Optimal parametric quantum channels for quantum computing

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replacing (unitary) circuits by non-unitary protocols, $U \to \mathcal{E}$

Even with fault-tolerance, some problems involve a mixed state preparation/evolution (for example for thermal states or for modeling open systems).

From unitary to non-unitary channels

Ideally

Unitary circuit: pure to pure

$$|\psi\rangle \to |\psi'\rangle = U|\psi\rangle$$

Realistically

Quantum channel: mixed/pure to mixed

$$\rho \to \rho' = \mathcal{E}(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$$

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Quantum channels can also be engineered through partial measurements and/or stochastic sampling, not only noise.

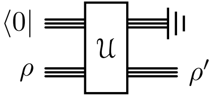
Some useful channel representations

One can engineer a channel through Stinespring representation or via statistical sampling of circuit ensembles (mixed-unitary channels).

Stinespring representation

Unitary on extended space $\mathcal{A} \otimes \mathcal{H}$ without measurement on ancilla register $\mathcal{A}$:

$$\rho \to \rho' = \mathrm{Tr}_{\mathcal{A}}[\mathcal{U}(|0\rangle\langle 0|_{\mathcal{A}} \otimes \rho)],$$
$$\mathcal{U} \in \mathrm{SU}(2^{q+q_a})$$

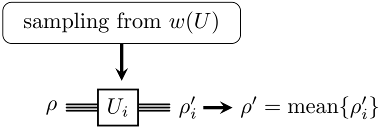


Stochastic sampling

Sample from circuits ensamble $\{U_i\} \sim w(U)$:

$$\rho \to \rho' \simeq \frac{1}{M} \sum_{i=1}^{M} U_i \rho U_i^\dagger$$

$$\rho' = \int_{\mathrm{SU}(2^q)} dU \; w(U) U \rho U^\dagger$$



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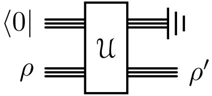

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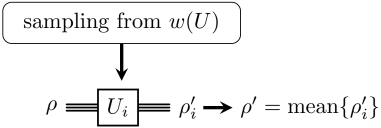

In either representation, we can parameterize the protocol ($\mathcal{U}_{\vec{\theta}}$ or $w_{\vec{\theta}}(U)$).

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Unitary task:
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Our aim is then to minimize the difference between target and real parameterized channel

$$C(\vec{\theta}) \equiv d(\mathcal{E}_U, \widetilde{\mathcal{E}}_U^{(\vec{\theta})})$$

Proper distance between channels: the diamond norm

[A. Yu. Kitaev and A. Shen, *Classical and quantum computation* (2002)]

[M. Kliesch, R. Kueng, J. Eisert and D. Gross, *Improving Compressed Sensing With the Diamond Norm* (2016)]

A proper definition of distance between two quantum channels $\mathcal{E}_1$, $\mathcal{E}_2$ is the **diamond norm** of their difference:

$$d_\diamond(\mathcal{E}_1, \mathcal{E}_2) \equiv \frac{1}{2} \sup_{n, \xi \geq 0} \mathrm{Tr} \left| (\mathcal{I}_n \otimes \mathcal{E}_1)(\xi) - (\mathcal{I}_n \otimes \mathcal{E}_2)(\xi) \right|.$$

Each channel $\mathcal{E}_i$ in $\mathcal{H}$ is *trivially* extended to an operator $\mathcal{I}_n \otimes \mathcal{E}_i$ in $\mathcal{A} \otimes \mathcal{H}$ ($n = \dim \mathcal{A}$), but states ξ can have *non-zero entanglement entropy* between $\mathcal{A}$ and $\mathcal{H}$ subsystems.

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Fortunately, by *convexity*, $d_\diamond$ is saturated by ξ pure states
$\Longrightarrow$ possible preparation via unitaries: $\xi = V \left|0\right\rangle\!\left\langle 0\right| V^\dagger$. Trace distance can be estimated, e.g., via randomized measurements.

Known error mitigation techniques as particular cases

Mixed protocol can be implemented in different flavours, some specializing into already well known mitigation techniques:

- unitary mitigation channels as optimal encoders-decoders (e.g., **dynamical decoupling** in the case of identity maps);
- optimal sampling of ideally equivalent circuit ensembles (similar to **randomized compiling**);
- non-unitary mitigation channels as optimal channel encoders-decoders;
- non-unitary mitigation channel as embedded gate sampling;
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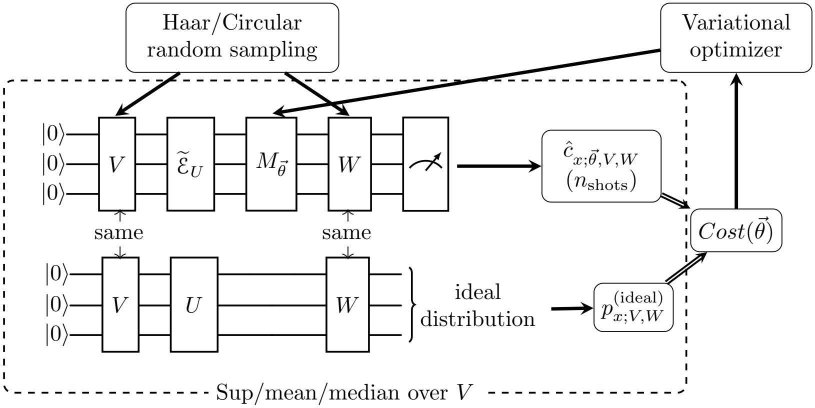
The more the freedom in modeling $\widetilde{\mathcal{E}}^{(\vec{\theta})}$ the higher the training costs: some tradeoff is in order.

