The first $(0^+;0)$ excited state of ^{4}He and the ground state of ^{6}Li are then obtained by means of NCSM calculations in several spaces and frequencies. After we remove the harmonic-oscillator frequency dependence, we predict for ^{4}He an energy level for the first $(0^+;0)$ excited state in remarkable agreement with the experimental value. The corresponding ^{6}Li binding energy is about 70% of the experimental value, consistent with the expansion parameter of the EFT.

PACS: 21.30.-x; 21.60.Cs; 24.10.Cn; 45.50.Jf

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Outline

- Nuclear Physics from QCD
- Pionless Effective Field Theory
- Improved Actions
- Light-Medium Nuclei
- Prospects and conclusion

with

<u>Contessi</u>, Pavón
<u>Contessi</u>, Schäfer
<u>Contessi</u>, Gnech, Lovato, Schäfer

Nuclear Physics from QCD

"Folk Theorem"

Weinberg '79

The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general S matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and those symmetries, with no further physical content.

high-momentum/short-distance details in coefficients

regulator = infinite series of interactions with related coefficients

e.g. $\delta_\Lambda(\vec{r}) = \left(\frac{\Lambda}{2\sqrt{\pi}}\right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4}\right)$

NOT most general → renormalization

Observables expansions in $\frac{Q}{M_{\rm hi}}$

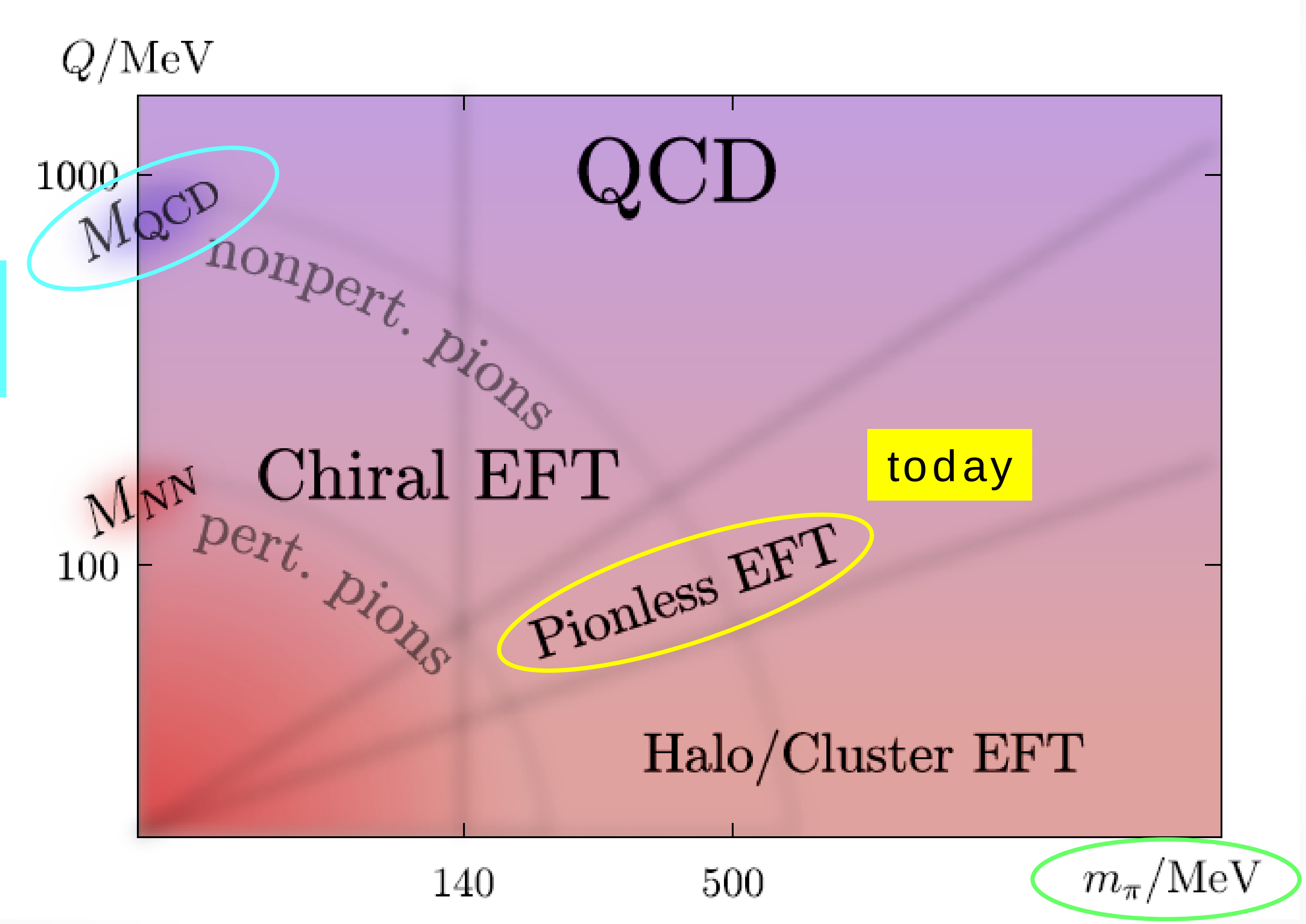
Q: typical momentum & low-energy scales

$M_{\rm hi}$: scale of underlying theory

Landscape of Nuclear Effective Field Theories

typical momentum

chiral-symmetric QCD scale



today

More generally, Short-Range EFT

pion mass

chiral-symmetry breaking QCD scale

Hammer, König, vK, *Rev. Mod. Phys. 92* (2019) 025004

Pionless EFT

$$M_{\rm hi} = \mathcal{O}(m_\pi)$$

pion mass

$$= \mathcal{O}(2/r_2)$$

two-body effective range

two-nucleon scattering
to high order:

