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Quantum-Enhanced Machine Learning for Classifying Quantum Phases of Matter

Mehran Khosrojerdi
mehran.khosrojerdi@unifi.it

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Learning to Classify Quantum Phases of Matter with a Few Measurements

arXiv: 2409.05188

Mehran Khosrojerdi, Jason L. Pereira, Alessandro Cuccoli, Leonardo Banchi



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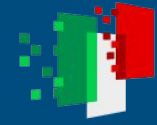
- Quantum Machine Learning
- Quantum Model: Quantum Support vector Machine (QSVM)
- Dataset: Tensor Networks
- Axial Next-Nearest-Neighbour Ising (ANNNI) example
- Haldane chain example



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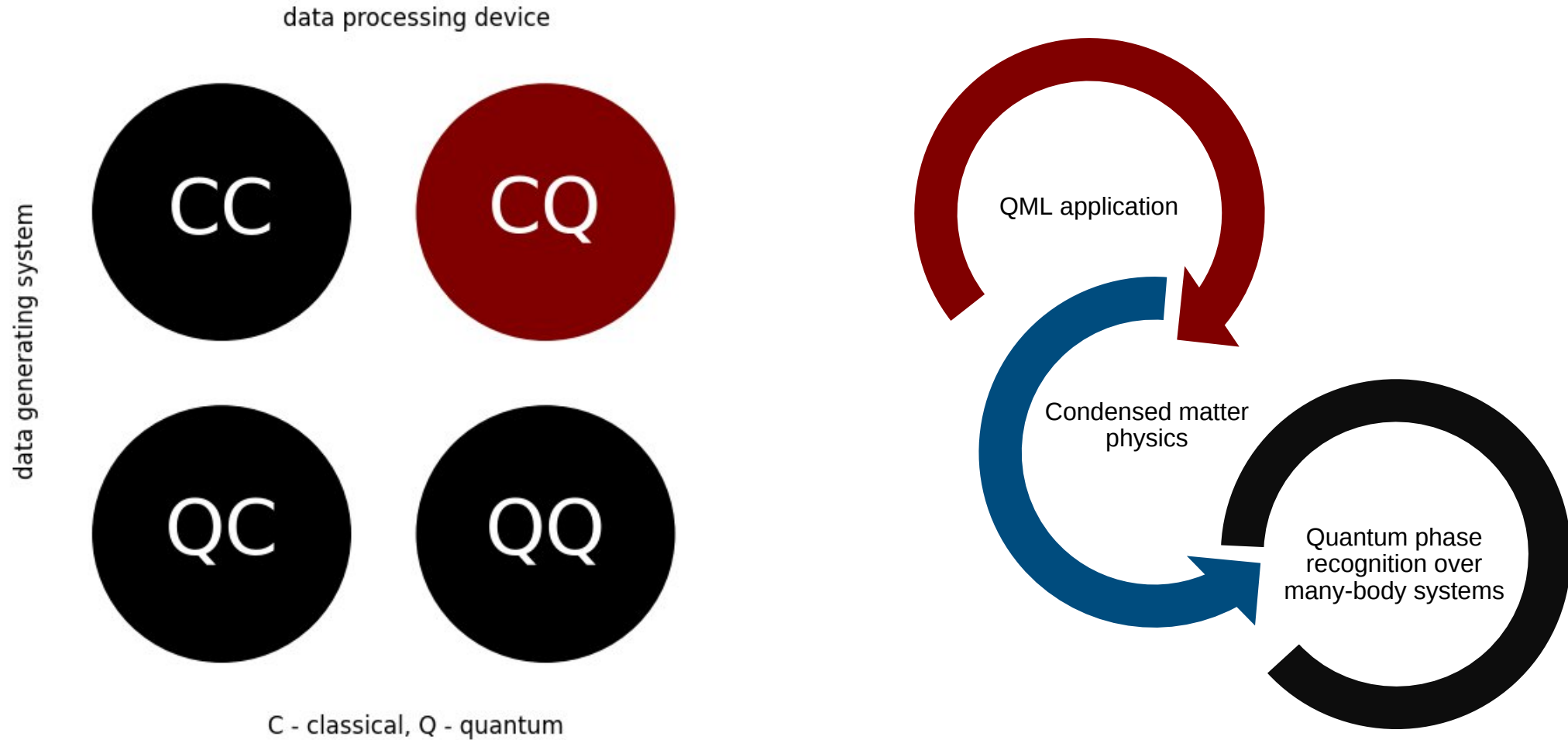
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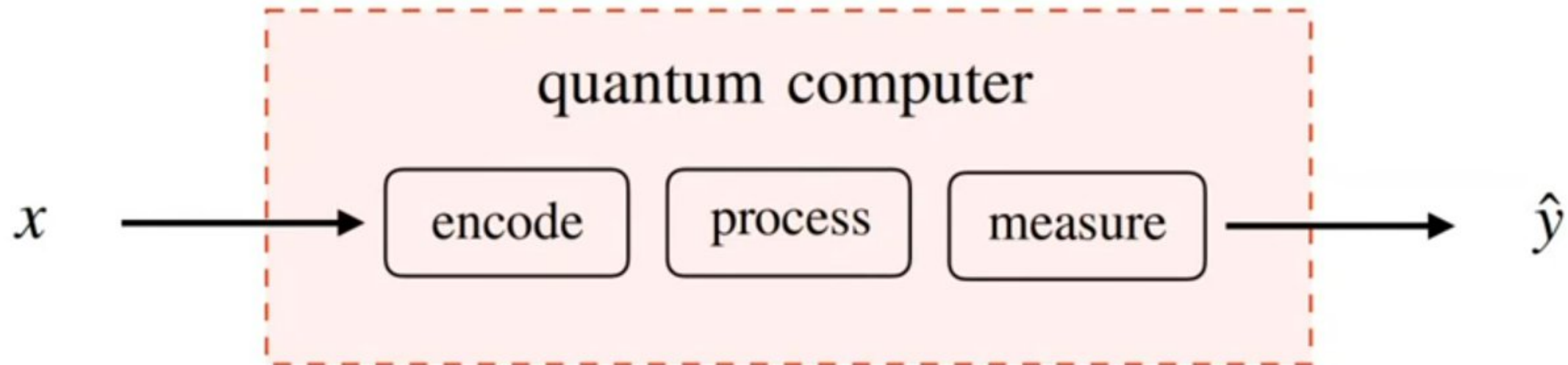
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The QML Flow
Credit: Maria Schuld



