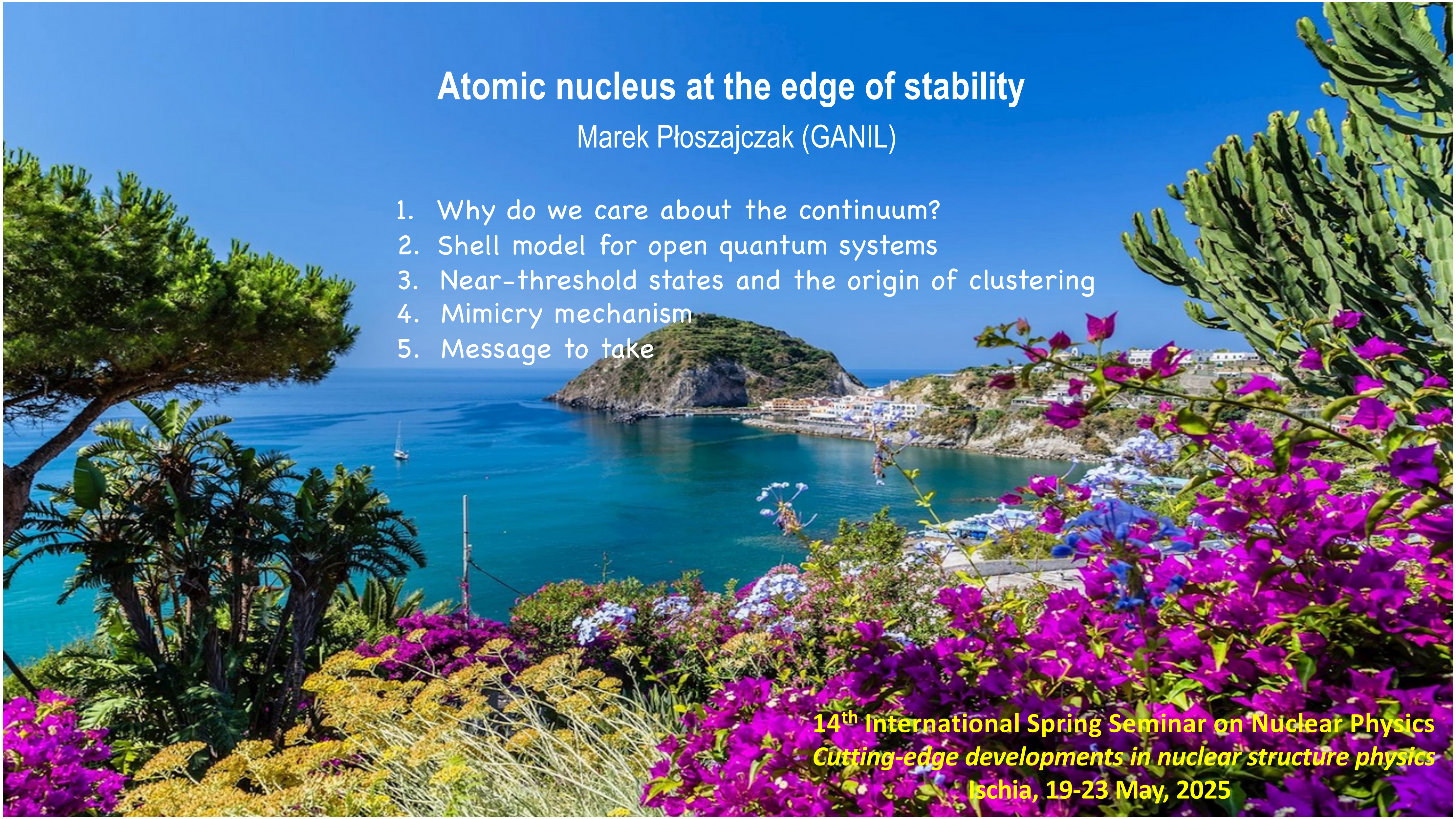


Atomic nucleus at the edge of stability

Marek Płoszajczak (GANIL)

1. Why do we care about the continuum?
2. Shell model for open quantum systems
3. Near-threshold states and the origin of clustering
4. Mimicry mechanism
5. Message to take

14th International Spring Seminar on Nuclear Physics
Cutting-edge developments in nuclear structure physics
Ischia, 19-23 May, 2025

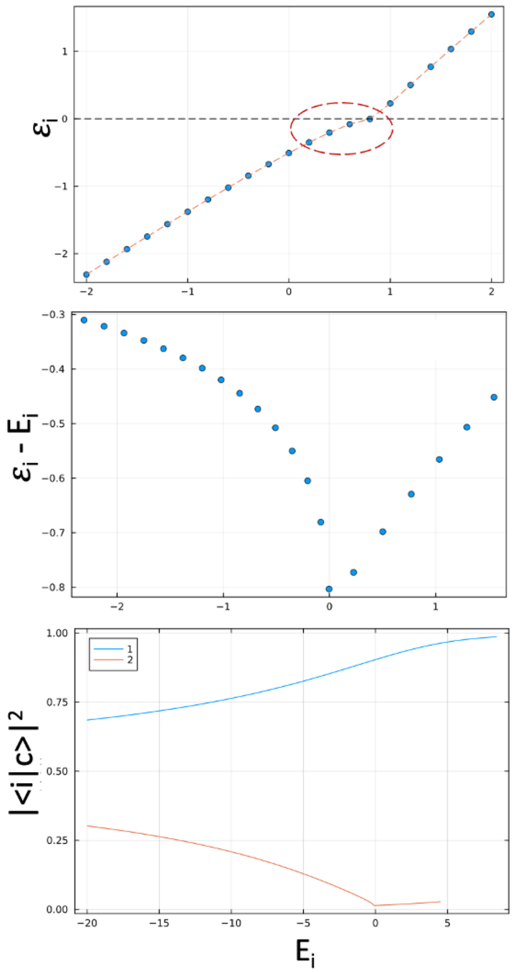
Why do we care about the continuum?

- Behavior of ℓ=0 states in loosely bound/unbound is qualitatively different from other ℓ values

J. Okolowicz, M. Ploszajczak, I. Rotter, Phys. Rep. 374, 271 (2003)
J. Okolowicz, M. Ploszajczak, W. Nazarewicz, Prog. Theor. Phys. Suppl., 196, 230 (201
I. Hamamoto, B.R. Mottelson, C. R. Acad. Sci., Ser IV: Phys., Astrophys. 4, 433 (2003)
I. Hamamoto, PRC 72, 024301 (2005)

Picket fence model (ℓ=0)

A. Volya, M. Ploszajczak (2025)



- Accumulation of states near threshold
- One state appears almost at the threshold
- Attractive energy correction for near-threshold states
- In the vicinity of the threshold, one (collective) state exhausts almost 100% of continuum coupling and aligns with the decay threshold

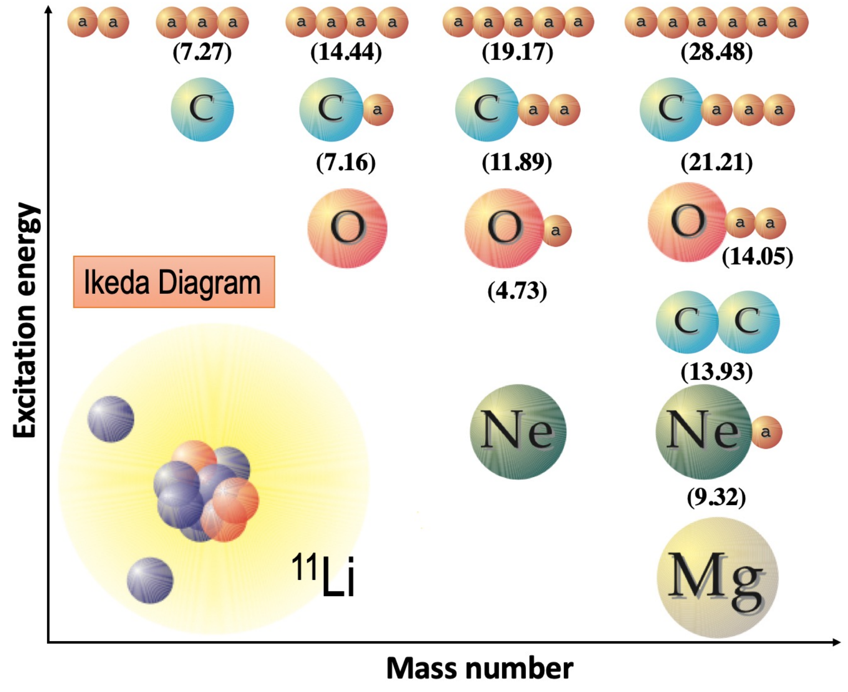
Why do we care about the continuum?

- Behavior of ℓ=0 states in loosely bound/unbound is qualitatively different from other ℓ values

  J. Okolowicz, M. Ploszajczak, I. Rotter, Phys. Rep. 374, 271 (2003)
  J. Okolowicz, M. Ploszajczak, W. Nazarewicz, Prog. Theor. Phys. Suppl., 196, 230 (2012)
  I. Hamamoto, B.R. Mottelson, C. R. Acad. Sci., Ser IV: Phys., Astrophys. 4, 433 (2003)
  I. Hamamoto, PRC 72, 024301 (2005)

- Appearance of exotic correlations and clusterings

  K. Ikeda, et al., Prog. Theor. Phys. Suppl. E68, 464 (1968)
  J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)



Why do we care about the continuum?

