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Noise in superconducting quantum computing devices a very partial overview of old and new results

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taliadomani

Superconducting hardware

- Qubits based on Josephson Ju ctions (JJ)
	- ❍ Wirable artificial atoms
	- ❍ Flexibility in design solutions
	- ❍ All-superconducting & hybrid systems
	- ❍ Prototype circuit: rf-SQUID

$$
H_0 = \frac{(Q - Q_x)^2}{2C} - E_J \cos \frac{2\pi \Phi}{\Phi_0} + \frac{(\Phi - \Phi_x)^2}{2L}
$$

Resonators

- ◯ Superconducting microstrip transmission line
- ❍ JJ-based LC circuit
- ❍ Superinductors

De Leon et al., Science 2021

Decoherence

What: loss of coherence in an open system

- ❍ Physically due to additional degrees of freedom to be traced out
- ❍ Reduction to a DM, in the basis of the observable coupling to the "pointer"
- ❍ General treatments
- Trend in one and two qubits
	- \bigcirc Single qubit coher. times > 1 ms
		- ◊ rather well understood, optimized device design
	- ❍ scalable manufacturing
	- **O** Upscaling: NISO, what about \mathcal{W} EC ?
- **Trend in Resonators**
	- \overline{O} Larger coherence times \rightarrow cat state qubits

Kjaergaard et al., Annu. Rev. Condens. Matter Phys. 2020

Markovian Master Equation → single-qubit noise metrics

Quantum Markovian ME (Bloch, Wangness, Redfield) $H = H_q + \frac{1}{2}\sigma_z \otimes \hat{B} + H_B$

 \circ von Neumann + **Born-Markov** approx ($b\tau_c \ll 1$)

❍ secular approximation

◊ **Relaxation**

◊ Non-secular **dephasing**

$$
\left(\frac{1/T_{\phi}=\dfrac{1}{2}\,\cos\theta\,S(0)}{1/T_1=\dfrac{1}{2}\,\sin\theta\,S(\Omega)}\right)
$$

 $\frac{d\rho_{ij}(t)}{dt} \approx -i\omega_{ij}\rho_{ij}(t) + \sum_i \mathcal{R}_{ijlm}\rho_{lm}(t)$

$$
\left(1/T_2 = 1/(2T_1) + 1/T_{\phi}\right)
$$

❍ FT of the symmetrized correlation functions ↔ **noise power spectrum**

$$
S(\omega) = \frac{1}{2} \langle \hat{B}(t)\hat{B}(0) + \hat{B}(0)\hat{B}(t) \rangle_{\omega} = J(\omega) \coth \frac{\omega}{2k_BT}
$$

- $-1/f$ noise \rightarrow failure of Born-Markov
	- Ω Non-secular dephasing rate $\propto S(0)$ diverges
	- \circ "phase diffusion" dephasing time T_{ϕ} calculated beyond Born-Markov

○ Tansverse noise $\theta = \pi/2$ pure **dephasing is minimized → sweet spot**

 σ_z

 θ

Phenomenological Hamiltonian

Tunability **opens "ports"** to decoherence

 $H_{tot} = H_0 + H_c(t) + H_n(t) + H_B + H_{int}$ Convenient sufficiently general form

 S ystem $H_0[\mathbf{q}] = \sum_i \epsilon_i(\mathbf{q}) |\phi_i(\mathbf{q})\rangle\langle\phi_i(\mathbf{q})|$ \circ **Bias** parameters **q** and **coupling constants** ϕ eg. SQUID design $H_0 = \frac{(Q - Q_x)^2}{2C} - E_J \cos \frac{2\pi \Phi}{\Phi_0} + \frac{(\Phi - \Phi_x)^2}{2L}$ \bullet Control $\mathbf{q} \to \mathbf{q}(t)$; $H_0[\mathbf{q}(t)] =: H_0[\mathbf{q}] + H_c(t)$

