

Exponential optimization of adiabatic quantum-state preparation

Davide Cugini

D. Nigro, M. Bruno, D. Gerace

Simulations of physical systems

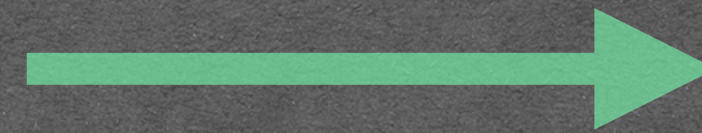
Observation of the phenomenon



Simulations of physical systems

Observation of the phenomenon

Mathematical model

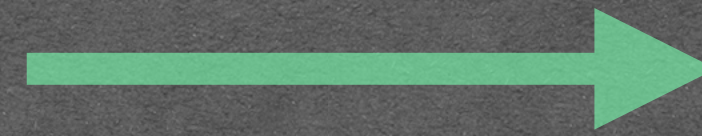


$$H = \dots$$

Simulations of physical systems

Observation of the phenomenon

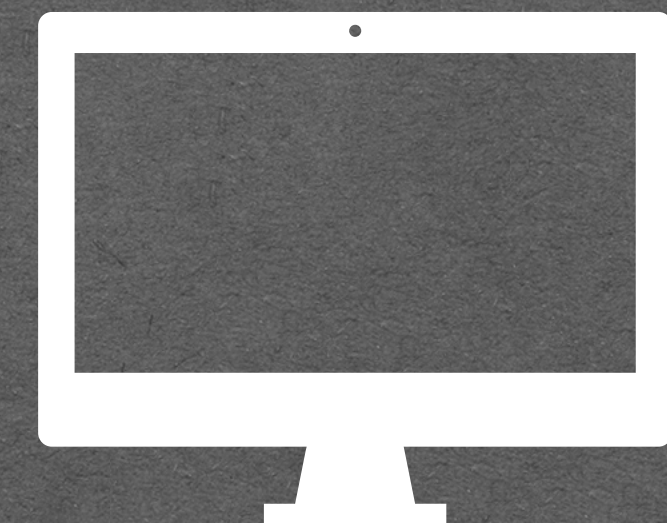
Mathematical model



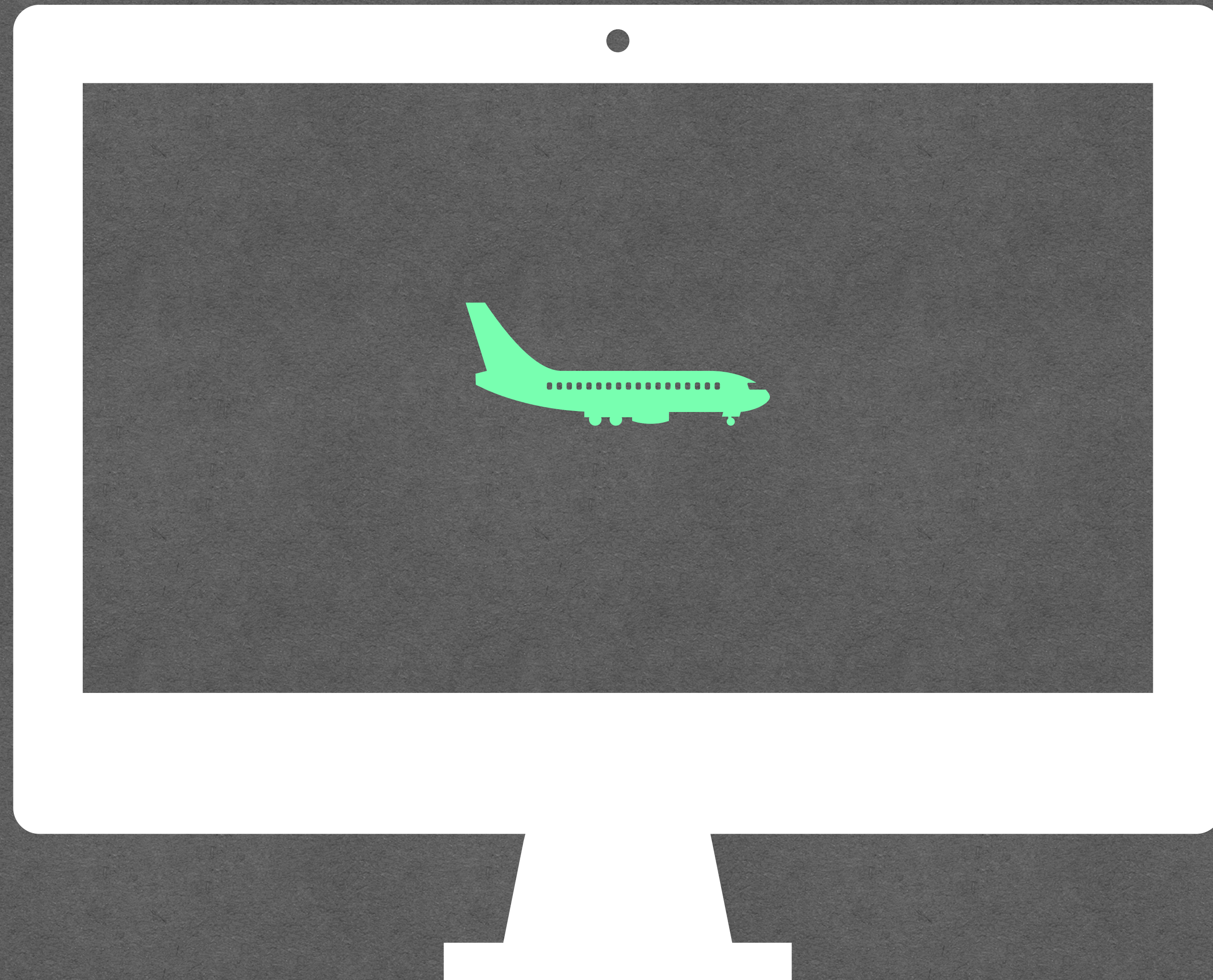
$$H = \dots$$

Analytical
calculation

Numerical
Calculus

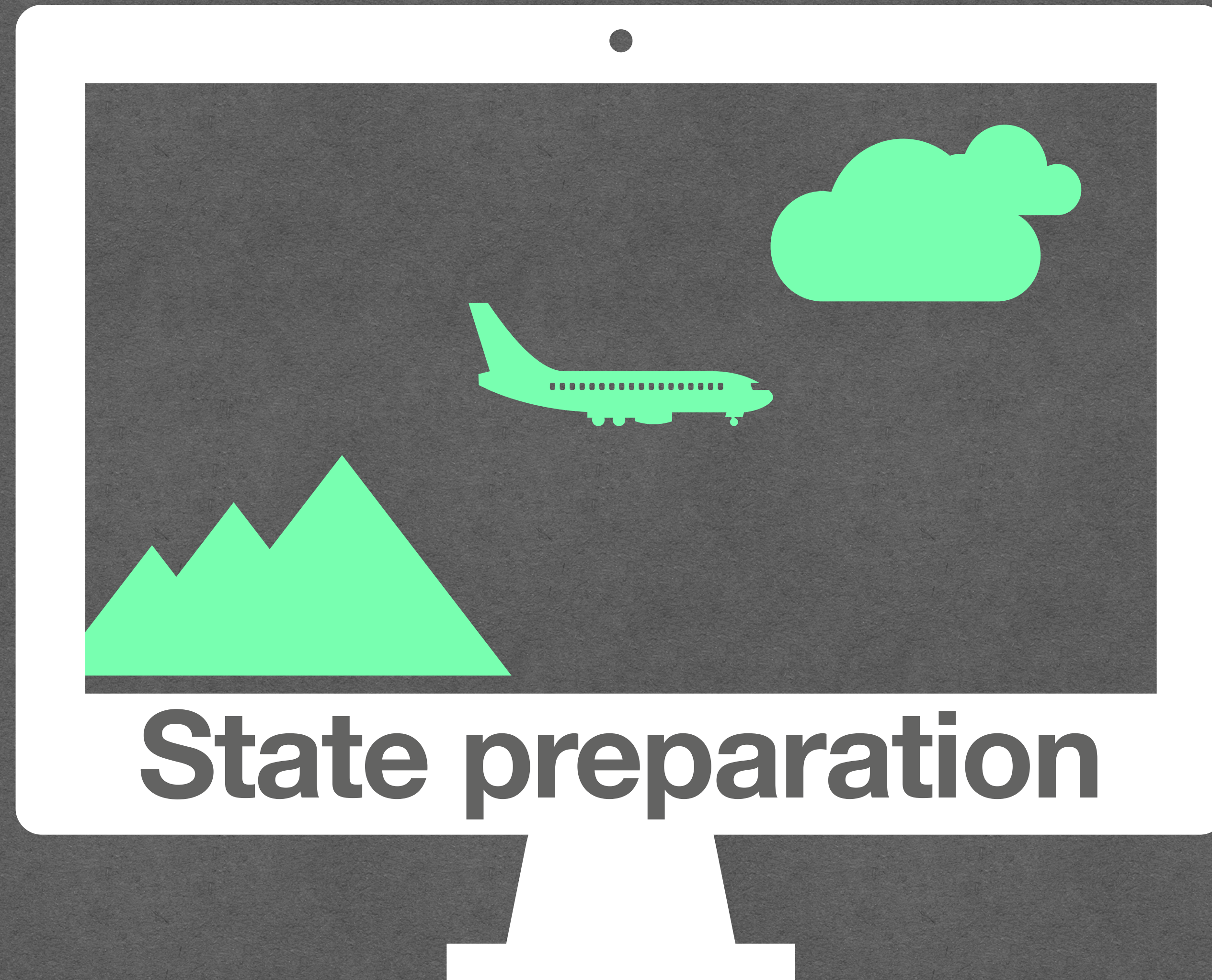


Simulations of physical systems



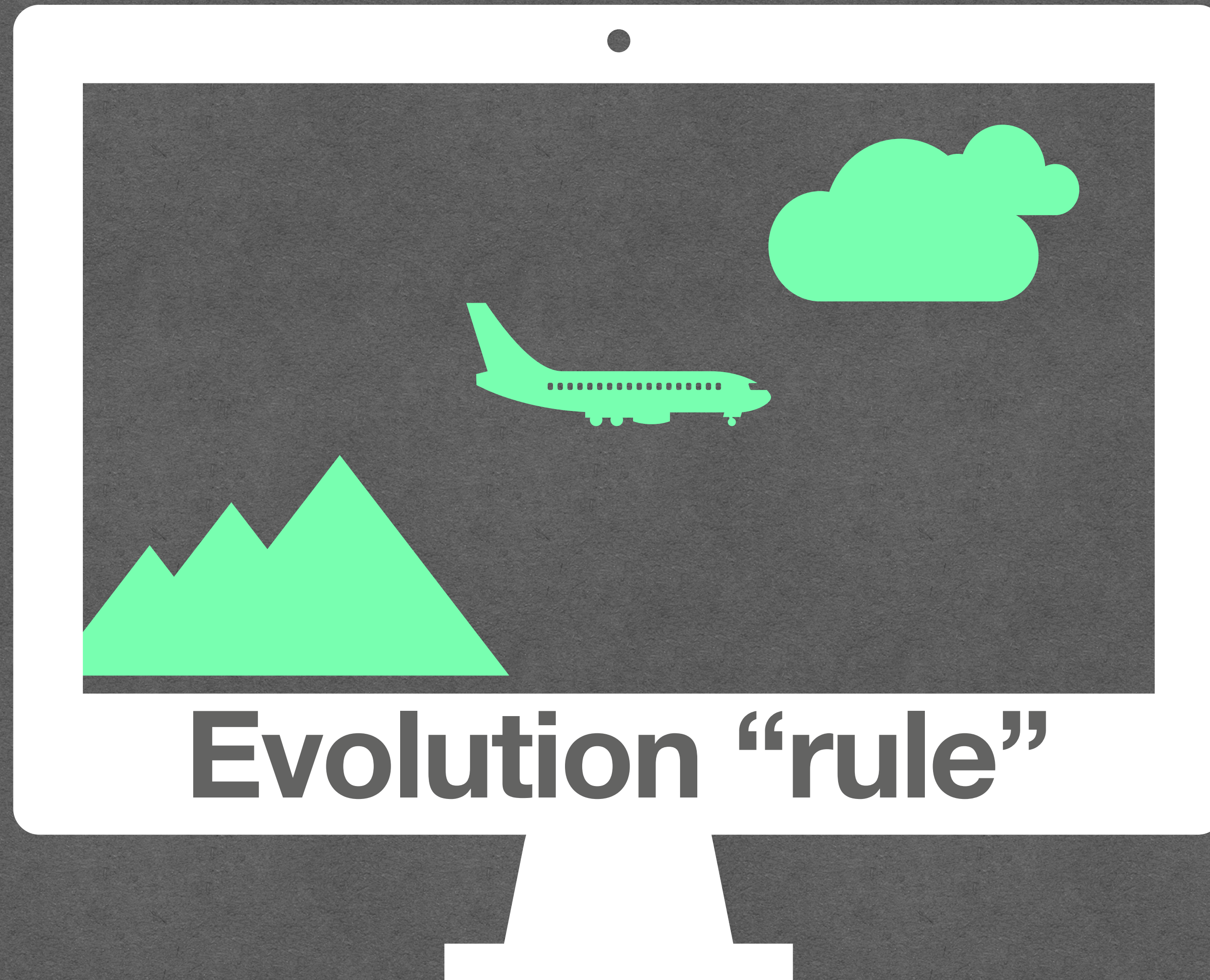
Simulations of physical systems

#1



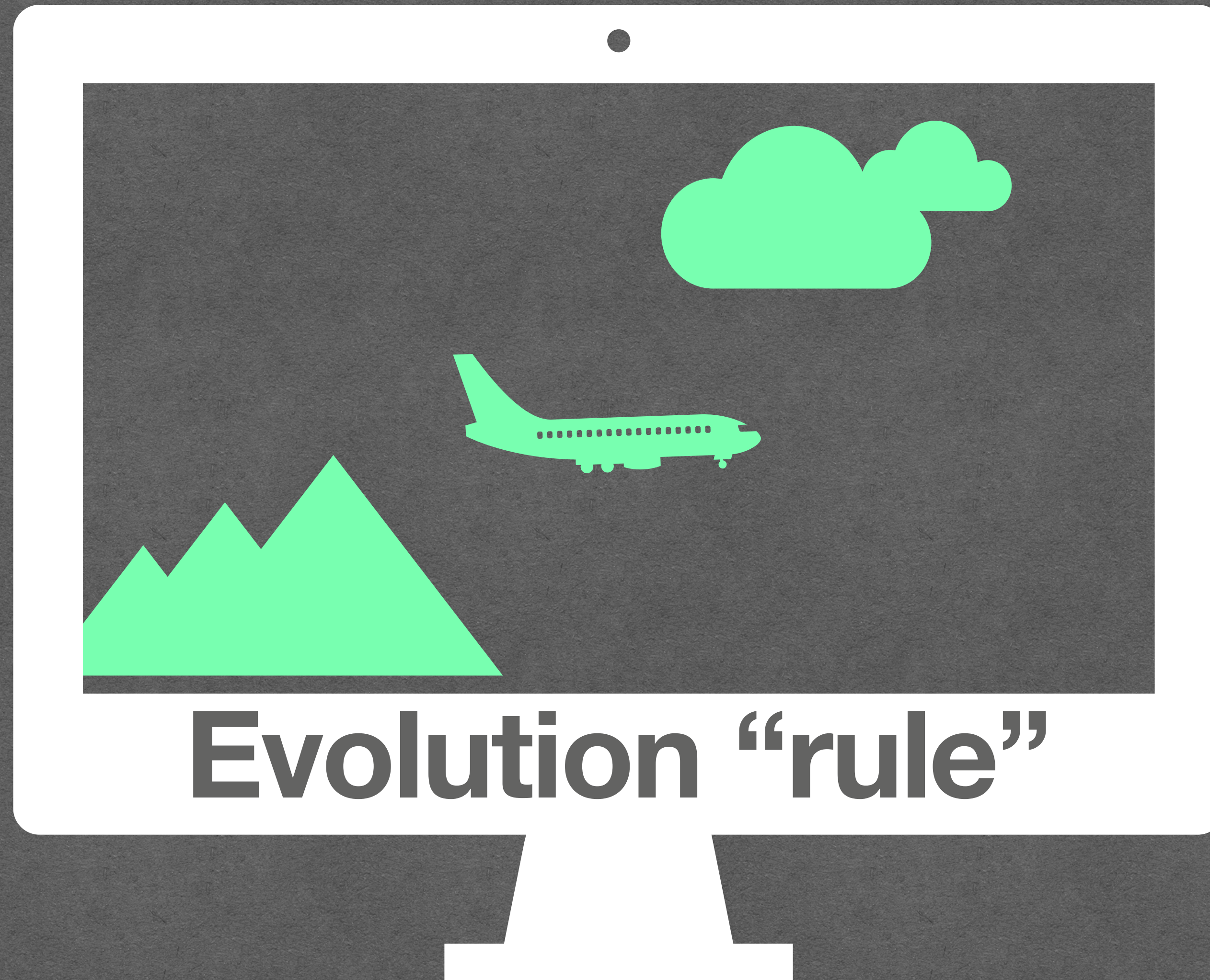
Simulations of physical systems

#2



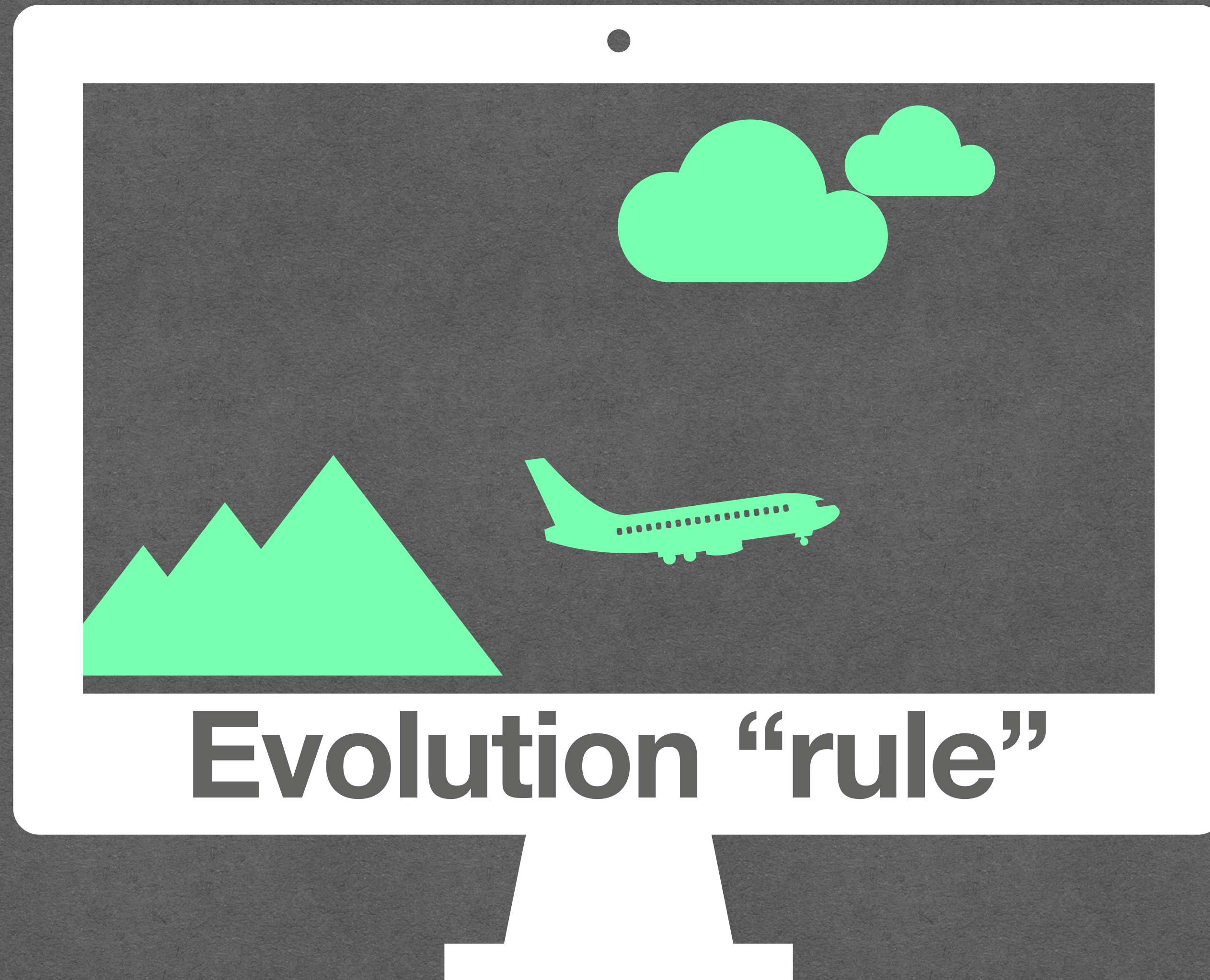
Simulations of physical systems

#2



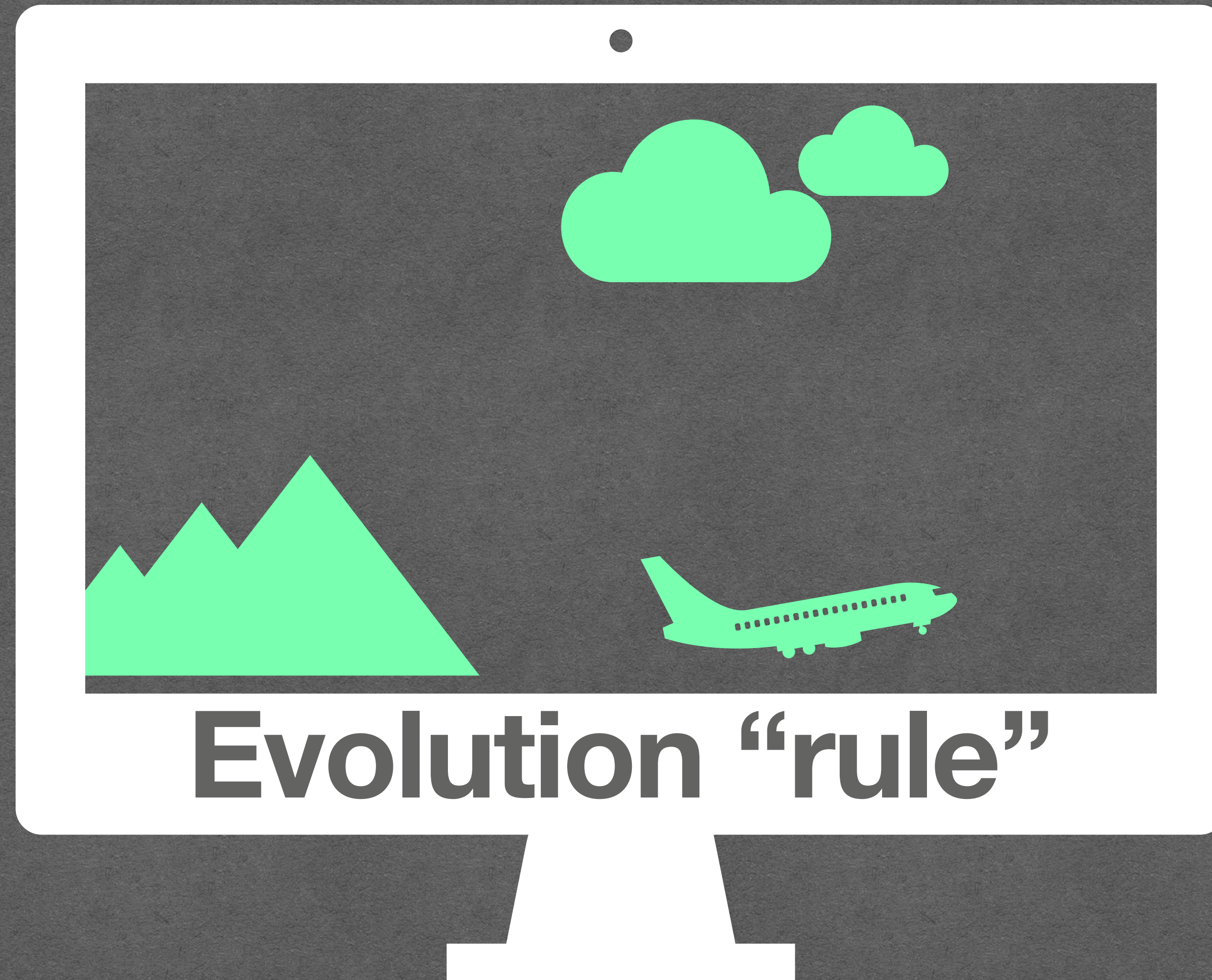
Simulations of physical systems

#2

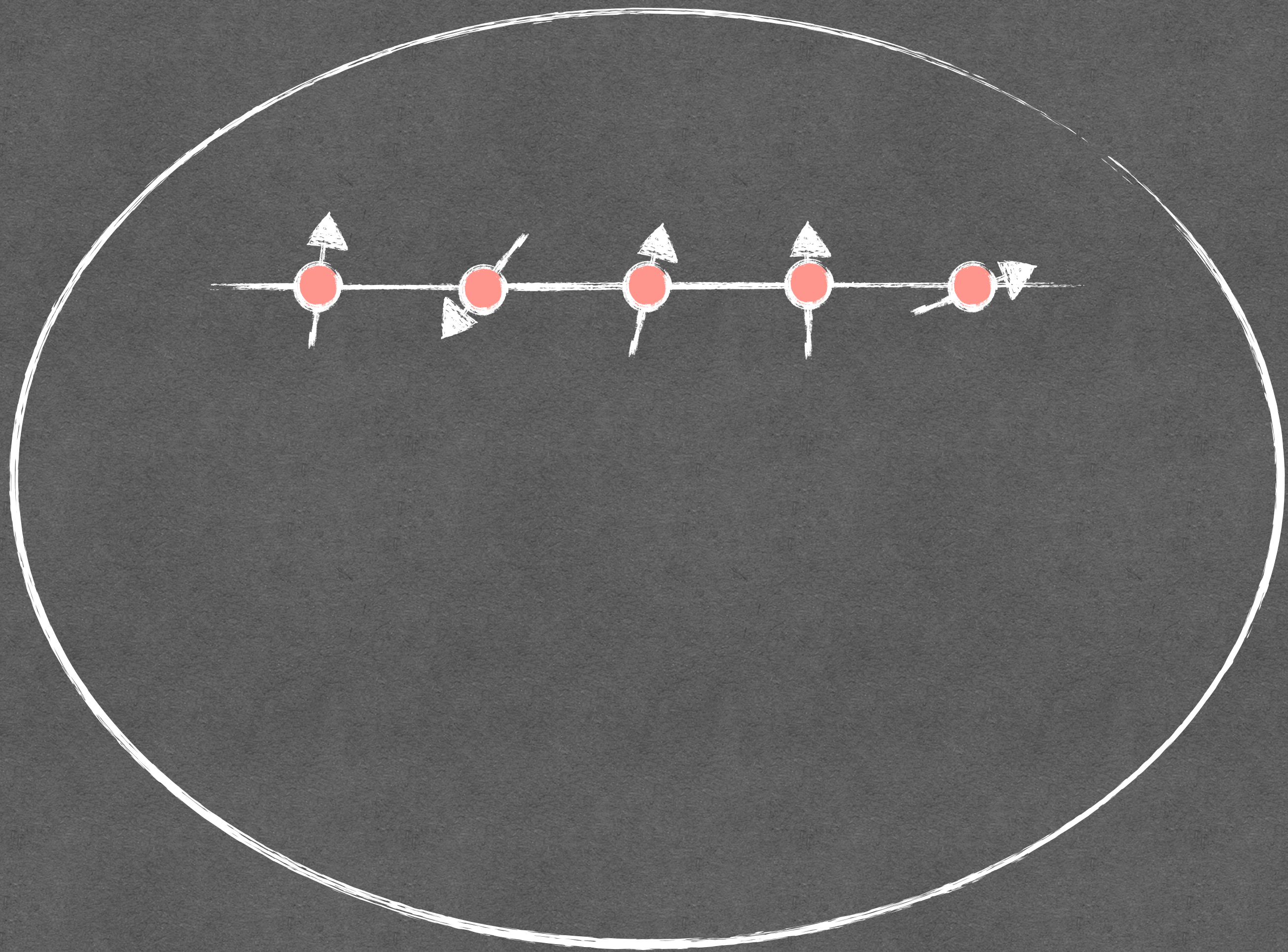


Simulations of physical systems

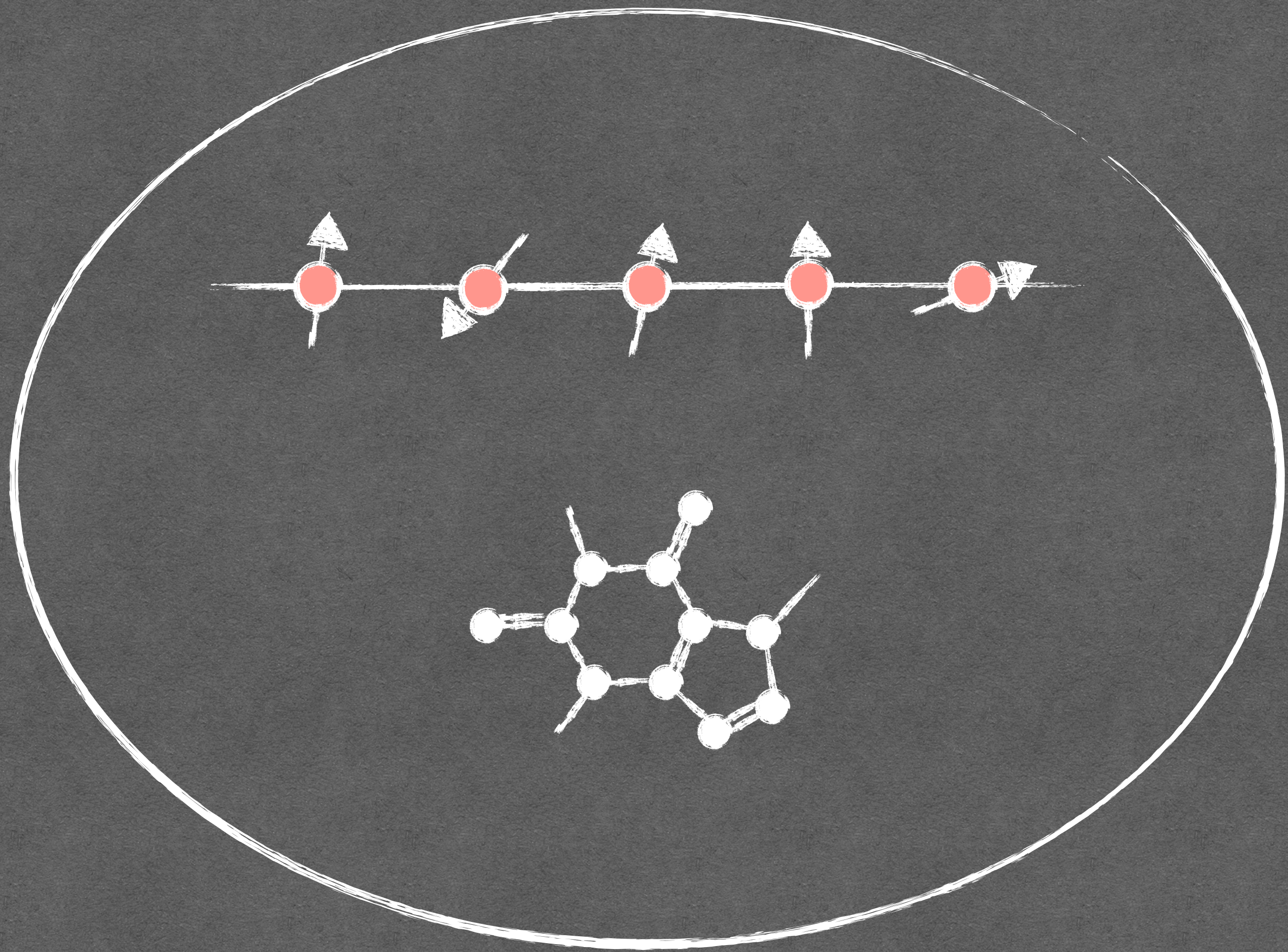
#2



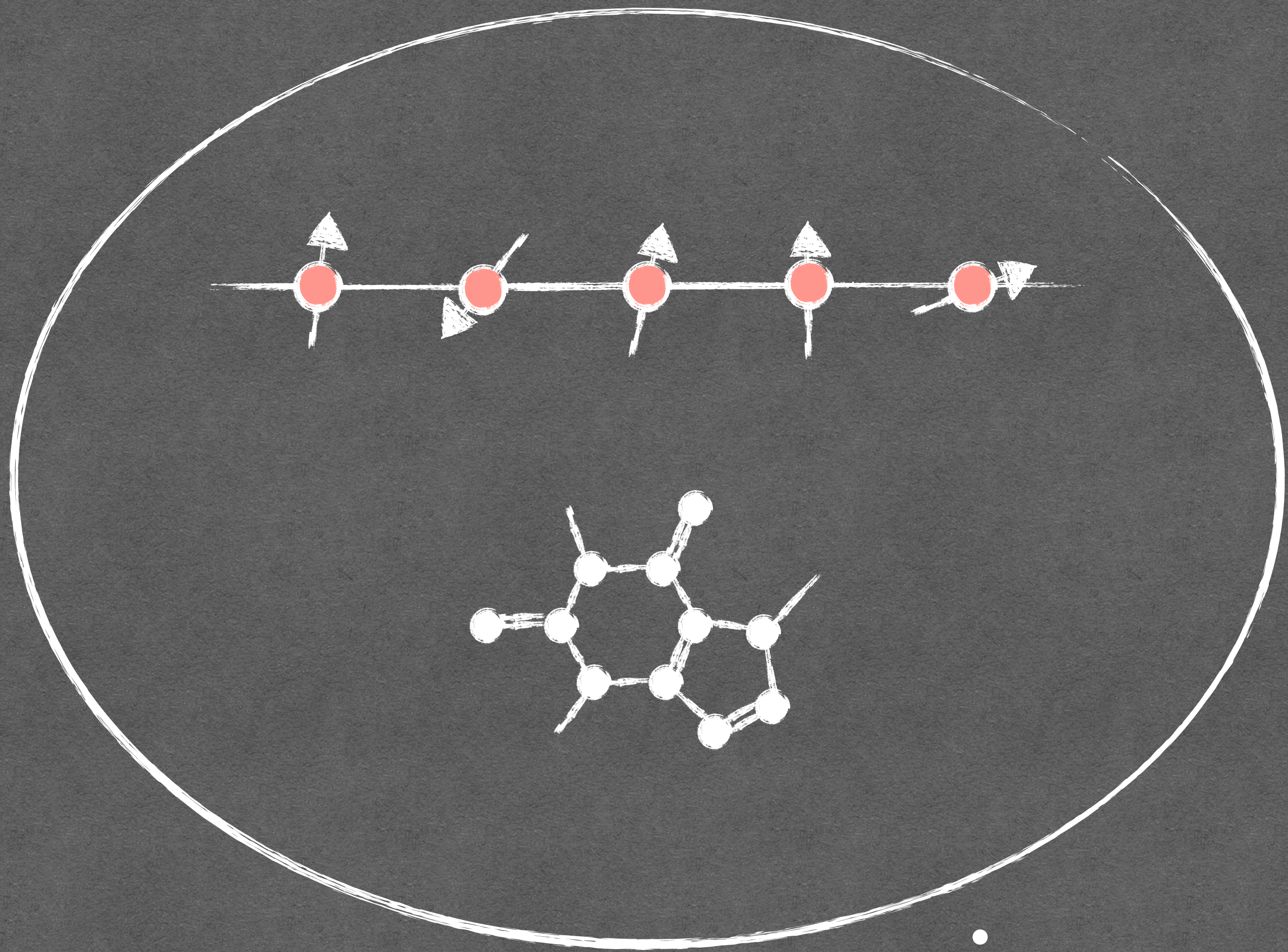
Quantum systems



Quantum systems

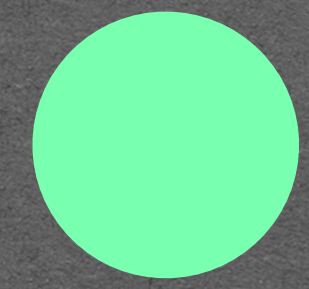
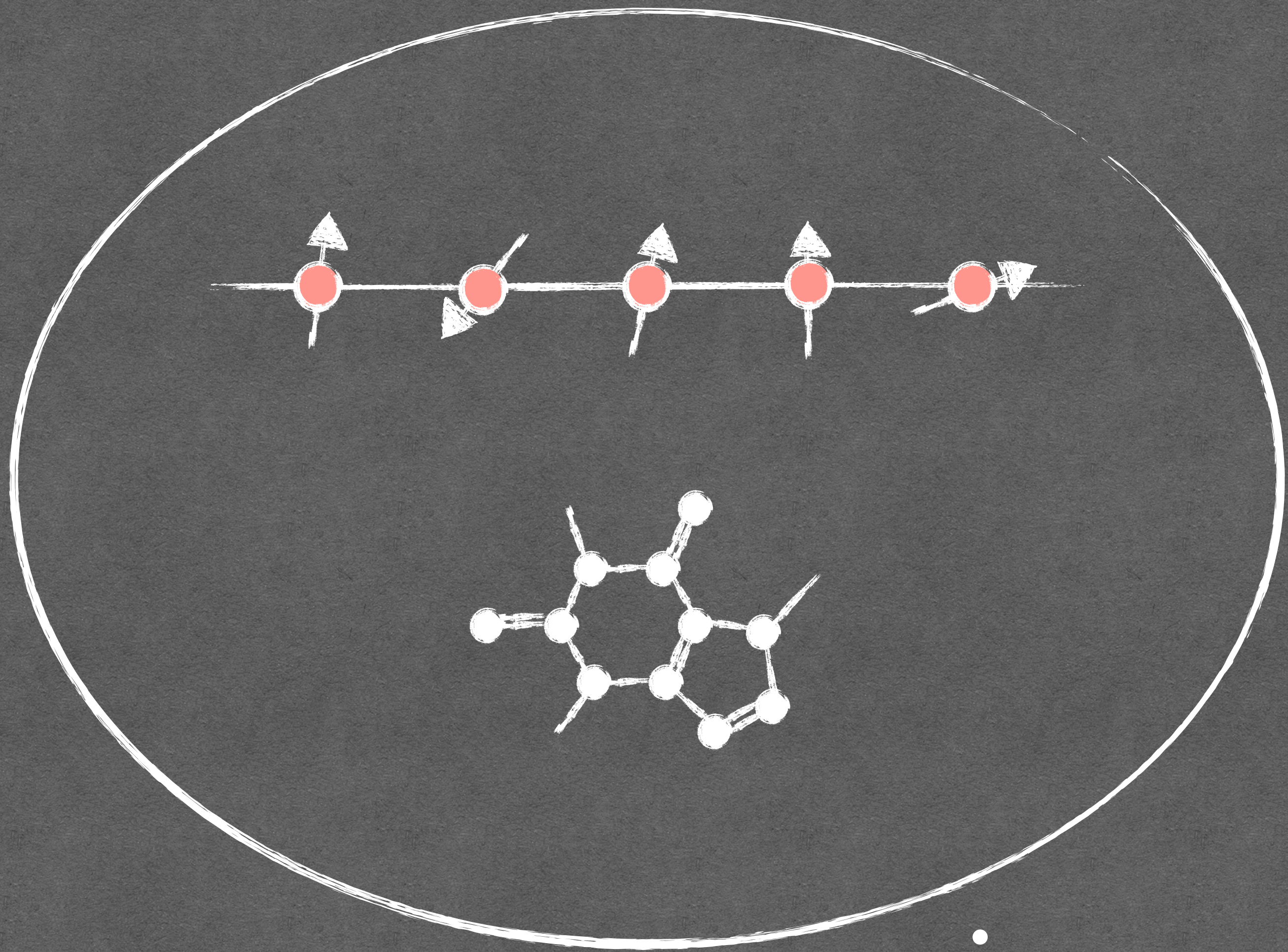


