# Quantum & Quantum-inspired Technologies for High-Energy Theories

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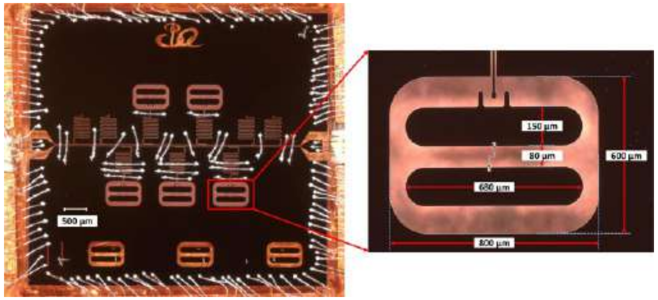
quantum.dfa.unipd.it

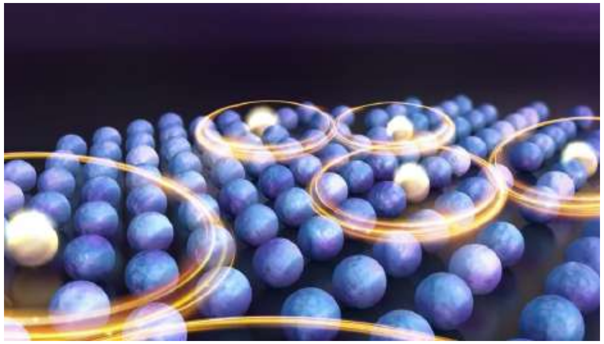
qtech.unipd.it

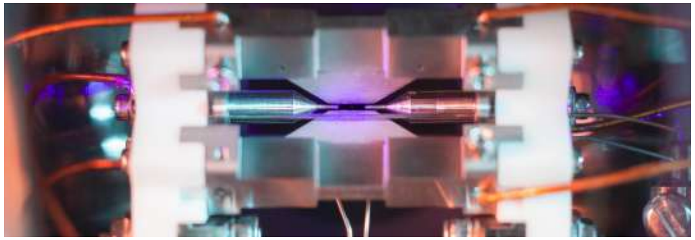
# Quantum Devices

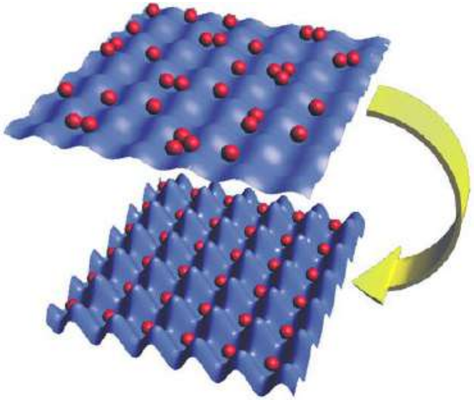
They harness the power of the quantum world:
*Superposition*, *Interference*, *Entanglement*

- Superconducting Qubits
- Rydberg Atoms in Optical Tweezers
- Trapped Ions
- Optical Lattices
- Photonic Circuits
- Nitrogen-Vacancy Centers in Diamond
- … and more!

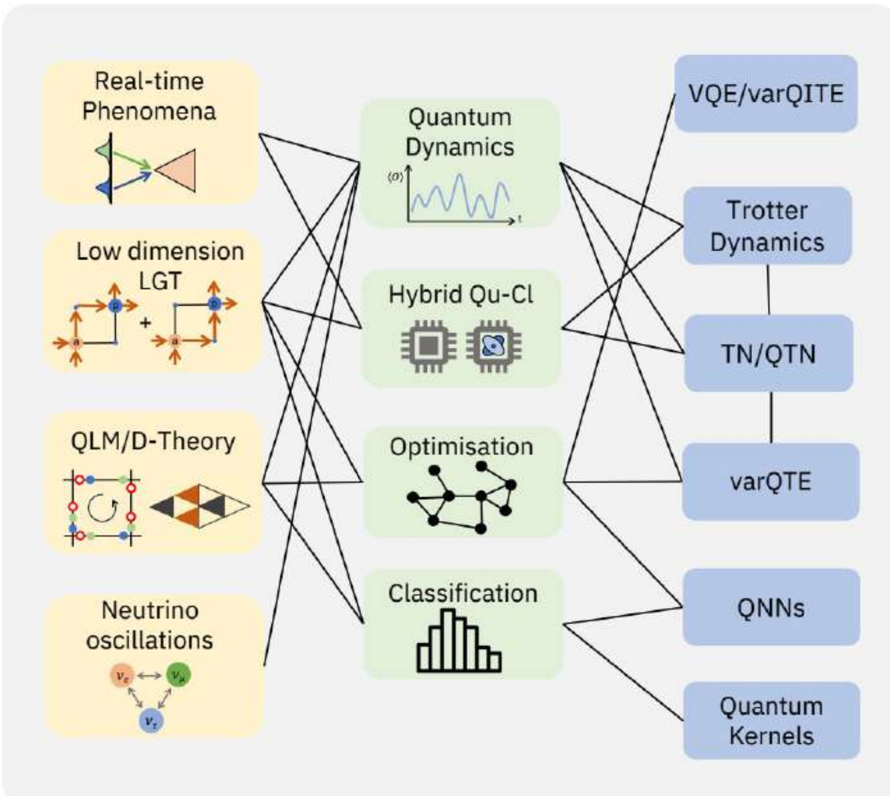


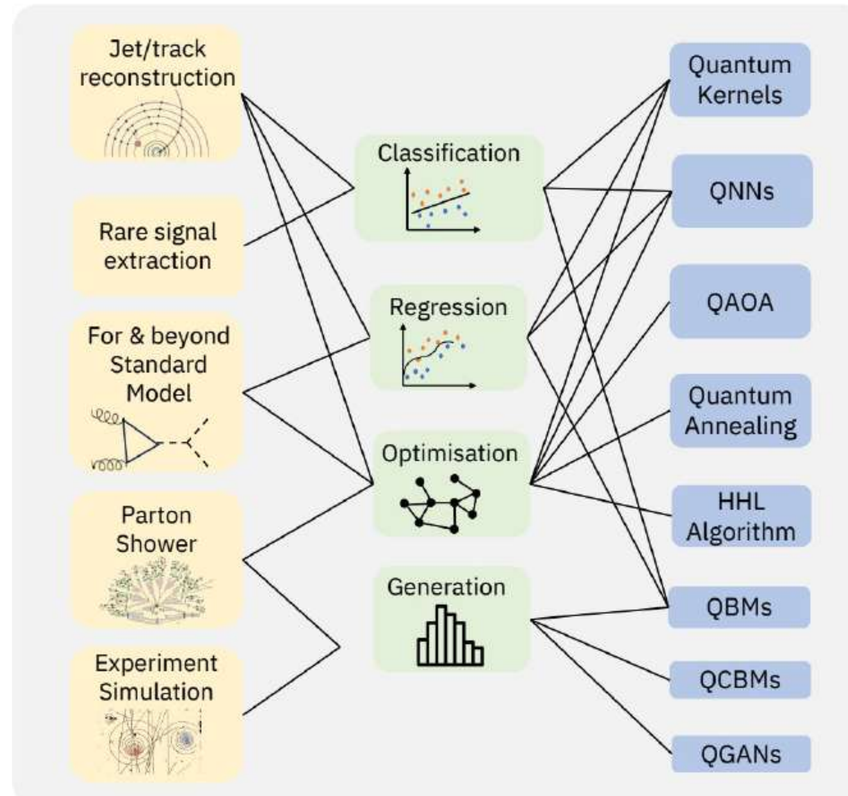




# Quantum for High–Energy Physics



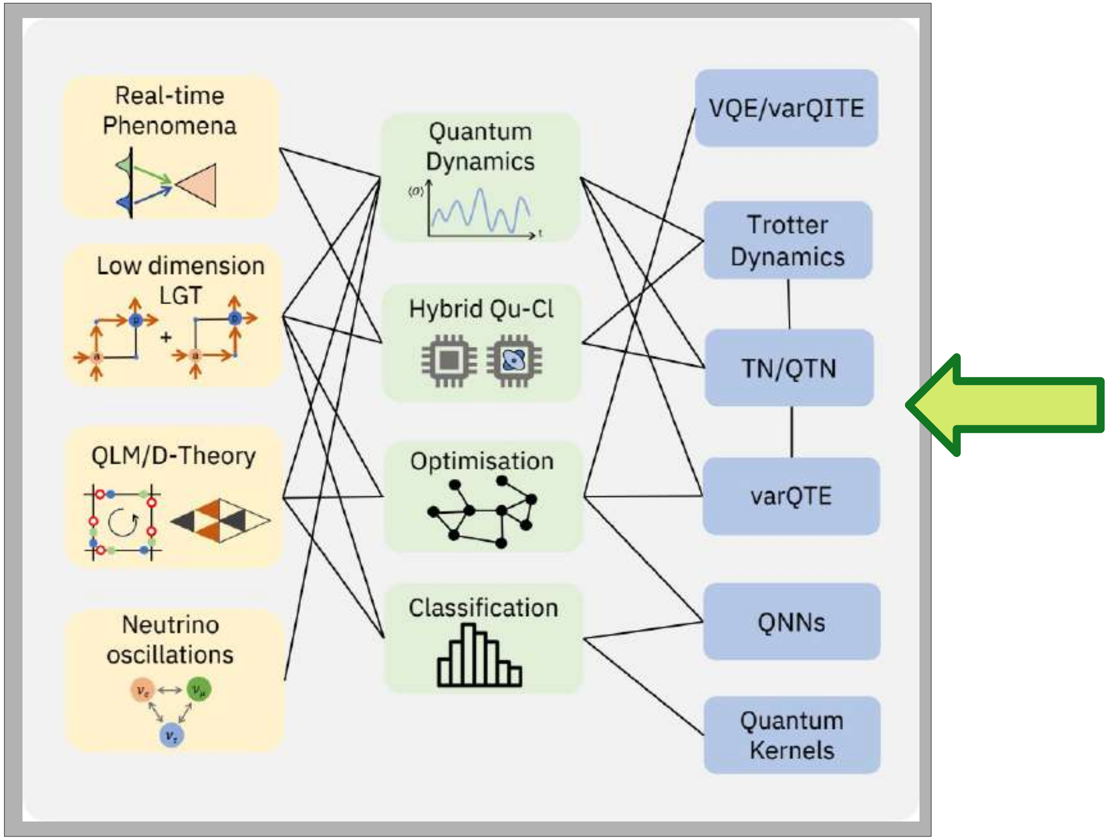

Quantum for HEP Theory




Quantum for HEP Experiment

PRX Quantum **5**, 037001 (2024)

# Quantum for High-Energy Physics

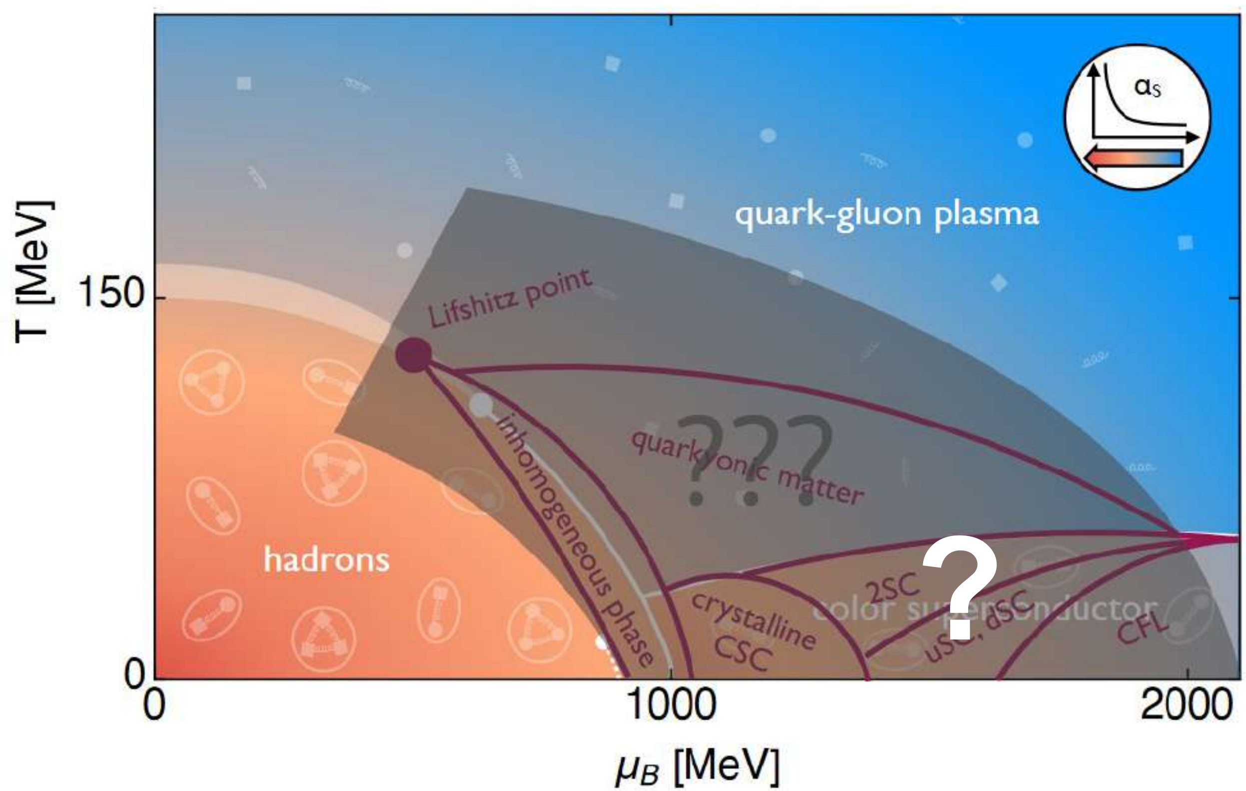



Quantum for HEP Theory

Focus of this Talk

PRX Quantum **5**, 037001 (2024)

# Quantum for HEP – Theory




**TASK:**
Quantitative Prediction of *Emergent Collective Phenomena*

Useful especially in regimes difficult for traditional methods (MC sign-problem)

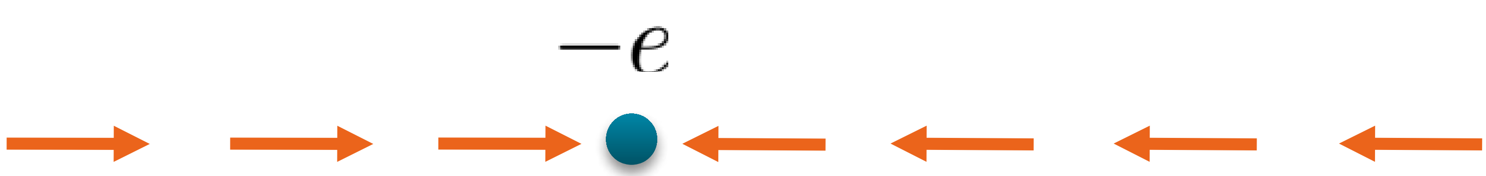
Encode *Quantum Field Theory* problems in the language of *Quantum Devices*

# Lattice Gauge Theories (LGT)

1) Quantum Matter and Quantum Fields

2) Local symmetries, e.g. Gauss's law in QED

$$\nabla \cdot \mathbf{E} = \rho$$



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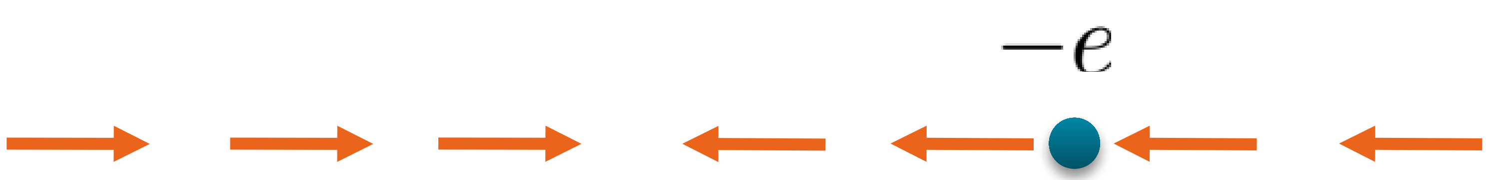

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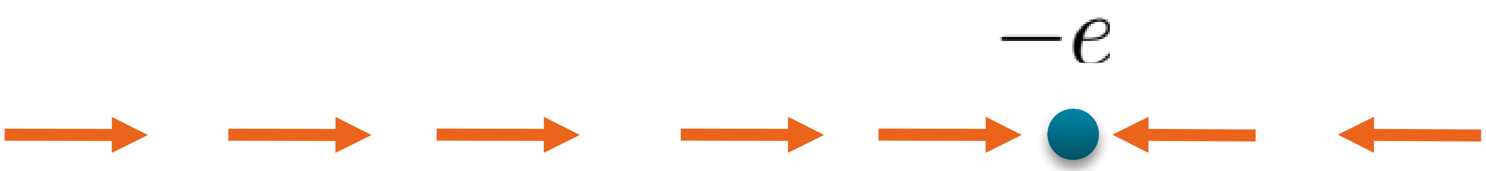
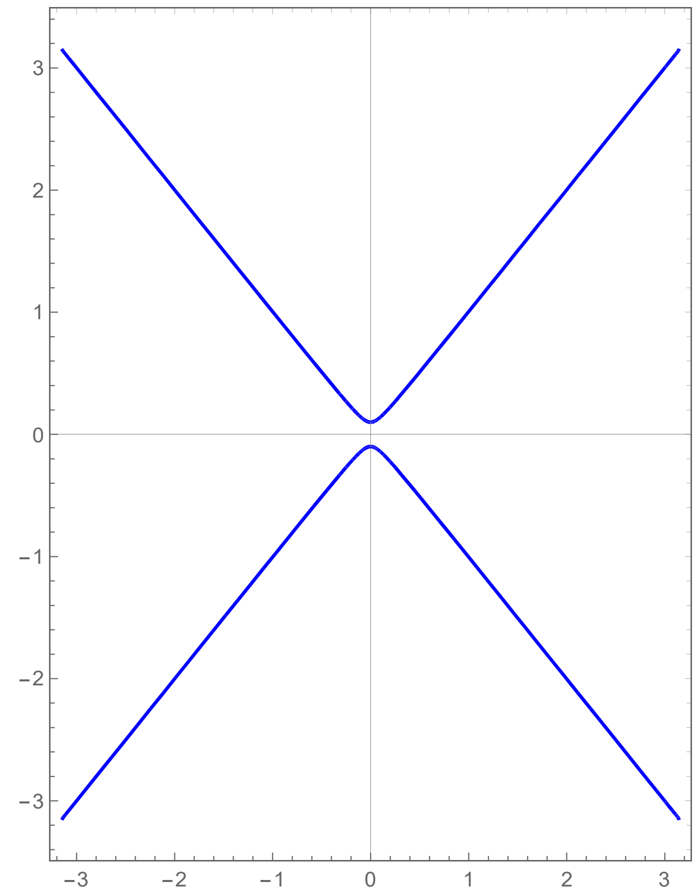


# Hamiltonian LGT from scratch

$$H_{\mathrm{Dirac},1} = c\gamma^0\vec{\gamma}\cdot\vec{p} + mc^2\gamma^0$$

Dirac Hamiltonian (Single fermion)

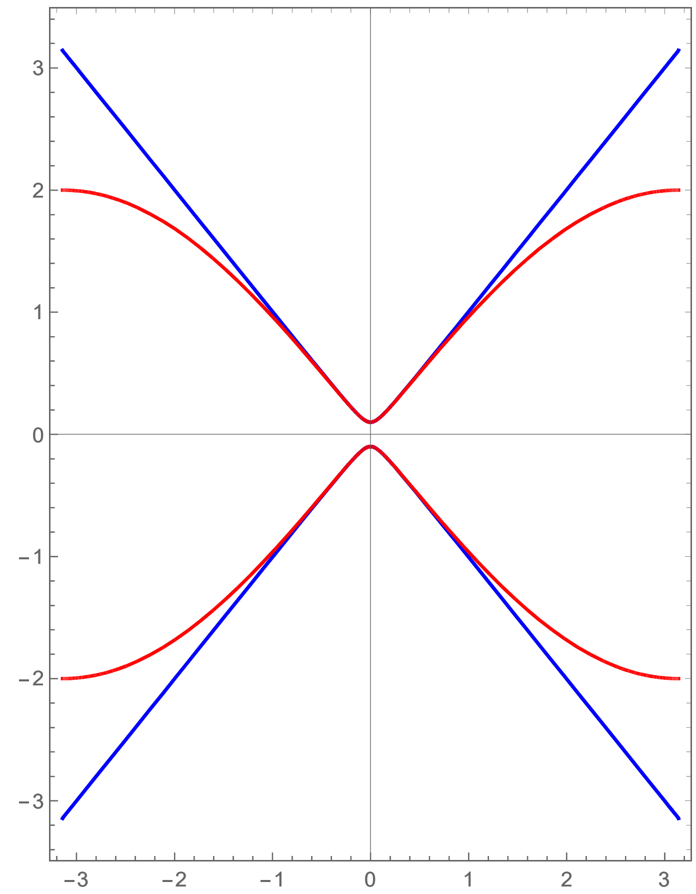


# Hamiltonian LGT from scratch

$$H_{\text{Dirac},1} = c\gamma^0\vec{\gamma}\cdot\vec{p} + mc^2\gamma^0$$

Replicate the low-k single particle dispersion with a free fermion theory

$$H_{\text{Suss}} = mc^2\sum_j(-1)^j\psi_j^\dagger\psi_j + \frac{c\hbar}{2a}\sum_j e^{i\eta_{j\mu}}\psi_j^\dagger\psi_{j+\mu} + \text{H.c.}$$




Many ways to do this.
Some symmetry gets broken

Susskind; PRD **16**, 3031 (1977)

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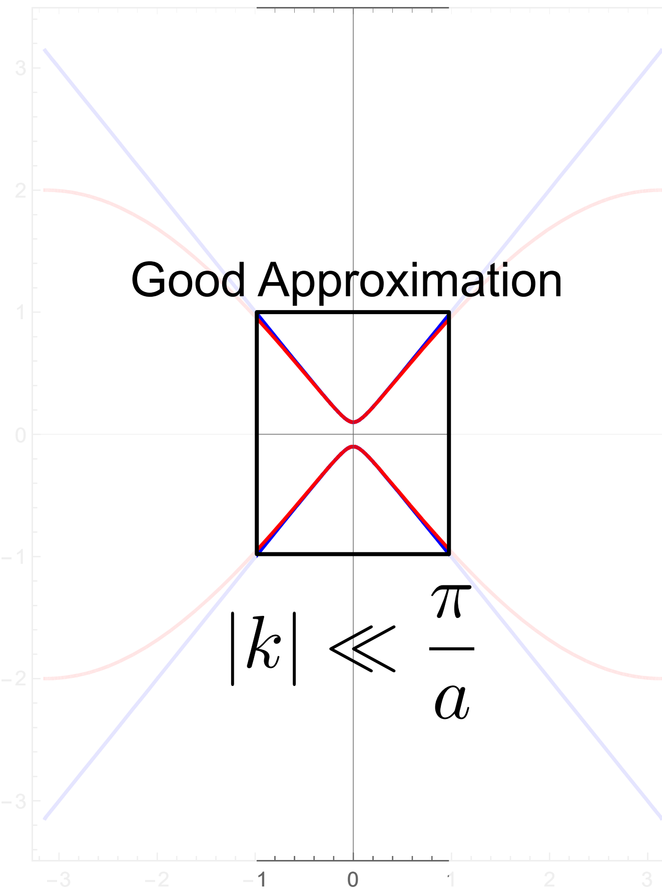


