

IMPROVED ACTIONS FOR NUCLEAR EFFECTIVE FIELD THEORY

Bira van Kolck



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Properties of ^{12}C in the *Ab Initio* Nuclear Shell Model

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We obtain properties of ^{12}C in the *ab initio* no-core nuclear shell model. The effective Hamiltonians are derived microscopically from the realistic CD Bonn and the Argonne V8' nucleon-nucleon (NN) potentials as a function of the finite harmonic oscillator basis space. Binding energies, excitation spectra, and electromagnetic properties are presented for model spaces up to $5\hbar\Omega$. The favorable comparison with available data is a consequence of the underlying NN interaction rather than a phenomenological fit.

PACS numbers: 21.60.Cs, 21.30.Fe, 27.20.+n

A deep but unpretentious
thinker

A noble
friend and colleague



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No-core shell model in an effective-field-theory framework

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Abstract

We present a new approach to the construction of effective interactions suitable for many-body calculations by means of the no-core shell model (NCSM). We consider an effective field theory (EFT) with only nucleon fields directly in the NCSM model spaces. In leading order, we obtain the strengths of the three contact interactions from the condition that in each model space the experimental ground-state energies of ^2H , ^3H and ^4He be exactly reproduced. The first $(0^+; 0)$ excited state of ^4He and the ground state of ^6Li are then obtained by means of NCSM calculations in several spaces and frequencies. After we remove the harmonic-oscillator frequency dependence, we predict for ^4He an energy level for the first $(0^+; 0)$ excited state in remarkable agreement with the experimental value. The corresponding ^6Li binding energy is about 70% of the experimental value, consistent with the expansion parameter of the EFT.

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PACS: 21.30.-x; 21.60.Cs; 24.10.Cn; 45.50.Jf

2025 PROGRAM OF ACTIVITIES

FEBRUARY 17-21	Key Reactions in Nuclear Astrophysics A. TUMINO (Università degli Studi di Enna "Kore" & INFN-LNS), C. BERTULANI (Texas A&M University-Commerce), R. DIEHL (Max Planck Institut für Experimentelle Physik), J. JORIS (Technical University of Catalonia), L. TRACHE (IFIN-HH)	21-25	Penetrating Probes of Hot High-μB Matter: Theory Meets Experiment E. SCOMPARIN (INFN Torino), T. GALATYUK (TU Darmstadt), M. LOMBARDO (INFN Firenze), R. RAPP (Texas A&M University), G. USAI (Università di Cagliari)
MARCH 3-7	Scale Setting: Precision Lattice QCD for Particle and Nuclear Physics A. BAZAVOV (Michigan State University), S. COLLINS (University of Regensburg), R. SOMMER (DESY & Humboldt-Universität zu Berlin)	JUL/AUG 28-01	New Perspectives in the Charge Radii Determination for Light Nuclei F. HAGELSTEIN (JGU Mainz & PSI Villigen), L. GASTALDO (Kierchhoff-Institute for Physics), N. PAUL (Laboratoire Kastler Brossel), R. POHL (JGU Mainz)
24-28	Holographic Perspectives on Chiral Transport and Spin Dynamics M. KAMINSKI (University of Alabama), K. LANDSTEINER (IFT Madrid), D. KHARZEEV (Stony Brook University/BNL), U. GURSOY (Utrecht University)	25-29	Bridging Analytical and Numerical Methods for Quantum Field Theory A. CHERMAN (University of Minnesota), A. ATHENODOROU (The Cyprus Institute), T. JACOBSON (University of California, Los Angeles), M. CASELLE (University of Torino)
MAR/APR 31-04	Mechanical Properties of Hadrons: Structure, Dynamics, Visualization P. SCHWEDTZER (University of Connecticut), S. DIEHL (University Gießen), K. JOO (University of Connecticut), C. LORCE (École Polytechnique), B. PASQUINI (Università di Pavia), C. WEISS (JLab)	SEPTEMBER 01-05	Hamiltonian Lattice Gauge Theories: Status, Novel Developments and Applications C. URBACH (University of Bonn), K. JANSEN (DESY Zeuthen / COTA), R. LEWIS (York University), G. MAGNIFICO (University of Bari), S. ROMITI (University of Bern), E. RICO ORTEGA (ikerbasque)
APRIL 14-18	Lepton Flavour Change in Nuclei K. BENNAEUR (IP2I, IN2P3), S. DAVIDSON (LUPM, IN2P3)	08-12	Analytic Structure of QCD and Yang-Lee Edge Singularity V. SKOKOV (North Carolina State University), G. BAŞAR (University of North Carolina Chapel Hill), C. SCHMIDT (Universität Bielefeld)
MAY 05-09	Quantum Science Generation 2025 A. BALDIZZI (University of Trento), C. BENAVIDES-RIVEROS (INO-CNR Pitaevskii BEC Center), C. CAPECCI (UnitN, Pitaevskii BEC Center), D. DE BERNARDIS (INO-CNR Pitaevskii BEC Center), C. JOHANSEN (INO-CNR Pitaevskii BEC Center), F. MANTEGAZZINI (FBK) et al.	22-26	Attractors and Thermalization in Nuclear Collisions and Cold Quantum Gases M. HELLER (Ghent University), J. BERGES (Heidelberg University), J. BREWER (University of Oxford), T. LAPPI (University of Jyväskylä), M. SPALINSKI (National Centre for Nuclear Research)
12-16	Nonequilibrium Phenomena in Superfluid Systems: Atomic Nuclei, Liquid Helium, Ultracold Gases, and Neutron Stars P. MAGIERSKI (Warsaw University of Technology), B. HASKELL (Nicolaus Copernicus Astronomical Center), G. ROATI (University of Florence, LENS), G. WLĄZŁOWSKI (Warsaw University of Technology)	OCTOBER 06-09	Superconducting Devices for Quantum Optics and Quantum Simulations F. MANTEGAZZINI (FBK), I. CARUSOTTO (CNR-INO), N. ROCH (CNRS-Institut Neel), M. ESPOSITO (CNR-SPIN), N. CRESCINI (FBK), F. AHNENS (FBK)
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07-09	Theory Service for the Low Energy Nuclear Physics Community: a Hands-on Workshop G. COLO (University of Milano and INFN), J. DUDEK (UdS and INFN-IN2P3/CNRS, Strasbourg), M. RODRIGUEZ-GALLARDO (University of Sevilla)	DECEMBER 01-05	Multi-Canonical Methods and Lattice Field Theory G. KANIWAR (University of Edinburgh), G. BONANNO (IFT UAM/CSIC), D. HACKETT (Fermilab), B. LUCINI (Swansea University), A. NADA (University of Turin), J. URBAN (MIT)
14-18	Next Generation Ab Initio Nuclear Theory C. BARBIERI (Università di Milano), E. EPELBAUM (Ruhr Universität Bochum), R. FURNSTADL (Ohio State University), S. PASTORE (Washington University in St. Louis)		