Classical noise: add $\delta \mathbf{q}_{sl}(t) + \delta \mathbf{q}_{f}(t)$ \rightarrow $H_0[\mathbf{q}(t) + \delta \mathbf{q}_{sl}(t)] + H_f(t)$

 \blacksquare Environment: add a "quantized" $H_c(t) \rightarrow \sum \hat{Q}_\alpha \,\hat{B}_\alpha + H_B$ $\overline{}$ eg. flux noise in SQUID $\overline{} \rightarrow \hat{\Phi} \hat{B}_{\alpha} + H_{B}$ α

Paladino, Galperin, GF, Altshuler, RMP 2014

Noise sources in superconductin q-circuits

 \Rightarrow Physical source \rightarrow noise injected from a "port" of the device

 (a) 3

 $\overline{\nabla}$ /3

- \rightarrow Impurities (coherent, incoherent \rightarrow 1/f)
	- ❍ In the junction 1/f, offset drift
- Dielectric loss
	- \overline{O} Electric field in the dielectric \leftrightarrow capacitance fluctuations
	- ❍ Via two-level fluctuator in the (glassy) substrate
	- ❍ Relevant for resonators
- Quasiparticles
	- ❍ Nonequilibrium & leakage

Upscale architectures: leakage from computational space

Paladino, Hakkonen, GAF, Enc. Cond Mat. Phys. 2024

Non-Markovian environment: coherent impurities

In the **tunnel oxide** – may couple resonantly to the qubit

Q-coherent *impurities* entangled with the device

Experiments in phase qubit: Cooper et al. PRL 04, Simmonds et al. PRL 04, Johnson et al. PRL 05, Lisenfeld et al. PRB '10 see also Paladino et al. RMP 2014

 \blacksquare Fluctuating $E_J \to$ critical current noise

❍ Reducing junction size

In the graphene layer of a **hybrid JGJ gatemon**

Pellegrino, GAF, Paladino, Comm. Phys. 202O & 2022

avoided crossings \leftrightarrow resonant quantum impurities

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PHYSICAL REVIEW LETTERS

3 JUNE 2002

Decoherence and $1/f$ Noise in Josephson Qubits

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We propose and study a model of dephasing due to an environment of bistable fluctuators. We apply our analysis to the decoherence of Josephson qubits, induced by background charges present in the substrate, which are also responsible for the $1/f$ noise. The discrete nature of the environment leads to a number of new features which are mostly pronounced for slowly moving charges. Far away from the degeneracy this model for the dephasing is solved exactly.

Gaussian vs non-Gaussian

- Effects of noise involve bath equilibrium **correlations higher than second** ↔ The **Markovian ME** describes **no effect of non-Gaussianity** of the environment
- Example: incoherent impurities in the surface oxides or in the substrates
	- Fano-Anderson impurity **PHENOMENOLOGICAL** description of **non-Gaussian QUANTUM** environment ↔ Caldeira Legget $H = -\frac{\Omega}{2}\,\sigma_3 + \frac{v}{2}\,\sigma_z\tau_z + \left[\tau_+\sum_k T_k c_k + \text{h.c.}\right] + \sum_k \xi_k c_k^{\dagger} c_k$ **O** Semiclassical counterpart: RTN $H = -\frac{\Omega}{2} \sigma_3 + \frac{v}{2} \sigma_z x(t)$ $x(t) = \pm 1$ γ_{\pm} $\left[\begin{matrix} \gamma_{--} & \gamma_{+} \end{matrix}\right]$ $\gamma_+ = e^{-\beta E} \gamma_- \quad \gamma_+ + \gamma_- =: \gamma$

$$
\langle x(t)x(0)\rangle = e^{-\gamma t} \qquad S_x(\omega) = \frac{x_+ - x_-}{4\cosh(E/2k_BT)} \mathcal{L}_{\gamma}(\omega)
$$