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Susskind; PRD **16**, 3031 (1977)

# Coupling to gauge fields

$$\vec{p} \longrightarrow (\vec{p} - q\vec{A})$$ Minimal coupling

Translates Into

$$\psi_j^\dagger \psi_{j+\mu} \longrightarrow \psi_j^\dagger e^{-i\frac{aq}{\hbar}A_\mu} \psi_{j+\mu}$$

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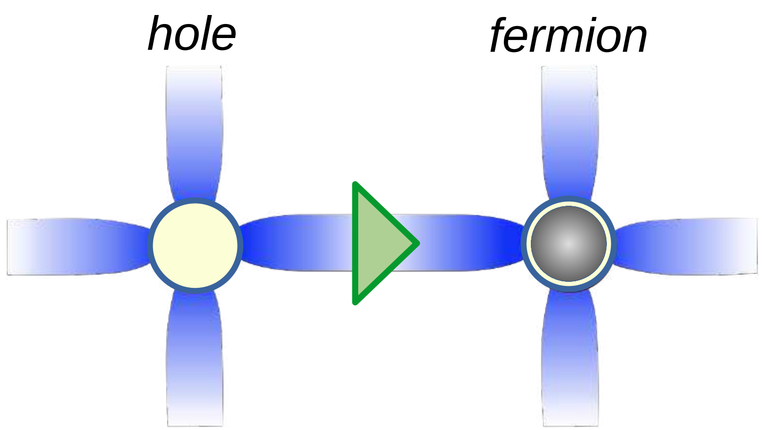


Quantum of Electric Field

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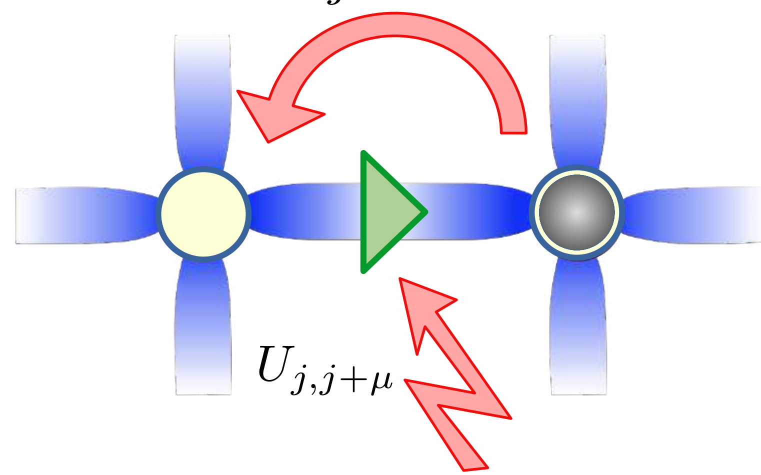
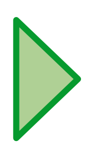
$$\psi_j^\dagger U_{j,j+\mu} \psi_{j+\mu}$$

While the fermion hops...

$$\psi_j^\dagger \psi_{j+\mu}$$




...the gauge field updates



Quantum of Electric Field

# Coupling to gauge fields

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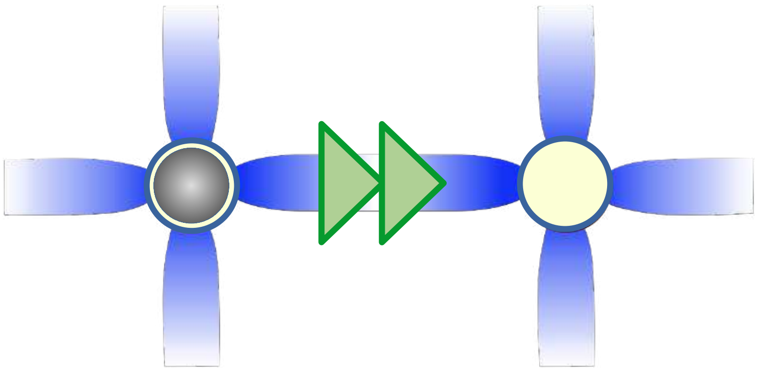

Quantum of Electric Field

# The Pure Gauge Hamiltonian

$$\frac{1}{2}(E^2 + B^2)$$

...for QED

# The Pure Gauge Hamiltonian

$$H = -t \sum_{x,\mu} \left( \hat{\psi}_x^\dagger \hat{U}_{x,\mu} \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$

$$+m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$

$$-\frac{g_m^2}{2} \sum_x \left( \hat{U}_{x,\mu_x} \hat{U}_{x+\mu_x,\mu_y} \hat{U}_{x+\mu_y,-\mu_x} \hat{U}_{x,-\mu_y} + \text{H.c.} \right)$$

$$\frac{1}{2}(E^2 + B^2)$$

Many ways to do this

Kogut, Susskind; PRD **11**, 395 (1975)

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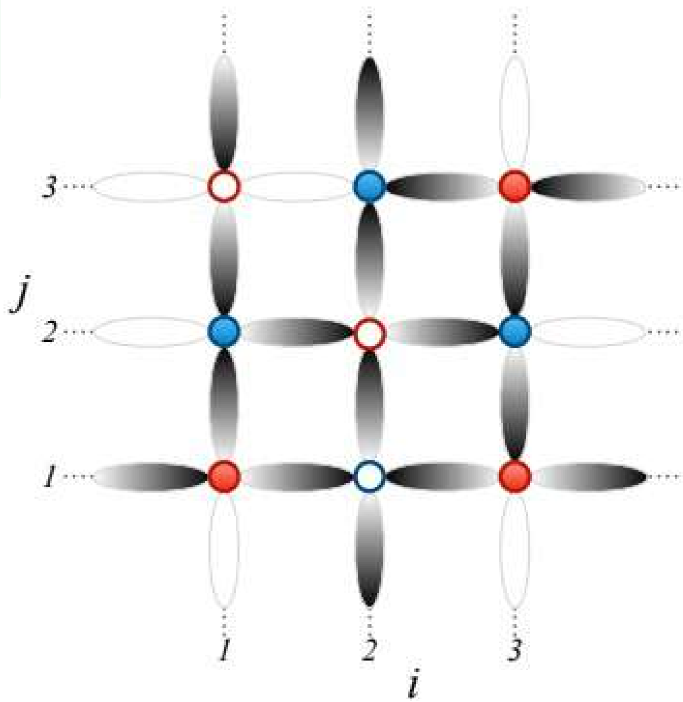
$$\frac{1}{2}(E^2 + B^2)$$

$E^2$ on-site term

$B^2$ plaquette term



# The Pure Gauge Hamiltonian

$$H = -t \sum_{x,\mu} \left( \hat{\psi}_x^\dagger \hat{U}_{x,\mu} \hat{\psi}_{x+\mu} + \text{H.c.} \right) + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 - \frac{g_m^2}{2} \sum_x \left( \hat{U}_{x,\mu_x} \hat{U}_{x+\mu_x,\mu_y} \hat{U}_{x+\mu_y,-\mu_x} \hat{U}_{x,-\mu_y} + \text{H.c.} \right)$$

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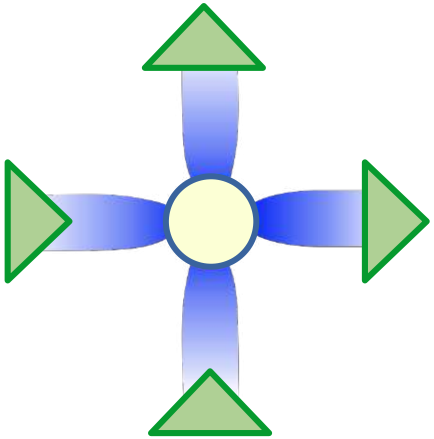
Gauge Symmetry: PROTECTED

**QED**
(Abelian)

$$\vec{\nabla} \cdot \vec{E} \propto \rho$$



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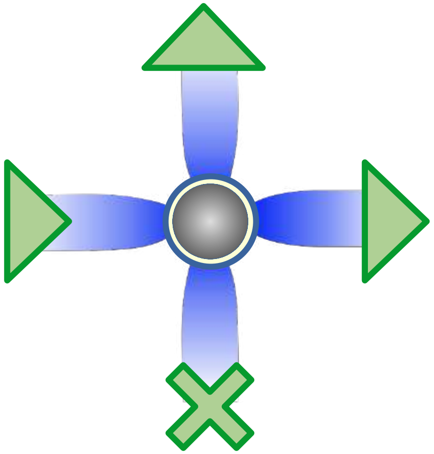

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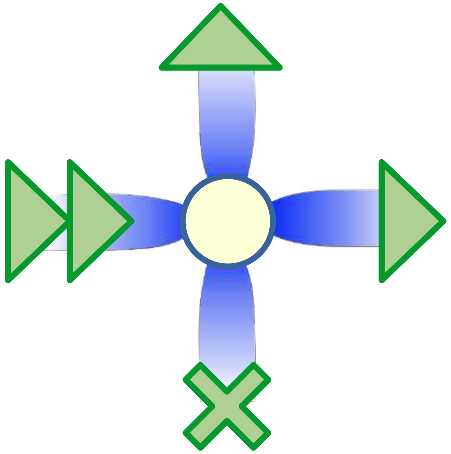
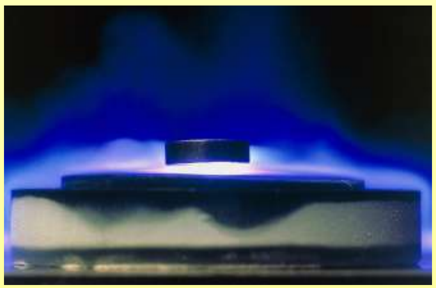
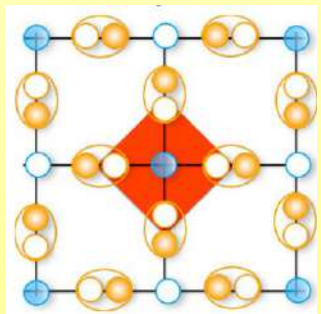


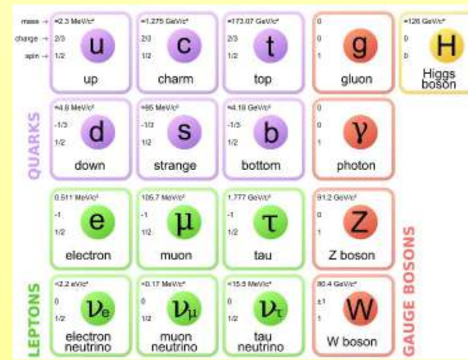
LGT are almost everywhere in theoretical physics!

As emergent theories in condensed matter: **high-Tc superconductors**, **frustrated systems, spin liquids.**





As fundamental description in particle physics: **Standard Model**




- They are **<u>extremely demanding</u>** from a numerical point of view.
- Powerful numerical methods, such as Monte Carlo, fail in several regimes of finite-density or for non-equilibrium phenomena (sign-problem).
- Ideal goal for quantum-inspired efficient algorithms and quantum simulation/computation!

# Quantum Technologies for LGT

Review 2020

Eur. Phys. J. D (2020) 74: 165
https://doi.org/10.1140/epjd/e2020-100571-8

THE EUROPEAN PHYSICAL JOURNAL D

Colloquium

## Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

Roadmap 2024

PRX QUANTUM 5, 037001 (2024)

Roadmap

## Quantum Computing for High-Energy Physics: State of the Art and Challenges

Alberto Di Meglio,[1,*] Karl Jansen,[2,3,†] Ivano Tavernelli,[4,‡] Constantia Alexandrou,[3,5] Srinivasan Arunachalam,[6] Christian W. Bauer,[7] Kerstin Borras,[8,9] Stefano Carrazza,[1,10] Arianna Crippa,[2,11] Vincent Croft,[12] Roland de Putter,[6] Andrea Delgado,[13] Vedran Dunjko,[12] Daniel J. Egger,[4] Elias Fernández-Combarro,[14] Elina Fuchs,[1,15,16] Lena Funcke,[17] Daniel González-Cuadra,[18,19] Michele Grossi,[1] Jad C. Halimeh,[20,21] Zoë Holmes,[22] Stefan Kühn,[2] Denis Lacroix,[23] Randy Lewis,[24] Donatella Lucchesi,[1,25] Miriam Lucio Martinez,[26,27] Federico Meloni,[8] Antonio Mezzacapo,[6] Simone Montangero,[1,25] Lento Nagano,[28] Vincent R. Pascuzzi,[6] Voica Radescu,[29] Enrique Rico Ortega,[30,31,32,33] Alessandro Roggero,[34,35] Julian Schuhmacher,[4] Joao Seixas,[36,37,38] Pietro Silvi,[1,25] Panagiotis Spentzouris,[39] Francesco Tacchino,[4] Kristan Temme,[6] Koji Terashi,[28] Jordi Tura,[12,40] Cenk Tüysüz,[2,11] Sofia Vallecorsa,[1] Uwe-Jens Wiese,[41] Shinjae Yoo,[42] and Jinglei Zhang[43,44]

Let us see a few *Selected Highlights*

Digital Quantum Simulation

Analog Quantum Simulation

Quantum-Inspired Simulation

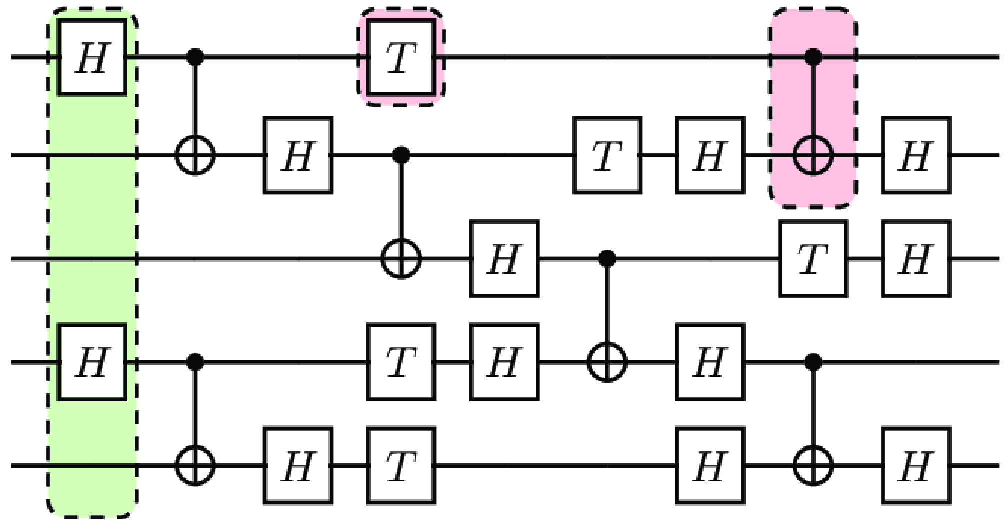
**Digital Quantum Simulation**

Analog Quantum Simulation

Quantum-Inspired Simulation

# The Digital Quantum Simulation Paradigm

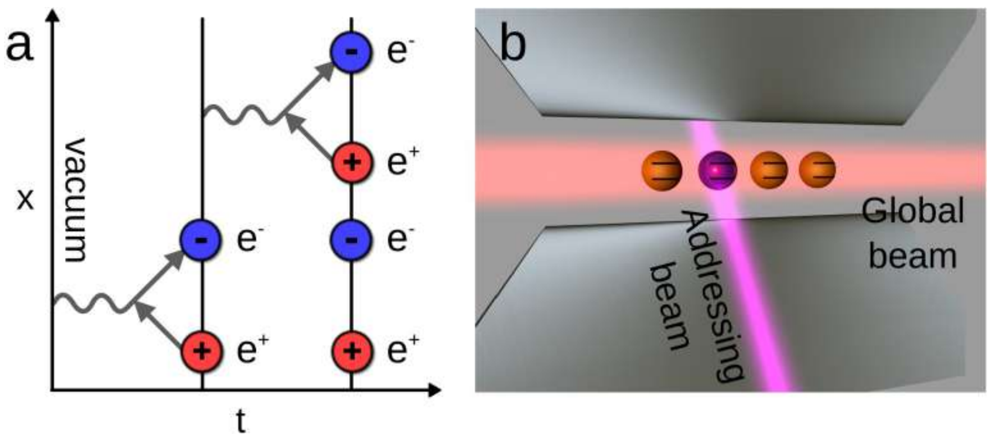
$$\exp\left(-\frac{it}{\hbar}H_{\mathrm{LGT}}\right) =$$

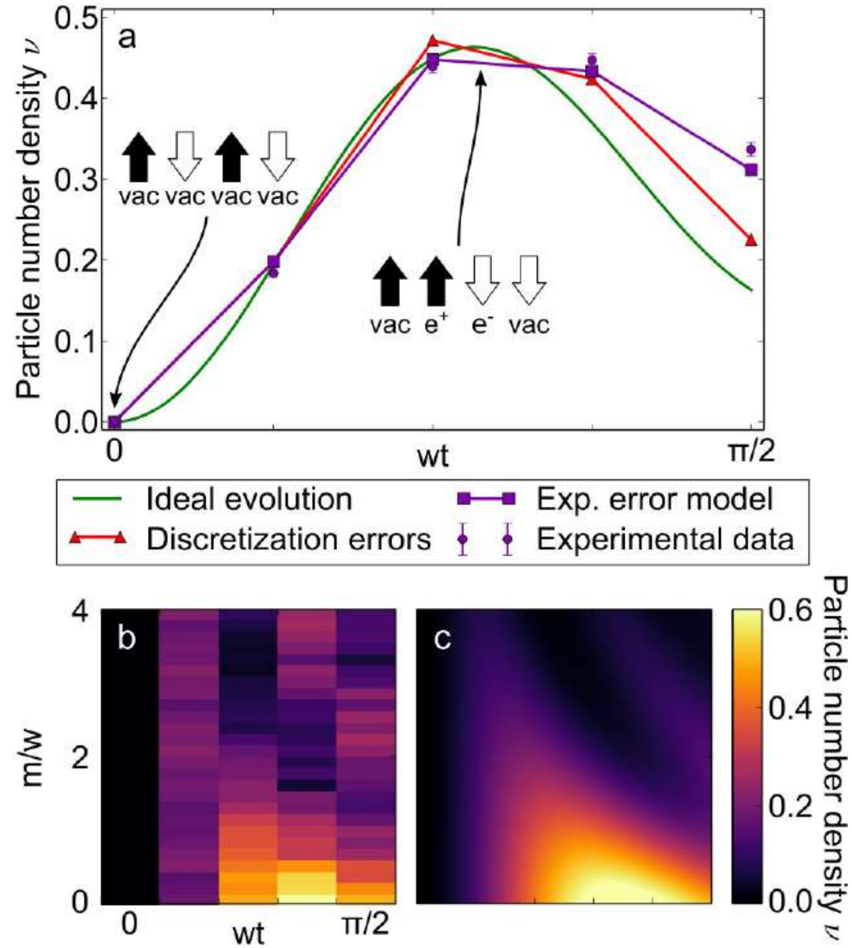



Using the natural quantum processing resources of your quantum hardware

- Flexible
  Ideal for Quantum Computers
- Expensive
  Prone to Errors

# Experiment: Digital Qsim of QED1D (2016)



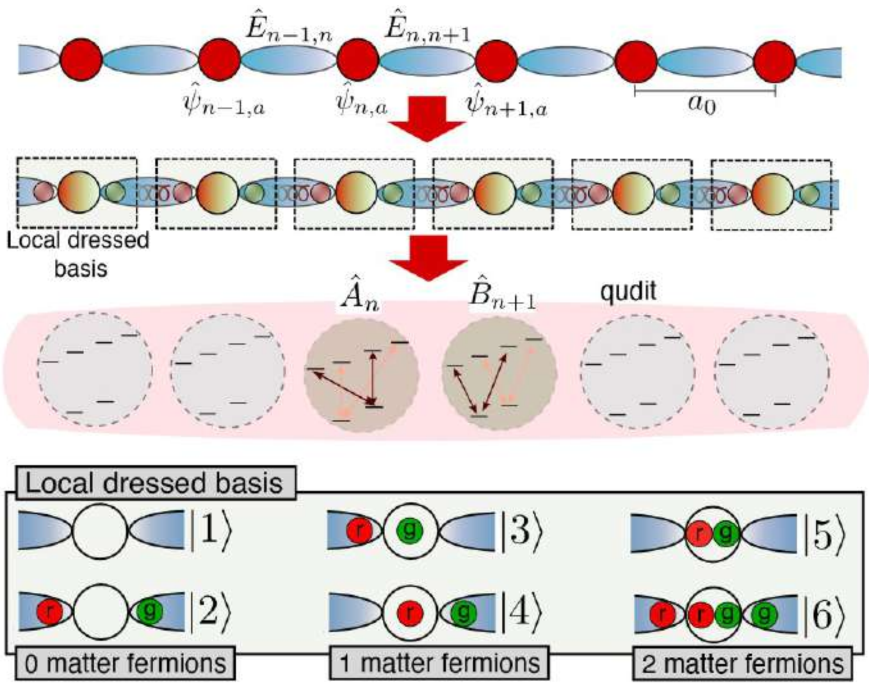

Hardware: Trapped Ion qubits ($^{40}Ca^{+}$)

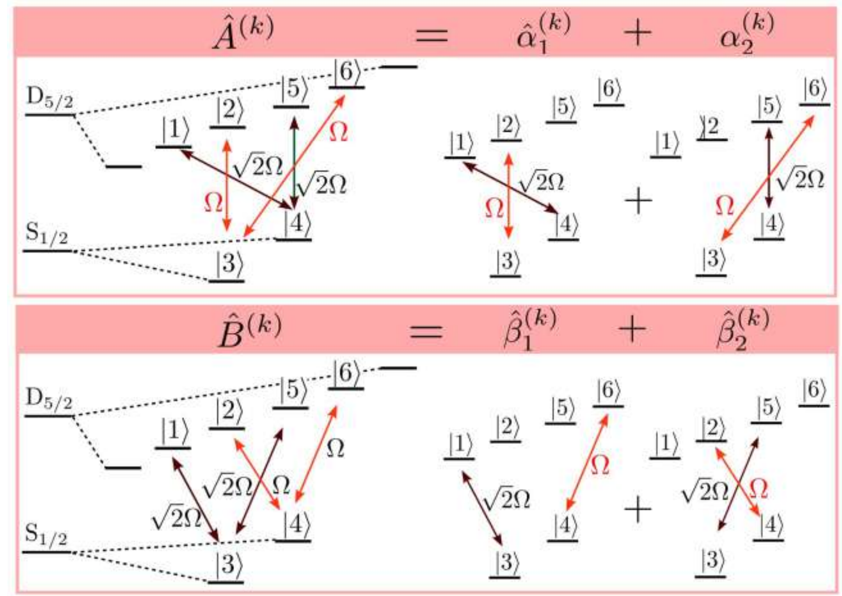
Details: Gauge fields integrated out (One-dimension only)