- Behavior of ℓ=0 states in loosely bound/unbound is qualitatively different from other ℓ values

  J. Okolowicz, M. Ploszajczak, I. Rotter, Phys. Rep. 374, 271 (2003)
  J. Okolowicz, M. Ploszajczak, W. Nazarewicz, Prog. Theor. Phys. Suppl., 196, 230 (2012)
  I. Hamamoto, B.R. Mottelson, C. R. Acad. Sci., Ser IV: Phys., Astrophys. 4, 433 (2003)
  I. Hamamoto, PRC 72, 024301 (2005)

- Appearance of exotic correlations and clusterings

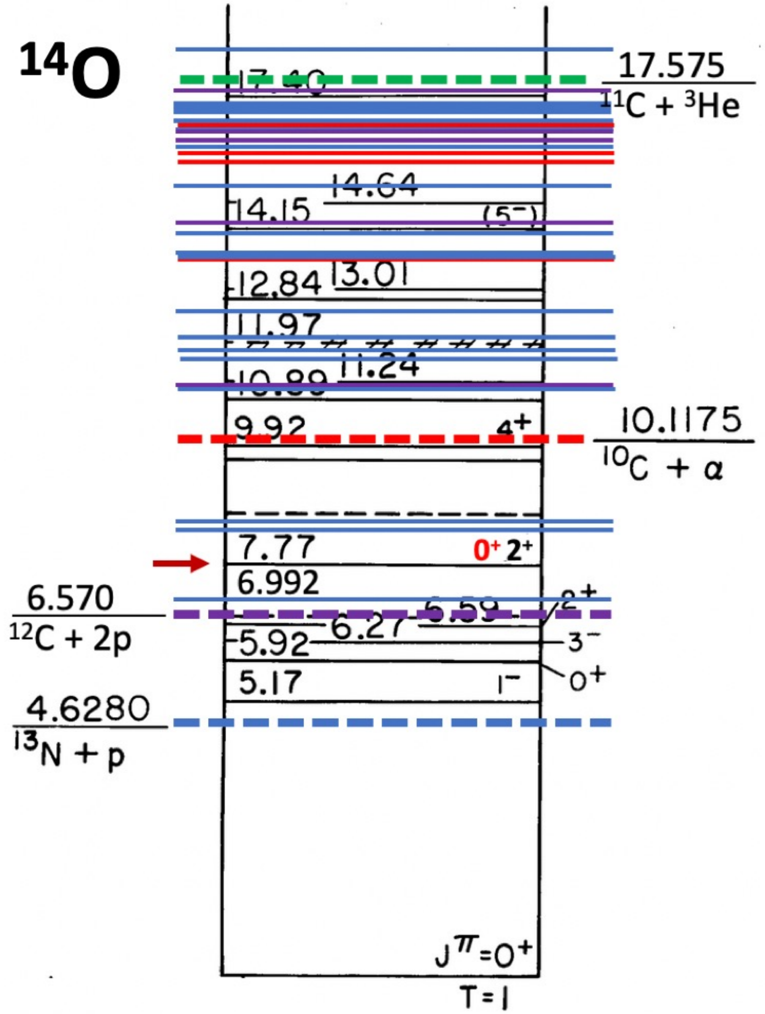
  K. Ikeda, et al., Prog. Theor. Phys. Suppl. E68, 464 (1968)
  J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

- Nuclear states are *embedded in the scattering continuum*

  J. Okołowicz, M. Ploszajczak, I. Rotter, Physics Reports 374, 271 (2003)

- Couplings to various particle emission channels are crucial for the properties of near-threshold states

  J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)



Why do we care about the continuum?

- Behavior of ℓ=0 states in loosely bound/unbound is qualitatively different from other ℓ values
  J. Okolowicz, M. Ploszajczak, I. Rotter, Phys. Rep. 374, 271 (2003)
  J. Okolowicz, M. Ploszajczak, W. Nazarewicz, Prog. Theor. Phys. Suppl., 196, 230 (2012)
  I. Hamamoto, B.R. Mottelson, C. R. Acad. Sci., Ser IV: Phys., Astrophys. 4, 433 (2003)
  I. Hamamoto, PRC 72, 024301 (2005)

- Appearance of exotic correlations and clusterings
  K. Ikeda, et al., Prog. Theor. Phys. Suppl. E68, 464 (1968)
  J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

- Nuclear states are *embedded in the scattering continuum*
  J. Okołowicz, M. Ploszajczak, I. Rotter, Physics Reports 374, 271 (2003)

- Couplings to various particle emission channels are crucial for the properties of near-threshold states
  J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

➡ • Thresholds are *branching points* ➡ *nonanalytic behavior*

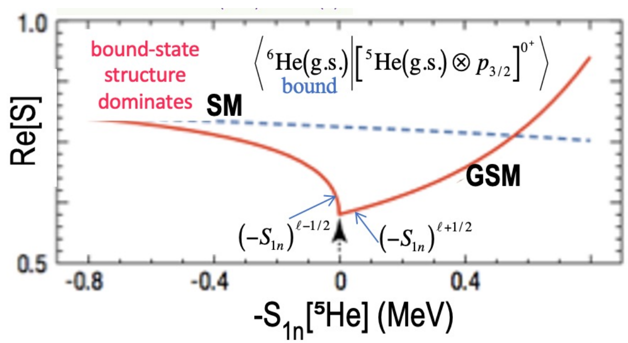
  - Wigner threshold law for *elastic and total cross-sections*
    E.P. Wigner, PR 73, 1002 (1948)

$\sigma(i \to j) \sim (k_j)^{2\ell_j+1} \sim (E_j)^{\ell_j+1/2}$ endoergic reactions: *production* of neutral particles

$\sigma(i \to j) \sim (k_i)^{2\ell_i-1} \sim (E_i)^{\ell_i-1/2}$ exoergic reactions: *absorption* of neutral particles

  - Analogous law for *spectroscopic factors*
    N. Michel, W. Nazarewicz., M. Ploszajczak, PRC 75, 031301(R) (2007)

  ➡ *Discrete and continuous aspects of the nuclear many-body problem converge at the particle emission threshold*



$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1}$

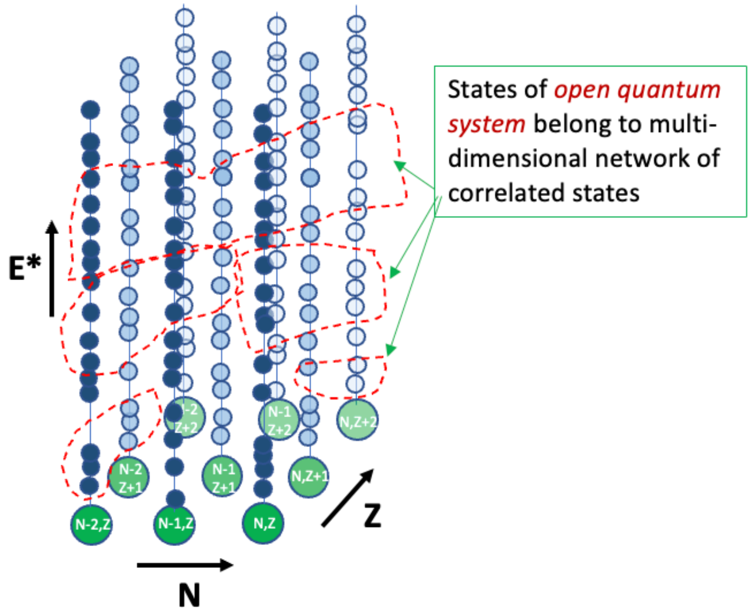
$X(a,b)Y : \sigma_\ell \sim k^{2\ell+1}$

⇕

$(-S_n)^{\ell-1/2}$ for $S_n < 0$

$(-S_n)^{\ell+1/2}$ for $S_n > 0$

Why do we care about the continuum?



Separation energy — Chemical potential — Pairing gap

$$S_{1n} \approx -\lambda - \Delta$$

openness — mean-field — correlations

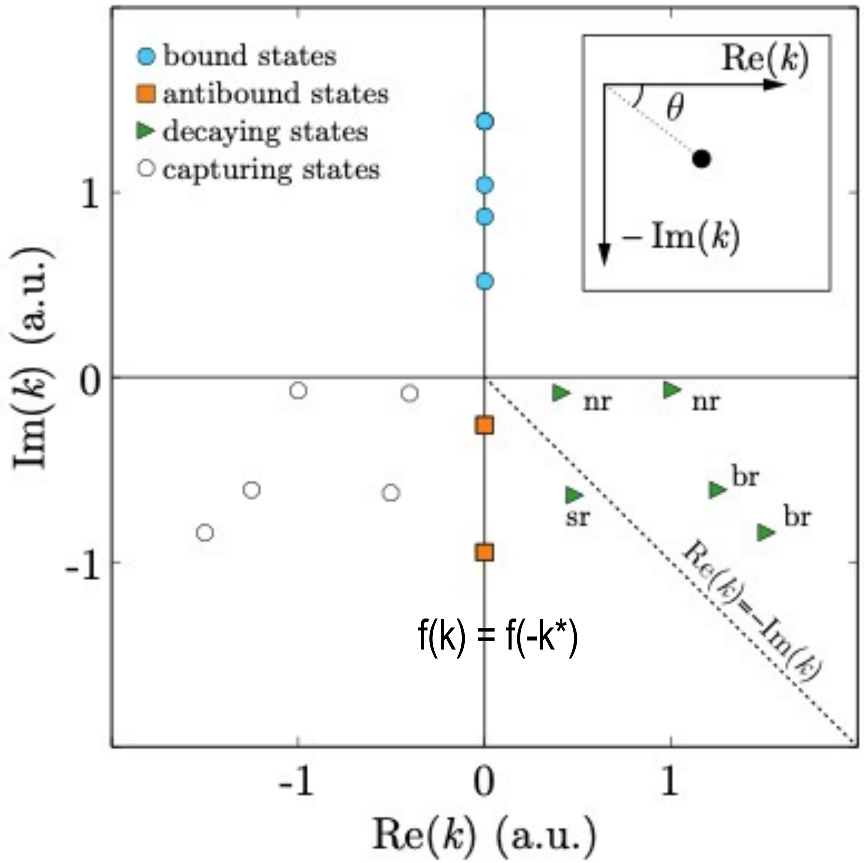
In a vicinity of n/p-drip lines: Correlation > Mean field
Beyond p-dripline: Coulomb ≅ Nuclear

- States in a vicinity of reaction channel thresholds provide the example of new (quantal) regime of nuclear system

Shell model for open quantum systems

Shell model for open quantum systems

Quasi-stationary extension in the complex k-plane: Gamow poles



$$i\hbar\frac{\partial}{\partial t}\Phi(r,t) = \hat{H}\Phi(r,t) \ ; \quad \Phi(r,t) = \tau(t)\Psi(r)$$

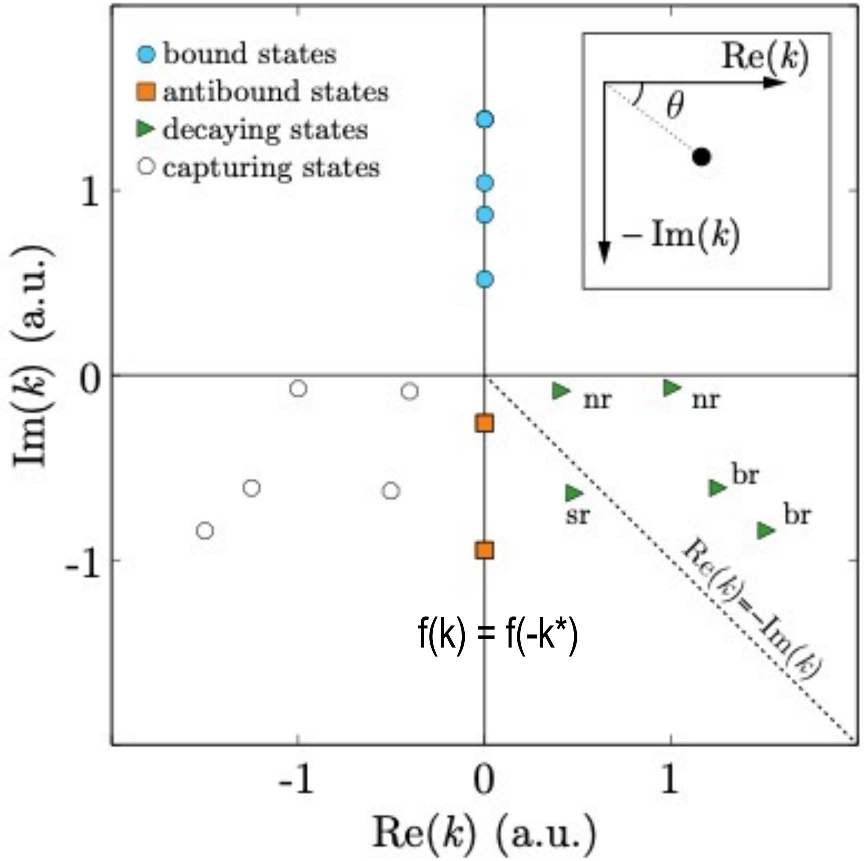
$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \quad \Rightarrow \quad \tau(t) = \exp\left(-i\left(e - i\frac{\Gamma}{2}\right)\right)$$