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We are
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for 2026!

Deadline May 26



Outline

- Nuclear Physics from QCD
- Pionless Effective Field Theory
- Improved Actions
- Light-Medium Nuclei
- Prospects and conclusion

with

Contessi, Pavón
Contessi, Schäfer
Contessi, Gnech,
Lovato, Schäfer

Nuclear Physics from QCD

“Folk Theorem”

Weinberg '79

The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general S matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and those symmetries, with no further physical content.

high-momentum/short-distance details in coefficients

regulator = infinite series of interactions
with related coefficients

e.g. $\delta_{\Lambda}(\vec{r}) = \left(\frac{\Lambda}{2\sqrt{\pi}} \right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4} \right)$

NOT
most general



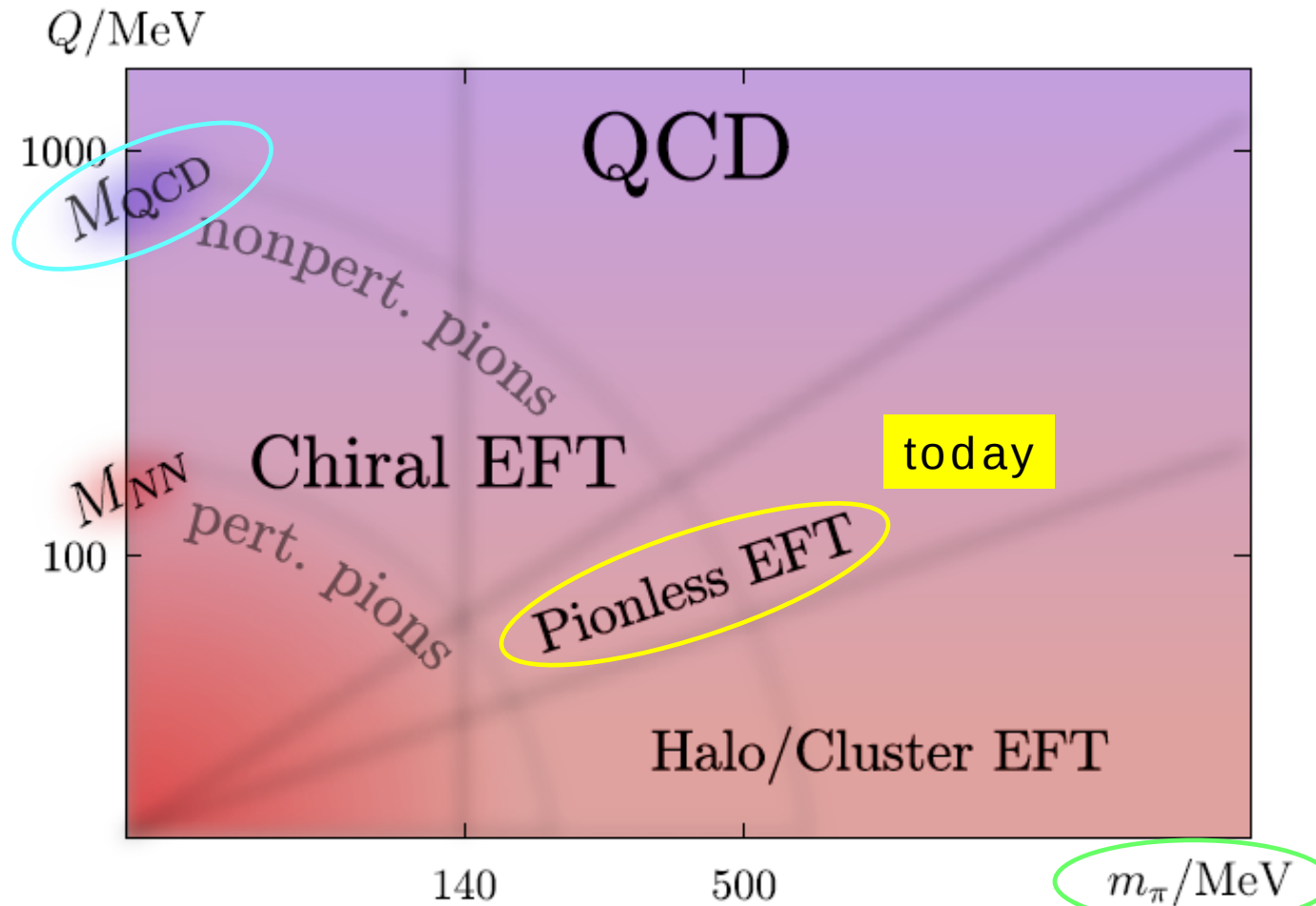
renormalization

Observables expansions in $\frac{Q}{M_{\text{hi}}}$ typical momentum & low-energy scales
scale of underlying theory

