The multi-shell connection between the fundamental structure models

J. Cseh

HUN-REN ATOMKI, Debrecen, Hungary
cseh@atomki.hu

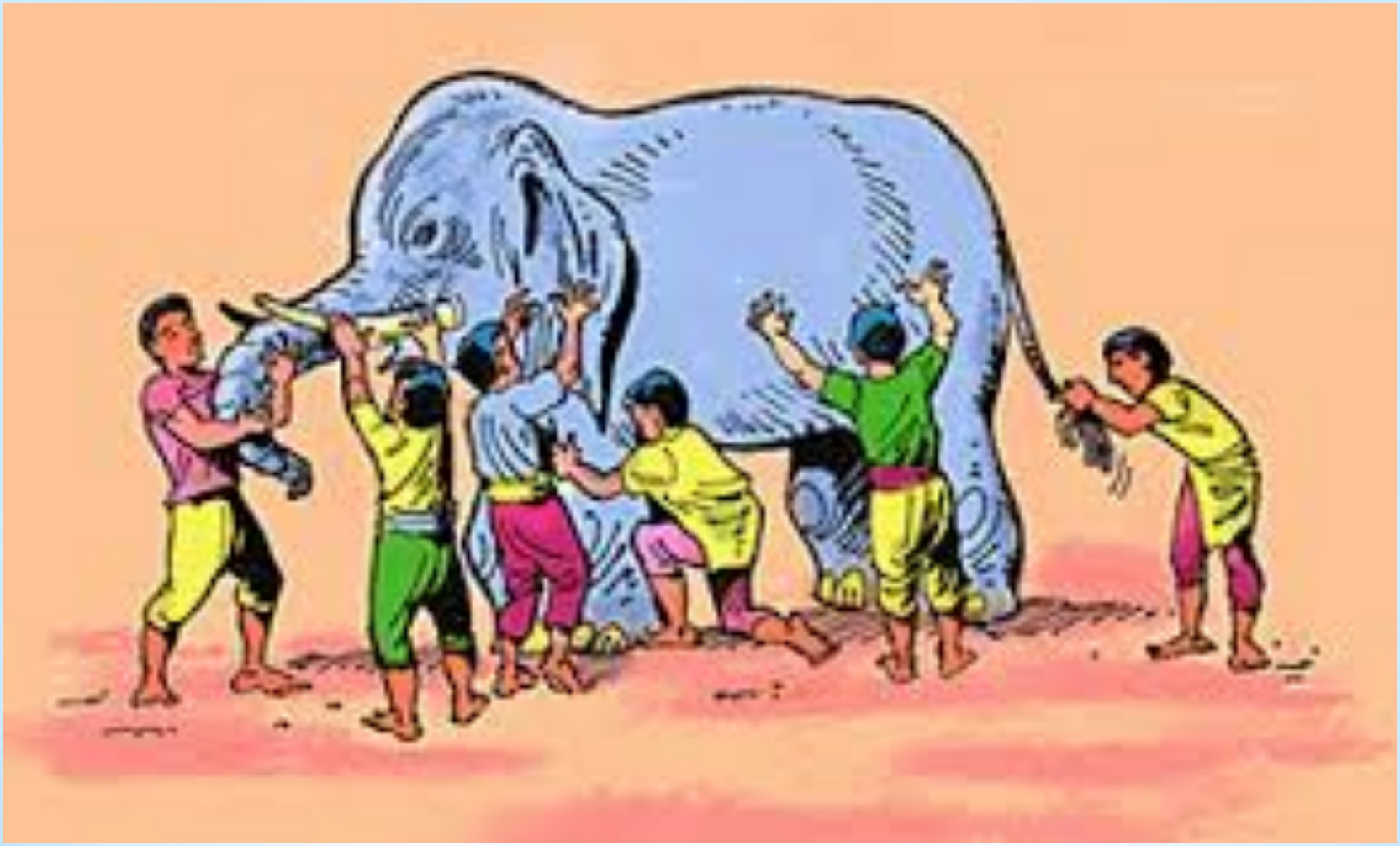
14th International Spring Seminar on Nclear Physics:

”Cutting-edge developments in nuclear structure physics”

Ischia, May 19-23, 2025

Nucleus: too many nucleons,
but not enough for pure statistical treatment.

Shell, collective and cluster degrees of freedom.



The elephant is like

snout: a thick snake
ear: a fan
leg: a pillar…

The nucleus is like

magic numbers: small atom
rotational bands: liquid drop
decay: molecule, cluster…

I. Shell, collective and cluster degrees of freedom

II. Connection (1958) U(3) dyn. symm.: single shell

III. Multi-shell: Us(3) x Ue(3) MUSY

IV. Features of MUSY

V. Applications, predictions

VI. Summary and conclusions

II. Connection 1958

Elliott: *Proc. Roy. Soc. A 245, 28,562 (1958)*

Spectra of light nuclei,
deformation + rotation from spherical shell model:
SU(3).

From shell model to cluster model:
Wildermuth-Kanellopoulos: *Nucl. Phys. 7, 150 (1958)*
Harm. osc. appr.

Cluster-shell connection: SU(3).
Bayman-Bohr: *Nucl. Phys. 9, 596 (1958/59).*

A quadrupole collective or a cluster band is picked up from the spherical shell model basis by their special SU(3) symmetry.

For a single major-shell problem the connection between the shell, collective and cluster models is provided by an SU(3) dynamical symmetry:

$$U(3) \supset SU(3) \supset SO(3) .$$

How about the multi-shell problem?

III. Multiconfigurational dynamical symmetry (MUSY)

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Extension of the U(3) connection from 1958
(J. Cseh, Phys. Rev. C 103 (2021) 064322.)

Intersection of the algebraic
- shell
- collective
- cluster models

of multi-shell problems, with microscopic spaces.

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

$U_s(3)$: symmetry of the valence shell
symmetry of the internal cluster structure

$U_e(3)$: major shell excitations

Since 1958

multi-shell models
- with algebraic structure and
- microscopic model space

in the
- shell
- collective
- cluster pictures.

$$G_{sym} = U_s(3) \otimes U_e(3)$$

Symmetry group of the many-shell space.

G_{sym}

may or may not be embedded into a larger group

$G_{dyn} \supset G_{sym}$

a dynamical group, which can generate the spectrum, too.

Shell:

SA - NCSM

Dytrych et al. J. Phys. G 35 (2008) 123101

SA - Quartet Model

J. Cseh, Phys. Lett. B **743**, 213 (2015).

Cluster:

K.T.. Hecht Nucl. Phys. A 283 (1977) 223.

$$G_{sym} = U_s(3) \otimes U_e(3)$$

Symmetry group of the many-shell space.

G_{sym}

may or may not be embedded into a larger group

$G_{dyn} \supset G_{sym}$

a dynamical group, which can generate the spectrum, too.