Result: Bare vacuum oscillations

Nature **534**, 516 (2016)

# Design: Digital Qsim of SU(2) YM 1D (2024)

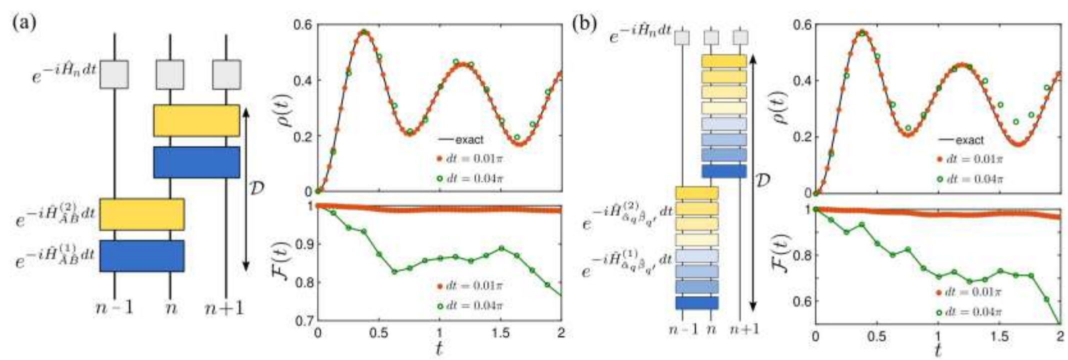



<u>Hardware</u>: Trapped Ion *qudits*

<u>Details</u>: The qudit encodes both matter and gauge fields

<u>Result</u>: Baryon-Meson dynamics,
Baryon string breaking




PRX Quantum **5**, 040309 (2024)

Digital Quantum Simulation

**Analog Quantum Simulation**

Quantum-Inspired Simulation

# The Analog Quantum Simulation Paradigm

$$\exp\left(-\frac{it}{\hbar}H_{\mathrm{LGT}}\right) = \exp\left(-\frac{it}{\hbar}H_{\mathrm{Experiment}}\right)$$

for some specific time *t,*
or from some timescale $t > t_0$

Engineering the desired “instantaneous” dynamics onto a specific Q-hardware

Efficient use of resources
Faster, thus more coherent

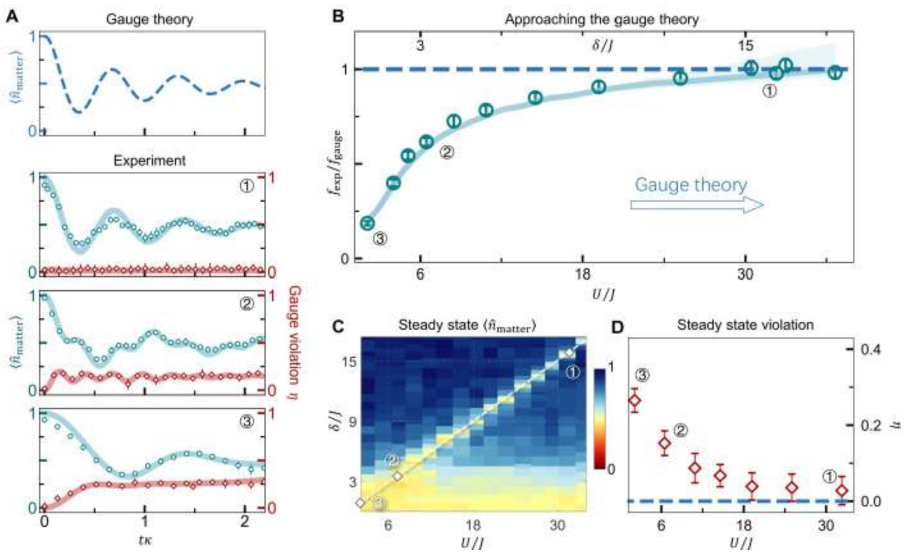
Non-Universal
3-Body interactions are unnatural

# Experiment: Analog Qsim of QED1D (2022)

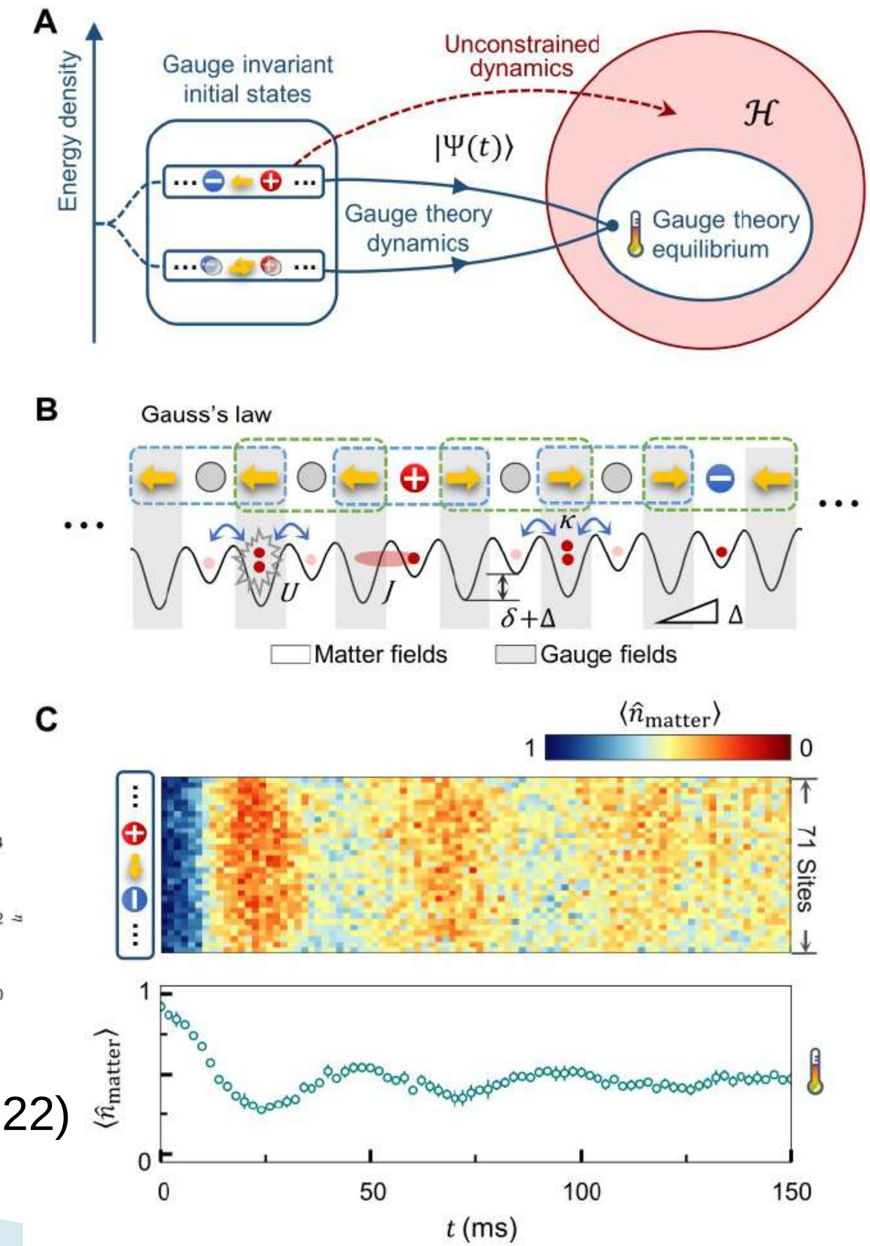
Hardware: Bosonic atoms ($^{87}Rb$) on multimode optical lattice

Details: Two sublattices encode gauge and matter fields resp.

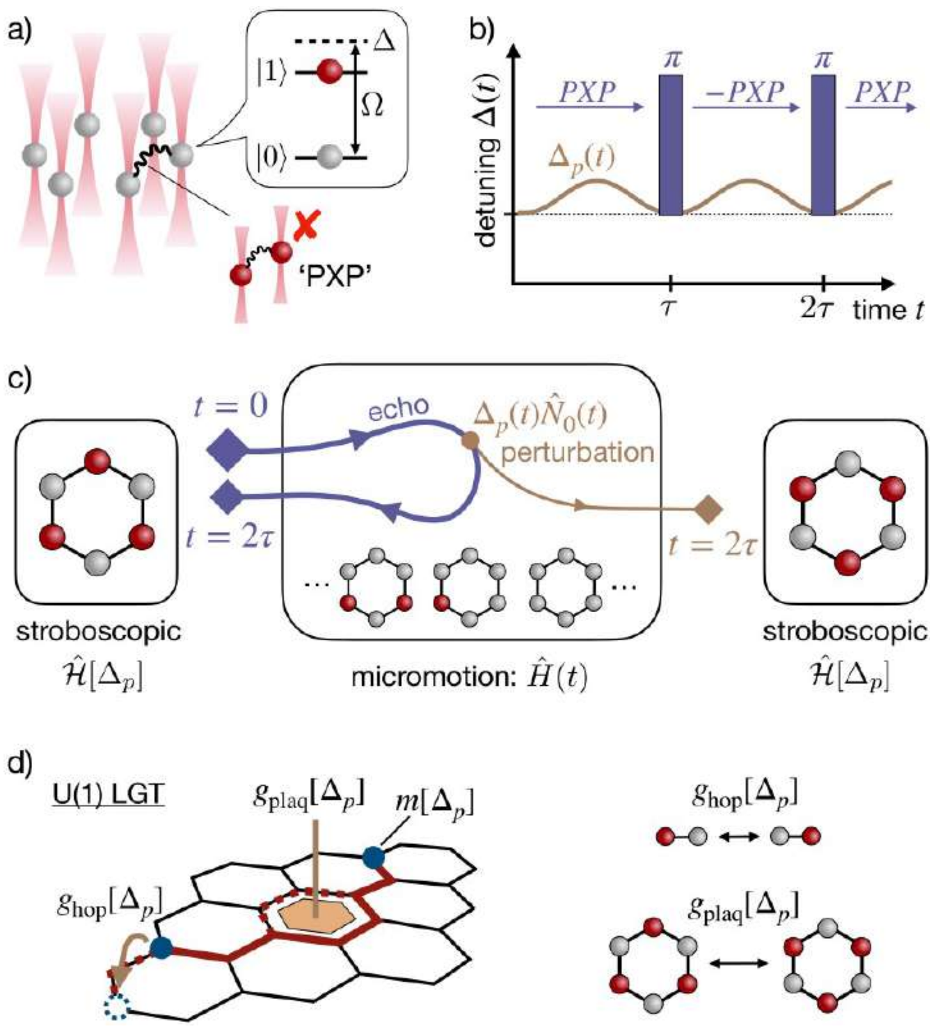
Result: Emergent Thermalization Dynamics

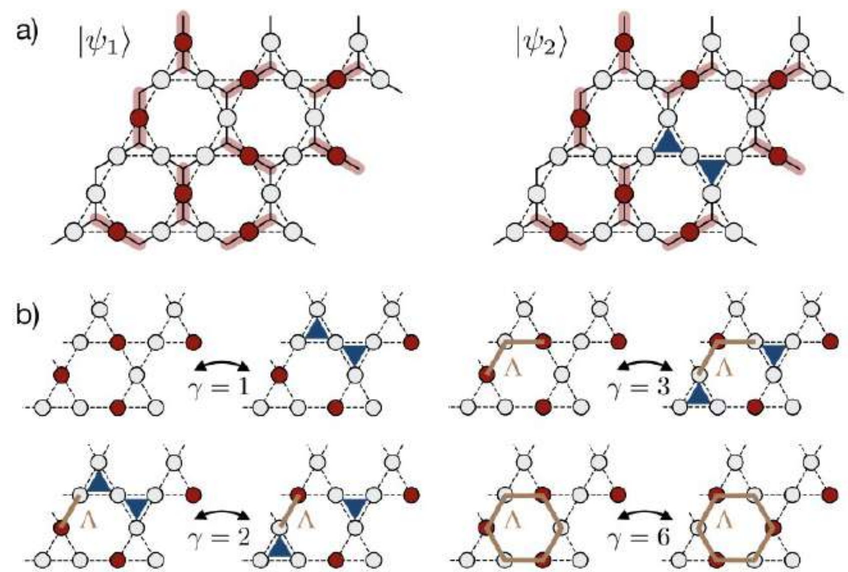



Science **377**, 311 (2022)



# Design: Analog Qsim of Kagome-PXP (2024)




<u>Hardware</u>: Rydberg Atoms in optical tweezer arrays

<u>Details</u>: the PXP model is a U(1) gauge (one excitation per triangle), Floquet protocol for 3-body terms

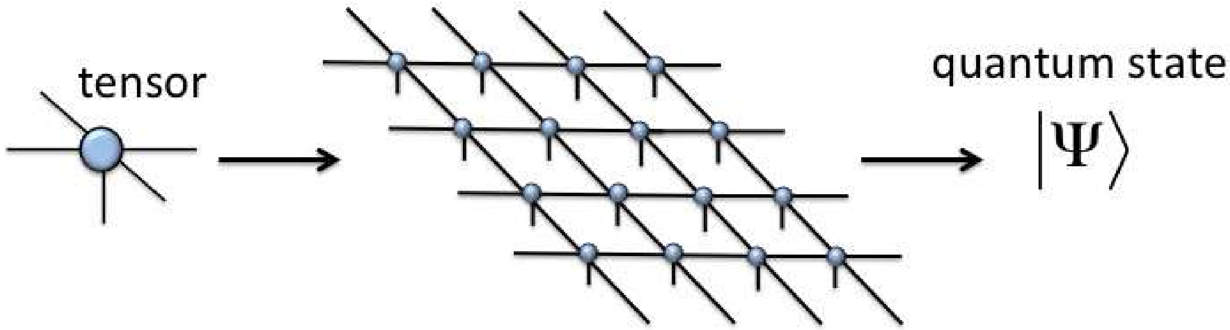
*arXiv:2408.02733* (2024)

Digital Quantum Simulation

Analog Quantum Simulation

**Quantum-Inspired Simulation**

# Tensor Networks




The wave function is described by a network of interconnected tensors.

The network pattern represents directly the amount of entaglement of the state.

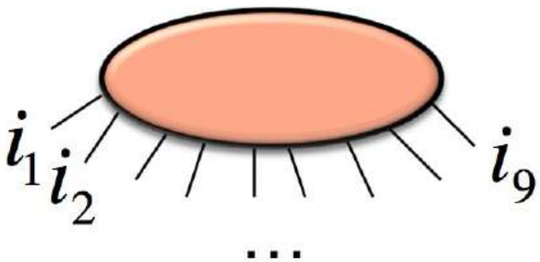
R. Orus, Annals of Physics 349 (2014) 117-158

$$|\Psi\rangle = \sum_{i_1 i_2 .. i_N} C_{i_1 i_2 \dots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

d-level systems

Tensor (multidimensional array of complex numbers)

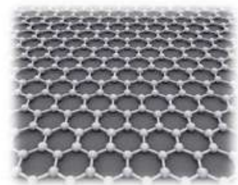
$C_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$

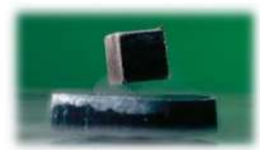



representation, exponentially large in the system size. Inefficient.

**System of $N$ spins 1/2**

Everyday material, $N$ ~ Avogadro number ~ $O(10^{23})$

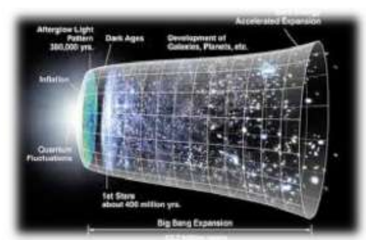




**Number of basis states in the Hilbert space ~ $O(10^{10^{23}})$**

Compare to…

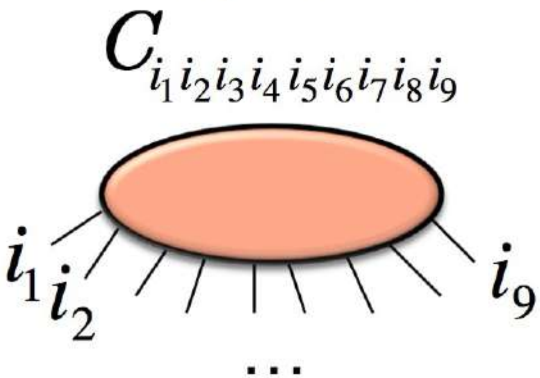
**Number atoms in the observable universe ~ $O(10^{80})$**



$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} C_{i_1 i_2 \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

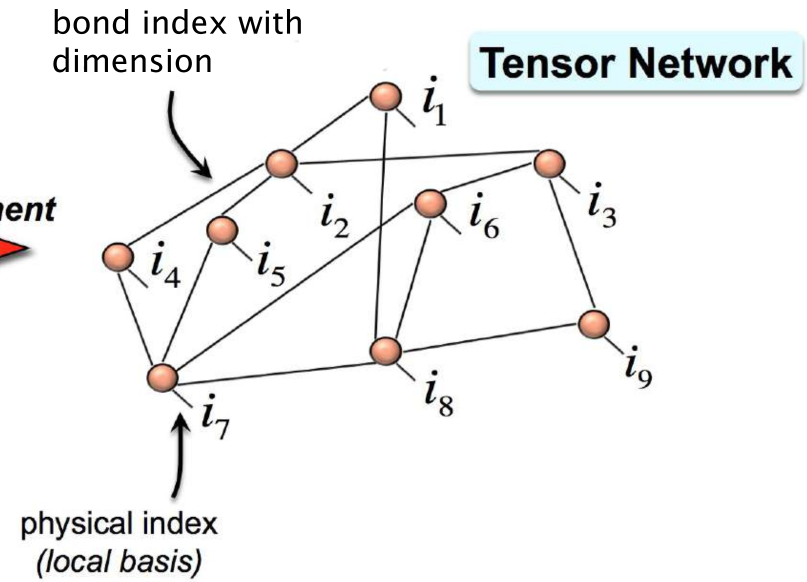
d-level systems

Tensor (multidimensional array of complex numbers)



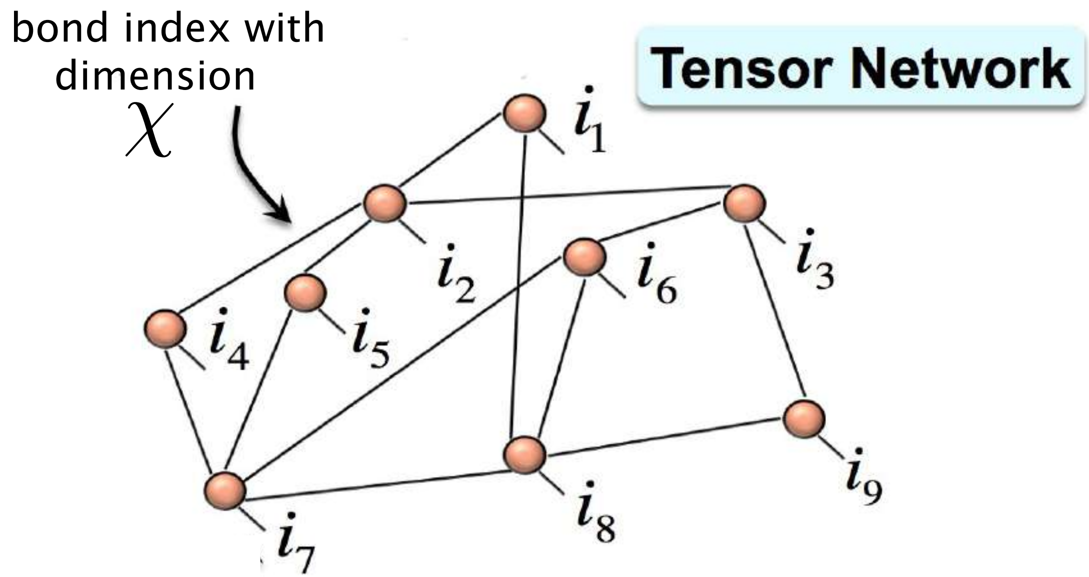

***replacement***

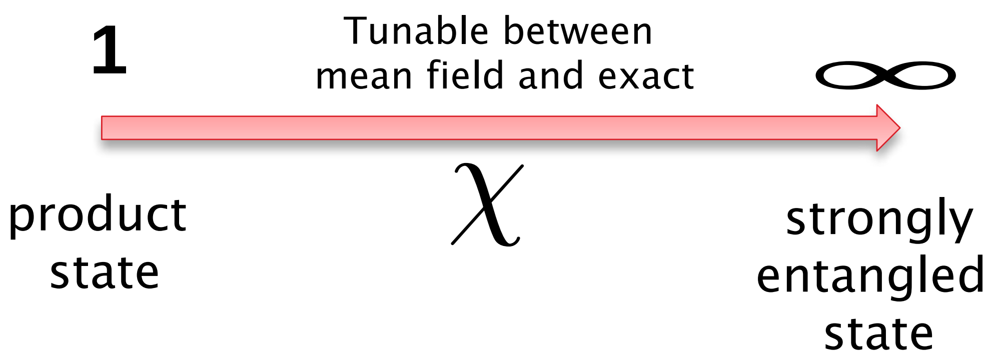
multiple SVD



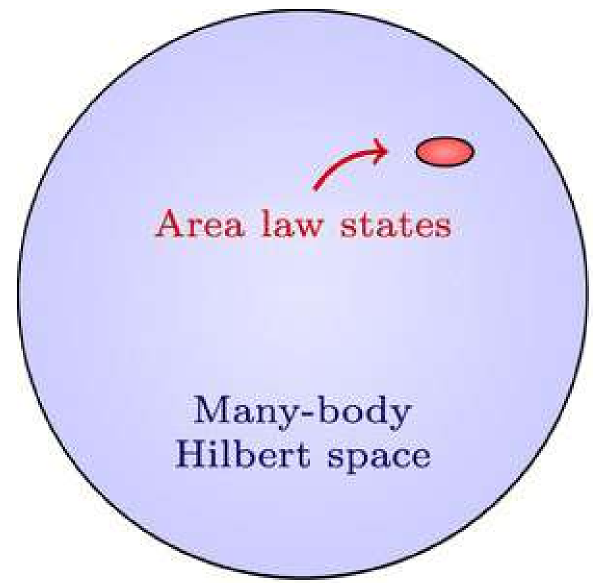

representation, exponentially large in the system size. Inefficient.