Quantum systems



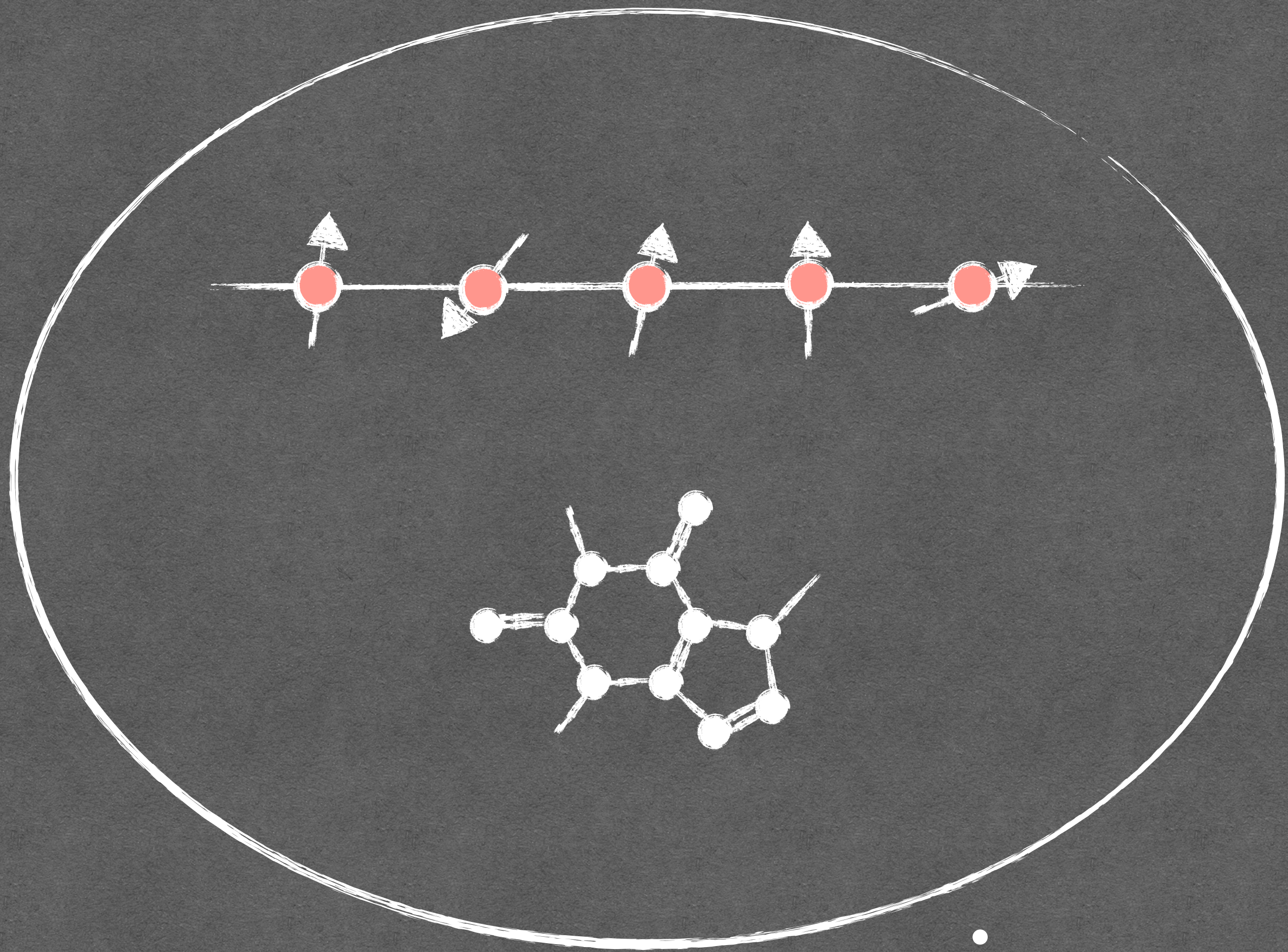
$$\mathcal{H} \sim 2^{\text{size}}$$

Quantum systems

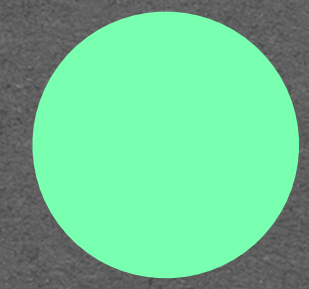


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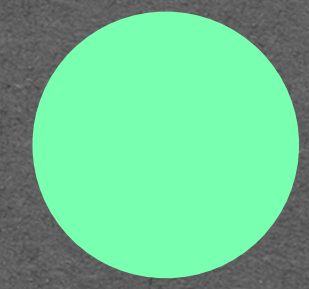
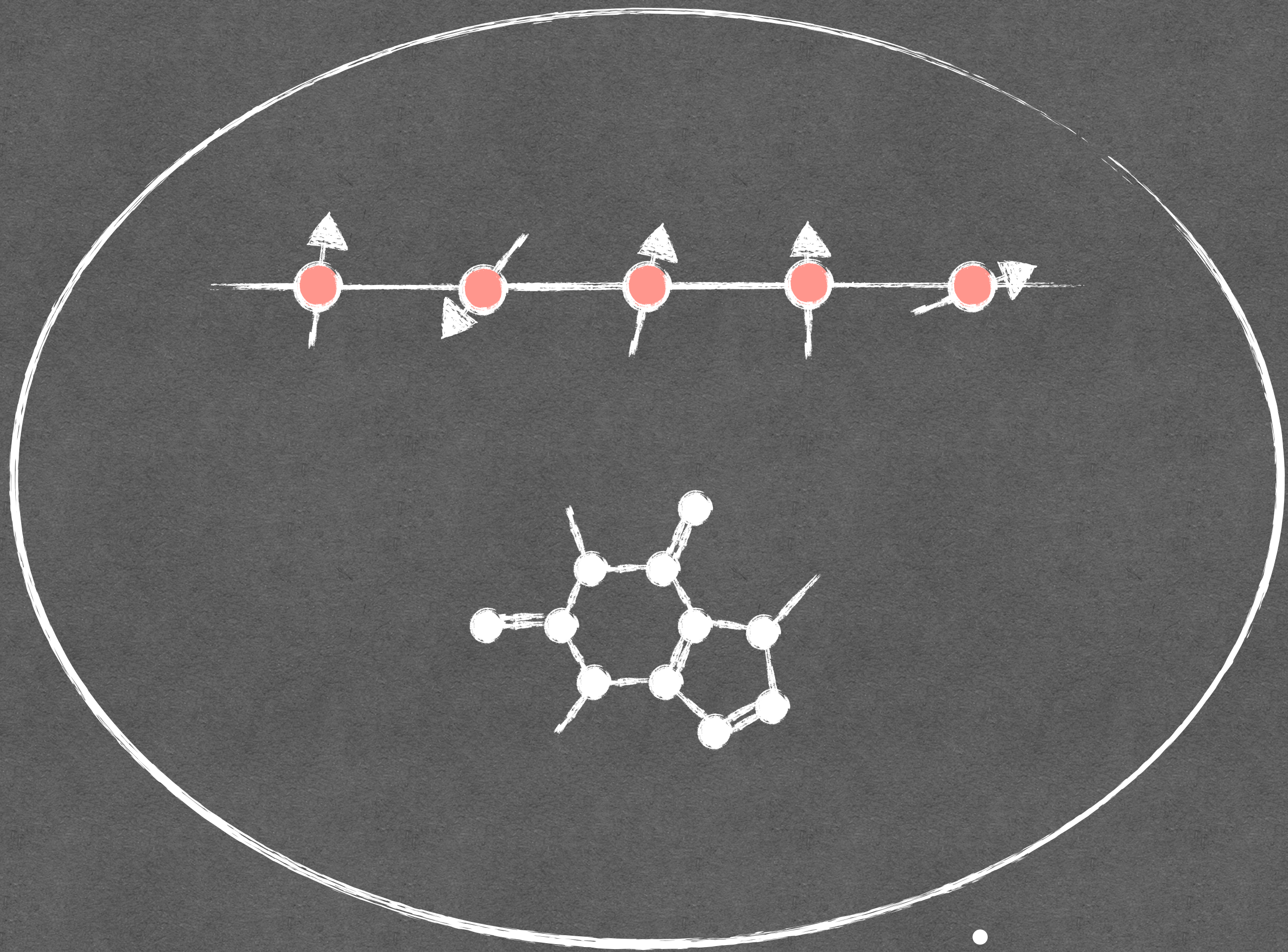


$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



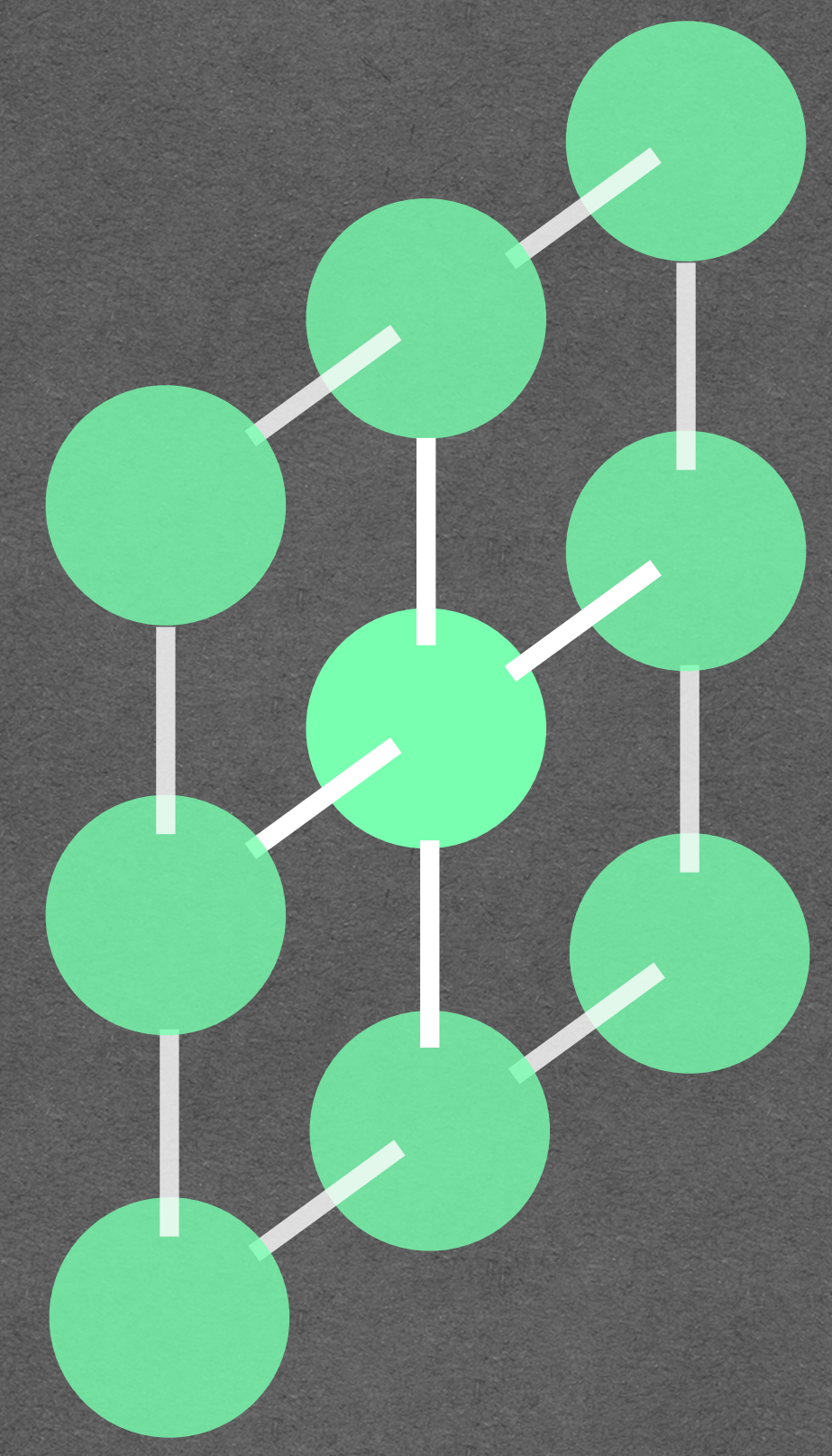
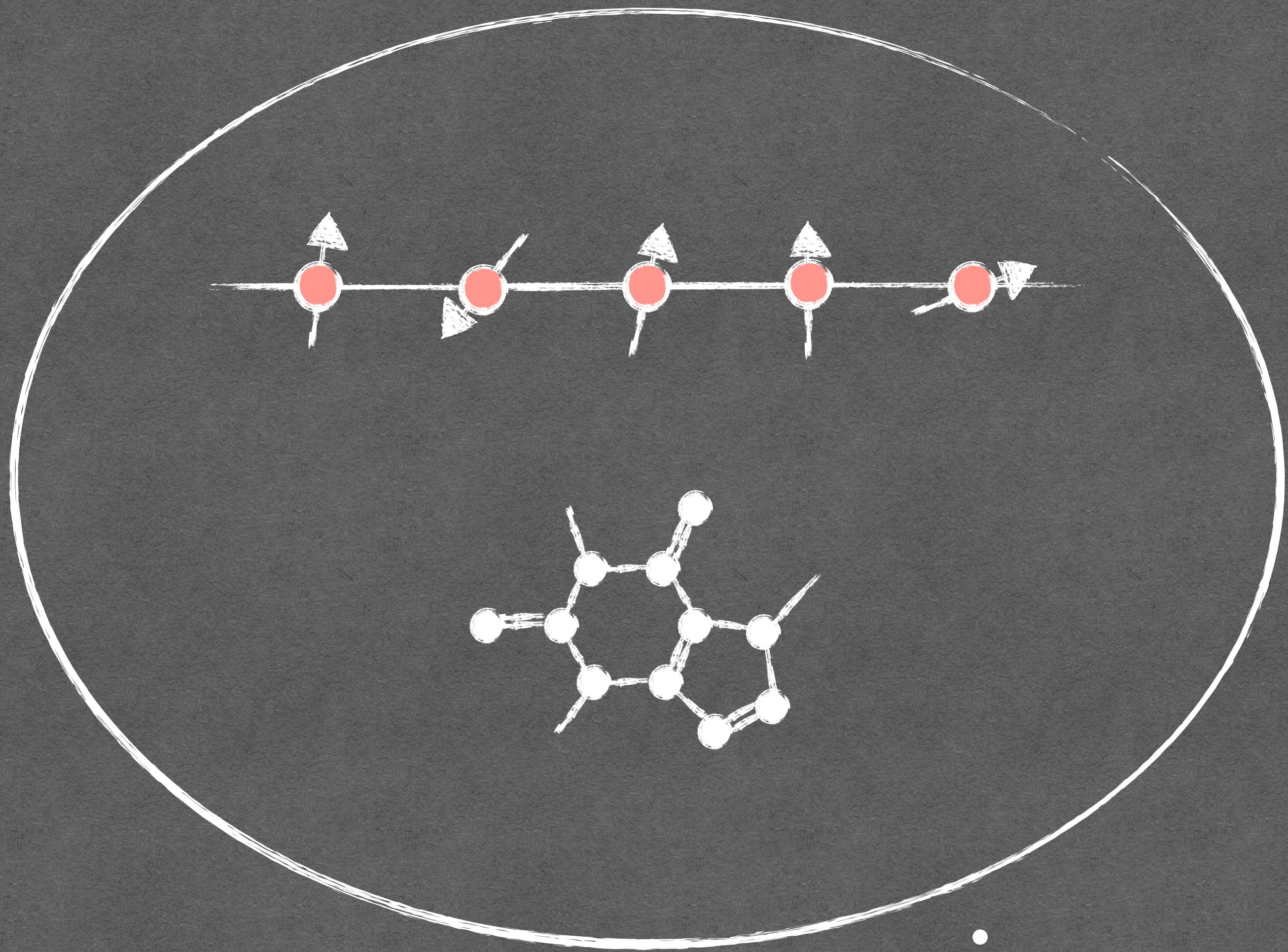
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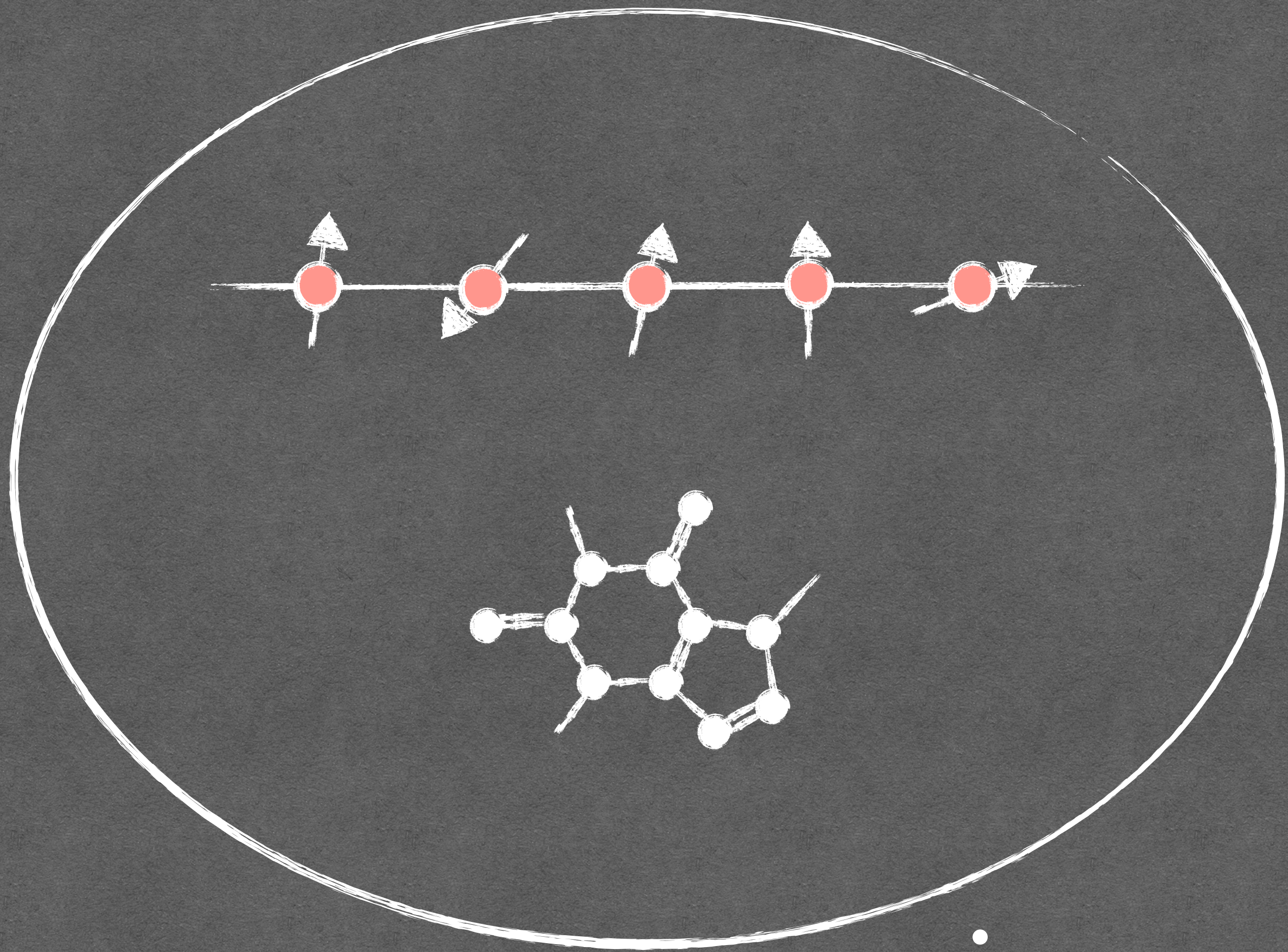
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Quantum systems

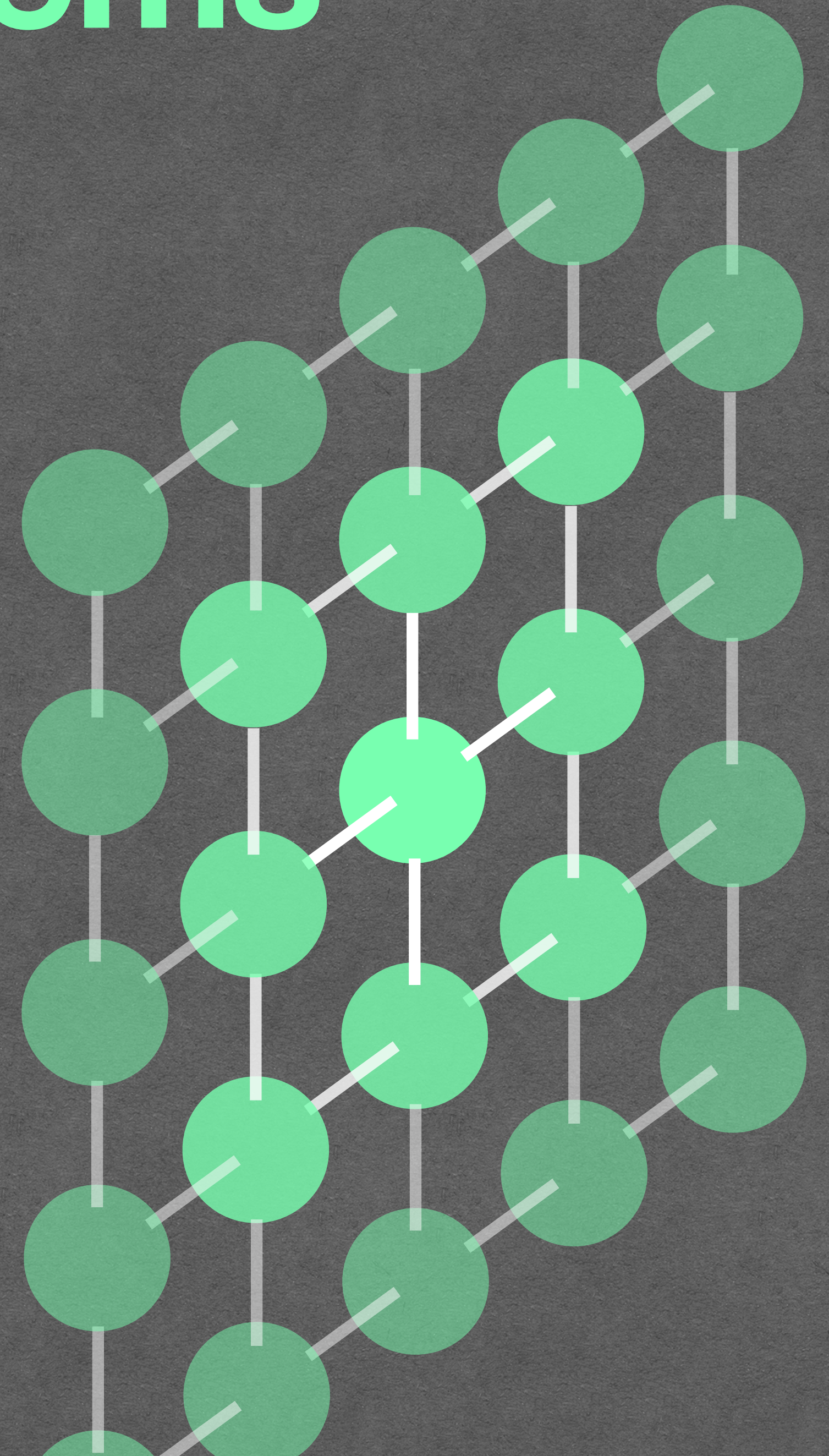


$$\mathcal{H} \sim 2^{\text{size}}$$

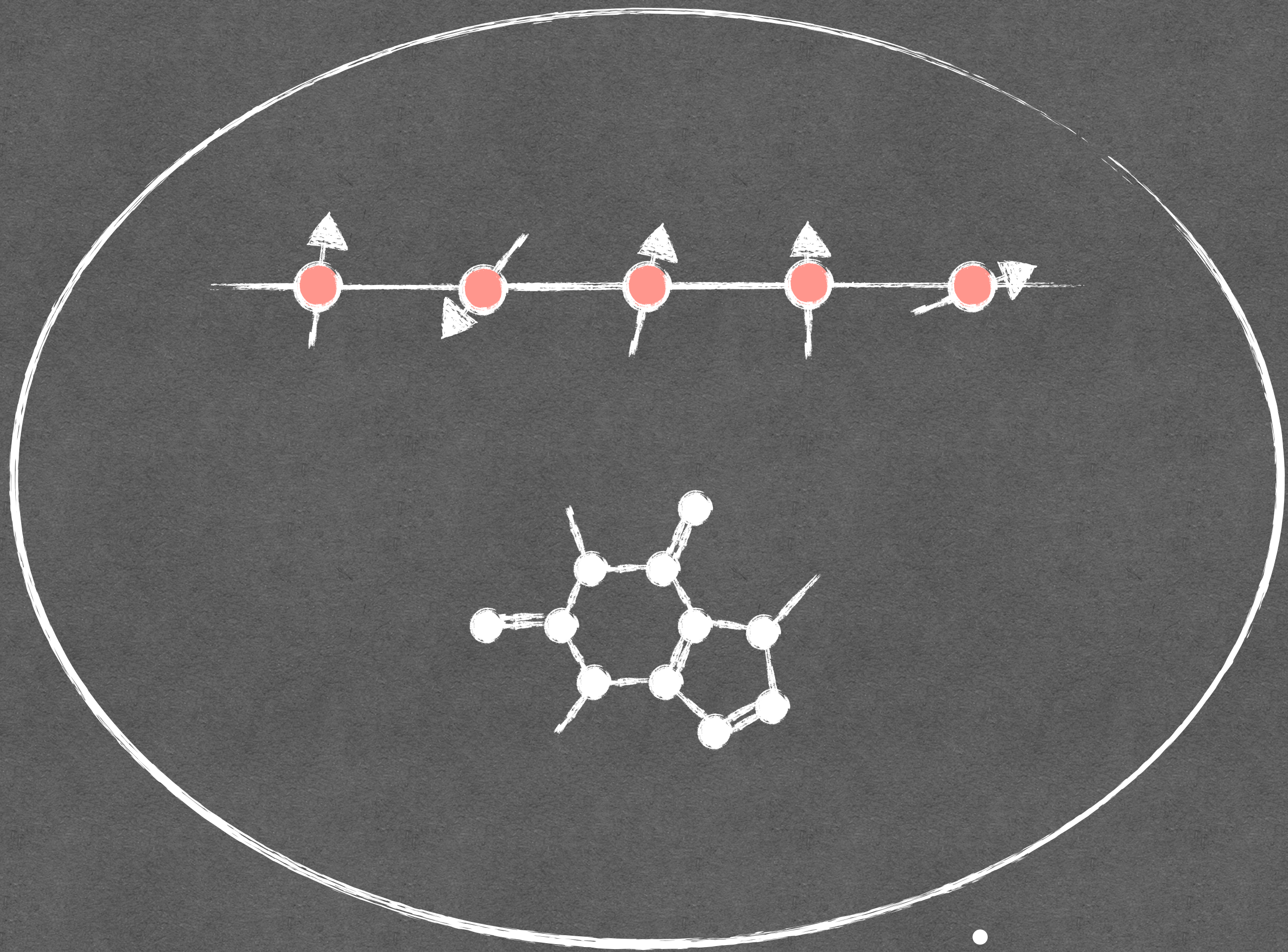
Quantum systems



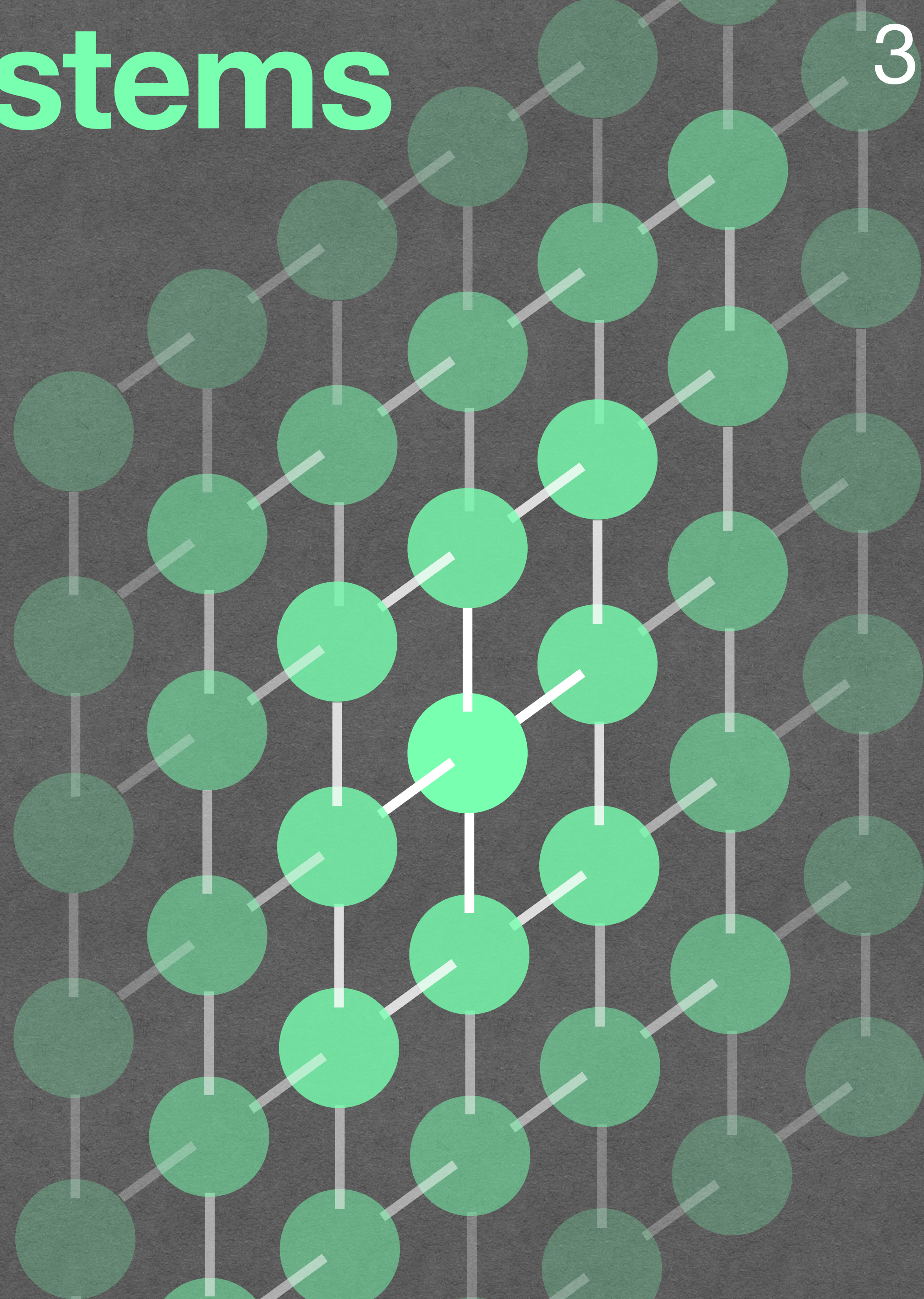
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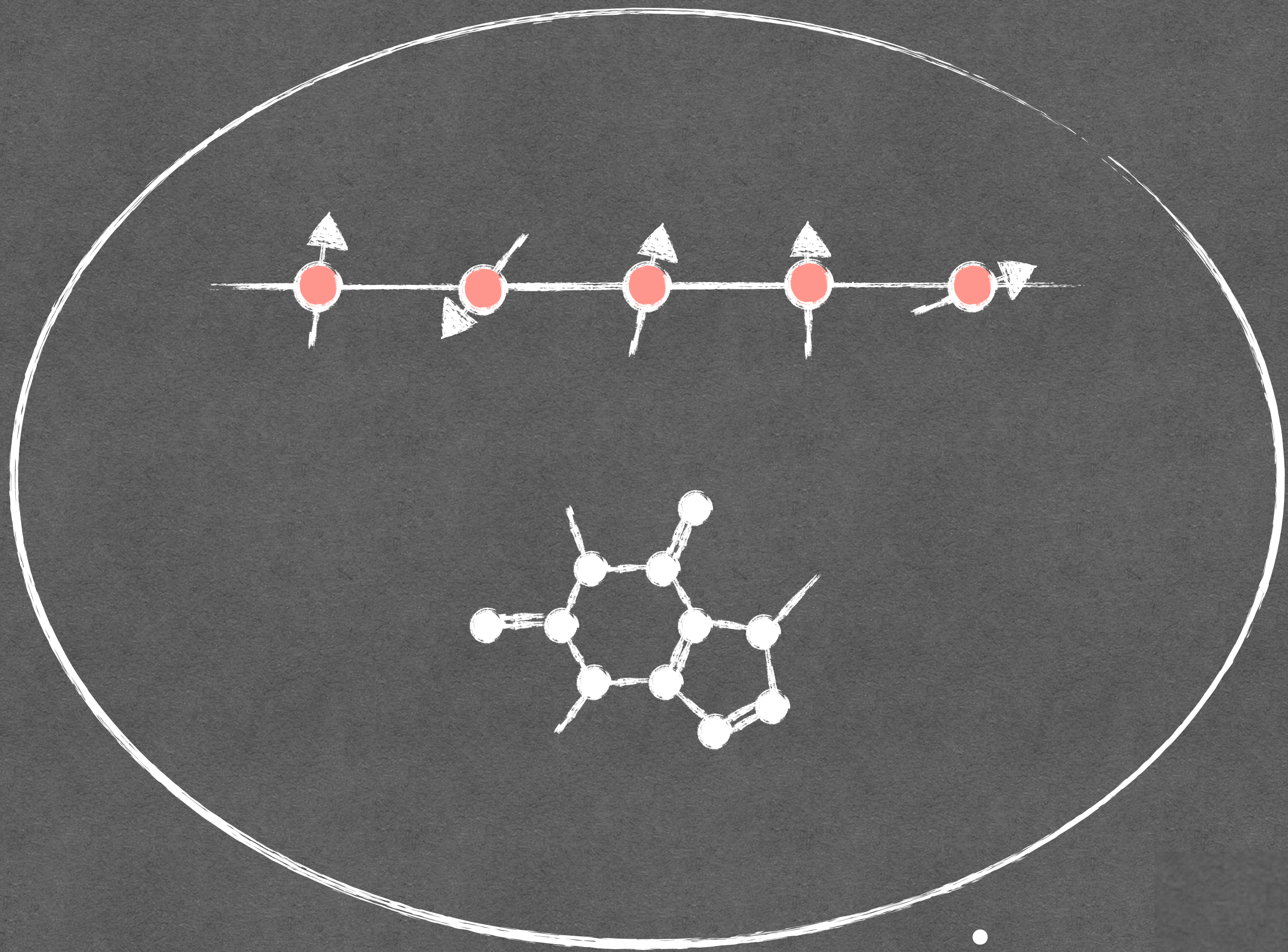
Quantum systems