G. Gamow (1928)

$$\Psi(0,k) = 0 \ , \quad \begin{cases} \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} O_l(kr) \\ \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} I_l(kr) + O_l(kr) \end{cases}$$

Shell model for open quantum systems

Quasi-stationary extension in the complex k-plane: Gamow poles



$$i\hbar\frac{\partial}{\partial t}\Phi(r,t) = \hat{H}\Phi(r,t)\ ;\quad \Phi(r,t) = \tau(t)\Psi(r)$$

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \quad \Longrightarrow \quad \tau(t) = \exp\left(-i\left(e - i\frac{\Gamma}{2}\right)\right)$$

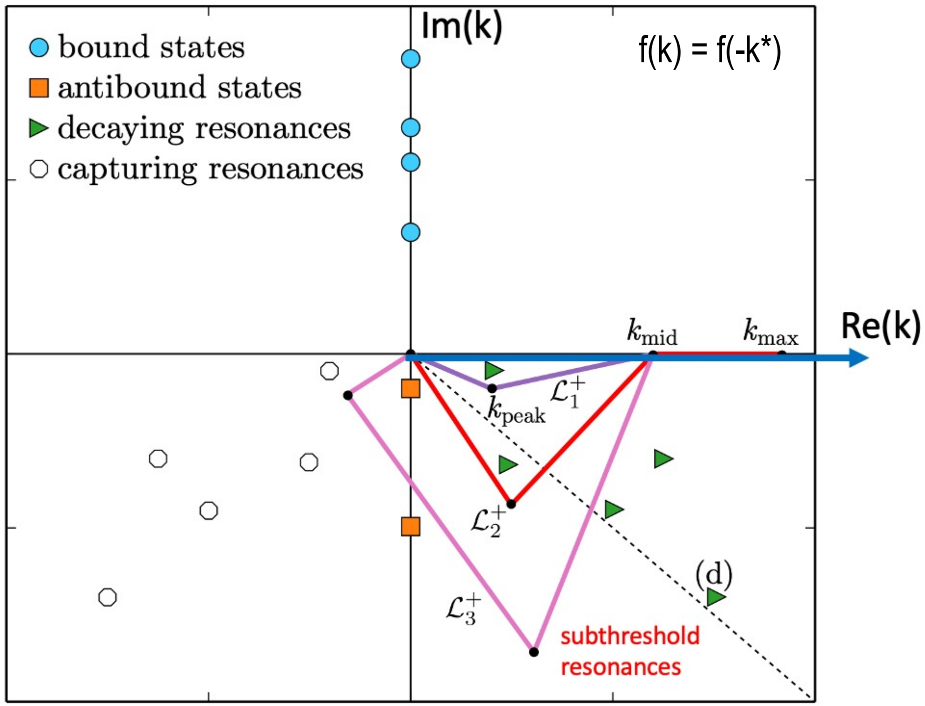
G. Gamow (1928)

$$\Psi(0,k) = 0\ ,\quad \begin{cases} \Psi(\vec{r},k) \underset{r\to\infty}{\rightarrow} O_l(kr) \\ \Psi(\vec{r},k) \underset{r\to\infty}{\rightarrow} I_l(kr) + O_l(kr) \end{cases}$$

- **np** bound state (deuteron): $k=+i0.2315\ \text{fm}^{-1}$ **T=0**
- **np** virtual state (deuteron): $k=-i0.044\ \text{fm}^{-1}$ **T=1**
- **nn** virtual state: $k=-i0.0559(33)\ \text{fm}^{-1}$ **T=1**
  V.A. Babenko, N.M. Petrov, Phys. At. Nucl. 76, 684 (2013)
- **pp** threshold resonant state: $k=(0.0647-i0.0870)\ \text{fm}^{-1}$ **T=1**
  L.P. Kok, Phys. Rev. Lett. 45, 427 (1980)

Shell model for open quantum systems

Quasi-stationary extension in the complex k-plane: Generalized completeness relation

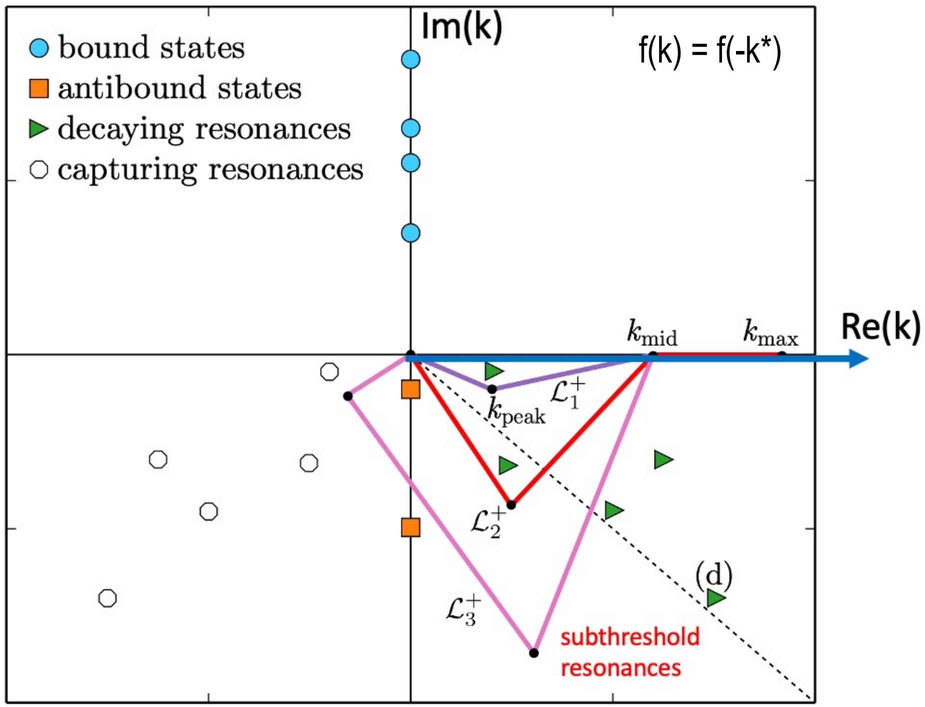


$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|\,dk = 1 \;;\; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
K. Maurin, Generalized Eigenfunction Expansion, Polish Scientific Publishers, Warsaw (1968)
T. Lind, Phys. Rev. C47, 1903 (1993)

Shell model for open quantum systems

Gamow shell model (GSM)



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1 \ ; \ \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
K. Maurin, Generalized Eigenfunction Expansion, Polish Scientific Publishers, Warsaw (1968)
T. Lind, Phys. Rev. C47, 1903 (1993)

$$|SD_i\rangle = |u_{i_1} ... u_{i_A}\rangle \Rightarrow \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

N. Michel et al, PRL 89, 042502 (2002)
N. Michel, et al, J. Phys. G37, 064042 (2010)

- *Complex-symmetric* eigenvalue problem for *hermitian* Hamiltonian
- Calculation in the relative coordinates of core cluster SM coordinates Y. Suzuki, K. Ikeda, PRC 38 (1988) 410
- Center-of-mass handled by recoil term:

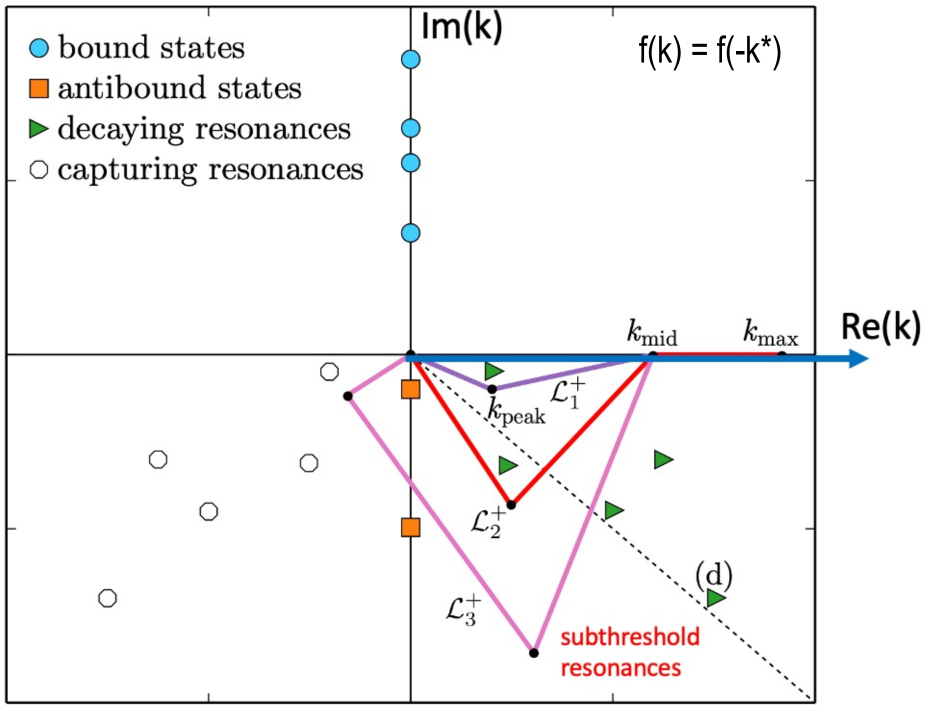
$$H \to H + \frac{1}{M_{\rm core}} \sum_{(i<j)\in {\rm val}} \mathbf{p}_i \cdot \mathbf{p}_j$$

in the Hamiltonian

- *Unitary formulation* of the nuclear Shell Model
- *No identification of reaction channels*
  → GSM in this representation is a tool *par excellence* for nuclear structure studies

