Updates on the relativistic nuclear field theory: Refining dynamical kernels of the nuclear response

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Outline: The nuclear many-body problem

- Nuclear structure theory has strived for decades to achieve accurate computation of nuclear spectra, but this is still difficult

- We address this problem by developing the model-independent Equation of Motion (EOM) framework in the universal QFT language, quantifying fermionic correlation functions (FCFs)

- The theory (i) is transferrable across the energy scales, (ii) capable of identifying the bottlenecks of the existing nuclear structure approaches, and (iii) opens the door to spectroscopic accuracy

- In this formulation, dynamical interaction kernels of the FCF EOMs are the source of the richness of the nuclear wave functions in terms of their configuration complexity and the major ingredient for an accurate description with quantified uncertainties

- Benchmarks and predictions: selected highlights for nuclear excited states below m_π

- Open problems

Exact "ab initio" (EOM) for the two-fermion correlation function with an unspecified NN interaction

$$H = \sum\!\!\!\!\!\!\int_{12} \bar\psi_1(-i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + M)_{12}\psi_2 + \frac{1}{4}\sum\!\!\!\!\!\!\int_{1234} \bar\psi_1\bar\psi_2\bar v_{1234}\psi_4\psi_3 \;+\ldots = T + V^{(2)} + \ldots$$

Bare (Relativistic) **Hamiltonian**

Particle-hole correlator two-time propagator, (response function):

$$R_{12,1'2'}(t-t') = -i\langle T(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle$$

In-medium scattering amplitude spectra of excitations, decays, …

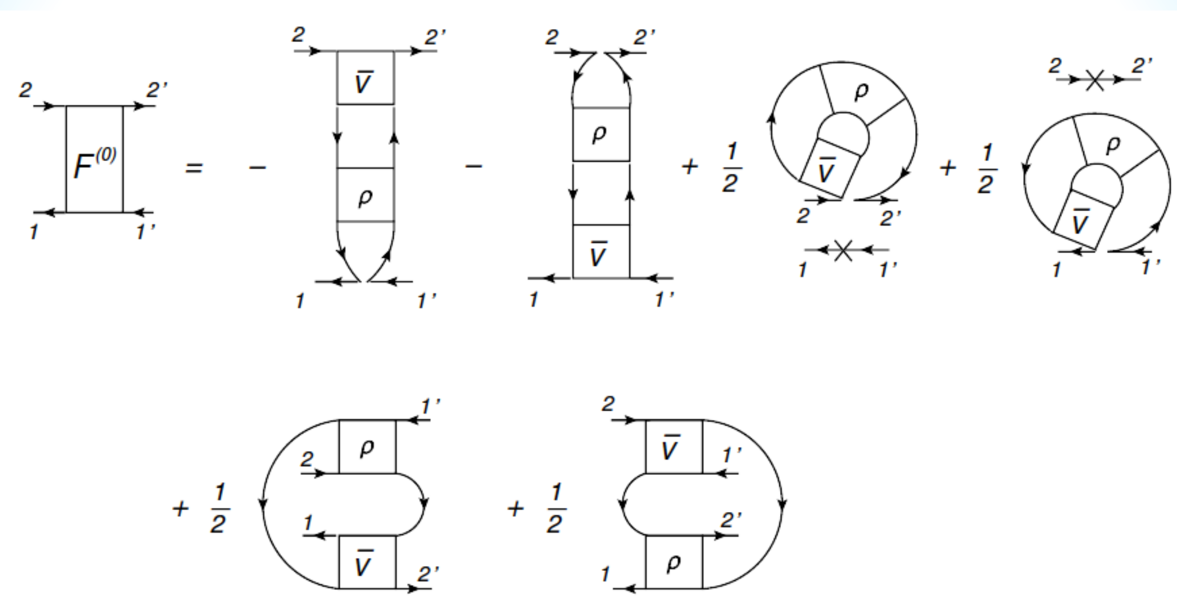
EOM: *Bethe-Salpeter-(Dyson) Eq. (**)*

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega)$$

Irreducible kernel (exact): in-medium "interaction"

$$F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Instantaneous term ("bosonic" mean field):
Short-range correlations

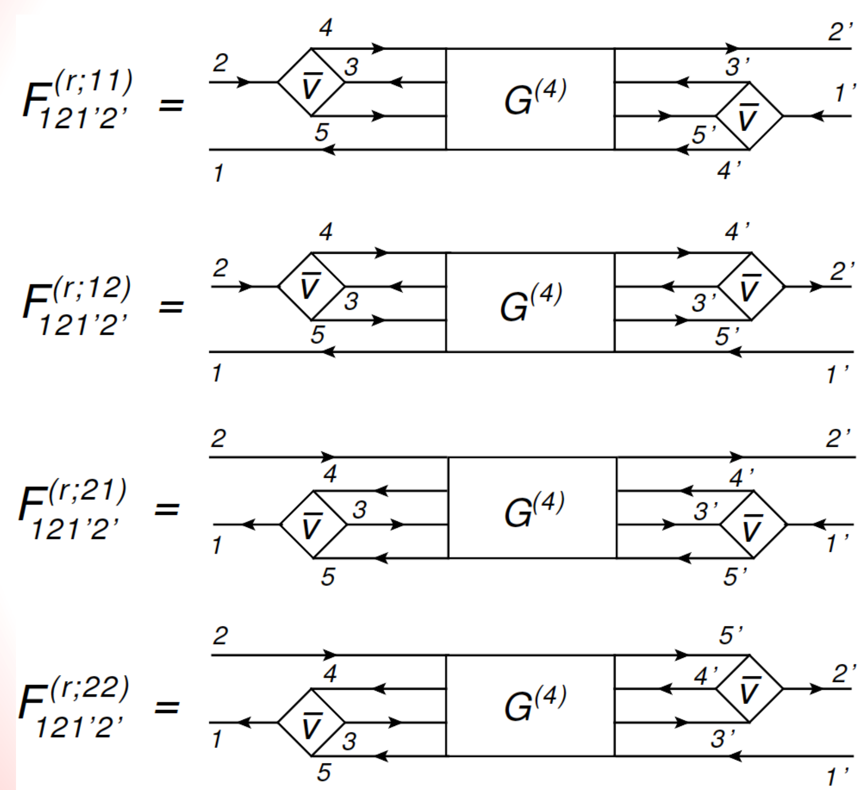


Self-consistent mean field $F^{(0)}$, where

$$\rho_{12,1'2'} = \delta_{22'}\rho_{11'} - i \lim_{t'\to t+0} R_{2'1,21'}(t-t')$$

contains the full solution of (**) ***including the dynamical term!***

t-dependent (dynamical) term:
Long-range correlations:
Couples to higher-rank FCFs



Symmetric form:

P. Schuck, J. Dukelsky, S. Adachi, et al.

Non-Symmetric form: see

J. Schwinger, F. Dyson, et al.

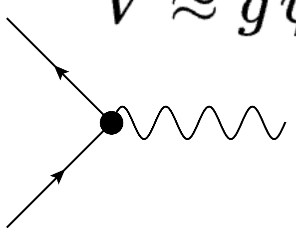
E.L. & P. Schuck, PRC 100, 064320 (2019)

Language remarks

Ab initio…

- *Ab initio = "from the beginning" (lat.), in science and engineering "from first principles"*
- ***First-principle physics theory does not exist***
  Even the Standard Model (SM) is an effective (field) theory:

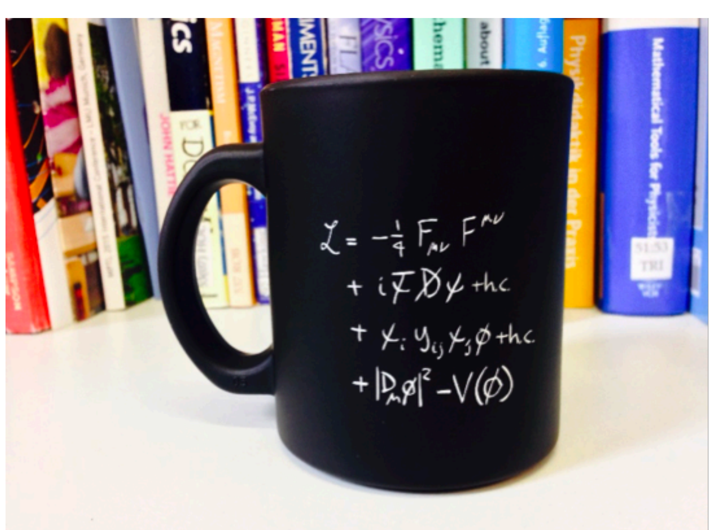
$$V \approx g\bar{\psi}\,\phi\,\psi$$

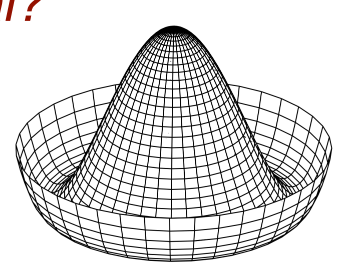


~20 parameters:
masses, coupling constants, etc.

Open questions (besides GR):

- *Why the SM interactions are as they are?*
- *What is the origin of the Higgs potential?*
- *Dark matter, neutrinos,… ???*



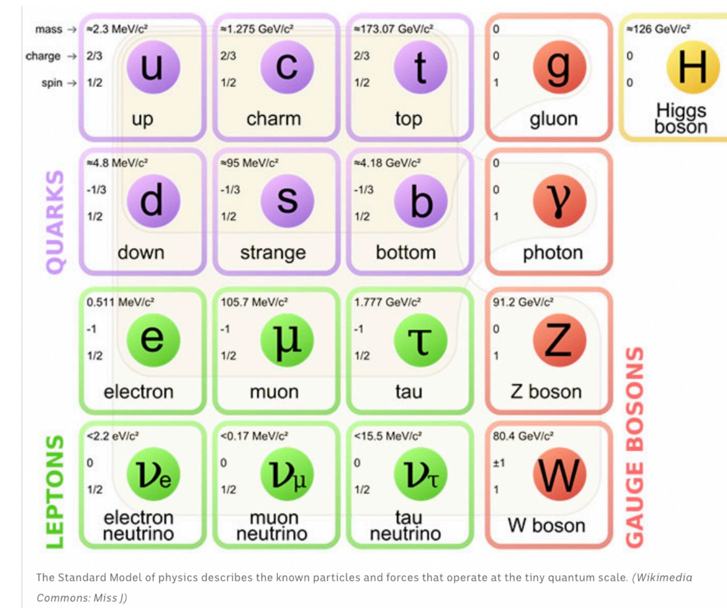


Higgs mechanism of mass generation

$$\mathcal{L} = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - V(\phi),$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

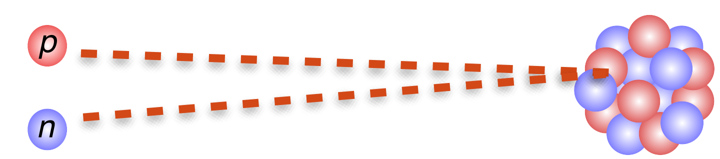
The Standard Model



"Bare" ***Interaction*** *(aka forces, potentials):*

$$V(\mathbf{r}_1 - \mathbf{r}_2)$$

Ab initio…



- *In **Theoretical Physics**: knowing the bare (vacuum) interaction between two particles, quantitatively describe the many-particle system*
- *"Ab initio = UV-complete" as opposed to "effective"*
- *Interactions adjusted to many-particle systems (e.g., finite nuclei) are not ab initio but effective*

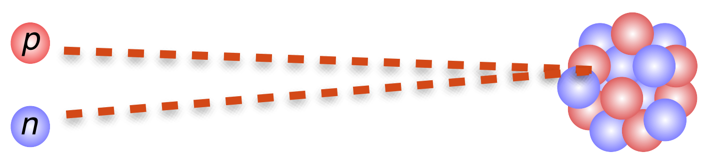
Language remarks

The narratives (i) "There is an ab initio nuclear theory as opposed to "traditional" nuclear theory"
(ii) There are "models" and there is "the theory"

"Bare" **Interaction** (aka forces, potentials):

$V(\mathbf{r}_1 - \mathbf{r}_2)$

Ab initio…



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Formulation of the theory and its implementation

- *There is a long way from formulation to implementation. We can formulate an ab initio theory, but it is difficult to implement accurately. However, we still need an ab initio theory to constrain concrete implementations, e.g., for maintaining their algebraic structure.*

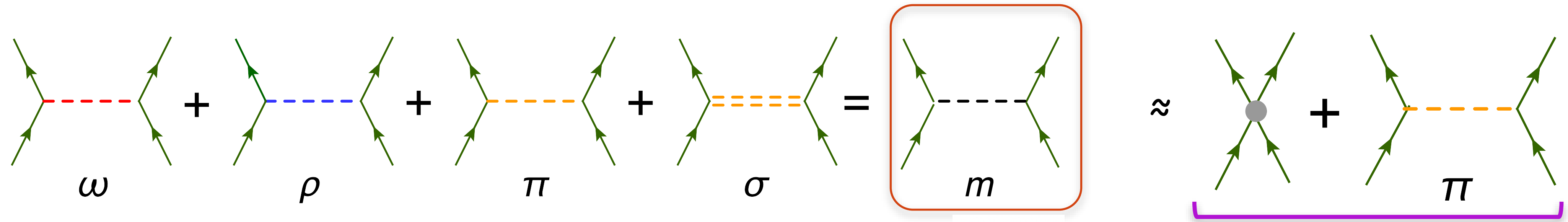
- *All "traditional" microscopic many-body models employing effective interactions can be derived ab initio and represent various approximations to the static and dynamical kernels of the same EOMs.*

- *Paraphrasing Steven Weinberg, saying "all non-renormalizable theories are as renormalizable as renormalizable theories", we find that "traditional" theories are as ab initio as ab initio theories :)*

Linking the nucleon-nucleon (NN) interaction and the many-body theory

Leading order:

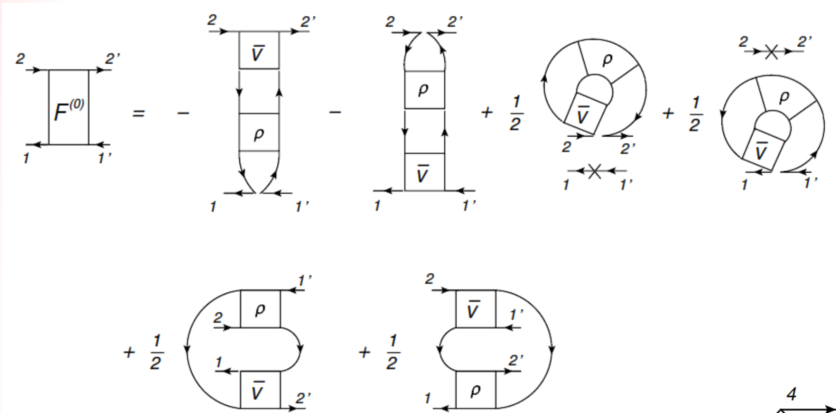
Quantum Hadrodynamics (QHD)

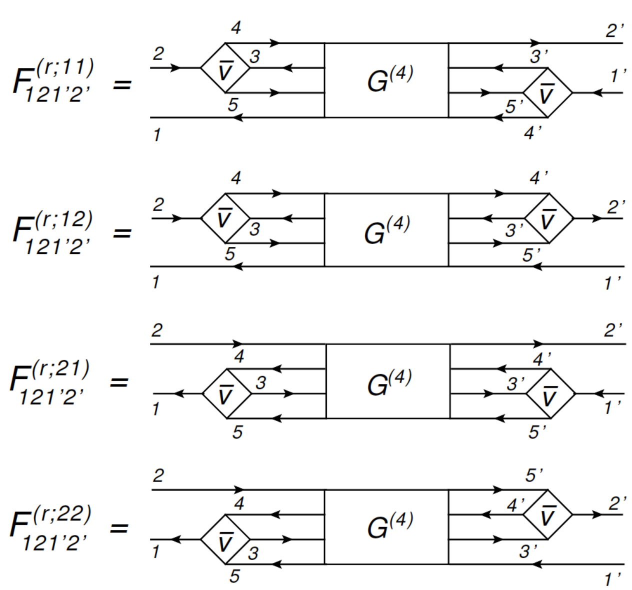


heavy light ~2π $\bar{v}_{1234}$ χEFT

Beyond the leading order:

Relativistic Nuclear Field Theory (this work): non-PT, in-medium



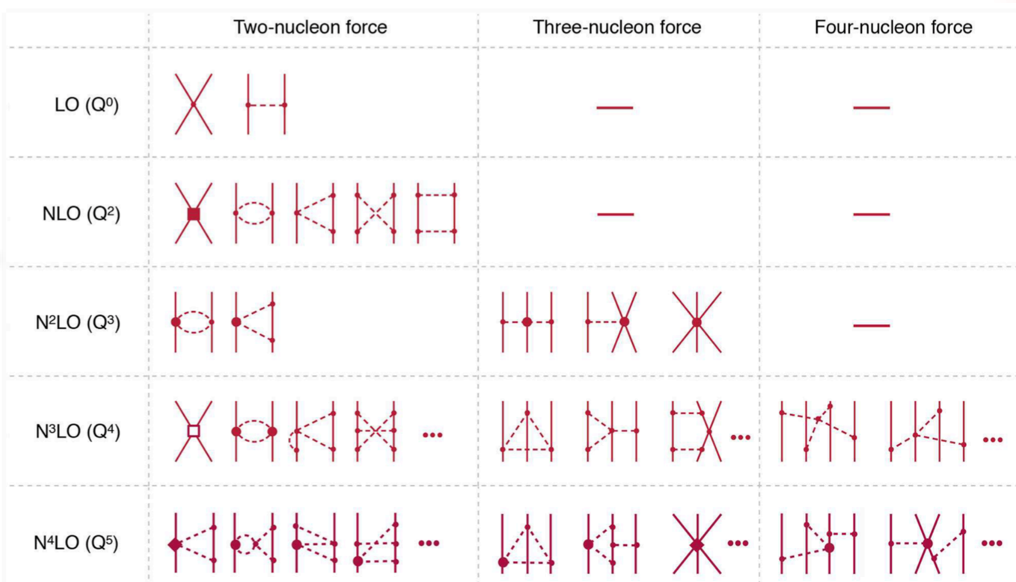


Static

+ Dynamical

cf. χEFT: PT in the vacuum

E. Epelbaum et al., Front. Phys. 8, 98 (2020)



+ Dynamical

Many-body… "solver" (?)

