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Certifying spectral gaps of 1D many-body systems using semidefinite programming

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Certifying the presence of a spectral gap of a many-body quantum systems is a fundamental challenge that finds numerous applications across very different fields. In many-body physics, a non-zero gap in the thermodynamic limit indicates properties such as area law scaling of entanglement and exponential decay of correlations, both in 1D [1] and (partially) in 2D [2]. In quantum computing, the size of the spectral gap along the path is in one-to-one correspondence with the efficiency of the adiabatic algorithm [3]. Furthermore, classical Monte-Carlo algorithms that are rapidly-mixing can be directly mapped to gapped many-body quantum hamiltonians [4].

However, estimating the spectral gap of a many-body quantum systems is a very challenging task, and an undecidable problem in general [5]. Nonetheless, given the relevance of the problem, several methods are known to lower bound the spectral gap in the thermodynamic limit. Two main exponents are the martingale method [6] and finite-size criteria [7], which have allowed to prove the existence of a spectral gap in a plethora of physically-relevant cases. Yet, each of those results required carefully tailoring the method to the specific model of interest.

I will present a novel and general approach to certify the gap of local Hamiltonians in the thermodynamic limit. We leverage the fact that the gap can be estimated as a minimisation problem under the constraint that a degree-two polynomial of the hamiltonian is positive semidefinite. By taking ideas from sum-of-squares proof of positivity, we introduce a relaxation of the minimisation problem that provides lower bounds to the spectral gap. The quality of the lower bound can be systematically improved by controlling a single parameter in our method. Being based on semidefinite programming, our technique provides an efficient and reliable numerical algorithm that can be applied flexibly to any local, frustration-free hamiltonian. Lastly, our method recovers many previous approaches as a special case.

We benchmark our method on several 1D models and show a clear improvement with respect to previous approaches. In all the observed cases, our technique allows to estimate a noticeably larger gap and to prove the existence of a gap in parameter regimes that are inaccessible to other methods. When combined with variational estimations of the gap, our lower bounds often match the corresponding upper bound, providing an exact calculation of the gap. Finally, we discuss extensions to 2D systems and future prospects of the method.

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