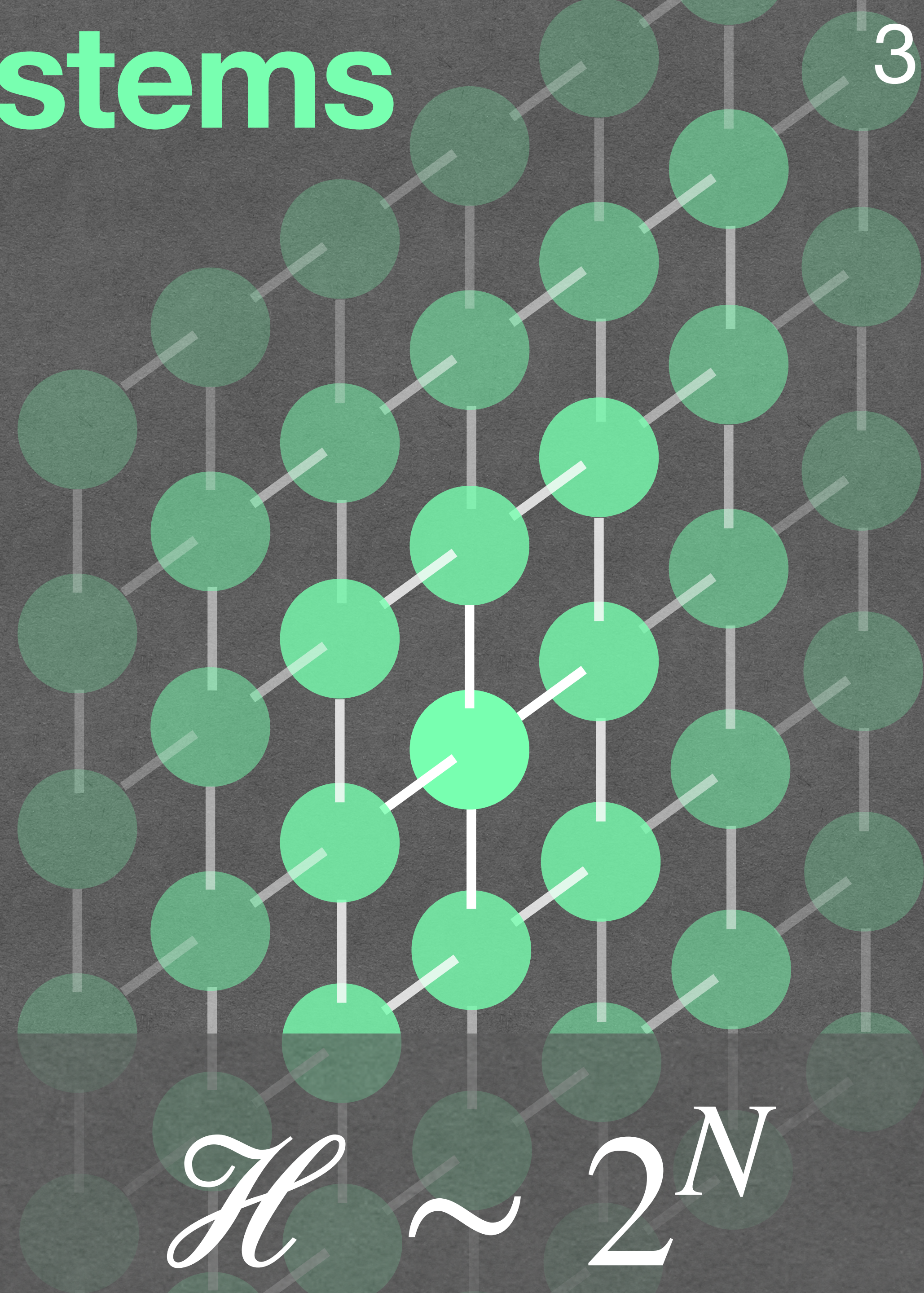
$$\mathcal{H} \sim 2^{\text{size}}$$



Quantum systems

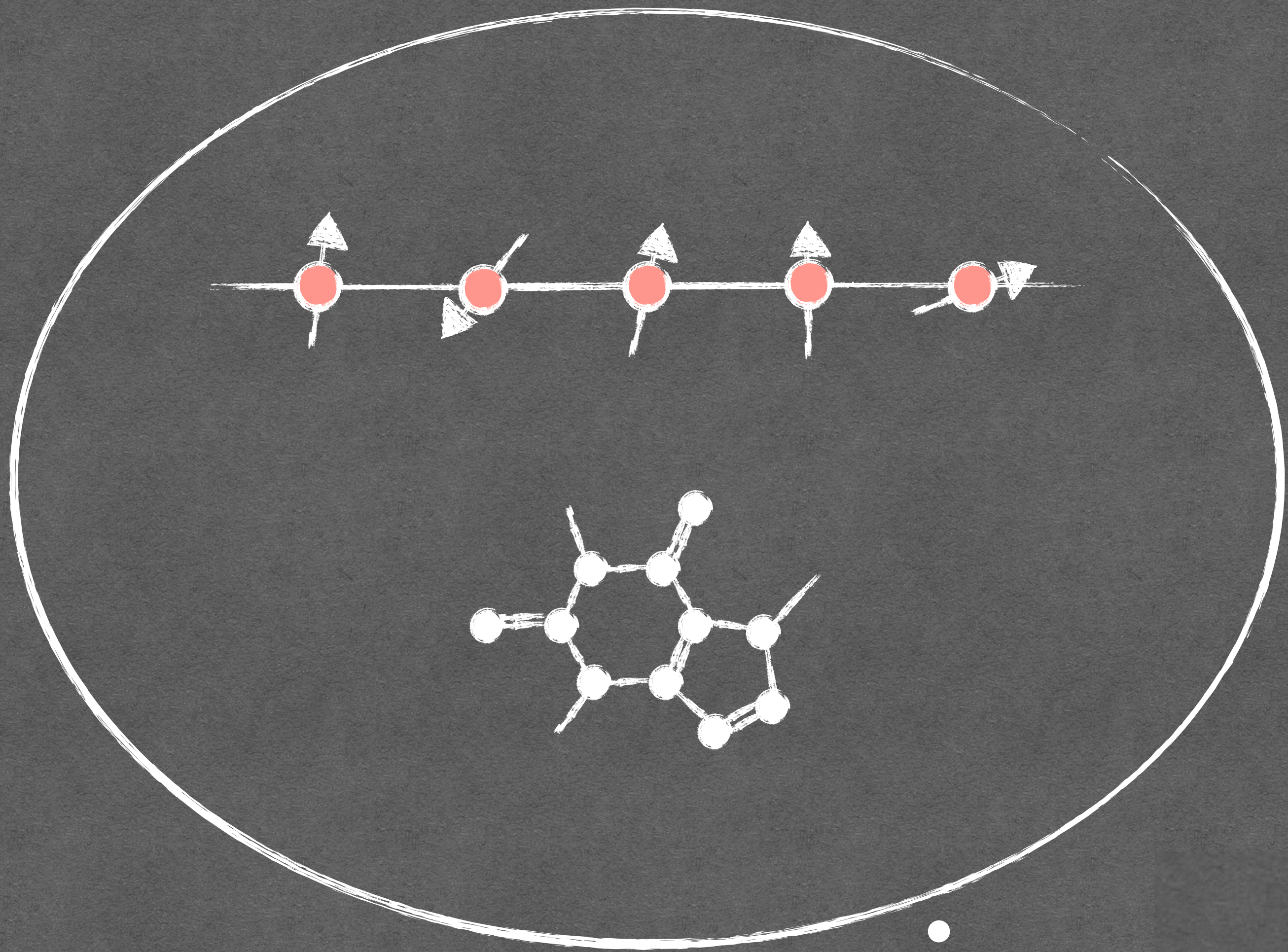


$$\mathcal{H} \sim 2^{\text{size}}$$

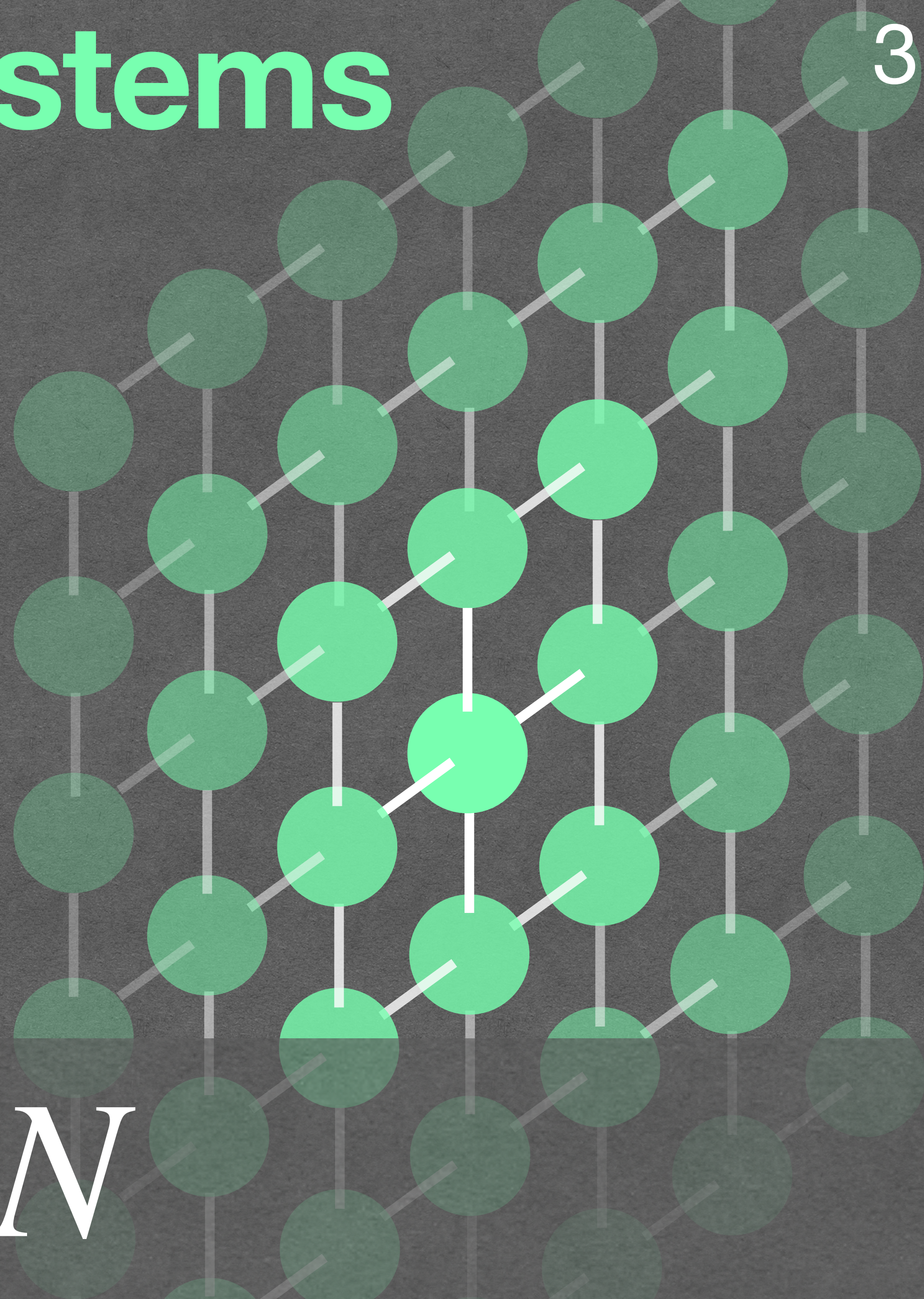


$$\mathcal{H} \sim 2^N$$

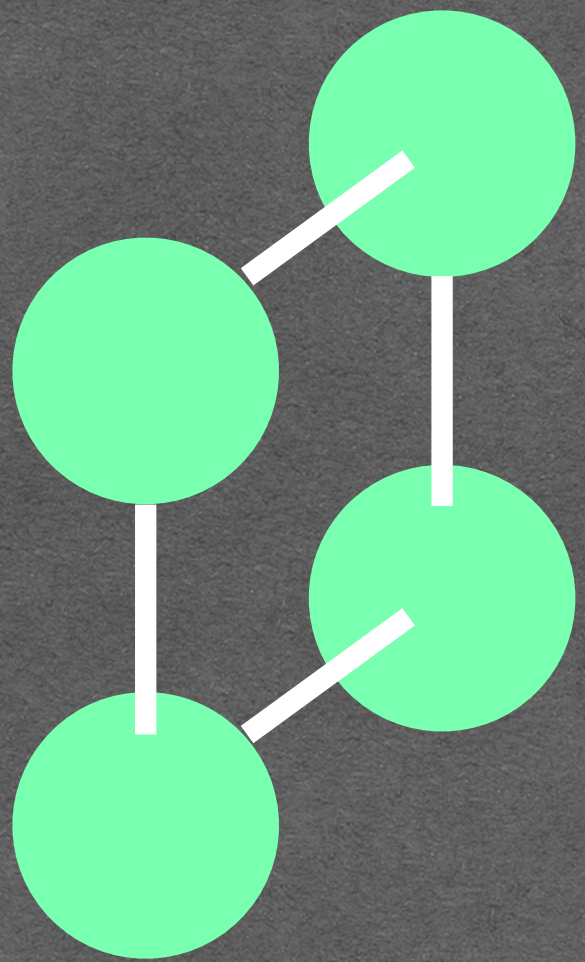
Quantum systems



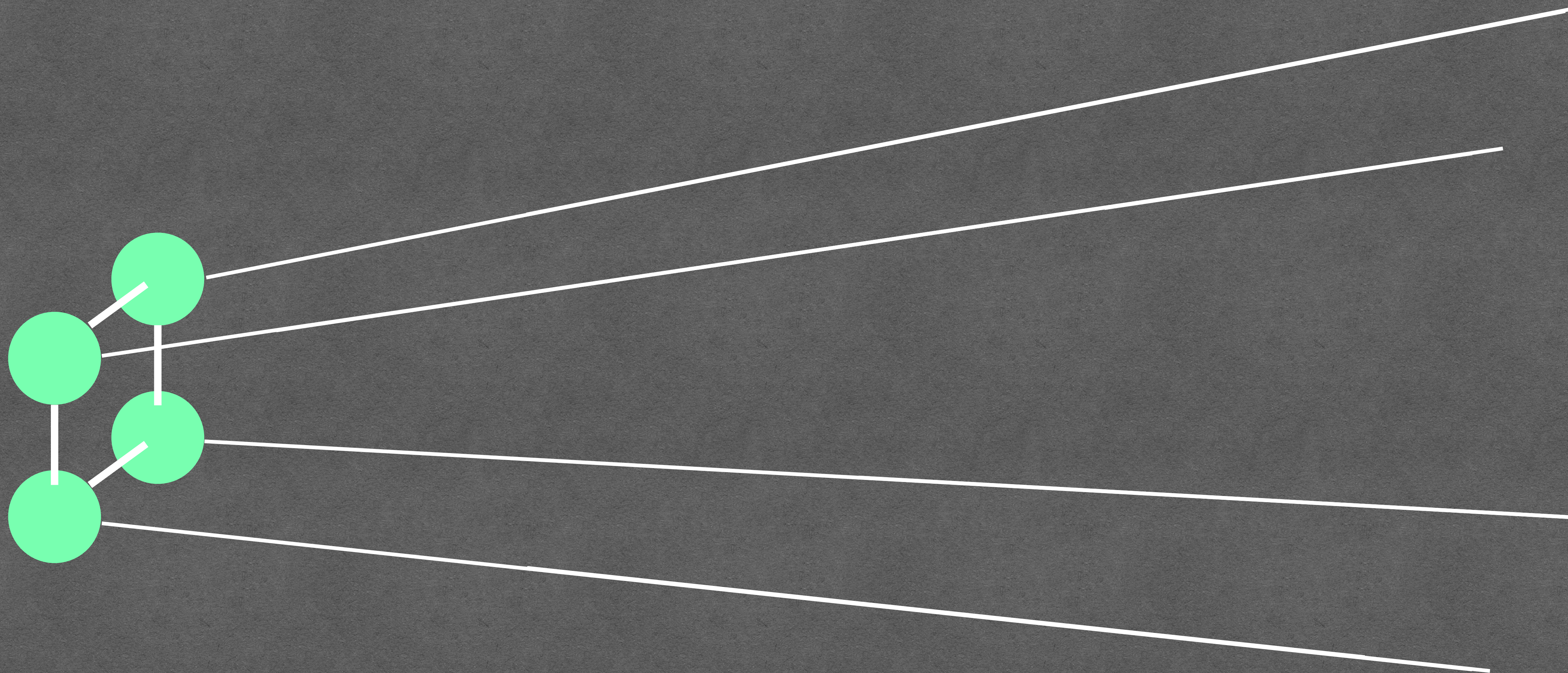
size $\sim N$



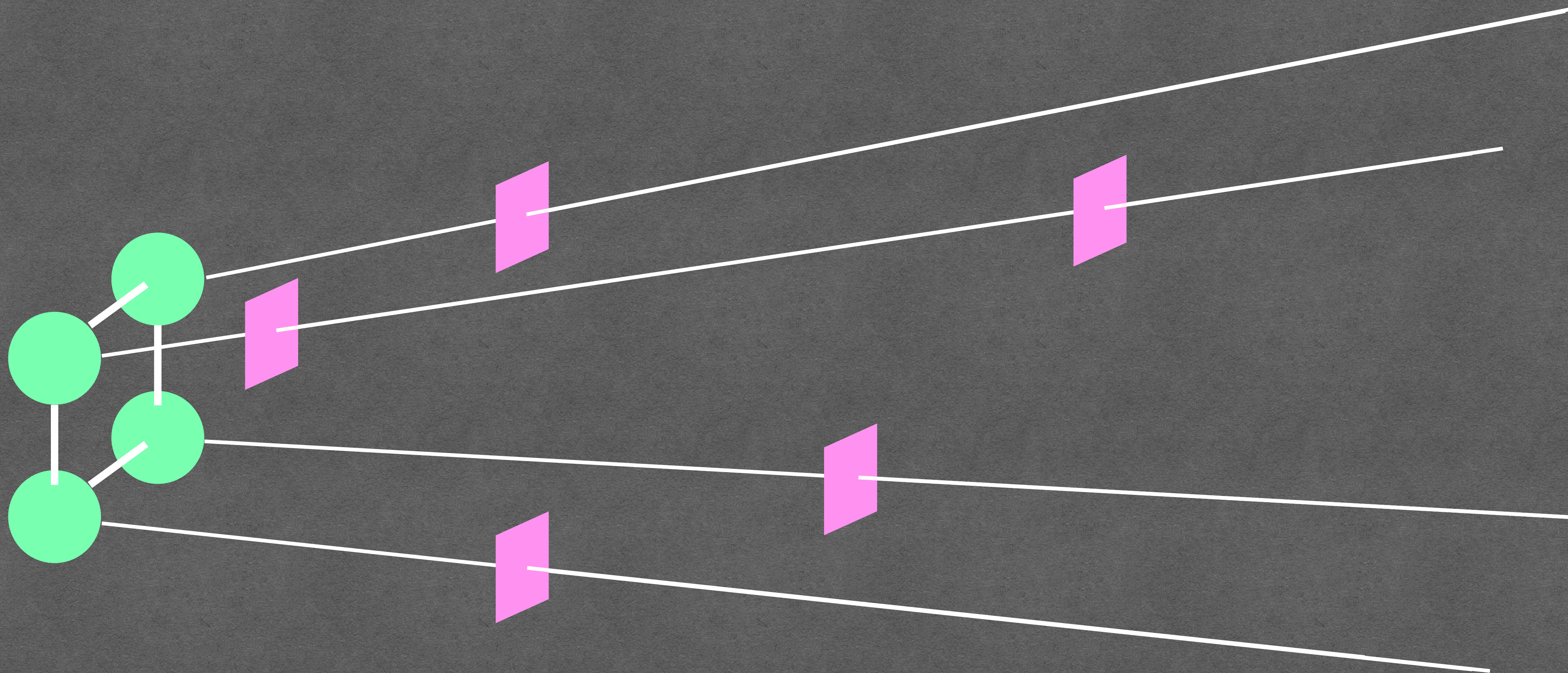
Quantum-state preparation



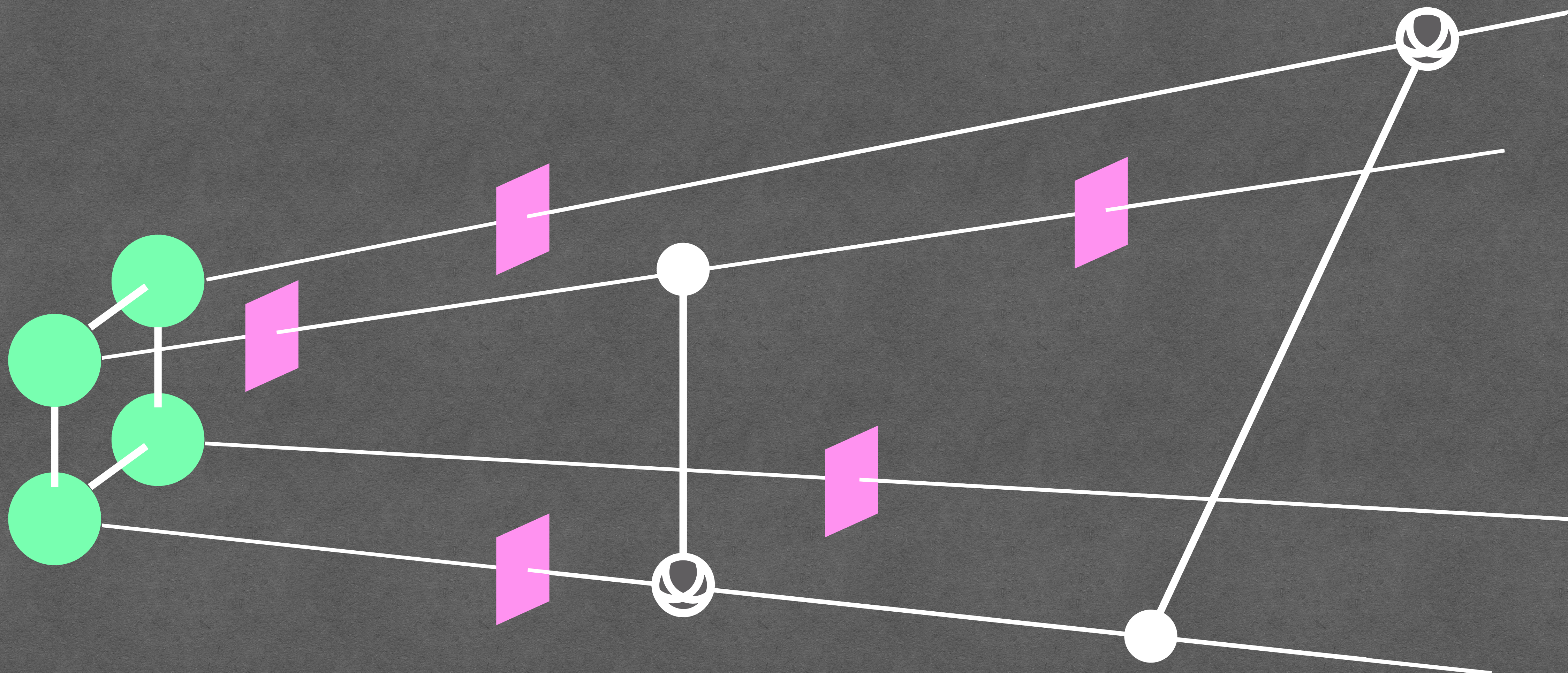
Quantum-state preparation



Quantum-state preparation



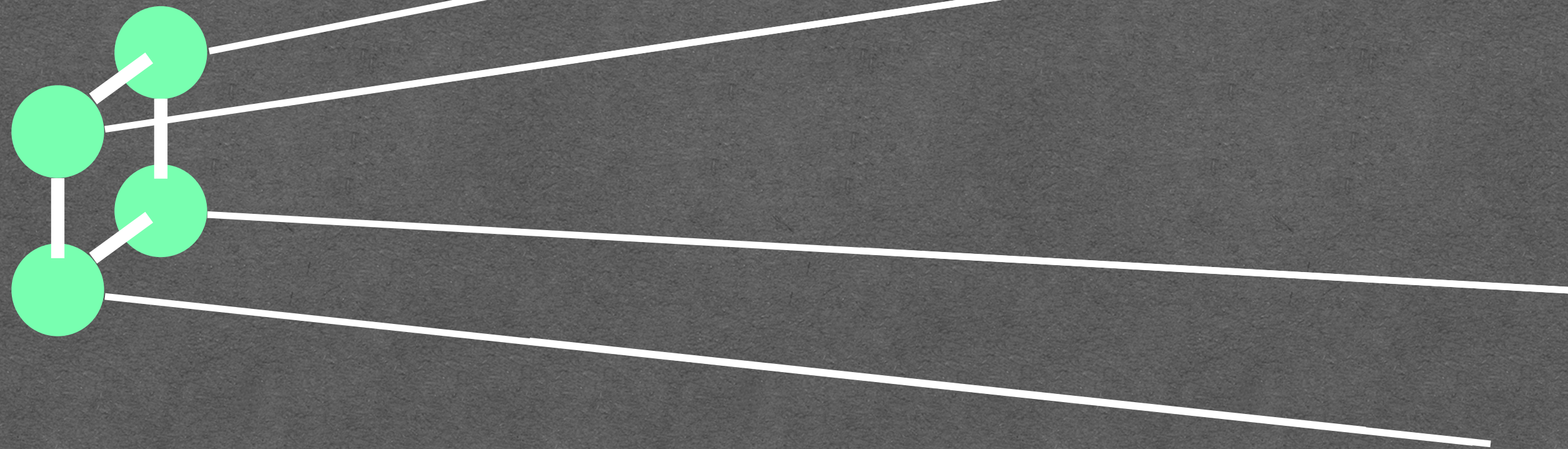
Quantum-state preparation



State preparation challenge

Default state

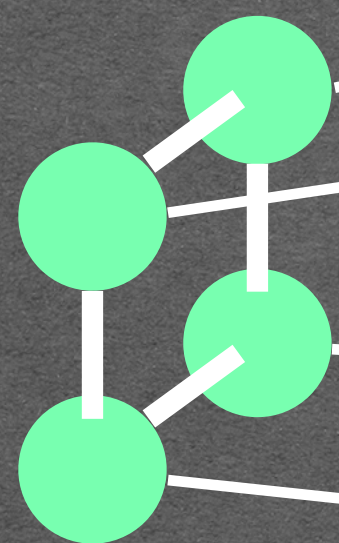
$$|0\rangle^{\otimes N}$$



State preparation challenge

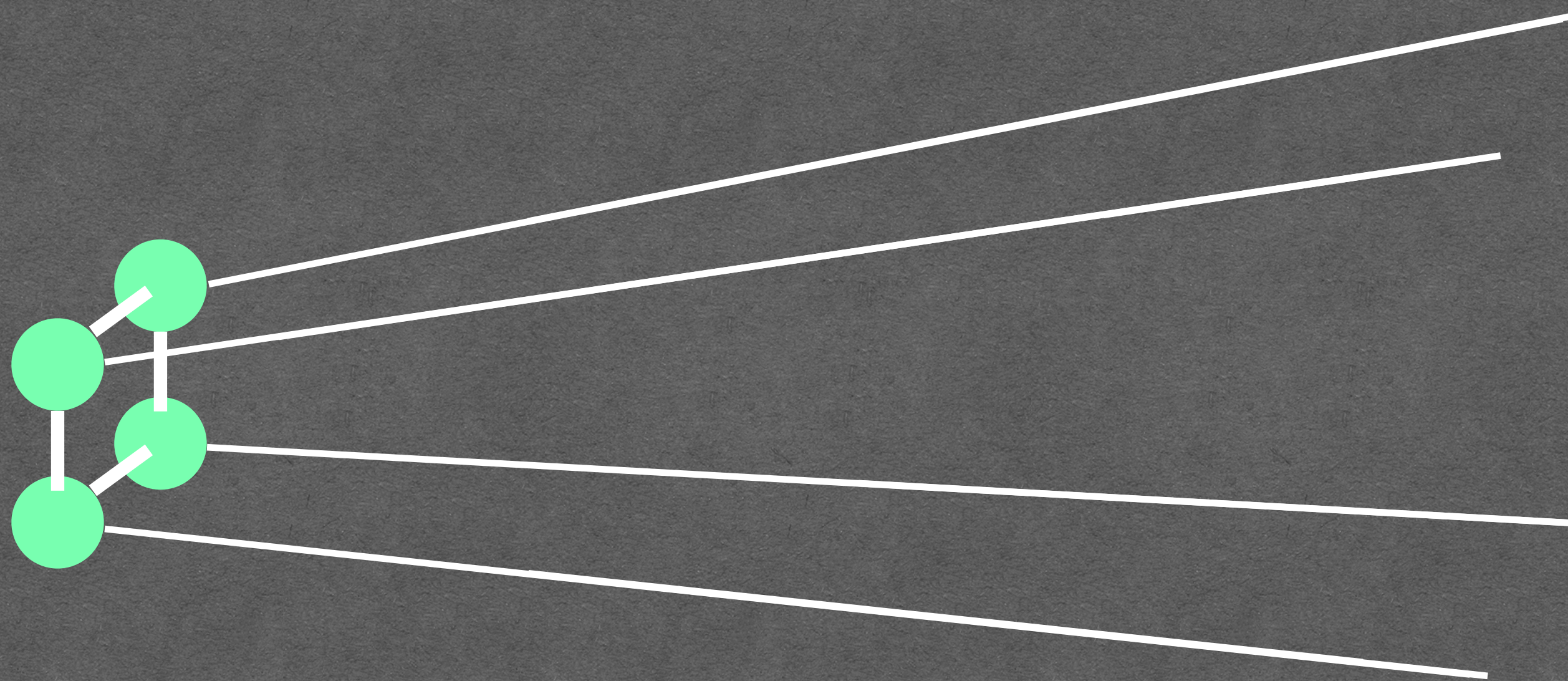
Default state

$$|0\rangle^{\otimes N}$$



Target state

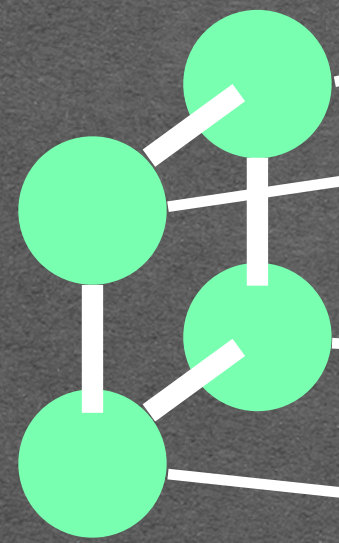
$$|\Psi\rangle$$



State preparation challenge

Default state

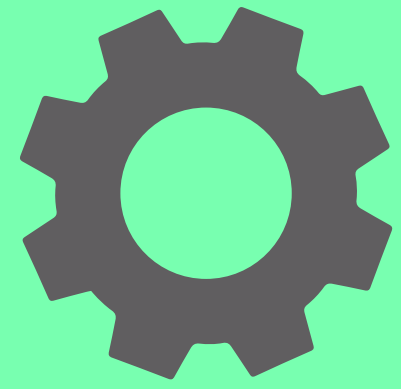
$$|0\rangle^{\otimes N}$$



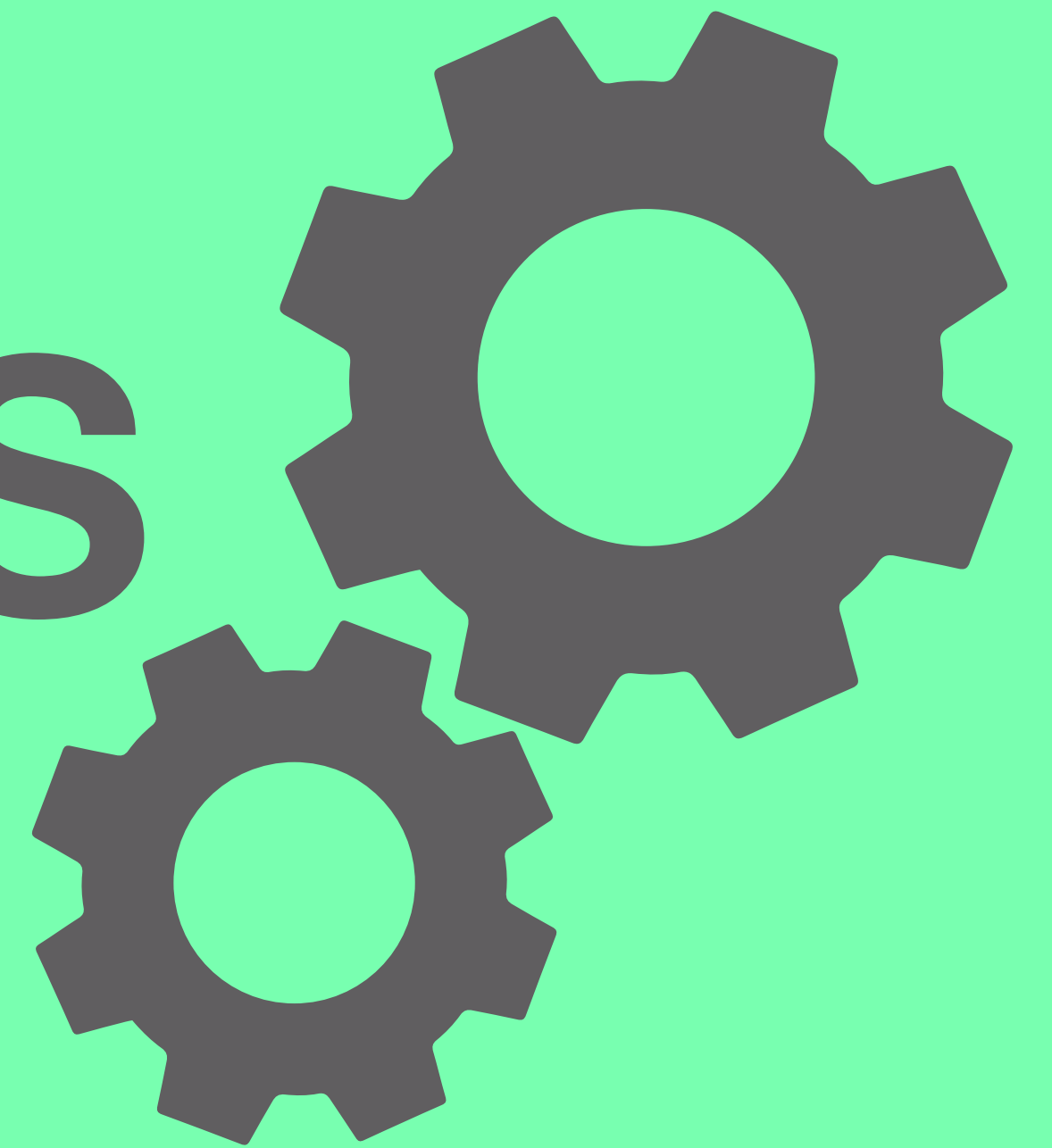
Target state

$$|\Psi\rangle$$

Fixed depth (# )