- *"Solvers" are too simplistic*
- *Bare NN and many-body theory should be linked consistently*
- ***The vacuum power counting is irrelevant in the strongly correlated medium of medium-heavy nuclei***

The in-medium power counting is associated with emergent scales in large systems

Dynamical kernels: bridging the scales

- ***Quantum many-body problem in a nutshell:*** *Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy or Schwinger-Dyson equations. $N_{Equations} \sim N_{Particles}$ & Coupled 🙈 !!!*
- ***Non-perturbative solutions:***

Cluster decomposition [QFT, S. Weinberg]

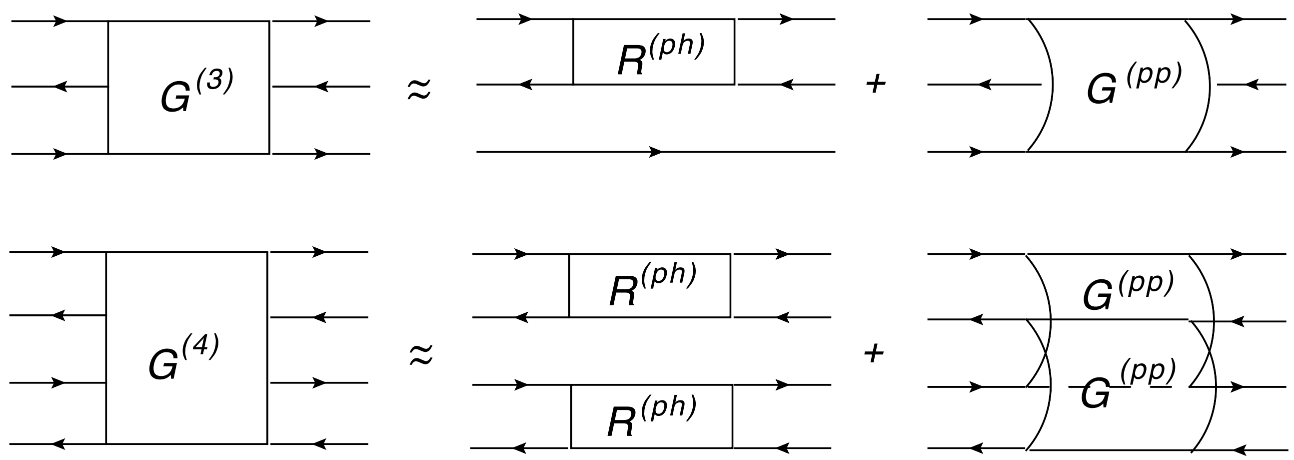
$$✦\, G^{(3)} = G^{(1)}G^{(1)}G^{(1)} + G^{(2)}G^{(1)} + \Xi^{(3)}$$

$$✦\, G^{(4)} = \boxed{G^{(1)}G^{(1)}G^{(1)}G^{(1)}} + \boxed{G^{(2)}G^{(2)}} + \boxed{G^{(3)}G^{(1)} + \Xi^{(4)}}$$

Leading at: ***weak*** / ***intermediate*** / ***strong*** *coupling*

- weak: *self-consistent GFs, second RPA, shell-model etc.*
- intermediate: *phonon coupling* **this work**
- strong: **Faddeev +** **future work**

Beyond weak coupling:

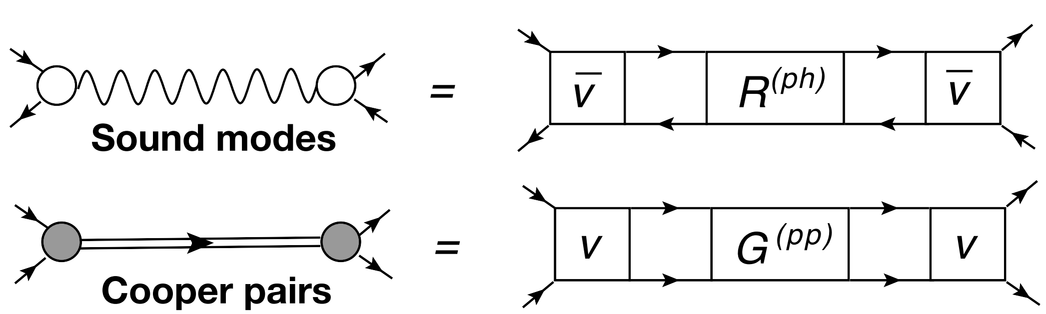


- *P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).*
- *N. Vinh Mau, Trieste Lectures 1069, 931 (1970)*
- *P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)*
- …

Exact mapping: particle-hole (2q) quasibound states

Emergence of effective "bosons" (phonons, vibrations):

Emergence of superfluidity:



Quasiparticle-vibration coupling (qPVC) in nuclei. Cf. NFT

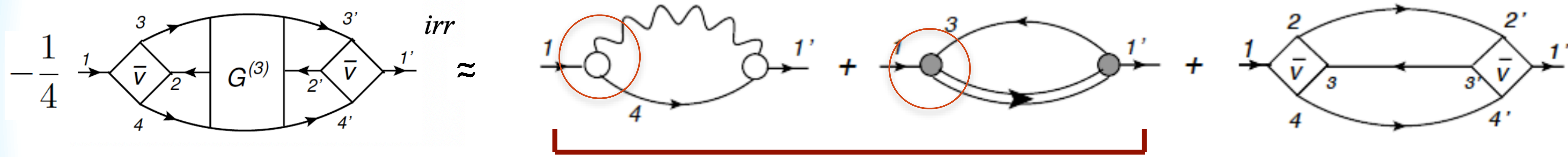
Diquarks in hadrons

Cf. C. Popovici, P. Watson, and H. Reinhardt [PRD83, 025013 (2011)]

Emergence of effective degrees of freedom at intermediate coupling

Dynamical self-energy $\Sigma^{(r)}$:

"Radiative-correction" "Second-order"



Can be treated in a unified way (superfluid theory):
qPVC in nuclei, correlations in hadrons / matter / solid state

Emergent phonon vertices and propagators: calculable from the underlying H, which does not contain phonon degrees of freedom

Cf. Nuclear Field Theory

R.A. Broglia, D. Bes, P.-F. Bortignon, R. Liotta, G. Colò, E. Vigezzi, F. Barranco, G. Potel, et al.

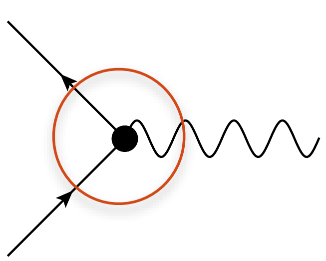
"Ab-initio"

$$H = \sum_{12} h_{12}\psi_1^\dagger\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}\psi_1^\dagger\psi_2^\dagger\psi_4\psi_3$$

"Effective" qPVC

$$H = \sum_{12} \tilde{h}_{12}\psi_1^\dagger\psi_2 + \sum_{\lambda\lambda'}\mathcal{W}_{\lambda\lambda'}Q_\lambda^\dagger Q_{\lambda'} + \sum_{12\lambda}\left[\Theta^\lambda_{12}\psi_1^\dagger Q_\lambda^\dagger\psi_2 + h.c.\right]$$

Cf.: The Standard Model elementary interaction vertices: boson-exchange interaction is the input:



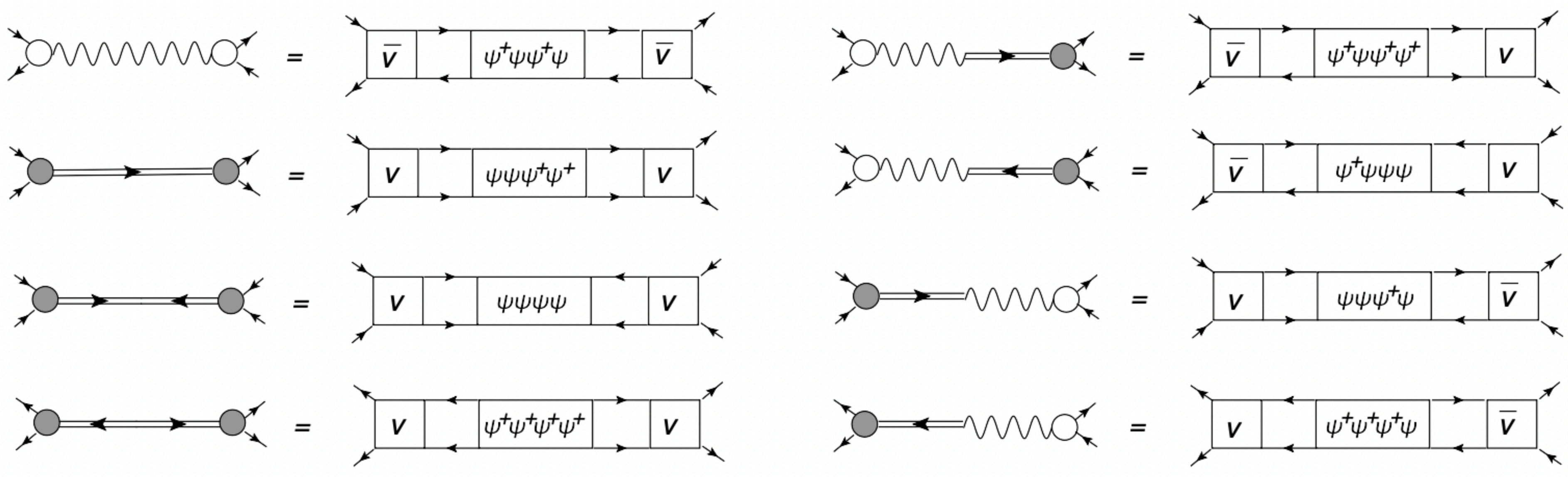
$$\gamma, g, W^\pm, Z^0$$

E.L., P. Schuck, PRC 100, 064320 (2019)
E.L., Y. Zhang, PRC 104, 044303 (2021)

“Ab-initio” qPVC in superfluid systems

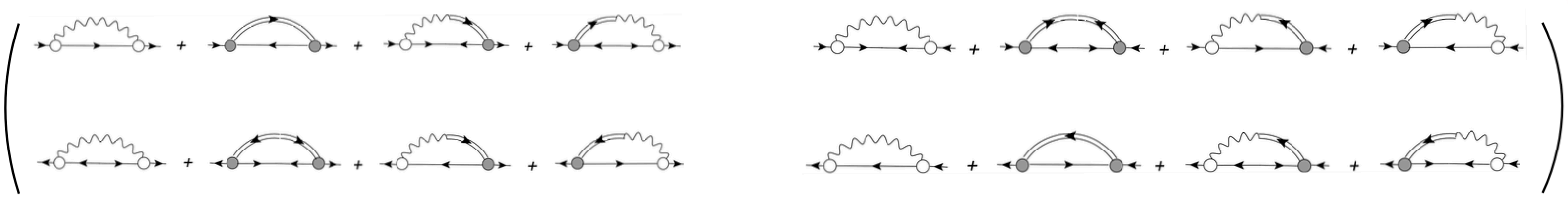
Superfluid dynamical kernel: adding particle-number violating contributions

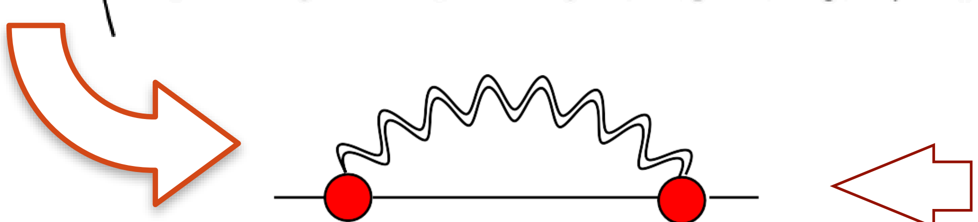
Mapping on the qPVC in the canonical basis



Quasiparticle dynamical self-energy (matrix): normal and pairing phonons are unified

$\hat{\Sigma}^{r}$ =





$\mathcal{W}$

Bogoliubov transformation

Compact form, (almost) as simple as non-superfluid

Cf.: Quasiparticle static self-energy (matrix) in HFB

$$\hat{\Sigma}^{0} = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta^{*}_{11'} & -\tilde{\Sigma}^{T}_{11'} \end{pmatrix}$$

E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

Currently formulated in the HFB basis keeping 4x4 block matrix structure at finite temperature: S. Bhattacharjee, E.L., arXiv:2412.20751

Toward concrete implementations: the elefant(s) in the room

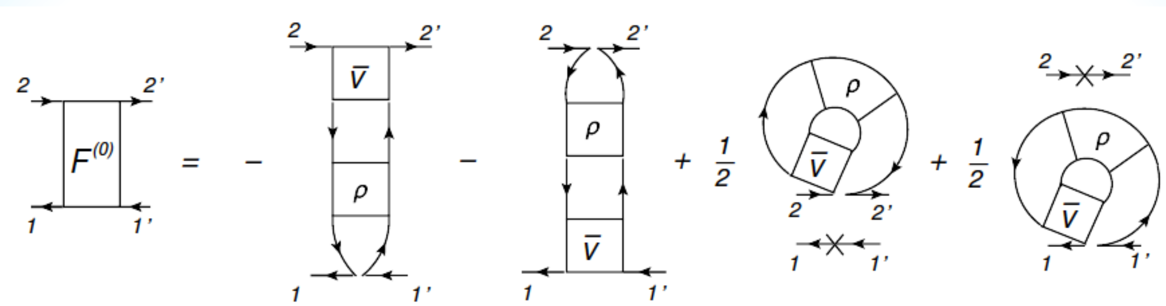
Particle-hole propagator (response function):

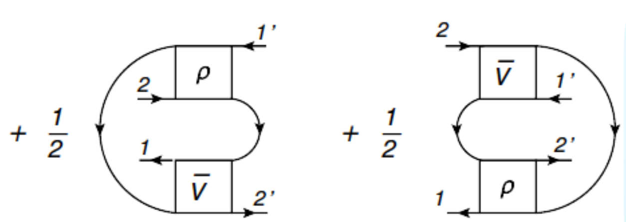
$$R_{12,1'2'}(t-t') = -i\langle T(\bar{\psi}_1\psi_2)(t)(\bar{\psi}_{2'}\psi_{1'})(t')\rangle$$

spectra of excitations, masses, decays, …

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \qquad (^{**}) \qquad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Static kernel (exact "bosonic" mean field):
Short-range, Constant





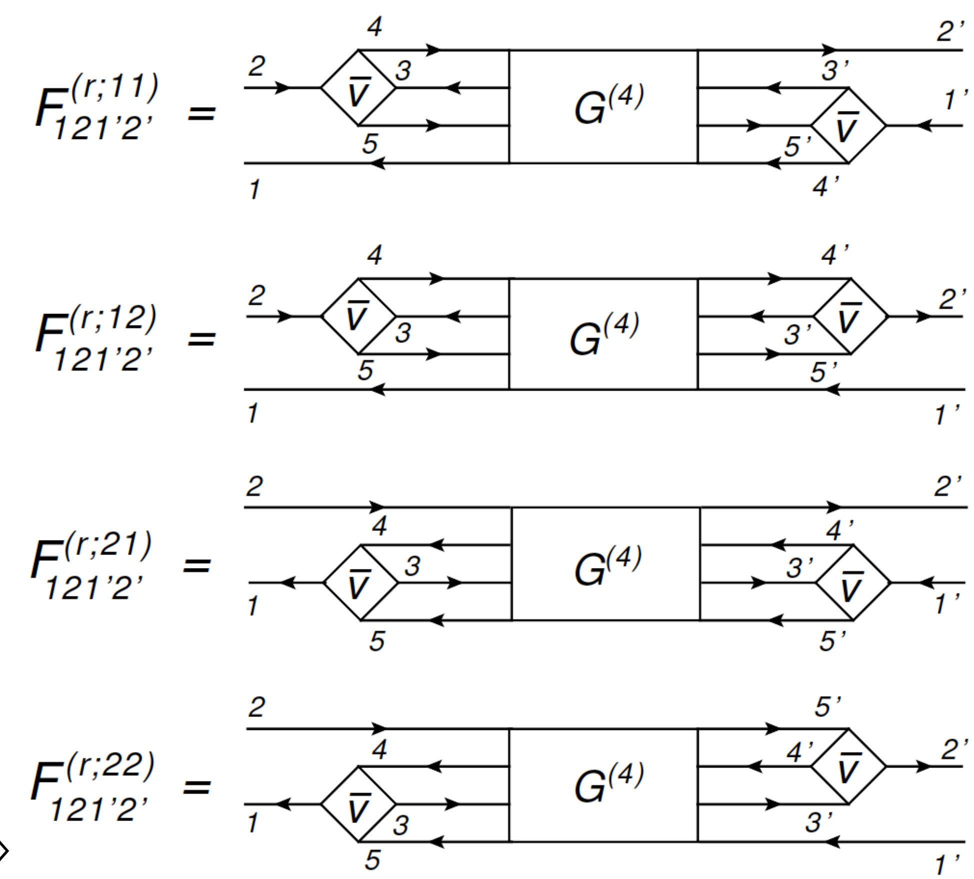
$$\sim \sum_{jkl} \bar{v}_{2jkl}\rho_{kl,2'j}$$

Static & regular

Major problems with the ab initio implementation:

1) $F^{(0)}$ is *not calculable* as $\rho^{(2)}$ is unknown a priori
2) In approximate treatments (G-matrix, SRG, etc.) *the delicate balance between $F^{(0)}$ and $F^{(r)}$ is violated*
3) $F^{(0)}$ approximated by an *effective interaction* (e.g., meson couplings adjusted to finite nuclei) works much better quantitatively, *but the ab-initio character is lost*
4) Artifacts of the *localizing potentials* are difficult to remove

t-dependent (energy-dependent) kernel:
Long-range, consists of poles (= singularities) when Fourier transformed



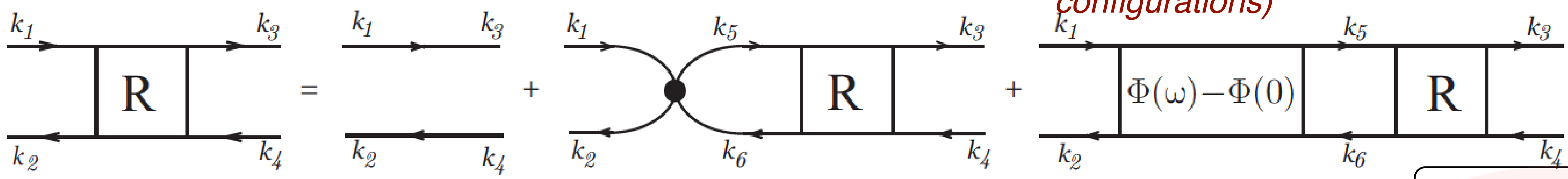
$$\sim \sum_{345}\sum_{3'4'5'} \bar{v}_{4513}G(325'4', 3'2'45)\bar{v}_{3'1'4'5'}$$

t(ω)-dependent & singular

Nuclear response: toward a complete theory

toward ab initio <= static kernel V

dynamic kernel => toward more complex configurations)



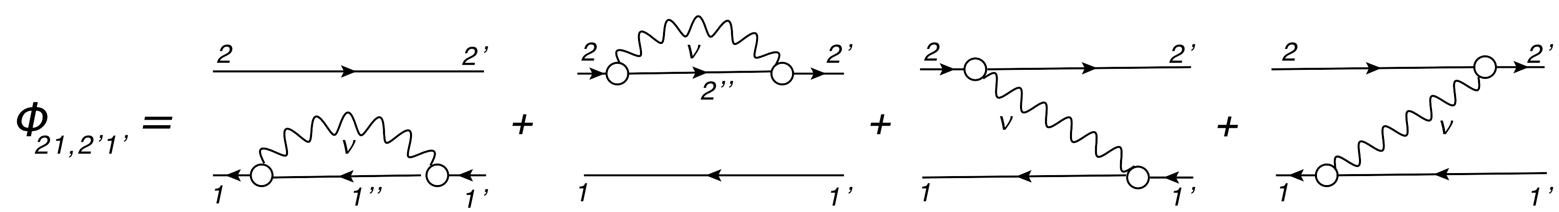
Dyson-Bethe-Salpeter Equation:

$$R(\omega) = R^0(\omega) + R^0(\omega)\,[V+\Phi(\omega)-\Phi(0)\,]\,R(\omega)$$

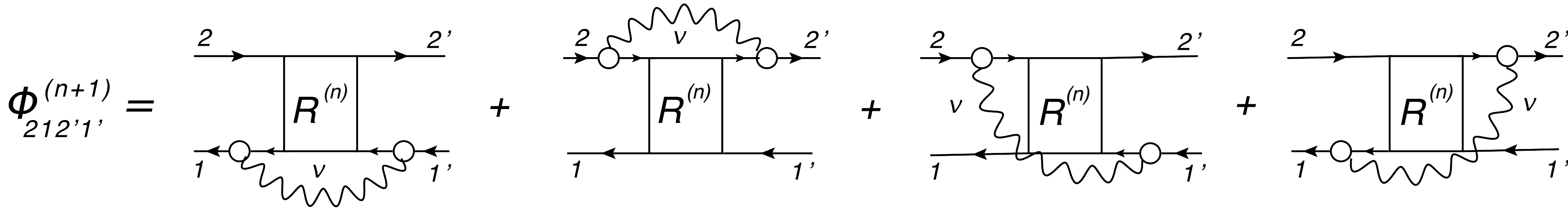
Subtraction [V.Tselyaev 2013]

for effective interactions V [P. Ring, A.Afanasjev, et al: CEDF]

NFT is reproduced as the leading approximation



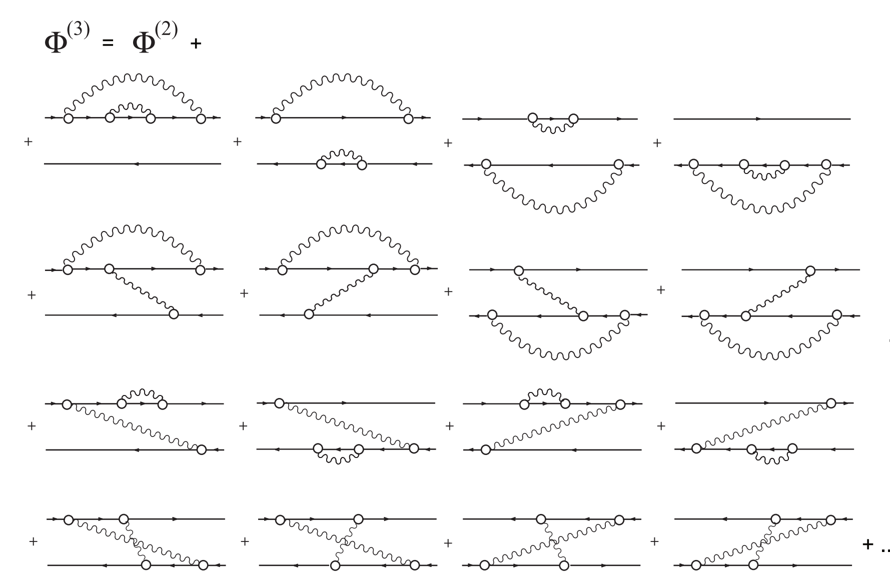
Extended NFT: closed cycle



Generalized approach for the correlated propagators

n-th order: E.L. PRC 91, 034332 (2015)

