Quantum Many-Body Scars in Non-Abelian Lattice Gauge Theories

ArXiv:2405.13112

Quantum Many-Body Scarring in a Non-Abelian Lattice Gauge Theory

Giuseppe Calajó*,[1] Giovanni Cataldi*,[1,2,3] Marco Rigobello,[1,2,3] Darvin Wanisch,[1,2,3]
Giuseppe Magnifico,[4,5] Pietro Silvi,[1,2,3] Simone Montangero,[1,2,3] and Jad C. Halimeh [6,7,8,9]

Quantum many-body scarring (QMBS) is an intriguing mechanism of ergodicity breaking that has recently spurred significant attention. Particularly prominent in Abelian lattice gauge theories (LGTs), an open question is whether QMBS nontrivially arises in non-Abelian LGTs. Here, we present evidence of robust QMBS in a non-Abelian SU(2) LGT with dynamical matter. Starting in product states that require little experimental overhead, we show that prominent QMBS arises for certain quenches, facilitated through meson and baryon-antibaryon excitations, highlighting its non-Abelian nature. The uncovered scarred dynamics manifests as long-lived coherent oscillations in experimentally accessible local observables as well as prominent revivals in the state fidelity. Our findings bring QMBS to the realm of non-Abelian LGTs, highlighting the intimate connection between scarring and gauge symmetry, and are amenable for observation in a recently proposed trapped-ion qudit quantum computer.


G. Calajò


M. Rigobello


D. Wanisch

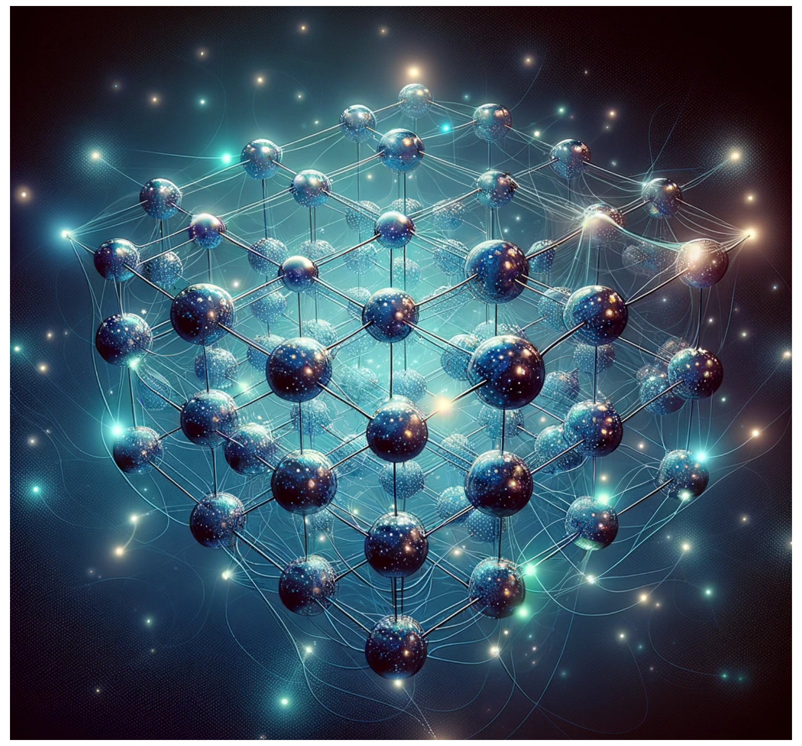

G. Magnifico


P. Silvi


S. Montangero

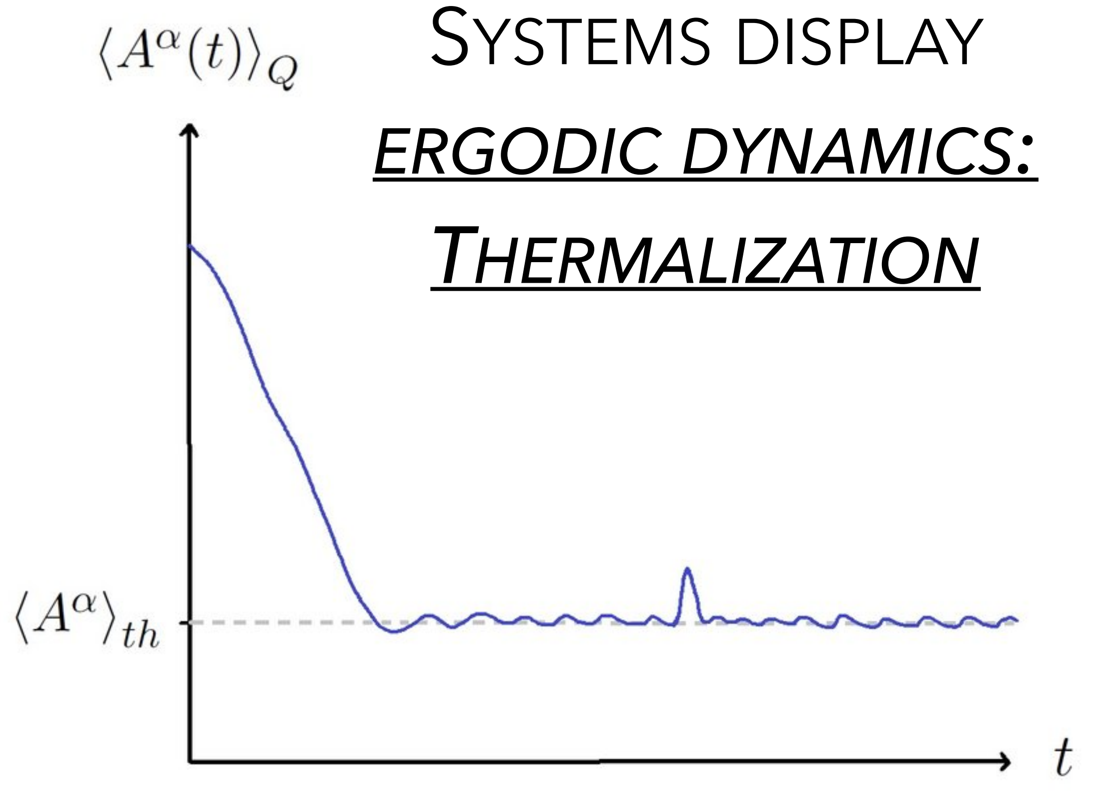

J.C. Halimeh

OVERVIEW

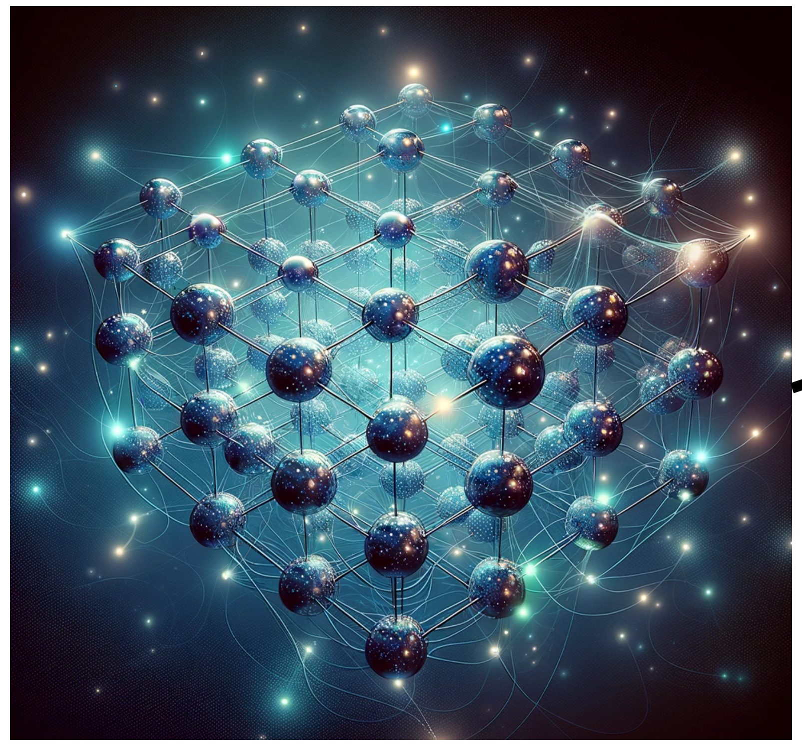


THE MAJORITY OF QMB SYSTEMS DISPLAY

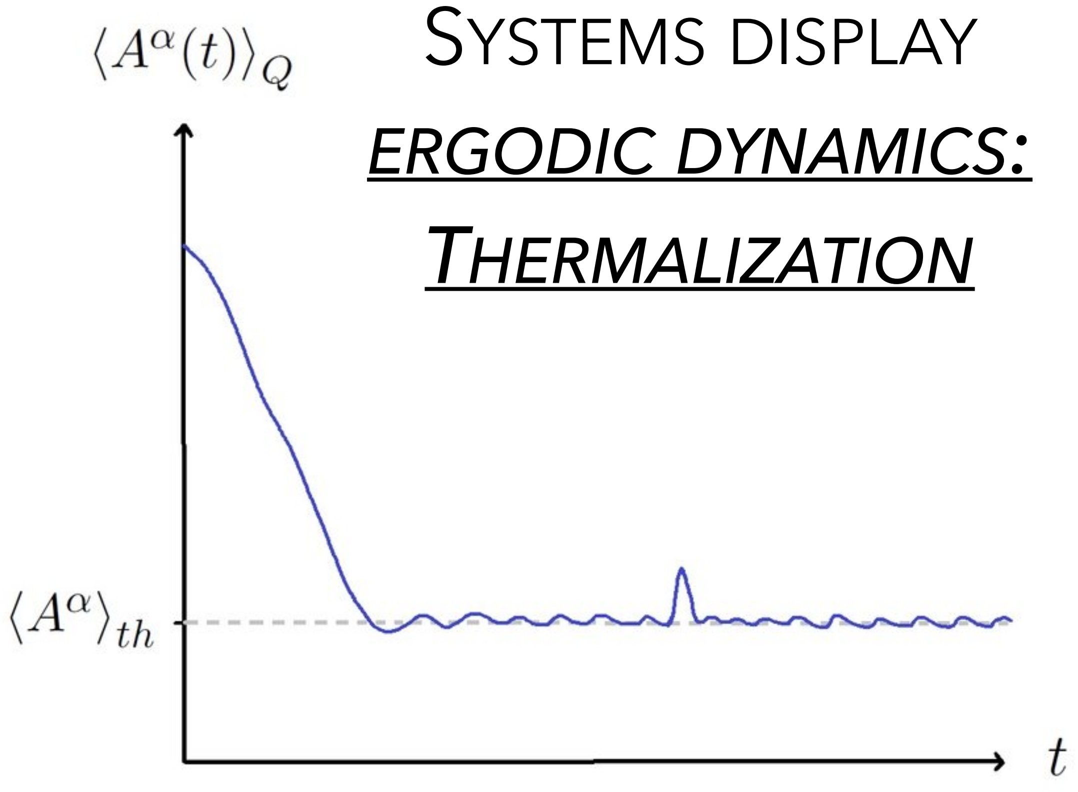
ERGODIC DYNAMICS:
THERMALIZATION

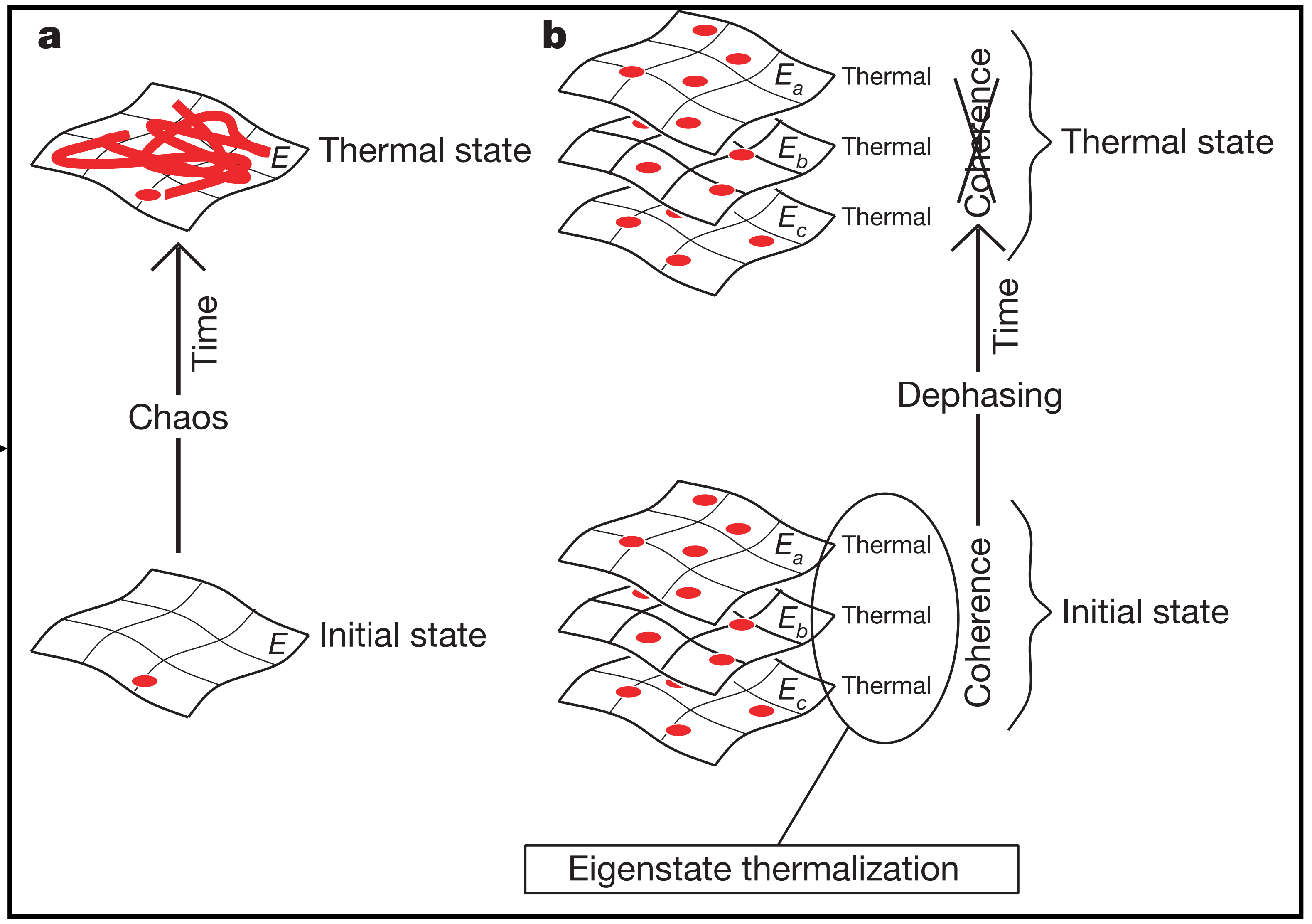


OVERVIEW



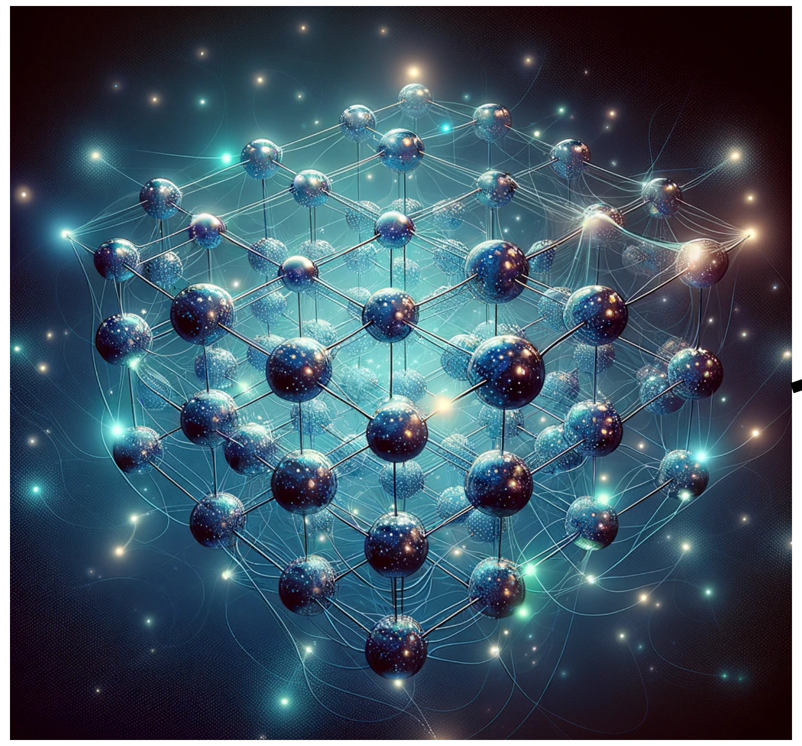
THE MAJORITY OF QMB SYSTEMS DISPLAY ***ERGODIC DYNAMICS: THERMALIZATION***



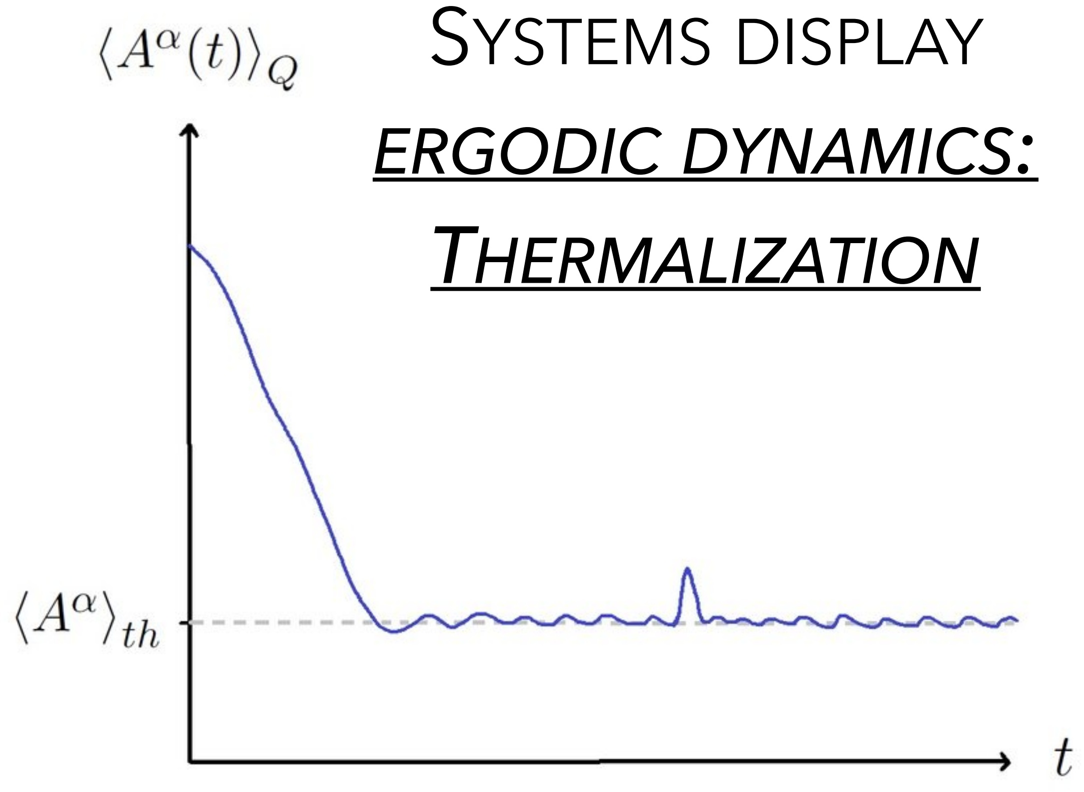


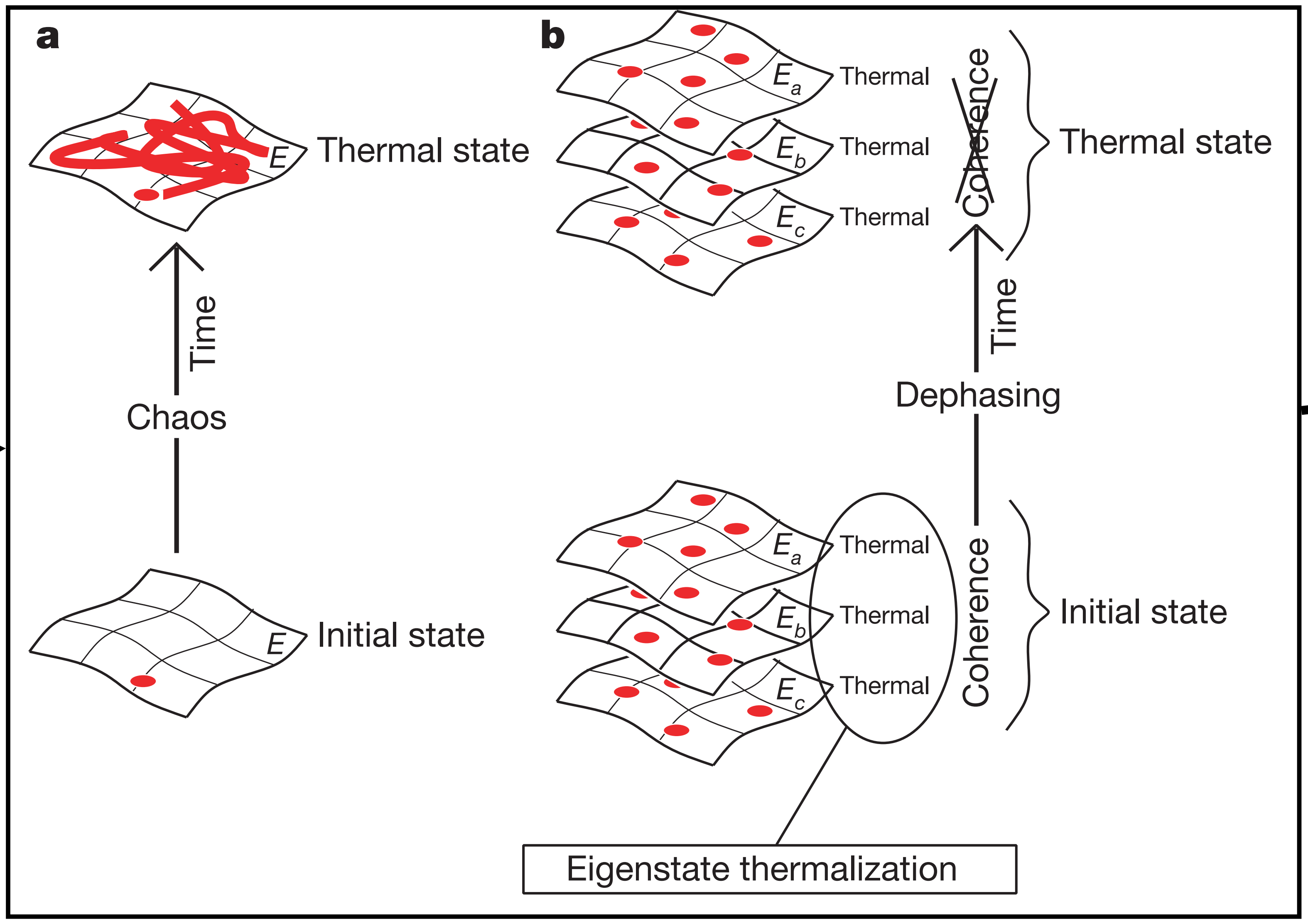
ERGODIC DYNAMICS IS EXPLAINED VIA ***EIGENSTATE THERMALIZATION HYPOTHESIS (ETH)***

OVERVIEW



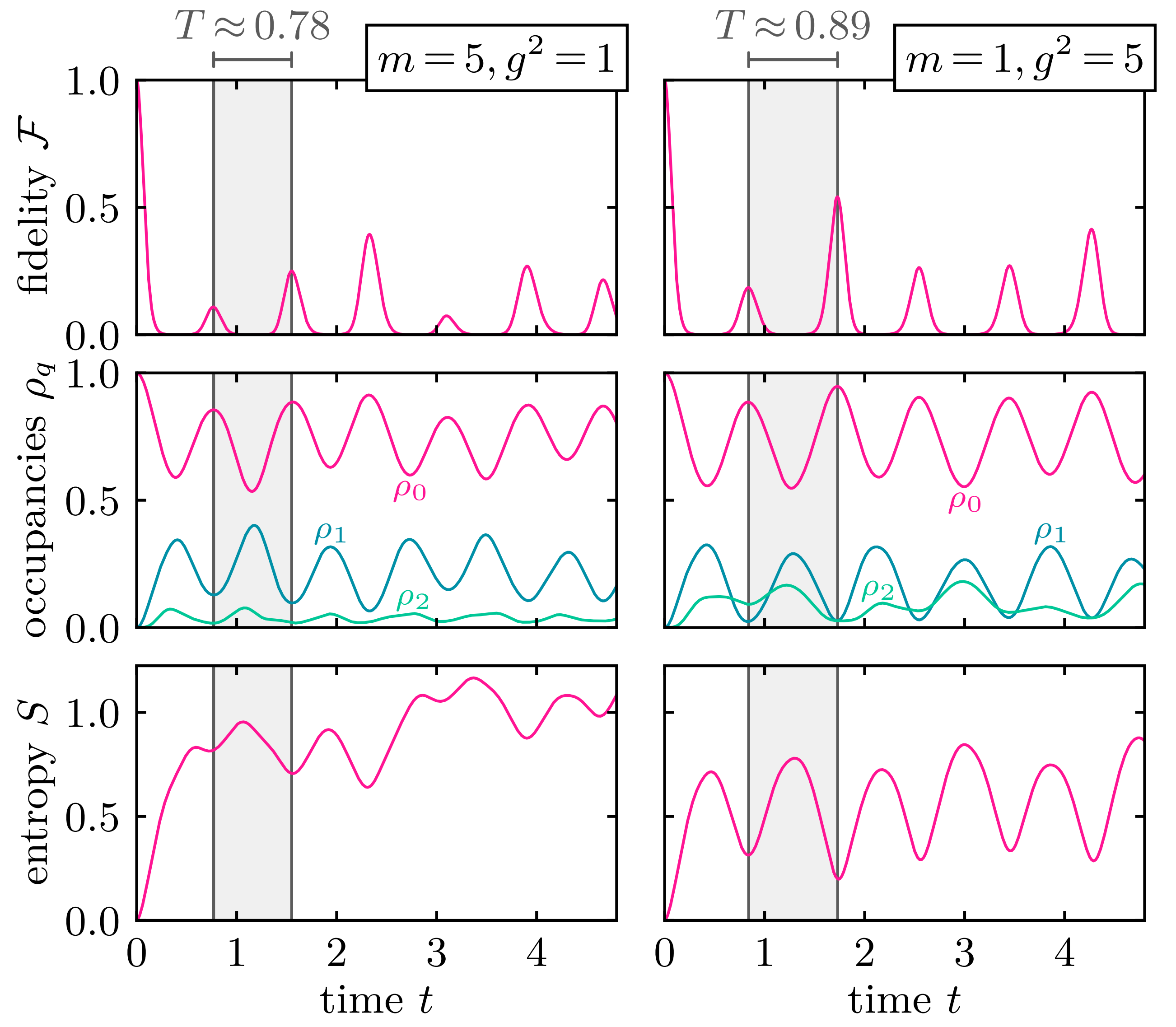
THE MAJORITY OF QMB SYSTEMS DISPLAY ***ERGODIC DYNAMICS: THERMALIZATION***



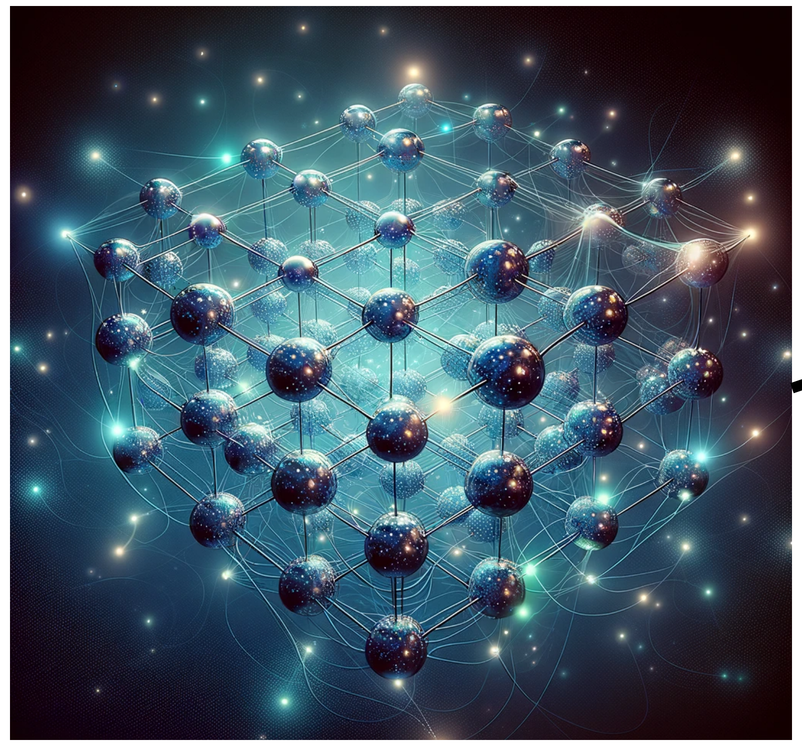


ERGODIC DYNAMICS IS EXPLAINED VIA ***EIGENSTATE THERMALIZATION HYPOTHESIS (ETH)***

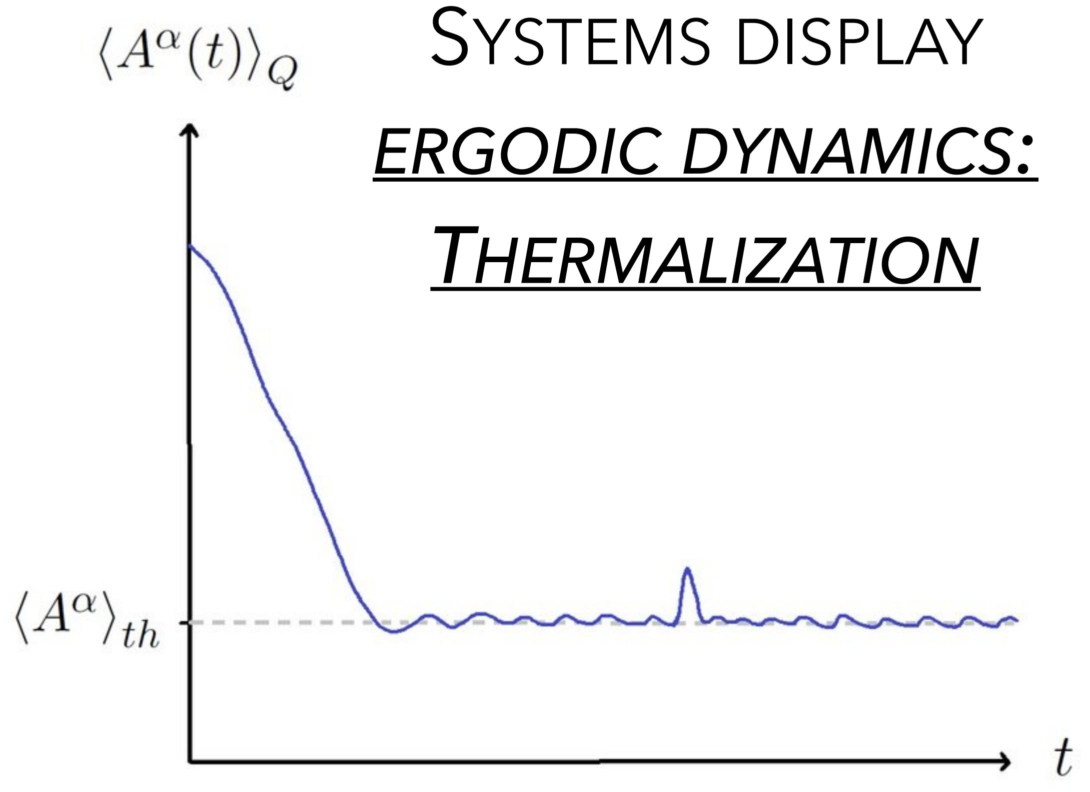
QMB SCARS VIOLATE ETH WITH ***LONG-LIVED OSCILLATIONS***

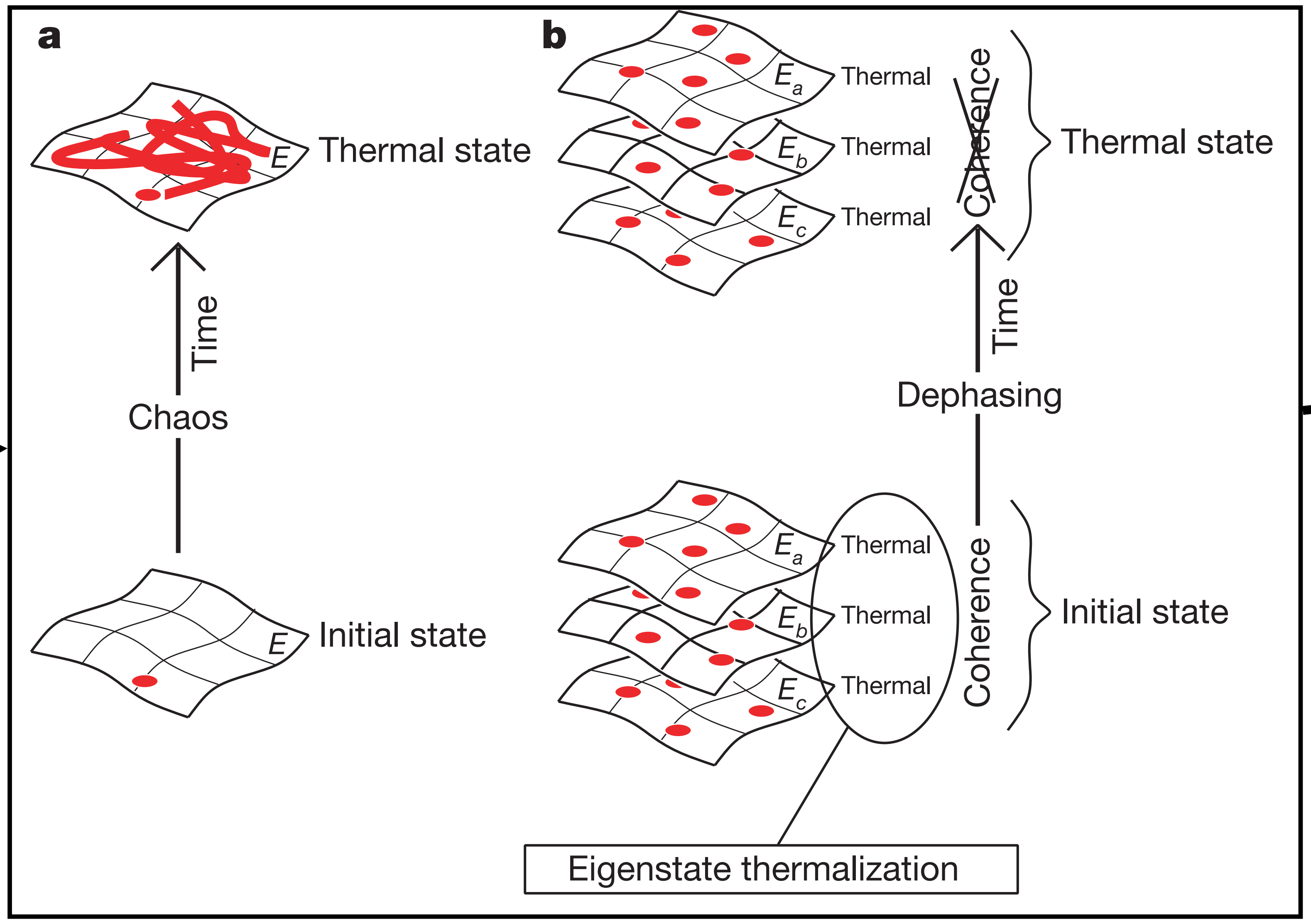


OVERVIEW



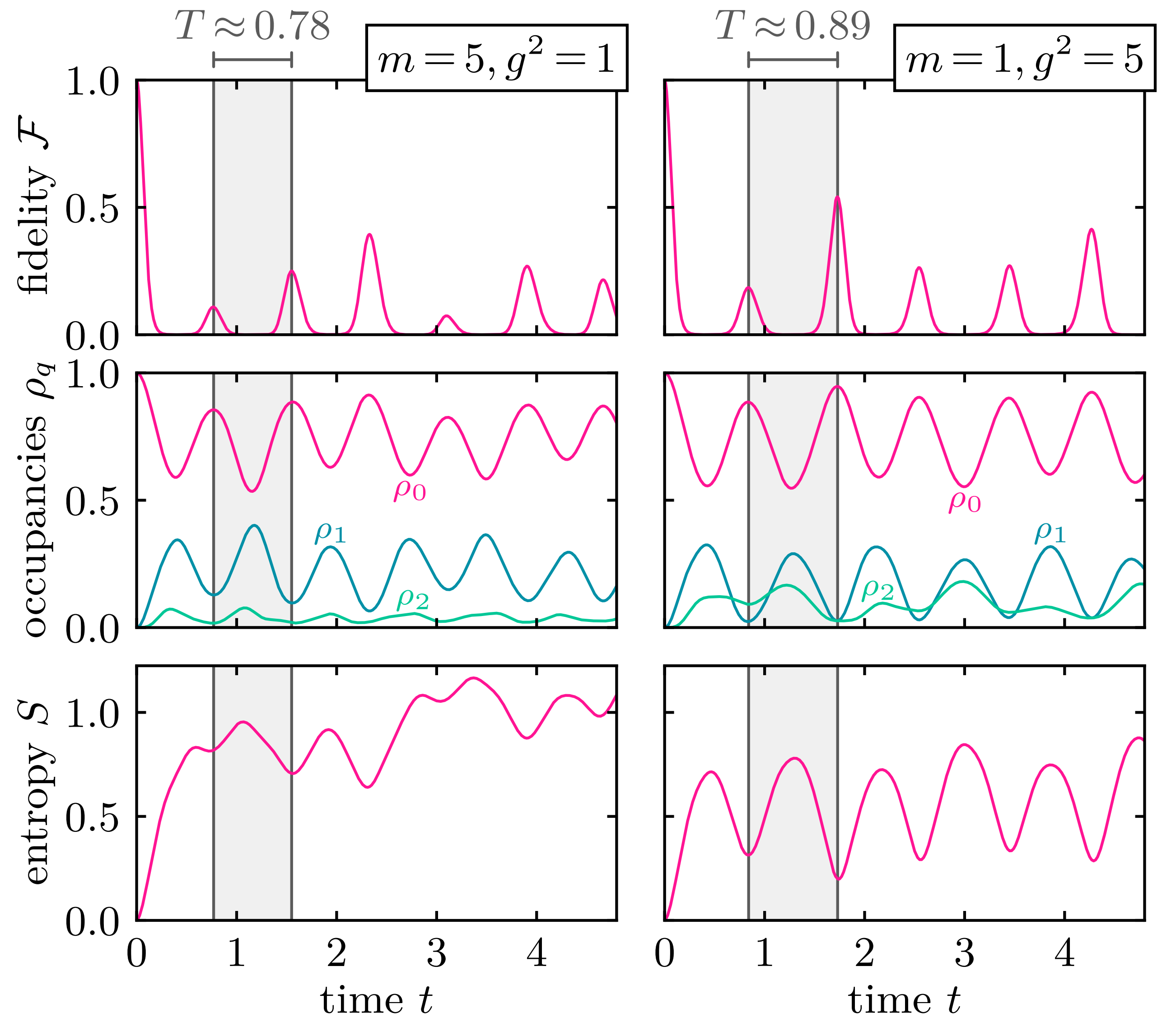
THE MAJORITY OF QMB SYSTEMS DISPLAY ***ERGODIC DYNAMICS: THERMALIZATION***

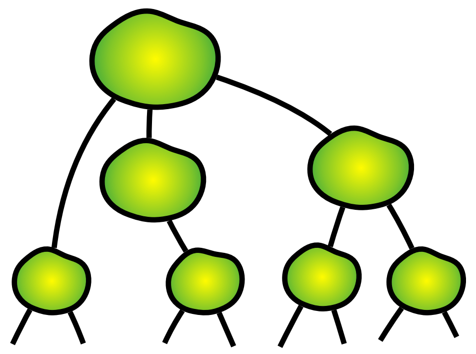


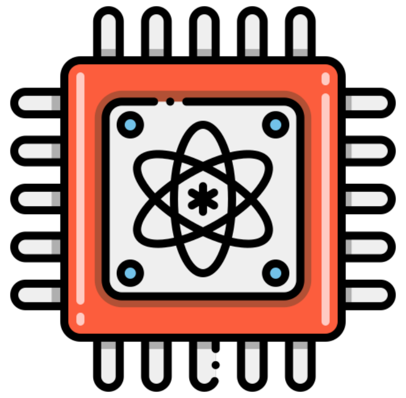


ERGODIC DYNAMICS IS EXPLAINED VIA ***EIGENSTATE THERMALIZATION HYPOTHESIS (ETH)***

QMB SCARS VIOLATE ETH WITH ***LONG-LIVED OSCILLATIONS***

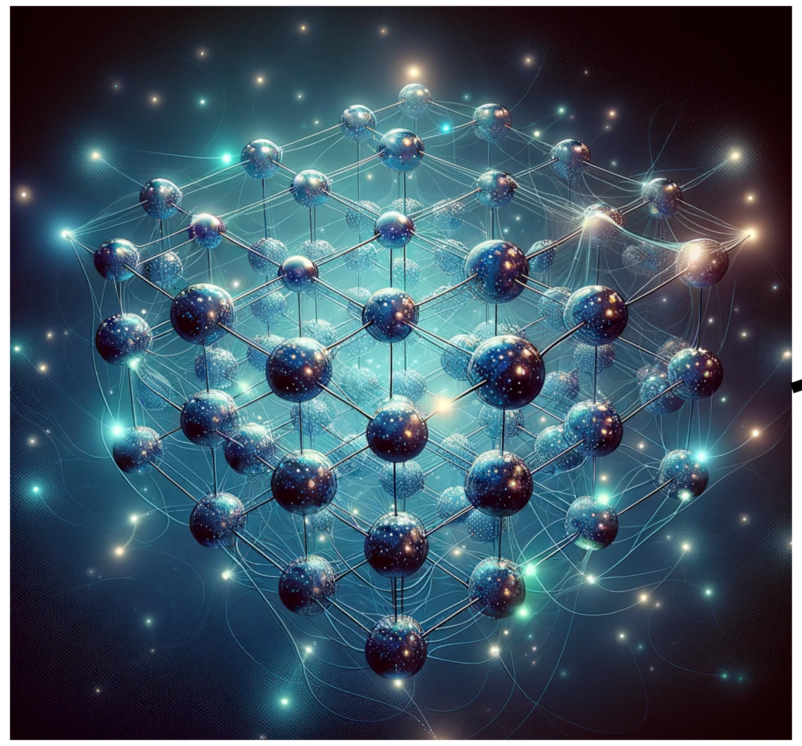




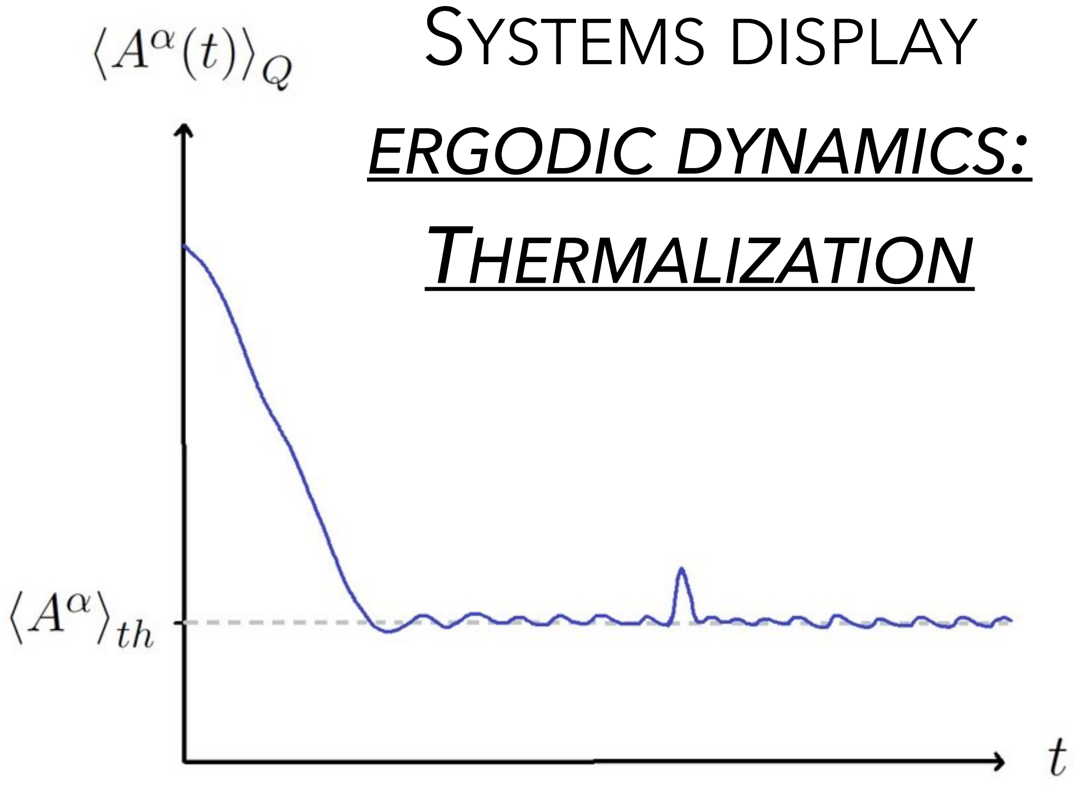


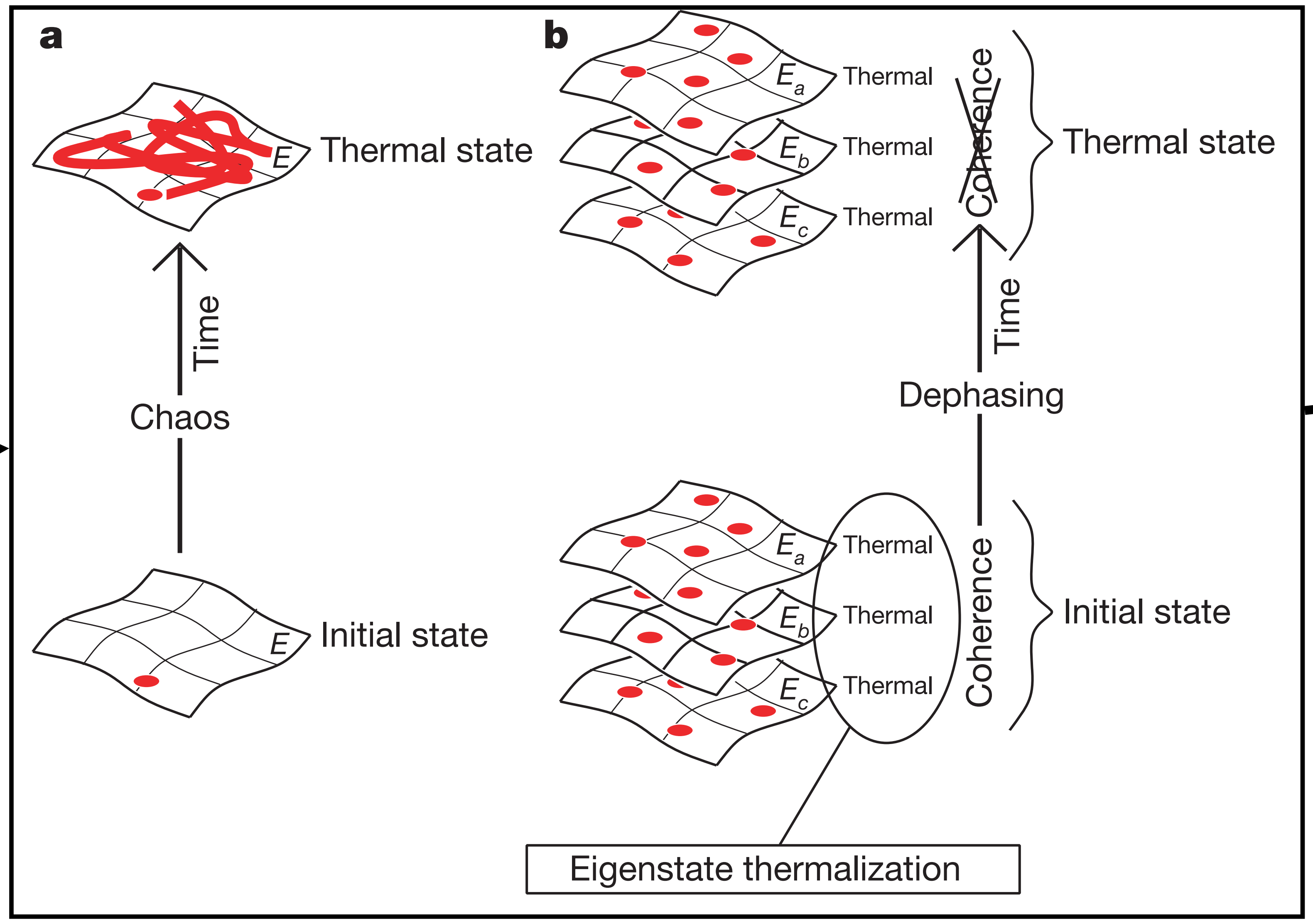
VISIBLE IN CLASSICAL AND QUANTUM SIMULATIONS!

OVERVIEW



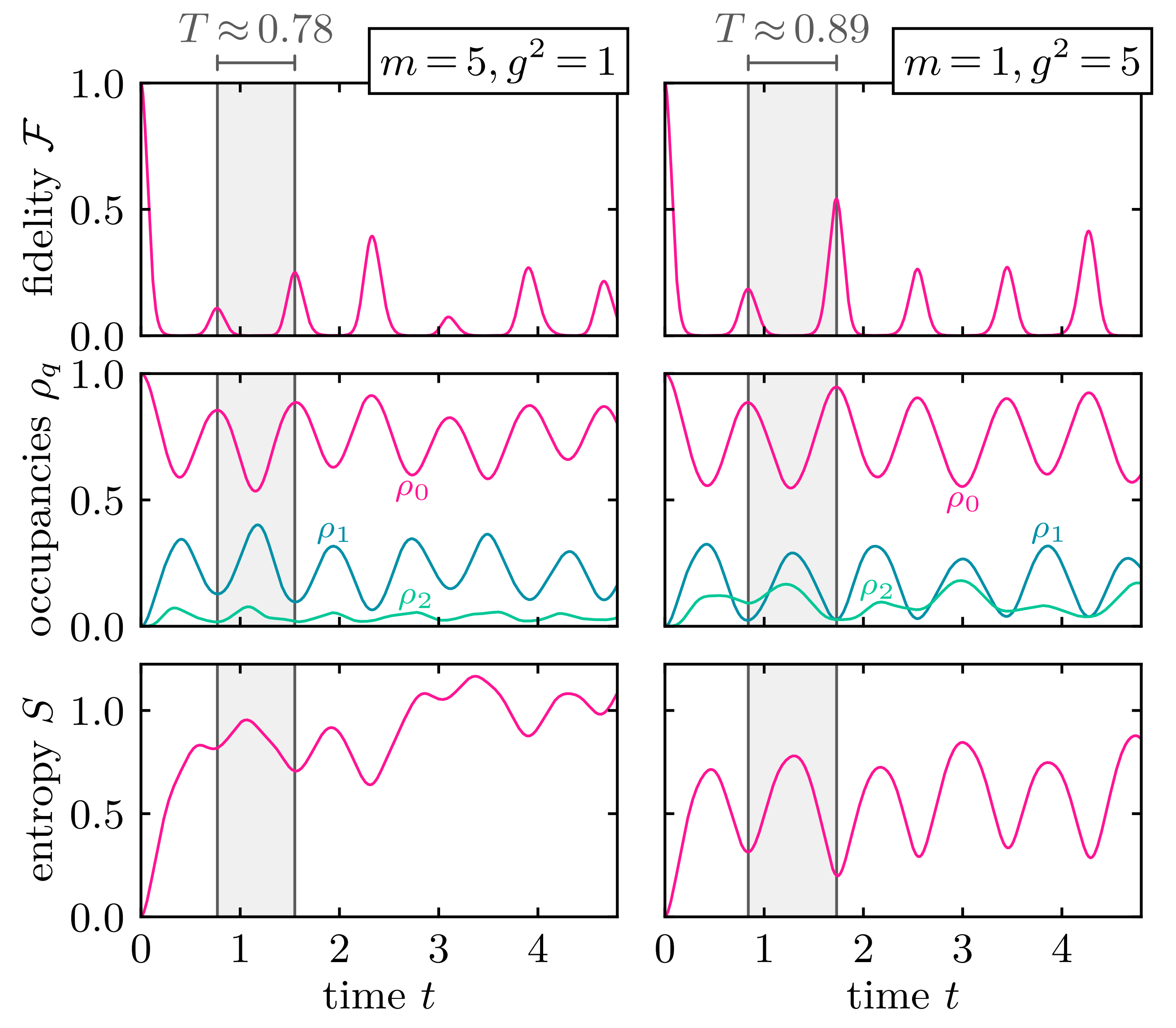
The Majority of QMB Systems display ***Ergodic dynamics: Thermalization***

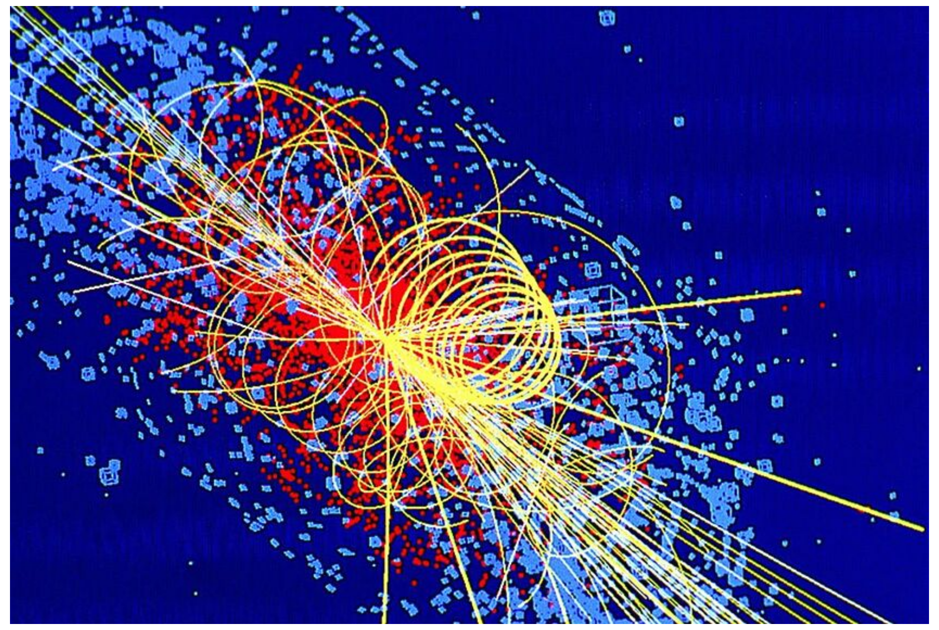


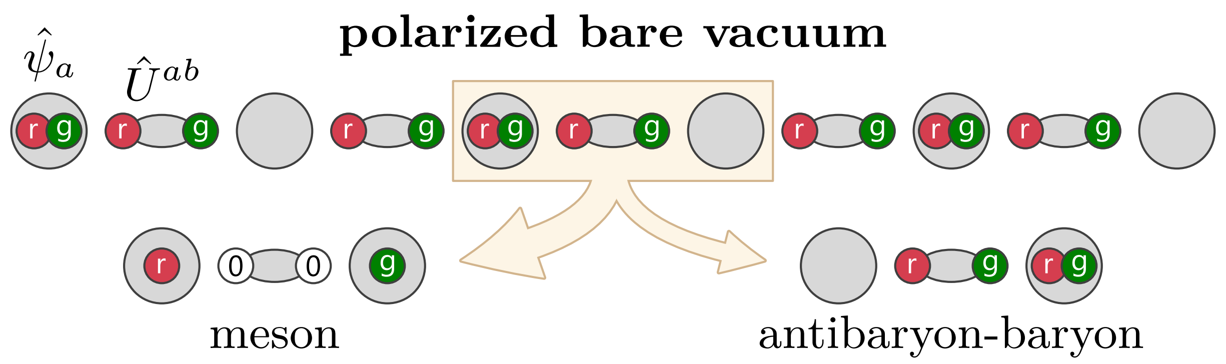


Ergodic Dynamics is explained via ***Eigenstate Thermalization Hypothesis (ETH)***

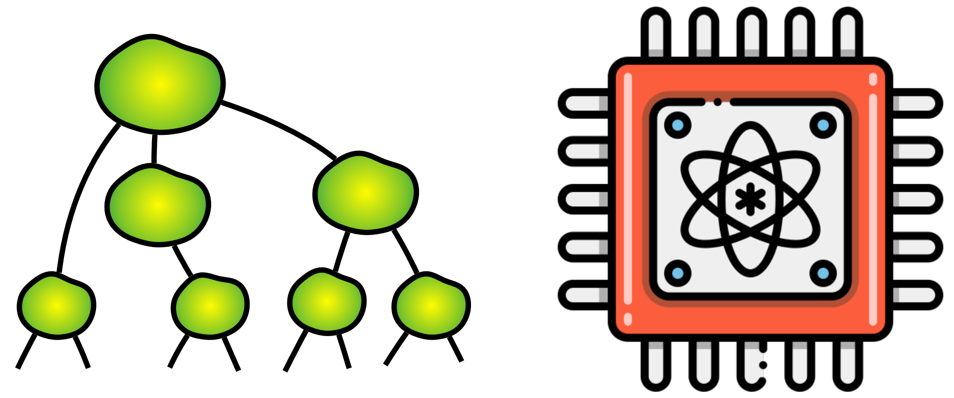
QMB Scars violate ETH with ***long-lived oscillations***





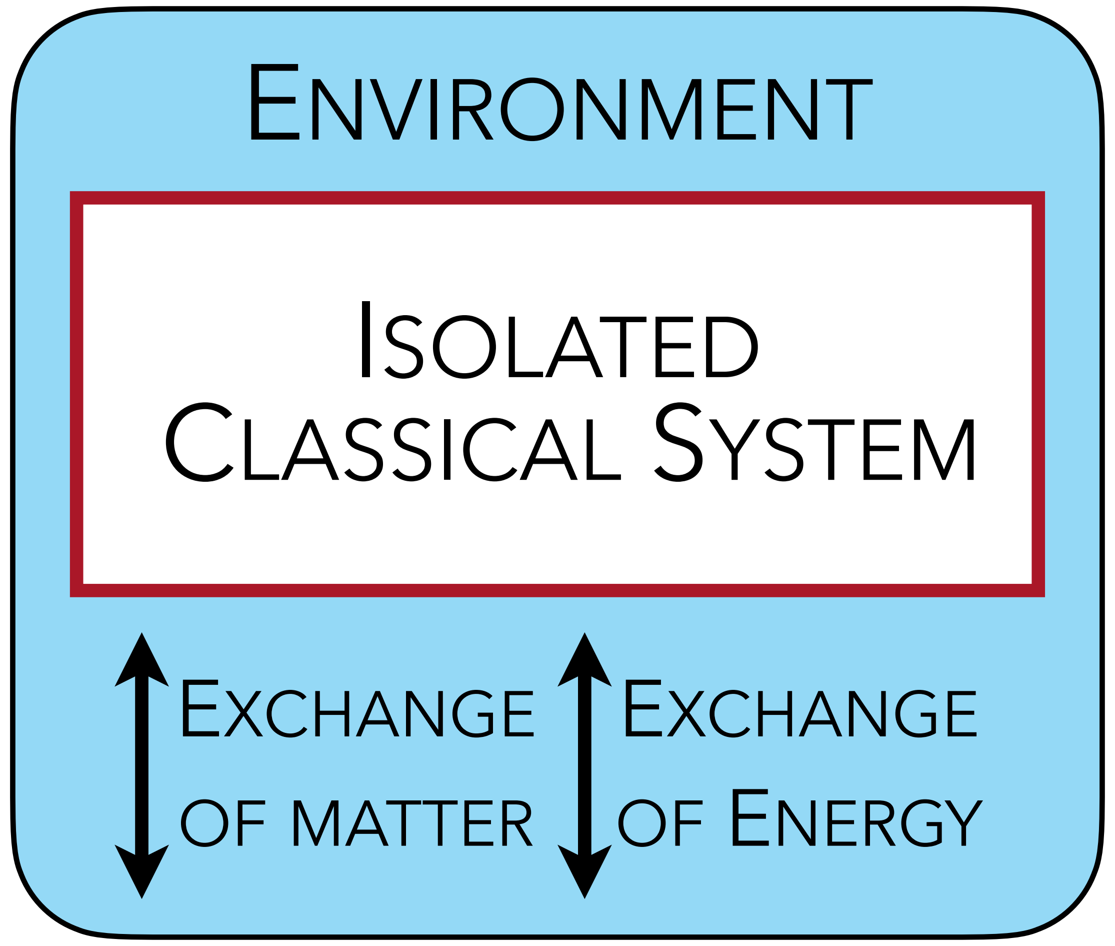


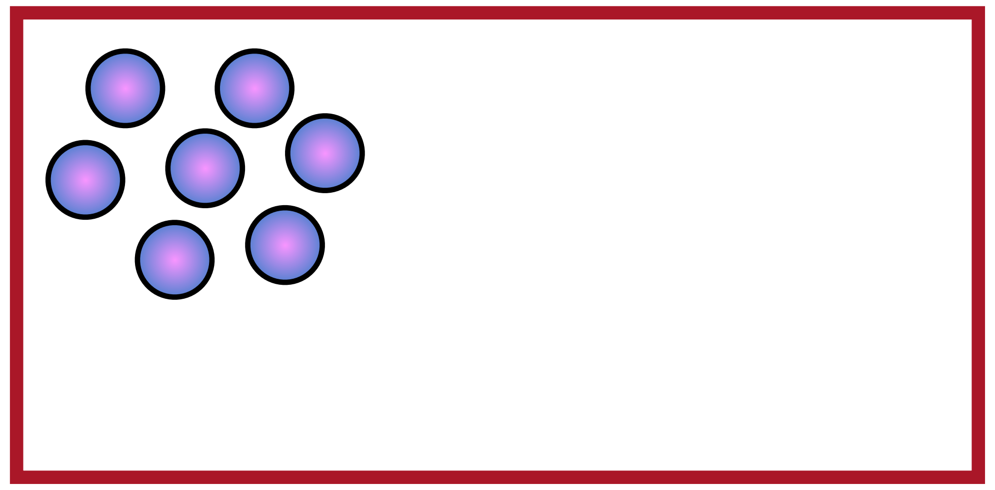
In Lattice Gauge Theories Scars can be explained via ***Particle Anti-particle Phenomena***



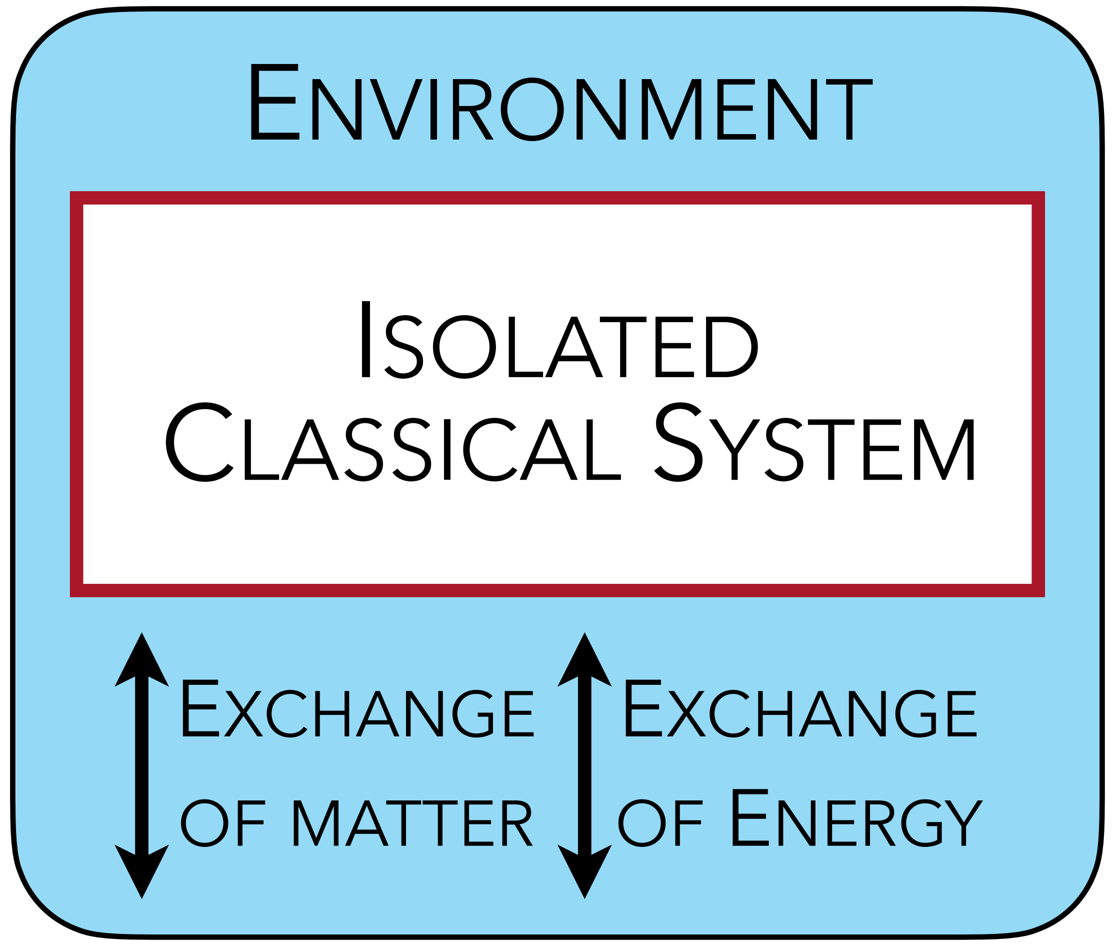
Visible in Classical and Quantum Simulations!

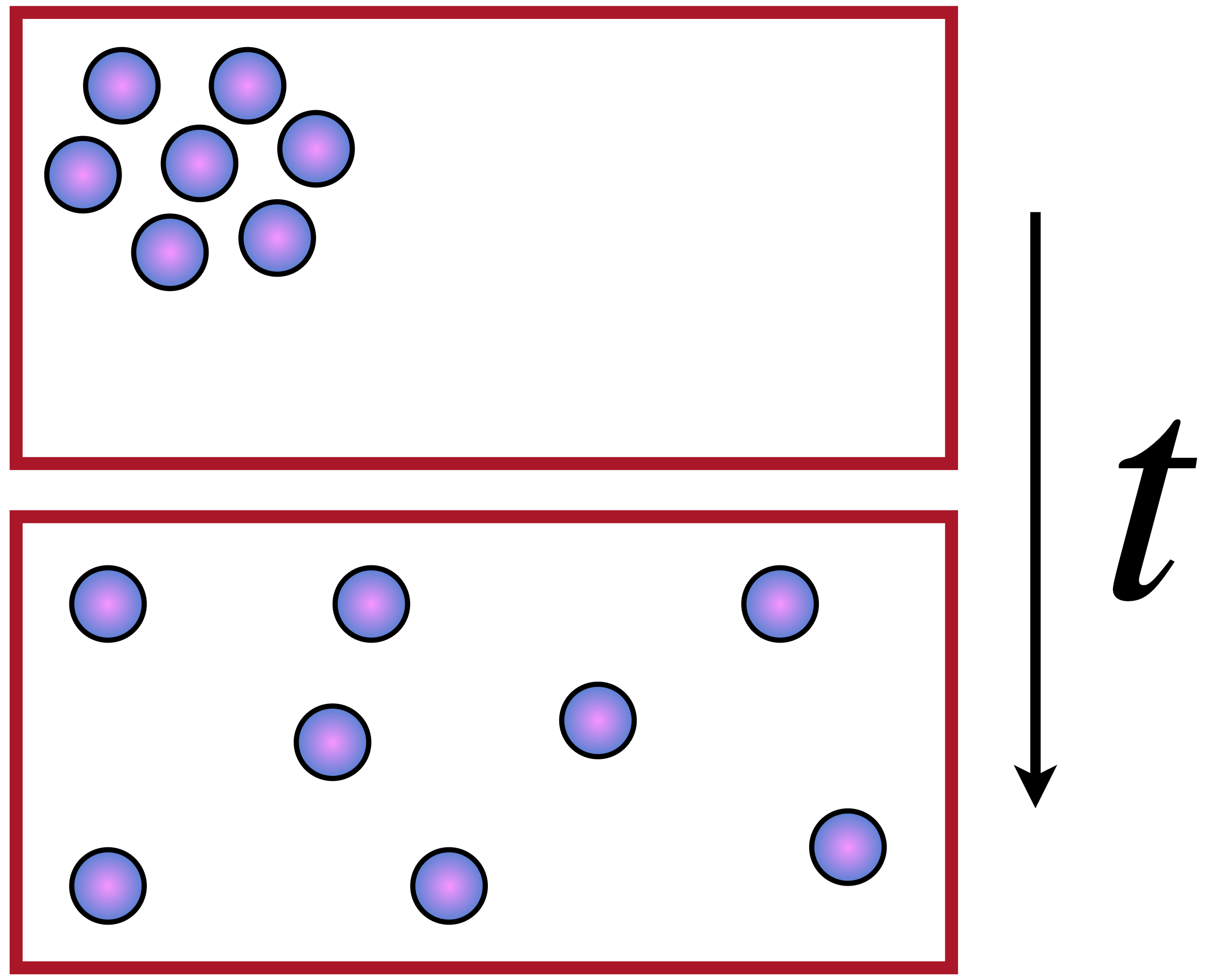
THERMALIZATION IN CLASSICAL SYSTEMS



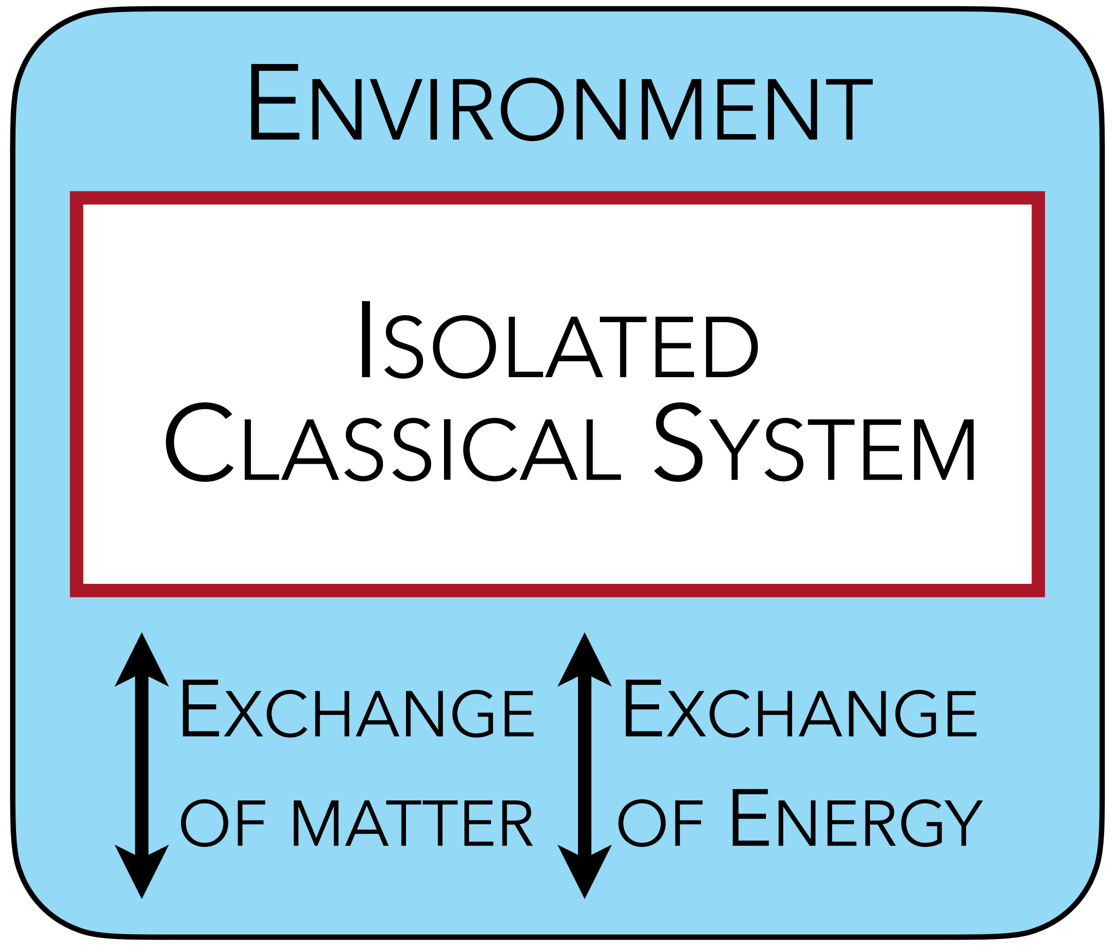


THERMALIZATION IN CLASSICAL SYSTEMS



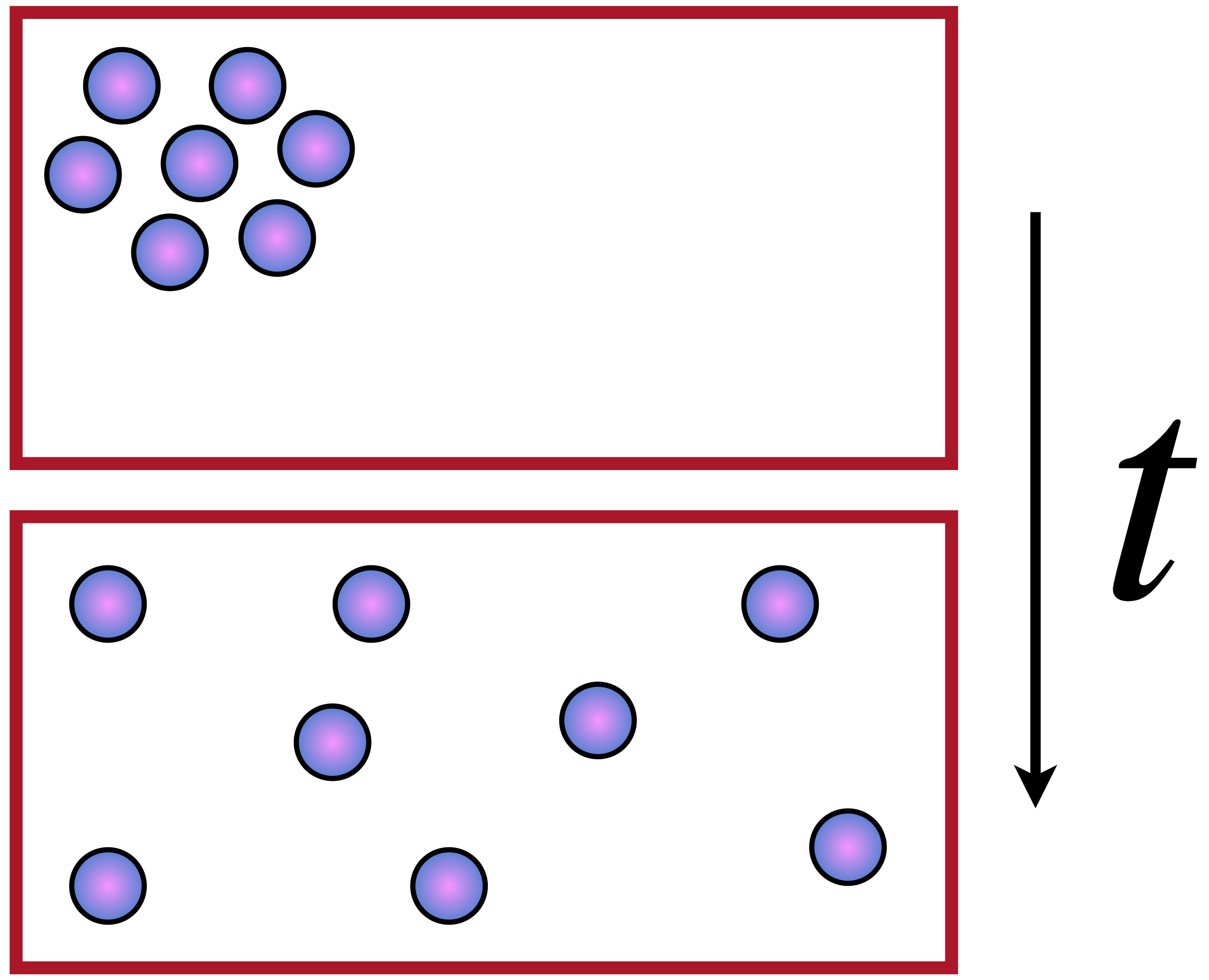


THERMALIZATION IN CLASSICAL SYSTEMS



Ergodic System

Model covering the whole phase space after a long enough time



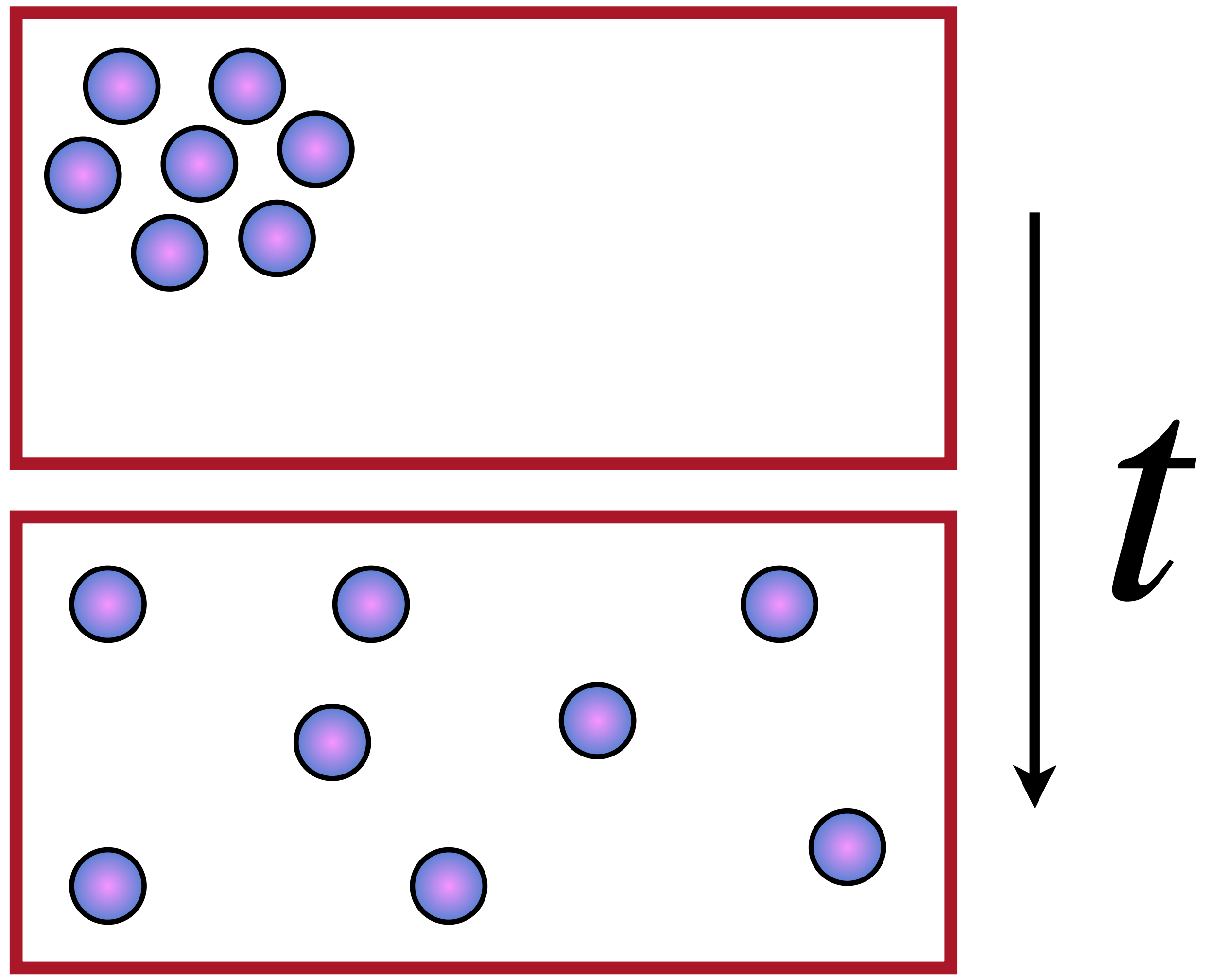
THERMALIZATION IN CLASSICAL SYSTEMS

ENVIRONMENT

ISOLATED CLASSICAL SYSTEM

EXCHANGE OF MATTER

EXCHANGE OF ENERGY

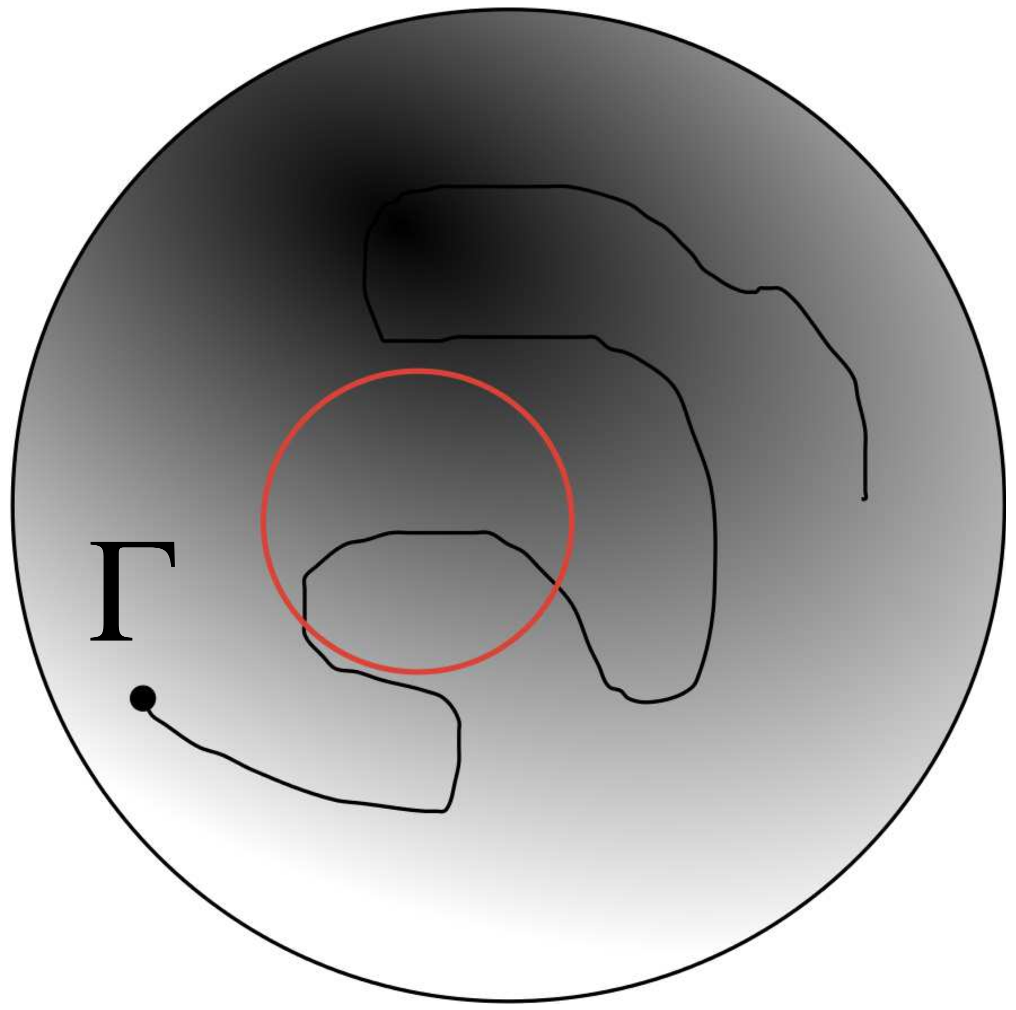


ERGODIC SYSTEM

MODEL COVERING THE WHOLE PHASE SPACE AFTER A LONG ENOUGH TIME

REP. PROG. PHYS. 81 082001 (2018)

$$\Gamma = \{x_1, \ldots, x_N, p_1, \ldots, p_N\}$$



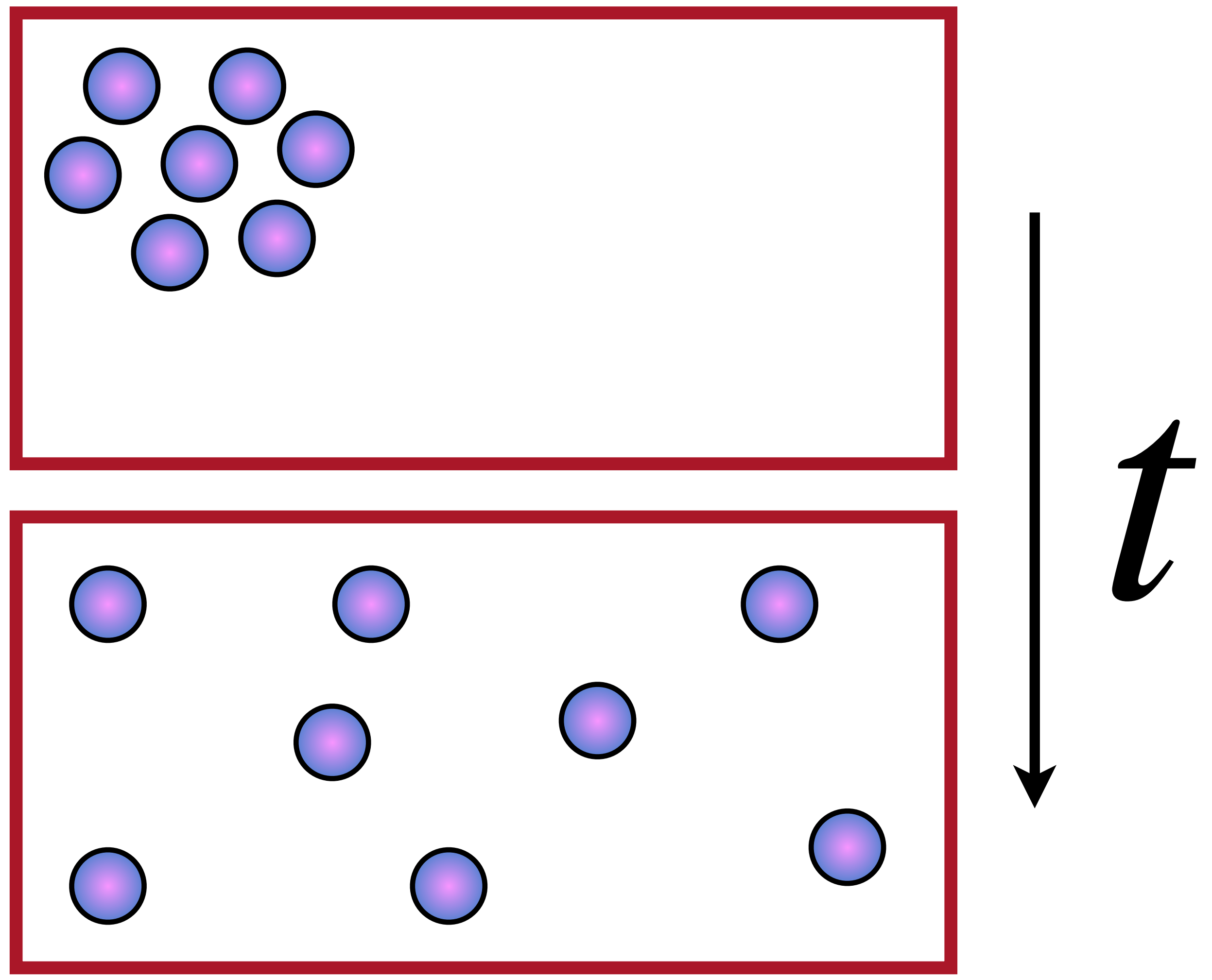
PHASE SPACE S AT CONSTANT ENERGY

THERMALIZATION IN CLASSICAL SYSTEMS

Environment

Isolated Classical System

↕ Exchange of matter ↕ Exchange of energy



Ergodic System

Model covering the whole phase space after a long enough time

Thermalization

Long Time Average

$$\langle O \rangle_t = \lim_{T\to\infty} \int_T dt O(\Gamma(t))$$

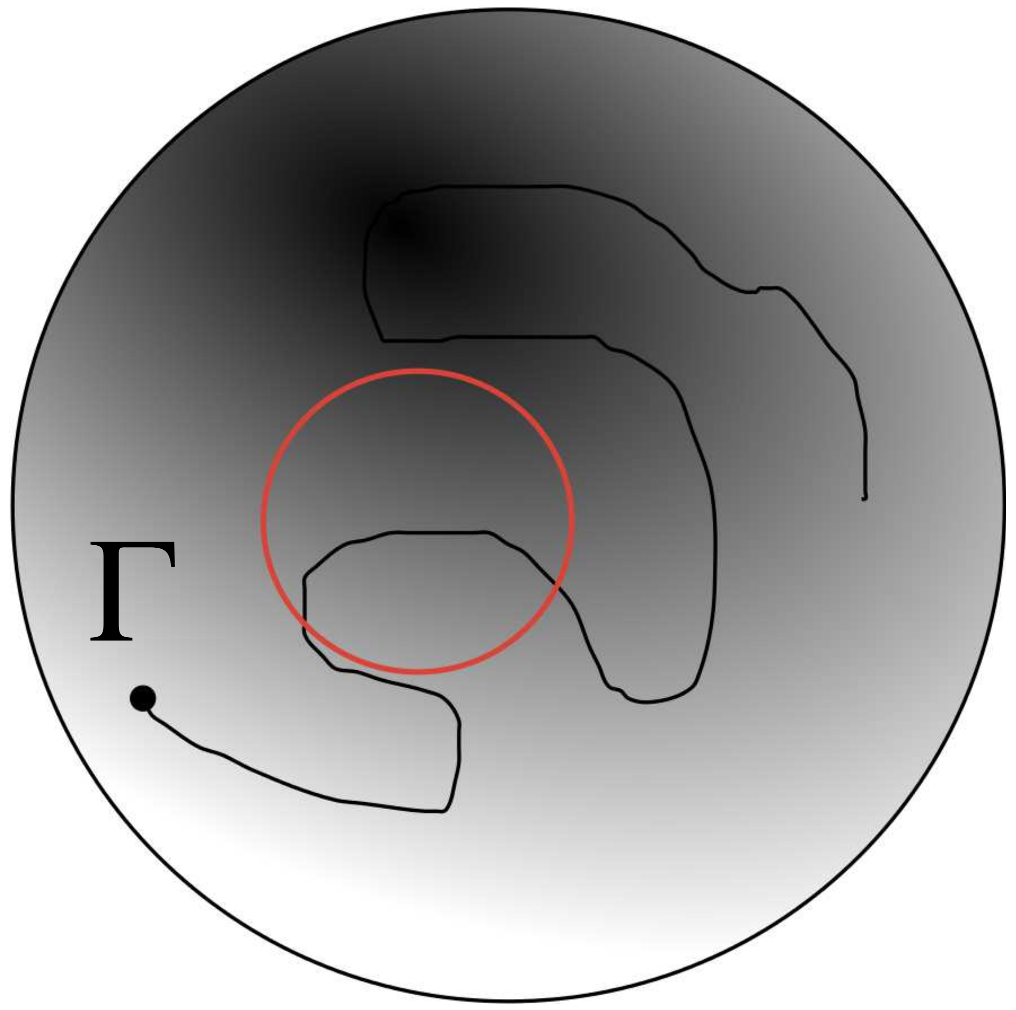
=

Microcanonical Ensemble

$$\langle O \rangle_t = \frac{\int_S O(\Gamma) d\Gamma}{\int_S d\Gamma}$$

Rep. Prog. Phys. 81 082001 (2018)