Landscape of Nuclear Effective Field Theories

typical
momentum

chiral-symmetric
QCD scale



More generally,
Short-Range EFT

pion mass

chiral-symmetry
breaking
QCD scale

Pionless EFT

$$M_{\text{hi}} = \mathcal{O}(m_\pi)$$

pion mass

$$= \mathcal{O}(2/r_2)$$

two-body effective range

two-nucleon scattering
to high order:

$$M_{\text{hi}} \simeq 1.4 m_\pi$$

Ekström, Platter '24

$$Q \sim \sqrt{2\mu E}$$

relative momentum
in two-body scattering

$$\propto \sqrt{2m E_A/A} \equiv Q_A$$

A-body
binding momentum

universality
near two-body unitarity

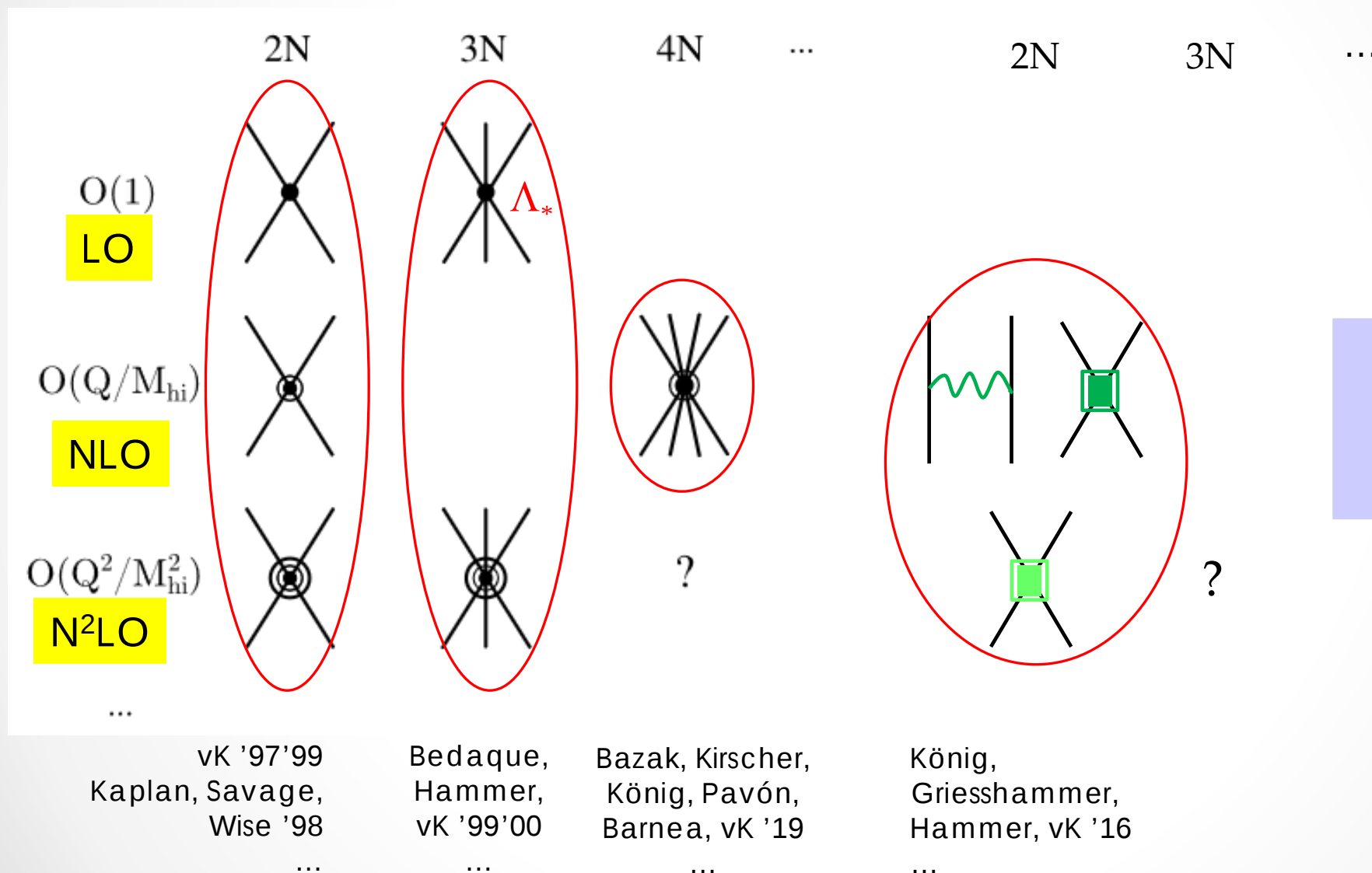
$$E_2/2 \ll (E_A/A)_{A \geq 3}$$

$$\frac{Q}{M_{\text{hi}}} \sim \frac{Q_3}{1.4 m_\pi} \sim \frac{1}{3}$$

Potential

isospin-symmetric

isospin-violating

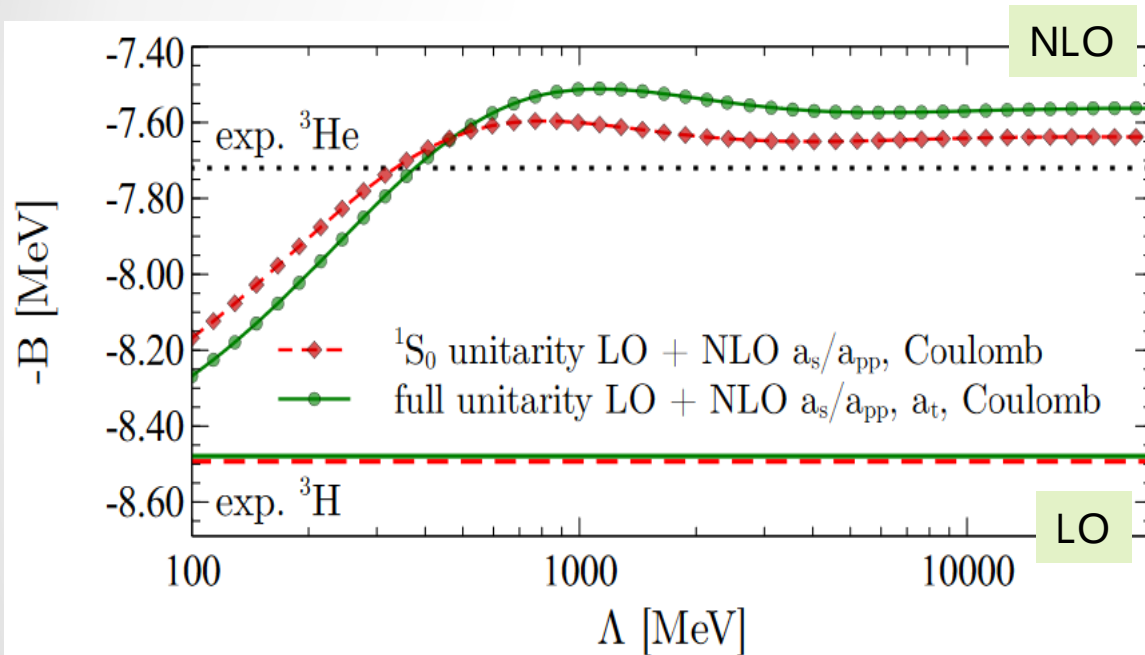


discrete
scale
invariance!