Symplectic shell model

G. Rosensteel, D. Rowe, PRL 38 (1977) 10

Extension of Elliott, microscopic coll. model

Also in no-core vesion

$$G_{dyn} = Sp(6,R)$$

Contracted symplectic model

(D.J. Rowe, G. Rosensteel, Phys. Rev. C 25 (1982) 3236(R);
O. Castanos, J. P. Draayer, Nucl. Phys. A 491 (1989) 349.)

$$G_{dyn} = U_s(3) \otimes U_e(6)$$

Simpler mathematical structure,
bosonized description,
algebraic collective model of the multi-shell problem.

Vibron model U(4) for dipole coll.: relative motion

F. Iachello, Phys. Rev. **23**, 2778 (1981)

Fully algebraic description of clusterization:
not only the states, but also the operators carry group symmetries.

Semimicroscopic algebraic cluster model

J. Cseh, G. Lévai, Ann. Phys. 230 (1994) 165.

$$G_{dyn} = U_{C_1}(3) \otimes U_{C_2}(3) \otimes U_R(4)$$

Microscopic model space, algebraic operators.
Internal cluster structure: Elliott model,
relative motion: modified vibron.

Spin-isospin sector:
in
symplectic
contarcted symplectic
semimicroscopic algebraic cluster models

$$U^{ST}(4) \supset U^{S}(2) \otimes U^{T}(2)$$

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Identical symmetry algebras of the
- shell
- collective
- cluster models,

embedded into different dynamical algebras.

IV. Features of MUSY I

(i) Composite symmetry of composite systems.
J. Cseh, Symmetry 12 (2023) 371.

System: more than one configuration.
Symmetry:
1. dynamical symmetry in each configuration +
2. tranformations (symmetries) between the configurations.

Analog: nuclear SUSY
1. dynamical symmetry in boson, fermion sectors
2. supertransformations.

Interesting features of MUSY II

(ii) Identical spectra both with invariant and with non-invariant Hamiltonians.

J. Cseh, Symmetry 12 (2023) 371.

(iii) Dual symmetry-breaking

J. Cseh, Eur. Phys. J. ST 229 (2020) 2543.

Dynamical breaking of SU(3)

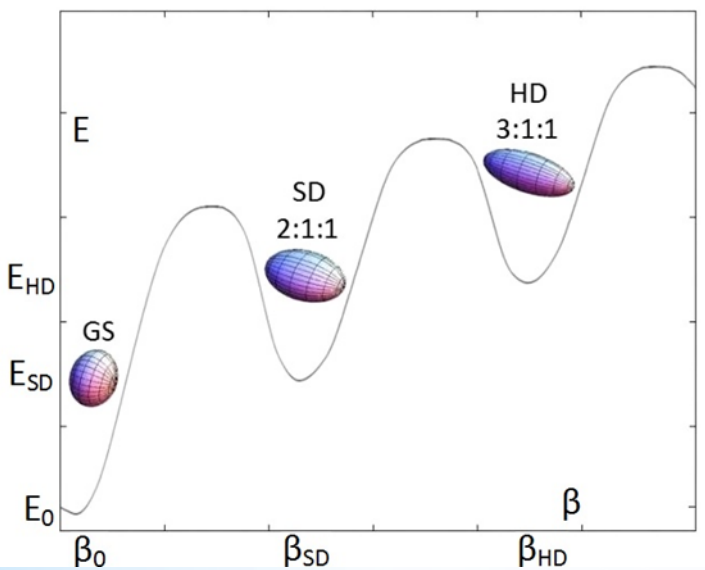
Spontaneous breaking of SO(3): deformation

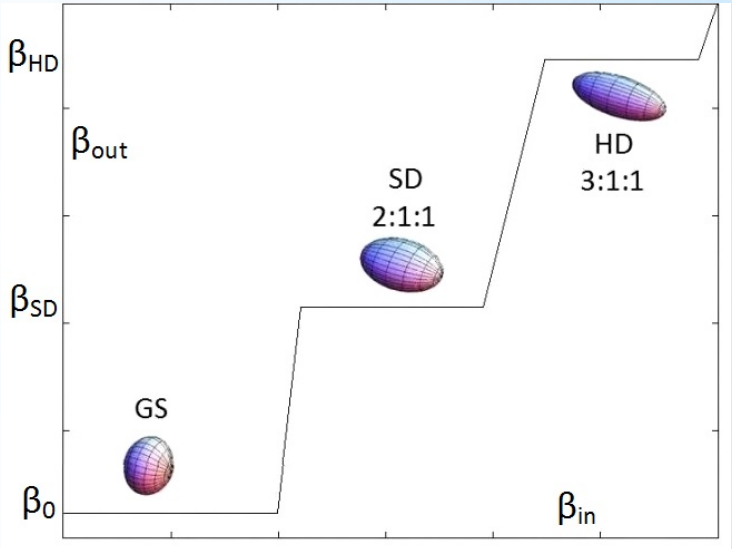
J. Cseh, Phys. Lett. B 793 (2019) 59.

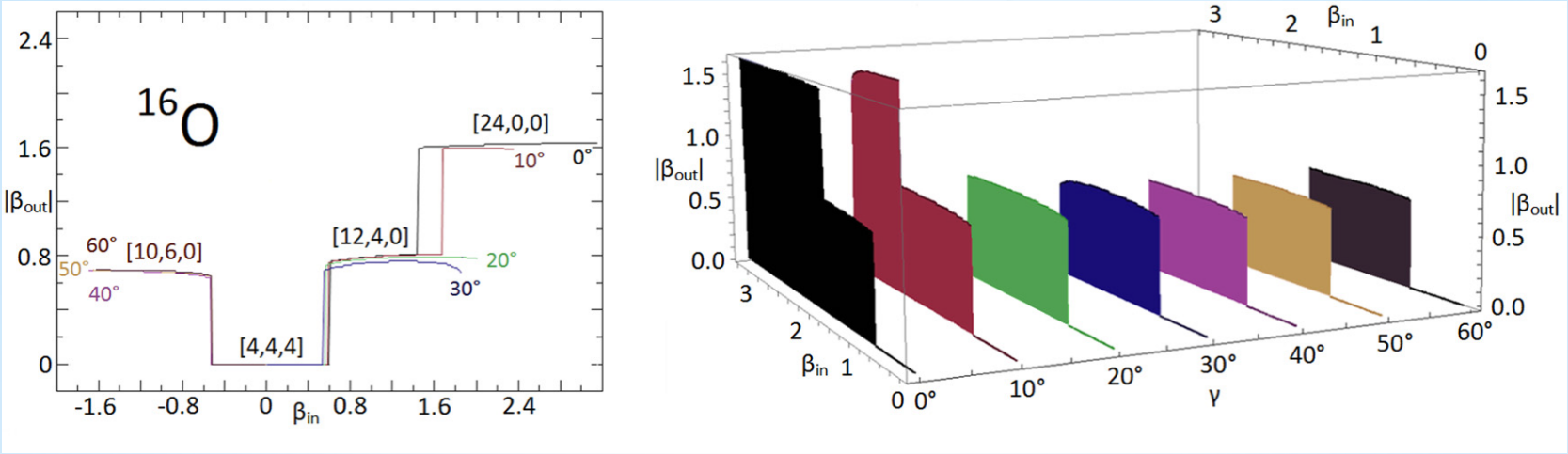
V. Applications, predictions

A) Shape isomers from the stability and selfconsistency of the (quasidynamiclal) U(3)