$$\Gamma = \{x_1, \ldots, x_N, p_1, \ldots, p_N\}$$



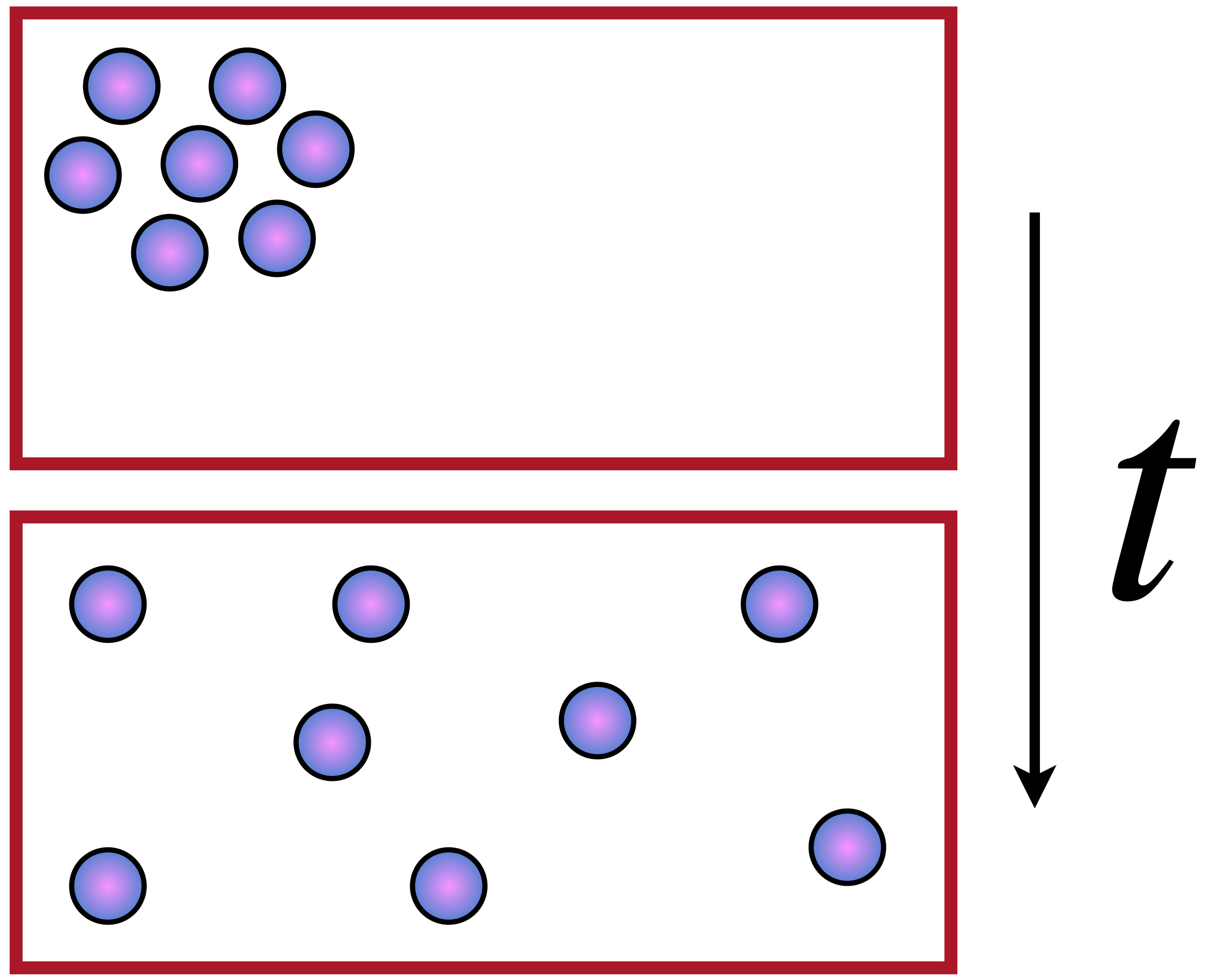
Phase Space S at constant energy

THERMALIZATION IN CLASSICAL SYSTEMS

ENVIRONMENT

ISOLATED CLASSICAL SYSTEM

EXCHANGE OF MATTER — EXCHANGE OF ENERGY



ERGODIC SYSTEM

MODEL COVERING THE WHOLE PHASE SPACE AFTER A LONG ENOUGH TIME

THERMALIZATION

LONG TIME AVERAGE

$$\langle O \rangle_t = \lim_{T\to\infty} \int_T dt O(\Gamma(t))$$

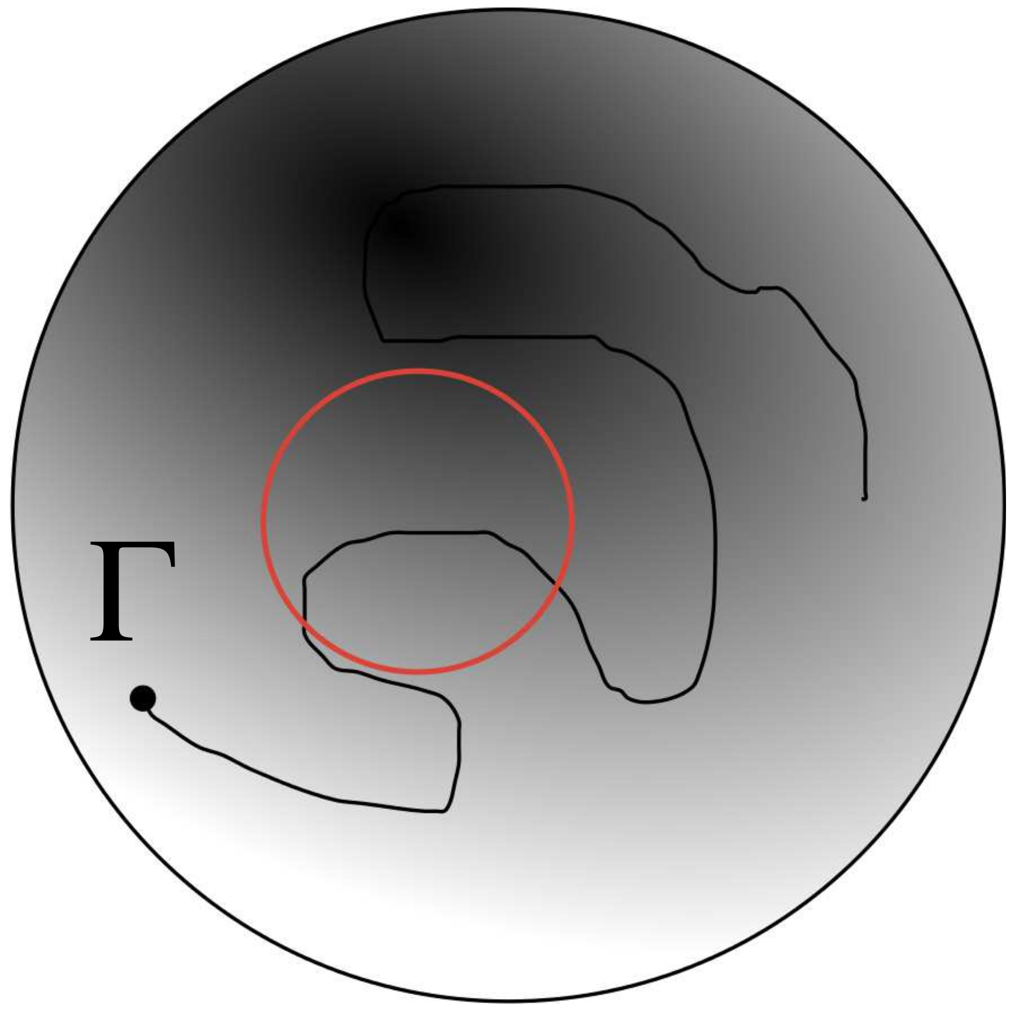
=

MICROCANONICAL ENSEMBLE

$$\langle O \rangle_t = \frac{\int_S O(\Gamma) d\Gamma}{\int_S d\Gamma}$$

REP. PROG. PHYS. 81 082001 (2018)

$$\Gamma = \{x_1, \ldots, x_N, p_1, \ldots, p_N\}$$



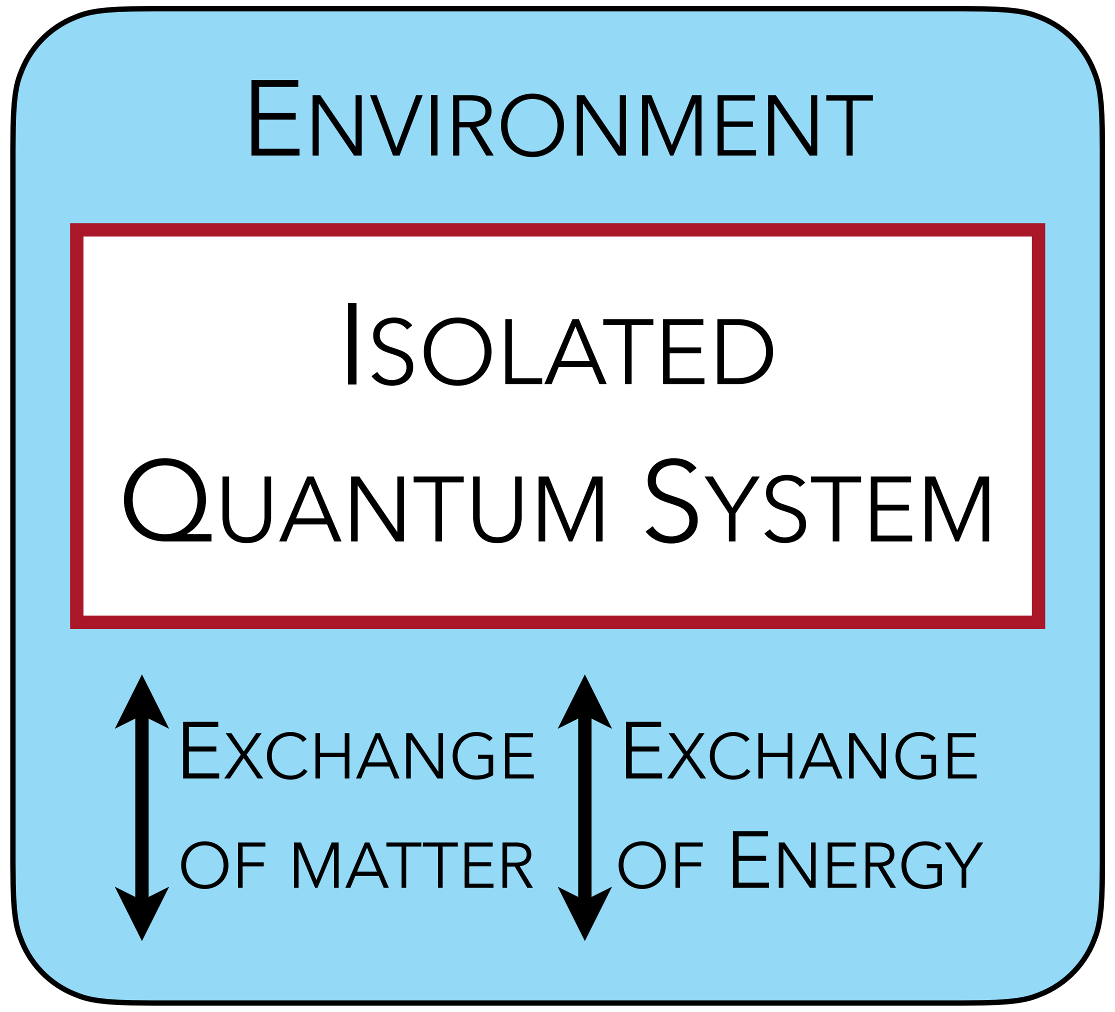
PHASE SPACE S AT CONSTANT ENERGY

INTEGRABLE SYSTEM

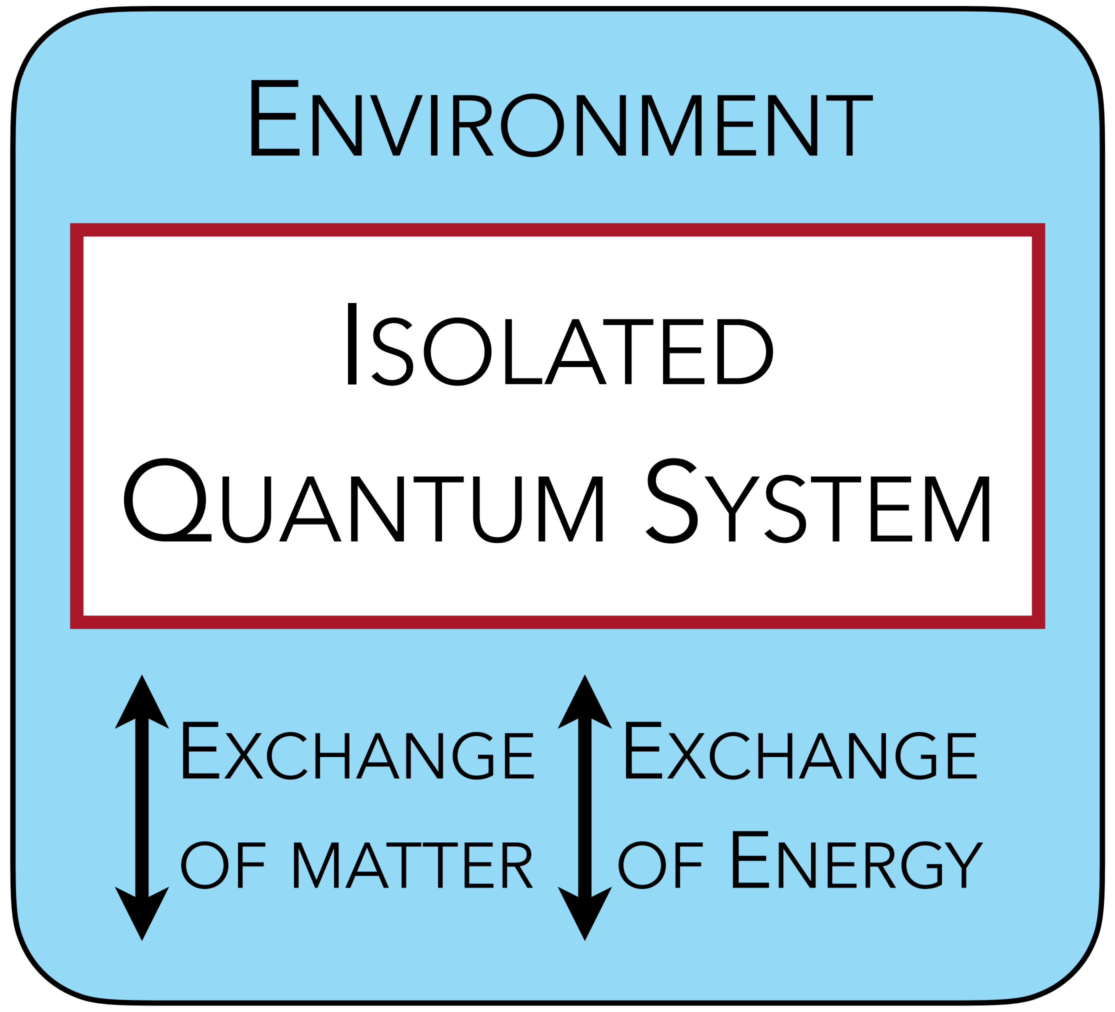
MODEL WITH N INVARIANTS OF MOTION DISPLAYS PERIODICITY. PHASE SPACE IS PARTIALLY COVERED

→ NO THERMALIZATION

THERMALIZATION IN QUANTUM SYSTEMS



THERMALIZATION IN QUANTUM SYSTEMS



$$H = \sum_{\alpha} E_{\alpha} | \Phi_{\alpha} \rangle$$

$$| \Psi(0) \rangle = \sum_{\alpha} C_{\alpha} | \Phi_{\alpha} \rangle$$

$$\longrightarrow$$

Schrodinger Equation

$$i\hbar \frac{\partial}{\partial t} | \Psi(t) \rangle = H | \Psi(t) \rangle$$

THERMALIZATION IN QUANTUM SYSTEMS

ENVIRONMENT

ISOLATED QUANTUM SYSTEM

EXCHANGE OF MATTER

EXCHANGE OF ENERGY

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

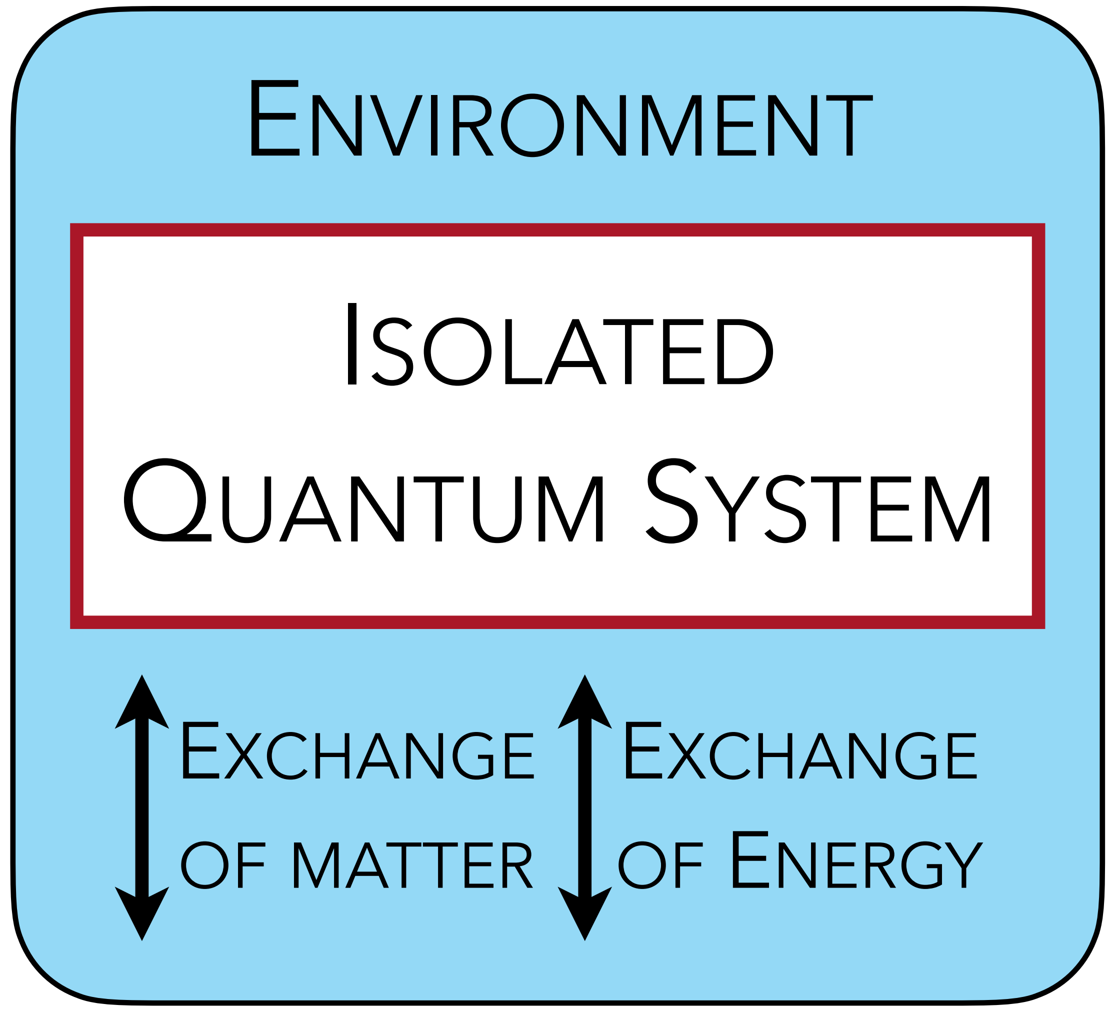
SCHRODINGER EQUATION

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

$$\langle\hat{O}\rangle_t \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\langle\Psi(t)|\hat{O}|\Psi(t)\rangle$$

LONG TIME AVERAGE

THERMALIZATION IN QUANTUM SYSTEMS



$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

Schrodinger Equation

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

$$\langle \hat{O} \rangle_t \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \Psi(t) | \hat{O} | \Psi(t) \rangle$$

Long Time Average

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | e^{it\hat{H}} \hat{O} e^{-it\hat{H}} | \Phi_\beta \rangle$$

$$= \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | \hat{O} | \Phi_\beta \rangle \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i(E_\alpha - E_\beta)t} dt$$

THERMALIZATION IN QUANTUM SYSTEMS

ENVIRONMENT

ISOLATED QUANTUM SYSTEM

EXCHANGE OF MATTER

EXCHANGE OF ENERGY

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

SCHRODINGER EQUATION

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

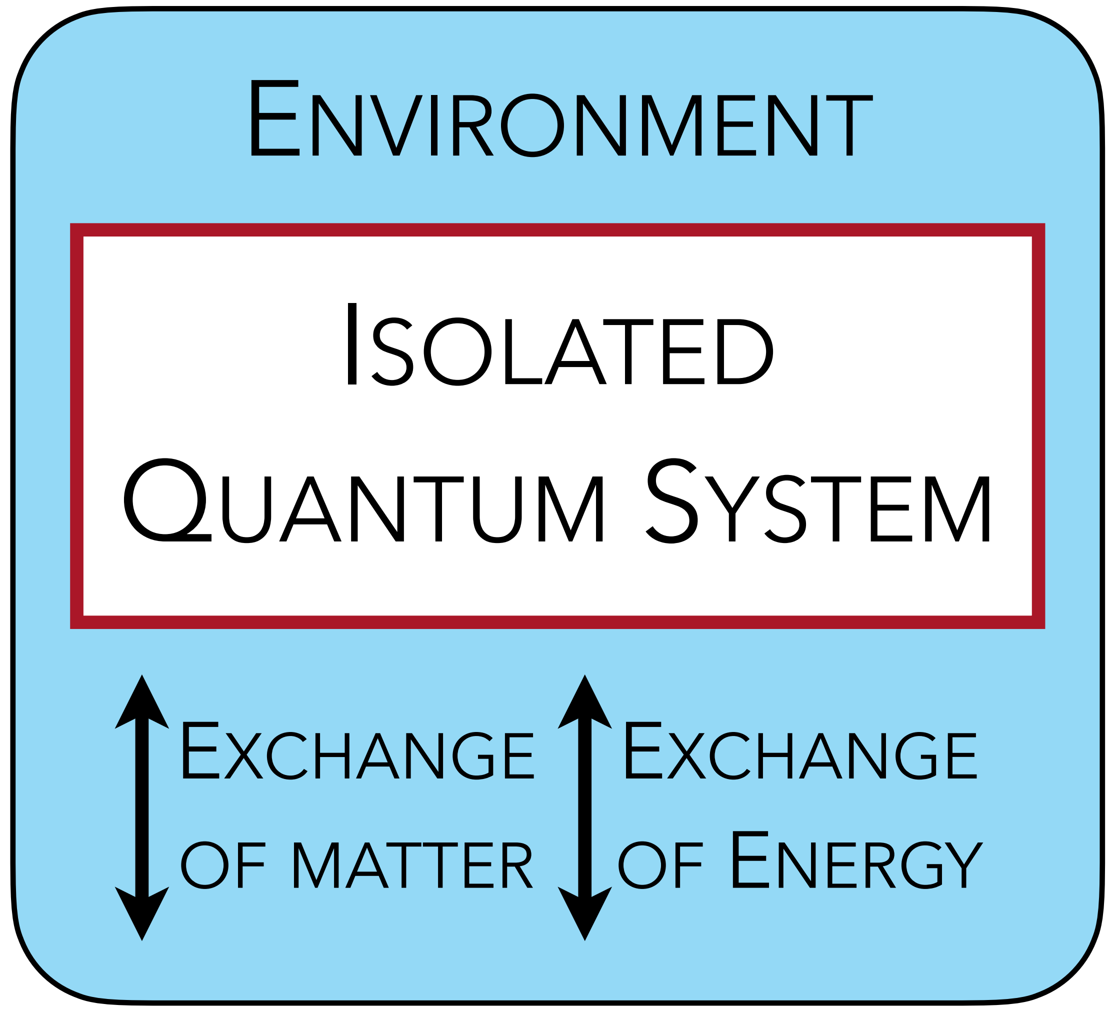
$$\langle \hat{O} \rangle_t \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \Psi(t) | \hat{O} | \Psi(t) \rangle$$

LONG TIME AVERAGE

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | e^{it\hat{H}} \hat{O} e^{-it\hat{H}} | \Phi_\beta \rangle$$

$$= \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | \hat{O} | \Phi_\beta \rangle \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i(E_\alpha - E_\beta)t} dt$$

THERMALIZATION IN QUANTUM SYSTEMS



$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

$\longrightarrow$

SCHRODINGER EQUATION

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

$$\langle \hat{O} \rangle_t \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \Psi(t) | \hat{O} | \Psi(t) \rangle$$ LONG TIME AVERAGE

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | e^{it\hat{H}} \hat{O} e^{-it\hat{H}} | \Phi_\beta \rangle$$

$$= \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | \hat{O} | \Phi_\beta \rangle \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i(E_\alpha - E_\beta)t} dt$$

$$= \sum_\alpha |C_\alpha|^2 \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{\mathrm{DE}}$$

DIAGONAL ENSAMBLE AVERAGE

THERMALIZATION IN QUANTUM SYSTEMS

ENVIRONMENT

ISOLATED QUANTUM SYSTEM

EXCHANGE OF MATTER

EXCHANGE OF ENERGY

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

SCHRODINGER EQUATION

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

$$\langle \hat{O} \rangle_t \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \Psi(t) | \hat{O} | \Psi(t) \rangle$$

LONG TIME AVERAGE

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \sum_{\alpha,\beta} C^*_\alpha C_\beta \langle \Phi_\alpha | e^{it\hat{H}} \hat{O} e^{-it\hat{H}} | \Phi_\beta \rangle$$

$$= \sum_{\alpha,\beta} C^*_\alpha C_\beta \langle \Phi_\alpha | \hat{O} | \Phi_\beta \rangle \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i(E_\alpha - E_\beta)t} dt$$

$$= \sum_\alpha |C_\alpha|^2 \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{\mathrm{DE}}$$

DIAGONAL ENSAMBLE AVERAGE

DEPENDS ON THE INITIAL STATE!

THERMALIZATION IN QUANTUM SYSTEMS

ENVIRONMENT

ISOLATED QUANTUM SYSTEM

↕ EXCHANGE OF MATTER ↕ EXCHANGE OF ENERGY

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

→

SCHRODINGER EQUATION

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

$$\langle \hat{O} \rangle_t \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \Psi(t) | \hat{O} | \Psi(t) \rangle$$

LONG TIME AVERAGE

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | e^{it\hat{H}} \hat{O} e^{-it\hat{H}} | \Phi_\beta \rangle$$

$$= \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | \hat{O} | \Phi_\beta \rangle \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i(E_\alpha - E_\beta)t} dt$$

$$= \sum_\alpha |C_\alpha|^2 \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{\mathrm{DE}}$$

DIAGONAL ENSAMBLE AVERAGE

DEPENDS ON THE INITIAL STATE!

SMALL ENERGY SHELL

$$\delta E = \sqrt{\langle \Psi | \hat{H}^2 | \Psi \rangle - \langle \Psi | \hat{H} | \Psi \rangle^2}$$

STATISTICAL MECHANICS

$$\langle \hat{O} \rangle_{\mathrm{ME}} \equiv \mathrm{Tr}\{\hat{O}\hat{\varrho}_{\mathrm{ME}}\} = \sum_{\substack{\alpha \\ |E_\alpha - E| < \delta E}} \frac{\langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle}{N_{E,\delta E}}$$

MICROCANONICAL ENSAMBLE AVERAGE

THERMALIZATION IN QUANTUM SYSTEMS

ENVIRONMENT

ISOLATED QUANTUM SYSTEM

↕ EXCHANGE OF MATTER ↕ EXCHANGE OF ENERGY

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

→

SCHRODINGER EQUATION

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

$$\langle\hat{O}\rangle_t \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\langle\Psi(t)|\hat{O}|\Psi(t)\rangle$$ LONG TIME AVERAGE

$$= \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\sum_{\alpha,\beta}C^*_\alpha C_\beta\langle\Phi_\alpha|e^{it\hat{H}}\hat{O}e^{-it\hat{H}}|\Phi_\beta\rangle$$

$$= \sum_{\alpha,\beta}C^*_\alpha C_\beta\langle\Phi_\alpha|\hat{O}|\Phi_\beta\rangle \lim_{T\to\infty}\frac{1}{T}\int_0^T e^{i(E_\alpha-E_\beta)t}dt$$

$$= \sum_\alpha |C_\alpha|^2\langle\Phi_\alpha|\hat{O}|\Phi_\alpha\rangle = \langle\hat{O}\rangle_{\mathrm{DE}}$$

DIAGONAL ENSAMBLE AVERAGE

DEPENDS ON THE INITIAL STATE!

SMALL ENERGY SHELL

$$\delta E = \sqrt{\langle\Psi|\hat{H}^2|\Psi\rangle - \langle\Psi|\hat{H}|\Psi\rangle^2}$$

STATISTICAL MECHANICS

$$\langle\hat{O}\rangle_{\mathrm{ME}} \equiv \mathrm{Tr}\{\hat{O}\hat{\varrho}_{\mathrm{ME}}\} = \sum_{\substack{\alpha \\ |E_\alpha - E| < \delta E}} \frac{\langle\Phi_\alpha|\hat{O}|\Phi_\alpha\rangle}{N_{E,\delta E}}$$

MICROCANONICAL ENSAMBLE AVERAGE

LONG-TIME AVERAGE $\neq$ STATISTICAL MECHANICS

THERMALIZATION IN QUANTUM SYSTEMS

ENVIRONMENT

ISOLATED QUANTUM SYSTEM

EXCHANGE OF MATTER

EXCHANGE OF ENERGY

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

SCHRODINGER EQUATION

$$i\hbar \frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

$$\langle \hat{O} \rangle_t \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt \langle \Psi(t)|\hat{O}|\Psi(t)\rangle$$

LONG TIME AVERAGE

$$= \lim_{T\to\infty} \frac{1}{T}\int_0^T dt \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | e^{it\hat{H}} \hat{O} e^{-it\hat{H}} | \Phi_\beta \rangle$$

$$= \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | \hat{O} | \Phi_\beta \rangle \lim_{T\to\infty} \frac{1}{T}\int_0^T e^{i(E_\alpha - E_\beta)t} dt$$

$$= \sum_\alpha |C_\alpha|^2 \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{DE}$$

DIAGONAL ENSAMBLE AVERAGE

DEPENDS ON THE INITIAL STATE!

SMALL ENERGY SHELL

$$\delta E = \sqrt{\langle \Psi | \hat{H}^2 | \Psi \rangle - \langle \Psi | \hat{H} | \Psi \rangle^2}$$

STATISTICAL MECHANICS

$$\langle \hat{O} \rangle_{ME} \equiv \mathrm{Tr}\{\hat{O}\hat{\varrho}_{ME}\} = \sum_{\substack{\alpha \\ |E_\alpha - E| < \delta E}} \frac{\langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle}{N_{E,\delta E}}$$

MICROCANONICAL ENSAMBLE AVERAGE

LONG-TIME AVERAGE ≠ STATISTICAL MECHANICS

BUT THERMALIZATION IS OBSERVED EXPERIMENTALLY IN ISOLATED COLD ATOMIC GASES FOR MANY INITIAL STATES!

REP. PROG. PHYS. 81 082001 (2018)

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

N BODY SYSTEM WITH NON-DEGENERATE SPECTRUM

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

N BODY SYSTEM WITH NON-DEGENERATE SPECTRUM

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

HYPOTHESIS II

$O_{\alpha\alpha}$ IS THERMAL ~ CONSTANT VALUE IN A SMALL WINDOW $\forall \alpha \; |E_\alpha - \bar{E}| < \Delta E$

$$O_{\alpha\alpha} = \langle \Phi_\alpha | O | \Phi_\alpha \rangle = \langle O \rangle_{\text{thm}}$$

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

N BODY SYSTEM WITH NON-DEGENERATE SPECTRUM

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

HYPOTHESIS II

$O_{\alpha\alpha}$ IS THERMAL ~ CONSTANT VALUE IN A SMALL WINDOW $\forall \alpha \; |E_\alpha - \bar{E}| < \Delta E$

$$O_{\alpha\alpha} = \langle \Phi_\alpha | O | \Phi_\alpha \rangle = \langle O \rangle_{\text{thm}}$$

CONSEQUENCE

$$\langle O(t) \rangle = \sum_\alpha^N |c_\alpha|^2 O_{\alpha\alpha} \sim \langle O \rangle_{\text{thm}} \sim \sum_\alpha^{N_{E,\Delta E}} \frac{O_{\alpha\alpha}}{N_{E,\Delta E}} = \langle O \rangle_{\text{ME}}$$

DIAGONAL = MICROCANONICAL

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

N BODY SYSTEM WITH NON-DEGENERATE SPECTRUM

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

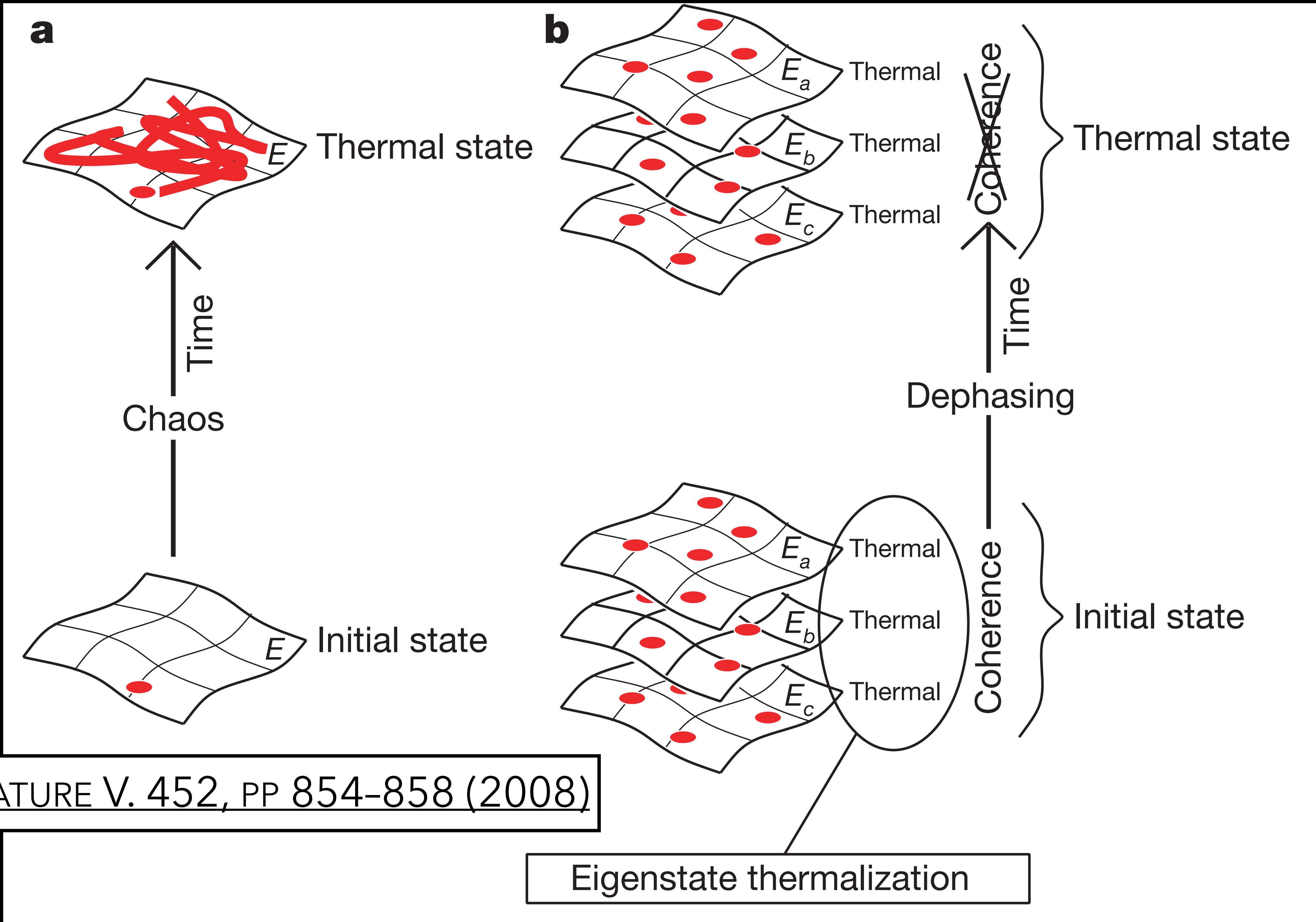
$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

HYPOTHESIS II

$O_{\alpha\alpha}$ IS THERMAL ~ CONSTANT VALUE IN A SMALL WINDOW $\forall \alpha \; |E_\alpha - \bar{E}| < \Delta E$

$$O_{\alpha\alpha} = \langle \Phi_\alpha | O | \Phi_\alpha \rangle = \langle O \rangle_{\text{thm}}$$



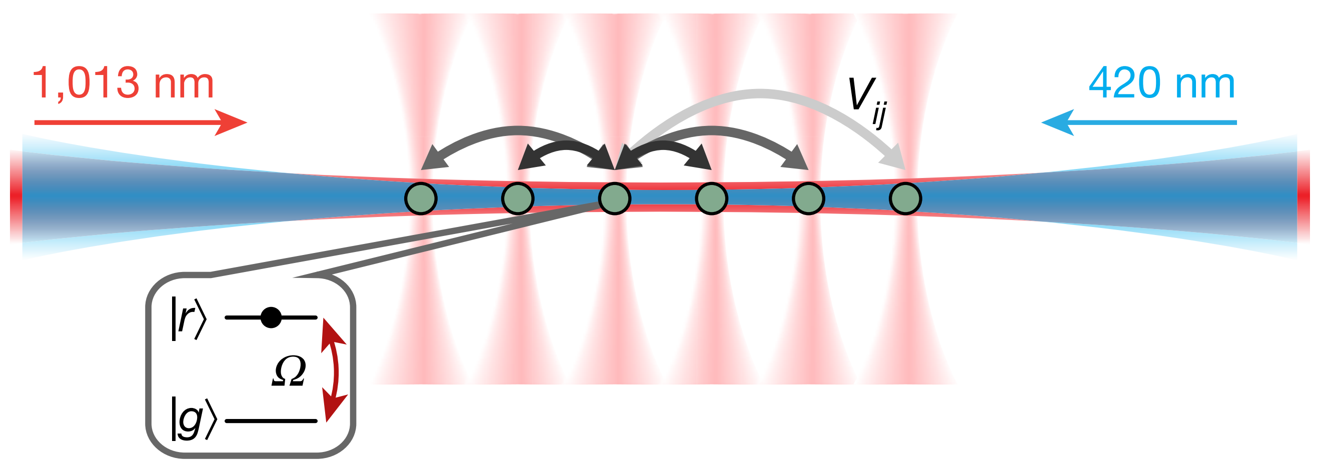
NATURE V. 452, PP 854-858 (2008)

CONSEQUENCE

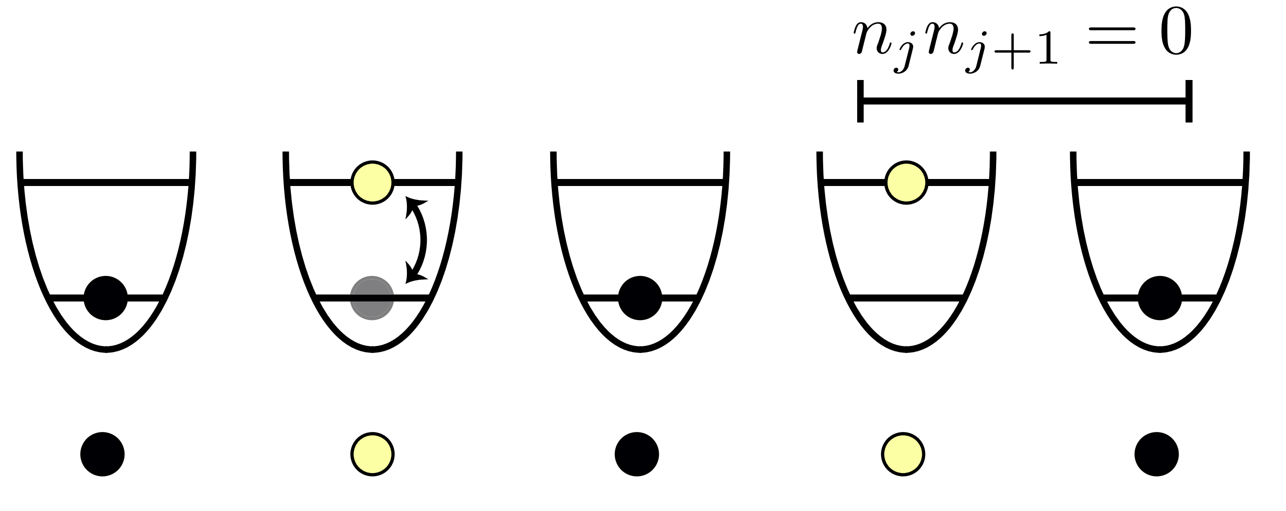
$$\langle O(t) \rangle = \sum_\alpha^N |c_\alpha|^2 O_{\alpha\alpha} \sim \langle O \rangle_{\text{thm}} \sim \sum_\alpha^{N_{E,\Delta E}} \frac{O_{\alpha\alpha}}{N_{E,\Delta E}} = \langle O \rangle_{\text{ME}}$$

DIAGONAL = MICROCANONICAL

QUANTUM MANYBODY SCARS

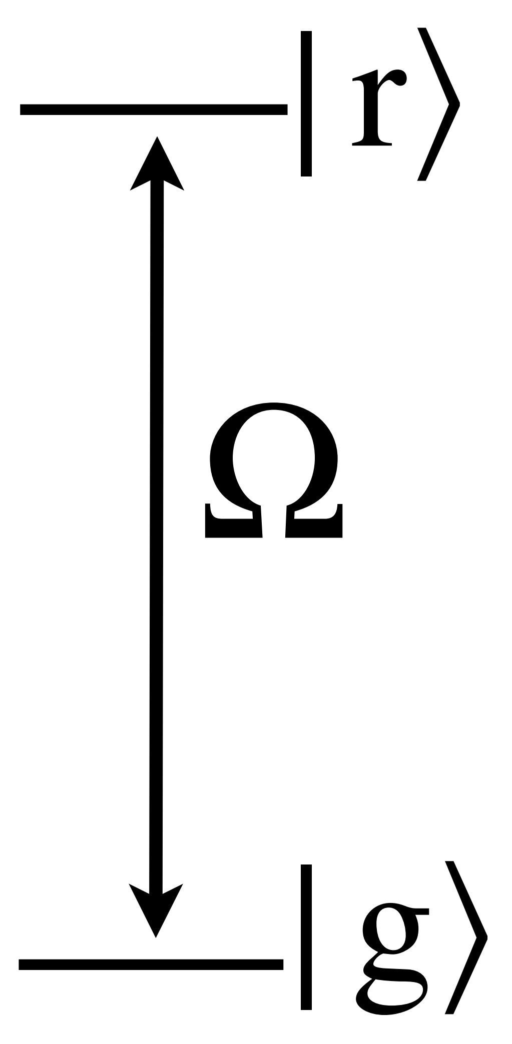


$$H_{\mathrm{Ryd}} = \sum_j (\Omega\sigma_j^x + \Delta\sigma_j^z) + \sum_{j<k} V_{jk} n_j n_k$$

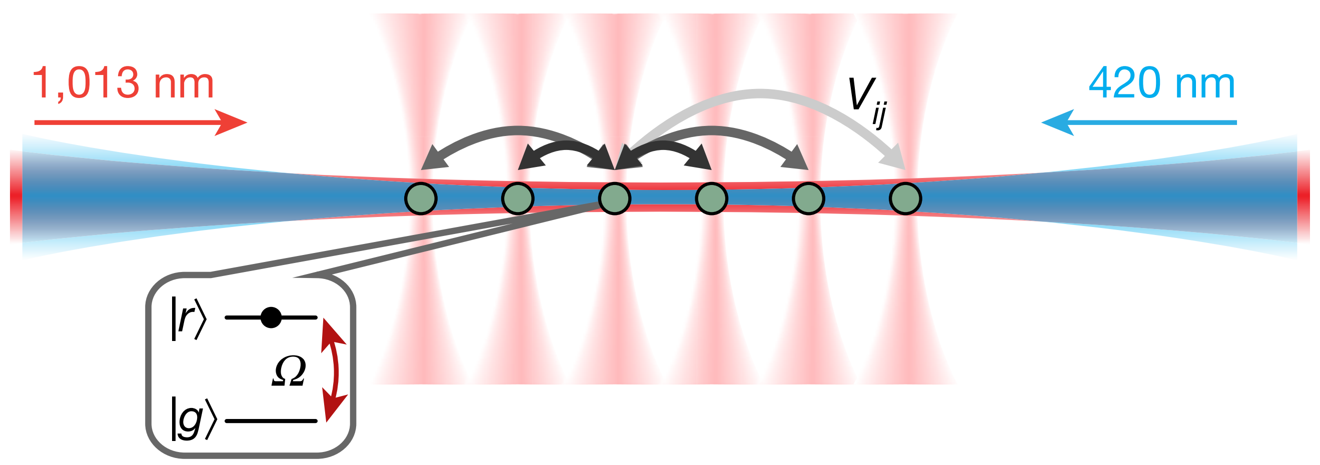


BLOCKADE: $V_{jk} \gg \Omega, \Delta$

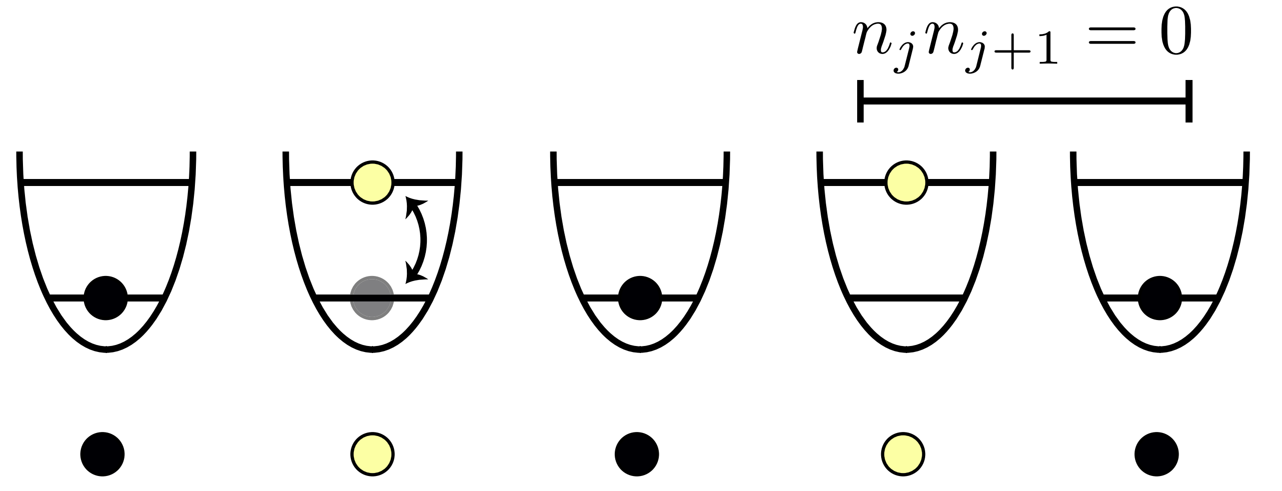
$$H_{\mathrm{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j)$$



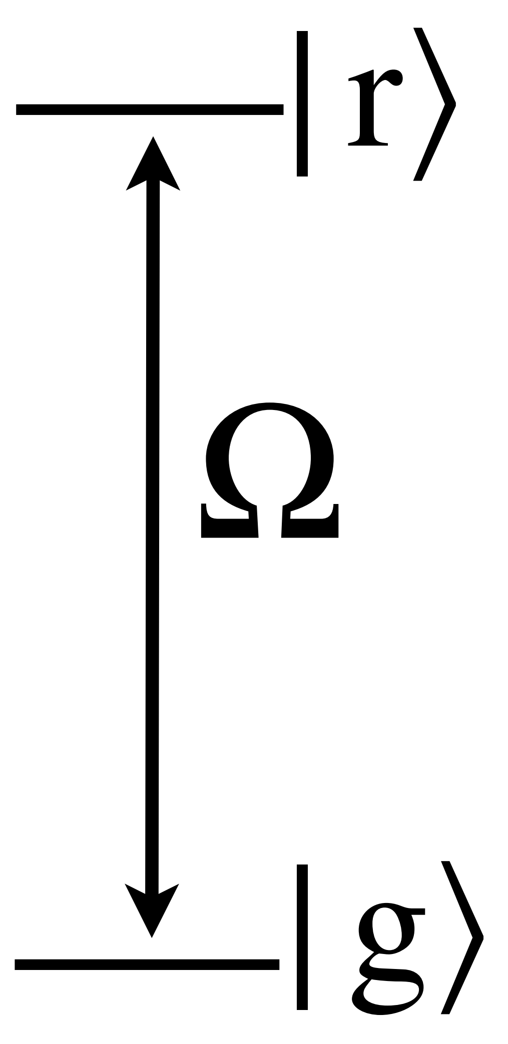
QUANTUM MANYBODY SCARS



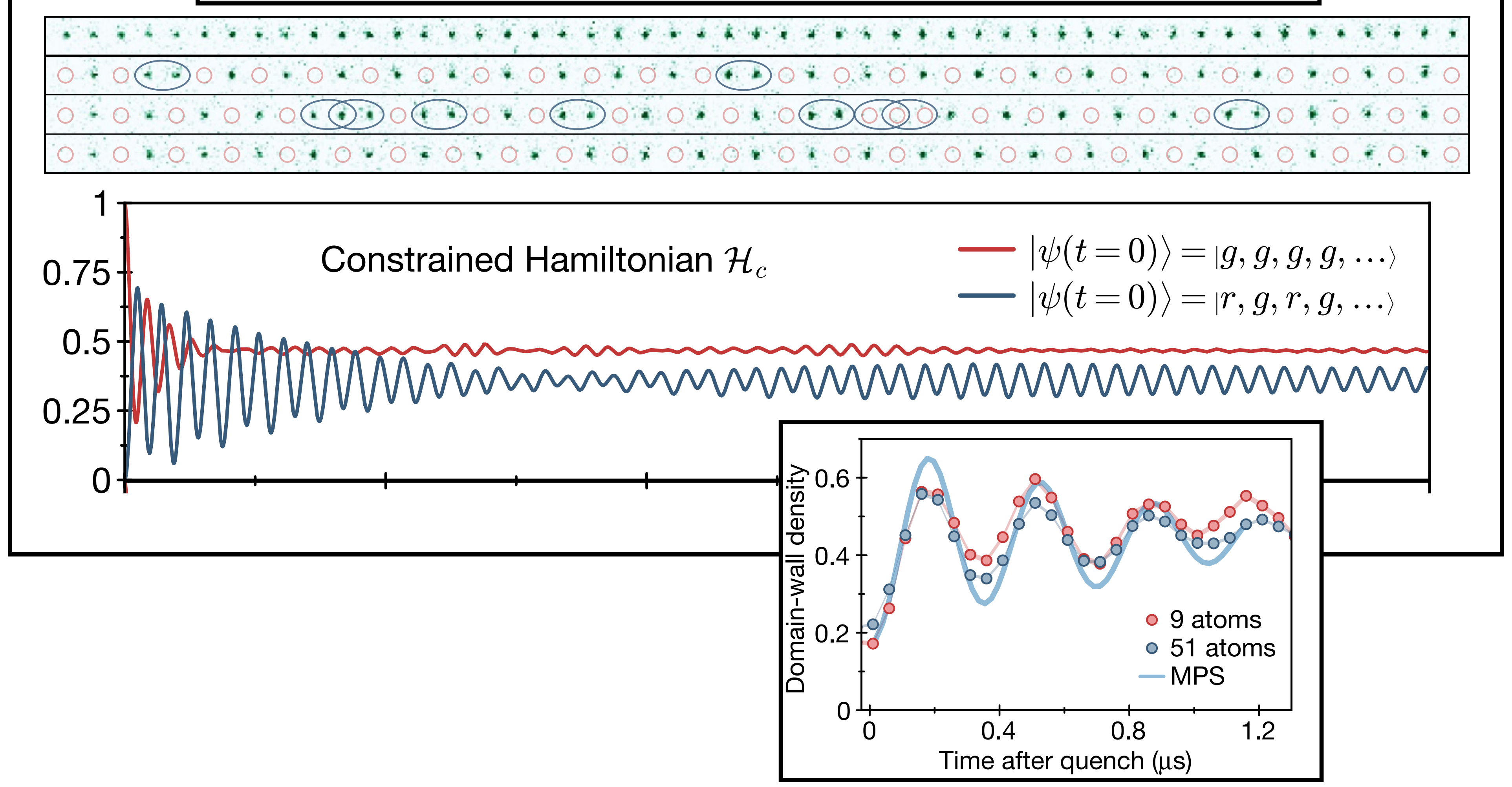
$$H_{\text{Ryd}} = \sum_j (\Omega\sigma_j^x + \Delta\sigma_j^z) + \sum_{j<k} V_{jk} n_j n_k$$



BLOCKADE: $V_{jk} \gg \Omega, \Delta$ $\quad H_{\text{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j)$



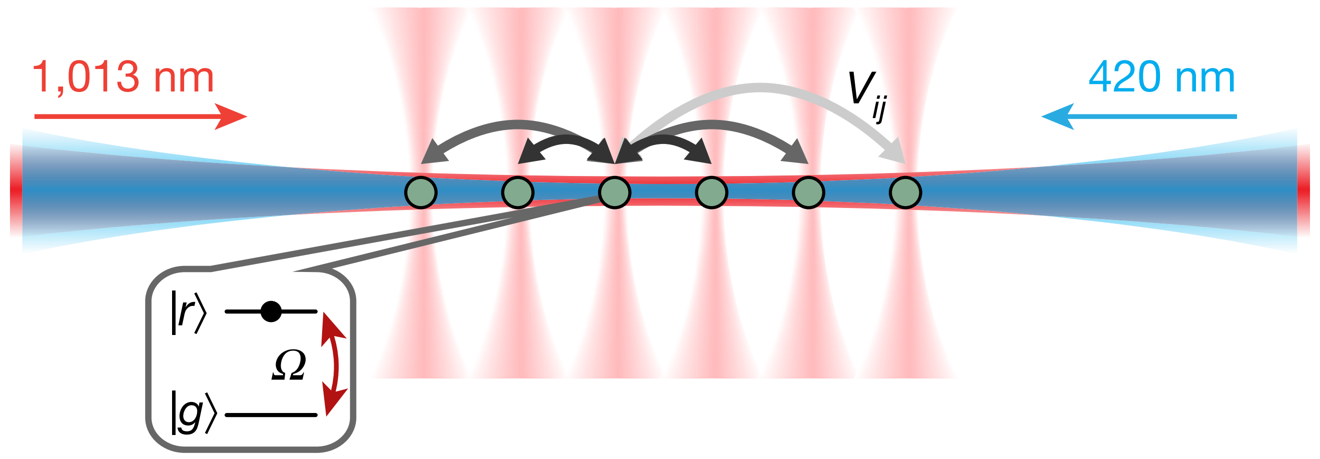
BERNIEN ET AL, NATURE 551, PP 579-584 (2017)



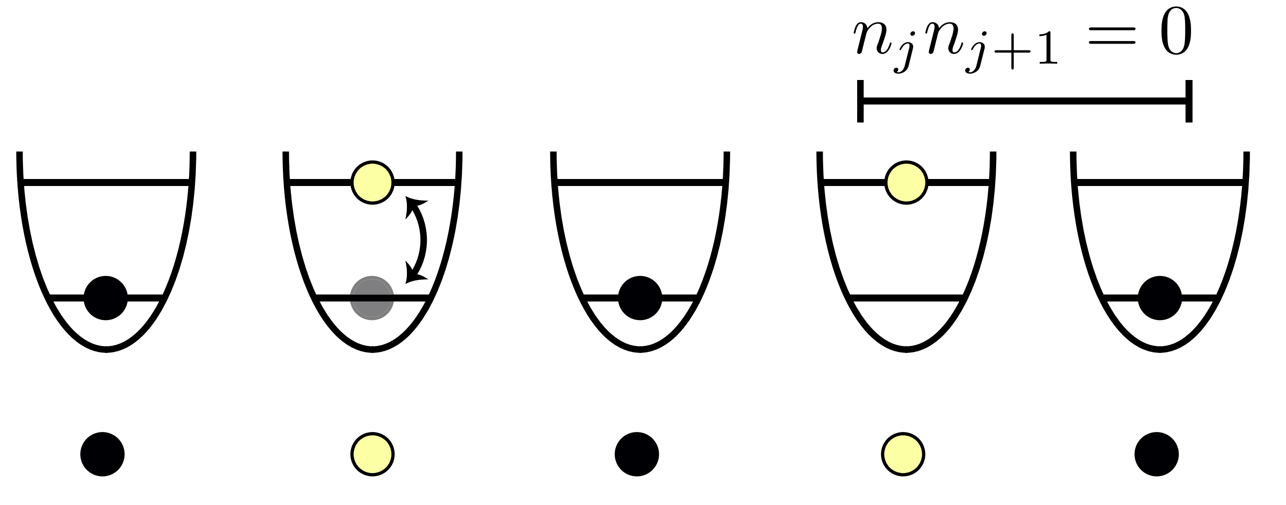
QUENCH DYNAMICS FROM $|0\rangle = |\bullet\bullet\bullet\bullet\bullet\bullet\ldots\rangle$: THERMALIZATION

QUENCH DYNAMICS FROM $|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\ldots\rangle$: LONG-LIVED OSCILLATIONS

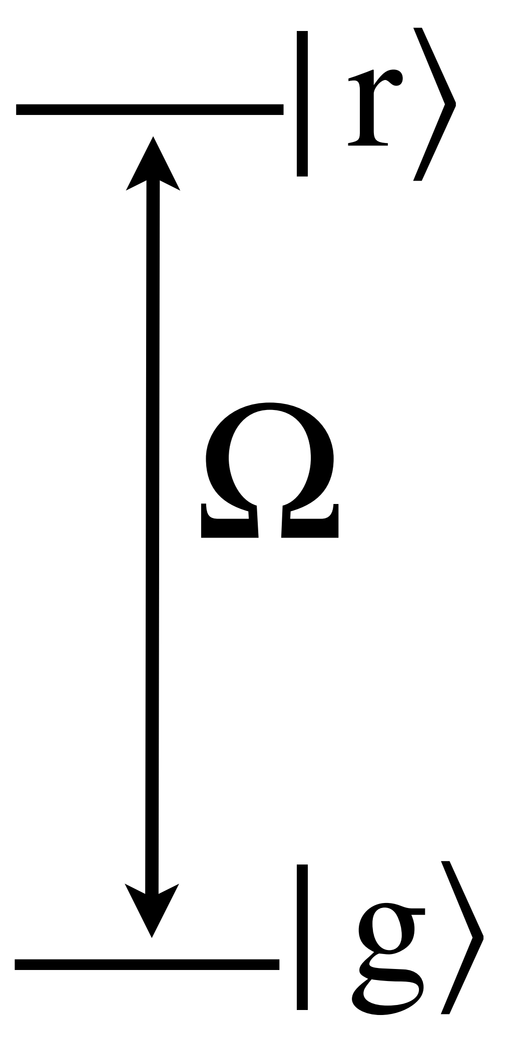
QUANTUM MANYBODY SCARS



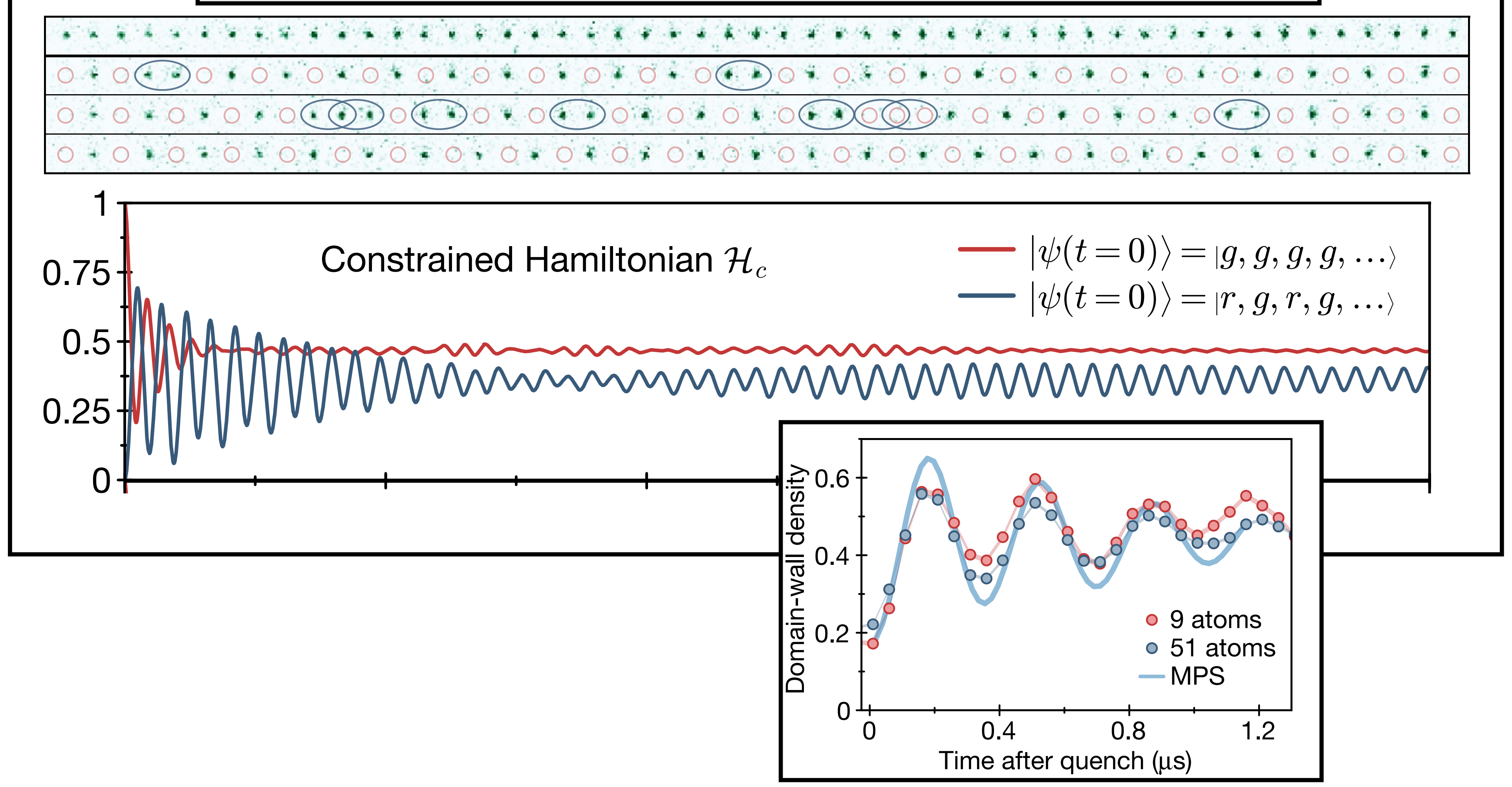
$$H_{\text{Ryd}} = \sum_j (\Omega\sigma_j^x + \Delta\sigma_j^z) + \sum_{j<k} V_{jk} n_j n_k$$



BLOCKADE: $V_{jk} \gg \Omega, \Delta$ $\quad H_{\text{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j)$



BERNIEN ET AL, NATURE 551, PP 579-584 (2017)

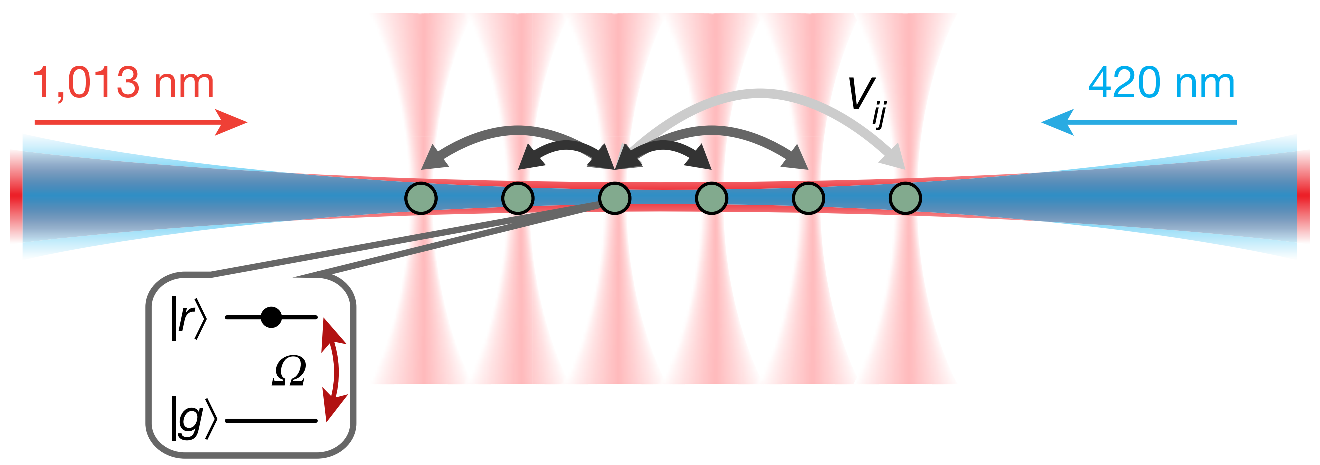


QUENCH DYNAMICS FROM
$|0\rangle = |\bullet\bullet\bullet\bullet\bullet\bullet\bullet\ldots\rangle$:
THERMALIZATION

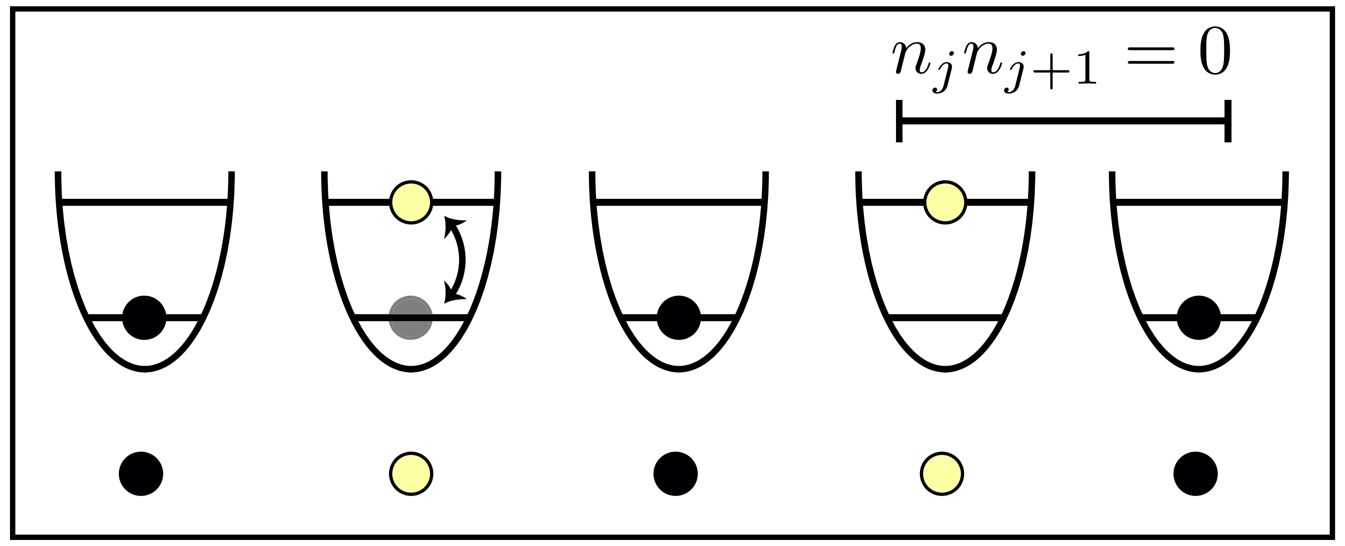
QUENCH DYNAMICS FROM
$|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\bullet\circ\ldots\rangle$:
LONG-LIVED OSCILLATIONS

[1] PERSISTENT DYNAMICS
SPECIFIC INITIAL STATES
FROM AN HILBERT SUBSPACE

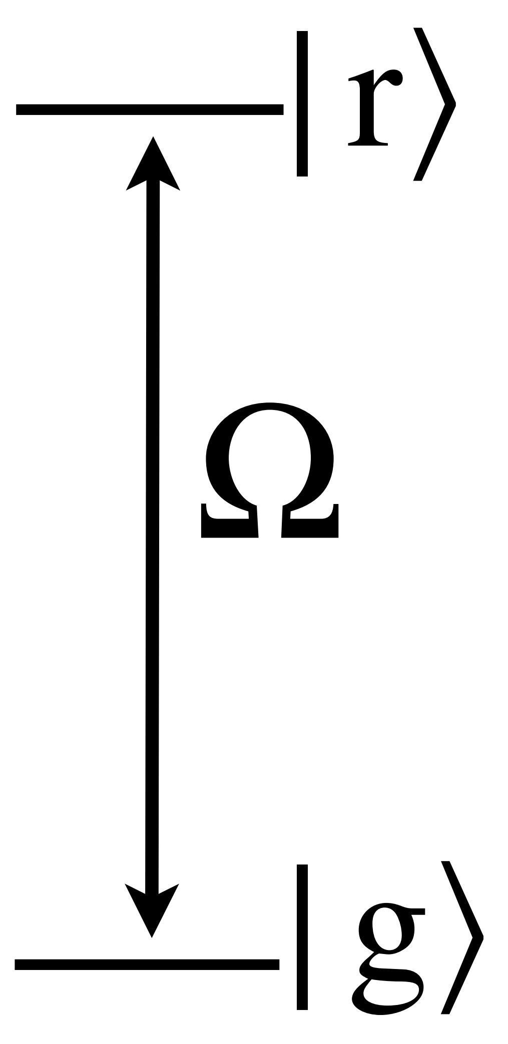
QUANTUM MANYBODY SCARS



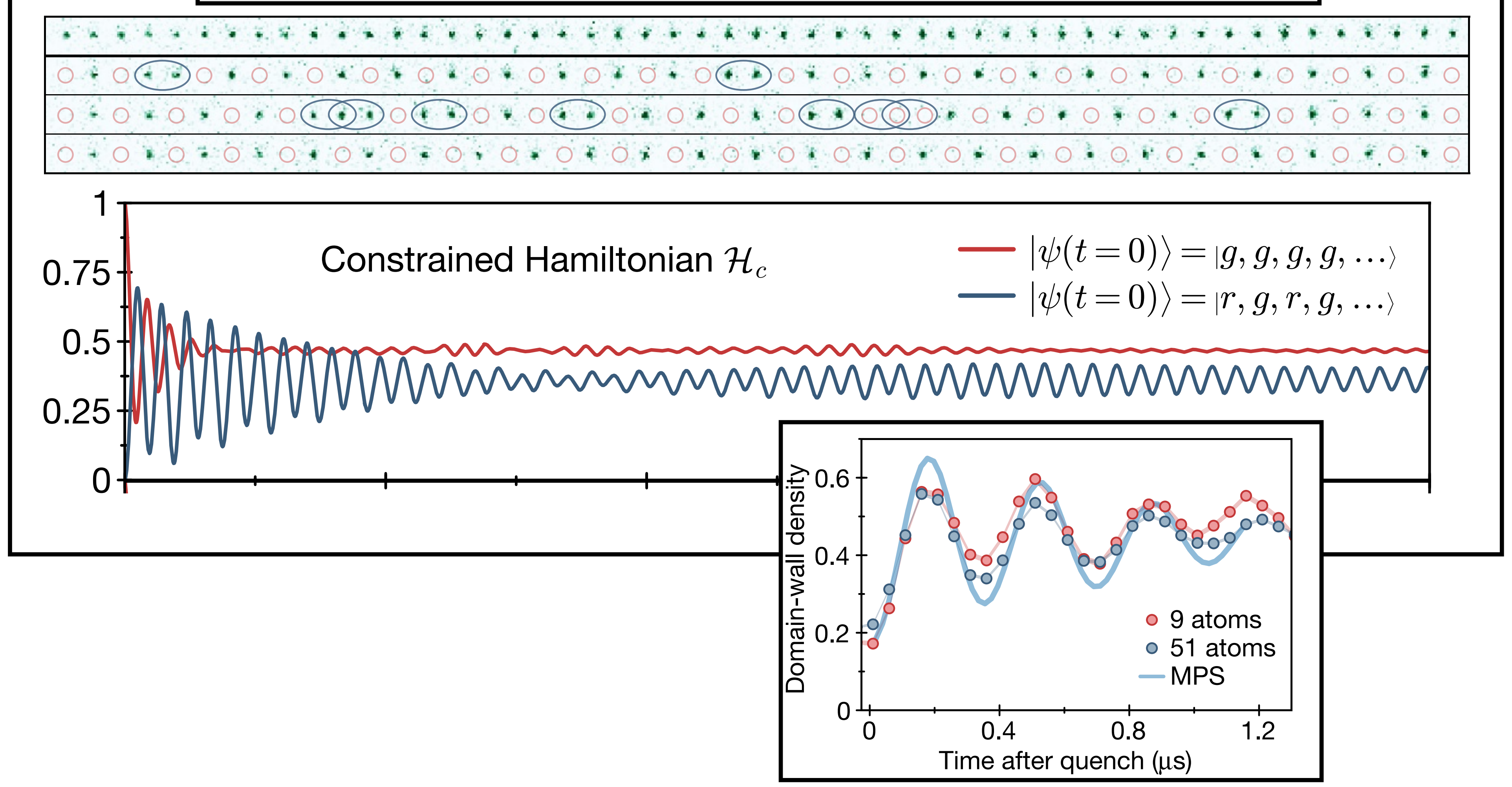
$$H_{\mathrm{Ryd}} = \sum_j (\Omega \sigma_j^x + \Delta \sigma_j^z) + \sum_{j<k} V_{jk} n_j n_k$$



BLOCKADE: $V_{jk} \gg \Omega, \Delta$ $\quad H_{\mathrm{FSS}} = \sum_j (\Omega \sigma_j^x + 2\Delta n_j)$



BERNIEN ET AL, NATURE 551, PP 579-584 (2017)

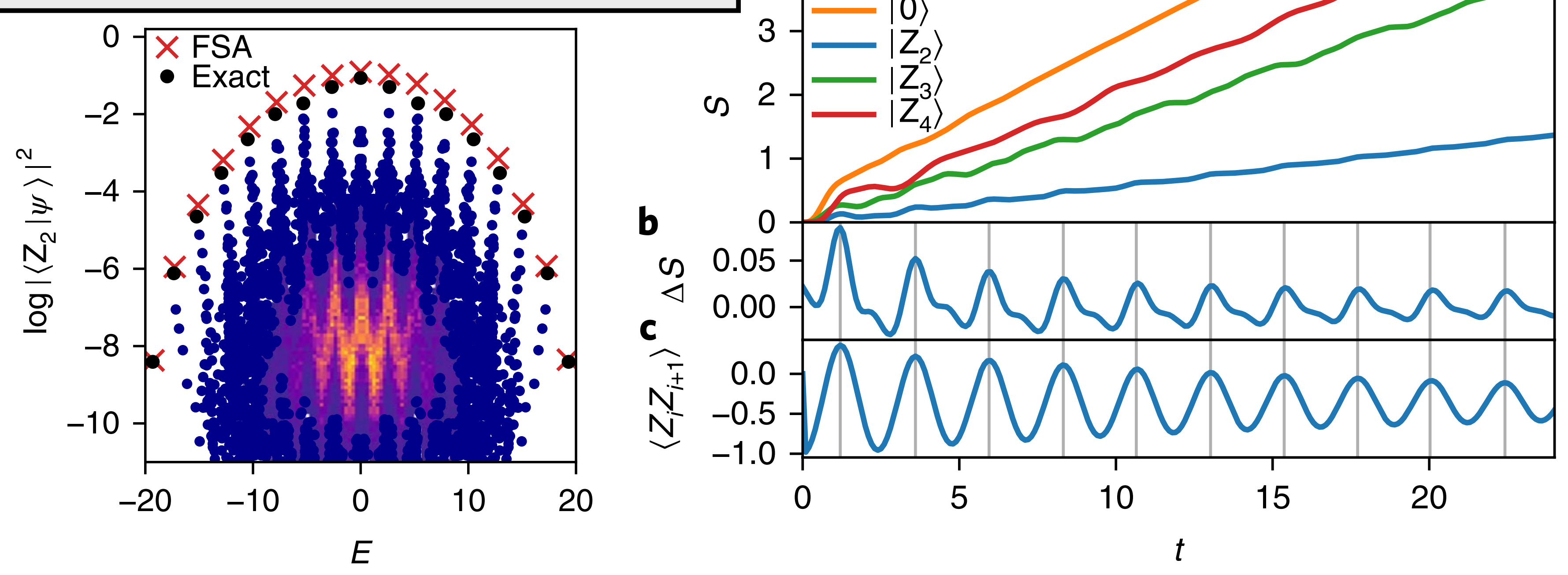


QUENCH DYNAMICS FROM
$|0\rangle = |\bullet\bullet\bullet\bullet\bullet\bullet\ldots\rangle$:
THERMALIZATION

QUENCH DYNAMICS FROM
$|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\ldots\rangle$:
LONG-LIVED OSCILLATIONS

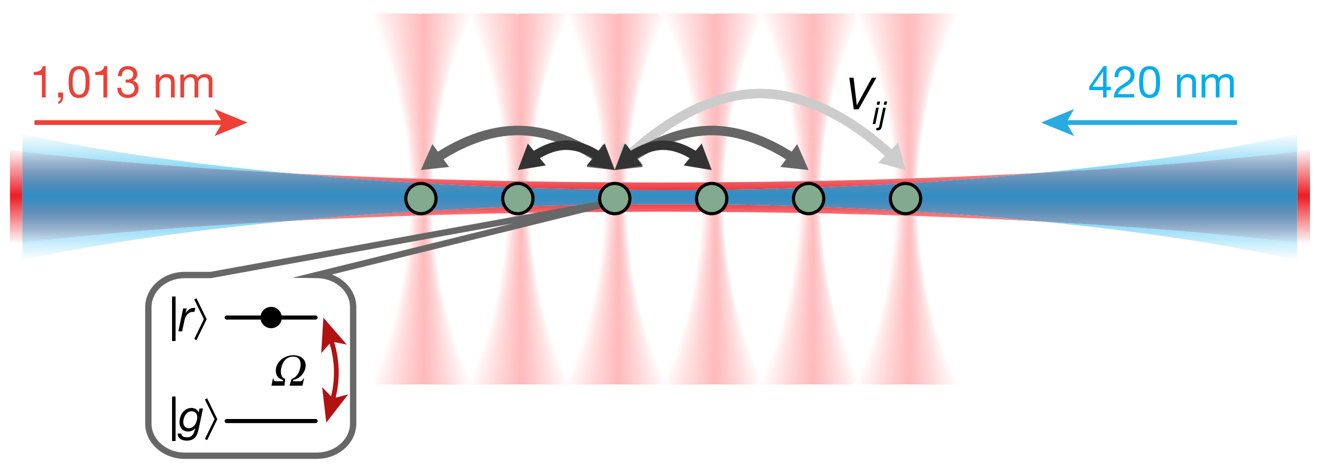
[1] PERSISTENT DYNAMICS
SPECIFIC INITIAL STATES
FROM AN HILBERT SUBSPACE

NAT. PHYS. 14, PP 745-749 (2018)

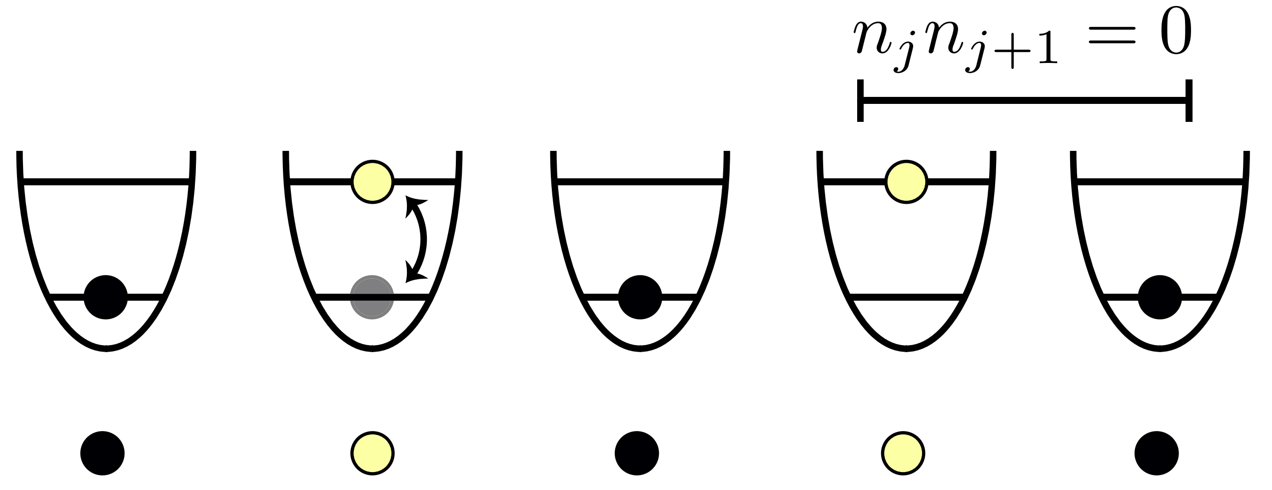


$$H = \sum_j P_j \sigma_{j+1}^x P_{j+2} \quad \text{with} \quad \mathrm{P} = (1 - \sigma^z)/2$$

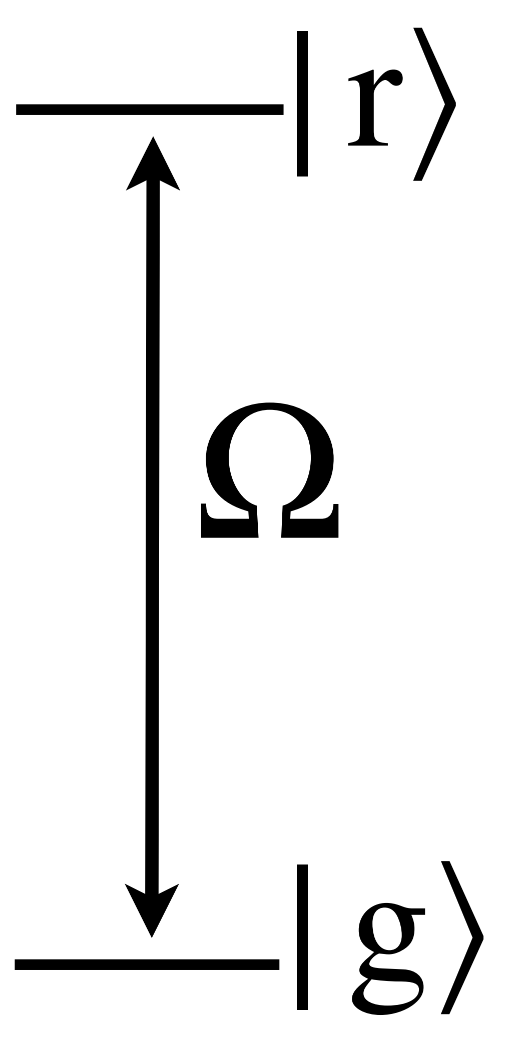
QUANTUM MANYBODY SCARS



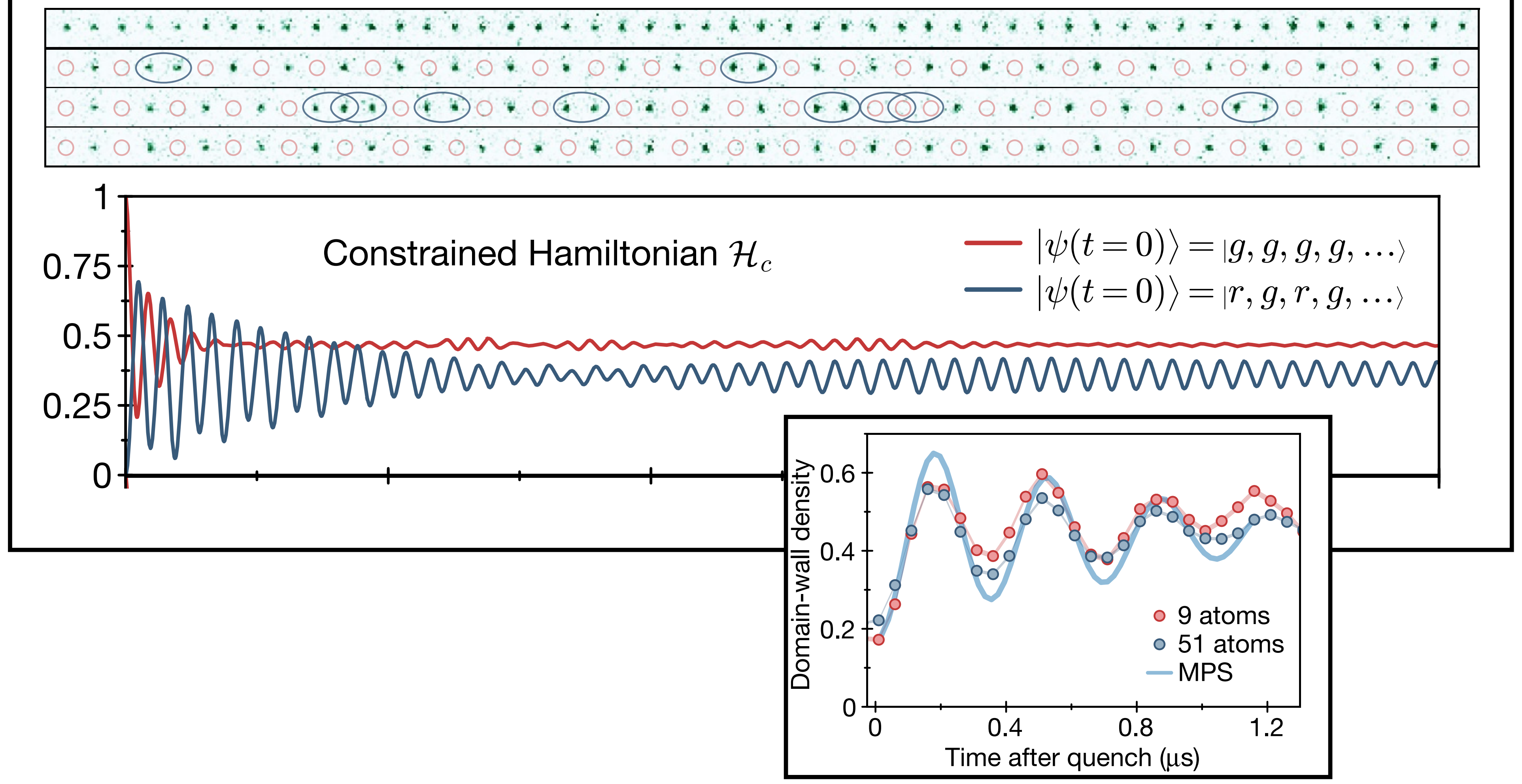
$$H_{\text{Ryd}} = \sum_j (\Omega\sigma_j^x + \Delta\sigma_j^z) + \sum_{j<k} V_{jk} n_j n_k$$