(distorted-wave)
perturbation
theory

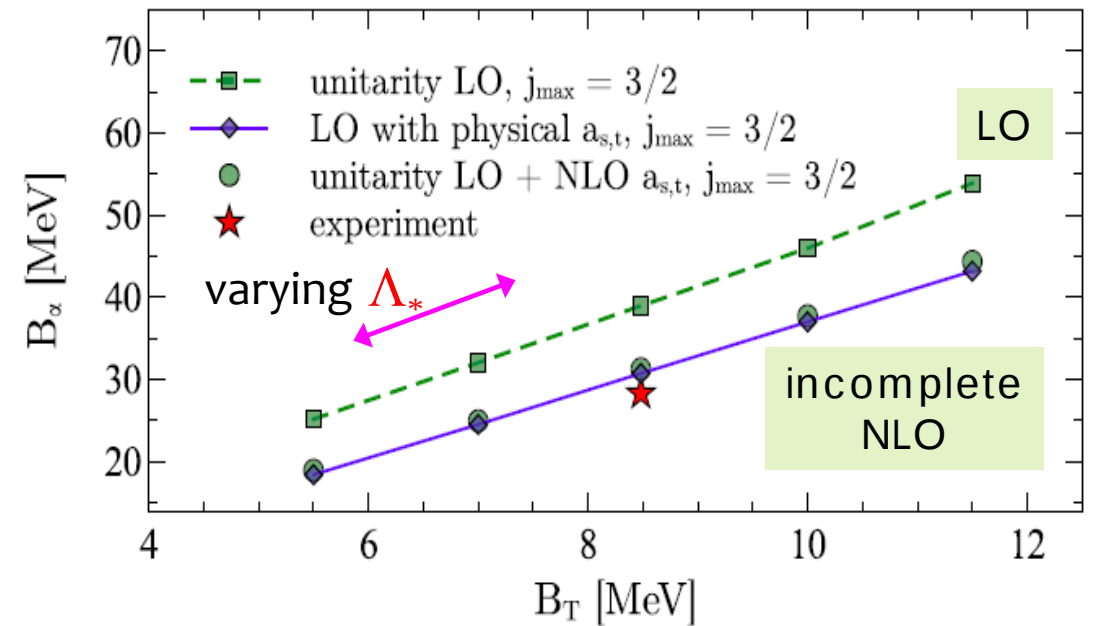
Good description up to alpha particle

$A = 3$



$A = 4$

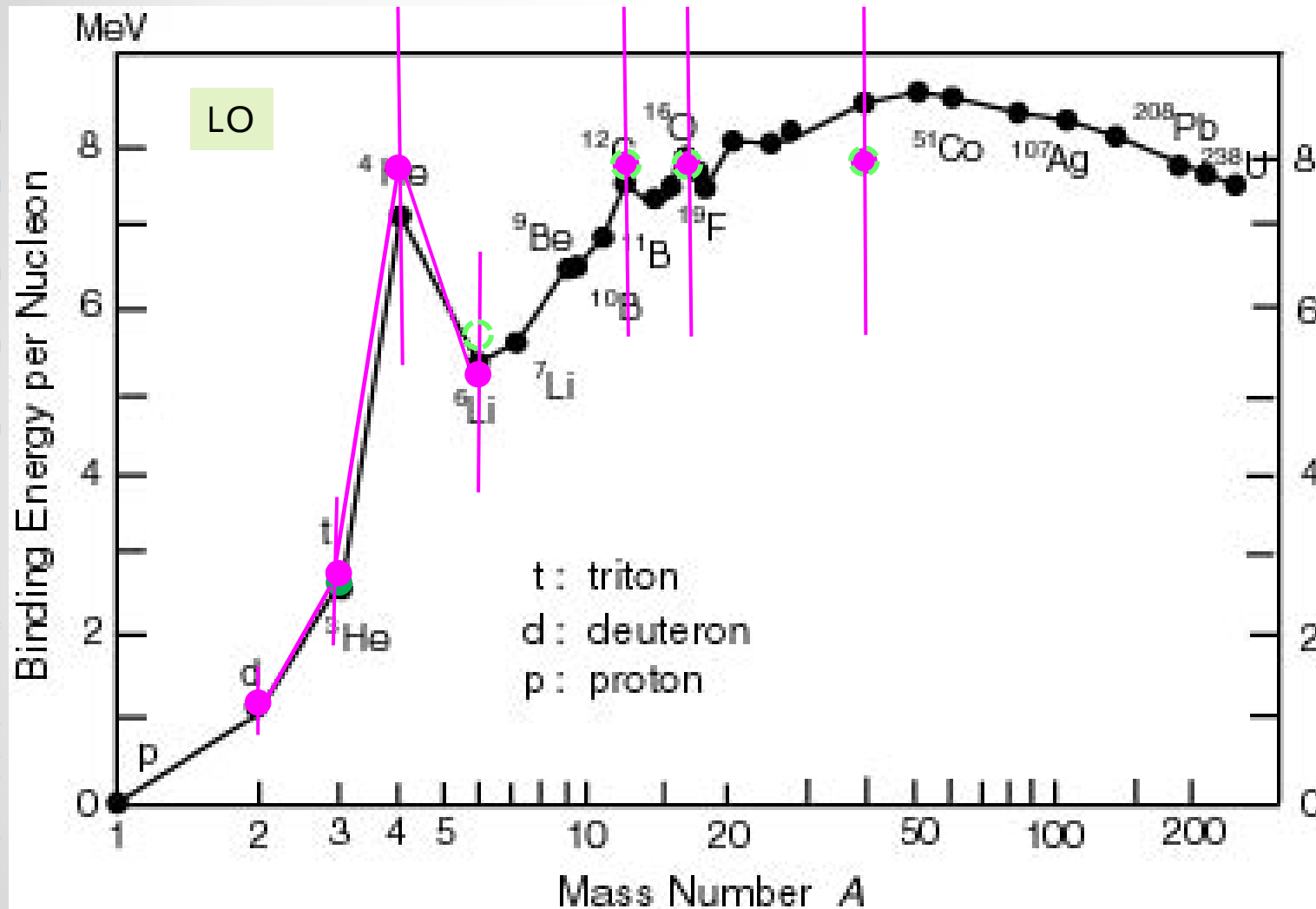
Tjon line



König, Grieshammer, Hammer, vK, *Phys. Rev. Lett.* 118 (2017) 202501

renormalization + near two-body unitarity: Efimov physics and corrections

Instability beyond alpha particle at LO



^6Li Stetcu, **Barrett**, vK '07
Contessi, Schäfer, Gnech,
Lovato, vK '25

but $n-\alpha$ scattering good
Bagnarol, Schäfer, Bazak, Barnea '23

^{12}C Contessi, Schäfer, Gnech,
Lovato, vK '25

^{16}O Contessi, Lovato, Pederiva,
Roggero, Kirscher, vK '17
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Jansen, Papenbrock '18
Contessi, Schäfer, Gnech,
Lovato, vK '25

^{40}Ca Bansal, Binder, Ekström, Hagen,
Jansen, Papenbrock '18

Dawkins, Carlson, vK, Gezerlis '20
Schäfer, Contessi, Kirscher, Mareš '20
Contessi, Schäfer, Kirscher, Lazauskas, Carbonell '23

renormalization + near two-body unitarity: clustering of multicomponent fermions

Improved Actions

Practical problem: how to do (distorted-wave) perturbation theory upon unstable states?

Solution: add to LO *some* subLO corrections, while maintaining renormalization

- no new *physical* parameter at LO
- effect no larger than NLO → removed perturbatively at NLO

$$V^{(0)}(\vec{r}; \Lambda) = C_0^{(0)}(\Lambda) \sum_{ij} \delta_{\Lambda}(\vec{r}_{ij}) + D_0^{(0)}(\Lambda) \sum_{ijk} \delta_{\Lambda}(\vec{r}_{ij}) \delta_{\Lambda}(\vec{r}_{ik}) \quad (\text{neglecting spin-isospin factors})$$

$$\Rightarrow \tilde{V}^{(0)}(\vec{r}; \Lambda, \tilde{R}_2, \tilde{R}_3) = V^{(0)}(\vec{r}; \Lambda) + \Delta V(\vec{r}; \Lambda, \tilde{R}_2, \tilde{R}_3)$$

“fake ranges”

within $\mathcal{O}(Q/M_{\text{hi}})$

$$\Delta V(\vec{r}; \Lambda, \tilde{R}_2, \tilde{R}_3) = \Delta V^{(1)}(\vec{r}; \Lambda, \tilde{R}_2, \tilde{R}_3) + \dots$$