Shell model for open quantum systems

GSM – Coupled-channel representation



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|\,dk = 1 \;;\; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
K. Maurin, Generalized Eigenfunction Expansion, Polish Scientific Publishers, Warsaw (1968)
T. Lind, Phys. Rev. C47, 1903 (1993)

$$|\Psi_M^J\rangle = \sum_c \int_0^{+\infty} |(c,r)_M^J\rangle \frac{u_c^{JM}(r)}{r} r^2\,dr$$

$$|(c,r)\rangle = \hat{\mathcal{A}}[|\Psi_T^{J_T}; N_T, Z_T\rangle \otimes |r\; L_{CM}\; J_{\rm int}\; J_P; n, z\rangle]_M^J$$

$$H\,|\Psi_M^J\rangle = E\,|\Psi_M^J\rangle \rightarrow \sum_c \int_0^{\infty} r^2 \left(H_{cc'}(r,r') - E N_{cc'}(r,r')\right) \frac{u_c(r)}{r} = 0$$

$$H_{cc'}(r,r') = \langle (c,r)|\,\hat{H}\,|(c',r')\rangle$$
$$N_{cc'}(r,r') = \langle (c,r)|(c',r')\rangle$$

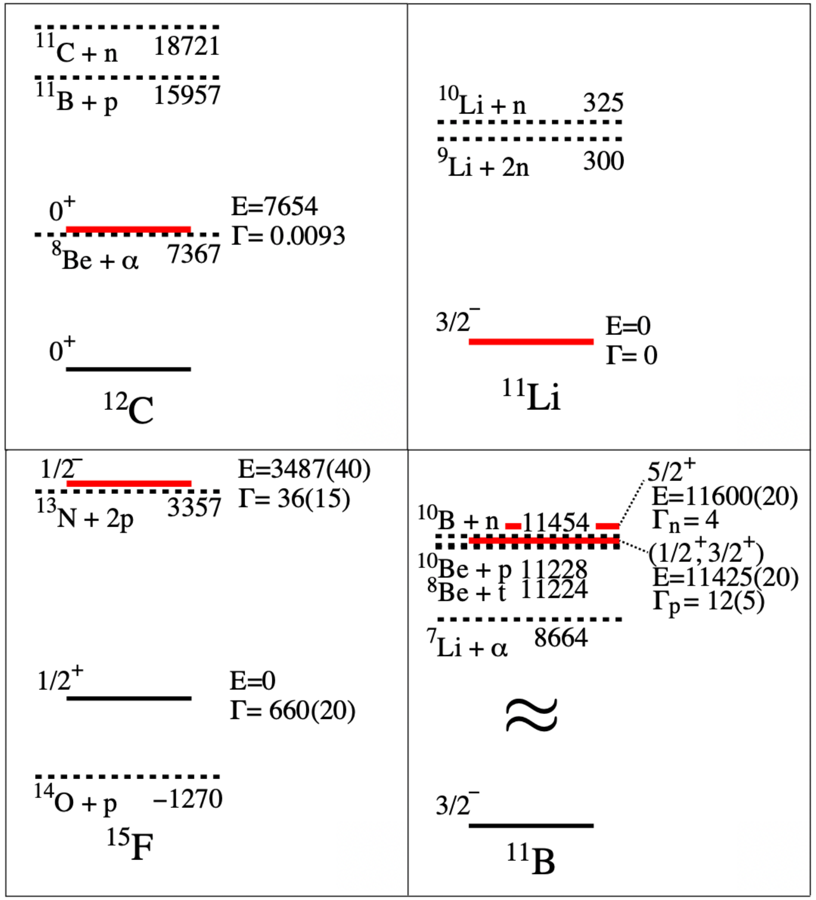
- Entrance and exit reaction channels defined
  → Unification of nuclear structure and reactions
- Reaction channels with different (binary) mass partitions
- Core is arbitrary

Y. Jaganathen et al, PRC 88, 044318 (2014)
K. Fossez et al., PRC 91, 034609 (2015)
A. Mercenne et al., PRC 99, 044606 (2019)

N. Michel, M.Płoszajczak,
«*Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions* »
Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021

Near-threshold states and origin of clustering

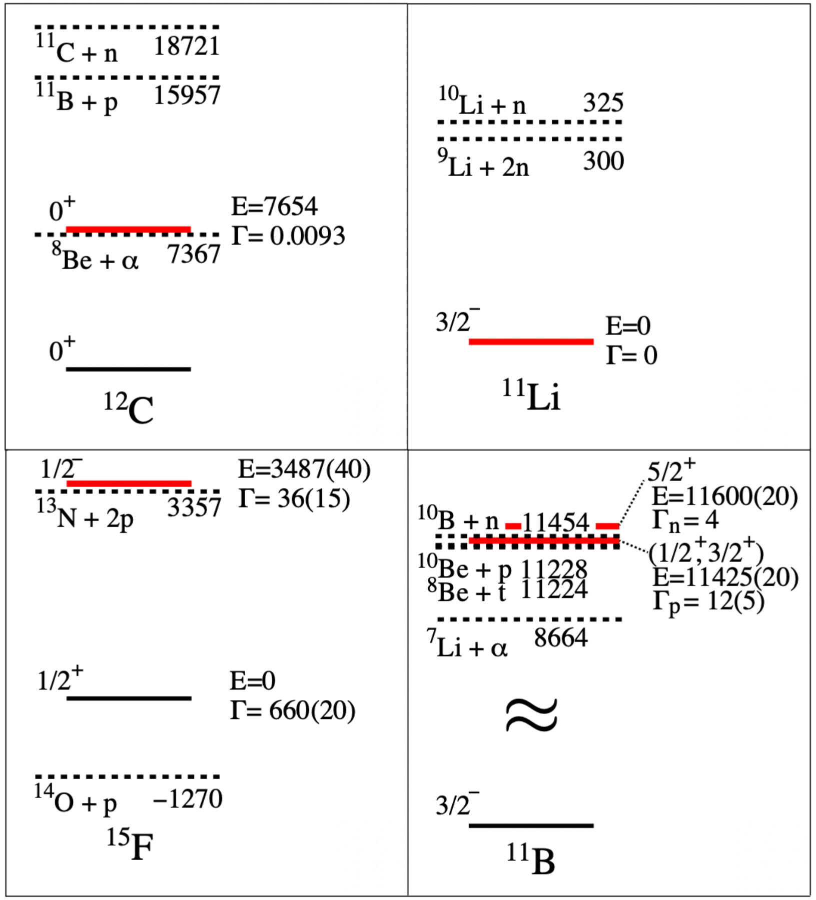
Threshold states in atomic nuclei



- Other cases: ^{6}He, ^{6}Li, ^{7}Be, ^{7}Li, ^{11}O, ^{11}C, ^{17}O, ^{20}Ne, ^{26}O, ^{26}Si …
- Various clusterings: ^{2}H, ^{3}He, ^{3}H, ^{4}He, 2p, 2n …

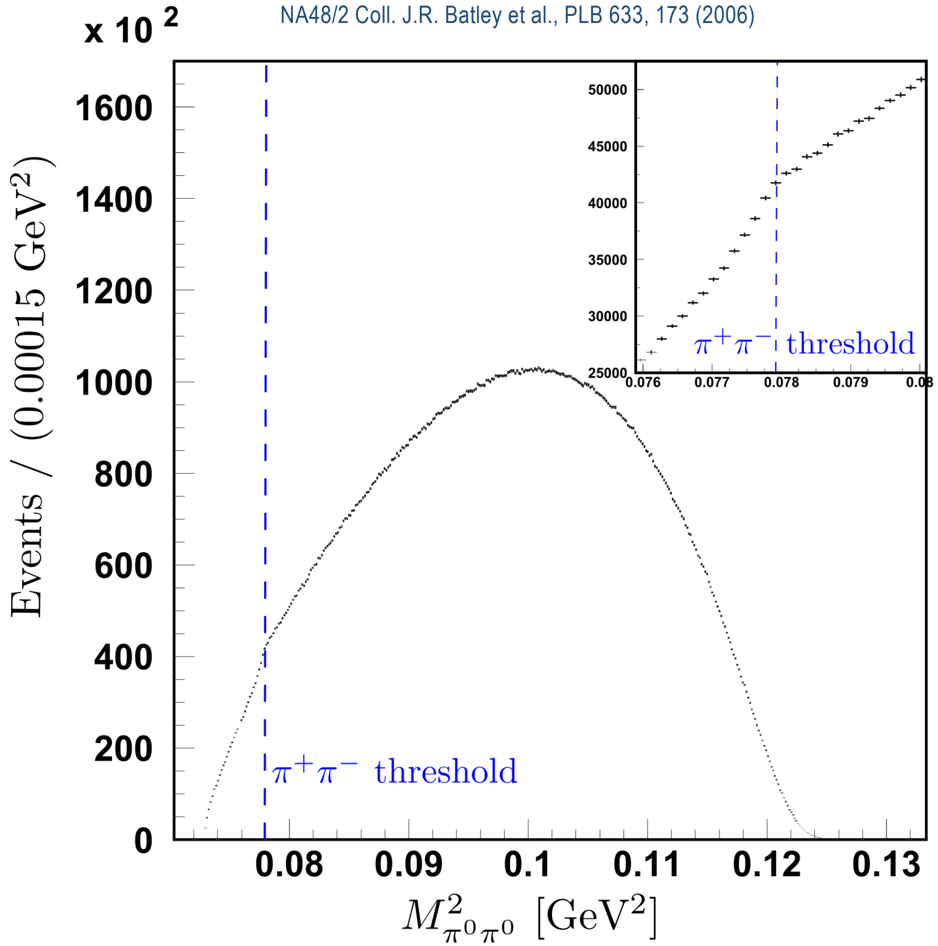
Near-threshold states and origin of clustering

Threshold states in atomic nuclei



- Other cases: ^{6}He, ^{6}Li, ^{7}Be, ^{7}Li, ^{11}O, ^{11}C, ^{17}O, ^{20}Ne, ^{26}O, ^{26}Si …
- Various clusterings: ^{2}H, ^{3}He, ^{3}H, ^{4}He, 2p, 2n …

Threshold effects in hadrons, hadronic molecules, multiquark systems



$K^+ \to \pi^+\pi^0\pi^0$

Near-threshold states and origin of clustering

Is the appearance of correlated states near open channels coincidental?