$$M_{\rm hi} \simeq 1.4\, m_\pi$$

Ekström, Platter '24

$$Q \sim \sqrt{2\mu E}$$

relative momentum
in two-body scattering

$$\propto \sqrt{2m\, E_A/A} \equiv Q_A$$

A-body
binding momentum

universality
near two-body unitarity

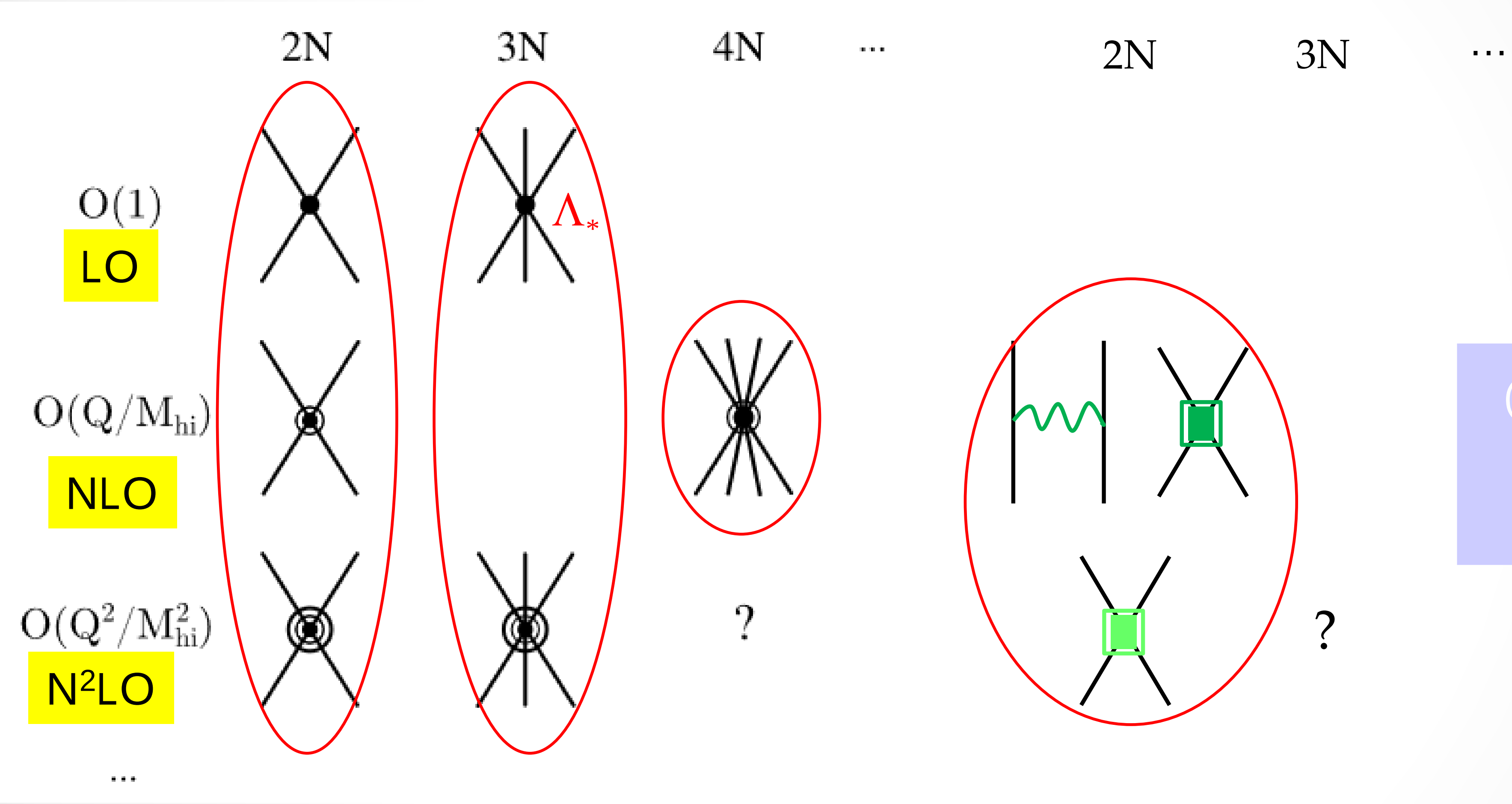
$$E_2/2 \ll \left(E_A/A\right)_{A\geq 3}$$

$$\frac{Q}{M_{\rm hi}} \sim \frac{Q_3}{1.4\, m_\pi} \sim \frac{1}{3}$$

Potential

isospin-symmetric

isospin-violating



vK '97'99
Kaplan, Savage,
Wise '98
...

Bedaque,
Hammer,
vK '99'00
...

Bazak, Kirscher,
König, Pavón,
Barnea, vK '19
...

König,
Griesshammer,
Hammer, vK '16
...