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$$y = \{+1, -1\}, \quad x \rightarrow \rho(x), \quad f(x) = \text{Tr} [W\rho(x)] + b = 0$$

$$W = \sum_{i=1}^M y_i \alpha_i \rho(x_i)$$

$$b = \sum_{i=1}^M \left(\frac{y_i - \sum_{j=1}^M y_j \alpha_j k(x_i, x_j)}{M} \right)$$

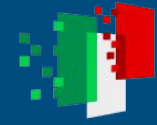
$$y_{\text{predicted}} = \text{sign} [f(x)] = \text{sign} \left[b + \sum_{i=1}^M y_i \alpha_i k(x_i, x) \right]$$



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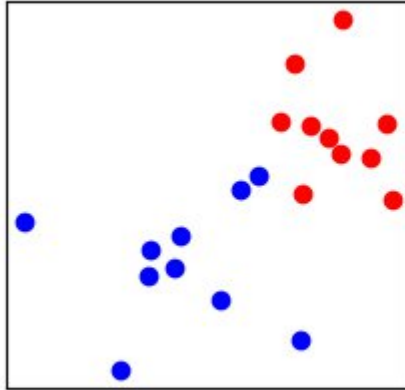


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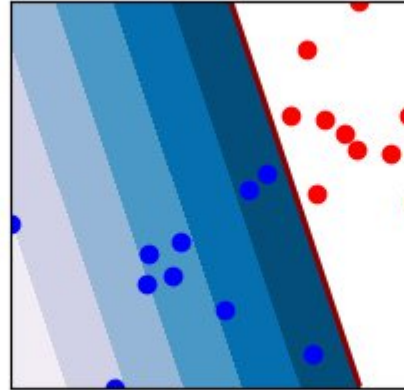


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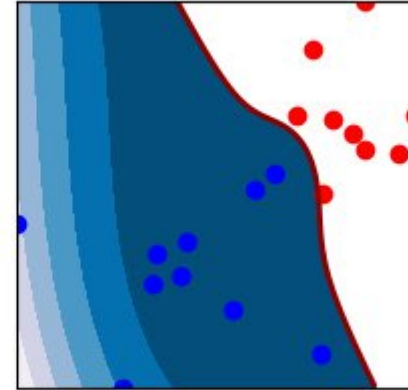
Before Applying Kernel