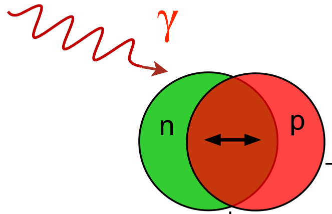
Ab-initio formulation,
$\Phi^{(3)}$ implementation; 2q+2phonon correlations:
E.L., P. Schuck, PRC 100, 064320 (2019)



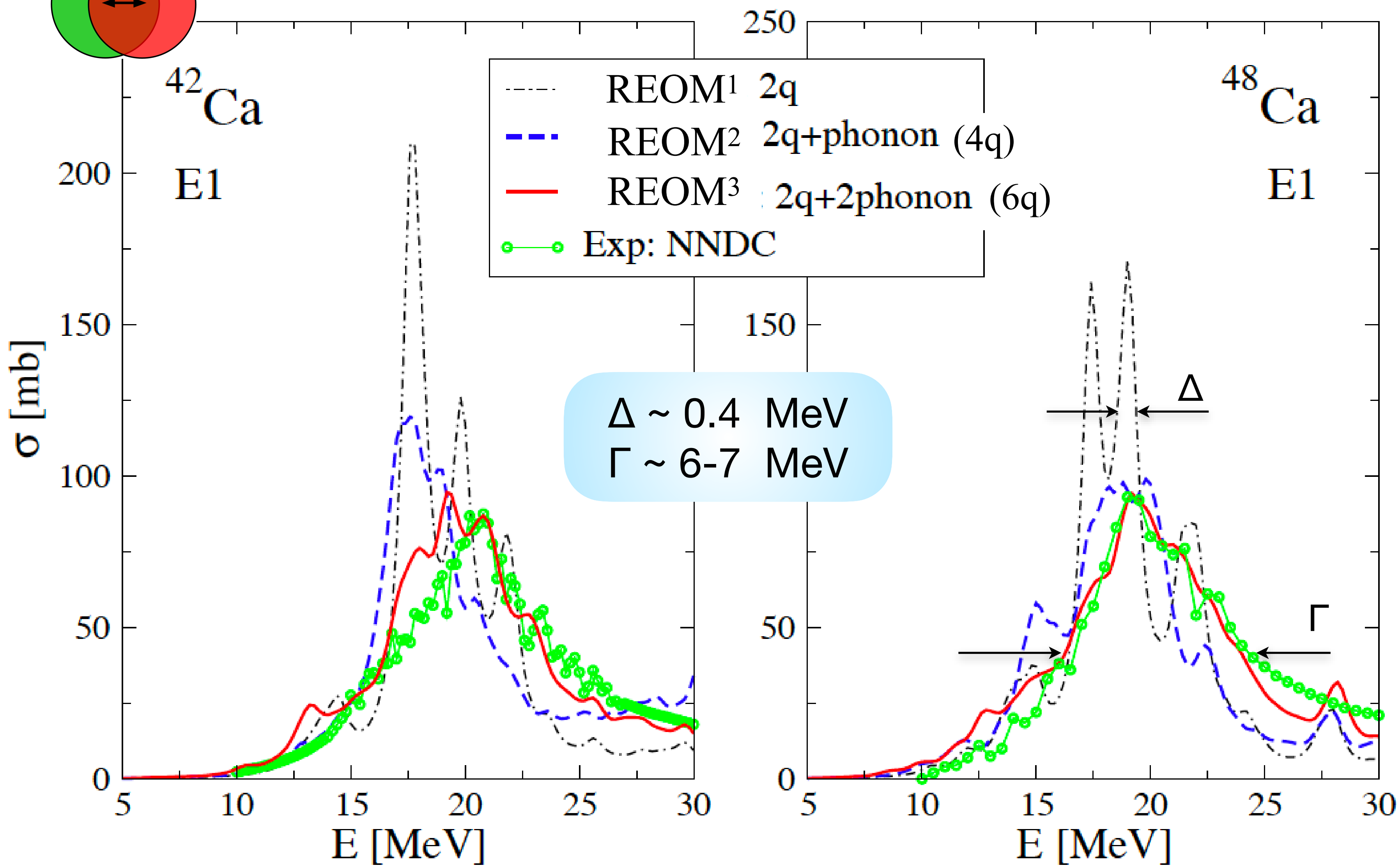
2q+2phonon configurations in the qPVC expansion:

Included in all orders (non-perturbatively)

Toward spectroscopically accurate theory: the relativistic EOM (REOM[1,2,3])



Giant Dipole Resonance (GDR) in Ca isotopes



- *On each iteration, the complex configurations enforce fragmentation and spreading toward higher and lower energies*
- *Exp. Data: V.A. Erokhova et al., Bull. Rus. Acad. Phys. 67, 1636 (2003)*
- *RQTBA³ demonstrates an overall systematic improvement of the description of nuclear excited states as compared to RQTBA² in a broad energy range*

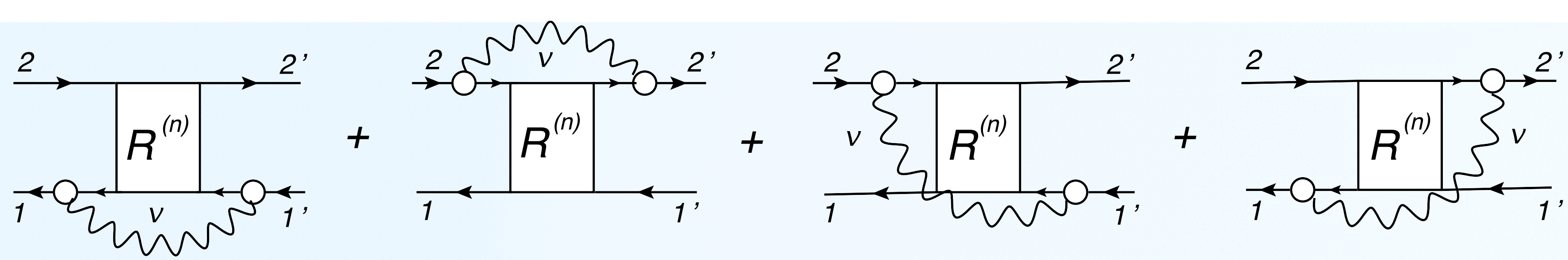
E.L., P. Schuck, PRC 100, 064320 (2019)

Basis: relativistic mean field (P. Ring et al.)

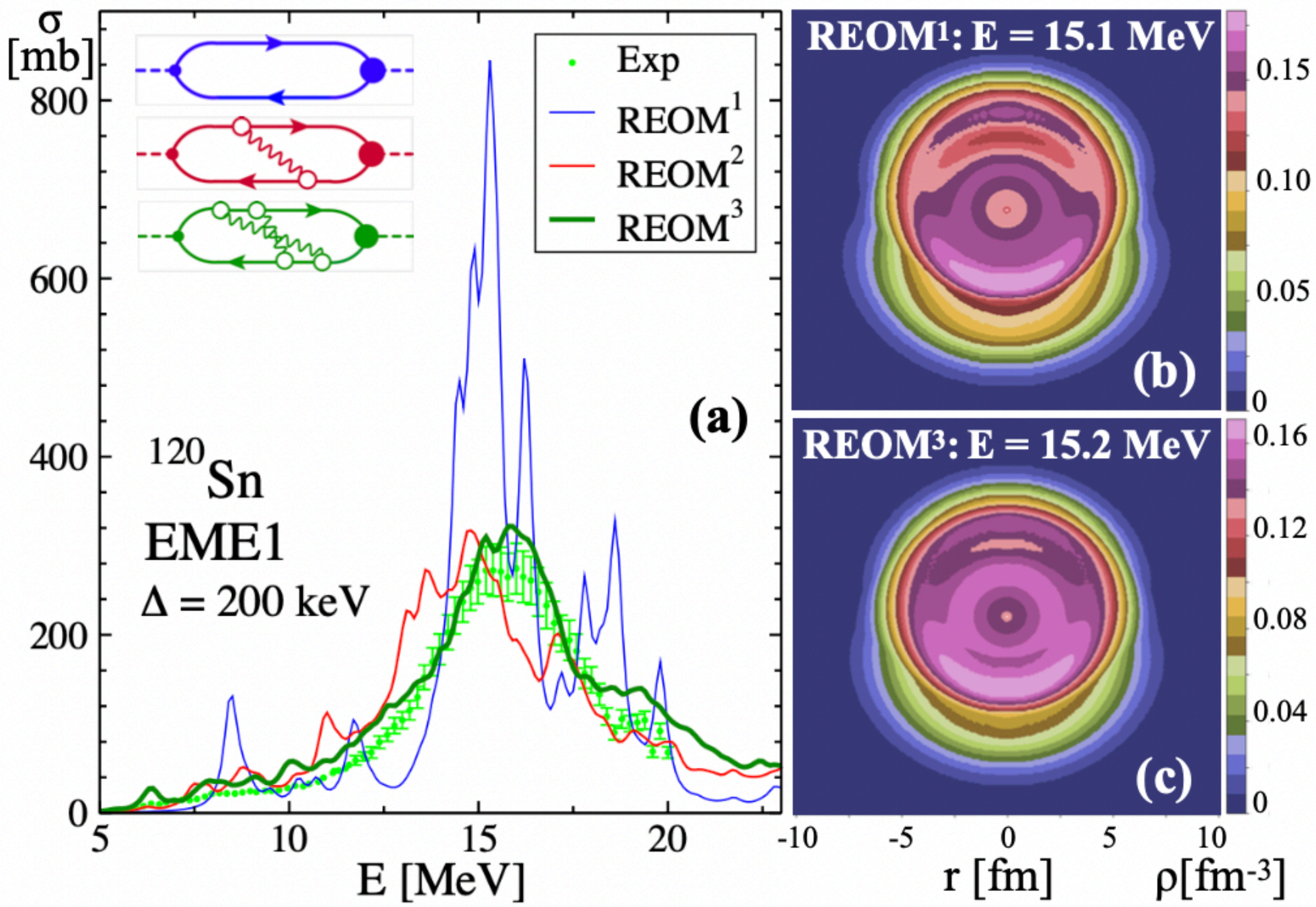
Each elementary 2q mode produces a multitude of states via fragmentation that repeats on the 4q level and so on. Cf.: Fractal self-similarity (nesting)

A hierarchy of the dynamical kernels:

n = 0 (no dynamical)
n = 1
n = 2

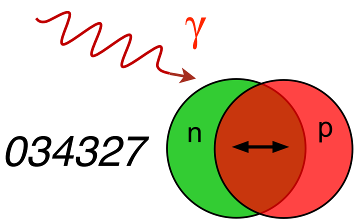


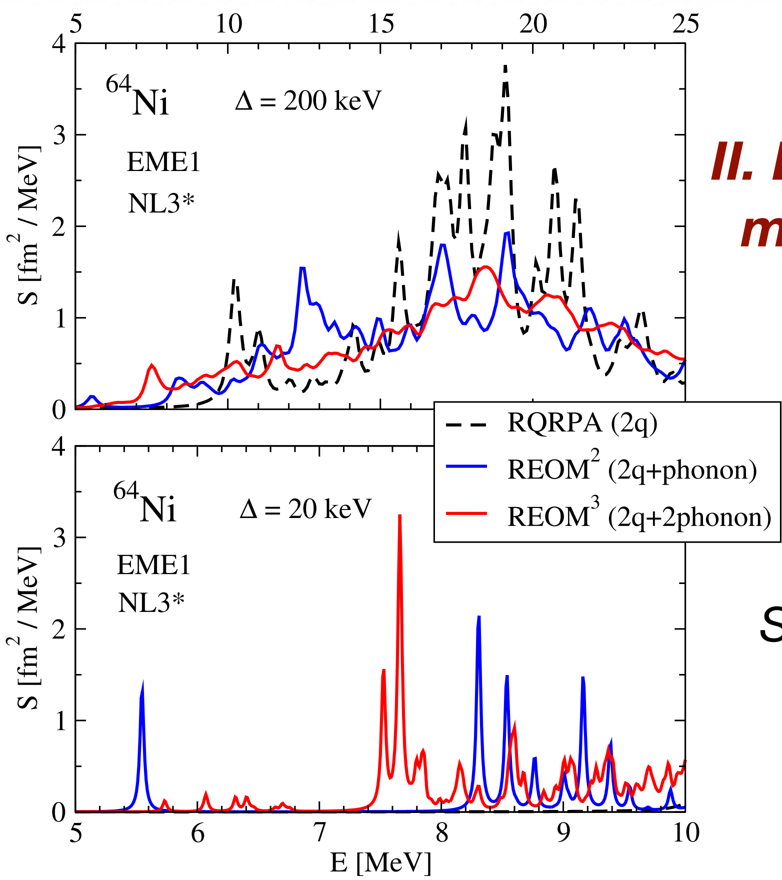
Heavy and deformed nuclei



I. Response of (medium)-heavy nuclei

- The complex configurations 2q+2phonon enforce fragmentation and spreading toward higher and lower energies
- Both giant and pygmy dipole resonances are affected; an improved description is achieved for the Sn-Sm mass region
- J. Novak, M. Hlatshwayo, E.L., arXiv:2405.02255
- Exp. Data: S. Bassauer, PRC102, 034327 (2020)



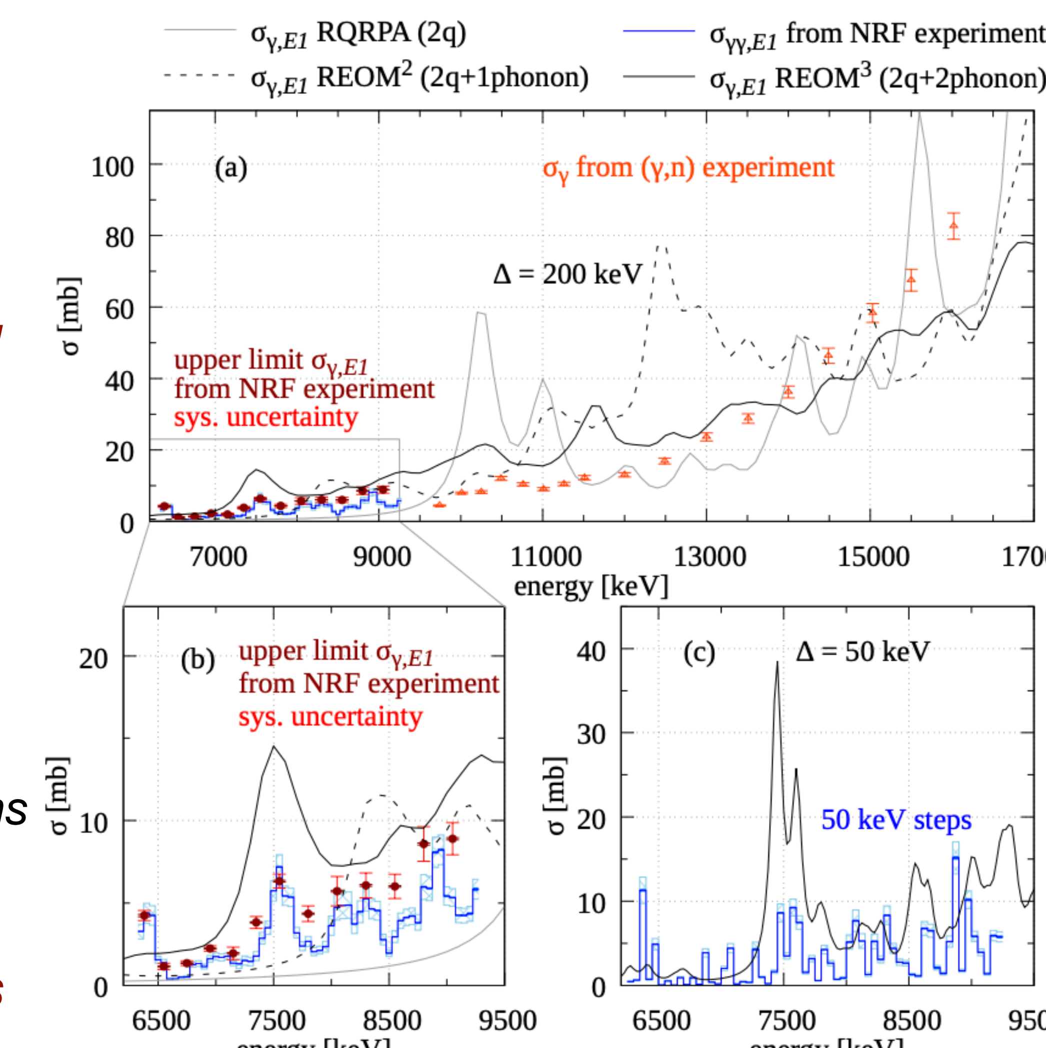


II. Description of transitional and moderately deformed systems

^{64}Ni [$\beta_2 \sim 0.16$]:
M. Müscher, A. Zilges, E.L. et al., PRC109, 044318 (2024)

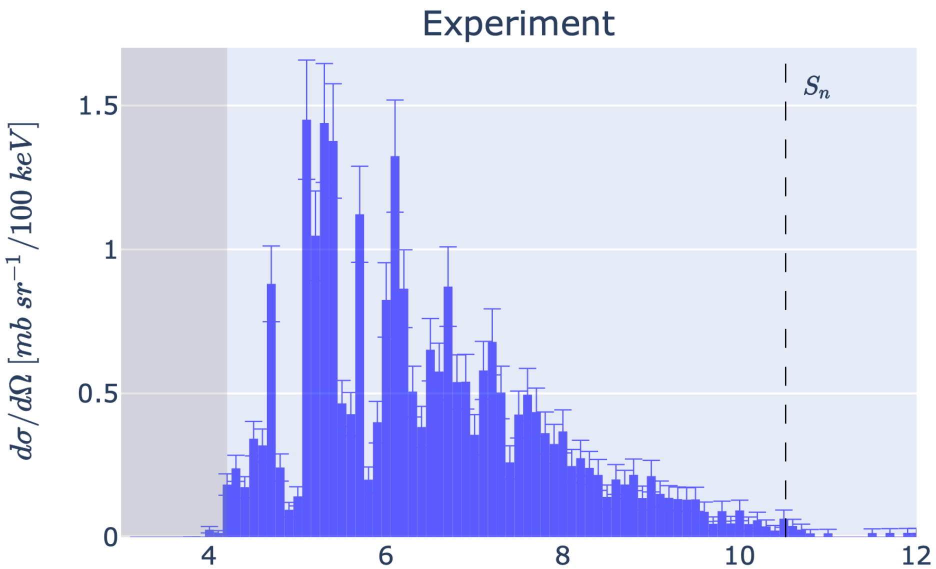
Spherical bases are used: correlations "generate" non-trivial geometry