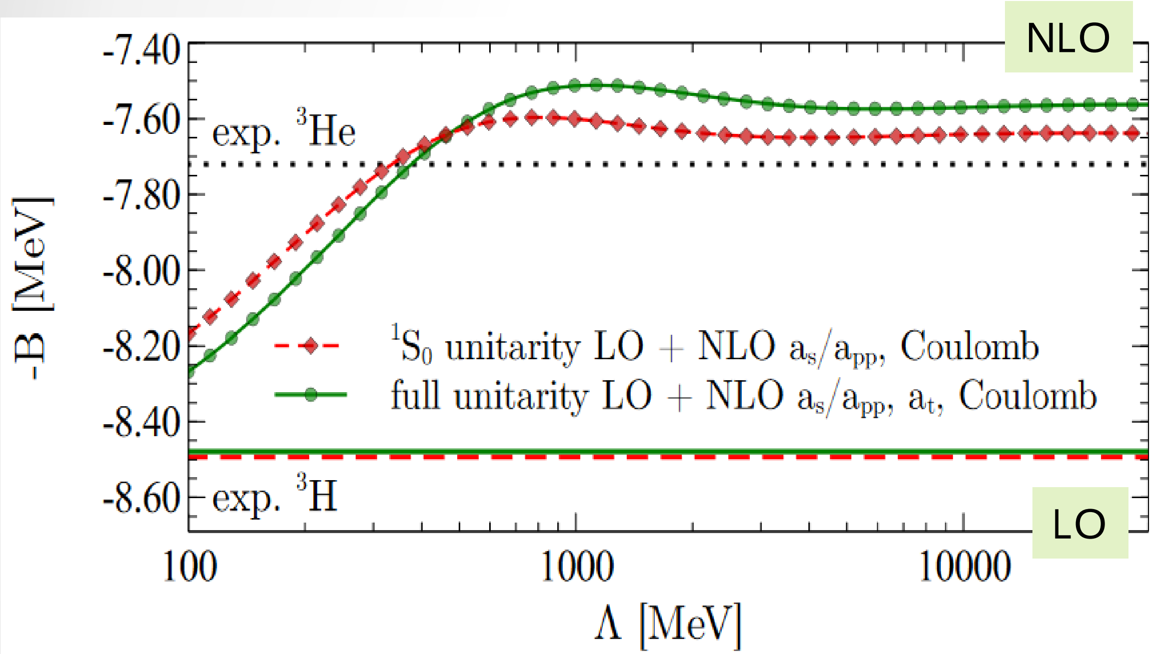
discrete scale invariance!