$$V^{(1)}(\vec{r}; \Lambda) = \dots \Rightarrow \tilde{V}^{(1)}(\vec{r}; \Lambda, \tilde{R}_2, \tilde{R}_3) = V^{(1)}(\vec{r}; \Lambda) - \Delta V^{(1)}(\vec{r}; \Lambda, \tilde{R}_2, \tilde{R}_3)$$

etc. within $\mathcal{O}(Q^2/M_{\text{hi}}^2)$

Examples

1) change of renormalization condition

fit to two-body scattering length $\Rightarrow$ fit to A -body binding energy

cf. NNLO_{sat} chiral pot

Ekström *et al.* '15

BUT difficult to discern a potential failure of the expansion as density increases

2) partial resummation of the two-body effective range

Phillips, Rupak, Savage '00

Contessi, Pavón, vK '24

introduce dimer field $\Rightarrow$ two-body coupling energy dependent

➤ convergence radius and *a priori* error estimates do not change

➤ central values for various orders within *a priori* error estimates

➤ central values at lower orders closer to exact for $\tilde{R}_2 \leq \tilde{R}_c$

where two-body effective range is reproduced

BUT difficult to apply to more bodies

3) partial resummation of a chain of ERE parameters

$$\Delta V(\vec{r}; \Lambda, \tilde{R}_2, \tilde{R}_3) = \sum_{ij} \left[\tilde{C}(\tilde{R}_2^{-1}) \delta_{\tilde{R}_2^{-1}}(\vec{r}_{ij}) - C_0^{(0)}(\Lambda) \delta_{\Lambda}(\vec{r}_{ij}) \right] \\ + \sum_{ijk} \left[\tilde{D}(\tilde{R}_2^{-1}, \tilde{R}_3^{-1}) \delta_{\tilde{R}_3^{-1}}(\vec{r}_{ij}) \delta_{\tilde{R}_3^{-1}}(\vec{r}_{ik}) - D_0^{(0)}(\Lambda) \delta_{\Lambda}(\vec{r}_{ij}) \delta_{\Lambda}(\vec{r}_{ik}) \right]$$

^4He

atomic clusters to NLO

$A \leq 5$

Contessi, Schäfer, vK '23

- central values at various improved orders:
 - slightly better than unimproved values
 - differ from unimproved results no more than change in order
- improvement at NLO nearly independent of fake range $\tilde{R}_2 \leq \tilde{R}_c$

Light-Medium Nuclei

Can we find a range of fake ranges that give stability at LO?