 \circ Slow impurities \rightarrow 1/f noise

Non-Markovian environment: incoherent impurities

 \blacksquare In the surface oxide or in the substrates

Decoherence

 Ω' visibility of the induced splitting

Weak coupling $g \ll 1$ Markovian $T_2^* = S(0)/2$

 $\rightarrow \atop{+}\rangle$ Ω $\overline{}$

Non-Markovian environment: incoherent impurities

 \blacksquare In the surface oxide or in the substrates

General working point & Markovianity

Paladino et al., Adv. Sol. State Phys. 2003 Paladino et al. RMP 2014

Dynamics at arbitrary working point

- Approximate solution with Fano-Anderson model
- **Threshold for strongly/weakly coupled** behavior

$$
g = \frac{\Omega_+ - \Omega_-}{\gamma} \quad \Omega_\pm = \sqrt{(\varepsilon \pm b)^2 + \Delta^2}
$$

also depends on the qubit's working point

- → **changing bias of the computer "activates" fluctuators**
- **Markovianity**
	- A **non-secular → Lindblad** approach
	- BLP, LFS, FT measures
	- Metrics for **upscaled systems**?

Chiatto, GAF et al. Preprint 2024

RTN → 1/f noise

REVIEWS OF MODERN PHYSICS, VOLUME 86, APRIL-JUNE 2014

$1/f$ noise: Implications for solid-state quantum information

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(published 3 April 2014)

1/f noise @ pure dephasing exact results

Ensemble of fluctuators with distribution of switching rates

$$
\frac{\rho_{+-}(t)}{\rho_{+-}(0)} = e^{-i\epsilon t} \prod_j \left\{ A_j e^{-\frac{Y_j}{2}(1-\alpha_j)t} + (1-A_j)e^{-\frac{Y_j}{2}(1+\alpha_j)t} \right\} = \lambda
$$

About nonequilibrium of the environment of slow fluctuators

- some of them thermalize during the whole measurement
- The slowest do not termalize but have little effect on decoherence
- Partial-equilibrium which depends on the whole procedure including preparation and measurement
	- \rightarrow non-Gaussianity provides an effective low-frequency cutoff

Paladino, Faoro, GF, Fazio PRL 2002 Paladino, Galperin, GF, Altshuler, RMP 2014

PRL 94, 167002 (2005)

PHYSICAL REVIEW LETTERS

week ending 29 APRIL 2005

Initial Decoherence in Solid State Oubits

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We study decoherence due to low frequency noise in Josephson qubits. Non-Markovian classical noise due to switching impurities determines inhomogeneous broadening of the signal. The theory is extended to include effects of high-frequency quantum noise, due to impurities or to the electromagnetic environment. The interplay of slow noise with intrinsically non-Gaussian noise sources may explain the rich physics observed in the spectroscopy and in the dynamics of charge based devices.

Adiabatic + Longitudinal + static path approx **Figure 10** GF et al. PRL 2005

- H1/f spectrum by slow fluctuators → **semiclassical Adiabatic approximation**
- Phase fluctuations accumulate in time **[→]** retain *only* fluctuations of the **length** of the Hamiltonian (**longitudinal approximation**)

$$
\rho_{ij}(\tau) = \rho_{ij}(0) \int \mathcal{D}[x(\tau')] P[x(\tau')] \exp \{i \int_0^{\tau'} d\tau'' \omega_{ij}[x(\tau'')]\}
$$

instantaneous eigen-splittings

• Static Path Approximation (SPA) $x(\tau) \rightarrow x(0) \equiv x$ inhomogeneous broadening