(distorted-wave) perturbation theory

Hammer, König, vK, *Rev. Mod. Phys.* 92 (2020) 025004

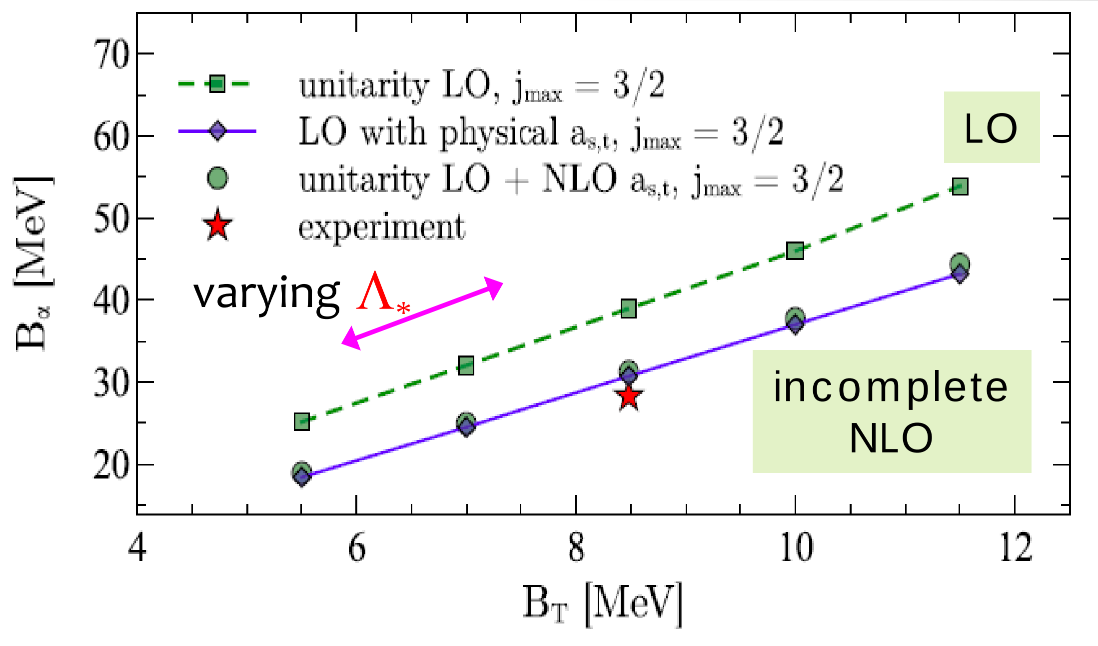
Good description up to alpha particle

$A = 3$



$A = 4$

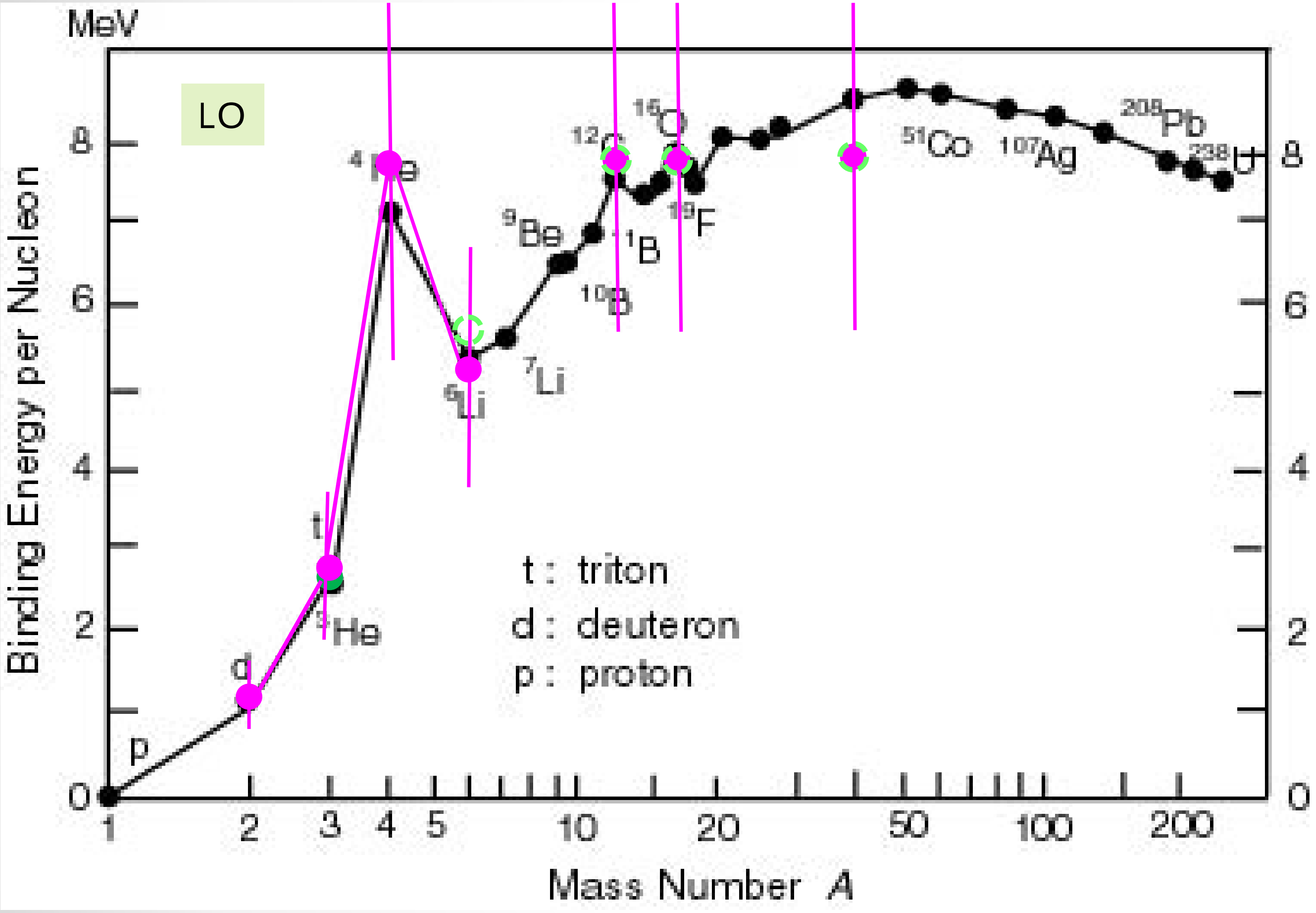
Tjon line



König, Grießhammer, Hammer, vK, *Phys. Rev. Lett.* 118 (2017) 202501

renormalization + near two-body unitarity: Efimov physics and corrections

Instability beyond alpha particle at LO



https://ne.phys.kyushu-u.ac.jp/seminar/MicroWorld3_E/3Part3_E/3P34_E/nuclear_fusion_E.htm

^{6}Li — Stetcu, Barrett, vK '07; Contessi, Schäfer, Gnech, Lovato, vK '25

but n-α scattering good — Bagnarol, Schäfer, Bazak, Barnea '23

^{12}C — Contessi, Schäfer, Gnech, Lovato, vK '25

^{16}O — Contessi, Lovato, Pederiva, Roggero, Kirscher, vK '17; Bansal, Binder, Ekström, Hagen, Jansen, Papenbrock '18; Contessi, Schäfer, Gnech, Lovato, vK '25

^{40}Ca — Bansal, Binder, Ekström, Hagen, Jansen, Papenbrock '18

Dawkins, Carlson, vK, Gezerlis '20
Schäfer, Contessi, Kirscher, Mareš '20
Contessi, Schäfer, Kirscher, Lazauskas, Carbonell '23

renormalization + near two-body unitarity: clustering of multicomponent fermions

Improved Actions

Contessi, Schäfer, vK '23
Contessi, Pavón, vK '24
Contessi, Schäfer, Gnech, Lovato, vK '25

Practical problem: how to do (distorted-wave) perturbation theory upon unstable states?