Physical indices with dimension $d$
Bond indices with dimension $\chi$
The number of parameters is



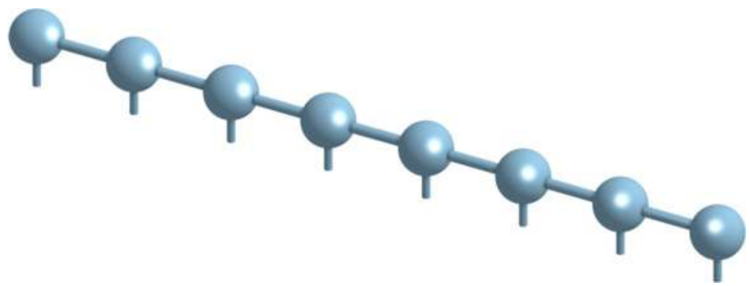




Equilibrium States




Rev. Mod. Phys. 82, 277 (2010)

# Examples



Matrix Product States (MPS)

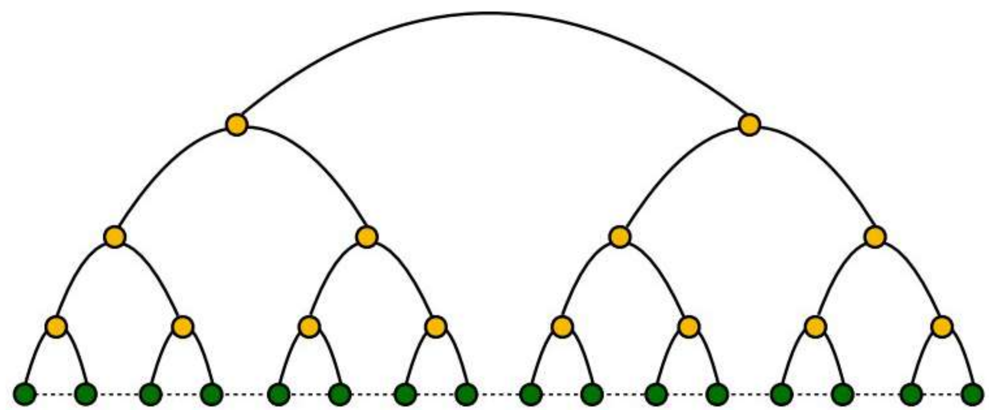
minimize $E = \langle \psi | H | \psi \rangle$

$O(\chi^3)$

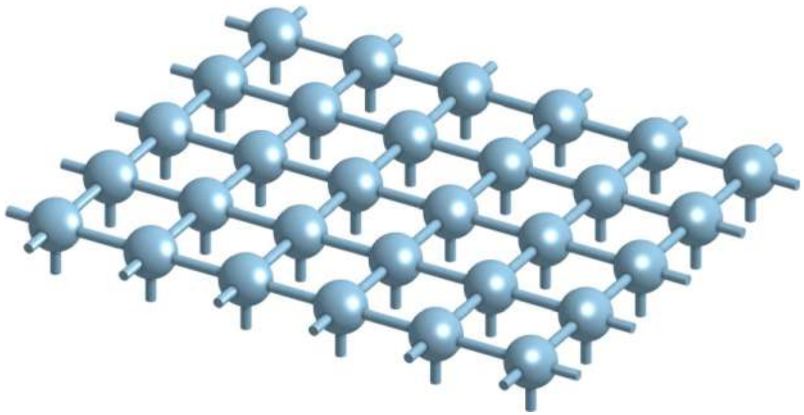
Tree Tensor Networks (TTN)

$O(\chi^4)$

- strong connectivity
- distance between two lattice sites scales logarithmically within the network



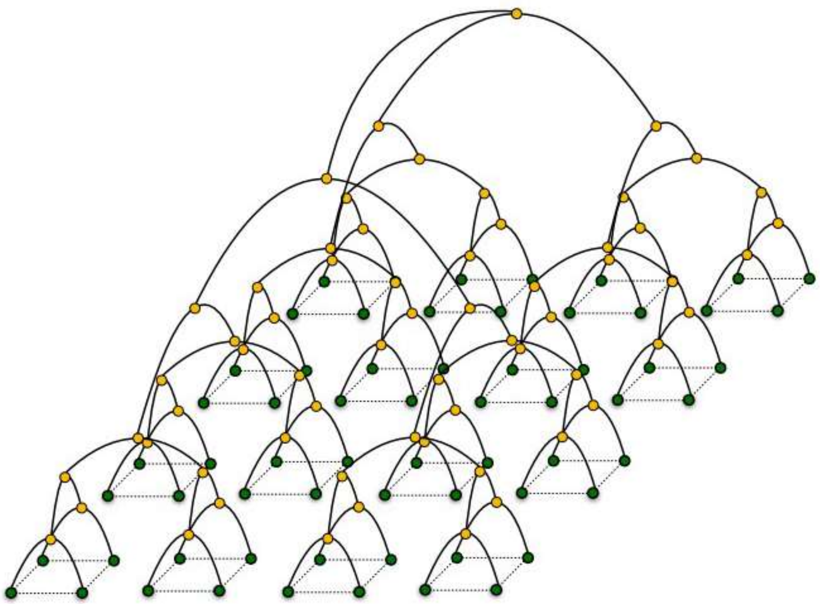
## Projected Entangled Pair States (PEPS)



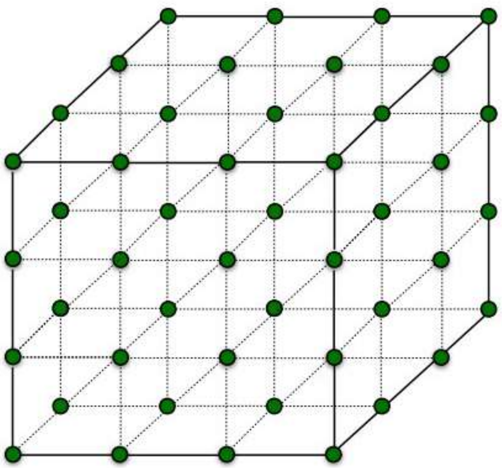
- they automatically reproduce the area-law of entanglement
- the optimization has a high complexity

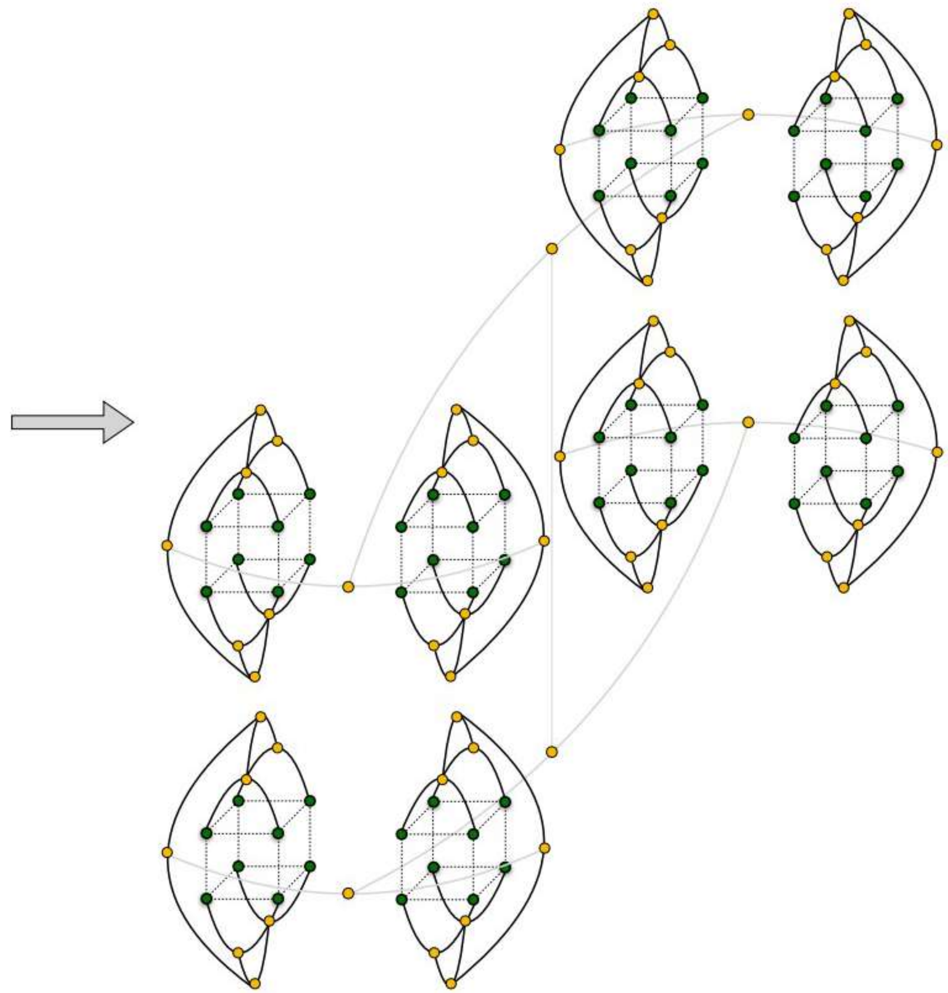
## Tree Tensor Networks (TTN)

- they do not automatically reproduce the area-law of entanglement
- the optimization has a poly complexity



# Tree Tensor Networks in 3D





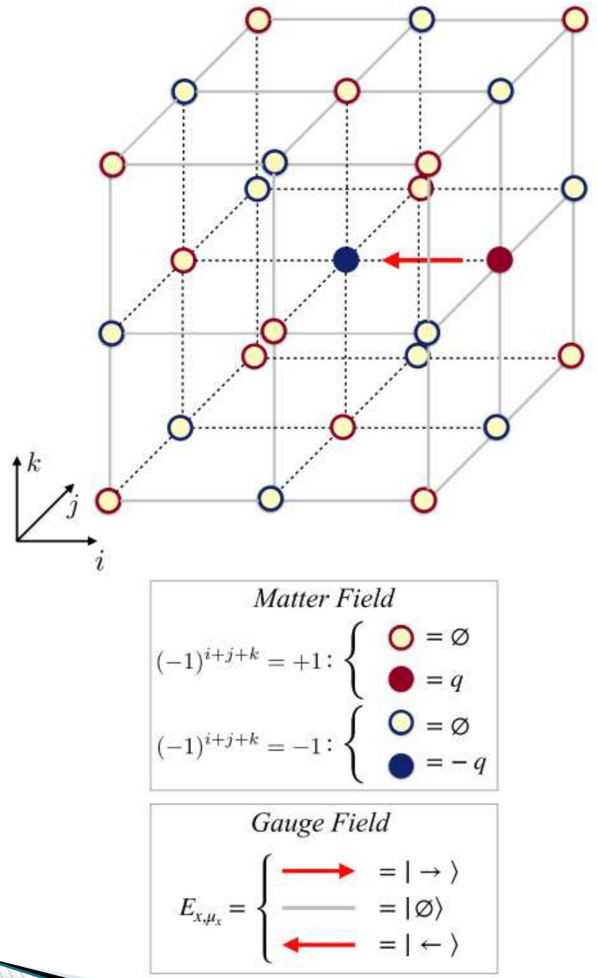
minimize

$$E = \langle \psi \,|H|\, \psi \rangle$$

Sign-problem-free approach!

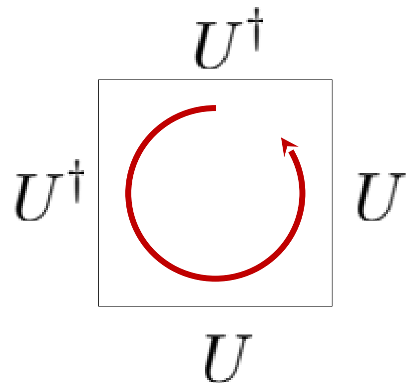
The optimization has polynomial complexity

# Lattice QED in (3+1)D

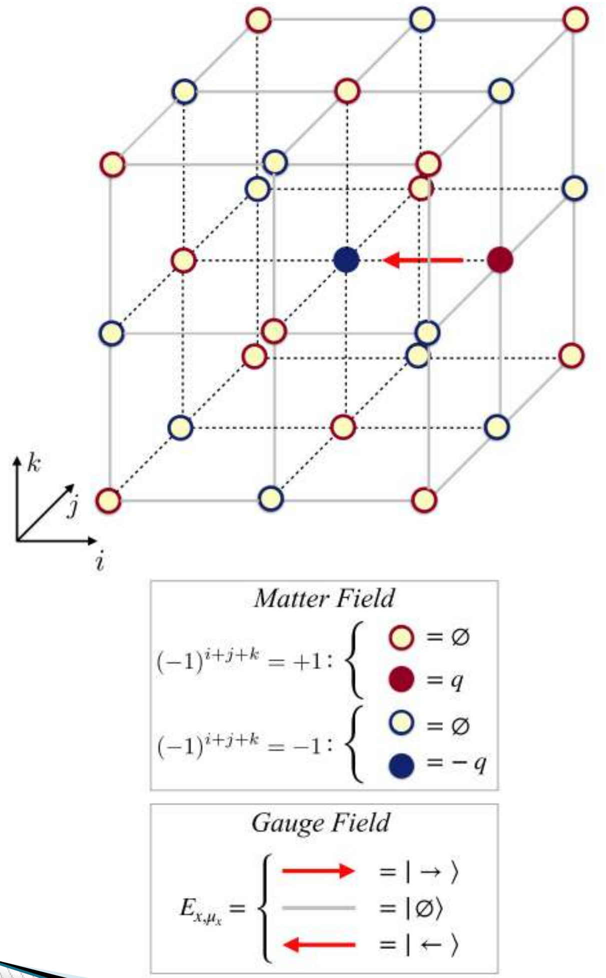



$$\hat{H} = -t \sum_{x,\mu} \left( \hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$
$$+m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$
$$-\frac{g_m^2}{2} \sum_x \left( \Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

$$\Box_{\mu_x,\mu_y} = U_{x,\mu_x} U_{x+\mu_x,\mu_y} U_{x+\mu_y,\mu_x}^\dagger U_{x,\mu_y}^\dagger$$



# Lattice QED in (3+1)D




$$\hat{H} = -t \sum_{x,\mu} \left( \hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right) + m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 - \frac{g_m^2}{2} \sum_x \left( \Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

Quantum Link Model
discretization of Gauge Fields

$$\hat{E}_{x,\mu} \rightarrow \hat{S}^z_{x,\mu}$$
$$\hat{U}_{x,\mu} \rightarrow \hat{S}^+_{x,\mu}/s,$$

Nature Communications **12**, 3600 (2021)

# Confinement Properties

Plaquette terms | Electric Field Energy

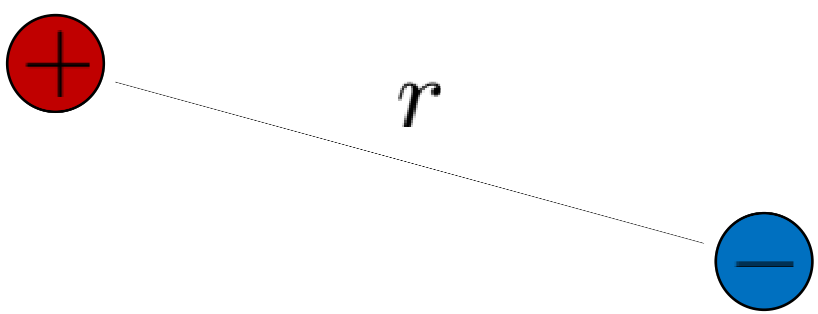
Weak-coupling → Strong-coupling $g_e$

$$V(r) \propto 1/r$$

Deconfined phase

$$V(r) \propto r$$

Confined phase




Kei-Ichi Kondo, Phys. Rev. D **58**, 085013 (1998)

# Confinement Properties

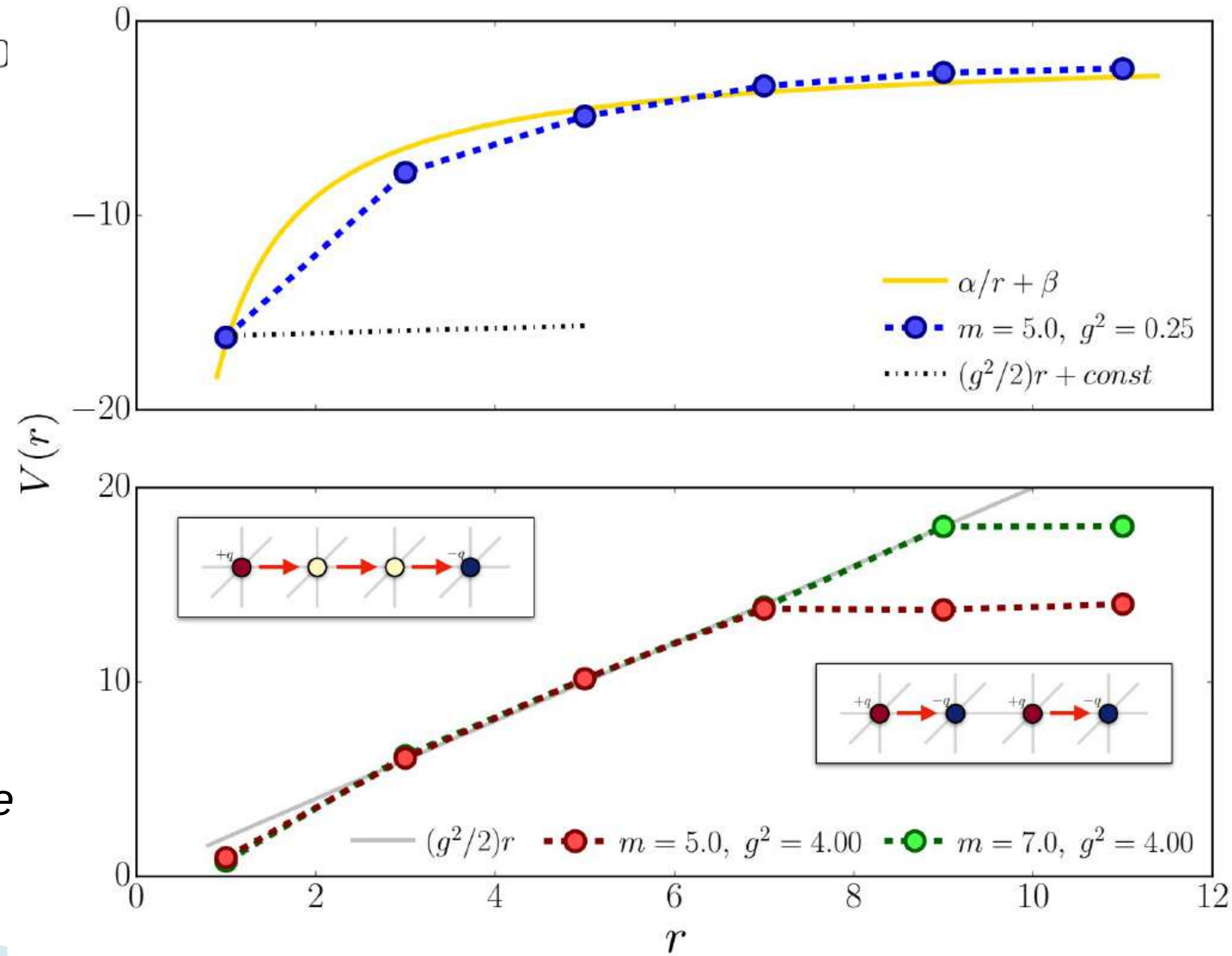
$$V(r) = E(r) - E_0$$

Weak-coupling

*Coulomb Regime*

Strong-coupling

*String-Break Regime*



# Take-Home Message

Quantum Science is useful to HEP-Theory because it can provide **Many-Body Quantitative Predictions** of Quantum Field Theories beyond traditional methods (MC)

- Digital Quantum Simulation
- Analog Quantum Simulation
- Quantum-Inspired Simulation

# Take-Home Message

Quantum Science is useful to HEP-Theory because it can provide **Many-Body Quantitative Predictions** of Quantum Field Theories beyond traditional methods (MC)

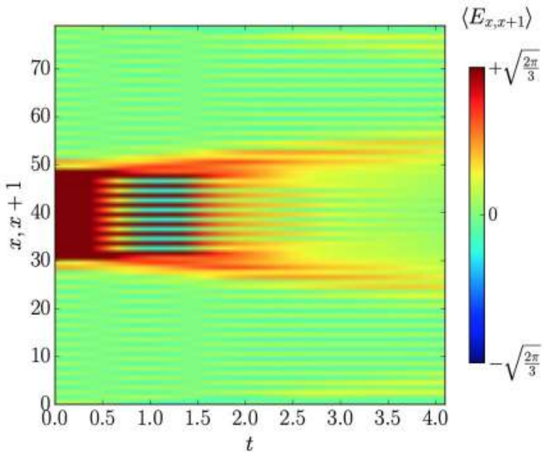
- Digital Quantum Simulation
- Analog Quantum Simulation
- Quantum-Inspired Simulation

**THANK YOU**

# Backup Slides

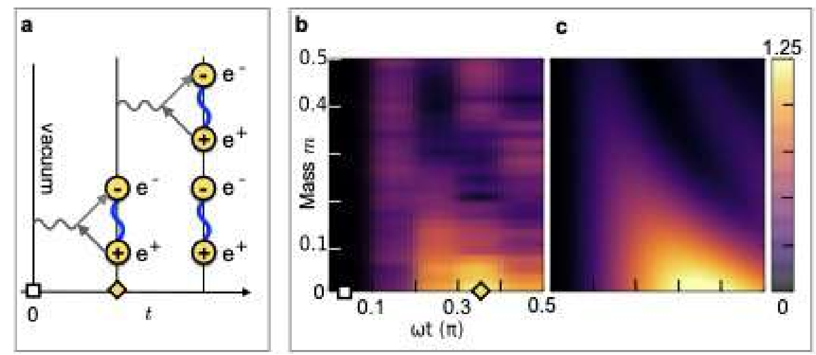
# Quantum Technologies for LGT

Efficient quantum-inspired algorithms:
Tensor Networks (no sign-problem)




Quantum **4**, 281 (2020)

First implementation of U(1) LGT on
digital quantum computer



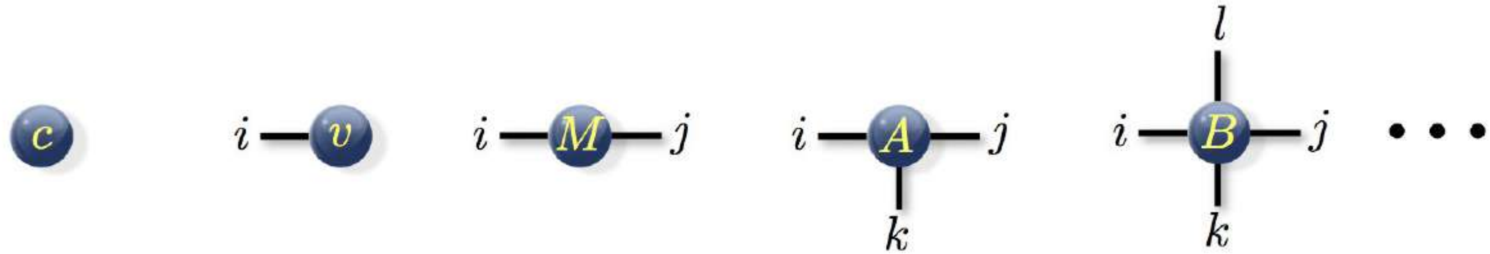

*Nature* **534,** 516–519 (2016).