BLOCKADE: $V_{jk} \gg \Omega, \Delta$ $\quad H_{\text{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j)$



Bernien et al, Nature 551, pp 579-584 (2017)

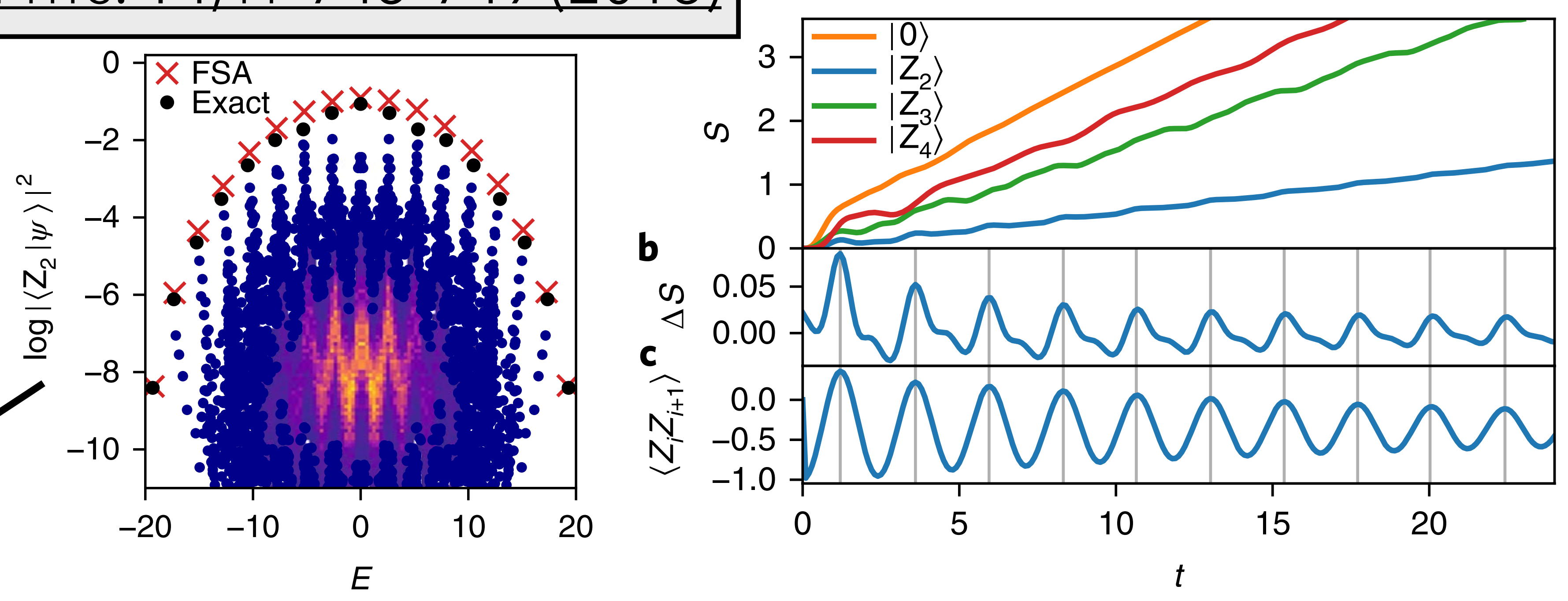


Quench Dynamics from
$|0\rangle = |\bullet\bullet\bullet\bullet\bullet\bullet\bullet\ldots\rangle$:
Thermalization

Quench Dynamics from
$|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\bullet\ldots\rangle$:
Long-lived Oscillations

[1] **Persistent Dynamics**
Specific initial states
from an Hilbert subspace

Nat. Phys. 14, pp 745-749 (2018)



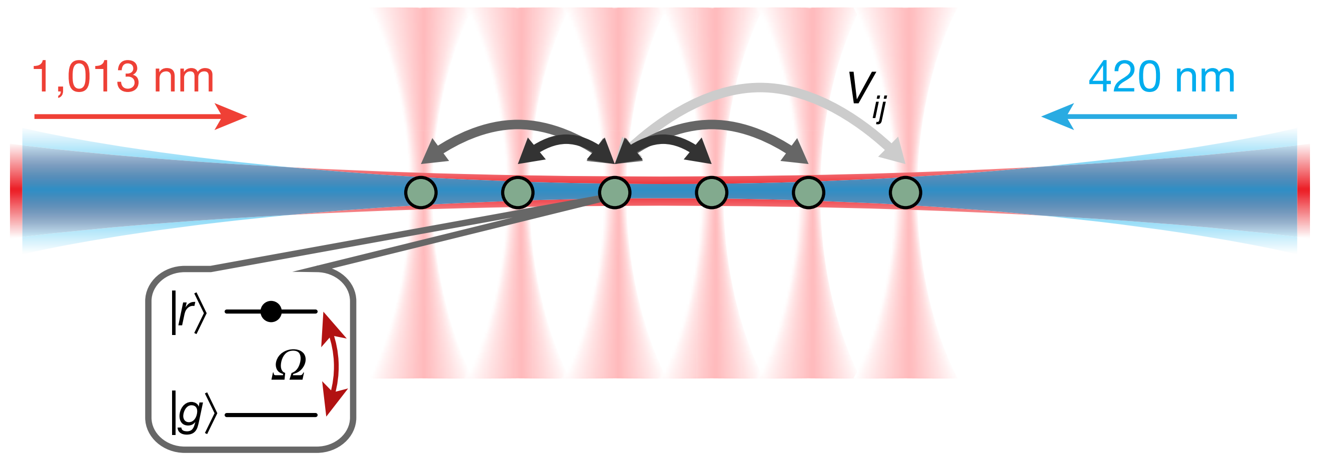
$$H = \sum_j P_j \sigma_{j+1}^x P_{j+2} \quad \text{with} \quad \text{P} = (1-\sigma^z)/2$$

[3] **Tower of states**
in the eigenstates
spectrum

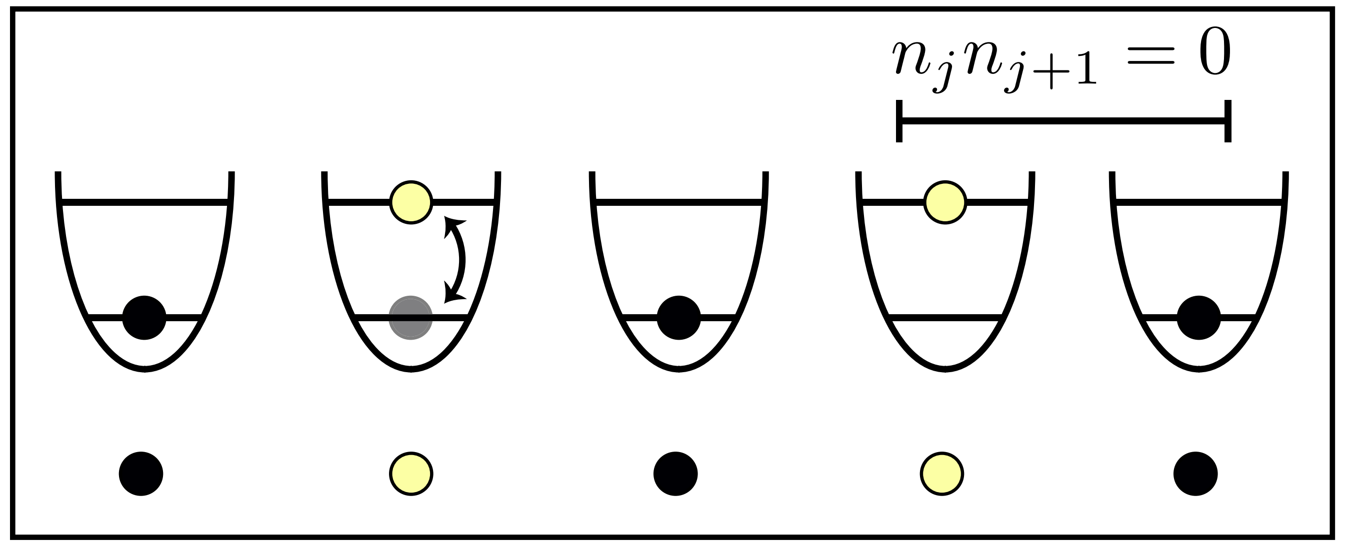
Phys. Rev. Lett. 122, 040603

Phys. Rev. Lett. 122, 173401

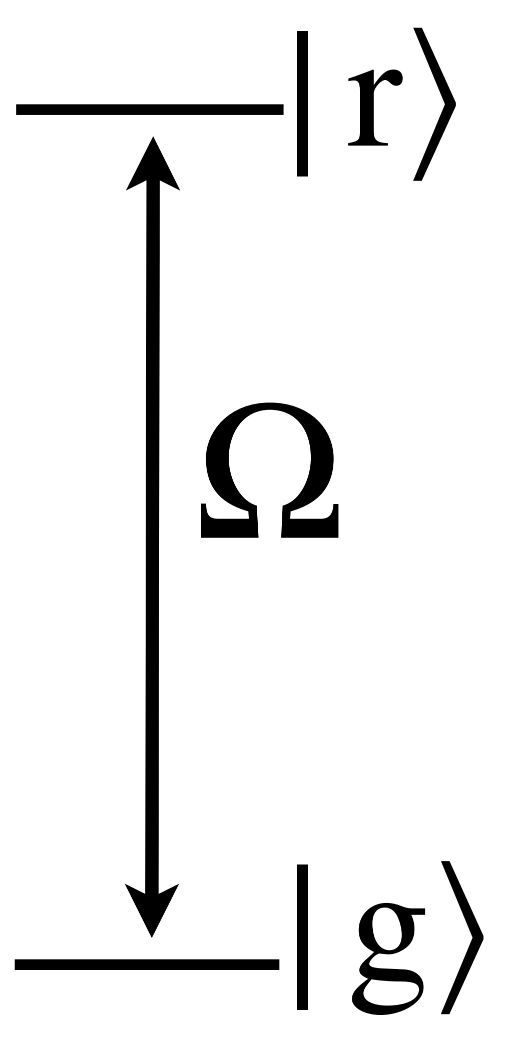
QUANTUM MANYBODY SCARS



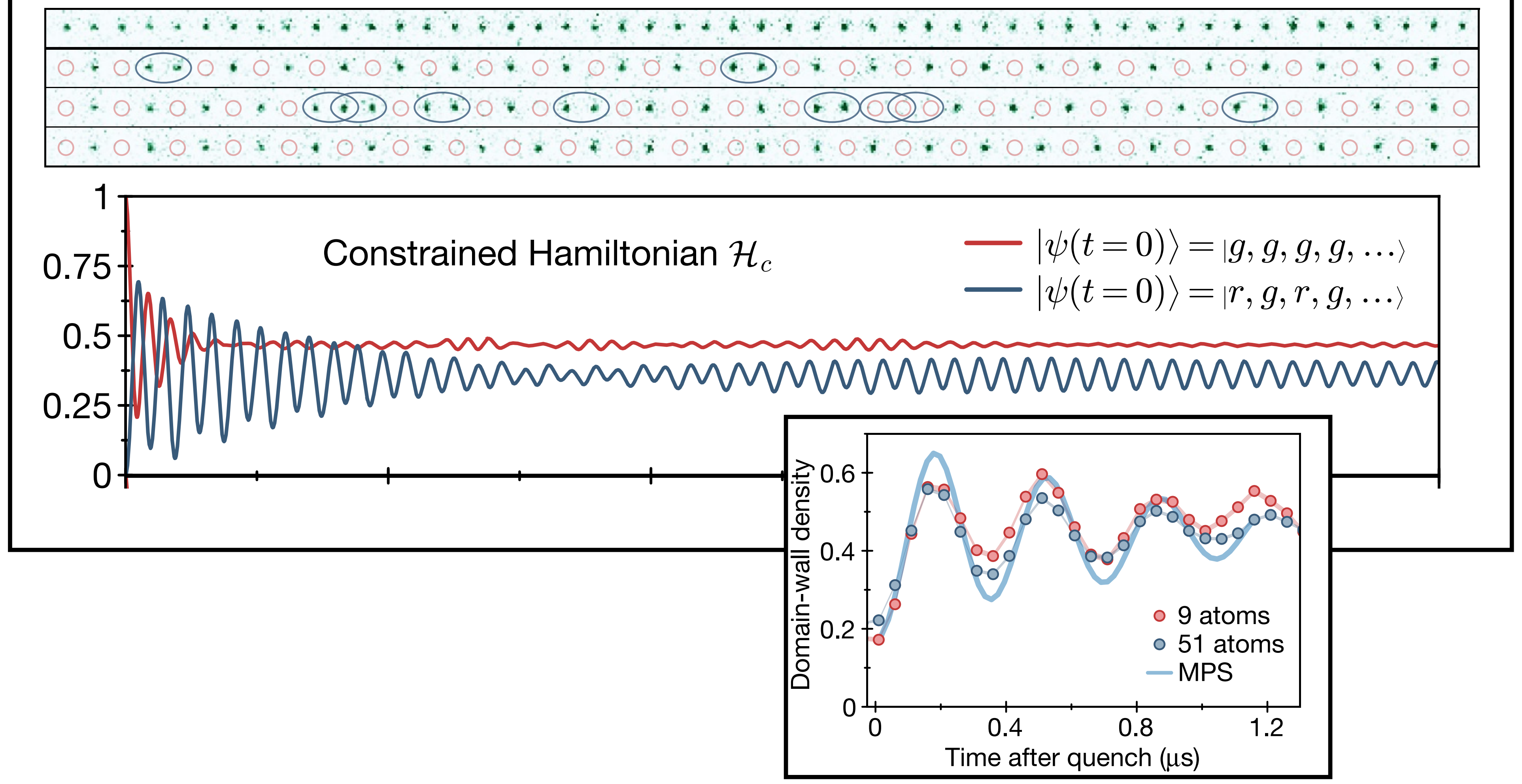
$$H_{\mathrm{Ryd}} = \sum_j (\Omega\sigma_j^x + \Delta\sigma_j^z) + \sum_{j<k} V_{jk} n_j n_k$$



BLOCKADE: $V_{jk} \gg \Omega, \Delta$ $\quad H_{\mathrm{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j)$



BERNIEN ET AL, NATURE 551, PP 579-584 (2017)

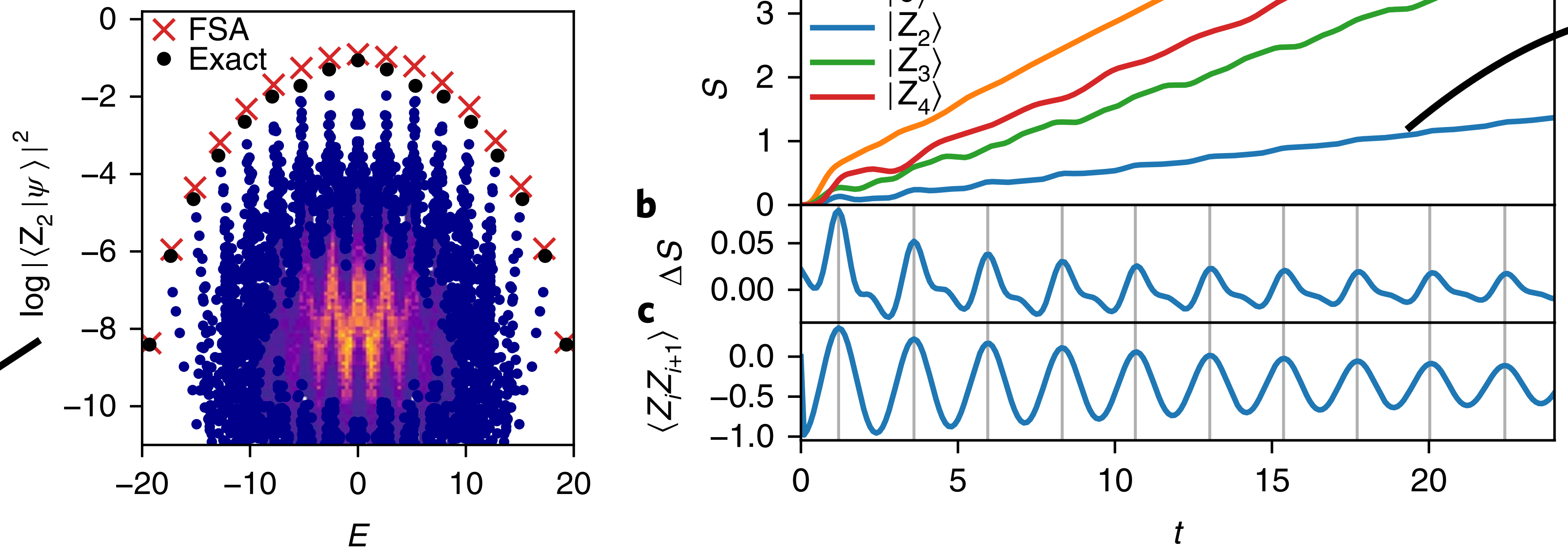


QUENCH DYNAMICS FROM
$|0\rangle = |\bullet\bullet\bullet\bullet\bullet\bullet\bullet\ldots\rangle$:
THERMALIZATION

QUENCH DYNAMICS FROM
$|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\bullet\ldots\rangle$:
LONG-LIVED OSCILLATIONS

[1] **PERSISTENT DYNAMICS**
SPECIFIC INITIAL STATES
FROM AN HILBERT SUBSPACE

NAT. PHYS. 14, PP 745-749 (2018)



$$H = \sum_j P_j \sigma_{j+1}^x P_{j+2} \quad \text{with} \quad \mathrm{P} = (1 - \sigma^z)/2$$

[3] **TOWER OF STATES**
IN THE EIGENSTATES
SPECTRUM

PHYS. REV. LETT. 122, 040603

PHYS. REV. LETT. 122, 173401

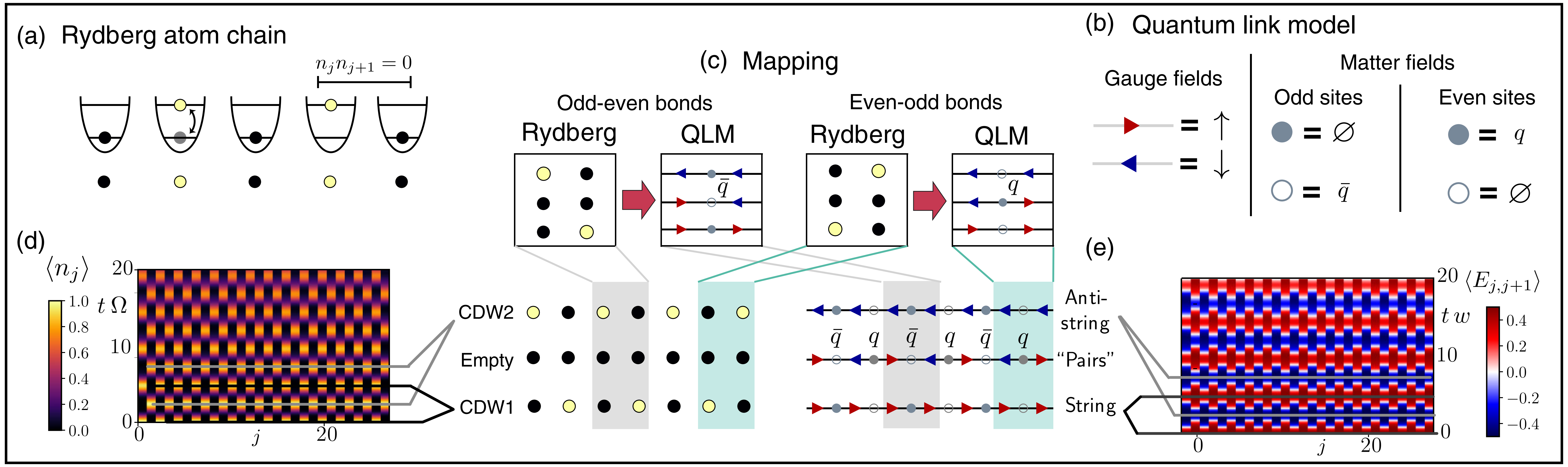
[2] SLOW OSCILLATING
GROWTH OF
ENTANGLEMENT ENTROPY

SCARS IN ABELIAN GAUGE THEORIES

$$H_{\mathrm{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j)$$

$\longleftrightarrow$

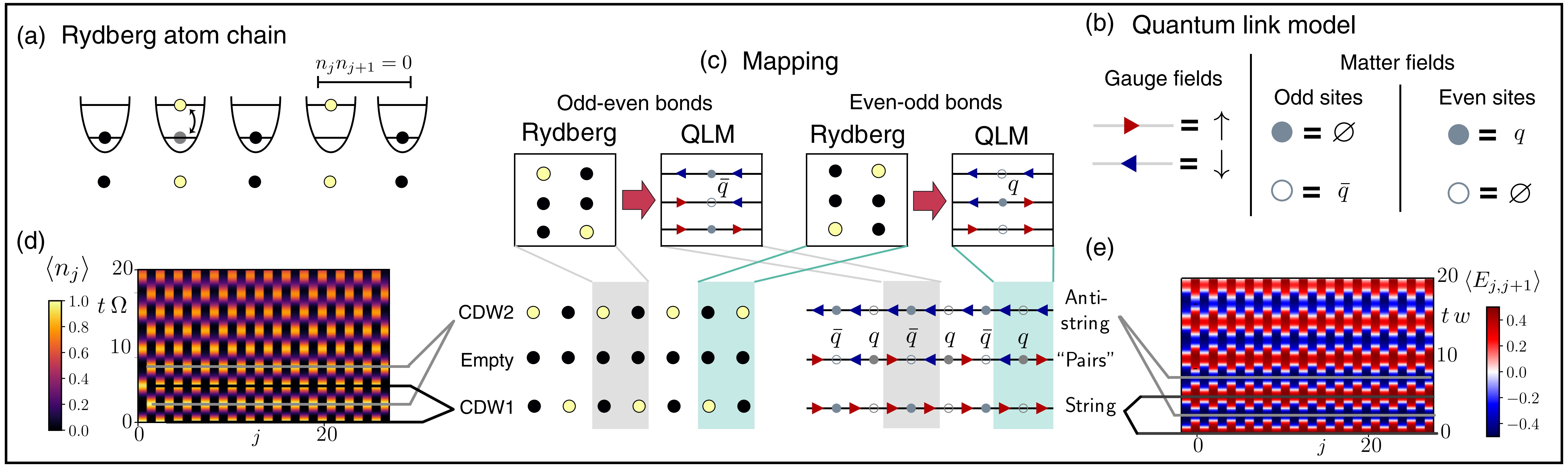
$$H_{\mathrm{QLM}}^{\mathrm{U(1)}} = -w \sum_{j,\mu} \left[\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \mathrm{H.c.}\right] + m \sum_j (-1)^j \psi_j^\dagger \psi_j + J \sum_j E_{j,j+\mu}^2$$

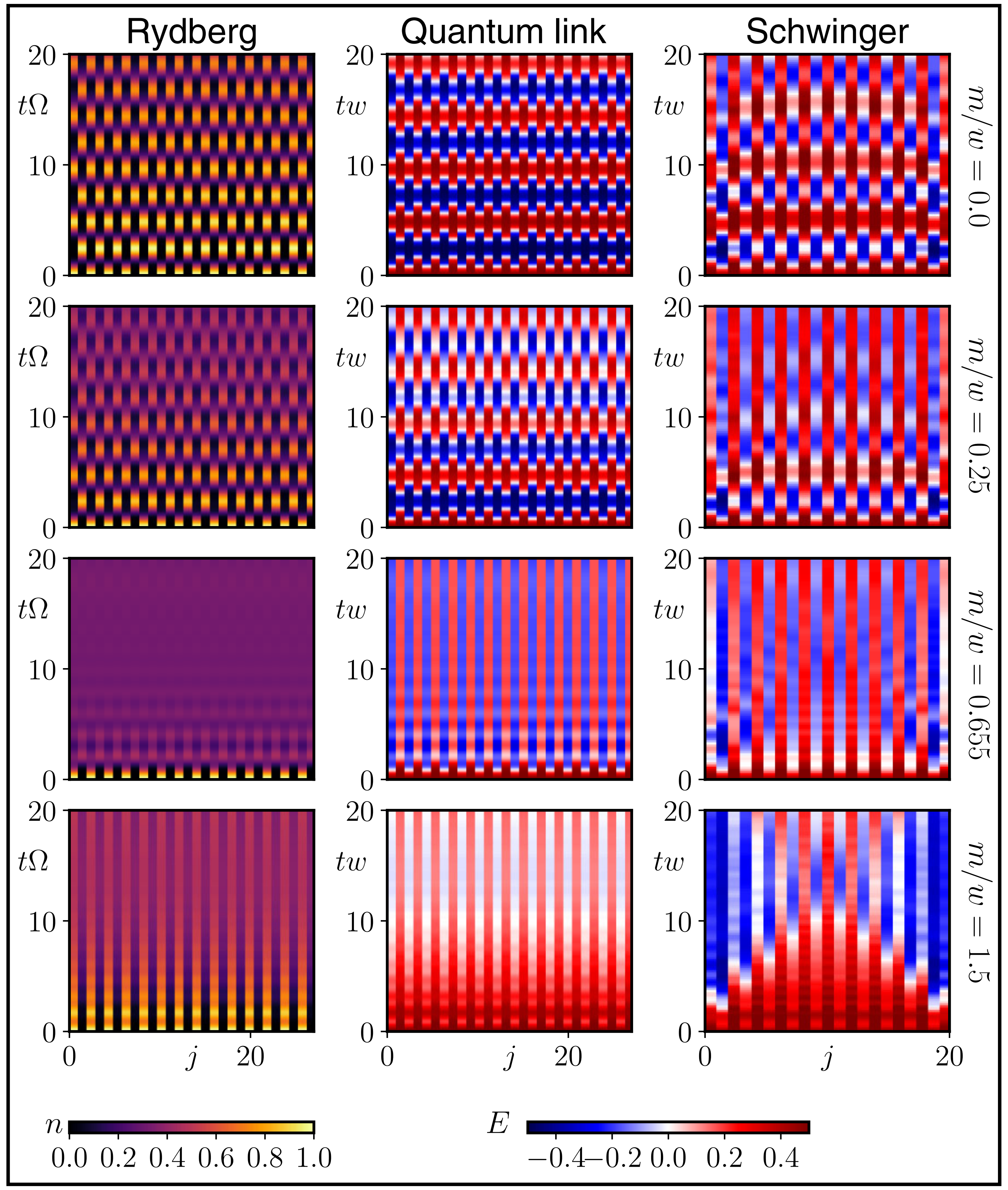


Surace et al, PRX 10, 021041 (2020)

SCARS IN ABELIAN GAUGE THEORIES

$$H_{\mathrm{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j) \longleftrightarrow H^{\mathrm{U(1)}}_{\mathrm{QLM}} = -w\sum_{j,\mu}\left[\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \mathrm{H.c.}\right] + m\sum_j (-1)^j\psi_j^\dagger\psi_j + J\sum_j E^2_{j,j+\mu}$$



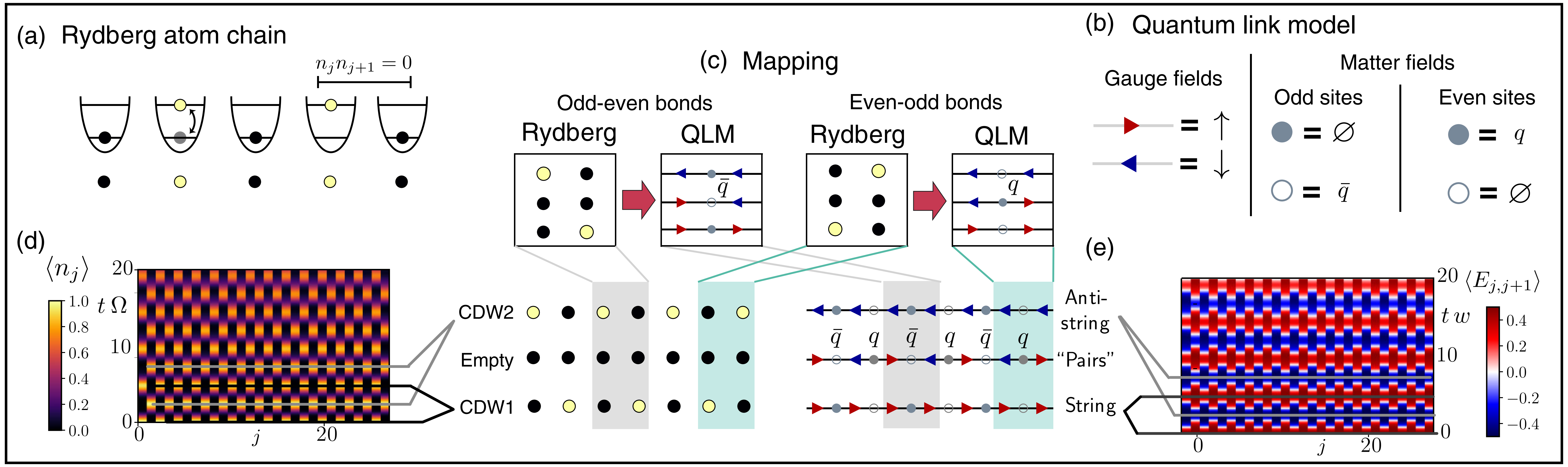


SURACE ET AL, PRX 10, 021041 (2020)

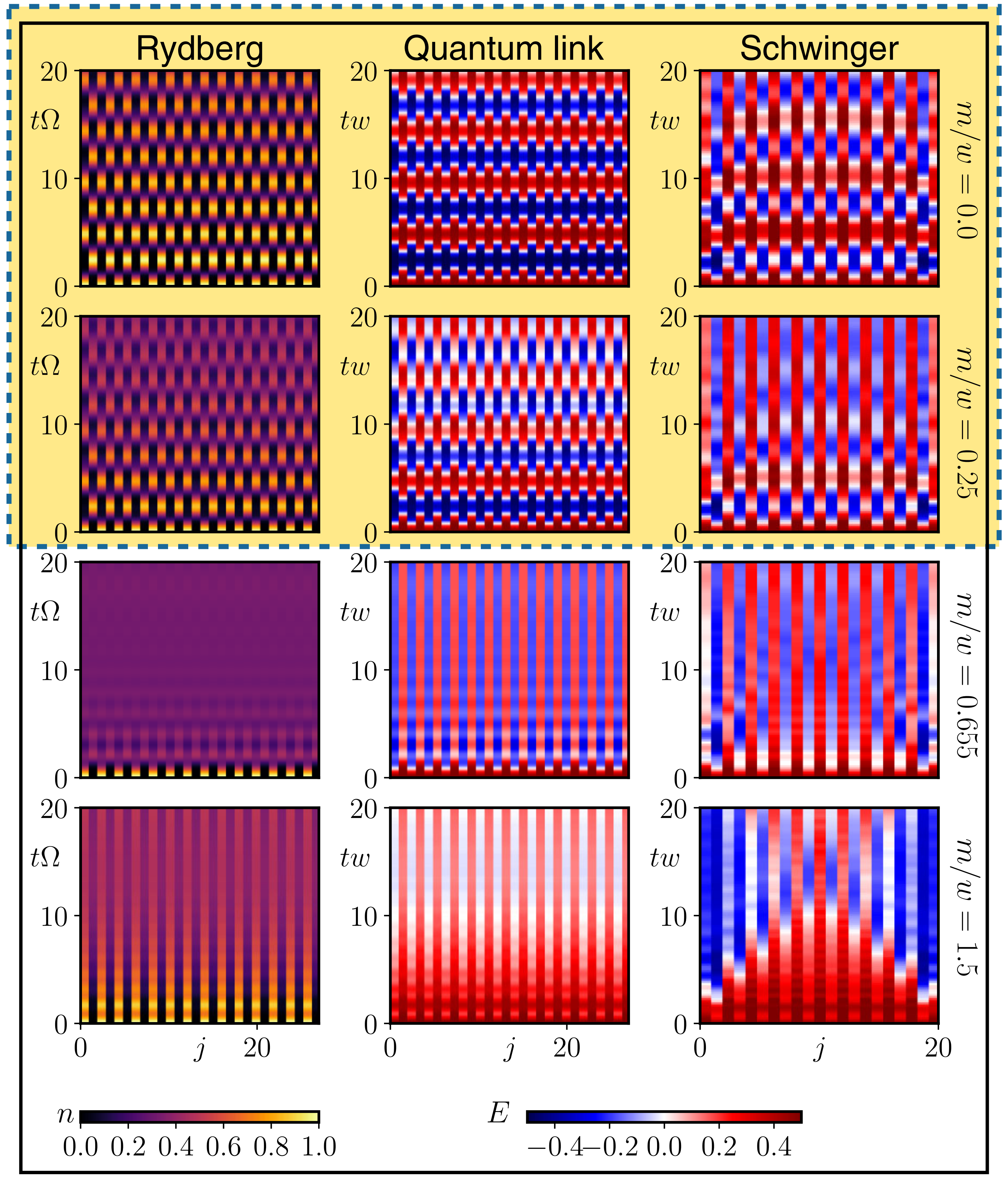
SCARS IN ABELIAN GAUGE THEORIES

$$H_{\mathrm{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j) \longleftrightarrow H_{\mathrm{QLM}}^{\mathrm{U(1)}} = -w\sum_{j,\mu}\left[\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \mathrm{H.c.}\right] + m\sum_j(-1)^j\psi_j^\dagger\psi_j + J\sum_j E_{j,j+\mu}^2$$

STRING INVERSION
THERMALIZATION SLOWDOWN



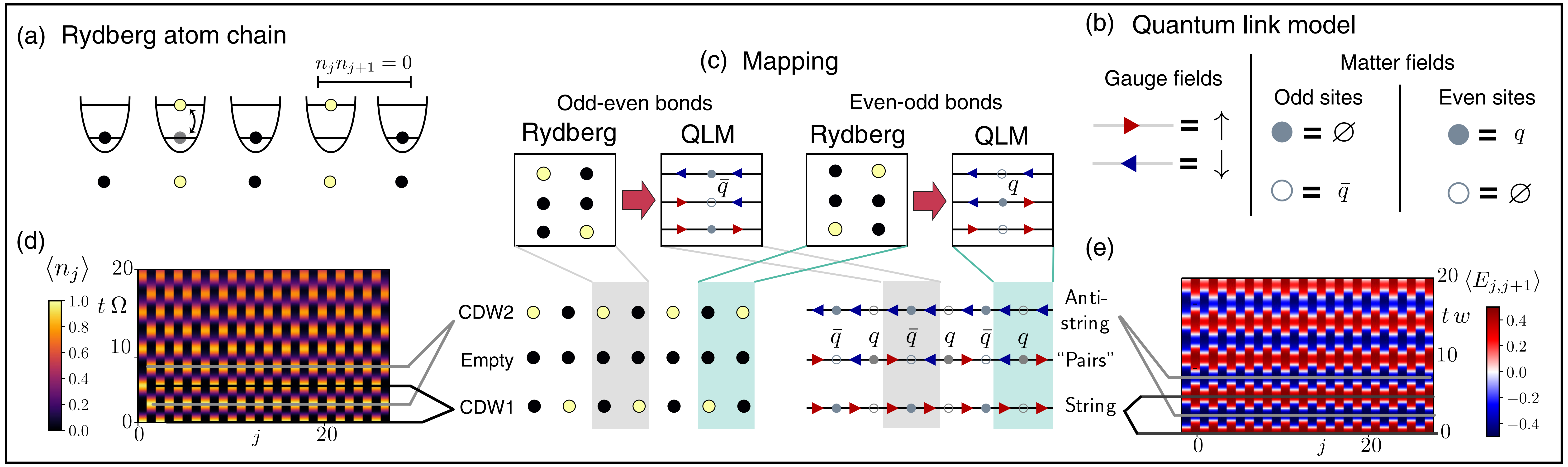
SURACE ET AL, PRX 10, 021041 (2020)



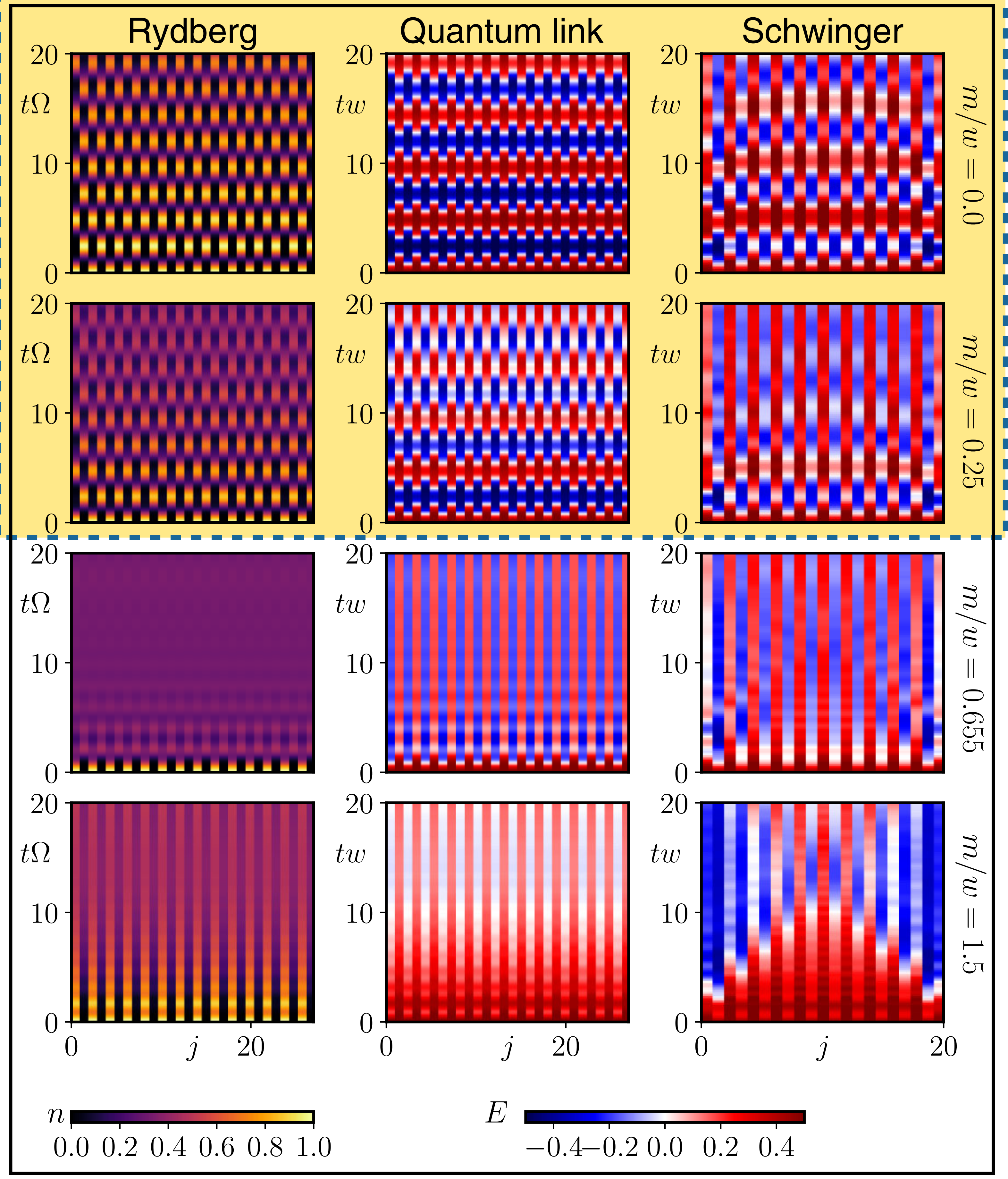
SCARS IN ABELIAN GAUGE THEORIES

$$H_{\rm FSS} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j) \longleftrightarrow H^{\rm U(1)}_{\rm QLM} = -w\sum_{j,\mu}\left[\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m\sum_j(-1)^j\psi_j^\dagger\psi_j + J\sum_j E^2_{j,j+\mu}$$

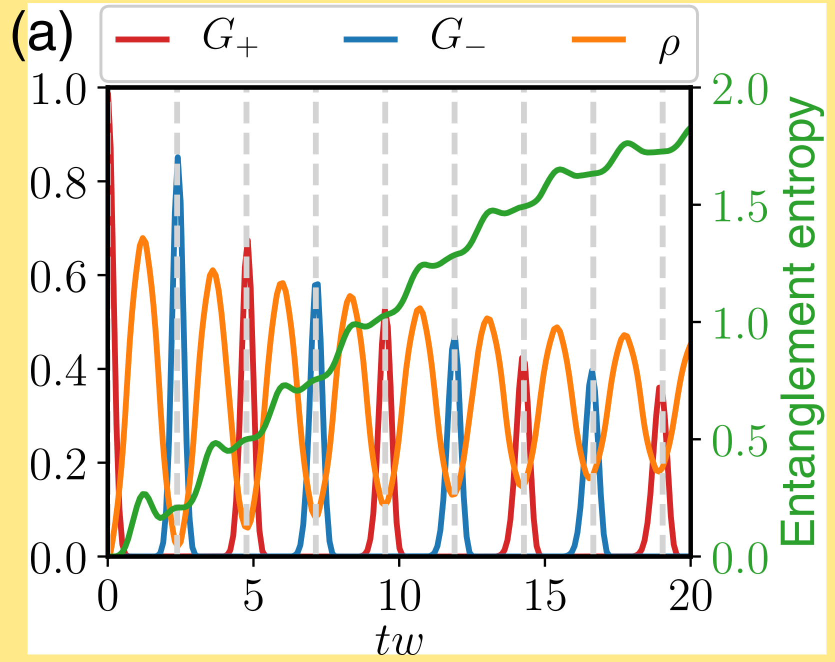
STRING INVERSION
THERMALIZATION SLOWDOWN



SURACE ET AL, PRX 10, 021041 (2020)



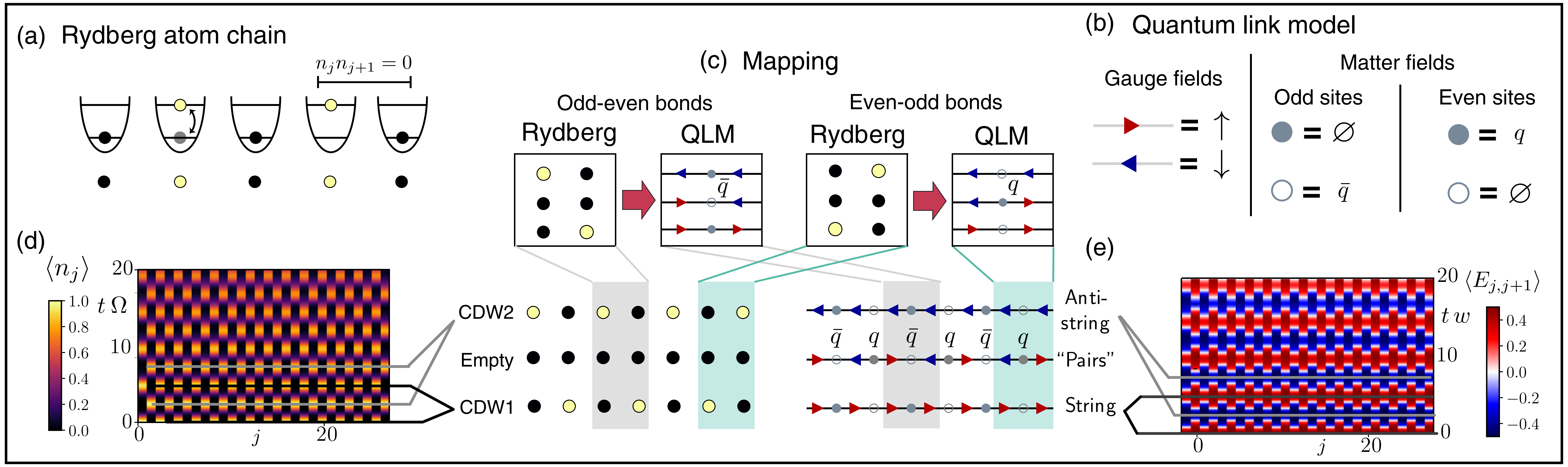
OSCILLATING FIDELITY &
ENTANGLEMENT ENTROPY



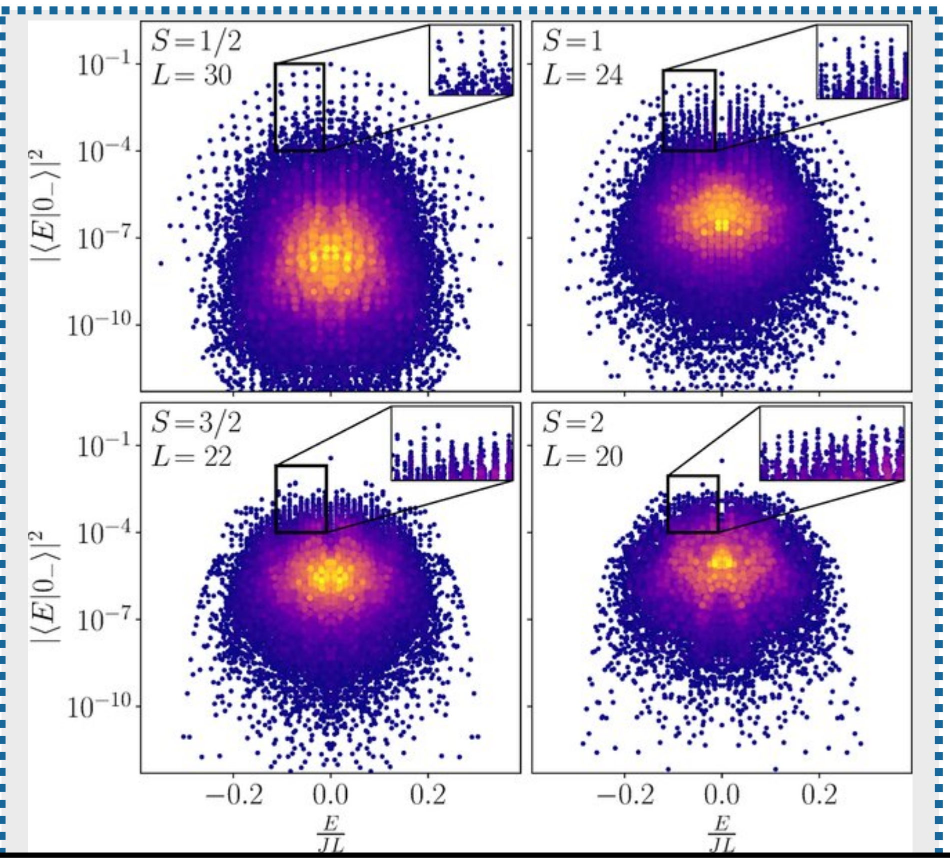
SCARS IN ABELIAN GAUGE THEORIES

$$H_{\mathrm{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j) \longleftrightarrow H_{\mathrm{QLM}}^{\mathrm{U(1)}} = -w\sum_{j,\mu}\left[\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \mathrm{H.c.}\right] + m\sum_j (-1)^j\psi_j^\dagger\psi_j + J\sum_j E_{j,j+\mu}^2$$

STRING INVERSION
THERMALIZATION SLOWDOWN

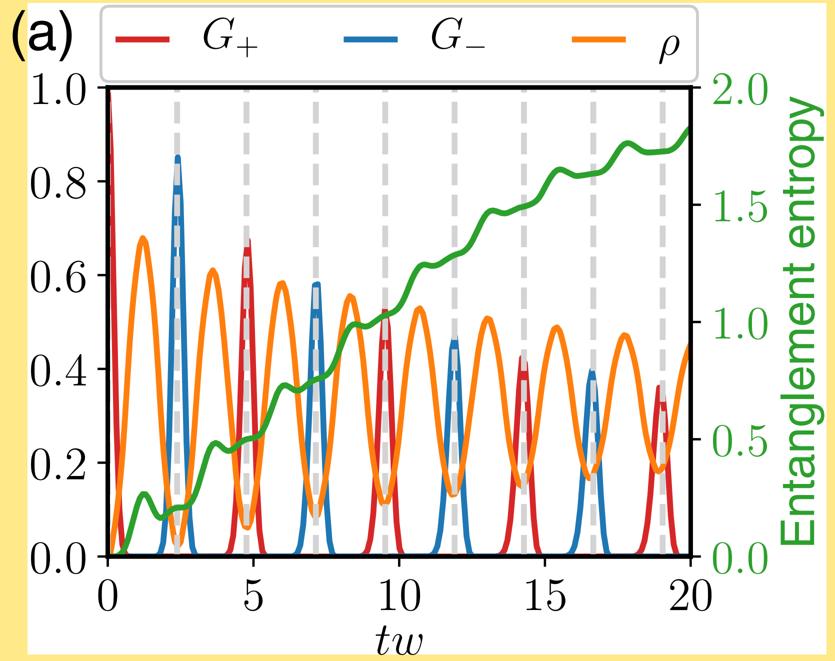


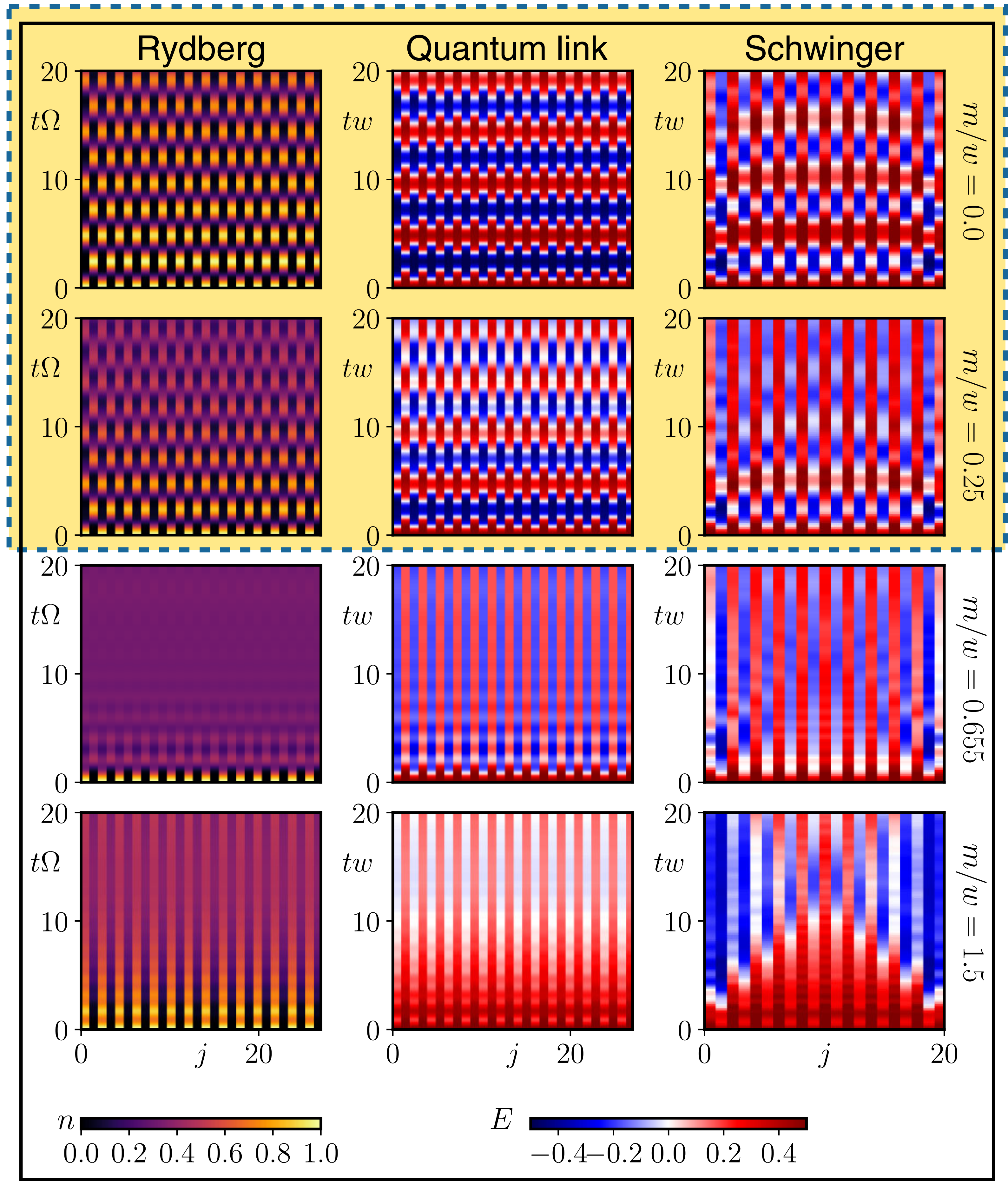
SURACE ET AL, PRX 10, 021041 (2020)



HALIMEH ET AL, PRB 107, L201105 (2023)

OSCILLATING FIDELITY &
ENTANGLEMENT ENTROPY

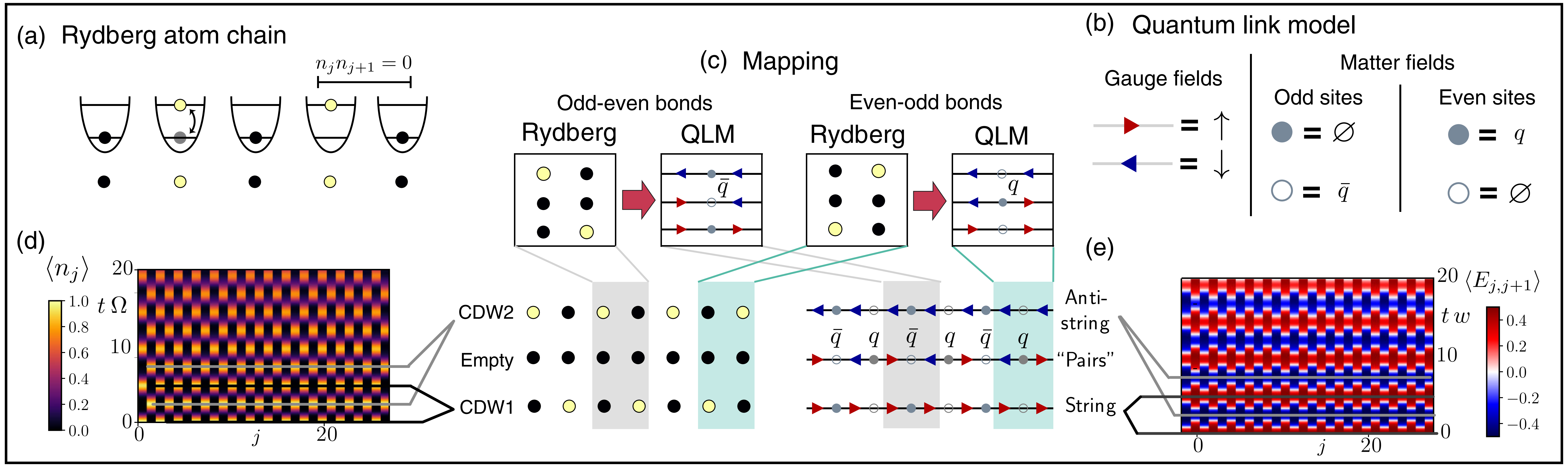




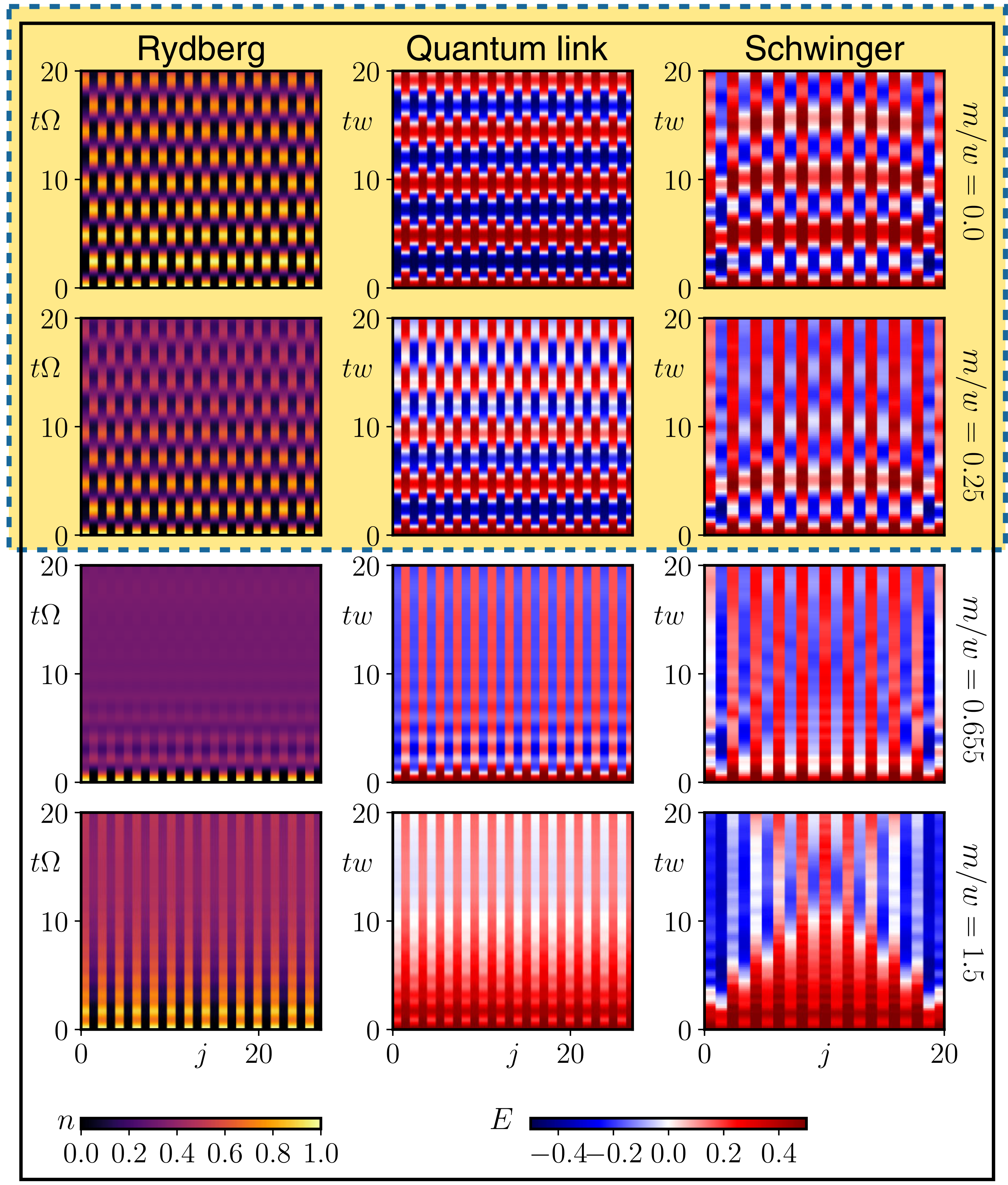
SCARS IN ABELIAN GAUGE THEORIES

$$H_{\mathrm{FSS}} = \sum_j \left(\Omega\sigma_j^x + 2\Delta n_j\right) \longleftrightarrow H_{\mathrm{QLM}}^{\mathrm{U}(1)} = -w\sum_{j,\mu}\left[\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \mathrm{H.c.}\right] + m\sum_j (-1)^j \psi_j^\dagger\psi_j + J\sum_j E_{j,j+\mu}^2$$

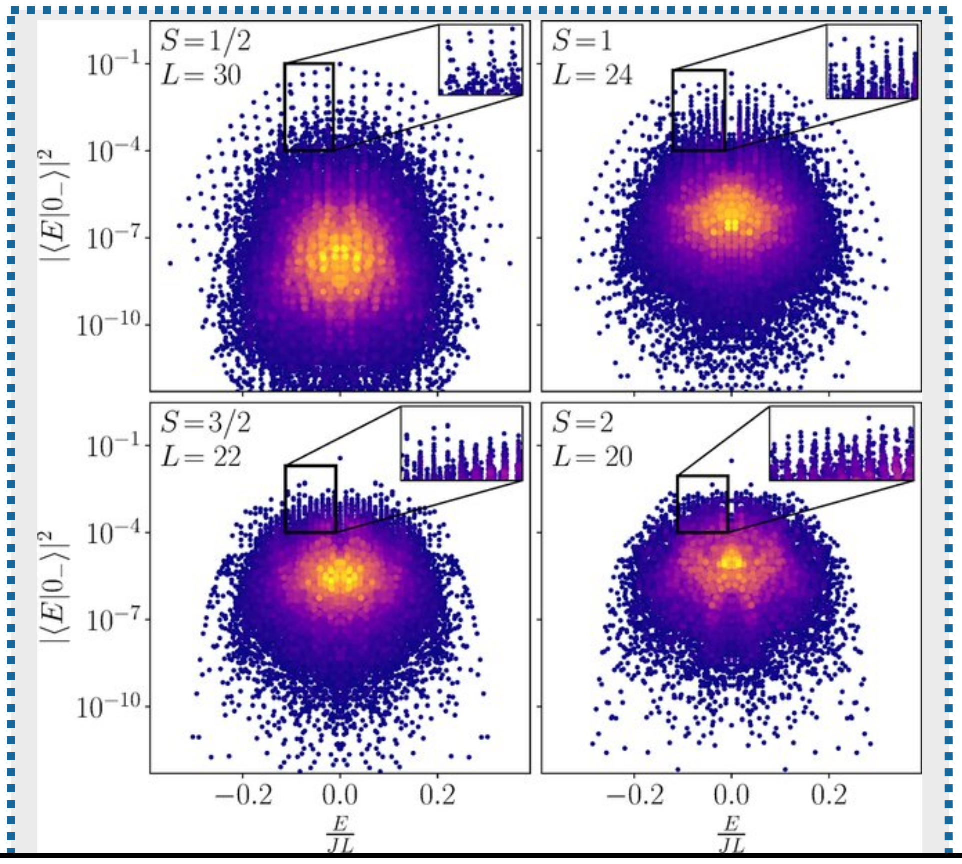
STRING INVERSION
THERMALIZATION SLOWDOWN



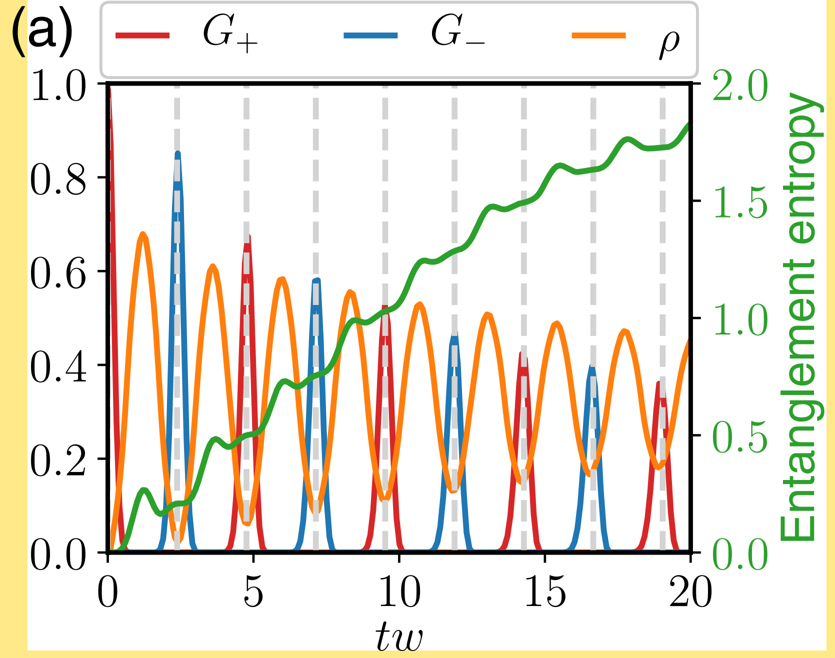
SURACE ET AL, PRX 10, 021041 (2020)



SCARS IN NONABELIAN LGTS?



OSCILLATING FIDELITY & ENTANGLEMENT ENTROPY

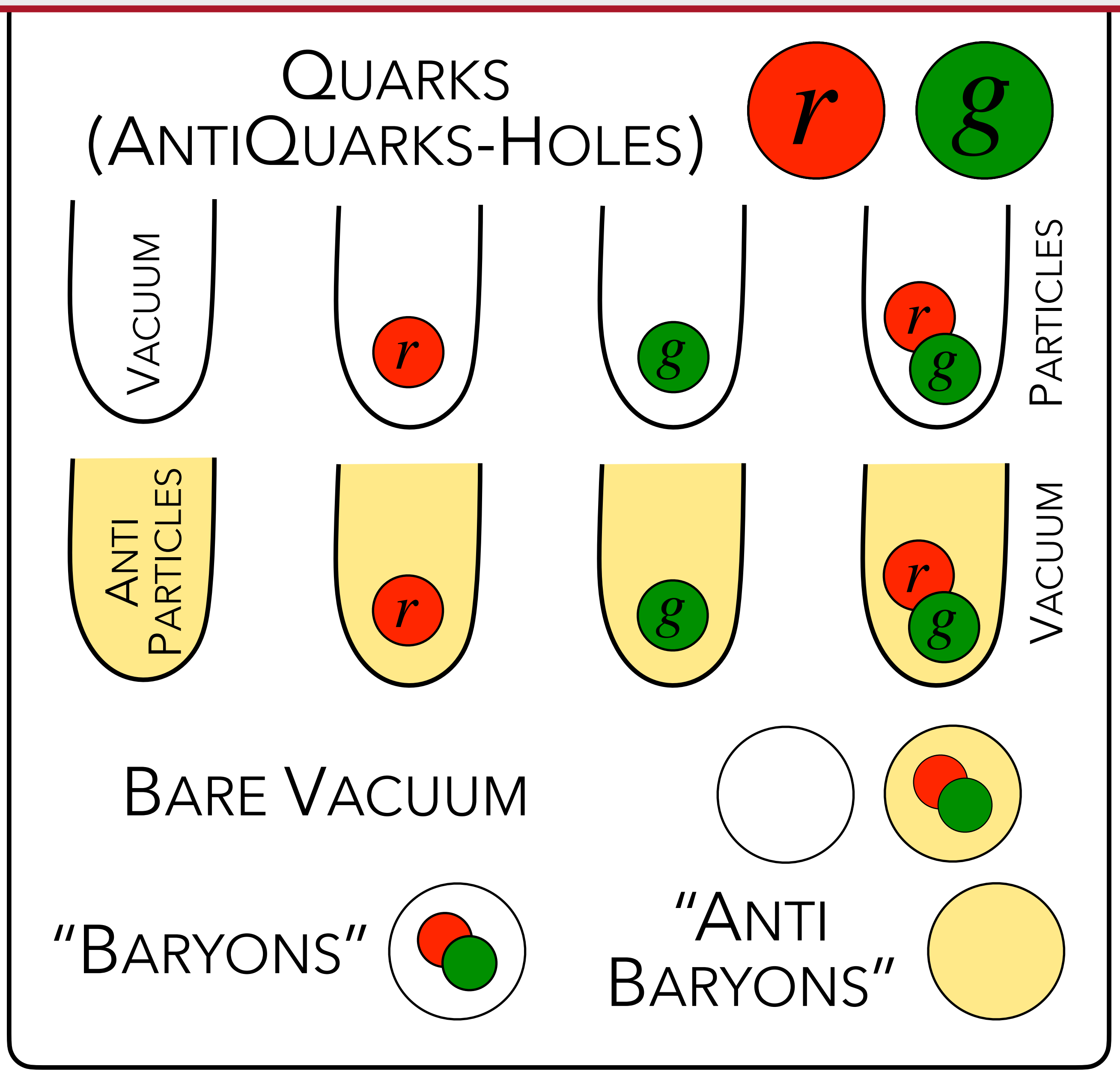


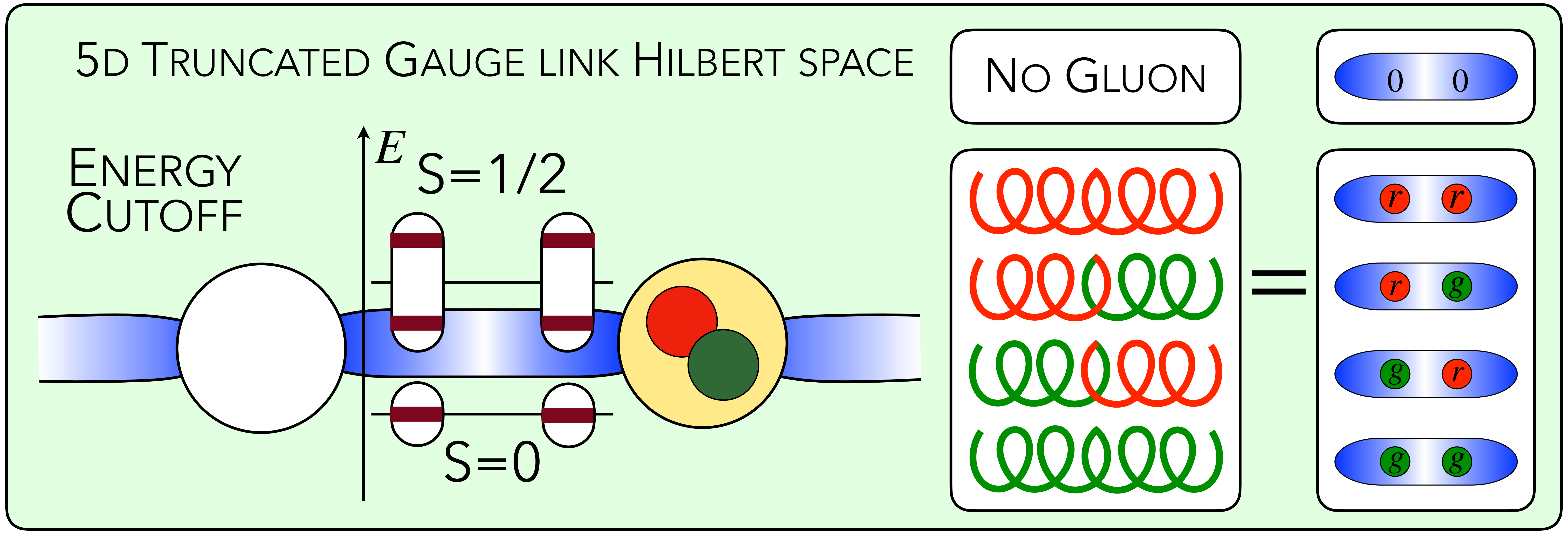
HALIMEH ET AL, PRB 107, L201105 (2023)

1D HARDCORE–GLUON SU(2) YANG MILLS LGT

$$H = \frac{1}{2a}\sum_{\alpha,\beta}\sum_{j,\mu}\left[\psi^{\dagger}_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m_0\sum_{j}(-1)^j\sum_{\alpha}\psi^{\dagger}_{j,\alpha}\psi_{j,\alpha} + \frac{ag^2}{2}\sum_{j}E^2_{j,j+\mu}$$

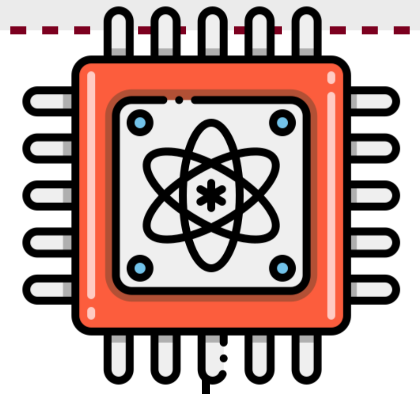
4D Matter Hilbert space
SU(2)-color 1/2 Flavorless Dirac Fermions

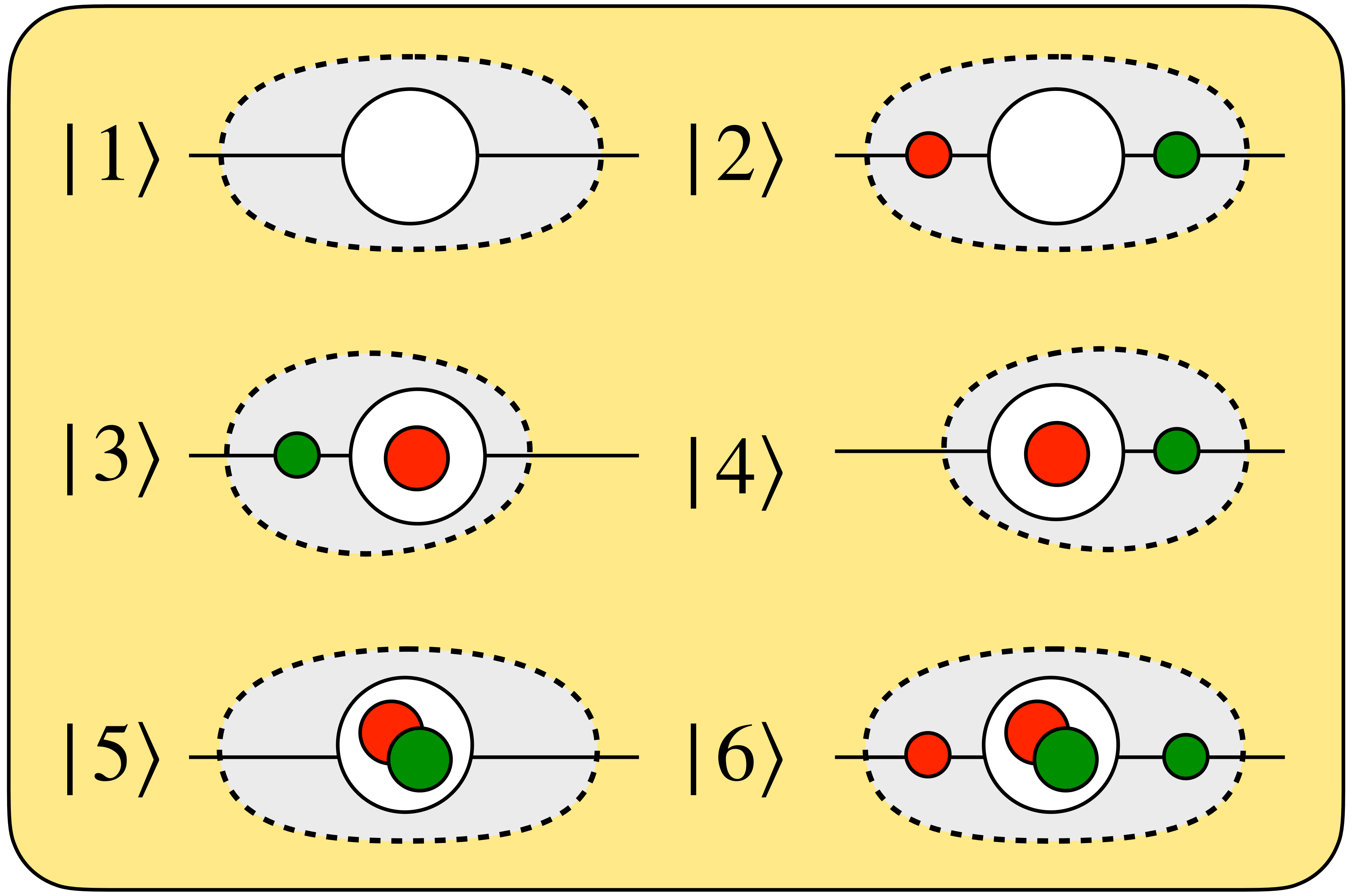




GAUGE INVARIANT QUDIT MODEL

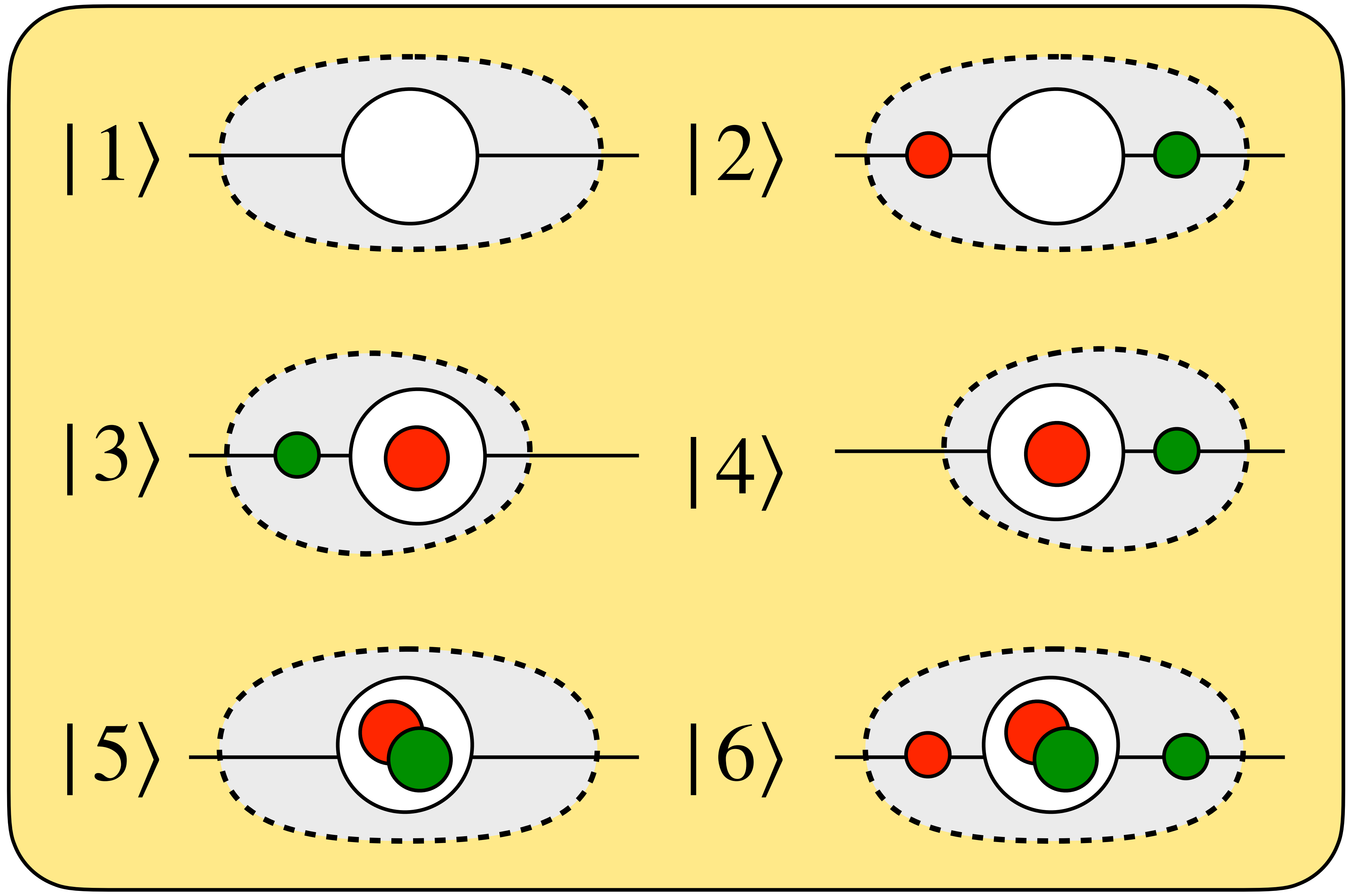
6 SU(2) Gauge Invariant States (color singlets)
6-QuDit Model on a Quantum Computer



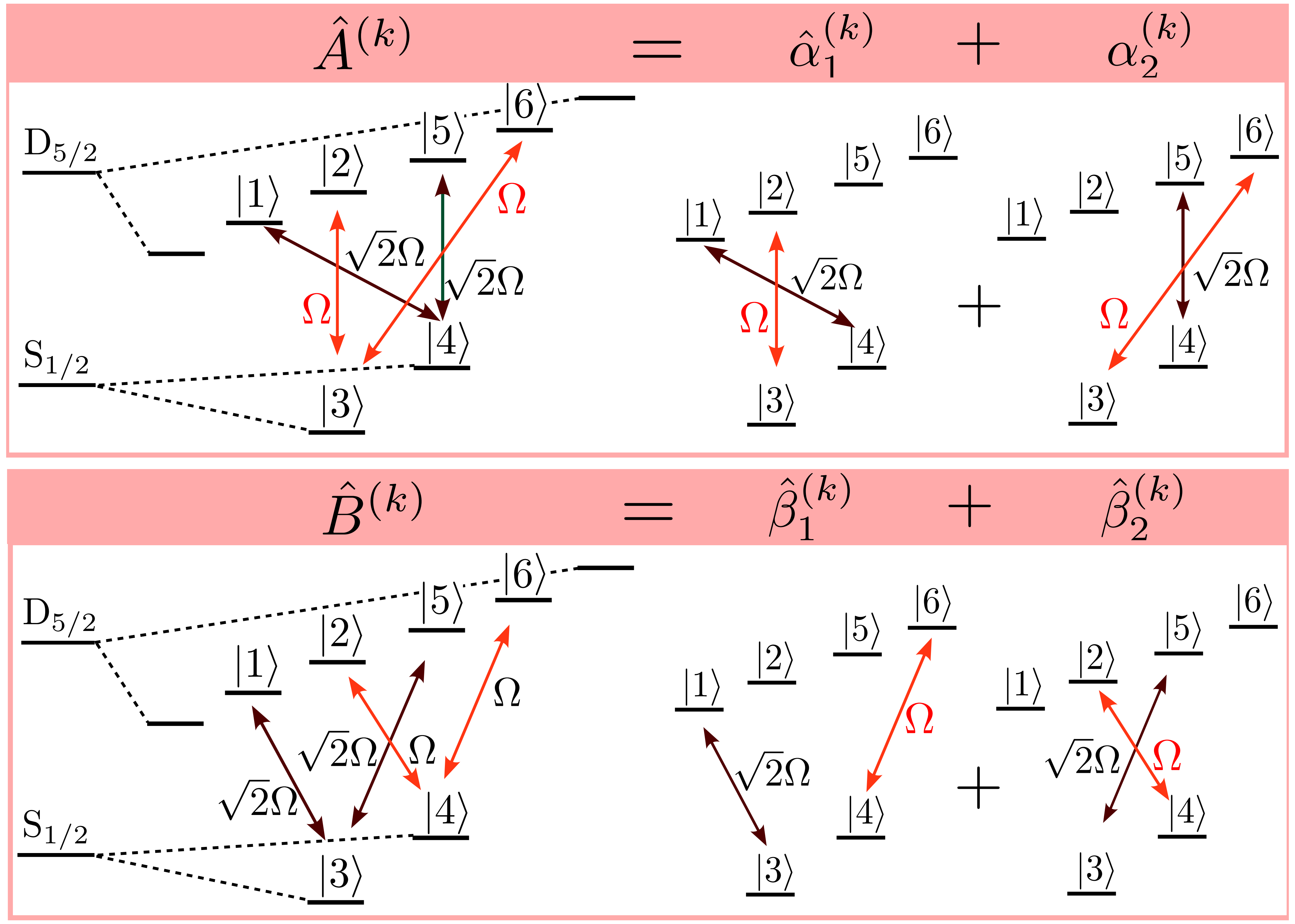


GAUGE INVARIANT QUDIT MODEL

6 SU(2) GAUGE INVARIANT STATES (COLOR SINGLETS)
6-QUDIT MODEL ON A QUANTUM COMPUTER

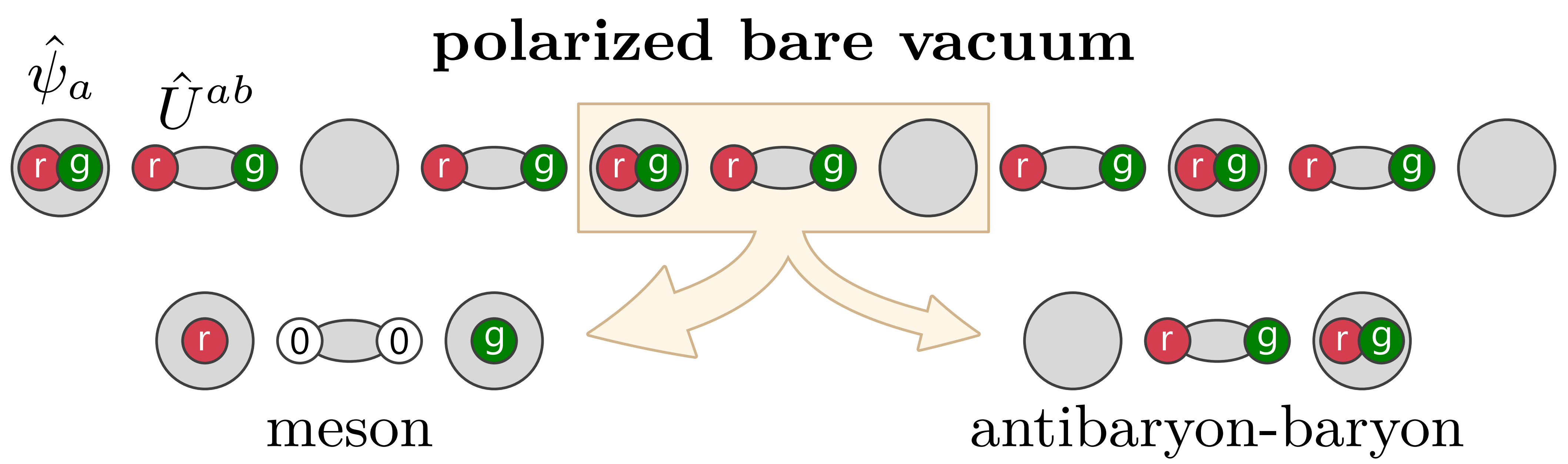


$^{40}Ca^+$



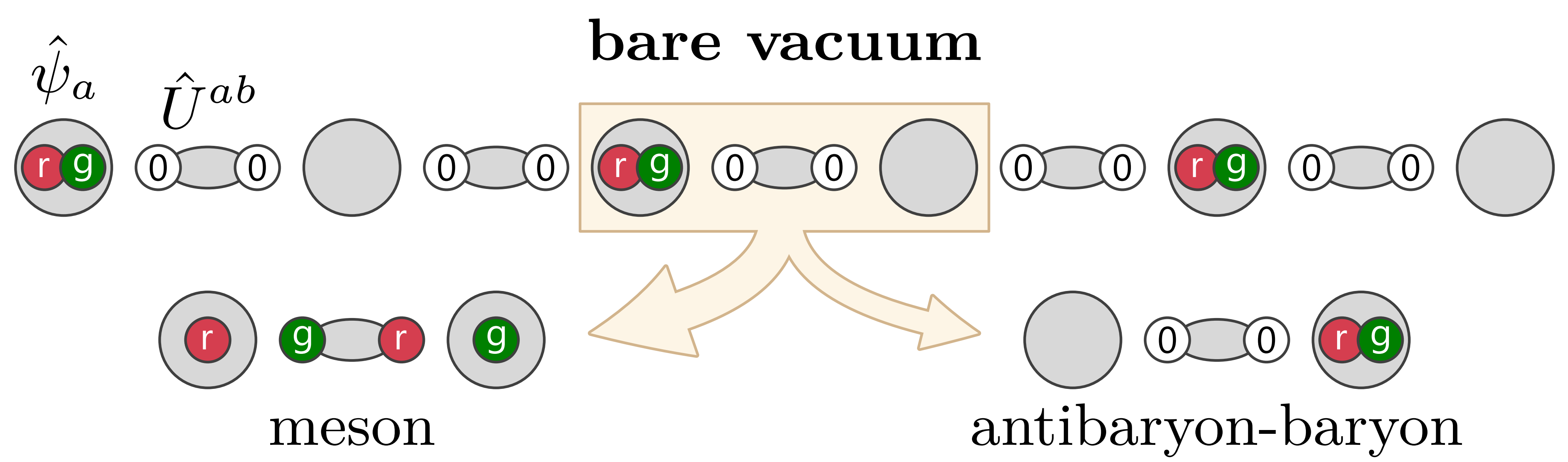
CALAJO' ET AL, ARXIV:2402.07987 (2024)