Solution: add to LO *some* subLO corrections, while maintaining renormalization

- no new *physical* parameter at LO
- effect no larger than NLO → removed perturbatively at NLO

$$V^{(0)}(\vec{r};\Lambda)=C_0^{(0)}(\Lambda)\sum_{ij}\delta_\Lambda(\vec{r}_{ij})+D_0^{(0)}(\Lambda)\sum_{ijk}\delta_\Lambda(\vec{r}_{ij})\delta_\Lambda(\vec{r}_{ik})$$

(neglecting spin-isospin factors)

$$\Rightarrow \tilde{V}^{(0)}(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3)=V^{(0)}(\vec{r};\Lambda)+\Delta V(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3)$$

"fake ranges"

within $\mathcal{O}\left(Q/M_{\text{hi}}\right)$

$$\Delta V(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3)=\Delta V^{(1)}(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3)+\ldots$$

$$V^{(1)}(\vec{r};\Lambda)=\ldots \quad \Rightarrow \quad \tilde{V}^{(1)}(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3)=V^{(1)}(\vec{r};\Lambda)-\Delta V^{(1)}(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3)$$

etc.

within $\mathcal{O}\left(Q^2/M_{\text{hi}}^2\right)$

Examples

1) change of renormalization condition

fit to two-body scattering length ⇒ fit to *A*-body binding energy

cf. $NNLO_{sat}$ chiral pot

Ekström *et al.* '15

BUT difficult to discern a potential failure of the expansion as density increases

2) partial resummation of the two-body effective range

Phillips, Rupak, Savage '00
Contessi, Pavón, vK '24

introduce dimer field ⇒ two-body coupling energy dependent

- convergence radius and *a priori* error estimates do not change
- central values for various orders within *a priori* error estimates
- central values at lower orders closer to exact for $\tilde{R}_2 \leq \tilde{R}_c$ where two-body effective range is reproduced

BUT difficult to apply to more bodies

3) partial resummation of a chain of ERE parameters

$$\Delta V(\vec{r};\Lambda,\tilde{R}_2,\tilde{R}_3) = \sum_{ij}\left[\tilde{C}(\tilde{R}_2^{-1})\,\delta_{\tilde{R}_2^{-1}}(\vec{r}_{ij}) - C_0^{(0)}(\Lambda)\,\delta_\Lambda(\vec{r}_{ij})\right]$$
$$+\sum_{ijk}\left[\tilde{D}(\tilde{R}_2^{-1},\tilde{R}_3^{-1})\,\delta_{\tilde{R}_3^{-1}}(\vec{r}_{ij})\delta_{\tilde{R}_3^{-1}}(\vec{r}_{ik}) - D_0^{(0)}(\Lambda)\delta_\Lambda(\vec{r}_{ij})\delta_\Lambda(\vec{r}_{ik})\right]$$

^{4}He atomic clusters to NLO $A \le 5$ Contessi, Schäfer, vK ’23

- central values at various improved orders:

  -- slightly better than unimproved values

  -- differ from unimproved results no more than change in order

- improvement at NLO nearly independent of fake range $\tilde{R}_2 \le \tilde{R}_c$

Contessi, Schäfer, Gnech, Lovato, vK '25

Light-Medium Nuclei

Can we find a range of fake ranges that give stability at LO?

(two two-body S-wave channels)

one possible improvement

$\tilde{R}_{2s} = x R_{2s}$ $\tilde{R}_{2t} = x R_{2t}$

fitted to corresponding effective ranges

$\tilde{R}_3 = x R_3$

fitted for maximal improvement of alpha binding energy

$$\Rightarrow \quad \tilde{V}^{(0)}(\vec{r};x) = V^{(0)}(\vec{r};\Lambda) + \Delta V(\vec{r};\Lambda, xR_{2s}, xR_{2t}, xR_3)$$