**Simulating Lattice Gauge Theories within Quantum Technologies**

M.C. Bañuls[1,2], R. Blatt[3,4], J. Catani[5,6,7], A. Celi[3,8], J.I. Cirac[1,2], M. Dalmonte[9,10], L. Fallani[5,6,7], K. Jansen[11], M. Lewenstein[8,12,13], S. Montangero[7,14 a], C.A. Muschik[3], B. Reznik[15], E. Rico[16,17 b], L. Tagliacozzo[18], K. Van Acoleyen[19], F. Verstraete[19,20], U.-J. Wiese[21], M. Wingate[22], J. Zakrzewski[23,24], and P. Zoller[3]

Eur. Phys. J. D **74,** 165 (2020)

# Tensor Network: notation



| number | vector $v_i$ | matrix $M_{ij}$ | $A_{ijk}$ | $B_{ijkl}$ |
|---|---|---|---|---|
| rank-0 tensor | rank-1 tensor | rank-2 tensor | rank-3 tensor | rank-4 tensor |

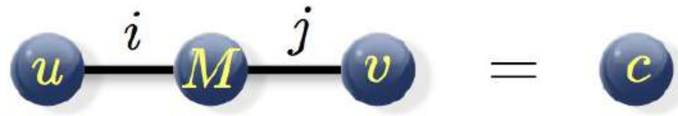
★ We don't need to write down formulas with tensors with many indices!

Example 1: 

$$\sum_j M_{ij} v_j = u_i$$

★ Connected lines: sum over corresponding indices!

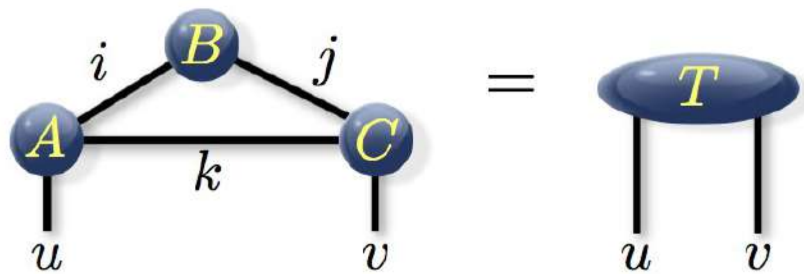
Example 2:




$$\sum_{ij} u_i M_{ij} v_j = c$$

★ sum over all connected indices: **contraction** of a tensor network

Example 3:




$$\sum_{ijk} A_{uik} B_{ij} C_{vjk} \quad = \quad T_{uv}$$

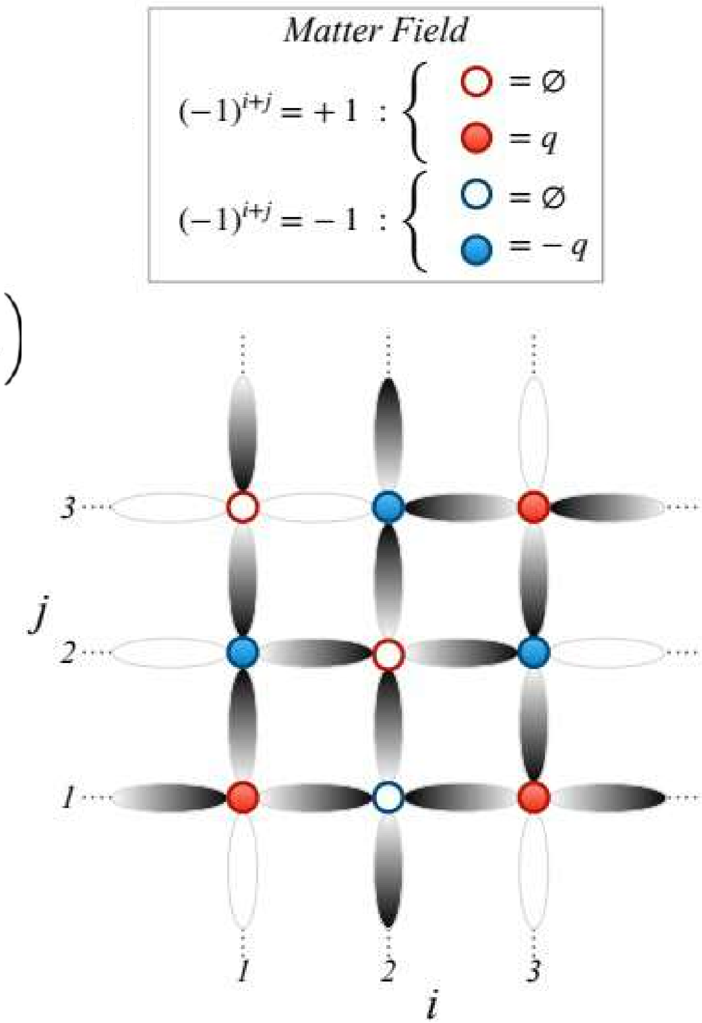
★ The rank of the resulting tensor corresponds to the number of open legs in the network

# Lattice QED in (2+1)D

$$H = -t \sum_{x,\mu} \left( \hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$

$$+ m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$

$$- \frac{g_m^2}{2} \sum_x \left( \hat{U}_{x,\mu_x} \hat{U}_{x+\mu_x,\mu_y} \hat{U}_{x+\mu_y,-\mu_x} \hat{U}_{x,-\mu_y} + \text{H.c.} \right)$$

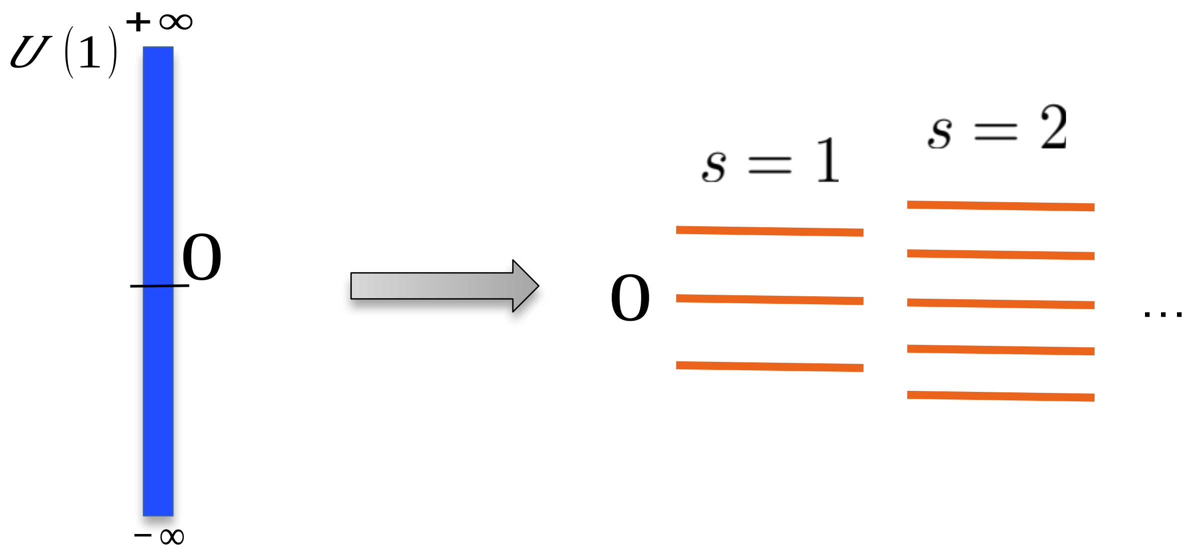


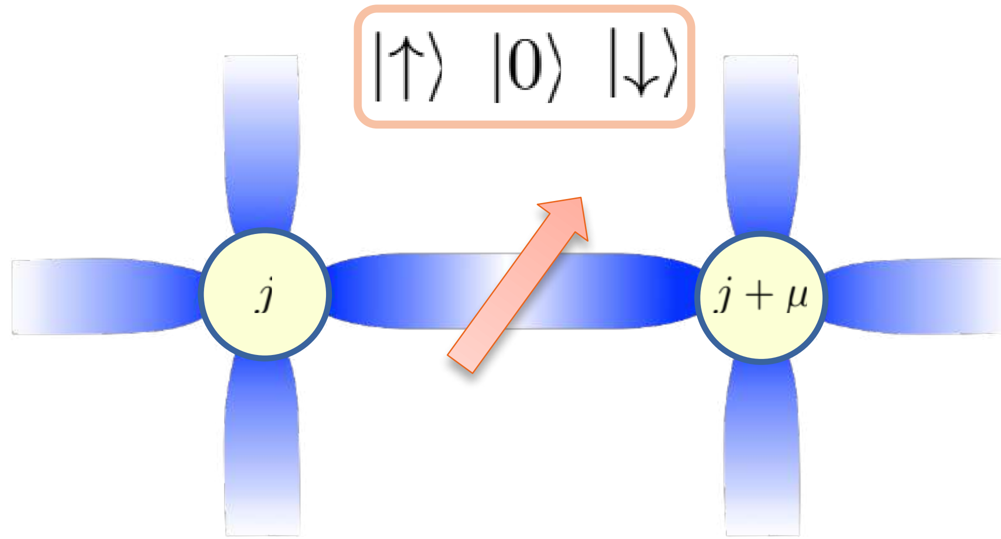

Quantum Link Model
discretization of Gauge Fields

$$\hat{E}_{x,\mu} \rightarrow \hat{S}_{x,\mu}^z$$

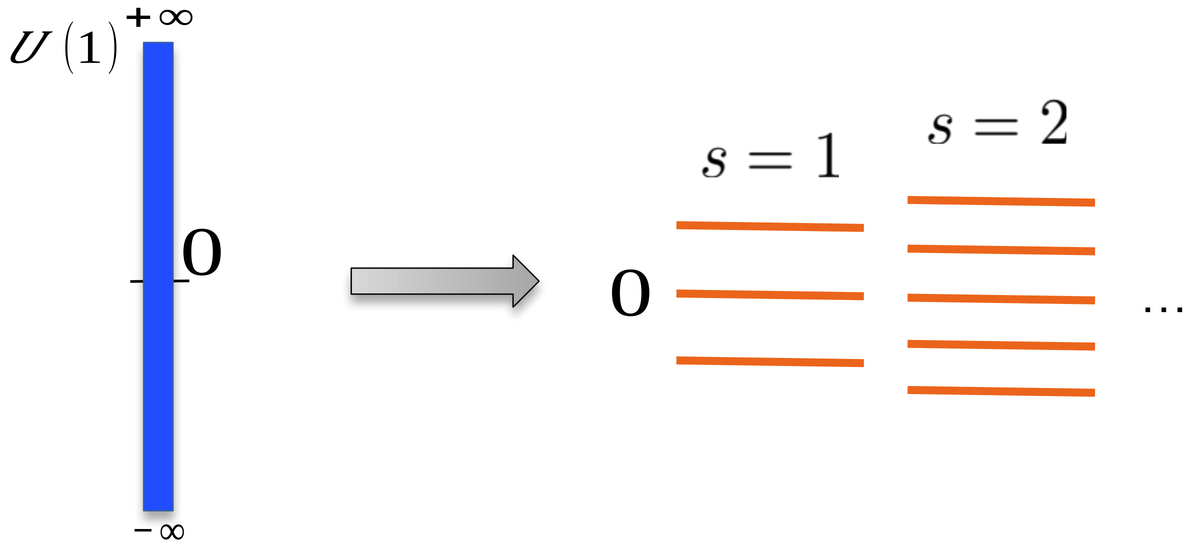
$$\hat{U}_{x,\mu} \rightarrow \hat{S}_{x,\mu}^+ / s,$$

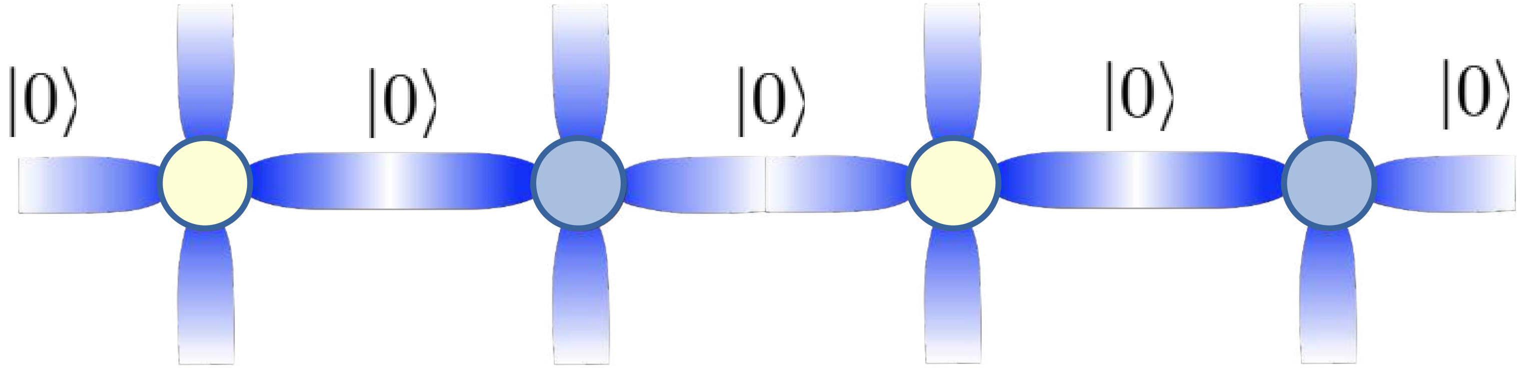
$$\hat{E}_{x,\mu} \to \hat{S}^z_{x,\mu} \qquad \hat{U}_{x,\mu} \to \hat{S}^+_{x,\mu}/s,$$



$$\hat{E}_{x,\mu} \to \hat{S}^z_{x,\mu} \qquad \hat{U}_{x,\mu} \to \hat{S}^+_{x,\mu}/s,$$



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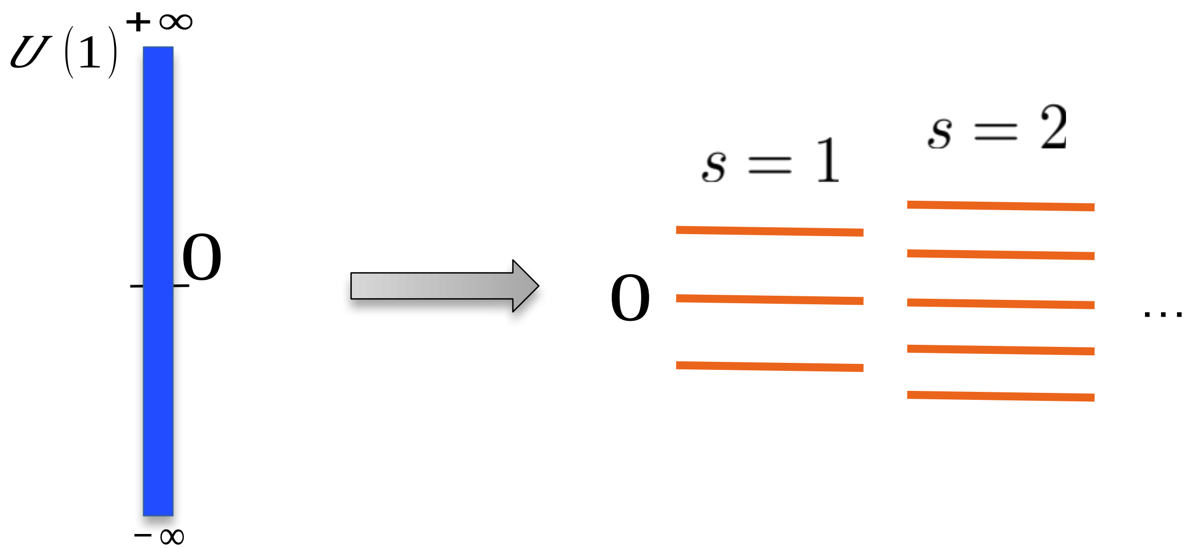


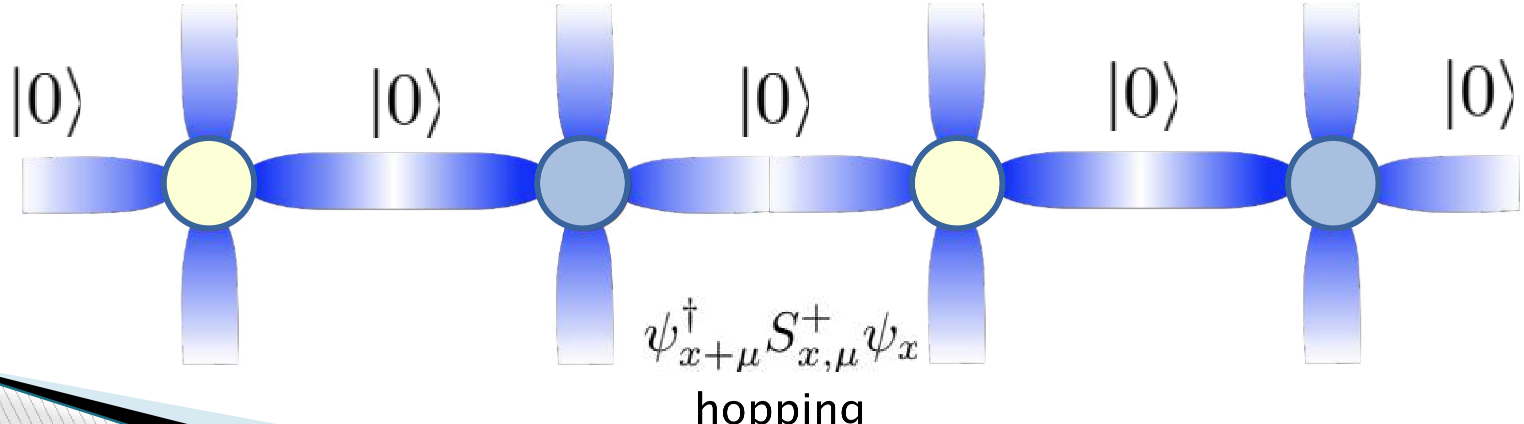

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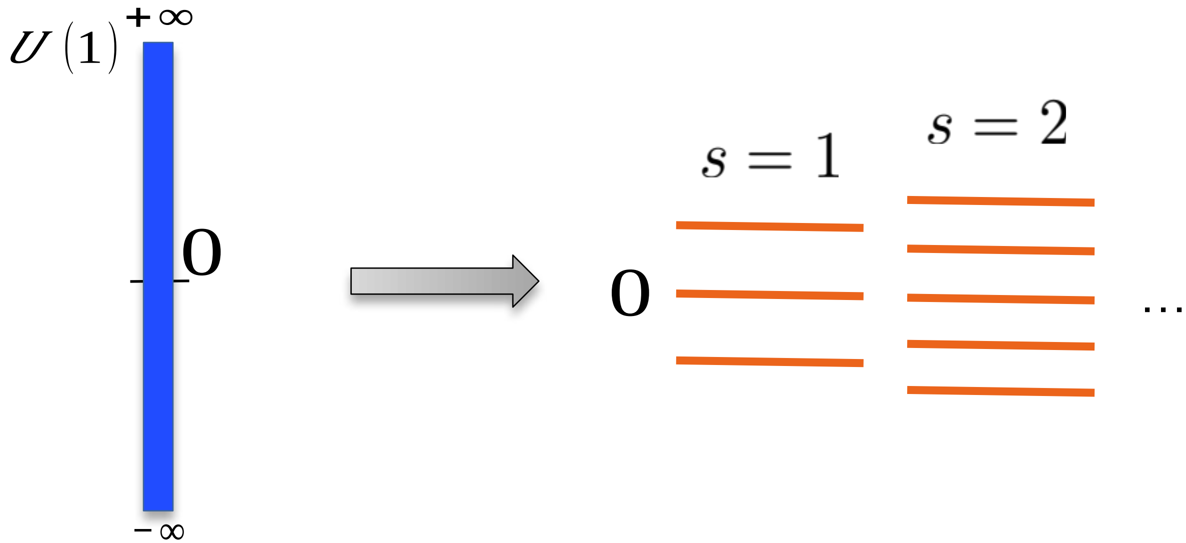