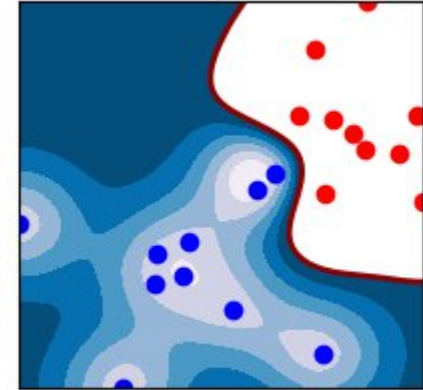
After Applying Linear Kernel



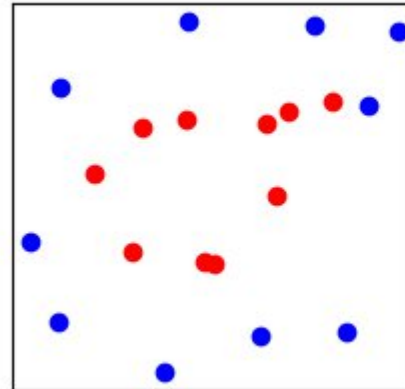
After Applying Polynomial Kernel



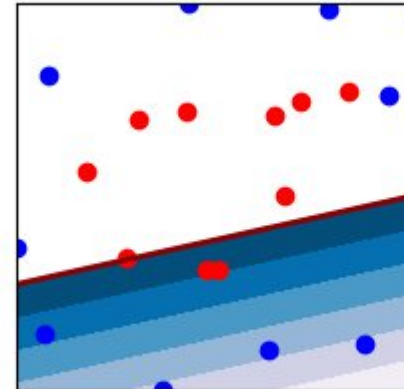
After Applying RBF Kernel



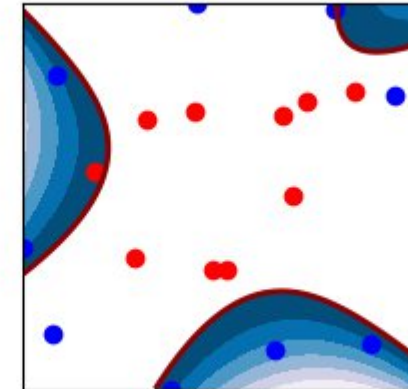
Before Applying Kernel



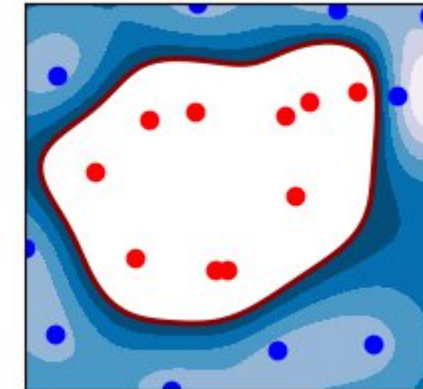
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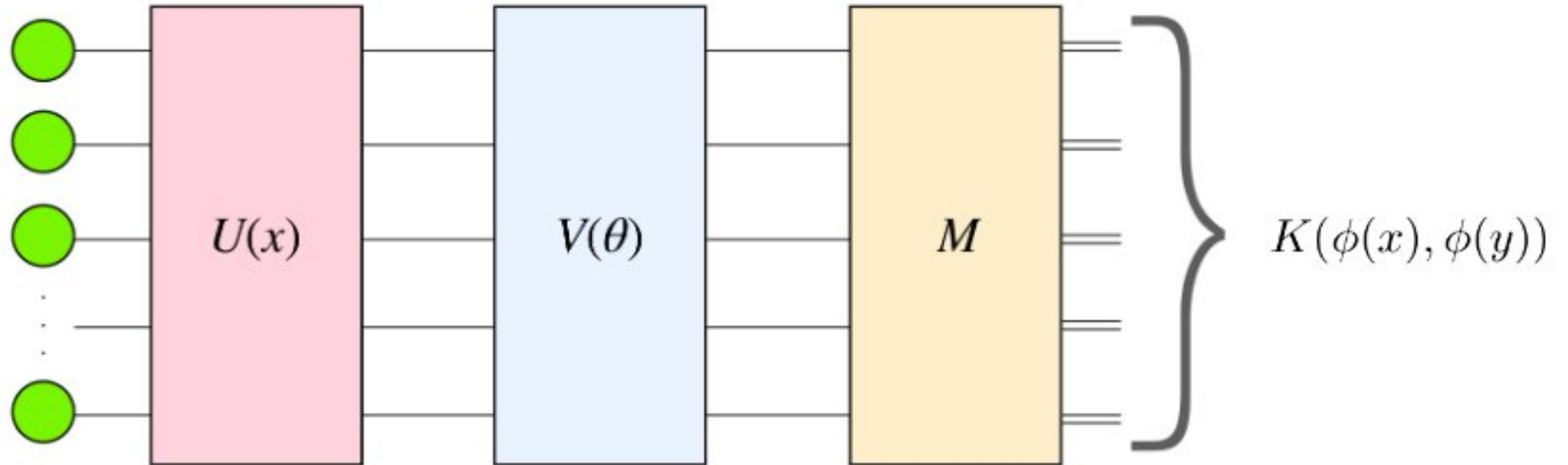