III. More cases are in progress



Nuclear response at low energy: Sm and Nd, ongoing

Low-lying dipole strength in Sm and Nd isotopes

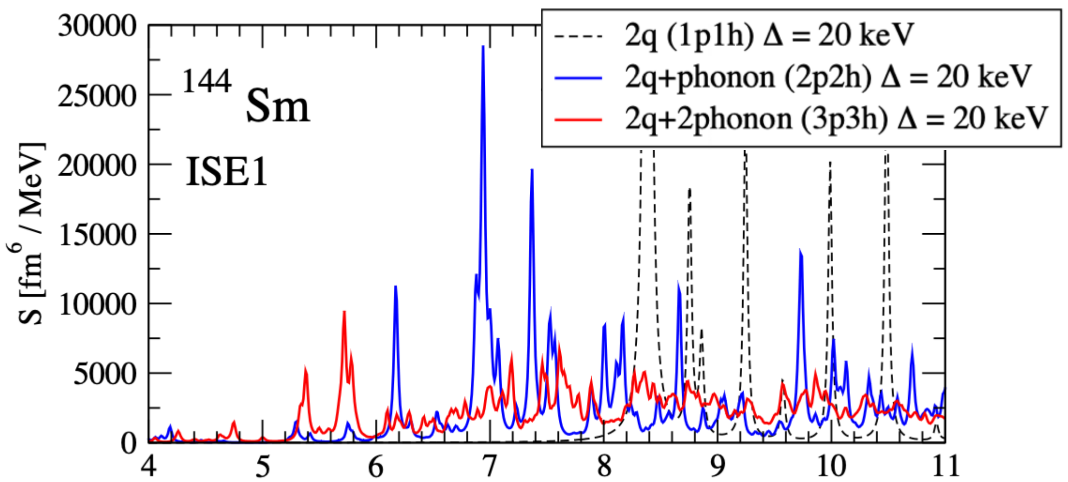


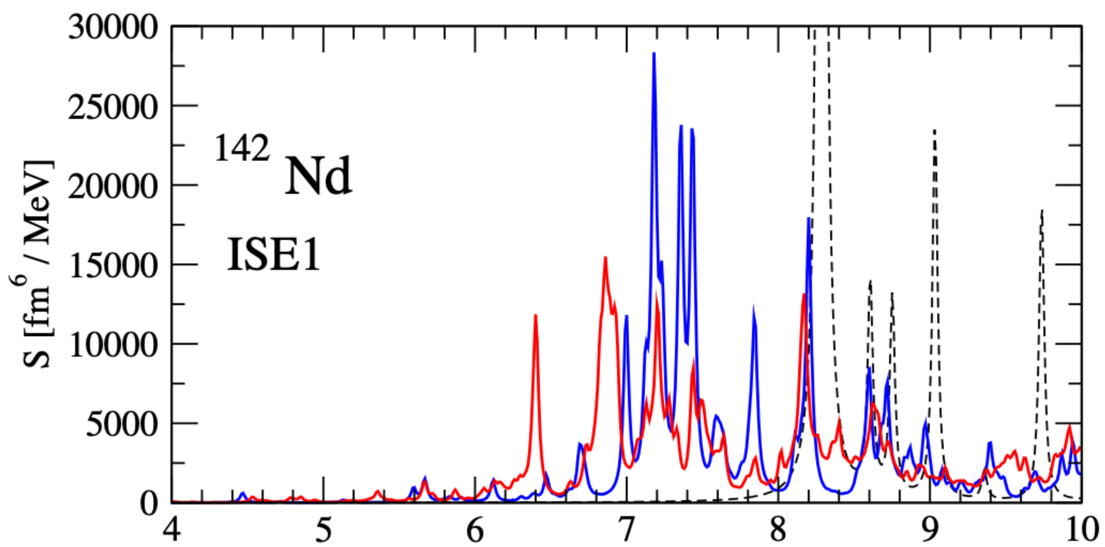
Measurement:

Luna Pellegri
&
Collaborators

Reaction:
Edoardo Lanza

Data
to be released



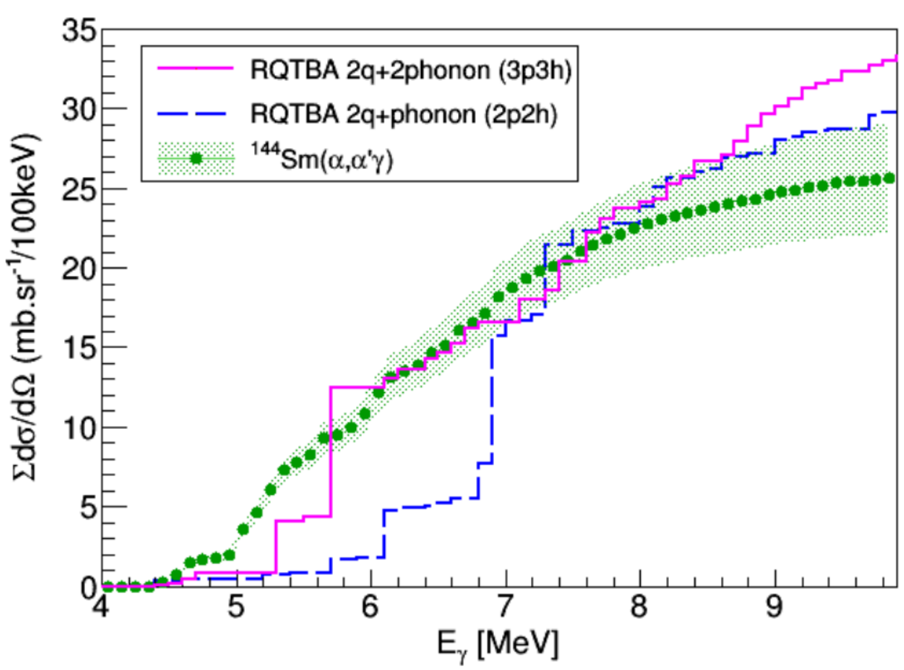


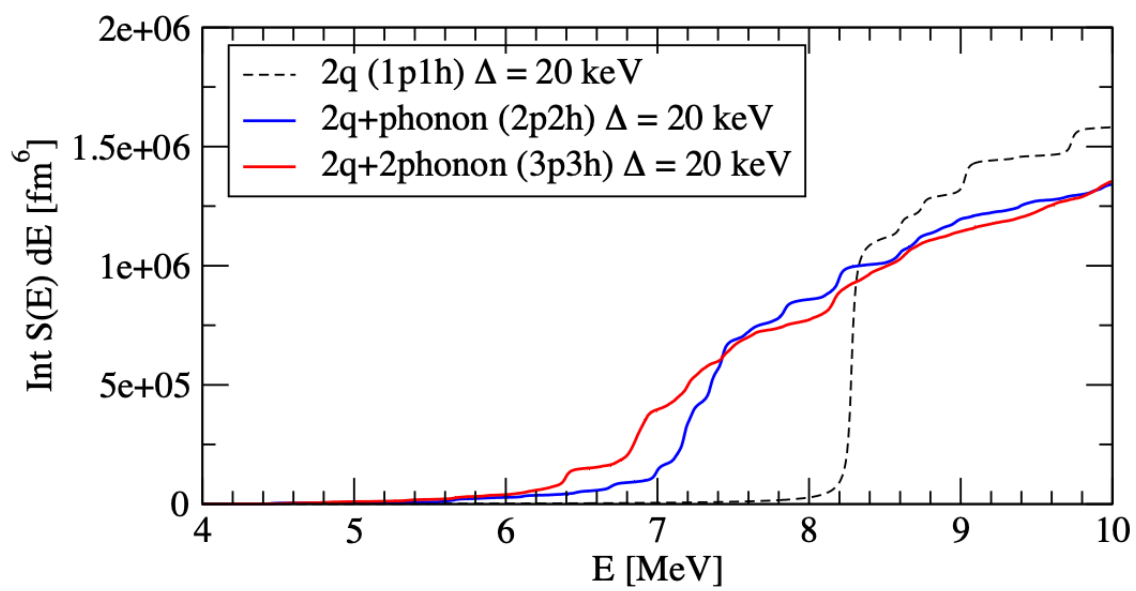
2q

4q

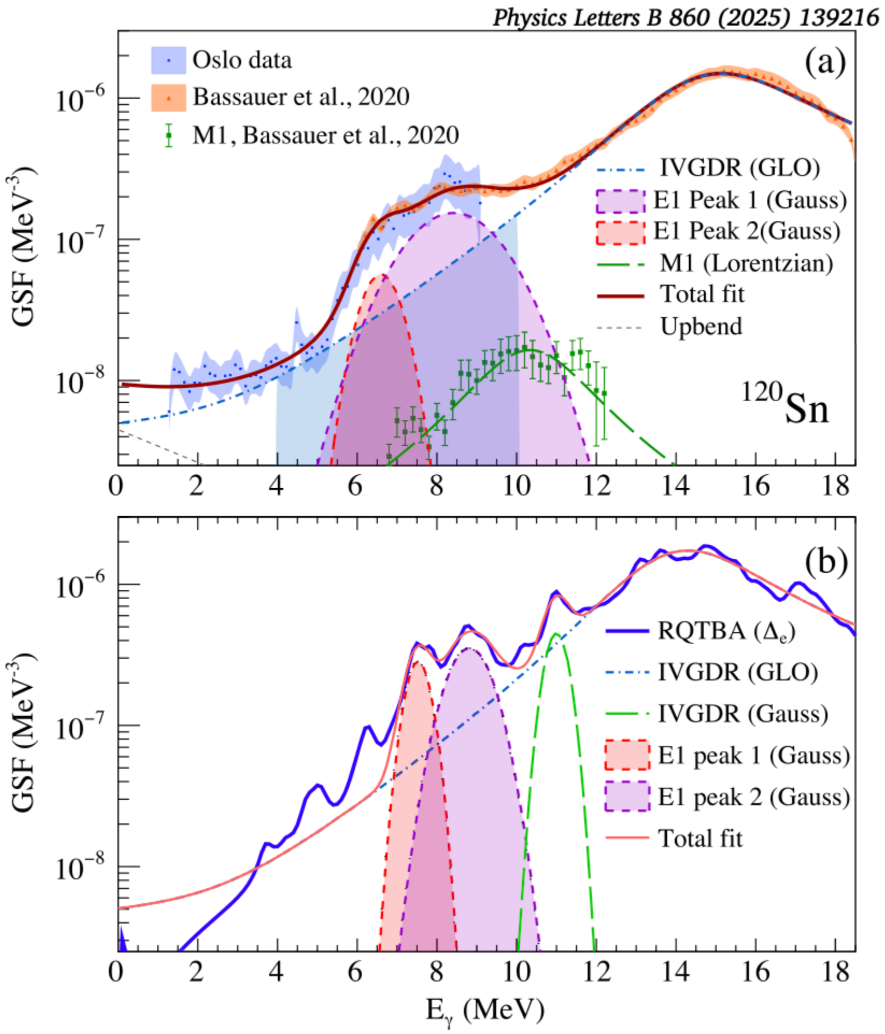
6q

Parameter-free



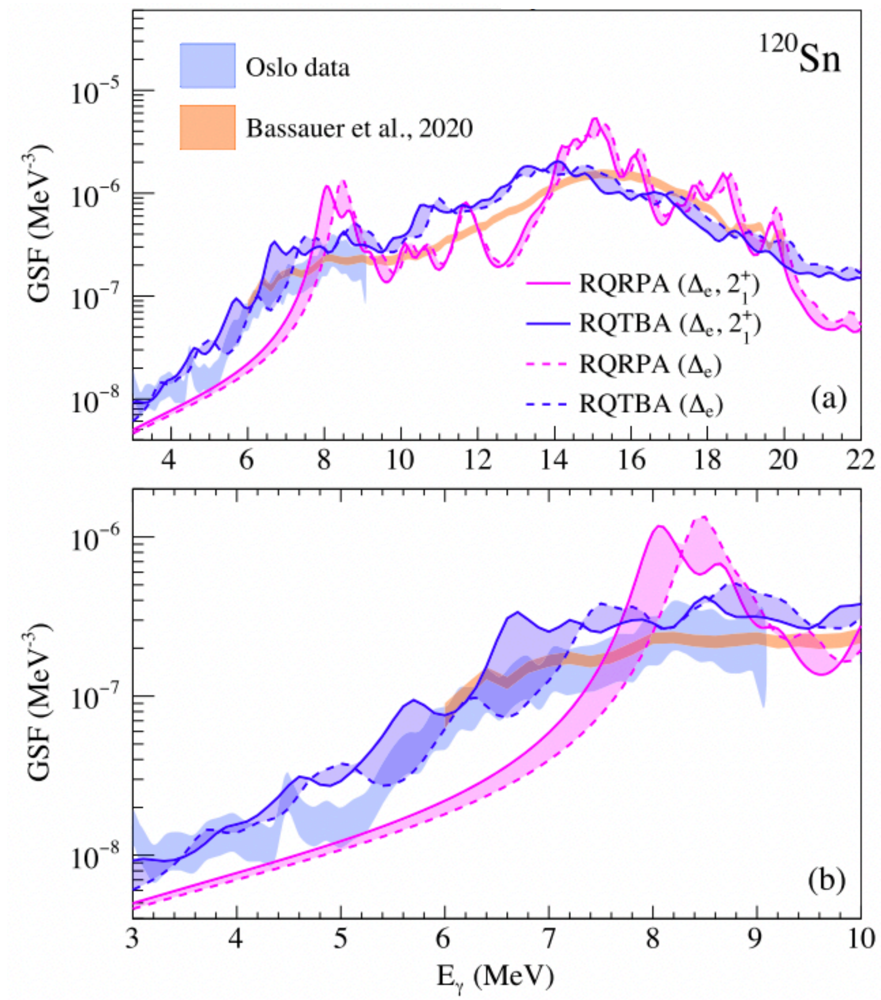


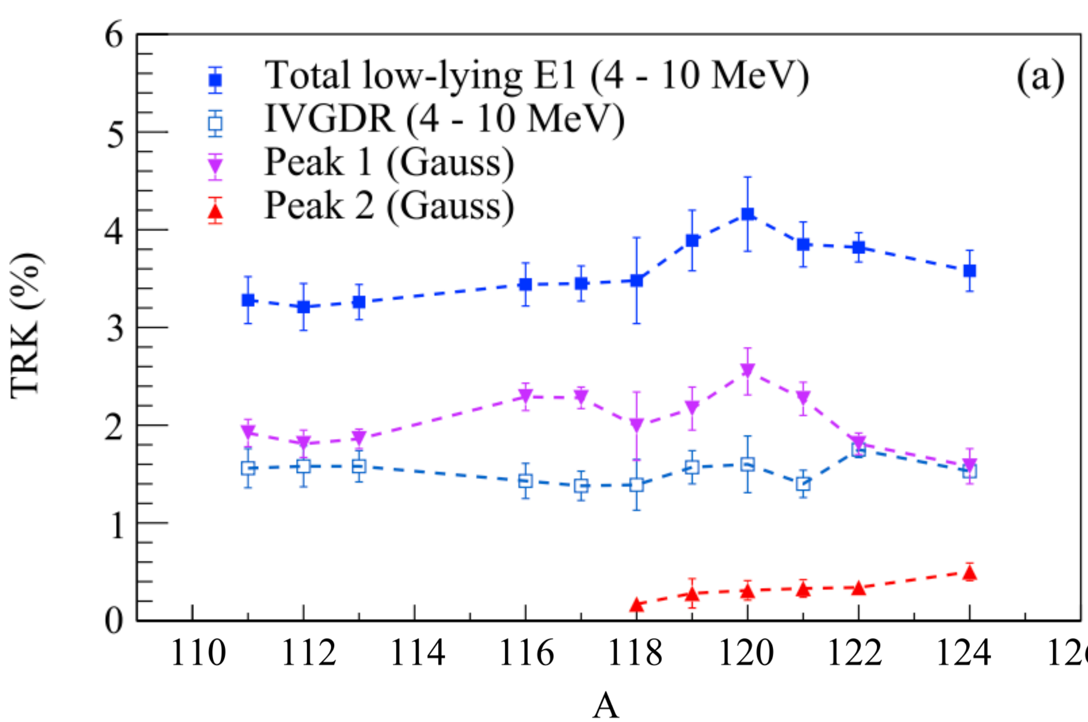
Pygmy dipole resonance: structural properties on the REOM² level

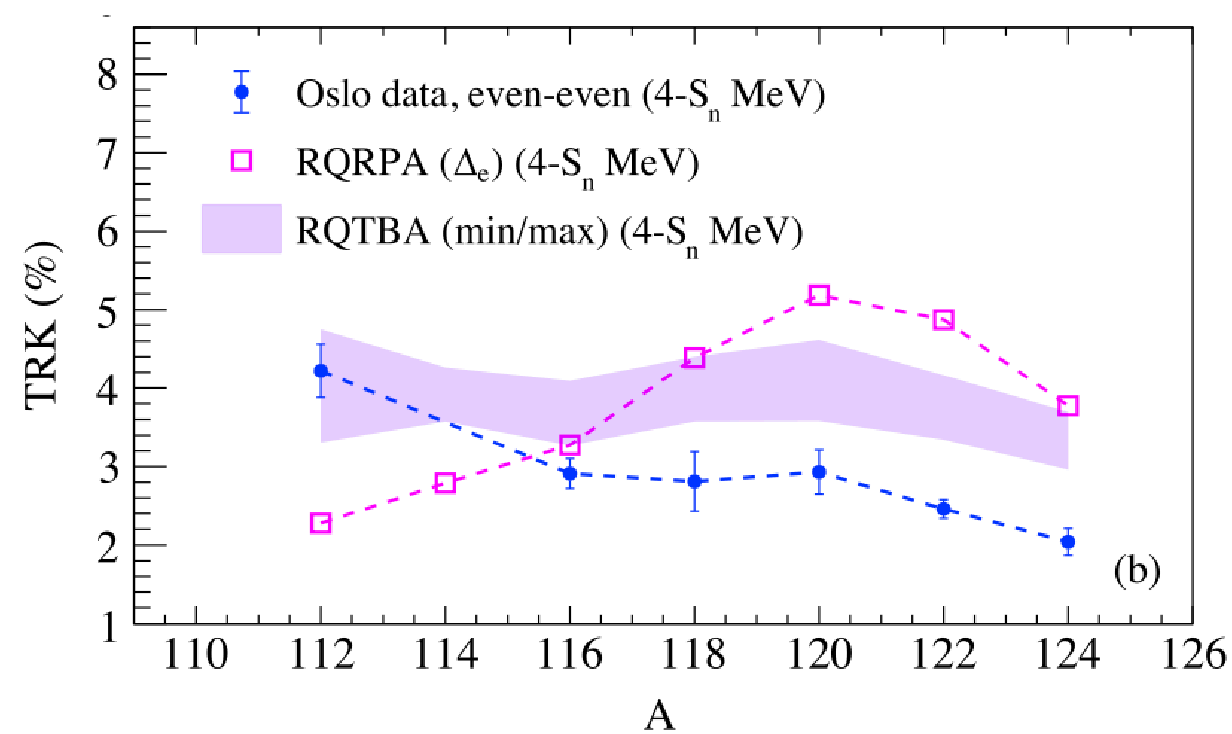


M. Markova, P. von Neumann-Cosel, E. L., PLB 860, 139216 (2025)

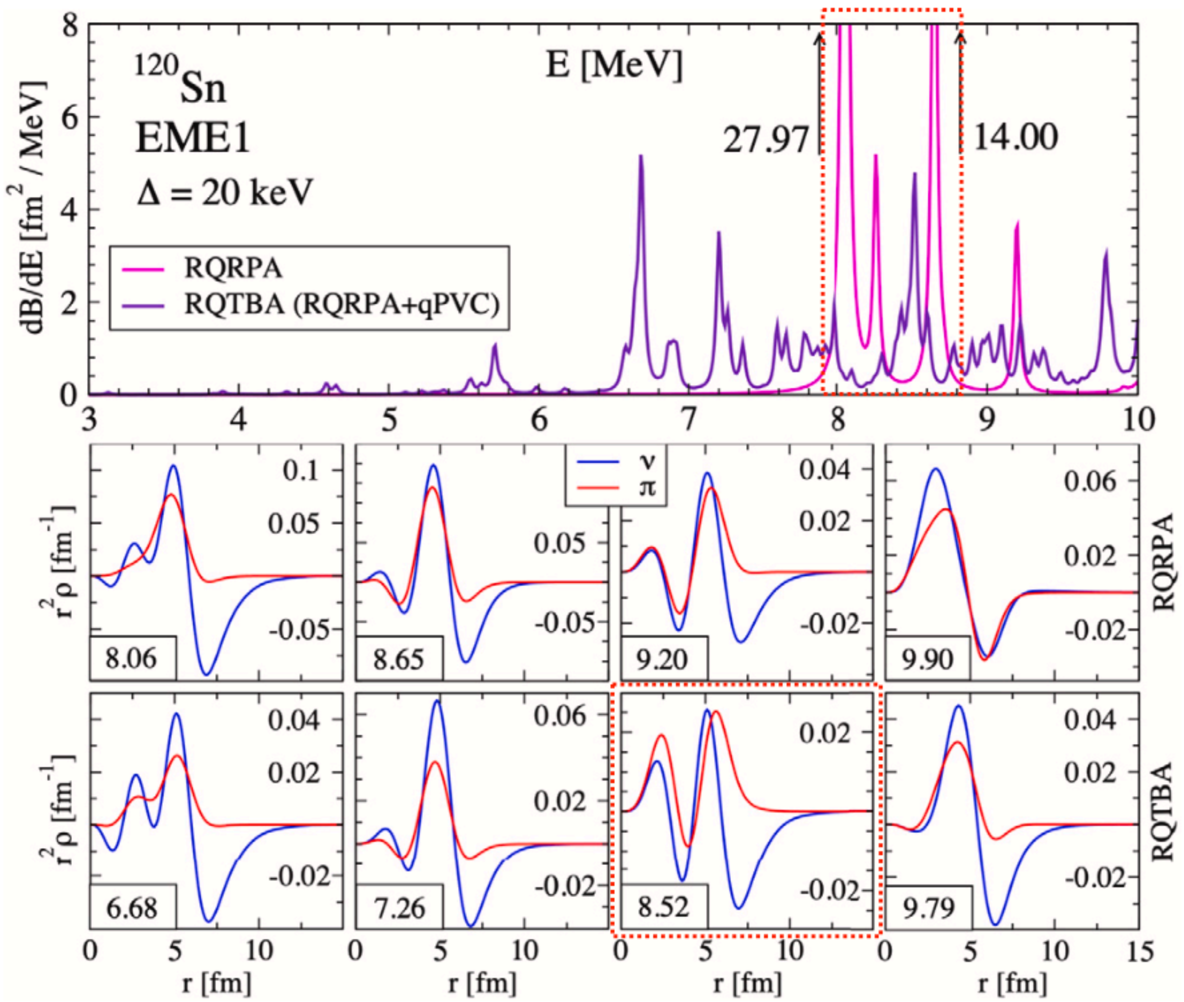
- Data analysis indicate the two-component structure of the low-energy dipole strength
- RQRPA does not describe the data well but remains a good reference point.
- 2q+phonon/RQTBA/REOM² is sensitive to the details of the superfluid pairing, particularly the pairing gap, especially at low energies.
- Pairing is the anomalous part of the mean field and also poorly constrained.
- Nevertheless, we could document the formation of the two-peak structure and reasonable total strength below S_n with theoretical uncertanties.