 $J\gamma_m$

Large

$$
\rho_{ij}(\tau) = \rho_{ij}(0) \int dx P(x) e^{i\tau \omega_{ij}(x)}
$$

Relevant cases central limit theorem \rightarrow Gaussian distributed $x \quad P(x) = \frac{e^{-\frac{x^2}{2\Sigma^2}}}{\sqrt{2\pi\Sigma}}$
large number of impurities $\Sigma = \int_{\gamma_m}^{\gamma_M} \frac{d\omega}{\pi} S(\omega) \qquad S(\omega) = \frac{\pi\Sigma^2}{\ln(\gamma_M/\gamma_m)} \frac{1}{\omega}$ Also their et al. PRB 2005

Experiments @ optimal points

Pulsed control → dynamical decoupling & noise sensing

Spin echo $t_{echo} = 2t_d$ cancels noise at frequencies $\omega \ll 1/t_d$

Bang-bang control

longer sequences t_{tot} = 2N t_{d} cancel noise at larger frequencies $\omega \ll 1/(Nt_d)$ for the same t_{tot} **PDD, CMPG, Uhrig sequences**

Filters out single qubit dephasing $\rho_{01}(t) = \rho_{01}(0)e^{-\Gamma_N(t) - i\Sigma_N(t)}$

Filter function → Noise sensing

Two-qubit decoupling while processing!

- Dynamical decoupling protocol for mitigating error of a two-qubit entangling-gate with longitudinal local noise while processing
- Magnus expansion for analytical expression of the gate error.
- Derivation of a generalized filter function theory: decoherence while processing
- Quantum sensing:
	- higher order cumulants: Gaussian vs non-Gaussian
	- **Spatial correlations**

D'Arrigo, Piccitto, GAF, Paladino Sci. Rep 2024

Coherent Transport by Adiabatic Passage

- 3 single-level qdots Hamiltonian coupled in the **Λ configuration** \bullet
	- \bullet "Stokes" and "pump" **tunneling amplitudes** Ω_k and **local bias** δ_k

$$
\mathbf{H} \equiv \begin{bmatrix} 0 & 0 & \frac{1}{2}\Omega_p^*(t) \\ 0 & \delta & \frac{1}{2}\Omega_s^*(t) \\ \frac{1}{2}\Omega_p^*(t) & \frac{1}{2}\Omega_s(t) & \delta_p \end{bmatrix}
$$

$$
\text{We know the following expression: } \delta := \delta_p - \delta_s = 0 \implies \text{trapped eigenstate} \quad |D\rangle = \frac{\Omega_s(t)|0\rangle - \Omega_p(t)|1\rangle}{\sqrt{|\Omega_s|^2 + |\Omega_p|^2}}
$$

Population **coherently trapped** in the subspace $span\{|0\rangle,|1\rangle\}$

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$$

@ two-photon resonance $\delta := \delta_p - \delta_s = 0$ \rightarrow **trapped** eigenstate $|D\rangle = \frac{\Omega_s(t)|0\rangle - \Omega_p(t)|1\rangle}{\sqrt{|\Omega_s|^2 + |\Omega_p|^2}}$

Switching couplings in **counterintuitive** sequence → **CTAP • 100% robust population transfer** $|0\rangle \rightarrow |1\rangle$ while $|2\rangle$ always empty ❍ Ac-driven 3LS in rot-frame → **STIRAP**

Vitanov, Rangelov, Shore, Bergmann,RMP (2017)

Menchon-Enrich, Greentree et al., Rep. Prog. Phys. (2016) Huneke, Platero, Koler, PRL (2013) Gullans & Petta, PRB (2020)

Noise classification in a 3-LS by supervised learning

\blacksquare CTAP/STIRAP + **pure dephasing** classical noise $\tilde{x}_i(t)$

- ❍ Simulate **fluctuations of the energy spectrum** of the system
- ❍ Effective model of the leading decoherence effects in most AAs

Goal: **classification** among 4/5 types of noise (**labels**)

• **Multilevel correlation parameter η**

• Non-Markovian quasistatic: Samples generated **O** correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta > 0$, by Stochastic **2** anti-correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta < 0$, Schrödinger equation **O** uncorrelated noise: $x_2(t)$ and $x_1(t)$, are independent from each other. • Markovian: Samples generated correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta > 0$, 4a.