After Applying Polynomial Kernel

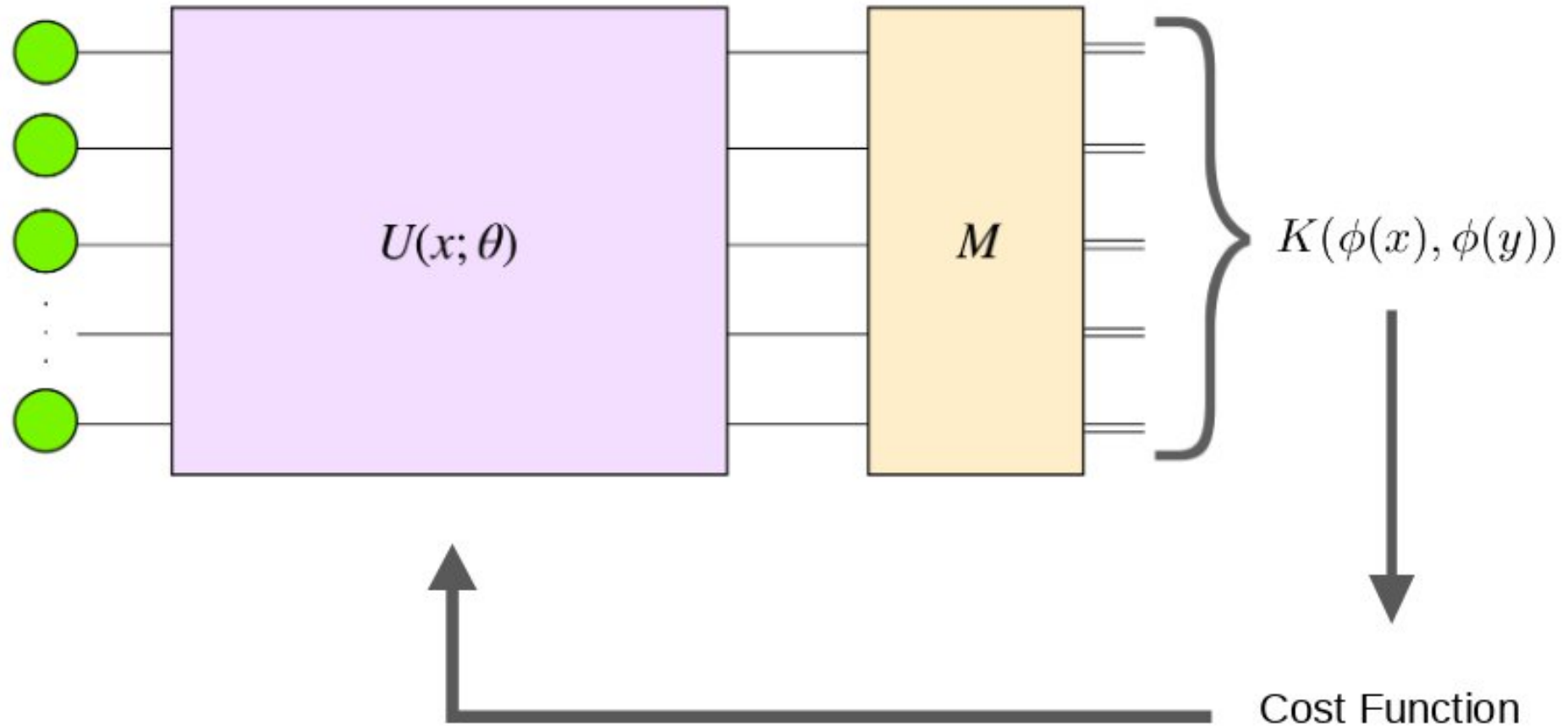


After Applying RBF Kernel

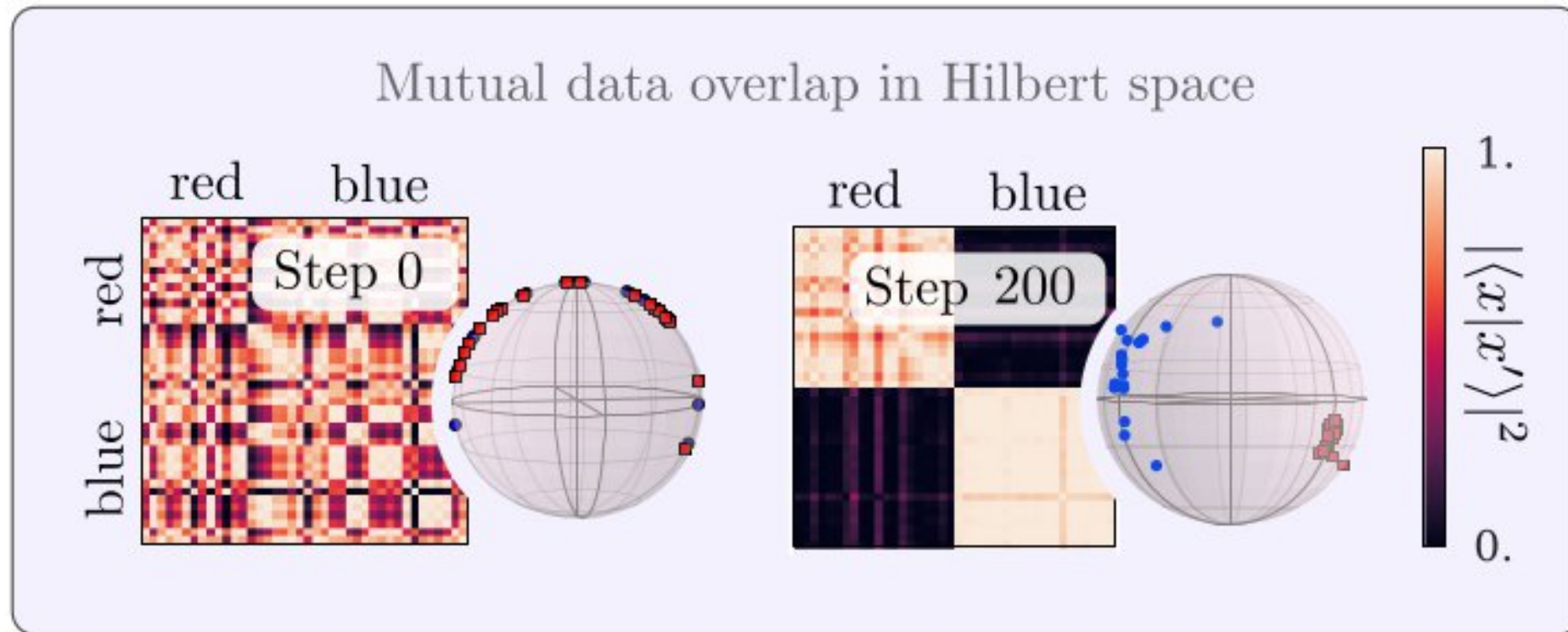
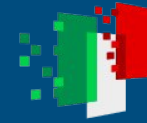




$$k(\phi(x_n), \phi(x_m)) = \text{Tr}[\rho(x_n)\rho(x_m)] = |\langle \phi(x_n) | \phi(x_m) \rangle|^2$$