(two two-body S-wave channels)

one possible
improvement

$$\tilde{R}_{2s} = x R_{2s} \quad \tilde{R}_{2t} = x R_{2t}$$

fitted to corresponding
effective ranges

$$\tilde{R}_3 = x R_3$$

fitted for maximal improvement
of alpha binding energy

$$\Rightarrow \tilde{V}^{(0)}(\vec{r}; x) = V^{(0)}(\vec{r}; \Lambda) + \Delta V(\vec{r}; \Lambda, x R_{2s}, x R_{2t}, x R_3)$$

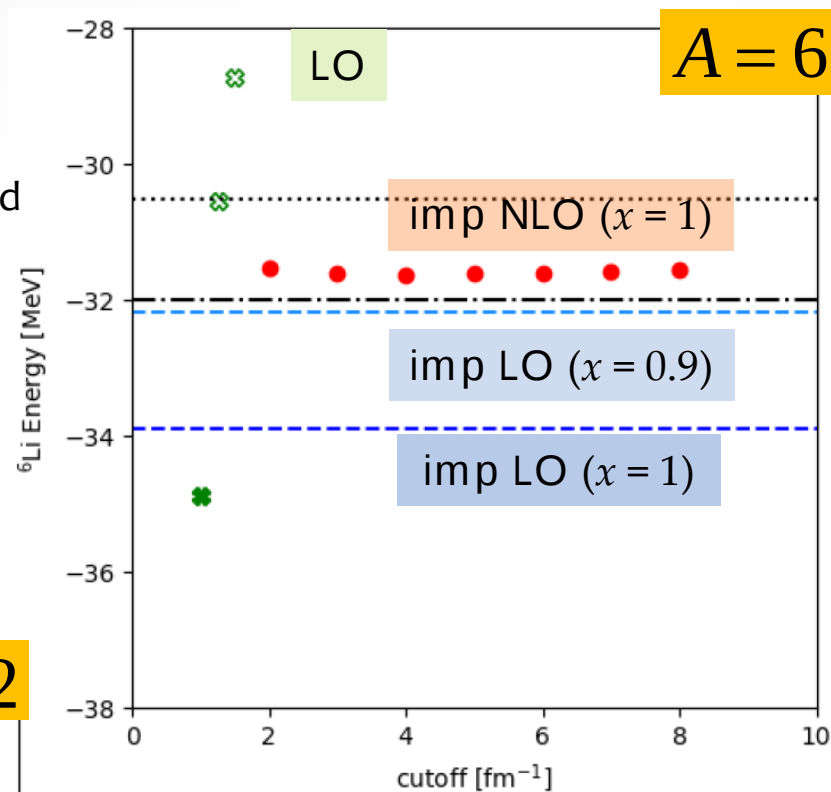
residual
cutoff
dependence
at NLO

$$E_A^{(1)}(\Lambda) = E_A^{(1)} \left(1 + \frac{q_A^{(1)}}{\Lambda} \right)$$

uncertainty from variation for $\Lambda \geq 2 \text{ fm}^{-1}$

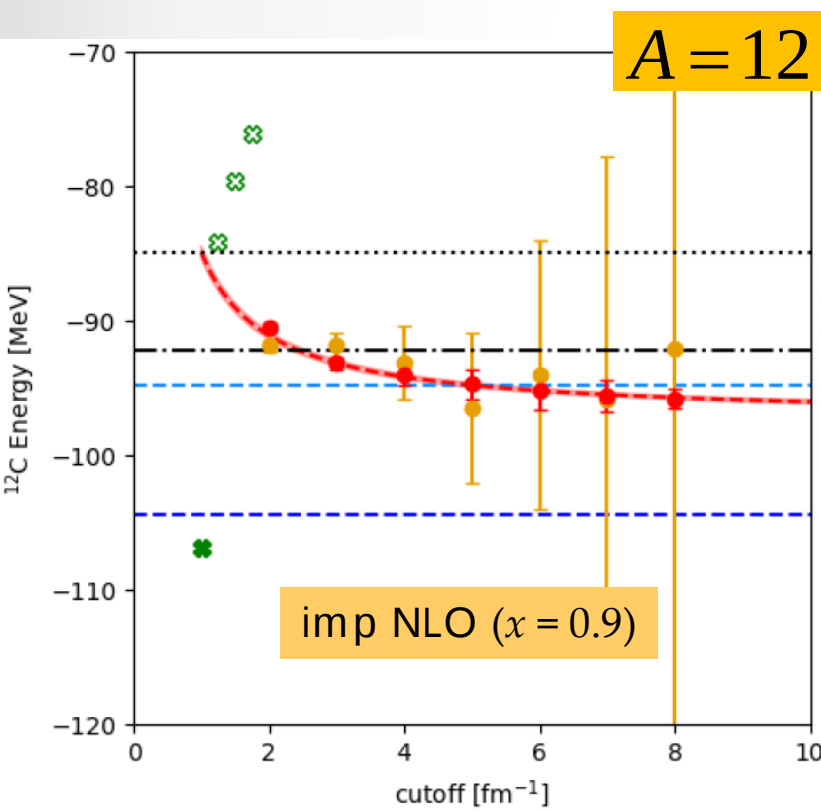
$$\sim \text{uncertainty from } \frac{Q}{M_{\text{hi}}} \sim \frac{Q_3}{1.4 m_\pi} \sim \frac{1}{3}$$