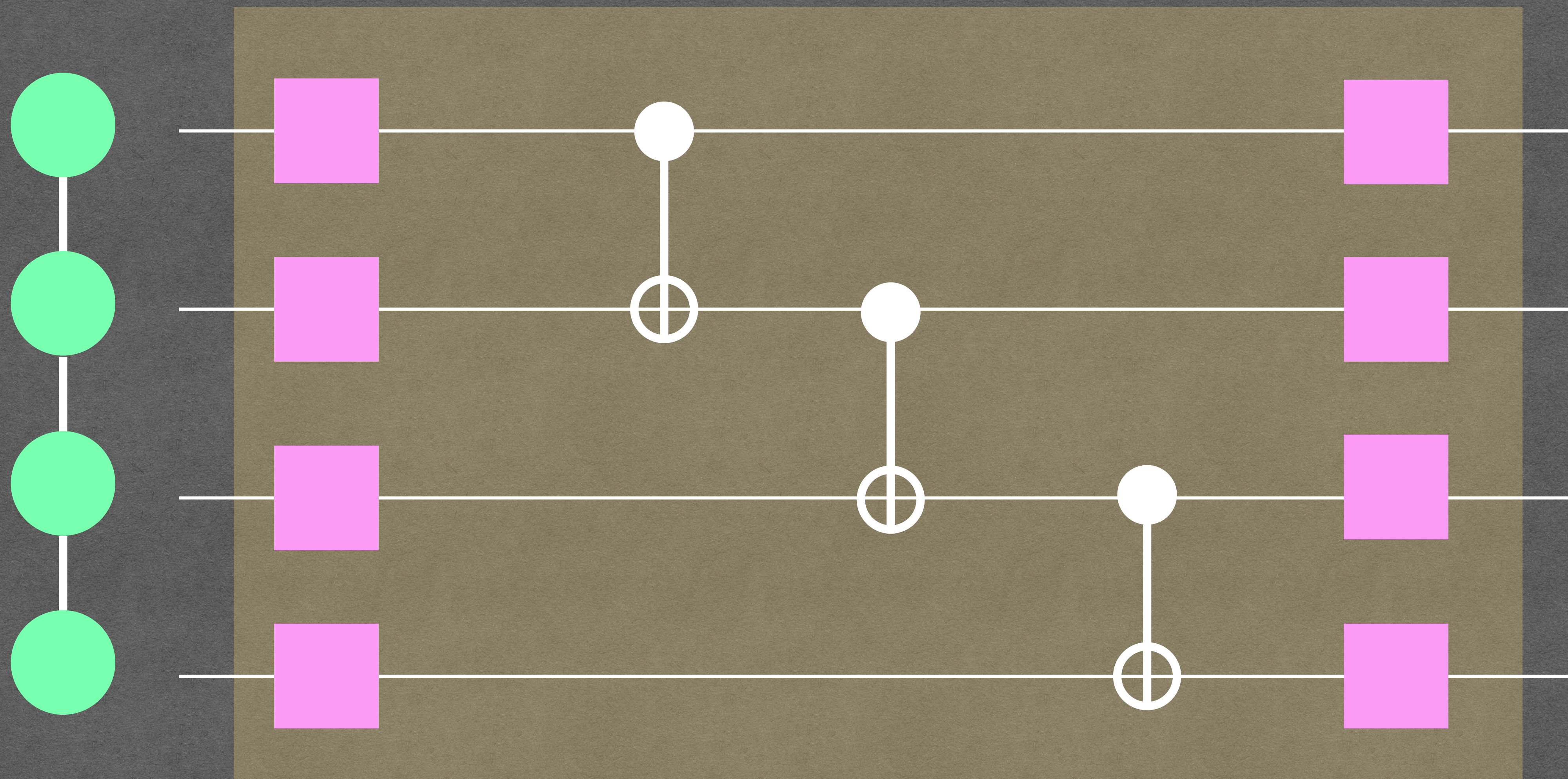
ALGORITHMS



H_1 : **Hamiltonian of the problem**

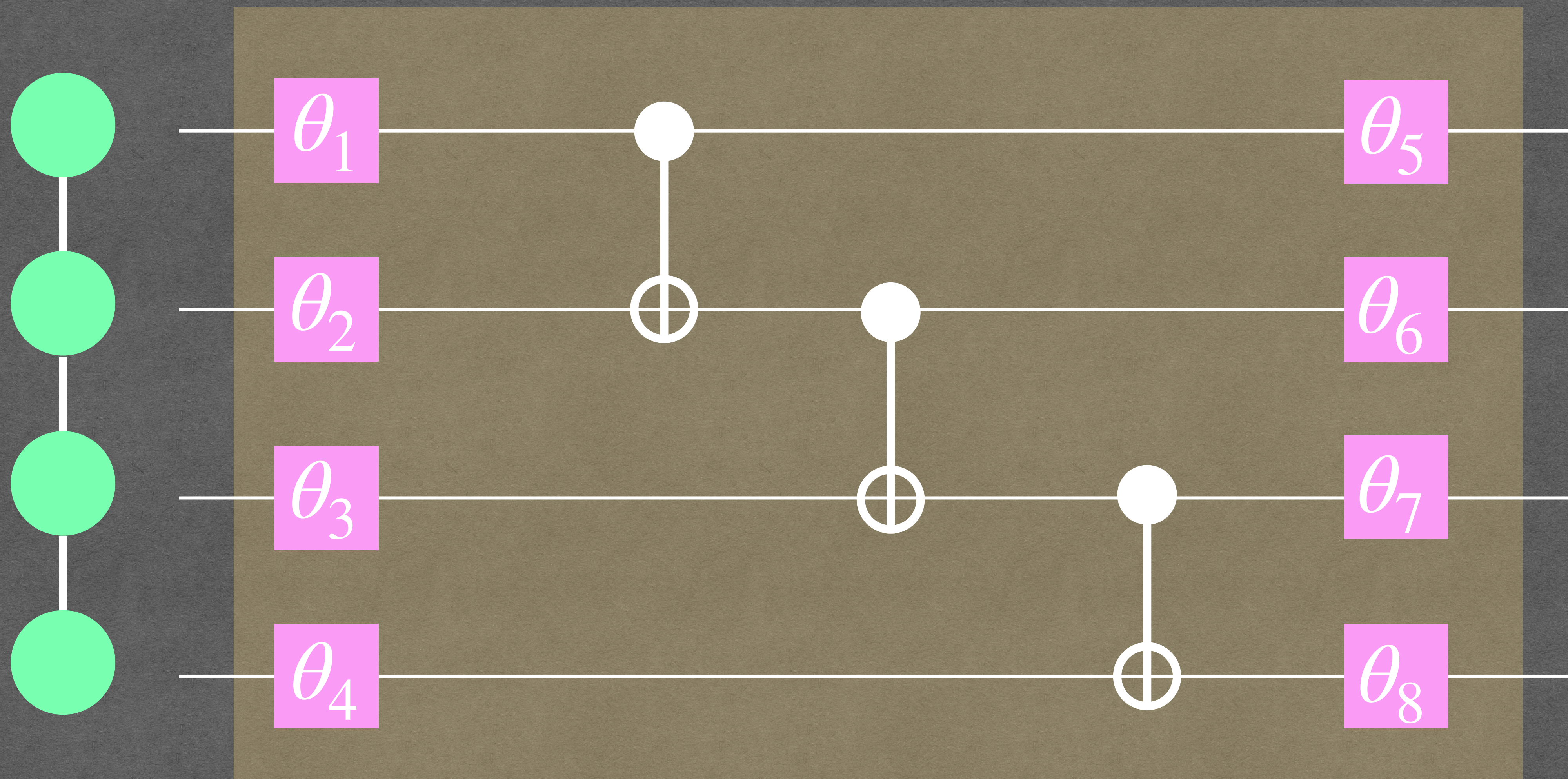
$|\Psi_n\rangle$: **n-th eigenstate (target)**

Variational Quantum Eigensolvers (VQE)



Fixed depth (# )

Variational Quantum Eigensolvers (VQE)



Fixed depth (# )

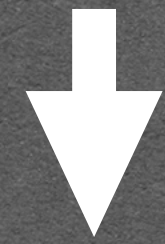
Main Problems of VQE

Shallow circuit

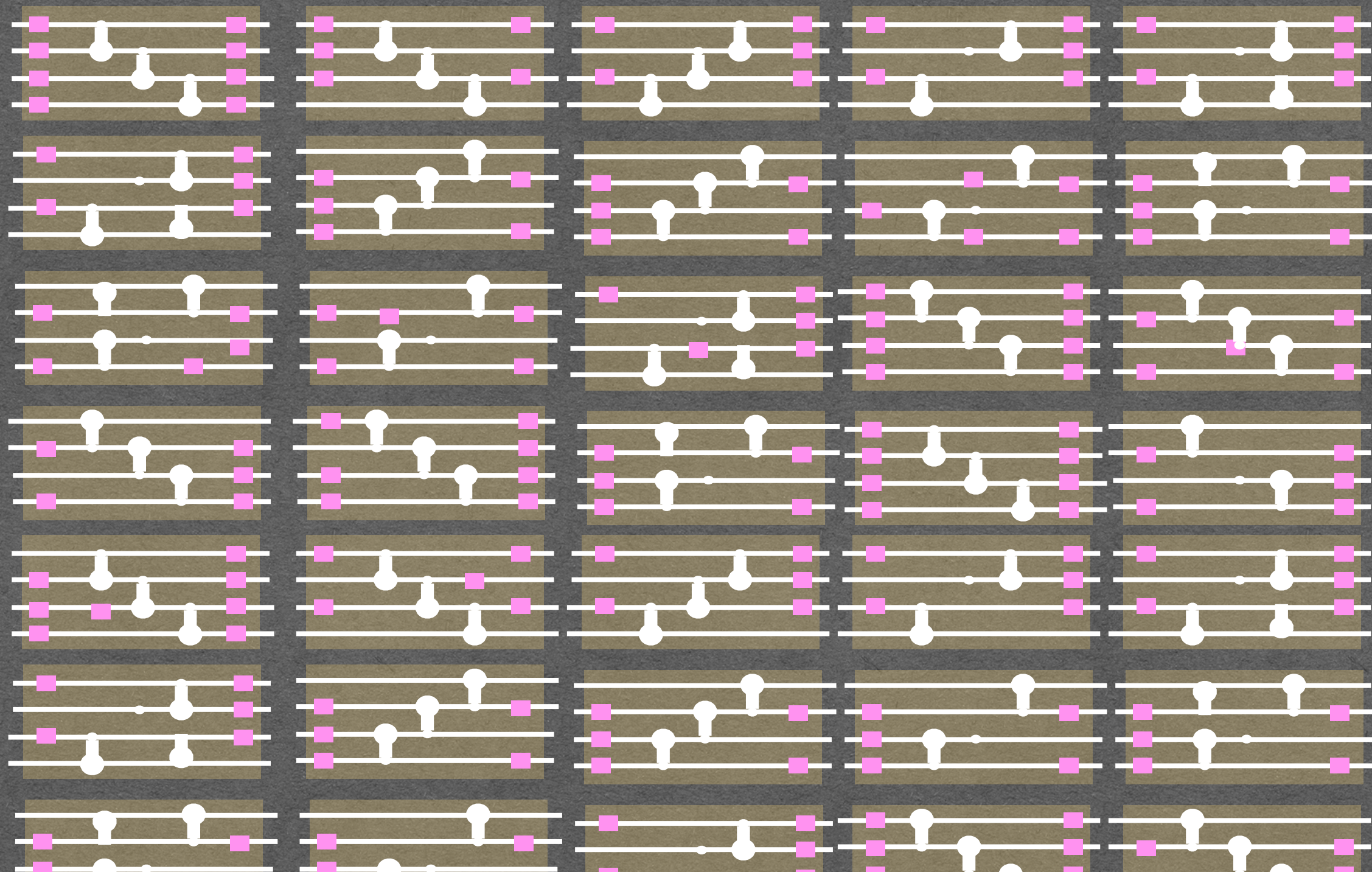


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Shallow circuit

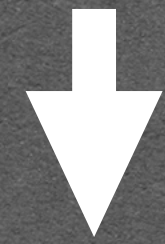


Ansatz choice

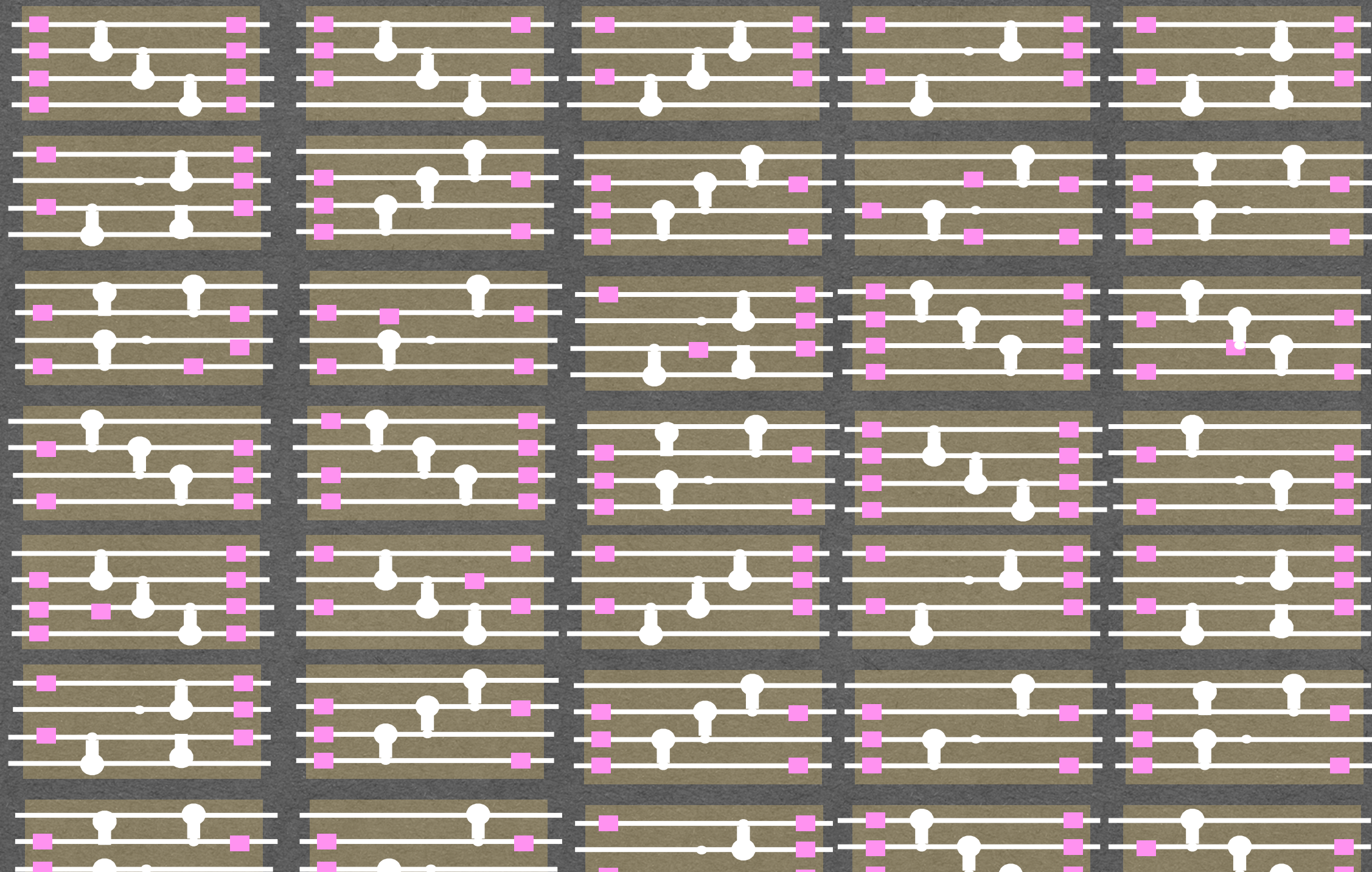


Main Problems of VQE

Shallow circuit



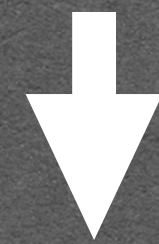
Ansatz choice



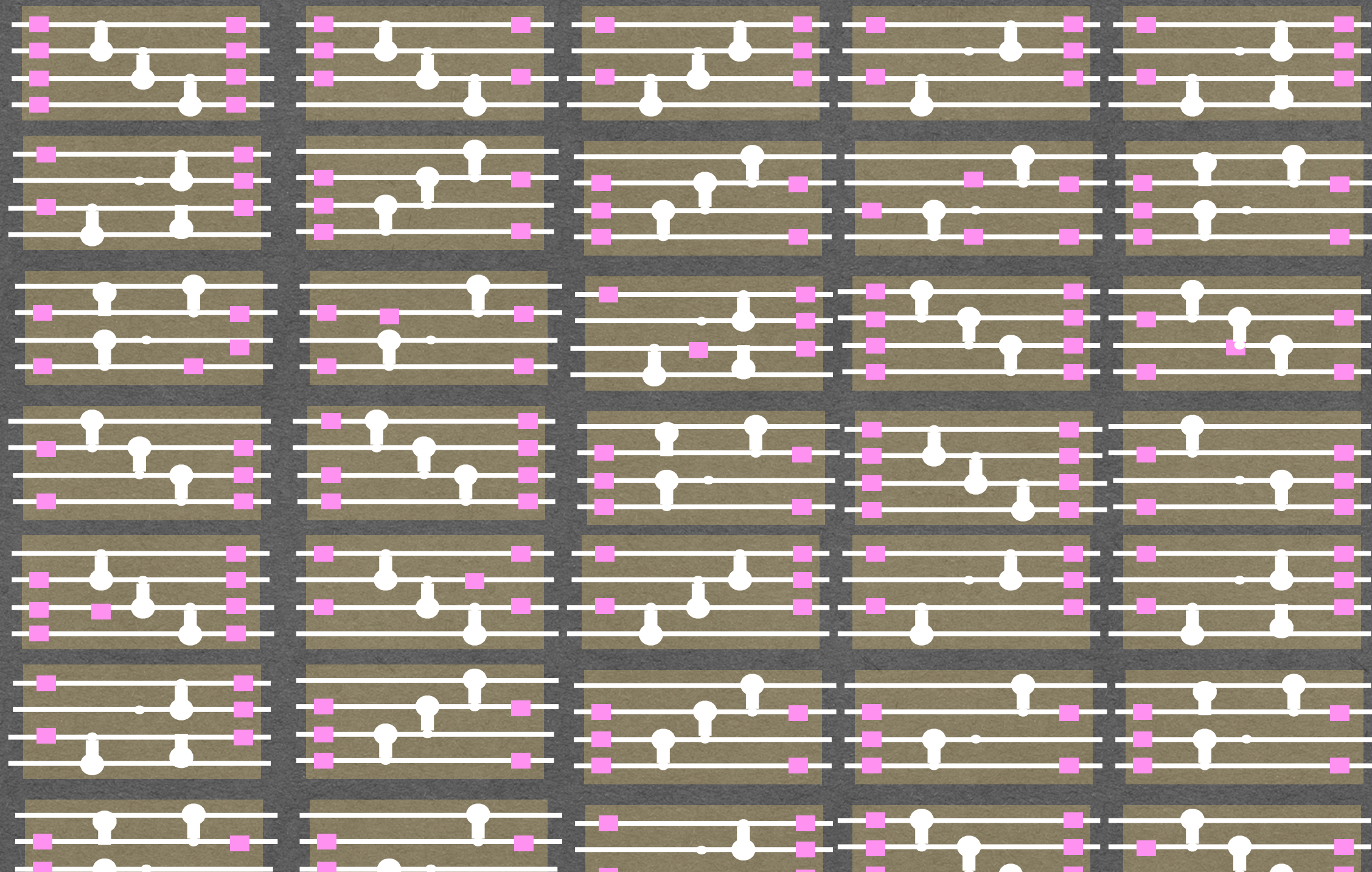
Large Hilbert spaces

Main Problems of VQE

Shallow circuit



Ansatz choice



Large Hilbert spaces

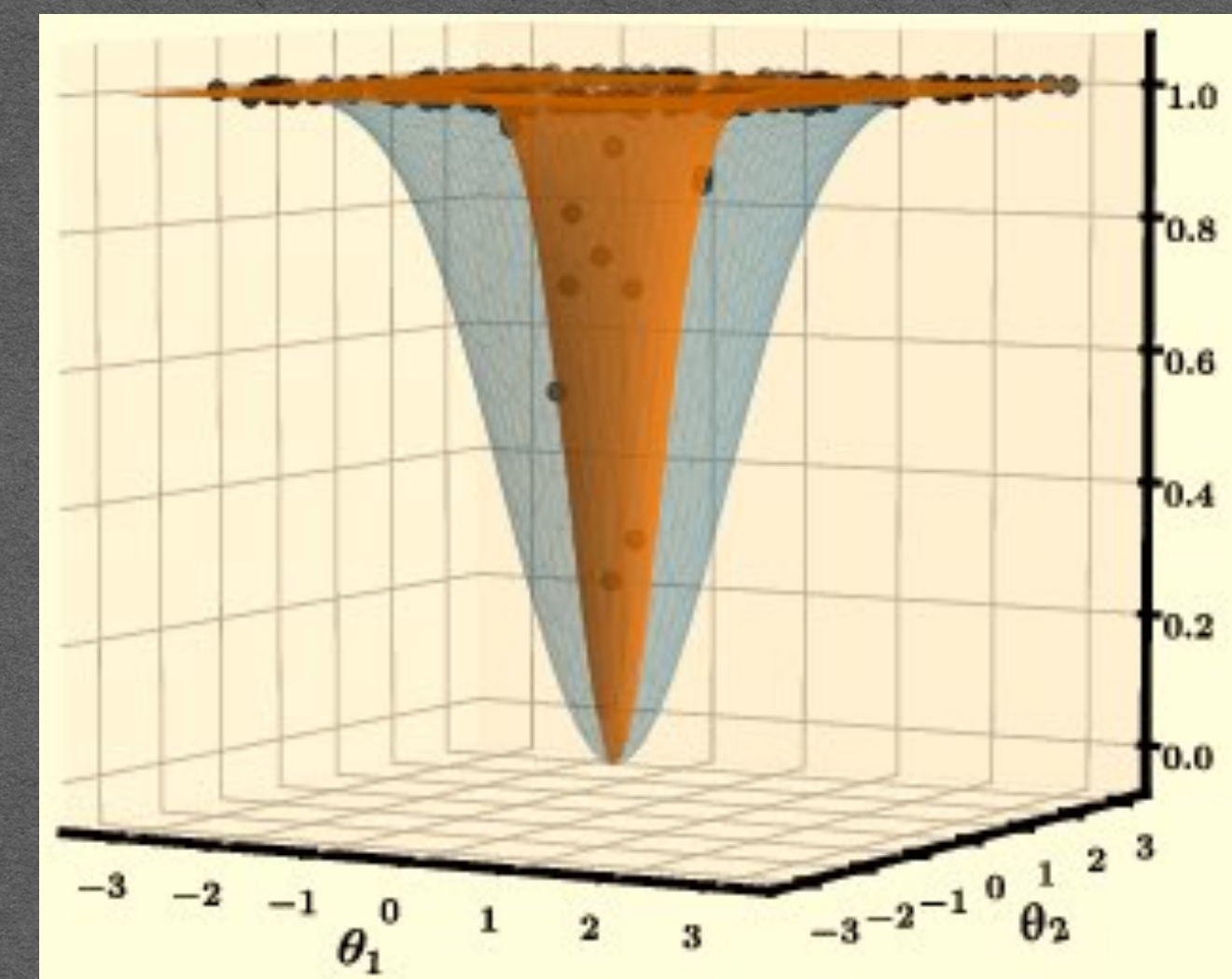


Barren Plateaus

$$\mathcal{H} \sim 2^N$$



Loss function



Adiabatic Preparation AP

H_1

—

—

—

—

Adiabatic Preparation AP

H_0

—

—

—

—

H_1

—

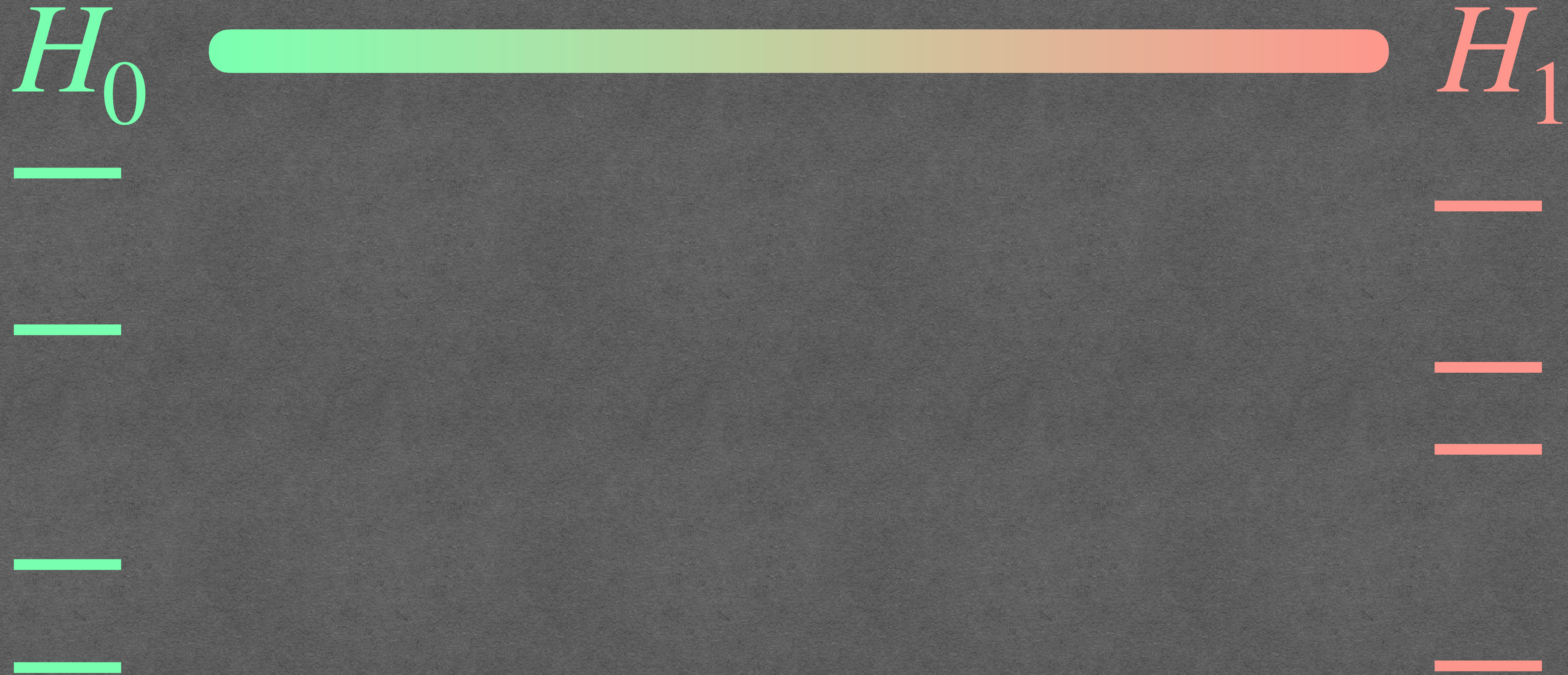
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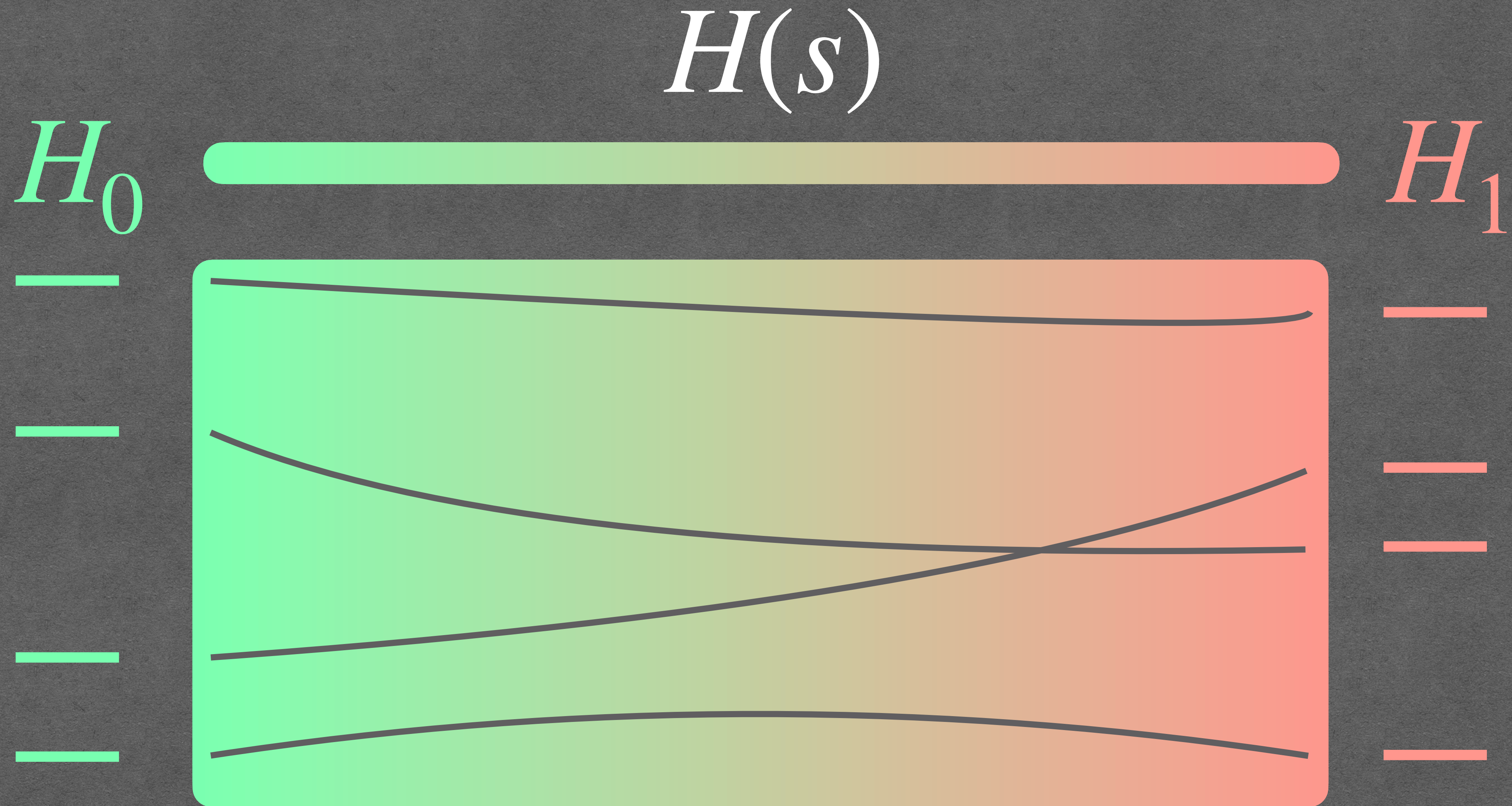
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Adiabatic Preparation AP