Simple example: unitary decoding

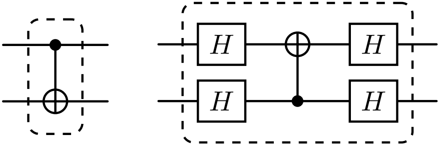
A single parameterized circuit $M_{\vec{\theta}}$ is trained to partially correct noise in $\widetilde{\mathcal{E}}_U$.



This also gives some explanation on how VQE algorithms are typically robust on noisy hardware.

Case study: stochastic CNOT with asymmetric noise

Let us consider $U = \mathrm{CNOT}$



$\mathcal{E}_1 \xrightarrow{noise} \widetilde{\mathcal{E}}_1$ $\quad$ $\mathcal{E}_2 \xrightarrow{noise} \widetilde{\mathcal{E}}_2$

Without noise $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_{\mathrm{CNOT}}$.
With asymmetric noise between the two qubits, the noisy version $\widetilde{\mathcal{E}}_i$ are different!

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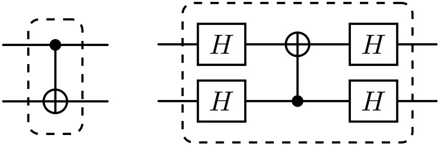

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One can convexly mix the two channels with weights w_i:

$$\widetilde{\mathcal{E}}^{(\vec{w})} = w_1\widetilde{\mathcal{E}}_1 + w_2\widetilde{\mathcal{E}}_2 = w_1\widetilde{\mathcal{E}}_1 + (1 - w_1)\widetilde{\mathcal{E}}_2.$$

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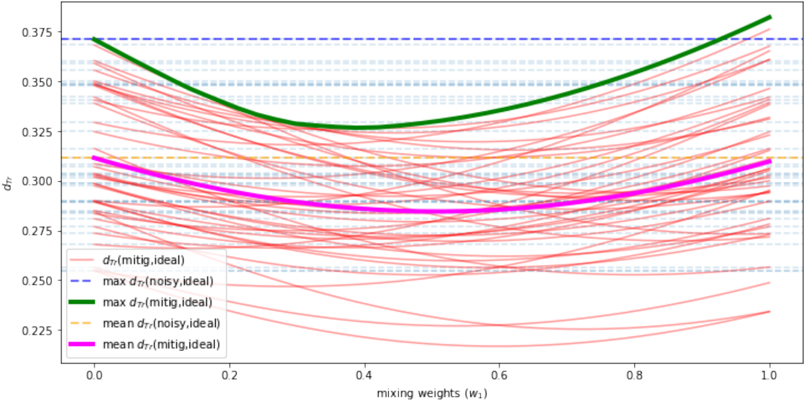
Case study – stochastic CNOT with asymmetric noise: distance

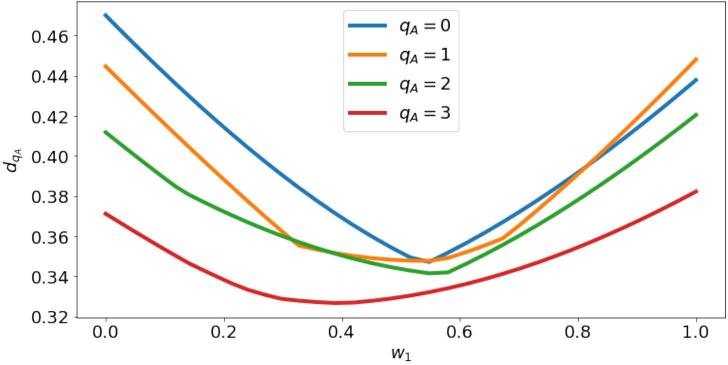
Cost as function of mixing weight w_1 for different input states



Minmaxing would select the best worst case $w_1^{(\text{opt})} \simeq 0.4$.

Case study – stochastic CNOT with asymmetric noise: extension dependence

Dependence on the extension dimension 2^{q_A} for diamond norm



In this case, no degradation on extension is observed.

Summary

Some main useful properties of mixed state quantum computation via parameterized quantum channels:

- generalizes unitary computing to NISQ era (give up ideal unitarity);
- it reduces to standard error mitigation techniques in some particular cases;
- can be realized in many different variants, depending on the task;
- it adapts to the specific noise properties of the QPU and gates/qubits involved.

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Thank you