exp threshold



$$E_6^{(1)} = -(31.57 \pm 0.02 \pm 0.3) \text{ MeV}$$

$$E_6^{(\text{exp})} = -31.994 \text{ MeV}$$

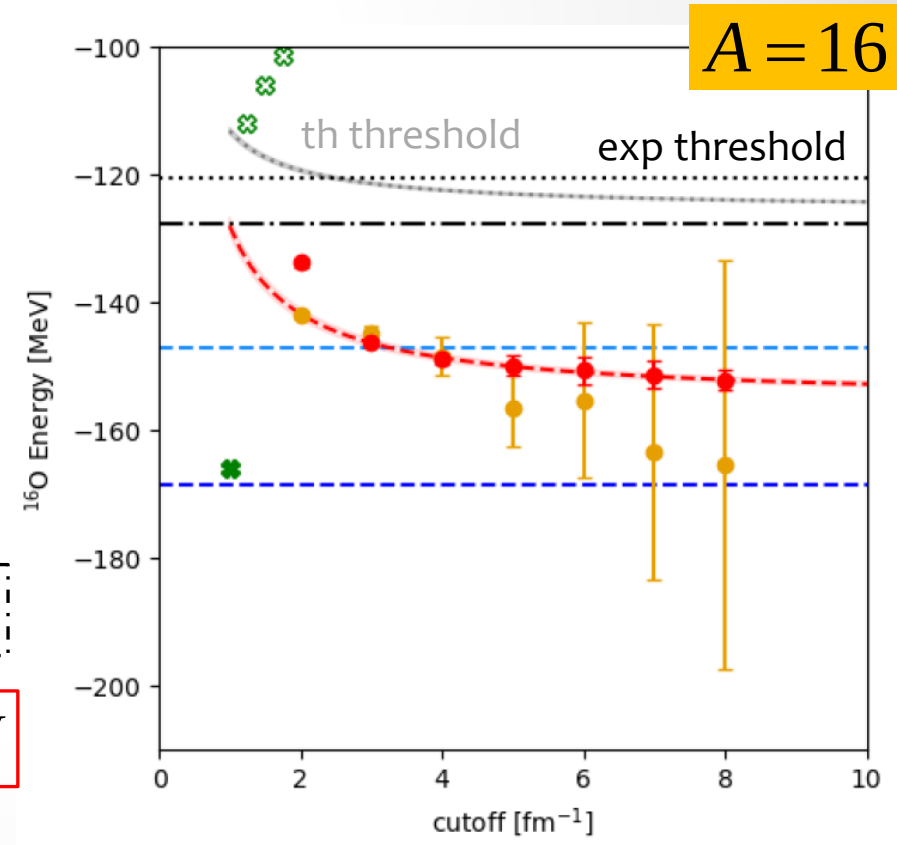


$$E_{12}^{(\text{exp})} = -92.162 \text{ MeV}$$

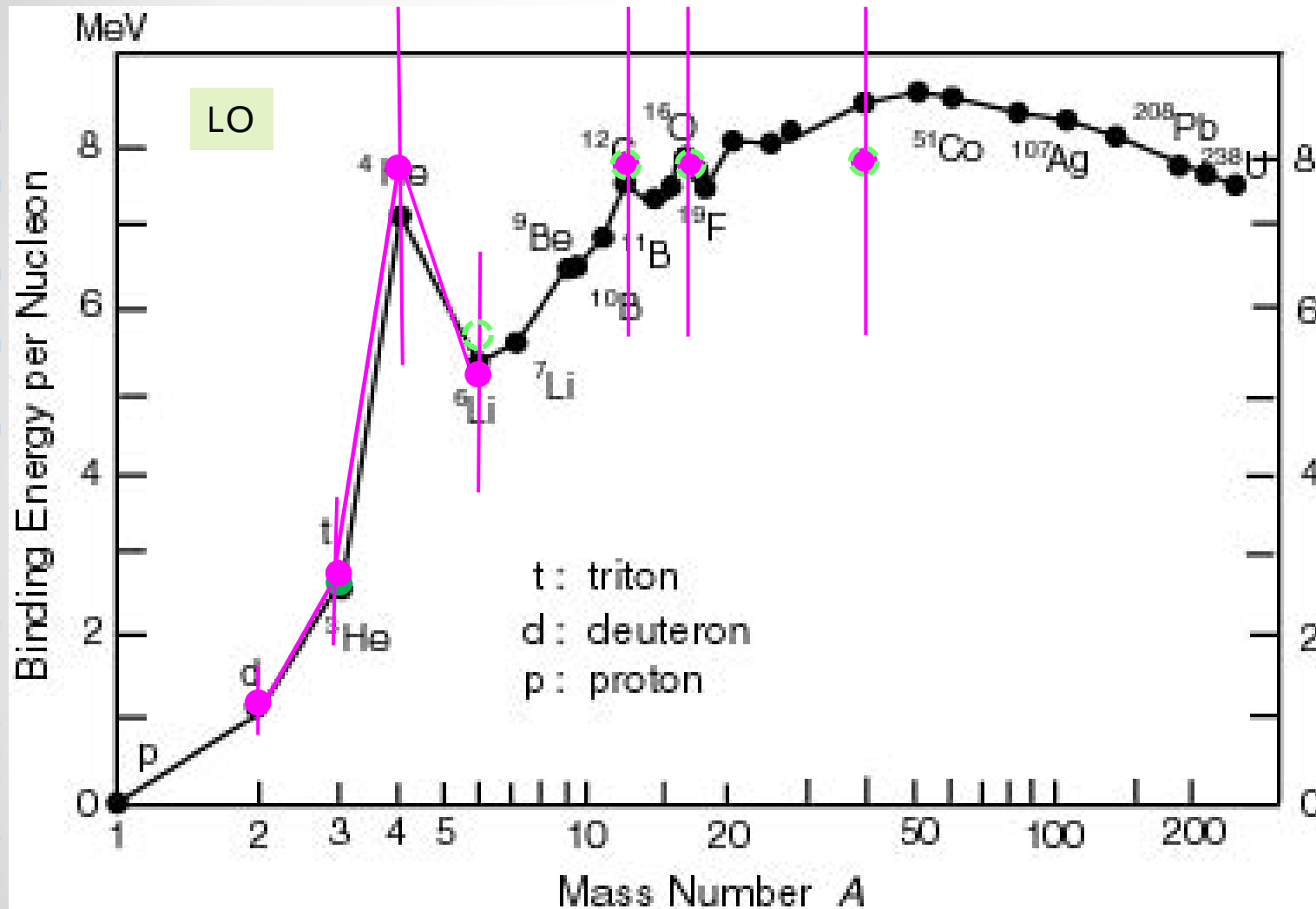
$$E_{12}^{(1)} = -(97.3 \pm 0.1 \pm 5) \text{ MeV}$$