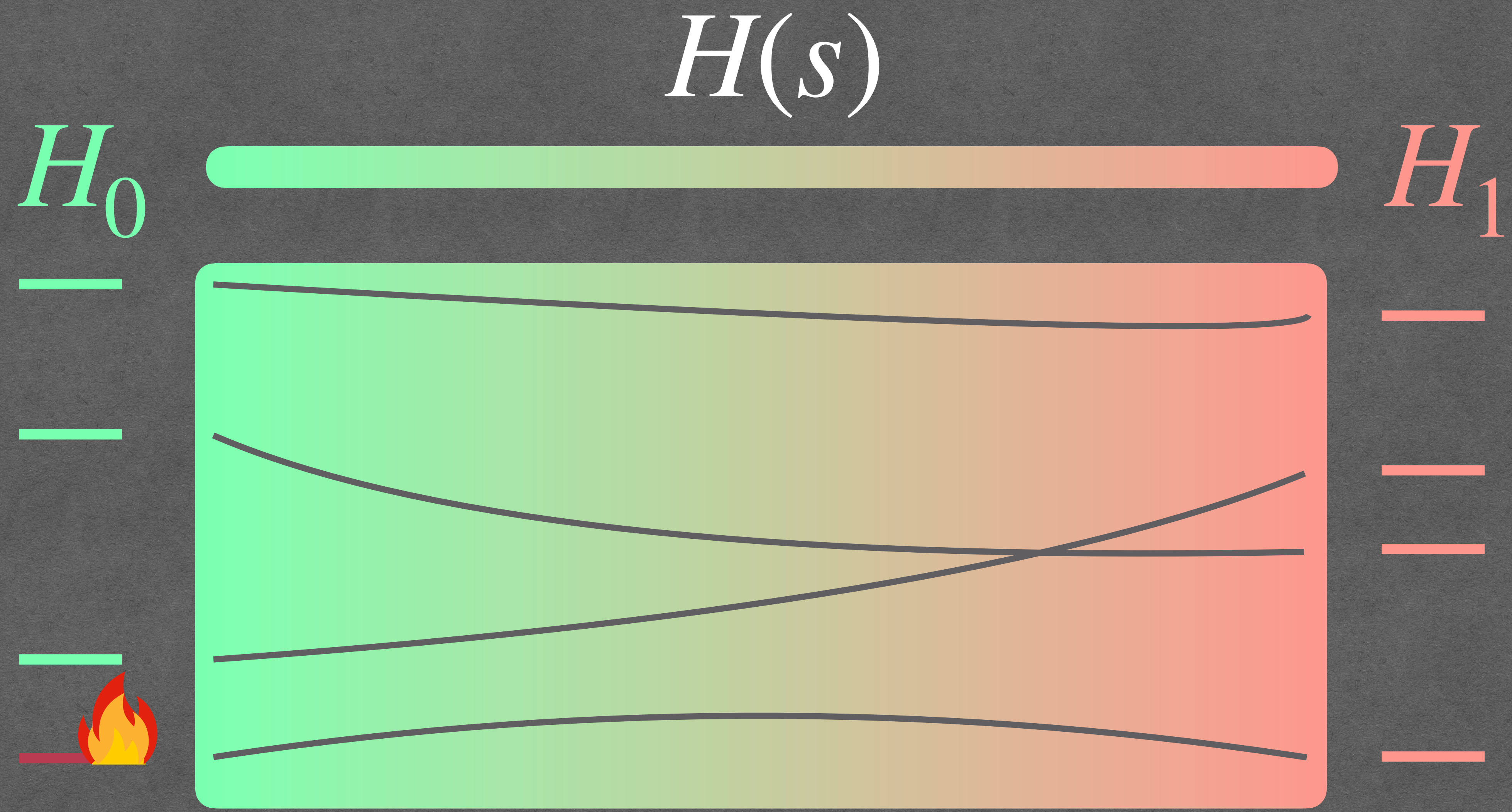
$$H(s) = H_0 + f(s)[H_1 - H_0]$$



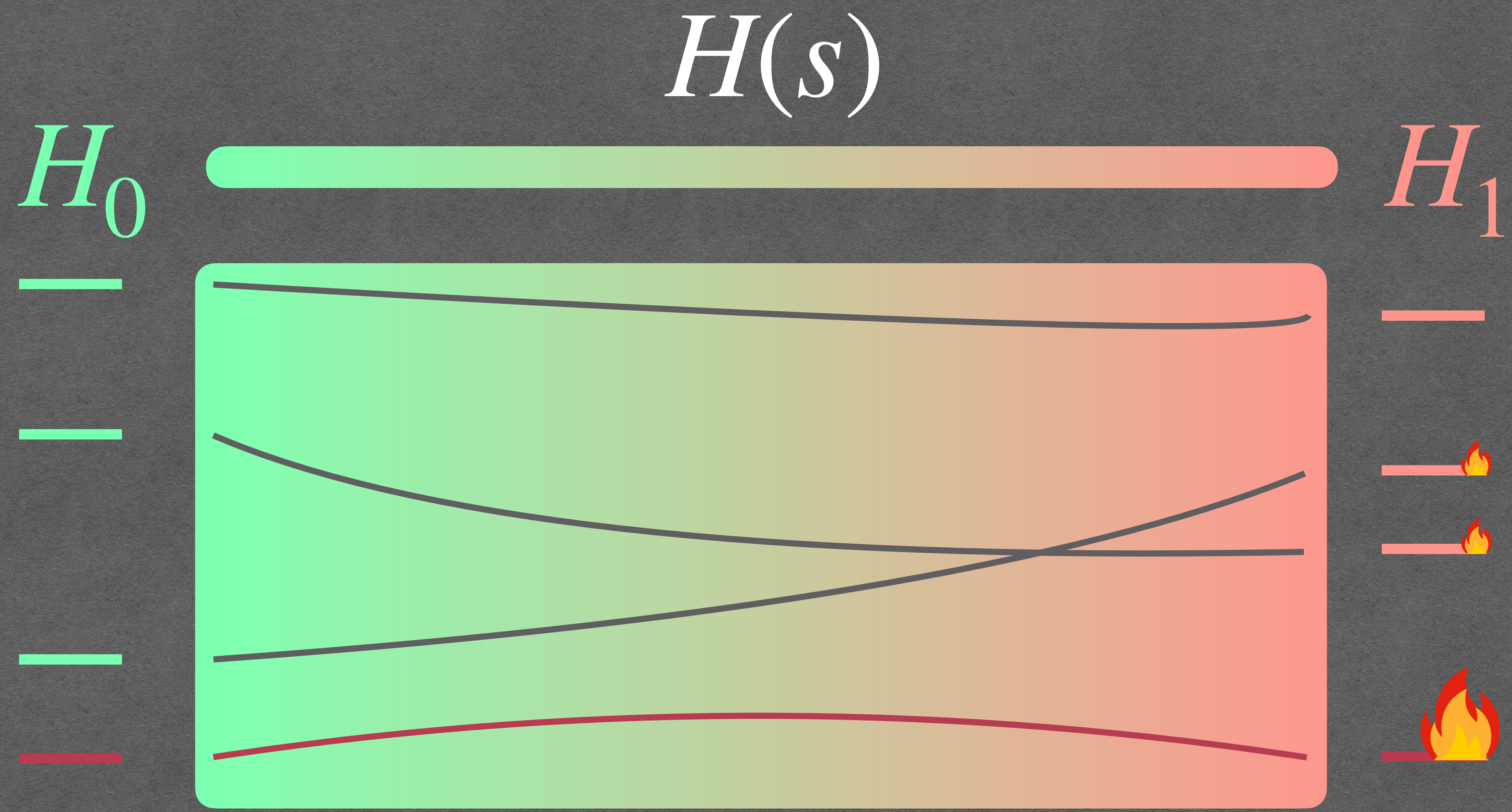
Adiabatic Preparation AP



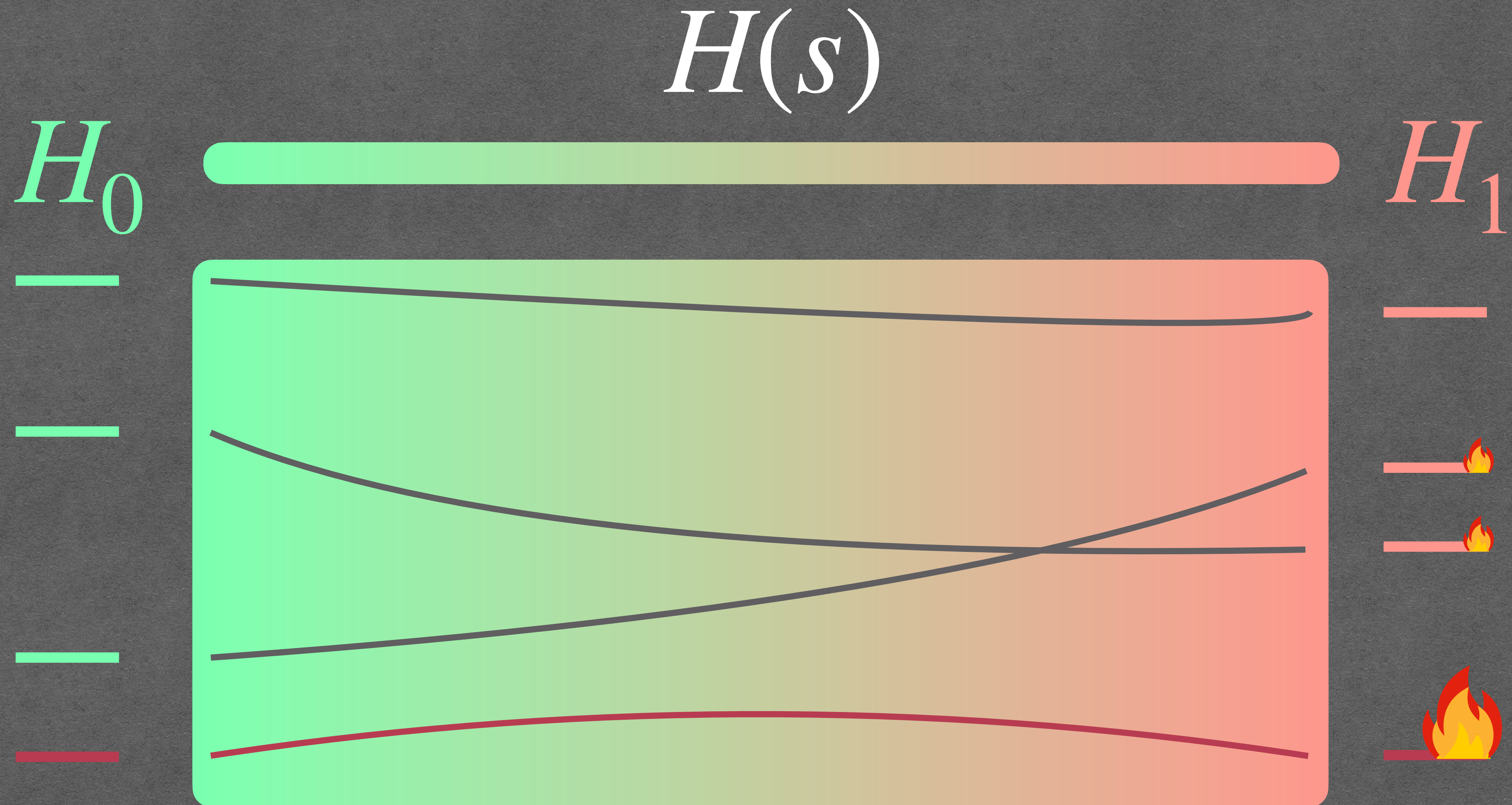
Adiabatic Preparation AP



Adiabatic Preparation AP



Adiabatic Preparation AP

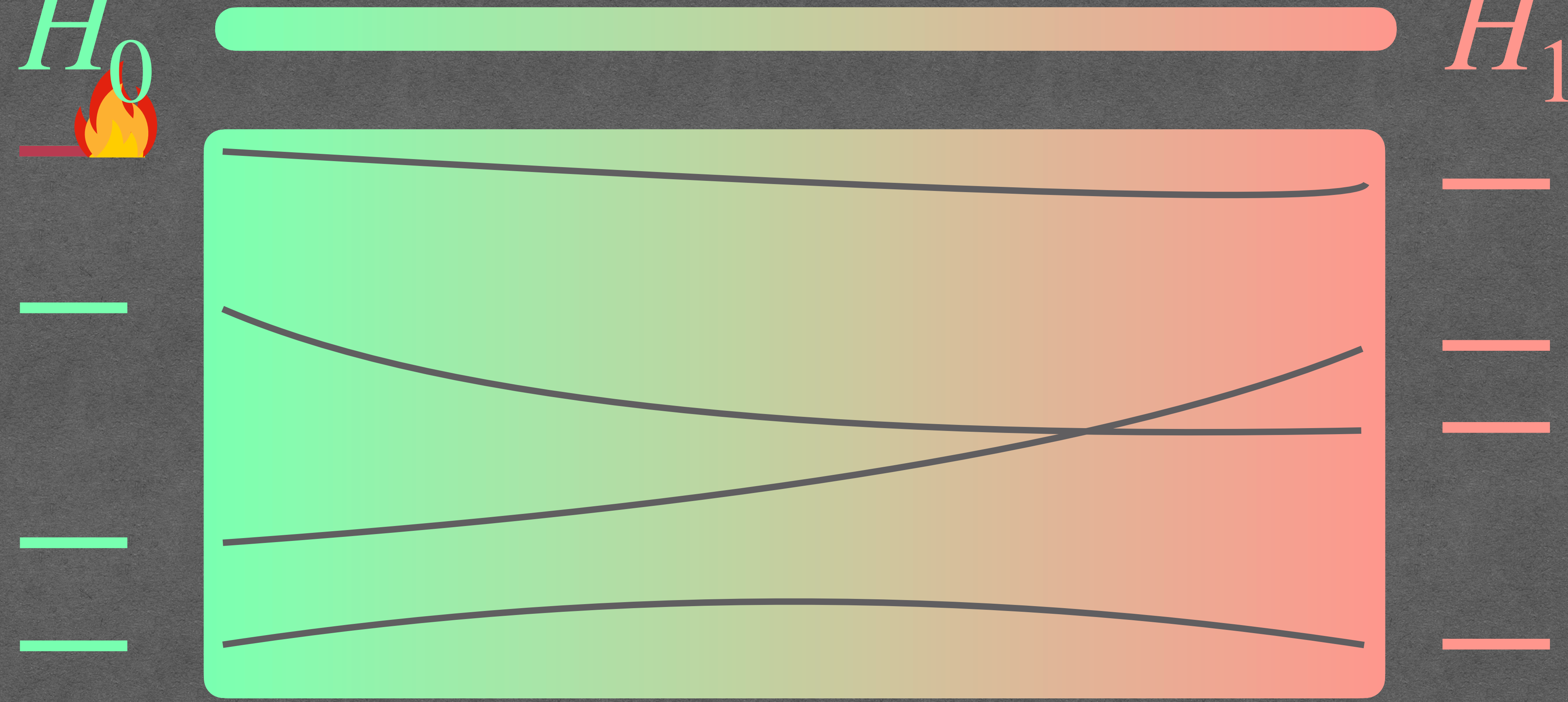


Adiabatic Preparation AP

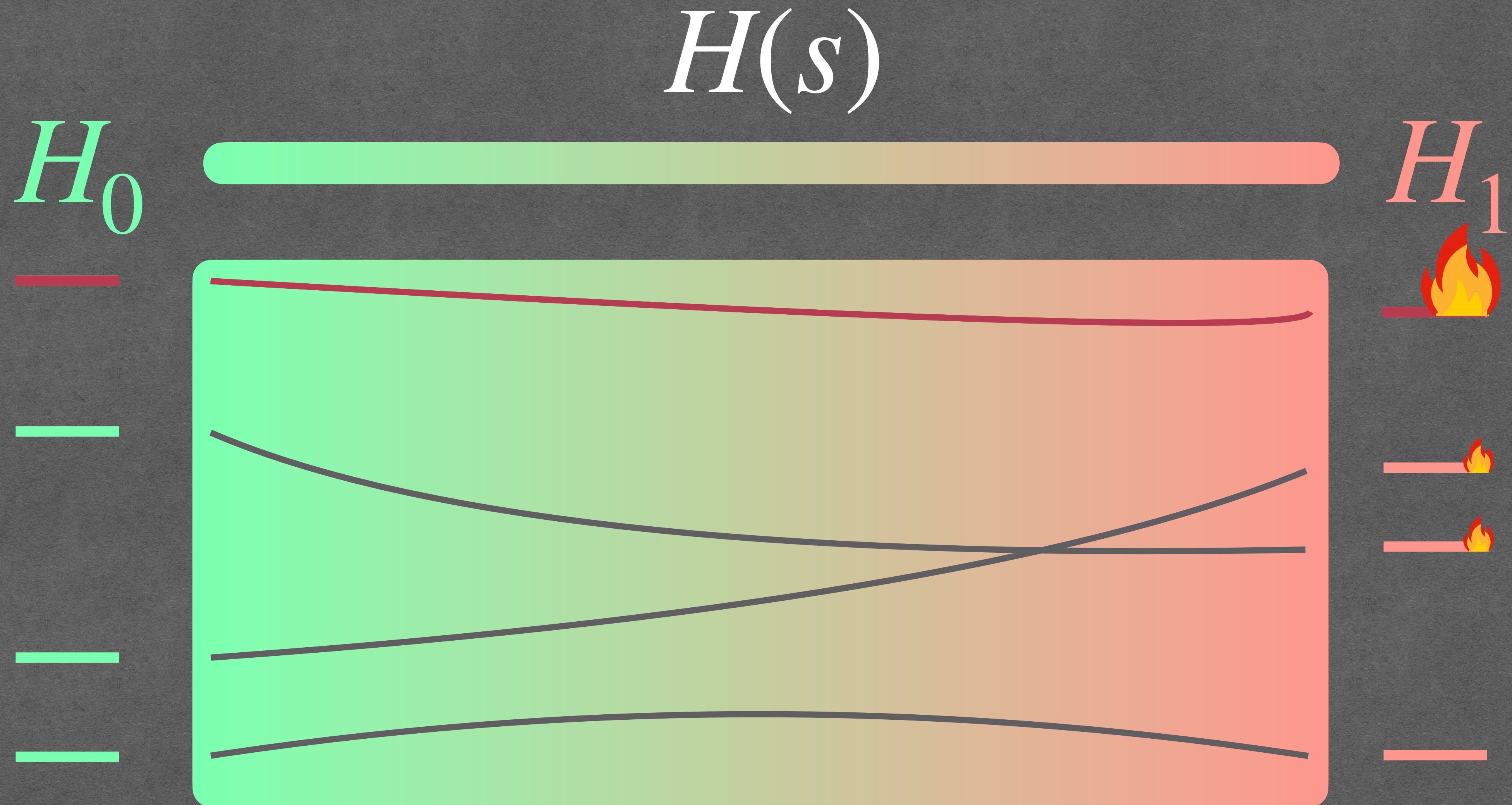
$H(s)$

H_0

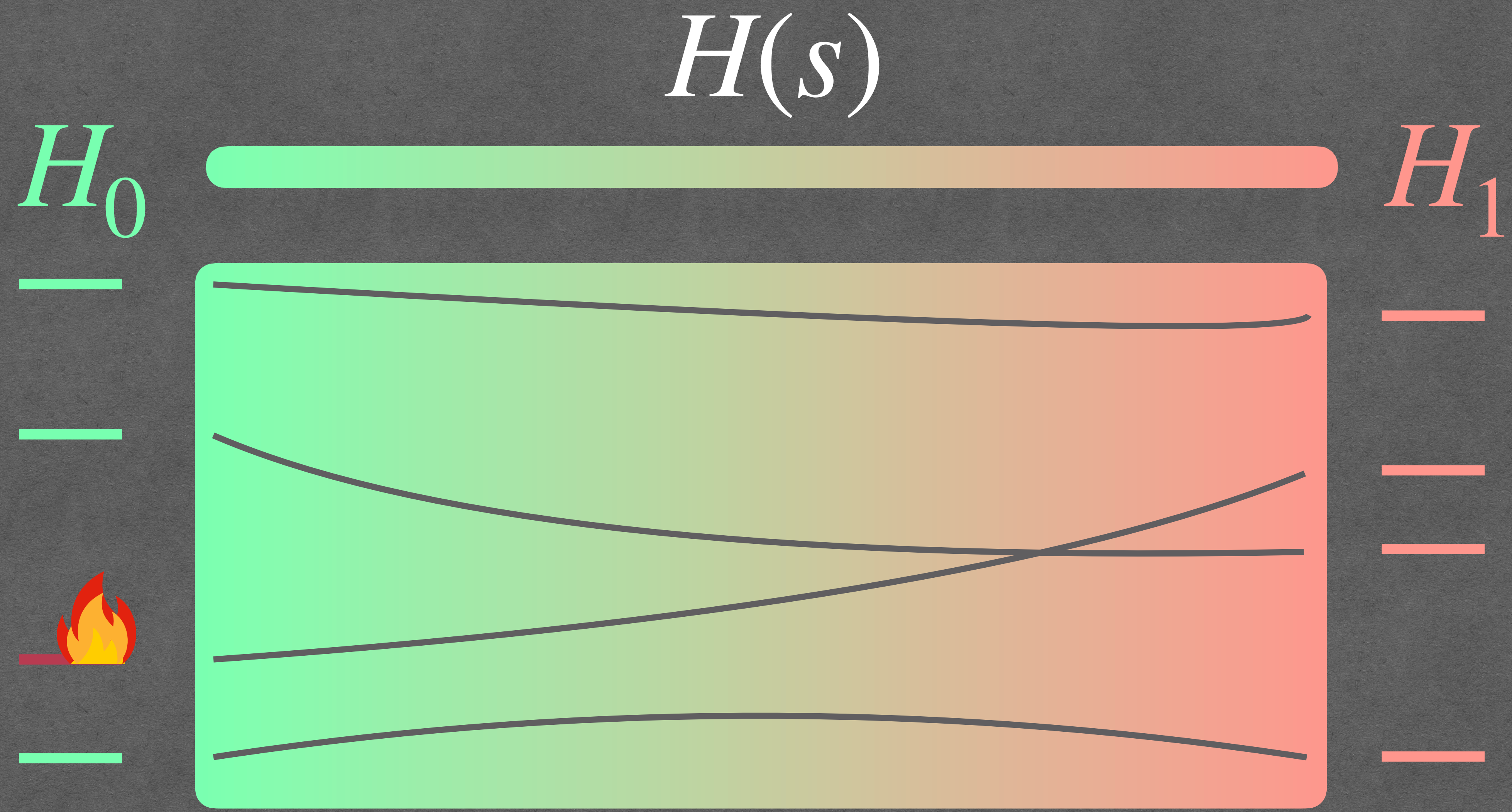

H_1



Adiabatic Preparation AP



Adiabatic Preparation AP

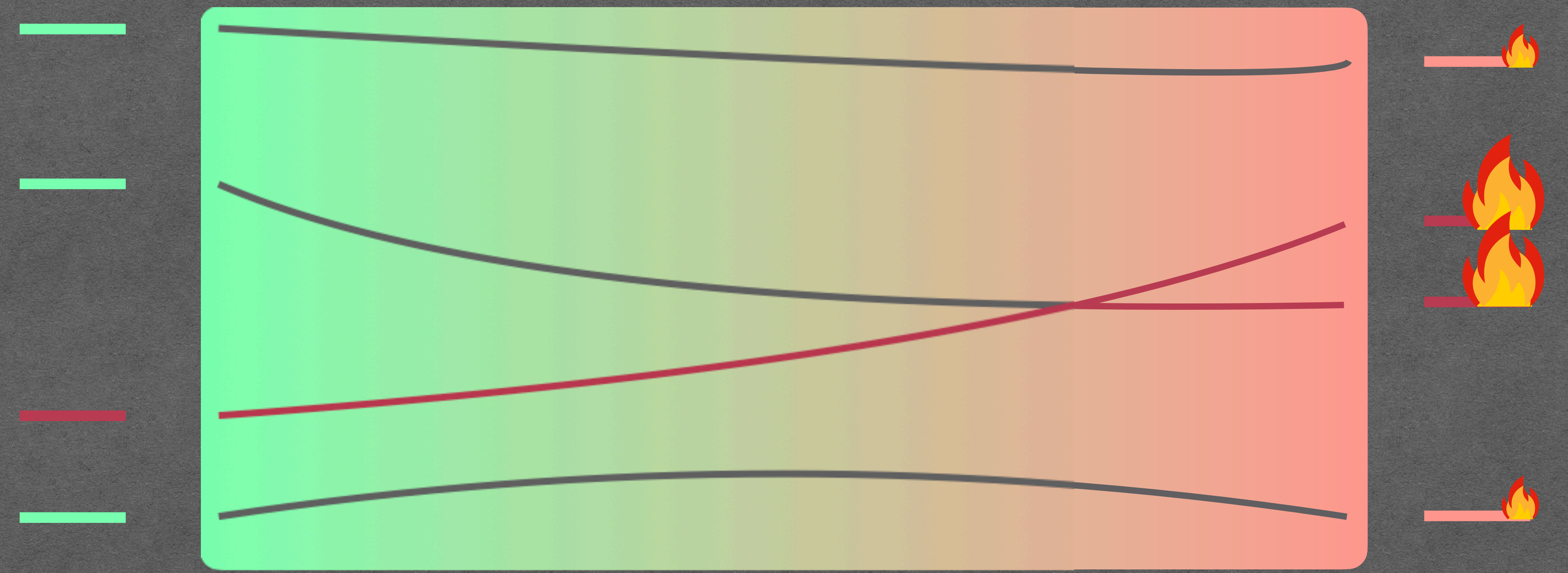


Adiabatic Preparation AP

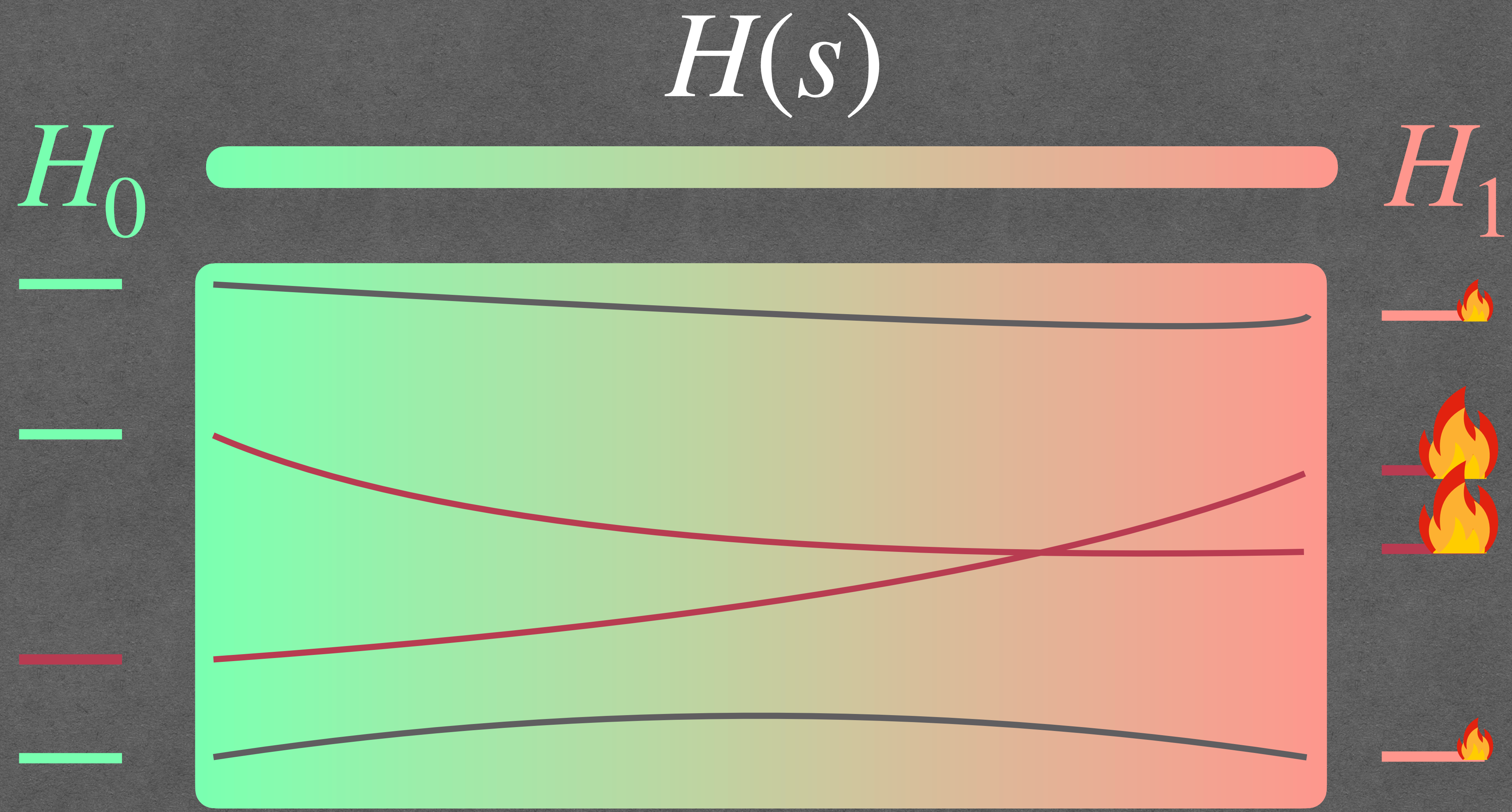
$$H(s)$$

H_0

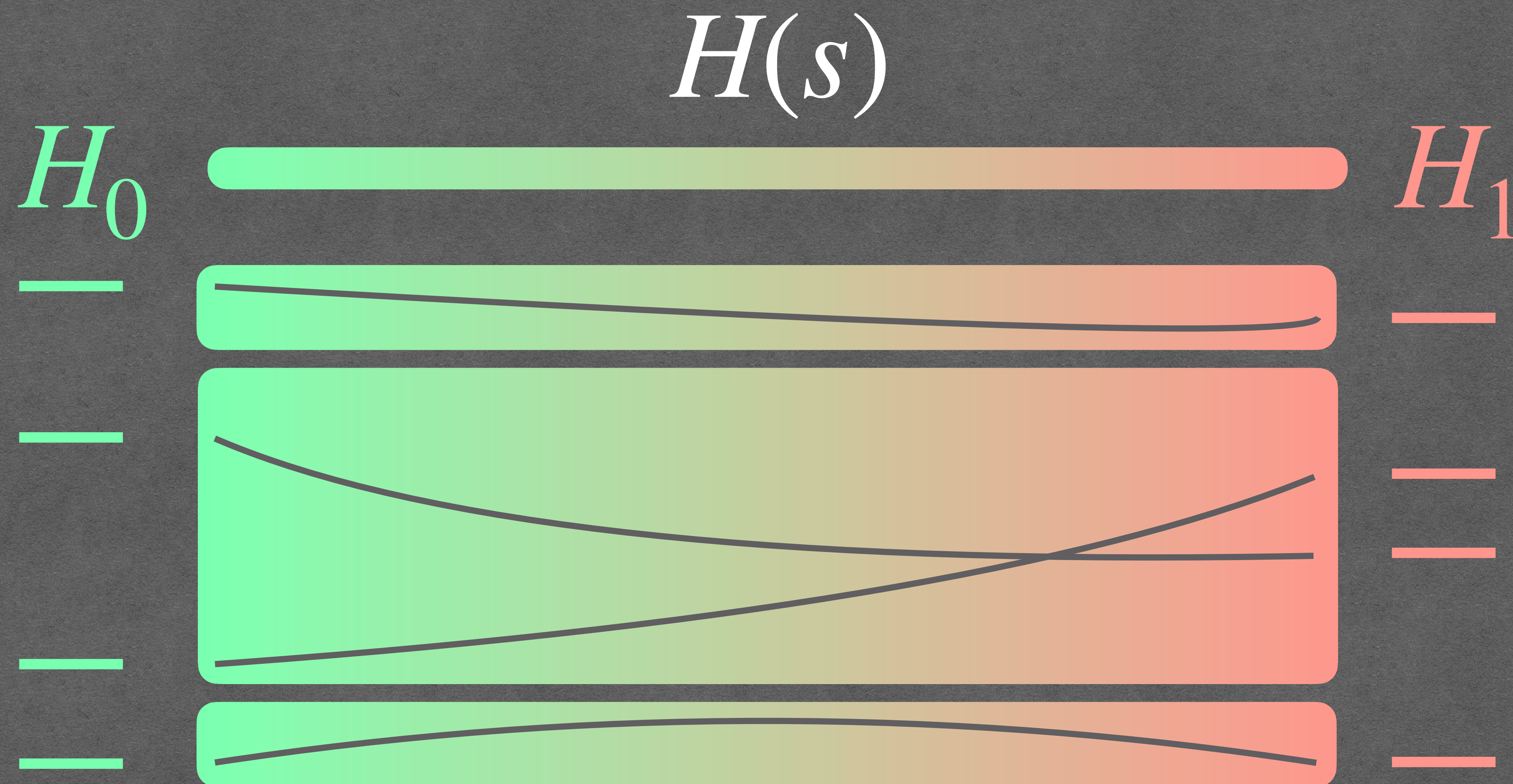
H_1



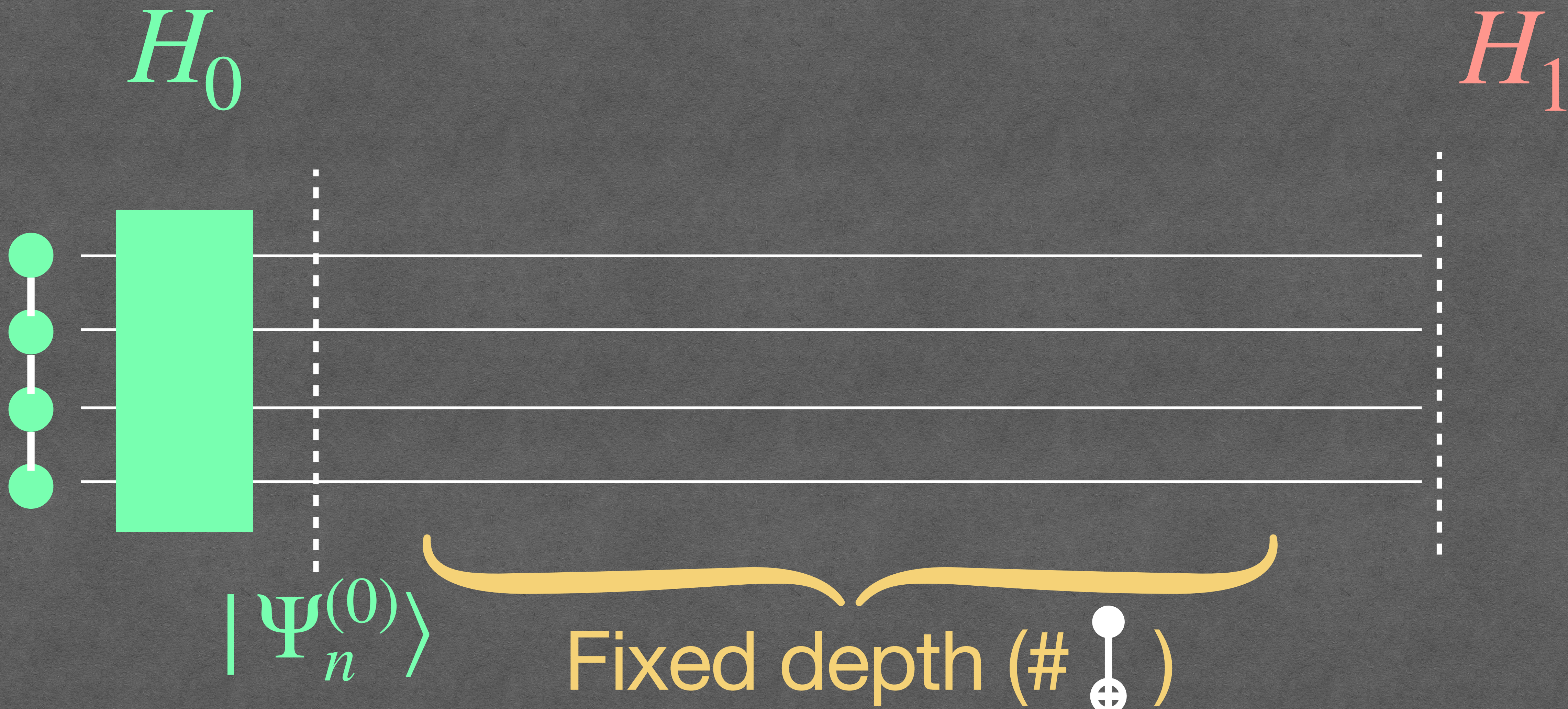
Adiabatic Preparation AP



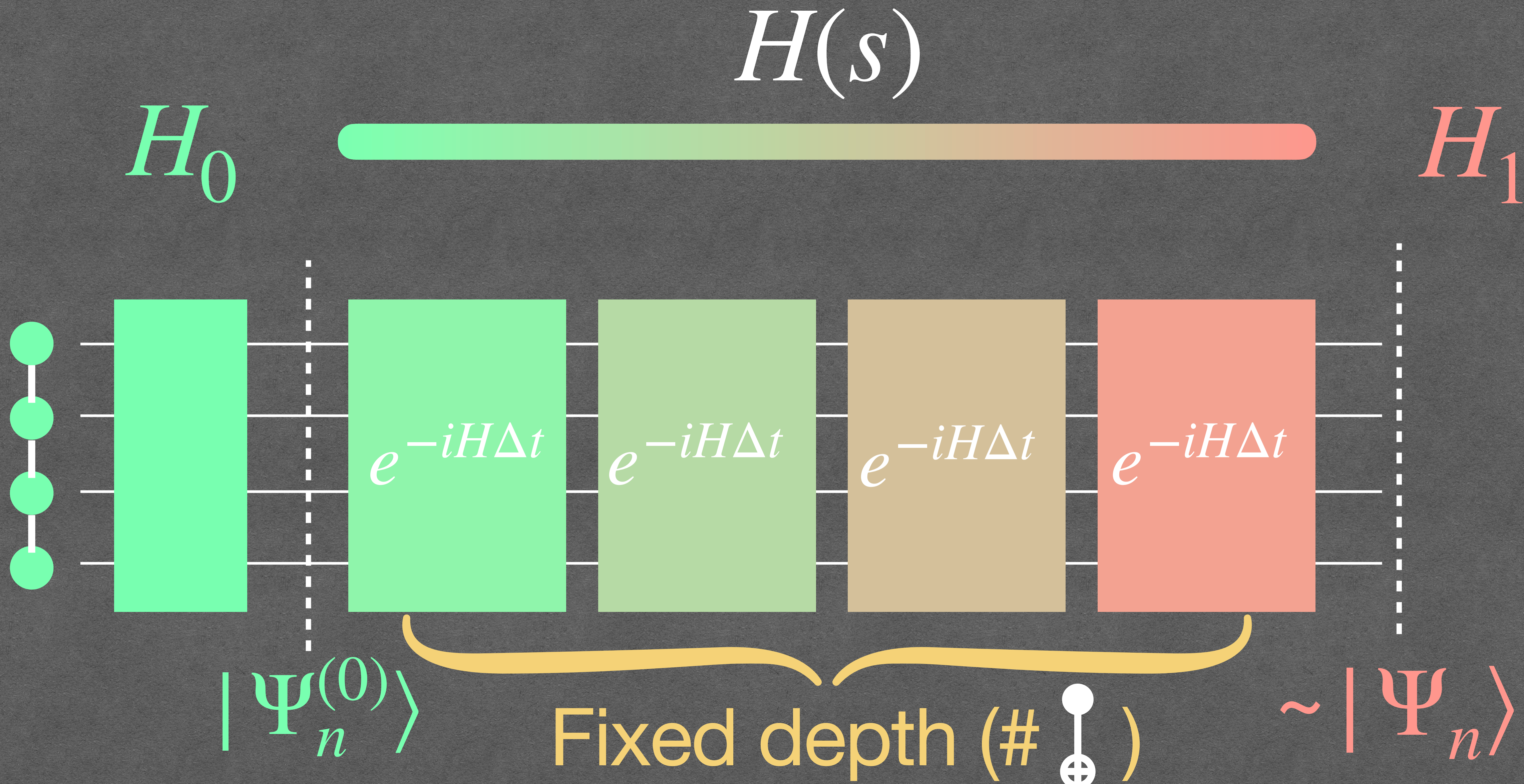
Adiabatic Preparation AP



AP on quantum computers



AP on quantum computers



Losses bound for AP

Linear Adiabatic Theory. Exponential Estimates

G. Nenciu

Losses bound for AP

Linear Adiabatic Theory. Exponential Estimates

G. Nenciu

$\exists C, \tilde{g} > 0$ such that

$$\left| \left| 1 - |\Psi_n\rangle\langle\Psi_n| \right| \right| \leq C \exp\left[-\tau/\tilde{g}\right]$$

τ : Duration of the adiabatic process

Losses bound for AP

Linear Adiabatic Theory. Exponential Estimates

G. Nenciu

$\exists C, \tilde{g} > 0$ such that

$$\text{🔥} \leq C \exp \left[-(\# \text{⊕}) / \tilde{g} \right]$$

Losses bound for AP

Linear Adiabatic Theory. Exponential Estimates

G. Nenciu

$\exists C, \tilde{g} > 0$ such that

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Fixed by the hardware

CAN WE REDUCE \tilde{g} ?