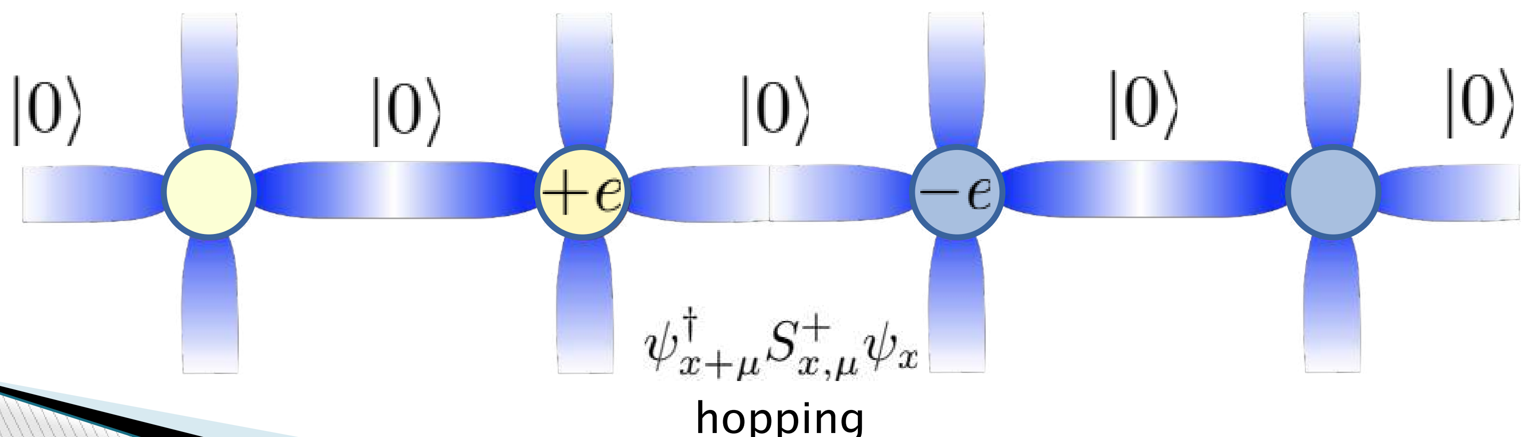

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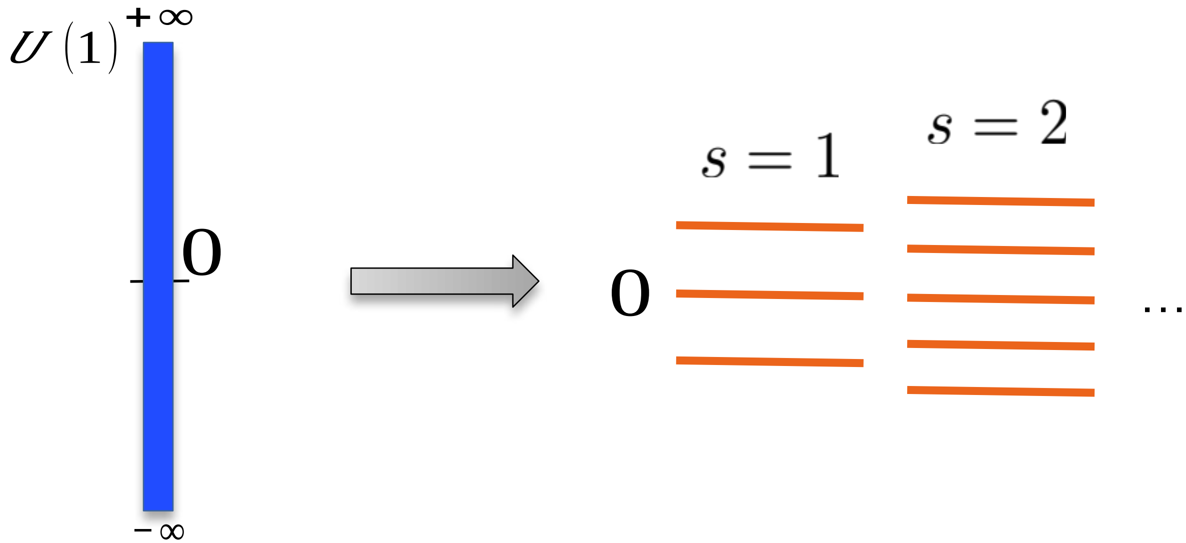
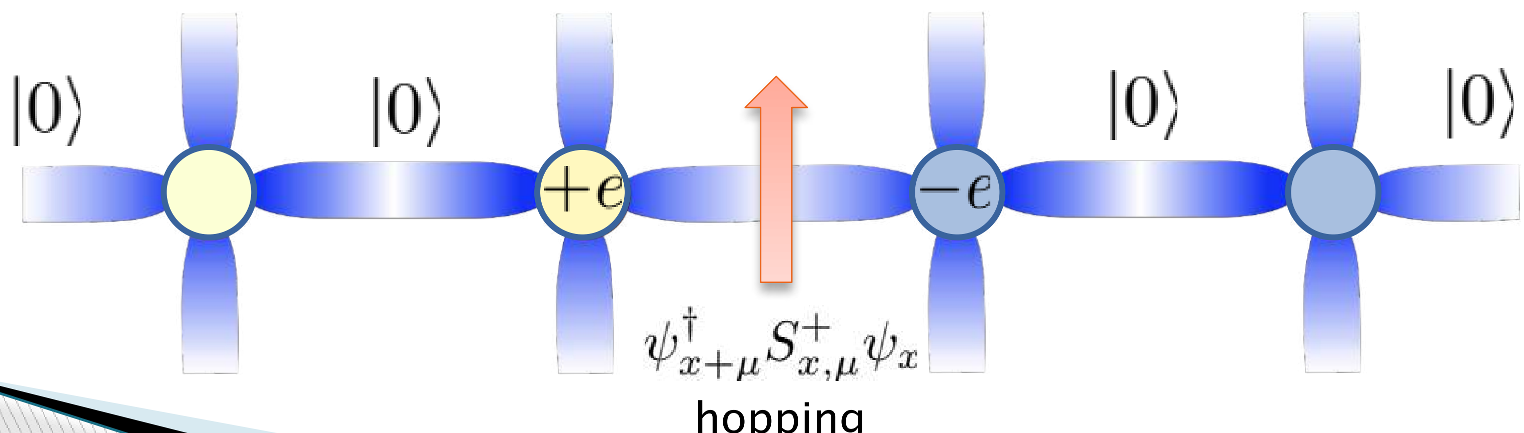
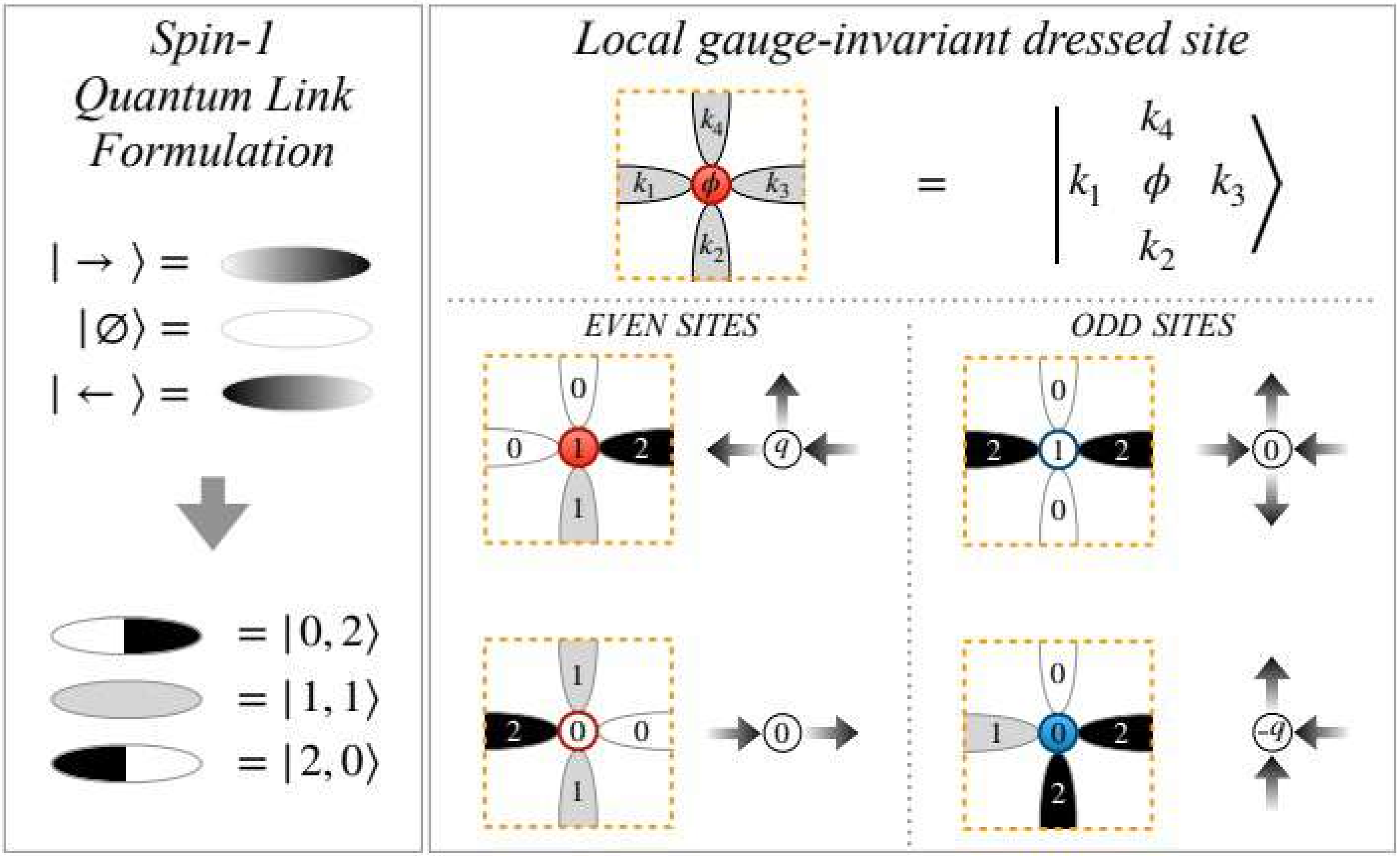


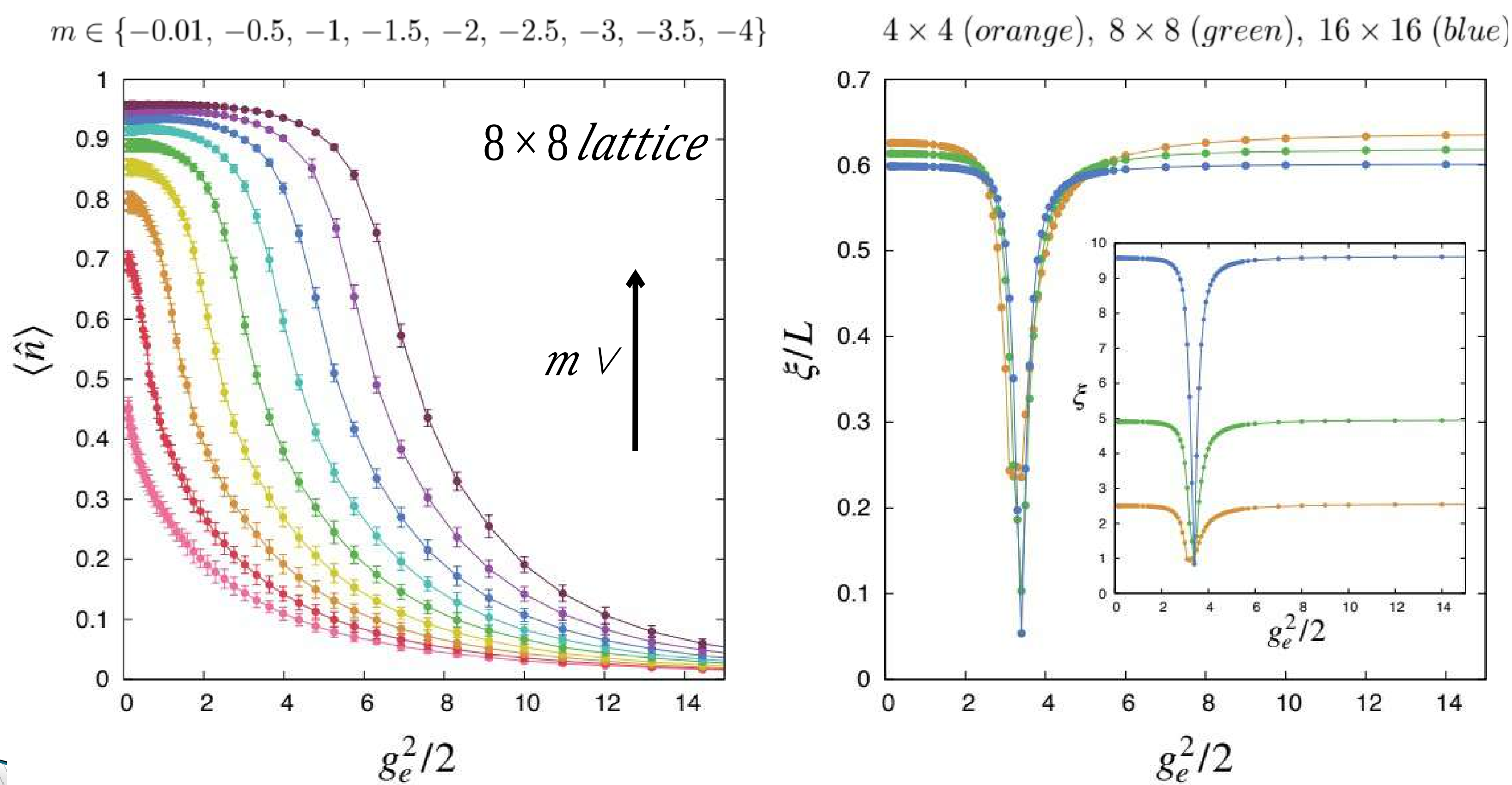



$$\hat{G}_x = \hat{\psi}_x^\dagger \hat{\psi}_x - \frac{1 - p_x}{2} - \sum_\mu \hat{E}_{x,\mu} \qquad \hat{G}_x|\Phi\rangle = 0 \; \forall x$$

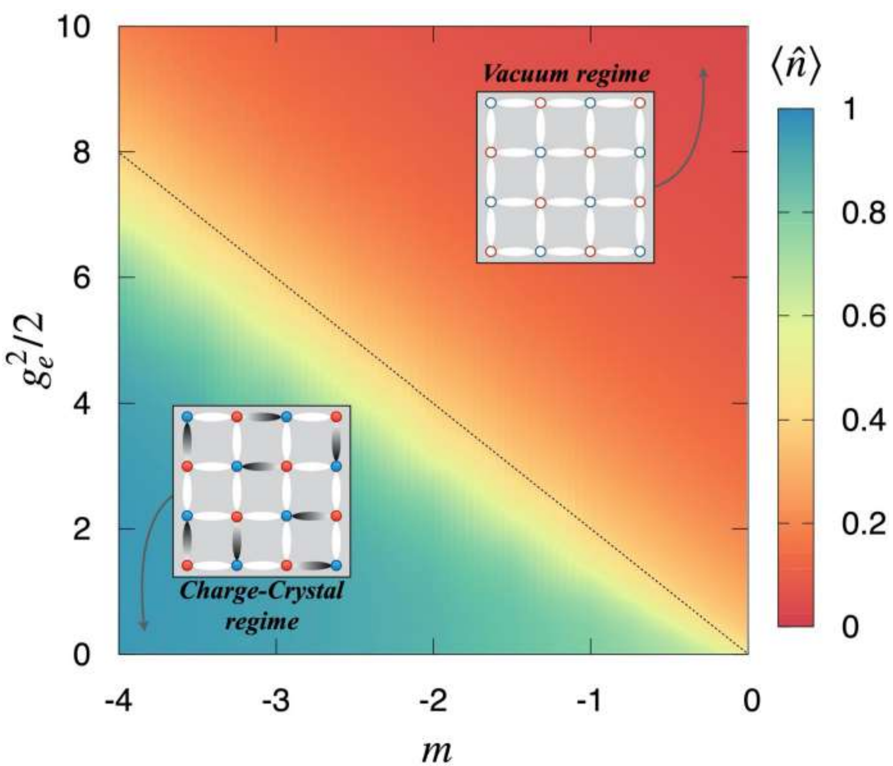



$$dim H_x = 35$$

# Ground-state properties as a function of  and without magnetic terms



## Ground-state properties as a function of  and without magnetic terms

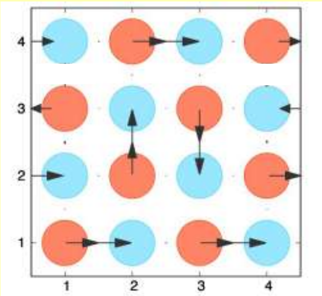



$$g_e^2/2 \gg 2|m|$$

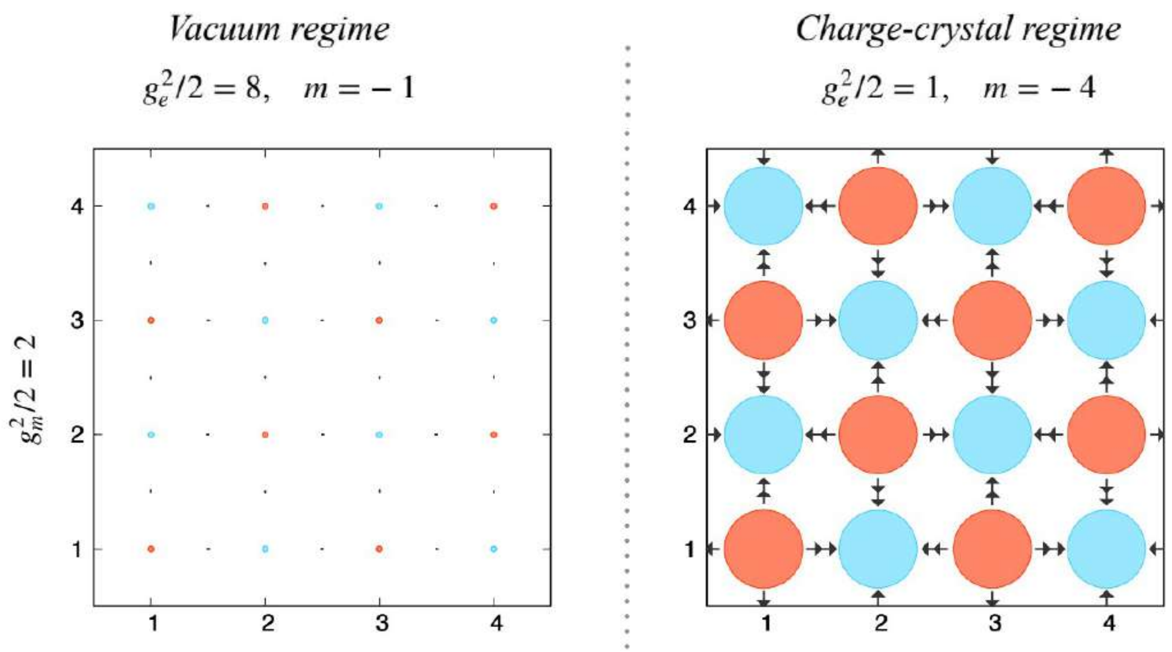
Vacuum phase: no particles, no field excitations

$$-2m \gg g_e^2/2 > 0$$

Charge-Crystal Phase: particle-antiparticle dimers



# Finite magnetic-coupling effects




$$g_e^2/2 \gg 2|m|$$

No changes affecting the vacuum configuration

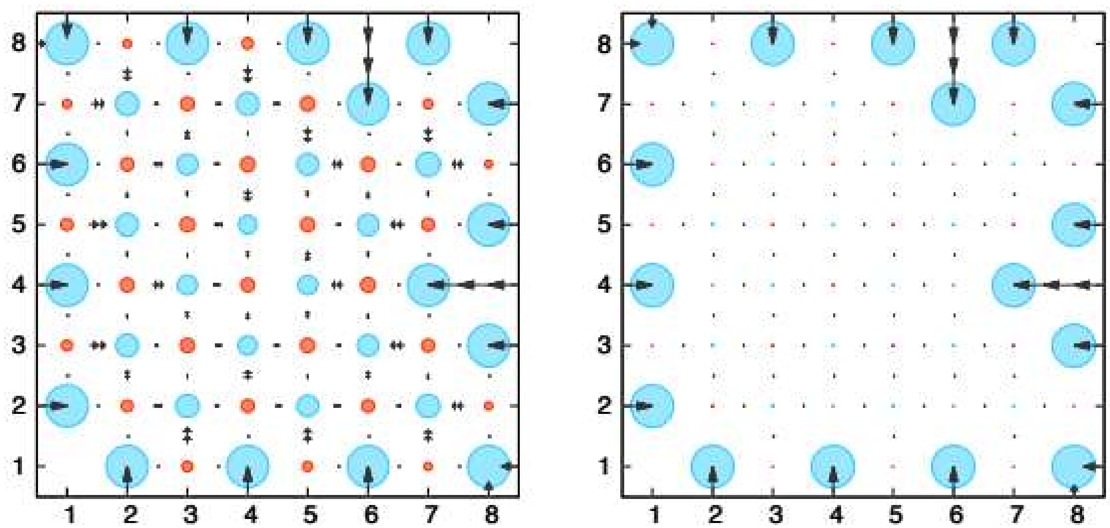
$$-2m \gg g_e^2/2 > 0$$

Nontrivial reorganisation of the electric fields, global entangled state of gauge fields

# Finite Charge Density Sector

$Q = -16$

Charge imbalance into the system: very challenging for Montecarlo techniques (sign problem)




$g_e^2/2 = 2,\ m = -1$

near the critical line

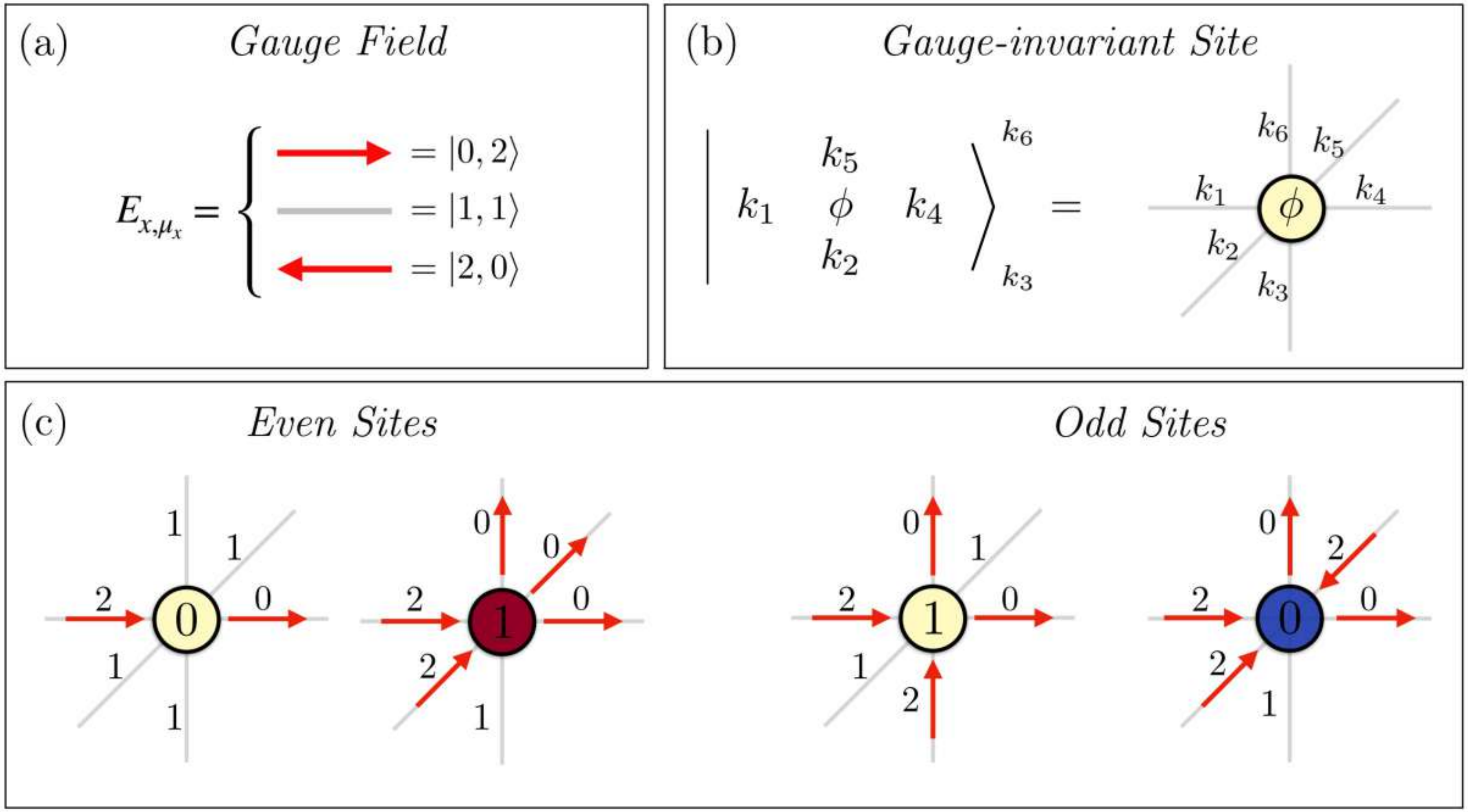
$g_e^2/2 = 2,\ m = 5$

deep in the vacuum phase

Charges forced to reach the boundaries to minimise the electric energy

Phys. Rev. X 10, 041040 (2020)

$$\hat{G}_x = \hat{\psi}_x^\dagger \hat{\psi}_x - \frac{1-p_x}{2} - \sum_\mu \hat{E}_{x,\mu} \qquad \hat{G}_x|\Phi\rangle = 0 \;\forall x$$