NO! In a random phenomenon, correlations/clustering in a state close to the threshold will not share common properties with the nearby reaction channel

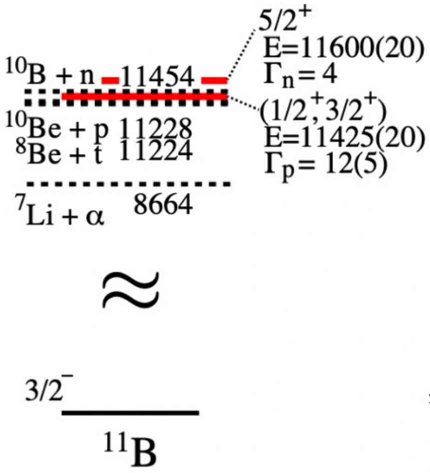
⇒ These states cannot result from any particular feature of the NN interaction or any dynamical symmetry of the nucleus

Near-threshold states and origin of clustering

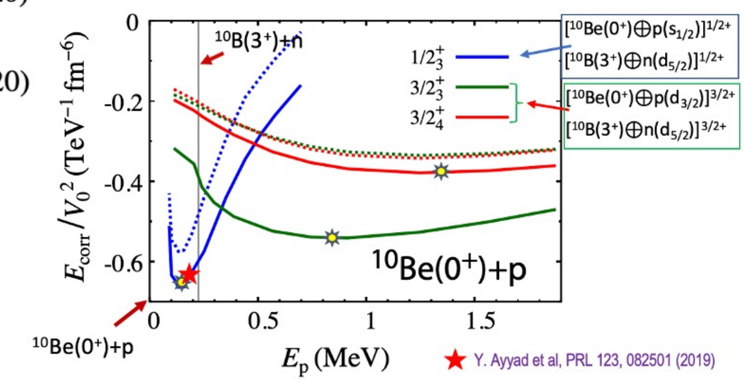
Is the appearance of correlated states near open channels coincidental?

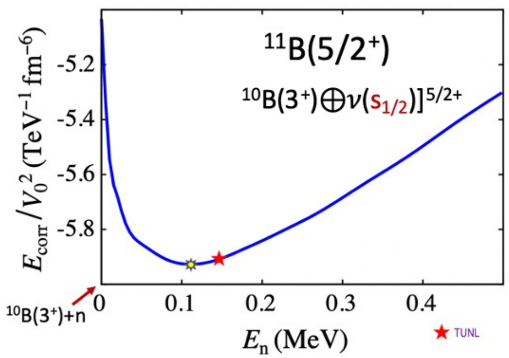
NO! In a random phenomenon, correlations/clustering in a state close to the threshold will not share common properties with the nearby reaction channel

➡ These states cannot result from any particular feature of the NN interaction or any dynamical symmetry of the nucleus



Continuum shell model





The appearance of correlated (cluster) states close to open channels is the generic open quantum system phenomenon related to the collective rearrangement of SM wave functions due to the coupling via the continuum

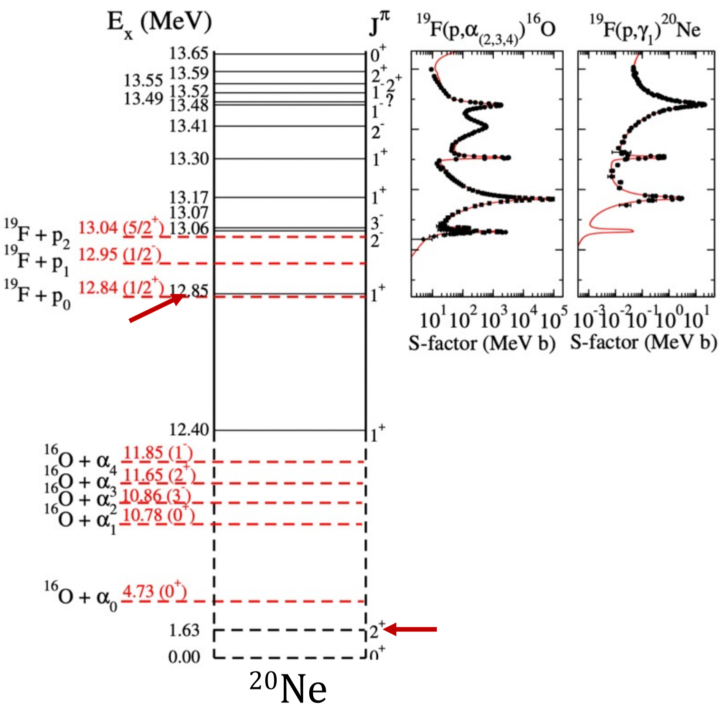
- *Point of strongest collectivity* is determined by the interplay of competing forces of repulsion and attraction

J. Okolowicz, W. Nazarewicz, M. P., PRL 124, 042502 (2020)

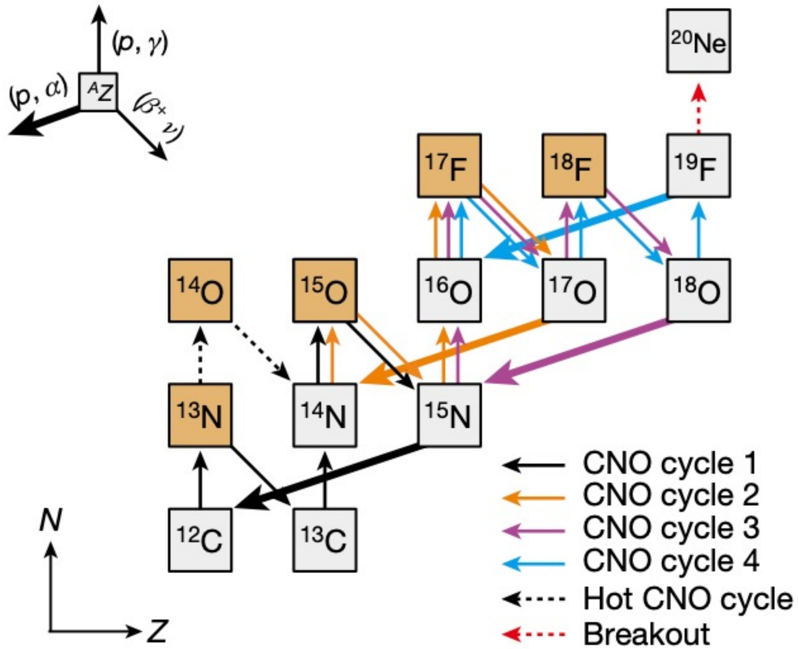
Near-threshold states and origin of clustering

Astrophysical relevance for α- and proton-capture reactions of nucleosynthesis

R.J. DeBoer et al, Nature 610, 656 (2022)



What is the effect of 1^+ resonance at ~10 keV above the proton emission threshold on the S-factor?



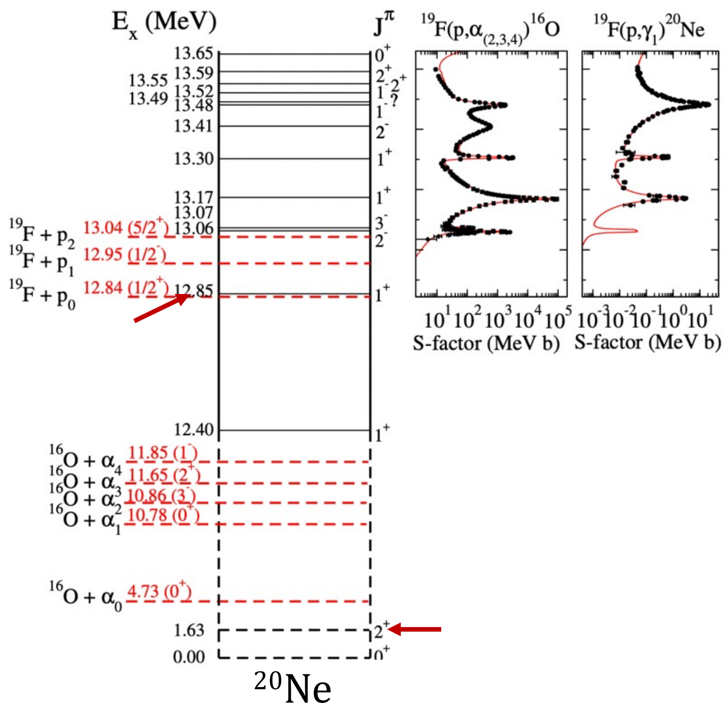
Liyong Zhang et al., Nature 610, 656 (2022)

Does $^{19}F(p,\gamma)^{20}Ne$ breakout reaction from the CNO cycle overcomes $^{19}F(p,\alpha)^{16}O$ back-process reaction cross section becoming a source of the Ca abondance in the first generation stars?

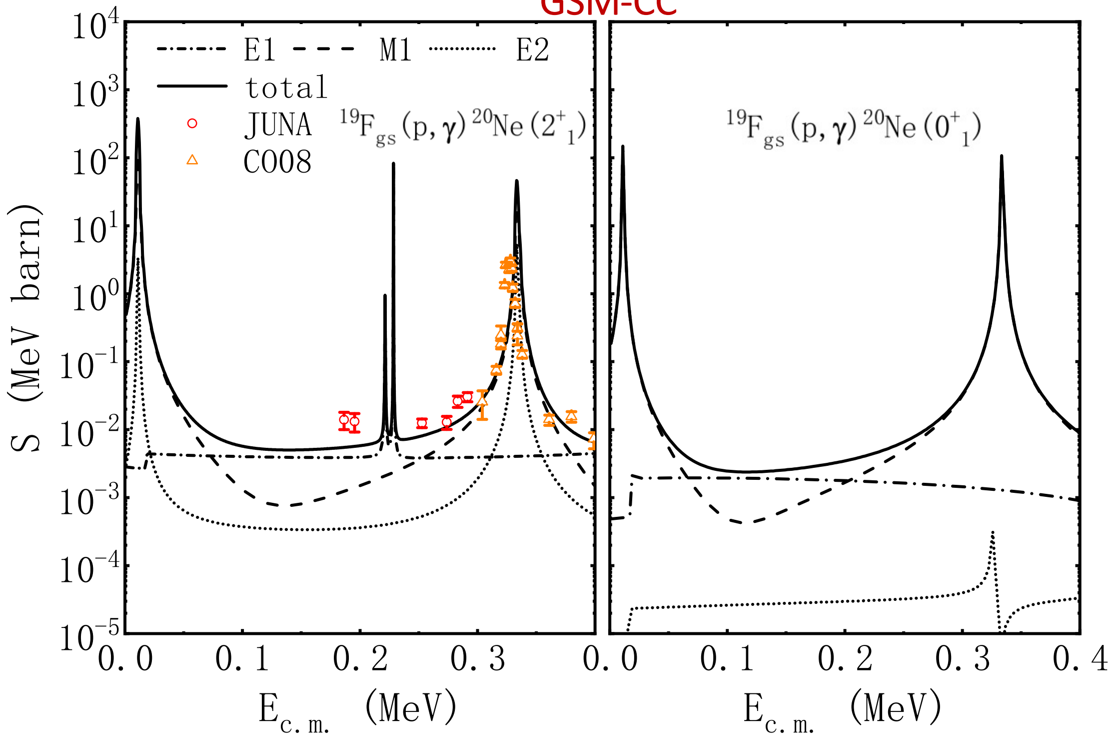
Near-threshold states and origin of clustering

Near-threshold resonances in ^{20}Ne and their role for $^{19}F(p,\gamma)^{20}Ne$ and and $^{19}F(p,\alpha)^{16}O$ reaction rates

R.J. DeBoer et al, Nature 610, 656 (2022)



GSM-CC



Exp: Liyong Zhang et al., Nature 610, 656 (2022)

What is the effect of 1^+ resonance at ~10 keV above the proton emission threshold on the S-factor?