Exponential optimization of quantum state preparation via adiabatic thermalization

Davide Cugini,¹ Davide Nigro,¹ Mattia Bruno,² and Dario Gerace¹

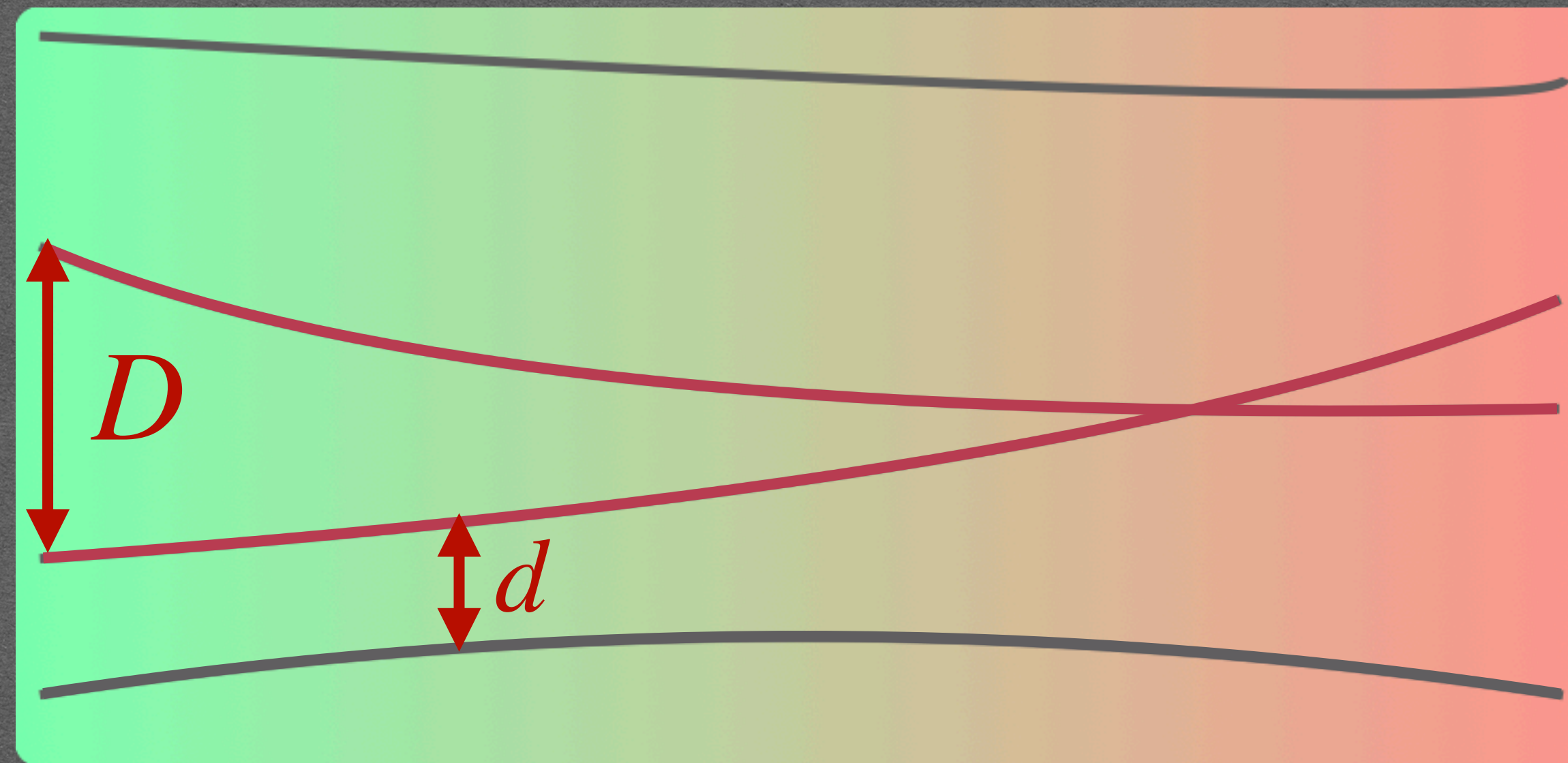
¹*Dipartimento di Fisica, Università di Pavia, via Bassi 6, 27100, Pavia, Italy*

²*Dipartimento di Fisica, Università di Milano-Bicocca, and INFN, sezione di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy*

$$\tilde{g} = \tilde{g}(\|H_1 - H_0\|, D, d)$$

Increasing function in terms of

$$\|\Delta\| = \|H_1 - H_0\|, \quad D, \quad d^{-1}$$



Exponential optimization of quantum state preparation via adiabatic thermalization

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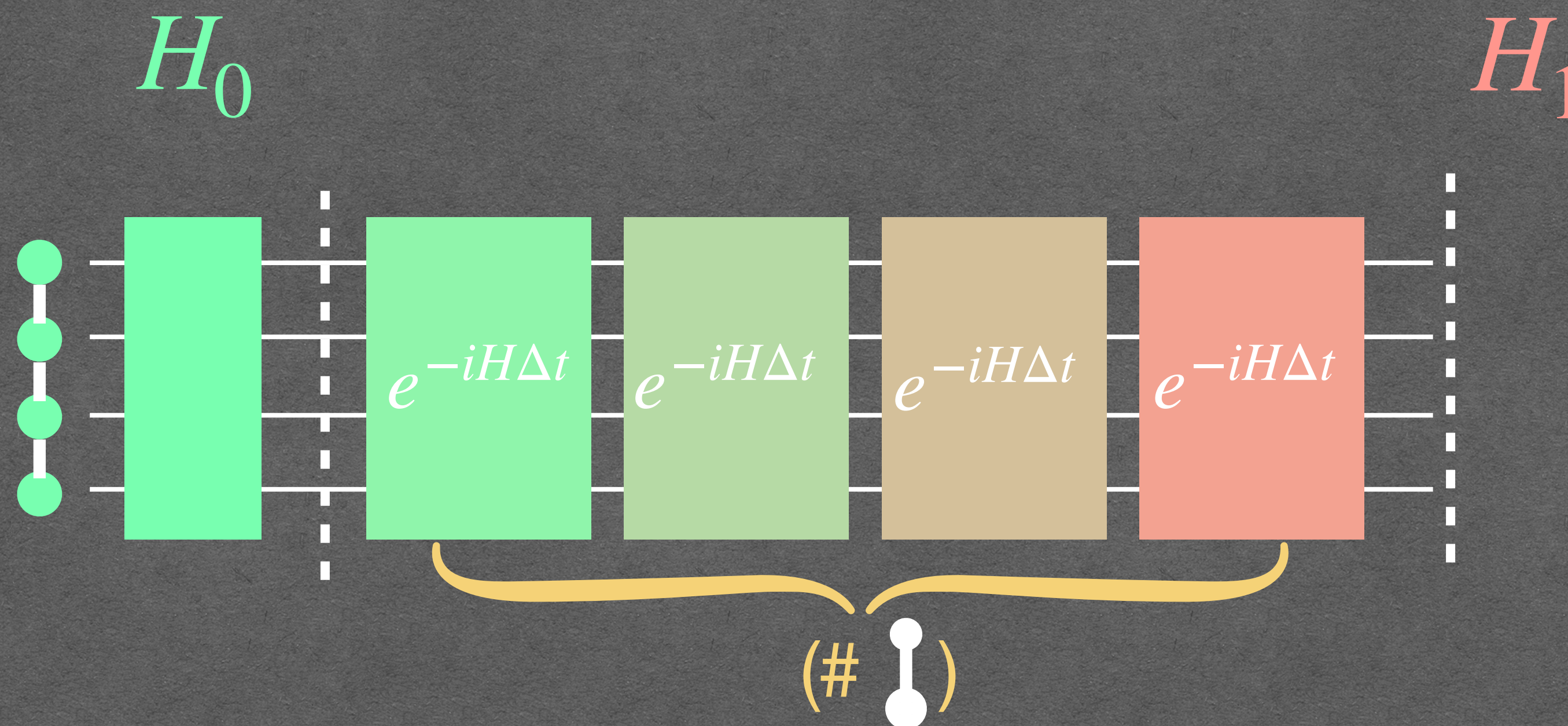
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Increasing function in terms of

$$\|\Delta\| = \|H_1 - H_0\|, \quad D, \quad d^{-1}$$

$$H_0 = \text{diag}(H_1)$$

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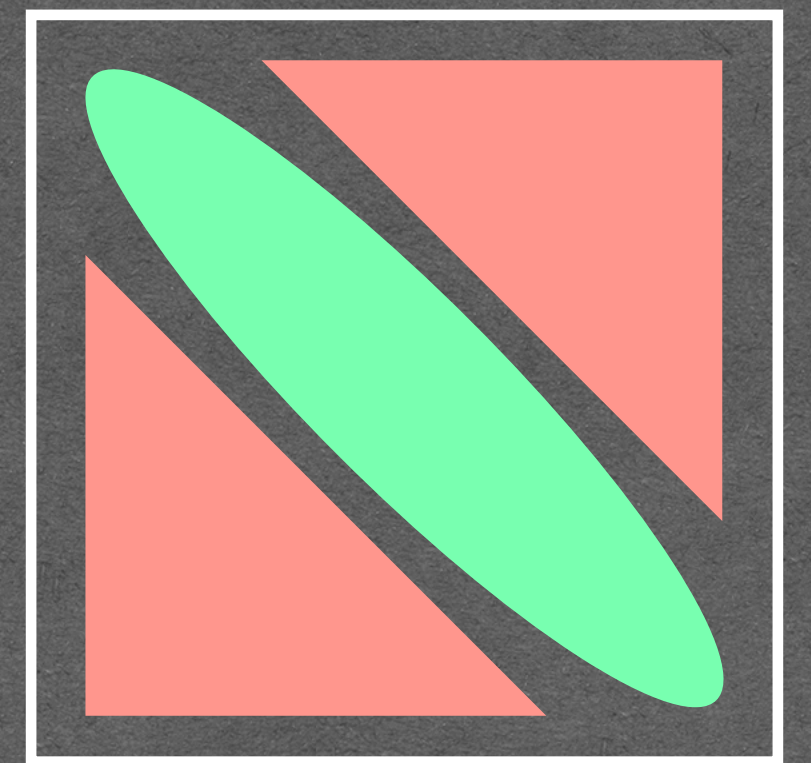
Increasing function in terms of

$$\|\Delta\| = \|H_1 - H_0\|, \quad D, \quad d^{-1}$$

$$H_0 = \text{diag}(H_1) + M(\alpha)$$

Exponential optimisation of AP

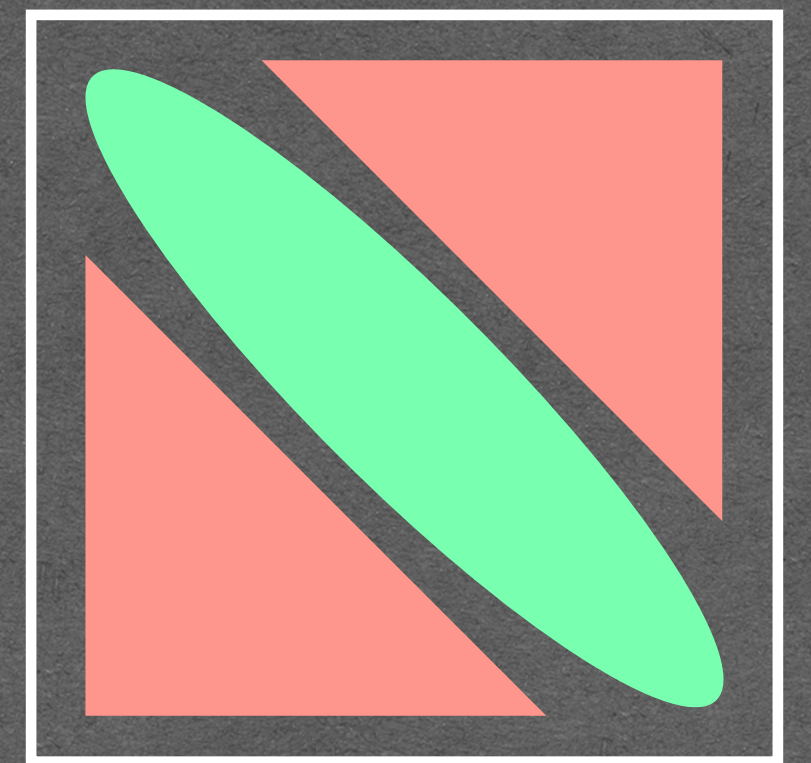
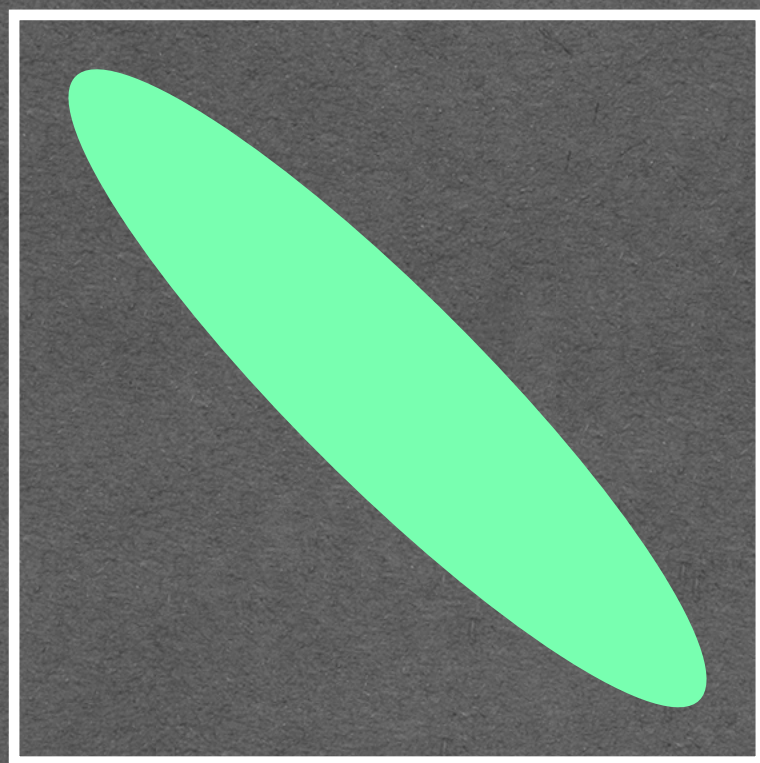
H_1



Exponential optimisation of AP

$$H_0 = \text{diag}(H_1)$$

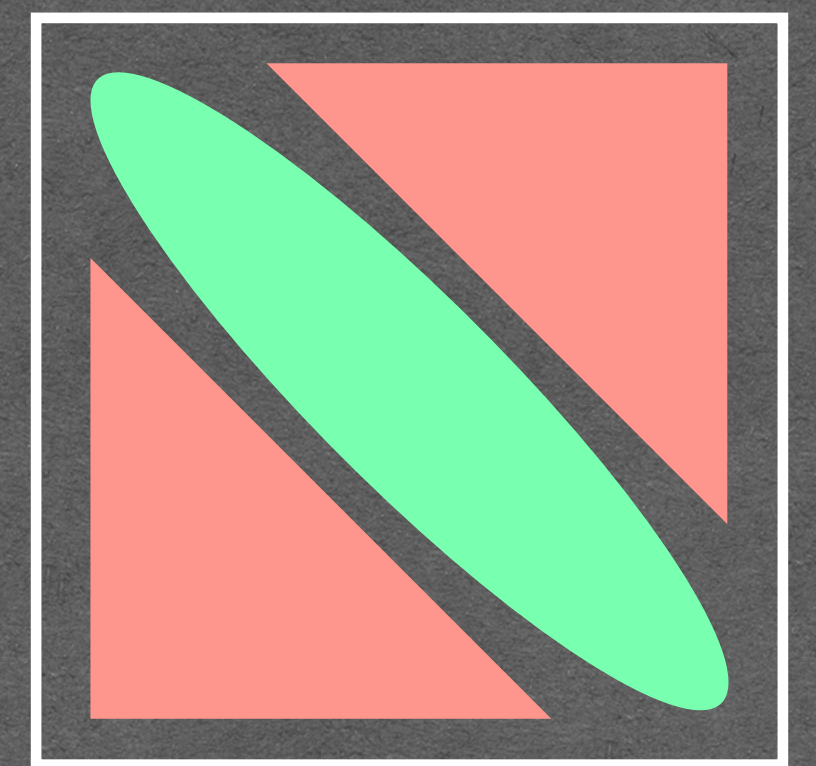
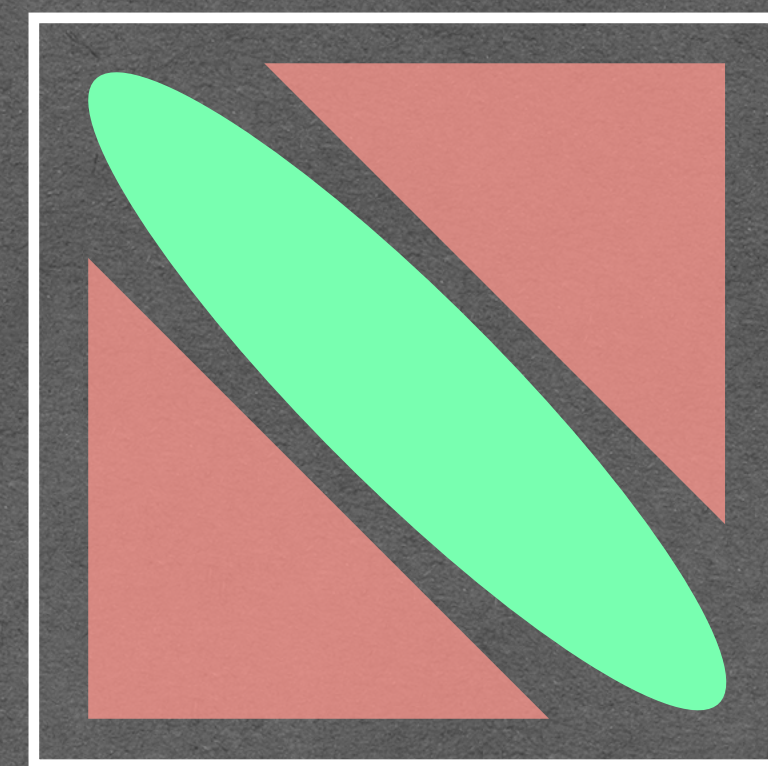
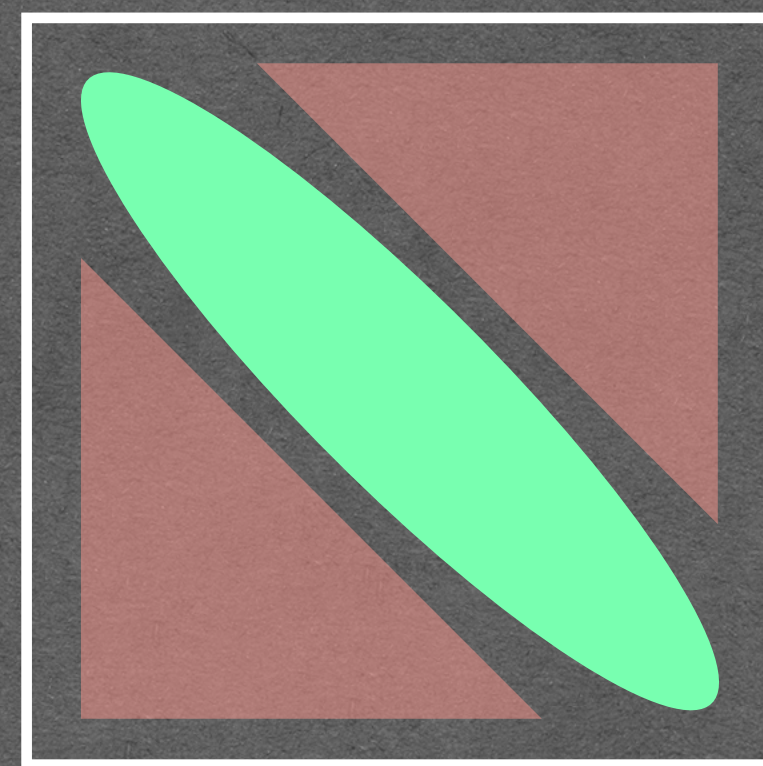
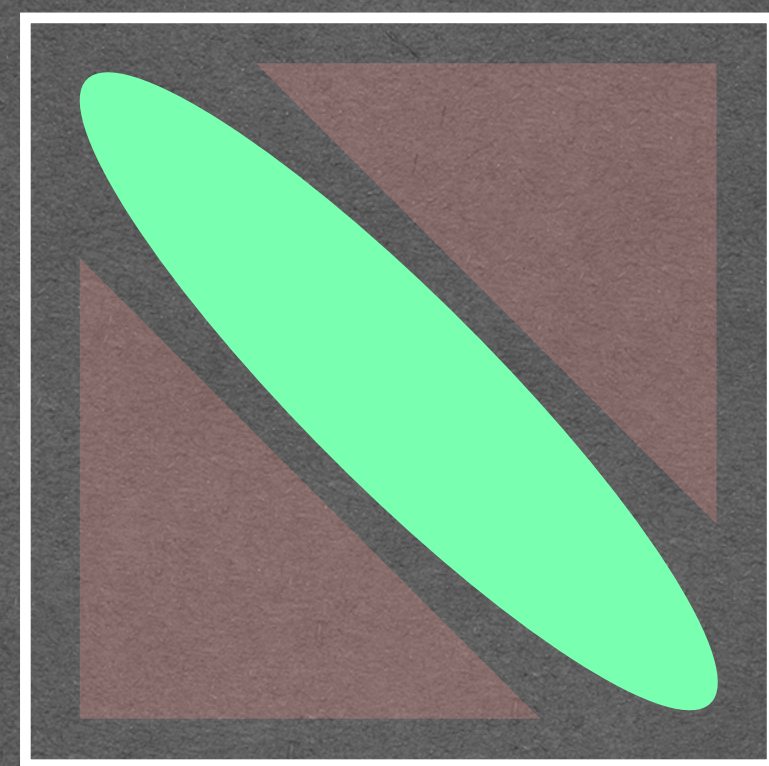
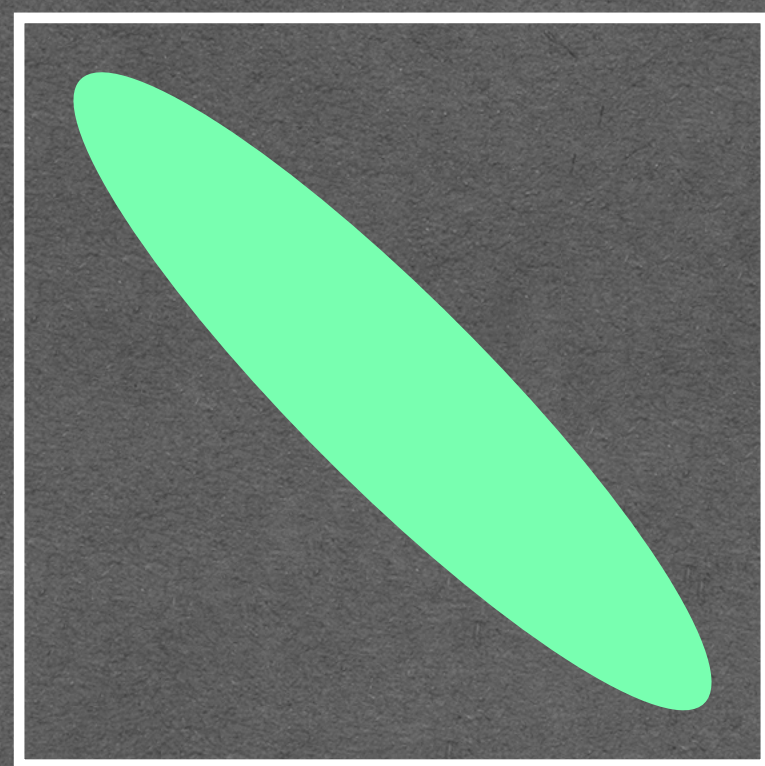
H_1



Exponential optimisation of AP

$$H_0 = \text{diag}(H_1)$$

H_1

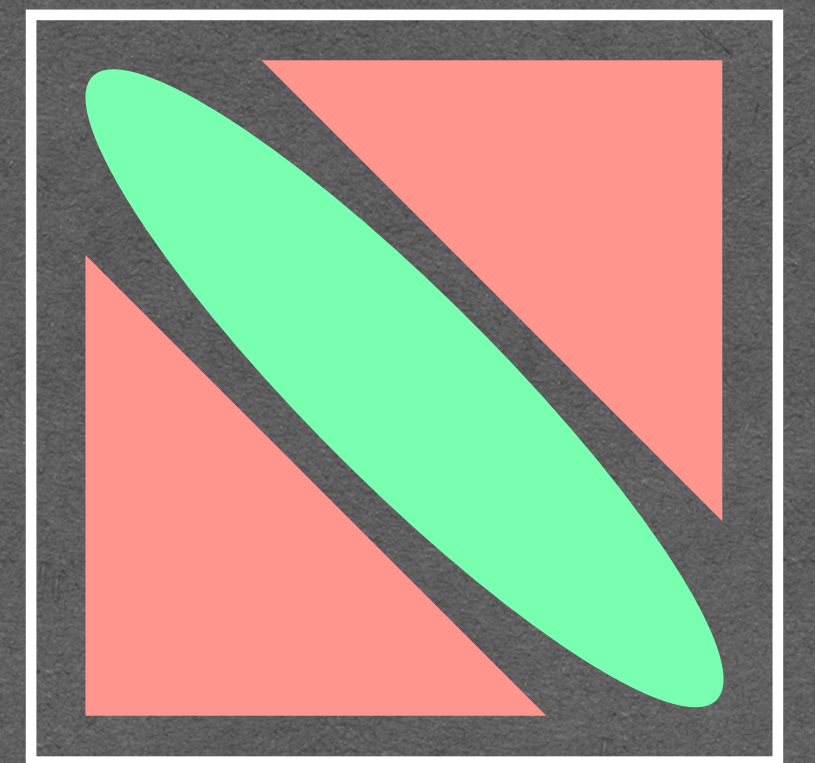
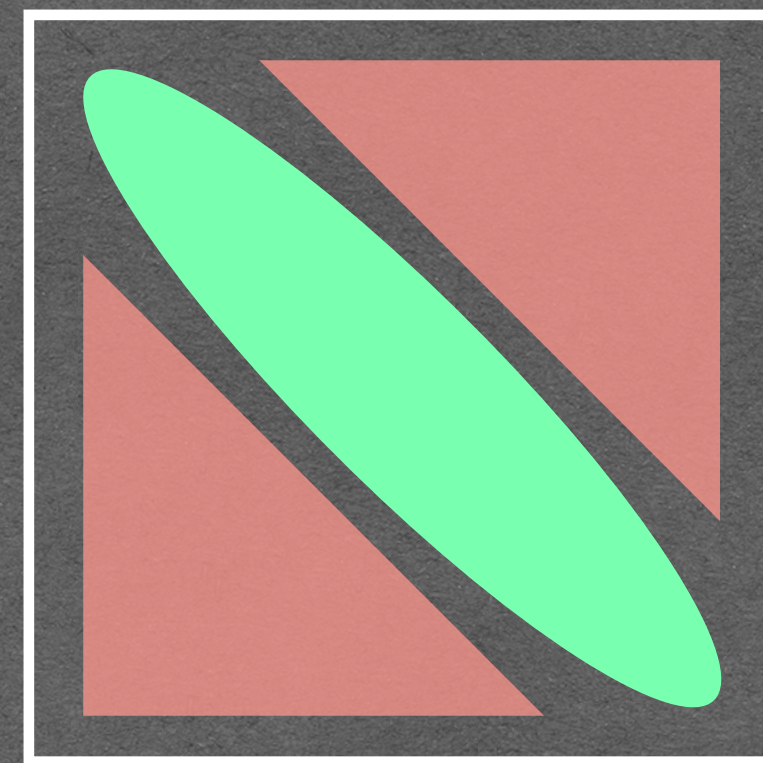
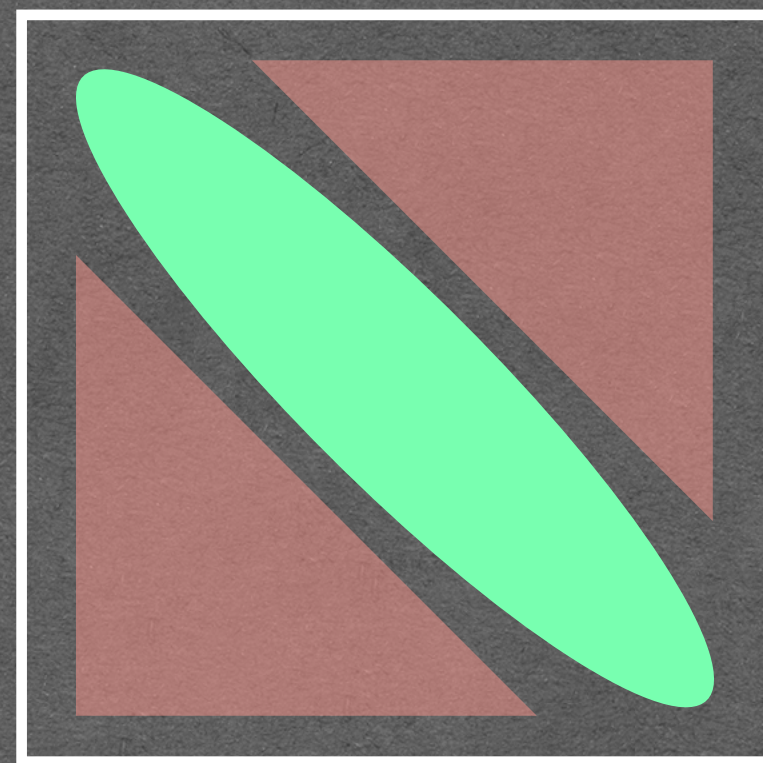
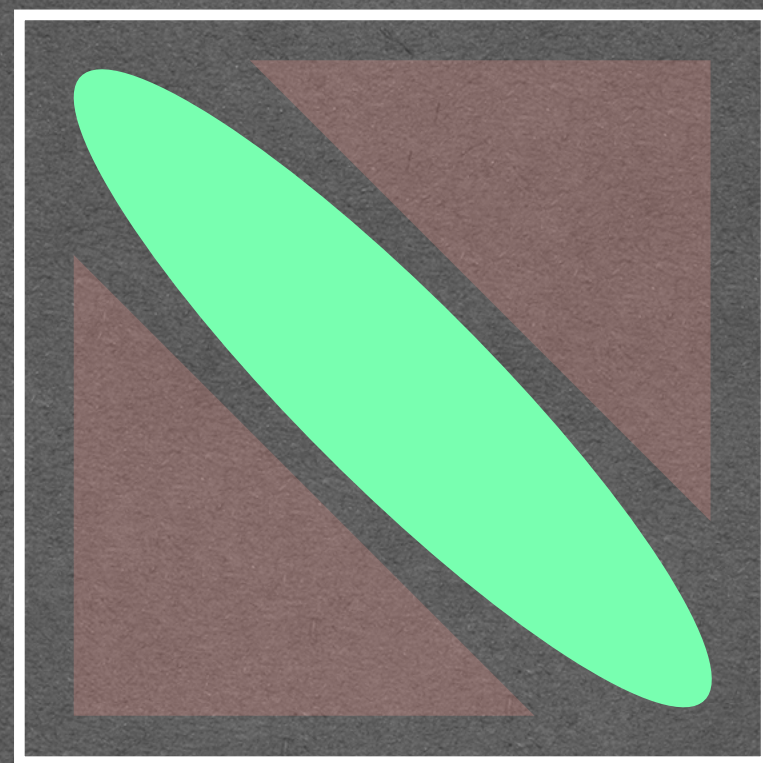
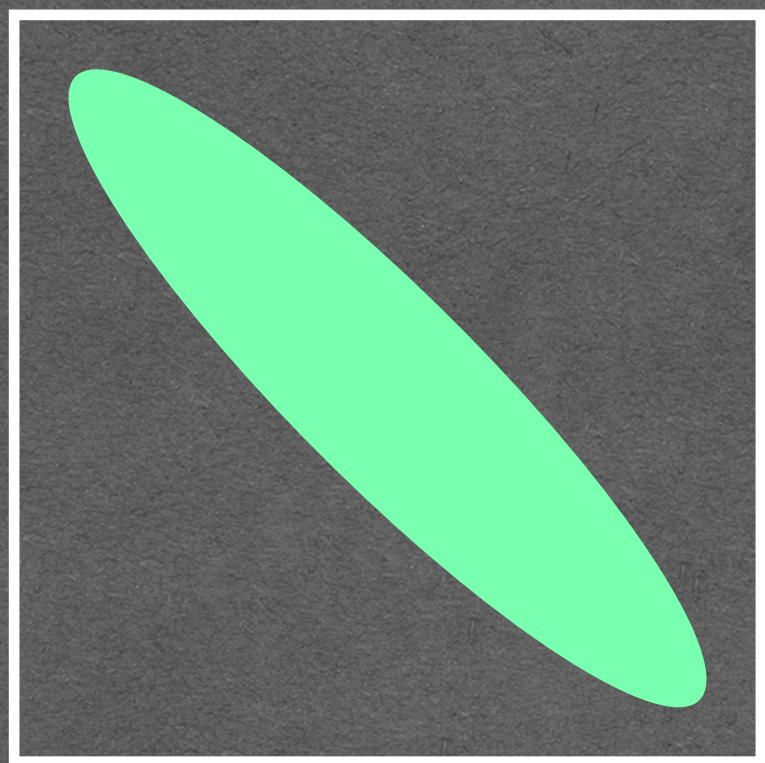