INITIAL STATE CANDIDATES FOR DYNAMICS



$$|\Psi_{\mathrm{PV}}\rangle = |\ldots 2020 \ldots\rangle_{\mathrm{m}} \left| \cdots \frac{rg - gr}{\sqrt{2}} \cdots \frac{rg - gr}{\sqrt{2}} \cdots \right\rangle_{\mathrm{g}}$$

$$= |6\rangle|2\rangle\ldots|6\rangle|2\rangle\ldots|6\rangle|2\rangle$$

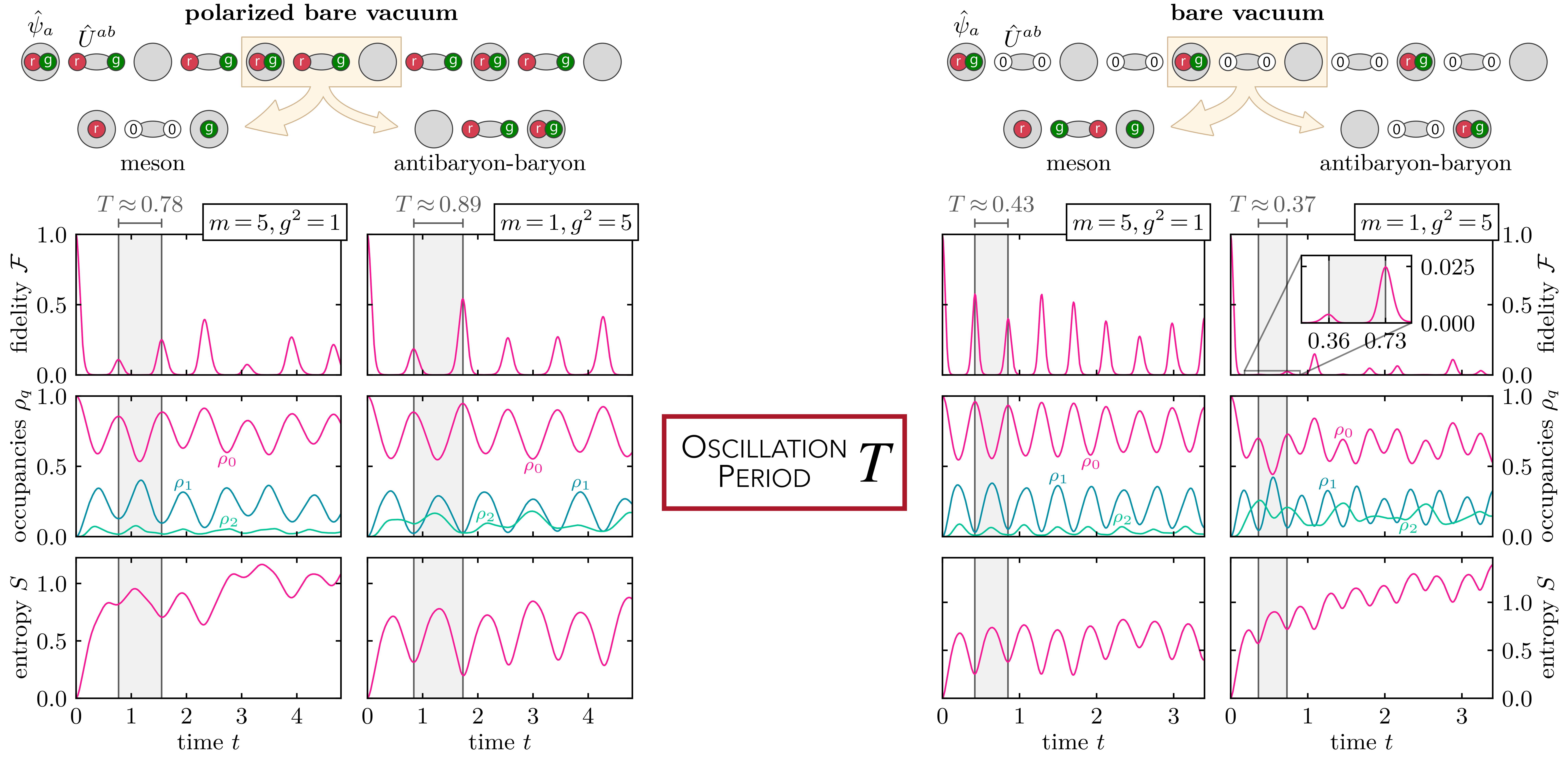


$$|\Psi_{\mathrm{V}}\rangle = |\ldots 2020 \ldots\rangle_{\mathrm{m}} \big| \ldots 000 \ldots \big\rangle_{\mathrm{g}}$$

$$= |5\rangle|1\rangle\ldots|5\rangle|1\rangle\ldots|5\rangle|1\rangle$$

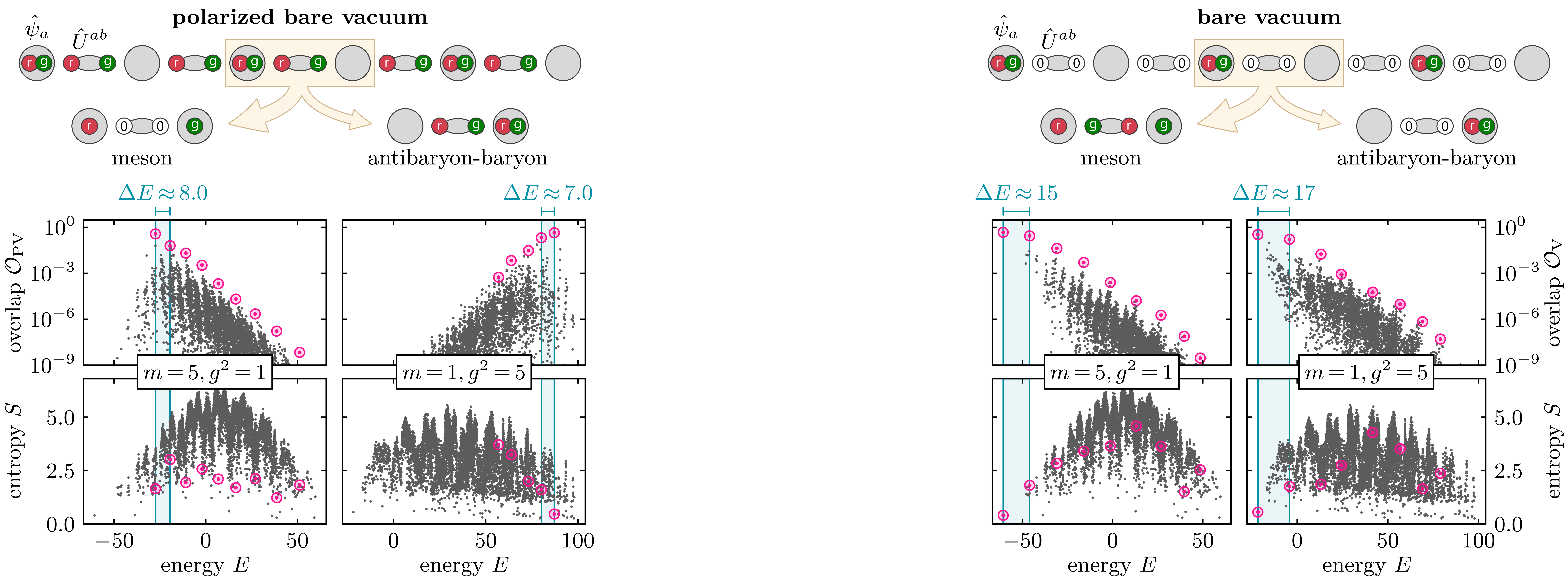
PRODUCT STATES: EASY FOR TENSOR NETWORKS & QUANTUM SIMULATORS

SCARRING DYNAMICS



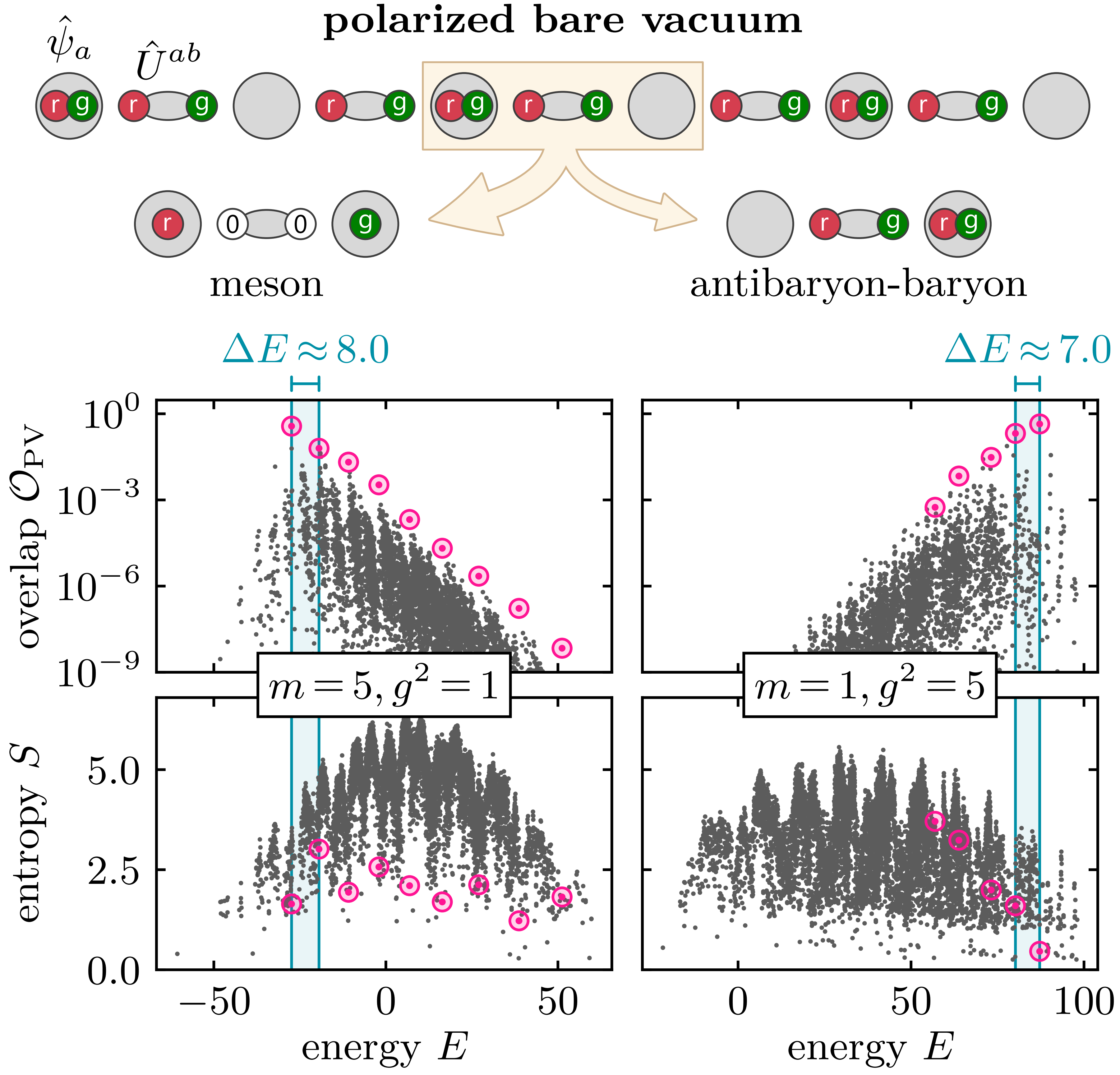
SCAR STATES IN THE SPECTRUM

The energy spectrum presents towers of equally spaced eigenstates with

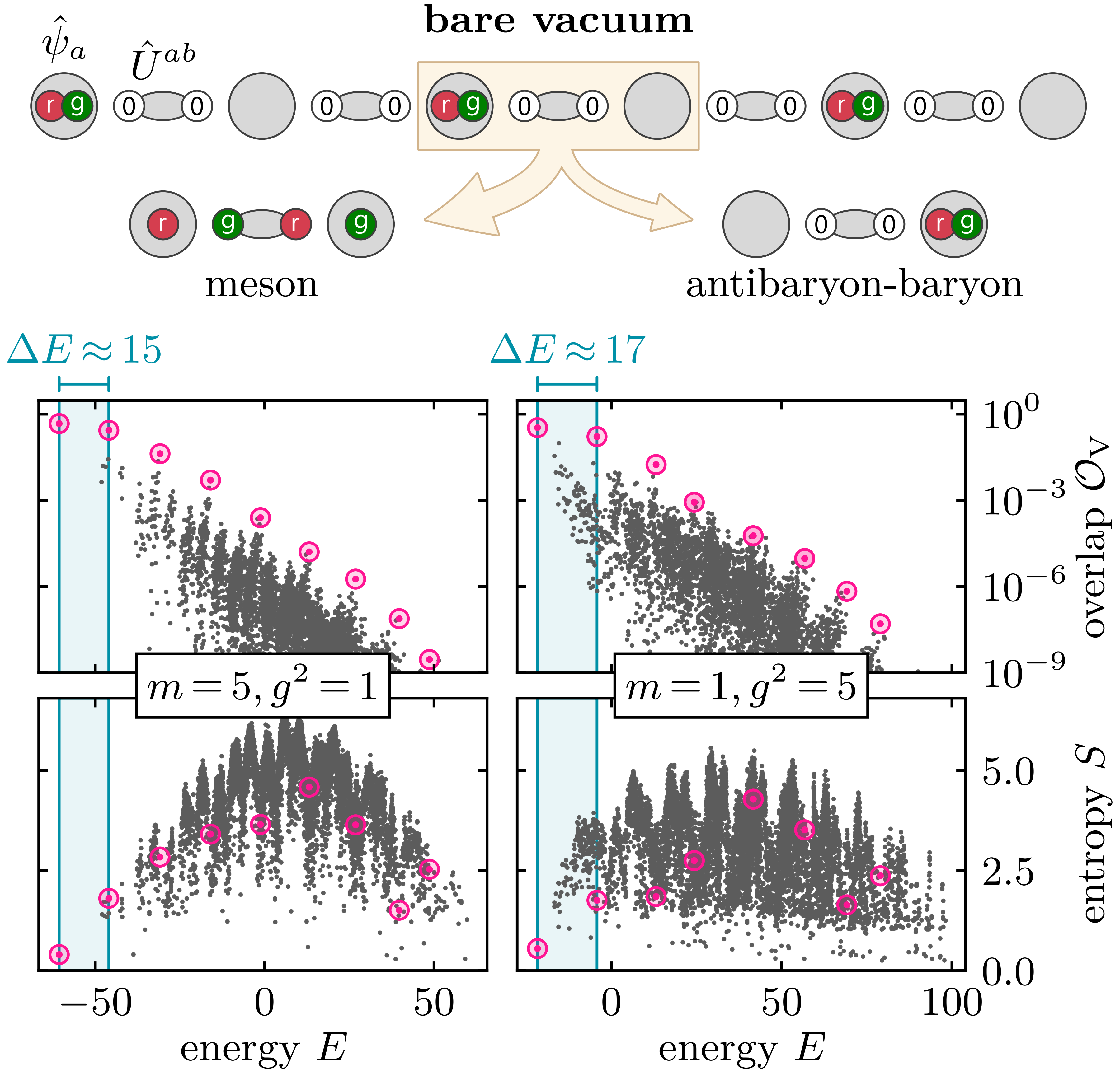


SCAR STATES IN THE SPECTRUM

The energy spectrum presents towers of equally spaced eigenstates with

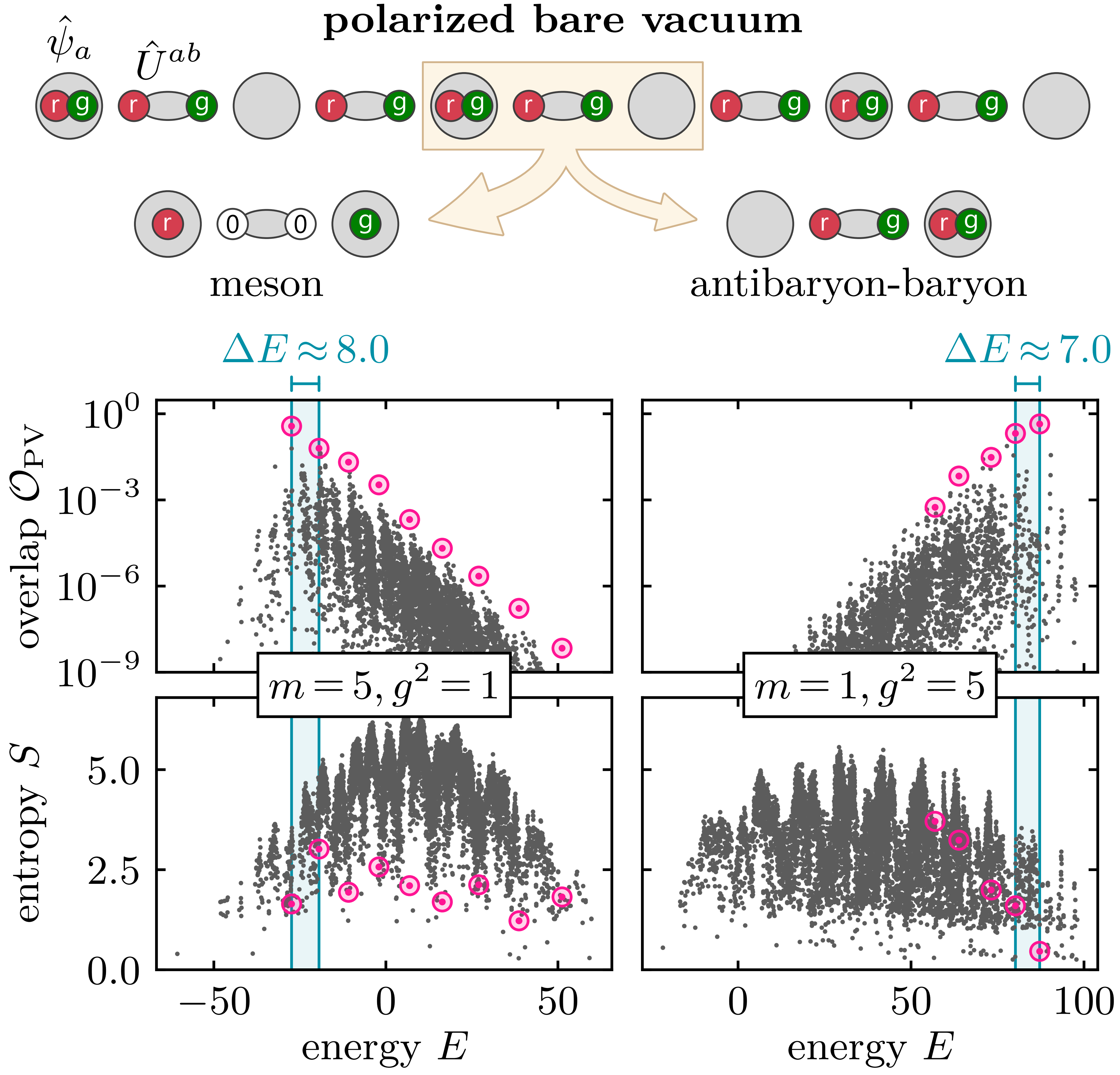


(1) HIGH OVERLAP
WITH THE INITIAL STATE
$\mathcal{O} = |\langle \Psi(0) | \Phi_s \rangle|^2$



SCAR STATES IN THE SPECTRUM

The energy spectrum presents towers of equally spaced eigenstates with

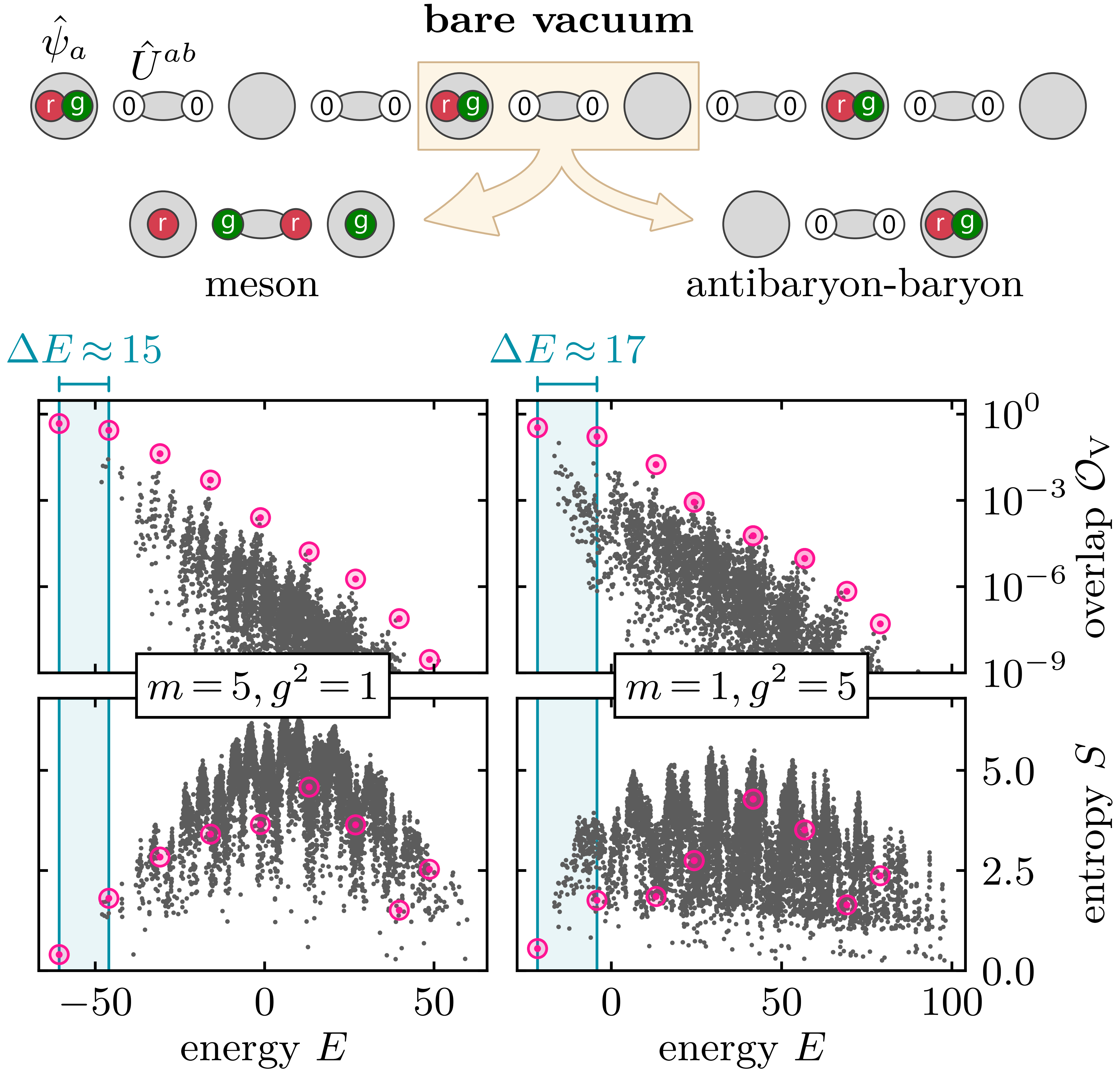


(1) HIGH OVERLAP WITH THE INITIAL STATE

$$\mathcal{O} = |\langle \Psi(0) | \Phi_s \rangle|^2$$

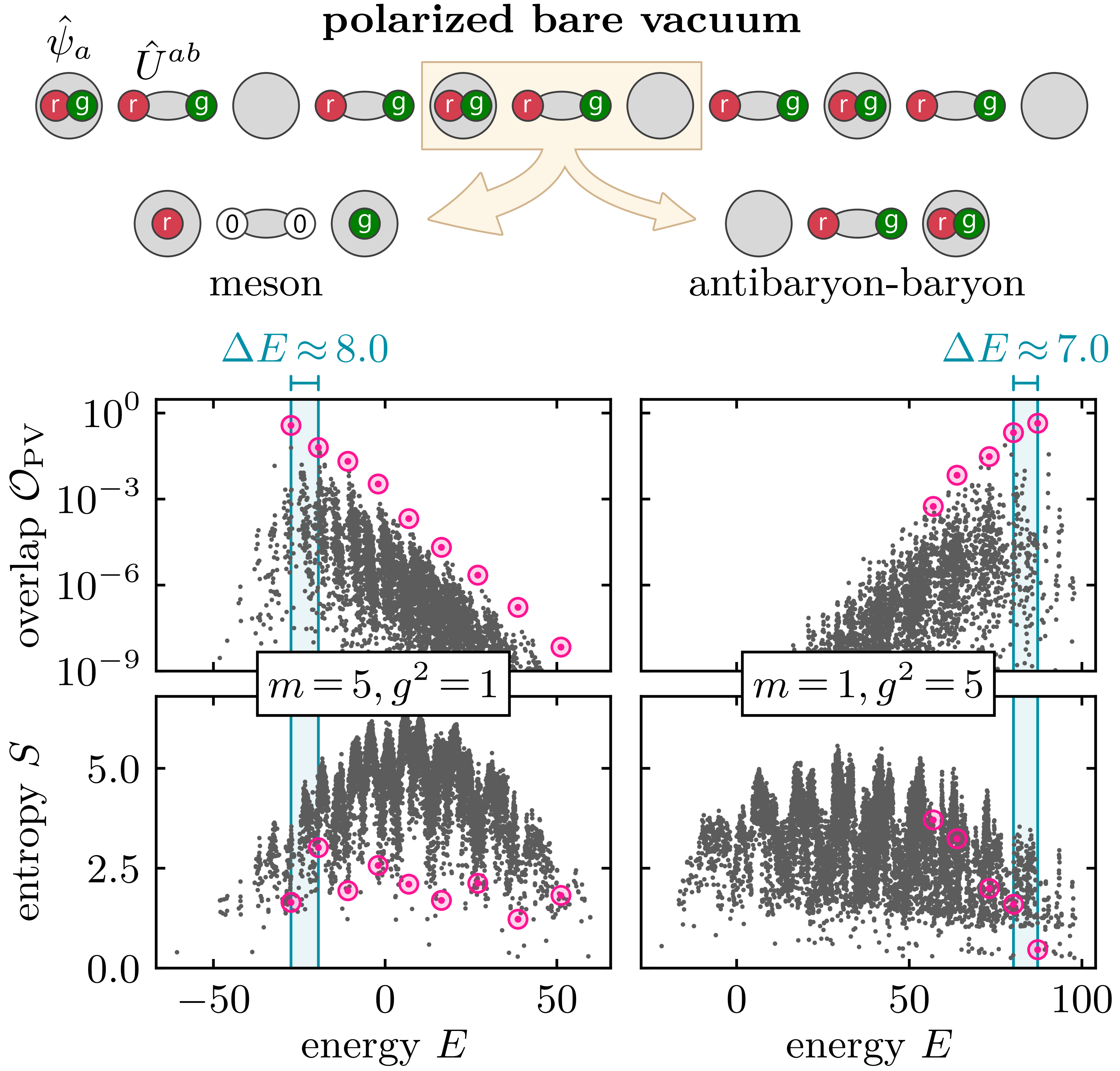
(2) LOW BIPARTITE ENTANGLEMENT ENTROPY

$$S = -\operatorname{Tr}(\rho_A \log \rho_A)$$



SCAR STATES IN THE SPECTRUM

The energy spectrum presents towers of equally spaced eigenstates with



(1) High Overlap with the initial state

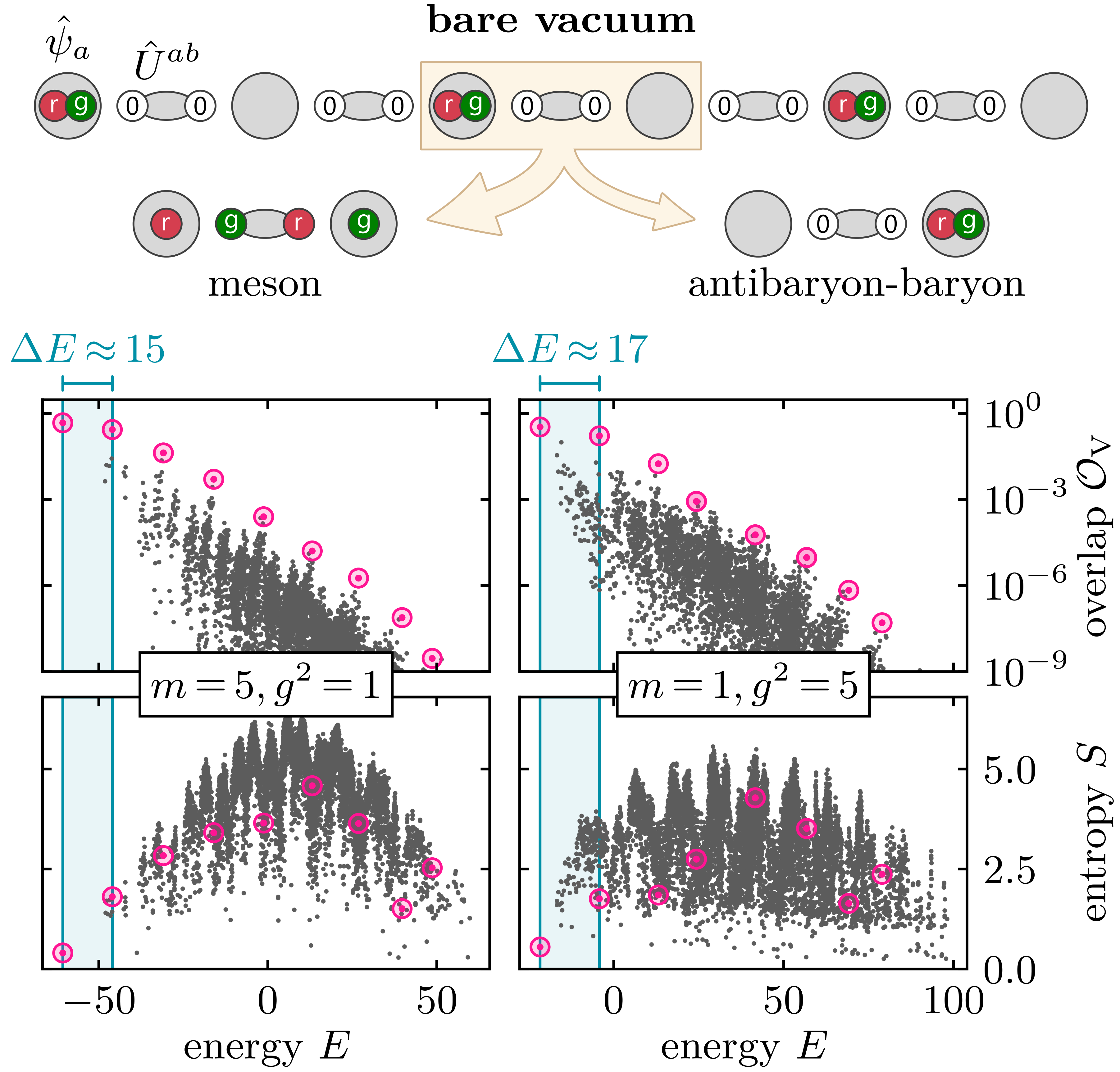
$$\mathcal{O} = |\langle \Psi(0) | \Phi_s \rangle|^2$$

(2) Low Bipartite Entanglement Entropy

$$S = -\,\mathrm{Tr}(\rho_A \log \rho_A)$$

(3) Energy gaps matching the frequency of revivals in dynamics

$$\Delta E \approx 2\pi/T$$



ERGODIC VS NON ERGODIC BEHAVIOR I

MICROCANONICAL ENSEMBLE (ME) STATE

$$|\Psi_{PV}^{ME}\rangle = \frac{1}{\sqrt{N_{E_{PV},\delta E}}} \sum_{\substack{s \\ |E_s - E_{PV}| < \delta E}} |\Phi_s\rangle$$

NUMBER OF EIGENSTATES IN THE SHELL

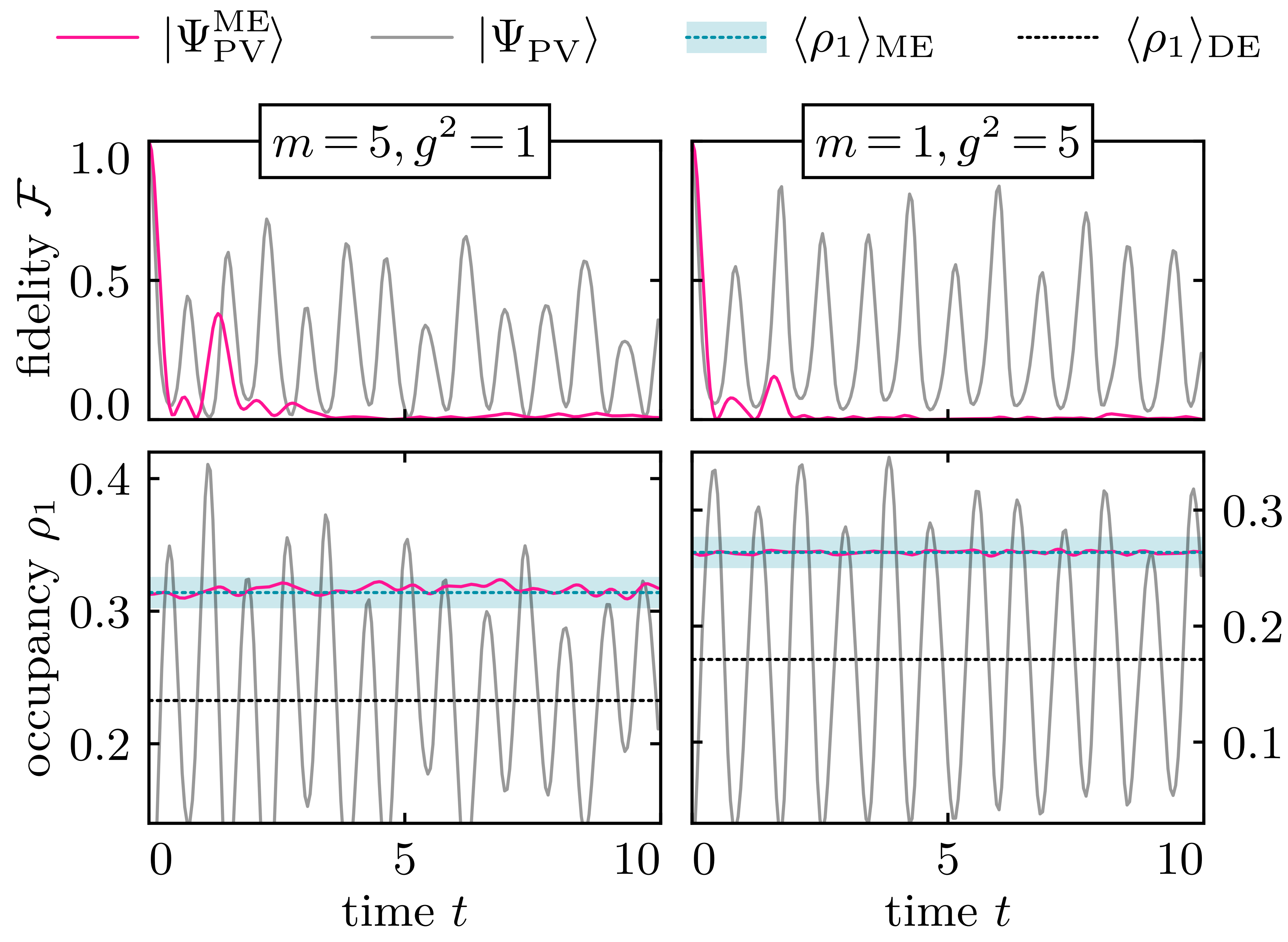
SMALL ENERGY SHELL AROUND E_{PV}

$$\delta E = \sqrt{\langle \Psi_{PV} | \hat{H}^2 | \Psi_{PV} \rangle - E_{PV}^2}$$

SPECTRUM EIGENSTATES

NO SCARS

FOR OTHER INITIAL STATES IN THE SAME PARAMETER REGIMES



ERGODIC VS NON ERGODIC BEHAVIOR I

MICROCANONICAL ENSEMBLE (ME) STATE

$$|\Psi_{PV}^{ME}\rangle = \frac{1}{\sqrt{N_{E_{PV},\delta E}}} \sum_{s \atop |E_s - E_{PV}| < \delta E} |\Phi_s\rangle$$

NUMBER OF EIGENSTATES IN THE SHELL

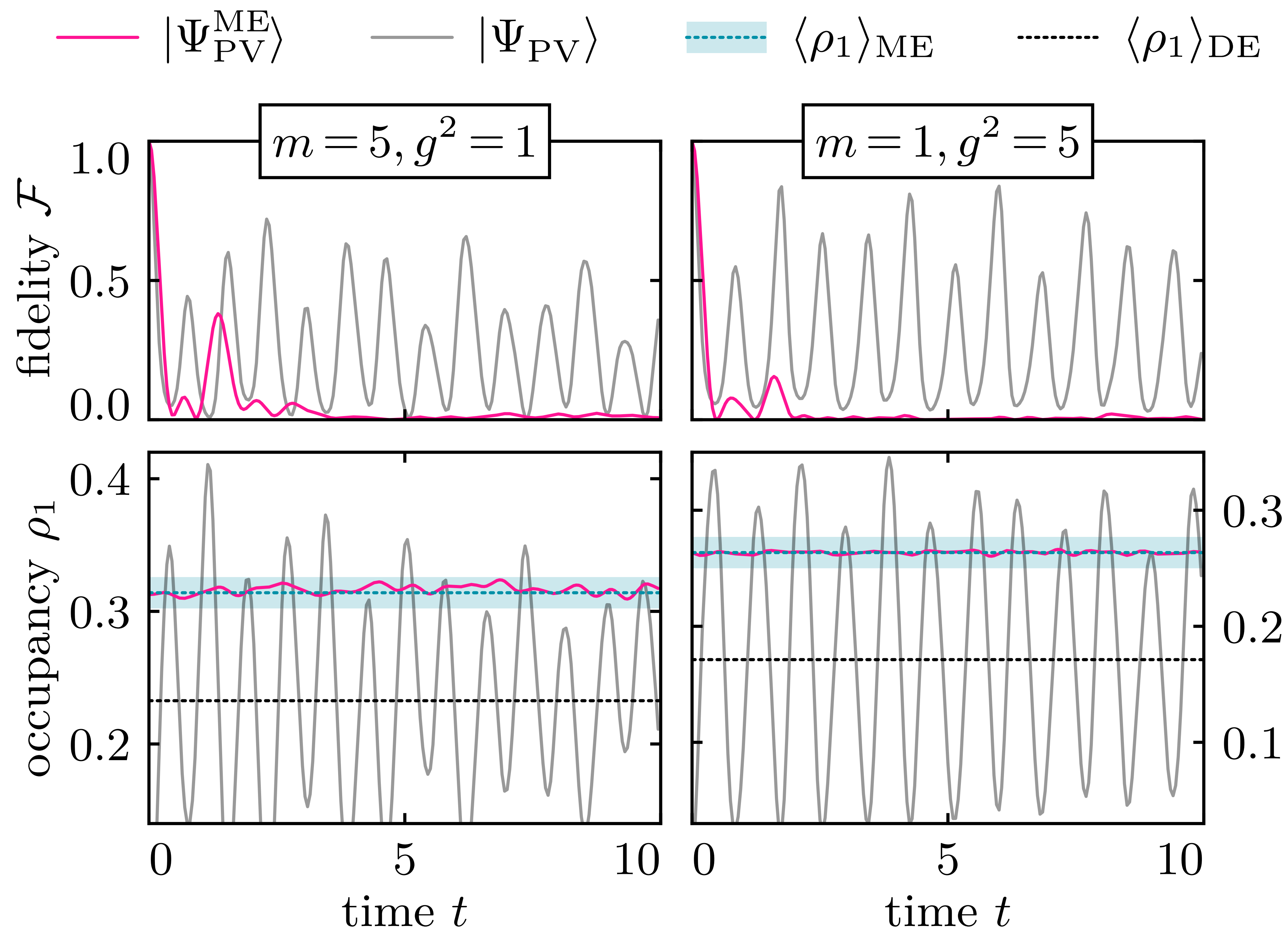
SMALL ENERGY SHELL AROUND E_{PV}

$\delta E = \sqrt{\langle \Psi_{PV} | \hat{H}^2 | \Psi_{PV} \rangle - E_{PV}^2}$

SPECTRUM EIGENSTATES

NO SCARS

FOR OTHER INITIAL STATES IN THE SAME PARAMETER REGIMES



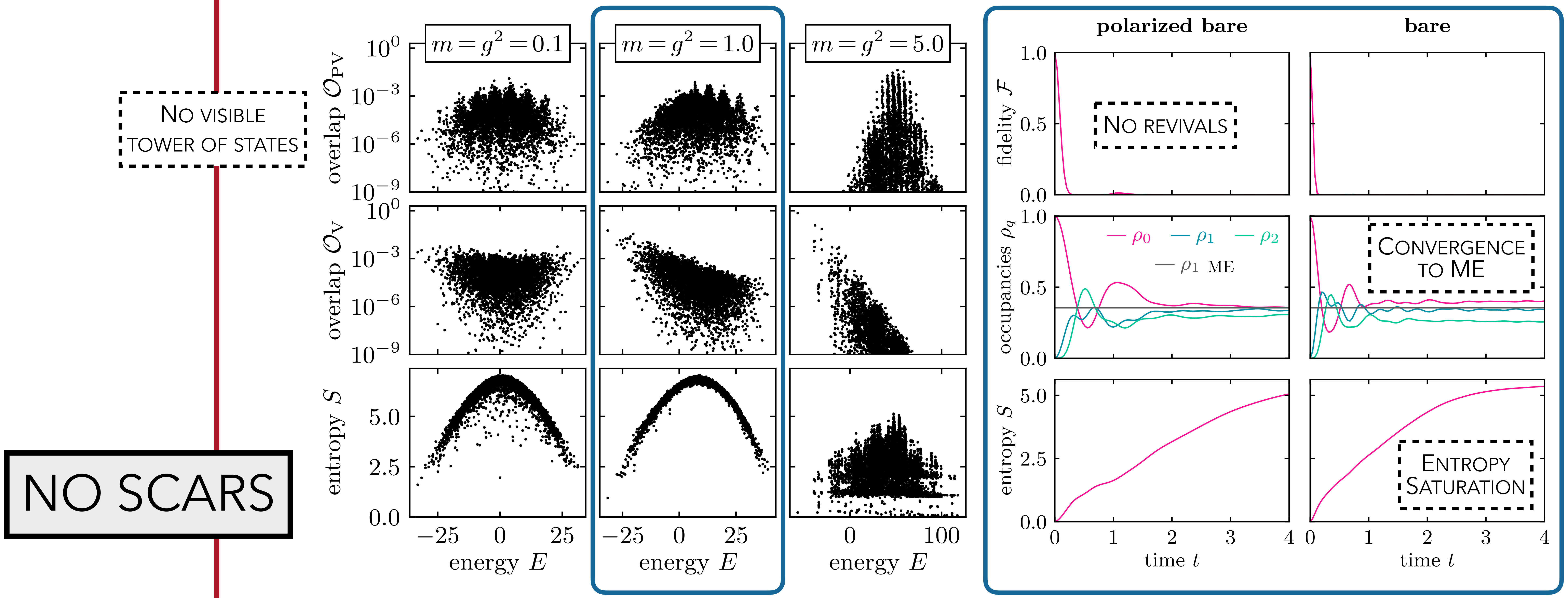
SCARRING DYNAMICS VIOLATES ETH (AND HENCE STATISTICAL MECHANICS)

ERGODIC VS NON ERGODIC BEHAVIOR II

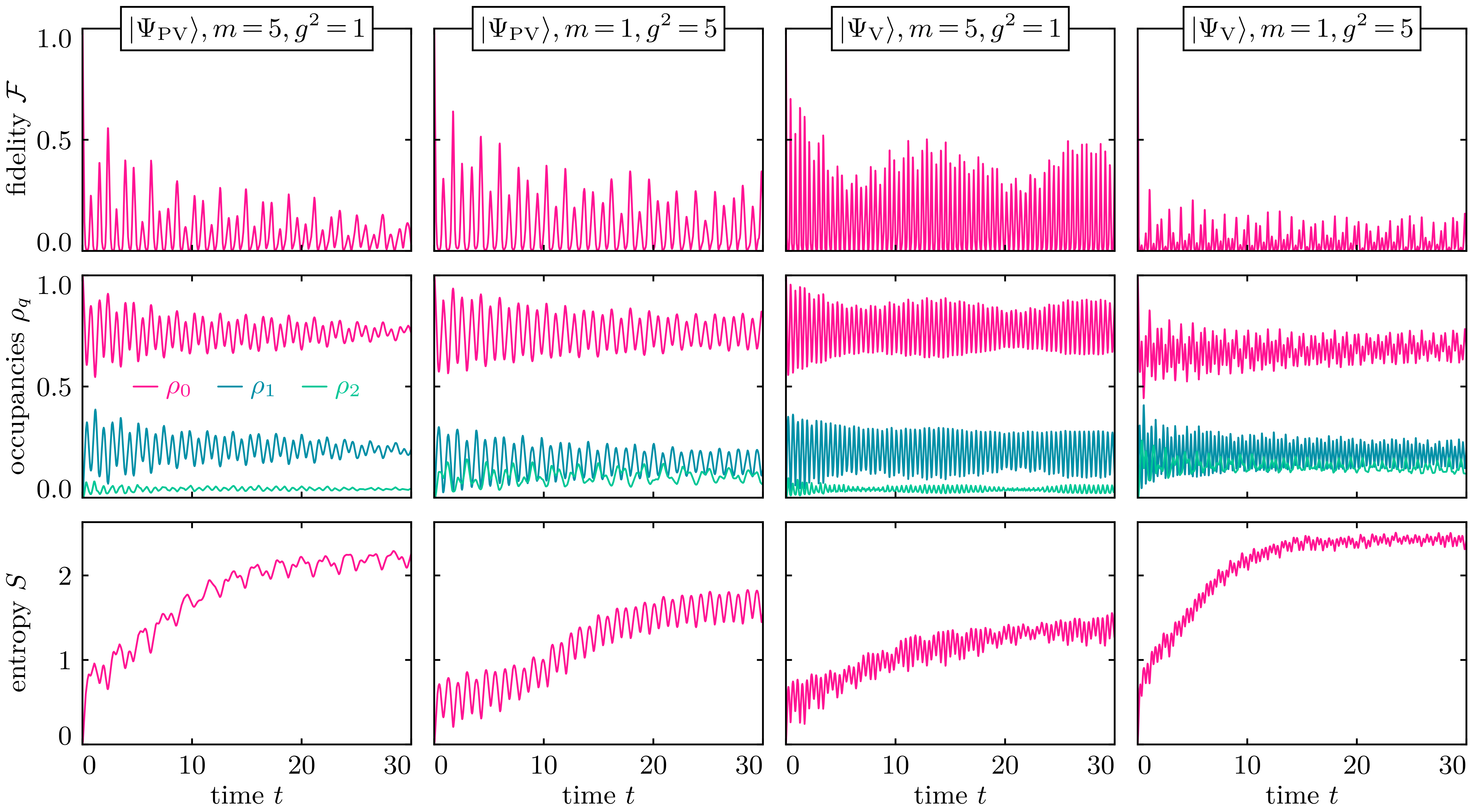
QMB Scars are **POLYNOMIAL** deviations from the ETH: they arise for specific initial states in specific parameter values!

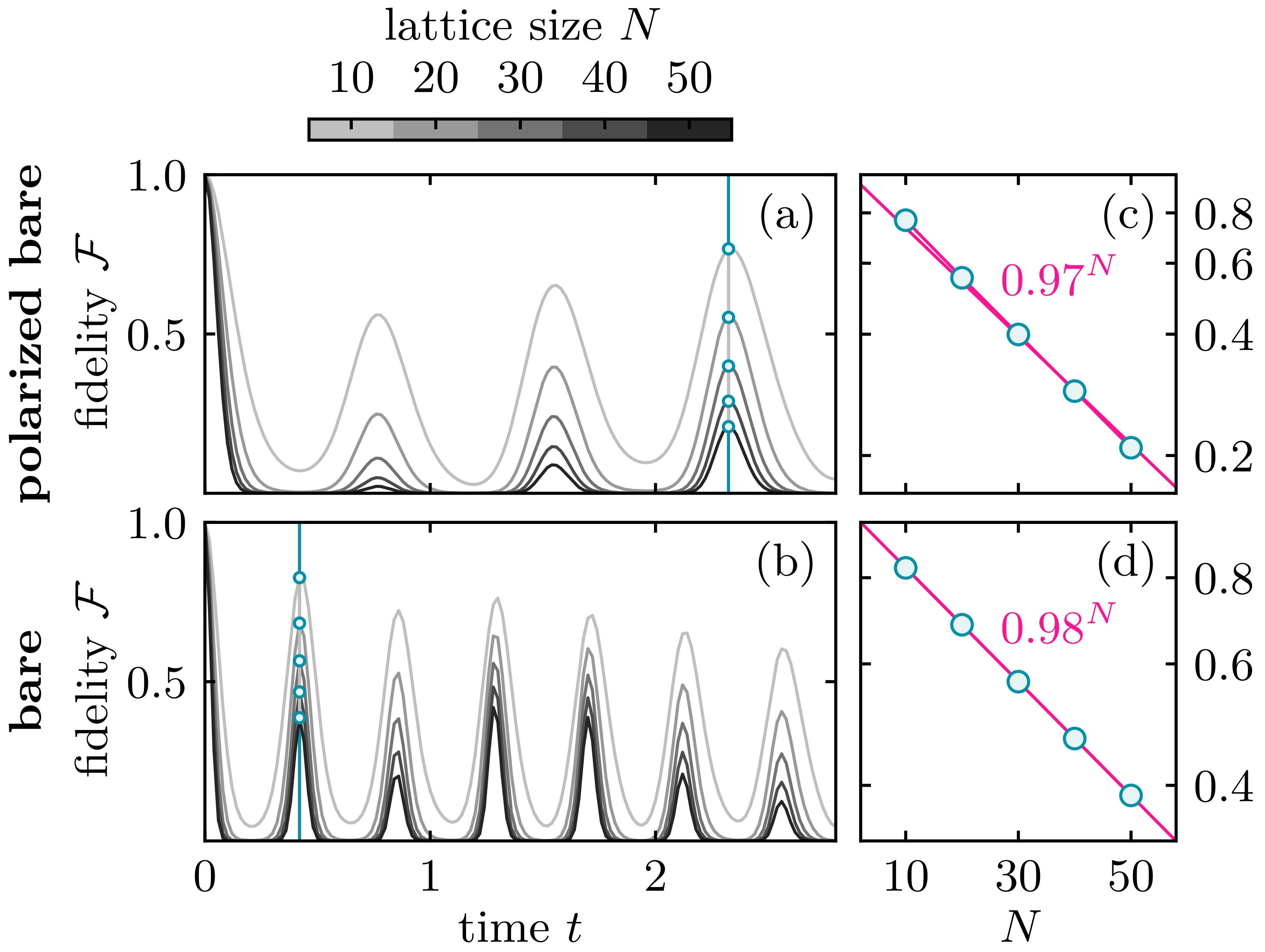
WEAK ETH BREAKING

FOR THE SAME INITIAL STATES IN OTHER PARAMETER REGIMES



LONGER TIMES, LARGER SYSTEMS SIZES





CONCLUSIONS

SUMMARY

- Standard Ergodic Behaviors
- Scarring characterization
- Scars in Abelian LGT
- Scars in NonAbelian LGT

Dipartimento di Fisica e Astronomia Galileo Galilei

INFN

QUANTUM

T-NiSQ

CONCLUSIONS

Summary

- Standard Ergodic Behaviors
- Scarring characterization
- Scars in Abelian LGT
- Scars in NonAbelian LGT

Future Research

- Scars in quasi-2D SU(2) LGT
- Disorder Free Localization

Dipartimento di Fisica e Astronomia Galileo Galilei

INFN

QUANTUM

T-NiSQ

CONCLUSIONS

- SUMMARY
  - Standard Ergodic Behaviors
  - Scarring characterization
  - Scars in Abelian LGT
  - Scars in NonAbelian LGT
- FUTURE RESEARCH
  - Scars in quasi-2D SU(2) LGT
  - Disorder Free Localization


G. CALAJÒ


M. RIGOBELLO


D. WANISCH


G. MAGNIFICO


P. SILVI


S. MONTANGERO


J.C. HALIMEH

Dipartimento di Fisica e Astronomia Galileo Galilei

INFN

QUANTUM

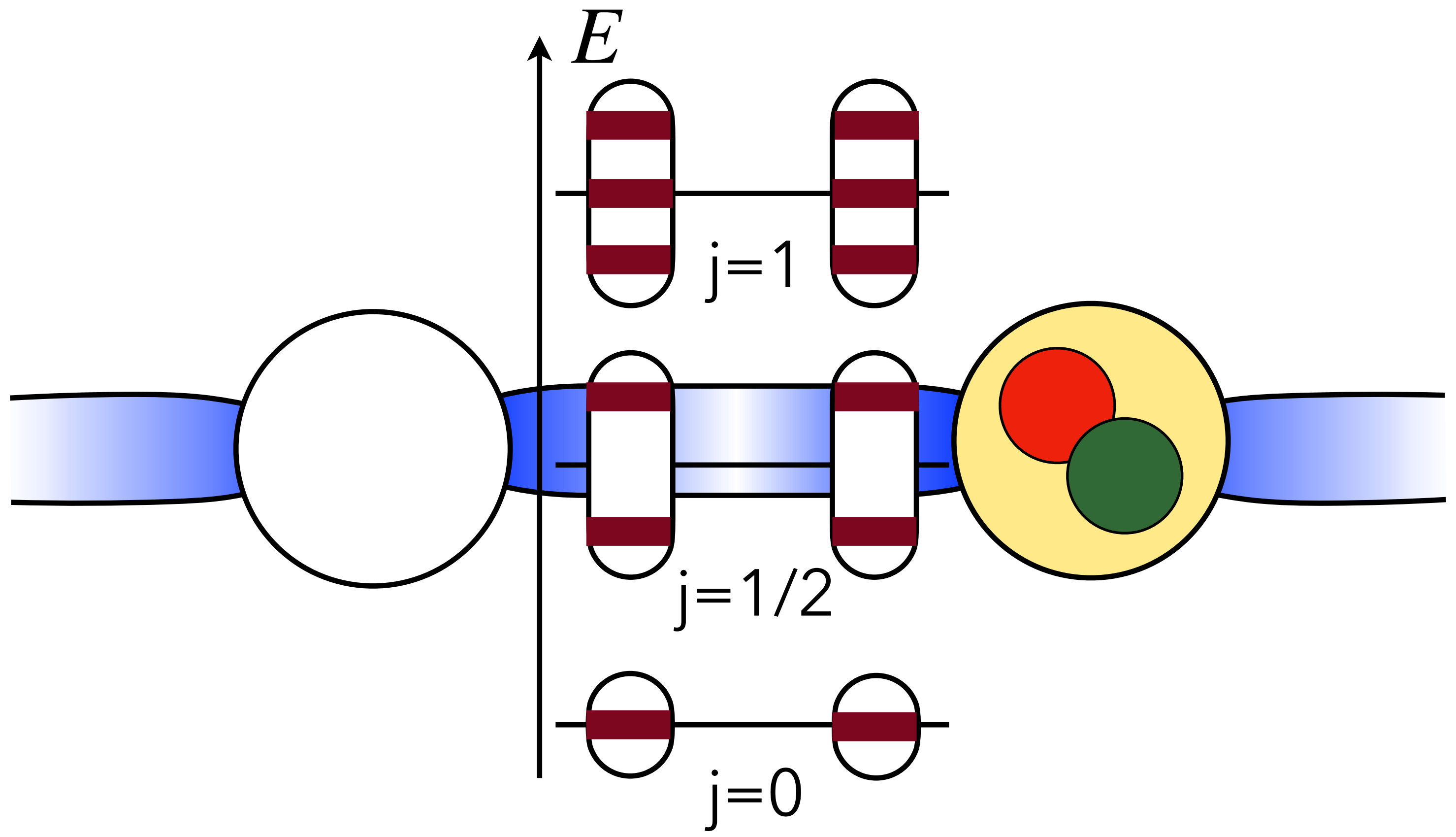
T-NiSQ

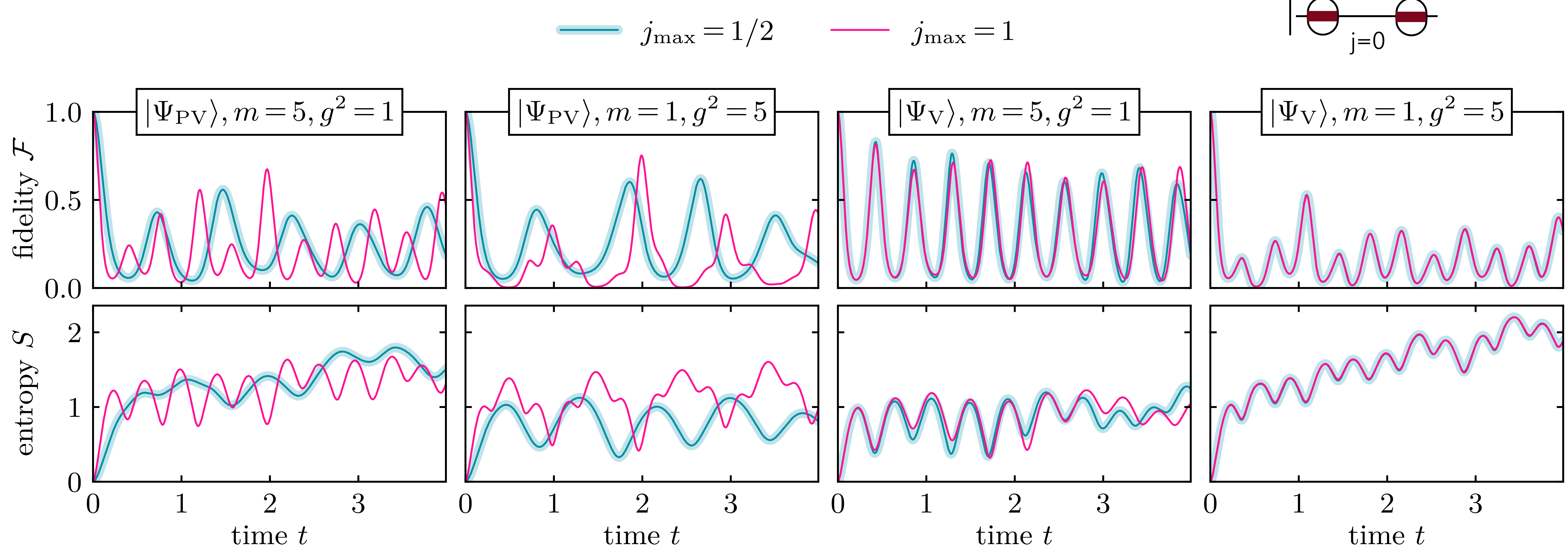
BACKUP SLIDES

LARGER SPIN-TRUNCATIONS OF SU2

Do Scars appear at SU(2) with a larger truncation?

For $j_{\max} = 1$, we get a 10-dimensional qudit model:

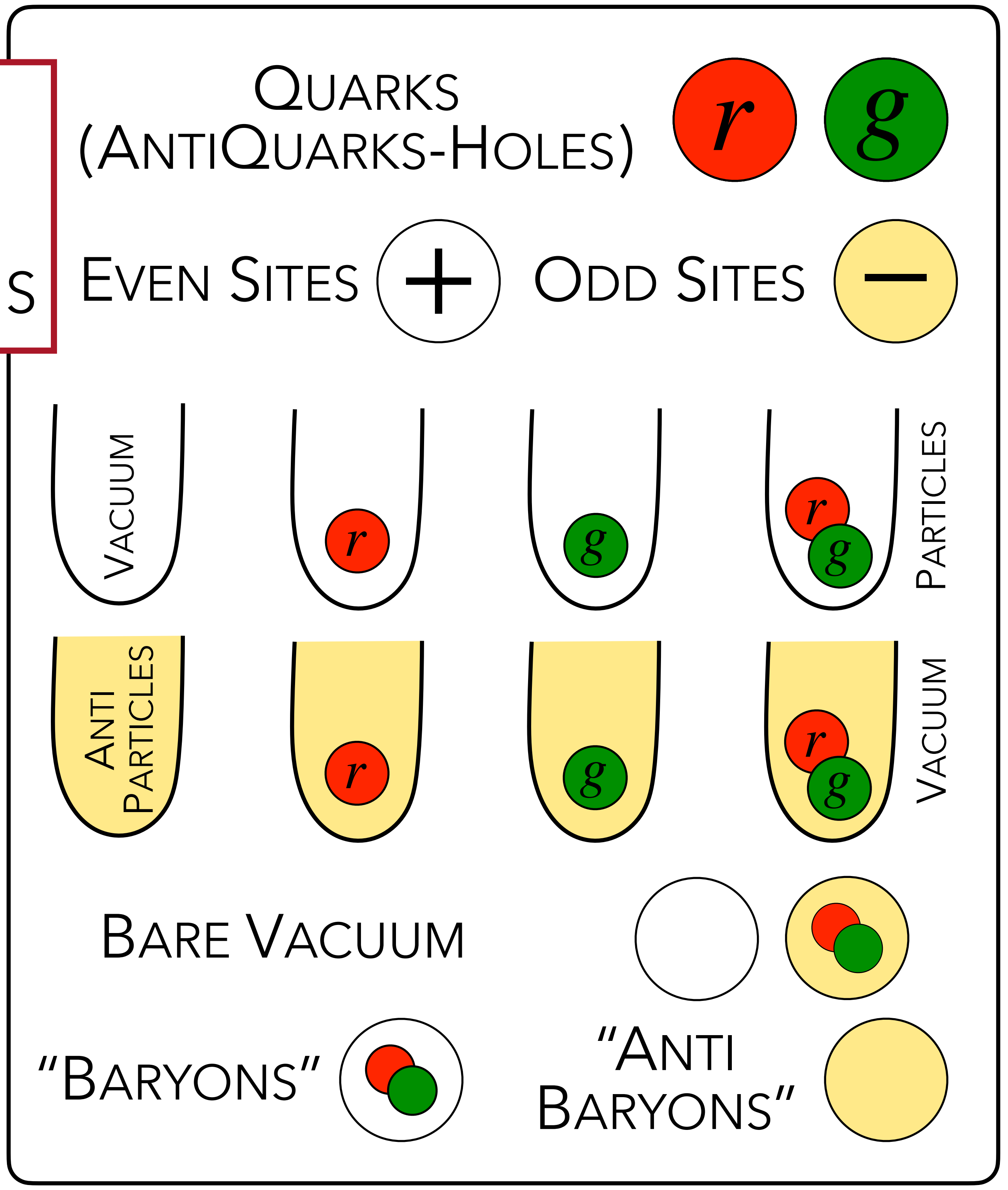


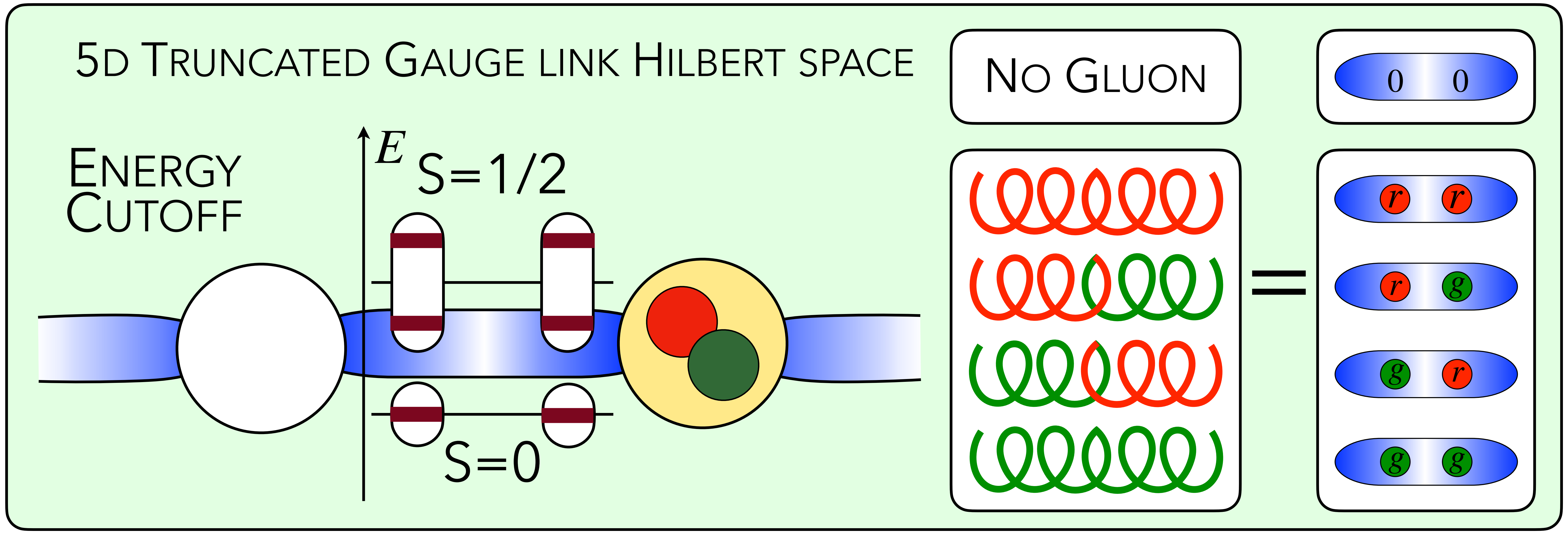


(1+1)D HARDCORE SU(2) YANG MILLS LGT

$$H = \frac{1}{2a}\sum_{\alpha,\beta}\sum_{j,\mu}\left[\psi^{\dagger}_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m_0\sum_{j}(-1)^j\sum_{\alpha}\psi^{\dagger}_{j,\alpha}\psi_{j,\alpha} + \frac{ag^2}{2}\sum_{j}E^2_{j,j+\mu}$$

4D MATTER HILBERT SPACE
SU(2)-COLOR 1/2
FLAVORLESS DIRAC FERMIONS



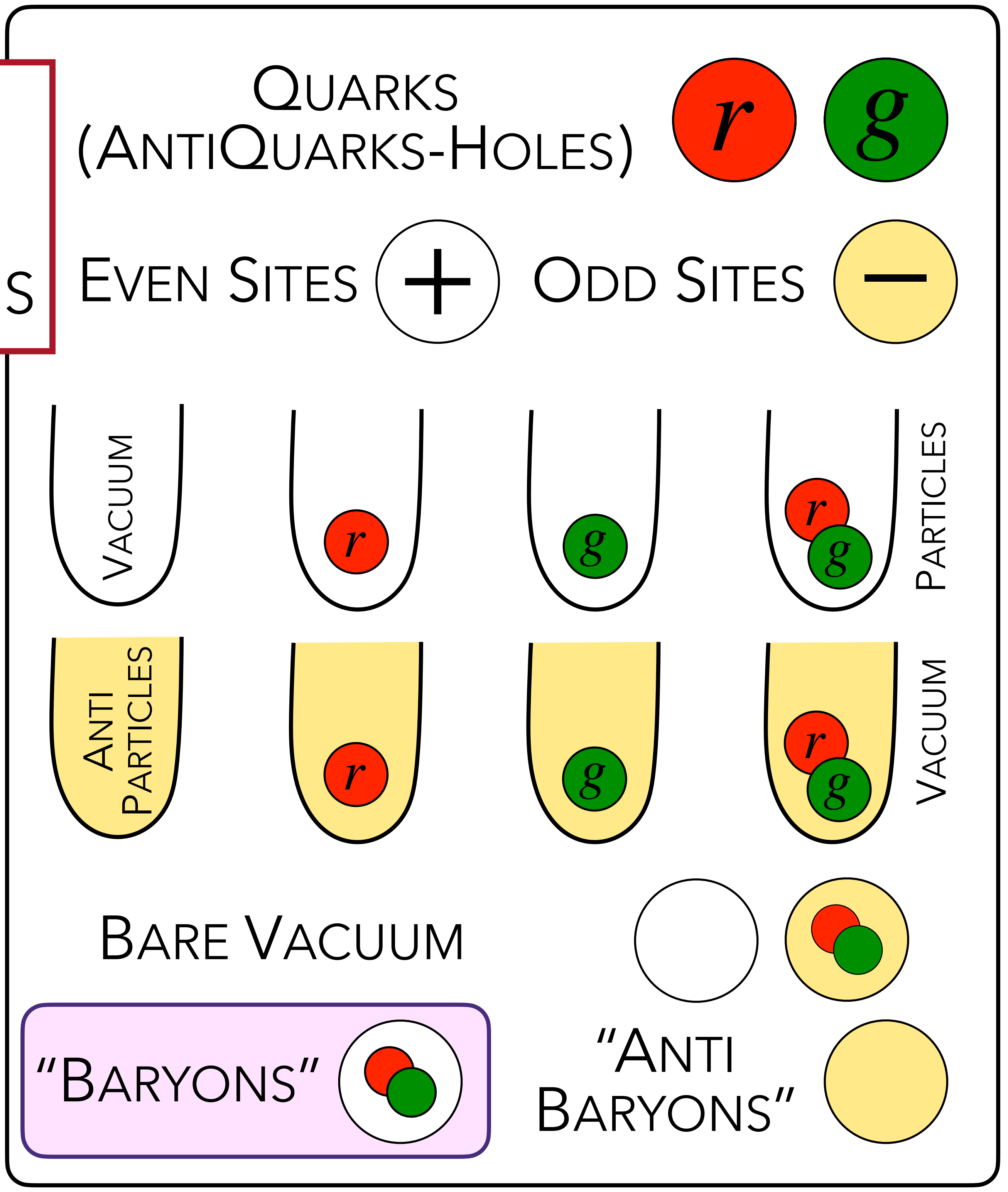


(1+1)D HARDCORE SU(2) YANG MILLS LGT

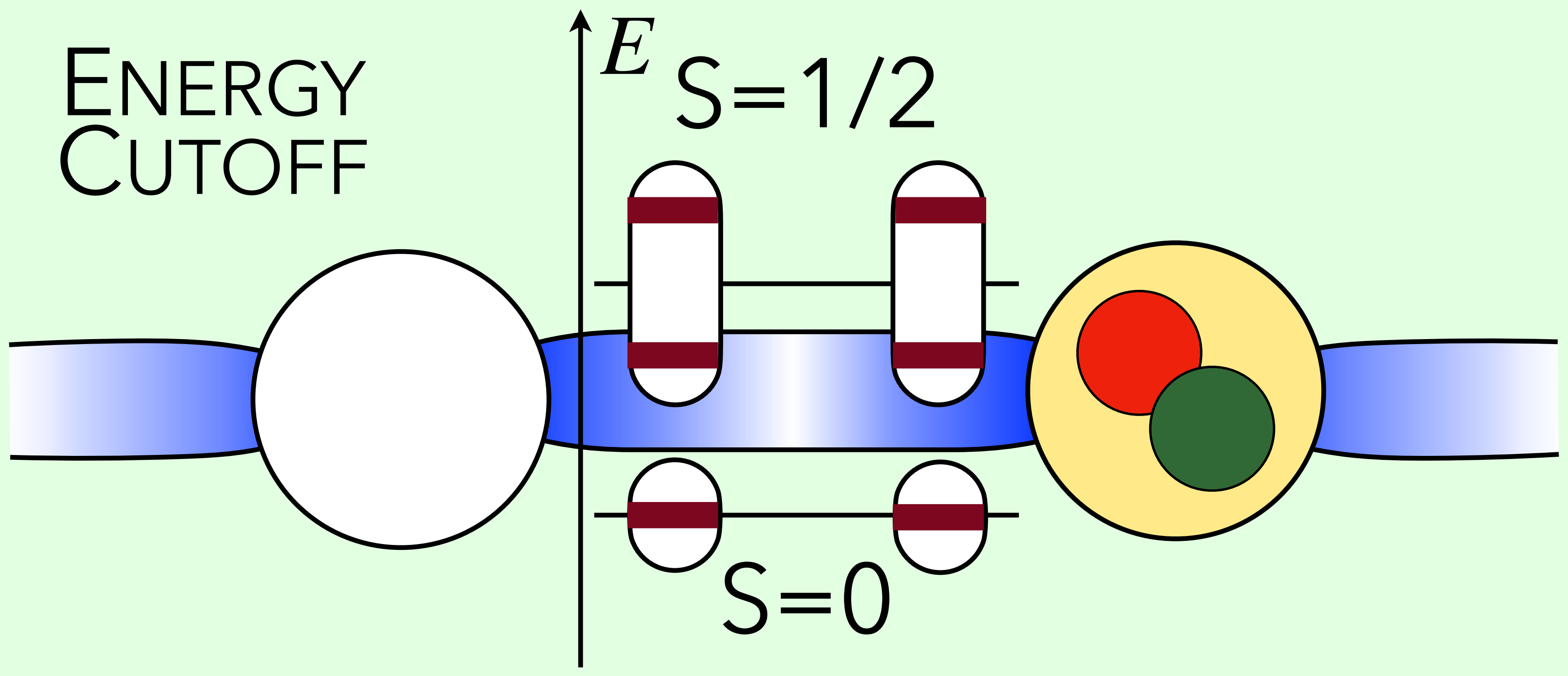
$$H = \frac{1}{2a}\sum_{\alpha,\beta}\sum_{j,\mu}\left[\psi^\dagger_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \mathrm{H.c.}\right] + m_0\sum_j(-1)^j\sum_\alpha \psi^\dagger_{j,\alpha}\psi_{j,\alpha} + \frac{ag^2}{2}\sum_j E^2_{j,j+\mu}$$

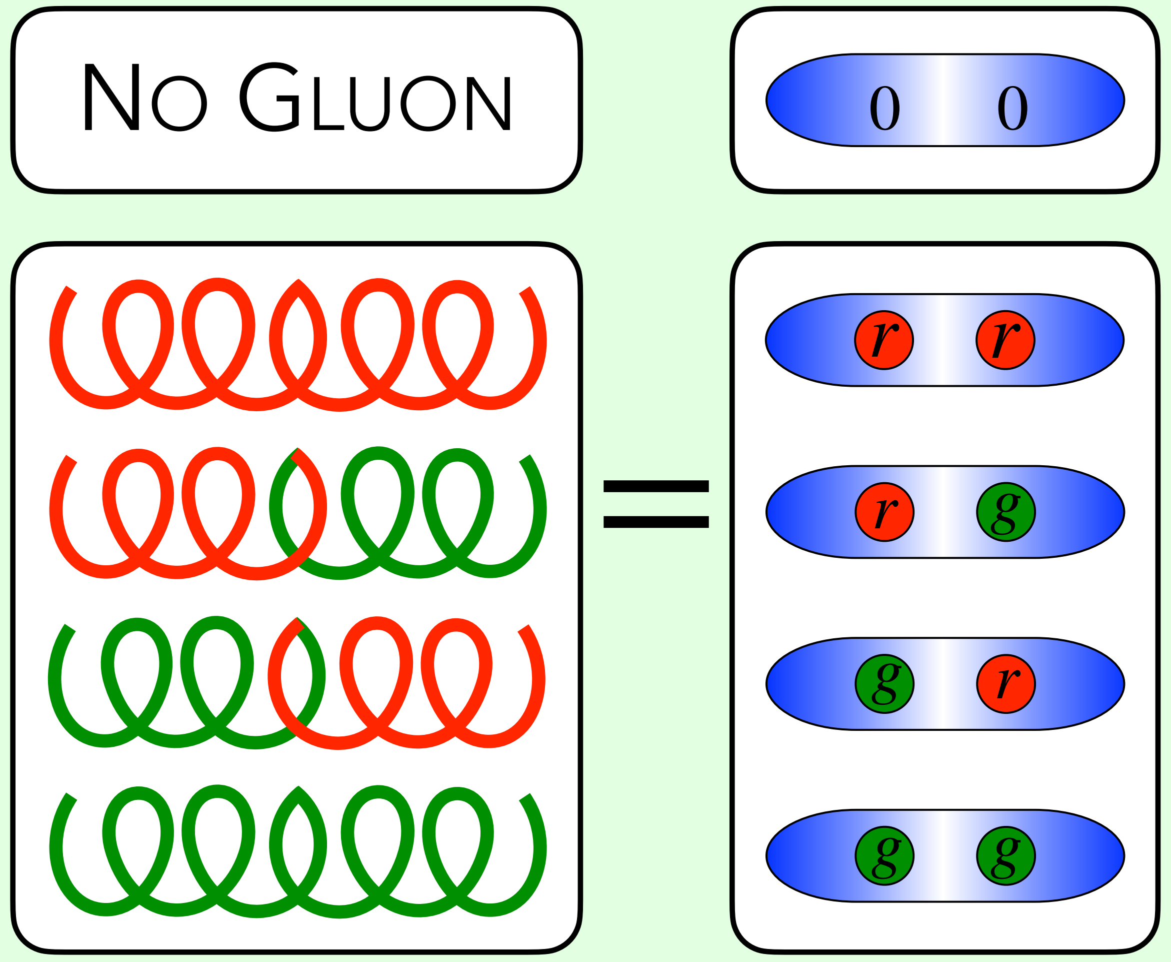
STAGGERED MASS TERM

4D MATTER HILBERT SPACE
SU(2)-COLOR 1/2
FLAVORLESS DIRAC FERMIONS



5D TRUNCATED GAUGE LINK HILBERT SPACE





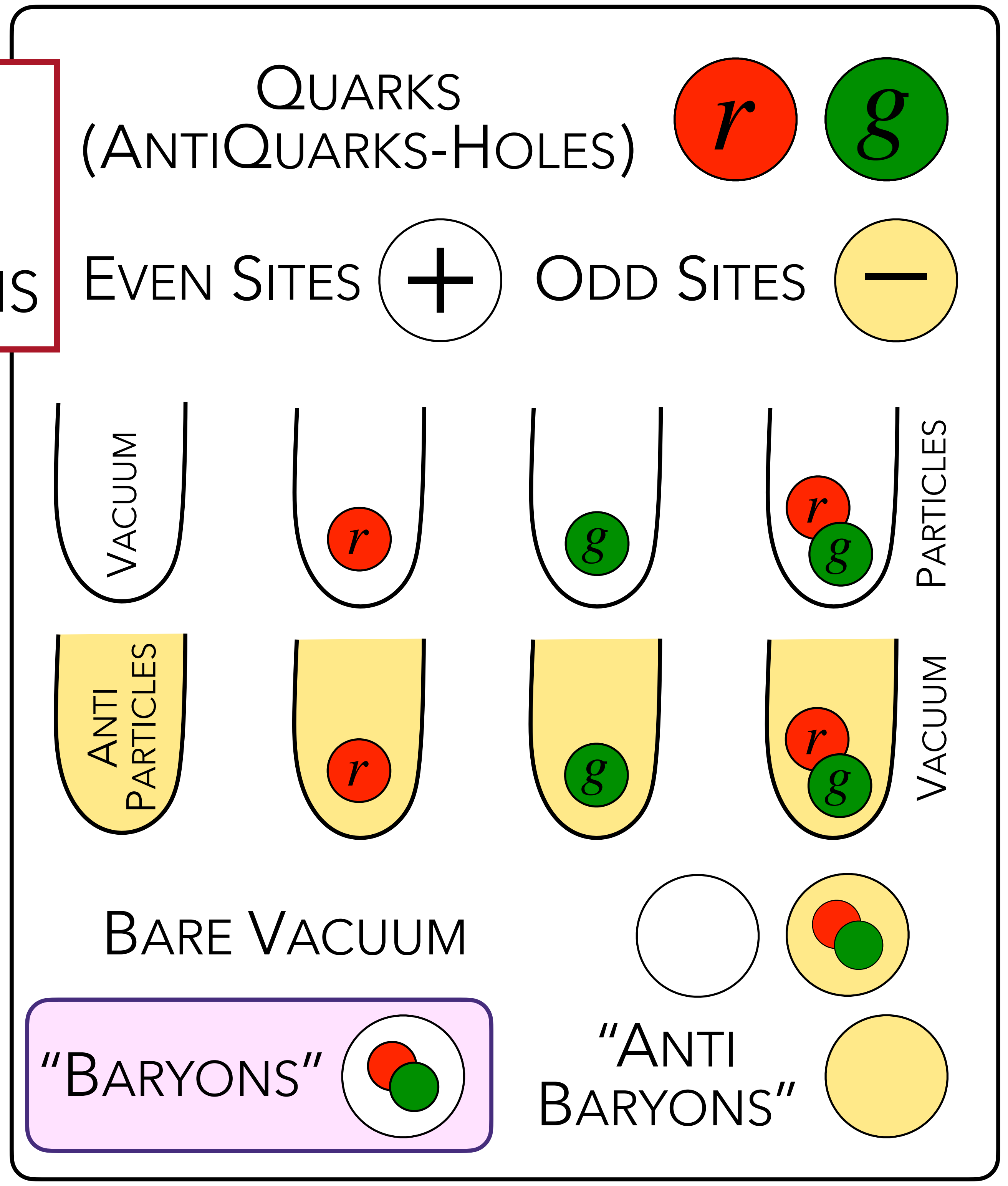
(1+1)D HARDCORE SU(2) YANG MILLS LGT

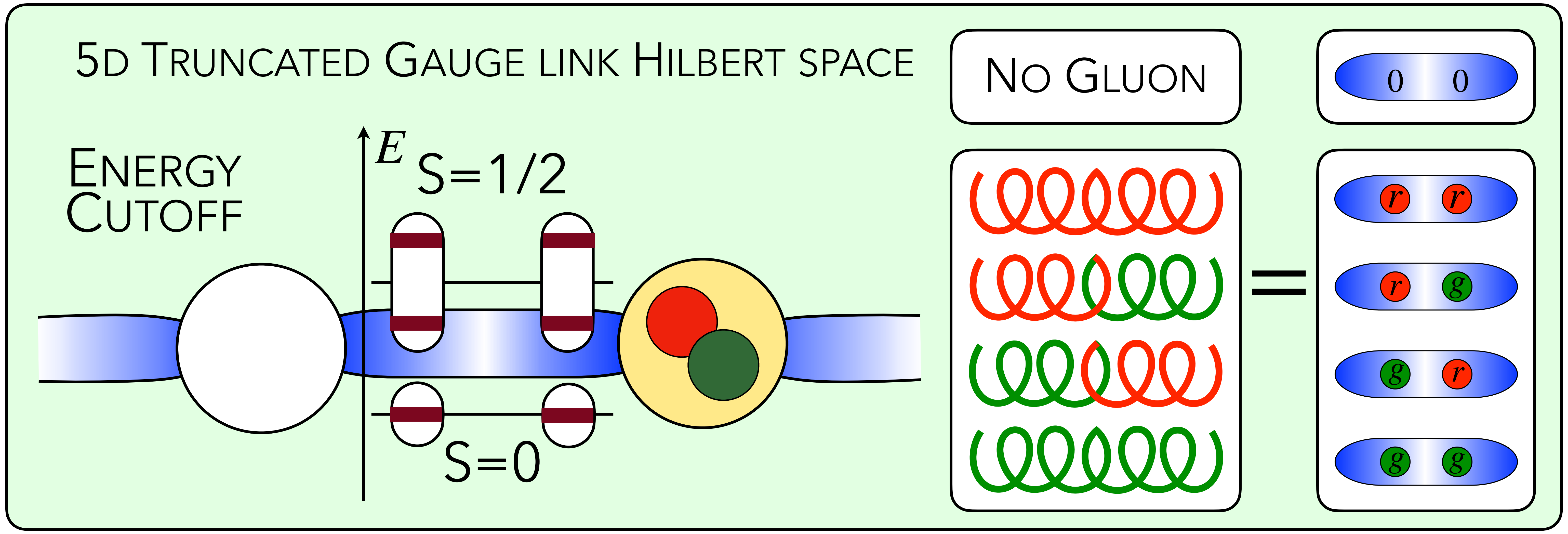
$$H = \frac{1}{2a}\sum_{\alpha,\beta}\sum_{j,\mu}\left[\psi^{\dagger}_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m_0\sum_{j}(-1)^j\sum_{\alpha}\psi^{\dagger}_{j,\alpha}\psi_{j,\alpha} + \frac{ag^2}{2}\sum_{j}E^2_{j,j+\mu}$$

MATTER-GAUGE HOPPING TERM

STAGGERED MASS TERM

4D MATTER HILBERT SPACE
SU(2)-COLOR 1/2
FLAVORLESS DIRAC FERMIONS





(1+1)D HARDCORE SU(2) YANG MILLS LGT

$$H = \frac{1}{2a}\sum_{\alpha,\beta}\sum_{j,\mu}\left[\psi^{\dagger}_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m_0\sum_{j}(-1)^j\sum_{\alpha}\psi^{\dagger}_{j,\alpha}\psi_{j,\alpha} + \frac{ag^2}{2}\sum_{j}E^2_{j,j+\mu}$$

SU(2) ELECTRIC ENERGY

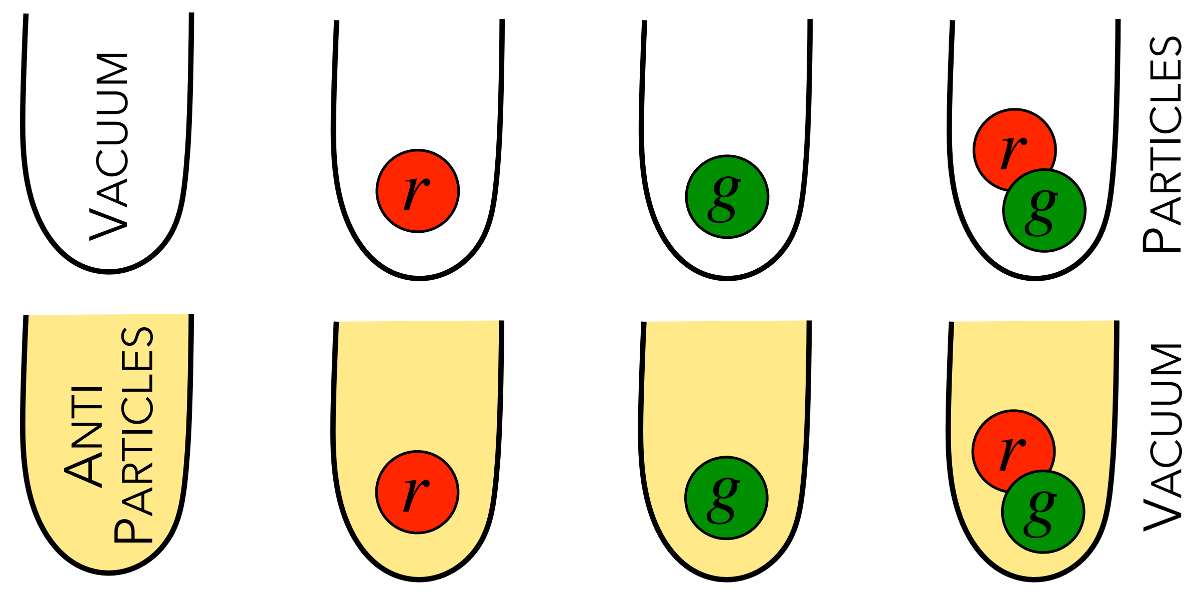
MATTER-GAUGE HOPPING TERM

STAGGERED MASS TERM

4D MATTER HILBERT SPACE
SU(2)-COLOR 1/2
FLAVORLESS DIRAC FERMIONS

QUARKS (ANTIQUARKS-HOLES) r g

EVEN SITES (+) ODD SITES (−)



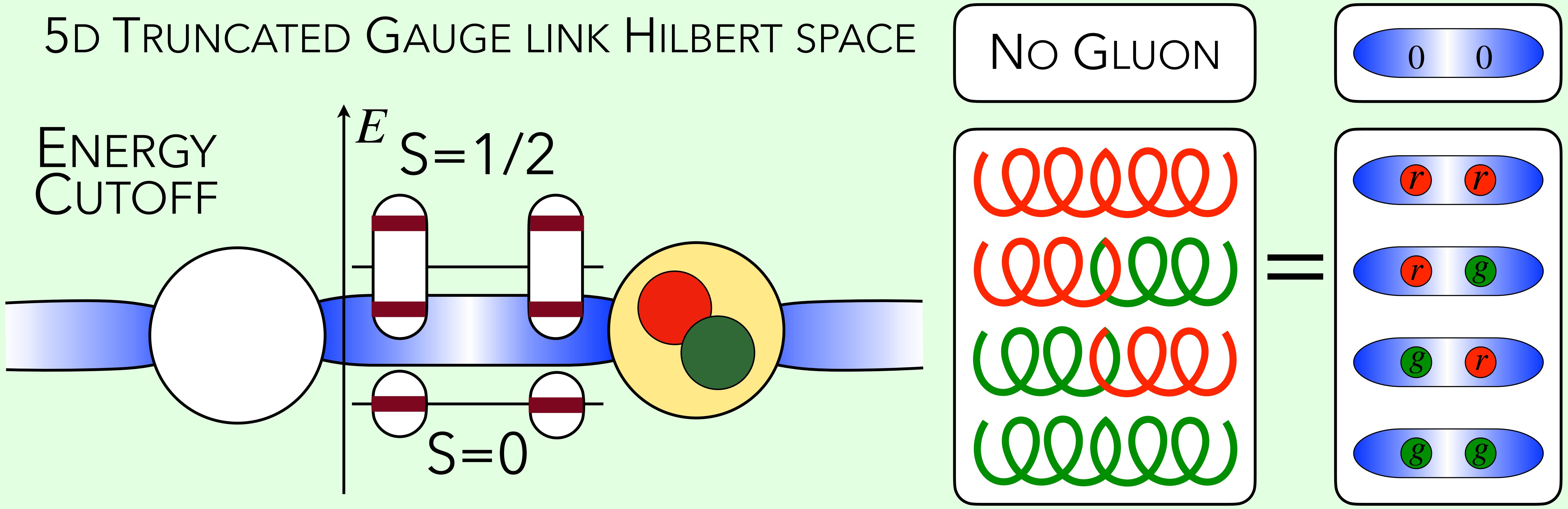
BARE VACUUM

"BARYONS"