- Sancho-Lorente, Teresa, Juan Román-Roche, and David Zueco. "Quantum kernels to learn the phases of quantum matter." *Physical Review A* 105.4 (2022): 042432.
- Lloyd, Seth, et al. "Quantum embeddings for machine learning." *arXiv preprint arXiv:2001.03622* (2020).



credit by: Lloyd, Seth, et al. "Quantum embeddings for machine learning." *arXiv preprint arXiv:2001.03622* (2020).



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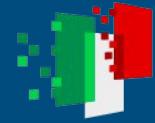


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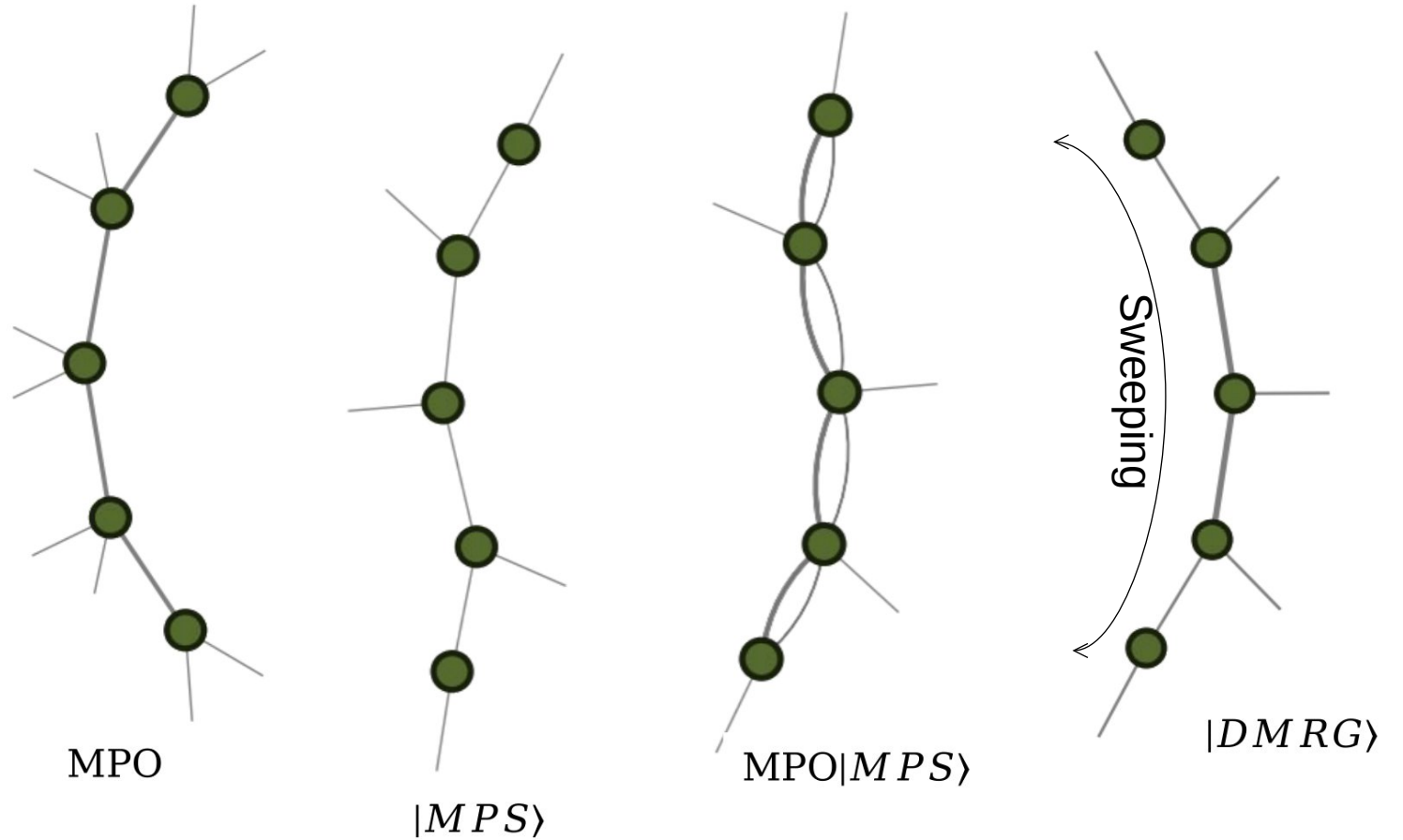
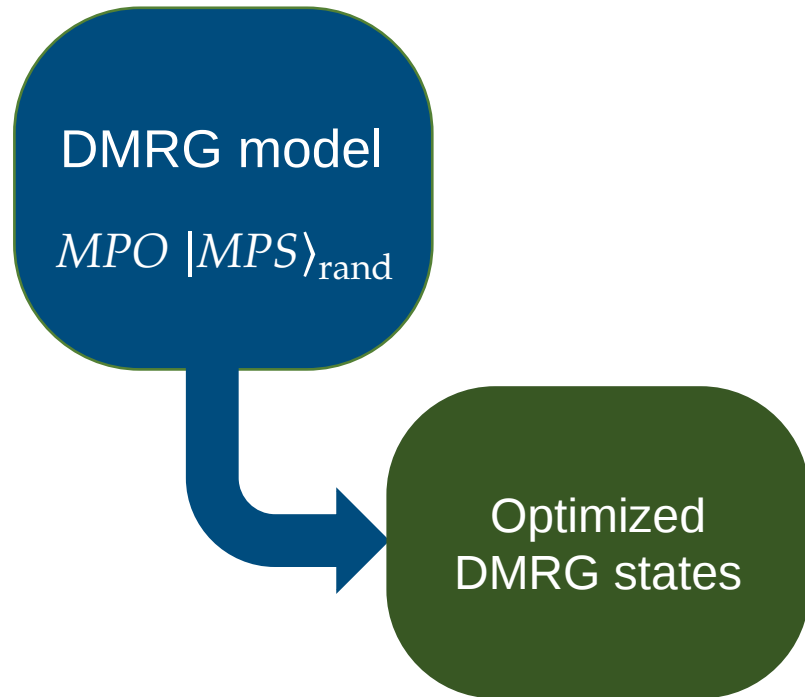
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$$H(p_1, p_2, \dots, p_n) = \text{MPO}(p_1, p_2, \dots, p_n)$$