anti-correlated noise: $x_2(t) = \eta x_1(t)$, with $\eta < 0$. 4**b.** 0_R

by Markovian Master equation

correlated noise: $x_2(t) = \eta x_1(t)$, with no conditions on η .
Mukharjee, GAF, Giannelli et al., Machine Learning: Science and Technology 2024 \overline{a}

$$
\mathbf{H} \equiv \begin{bmatrix} 0 & 0 & \frac{1}{2}\Omega_p(t) \\ 0 & \delta + \tilde{x}_1(t) & \frac{1}{2}\Omega_s(t) \\ \frac{1}{2}\Omega_p(t) & \frac{1}{2}\Omega_s(t) & \delta_p + \tilde{x}_2(t) \end{bmatrix}
$$

Dataset generation → output

We exploit sensitivity of CTAP/STIRAP to **asymmetries in the peak amplitudes**

- For each **label** $i = 1, \ldots, 5$
	- ◊ Extract correlation coefficient **η** (↔ device) **in the proper range**
	- \circ Run STIRAP with **3 combinations of amplitudes** $\Omega_{\rm p}(t) = \Omega_{\rm p}^{\rm (max)} e^{-\left(\frac{t-\tau}{T}\right)^2}$,

- ◊ Take 3 efficiencies {**ξ**} as the **NN input**
- \lozenge Train the NN to maximize probabilities $\{P_i\}$ of correct output for each label $i = 1, \ldots, 5$
- Each dataset has 500 samples

Mukharjee, GAF, Giannelli et al., Machine Learning: Science and Technology 2024

Results for supervised learning

Discriminates **Markovian/non-Markovian**

❍ ~ 100% accuracy in discrimination of **correlations**

of non-Markovian noises

- ❍ Unable to discriminate correlations of Markovian noises
- ❍ Ideal situation: each sample from averaging an **infinite set** of identical experiments
- ❍ Each sample from averaging over a **finite set** of (simulated) identical experiments

 \sim ue label $\frac{3}{3}$

- **Amazing** sensitivity to amplitude's asymmetry
- developing **tailored clustering algorithms for unsupervised learning** → ICSC

 $\overline{4n}$

predicted label

Design of quantum devices specific for sensing

Mukharjee, GAF, Giannelli et al., Machine Learning: Science and Technology 2024

Modular computing ↔ USC quantum interconnect

- Post-NISQ **modular architectures** combine quantum gates with communication between separate quantum cores
	- \circ Complexity of control, cooling and power infrastructure, crosstalk, component reliability, upscaled decoherence, …
	- O Developed on **various platforms:** semiconductor, superconductor, impurities
	- Prototype model: **mutiqubit nodes interconnected by quantized harmonic modes**

A **USC coupled** interconnect → **ultrafast intercore q-operations** ?

USC achieved in semi/superconductor architectures

Theory: Ciuti et al. (2005), Experiments: Aanappara et al., PRB (2009) Niemczyck, et al., Nat.Phys. (2010) Reviews: Kockum, Nori et al., Nat. Phys Rev. (2019) Forn-Diaz et al. Rev. Mod. Phys (2020)

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USC achieved in semi/superconductor architectures

Faster dynamics **costs fidelity**!

in USC fundamental limitation: **Dynamical Casimir Effect** (DCE)

Benenti, D'Arrigo et al. , PRA (2014) Hoeb, Benenti et al. PRA (2017)

Mode as quantum interconnect: USC

Mode as quantum interconnect: USC

Fidelity **drastically deteriorates** for faster dynamics

Benenti, D'Arrigo et al. , PRA (2014) Hoeb, Benenti et al. PRA (2017)

Mode as quantum interconnect: USC

Our work: quantized mode as a **virtual quantum interconnect** by CTAP \circ does it **suppress DCE** → Ideal clock scaling as $T \sim \pi/g$?