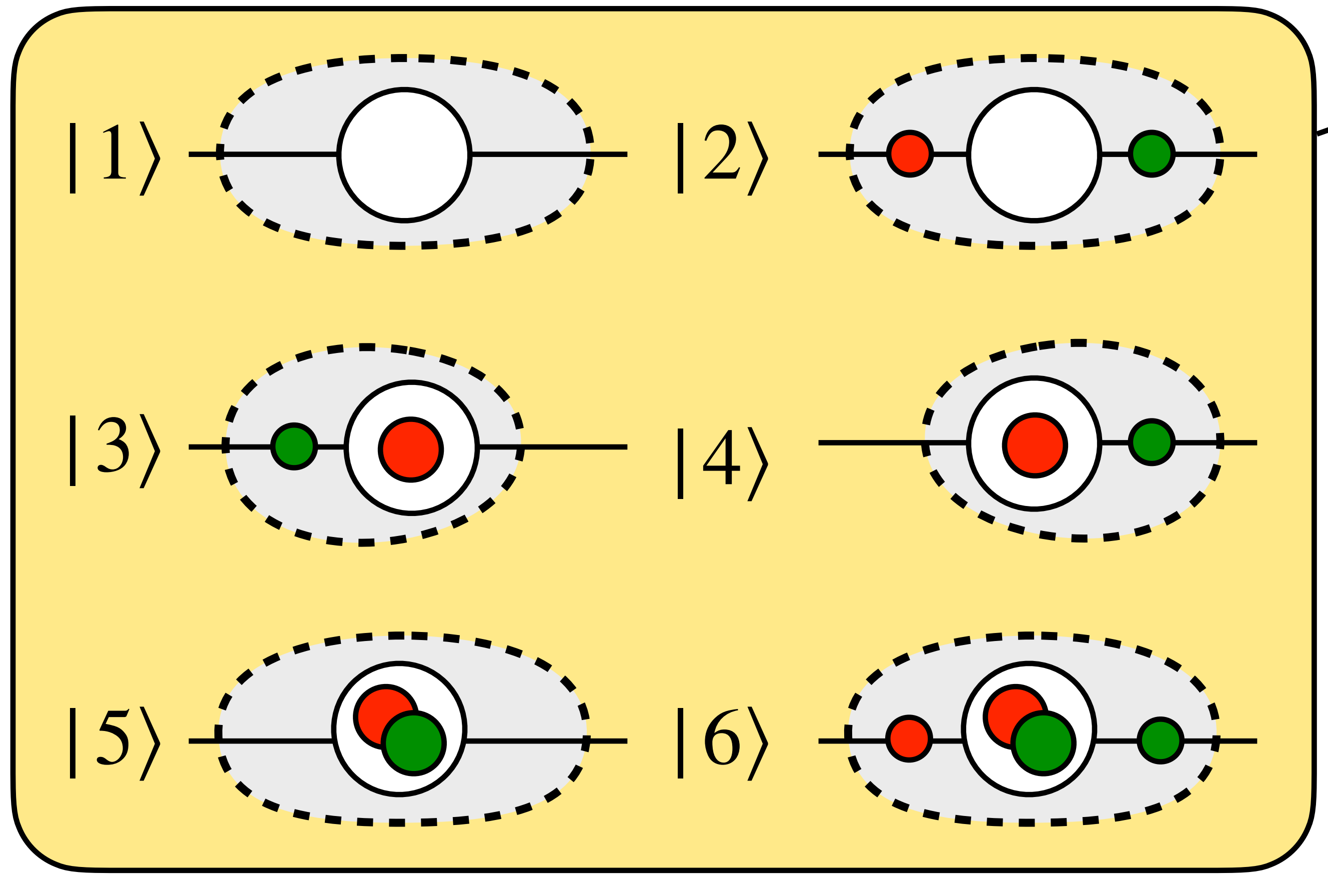
"ANTI BARYONS"

5D TRUNCATED GAUGE LINK HILBERT SPACE

NO GLUON

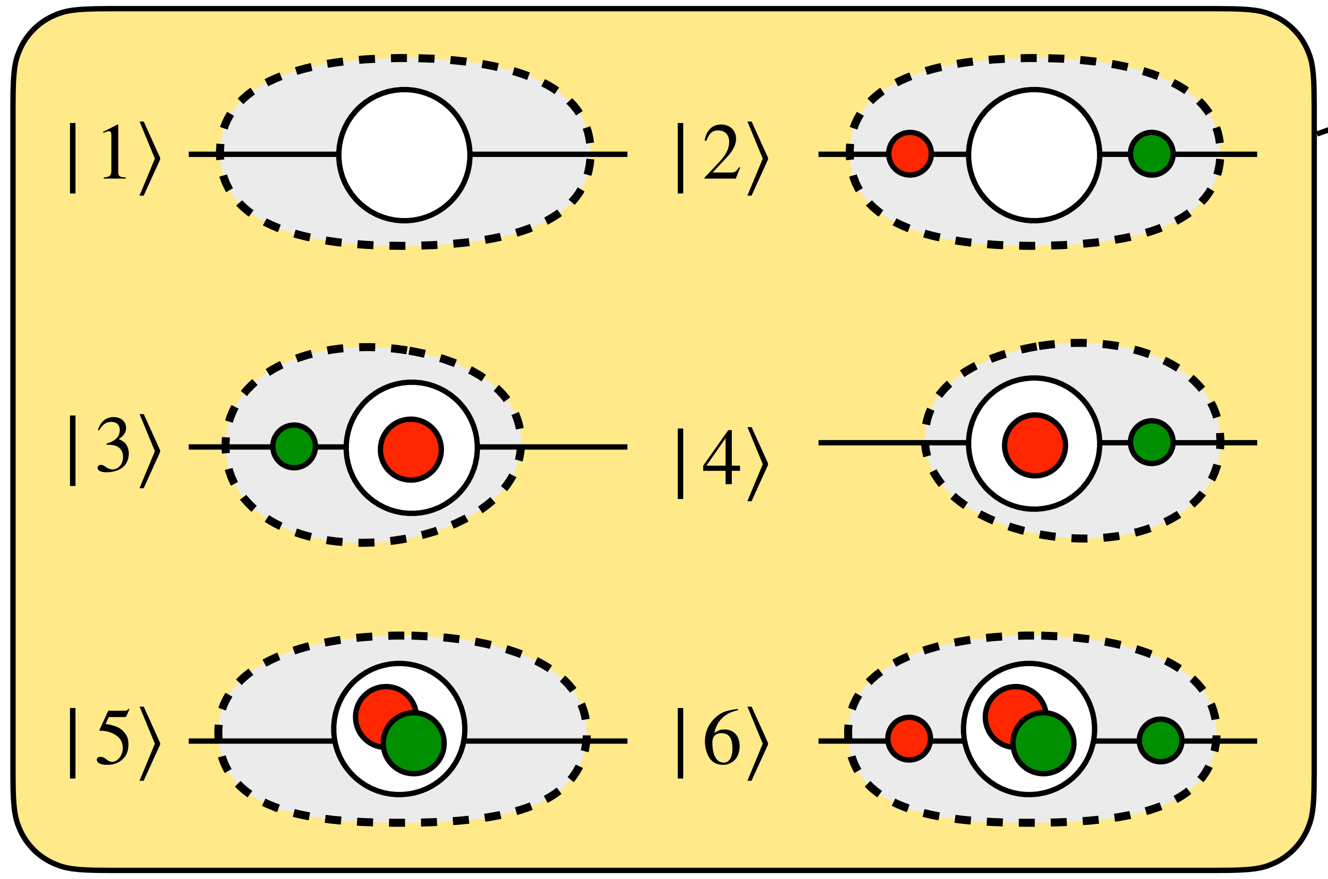


GAUSS LAW AND DEFERMIONIZATION



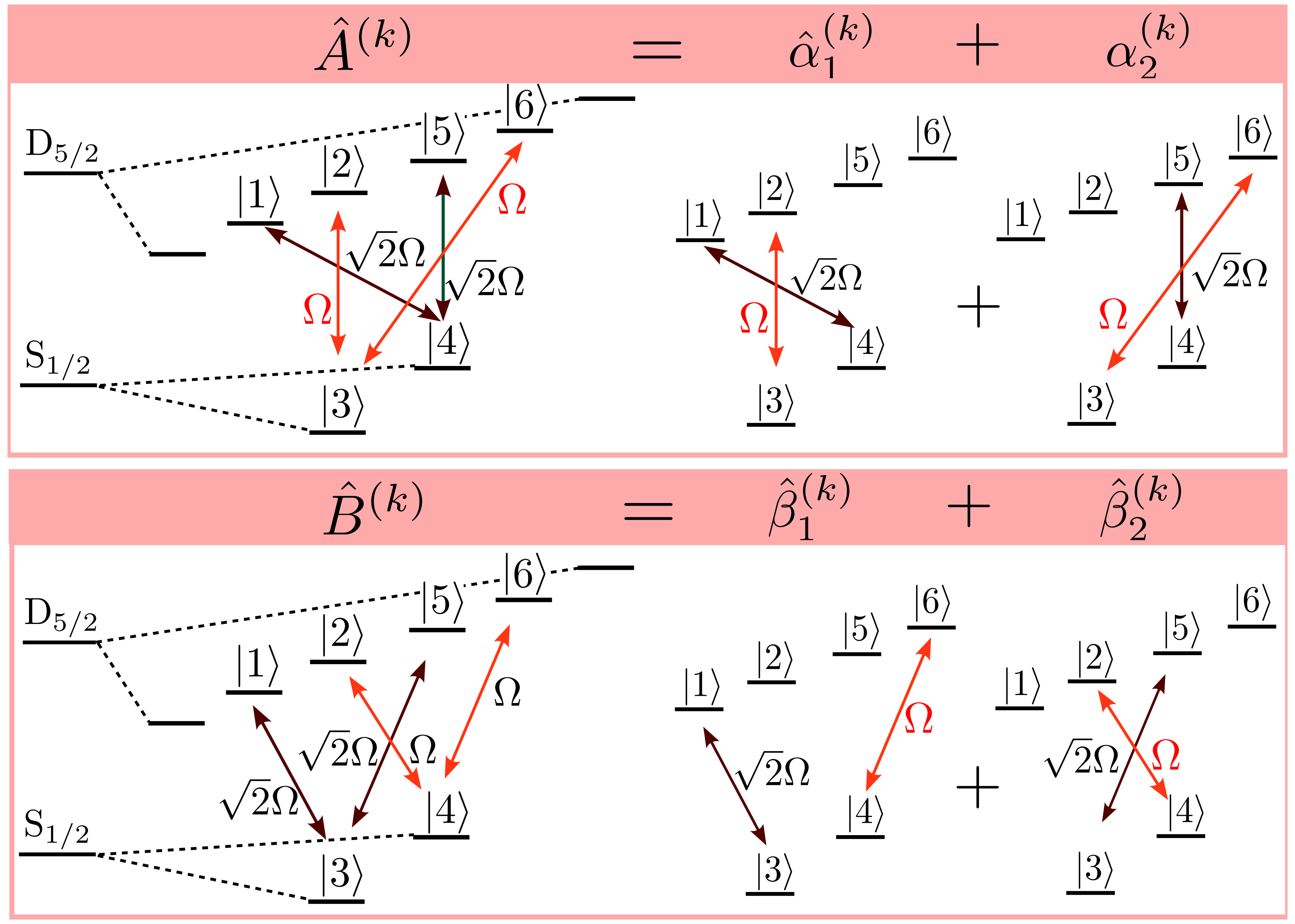
6 SU(2) Gauge Invariant States (color singlets)
6-QuDit Model on a Quantum Computer

GAUSS LAW AND DEFERMIONIZATION



6 SU(2) GAUGE INVARIANT STATES (COLOR SINGLETS)

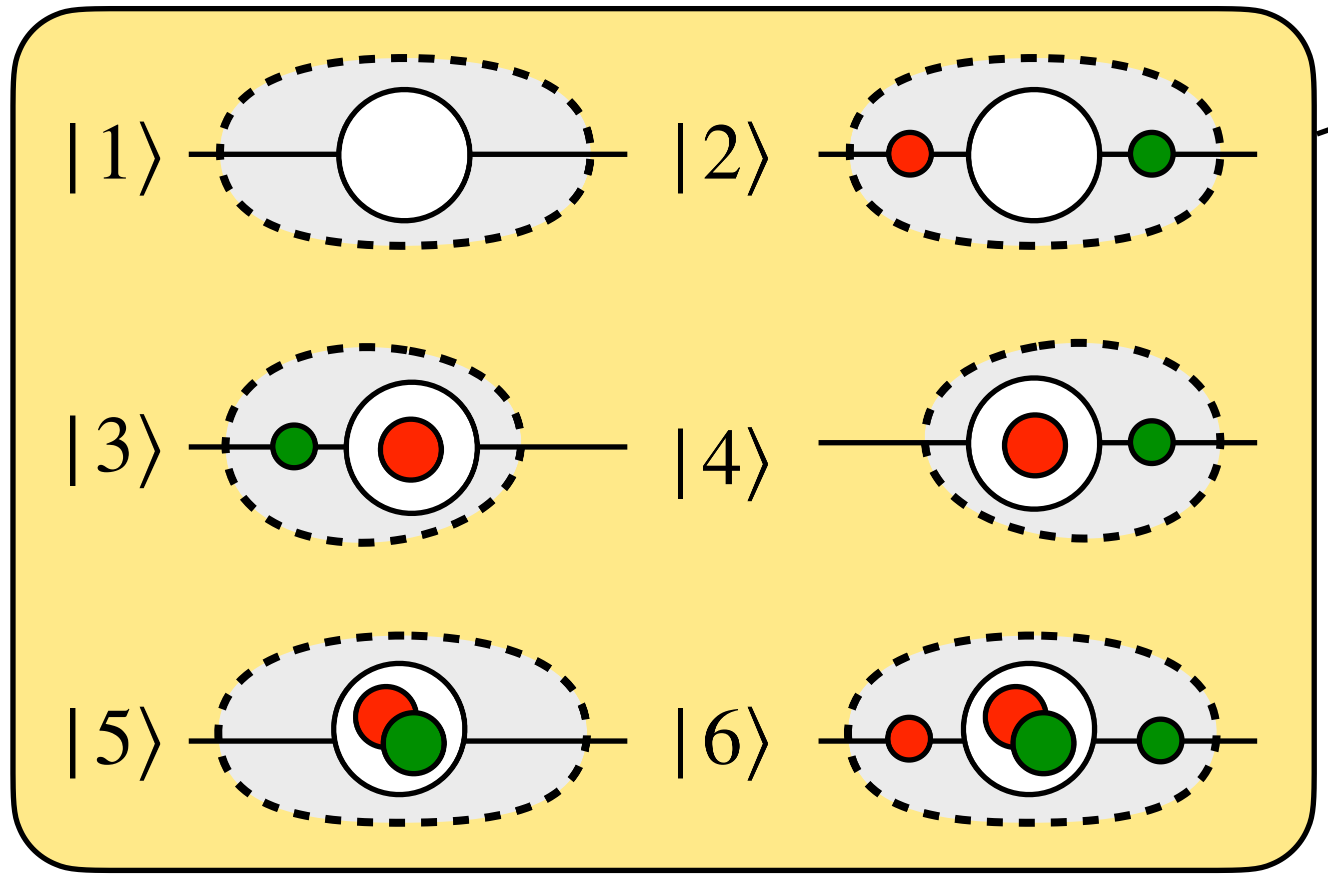
6-QUDIT MODEL ON A QUANTUM COMPUTER



$^{40}Ca^{+}$

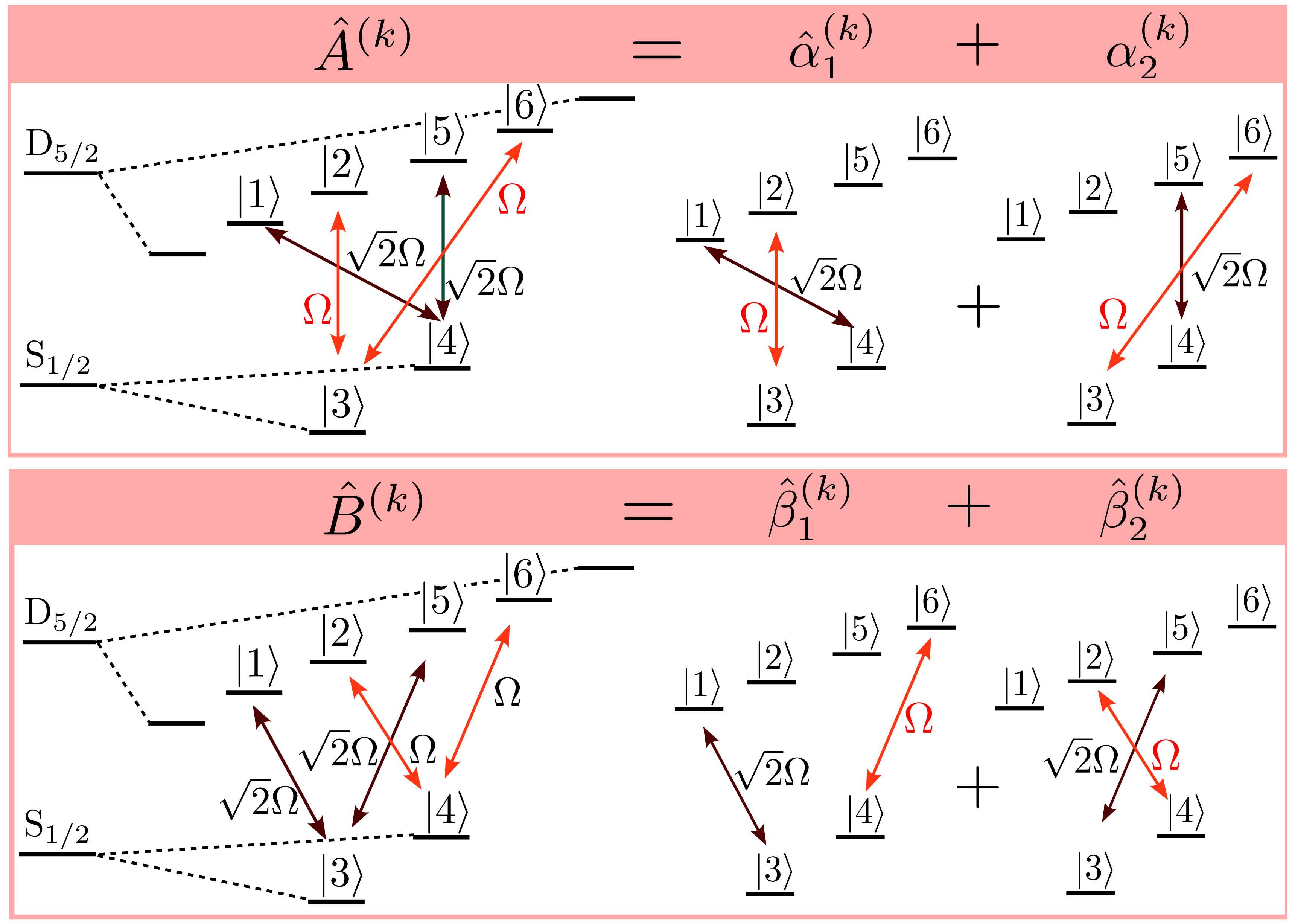
CALAJO' ET AL, ARXIV:2402.07987 (2024)

GAUSS LAW AND DEFERMIONIZATION



6 SU(2) GAUGE INVARIANT STATES (COLOR SINGLETS)
6-QUDIT MODEL ON A QUANTUM COMPUTER

$^{40}Ca^+$



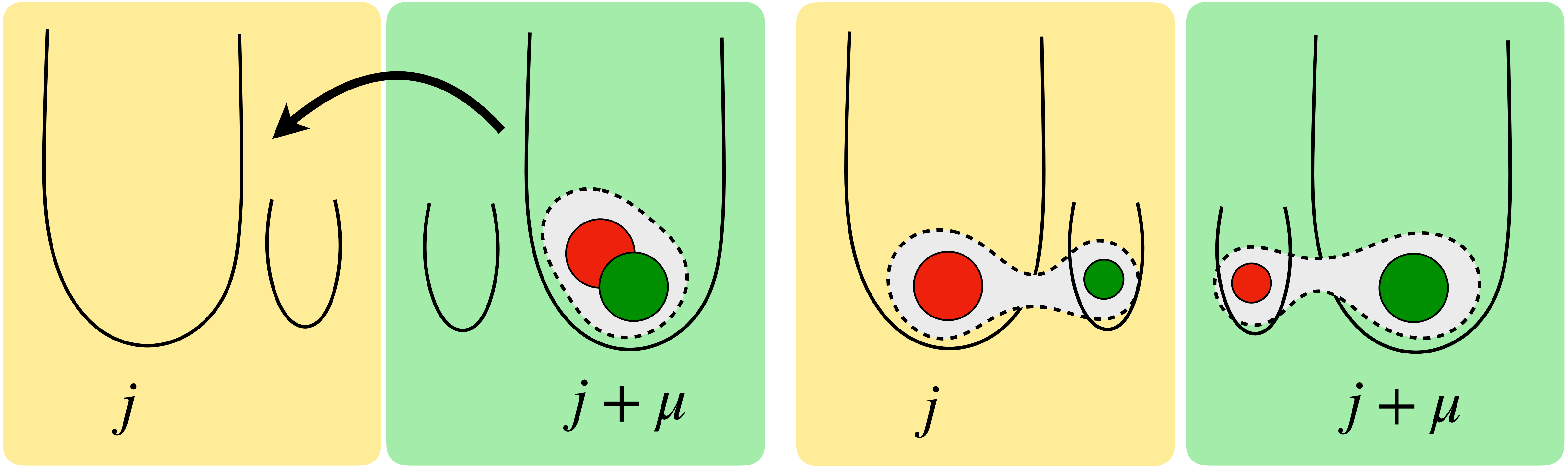
CALAJO' ET AL, ARXIV:2402.07987 (2024)

ZOAHR & CIRAC, PRB 98, 075119 (2018)

ZOAHR & CIRAC, PRD 99, 114511 (2019)

EVERY HAMILTONIAN TERM IS MADE OF AN **EVEN NUMBER OF FERMION OPERATORS**:
WE GOT A **BOSONIC** THEORY!

$$\psi^\dagger_{j,r}\, U^{r,r}_{j,j+\mu}\, \psi_{j+\mu,r} \longrightarrow \psi^\dagger_{j,r}\, \zeta^r_{j,\mu}\; \zeta^{r\dagger}_{j+\mu,-\mu}\, \psi_{j+\mu,r}$$



EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

NON-INTEGRABLE MODEL OF N BODY SYSTEM: WITH NON-DEGENERATE SPECTRUM

$$H = \sum_{\alpha} E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_{\alpha} C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

NON-INTEGRABLE MODEL OF N BODY SYSTEM: WITH NON-DEGENERATE SPECTRUM

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

HYPOTHESIS II

$$O_{\alpha\alpha} = \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{\text{thm}} + \Delta_\alpha$$

$$\bar{\Delta}_\alpha = 0 \qquad \Delta_\alpha^2 \sim \langle \hat{O}^2 \rangle_{\text{MC},\alpha} e^{-S(E)}$$

$O_{\alpha\alpha}$ IS THERMAL ~ CONSTANT VALUE IN A SMALL WINDOW

$\forall \alpha$ with $|E_\alpha - \bar{E}| < \Delta E$

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

Hypothesis I

Non-integrable model of N body system: with non-degenerate spectrum

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

Hypothesis II

$$O_{\alpha\alpha} = \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{\text{thm}} + \Delta_\alpha$$

$$\bar{\Delta}_\alpha = 0 \qquad \Delta_\alpha^2 \sim \langle \hat{O}^2 \rangle_{\text{MC},\alpha} e^{-S(E)}$$

$O_{\alpha\alpha}$ is Thermal ~ constant value in a small window

$\forall \alpha$ with $|E_\alpha - \bar{E}| < \Delta E$

Consequence

$$\langle O(t) \rangle = \sum_\alpha^N |c_\alpha|^2 O_{\alpha\alpha} \sim \langle O \rangle_{\text{thm}} \sum_\alpha^N |c_\alpha|^2 = \langle O \rangle_{\text{thm}}$$

$$\langle O \rangle_{\text{MC}} = \sum_\alpha^{N_{E,\Delta E}} \frac{O_{\alpha\alpha}}{N_{E,\Delta E}} \sim \langle O \rangle_{\text{thm}} \sum_\alpha^{N_{E,\Delta E}} \frac{1}{N_{E,\Delta E}} = \langle O \rangle_{\text{thm}}$$

Diagonal = Microcanonical

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

NON-INTEGRABLE MODEL OF N BODY SYSTEM: WITH NON-DEGENERATE SPECTRUM

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

HYPOTHESIS II

$$O_{\alpha\alpha} = \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{\text{thm}} + \Delta_\alpha$$

$$\bar{\Delta}_\alpha = 0 \qquad \Delta_\alpha^2 \sim \langle \hat{O}^2 \rangle_{\text{MC},\alpha} e^{-S(E)}$$

$O_{\alpha\alpha}$ IS THERMAL ~ CONSTANT VALUE IN A SMALL WINDOW

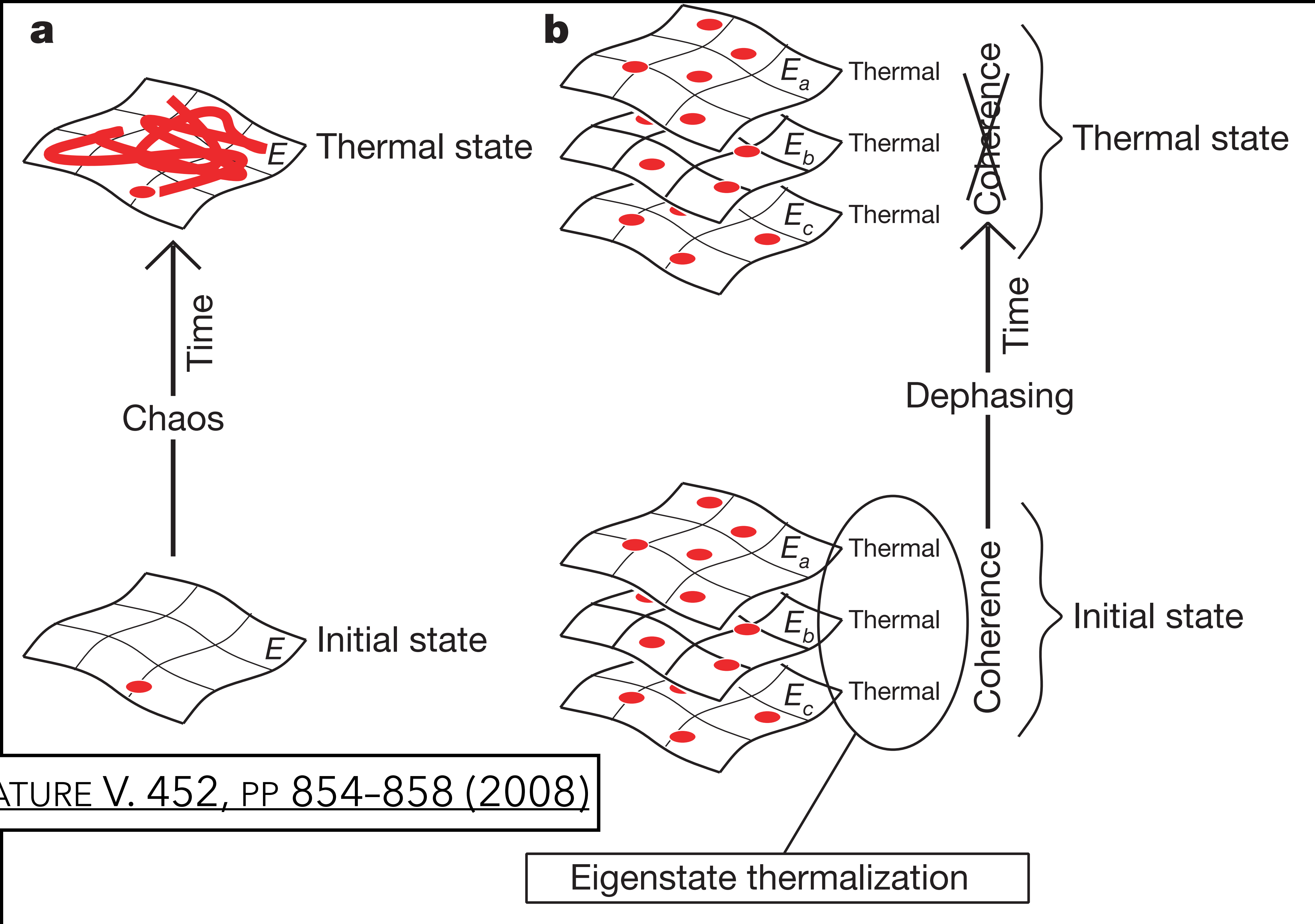
$\forall \alpha$ with $|E_\alpha - \bar{E}| < \Delta E$

CONSEQUENCE

$$\langle O(t) \rangle = \sum_\alpha^N |c_\alpha|^2 O_{\alpha\alpha} \sim \langle O \rangle_{\text{thm}} \sum_\alpha^N |c_\alpha|^2 = \langle O \rangle_{\text{thm}}$$

$$\langle O \rangle_{\text{MC}} = \sum_\alpha^{N_{E,\Delta E}} \frac{O_{\alpha\alpha}}{N_{E,\Delta E}} \sim \langle O \rangle_{\text{thm}} \sum_\alpha^{N_{E,\Delta E}} \frac{1}{N_{E,\Delta E}} = \langle O \rangle_{\text{thm}}$$

DIAGONAL = MICROCANONICAL



NATURE V. 452, PP 854-858 (2008)

HAMILTONIAN LATTICE GAUGE THEORIES

General Dirac Hamiltonian:

$$H = -i \int \psi^\dagger \gamma^k (\partial_k - gA_k)\psi + \int \psi^\dagger \gamma^0 m \psi - \int \left[E^2 + B^2 \right]$$

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

PURE GAUGE TERM

$$H = -i \int \psi^\dagger \gamma^k (\partial_k - gA_k)\psi + \int \psi^\dagger \gamma^0 m \psi - \int [E^2 + B^2]$$

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

MASS TERM

PURE GAUGE TERM

$$H = -i\int \psi^\dagger \gamma^k (\partial_k - gA_k)\psi + \int \psi^\dagger \gamma^0 m\psi - \int [E^2 + B^2]$$

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

HAMILTONIAN LATTICE GAUGE THEORIES

General Dirac Hamiltonian:

Mass term

Pure gauge term

$$H = -i\int \psi^\dagger \gamma^k (\partial_k - gA_k)\psi + \int \psi^\dagger \gamma^0 m \psi - \int [E^2 + B^2]$$

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

Matter-gauge int.

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

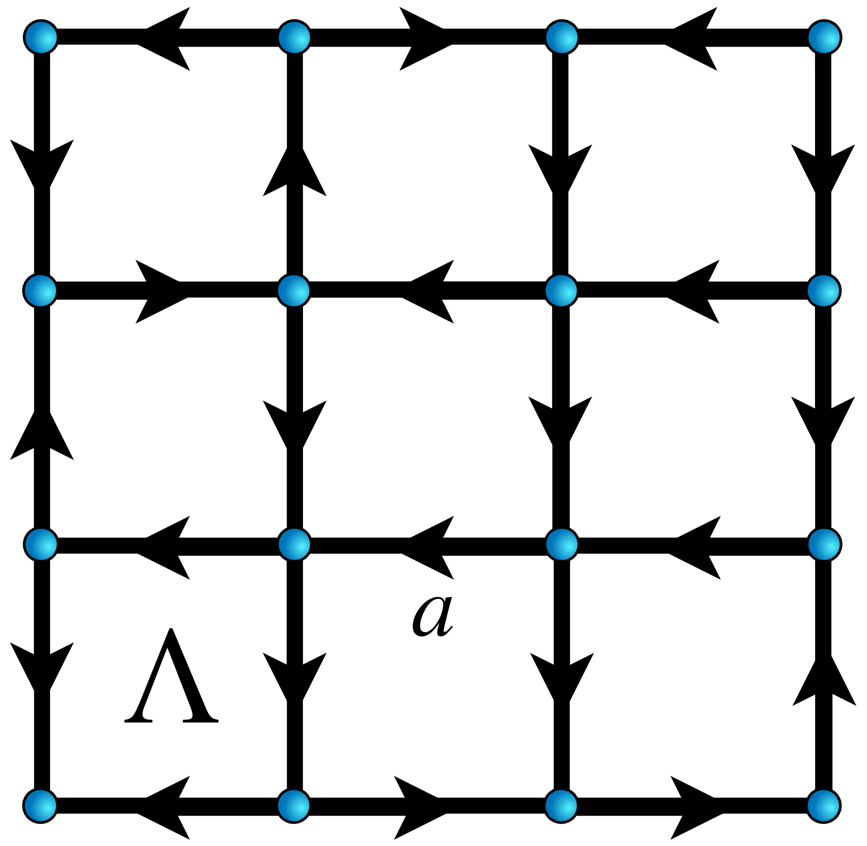
$$H = -i\int \psi^\dagger \gamma^k(\partial_k - gA_k)\psi + \int \psi^\dagger\gamma^0 m\psi - \int [E^2 + B^2]$$

MATTER-GAUGE INT. — MASS TERM — PURE GAUGE TERM

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

SPATIAL DISCRETIZATION: $x = ja,\ j \in \Lambda$. TIME IS CONTINUOUS



HAMILTONIAN LATTICE GAUGE THEORIES

General Dirac Hamiltonian:

Mass Term — Pure Gauge Term

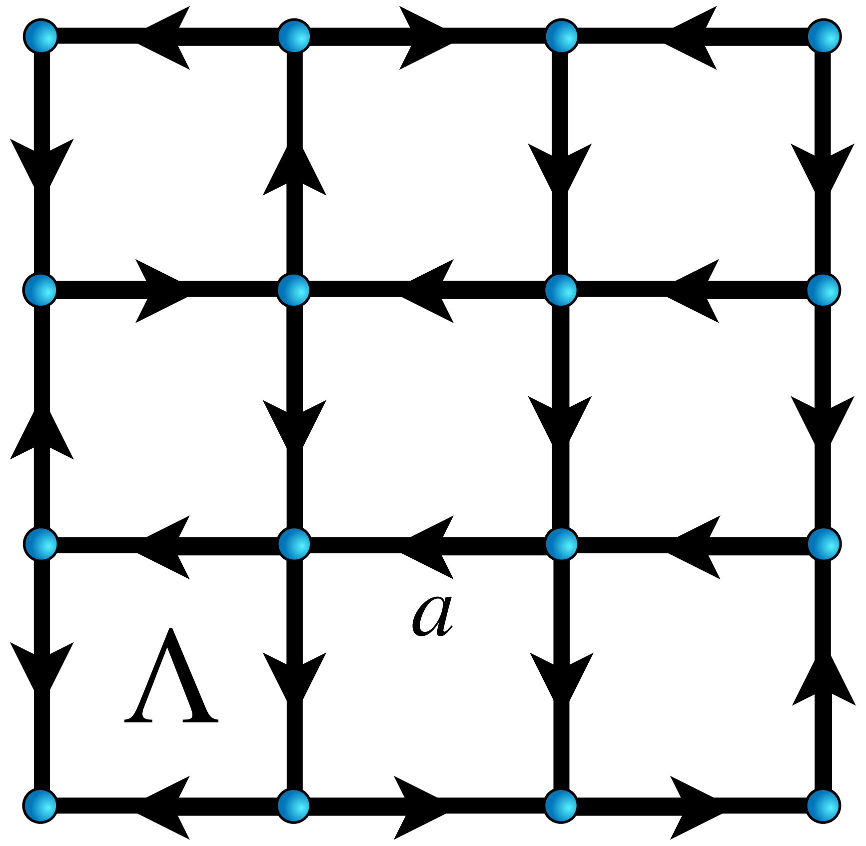
$$H = -i\int \psi^\dagger \gamma^k(\partial_k - gA_k)\psi + \int \psi^\dagger\gamma^0 m\psi - \int\left[E^2 + B^2\right]$$

Matter-Gauge int.

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

Spatial Discretization: $x = ja,\, j \in \Lambda$. Time is continuous



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\,\psi_j^\dagger\, U_{j,j+\mu}\,\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j(-1)^j\,\psi_j^\dagger\psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2c\hbar}{2a}\sum_j\left(E^2_{j,\rightarrow} + E^2_{j,\uparrow}\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\mathfrak{Re}\begin{pmatrix} \ulcorner & U^\dagger & \urcorner \\ U^\dagger & & U \\ \llcorner & U & \lrcorner \end{pmatrix}$$

HAMILTONIAN LATTICE GAUGE THEORIES

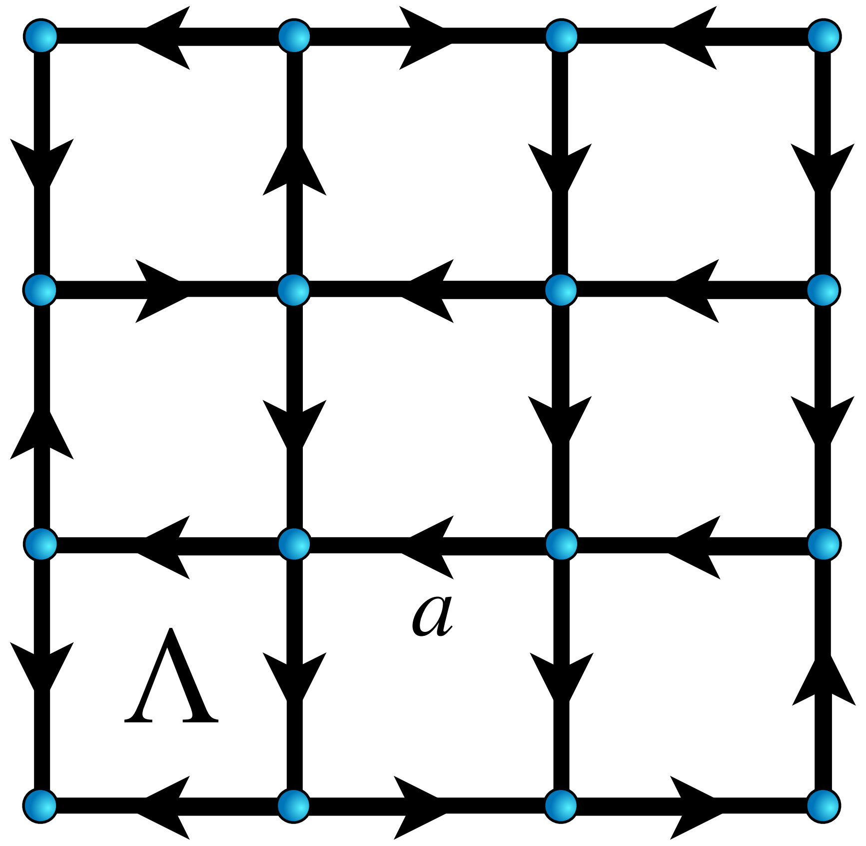
GENERAL DIRAC HAMILTONIAN:

$$H = -i\int \psi^\dagger \underbrace{\gamma^k(\partial_k - gA_k)}_{\text{Matter-Gauge int.}}\psi + \overbrace{\int \psi^\dagger\gamma^0 m\psi}^{\text{Mass Term}} - \overbrace{\int [E^2 + B^2]}^{\text{Pure Gauge Term}}$$

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

SPATIAL DISCRETIZATION: $x = ja,\ j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\,\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j(-1)^j\,\psi_j^\dagger\psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2c\hbar}{2a}\sum_j\left(E_{j,\rightarrow}^2 + E_{j,\uparrow}^2\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\Re e\left(\begin{array}{ccc} \ulcorner & U^\dagger & \urcorner \\ U^\dagger & & U \\ \llcorner & U & \lrcorner \end{array}\right)$$

PARALLEL TRANSPORT
$U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

$$H = -i \int \psi^\dagger \gamma^k (\partial_k - g A_k) \psi + \int \psi^\dagger \gamma^0 m \psi - \int [E^2 + B^2]$$

MATTER-GAUGE INT.

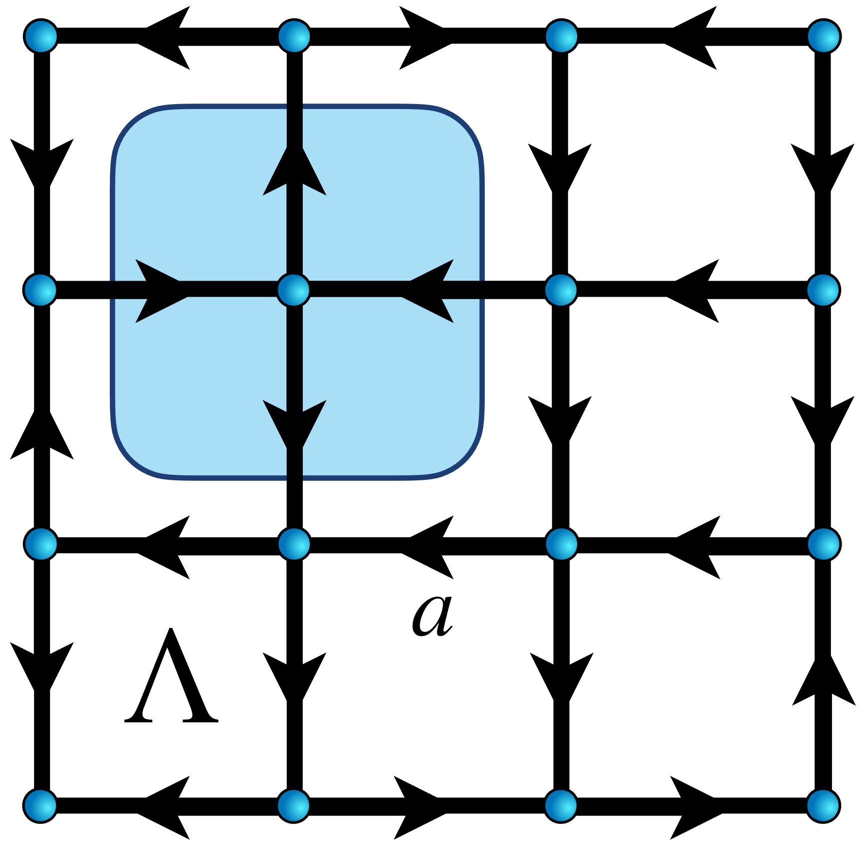
MASS TERM

PURE GAUGE TERM

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

SPATIAL DISCRETIZATION: $x = ja,\ j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a} \sum_{j,\mu} \left[e^{i\varphi(j,\mu)}\, \psi_j^\dagger U_{j,j+\mu} \psi_{j+\mu} + \text{H.c.} \right] + m_0 c^2 \sum_j (-1)^j \psi_j^\dagger \psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2 c\hbar}{2a} \sum_j \left(E^2_{j,\rightarrow} + E^2_{j,\uparrow} \right) - \frac{c\hbar}{2ag^2} \sum_{\square} \Re e \begin{pmatrix} & U^\dagger & \\ U^\dagger & & U \\ & U & \end{pmatrix}$$

PARALLEL TRANSPORT
$U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

ELECTRIC CONTRIBUTION

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

MASS TERM — PURE GAUGE TERM

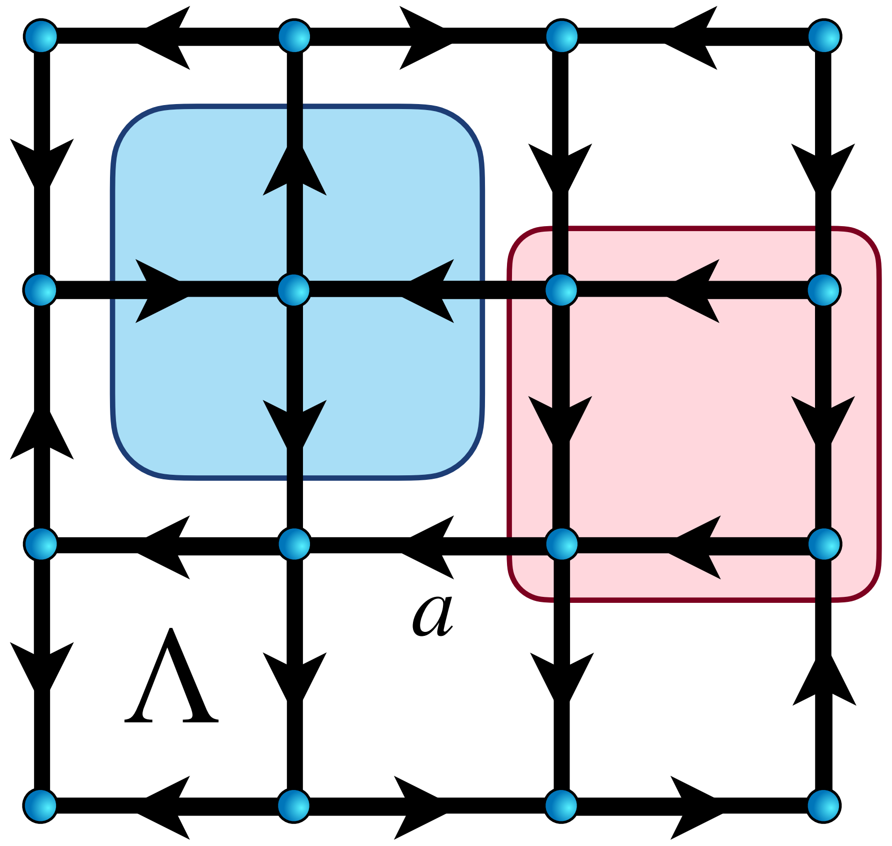
$$H = -i\int \psi^\dagger \gamma^k(\partial_k - gA_k)\psi + \int \psi^\dagger\gamma^0 m\psi - \int [E^2 + B^2]$$

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

MATTER-GAUGE INT.

SPATIAL DISCRETIZATION: $x = ja,\ j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\,\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j(-1)^j\psi_j^\dagger\psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2c\hbar}{2a}\sum_j\left(E^2_{j,\rightarrow} + E^2_{j,\uparrow}\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\Re e\left(\begin{matrix} & U^\dagger & \\ U^\dagger & & U \\ & U & \end{matrix}\right)$$

PARALLEL TRANSPORT $U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

ELECTRIC CONTRIBUTION

MAGNETIC CONTRIBUTION

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

$$H = -i\int \psi^{\dagger}\gamma^{k}(\partial_k - gA_k)\psi + \int \psi^{\dagger}\gamma^{0}m\psi - \int [E^2 + B^2]$$

MATTER-GAUGE INT.

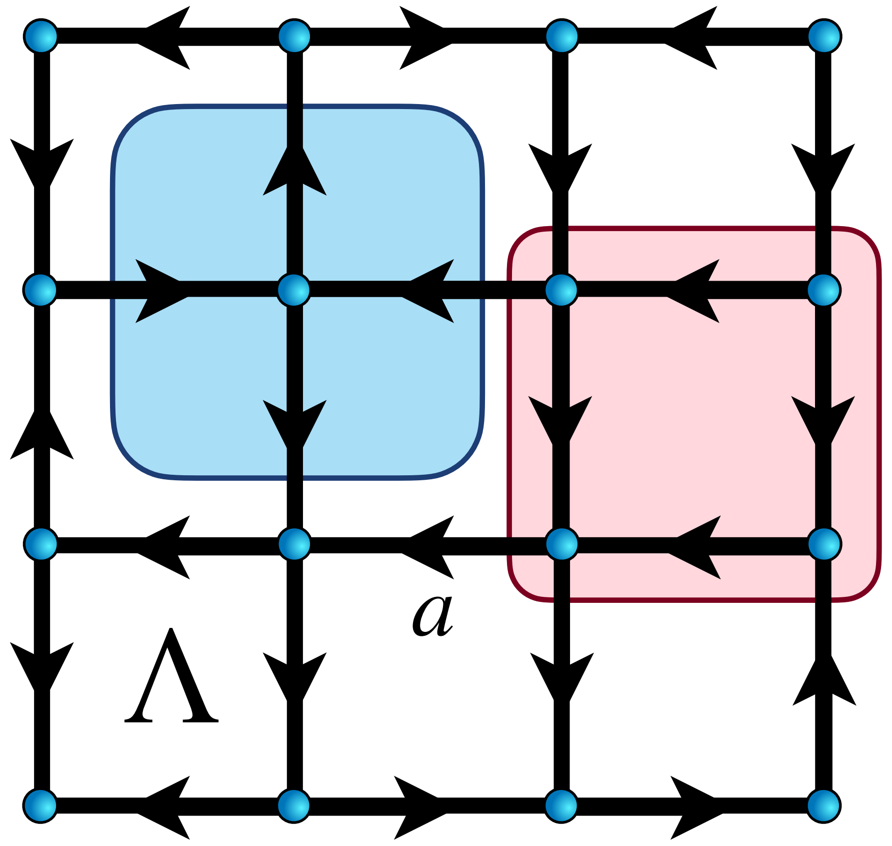
MASS TERM

PURE GAUGE TERM

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

SPATIAL DISCRETIZATION: $x = ja$, $j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\psi_j^{\dagger}U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j (-1)^j\psi_j^{\dagger}\psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2c\hbar}{2a}\sum_j\left(E_{j,\rightarrow}^2 + E_{j,\uparrow}^2\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\Re e\left(\begin{matrix} & U^{\dagger} & \\ U^{\dagger} & & U \\ & U & \end{matrix}\right)$$

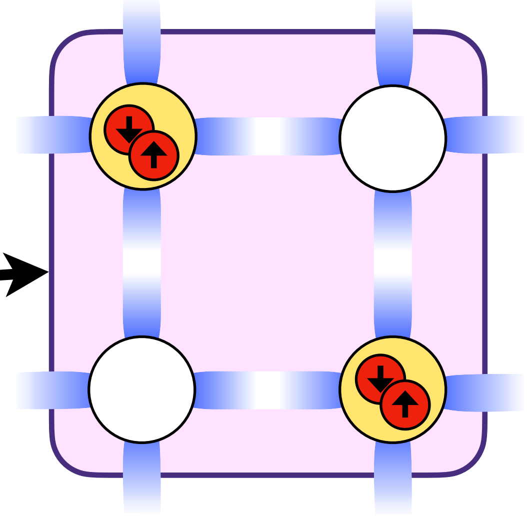
PARALLEL TRANSPORT $U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

ELECTRIC CONTRIBUTION

MAGNETIC CONTRIBUTION

REGULARIZING FERMIONS VIA STAGGERED FERMIONS

SUSSKIND (1977)



HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

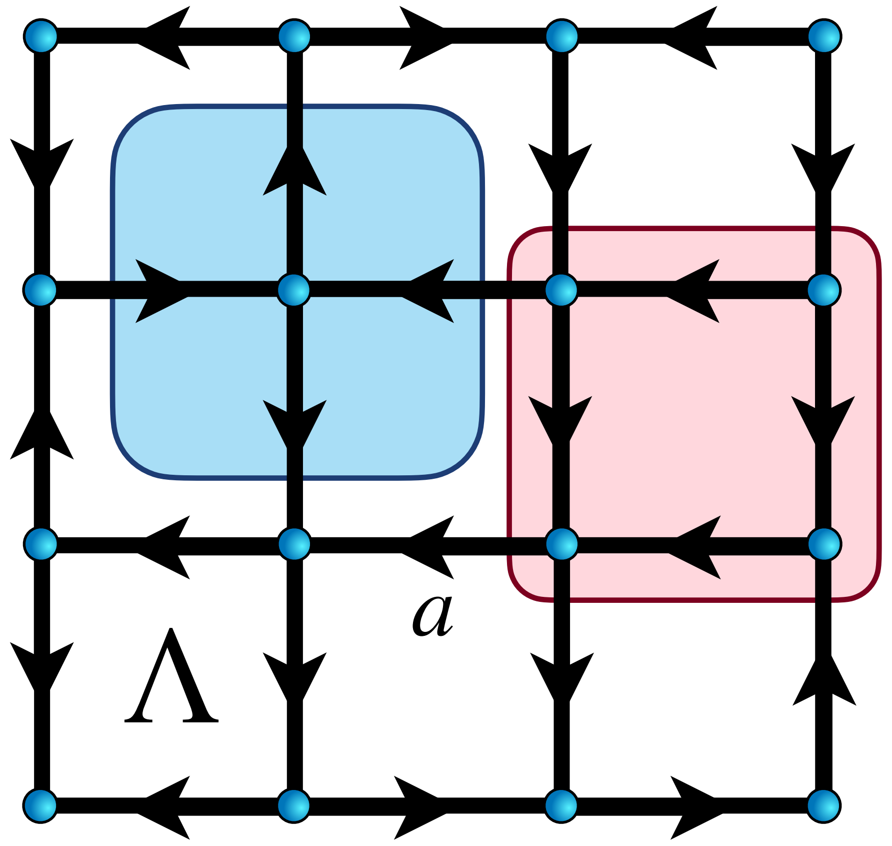
$$H = -i\int \psi^\dagger \gamma^k(\partial_k - gA_k)\psi + \int \psi^\dagger\gamma^0 m\psi - \int [E^2 + B^2]$$

MATTER-GAUGE INT. — MASS TERM — PURE GAUGE TERM

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

SPATIAL DISCRETIZATION: $x = ja$, $j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j (-1)^j\psi_j^\dagger\psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2c\hbar}{2a}\sum_j\left(E_{j,\rightarrow}^2 + E_{j,\uparrow}^2\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\mathfrak{Re}\begin{pmatrix} & U^\dagger & \\ U^\dagger & & U \\ & U & \end{pmatrix}$$

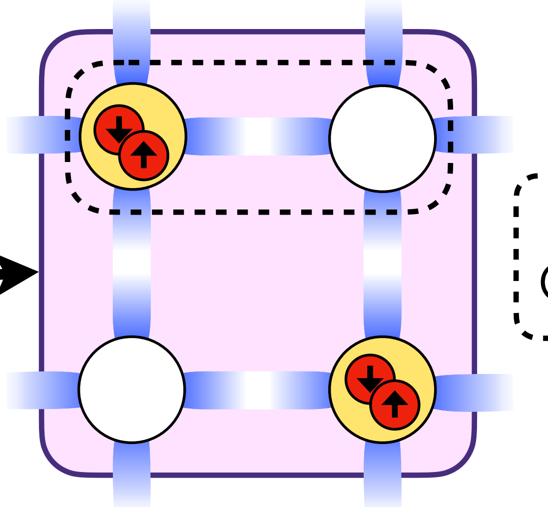
PARALLEL TRANSPORT $U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

ELECTRIC CONTRIBUTION

MAGNETIC CONTRIBUTION

REGULARIZING FERMIONS VIA STAGGERED FERMIONS

SUSSKIND (1977)



4 SPINOR COMPONENTS

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

MASS TERM — PURE GAUGE TERM

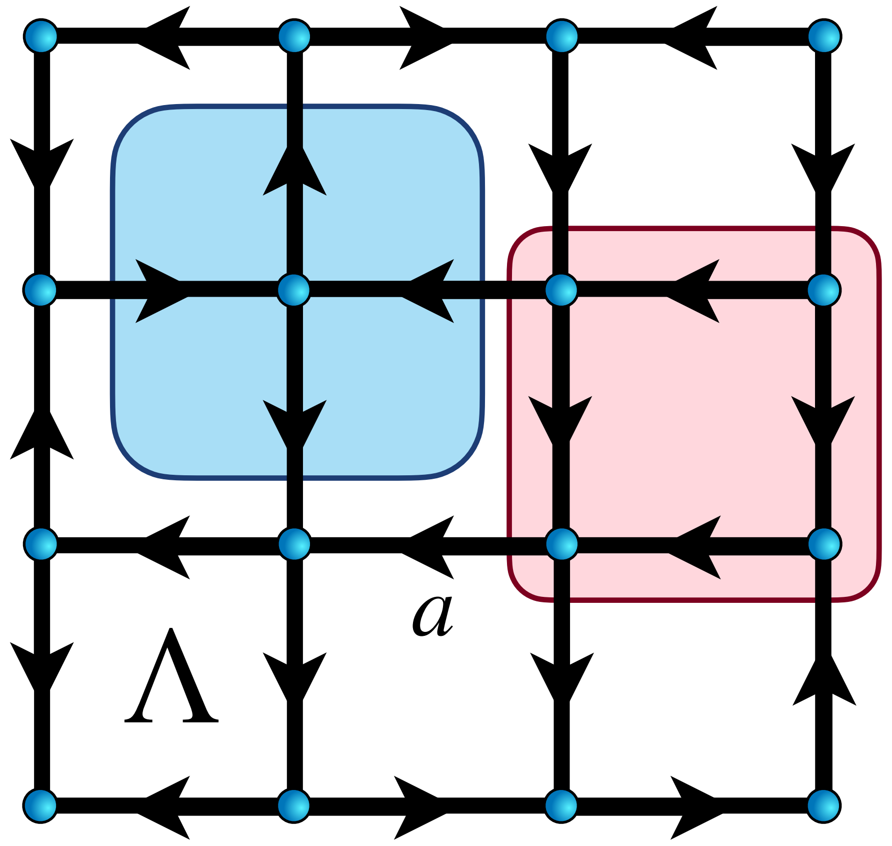
$$H = -i\int \psi^\dagger \gamma^k (\partial_k - gA_k)\psi + \int \psi^\dagger \gamma^0 m \psi - \int [E^2 + B^2]$$

MATTER-GAUGE INT.

$E_i = \dot{A}_i$

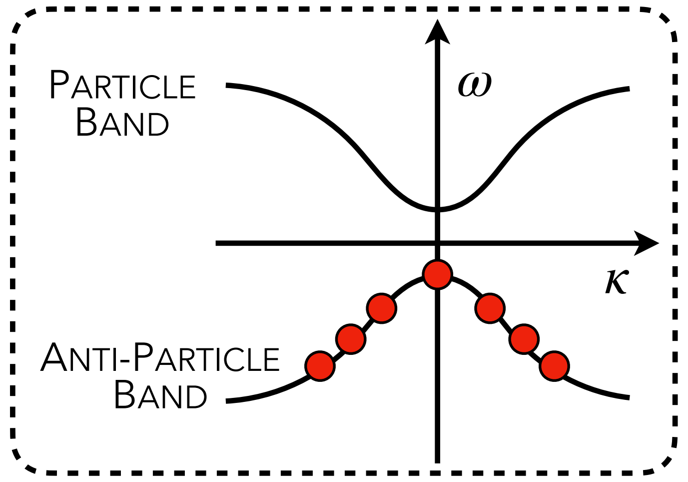
$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

SPATIAL DISCRETIZATION: $x = ja,\ j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j (-1)^j \psi_j^\dagger\psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2c\hbar}{2a}\sum_j\left(E_{j,\rightarrow}^2 + E_{j,\uparrow}^2\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\Re e\begin{pmatrix} & U^\dagger & \\ U^\dagger & & U \\ & U & \end{pmatrix}$$



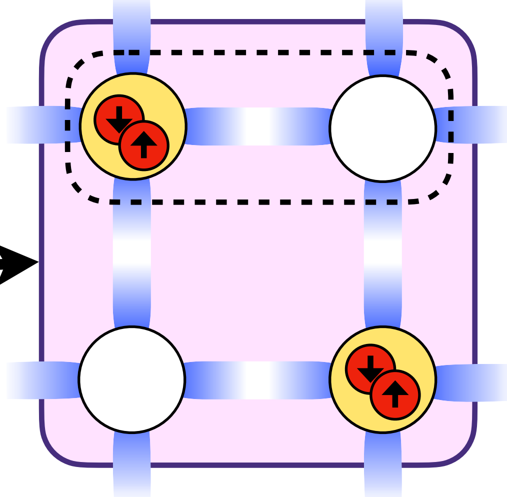
PARALLEL TRANSPORT $U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

ELECTRIC CONTRIBUTION

MAGNETIC CONTRIBUTION

REGULARIZING FERMIONS VIA STAGGERED FERMIONS

SUSSKIND (1977)



4 SPINOR COMPONENTS

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

MASS TERM — PURE GAUGE TERM

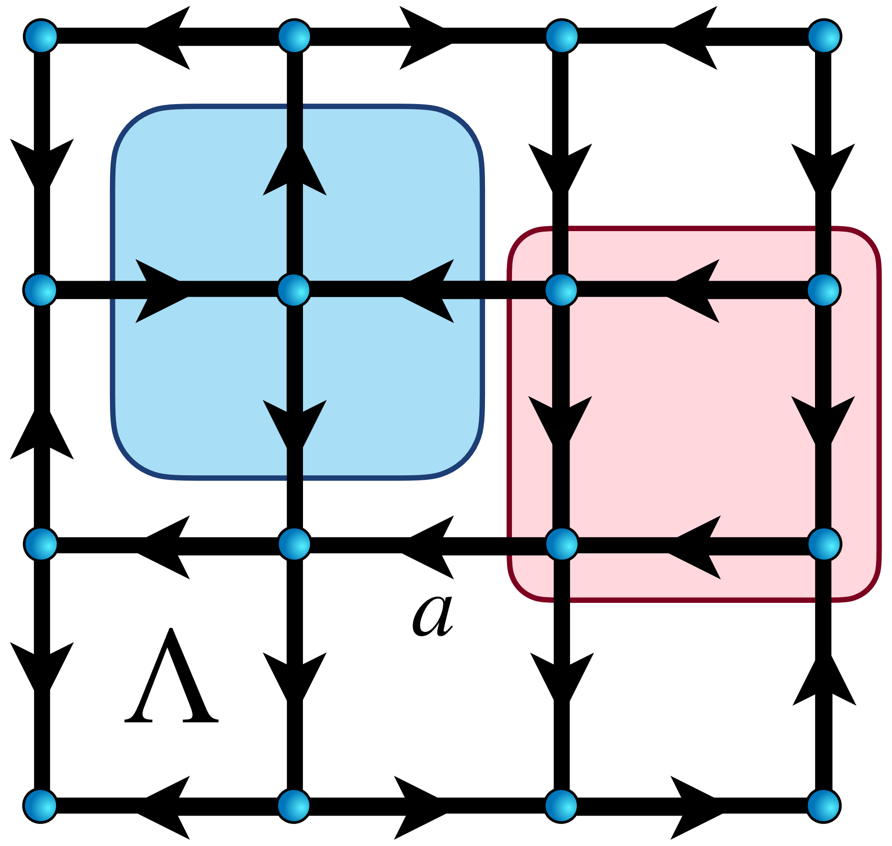
$$H = -i\int \psi^\dagger \gamma^k (\partial_k - gA_k)\psi + \int \psi^\dagger \gamma^0 m \psi - \int [E^2 + B^2]$$

MATTER-GAUGE INT.

$E_i = \dot{A}_i$

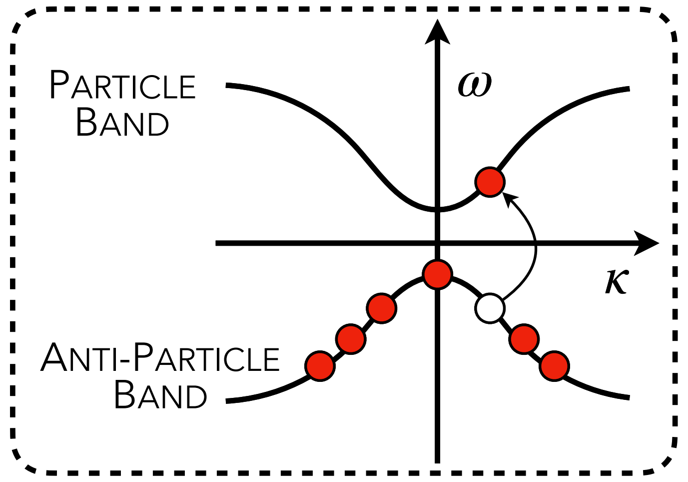
$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

SPATIAL DISCRETIZATION: $x = ja,\ j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j (-1)^j \psi_j^\dagger \psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2c\hbar}{2a}\sum_j\left(E^2_{j,\rightarrow} + E^2_{j,\uparrow}\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\Re e\begin{pmatrix} & U^\dagger & \\ U^\dagger & & U \\ & U & \end{pmatrix}$$



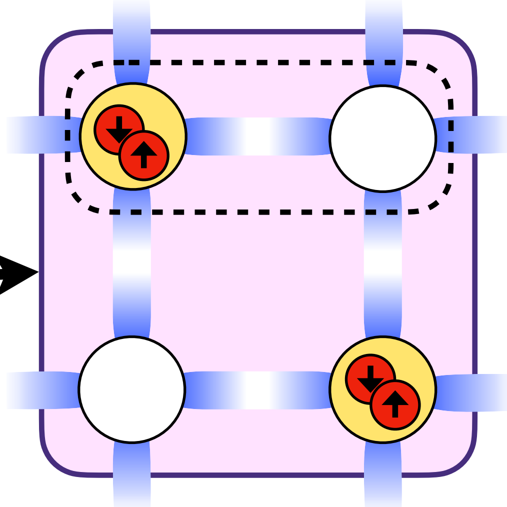
PARALLEL TRANSPORT $U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

ELECTRIC CONTRIBUTION

MAGNETIC CONTRIBUTION

REGULARIZING FERMIONS VIA STAGGERED FERMIONS

SUSSKIND (1977)



4 SPINOR COMPONENTS

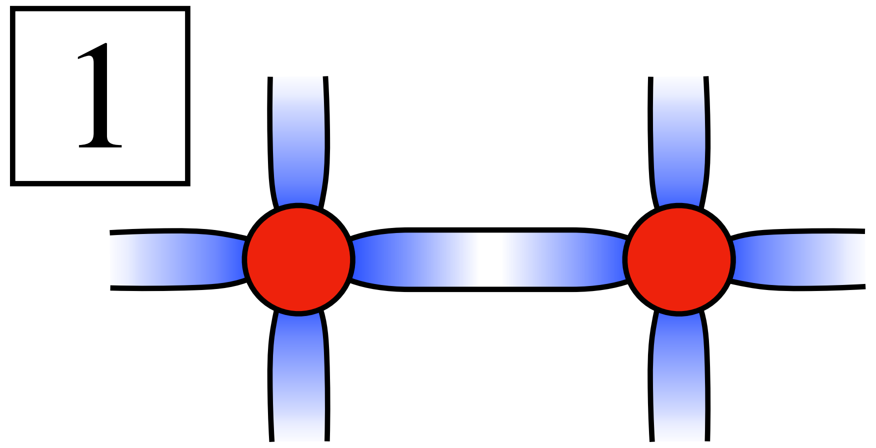
QUANTUM SIMULATIONS OF LGT

Chandrasekharan, Wiese (1997)

QUANTUM SIMULATIONS OF LGT

CHANDRASEKHARAN, WIESE (1997)

1. HILBERT SPACE: $\mathscr{H}_m \otimes \mathscr{H}_G$ WITH GAUGE-INVARIANT STATES



$\Psi = |\text{site}\rangle \otimes |\text{link}\rangle \otimes \dots$

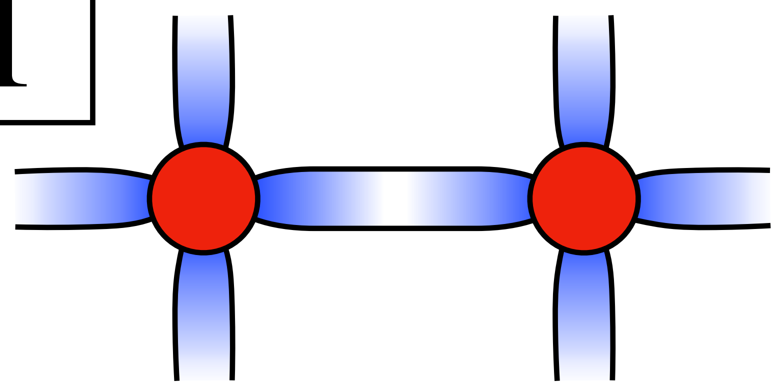
QUANTUM SIMULATIONS OF LGT

CHANDRASEKHARAN, WIESE (1997)

1. HILBERT SPACE: $\mathscr{H}_m \otimes \mathscr{H}_G$ WITH GAUGE-INVARIANT STATES

2. GAUGE D.O.F TO **SPIN-LIKE OPERATORS**: E.G. QED $(E, U) \rightarrow (S^z, S^-)$

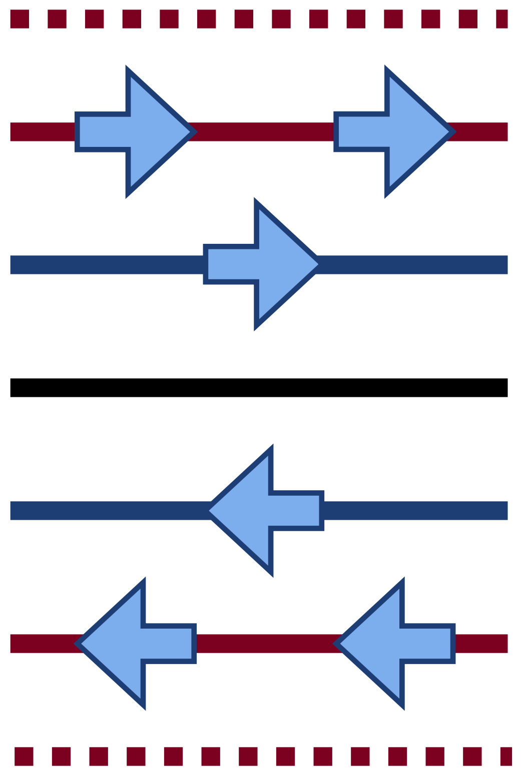
1



$\Psi = |\text{site}\rangle \otimes |\text{link}\rangle \otimes \dots$

2

$|\text{link}\rangle =$



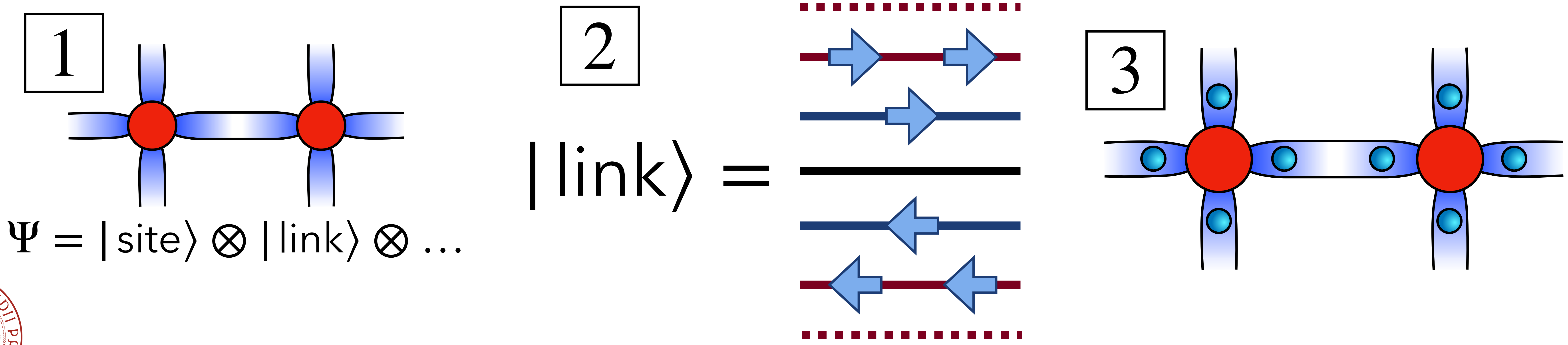
QUANTUM SIMULATIONS OF LGT

CHANDRASEKHARAN, WIESE (1997)

1. HILBERT SPACE: $\mathscr{H}_m \otimes \mathscr{H}_G$ WITH GAUGE-INVARIANT STATES

2. GAUGE D.O.F TO **SPIN-LIKE OPERATORS**: E.G. QED $(E, U) \to (S^z, S^-)$

3. DECOMPOSE GAUGE LINKS INTO PAIRS OF **FERMIONIC RISHONS**



QUANTUM SIMULATIONS OF LGT

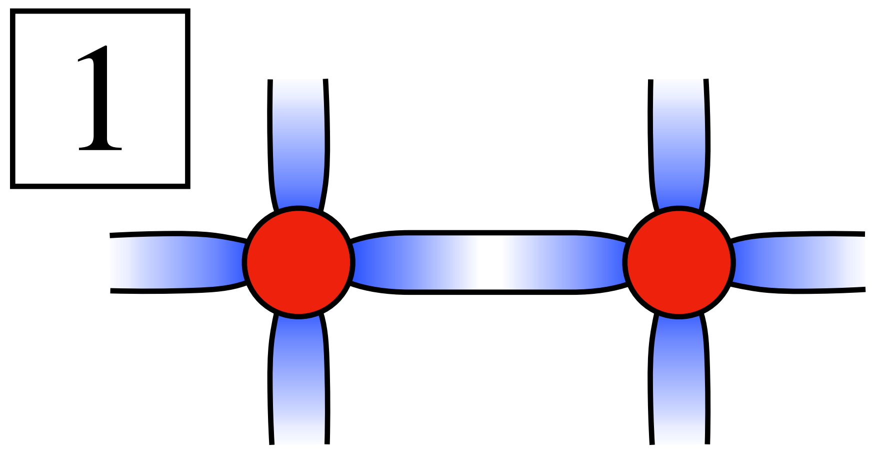
CHANDRASEKHARAN, WIESE (1997)

1. HILBERT SPACE: $\mathscr{H}_m \otimes \mathscr{H}_G$ WITH GAUGE-INVARIANT STATES

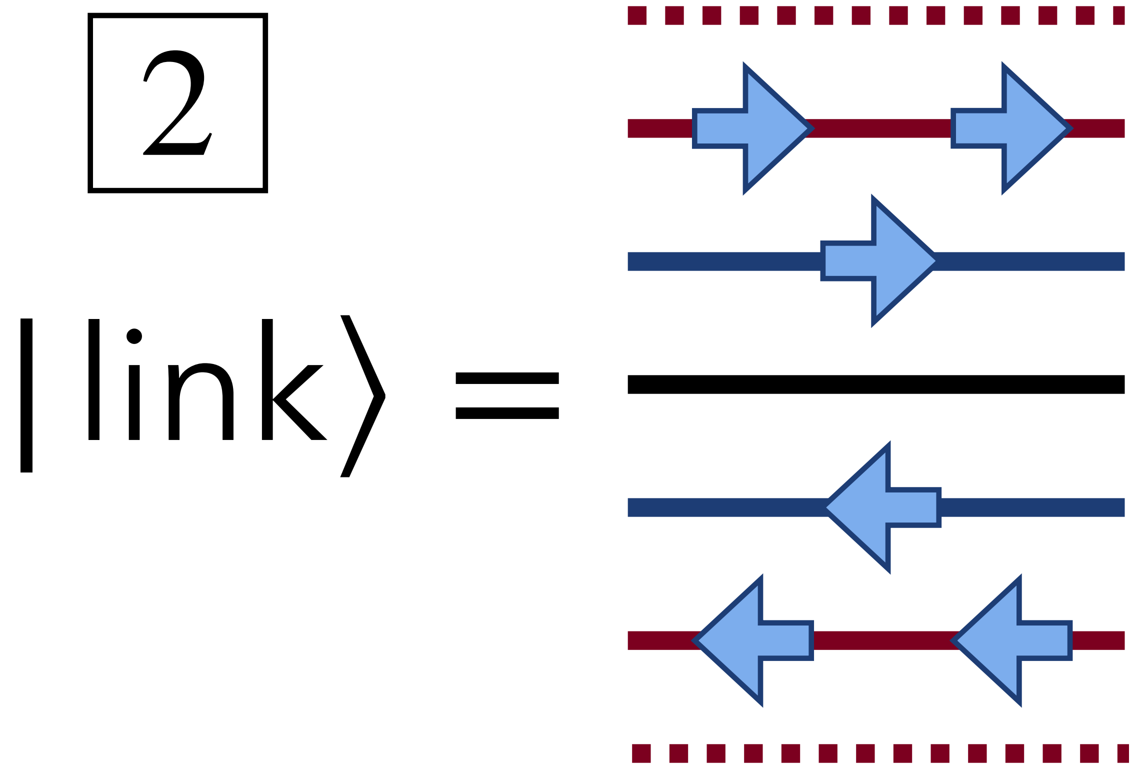
2. GAUGE D.O.F TO **SPIN-LIKE OPERATORS**: E.G. QED $(E, U) \rightarrow (S^z, S^-)$

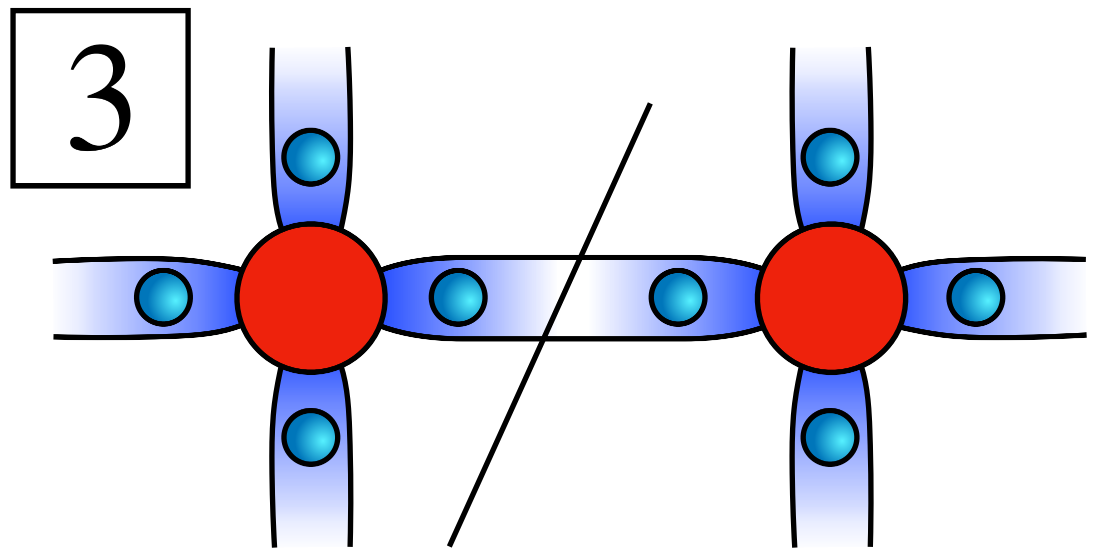
3. DECOMPOSE GAUGE LINKS INTO PAIRS OF **FERMIONIC RISHONS**

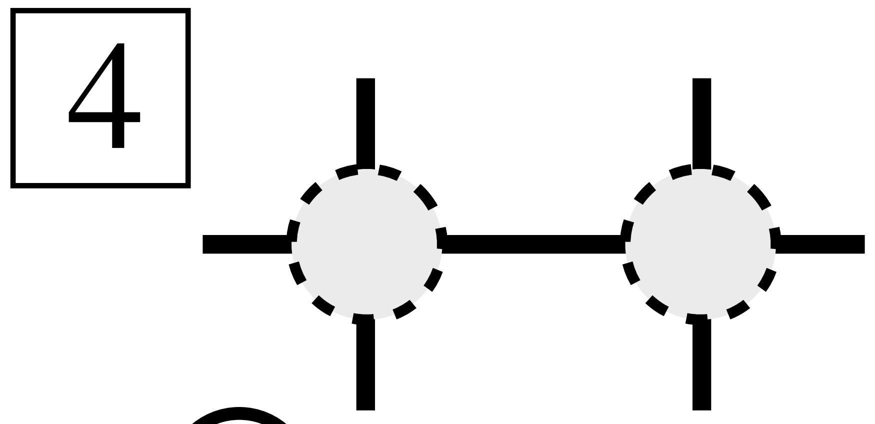
4. MERGE RISHONS & MATTER SITES INTO **BOSONIC GAUGE INVARIANT SITES**



$\Psi = |\text{site}\rangle \otimes |\text{link}\rangle \otimes \ldots$







$\Psi = \bigotimes_k |\text{dressed site}_k\rangle$

TRUNCATED SU(2) LINKS VIA RISHONS

Cataldi et al. arXiv:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

TRUNCATED SU(2) LINKS VIA RISHONS

Cataldi et al. arXiv:2307.09396 (2023)

The parallel transport in terms of fermionic rishons

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

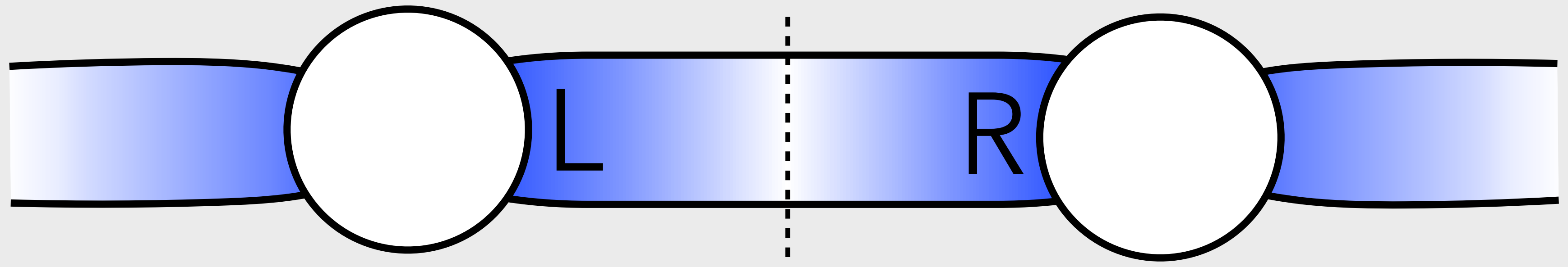
$$\zeta_r = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

Allowed link configurations satisfy

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) quasi-real representation

SU(2) Casimir operator



TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

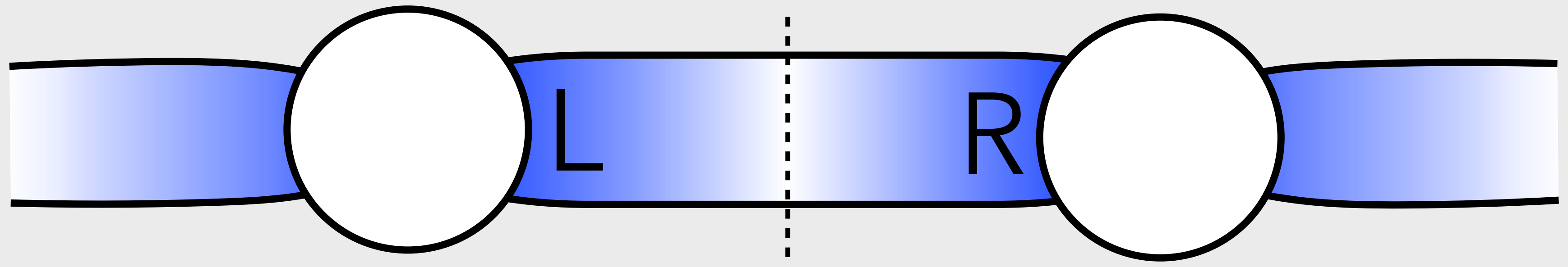
$$\zeta_{r} = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_{F} \qquad \zeta_{g} = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_{F}$$

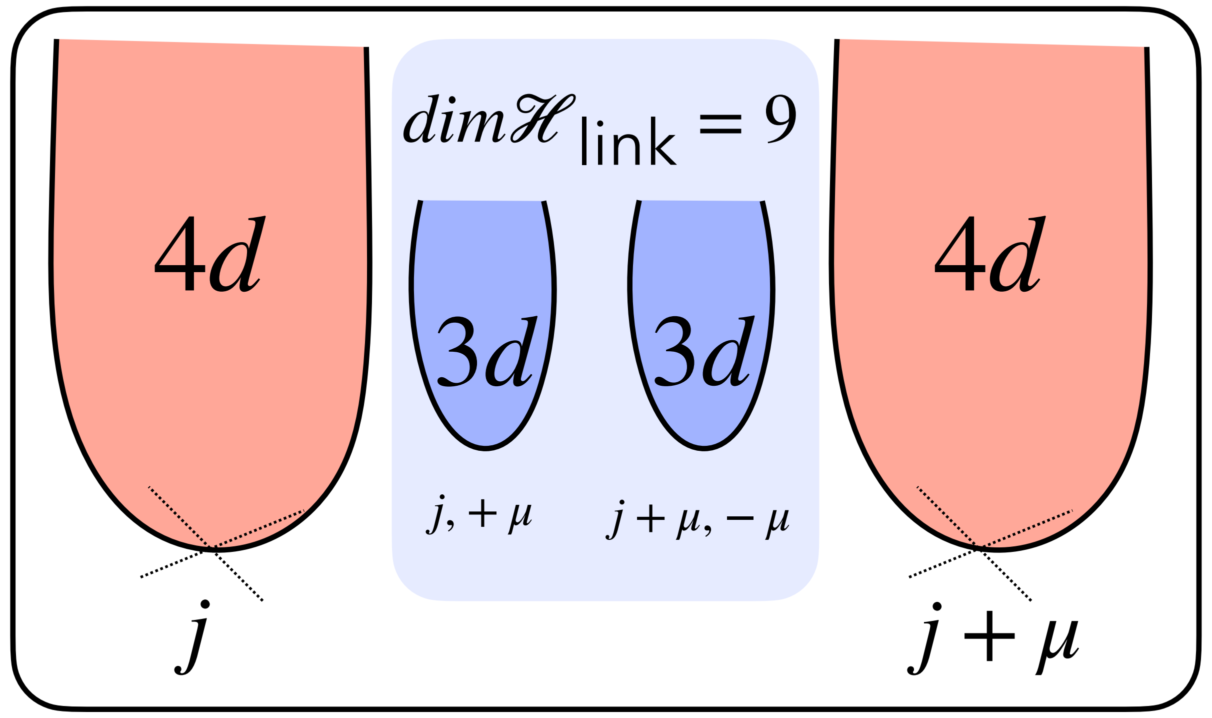
ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

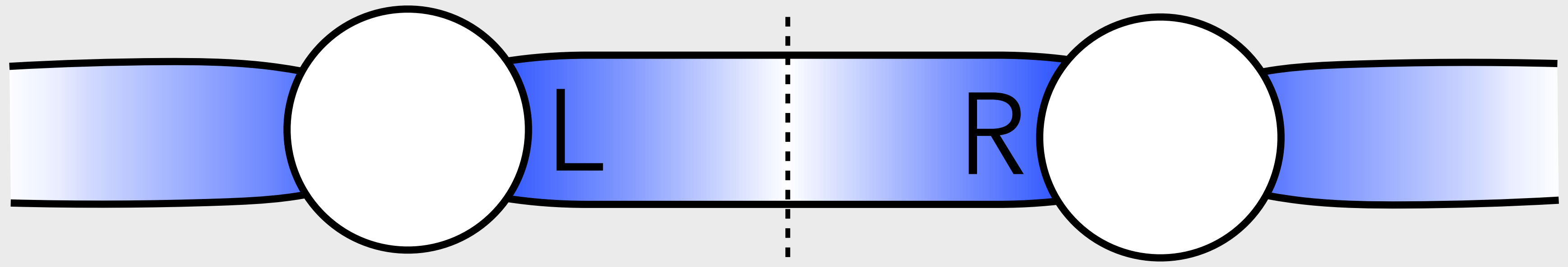
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

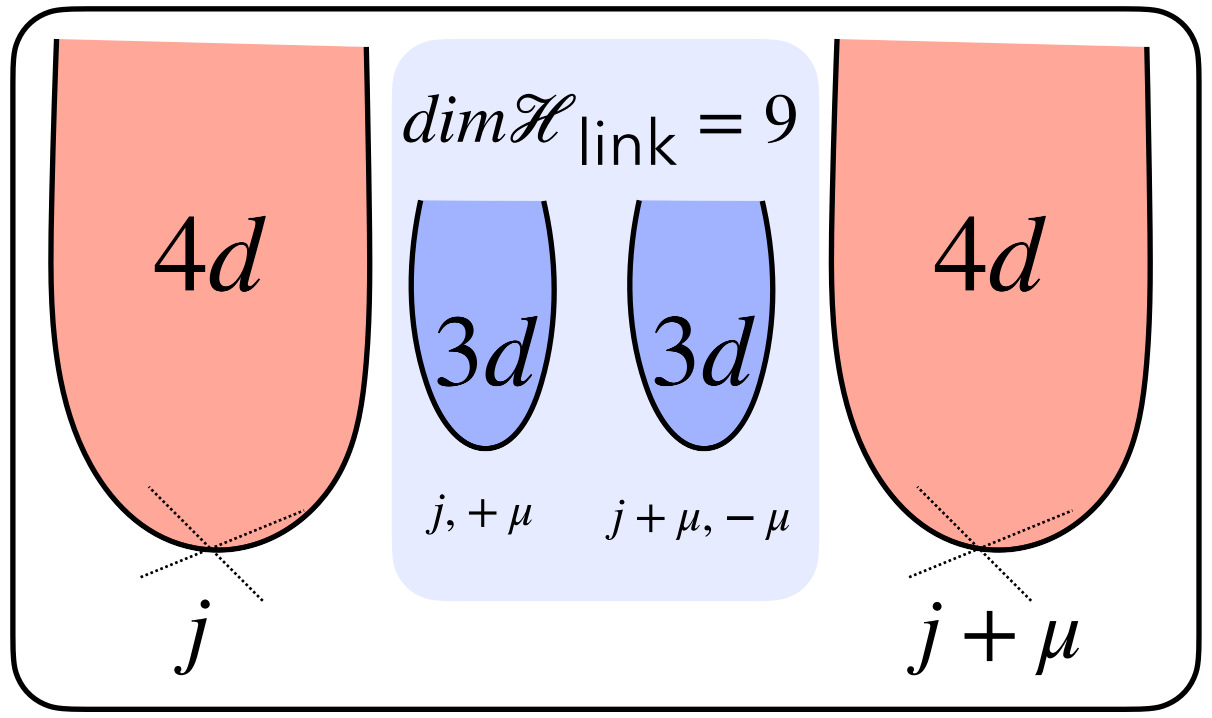
ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