J. Cseh, G. Riczu, J. Darai, Phys. Lett. B 795 (2019) 160.



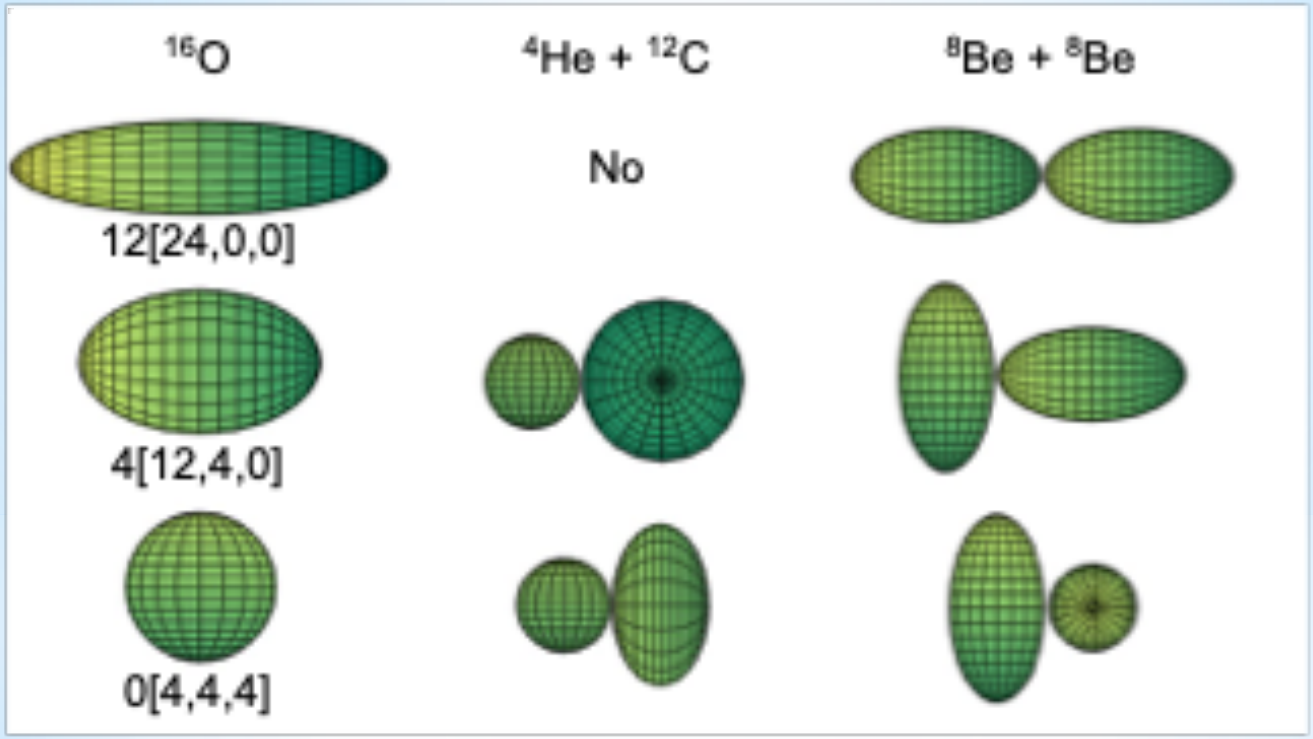


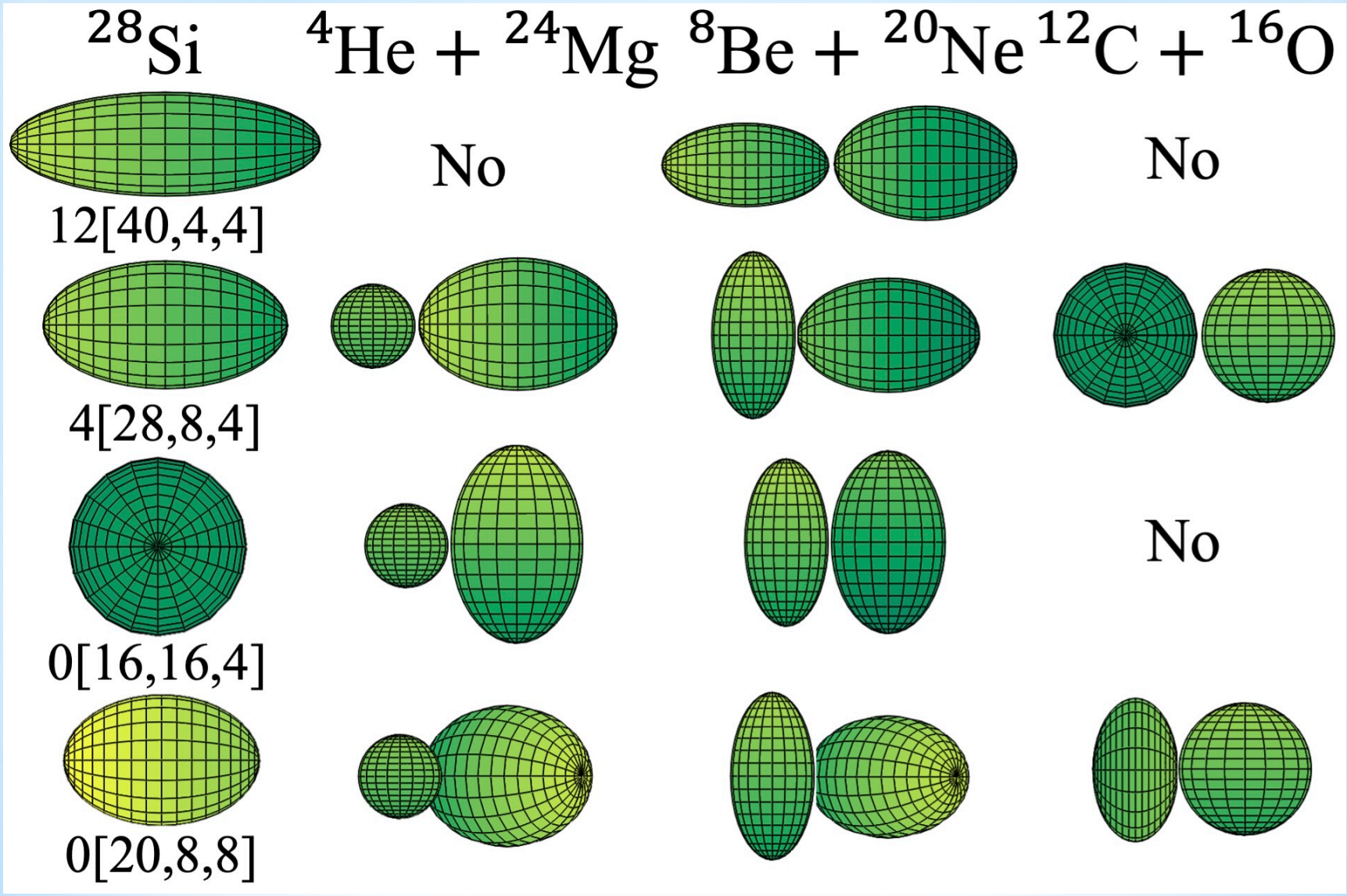

16O
[24,0,0]
10°
0°
60°
[10,6,0]
50°
40°
[12,4,0]
20°
30°
[4,4,4]
|βout|
βin
2.4
1.6
0.8
0
-1.6
-0.8
0
0.8
1.6
2.4
1.5
1.0
0.5
0.0
3
2
1
0
0°
10°
20°
30°
40°
50°
60°
γ