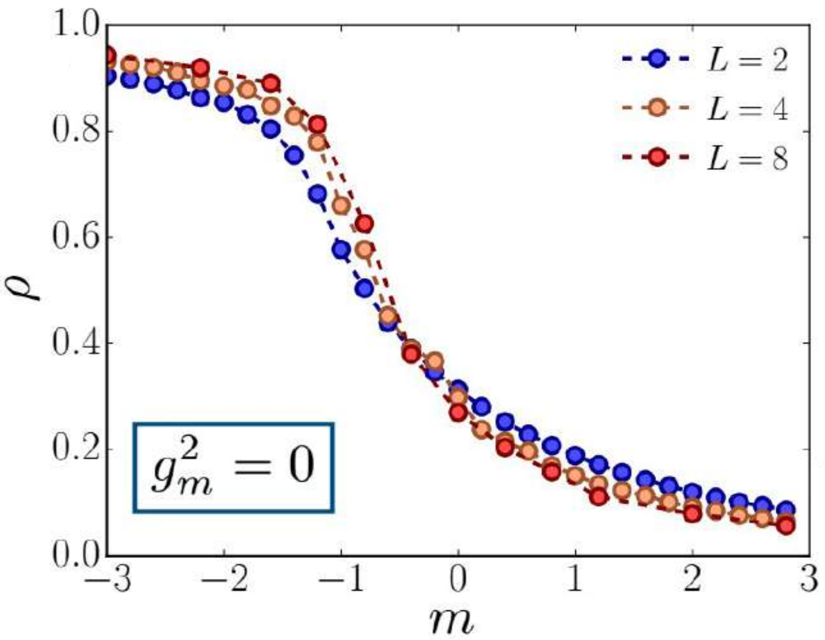

$$dim H_x = 267$$

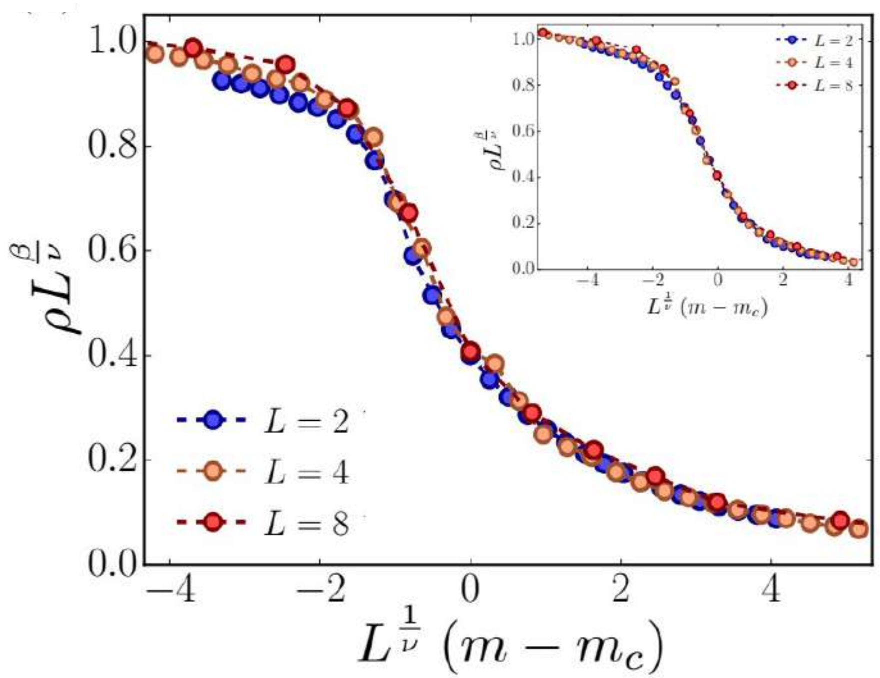
just for comparison, like a spin system with

# Ground state properties for

$$\rho = \frac{1}{L^3} \sum_x \langle GS|\hat{n}_x|GS\rangle$$

$$\rho L^{\frac{\beta}{\nu}} = \lambda\left(L^{\frac{1}{\nu}}(m - m_c)\right)$$



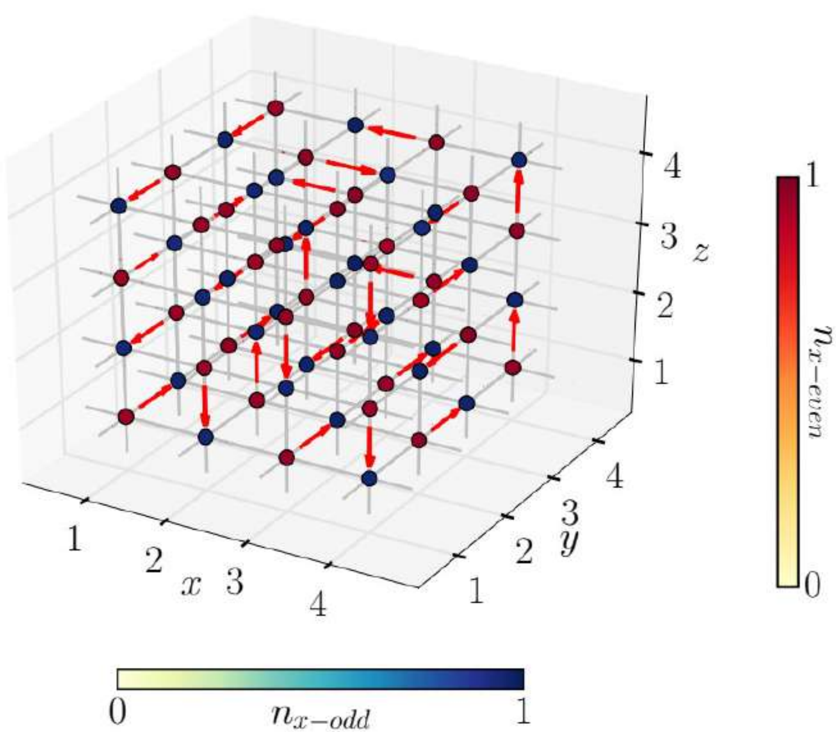




$$m_c = -0.39$$
$$\beta = 0.16 \quad \nu = 0.22$$

# Local configurations of matter and gauge fields

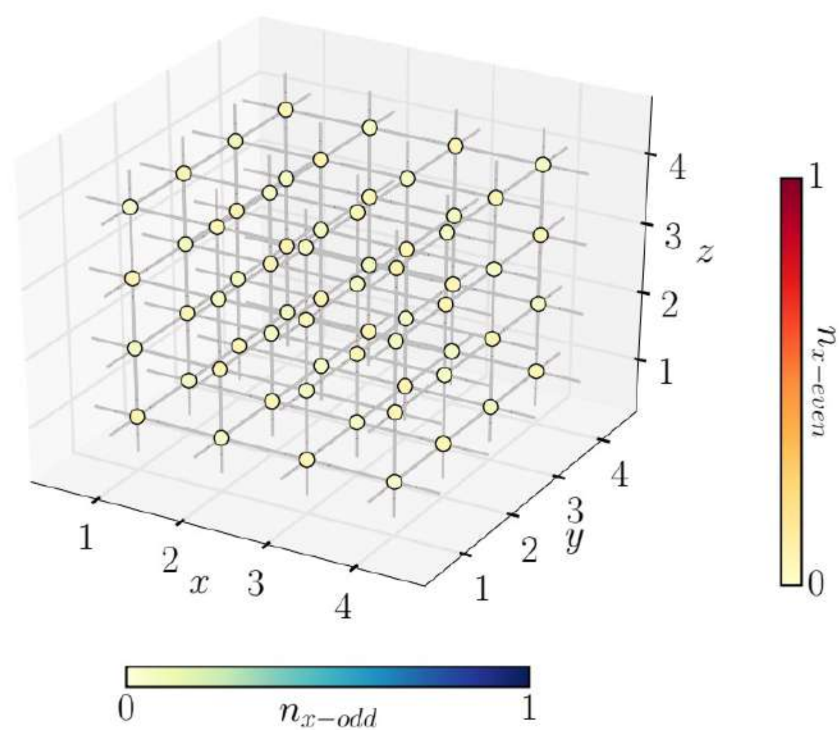
$$-2m \gg g_e^2/2 > 0$$

Charge-Crystal Phase:
particle-antiparticle dimers




$$g_e^2/2 \gg 2|m|$$

Vacuum phase: no particles, no field excitations



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$$\rho = \frac{1}{L^3} \sum_x \langle GS|\hat{n}_x|GS\rangle$$

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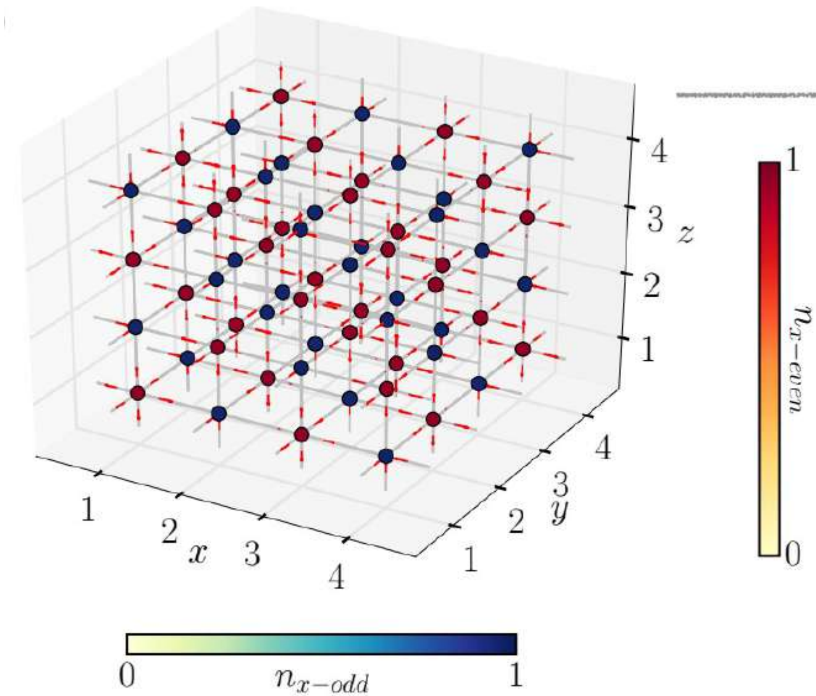
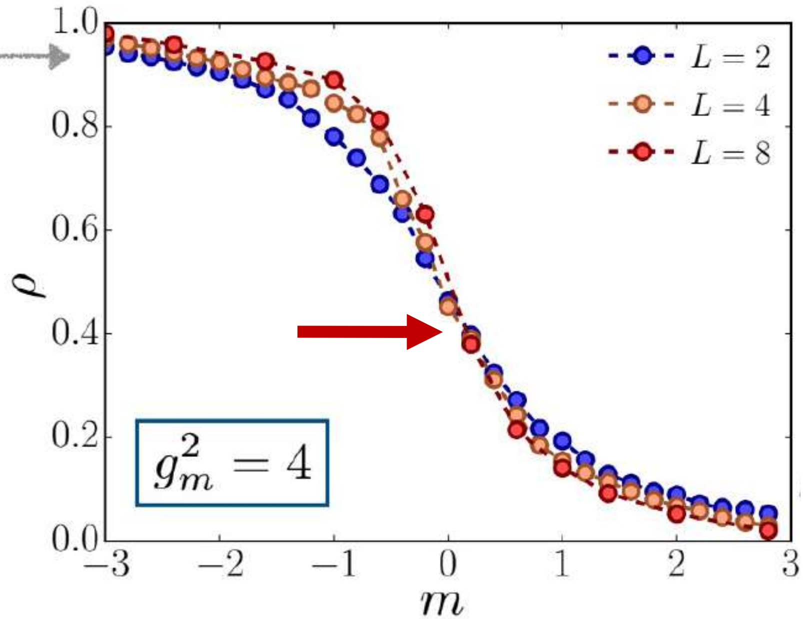


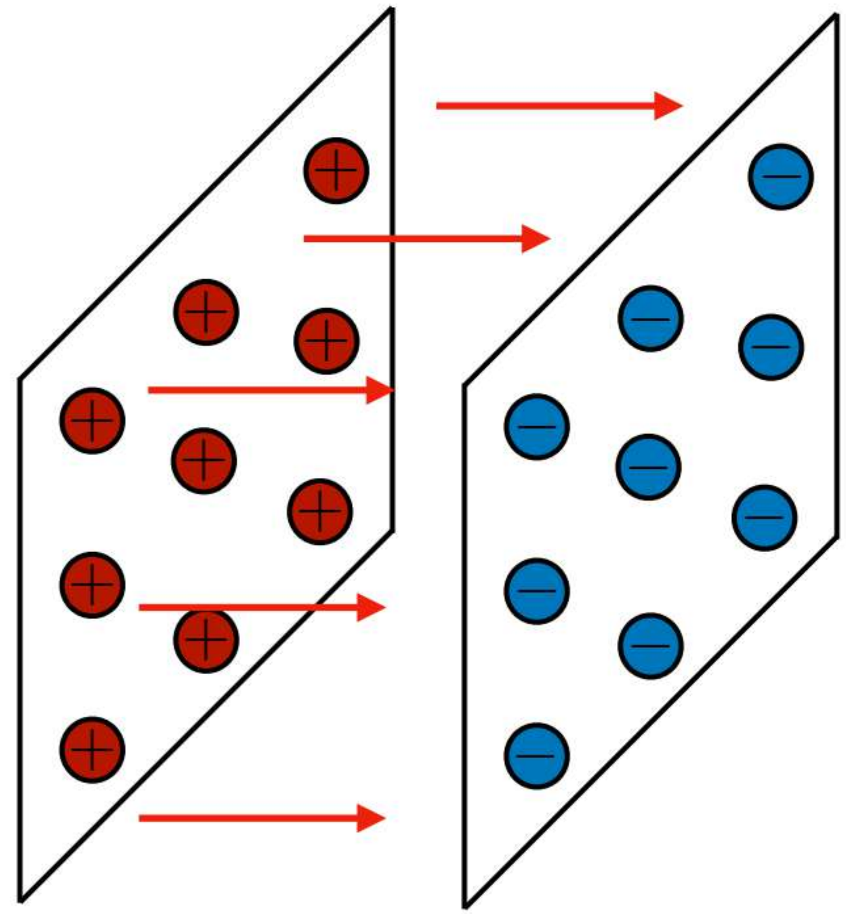




$$m_c = +0.22$$
$$\beta = 0.16 \quad \nu = 0.22$$

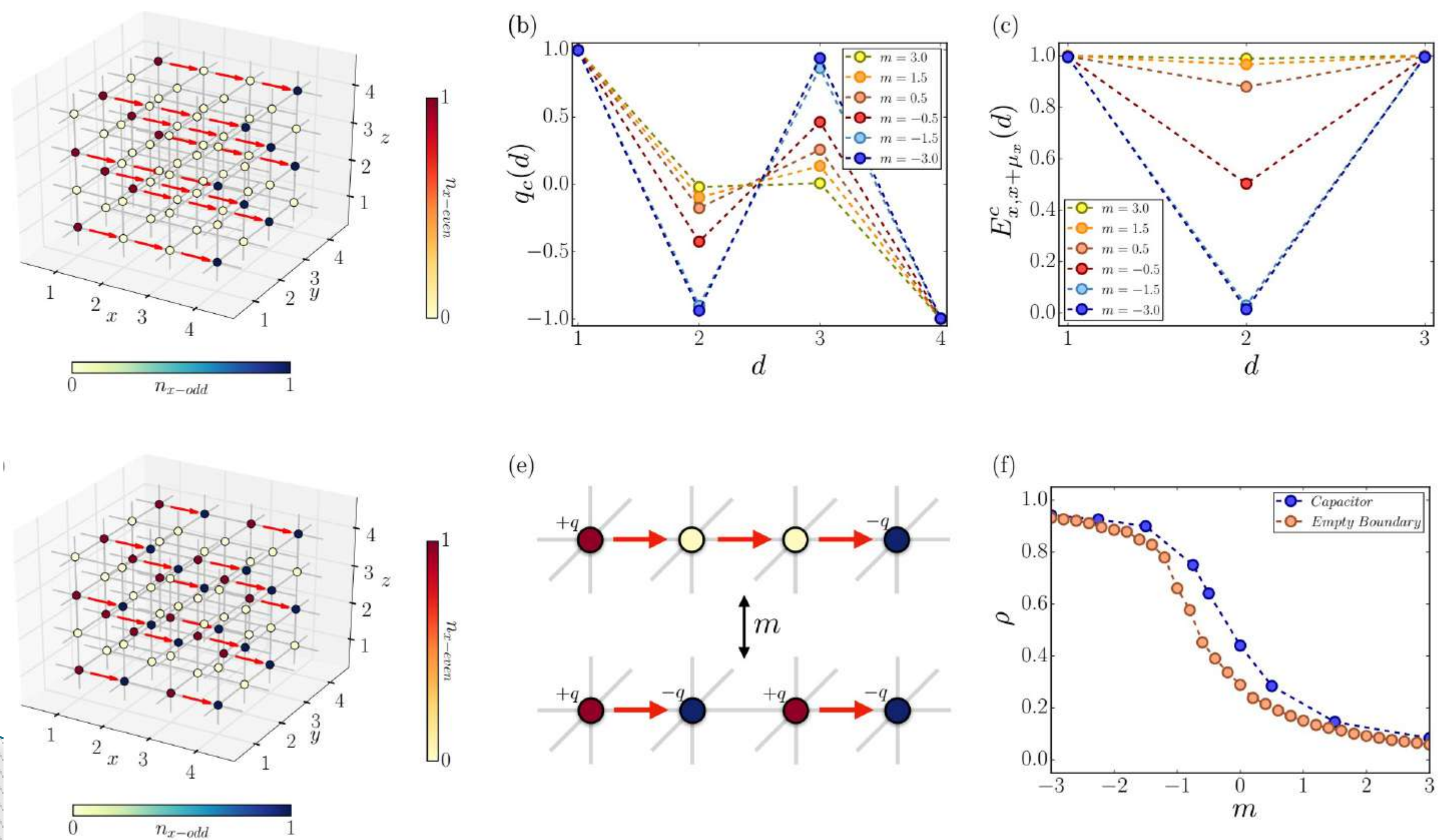
# Quantum Capacitor

Our approach is very flexible to simulate different geometries and charge-configurations.

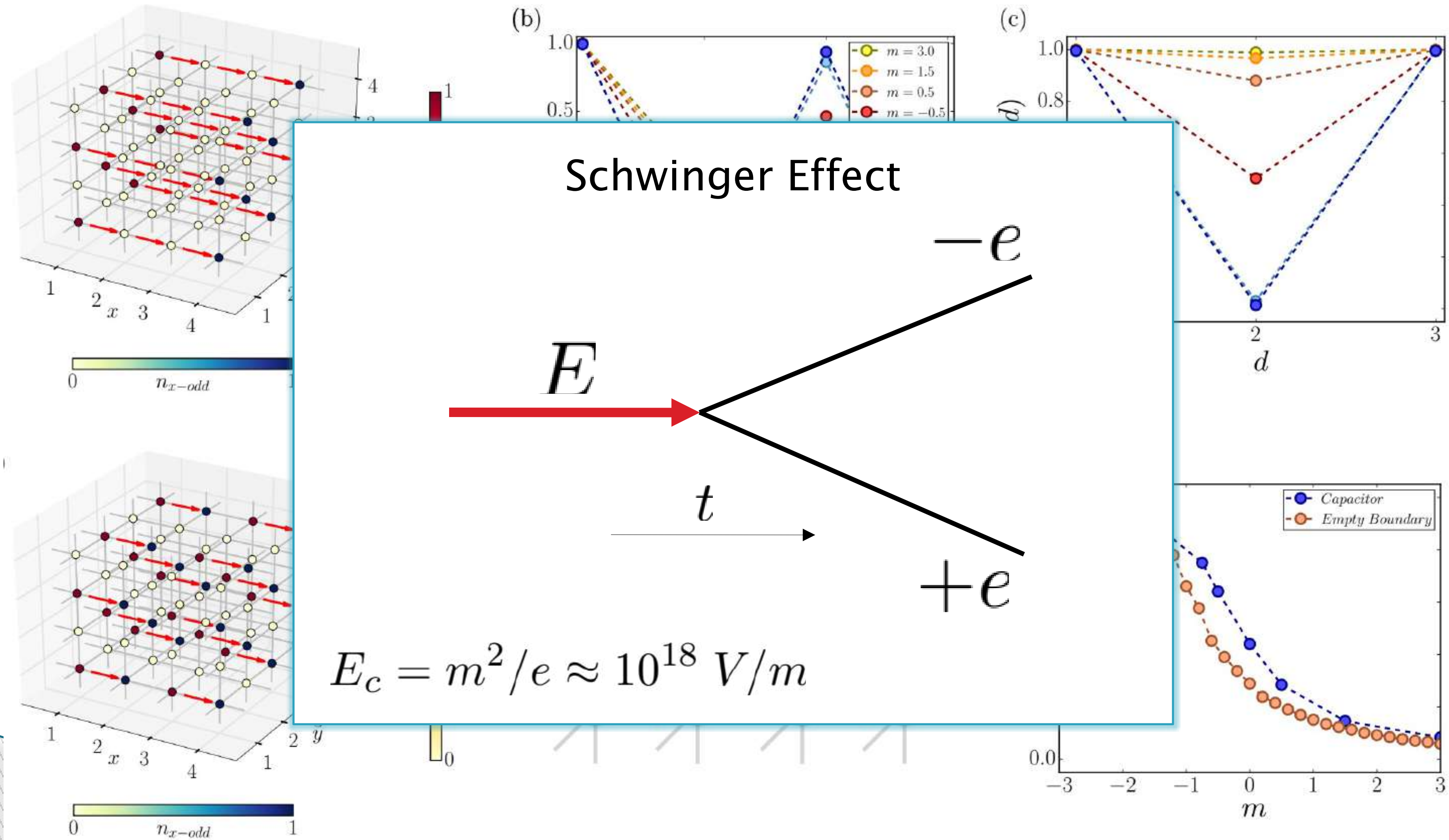
Field-screening and equilibrium string-breaking properties in presence of external field.