Exponential optimisation of AP

$$\text{🔥} \leq C \exp \left[- (\# \text{⊕}) / \tilde{g} \right]$$

$$H_0 = \text{diag}(H_1)$$

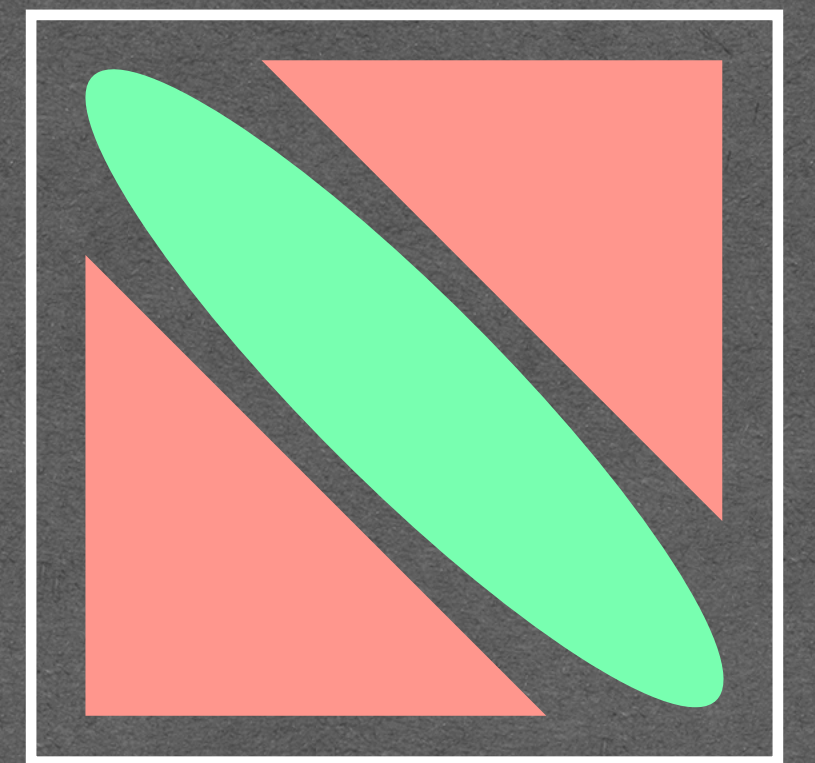
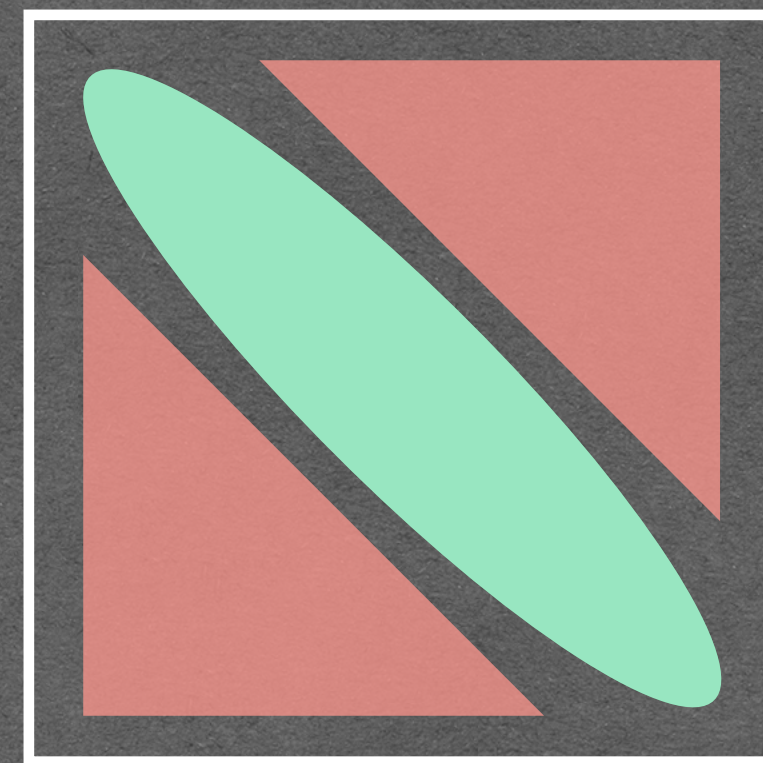
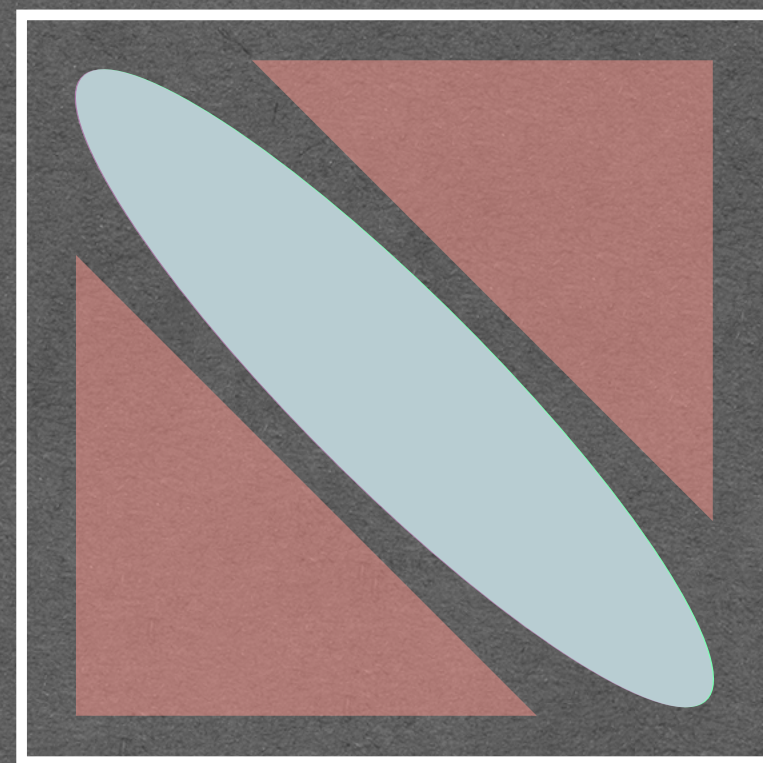
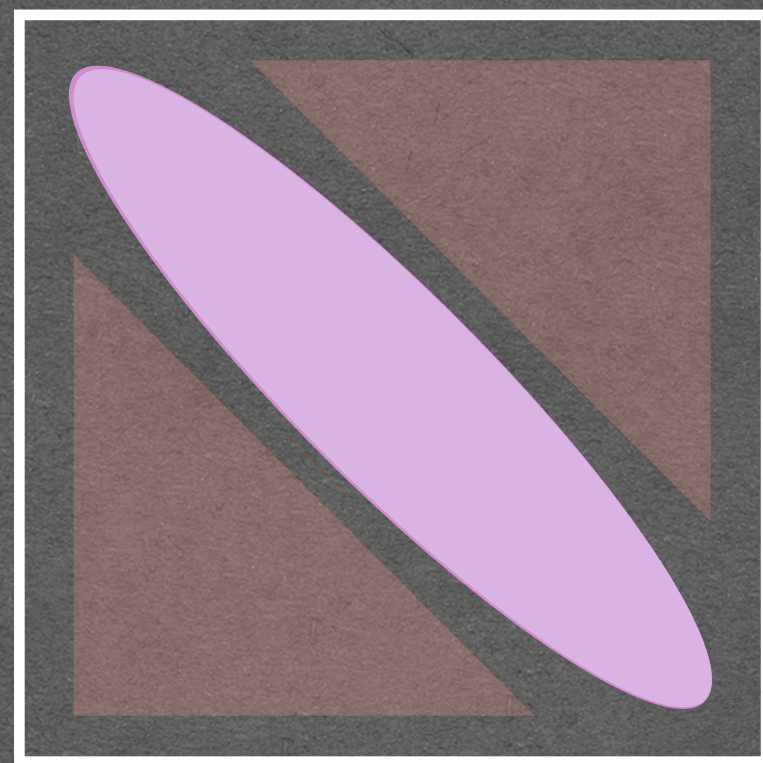
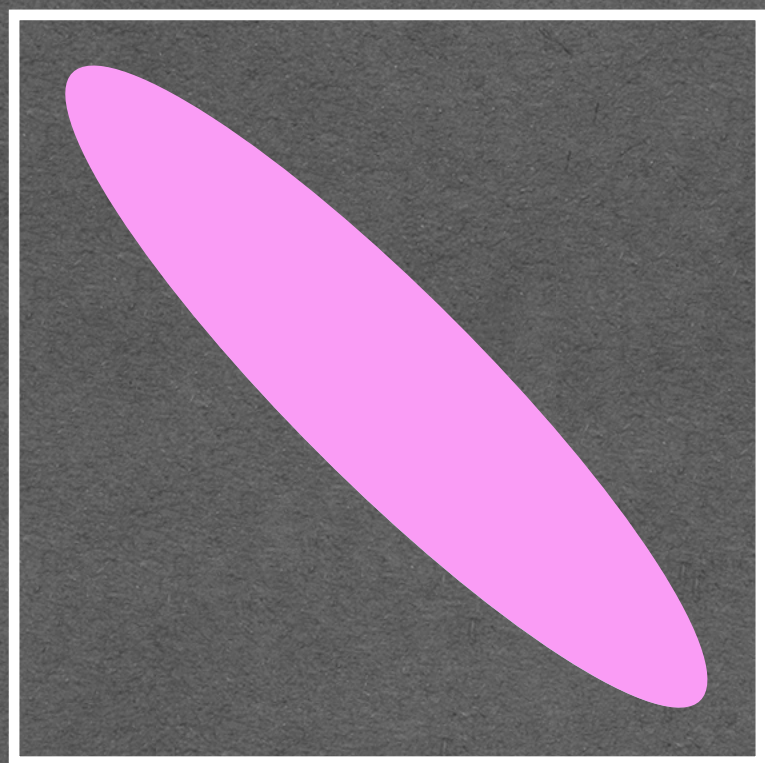
$$H_1$$

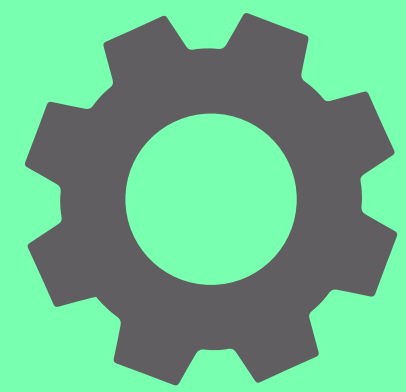


Exponential optimisation of AP

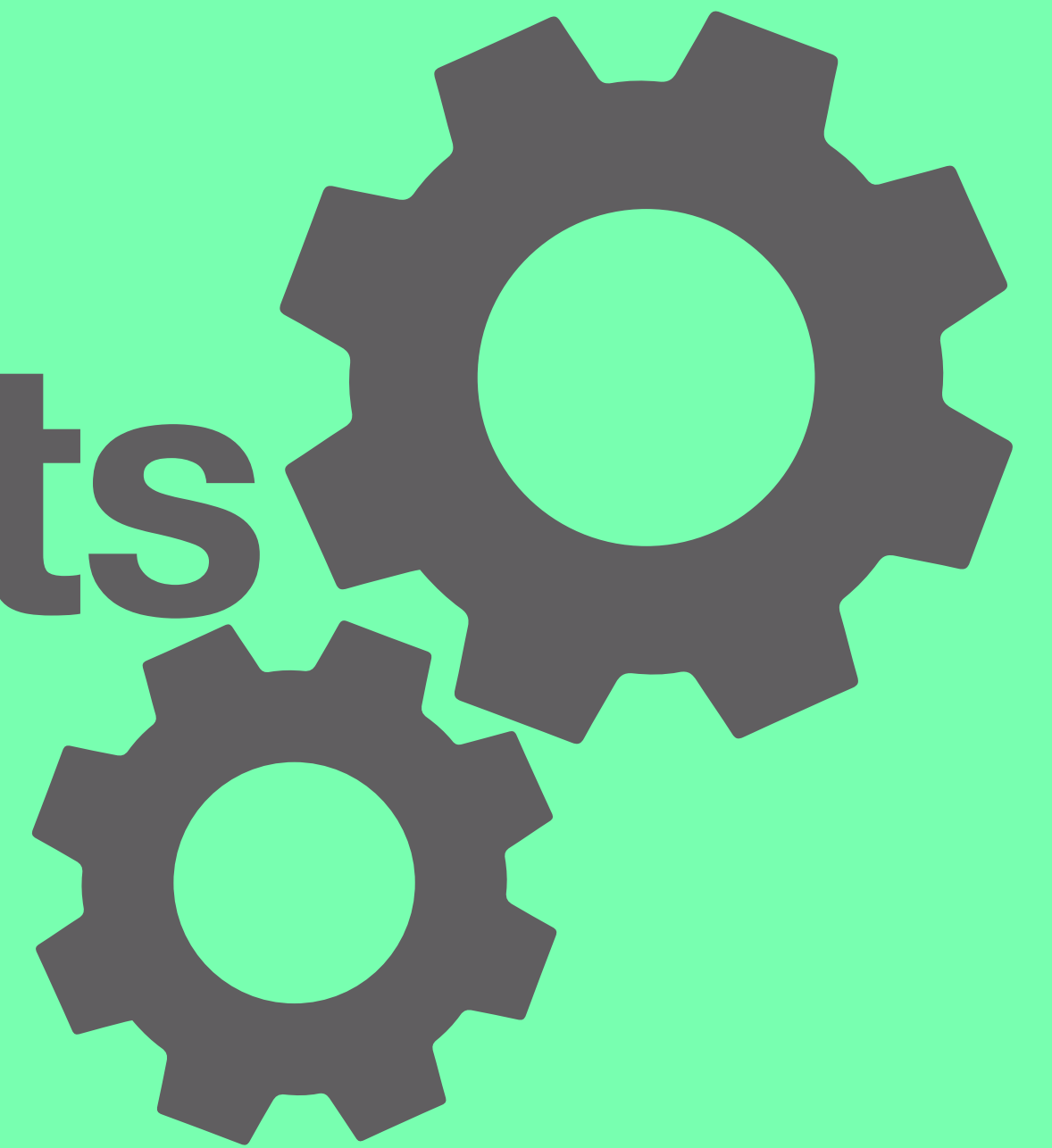
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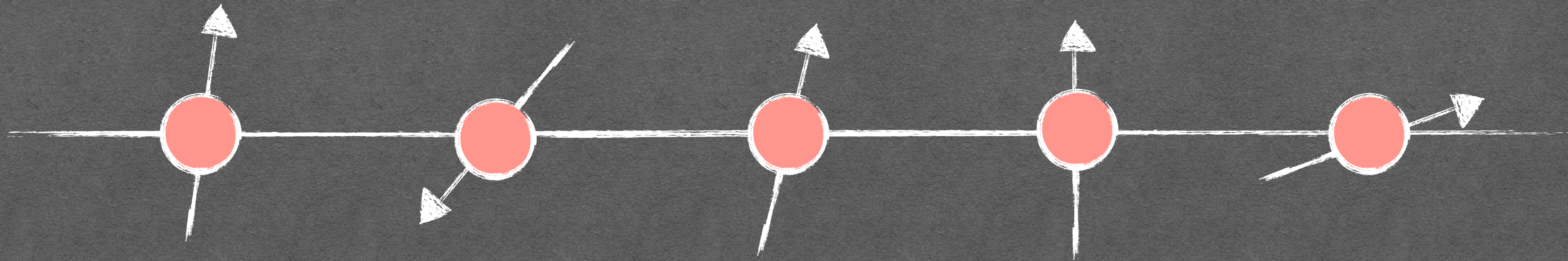




Numerical tests



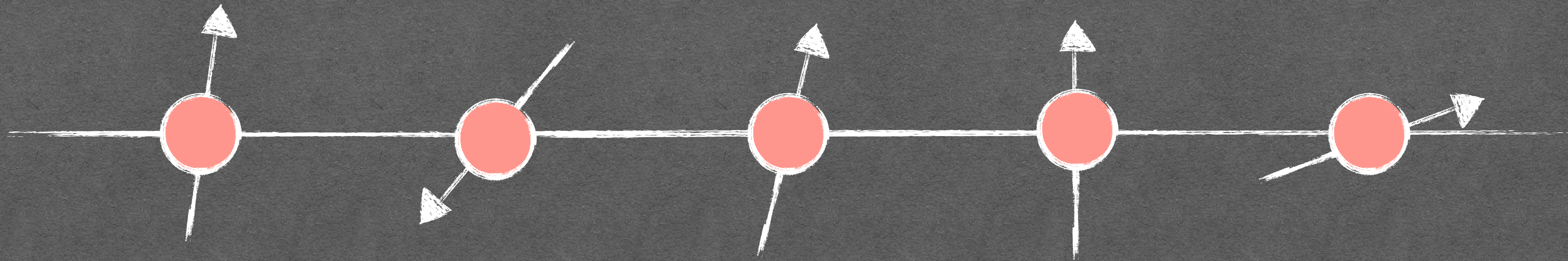
XZ Heisenberg model



$$H_1 = -\frac{1}{2} \sum_{\langle i,j \rangle} \left(J^z Z_i Z_j + J^x X_i X_j \right)$$

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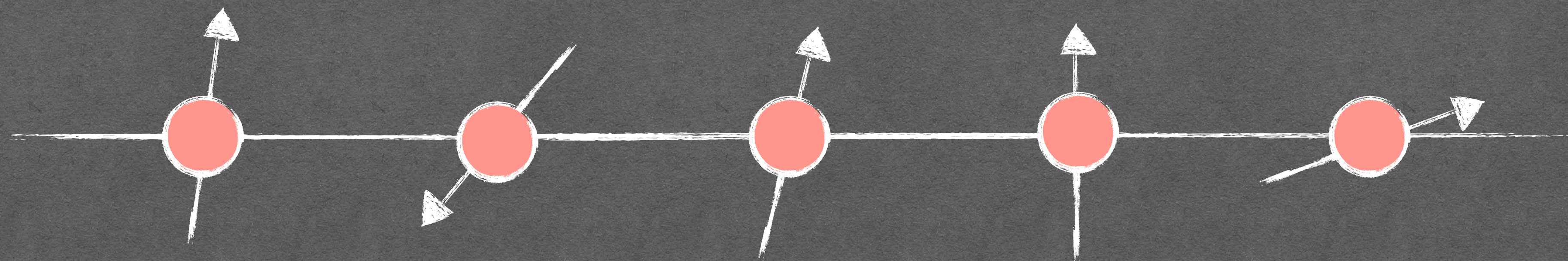
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XZ Heisenberg model

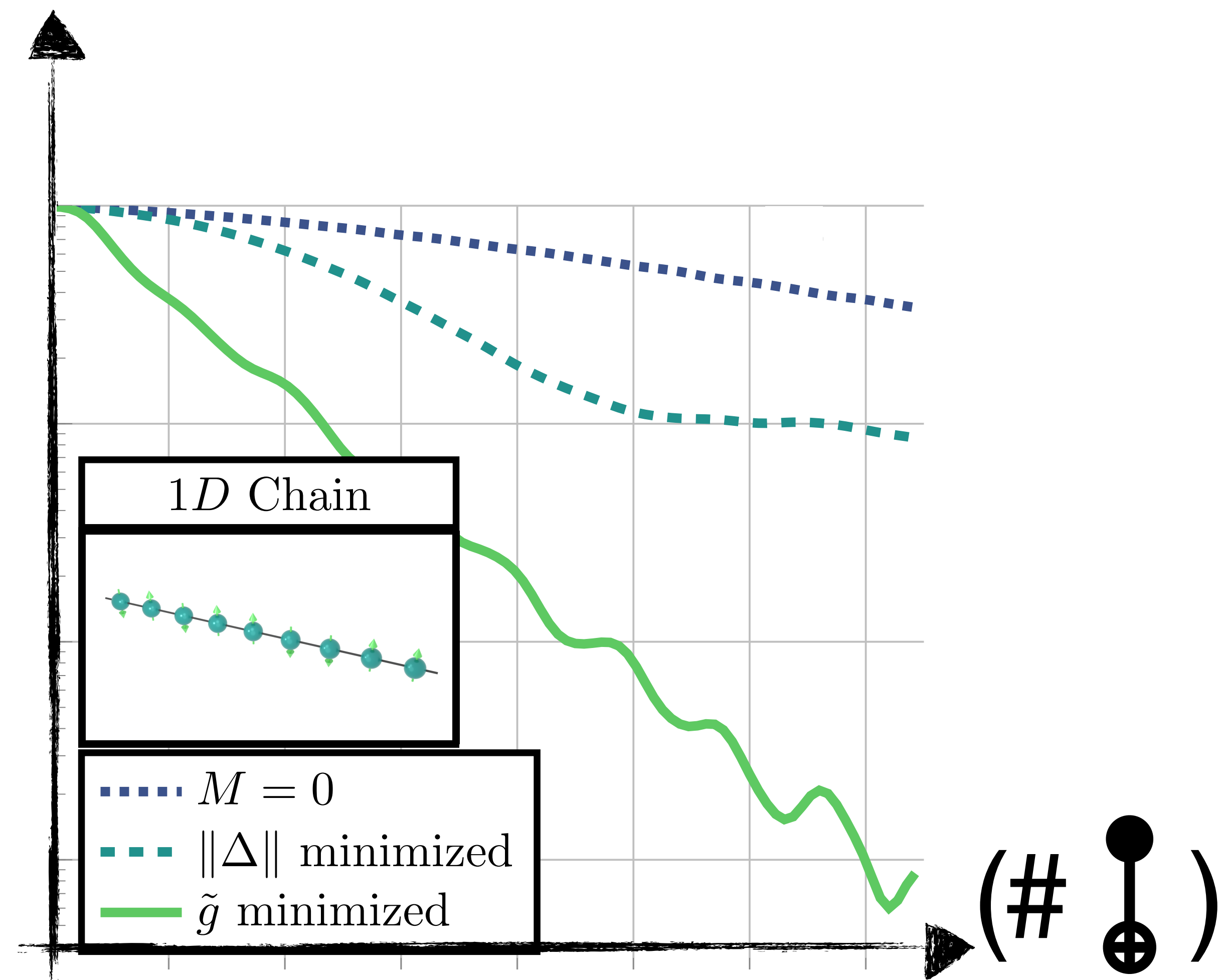
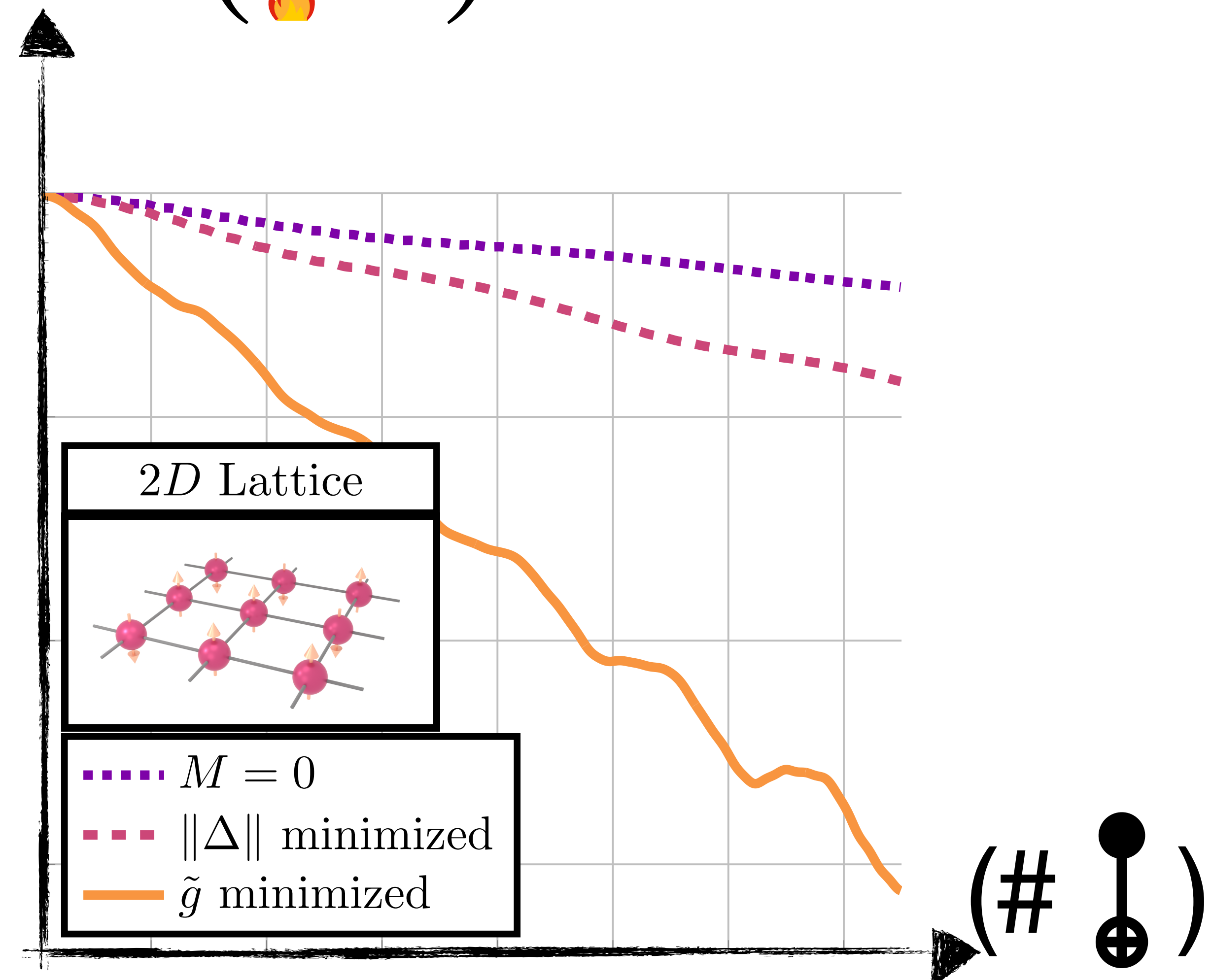


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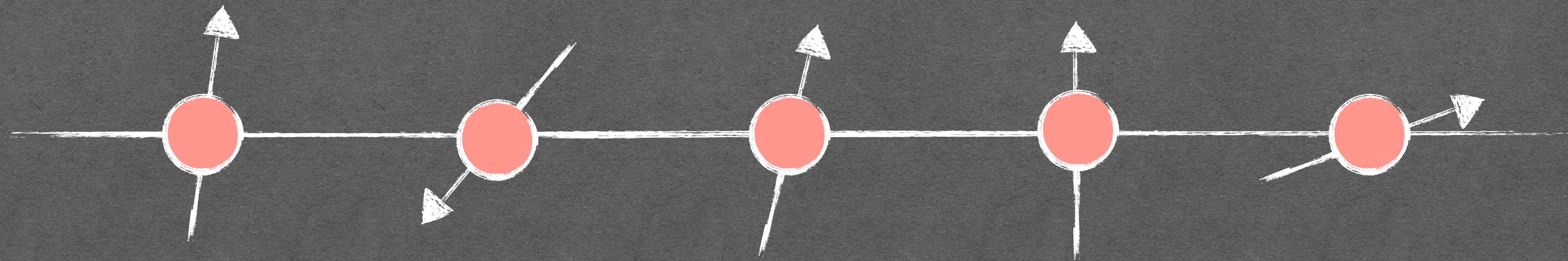
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COSTLESS

$$J^x = 5J^z$$

 $\ln(\text{🔥🔥🔥🔥🔥})$

 $\ln(\text{🔥🔥🔥🔥🔥})$


Ising model in a magnetic field

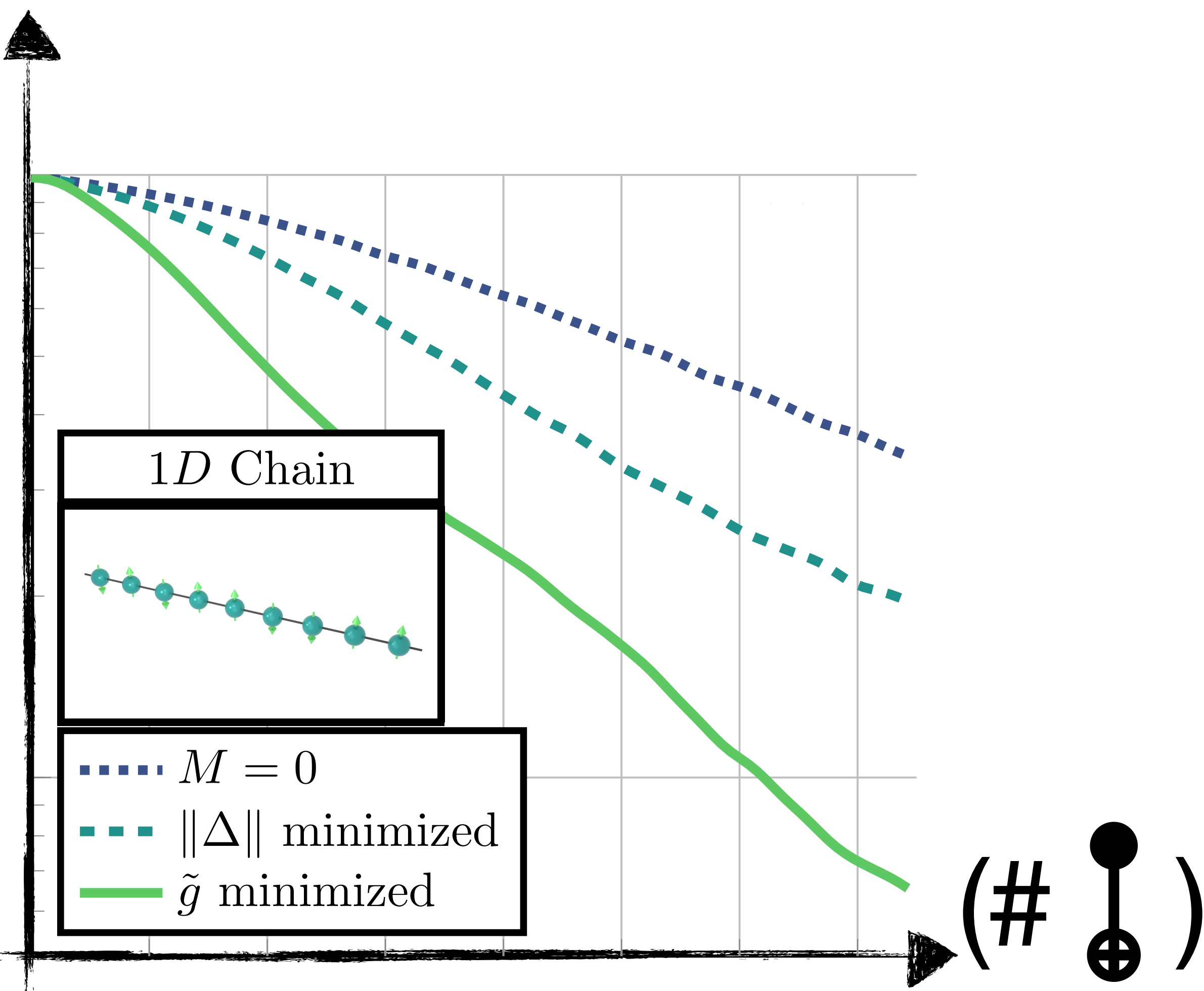


$$H_1 = -\frac{h}{2} \sum_i Z_i - \frac{J^x}{2} \sum_{\langle i,j \rangle} (X_i X_j)$$

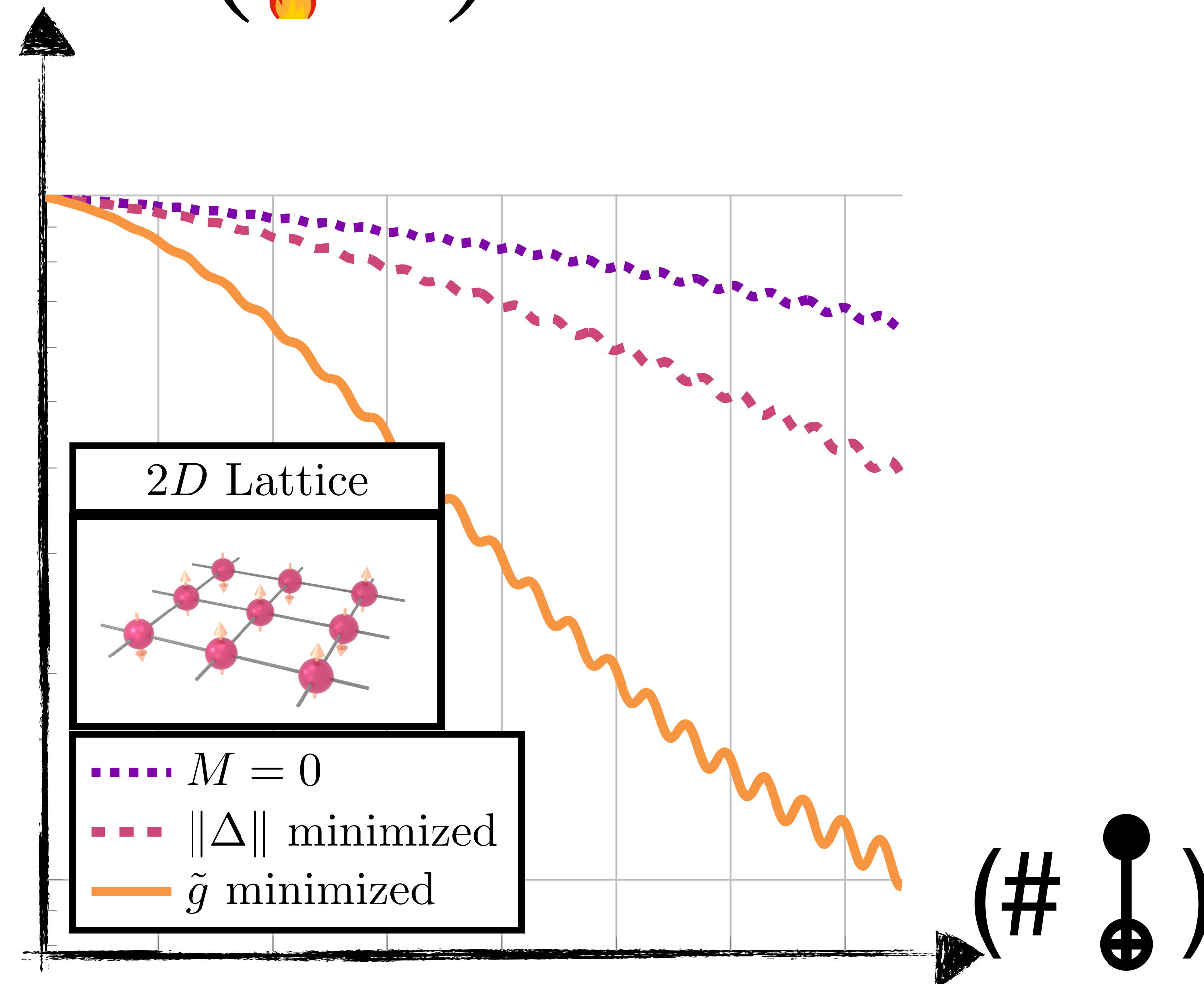
$$H_0 = -\frac{h}{2} \sum_i Z_i + \alpha \sum_i (Z_i)$$

$$J^x = 5J^z$$

$\ln(\text{🔥🔥🔥🔥})$



$\ln(\text{🔥🔥🔥🔥})$



- State preparation with **shallow circuits**
- AP error is **exponentially bounded**
- **Explicit formula** for the **characteristic time**
- Numerical tests for spin-systems
GS preparation