B) Clusterization of shape isomers

P. Dang, G. Riczu, J. Cseh, Phys. Rev. C 107 (2023) 044315

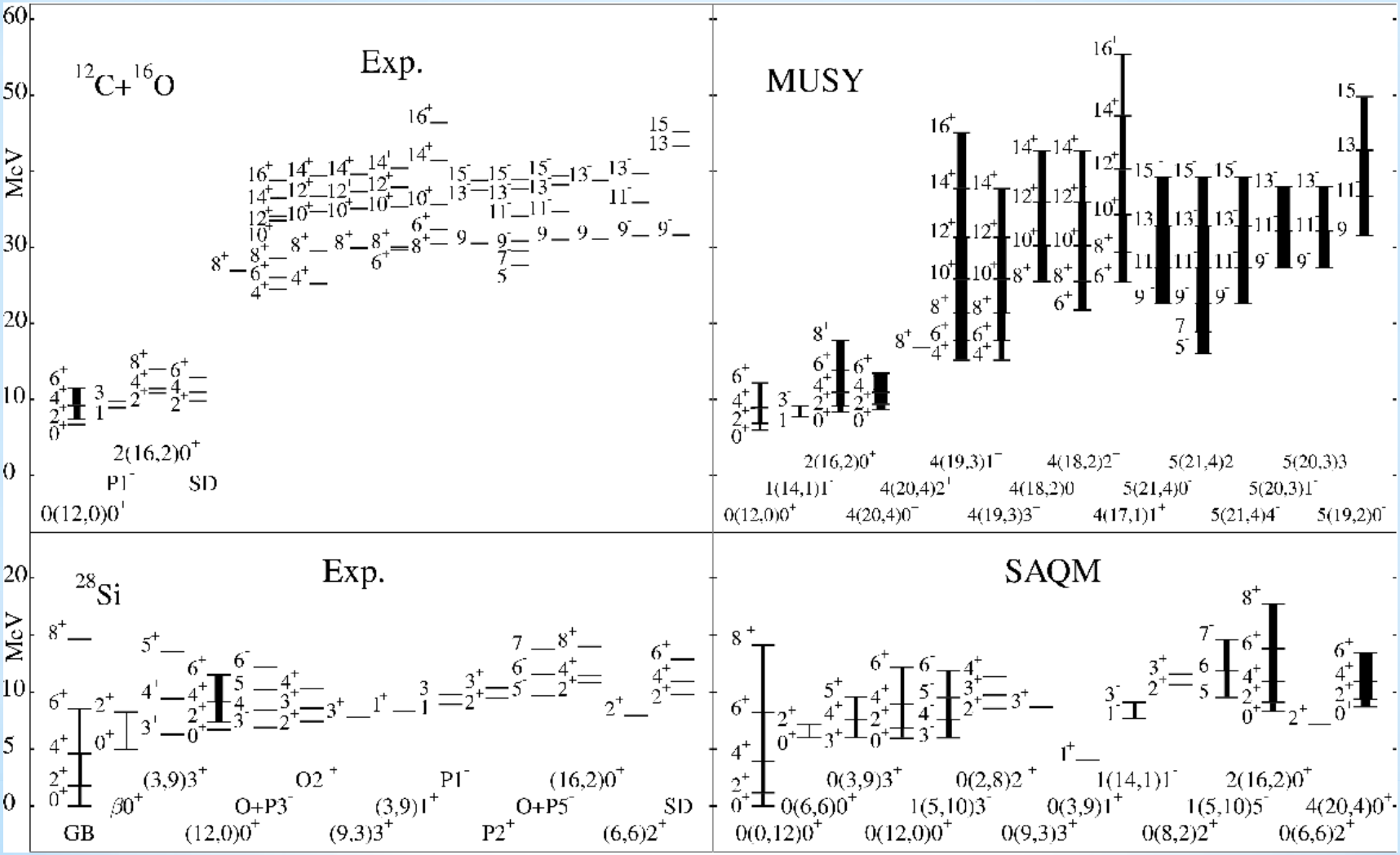
Shape isomers of α-like nuclei



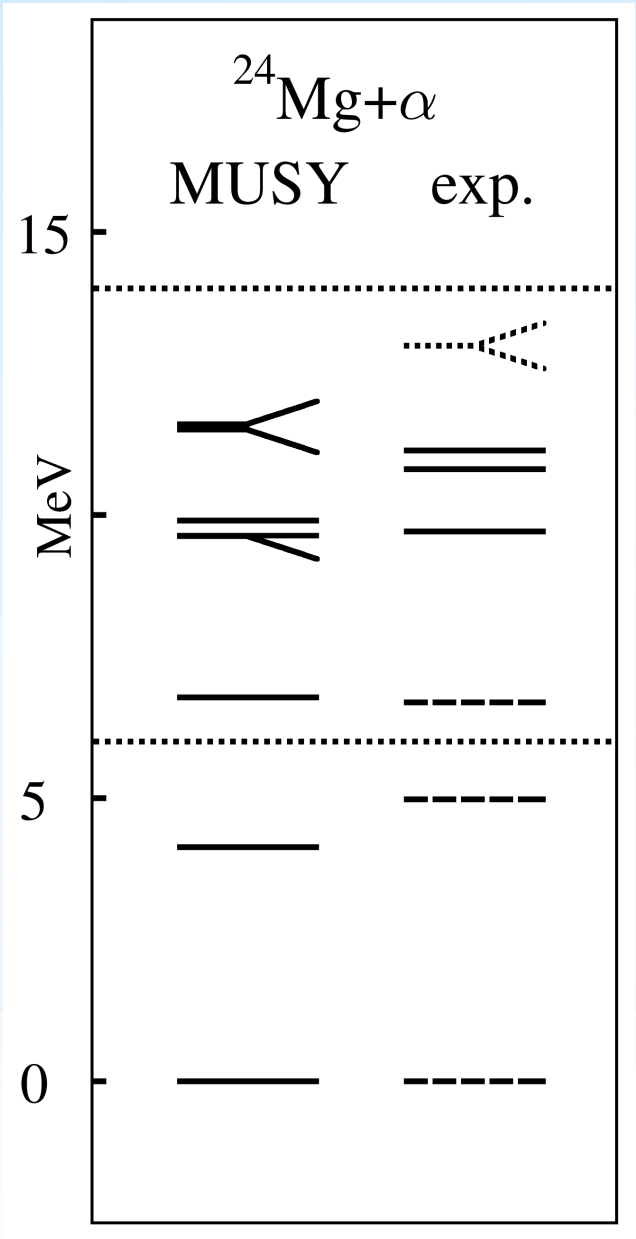

^{28}Si
^{4}He + ^{24}Mg
^{8}Be + ^{20}Ne
^{12}C + ^{16}O
No
No
No
12[40,4,4]
4[28,8,4]
0[16,16,4]
0[20,8,8]