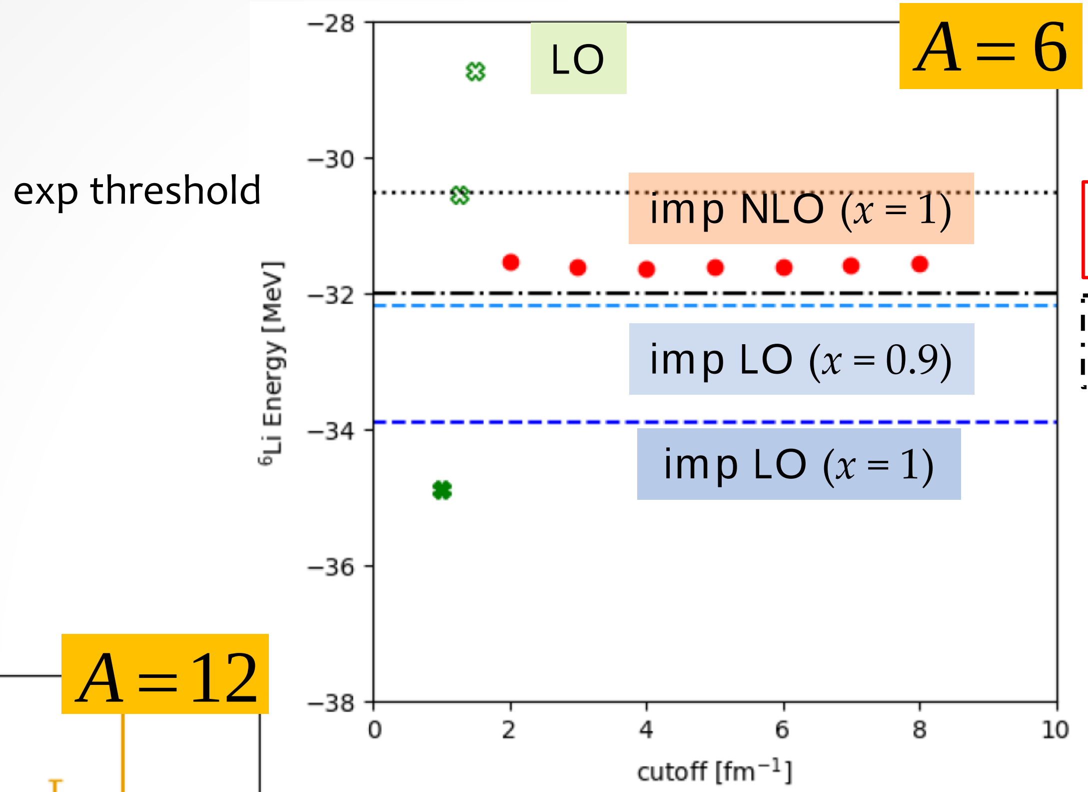
residual cutoff dependence at NLO

$$E_A^{(1)}(\Lambda) = E_A^{(1)}\left(1 + \frac{q_A^{(1)}}{\Lambda}\right)$$

uncertainty from variation for $\Lambda \geq 2\ \mathrm{fm}^{-1}$

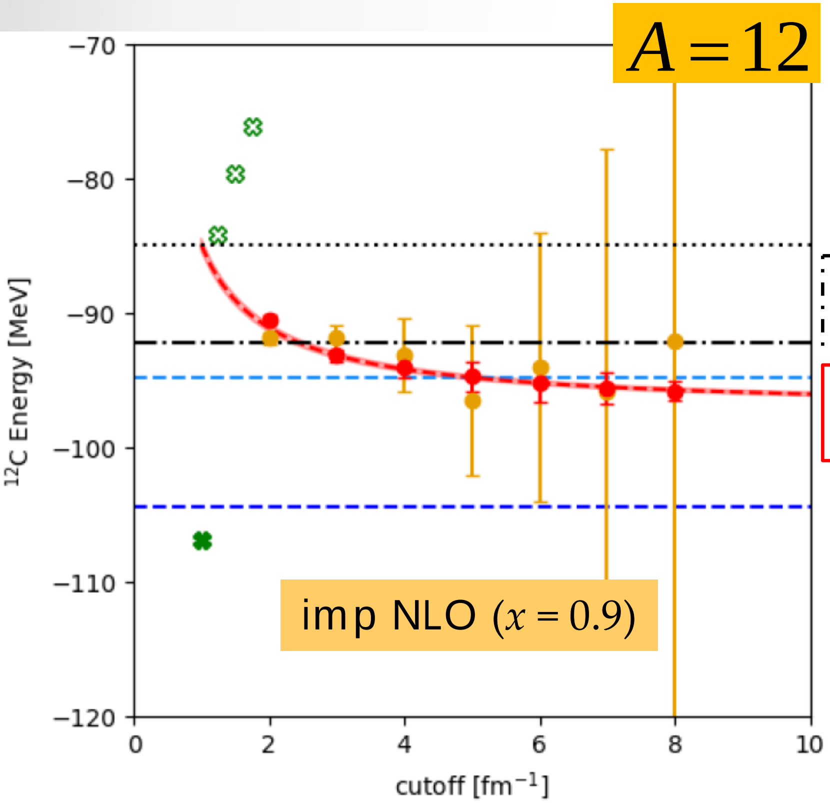
~ uncertainty from $\dfrac{Q}{M_{\mathrm{hi}}} \sim \dfrac{Q_3}{1.4\, m_\pi} \sim \dfrac{1}{3}$