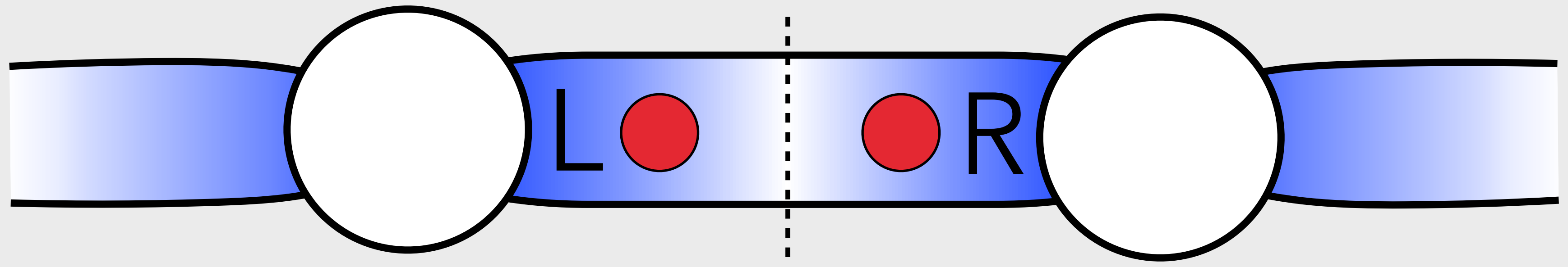
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

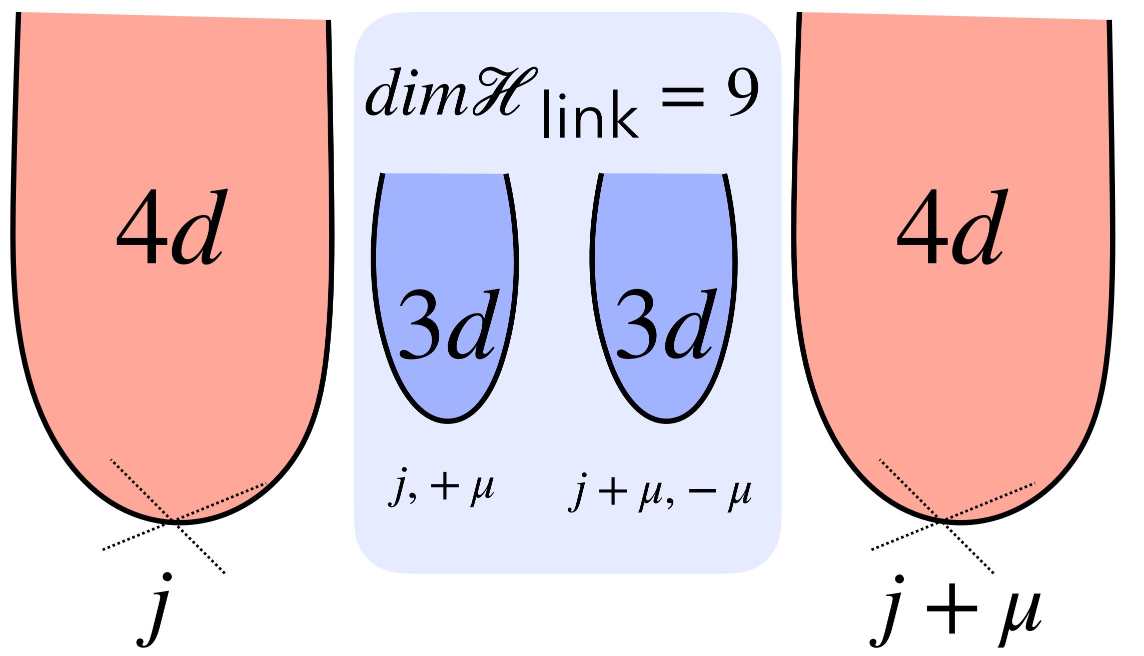
ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

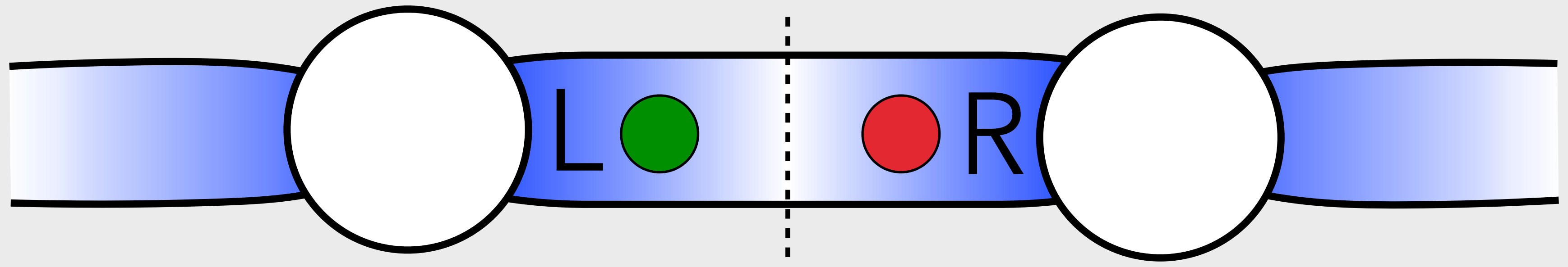
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

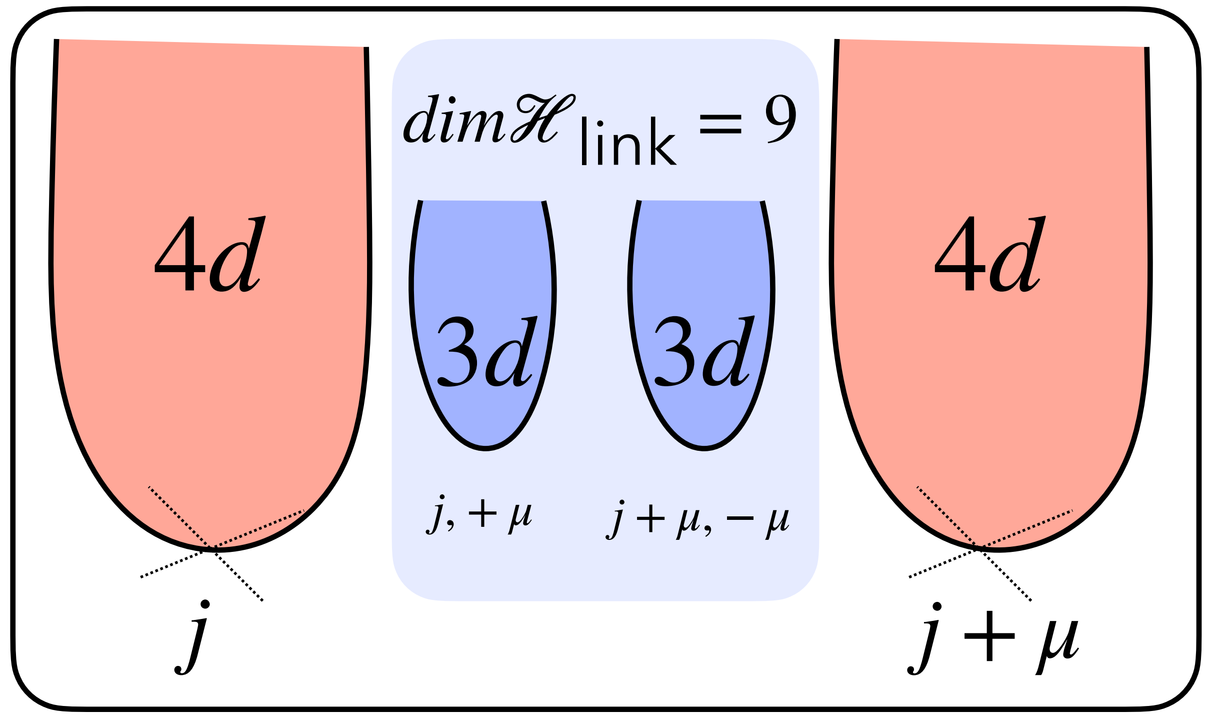
ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

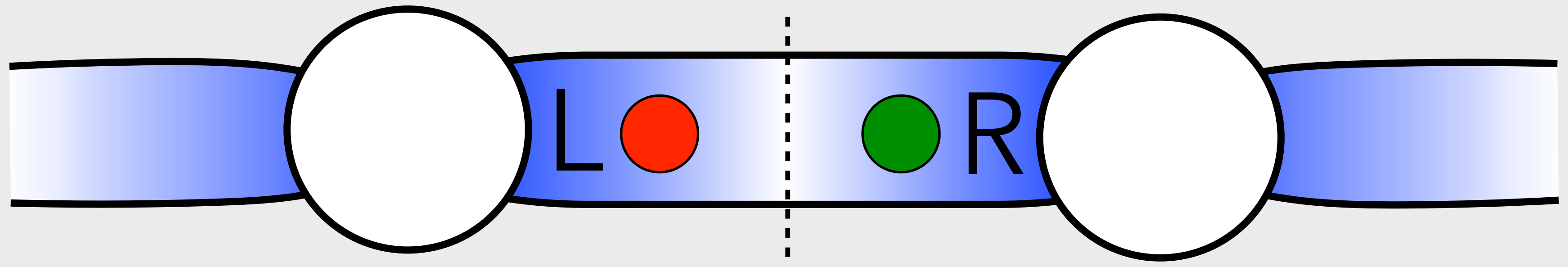
$$\zeta_r = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_F$$

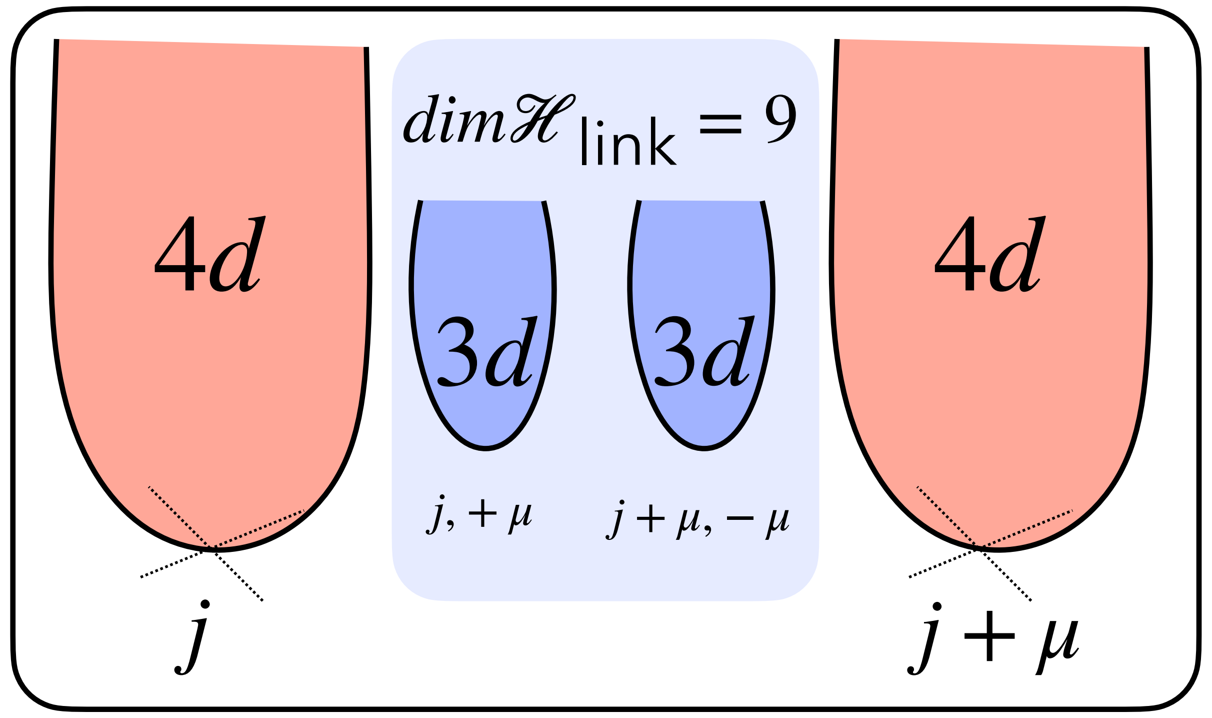
ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

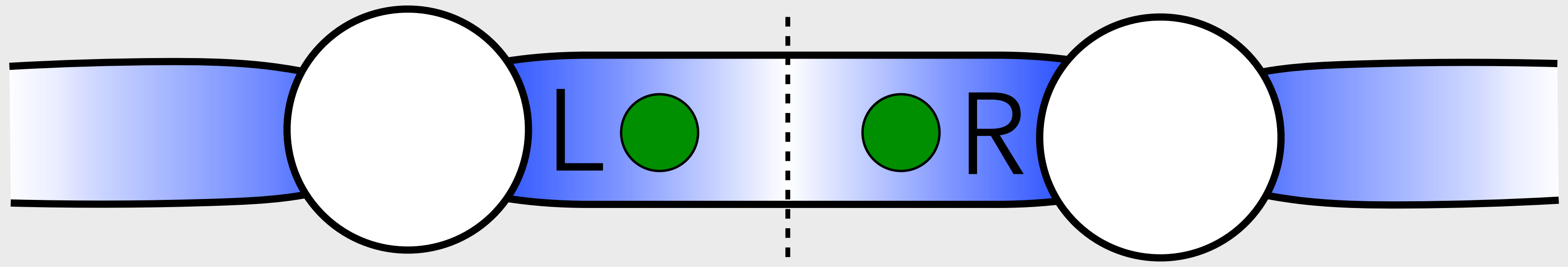
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

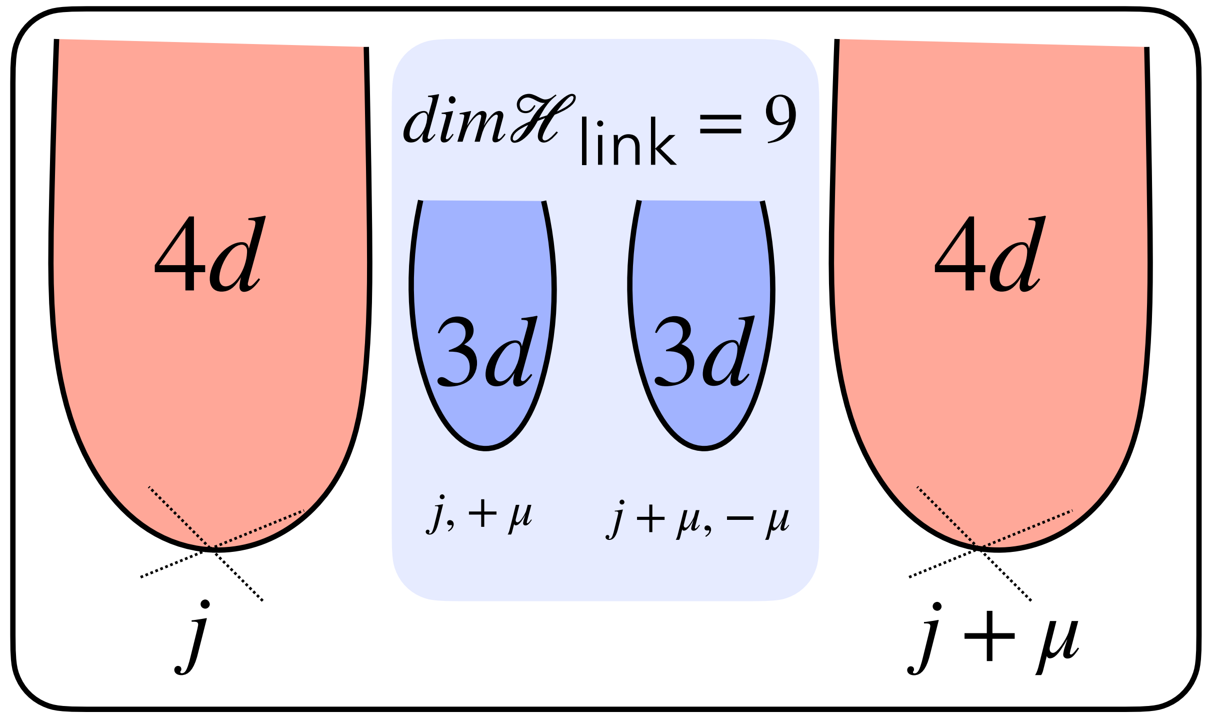
ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

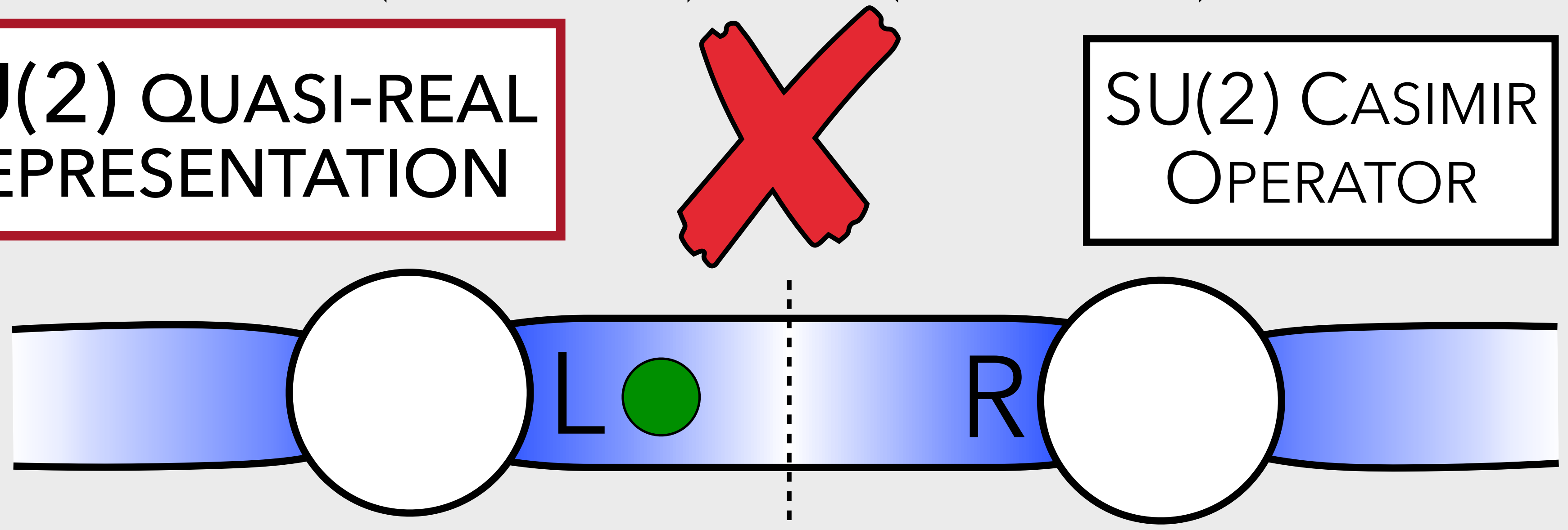
$$\zeta_{r} = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_{g} = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

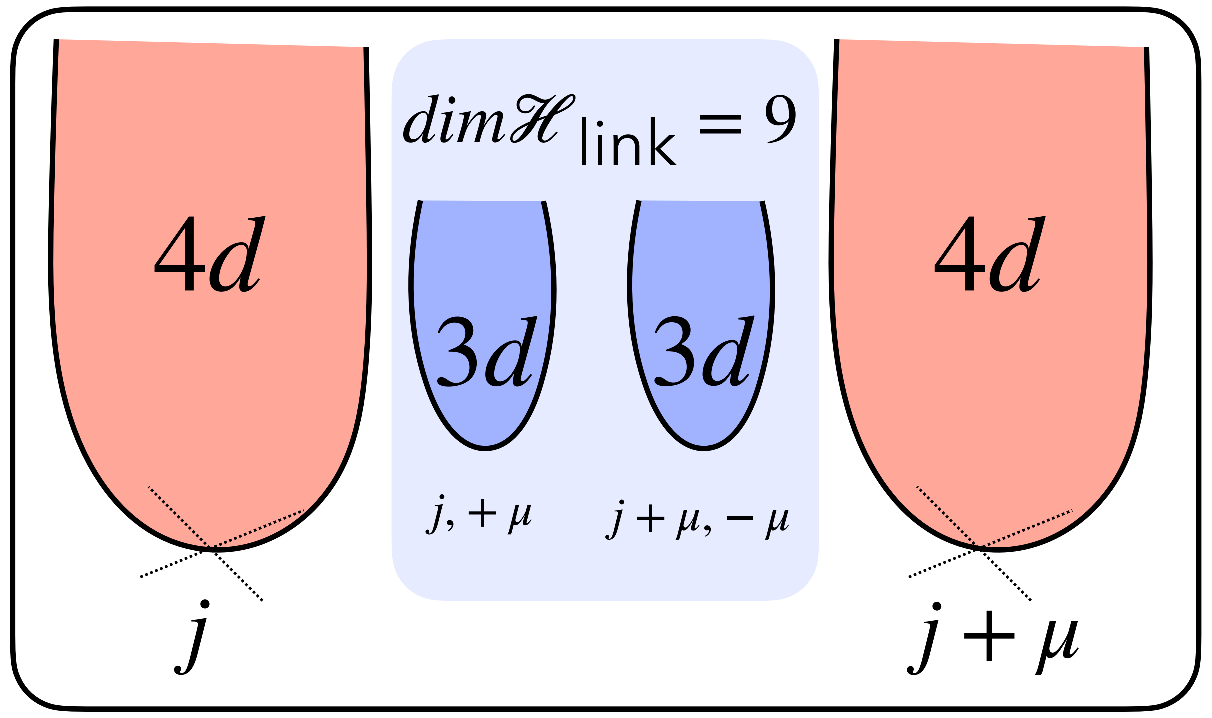
ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

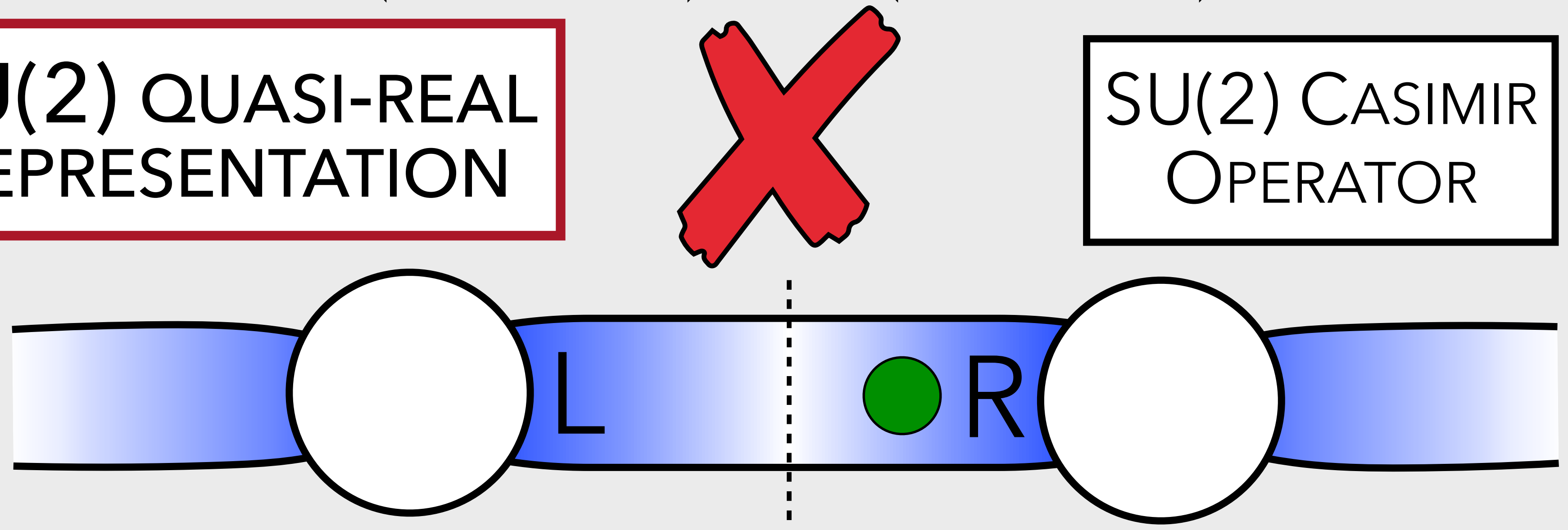
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

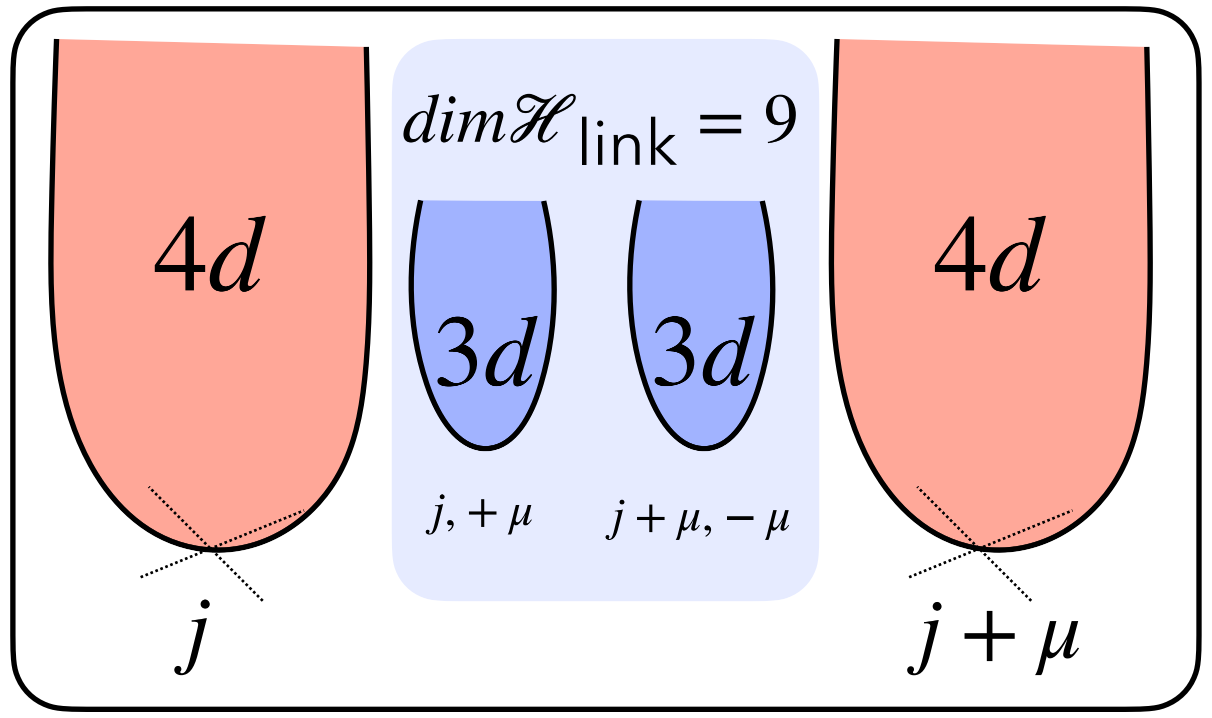
ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

Cataldi et al. arXiv:2307.09396 (2023)

The parallel transport in terms of fermionic rishons

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

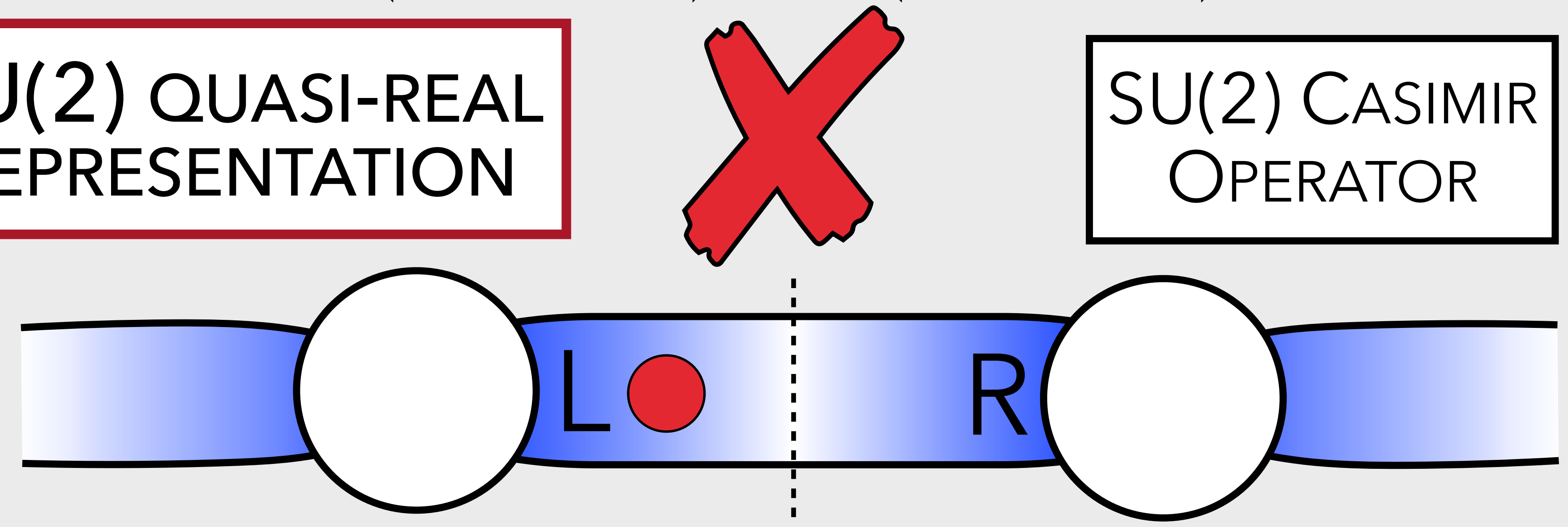
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

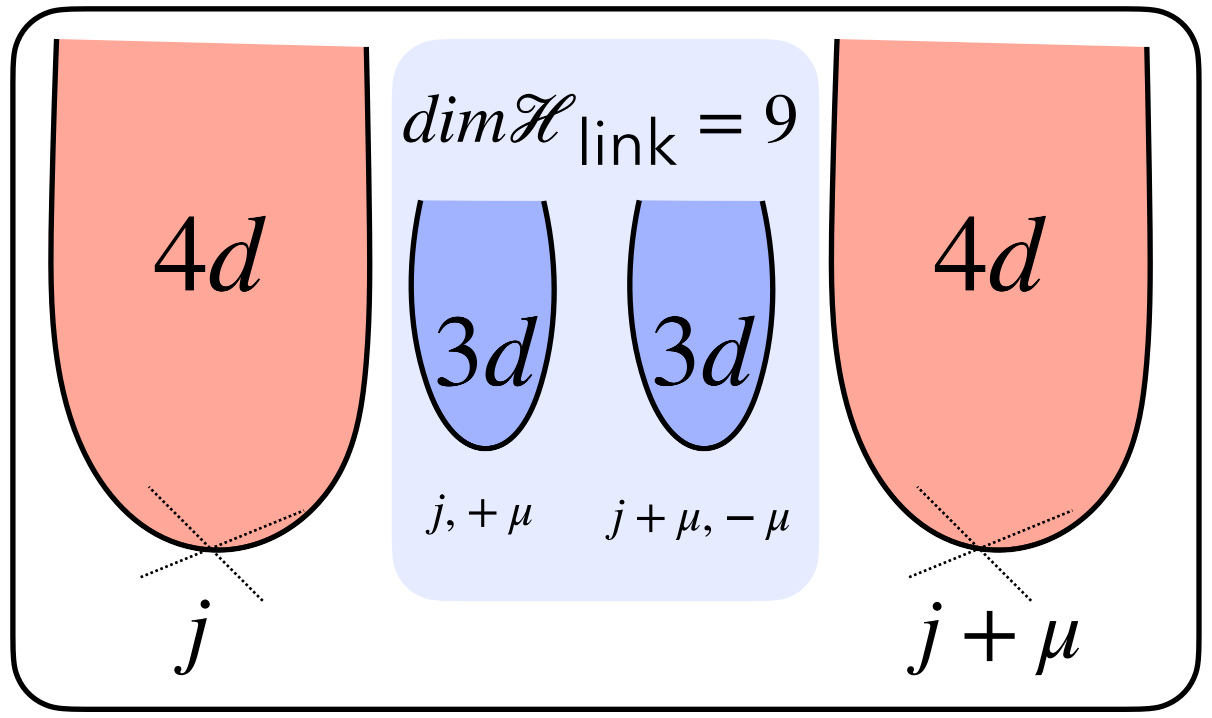
Allowed link configurations satisfy

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) quasi-real representation

SU(2) Casimir operator





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

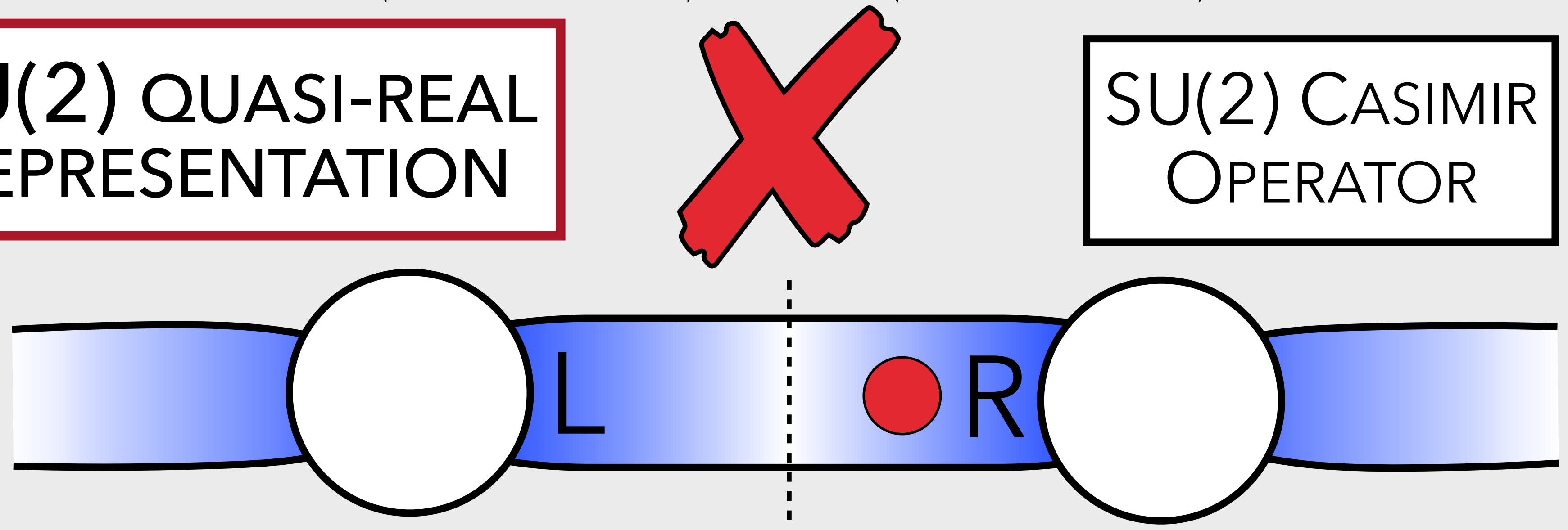
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

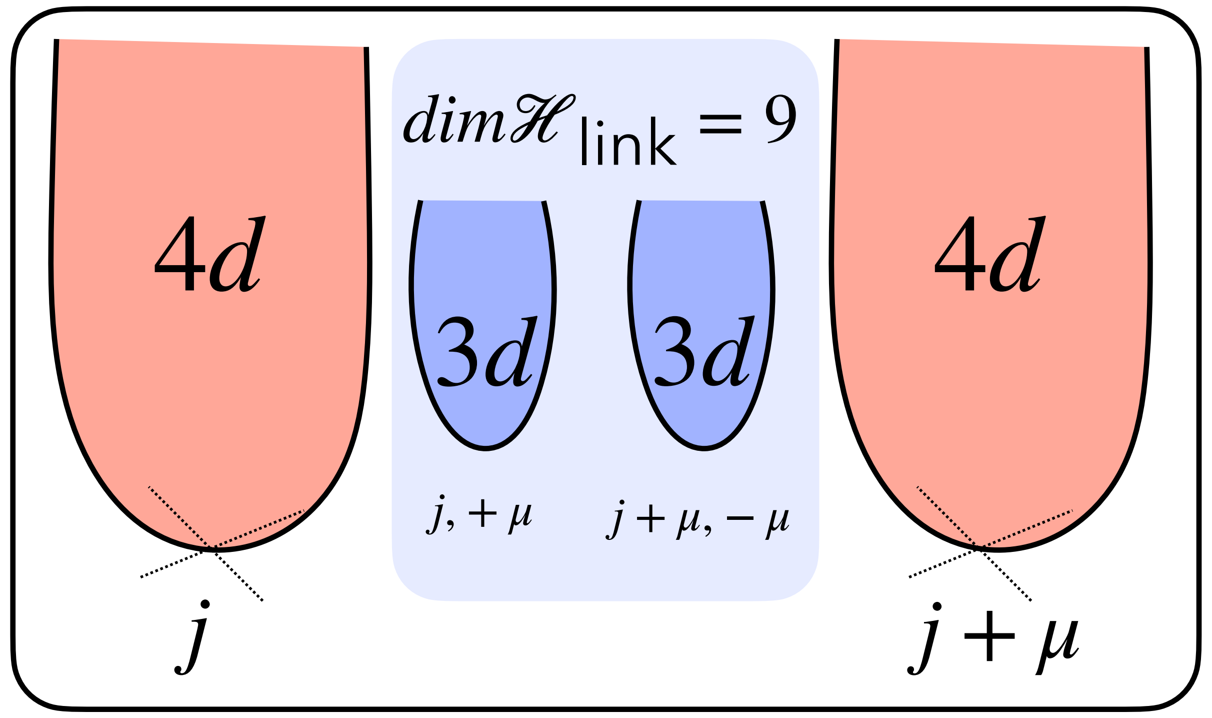
ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

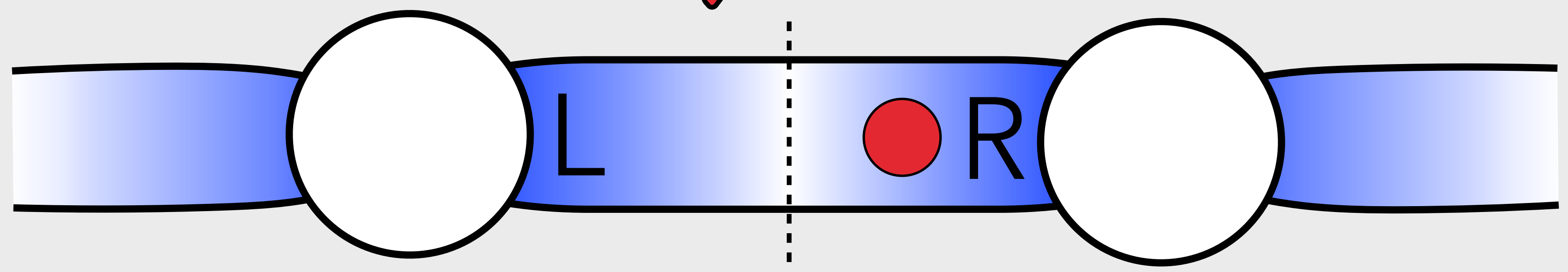
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

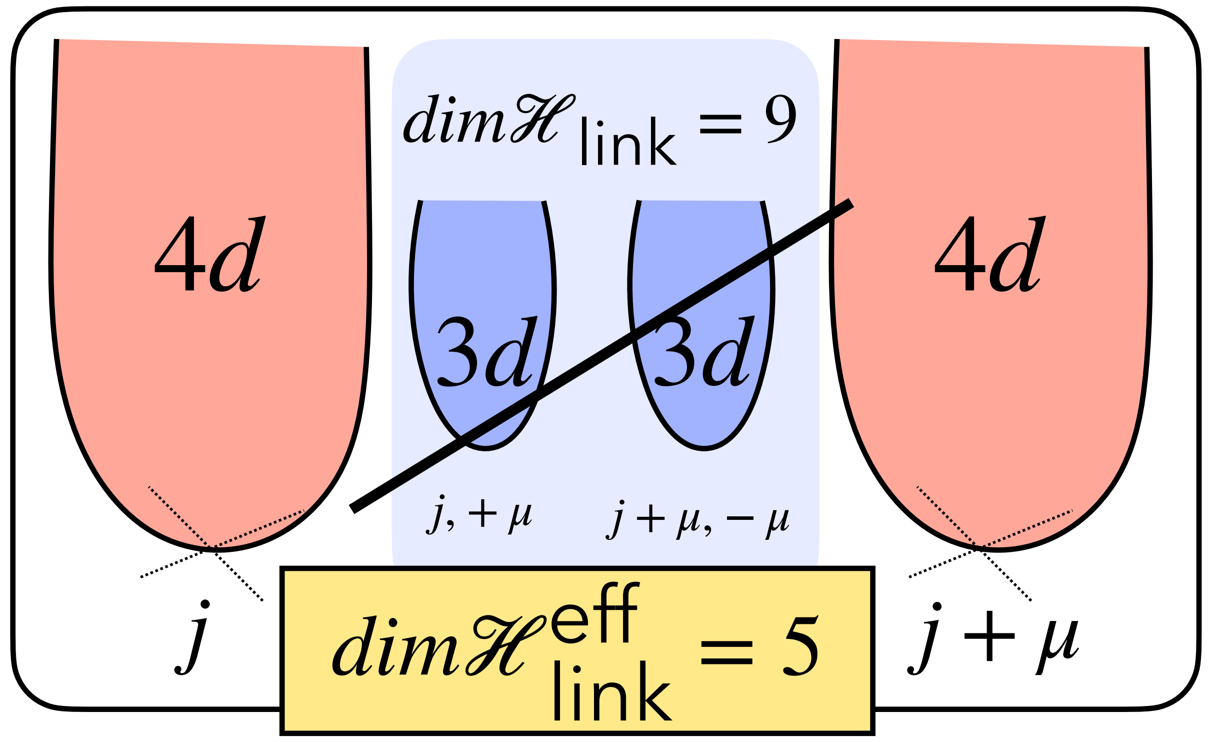
ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

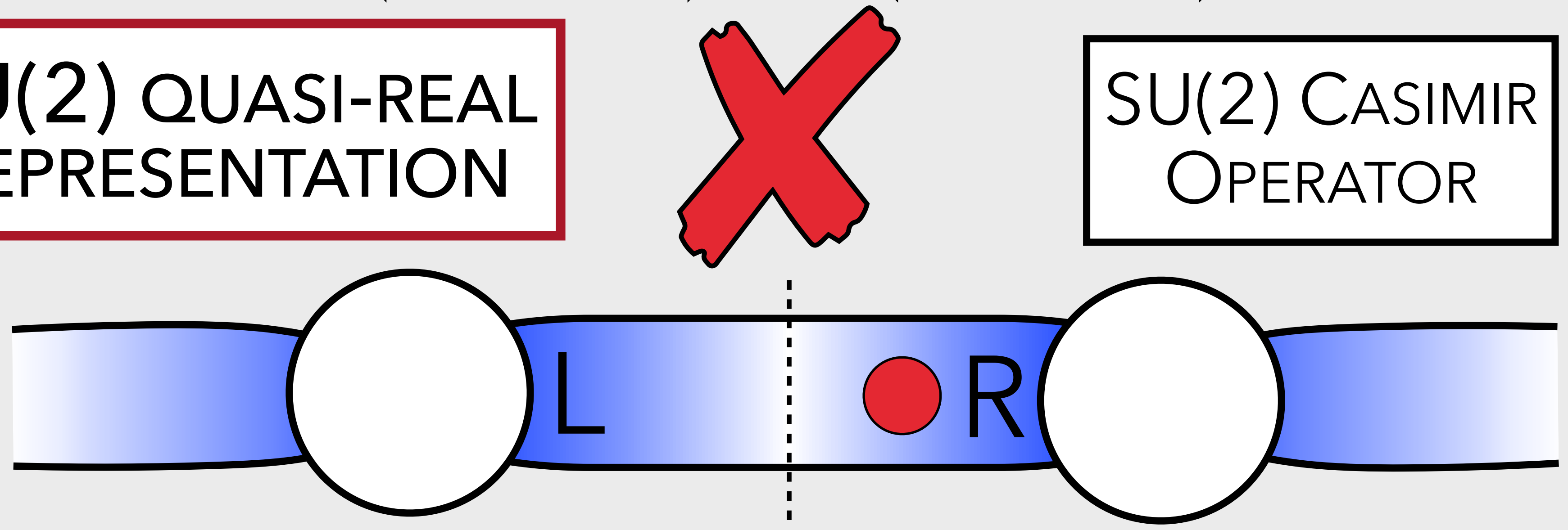
$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

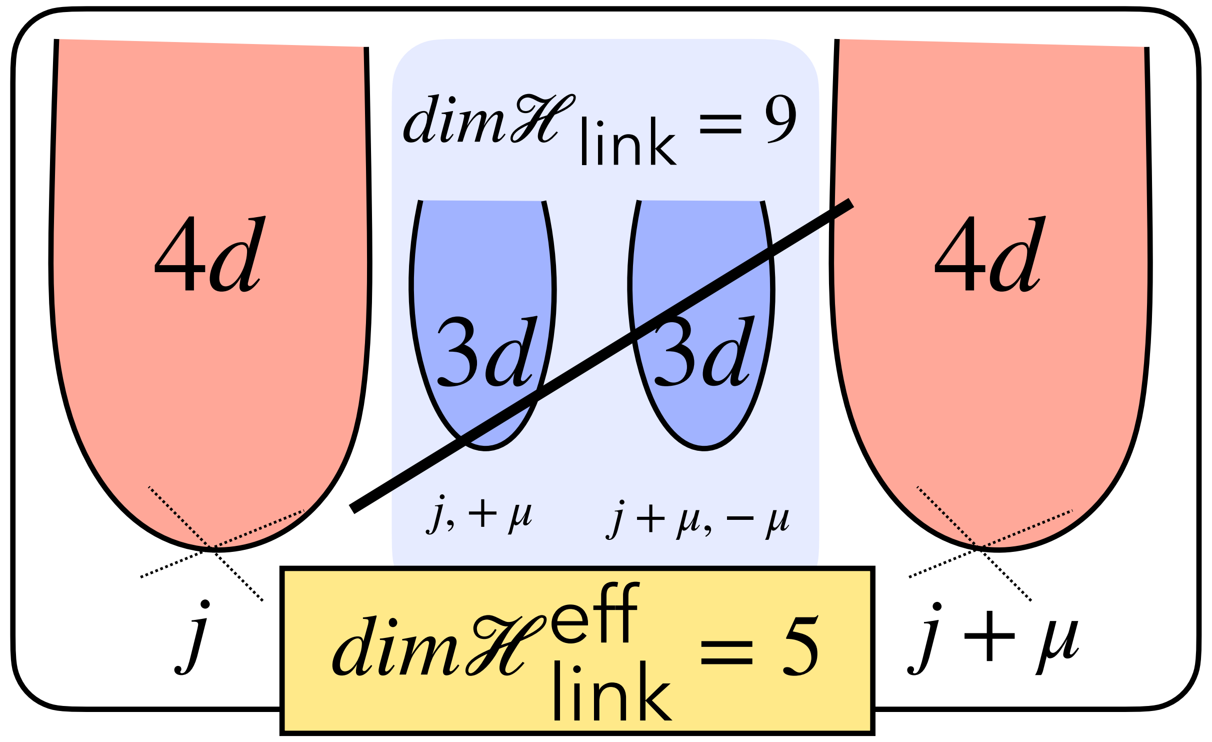
ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





ζ Rishon-based Parallel Transport ✓

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \frac{1}{2}\begin{pmatrix} 0 & -\delta_{g,\alpha}\delta_{r,\beta} & +\delta_{g,\alpha}\delta_{\downarrow\beta} & -\delta_{r,\alpha}\delta_{r,\beta} & +\delta_{r,\alpha}\delta_{g,\beta} \\ +\delta_{r,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ +\delta_{r,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \\ -\delta_{g,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ -\delta_{g,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

ZOHAR & BURRELLO, PRD 91, 054506 (2015)

TRUNCATED SU(2) LINKS VIA RISHONS

CATALDI ET AL. ARXIV:2307.09396 (2023)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

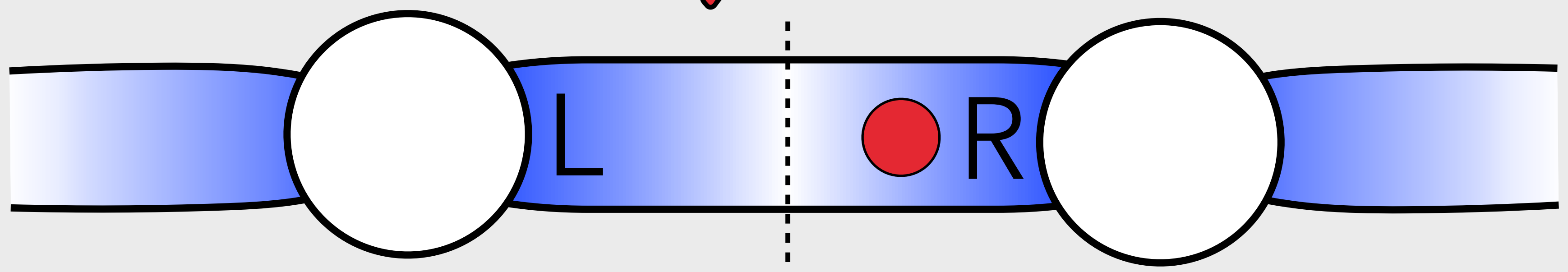
$$\zeta_{r} = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F \qquad \zeta_{g} = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

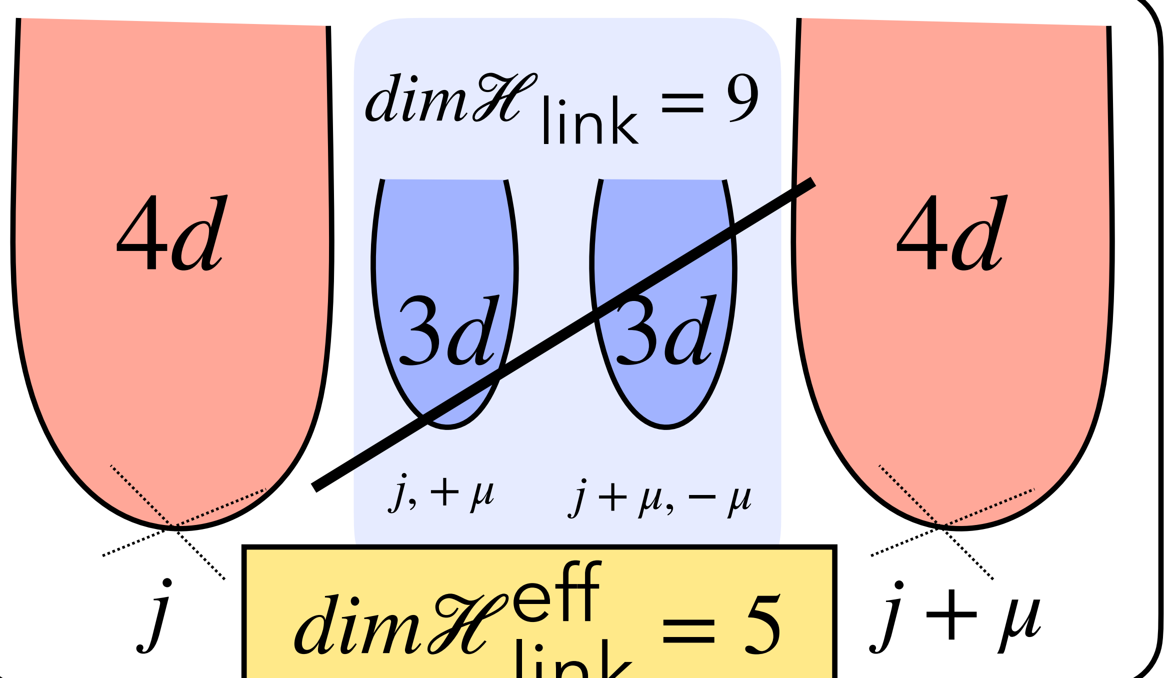
ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR





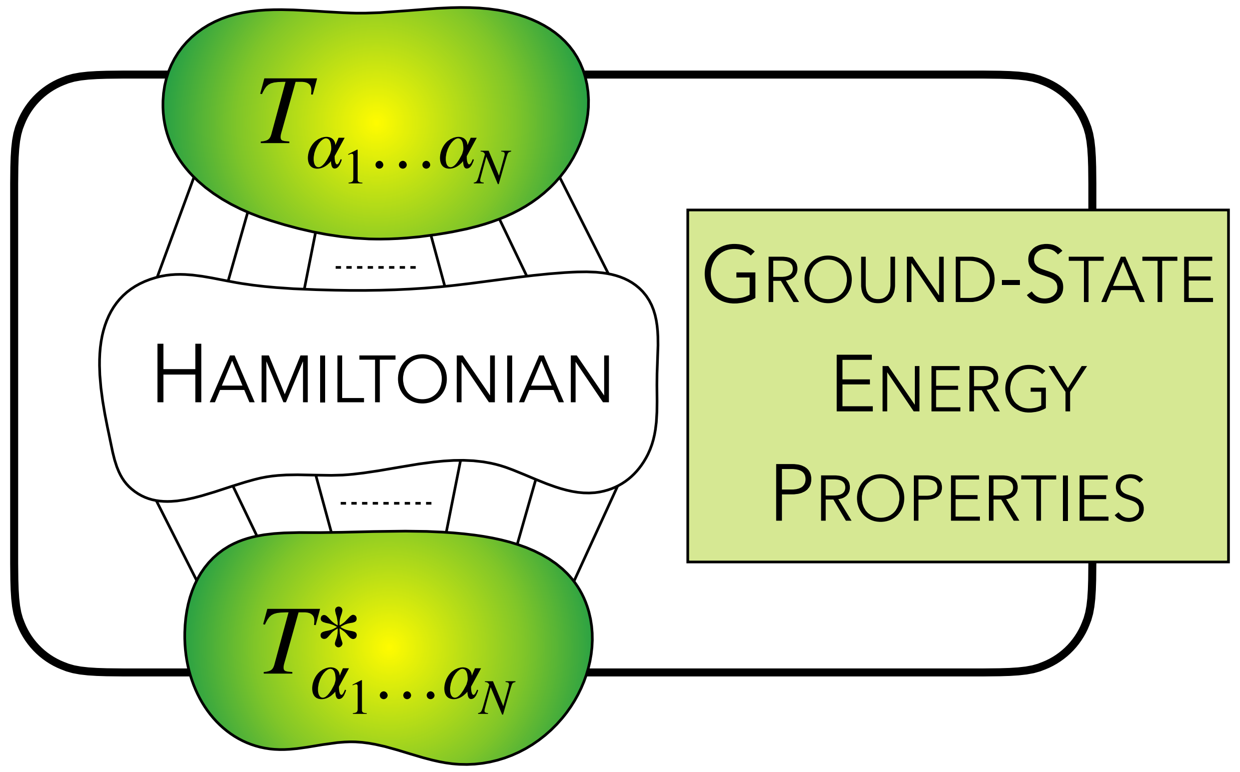
ζ Rishon-based Parallel Transport ✓

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \frac{1}{2}\begin{pmatrix} 0 & -\delta_{g,\alpha}\delta_{r,\beta} & +\delta_{g,\alpha}\delta_{\downarrow\beta} & -\delta_{r,\alpha}\delta_{r,\beta} & +\delta_{r,\alpha}\delta_{g,\beta} \\ +\delta_{r,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ +\delta_{r,\alpha}\delta_{r,\beta} & 0 & & & \\ -\delta_{g,\alpha}\delta_{g,\beta} & 0 & & & \\ -\delta_{g,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

WE CAN GENERALIZE IT TO ANY SPIN-REPRESENTATION!

ZOHAR & BURRELLO, PRD 91, 054506 (2015)

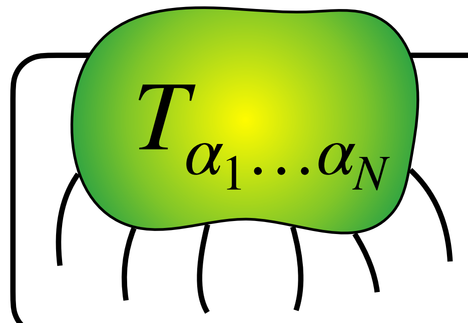
TENSOR NETWORK METHODS



The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

$$\dim \mathscr{H} = d^N$$



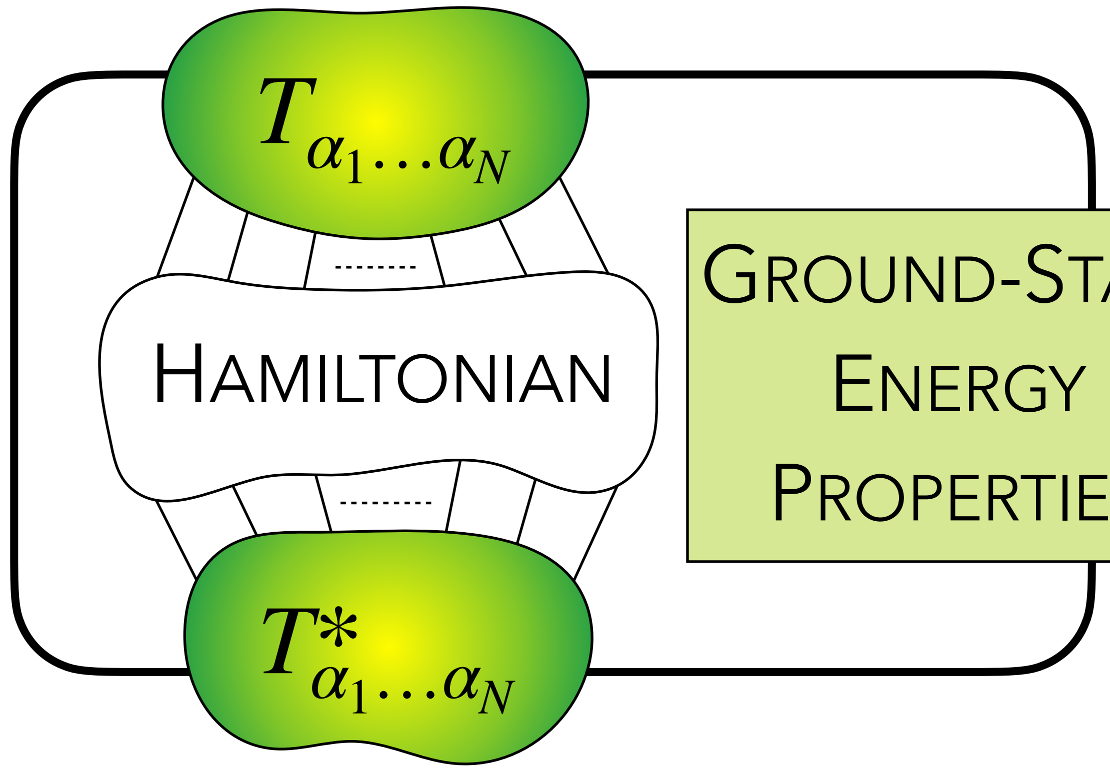
$$\left|\Psi_{\text{QMB}}\right\rangle = \sum_{\alpha_1 \ldots \alpha_N} T_{\alpha_1 \ldots \alpha_N} \left|\alpha_1 \ldots \alpha_N\right\rangle$$

TENSOR NETWORK METHODS

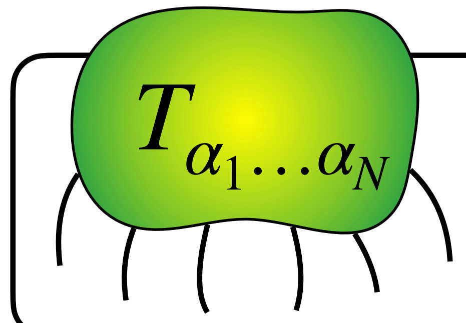
The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

$$\dim\mathscr{H} = d^N$$



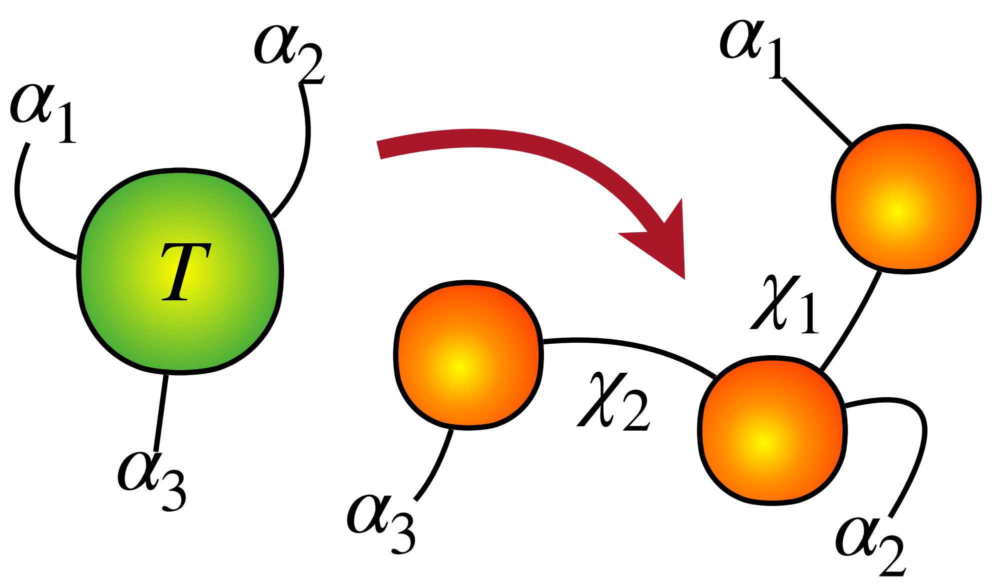
GROUND-STATE ENERGY PROPERTIES



$$\left|\Psi_{\mathrm{QMB}}\right\rangle = \sum_{\alpha_1\ldots\alpha_N} T_{\alpha_1\ldots\alpha_N} \left|\alpha_1\ldots\alpha_N\right\rangle$$

AUXILIARY LINKS χ "BOND DIMENSION" (ENTANGLEMENT)

PHYSICAL LINKS α_i

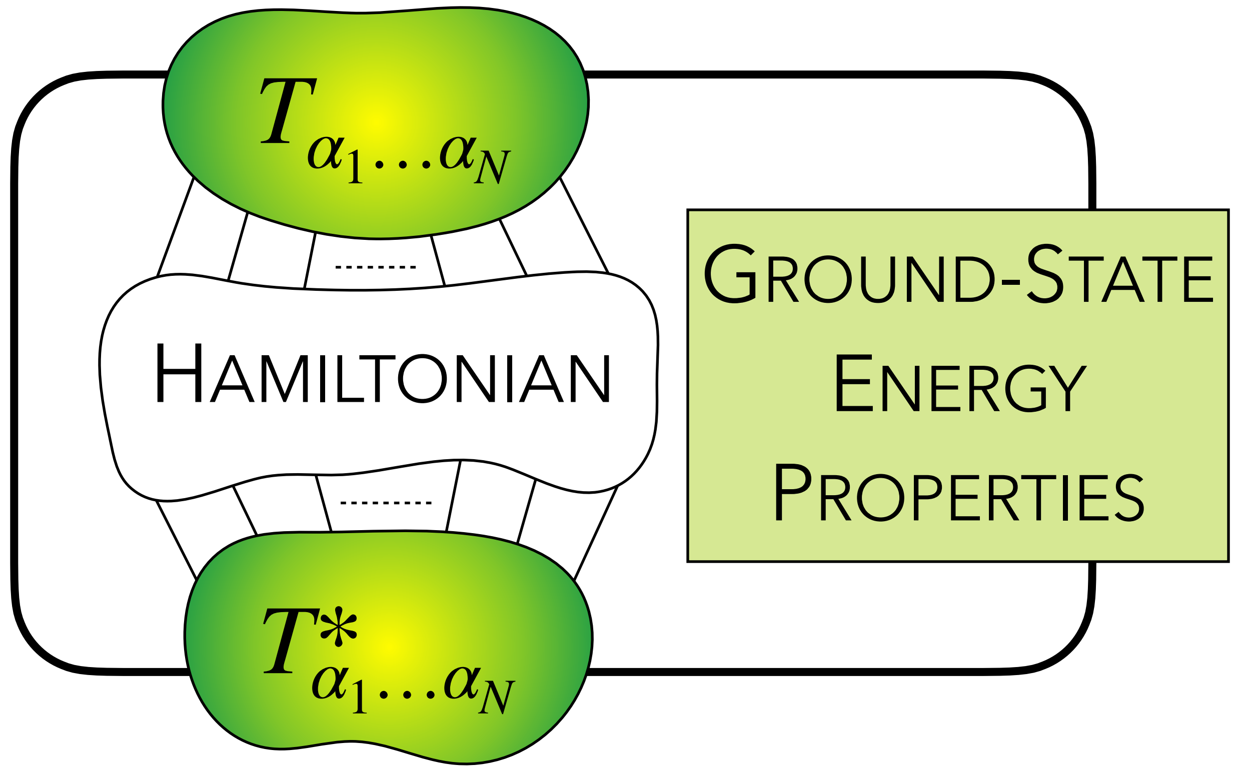


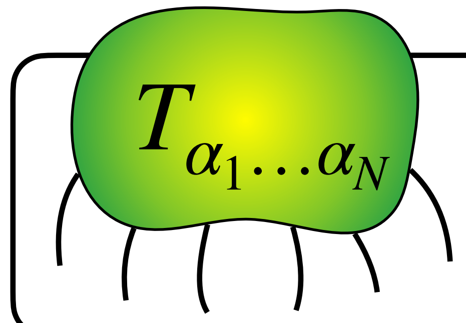
TENSOR NETWORK METHODS

The Hilbert space of N-Body Systems *grows exponentially*.

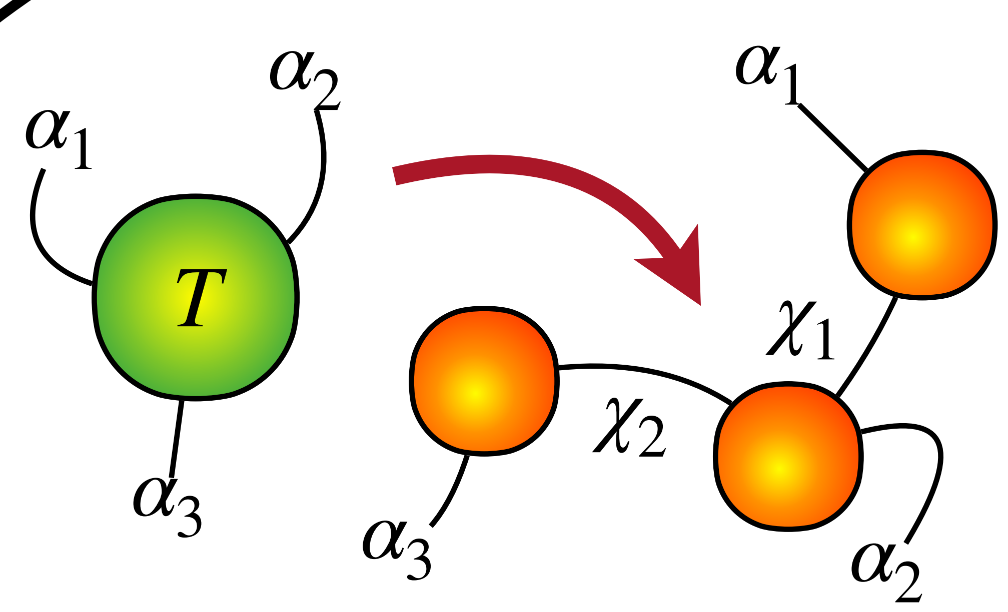
Exact Diagonalization (ED) is not sustainable for large N

$\dim\mathscr{H} = d^N$



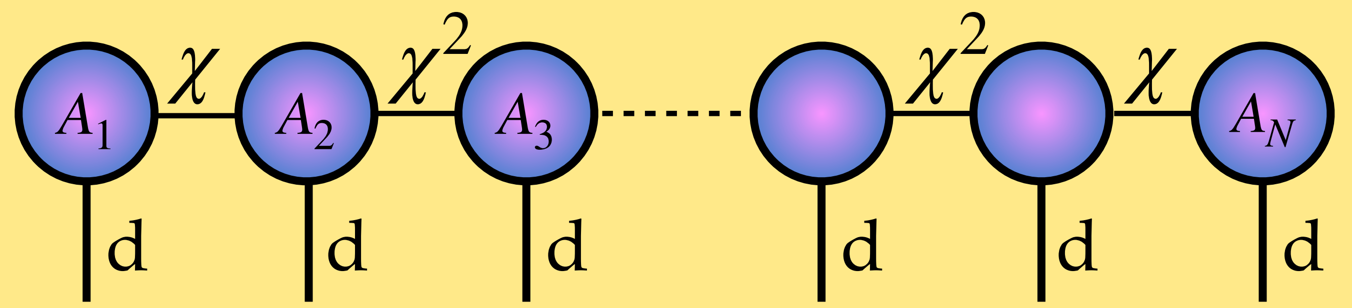


$$\left|\Psi_{\mathrm{QMB}}\right\rangle = \sum_{\alpha_1\ldots\alpha_N} T_{\alpha_1\ldots\alpha_N} |\alpha_1\ldots\alpha_N\rangle$$



AUXILIARY LINKS χ
"BOND DIMENSION"
(ENTANGLEMENT)

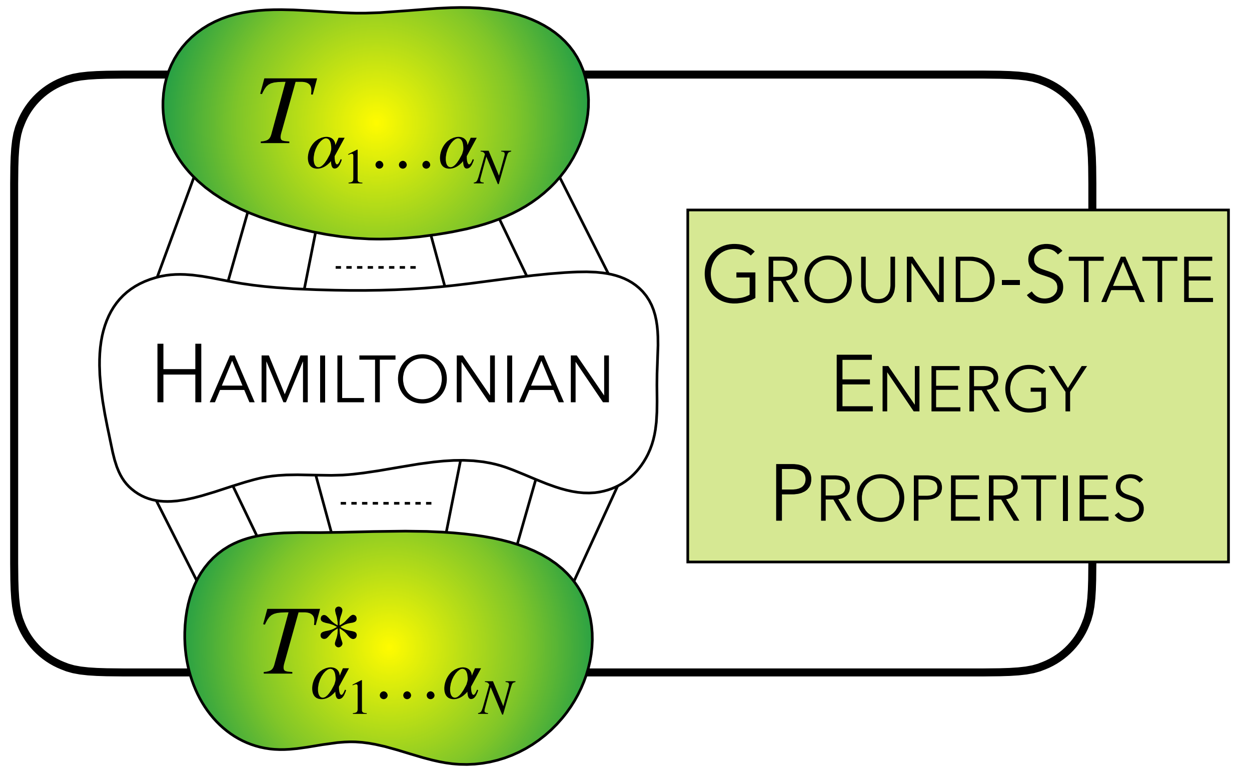
PHYSICAL LINKS α_i



$$\left|\Psi_{\mathrm{MPS}}\right\rangle = \sum_{\alpha_1\ldots\alpha_N} \mathrm{Tr}\left[A_1^{\alpha_1}\ldots A_N^{\alpha_N}\right] |\alpha_1\ldots\alpha_N\rangle$$

MATRIX PRODUCT STATE

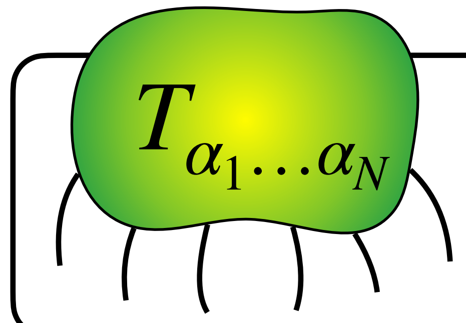
TENSOR NETWORK METHODS



The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

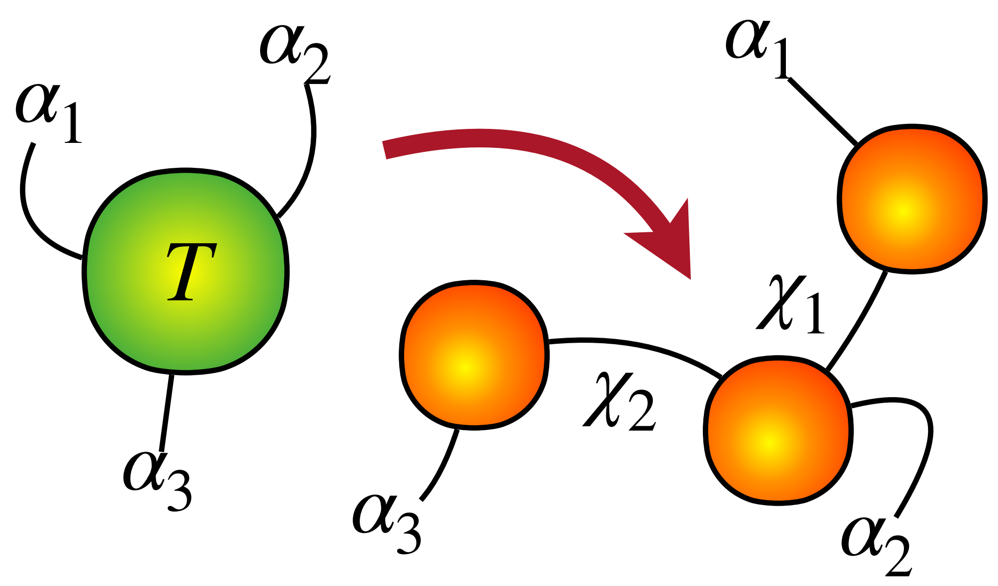
$$\dim \mathscr{H} = d^N$$

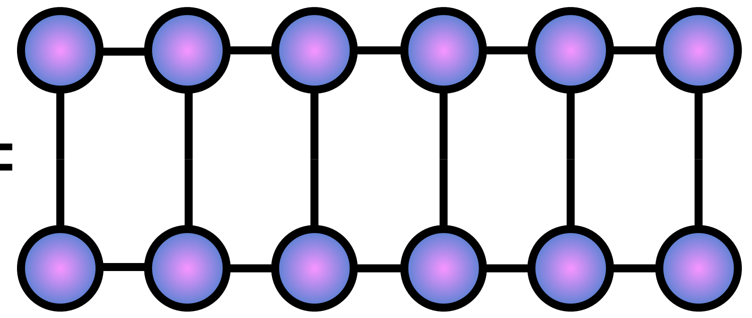


$$\left|\Psi_{\mathrm{QMB}}\right\rangle = \sum_{\alpha_1 \dots \alpha_N} T_{\alpha_1 \dots \alpha_N} \left|\alpha_1 \dots \alpha_N\right\rangle$$

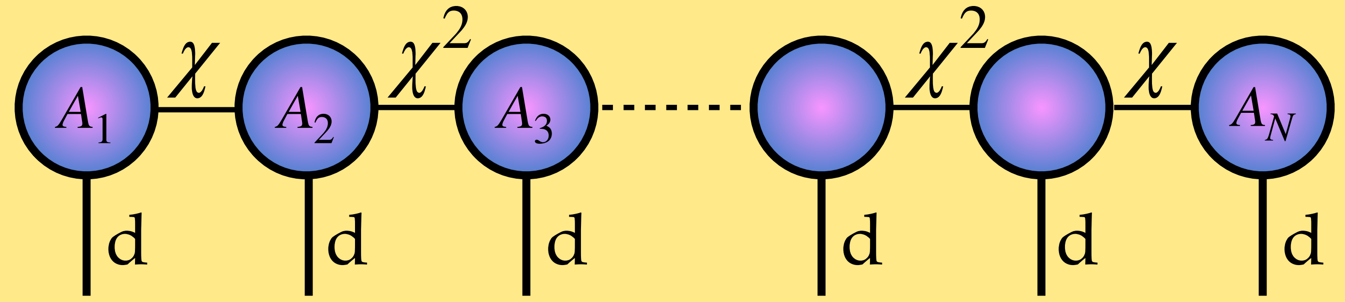
AUXILIARY LINKS χ
"BOND DIMENSION"
(ENTANGLEMENT)

PHYSICAL LINKS α_i





$$\langle \Psi | \Psi \rangle =$$



$$\left|\Psi_{\mathrm{MPS}}\right\rangle = \sum_{\alpha_1 \dots \alpha_N} \mathrm{Tr}\left[A_1^{\alpha_1} \dots A_N^{\alpha_N}\right] \left|\alpha_1 \dots \alpha_N\right\rangle$$