# Quantum Capacitor



# Quantum Capacitor




$$E_c = m^2/e \approx 10^{18}\ V/m$$

# Confinement Properties

$$g_m^2 = 8/g_e^2$$

$$\hat{H} = -t \sum_{x,\mu} \left( \hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + \text{H.c.} \right)$$
$$+ m \sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2$$
$$- \frac{g_m^2}{2} \sum_x \left( \Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \text{H.c.} \right)$$

Plaquette terms

Electric Field Energy

Weak-coupling

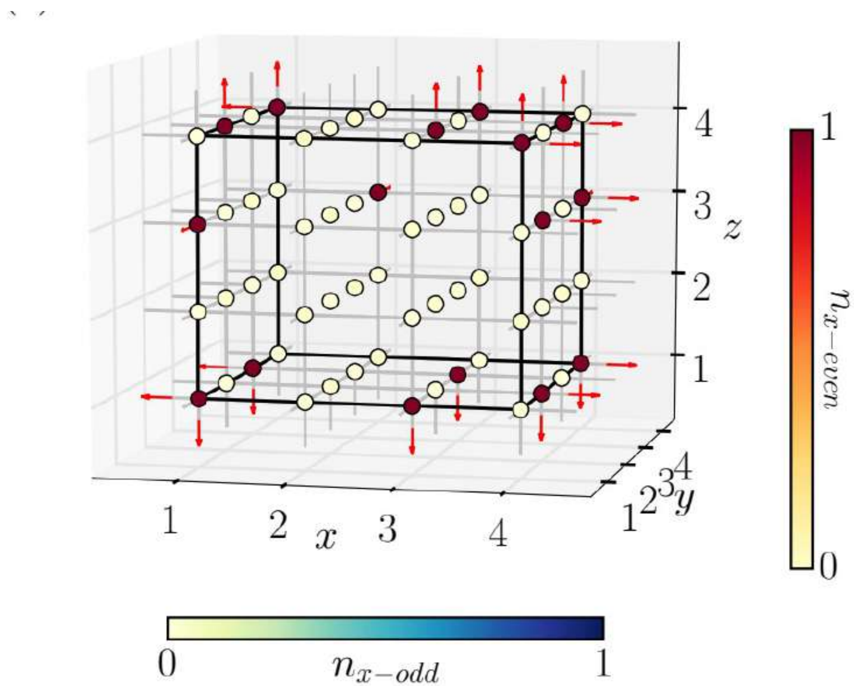
Strong-coupling

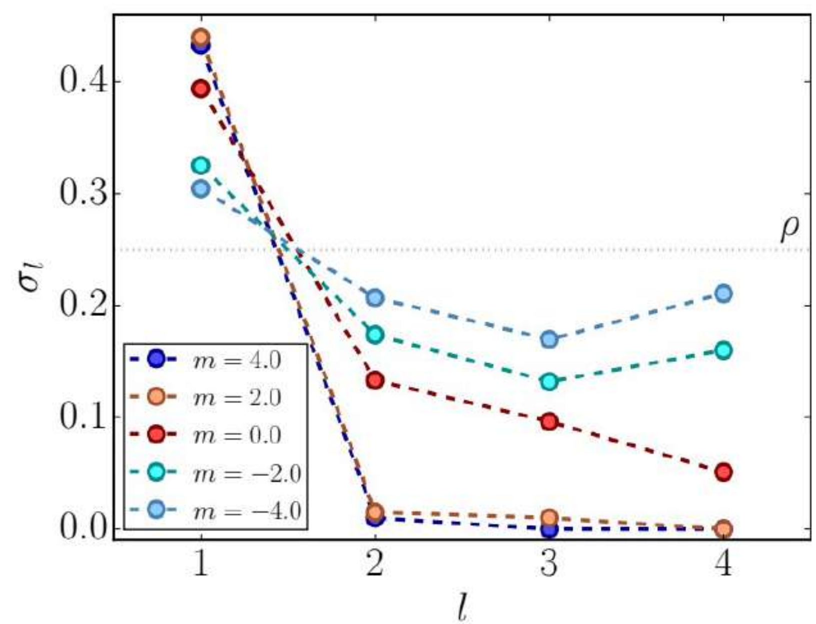
$g_e$

# Finite Density

$$L = 4\,, Q = 16\,, \rho = 1/4$$

$$L = 8\,, Q = 128\,, \rho = 1/4$$



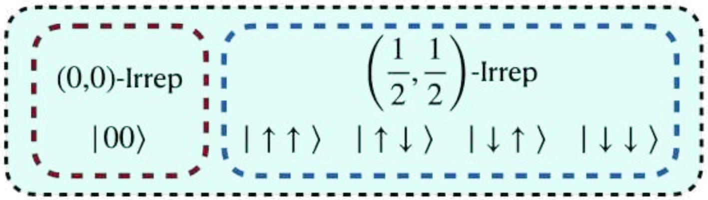



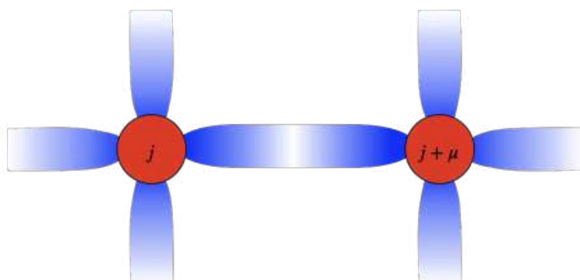

$$\sigma(l) = \frac{1}{A(l)} \sum_{x \in A(l)} \left\langle \hat{\psi}_x^\dagger \hat{\psi}_x \right\rangle$$

# What's next? SU(2) LGT

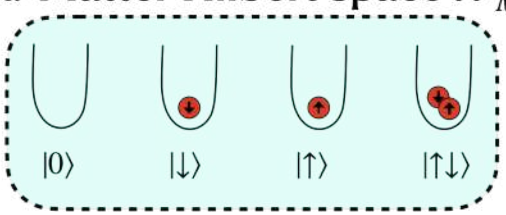
**GIOVANNI CATALDI**
University of Padua

$5d$ SU(2) Gauge Link Hilbert space $\mathscr{H}_G$



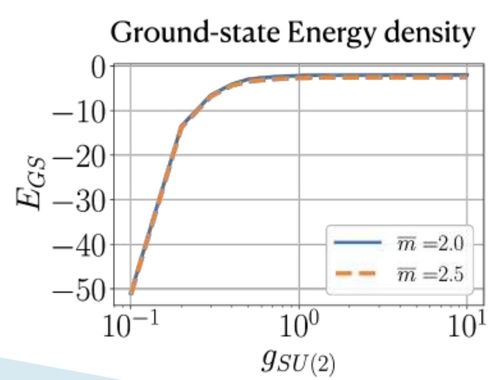

$4d$-Matter Hilbert space $\mathscr{H}_M$

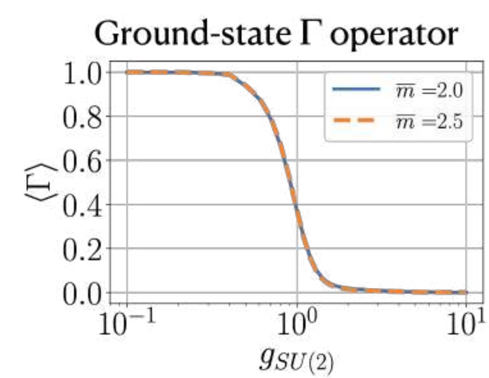



$$H^{2D}_{\text{LYM-SU(2)}} = \frac{c\hbar}{2a}\sum_j \left( -iQ^{\dagger}_{j,+\mu_x} Q_{j+\mu_x,-\mu_x} - (-1)^{j_x+j_y} Q^{\dagger}_{j,+\mu_y} Q_{j+\mu_y,-\mu_y} + \text{H.c.} \right) + mc^2 \sum_j (-1)^{j_x+j_y} D_j + H_{\text{pure}} + H_{\text{penalty}}$$

$$H_{\text{pure}} = \frac{3c\hbar g^2_{\text{SU(2)}}}{16a}\sum_j \Gamma_j - \frac{4c\hbar}{a g^2_{\text{SU(2)}}}\sum_j \left( C_{j;\mu_x,\mu_y} C_{j+\mu_x;\mu_y,-\mu_x} C_{j+\mu_x+\mu_y;-\mu_x,-\mu_y} C_{j+\mu_y;-\mu_y,\mu_x} + \text{H.c.} \right)$$

$$H_{\text{penalty}} = -\eta \sum_j \left[ W_{j,+\mu_x} W_{j+\mu_x,-\mu_x} - W_{j,+\mu_y} W_{j+\mu_y,-\mu_y} \right]$$



# Tensor Networks – Scattering dynamics

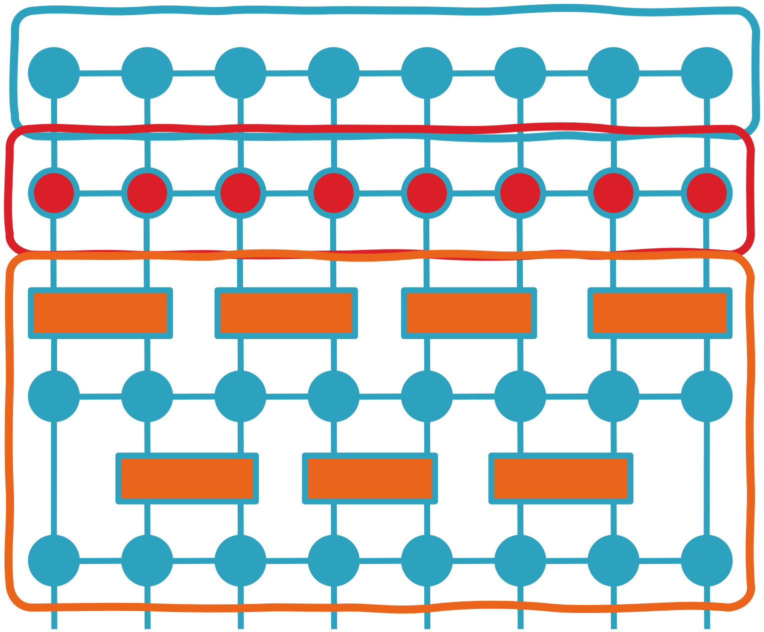
**MARCO RIGOBELLO**
University of Padua

$$\hat{H} = -t\sum_x \hat{\psi}_x^\dagger \hat{U}_{x,x+1}\hat{\psi}_{x+1} + \text{h.c.} + m\sum_x (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g^2}{2}\sum_x \hat{E}_{x,x+1}^2$$

interacting vacuum MPS via DMRG

initial state via wave packet creation **MPOs**

time evolution via TEBD & observables



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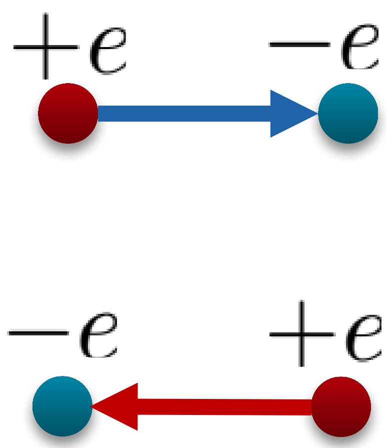
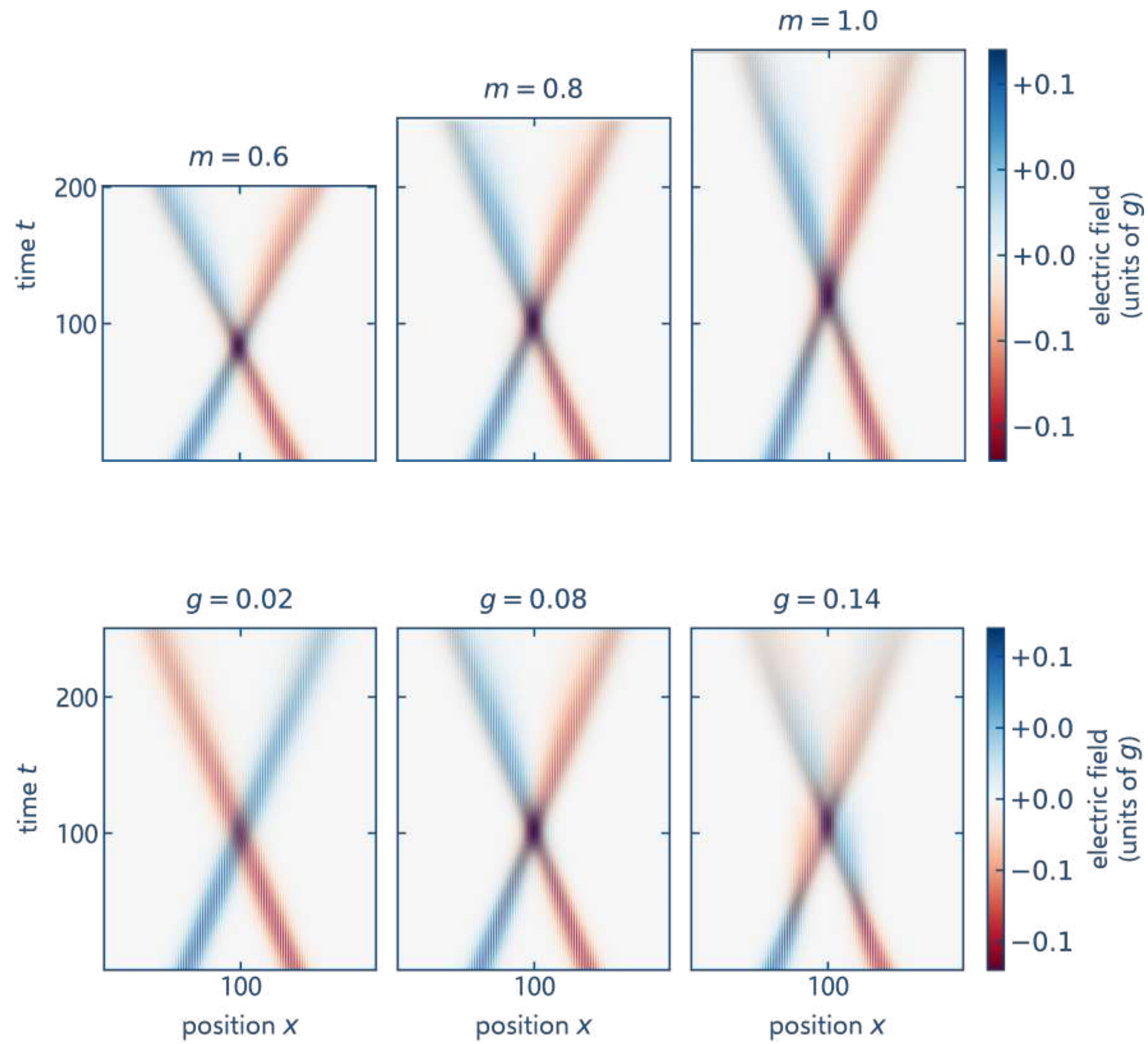
# Tensor Networks – Scattering dynamics

Mesons with opposite momenta and internal electric fields

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# Tensor Networks – Scattering dynamics

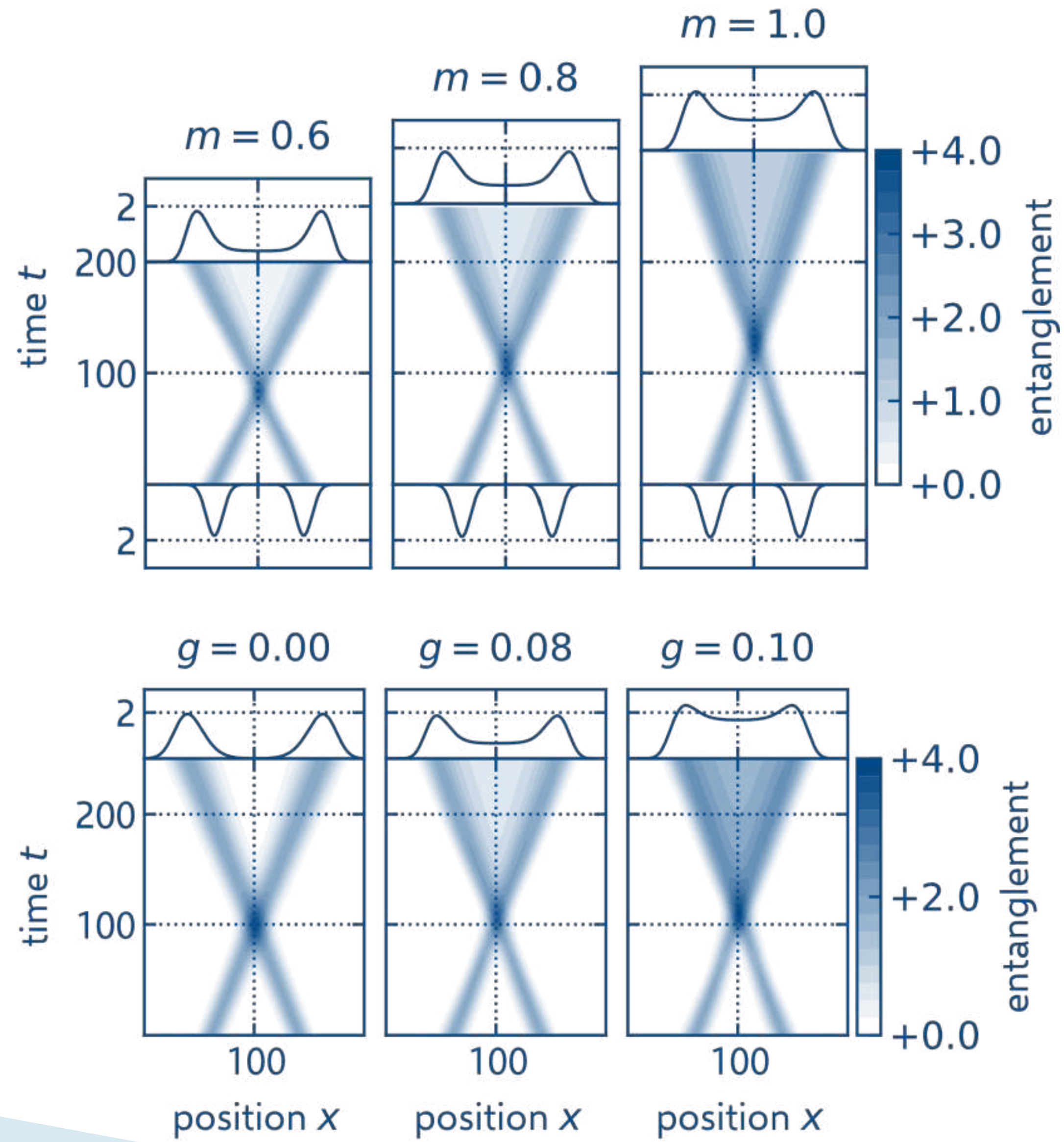
Vacuum

\+

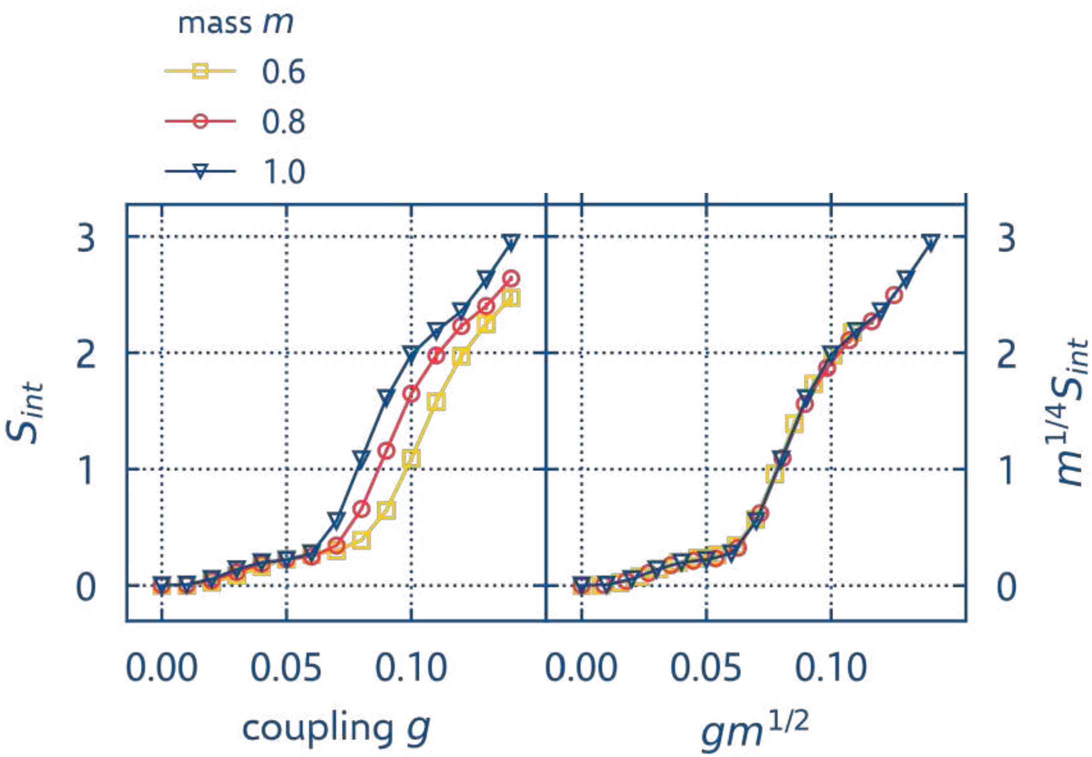
wave packets
intra-particle

\+

interactions
intra-particle



# Tensor Networks – Scattering dynamics




**scaling relation**

is independent

**two regimes**

appearance of new effective d.o.f.

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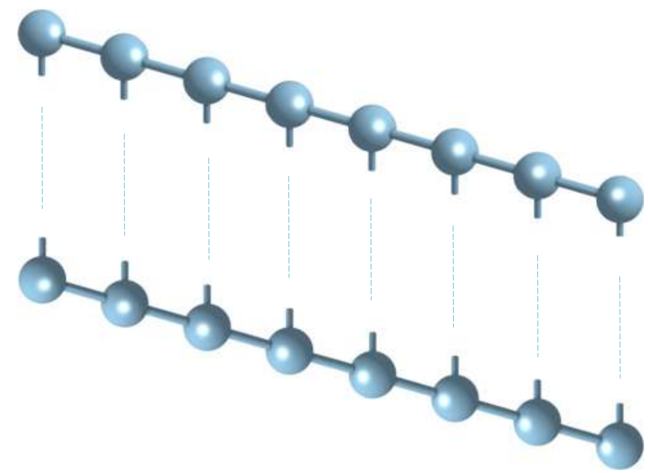
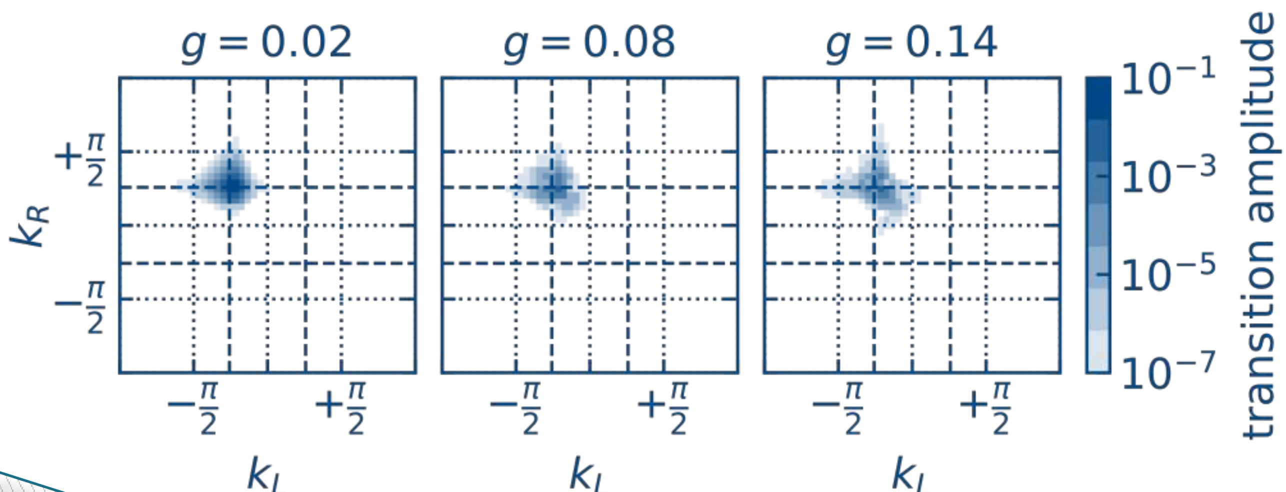
# Tensor Networks – Scattering dynamics

overlap of final state with
**pair of meson wave packet**

$|\psi(t_f)\rangle$

$\langle\psi(t_i)|$






~ S-matrix elements

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# What's next?

Quantum Many-Body Physics ⟷ High-Energy Physics domain

- Tensor networks can be used to develop, support, validate, quantum simulation and quantum computation protocols.
- Relevant interactions with High-Energy Physics.
- Long-term goal: tensor networks and quantum simulation of QCD.