$$|\psi\rangle = |\text{MPS}\rangle$$





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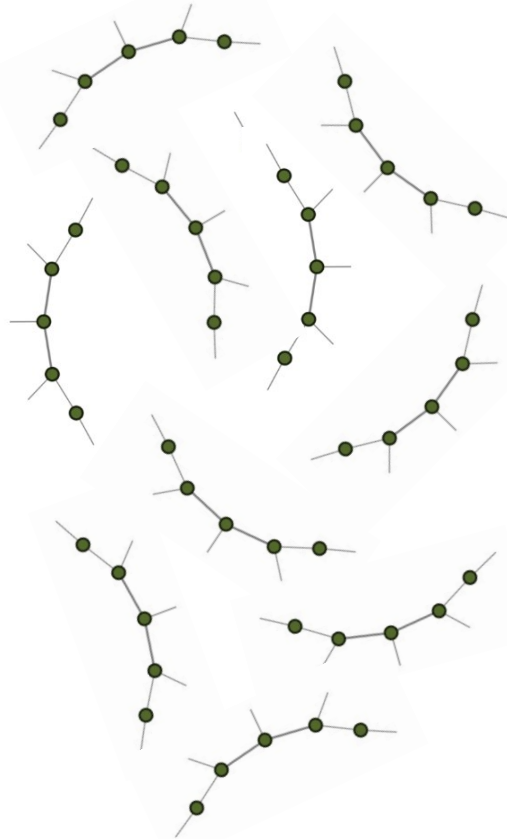
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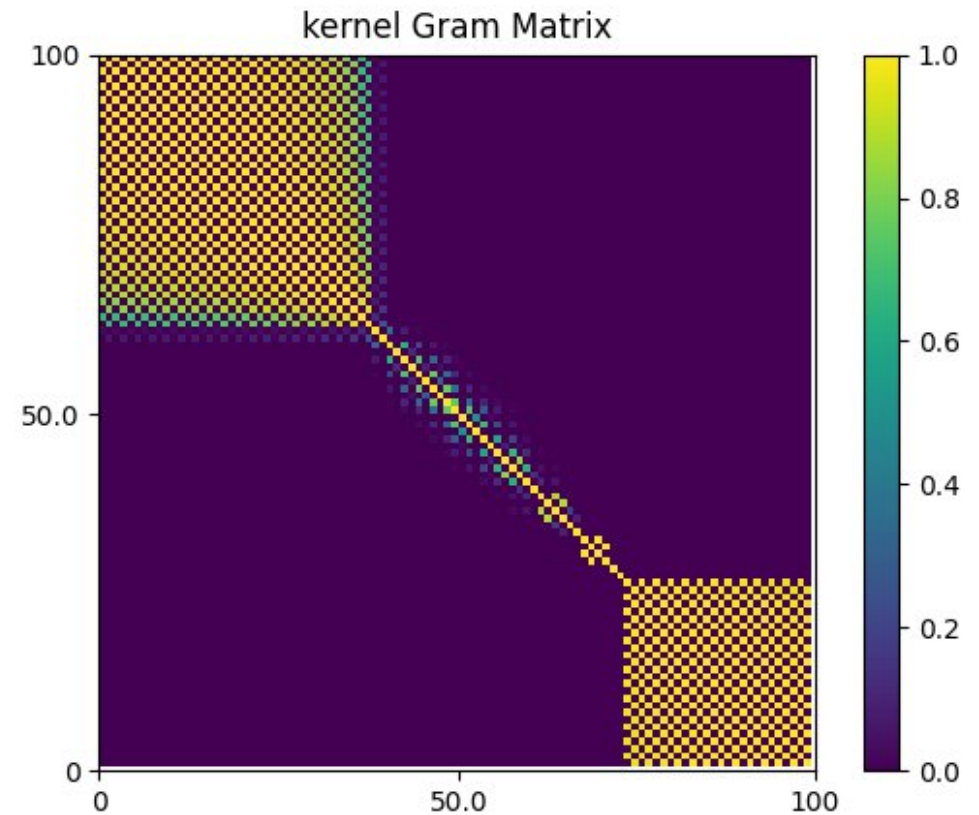
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$$k(x_n, x_m) = |\langle \psi(x_n) | \psi(x_m) \rangle|^2$$