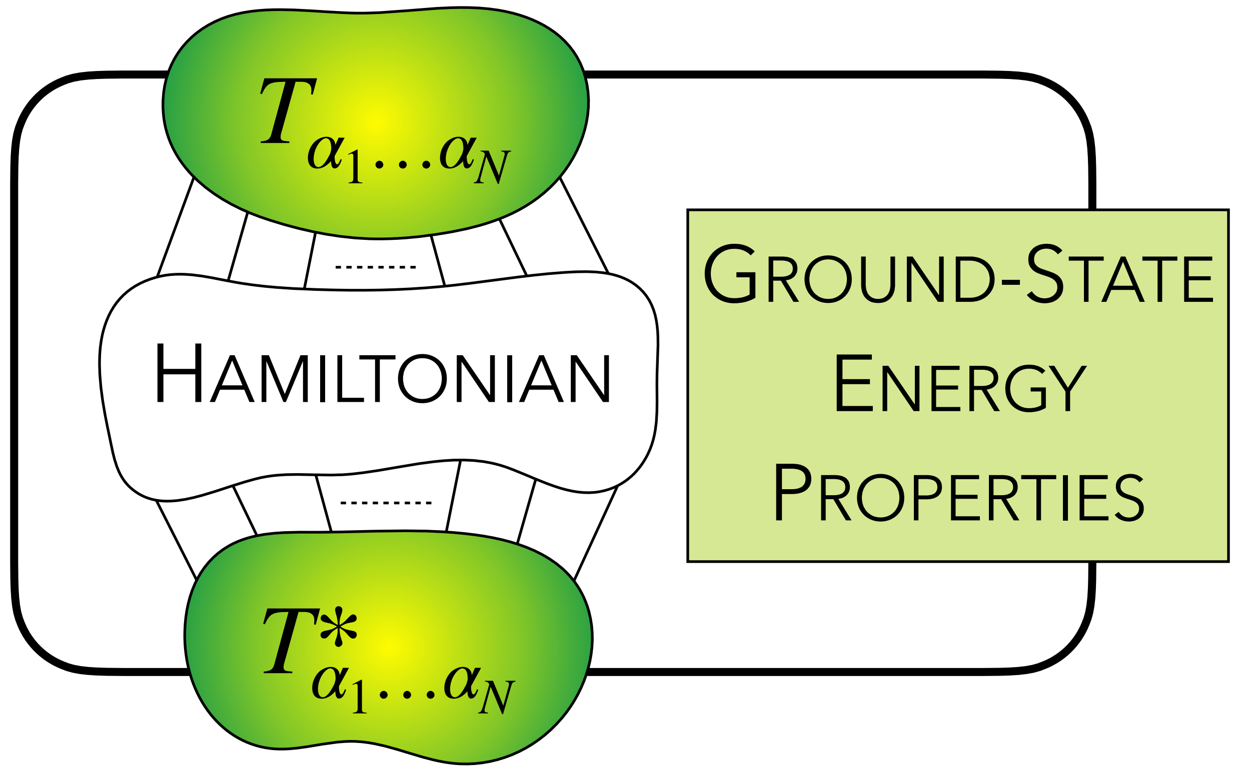
MATRIX PRODUCT STATE

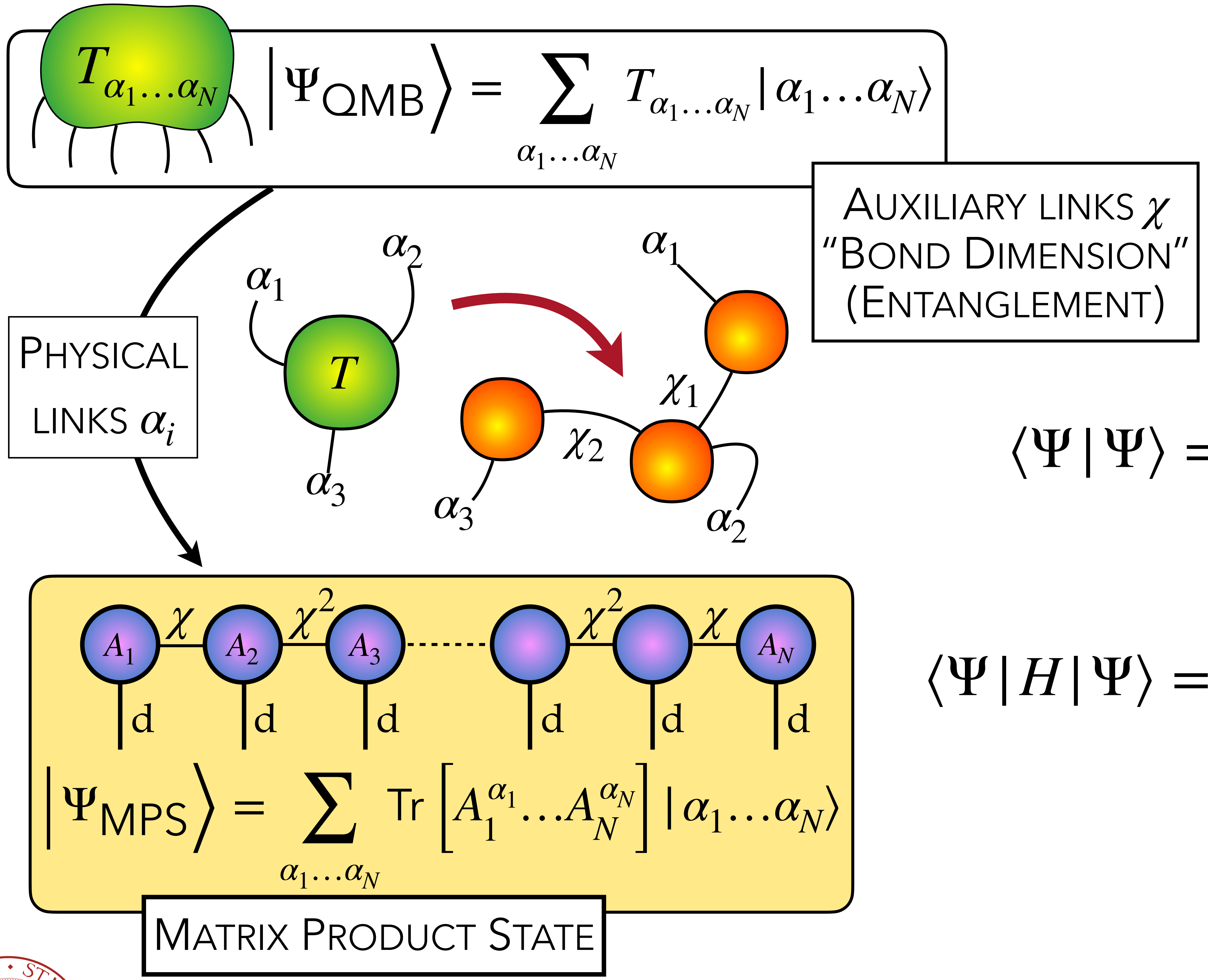
TENSOR NETWORK METHODS

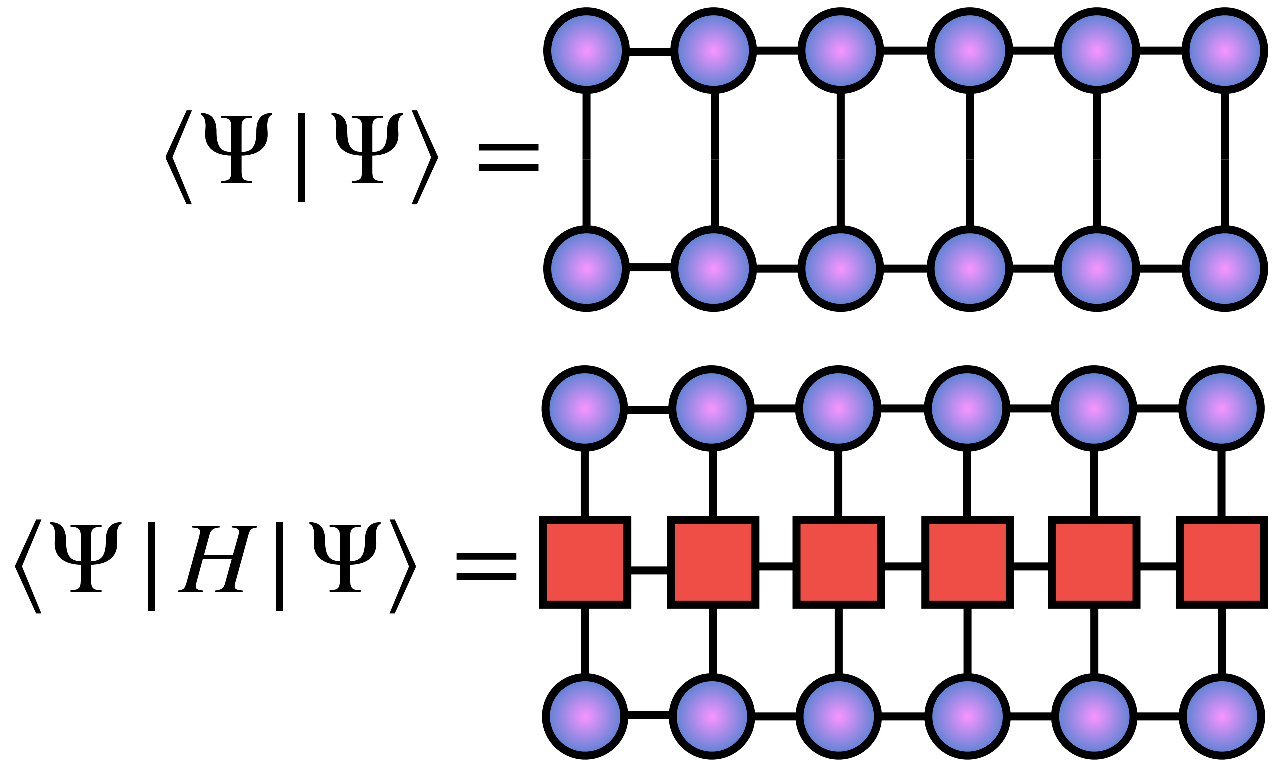
The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

$$\dim \mathscr{H} = d^N$$





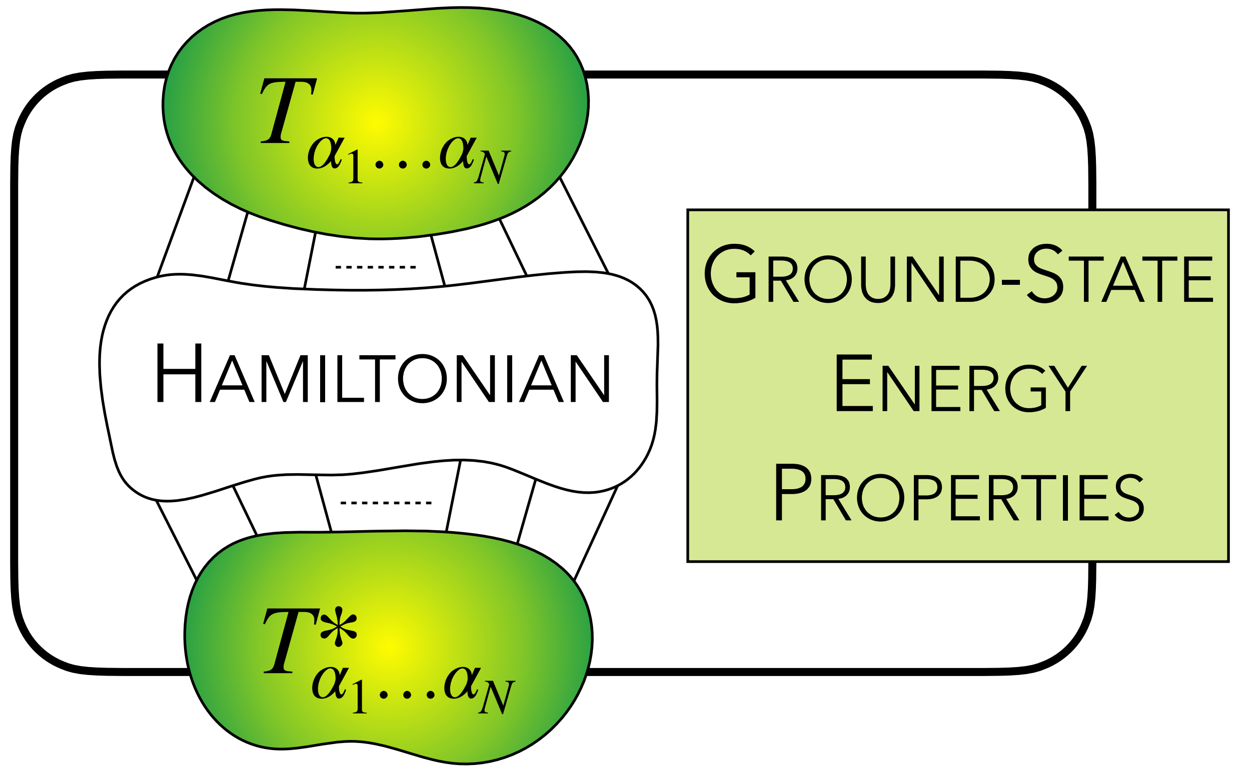


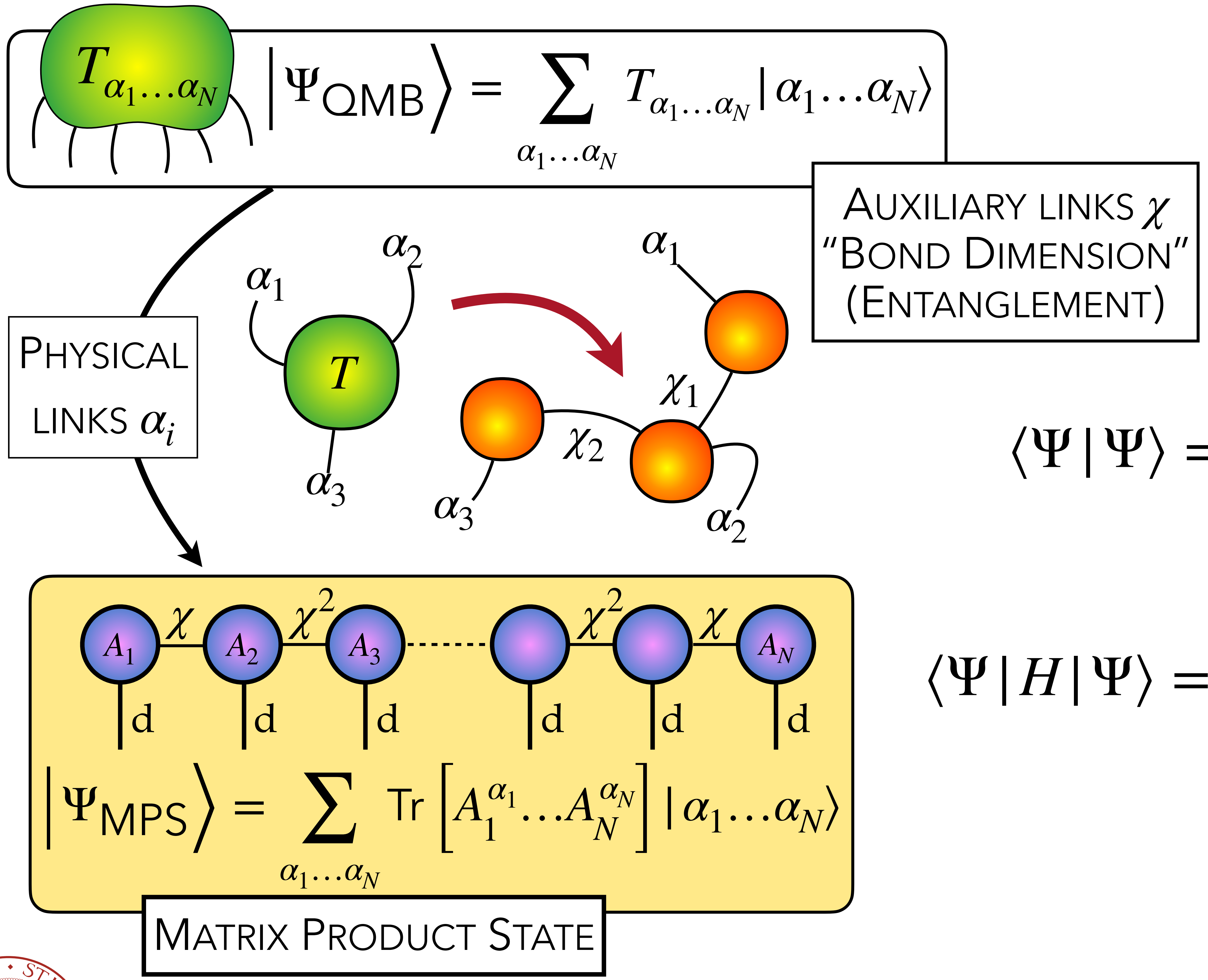
TENSOR NETWORK METHODS

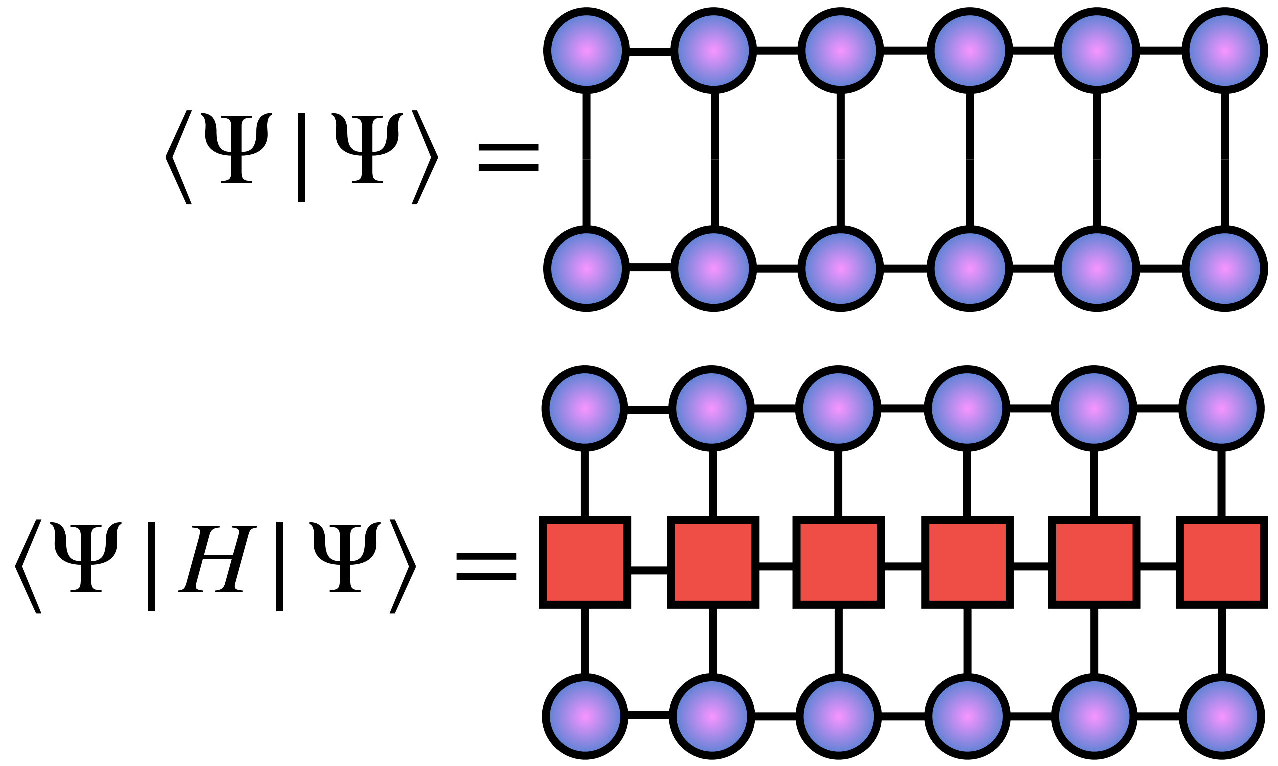
The Hilbert space of N-Body Systems *grows exponentially.*

Exact Diagonalization (ED) is not sustainable for large N

$$\dim \mathscr{H} = d^N$$







GROUND-STATE SEARCH VARIATIONAL ALGORITHM

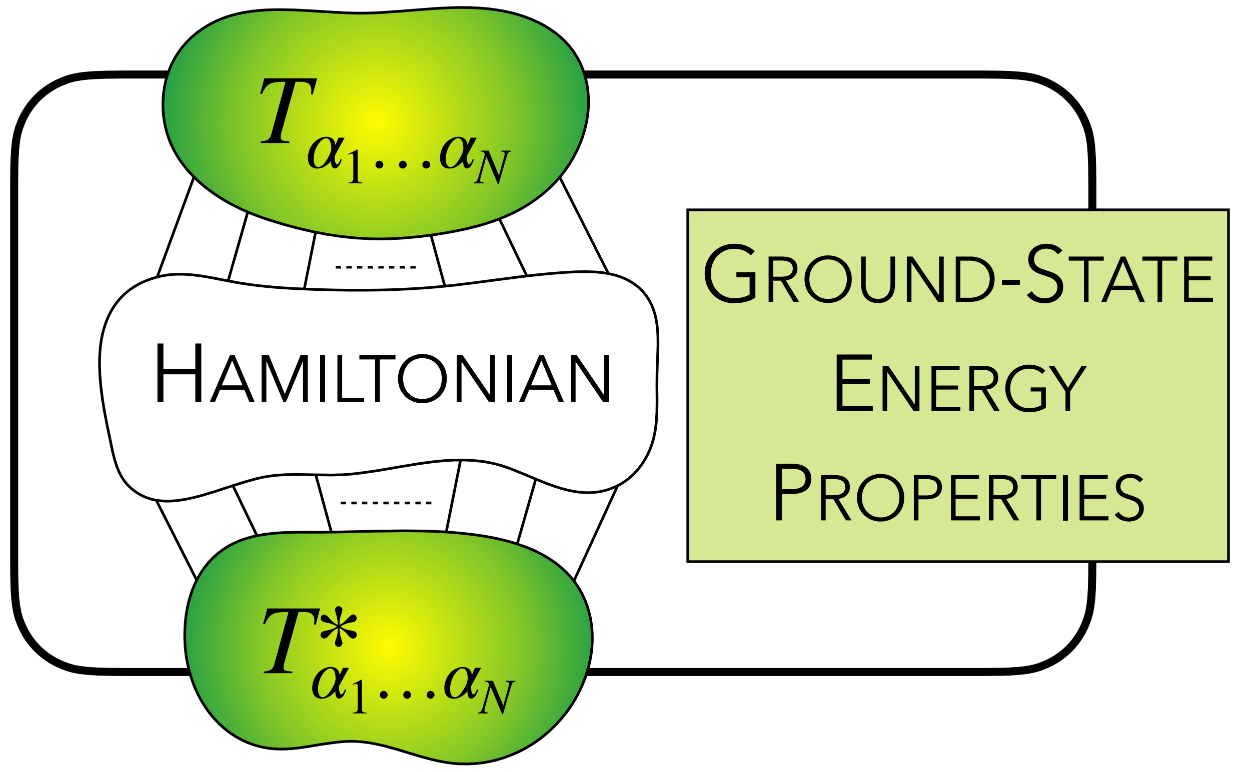
1. Initialize (randomly) $|\Psi\rangle = \{A_1 \ldots, A_N\}$

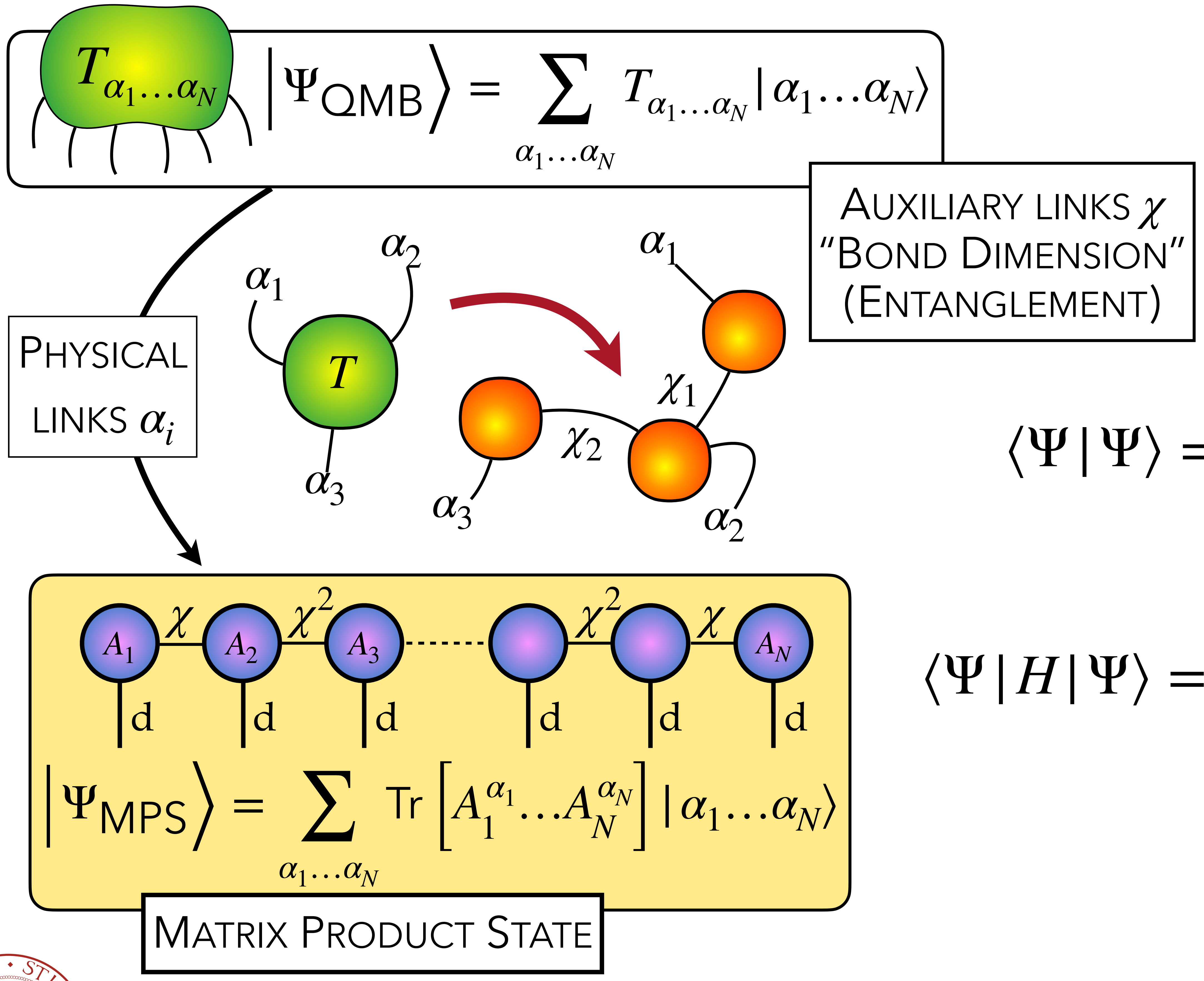
TENSOR NETWORK METHODS

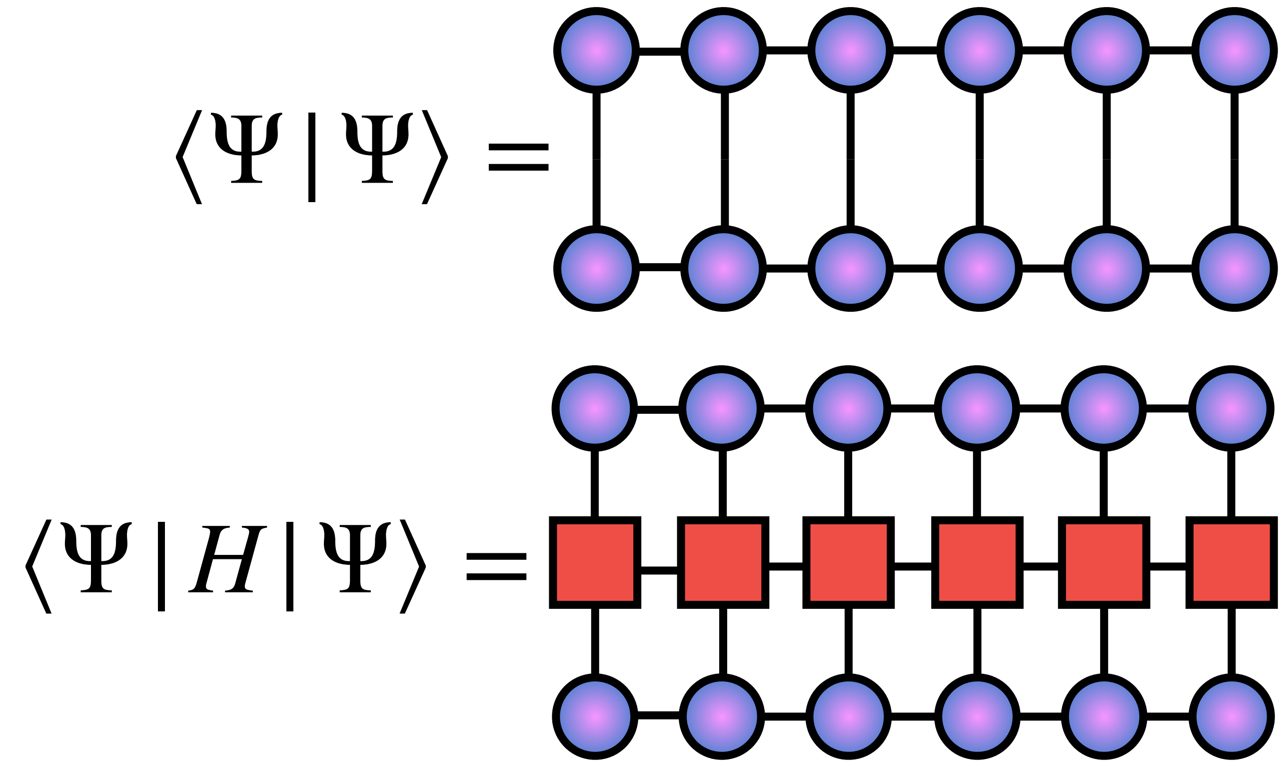
The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

$$\dim \mathscr{H} = d^N$$







GROUND-STATE SEARCH VARIATIONAL ALGORITHM

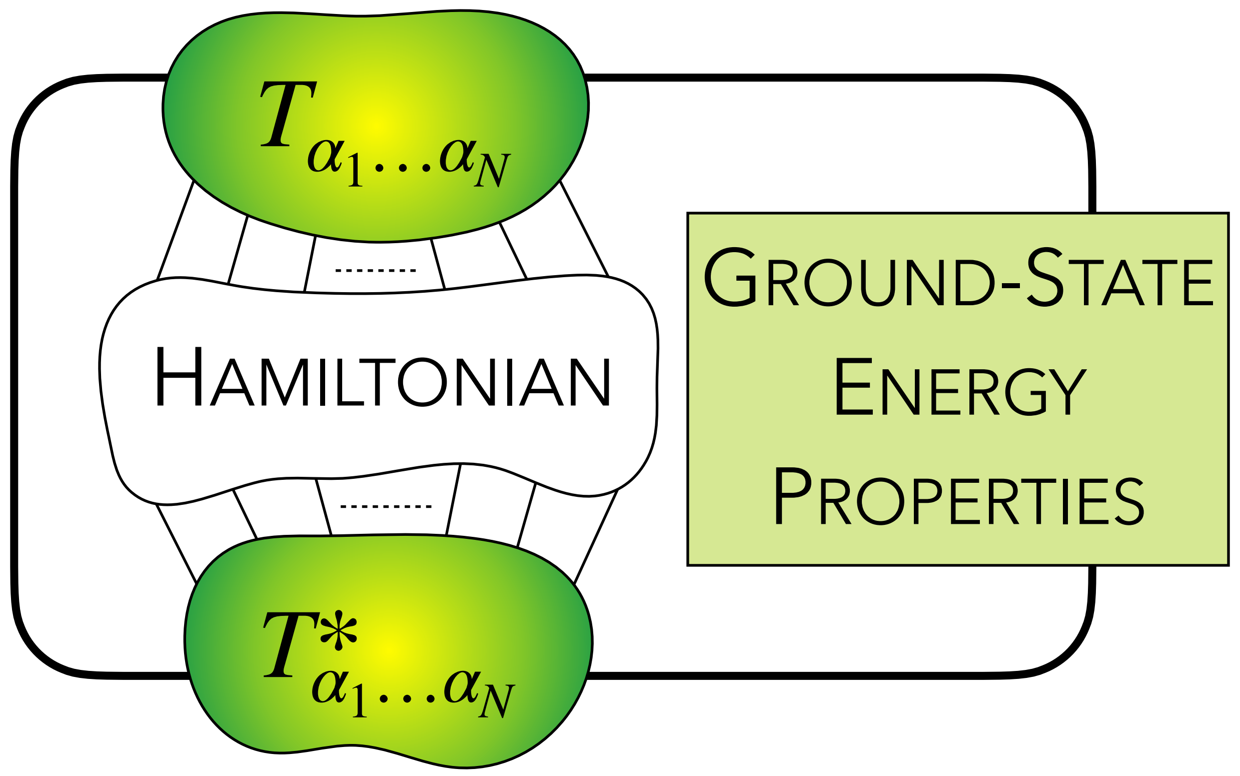
1. Initialize (randomly) $|\Psi\rangle = \{A_1 \dots, A_N\}$
2. Measure $E(0) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$

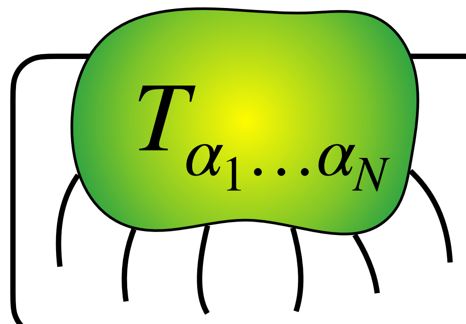
TENSOR NETWORK METHODS

The Hilbert space of N-Body Systems *grows exponentially*.

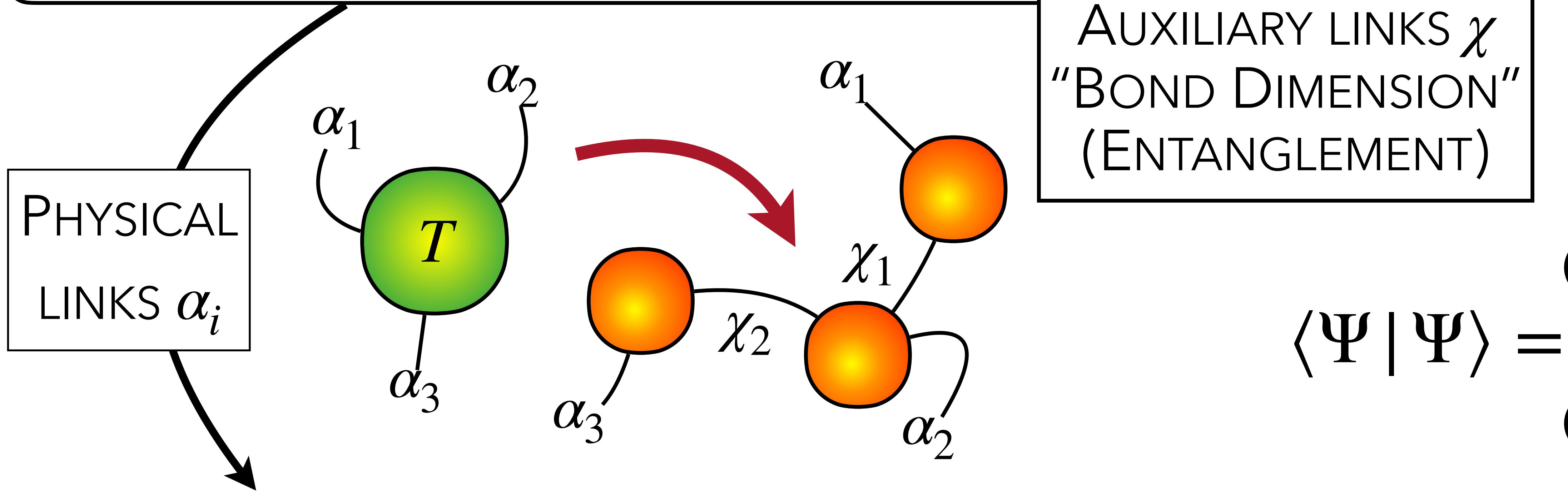
Exact Diagonalization (ED) is not sustainable for large N

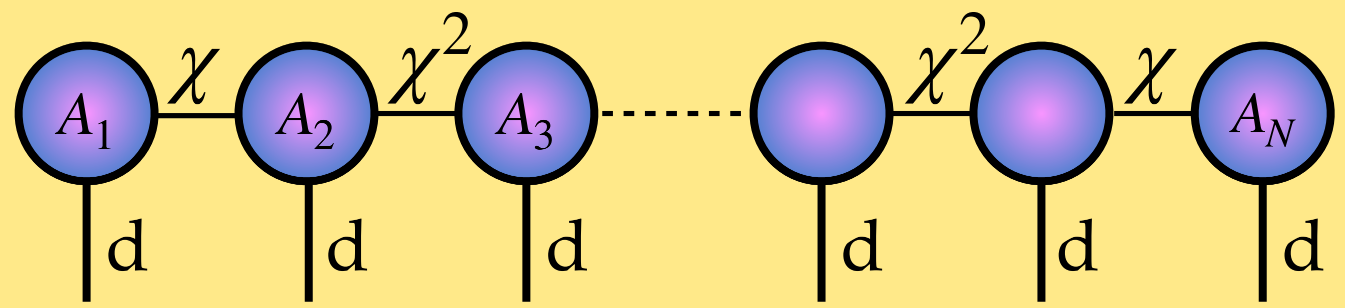
$\dim \mathcal{H} = d^N$





$$\left|\Psi_{\text{QMB}}\right\rangle = \sum_{\alpha_1 \dots \alpha_N} T_{\alpha_1 \dots \alpha_N} \left|\alpha_1 \dots \alpha_N\right\rangle$$

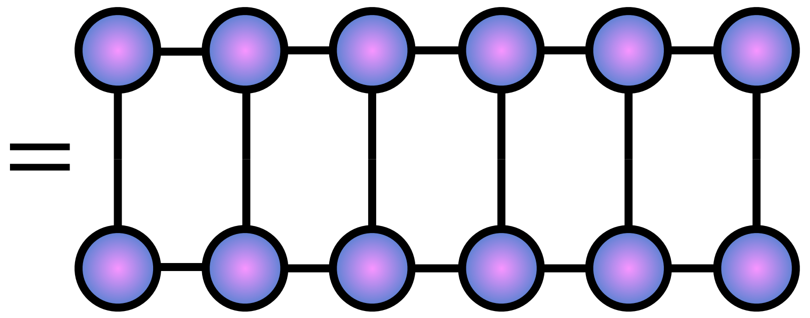




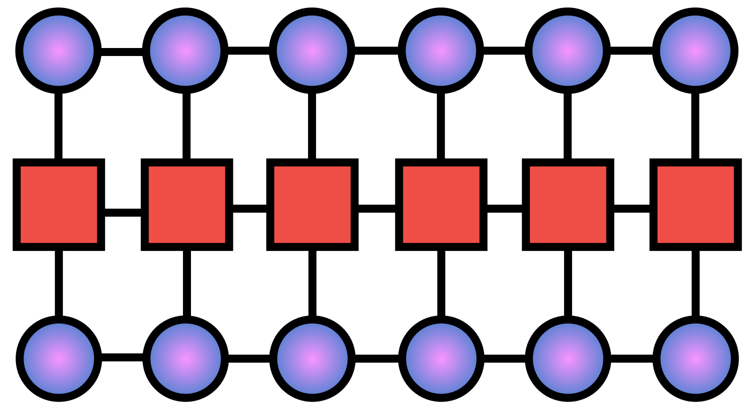
$$\left|\Psi_{\text{MPS}}\right\rangle = \sum_{\alpha_1 \dots \alpha_N} \text{Tr}\left[A_1^{\alpha_1} \dots A_N^{\alpha_N}\right] \left|\alpha_1 \dots \alpha_N\right\rangle$$

MATRIX PRODUCT STATE

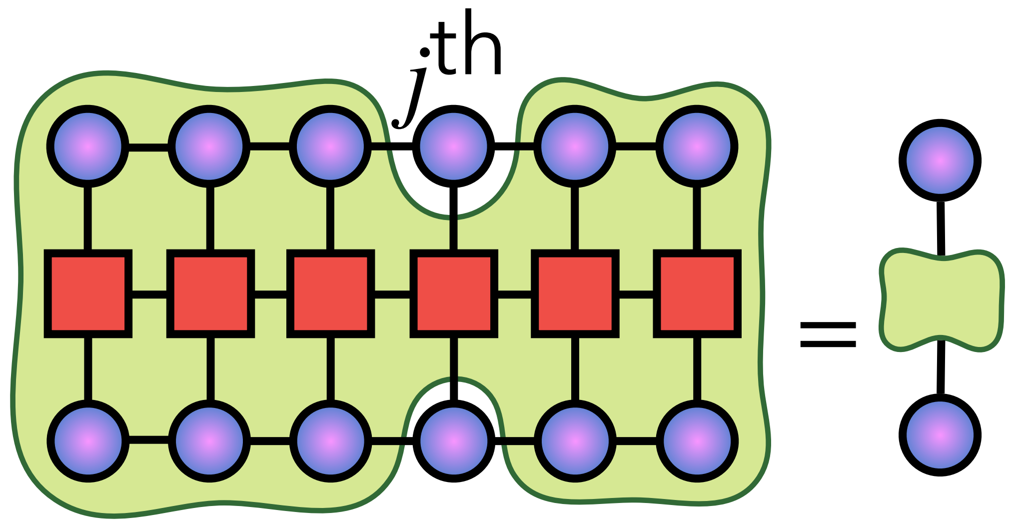
$\langle \Psi | \Psi \rangle =$



$\langle \Psi | H | \Psi \rangle =$



$\langle A_j | H_j^{\text{eff}} | A_j \rangle =$



GROUND-STATE SEARCH VARIATIONAL ALGORITHM

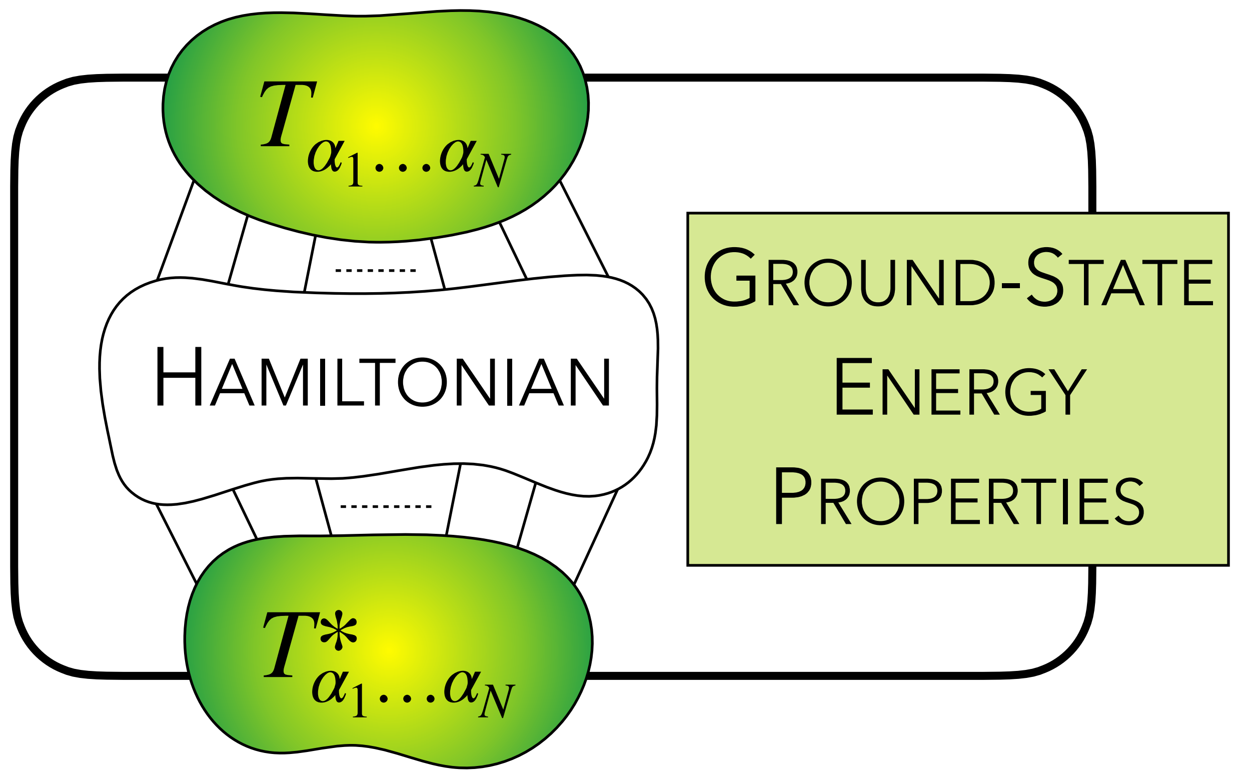
1. Initialize (randomly) $|\Psi\rangle = \{A_1 \dots, A_N\}$
2. Measure $E(0) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$
3. n^{th}Sweep Sequence: $\forall j \in \{1, \dots, N\}$, optimize the tensor A_j and update $|\Psi\rangle$

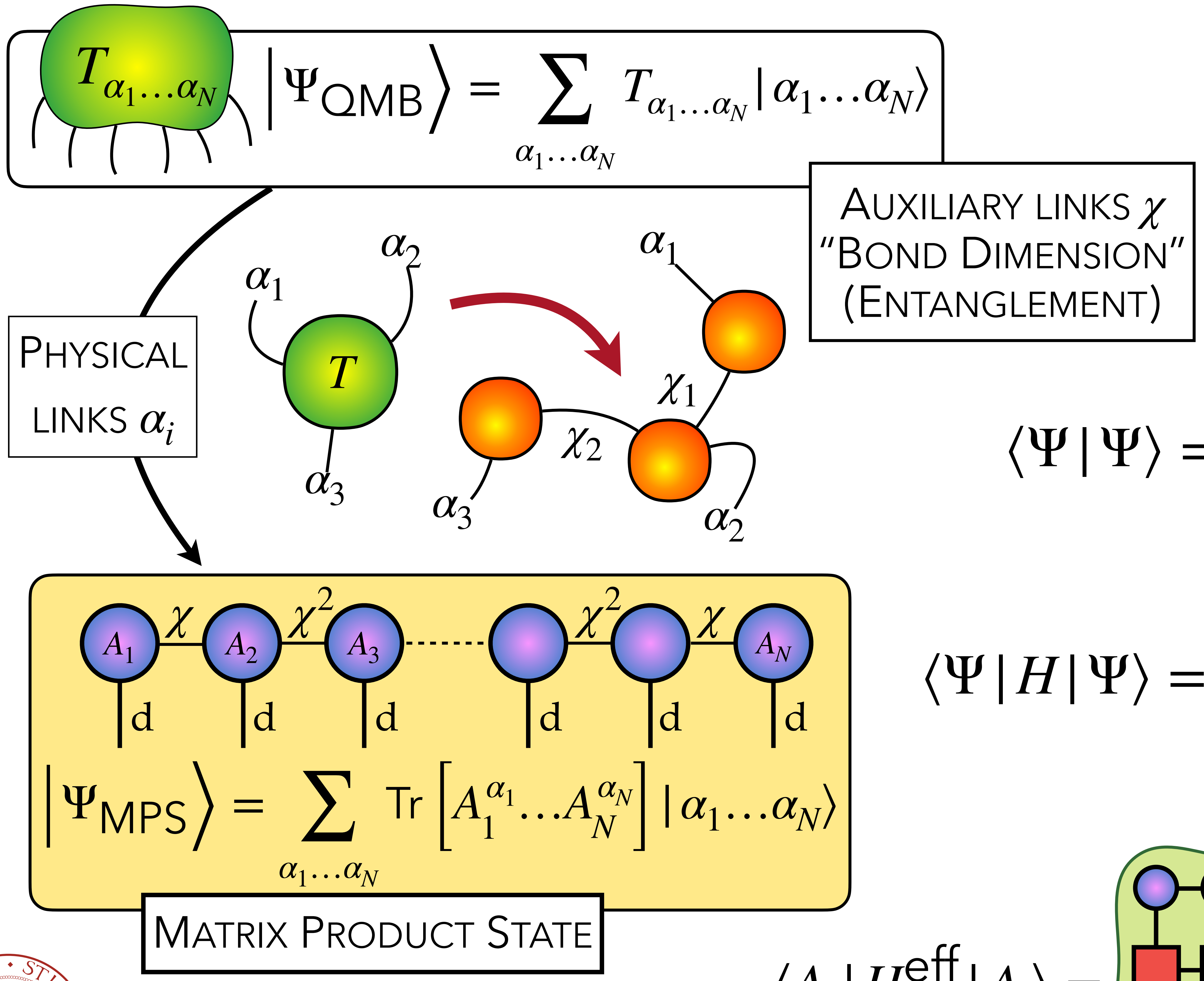
TENSOR NETWORK METHODS

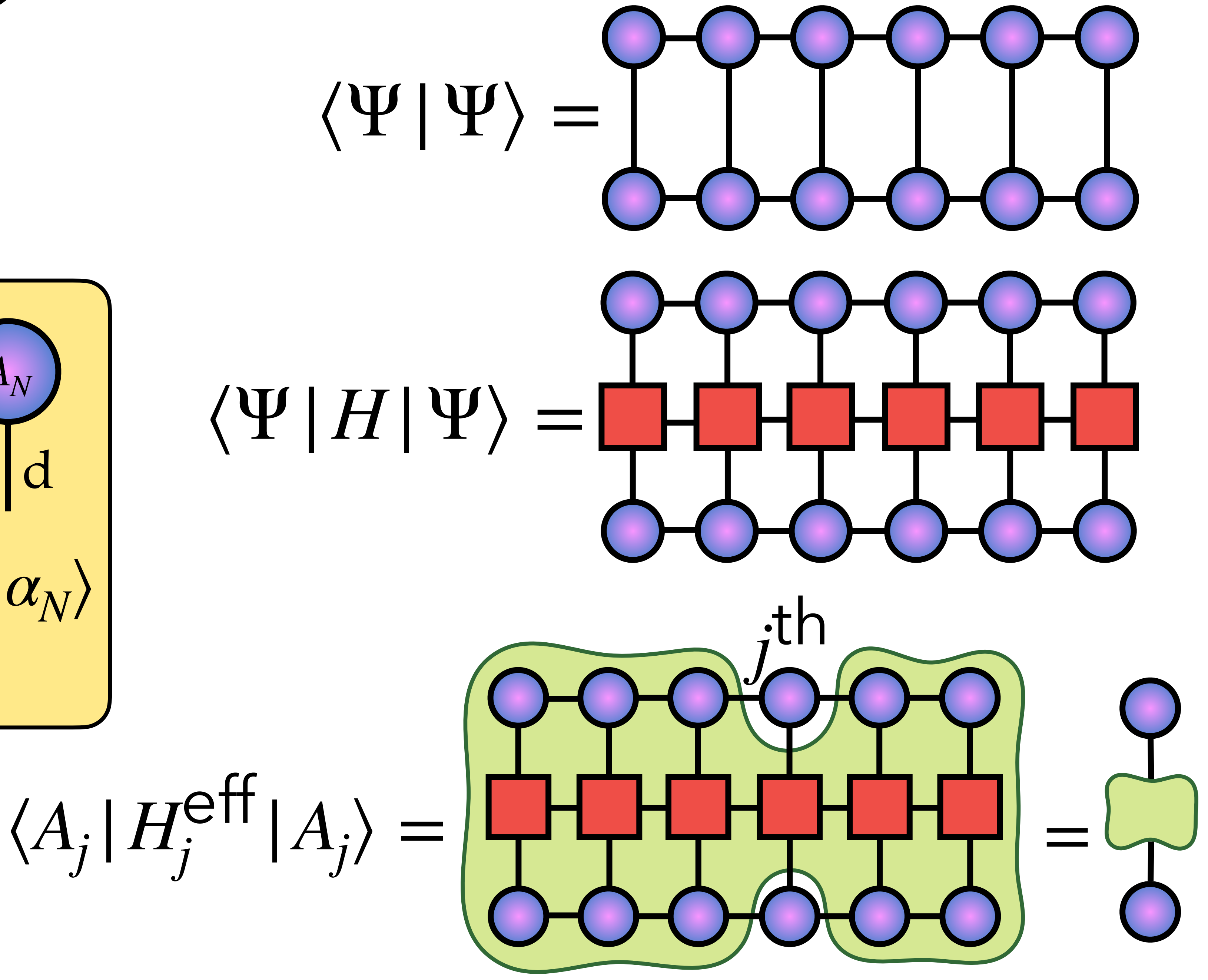
The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

$$\dim \mathscr{H} = d^N$$







GROUND-STATE SEARCH VARIATIONAL ALGORITHM

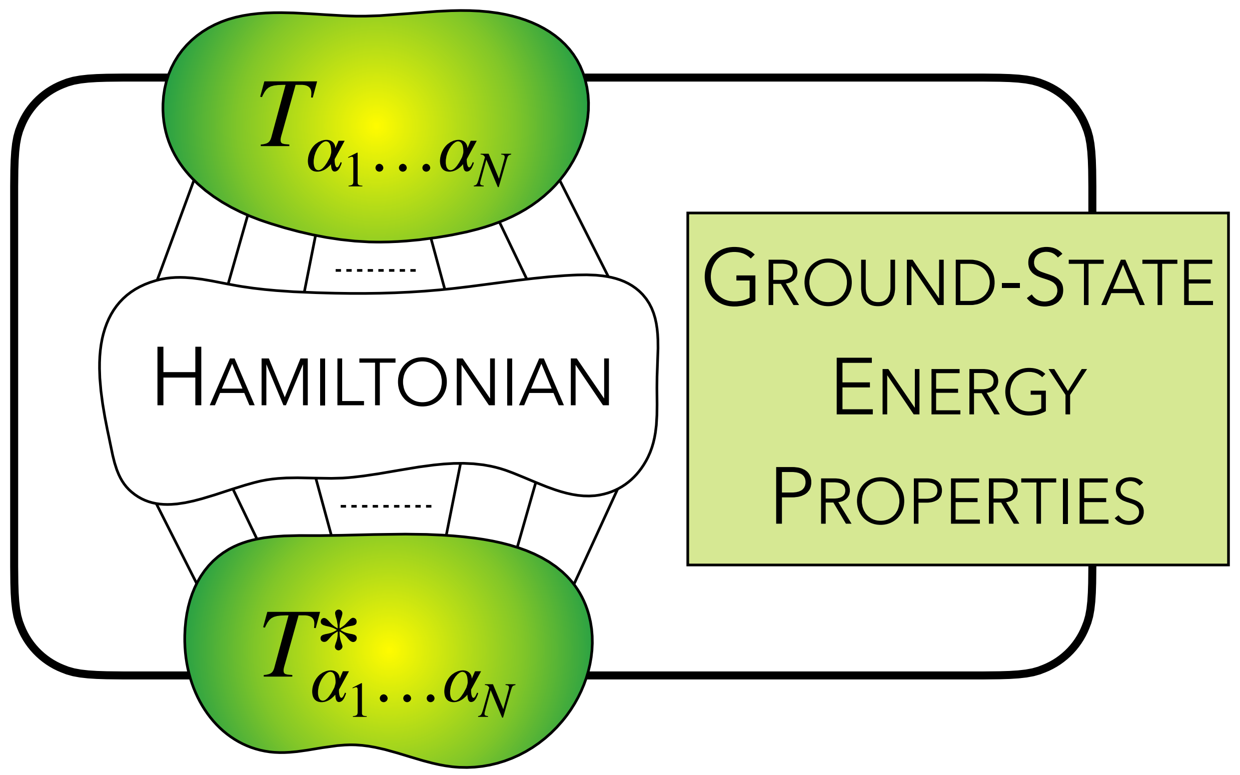
1. Initialize (randomly) $|\Psi\rangle = \{A_1, \dots, A_N\}$
2. Measure $E(0) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$
3. n^{th} Sweep Sequence: $\forall j \in \{1, \dots, N\}$, optimize the tensor A_j and update $|\Psi\rangle$
4. Measure $E(n) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$

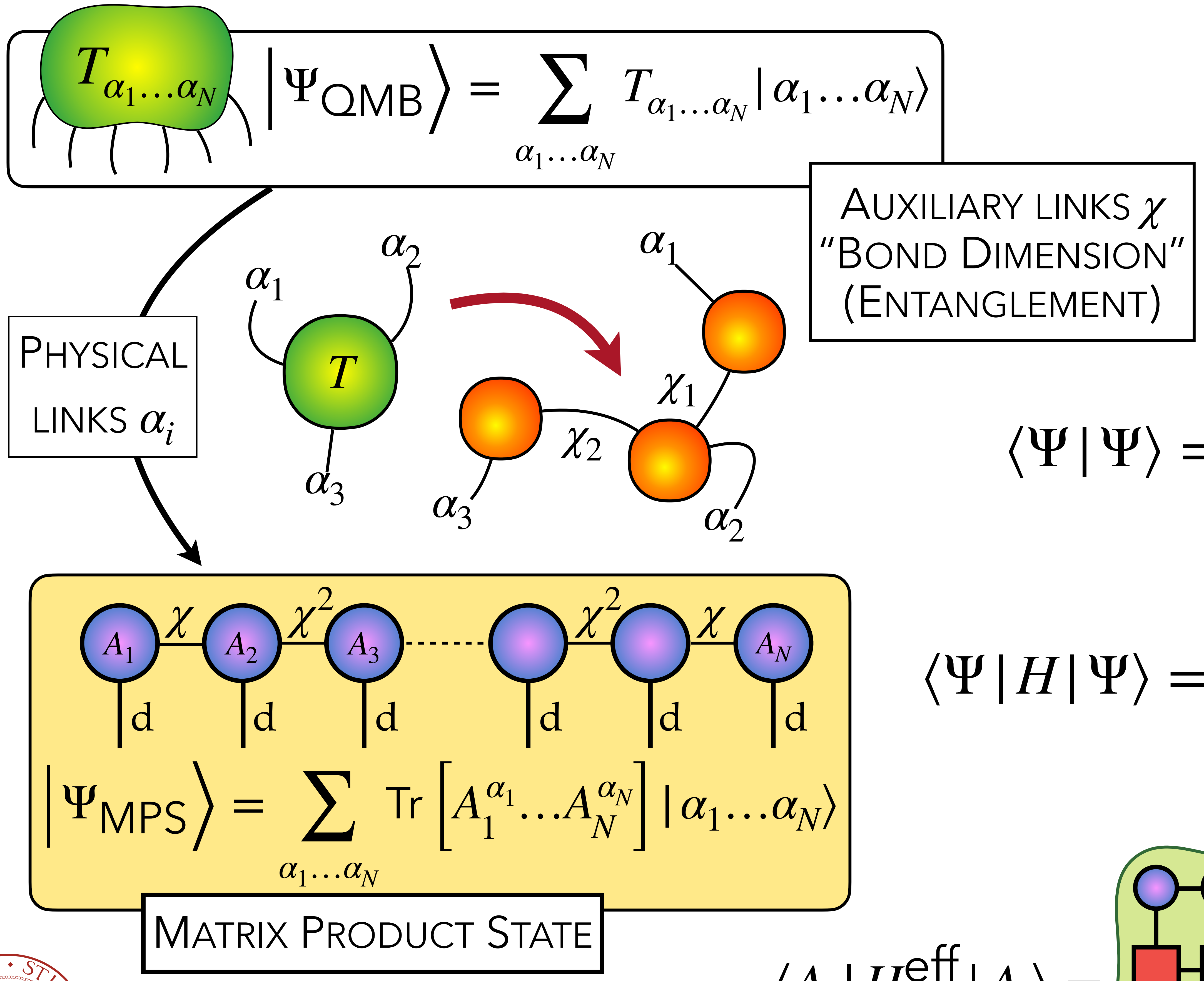
TENSOR NETWORK METHODS

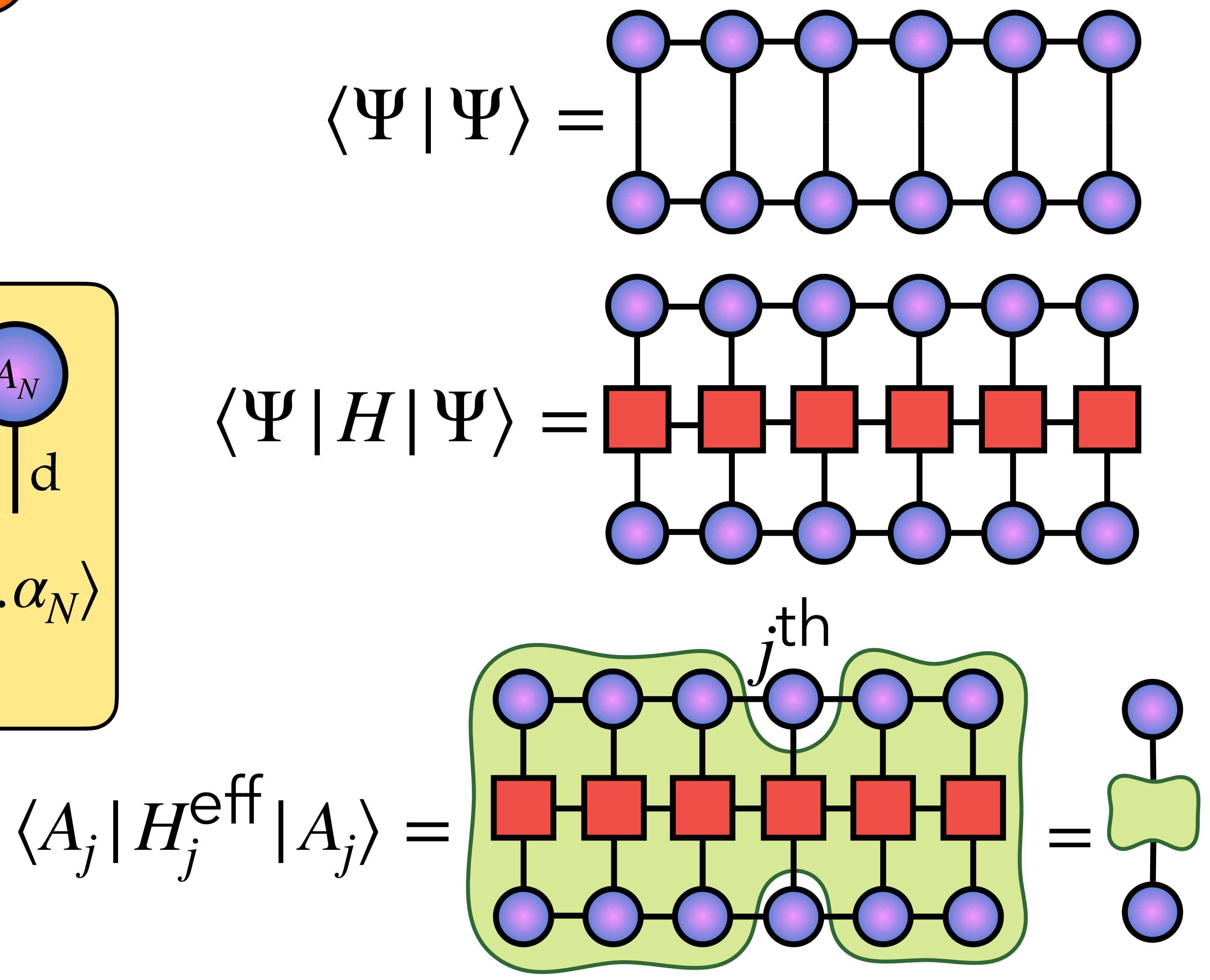
The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

$$\dim\mathcal{H} = d^N$$







GROUND-STATE SEARCH VARIATIONAL ALGORITHM

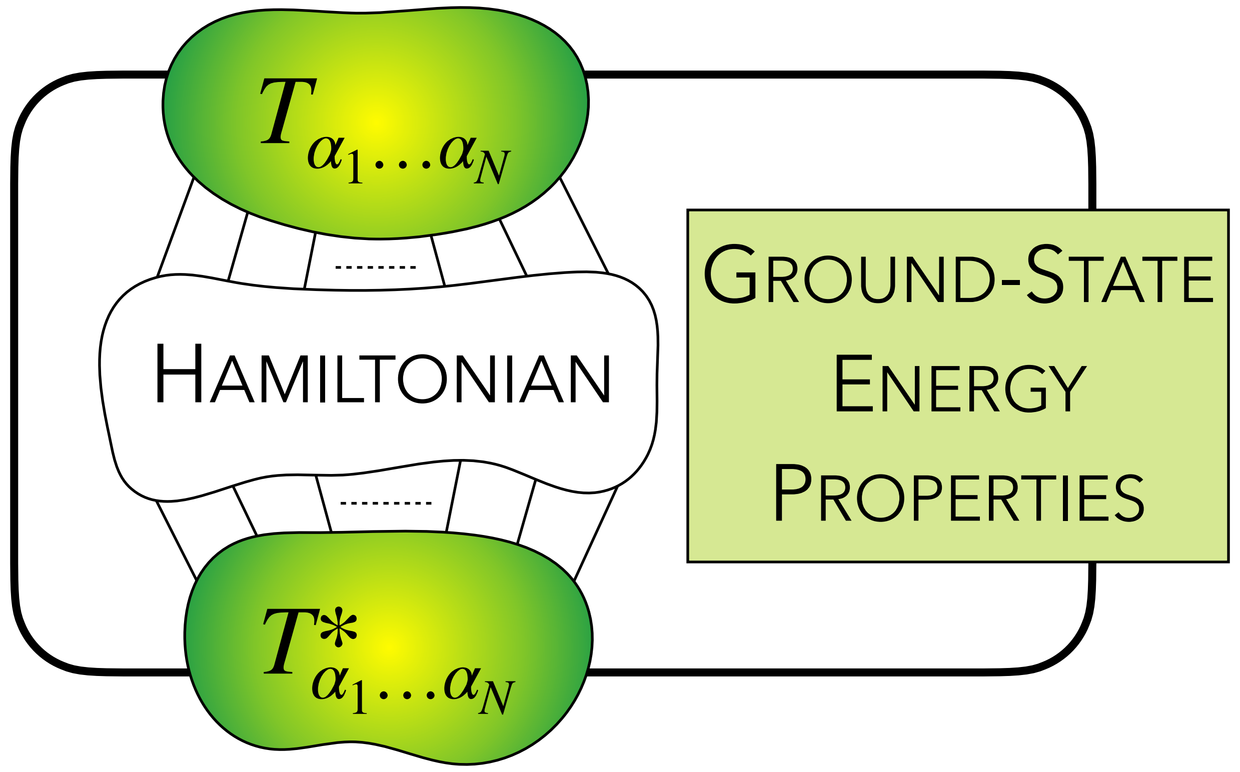
1. Initialize (randomly) $|\Psi\rangle = \{A_1\ldots, A_N\}$
2. Measure $E(0) = \langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle$
3. n^{th}Sweep Sequence: $\forall j \in \{1,\ldots,N\}$, optimize the tensor A_j and update $|\Psi\rangle$
4. Measure $E(n) = \langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle$
5. Compute $\delta E = E(n) - E(n-1)$

TENSOR NETWORK METHODS

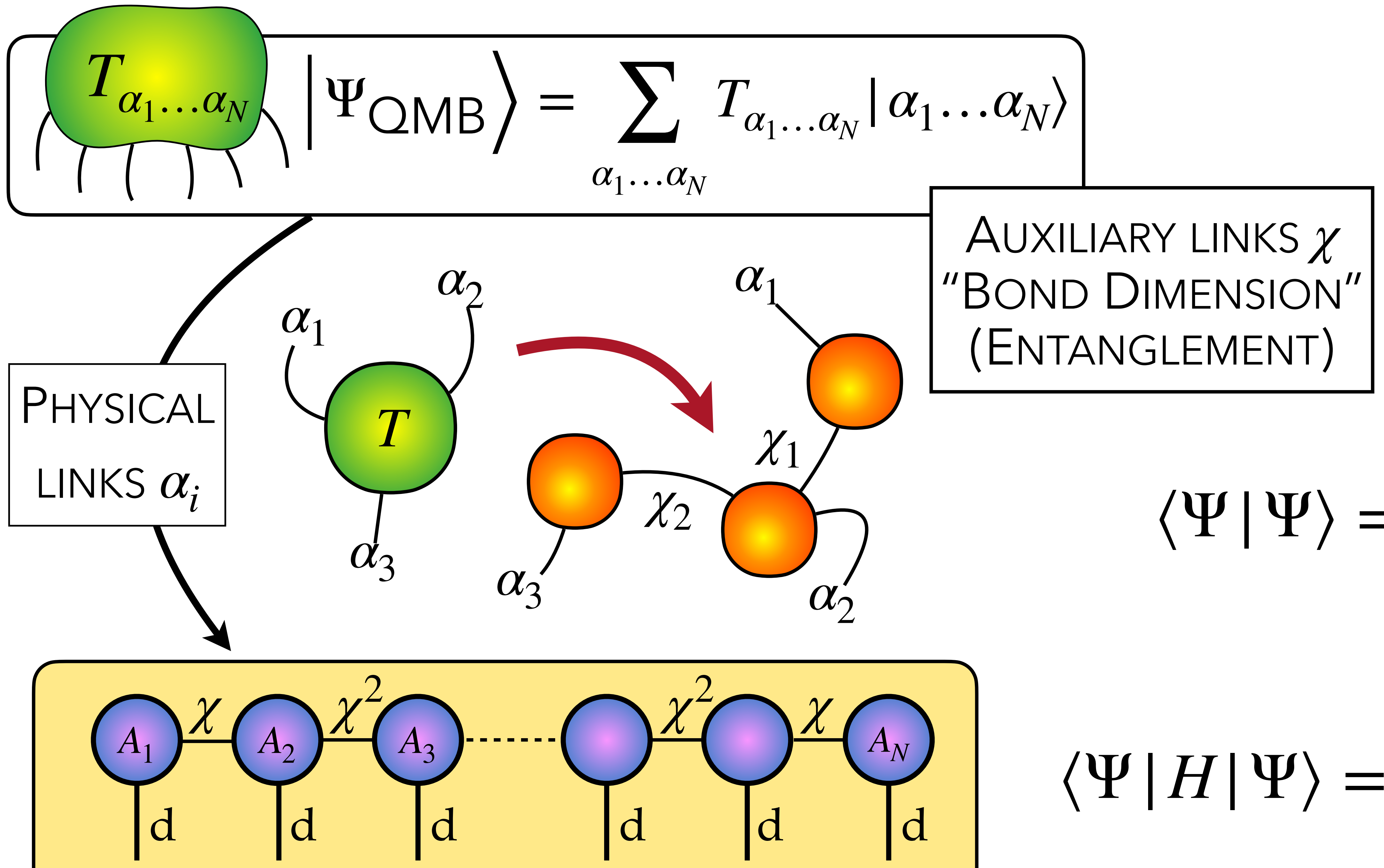
The Hilbert space of N-Body Systems *grows exponentially*.

Exact Diagonalization (ED) is not sustainable for large N

$$\dim \mathscr{H} = d^N$$

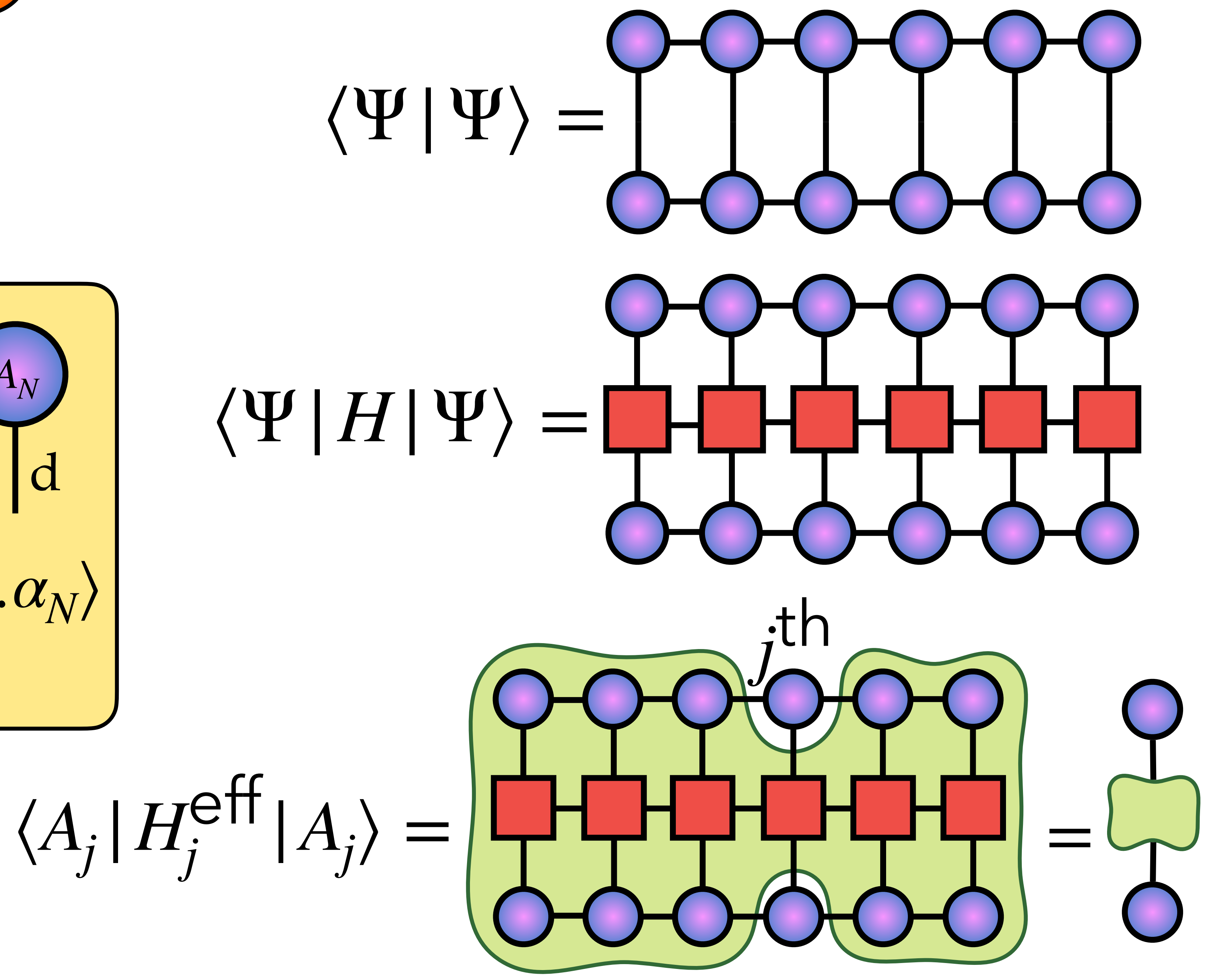


$$|\Psi_{\text{QMB}}\rangle = \sum_{\alpha_1 \dots \alpha_N} T_{\alpha_1 \dots \alpha_N} |\alpha_1 \dots \alpha_N\rangle$$



$$|\Psi_{\text{MPS}}\rangle = \sum_{\alpha_1 \dots \alpha_N} \text{Tr}\left[A_1^{\alpha_1} \dots A_N^{\alpha_N}\right] |\alpha_1 \dots \alpha_N\rangle$$

MATRIX PRODUCT STATE

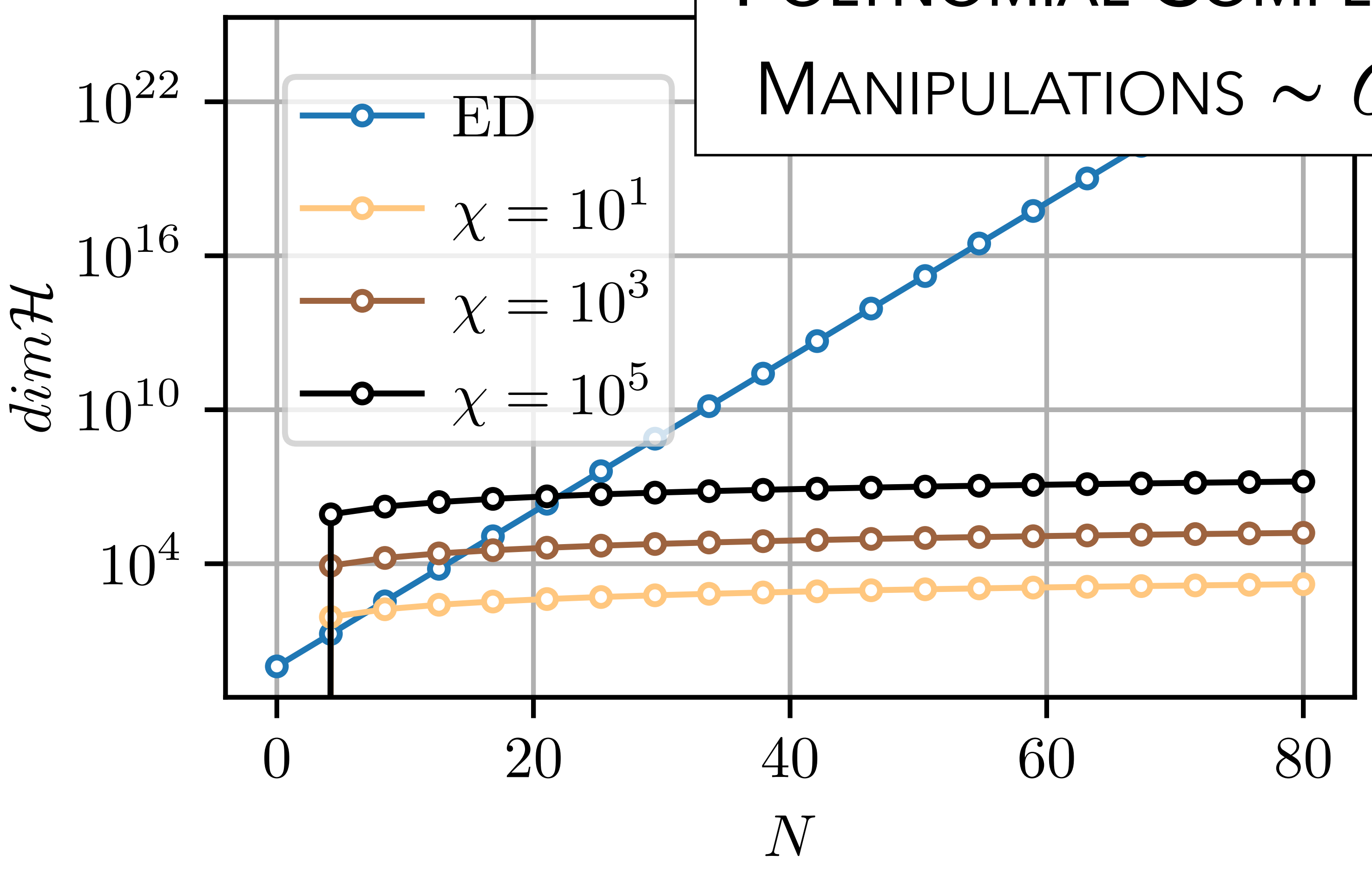


GROUND-STATE SEARCH VARIATIONAL ALGORITHM

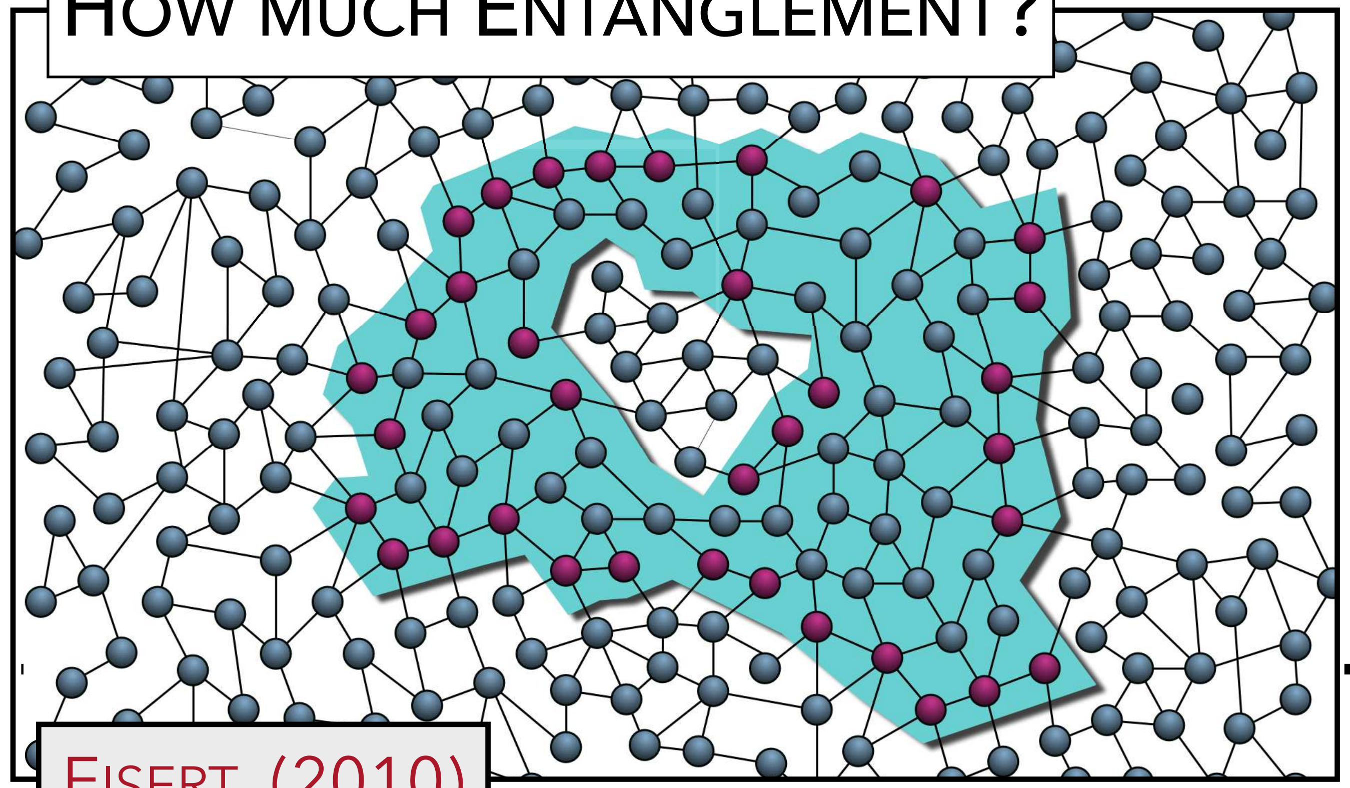
1. Initialize (randomly) $|\Psi\rangle = \{A_1 \dots, A_N\}$
2. Measure $E(0) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$
3. n^{th}Sweep Sequence: $\forall j \in \{1, \dots, N\}$, optimize the tensor A_j and update $|\Psi\rangle$
4. Measure $E(n) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$
5. Compute $\delta E = E(n) - E(n-1)$
6. If $\delta E < \varepsilon$, convergence is reached, otherwise, go back to step 2.

TENSOR NETWORK ANSÄTZE

Polynomial Complexity:
Manipulations $\sim \mathcal{O}(\chi^k)$



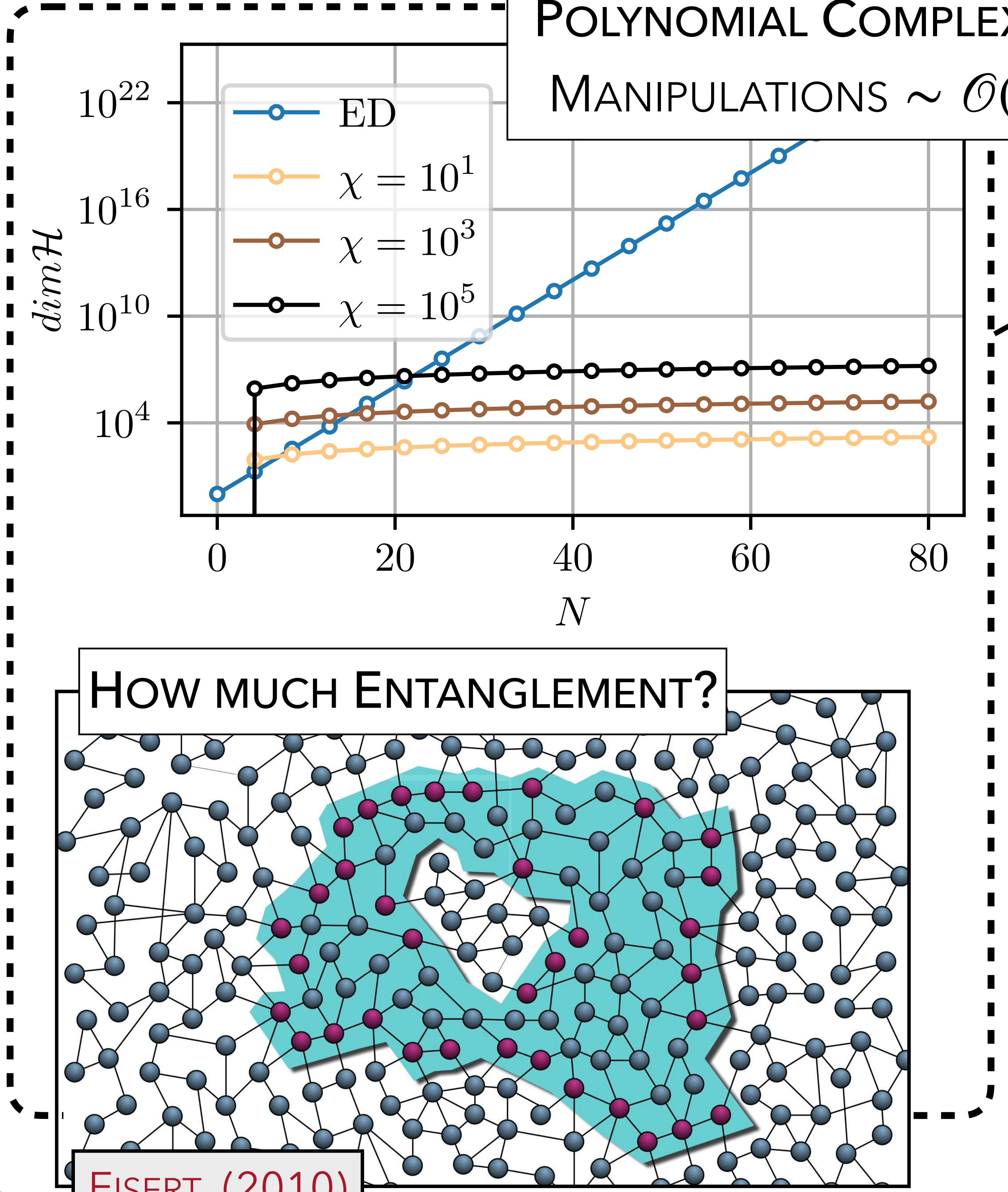
How much Entanglement?



Eisert, (2010)

TENSOR NETWORK ANSÄTZE

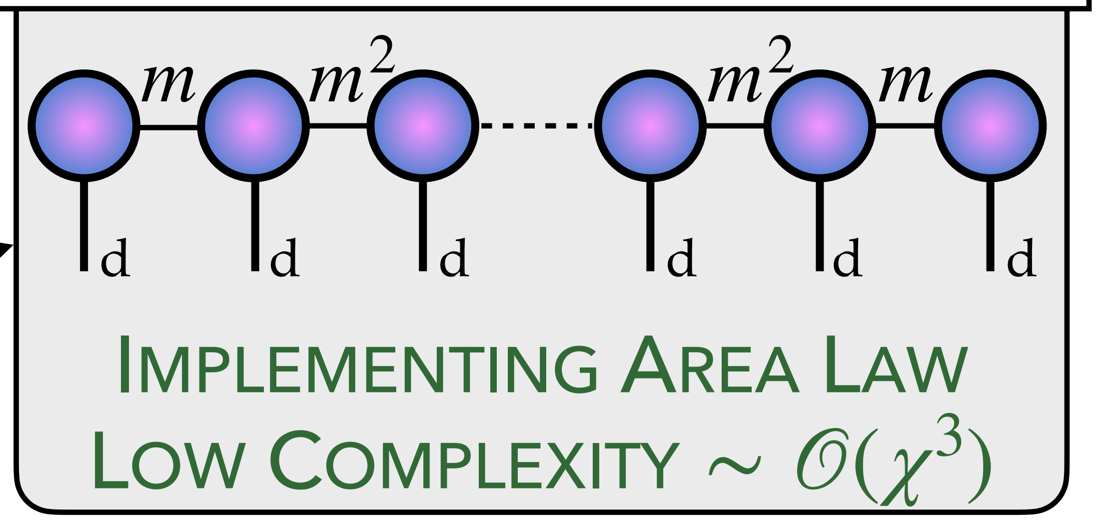
POLYNOMIAL COMPLEXITY:
MANIPULATIONS $\sim \mathcal{O}(\chi^k)$



HOW MUCH ENTANGLEMENT?

EISERT, (2010)

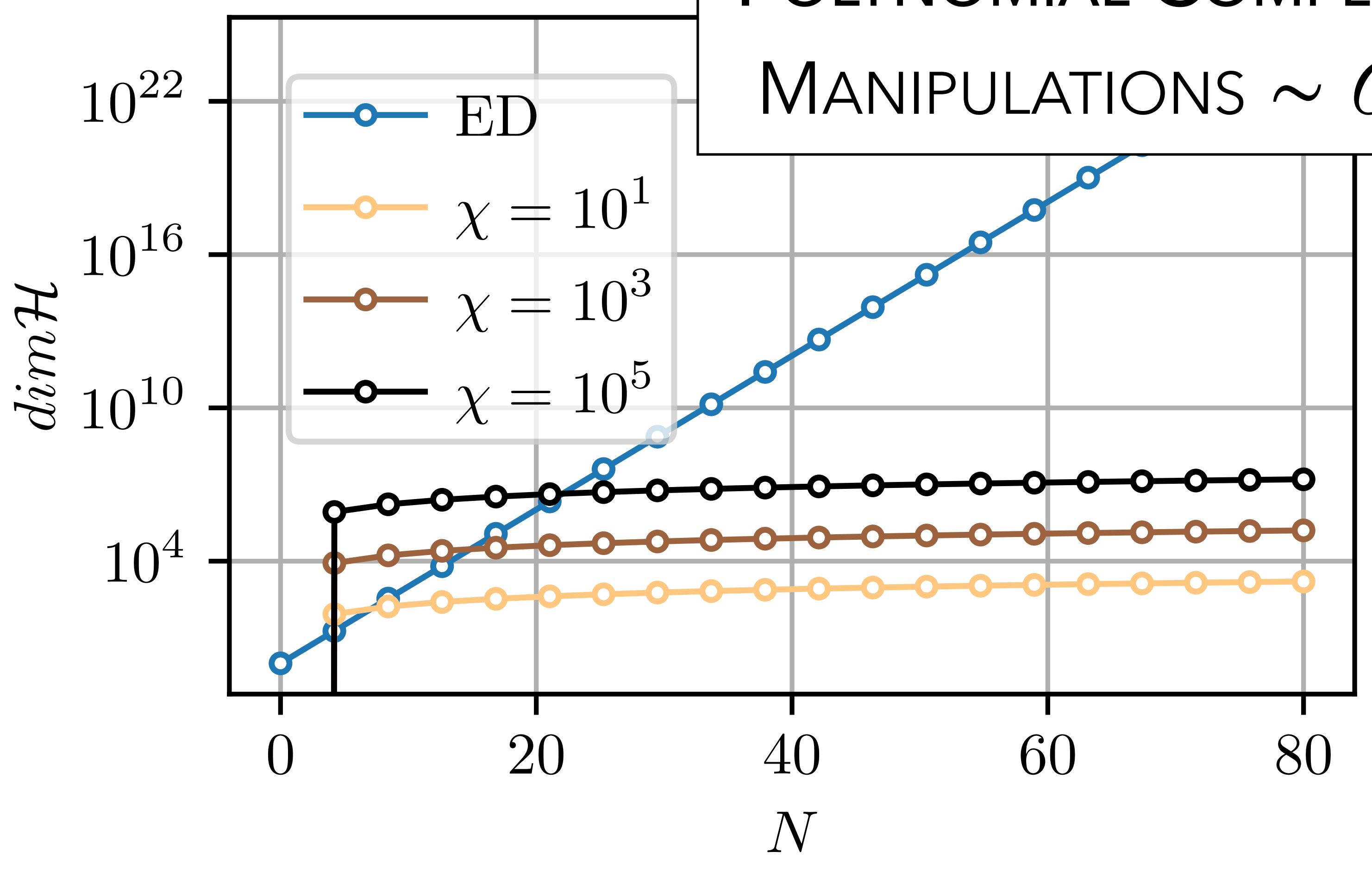
1D MATRIX PRODUCT STATES (MPS)



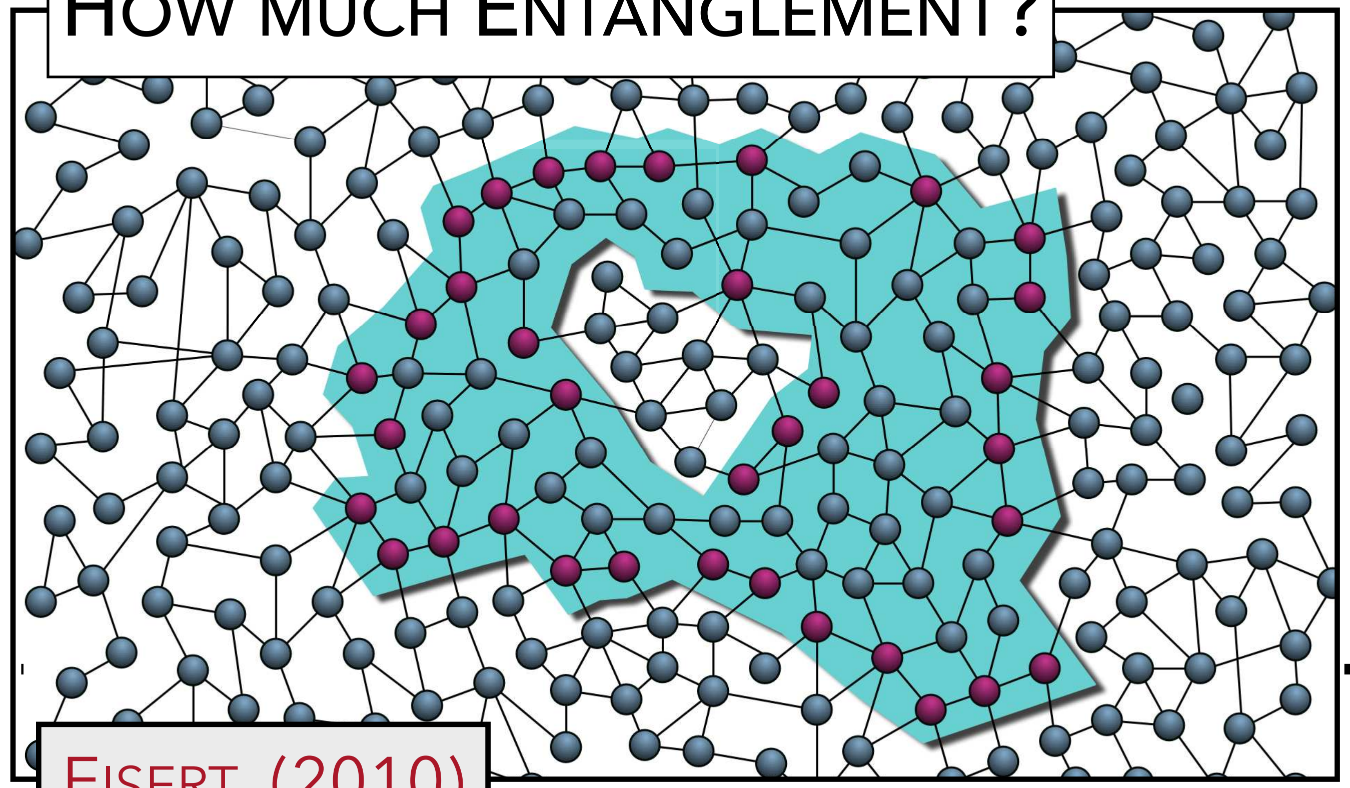
IMPLEMENTING AREA LAW
LOW COMPLEXITY $\sim \mathcal{O}(\chi^3)$

TENSOR NETWORK ANSÄTZE

POLYNOMIAL COMPLEXITY:
MANIPULATIONS $\sim \mathcal{O}(\chi^k)$

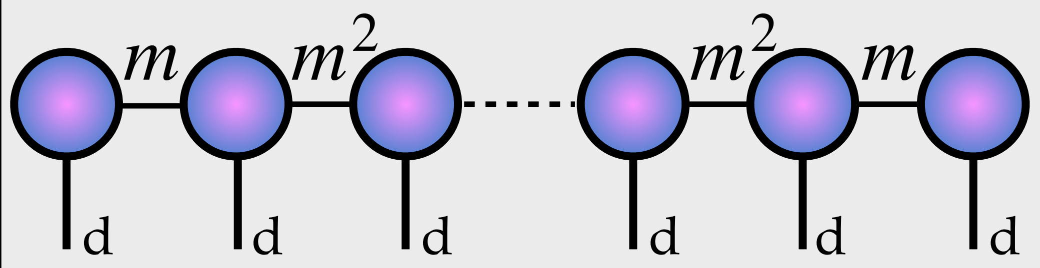


HOW MUCH ENTANGLEMENT?