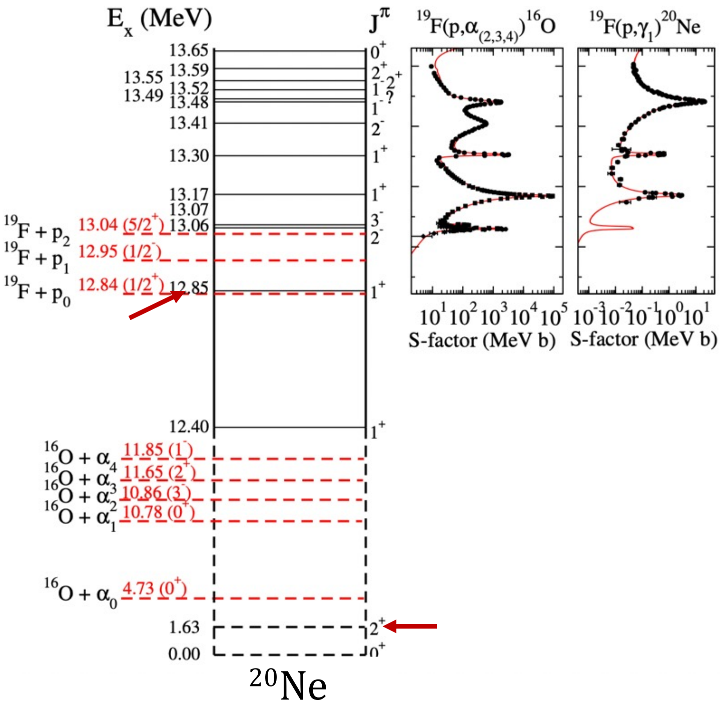
- S(0) astrophysical factor increases by more than 2 orders of magnitude!
- The decay to the 2+ first excited state in ^{20}Ne dominates

X.B. Wang, et al, Phys. Rev. C 110, L061601 (2024)

Near-threshold states and origin of clustering

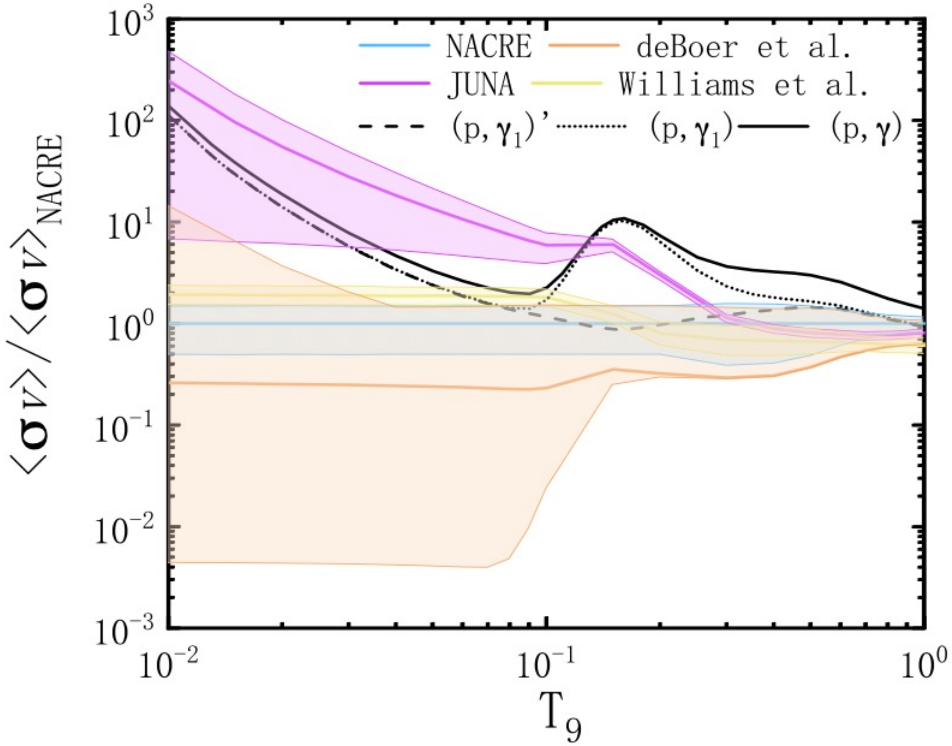
Near-threshold resonances in ^{20}Ne and their role for $^{19}F(p,\gamma)^{20}Ne$ and and $^{19}F(p,\alpha)^{16}O$ reaction rates

R.J. DeBoer et al, Nature 610, 656 (2022)



What is the effect of 1^+ resonance at ~10 keV above the proton emission threshold on the S-factor?

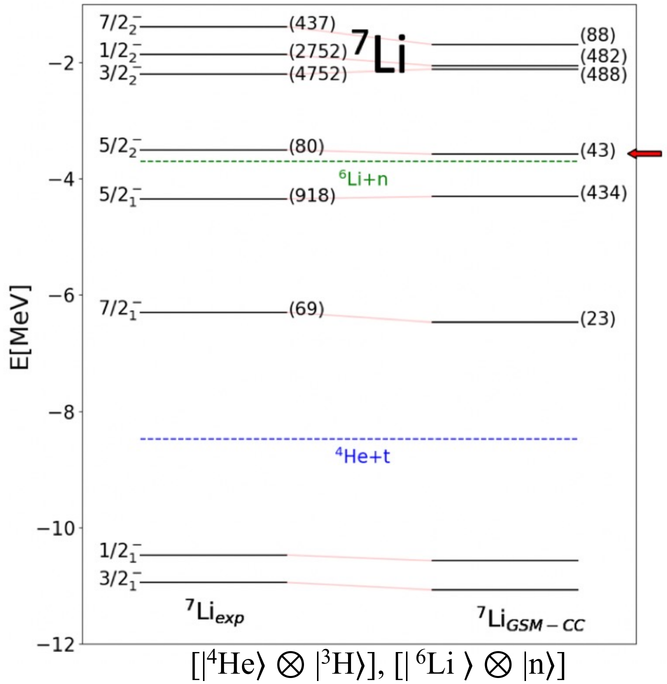
Exp: Liyong Zhang et al., Nature 610, 656 (2022)



- GSM-CC reaction rates are significantly larger than in NACRE and comparable with JUNA data
- $^{19}F(p,\alpha)^{16}O$ back-process reaction should be remeasued to verify the hypothesis of breaking from hot-CNO cycle

Mimicry mechanism

Chameleon nature of resonances



- Hamiltonian: 1-body potential, 2-body FHT interaction

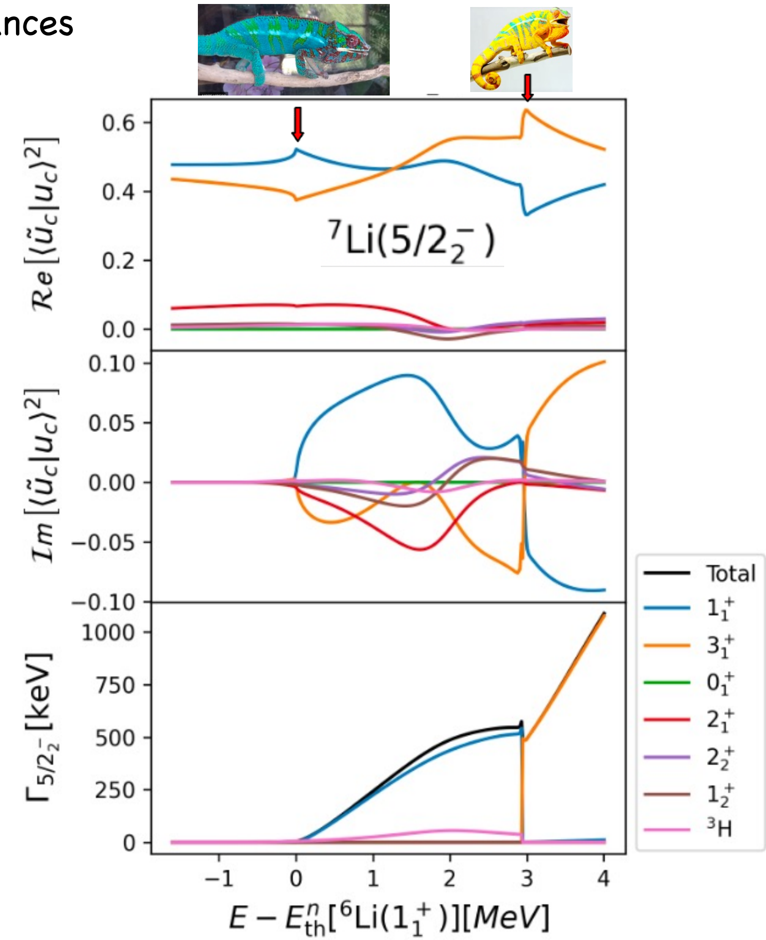
  H. Furutani et al, Prog. Theor. Phys. 62, 981 (1979)

  ^{3}H wave functions calculated using N^{3}LO$_{(2\text{-body})}$ interaction

- Channels: ^{6}Li(K^π): K^π=1_1^+, 1_2^+, 3_1^+, 0_1^+, 2_1^+, 2_2^+

  n: ℓ_j = $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$, $d_{5/2}$, $f_{5/2}$, $f_{7/2}$

  ^{3}H(L): L ≡ $^{2J_{int}+1}[L_{CM}]_{J^P}$ = $^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$, $^2D_{3/2}$, $^2D_{5/2}$, $^2F_{5/2}$, $^2F_{7/2}$