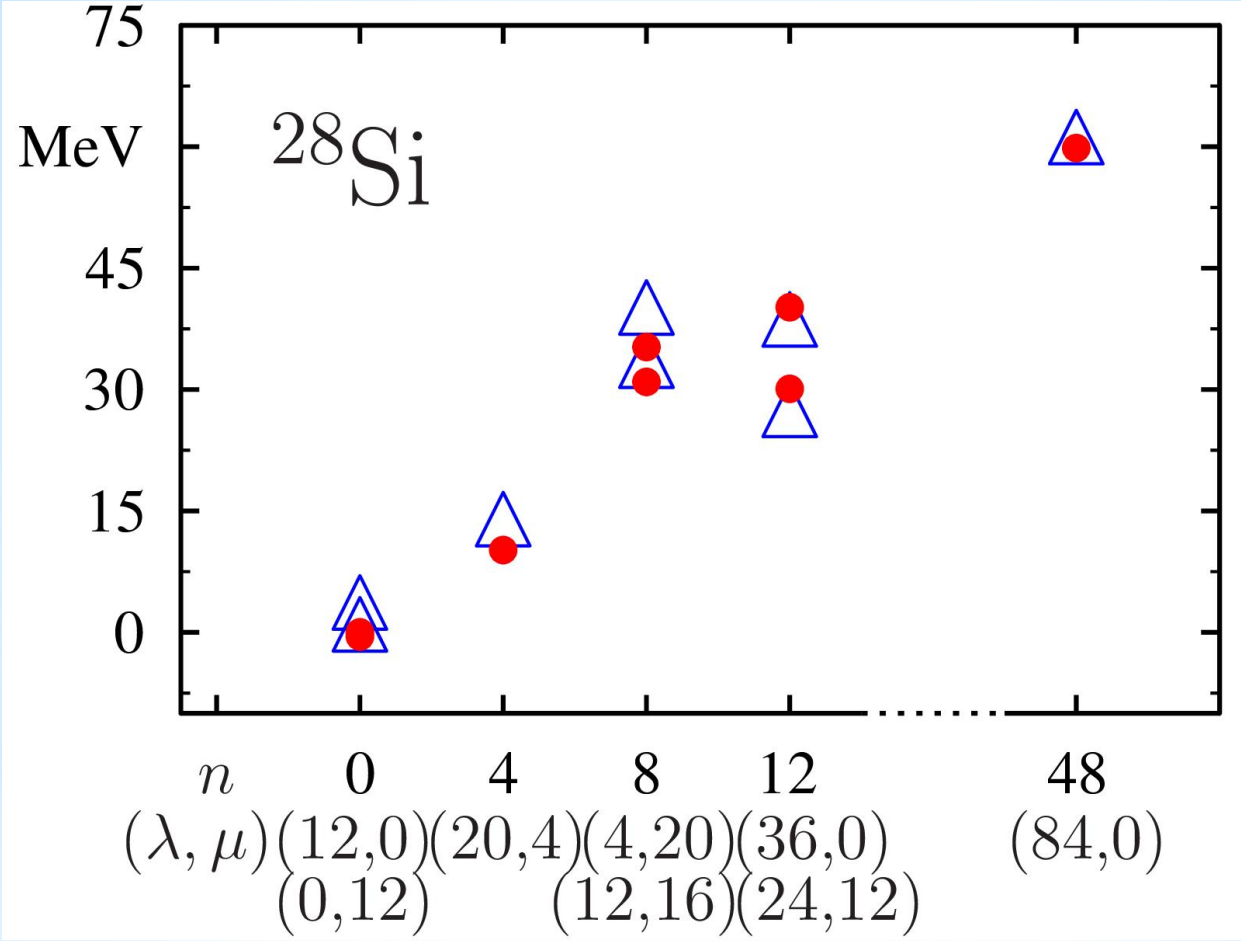
C) Spectrum

$$H = \epsilon n + \alpha C^2_{SU3} + \beta C^3_{SU3} + \gamma \frac{1}{2\theta} C^2_{SO3}$$



J. Cseh, G Riczu, Phys. Lett B **757**, 312 (2016)


^{24}Mg+α
MUSY
exp.
15
MeV
5
0


75
MeV
45
30
15
0
^{28}Si
n
0
4
8
12
48
(λ, μ)
(12,0)
(20,4)
(4,20)
(36,0)
(84,0)
(0,12)
(12,16)
(24,12)

Typical applications:

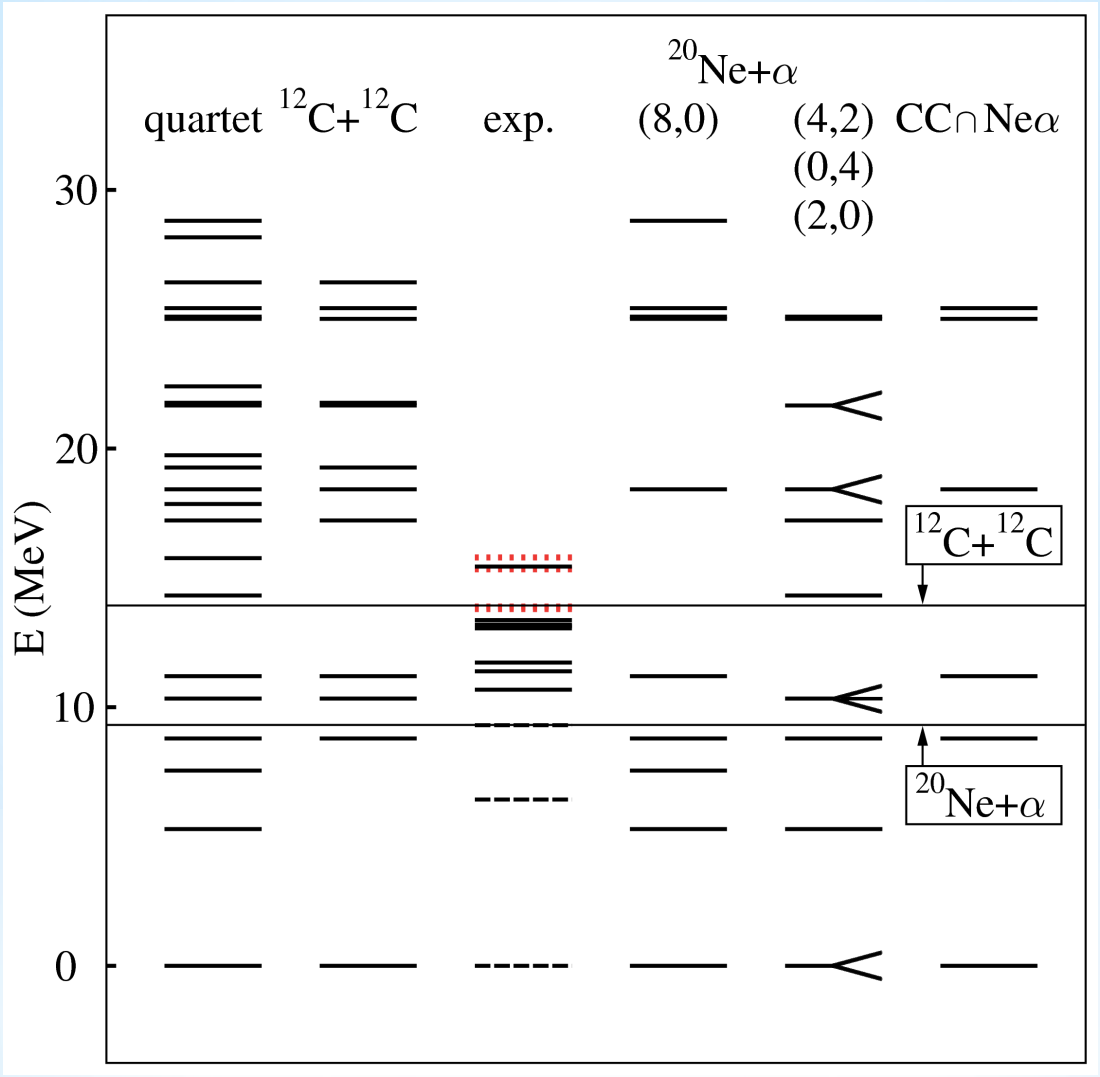
Detailed spectra in regions of
GS: shell model, core + α, HI resonances