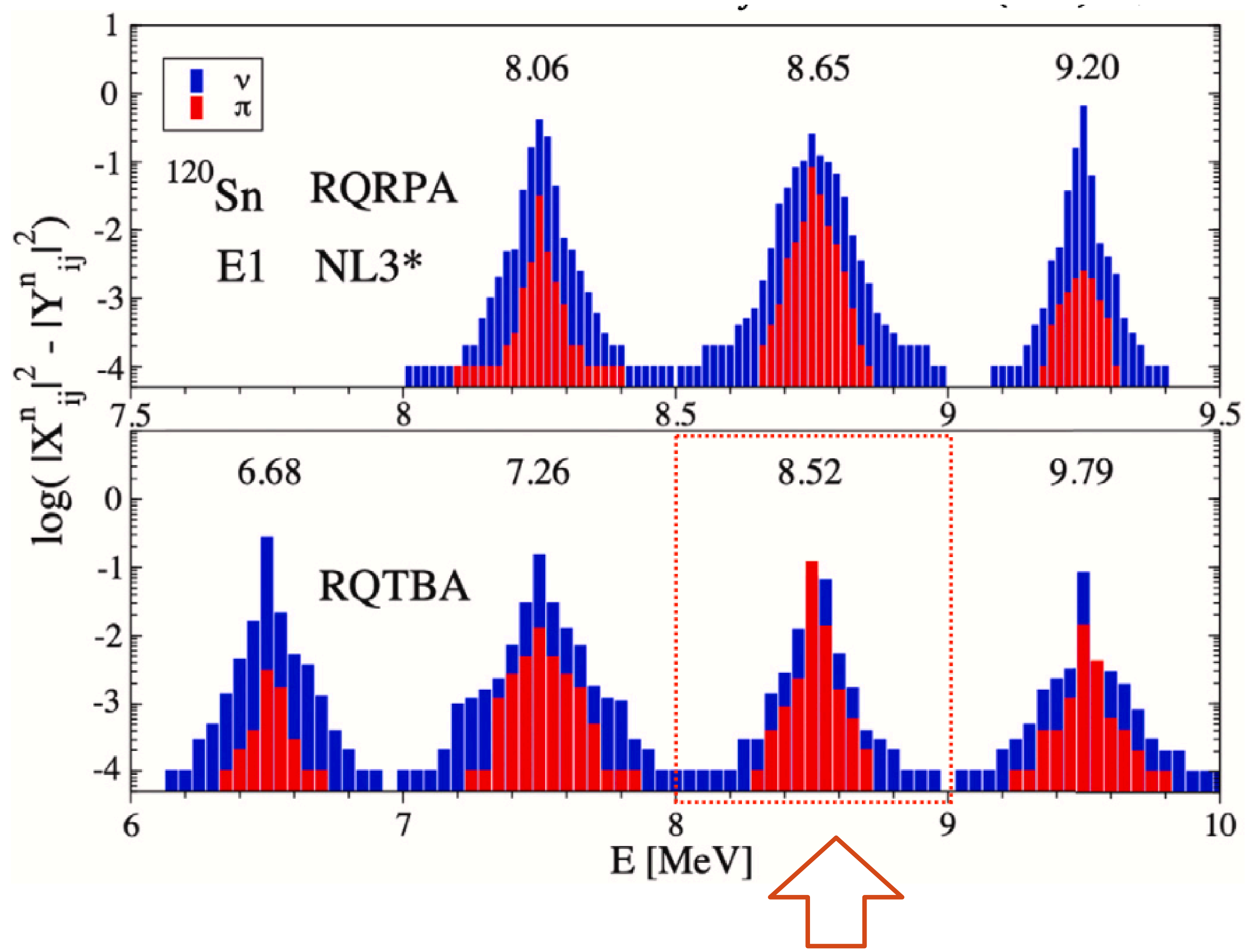






Low-energy dipole strength (LEDS): structural properties





Gamma transitions between excited states:

Dominate the upper component of LEDS

$$\mathcal{F}_{mn} = \langle m|\mathcal{F}|n\rangle = f_{ij}(\mathcal{X}^{m*}_{ik}\mathcal{X}^{n}_{jk} + \mathcal{Y}^{m*}_{jk}\mathcal{Y}^{n}_{ik})$$

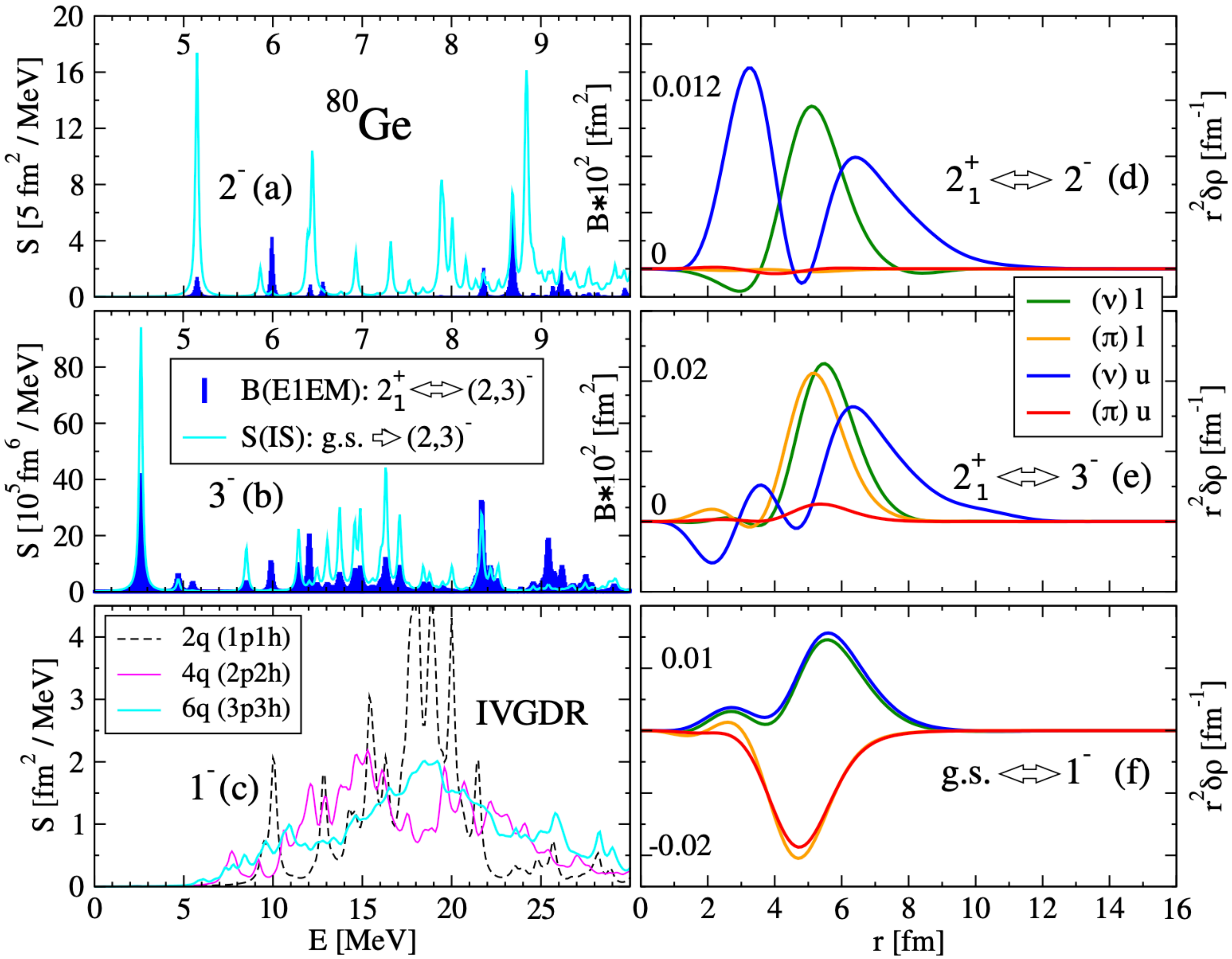
$$\mathcal{X}^n_{jk} = \langle 0|\alpha_k\alpha_j|n\rangle \qquad \mathcal{Y}^n_{jk} = \langle 0|\alpha^\dagger_j\alpha^\dagger_k|n\rangle$$

$$F_{LM} = e\sum_{i=1}^{Z} r_i{}^L Y_{LM}(\hat{\mathbf{r}}_i), \qquad L \geq 2$$

$$F_{1M} = \frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\hat{\mathbf{r}}_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\hat{\mathbf{r}}_i)$$

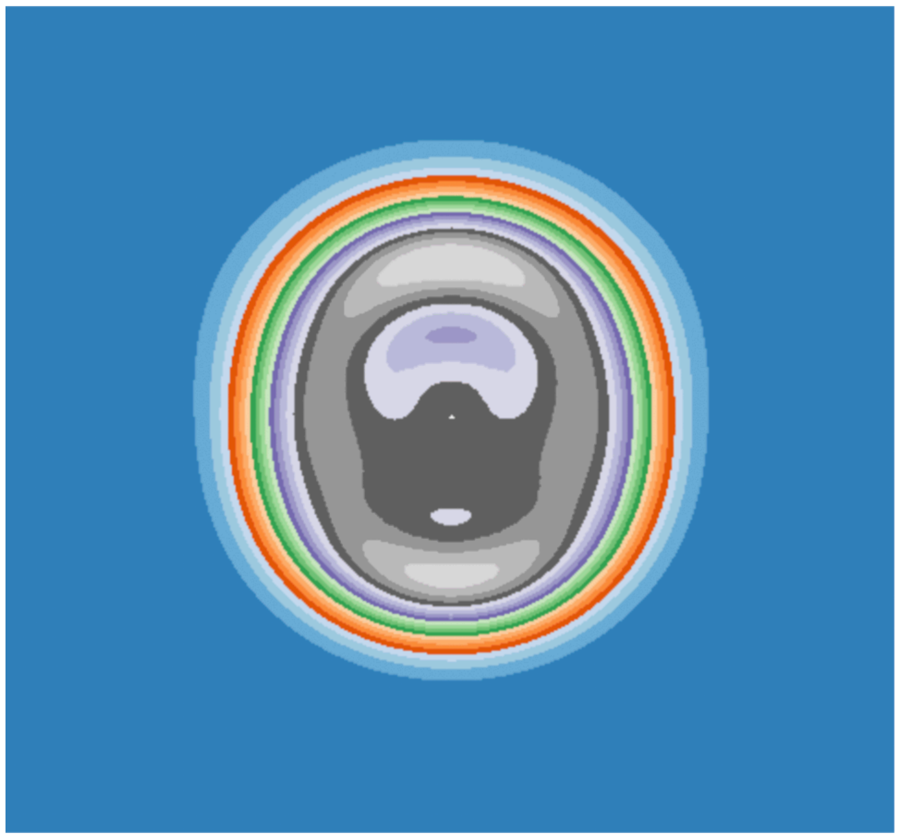
Couples exclusively to protons

Explicit transitions between excited states in REOM^3 : Dipole transitions $2^+ \Leftarrow\Rightarrow 3^-$ and $2^+ \Leftarrow\Rightarrow 2^-$



^{80}Ge

$2^+ \Rightarrow 3^-$



$$\mathcal{X}^n_{jk} = \langle 0|\alpha_k \alpha_j|n\rangle \qquad \mathcal{Y}^n_{jk} = \langle 0|\alpha^\dagger_j \alpha^\dagger_k|n\rangle$$

Partial amplitudes

$$|0\rangle \to |n\rangle \qquad |0\rangle \to |m\rangle$$

$$\mathcal{F}_{mn} = \langle m|\mathcal{F}|n\rangle = f_{ij}(\mathcal{X}^{m*}_{ik}\mathcal{X}^n_{jk} + \mathcal{Y}^{m*}_{jk}\mathcal{Y}^n_{ik})$$

Recoupling via 6j-symbol

Nuclei at the limits of existence: Finite-temperature response with the ph+phonon dynamical kernel

$$R_{12,1'2'}(t-t') = -i\langle \mathcal{T}(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle \rightarrow -i\langle \mathcal{T}(\bar\psi_1\psi_2)(t)(\bar\psi_{2'}\psi_{1'})(t')\rangle_T$$

$$< ... > \equiv < 0|...|0 > \quad \rightarrow \quad < ... >_T \equiv \sum_n exp\Bigl(\frac{\Omega - E_n - \mu N}{T}\Bigr) < n|...|n >$$

averages → *thermal averages*

Method: EOM for Matsubara Green's functions

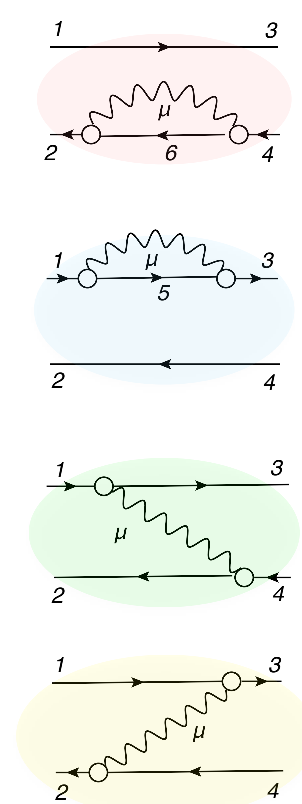
E.L., H. Wibowo, PRL 121, 082501 (2018)
H. Wibowo, E.L., PRC 100, 024307 (2019)

$$\mathcal{R}_{14,23}(\omega,T) = \tilde{\mathcal{R}}^0_{14,23}(\omega,T) + + \sum_{1'2'3'4'} \tilde{\mathcal{R}}^0_{12',21'}(\omega,T)\bigl[\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega,T)\bigr]\mathcal{R}_{3'4,4'3}(\omega,T)$$

$$\delta\Phi_{1'4',2'3'}(\omega,T) = \Phi_{1'4',2'3'}(\omega,T) - \Phi_{1'4',2'3'}(0,T)$$

T > 0: *Leading-order 1p1h+phonon dynamical kernel:* *T = 0:*

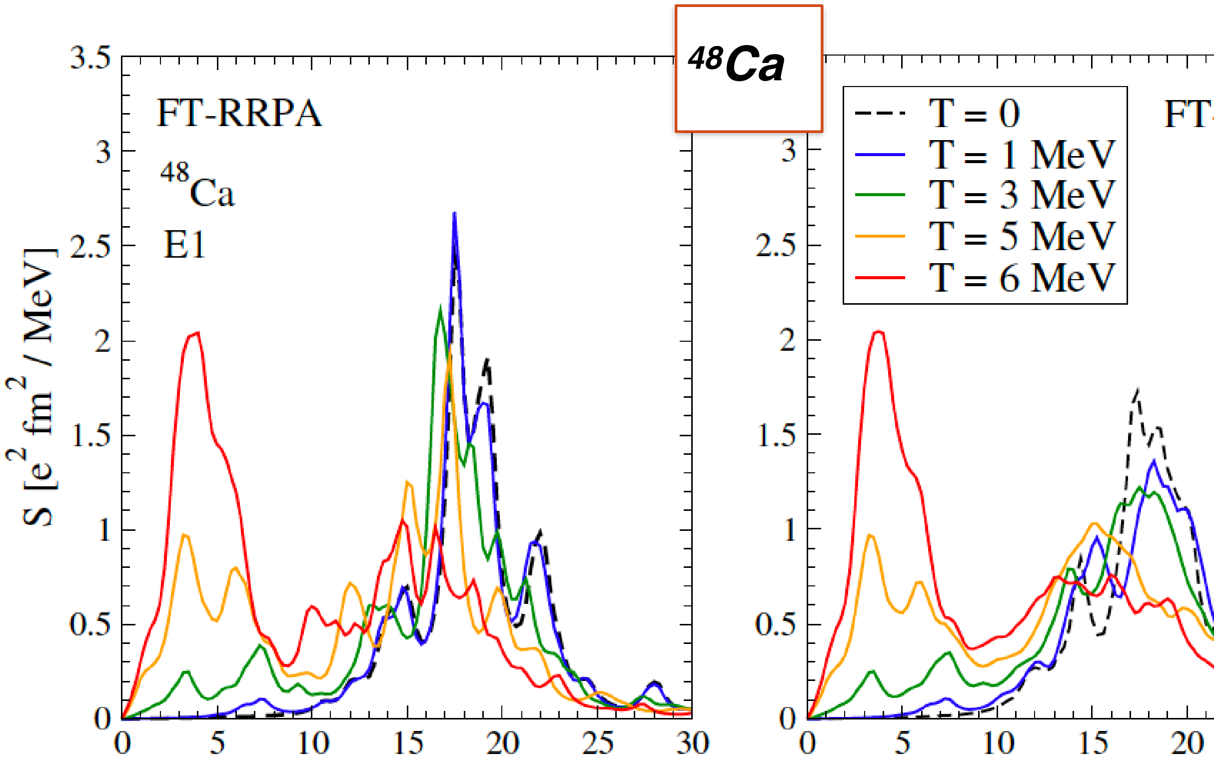
$$\Phi^{(ph)}_{14,23}(\omega,T) = \frac{1}{n_{43}(T)} \sum_{\mu,\eta_\mu=\pm1} \eta_\mu \Bigl[\delta_{13}\sum_6 \gamma^{\eta_\mu}_{\mu;62}\gamma^{\eta_\mu *}_{\mu;64} \times \times \frac{\bigl(N(\eta_\mu\Omega_\mu)+n_6(T)\bigr)\bigl(n(\varepsilon_6-\eta_\mu\Omega_\mu,T)-n_1(T)\bigr)}{\omega-\varepsilon_1+\varepsilon_6-\eta_\mu\Omega_\mu} + + \delta_{24}\sum_5 \gamma^{\eta_\mu}_{\mu;15}\gamma^{\eta_\mu *}_{\mu;35} \times \times \frac{\bigl(N(\eta_\mu\Omega_\mu)+n_2(T)\bigr)\bigl(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_5(T)\bigr)}{\omega-\varepsilon_5+\varepsilon_2-\eta_\mu\Omega_\mu} - - \gamma^{\eta_\mu}_{\mu;13}\gamma^{\eta_\mu *}_{\mu;24} \times \times \frac{\bigl(N(\eta_\mu\Omega_\mu)+n_2(T)\bigr)\bigl(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_3(T)\bigr)}{\omega-\varepsilon_3+\varepsilon_2-\eta_\mu\Omega_\mu} - - \gamma^{\eta_\mu *}_{\mu;31}\gamma^{\eta_\mu}_{\mu;42} \times \times \frac{\bigl(N(\eta_\mu\Omega_\mu)+n_4(T)\bigr)\bigl(n(\varepsilon_4-\eta_\mu\Omega_\mu,T)-n_1(T)\bigr)}{\omega-\varepsilon_1+\varepsilon_4-\eta_\mu\Omega_\mu}\Bigr],$$