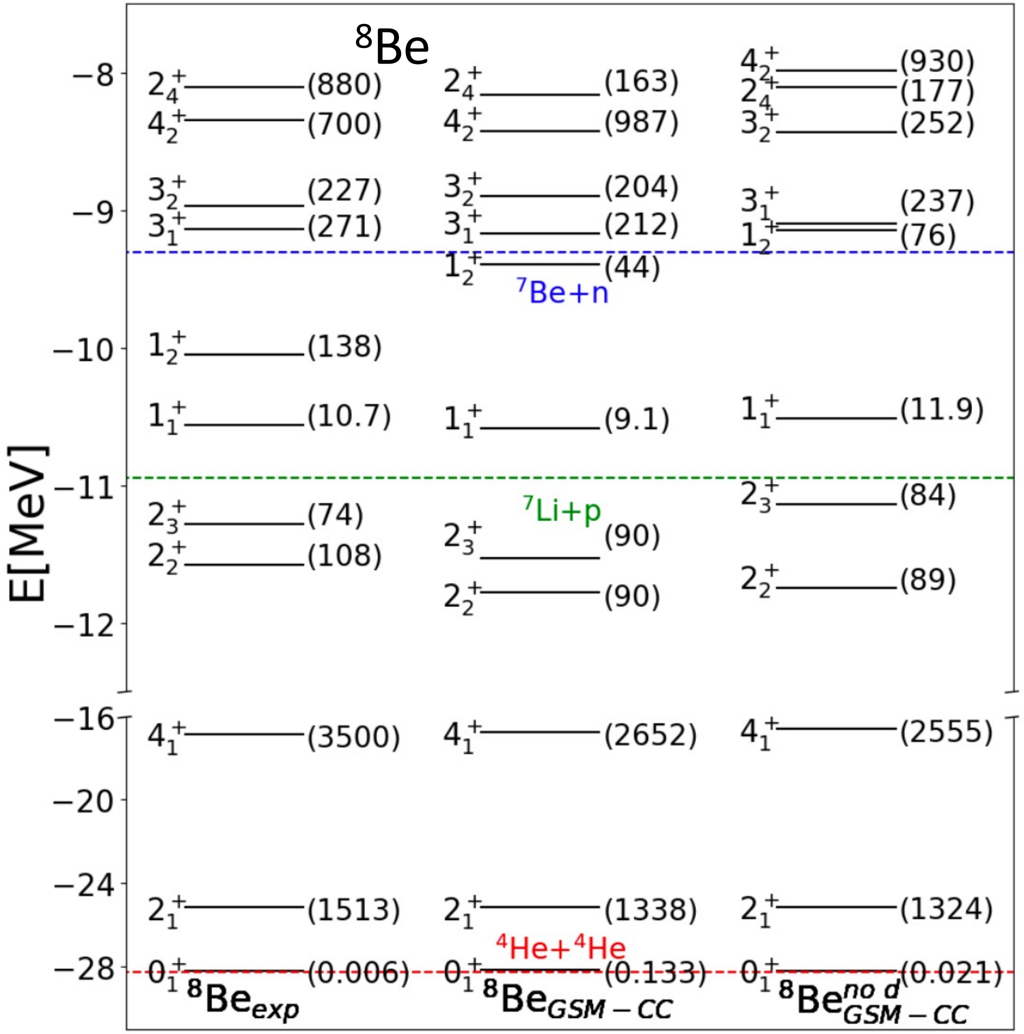


- The resonance (*chameleon*) changes its structure (*skin color*) as a result of the alignment (*mimicry*) with the nearby new reaction channel (*changing environment*)

  J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Mimicry mechanism

Near-threshold clustering in ^{8}Be



Mass partitions:

$[|^4\mathrm{He}\rangle \otimes |^4\mathrm{He}\rangle]$, $[|^7\mathrm{Li}\rangle \otimes |p\rangle]$, $[|^7\mathrm{Be}\rangle \otimes |n\rangle]$, $[|^6\mathrm{Li}\rangle \otimes |d\rangle]$

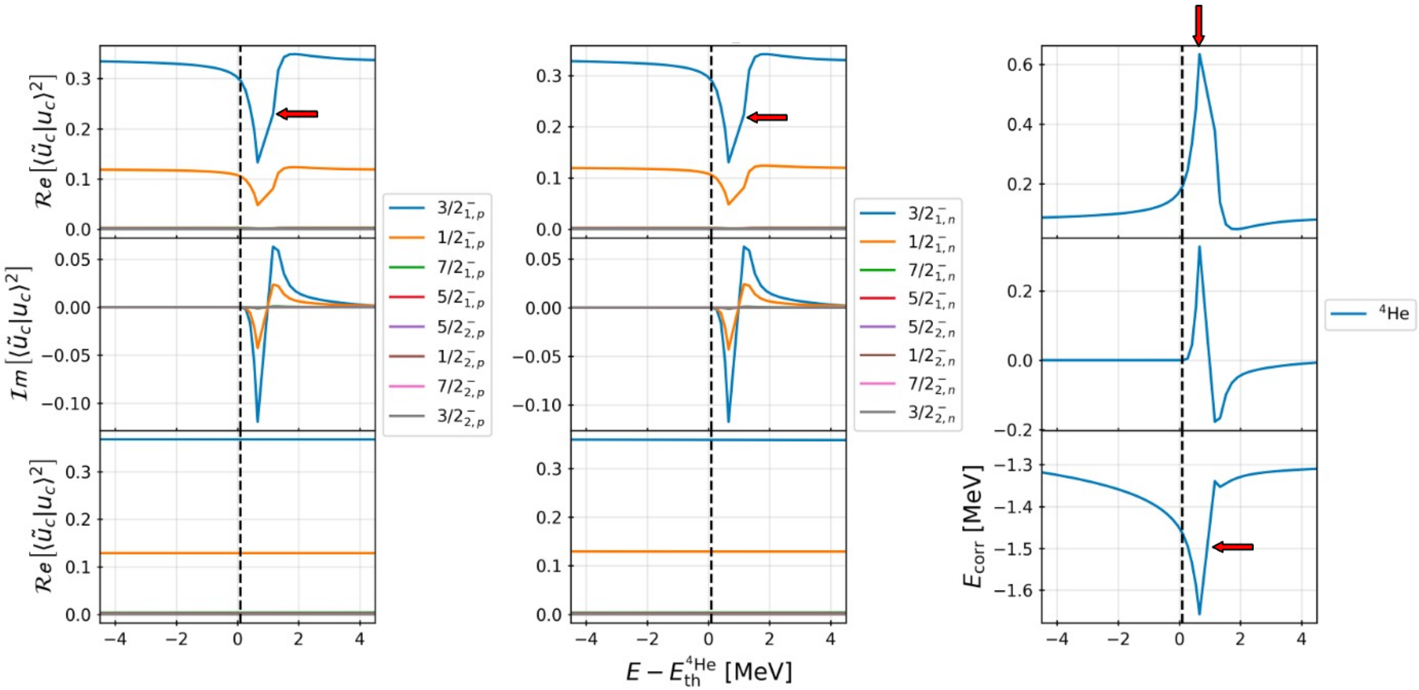
J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Mimicry mechanism

Near-threshold clustering in ^{8}Be

Continuum coupling correlation energy → $E^{(\mathrm{corr})}_{J^\pi,M} = \langle \tilde{\Psi}^J_M | H | \Psi^J_M \rangle - \langle \tilde{\Phi}^{J;(\alpha)}_M | H | \Phi^{J;(\alpha)}_M \rangle \equiv \mathcal{E}_{J^\pi,M} - \mathcal{E}^{(\alpha)}_{J^\pi,M}$

$$|\Phi^{J;(\alpha)}_M\rangle = \sum_{\mathrm{c;c}\neq\alpha} \int_0^{+\infty} |(\mathrm{c},r)^J_M\rangle \frac{\bar{u}^{JM}_{\mathrm{c}}(r)}{r} r^2\, dr$$



Near-threshold clustering is the *emergent phenomenon* in SM for *open* quantum systems

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Message to take

- Quantum systems in the vicinity of a particle emission threshold belong to the category of open quantum systems having unique properties which distinguish them from well-bound closed quantum systems

  The richness of nuclear interaction and the existence of nucleons in two distinct states (proton/neutron) make studies on the near-threshold phenomena in atomic nucleus unique.

- Deeper understanding of near-threshold phenomena in the shell model for open quantum systems will help to define new territory of nuclear spectroscopy studies involving resonances:
  - γ-selection rules for in- and out-band transitions in resonance bands
  - Coupling of collective and s.p. motion in the continuum
  - Violation of mirror symmetry/isospin symmetry
  - New kinds of near-threshold clustering: ^{2}H, ^{3}H, ^{3}He, ^{3}n, ^{4}n, ...
  - Effects of coalescing resonances in nuclear spectroscopy and reactions
  - ... ...

In collaboration with:

Nicolas	Michel	IMP/CAS Lanzhou/Beijin, China
Witek	Nazarewicz	MSU/FRIB East Lansing, USA
Jacek	Okołowicz	INP Kraków, Poland
Alexander	Volya	FSU Tallahassee, USA
Jose Pablo	Linares	LSU Baton Rouge, USA

14th International Spring Seminar on Nuclear Physics

Cutting-edge developments in nuclear structure physics

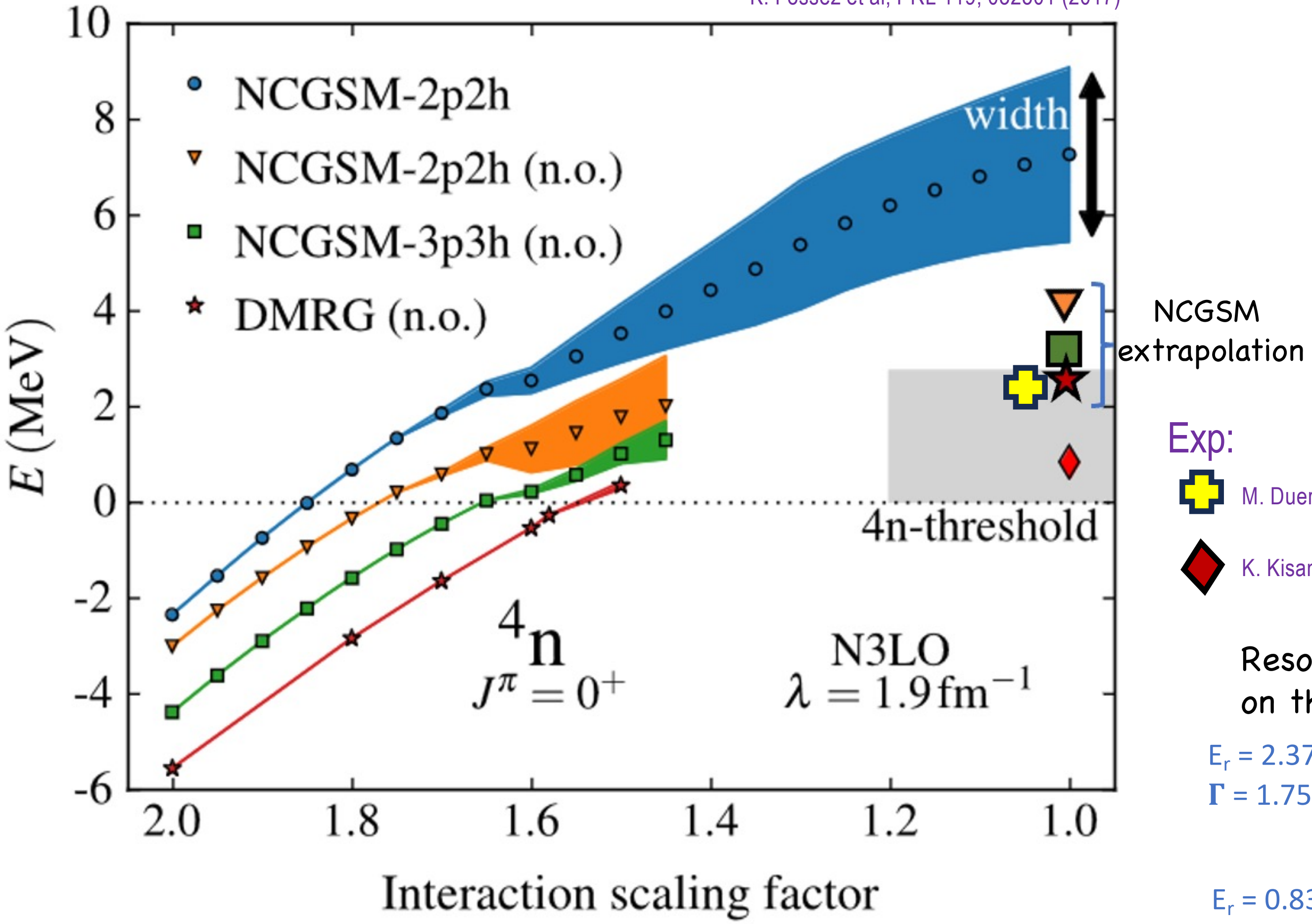
Ischia, 19-23 May, 2025

Backup slides

Near-threshold states and origin of clustering

Tetraneutron: Near-threshold transition phenomenon or the resonance?