Contessi, Schäfer, Gnech, Lovato, vK, arXiv:2505.09299



$$E_6^{(1)} = -(31.57 \pm 0.02 \pm 0.3)\ \mathrm{MeV}$$

$$E_6^{(\mathrm{exp})} = -31.994\ \mathrm{MeV}$$

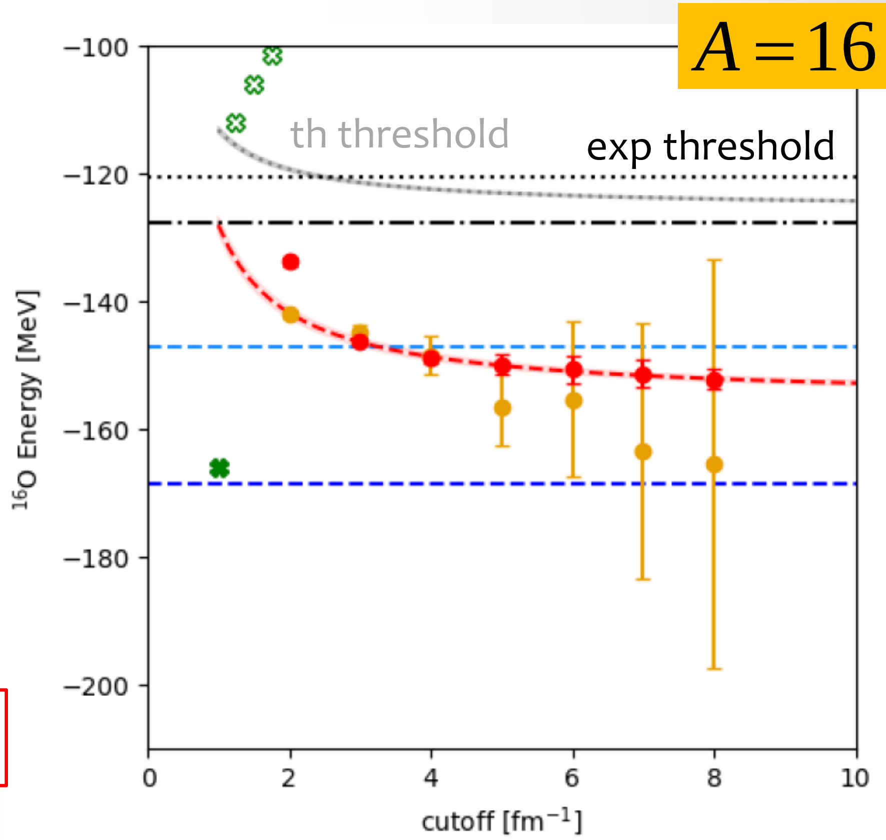


$$E_{12}^{(\mathrm{exp})} = -92.162\ \mathrm{MeV}$$

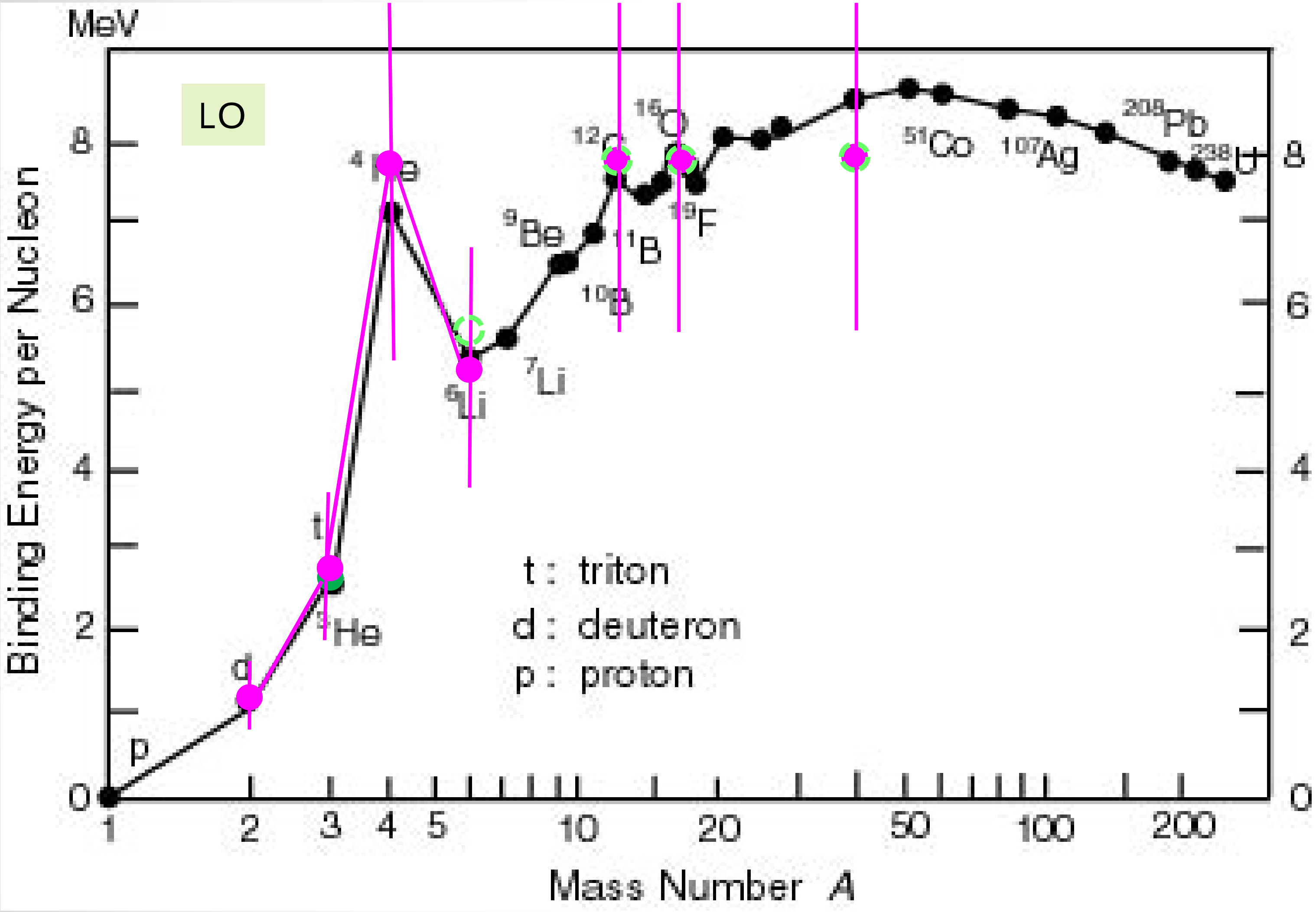
$$E_{12}^{(1)} = -(97.3 \pm 0.1 \pm 5)\ \mathrm{MeV}$$