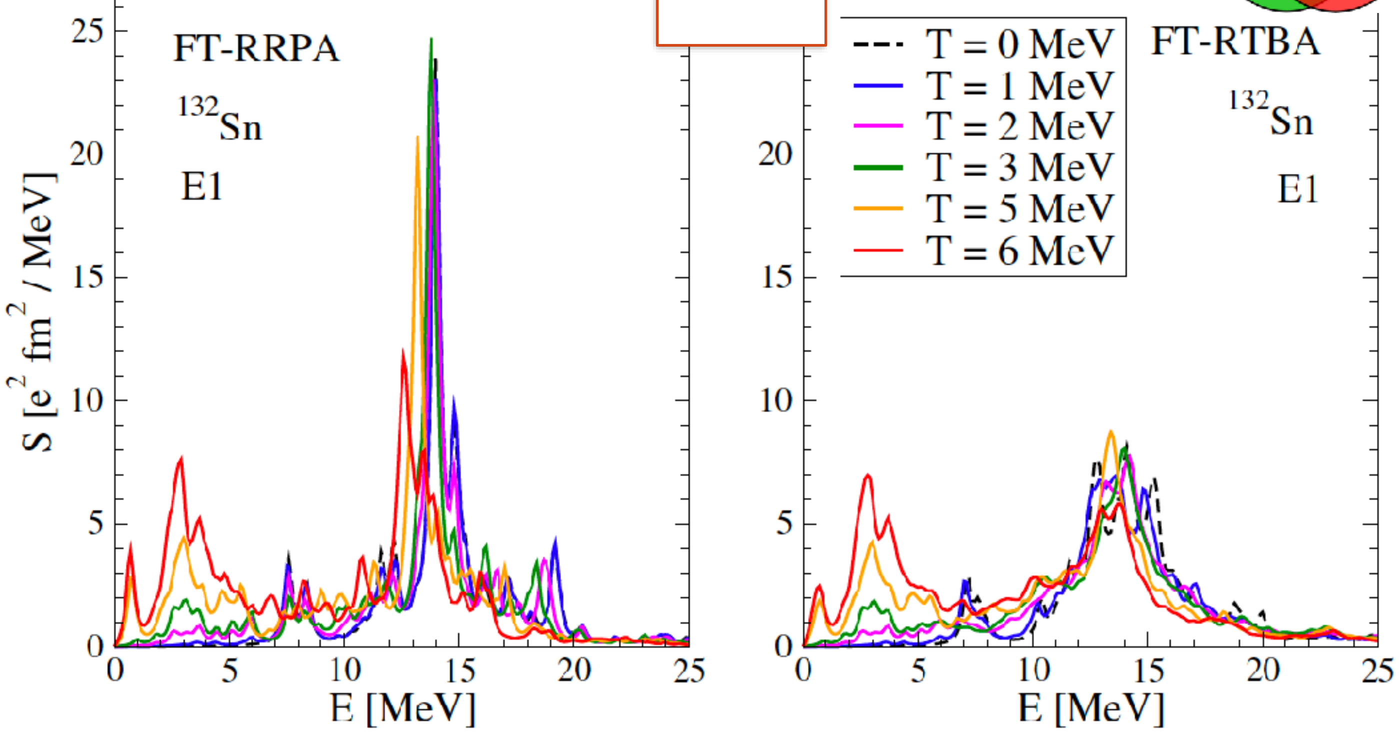


$$\Phi^{(ph,ph)}_{14,23}(\omega) = \sum_\mu \times \times\Bigl[\delta_{13}\sum_6 \frac{\gamma^{\mu}_{62}\gamma^{\mu *}_{64}}{\omega-\varepsilon_1+\varepsilon_6-\Omega_\mu} + +\delta_{24}\sum_5 \frac{\gamma^{\mu}_{15}\gamma^{\mu *}_{35}}{\omega-\varepsilon_5+\varepsilon_2-\Omega_\mu} - - \frac{\gamma^{\mu}_{13}\gamma^{\mu *}_{24}}{\omega-\varepsilon_3+\varepsilon_2-\Omega_\mu} - - \frac{\gamma^{\mu *}_{31}\gamma^{\mu}_{42}}{\omega-\varepsilon_1+\varepsilon_4-\Omega_\mu}\Bigr]$$

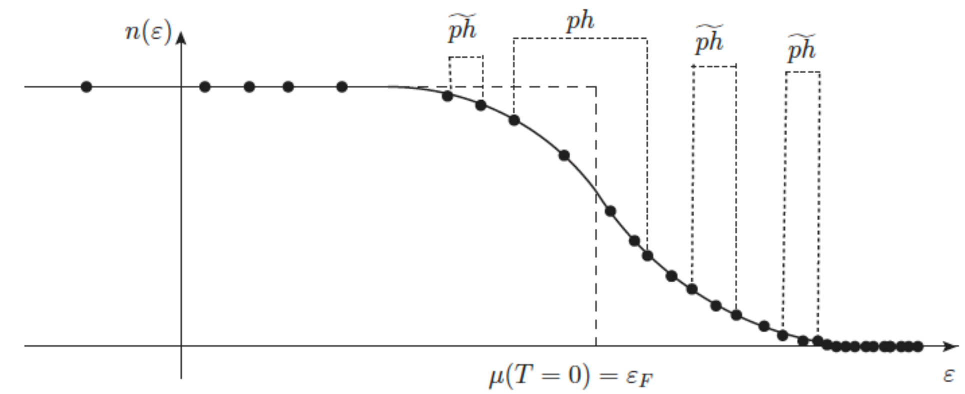
The GDR collectivity puzzle: Dipole strength at finite temperature (T>0): ^{48}Ca and ^{132}Sn

Static only (FT-REOM1) **Static + dynamic (FT-REOM2)**



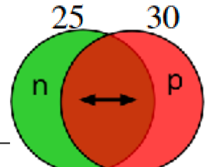


Thermal unblocking mechanism (simplified):



0th approximation:
Uncorrelated propagator

$$\tilde{R}^0_{14,23}(\omega) = \delta_{13}\delta_{24}\frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$



- *New transitions due to the thermal unblocking effects*
- *More collective and non-collective modes contribute to the PVC self-energy (~400 modes at T=5-6 MeV)*
- *Broadening of the resulting GDR spectrum*
- *Development of the low-energy part => a feedback to GDR*

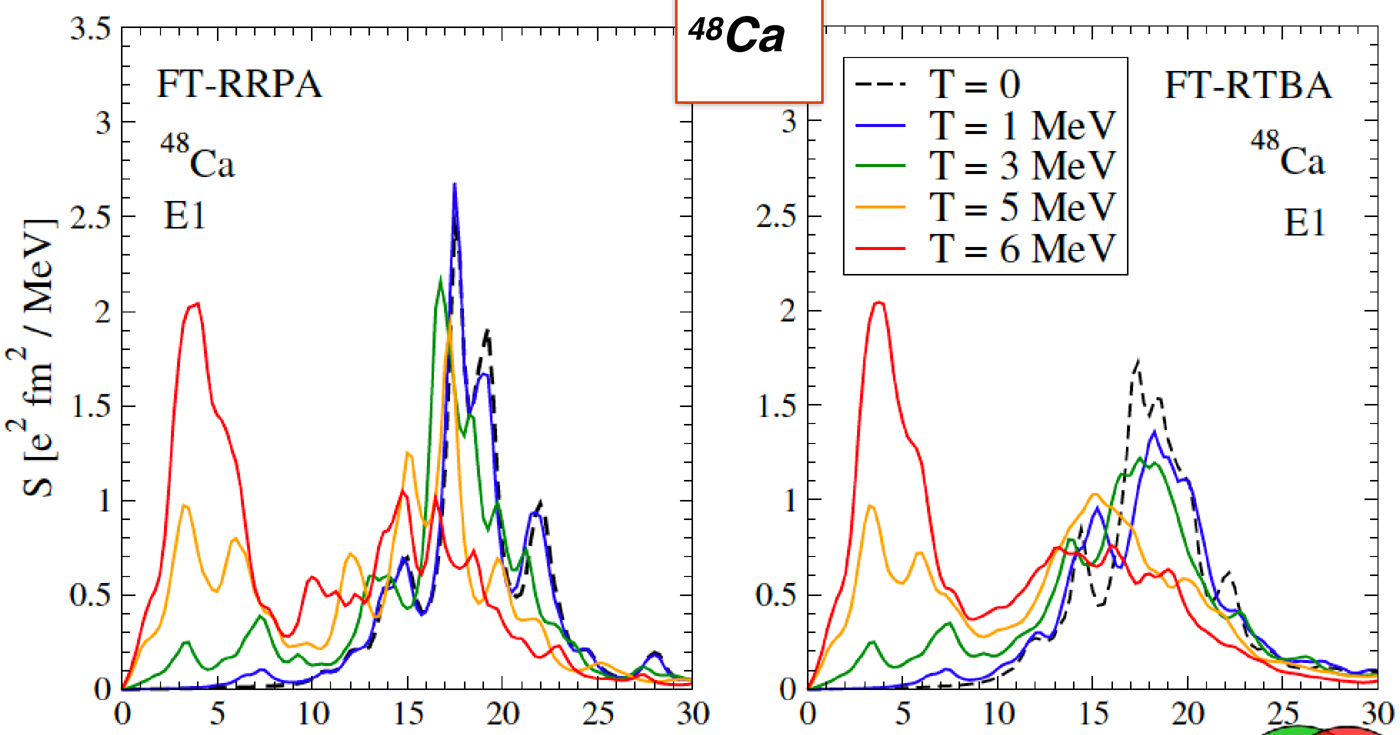
- *The spurious translation mode is properly decoupled as the mean field is modified consistently*
- *The role of the new terms in the Φ amplitude increases with temperature*
- *The role of dynamical correlations and fragmentation remain significant in the high-energy part*

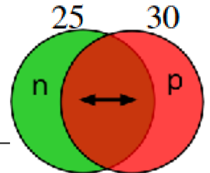
E.L., H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018)
H. Wibowo, E.L., Phys. Rev. C 100, 024307 (2019)

The GDR collectivity puzzle: Dipole Strength at T>0: ^{48}Ca and ^{132}Sn

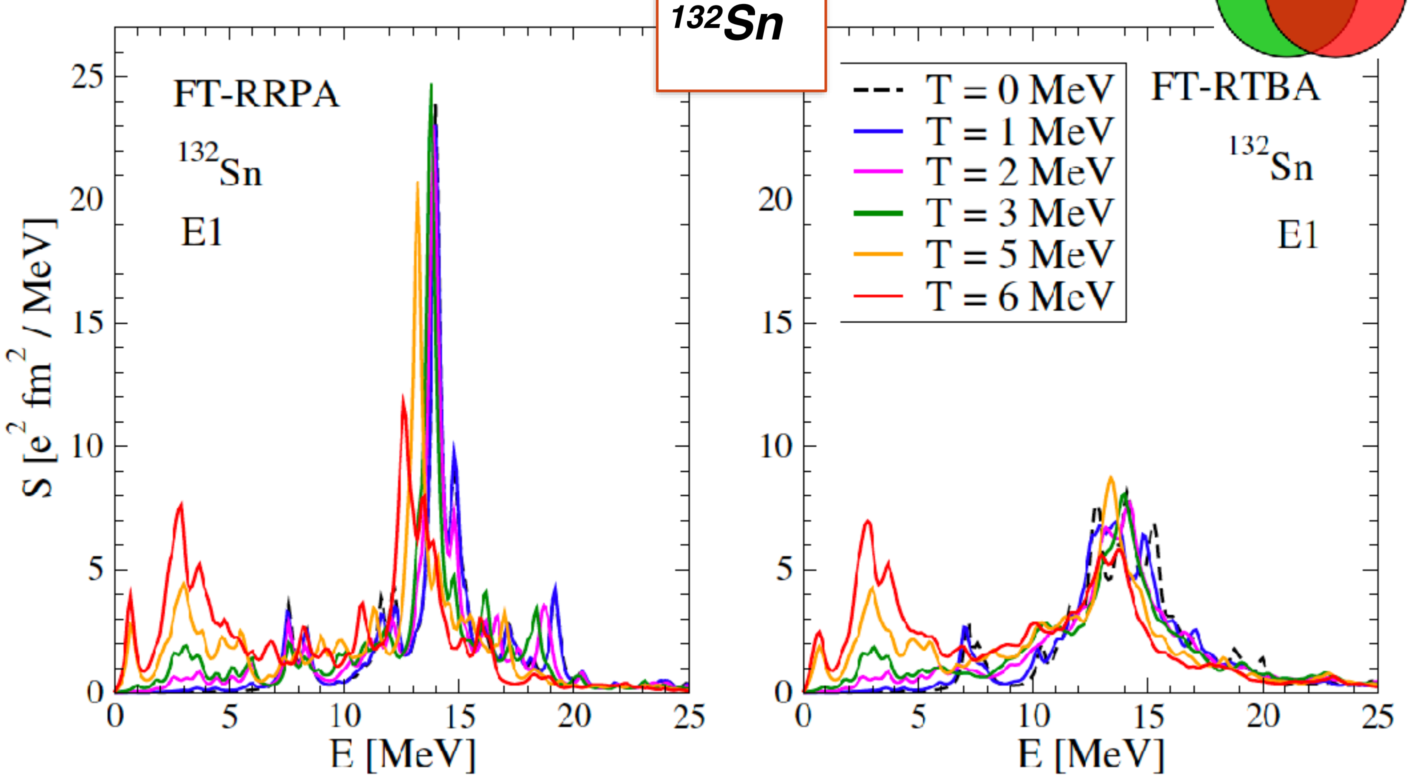
Static only (FT-REOM1) | **Static + dynamic (FT-REOM2)**

^{48}Ca

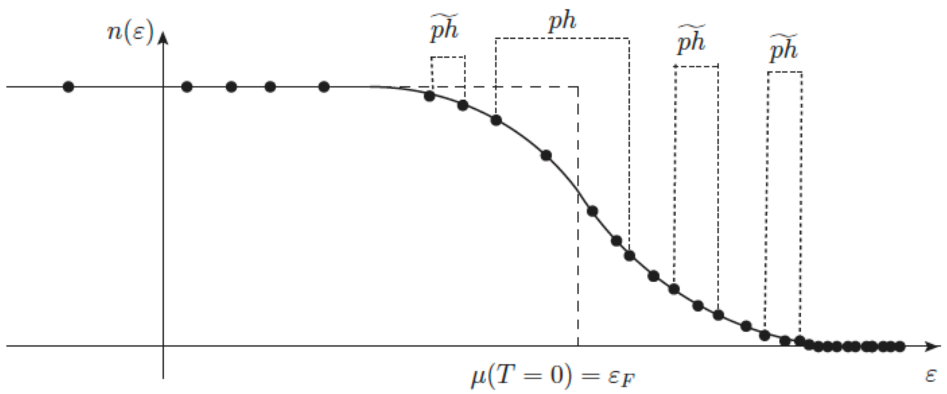




^{132}Sn



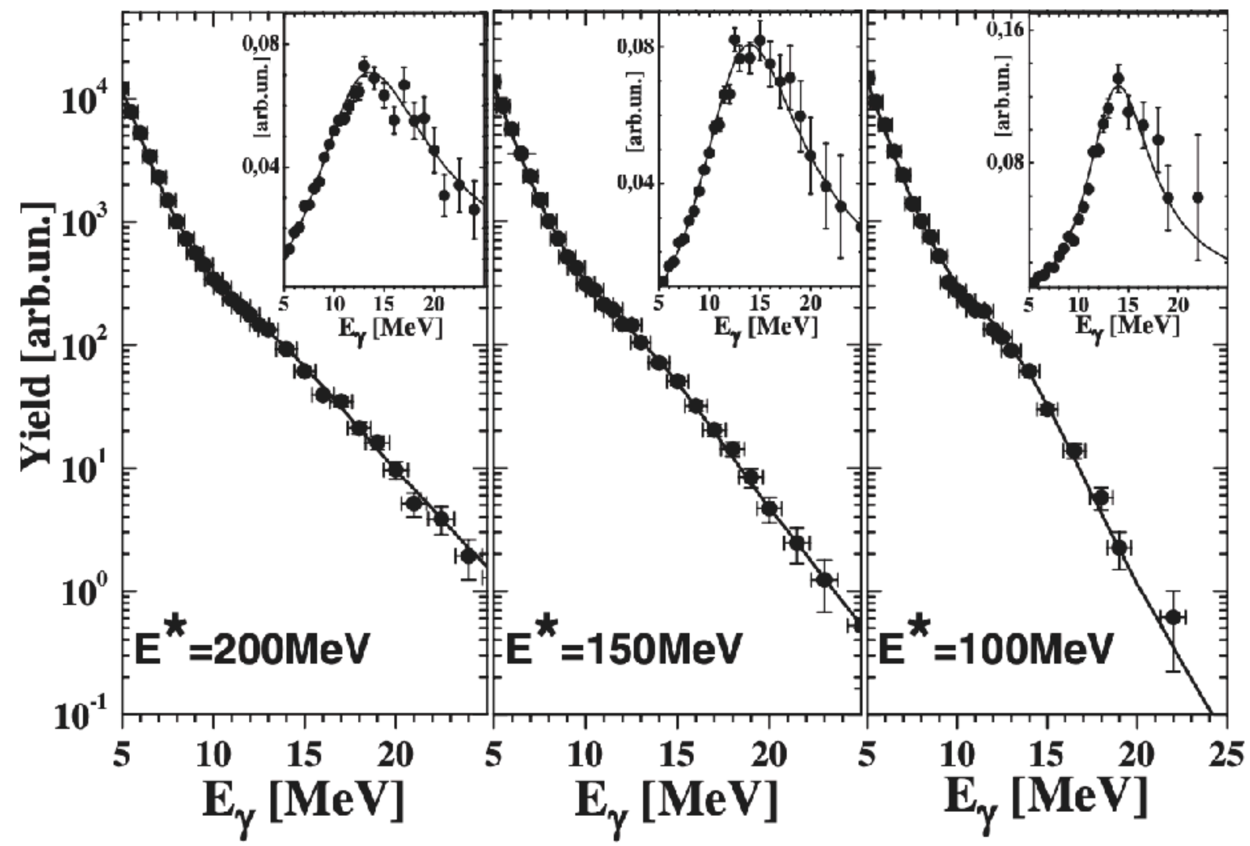
Thermal unblocking:



0th approximation:
Uncorrelated propagator

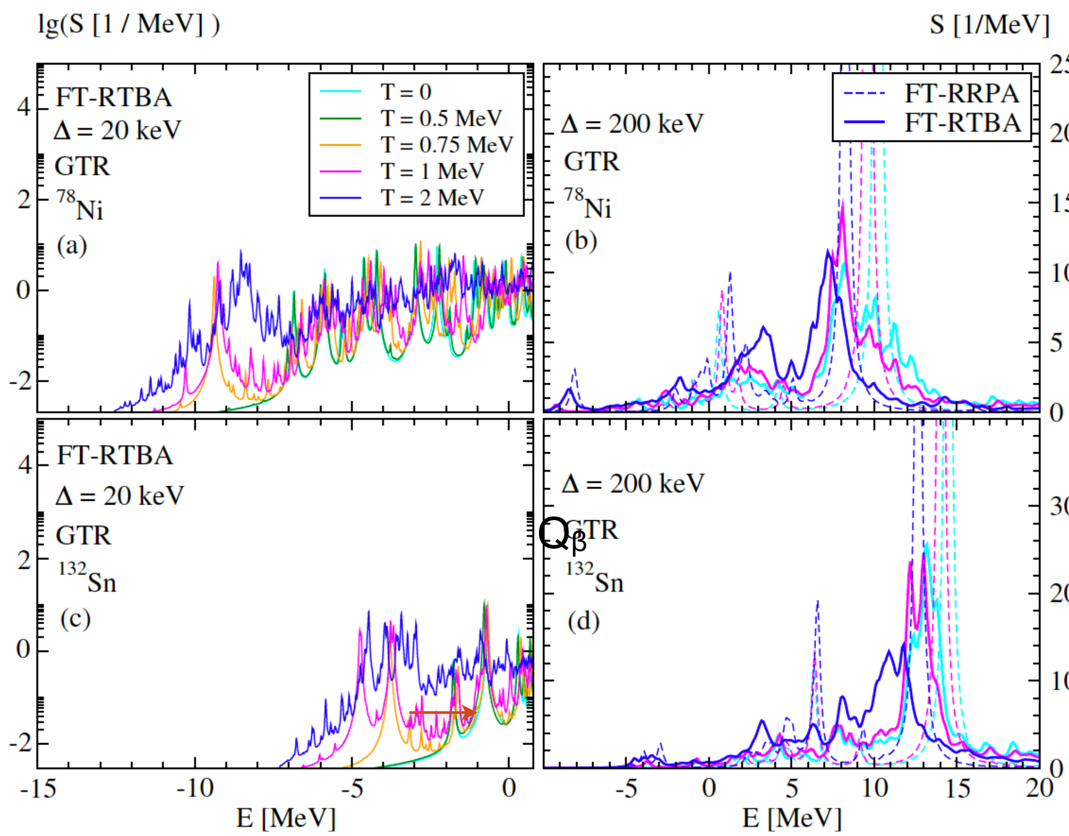
$$\tilde{R}^0_{14,23}(\omega) = \delta_{13}\delta_{24}\frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

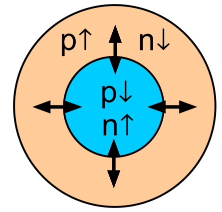
*O. Wieland et al., PRL **97,** 012501 (2006):*
GDR in ^{132}Ce