Single-excitation transfer: the building block

 $|0ge\rangle$

GAF, Giannelli, Benenti, Montangero, Paladino, 2024

Virtual interconnect protects from **DCE**

Stramacchia, et al. MDPI (2019) Giannelli, et al. Nuovo Cim. C (2022)

 $|0eg\rangle$

Ideal only up to $g^{\max}/\omega_d \approx 0.1$. What at **intermediate g**?

Single-excitation transfer: renormalization

Single-excitation transfer: renormalization

Optimization, state-transfer, resilience, ….

 \rightarrow Optimizing ϵ -modulation → populationtransfer **error ~ 10**⁻⁷

Theorem: in the absence of DCE & within a state-independent local unitary, the error for state-transfer is maximized by the **efficiency** of excitation transfer **→ fidelity of state-transfer**

Optimization, state-transfer, resilience, ….

 \rightarrow Optimizing ϵ -modulation → populationtransfer **error ~ 10**⁻⁷

Theorem: in the absence of DCE & within a state-independent local unitary, the error for state-transfer is maximized by the **efficiency** of excitation transfer **→ fidelity of state-transfer**

- Bottleneck: interconnect **decoherence** rather than USC
- Remarks on hardware and architectures
	- ❍ Operation **time scale: intercore ~ intracore**
		- \cdot In **noisy** superconducting q-circuits: infidelities $1 \mathcal{F} \geq 10^{-5}$ for a protocol duration $T_{tot} \sim 6T \approx 30 \, ns \, \omega_q \, \text{m}^{\text{max}}/\omega_c = 0.5$
	- ❍ Natural **unidirectional** photon emission/routing necessary in communication *Kannan et al., Nat. Phys. (2023)*
	- ❍ **Available switchable** components in flux-based superconducting circuits

Intercore communication: entanglement generation/sharing

Intercore communication: entanglement generation/sharing

ancillary qubit → entanglement **sharing/multipartite**

- ❍ One excitation coherently transferred at once qubit $0 \rightarrow (1,...,N) = W$ state $|0\rangle_c |g\rangle^{\otimes N} |e\rangle_0 \stackrel{\mathscr{S}^{\otimes N}}{\longrightarrow} |0\rangle_c \frac{1}{\sqrt{N}} [|egg \dots \rangle + |geg \dots \rangle + \dots + |g \dots ge \rangle] |g\rangle_0$
- Rescaling of interactions $q^0 \sim \sqrt{N} q^i \rightarrow$ **USC** q^0 necessary for N>>1
- ❍ **-modulated** protocol **mets** the benchmark of **N-conserving** system

Intercore computation

- Two-qubit remote computing
	- ❍ by entanglement teleportation
	- ❍ by remote SWAP+ local entangling gates

SWAP by **ancillary qubit**

- **O** 3 ϵ -modulated sequences \mathscr{S}_{02} \mathscr{S}_{21} \mathscr{S}_{10}
- ❍ Implemented by **4 composite pulses**
- ❍ **Error smaller than the N-conserving (RW) benchmark**

SWAP by **two-particle CTAP**

❍ Two excitations transferred at once→ SWAP

❍ Transfer of entangled states

A quantum theory of low-frequency noise

With Gaussian noise ↔ **spin-boson**

Non-Gaussian noise ↔ **spin-bath**

$$
J(\omega) = \sum_{j} \frac{c_j^2}{2\omega_j} \delta(\omega - \omega_j) = 2\alpha \frac{\omega^s}{\omega_c^{s-1}} e^{-\frac{\omega}{\omega_c}} \Theta(\omega - \omega_{\rm ir})
$$

- ❍ Coherent, incoherent, **pseudocoherent**
- ❍ negative s (highly non-Markovian):
	- ◊ Pseudocoherent region widens
	- ◊ Strong dependence on initial conditions