EISERT, (2010)

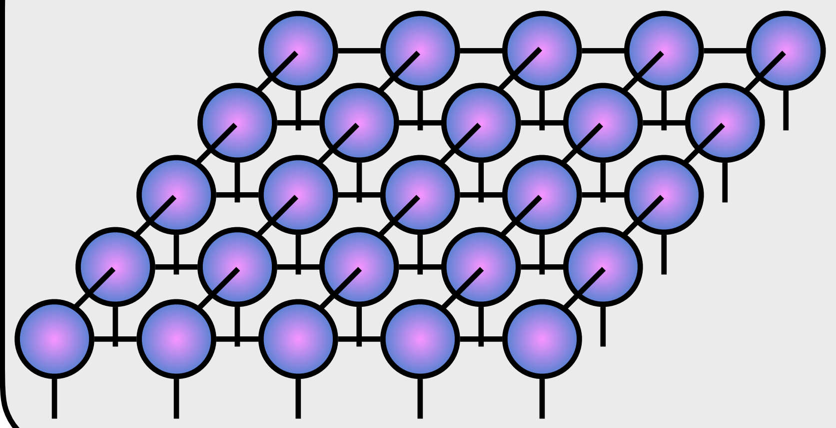
1D MATRIX PRODUCT STATES (MPS)



IMPLEMENTING AREA LAW
LOW COMPLEXITY $\sim \mathcal{O}(\chi^3)$

PROJECTED ENTANGLED PAIR STATES (PEPS)

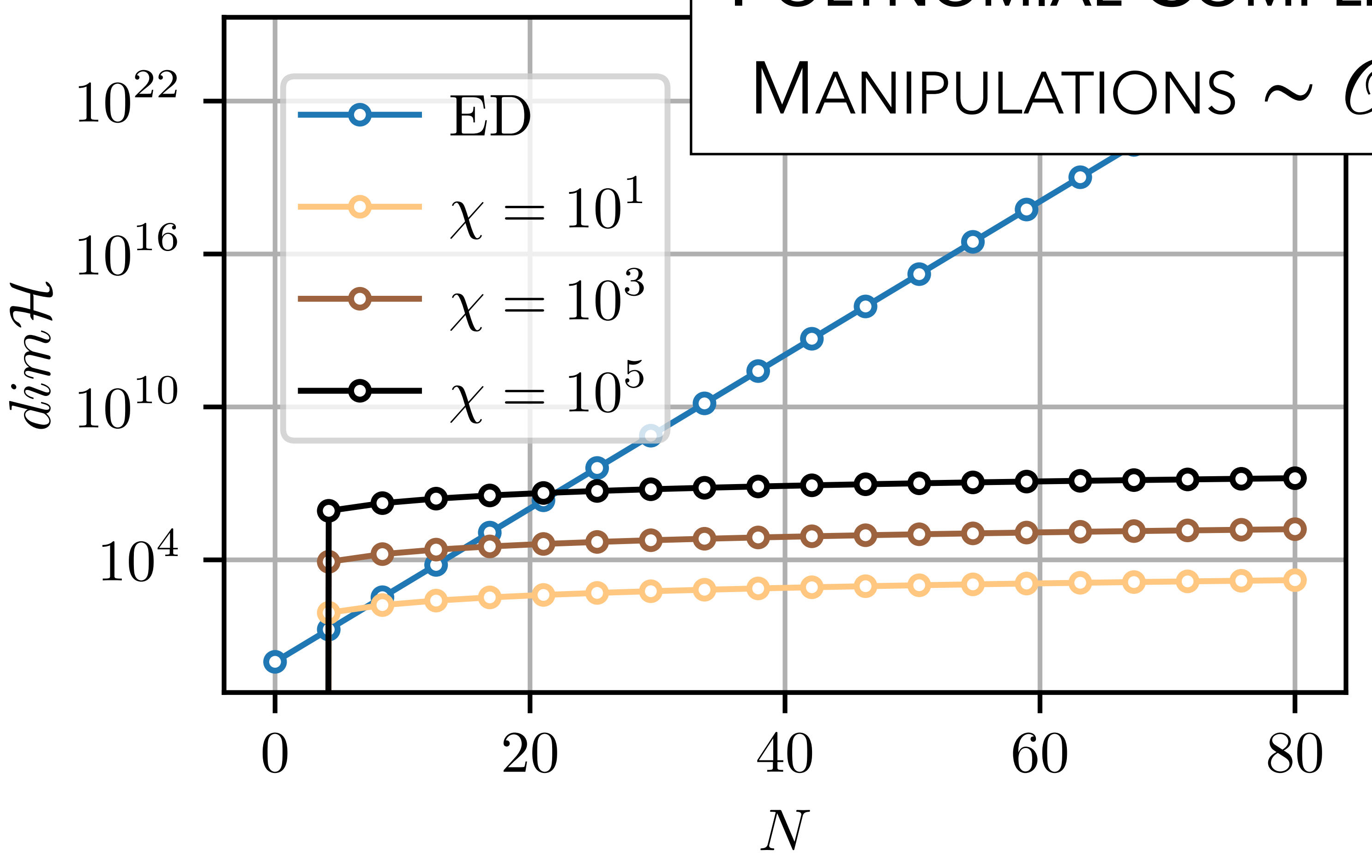
2D



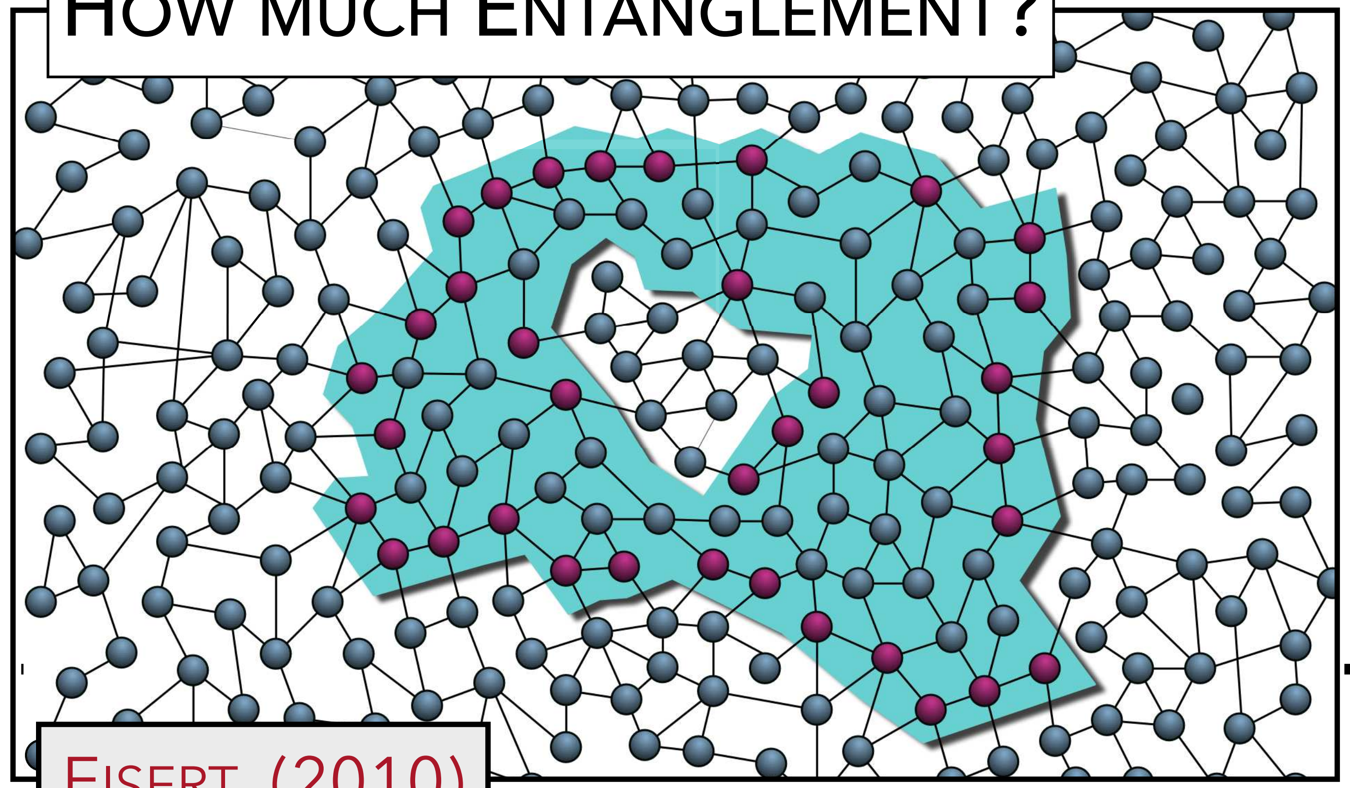
NATURALLY IMPLEMENTING ENTANGLEMENT AREA LAW
TOO HIGH ALGORITHMIC COMPLEXITY $\sim \mathcal{O}(\chi^8)$

TENSOR NETWORK ANSÄTZE

POLYNOMIAL COMPLEXITY:
MANIPULATIONS $\sim \mathcal{O}(\chi^k)$

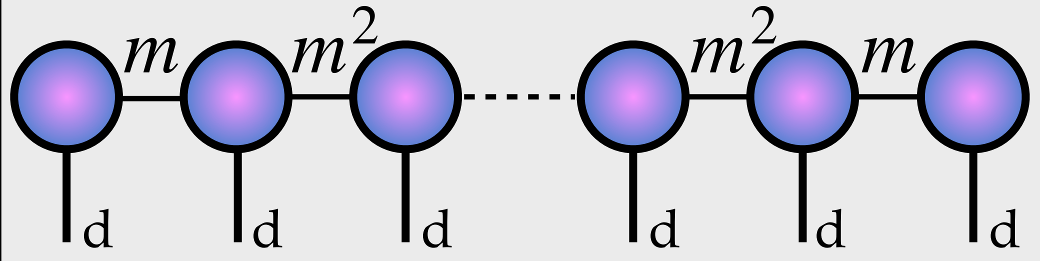


HOW MUCH ENTANGLEMENT?



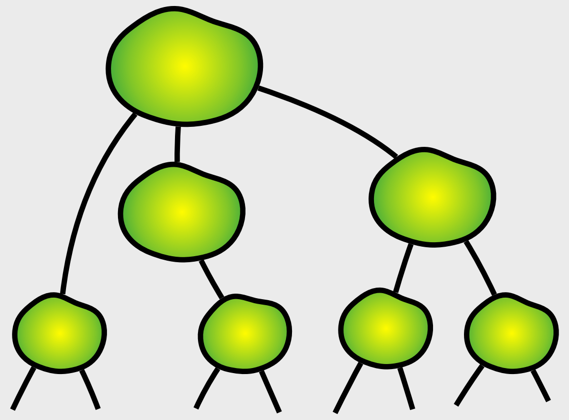
EISERT, (2010)

1D **MATRIX PRODUCT STATES (MPS)**



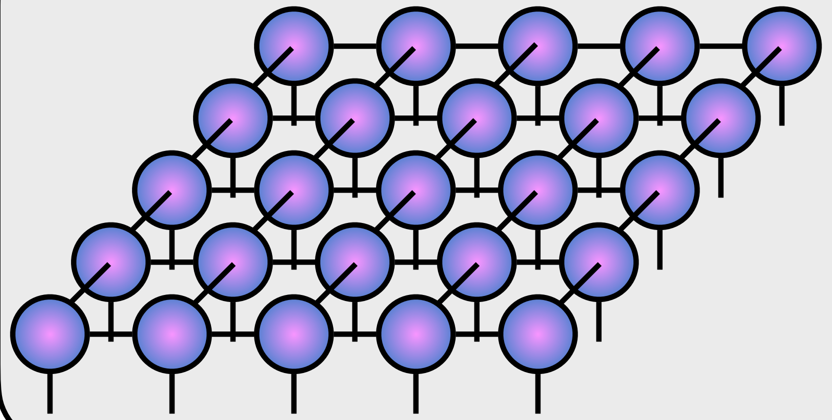
IMPLEMENTING AREA LAW
LOW COMPLEXITY $\sim \mathcal{O}(\chi^3)$

TREE TENSOR NETWORKS (TTN)



NOT IMPLEMENTING AREA LAW
LOW ALGORITHMIC COMPLEXITY $\sim \mathcal{O}(\chi^4)$

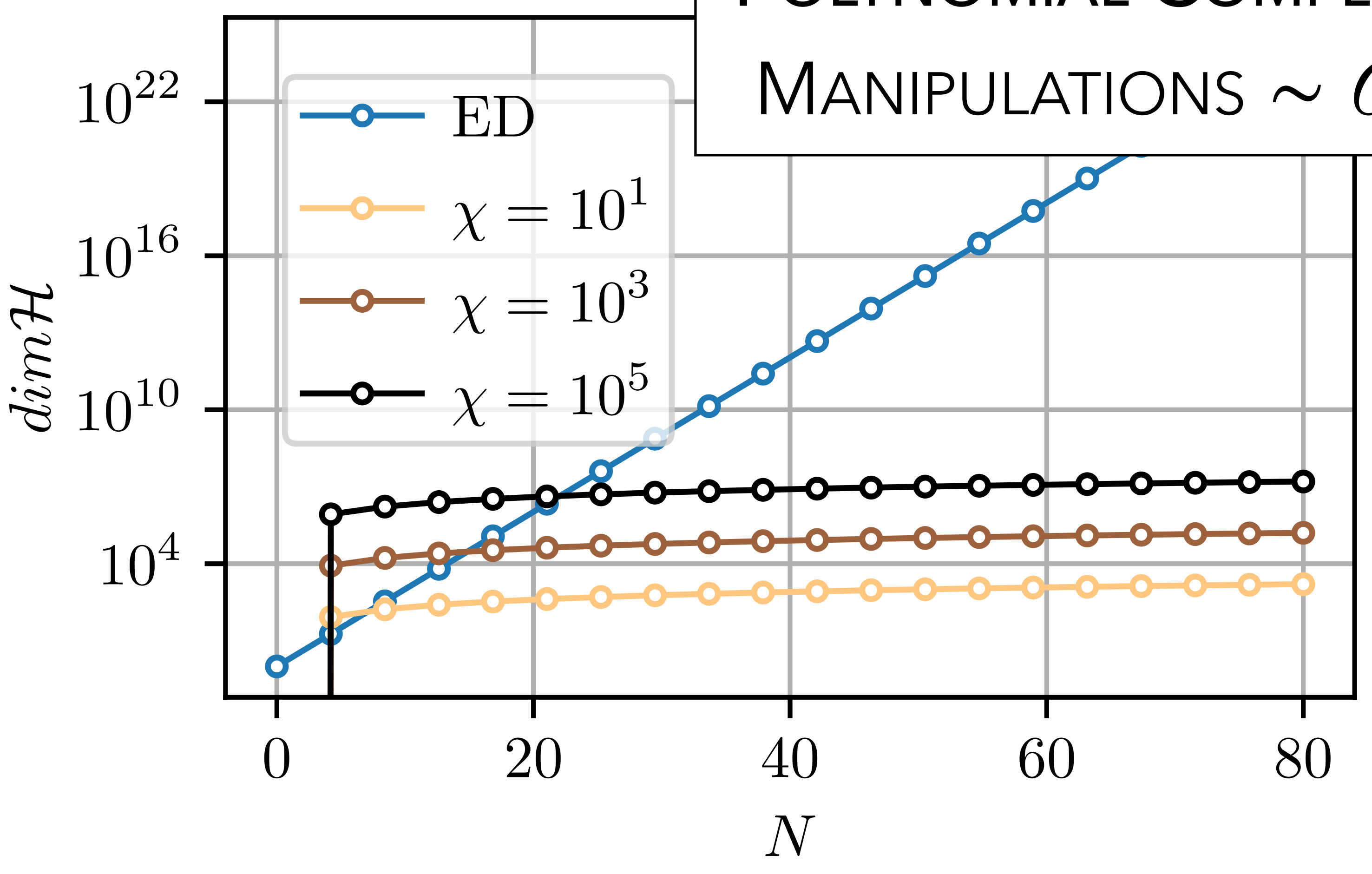
2D **PROJECTED ENTANGLED PAIR STATES (PEPS)**



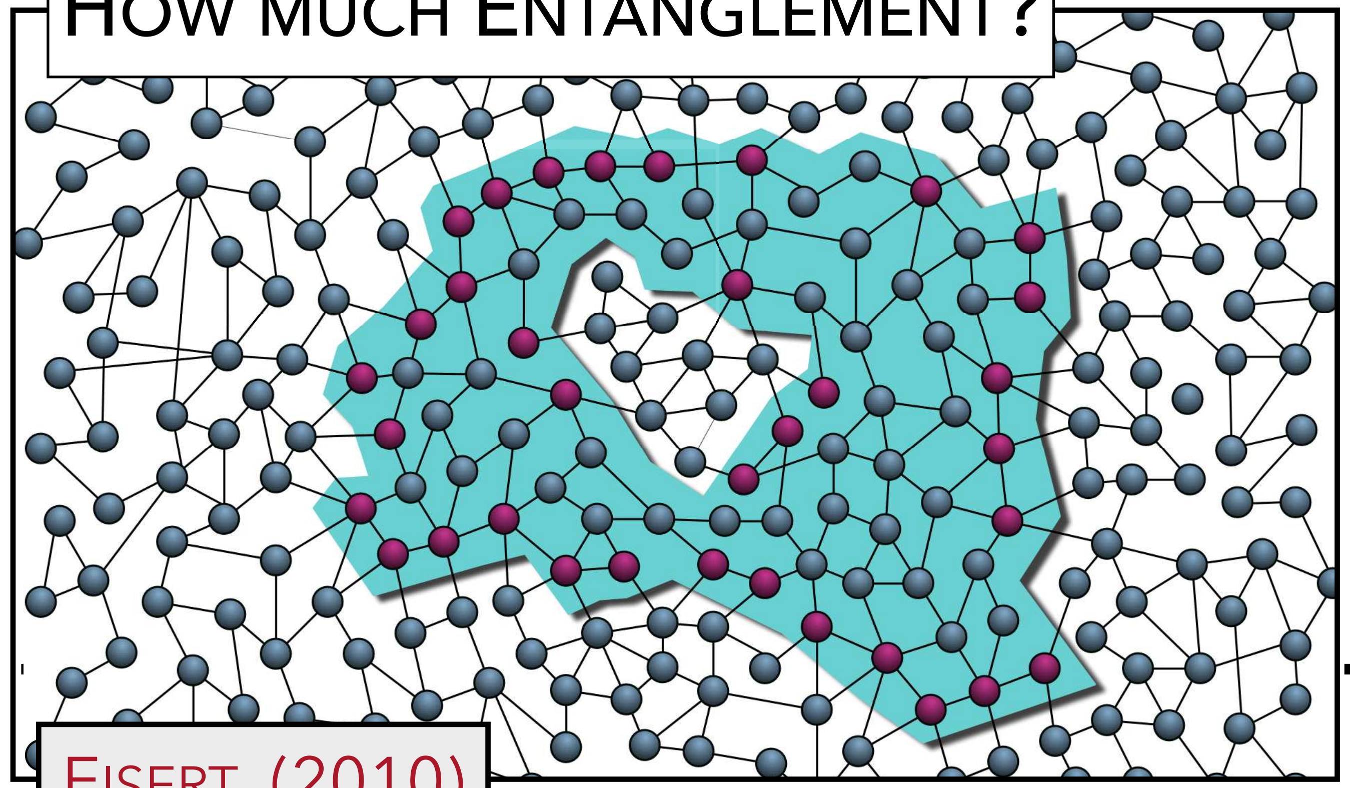
NATURALLY IMPLEMENTING ENTANGLEMENT AREA LAW
TOO HIGH ALGORITHMIC COMPLEXITY $\sim \mathcal{O}(\chi^8)$

TENSOR NETWORK ANSÄTZE

POLYNOMIAL COMPLEXITY:
MANIPULATIONS $\sim \mathcal{O}(\chi^k)$

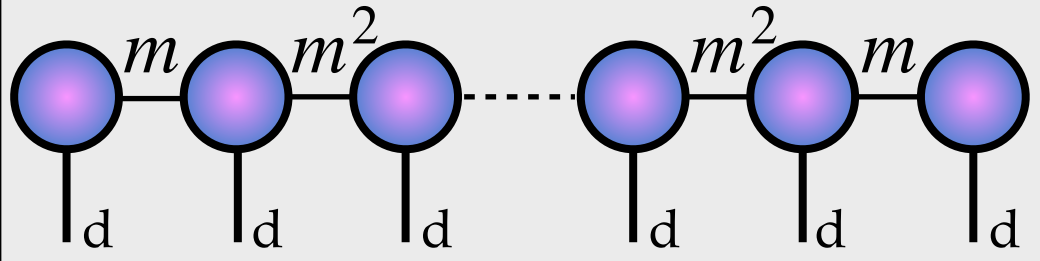


HOW MUCH ENTANGLEMENT?



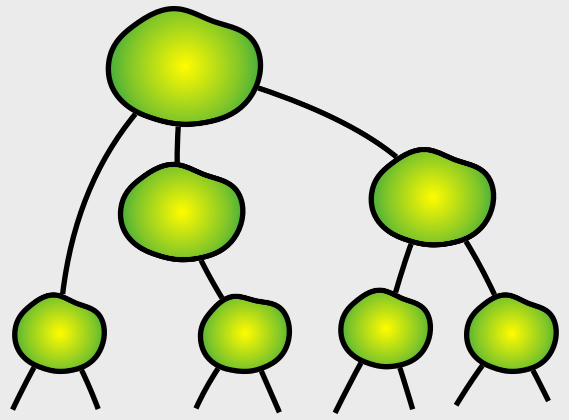
EISERT, (2010)

1D — MATRIX PRODUCT STATES (MPS)



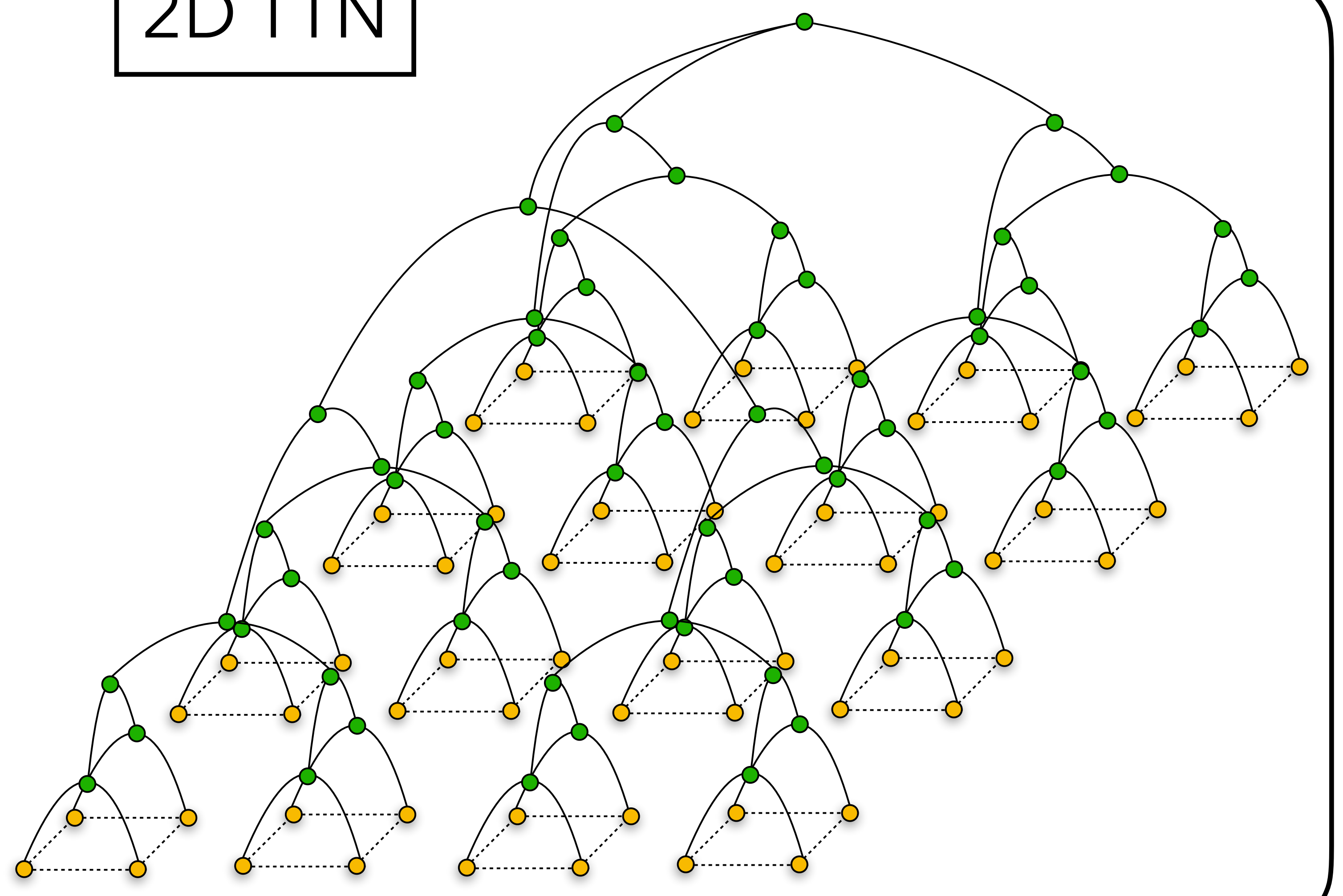
IMPLEMENTING AREA LAW
LOW COMPLEXITY $\sim \mathcal{O}(\chi^3)$

TREE TENSOR NETWORKS (TTN)

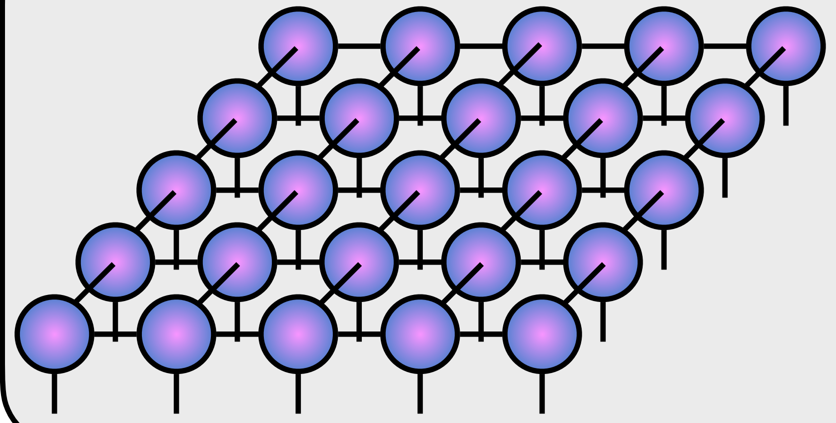


NOT IMPLEMENTING
AREA LAW
LOW ALGORITHMIC
COMPLEXITY $\sim \mathcal{O}(\chi^4)$

2D TTN



2D — PROJECTED ENTANGLED PAIR STATES (PEPS)



NATURALLY IMPLEMENTING
ENTANGLEMENT AREA LAW
TOO HIGH ALGORITHMIC
COMPLEXITY $\sim \mathcal{O}(\chi^8)$

SU(2) GAUGE LINKS VIA RISHONS

Zohar, Cirac (2018)

Zohar, Cirac (2019)

Express the parallel transporter with fermionic rishons

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

SU(2) GAUGE LINKS VIA RISHONS

Zohar, Cirac (2018)

Zohar, Cirac (2019)

Express the parallel transporter with fermionic rishons

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F$$

$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

SU(2) GAUGE LINKS VIA RISHONS

Zohar, Cirac (2018)

Zohar, Cirac (2019)

Express the parallel transporter with fermionic rishons

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F$$

$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

Rishon Parity

$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array} \right)$$

0 (integer) SU(2) Representation

1/2 (semi-integer) SU(2) Representation

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F$$

$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

SU(2) HAS A QUASI-REAL REPRESENTATION

RISHON PARITY

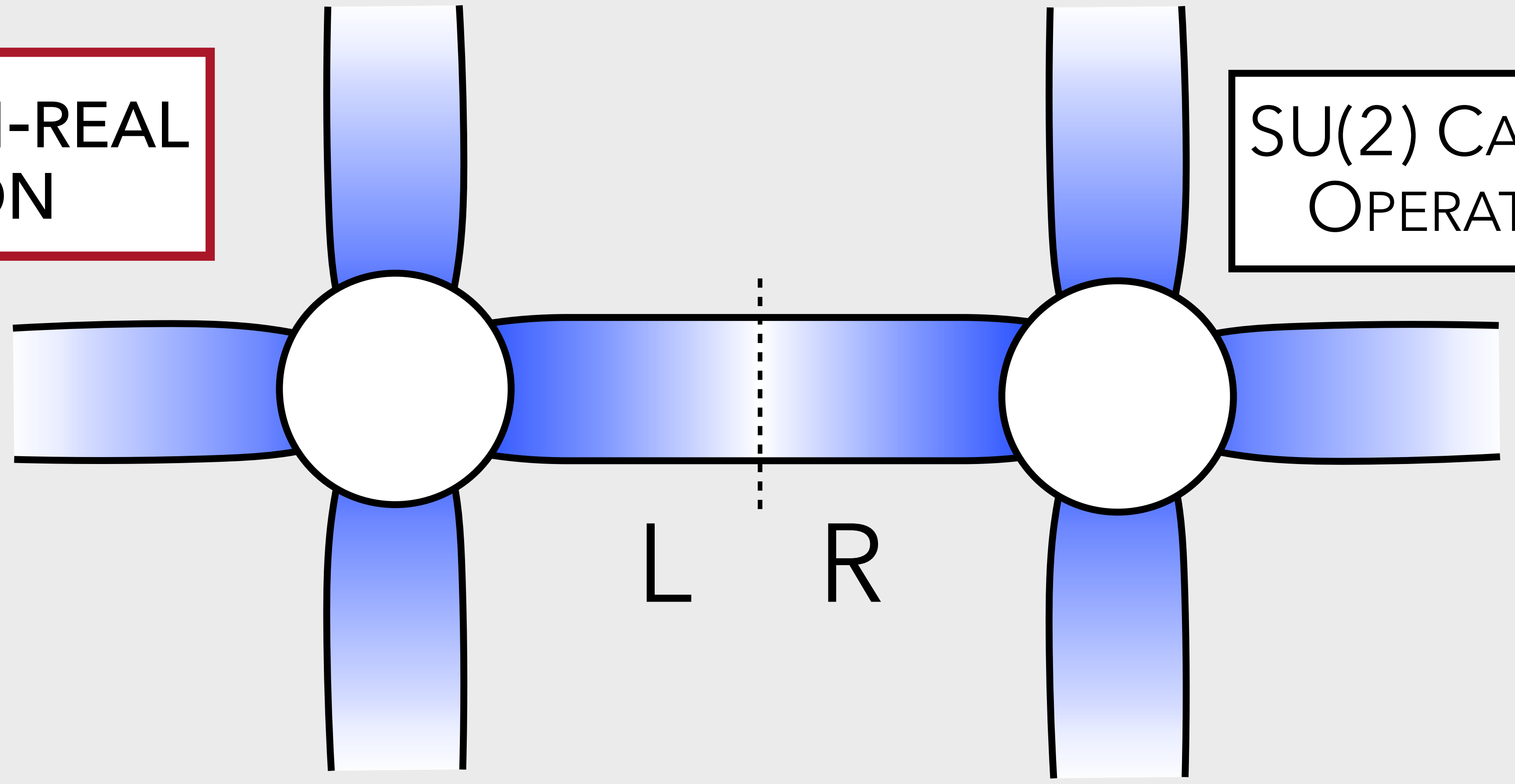
$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array} \right)$$

0 (INTEGER) SU(2) REPRESENTATION

1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$



SU(2) CASIMIR OPERATOR

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)
ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F$$

$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

SU(2) HAS A QUASI-REAL REPRESENTATION

RISHON PARITY

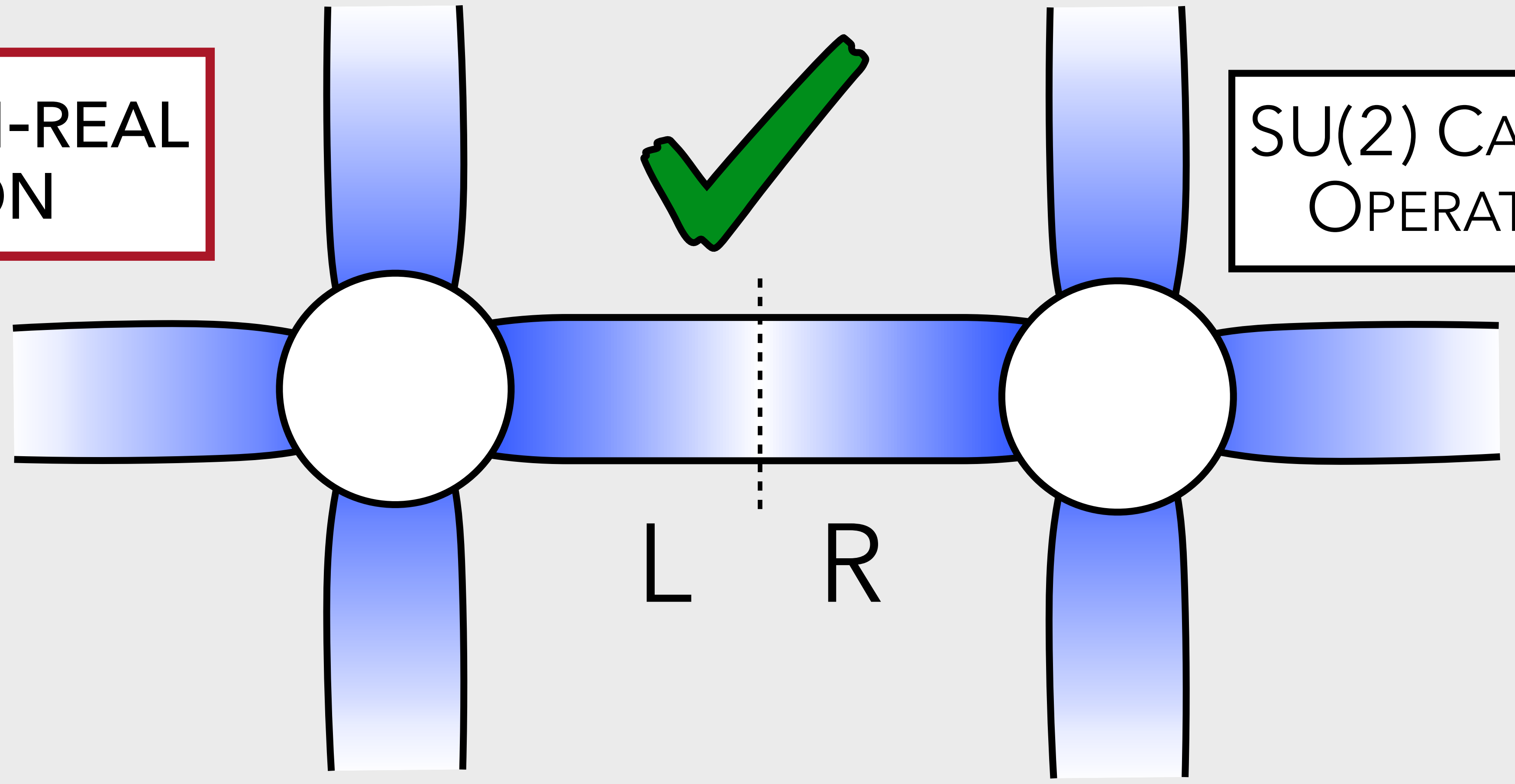
$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array}\right)$$

0 (INTEGER) SU(2) REPRESENTATION

1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$



SU(2) CASIMIR OPERATOR

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F$$

$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

RISHON PARITY

$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array} \right)$$

0 (INTEGER) SU(2) REPRESENTATION

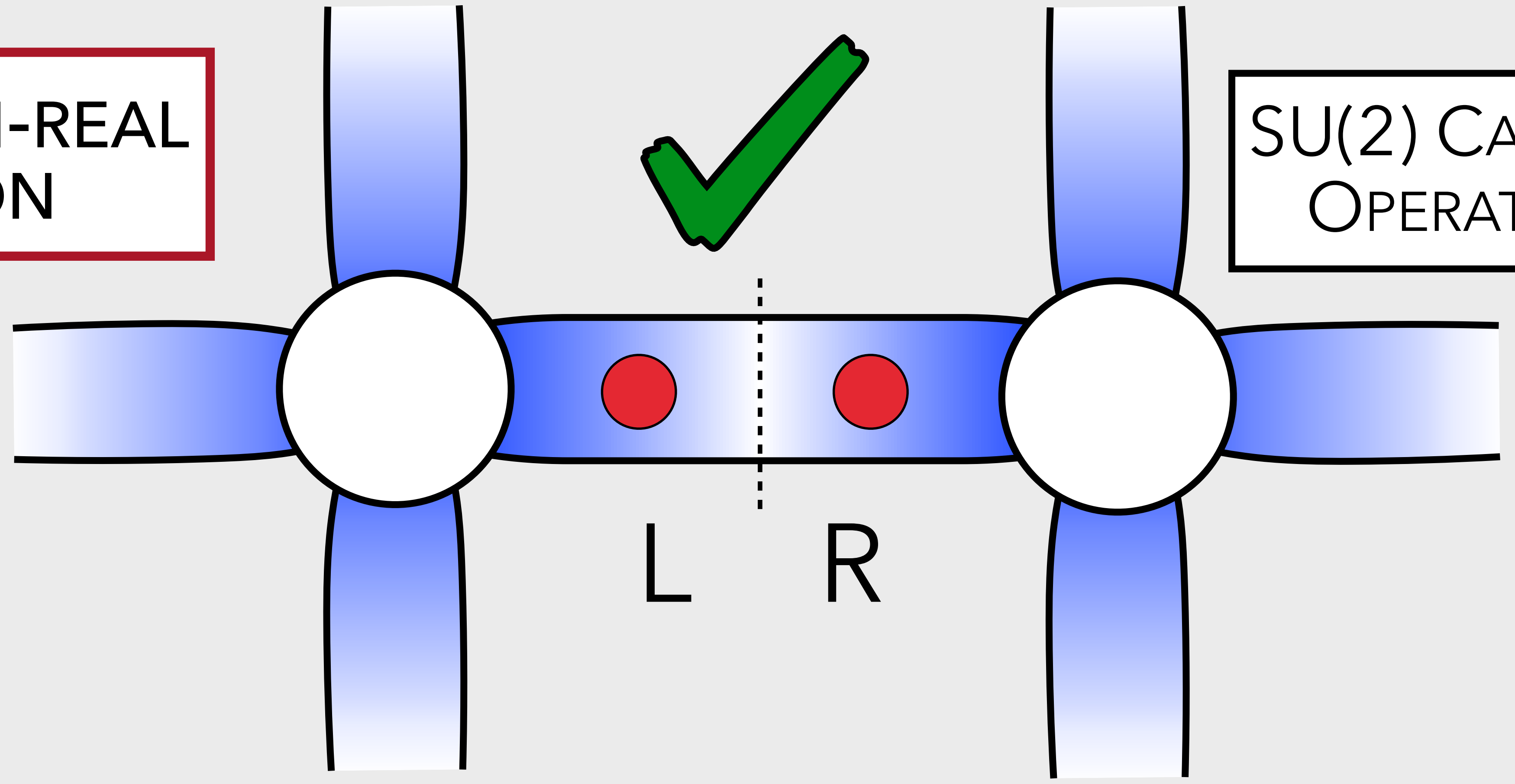
1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

SU(2) HAS A QUASI-REAL REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) CASIMIR OPERATOR



SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

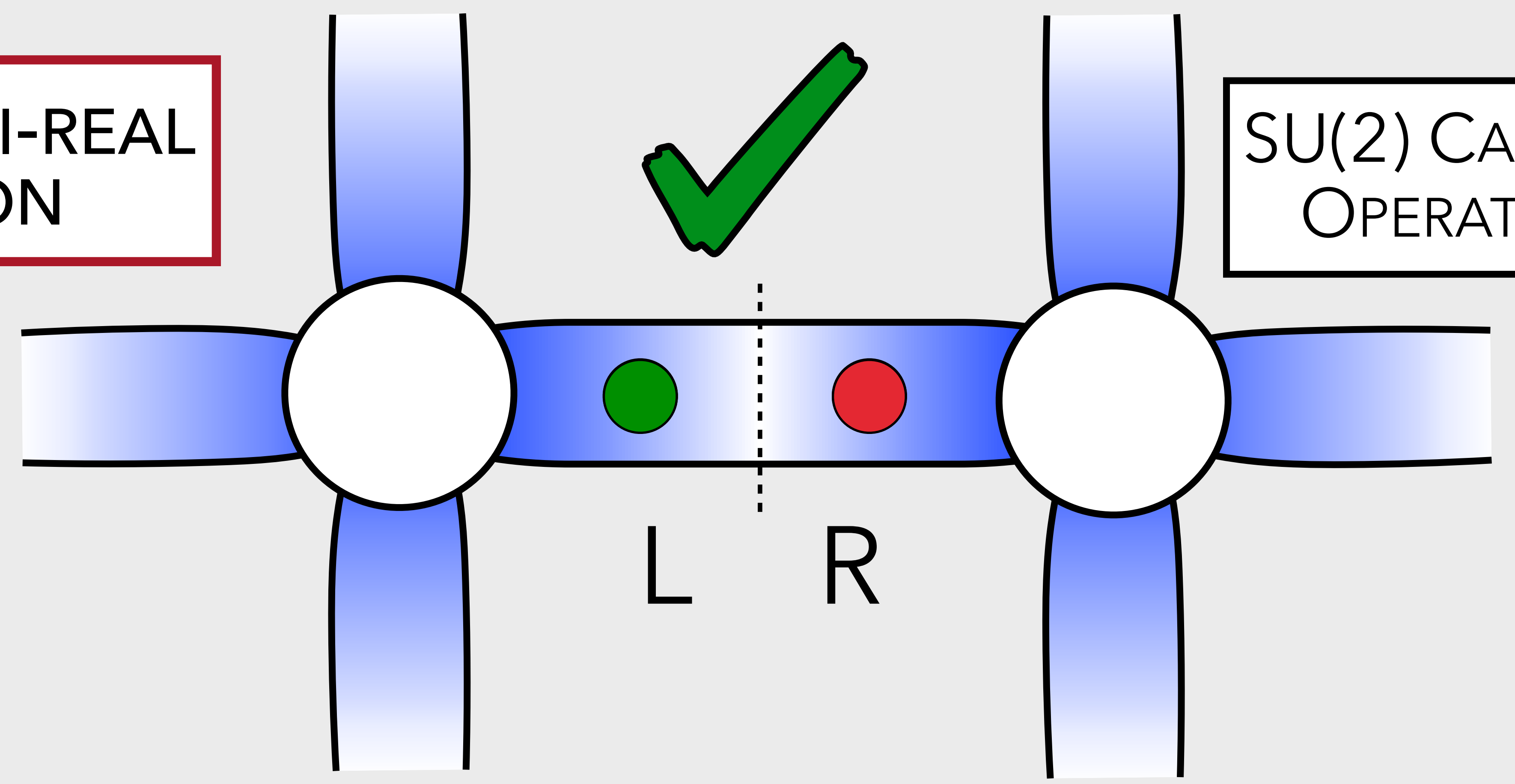
$$\zeta_r = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F$$

$$\zeta_g = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

SU(2) HAS A QUASI-REAL REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$



SU(2) CASIMIR OPERATOR

RISHON PARITY

$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array} \right)$$

0 (INTEGER) SU(2) REPRESENTATION

1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

SU(2) GAUGE LINKS VIA RISHONS

Zohar, Cirac (2018)

Zohar, Cirac (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F$$

$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

SU(2) HAS A QUASI-REAL REPRESENTATION

RISHON PARITY

$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array}\right)$$

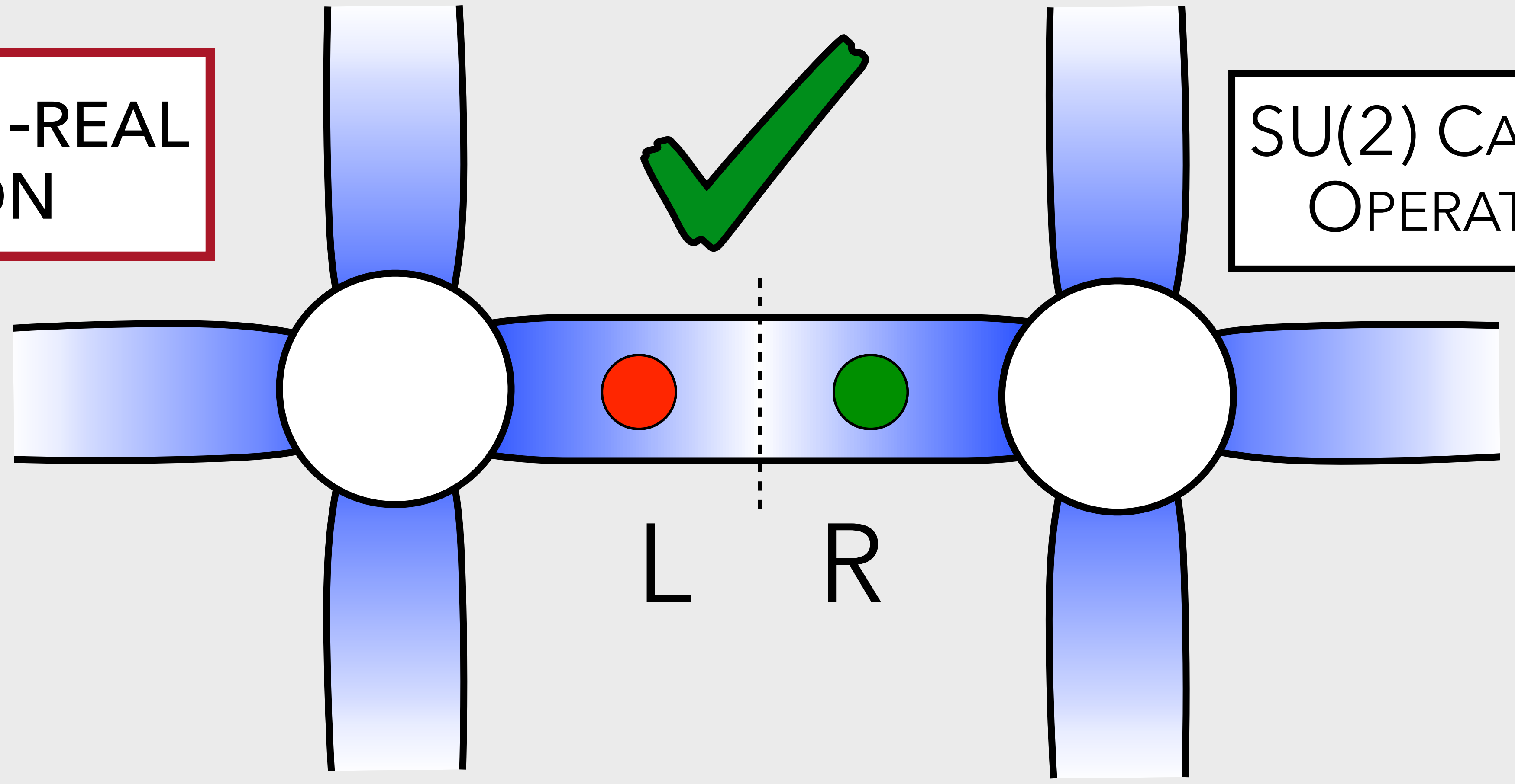
0 (INTEGER) SU(2) REPRESENTATION

1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) CASIMIR OPERATOR



SU(2) GAUGE LINKS VIA RISHONS

Zohar, Cirac (2018)

Zohar, Cirac (2019)

Express the parallel transporter with fermionic rishons

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_r = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F$$

$$\zeta_g = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

Rishon Parity

$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array} \right)$$

0 (integer) SU(2) representation

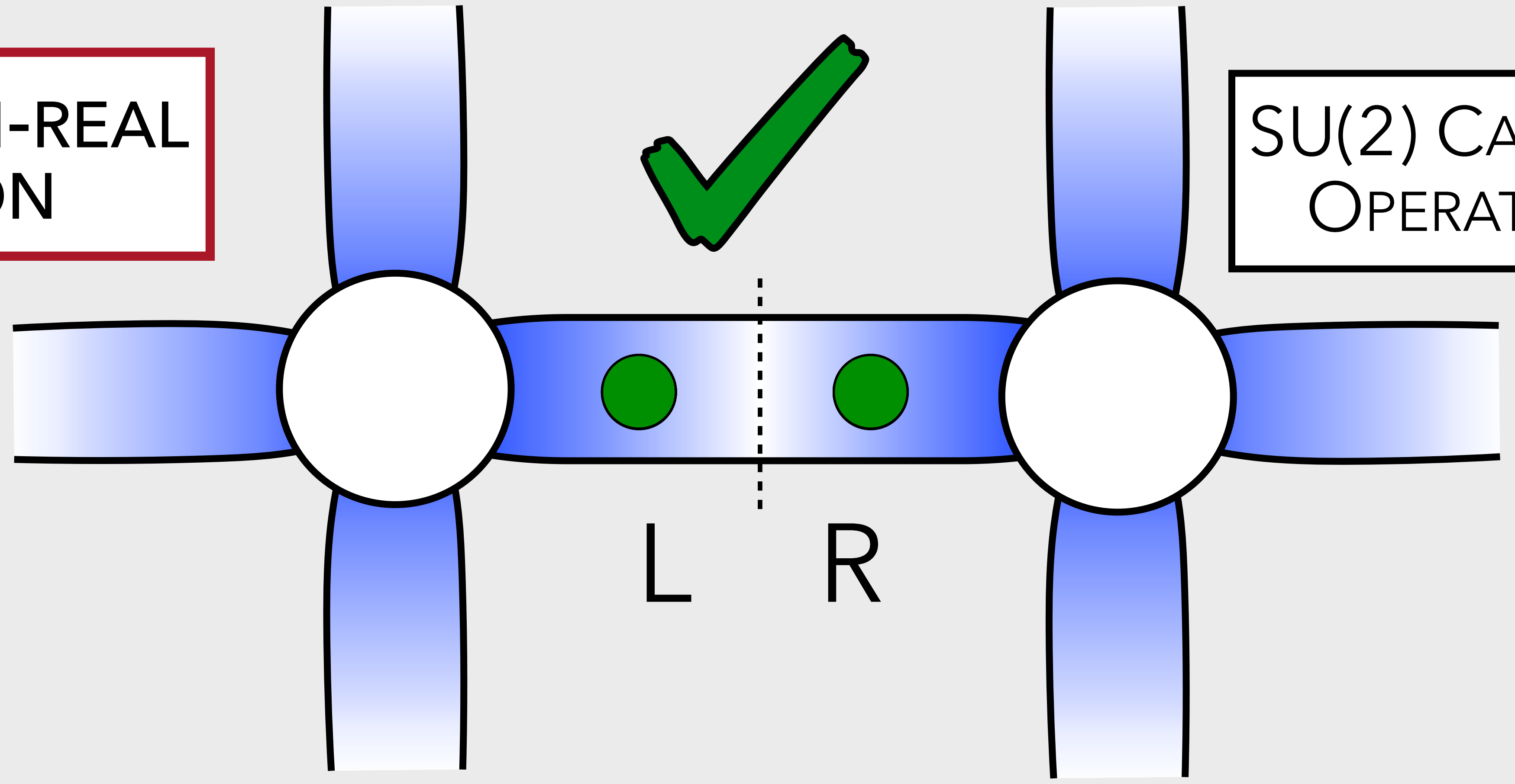
1/2 (semi-integer) SU(2) representation

SU(2) has a quasi-real representation

Allowed link configurations satisfy

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) Casimir operator



SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_r = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F$$

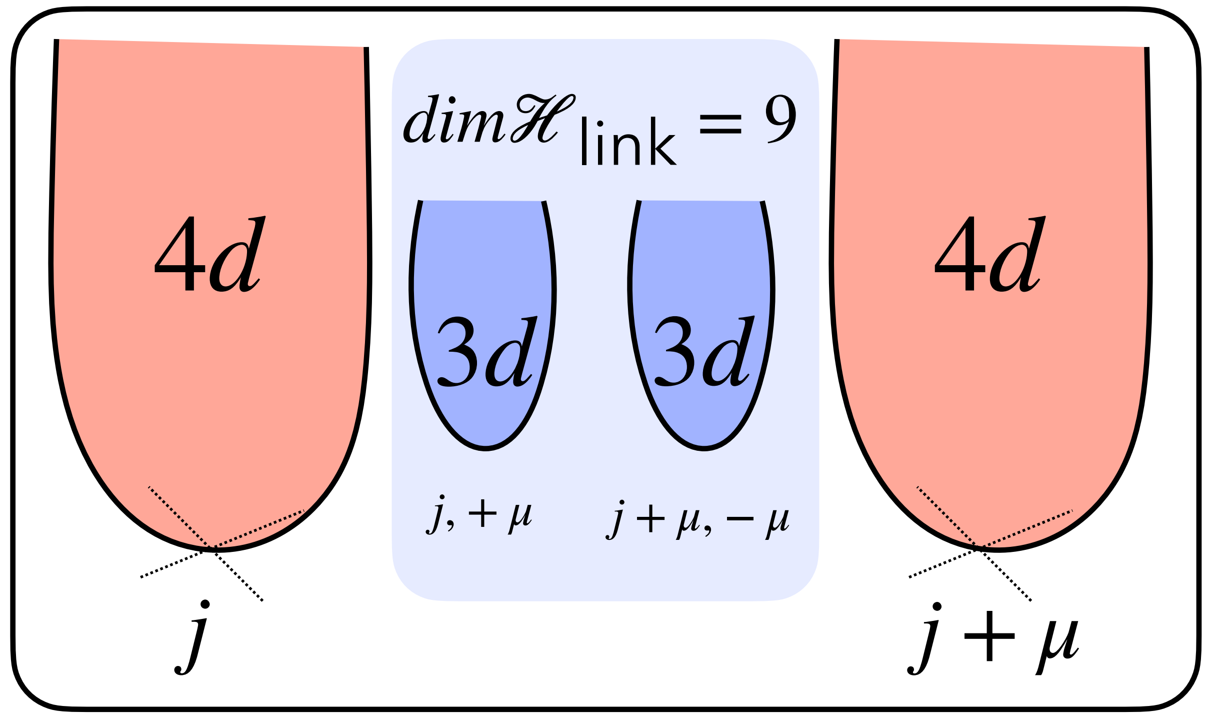
$$\zeta_g = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$

RISHON PARITY

$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array} \right)$$

0 (INTEGER) SU(2) REPRESENTATION

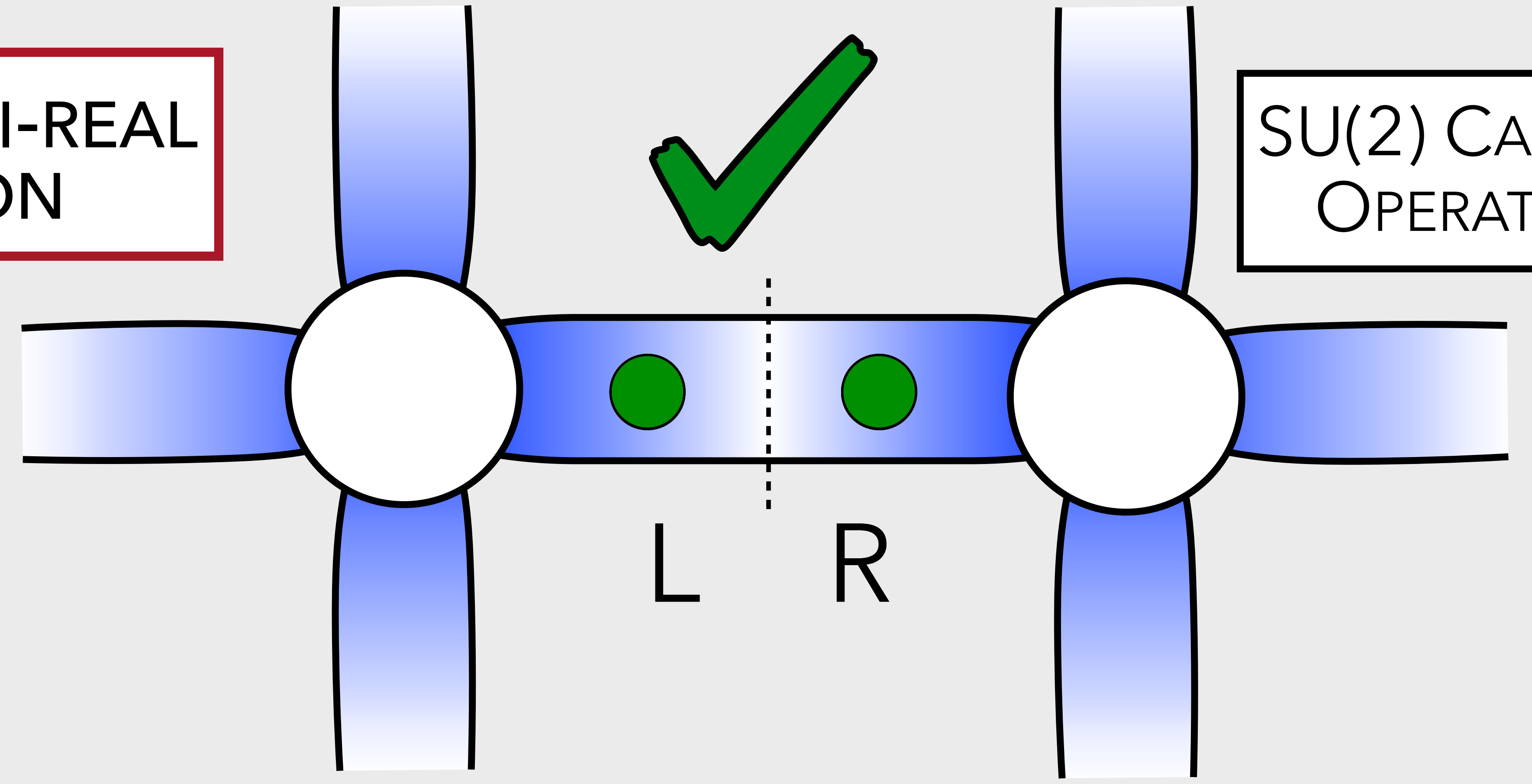
1/2 (SEMI-INTEGER) SU(2) REPRESENTATION



SU(2) HAS A QUASI-REAL REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$



SU(2) CASIMIR OPERATOR

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)
ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^\dagger_{R,\beta} = \zeta_\alpha \cdot P_\zeta \otimes \zeta^\dagger_\beta$$

$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F$$

$$\zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

RISHON PARITY

$$P_\zeta = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array}\right)$$

0 (INTEGER) SU(2) REPRESENTATION

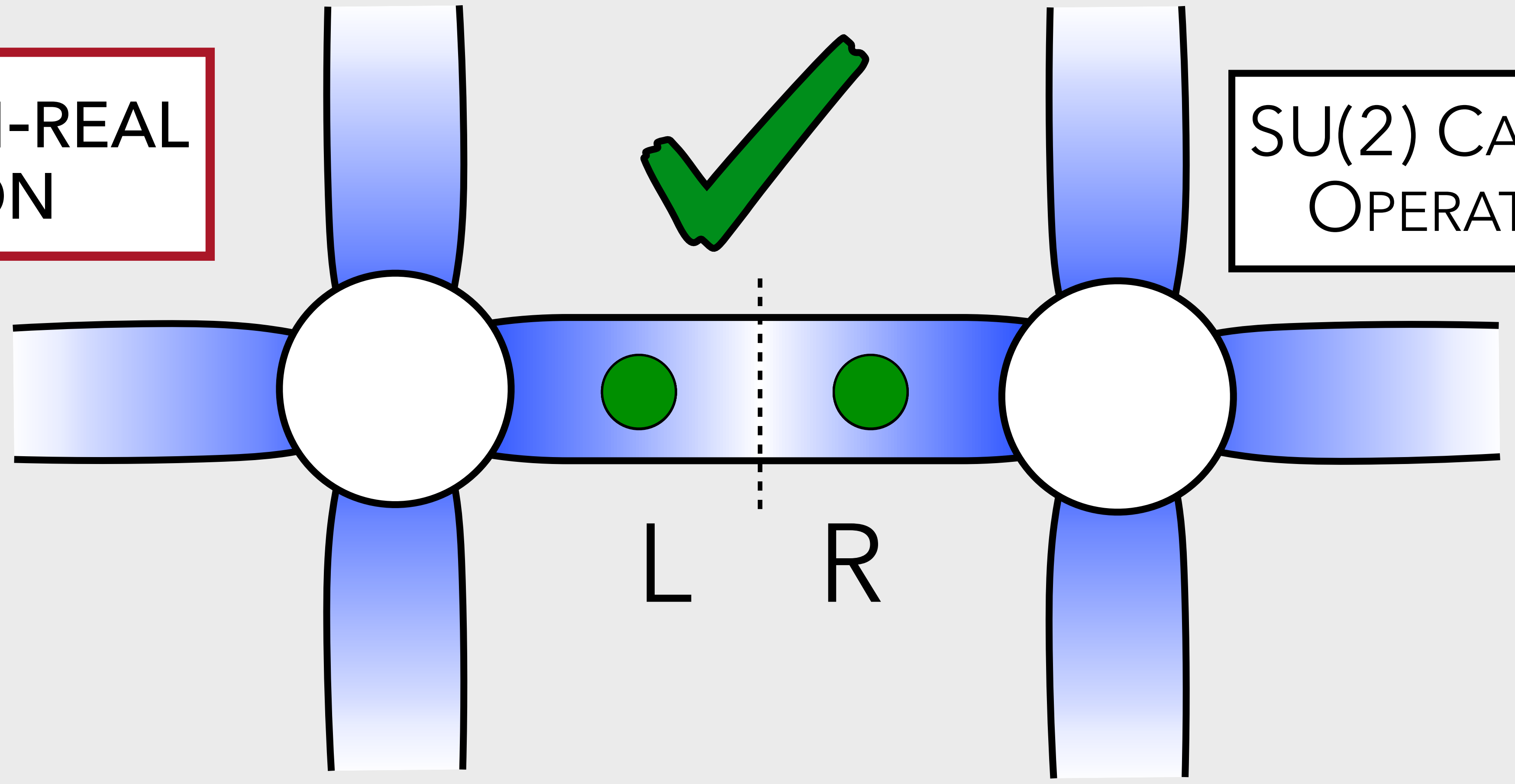
1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

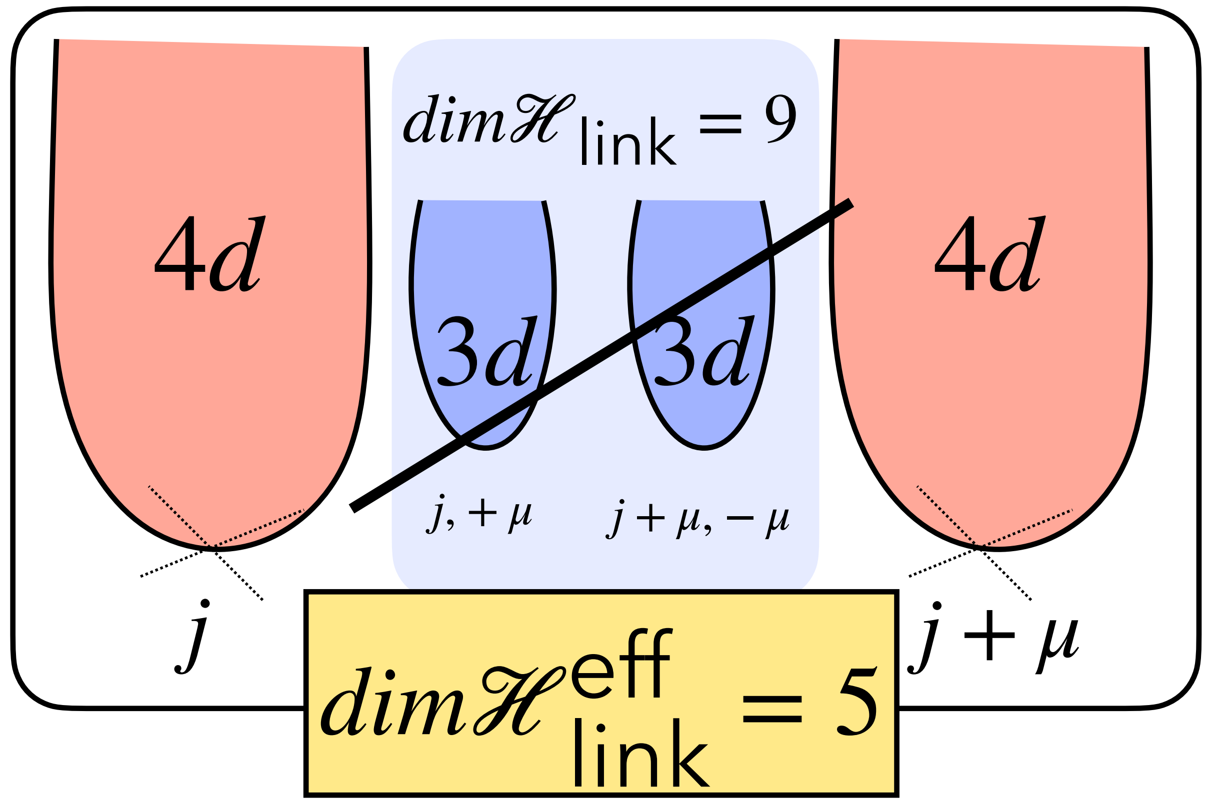
SU(2) HAS A QUASI-REAL REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) CASIMIR OPERATOR





SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F$$

$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

SU(2) HAS A QUASI-REAL REPRESENTATION

RISHON PARITY

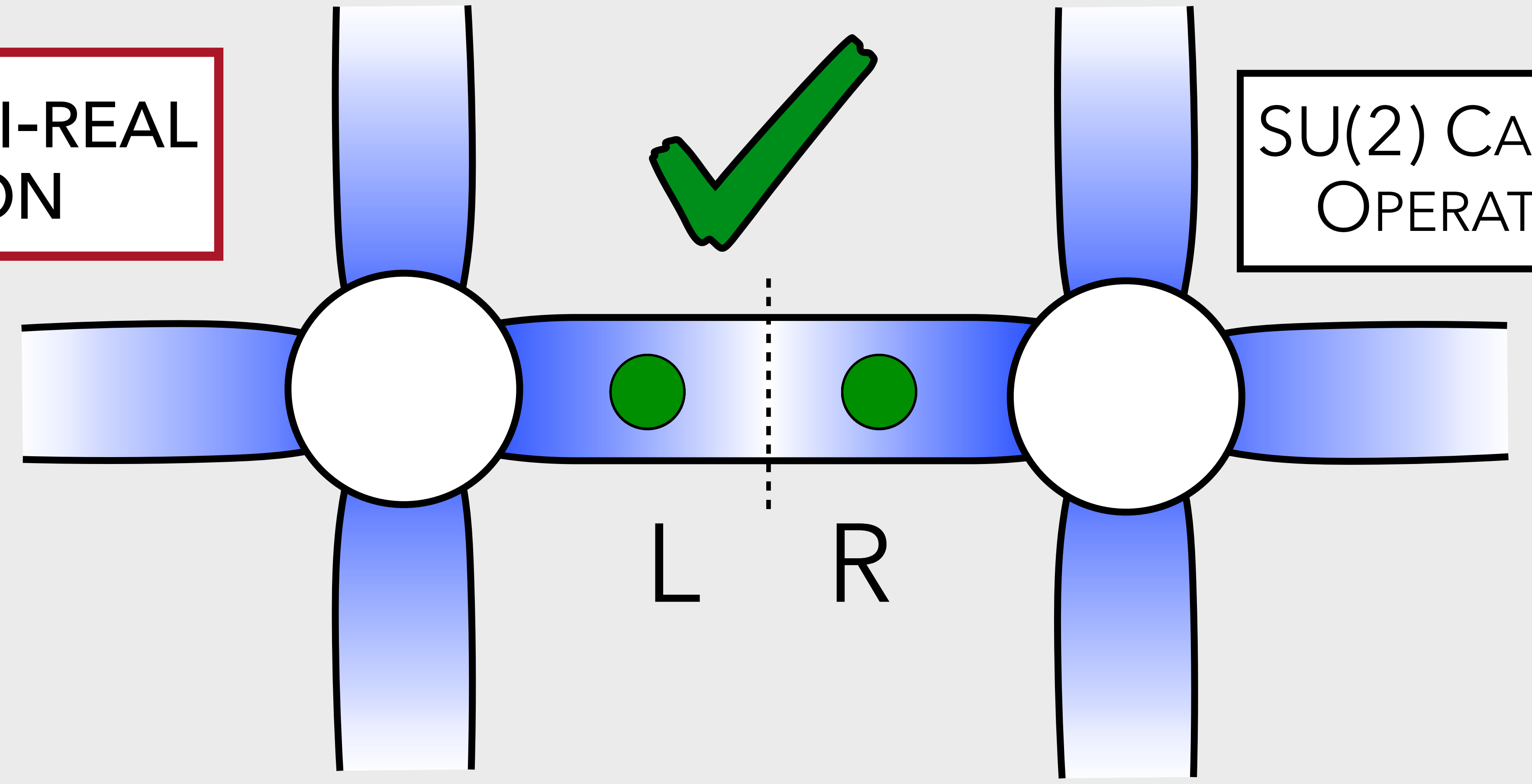
$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array}\right)$$

0 (INTEGER) SU(2) REPRESENTATION

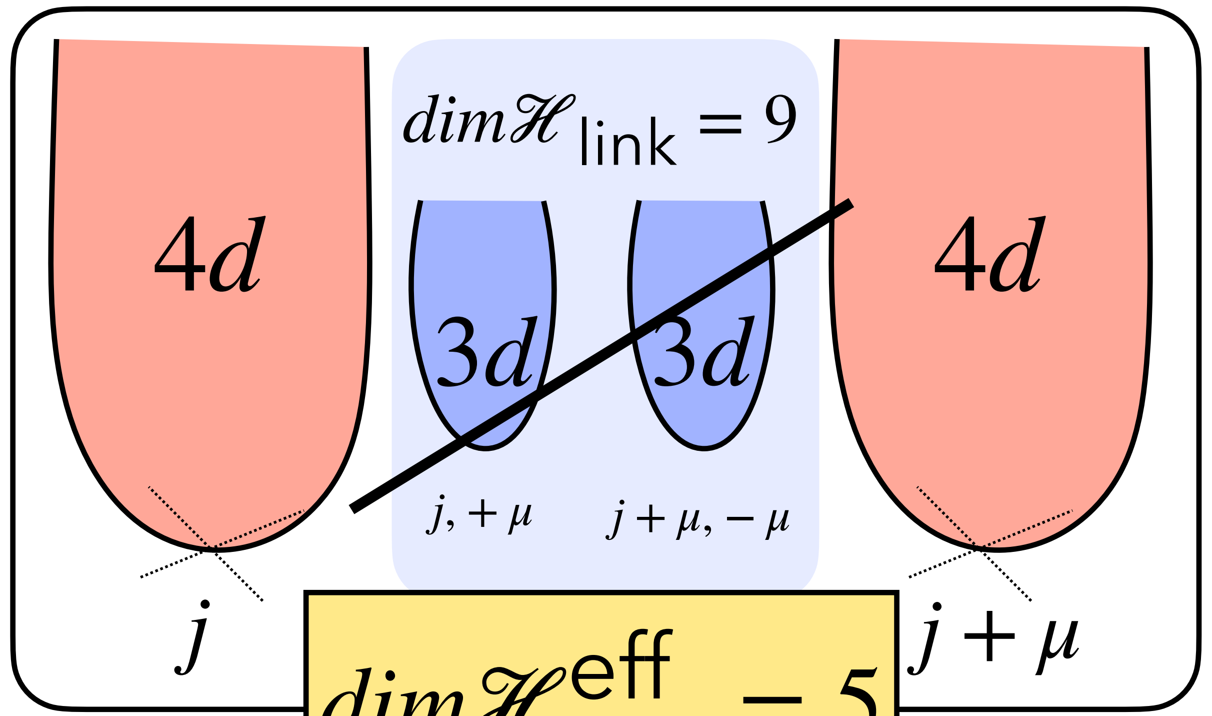
1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$



SU(2) CASIMIR OPERATOR



$$dim\mathscr{H}^{\text{eff}}_{\text{link}} = 5$$

ζ Rishon-based Parallel Transport ✓

$$\zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \frac{1}{2} \begin{pmatrix} 0 & -\delta_{g,\alpha}\delta_{r,\beta} & +\delta_{g,\alpha}\delta_{\downarrow\beta} & -\delta_{r,\alpha}\delta_{r,\beta} & +\delta_{r,\alpha}\delta_{g,\beta} \\ +\delta_{r,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ +\delta_{r,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \\ -\delta_{g,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ -\delta_{g,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_{r} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_{F}$$

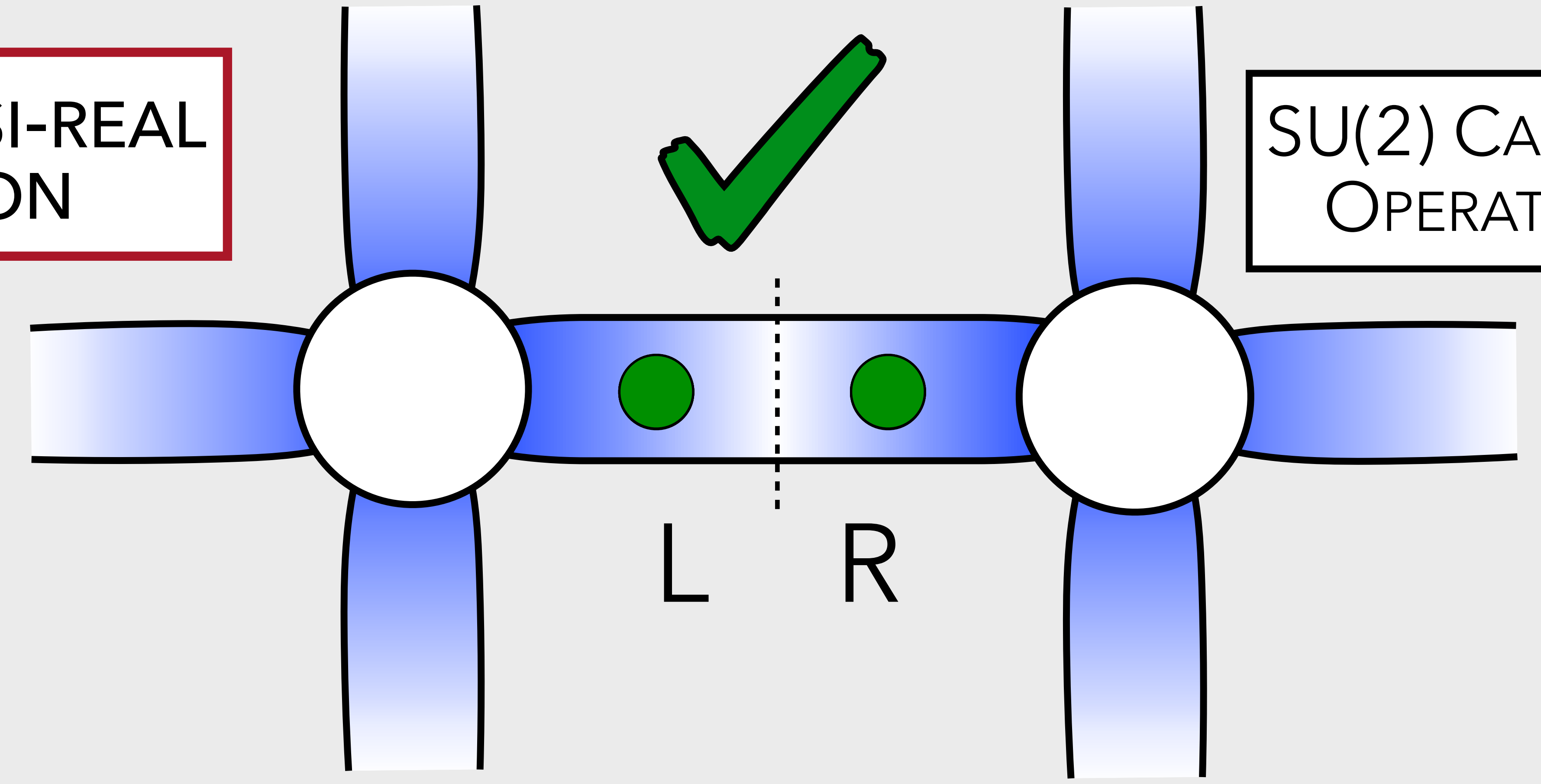
$$\zeta_{g} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_{F}$$

ALLOWED LINK CONFIGURATIONS SATISFY

$$S^2_L = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S^2_R$$

SU(2) HAS A QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR

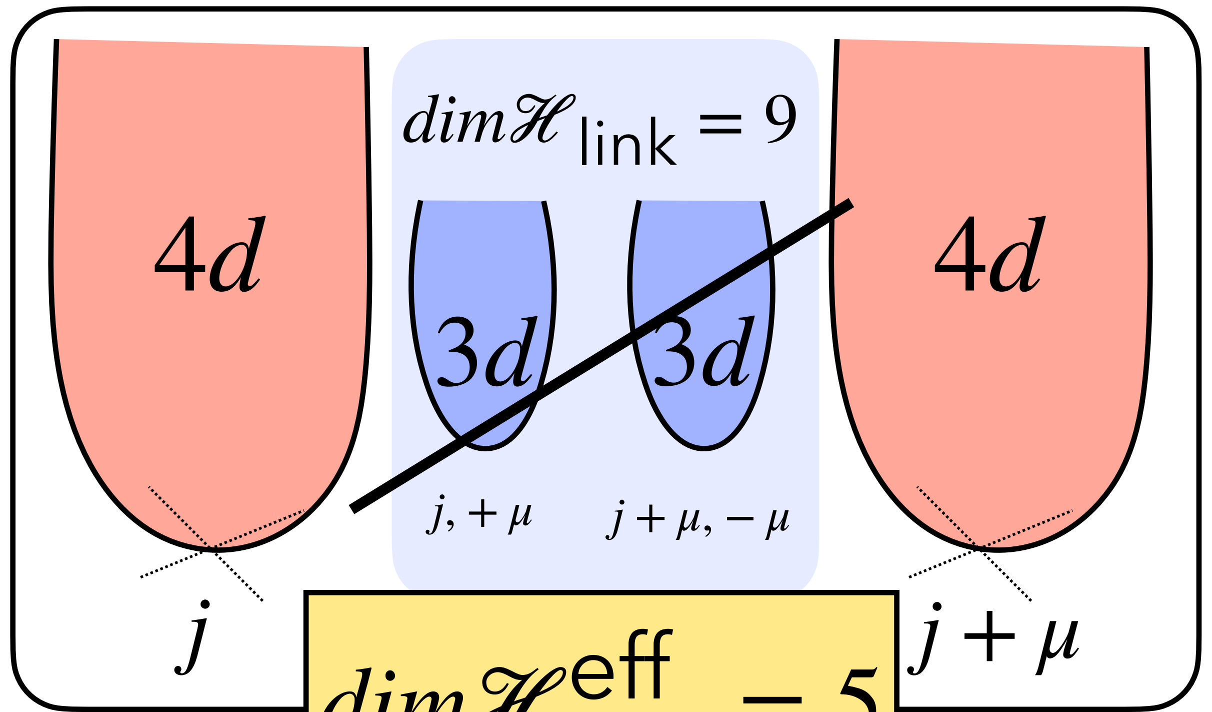


RISHON PARITY

$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array}\right)$$

0 (INTEGER) SU(2) REPRESENTATION

1/2 (SEMI-INTEGER) SU(2) REPRESENTATION



$\dim\mathscr{H}^{\text{eff}}_{\text{link}} = 5$

ζ Rishon-based Parallel Transport ✓

$$\zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \frac{1}{2}\begin{pmatrix} 0 & -\delta_{g,\alpha}\delta_{r,\beta} & +\delta_{g,\alpha}\delta_{\downarrow\beta} & -\delta_{r,\alpha}\delta_{r,\beta} & +\delta_{r,\alpha}\delta_{g,\beta} \\ +\delta_{r,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ +\delta_{r,\alpha}\delta_{r,\beta} & & & & \\ -\delta_{g,\alpha}\delta_{g,\beta} & & & & \\ -\delta_{g,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

WE CAN GENERALIZE IT TO ANY SPIN-REPRESENTATION!

FERMIONIC QMB OPERATORS

WHAT IS A FERMION?

BOSONS $\left[\hat{B}_i, \hat{B}_j\right] = 0$

FERMION $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

FERMIONIC QMB OPERATORS

How to encode Fermi-statistics in Matrix Notation?

$$\hat{A}_j = \begin{pmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \dots & a_{NN} \end{pmatrix}_F$$

What is a Fermion?

Bosons $\left[\hat{B}_i, \hat{B}_j\right] = 0$

Fermion $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

FERMIONIC QMB OPERATORS

WHAT IS A FERMION?

BOSONS $\left[\hat{B}_i, \hat{B}_j\right] = 0$

FERMION $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

HOW TO ENCODE FERMI-STATISTICS IN MATRIX NOTATION?

$$\hat{A}_j = \begin{pmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \dots & a_{NN} \end{pmatrix}_F$$

PARITY OPERATOR

$$P = P^{-1} = P^T$$

$$P^2 = 1$$

FERMIONIC QMB OPERATORS

How to encode Fermi-statistics in Matrix Notation?

What is a Fermion?

Bosons $\left[\hat{B}_i, \hat{B}_j\right] = 0$

Fermion $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

$$\hat{A}_j = \begin{pmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \dots & a_{NN} \end{pmatrix}_F$$

Gets **unchanged** by the action of **Boson** Operator

$$[P, B] = 0$$

Parity Operator

$$P = P^{-1} = P^T$$

$$P^2 = 1$$

FERMIONIC QMB OPERATORS

How to encode Fermi-statistics in Matrix Notation?

$$\hat{A}_j = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}_F$$

What is a Fermion?

Bosons $\left[\hat{B}_i, \hat{B}_j\right] = 0$

Fermion $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

Parity Operator
$P = P^{-1} = P^T$
$P^2 = 1$

Gets **unchanged** by the action of **Boson** Operator
$[P, B] = 0$

Gets **flipped** by the action of **Fermion** Operator
$\{P, F\} = 0$

FERMIONIC QMB OPERATORS

HOW TO ENCODE FERMI-STATISTICS IN MATRIX NOTATION?

$$\hat{A}_j = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}_F$$

What is a Fermion?

Bosons $\left[\hat{B}_i, \hat{B}_j\right] = 0$

Fermion $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

Gets **unchanged** by the action of **Boson** Operator

$$[P, B] = 0$$

Gets **flipped** by the action of **Fermion** Operator

$$\{P, F\} = 0$$

Parity Operator

$$P = P^{-1} = P^T$$

$$P^2 = 1$$

Dirac Fermion

$$\Psi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_F \qquad P = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$$

FERMIONIC QMB OPERATORS

How to encode Fermi-statistics in Matrix Notation?

What is a Fermion?

Bosons $\left[\hat{B}_i, \hat{B}_j\right] = 0$

Fermion $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

$$\hat{A}_j = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}_F$$

Gets **unchanged** by the action of **Boson** Operator
$[P, B] = 0$

Gets **flipped** by the action of **Fermion** Operator
$\{P, F\} = 0$

Parity Operator
$P = P^{-1} = P^T$
$P^2 = 1$

Dirac Fermion

$\Psi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_F \qquad P = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$

Bosons $\hat{B}_j = \mathbb{1} \otimes \ldots \underset{j-1}{\mathbb{1}} \otimes \underset{j}{B} \otimes \underset{j+1}{\mathbb{1}} \otimes \cdots \otimes \underset{N}{\mathbb{1}}$

Fermion $\hat{F}_j = P \otimes \ldots \underset{j-1}{P} \otimes \underset{j}{F} \otimes \underset{j+1}{\mathbb{1}} \otimes \cdots \otimes \underset{N}{\mathbb{1}}$

FERMIONIC QMB OPERATORS

HOW TO ENCODE FERMI-STATISTICS IN MATRIX NOTATION?

$$\hat{A}_j = \begin{pmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \dots & a_{NN} \end{pmatrix}_F$$

What is a Fermion?

Bosons $\left[\hat{B}_i, \hat{B}_j\right] = 0$

Fermion $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

Gets **unchanged** by the action of **Boson** Operator

$$[P, B] = 0$$

Gets **flipped** by the action of **Fermion** Operator

$$\{P, F\} = 0$$

Parity Operator

$$P = P^{-1} = P^T$$

$$P^2 = 1$$

Dirac Fermion

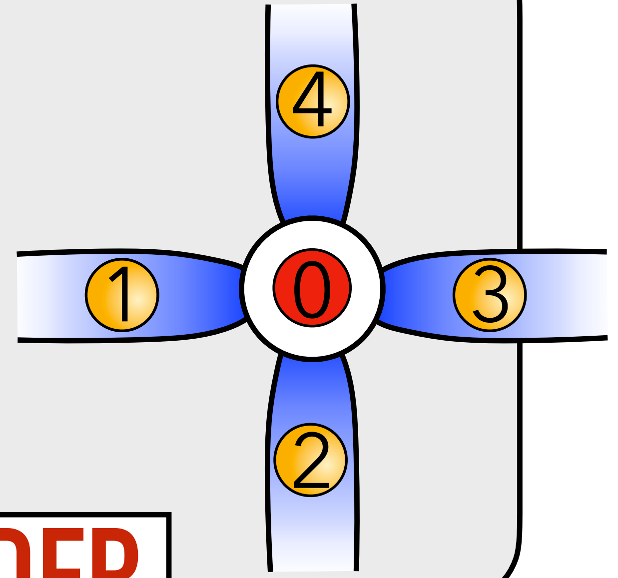
$$\Psi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_F \qquad P = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$$

Bosons $\hat{B}_j = \mathbb{1} \otimes \dots \underset{j-1}{\mathbb{1}} \otimes \underset{j}{B} \otimes \underset{j+1}{\mathbb{1}} \otimes \dots \otimes \underset{N}{\mathbb{1}}$

Fermion $\hat{F}_j = P \otimes \dots \underset{j-1}{P} \otimes \underset{j}{F} \otimes \underset{j+1}{\mathbb{1}} \otimes \dots \otimes \underset{N}{\mathbb{1}}$

THAT'S HOW A DRESSED SITE READS!

$$\left| \begin{matrix} & \zeta^4_{+\mu_y} & \\ \zeta^1_{-\mu_x} & \psi^0 & \zeta^3_{+\mu_x} \\ & \zeta^2_{-\mu_y} & \end{matrix} \right\rangle$$



CHOOSE AN INTERNAL ORDER

DIMENSIONAL ANALYSIS FOR CONTINUUM LIMIT

Gauge Theory in the Continuum

Electric Energy $H_{\text{elec}} = \frac{\epsilon_c}{2}\int \mathcal{E}^2(x)d^Dx$

Color Vacuum Permittivity $[\epsilon_c] = \frac{\text{charge}^2 \cdot \text{length}^{2-D}}{\text{energy}}$

Color-Electric Field $[\mathcal{E}] = \frac{\text{energy}}{\text{charge} \cdot \text{length}}$

Hamiltonian Discretization

$$\int (dx)^D \longrightarrow a^D \sum_{j,\mu}$$

$$\mathcal{E}^2(x) \longrightarrow \frac{q_c^2 a^{2-2D}}{\epsilon_c^2} E_{j,\mu}^2$$

The way to go back $0 \longleftarrow a$

Lattice Gauge Theory

$$H_{\text{elec}} = \frac{q_c^2 a^{2-D}}{2\epsilon_c}\sum_{j,\mu} E_{j,\mu}^2 = g^2\frac{c\hbar}{2a}\sum_{j,\mu} E_{j,\mu}^2$$

Gauge Coupling $g = q_c \frac{a^{\frac{3-D}{2}}}{\sqrt{\hbar c \epsilon_c}}$

D=2 $g = \sqrt{\frac{q_c a^2}{2\hbar c \epsilon_c}}$

Quark Ratio $\alpha_c = \frac{q_c^2}{2m_0c^2\epsilon_c} \propto \frac{q_c^2}{m_0}$

Continuum Limit at fixed α_c $g^2 = \alpha_c m \longrightarrow 0$

$$a \cdot H^{2D}_{SU(2)} = \frac{1}{2}\sum_{\alpha,\beta}\sum_{j,\mu}\left[e^{ik(j,\mu)}\,\psi^\dagger_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m\sum_j(-1)^{j_x+j_y}\sum_\alpha \psi^\dagger_{j\alpha}\psi_{j\alpha}$$

$$+\frac{g^2}{2}\sum_j\left(E^2_{j,j+\mu_x} + E^2_{j,j+\mu_y}\right) - \frac{8}{g^2}\sum_j\sum_{\alpha,\beta,\gamma,\delta}\Re e\begin{pmatrix} & U^\dagger_{\gamma\delta} & \\ U^\dagger_{\gamma\alpha} & & U_{\beta\gamma} \\ & U_{\alpha\beta} & \end{pmatrix}$$

USEFUL PROPERTIES OF SU(2) RISHONS

ζ-Rishons have a unique gauge transformation algebra, generated by $\overrightarrow{T} = \frac{1}{2}\sum_{a,b} \vec{\sigma}_{ab} c_a^\dagger c_b$

$\overrightarrow{T}$ is genuinely *local*
$\left[\overrightarrow{T}_1, \zeta_{2,a}\right] = \left[\overrightarrow{T}_1, \psi_{2,a}\right] = 0$

It is possible to check that ζ-Rishons transform covariantly under $\overrightarrow{T}$.

$$\left[\overrightarrow{T}, \zeta_a\right] = -\frac{1}{2}\sum_b \vec{\sigma}_{ab}\zeta_b \qquad \left[\overrightarrow{T}, \zeta_a^\dagger\right] = -\left[\overrightarrow{T}, \zeta_a\right]^\dagger = \frac{1}{2}\sum_b \zeta_b^\dagger \vec{\sigma}_{ba}$$

Left- and Right-generators of the gauge field at link $(j, j+\mu)$ can be expressed like

$$\overrightarrow{L}_{j,\mu} = \overrightarrow{T}_{j,+\mu} \otimes 1_{j+\mu,-\mu} \qquad \overrightarrow{R}_{j,\mu} = 1_{j,+\mu} \otimes \overrightarrow{T}_{j+\mu,-\mu}$$

$\zeta^a \zeta^{b\dagger}$ transforms like U^{ab} do with $\overrightarrow{L}$ and $\overrightarrow{R}$ generators!