Spin-Isospin response and beta decay in stellar environments (T>0)

Gamow-Teller GT^- response of ^{78}Ni and ^{132}Sn



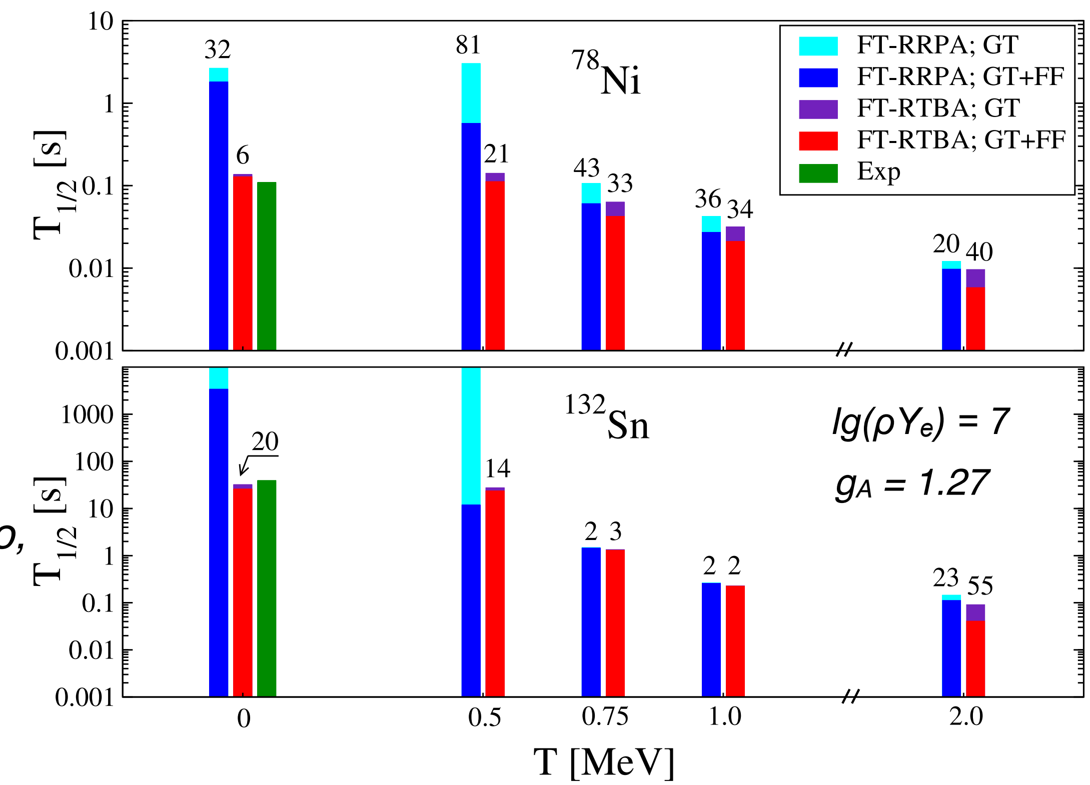


Thermal unblocking mechanism: similar but with proton-neutron pairs

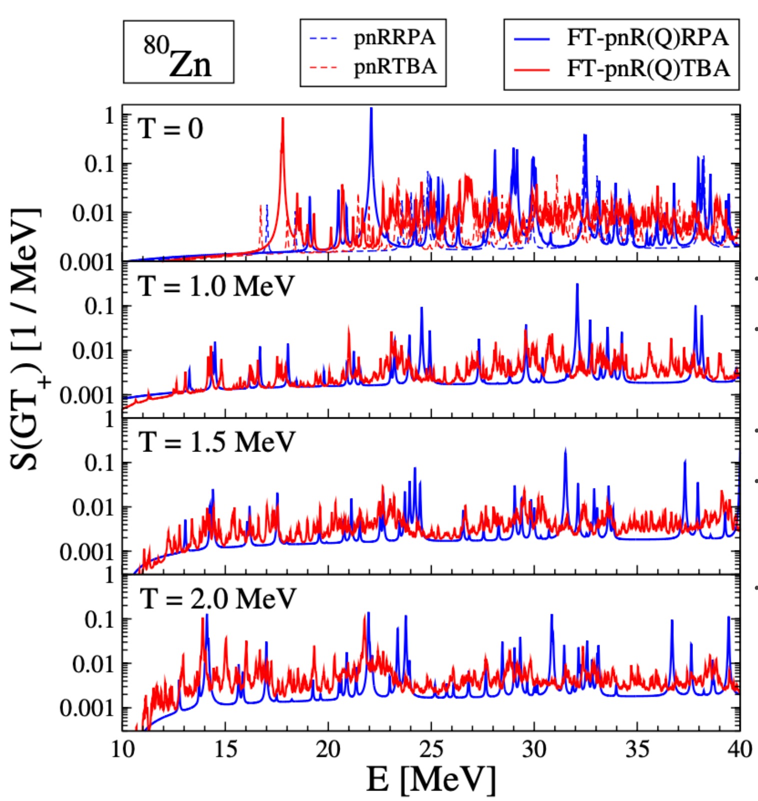
Beta decay in r-process at T > 0

E. L., C. Robin, H. Wibowo, PLB 800, 135134 (2020)

Beta decay half-lives in a stellar environment



GT+ response around ^{78}Ni

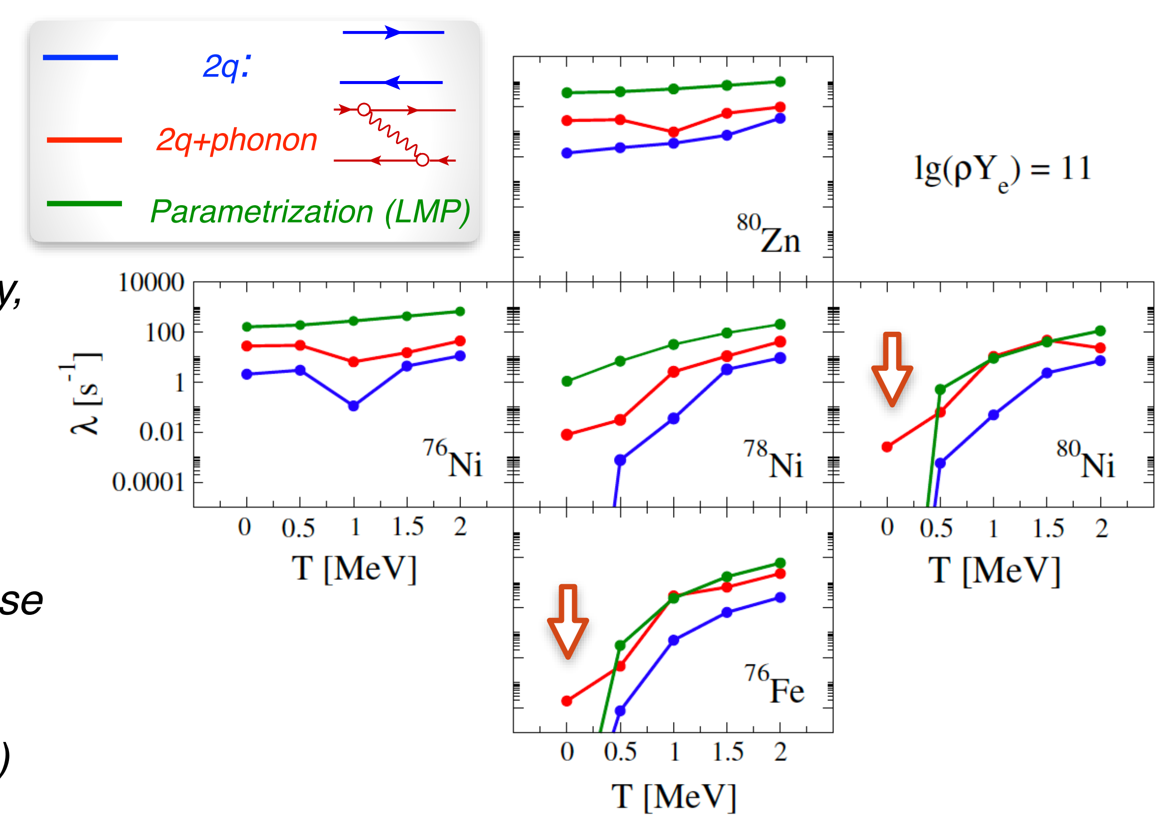


***Interplay** of superfluidity and collective effects in core-collapse supernovae:*

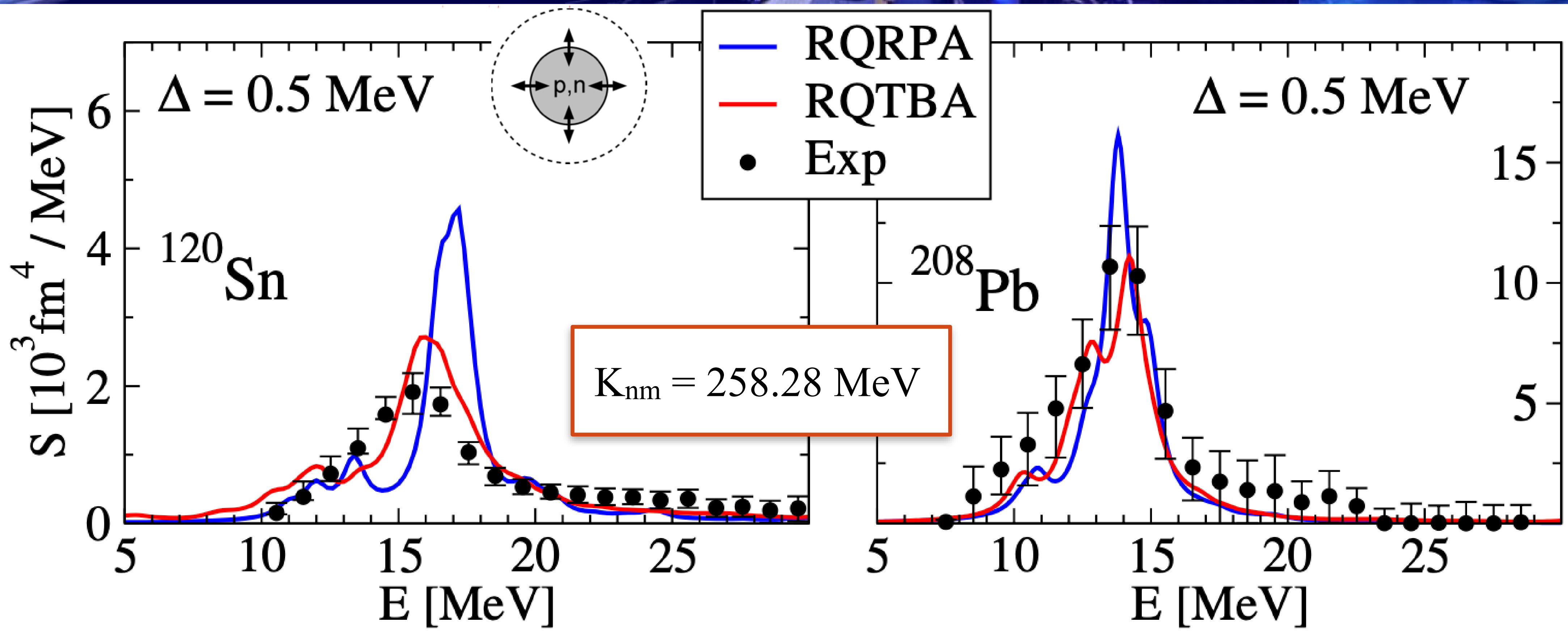
- *Amplifies the EC rates and, consequently,*
- *Reduces the electron-to-baryon ratio, leading to lower pressure*
- *Promotes the gravitational collapse*
- *Increases the neutrino flux and effective cooling*
- *Allows heavy nuclei to survive the collapse*

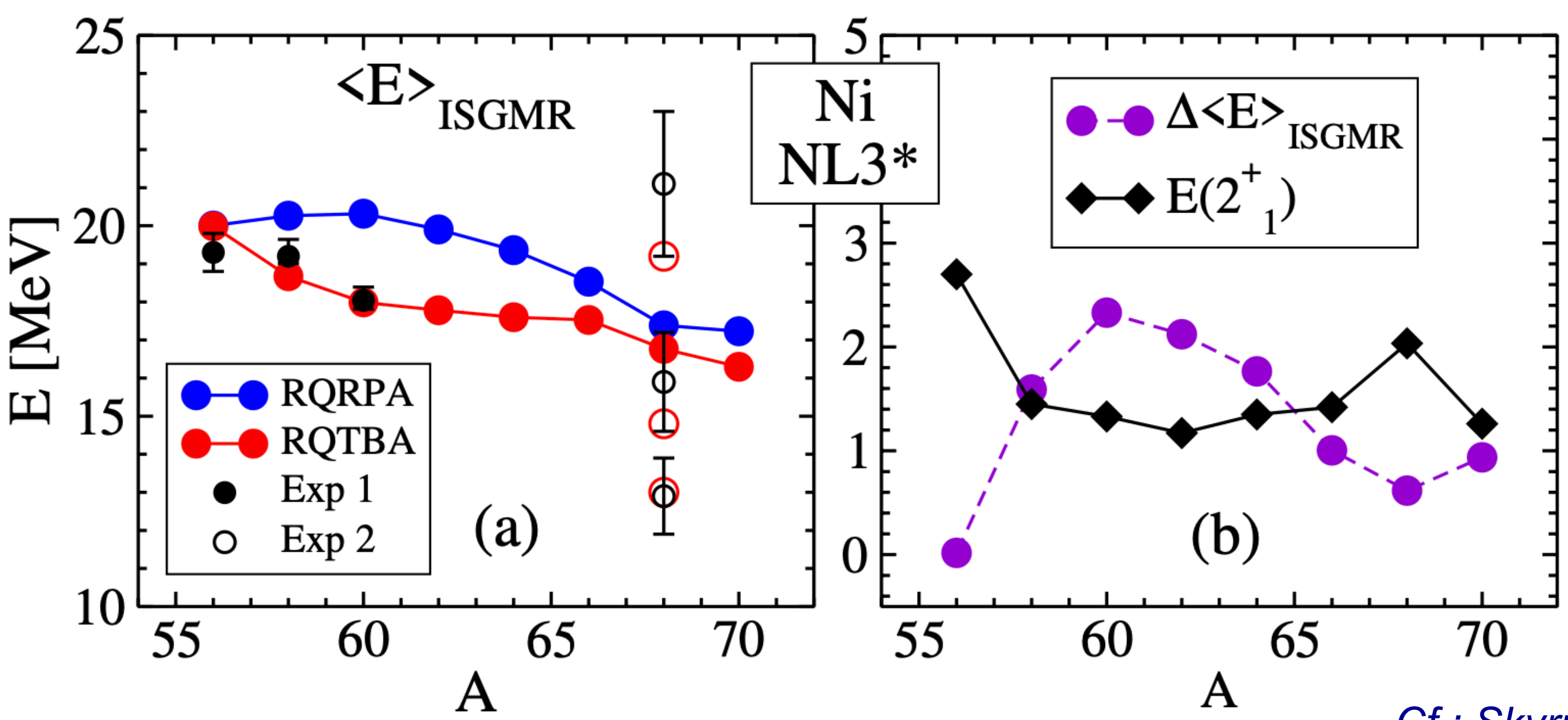
E.L., C. Robin, PRC 103, 024326 (2021)

Electron capture rates around ^{78}Ni



Isoscalar giant monopole resonance (ISGMR): The "fluffiness" puzzle





E.L., PRC **107**, L041302 (2023)

"Softness" increases:

- *with the neutron number*
- *with superfluidity*
- *with correlations beyond QRPA (q)PVC*

Stiffer EOS can be used

Cf.: Skyrme HFB+qPVC

*Z.Z. Li, Y. Niu, G. Colò, PRL **131**, 082501 (2023)*

Finite-temperature superfluid formalism (preliminary)

Theory is formulated in the HFB basis keeping 4x4 block matrix structure

S. Bhattacharjee, E.L., arXiv:2412.20751

Correlated 2q-propagator with Matsubara frequencies

$$\hat{\mathcal{R}}_{\mu\mu'\nu\nu'}(\omega_n) = \hat{\mathcal{R}}^0_{\mu\mu'\nu\nu'}(\omega_n) + \\ + \frac{1}{4}\sum_{\gamma\gamma'\delta\delta'} \hat{\mathcal{R}}^0_{\mu\mu'\gamma\gamma'}(\omega_n)\hat{\mathcal{K}}_{\gamma\gamma'\delta\delta'}(\omega_n)\hat{\mathcal{R}}_{\delta\delta'\nu\nu'}(\omega_n)$$

Uncorrelated 2q-propagator

$$\hat{\mathcal{R}}^0_{\mu\mu'\nu\nu'}(\omega_n) = [\omega_n - \hat{\Sigma}_3\hat{E}_{\mu\mu'}]^{-1}\hat{\mathcal{N}}_{\mu\mu'\nu\nu'}$$

The norm matrix

$$\hat{\mathcal{N}}_{\mu\mu'\nu\nu'} = \\ \left\langle \begin{pmatrix} [A_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [A_{\mu\mu'}, A_{\nu\nu'}] & [A_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [A_{\mu\mu'}, C_{\nu\nu'}] \\ [A^{\dagger}_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [A^{\dagger}_{\mu\mu'}, A_{\nu\nu'}] & [A^{\dagger}_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [A^{\dagger}_{\mu\mu'}, C_{\nu\nu'}] \\ [C_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [C_{\mu\mu'}, A_{\nu\nu'}] & [C_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [C_{\mu\mu'}, C_{\nu\nu'}] \\ [C^{\dagger}_{\mu\mu'}, A^{\dagger}_{\nu\nu'}] & [C^{\dagger}_{\mu\mu'}, A_{\nu\nu'}] & [C^{\dagger}_{\mu\mu'}, C^{\dagger}_{\nu\nu'}] & [C^{\dagger}_{\mu\mu'}, C_{\nu\nu'}] \end{pmatrix} \right\rangle$$

$$\langle O \rangle = \sum_i p_i \langle i|O|i\rangle \qquad p_i = \frac{e^{-E_i/T}}{\sum_j e^{-E_j/T}}$$

$$\mathcal{R}^{[lk]}_{\mu\mu'\nu\nu'}(\omega) = \sum_{fi}\sum_{\sigma=\pm} \sigma p_i \frac{\mathcal{Z}^{if[l\sigma]}_{\mu\mu'}\mathcal{Z}^{if[k\sigma]*}_{\nu\nu'}}{\omega - \sigma\omega_{fi}}$$

$$\mathcal{Z}^{if[1+]}_{\mu\mu'} = \mathcal{X}^{if}_{\mu\mu'} \qquad \mathcal{Z}^{if[2+]}_{\mu\mu'} = \mathcal{Y}^{if}_{\mu\mu'}$$
$$\mathcal{Z}^{if[3+]}_{\mu\mu'} = \mathcal{U}^{if}_{\mu\mu'} \qquad \mathcal{Z}^{if[4+]}_{\mu\mu'} = \mathcal{V}^{if}_{\mu\mu'}$$
$$\mathcal{Z}^{if[1-]}_{\mu\mu'} = \mathcal{Y}^{if*}_{\mu\mu'} \qquad \mathcal{Z}^{if[2-]}_{\mu\mu'} = \mathcal{X}^{if*}_{\mu\mu'}$$
$$\mathcal{Z}^{if[3-]}_{\mu\mu'} = \mathcal{V}^{if*}_{\mu\mu'} \qquad \mathcal{Z}^{if[4-]}_{\mu\mu'} = \mathcal{U}^{if*}_{\mu\mu'}$$

$$\mathcal{X}^{if}_{\mu\mu'} = \langle i|\alpha_{\mu'}\alpha_{\mu}|f\rangle \qquad \mathcal{Y}^{if}_{\mu\mu'} = \langle i|\alpha^{\dagger}_{\mu}\alpha^{\dagger}_{\mu'}|f\rangle$$
$$\mathcal{U}^{if}_{\mu\mu'} = \langle i|\alpha^{\dagger}_{\mu}\alpha_{\mu'}|f\rangle \qquad \mathcal{V}^{if}_{\mu\mu'} = \langle i|\alpha^{\dagger}_{\mu'}\alpha_{\mu}|f\rangle$$