K. Fossez et al, PRL 119, 032501 (2017)



Exp:

M. Duer et al, Nature 606, 678 (2022) ^{8}He(p,p^{4}He)

K. Kisamori et al, PRL 116, 052501 (2016) ^{4}He(^{8}He,^{8}Be)

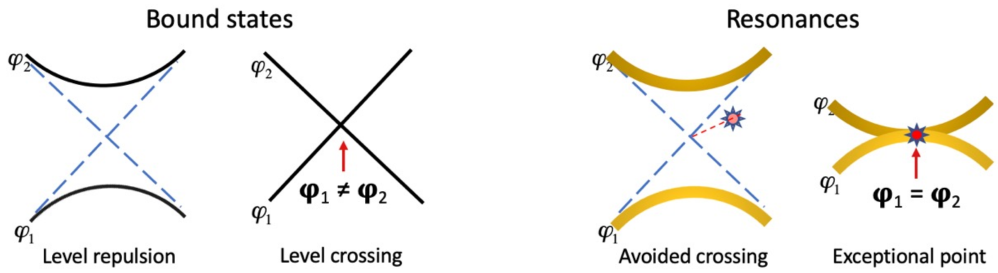
Resonance energy should not depend on the reaction used for its formation

$E_r = 2.37 \pm 0.38(\text{stat.}) \pm 0.44(\text{sys.})$ MeV
$\Gamma = 1.75 \pm 0.22(\text{stat.}) \pm 0.3(\text{sys.})$ MeV
^{8}He(p,p^{4}He)
M. Duer et al, Nature 606, 678 (2022)

$E_r = 0.83 \pm 0.65(\text{stat.}) \pm 1.25(\text{sys.})$ MeV
$\Gamma < 2.6$ MeV
^{4}He(^{8}He,^{8}Be)
K. Kisamori et al, PRL 116, 052501 (2016)

Shell model for open quantum systems

Configuration mixing in open quantum system



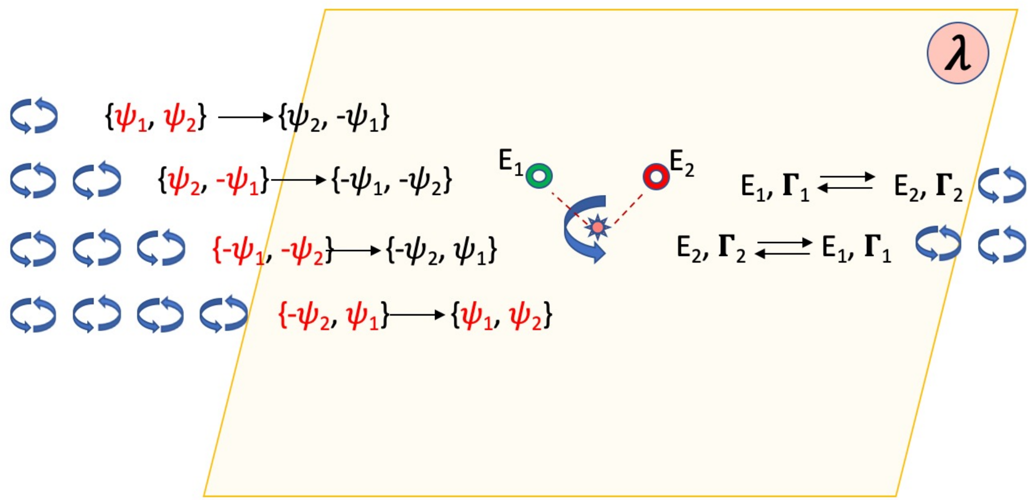
Exceptional points (EPs) and avoided crossings are responsible for the configuration mixing in the continuum

- Microwave cavity experiments
- Atoms coupled to radiation field
- Atom – cavity quantum composite
- Optical lattices
- Atomic nuclei

C. Dembowski et al., PRL 86 (2001) 787; PRL 90 (2003) 034101

J. Okołowicz, M. Ploszajczak PRC 80 (2009) 034619

Topological features of the EPs



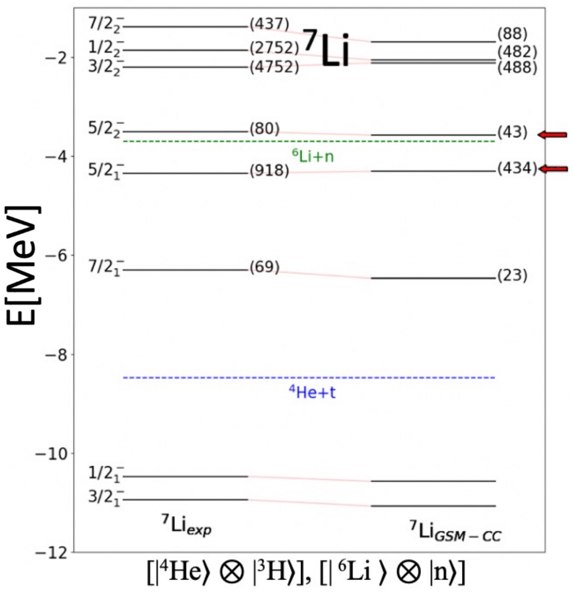
The presence of EPs in an eigenvalue spectrum will result in a *chirality*, i.e. the orientation of a loop in the parameter space determines which wave function will pick a sign during encircling of the EP:

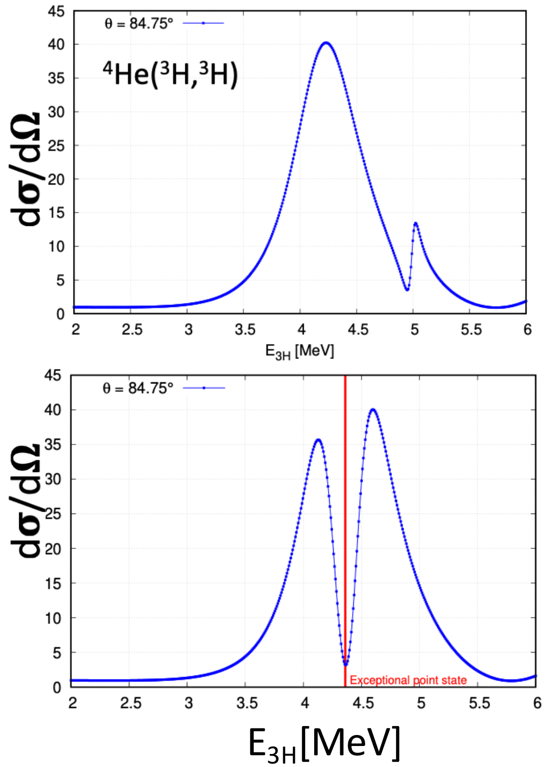
$$\begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix} \circlearrowleft \begin{Bmatrix} \psi_2 \\ -\psi_1 \end{Bmatrix} \text{ and } \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix} \circlearrowright \begin{Bmatrix} -\psi_2 \\ \psi_1 \end{Bmatrix}$$

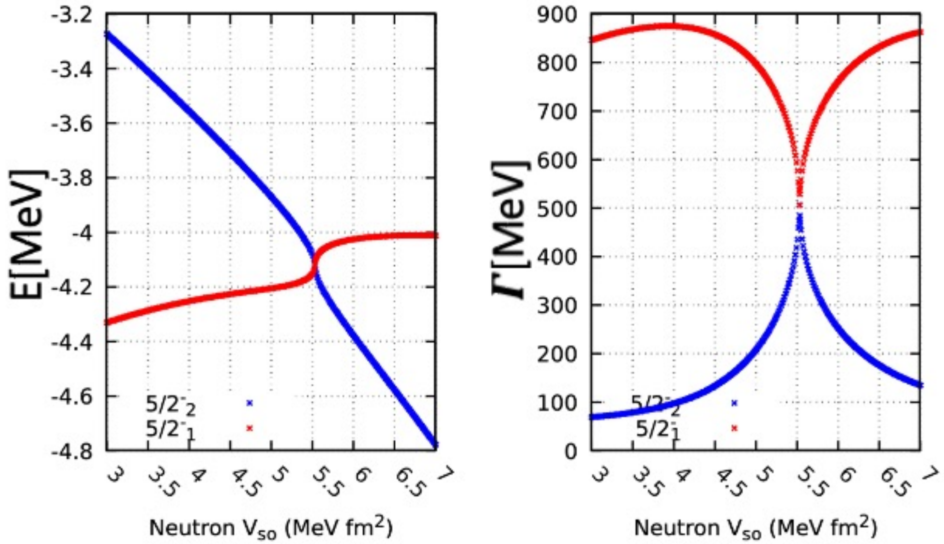
M.R. Zirnbauer et al., Nucl. Phys. A411 (1983) 161

Shell model for open quantum systems

Coalescence of eigenfunctions







- Hamiltonian: 1-body potential, 2-body FHT interaction
  H. Furutani et al, Prog. Theor. Phys. 62, 981 (1979)
  ^{3}H wave functions calculated using N^{3}LO$_{(2\text{-body})}$ interaction

- Channels: ^{6}Li(K^π): $K^\pi = 1_1^+, 1_2^+, 3_1^+, 0_1^+, 2_1^+, 2_2^+$
  n: ℓ_j = $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$, $d_{5/2}$, $f_{5/2}$, $f_{7/2}$
  ^{3}H(L): L ≡ $^{2J_{int}+1}[L_{CM}]_{JP}$ = $^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$, $^2D_{3/2}$, $^2D_{5/2}$, $^2F_{5/2}$, $^2F_{7/2}$