24Mg: GS, SD
28Si : GS, SD
36Ar : GS, SD, HD
44Ti : GS, SD

.

D) Distribution of the 12C+12C cluster states and the extension of the Hoyle-state paradigm

J. Cseh, G. Riczu, D. Jenkins, PRC 87 (2013) 067301.



E) E2 transition in superdeformed bands

G. Riczu, J. Darai, J. Cseh, Phys. Scr. 99 (2024) 045306

from the low-energy transitions.

V. Summary on MUSY

Composite symmetry

Dynamical symmetry in each configuration

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Symmetry connecting the configurations

Model spaces:

- microscopic
- huge (e.g. from GS to linear chain)

Operators (interactions)

- simple
- symmetric (e.g. doubly invariant U(3), U(N))

Connection of configurations: transparent

Complementer approach

Usual viewpoint:

- specific configuration - model
- description of data
- connection to other configurations ?

Unusual viewpoint:

- connection of configurations
- description of data

Unified description of spectra in different

- regions of energy
- deformation
- reactions
- configurations: cluster, shell

Predictive power

Cluster-shell coexistence

Disadvantage:

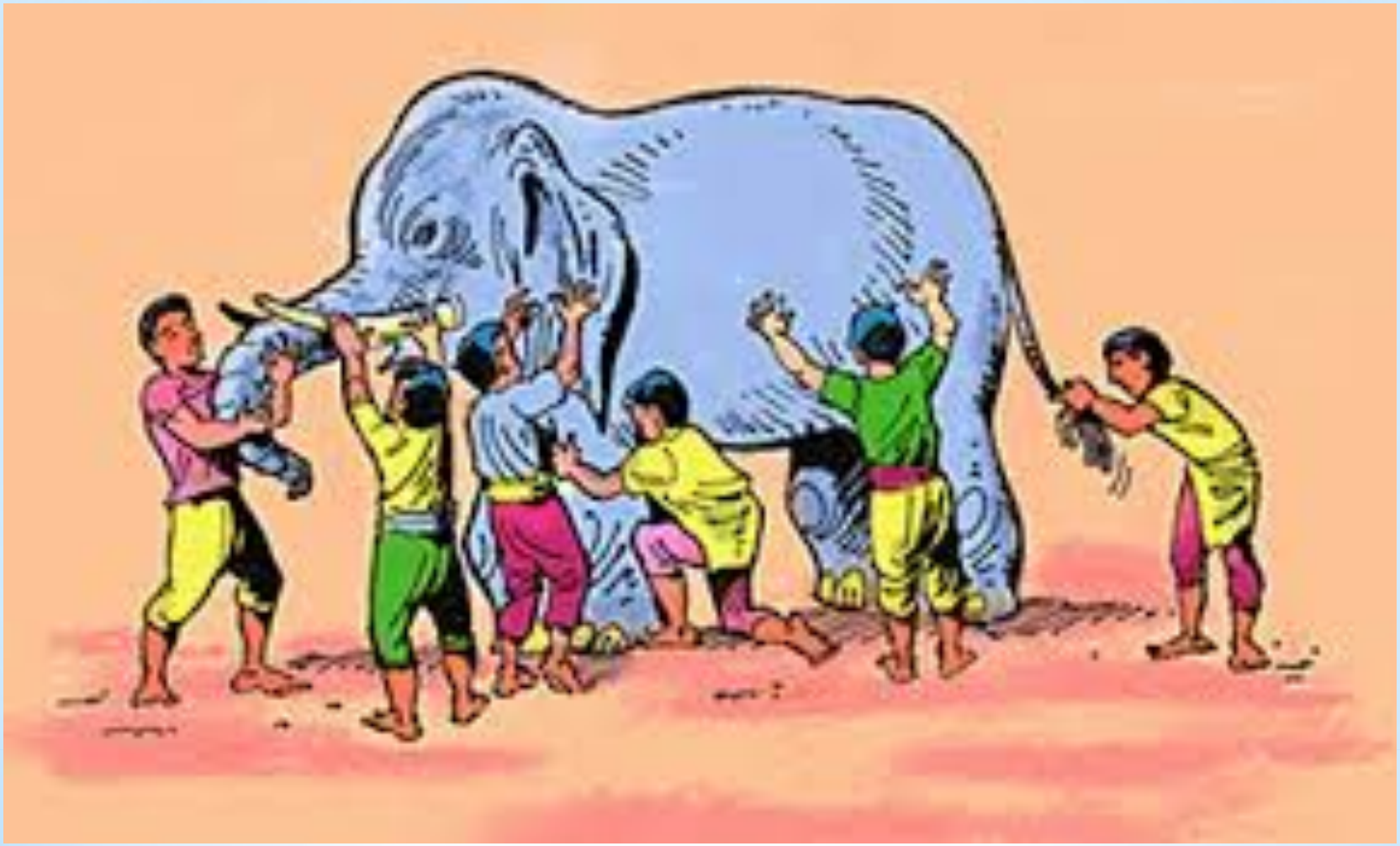
fine details

Advantage:

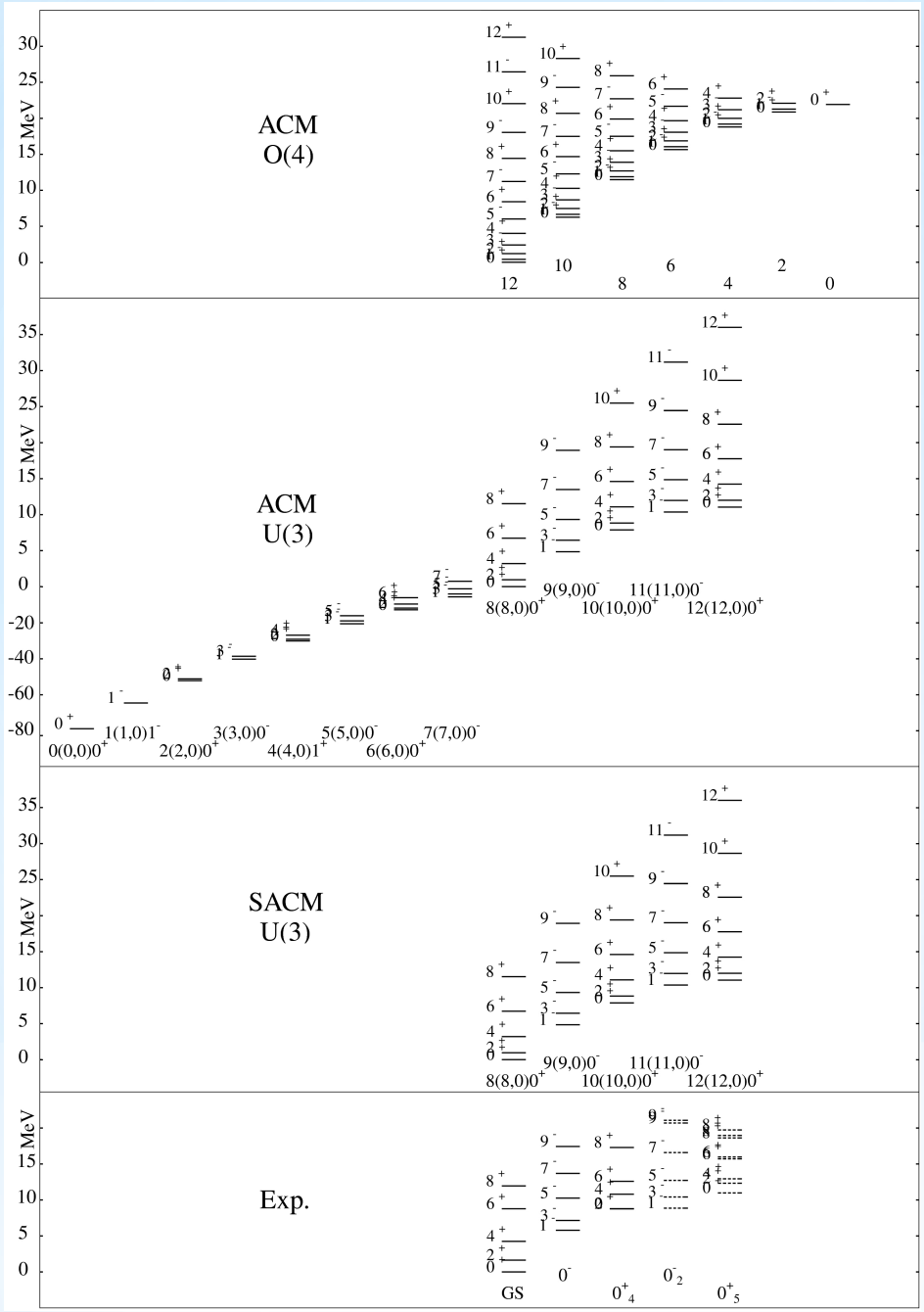
gross features

skeleton (of the elephant)