- Reberntrost, Patrick, Masoud Mohseni, and Seth Lloyd. "Quantum support vector machine for big data classification." *Physical review letters* 113.13 (2014): 130503.
- Cristianini, Nello, and John Shawe-Taylor. *An introduction to support vector machines and other kernel-based learning methods*. Cambridge university press, 2000.



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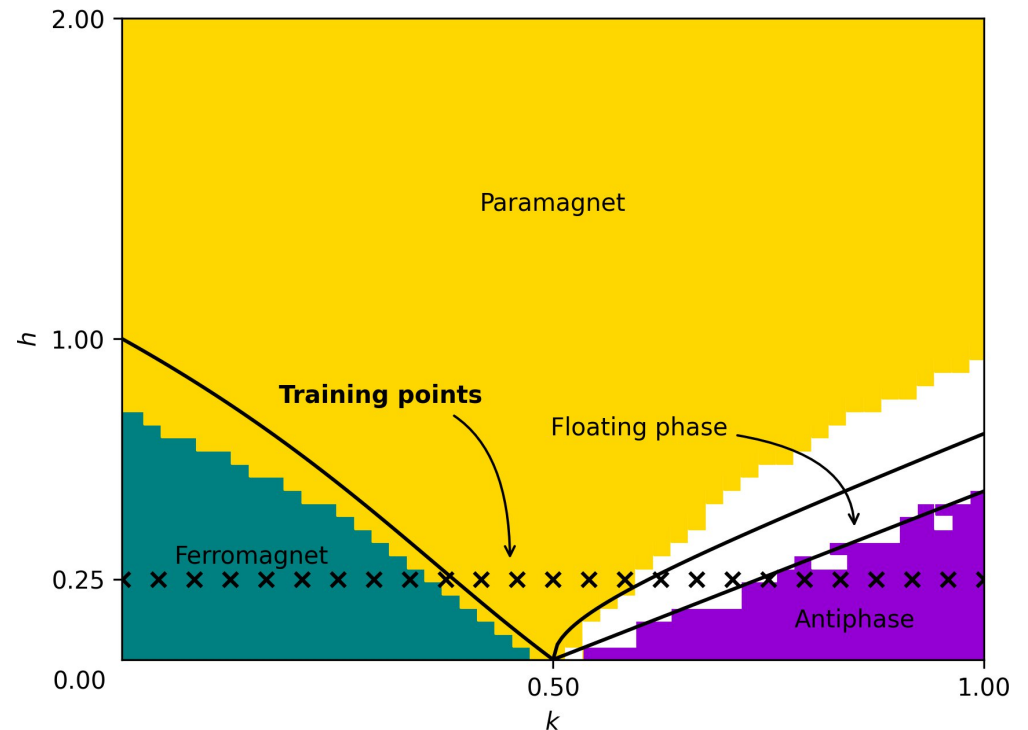
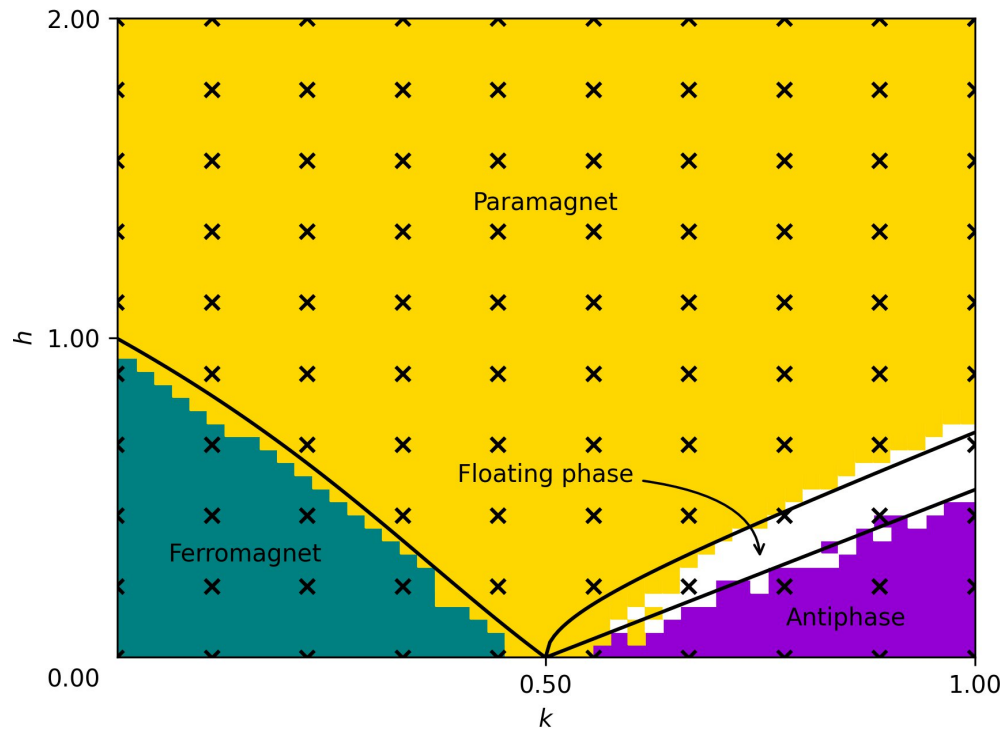
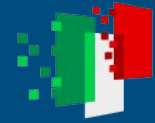