The dynamical kernel in the factorized form:

$$\mathcal{K}^{r[11]cc(a)}_{\mu\mu'\nu\nu'}(\omega_k) = \sum_{\gamma\delta nm} p_{i_n}p_{i_m}\left(1 - e^{-(\omega_n+\omega_m)/T}\right) \\ \times \Big[\frac{(\Gamma^{(11)n}_{\mu\gamma}\mathcal{X}^{m}_{\mu'\gamma} + \Gamma^{(02)\bar{n}*}_{\mu\gamma}\mathcal{U}^{m}_{\mu'\gamma})(\mathcal{X}^{m*}_{\nu'\delta}\Gamma^{(11)n*}_{\nu\delta} + \mathcal{U}^{\bar{m}*}_{\nu'\delta}\Gamma^{(02)\bar{n}}_{\nu\delta})}{\omega_k - \omega_n - \omega_m} \\ - \frac{(\Gamma^{(11)n*}_{\gamma\mu}\mathcal{Y}^{m*}_{\mu'\gamma} + \Gamma^{(02)\bar{n}}_{\gamma\mu}\mathcal{V}^{\bar{m}*}_{\mu'\gamma})(\mathcal{Y}^{m}_{\nu'\delta}\Gamma^{(11)n}_{\delta\nu} + \mathcal{V}^{m}_{\nu'\delta}\Gamma^{(02)\bar{n}*}_{\delta\nu})}{\omega_k + \omega_n + \omega_m}\Big]$$

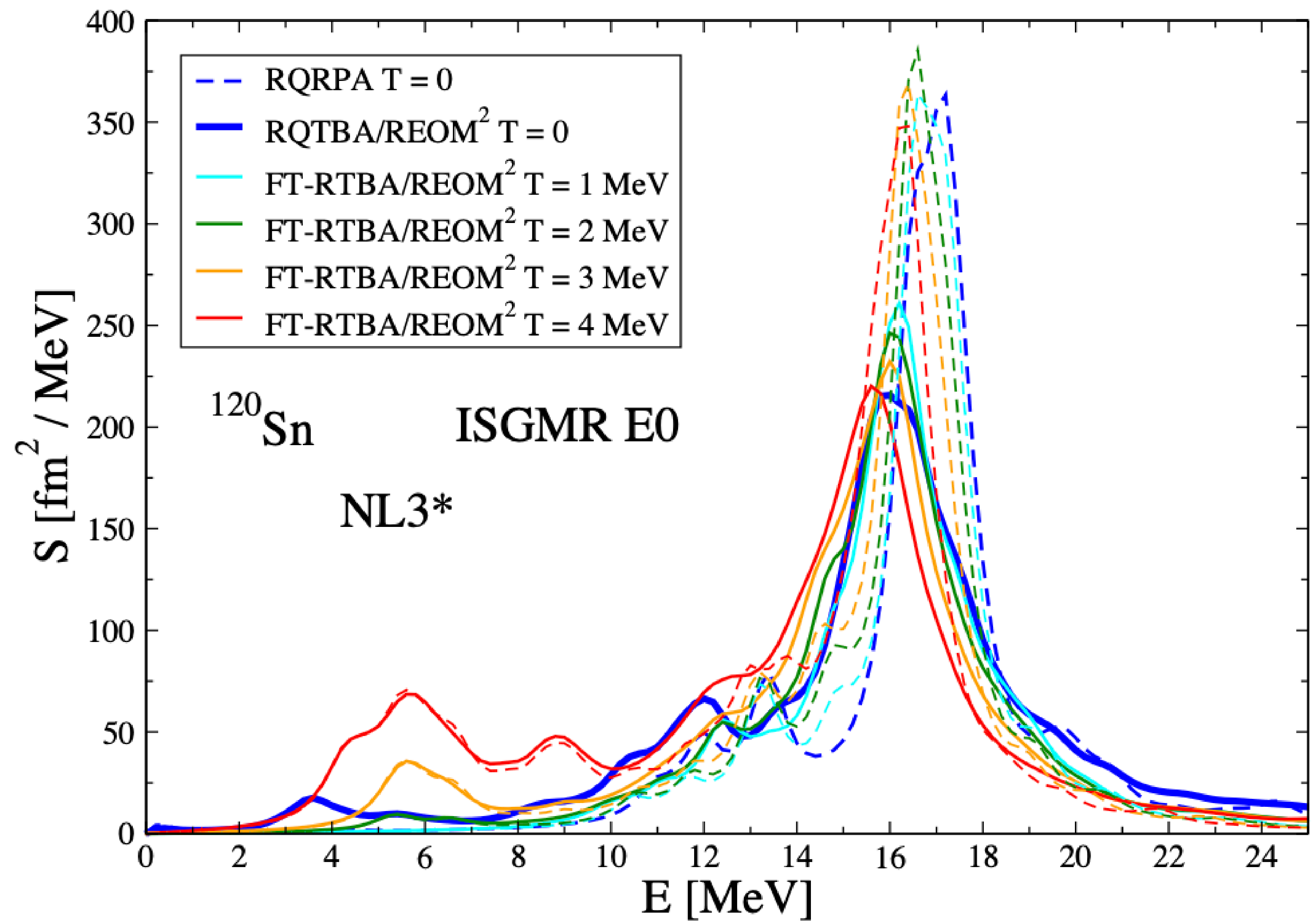
Temperature evolution of the "fluffiness" puzzle

$$\tilde{S}(E) = \frac{1}{1 - e^{-E/T}} S(E)$$

Not included here

Centroid

$$E_{\text{ISGMR}} = \sqrt{\frac{\hbar^2 A K_A}{m \langle r^2 \rangle_0}}$$



(In)compressibility:

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_\tau \left(\frac{N - Z}{A} \right)^2 + K_{Coul} Z^2 A^{-1/3}$$

Centroid shift:

ΔE = E(RQRPA) - E(REOM^2)

Decreases with temperature

U. Garg, G. Colò, Progress in Particle and Nuclear Physics 101 (2018) 55–95

Outlook

Summary:

- The emergent collective effects associated with the dynamical kernels of the fermionic EOMs renormalize interactions in correlated media, underly the spectral fragmentation mechanisms, affect superfluidity and weak decay rates.
- A hierarchy of converging growing-complexity approximations generates solutions of growing accuracy and quantify the uncertainties of the many-body theory.
- The recently enabled capabilities are higher configuration complexity, finite temperature, and quantum algorithms.

Open theoretical problems:

- Correct separation of and delicate relationship between the static and dynamical kernels in the practical approximate solutions.
- Artificial poles and virtual “particles” in approximate solutions.
- Consistency between direct and pairing channels in the dynamical kernels in practical implementations.
- Ambiguities of numerical implementations on the 2p2h and 3p3h levels of complexity.
- An adequate assessment of the quantitative role of fully connected multi-fermion correlators (~ irreducible three- and many-body “forces” in the medium).
- …

Many thanks

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D. Savran (GSI)
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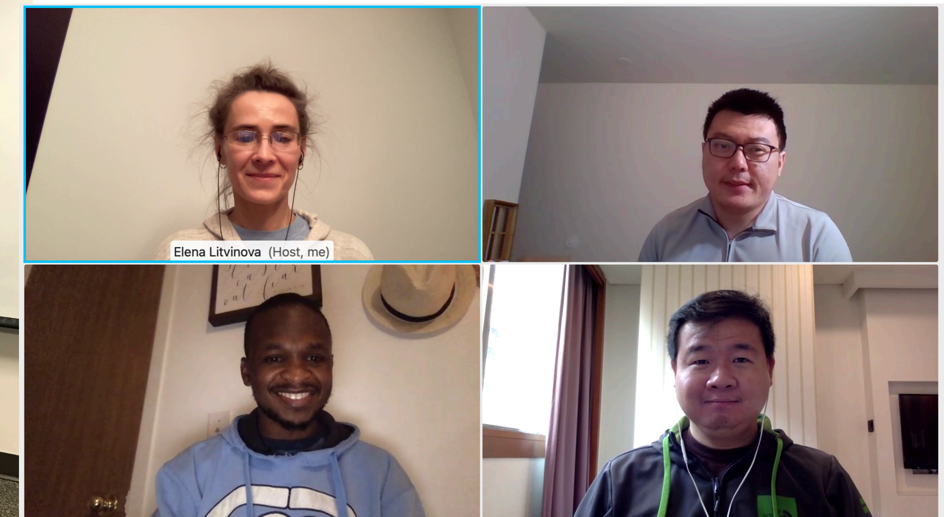
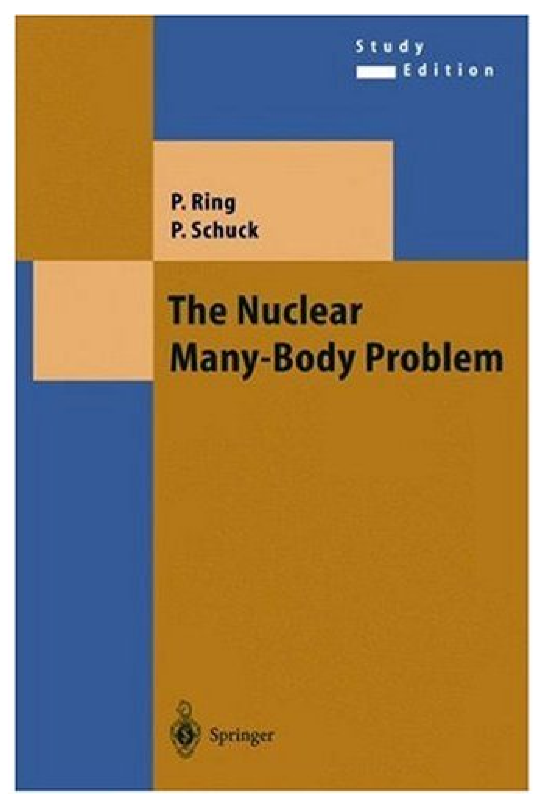
Kyle Morrisey (WMU)
Sumit Bhattacharjee (WMU)
Herlik Wibowo (U. York)
Caroline Robin (U. Bielefeld & GSI)



Peter Schuck (IPN Orsay)
Peter Ring (TU München)

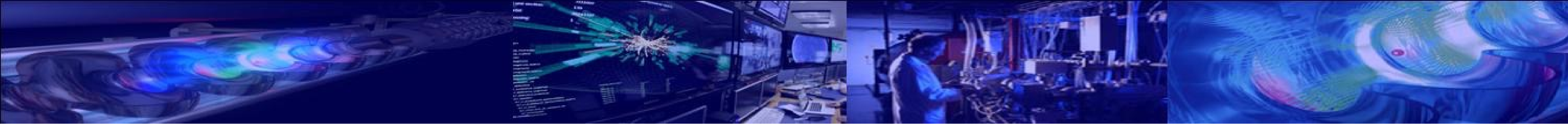
And beyond

US-NSF PHY-2209376 (2022-2025)

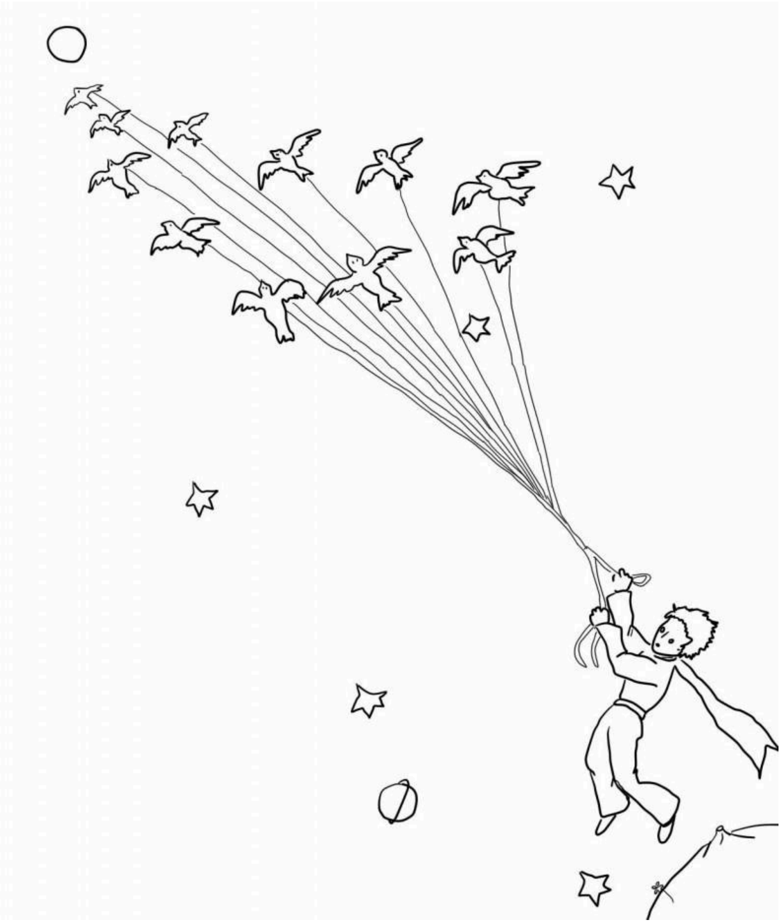


Kyle Morrisey

Sumit Bhattacharjee

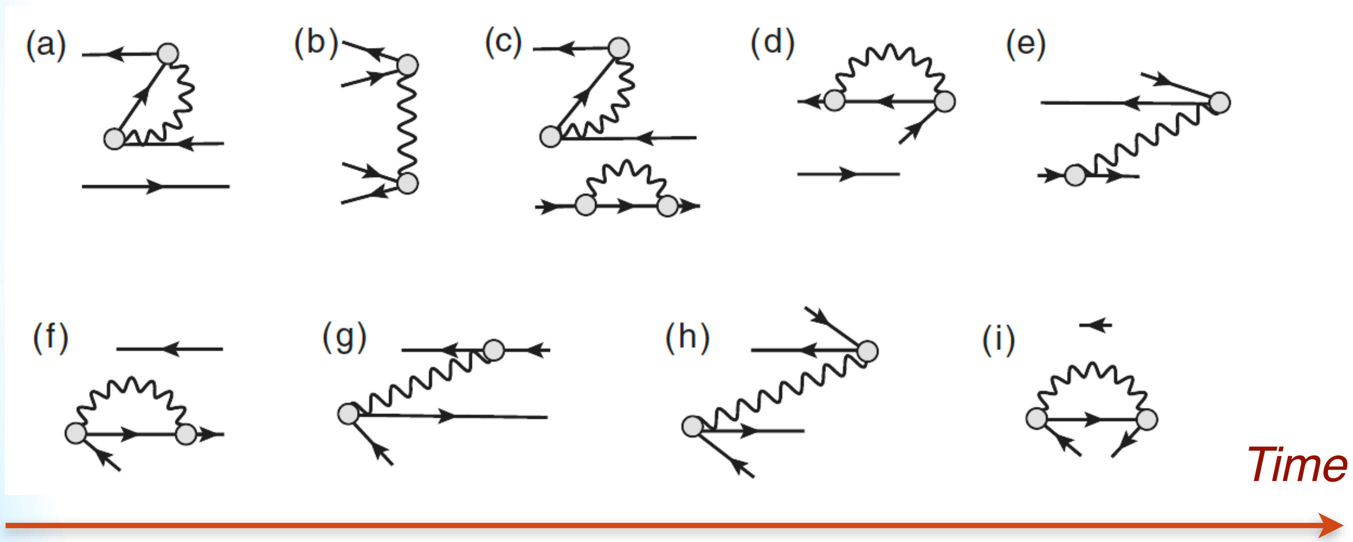


Thank you!

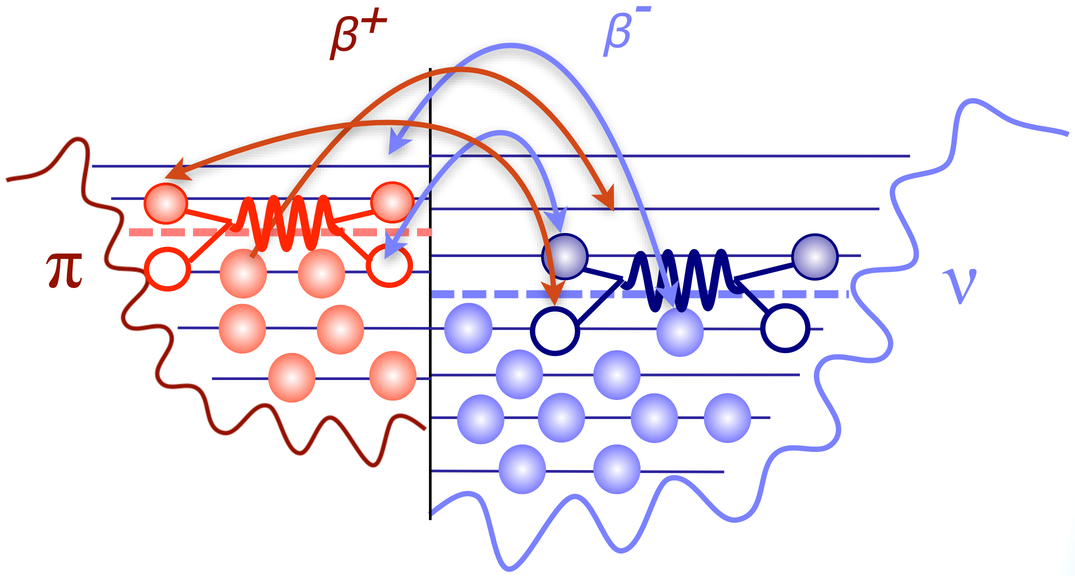


qPVC -induced ground state correlations: the leading mechanism of the β+ strength in neutron-rich nuclei; couple to two-body currents

Ground state correlations (GSC) induced by qPVC:
backward-going diagrams



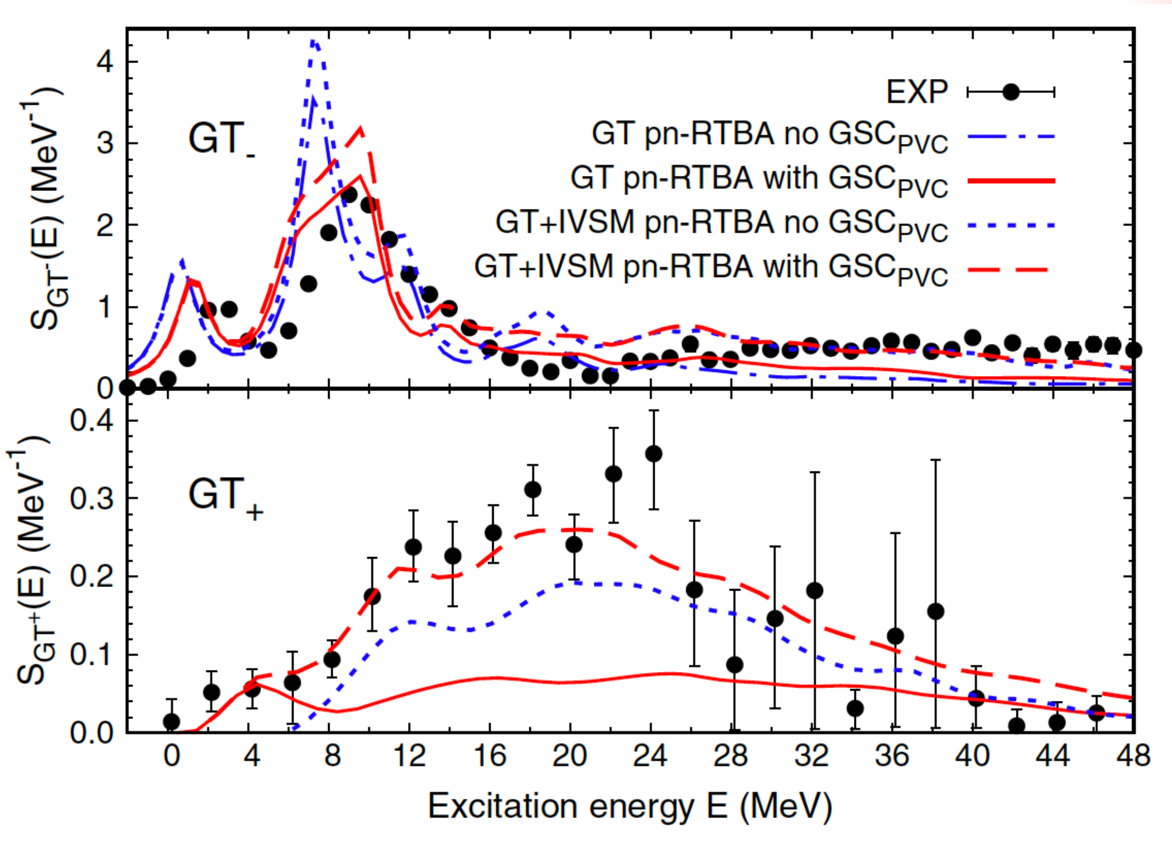
qPVC-GSC unblocking mechanism:



Emergent "time machine"

The backward-going diagrams are the leading mechanism of the β+ strength formation in neutron-rich nuclei

Gamow-Teller +IVSM strength in 90-Zr:



C. Robin, E.L., Phys. Rev. Lett. 123, 202501 (2019)