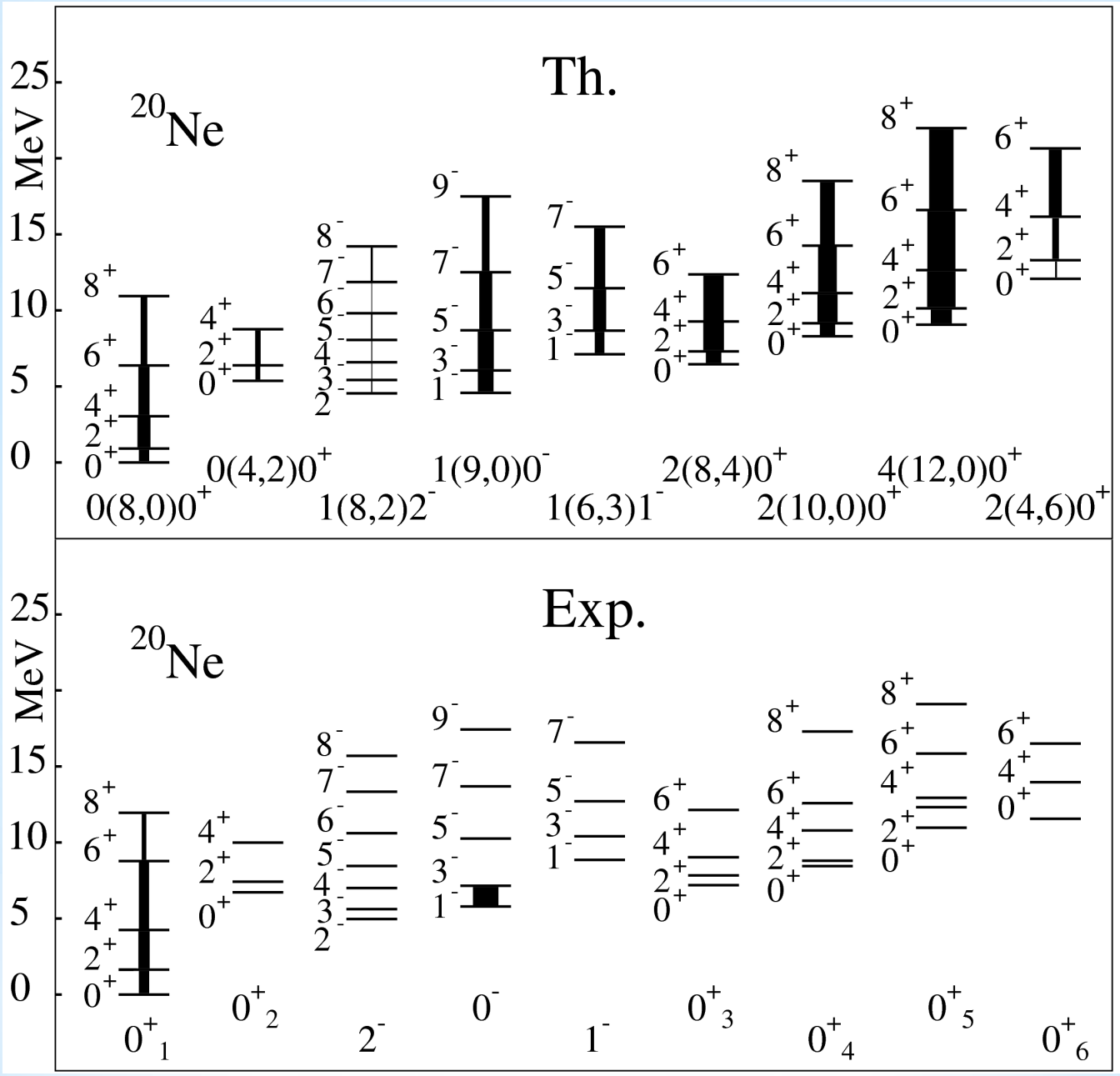
from foot to head



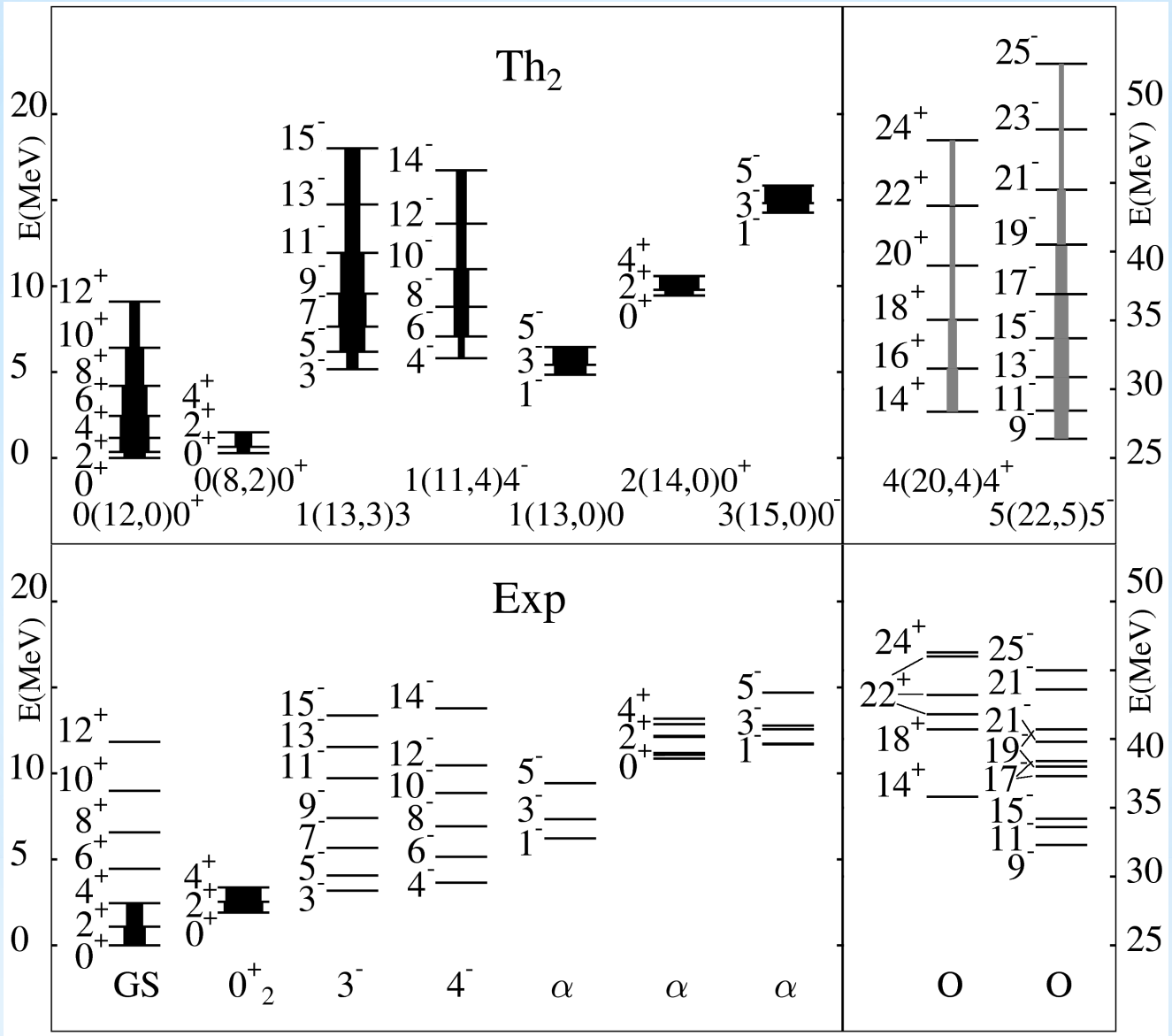
Thank you for your attention!



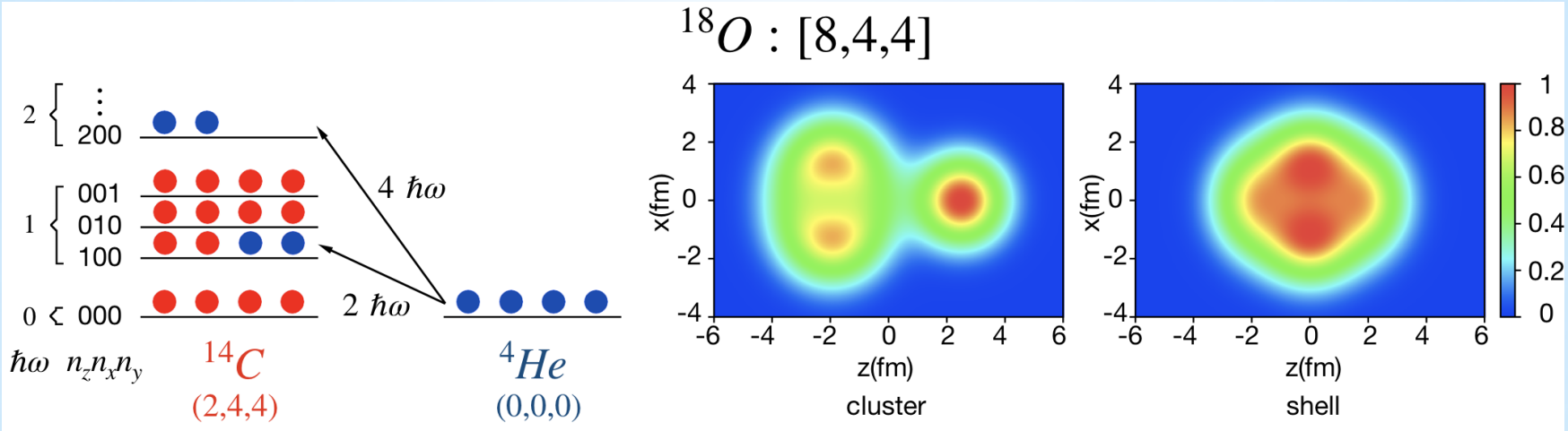
16O+alpha: experiment & dynamical symmetries



Band structure of 20Ne: experiment in comparison with MUSY..



The spectrum of 44Ti in comparison with that of a 2-parameter MUSY.



Harvey's prescription and the density plot for 14C+alpha.