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$$H_{ANNNI} = - \sum_{j=1}^{N-1} \sigma_j^x \sigma_{j+1}^x + k \sum_{j=1}^{N-2} \sigma_j^x \sigma_{j+2}^x - h \sum_{j=1}^N \sigma_j^z$$

- Khosrojerdi, Mehran, et al. "Learning to Classify Quantum Phases of Matter with a Few Measurements." *arXiv preprint arXiv:2409.05188* (2024).



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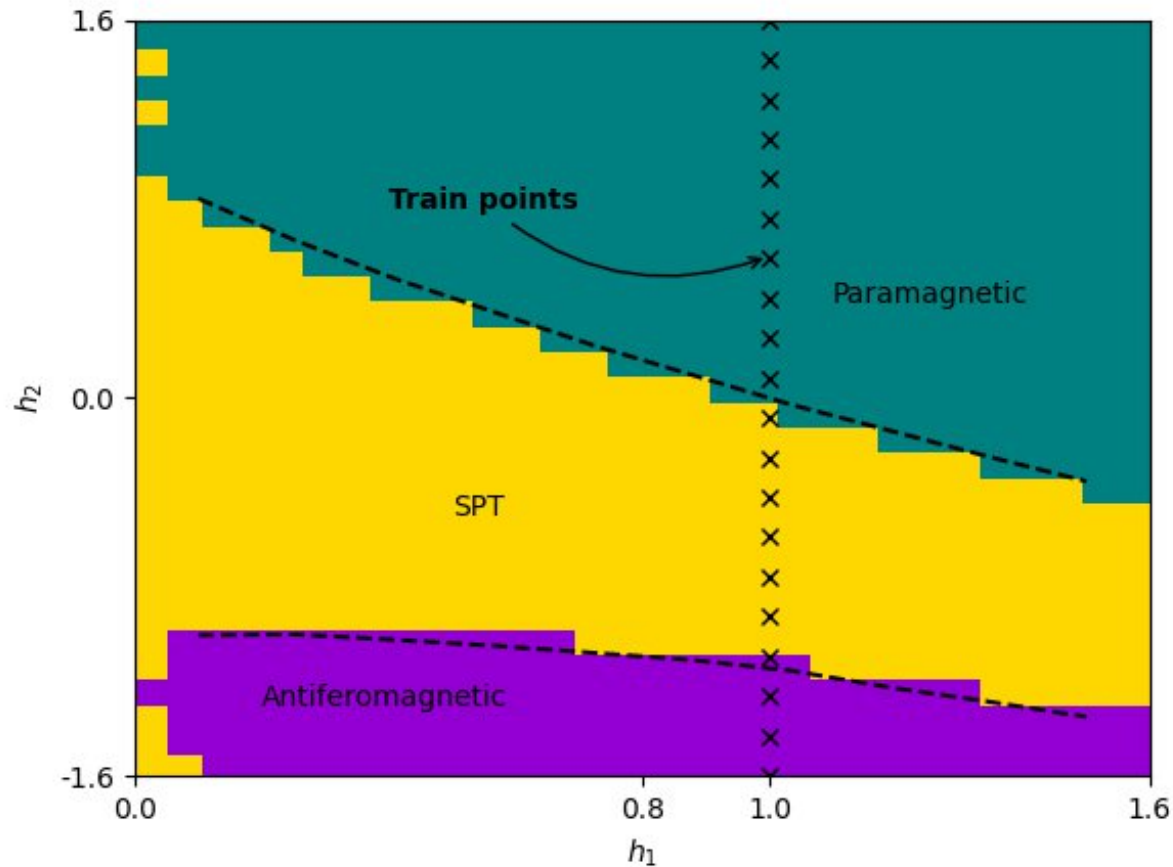
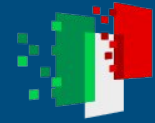


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$$H_{Haldan} = - \sum_{j=1}^{N-2} \sigma_j^z \sigma_{j+1}^x \sigma_{j+2}^z - h_2 \sum_{j=1}^{N-1} \sigma_j^x \sigma_{j+1}^x - h_1 \sum_{j=1}^N \sigma_j^x$$

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Conclusion:

- Tensor networks successfully work in classification problems
- This classifier can successfully interpolate and extrapolate phase diagram, accurately predicting quantum phases for new parameter sets.

Future:

- Implement this approach using unsupervised learning to tackle the clustering problem.
- Replace tensor networks with Quantum Variational Eigensolvers (QVE) and perform a comparative analysis.

Acknowledgements:

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Thank you for your time and attention!
Curious to hear your thoughts.