$$E_{16}^{(\text{exp})} = -127.619 \text{ MeV}$$

$$E_{16}^{(1)} = -(155.6 \pm 0.3 \pm 20) \text{ MeV}$$



Instability beyond alpha particle at LO



${}^6\text{Li}$ Stetcu, **Barrett**, vK '07
Contessi, Schäfer, Gnech,
Lovato, vK '25

but n - α scattering good
Bagnarol, Schäfer, Bazak, Barnea '23

${}^{12}\text{C}$ Contessi, Schäfer, Gnech,
Lovato, vK '25

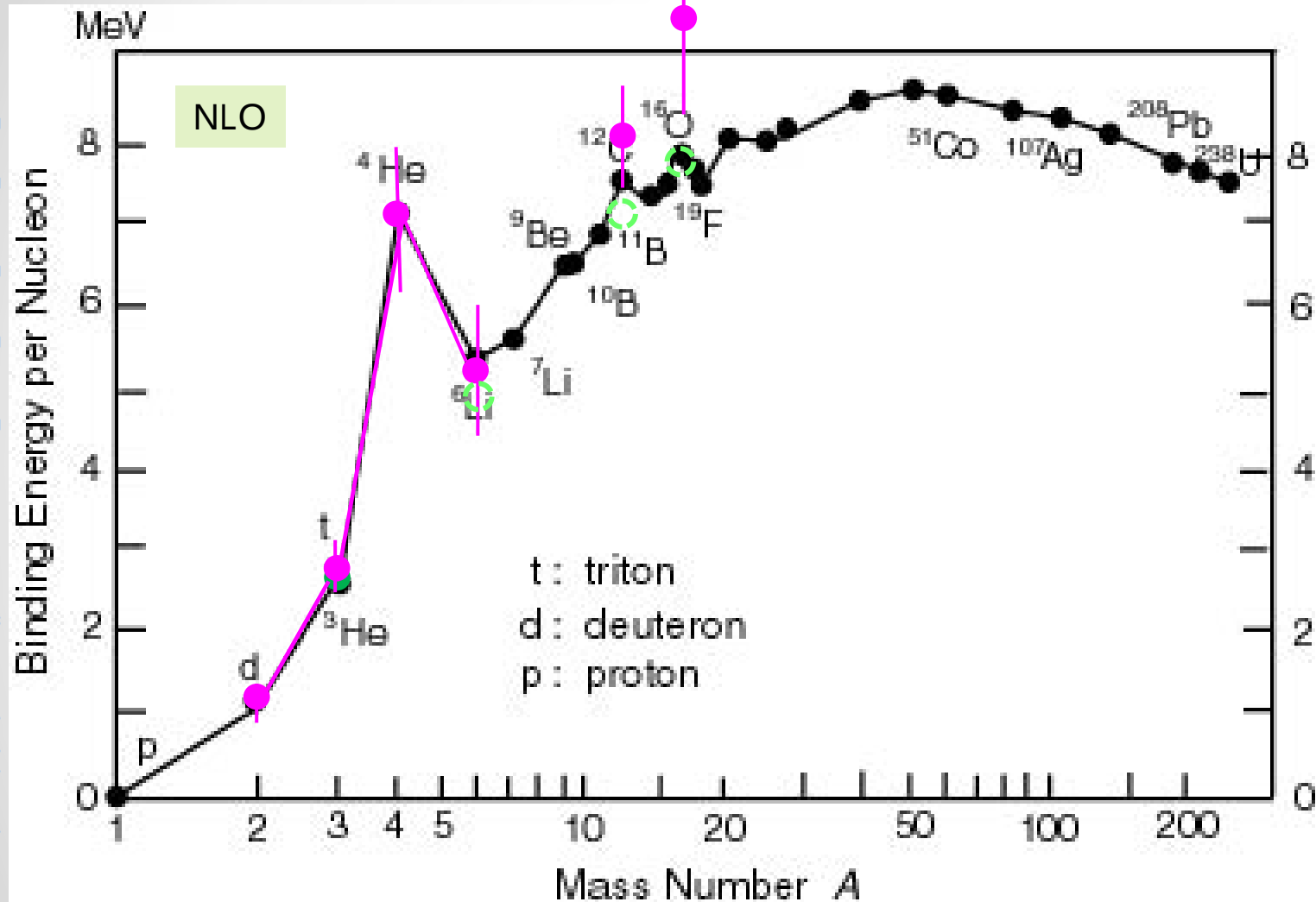
${}^{16}\text{O}$ Contessi, Lovato, Pederiva,
Roggero, Kirscher, vK '17
Bansal, Binder, Ekström, Hagen,
Jansen, Papenbrock '18
Contessi, Schäfer, Gnech,
Lovato, vK '25

${}^{40}\text{Ca}$ Bansal, Binder, Ekström, Hagen,
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Dawkins, Carlson, vK, Gezerlis '20
Schäfer, Contessi, Kirscher, Mareš '20
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renormalization + near two-body unitarity: clustering of multicomponent fermions

Stability beyond alpha particle at NLO



^6Li Contessi, Schäfer, Gnech, Lovato, vK '25

and n- α scattering good
Bagnarol, Schäfer, Bazak, Barnea '23

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renormalization + near two-body unitarity: clustering of multicomponent fermions

❑ cf. order-by-order weakening of Wigner bound

Beck, Bazak, Barnea '20

-- here more general

❑ cf. finite-range pionless potentials

Kievsky *et al.* '20
Recchia *et al.* '22

...

-- here preserving power counting

(# parameters, perturbative corrections, breakdown scale, ...)

Conclusion

Partial range effects can be included at LO without affecting power counting and model independence

Lower orders are improved without affecting convergence radius

Improvement is sufficient to provide stability for near-unitarity fermions at LO and allow perturbation theory for corrections

Light-medium binding energies are within $\sim 15\%$ of experiment at NLO

Need: more observables, heavier nuclei, higher orders