$$E_{16}^{(\mathrm{exp})} = -127.619\ \mathrm{MeV}$$

$$E_{16}^{(1)} = -(155.6 \pm 0.3 \pm 20)\ \mathrm{MeV}$$



Instability beyond alpha particle at LO



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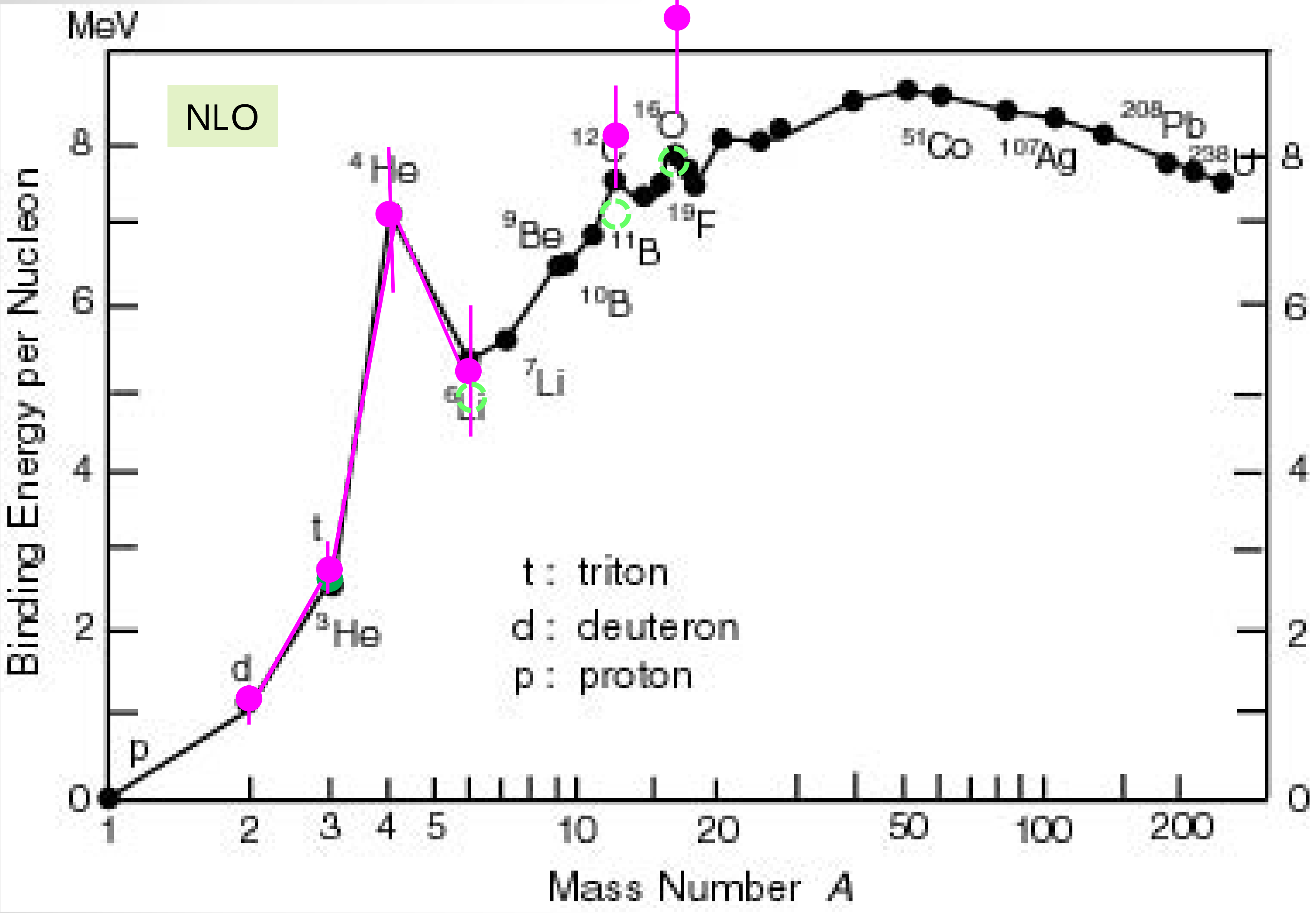
renormalization + near two-body unitarity: clustering of multicomponent fermions

Stability beyond alpha particle at NLO



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and n-α scattering good

Bagnarol, Schäfer, Bazak, Barnea ‘23

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renormalization + near two-body unitarity: clustering of multicomponent fermions

- cf. order-by-order weakening of Wigner bound

Beck, Bazak, Barnea '20

-- here more general

- cf. finite-range pionless potentials

Kievsky *et al.* '20
Recchia *et al.* '22
...

-- here preserving power counting

(# parameters, perturbative corrections, breakdown scale, ...)

Conclusion

Partial range effects can be included at LO
without affecting power counting and model independence

Lower orders are improved
without affecting convergence radius

Improvement is sufficient to
provide stability for near-unitarity fermions at LO
and allow perturbation theory for corrections

Light-medium binding energies are
within ~15% of experiment at NLO

Need: more observables, heavier nuclei, higher orders