**QR code
to the paper:**



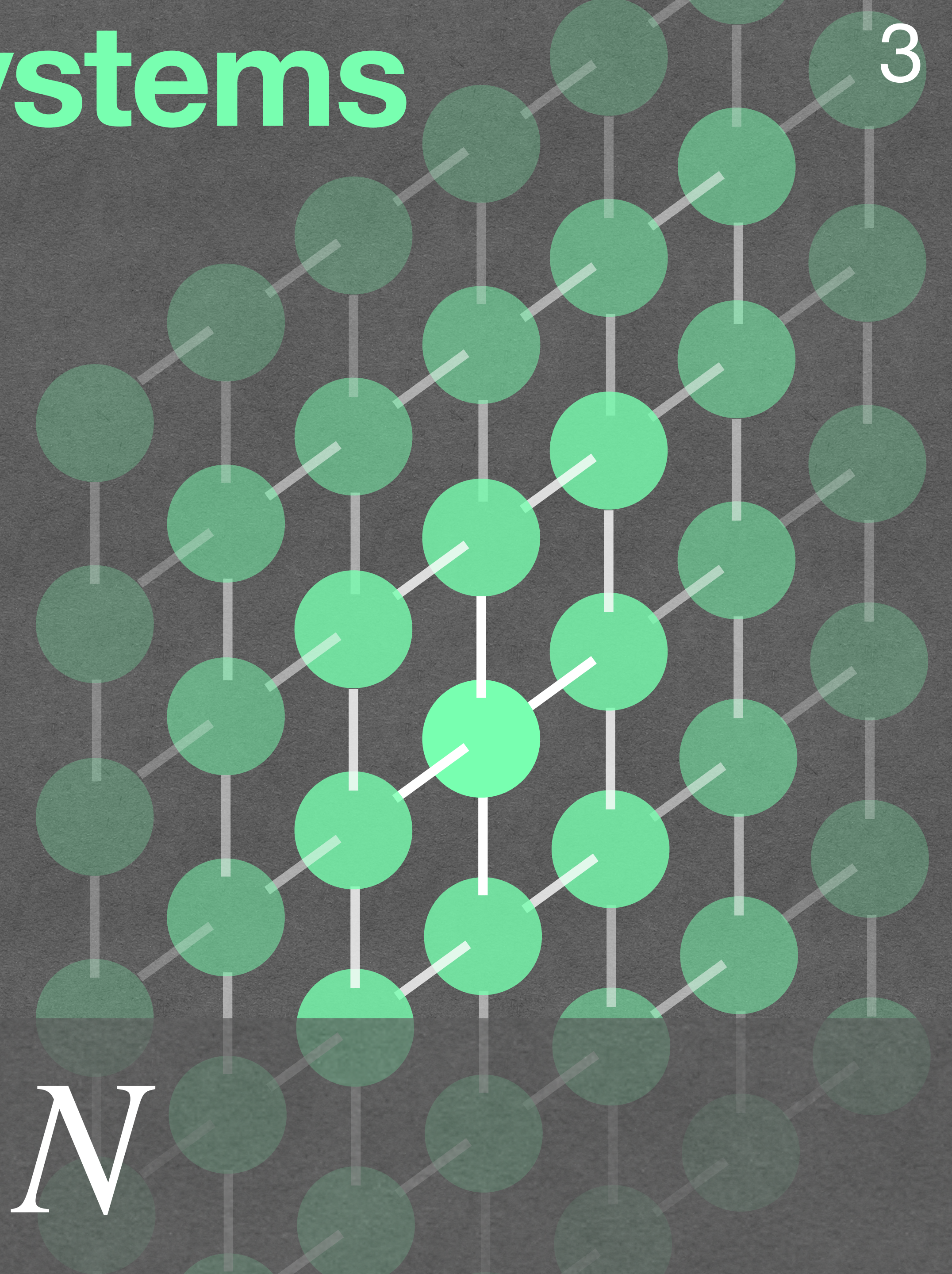
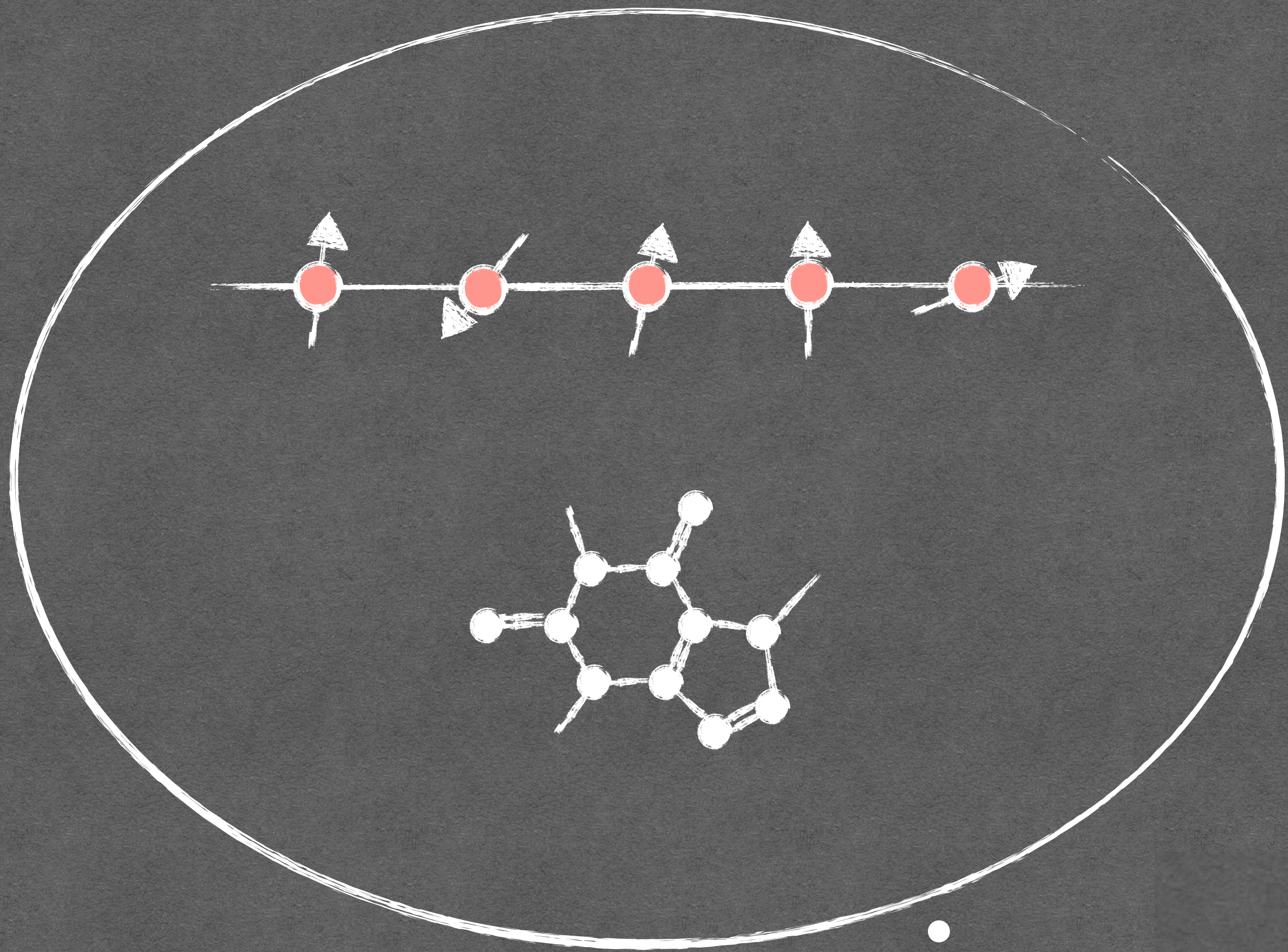
arXiv preprint arXiv:2405.03656

Exponential optimization of adiabatic quantum-state preparation

Davide Cugini

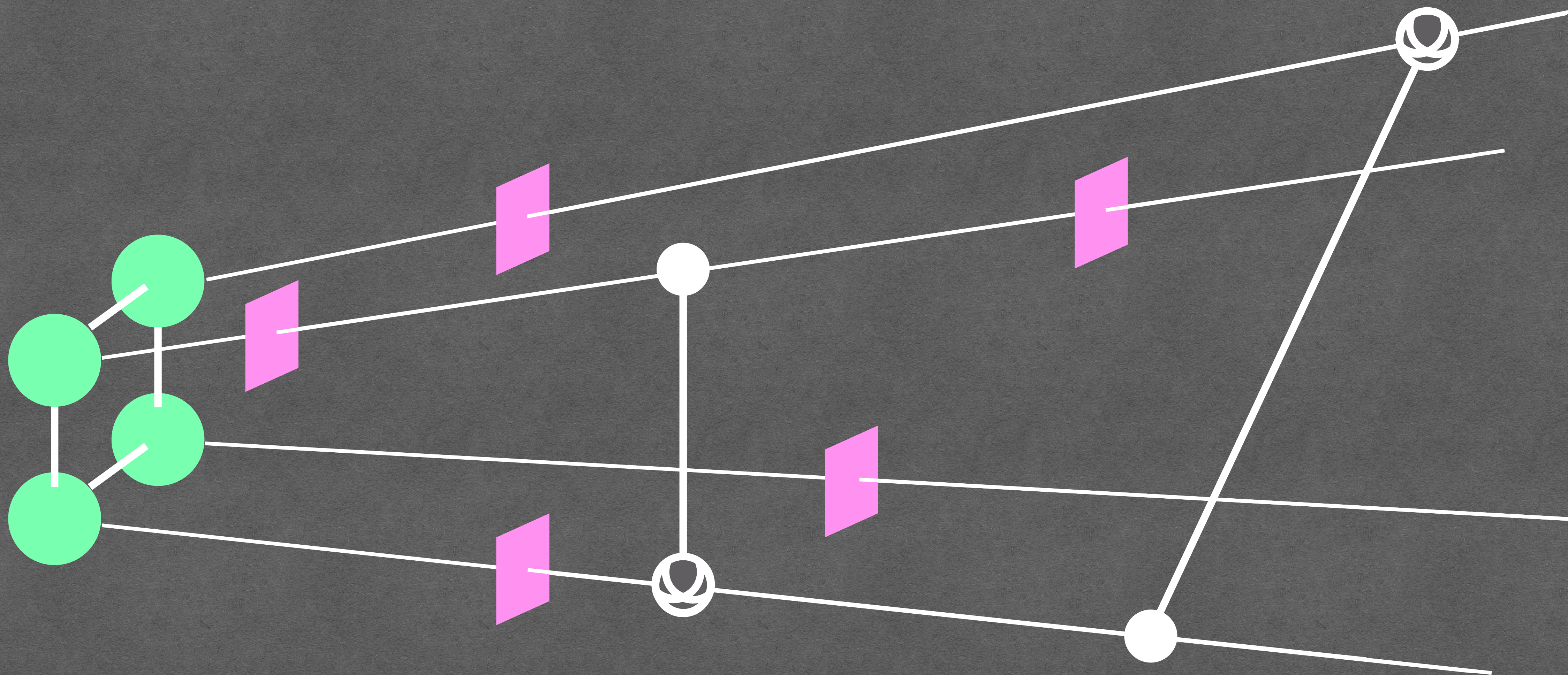
D. Nigro, M. Bruno, D. Gerace

Quantum systems



size $\sim N$

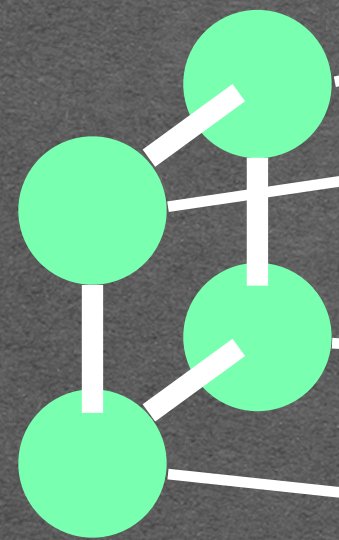
Quantum-state preparation



State preparation challenge

Default state

$$|0\rangle^{\otimes N}$$



Target state

$$|\Psi\rangle$$

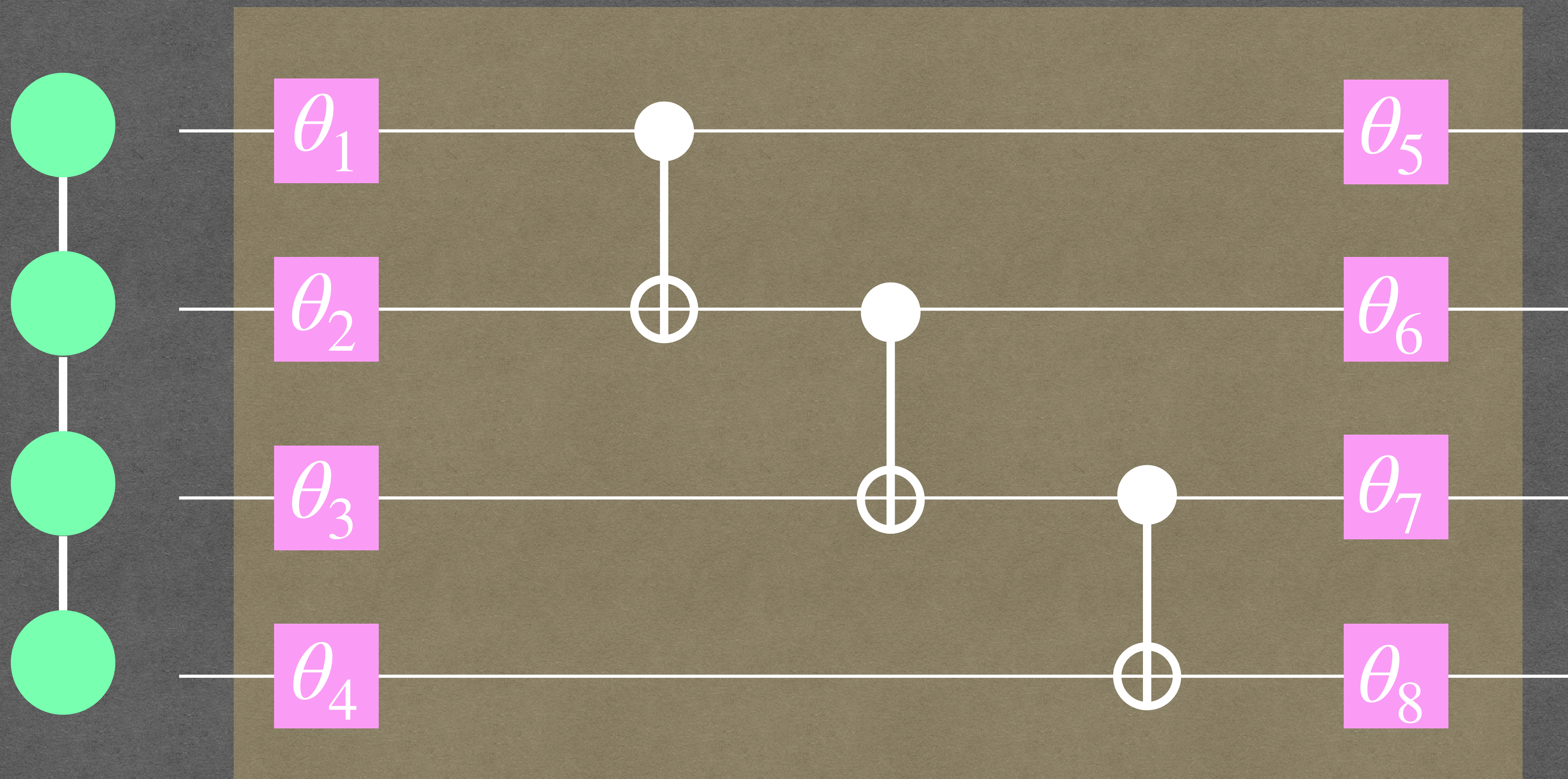


Fixed depth (# )

H_1 : **Hamiltonian of the problem**

$|\Psi_n\rangle$: **n-th eigenstate (target)**

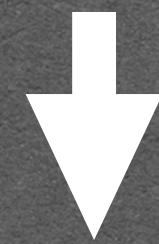
Variational Quantum Eigensolvers (VQE)



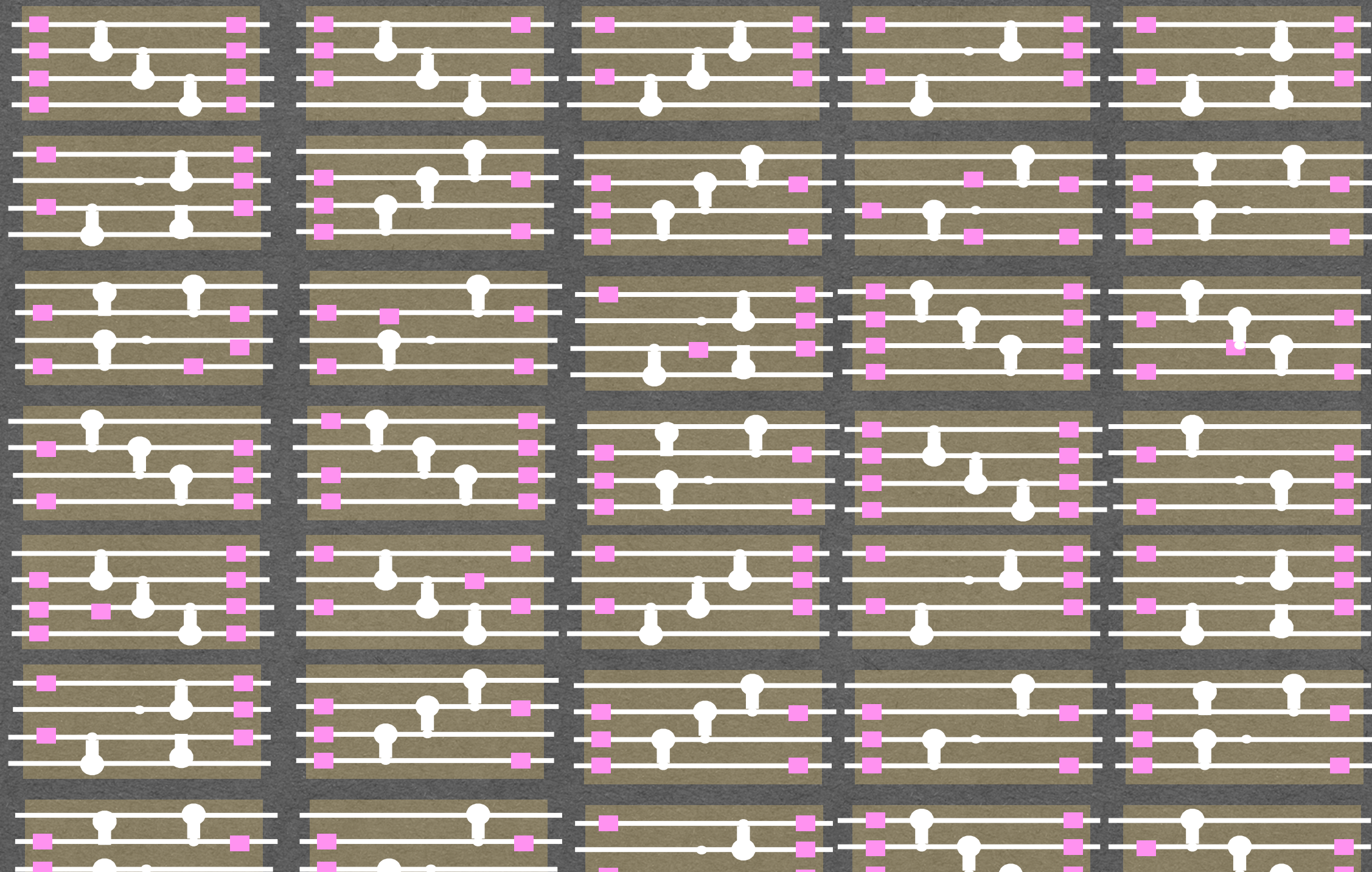
Fixed depth (# )

Main Problems of VQE

Shallow circuit



Ansatz choice

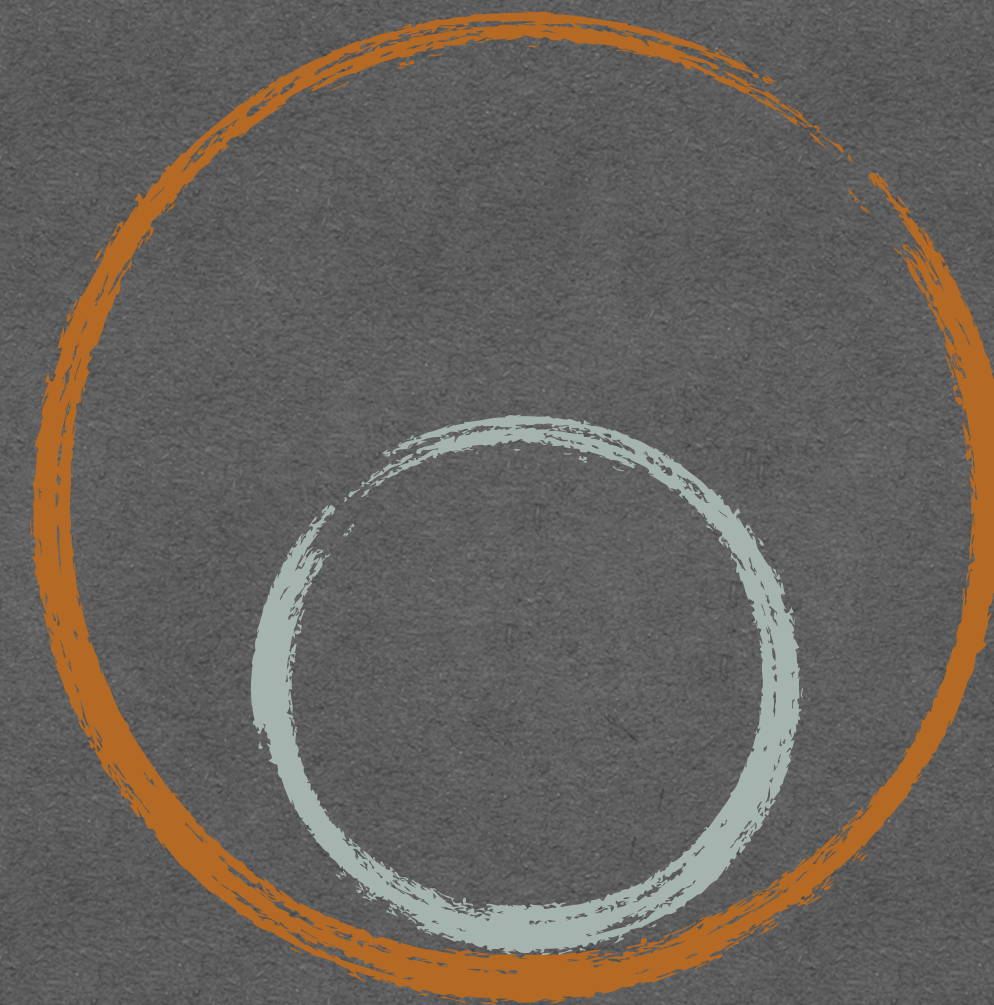


Large Hilbert spaces

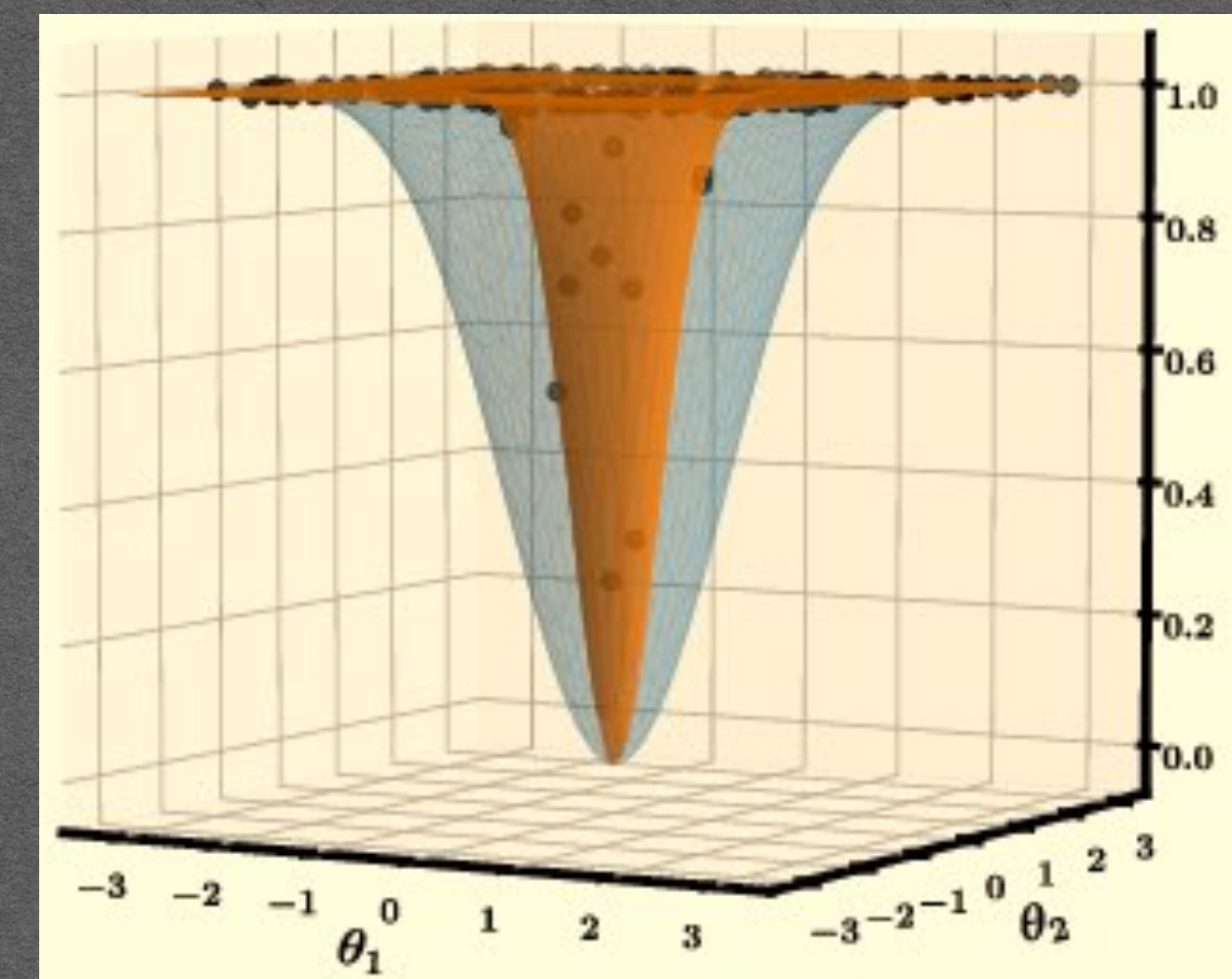


Barren Plateaus

$$\mathcal{H} \sim 2^N$$



Loss function



Eigenstates superposition

$$|SGS(i,j)\rangle \equiv \frac{1}{\sqrt{2}} \left(|\Psi_i\rangle + |\Psi_j\rangle \right)$$

Spectral Gap Superposition States

Davide Cugini,^{1,*} Francesco Ghisoni,^{1,†} Angela Rosy Morgillo,^{1,‡} and Francesco Scala^{1,§}

¹*Dipartimento di Fisica, Università degli Studi di Pavia, via A. Bassi 6, 27100 Pavia (Italy)*

(Dated: February 28, 2024)

Eigenstates superposition

$$|SGS(i, j)\rangle \equiv \frac{1}{\sqrt{2}} \left(|\Psi_i\rangle + |\Psi_j\rangle \right)$$

$$\langle O(t) \rangle = \frac{1}{2} \left(\langle \Omega_i | O | \Omega_i \rangle + \langle \Omega_j | O | \Omega_j \rangle \right) + \rho \cos \left(\Delta E_{ji} t + \theta \right)$$

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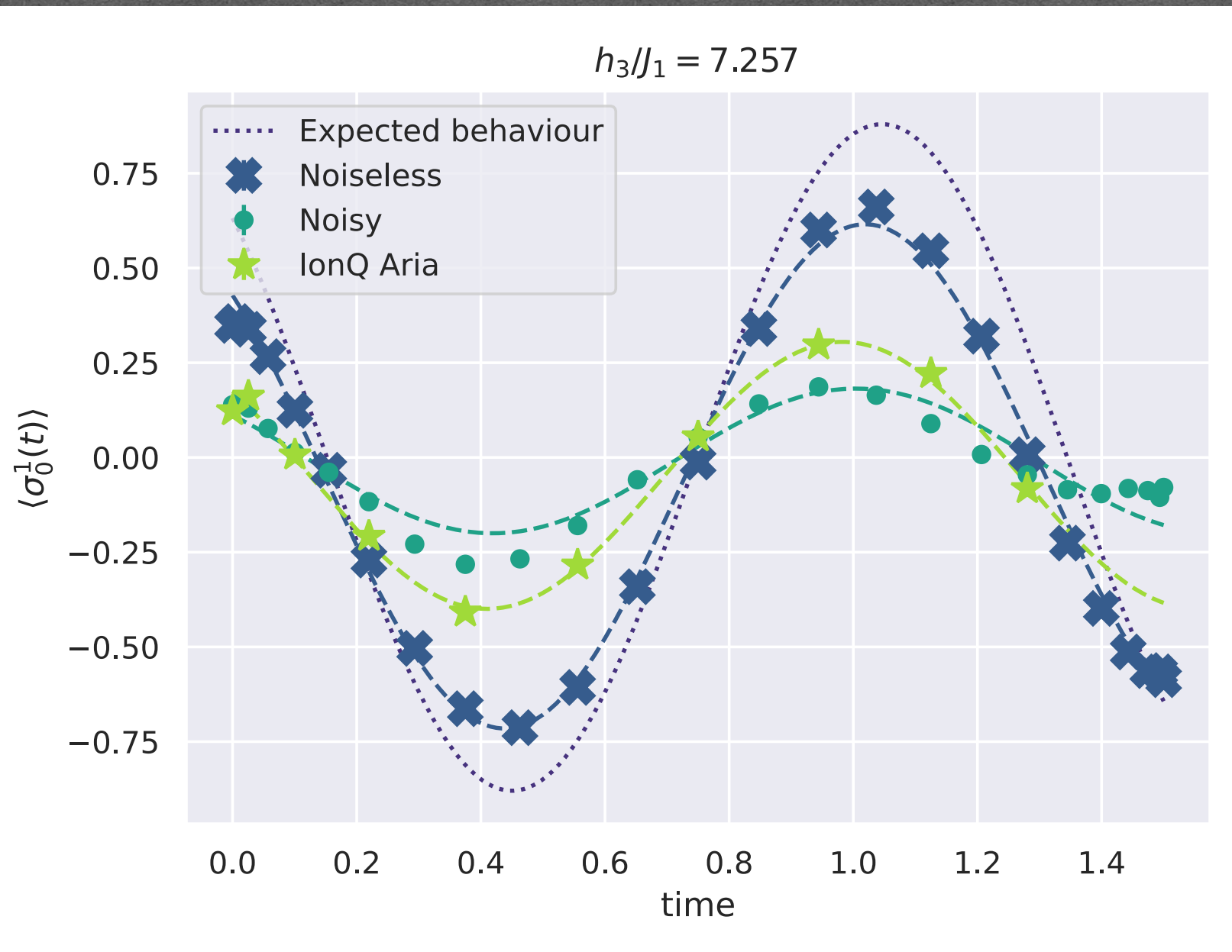
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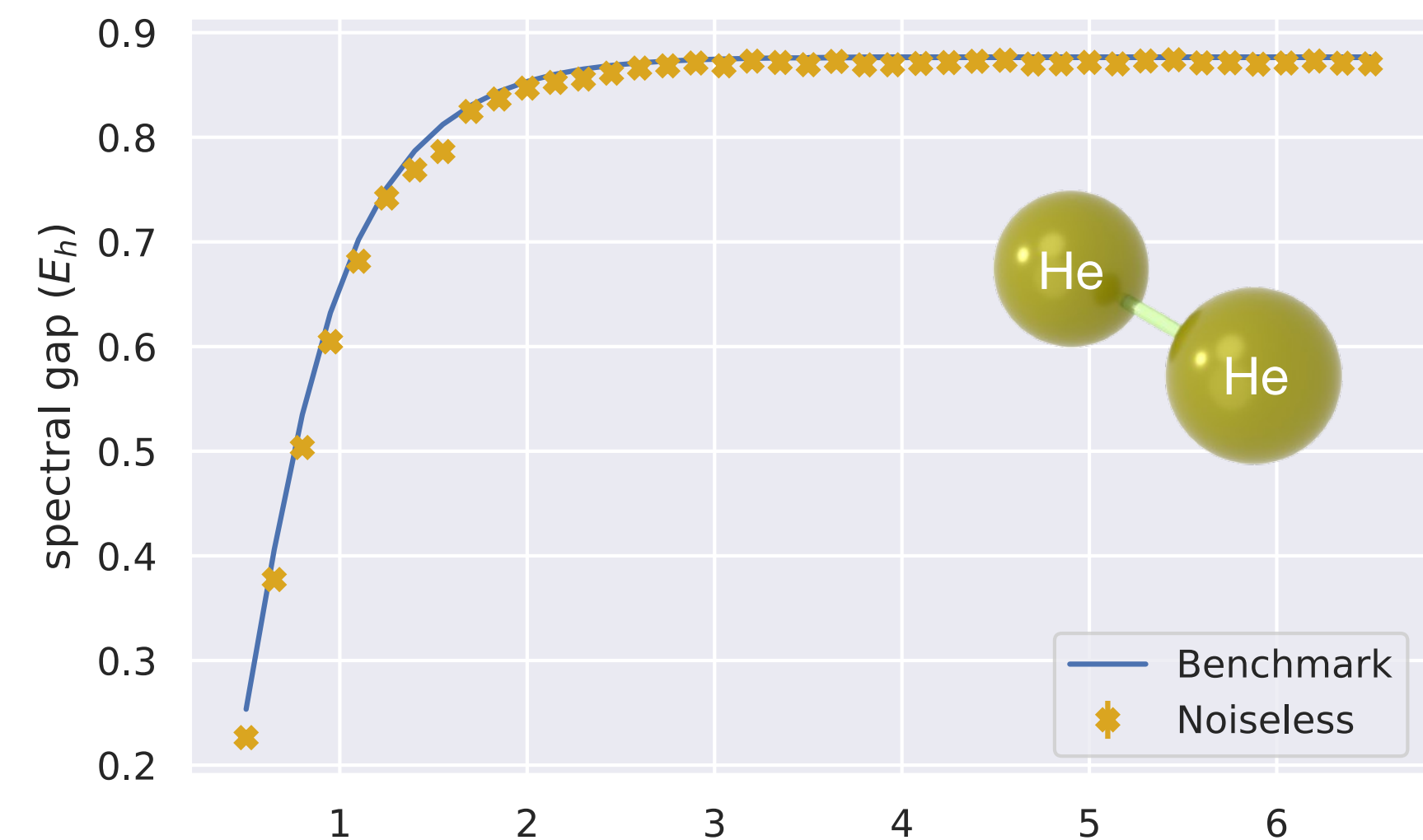
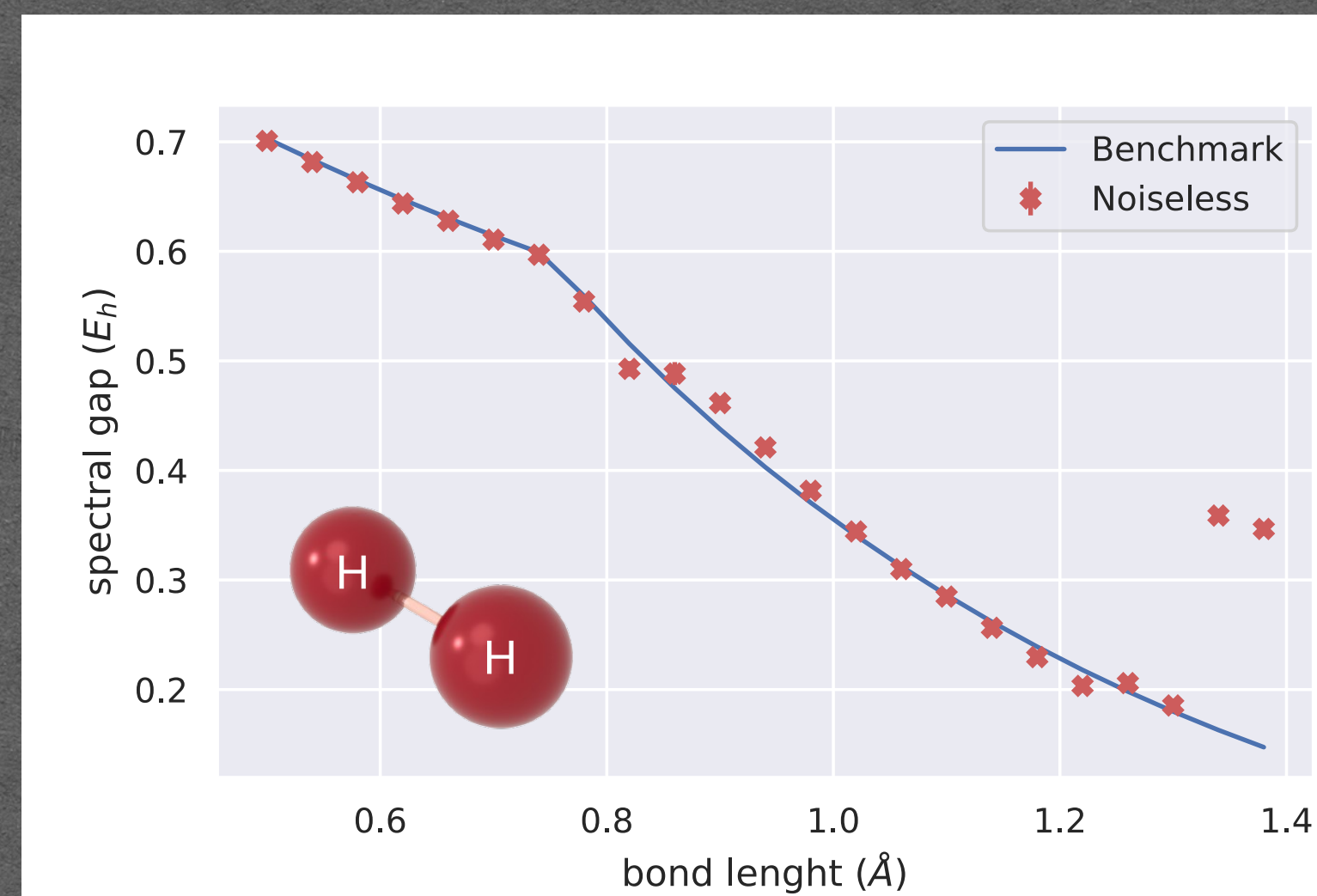
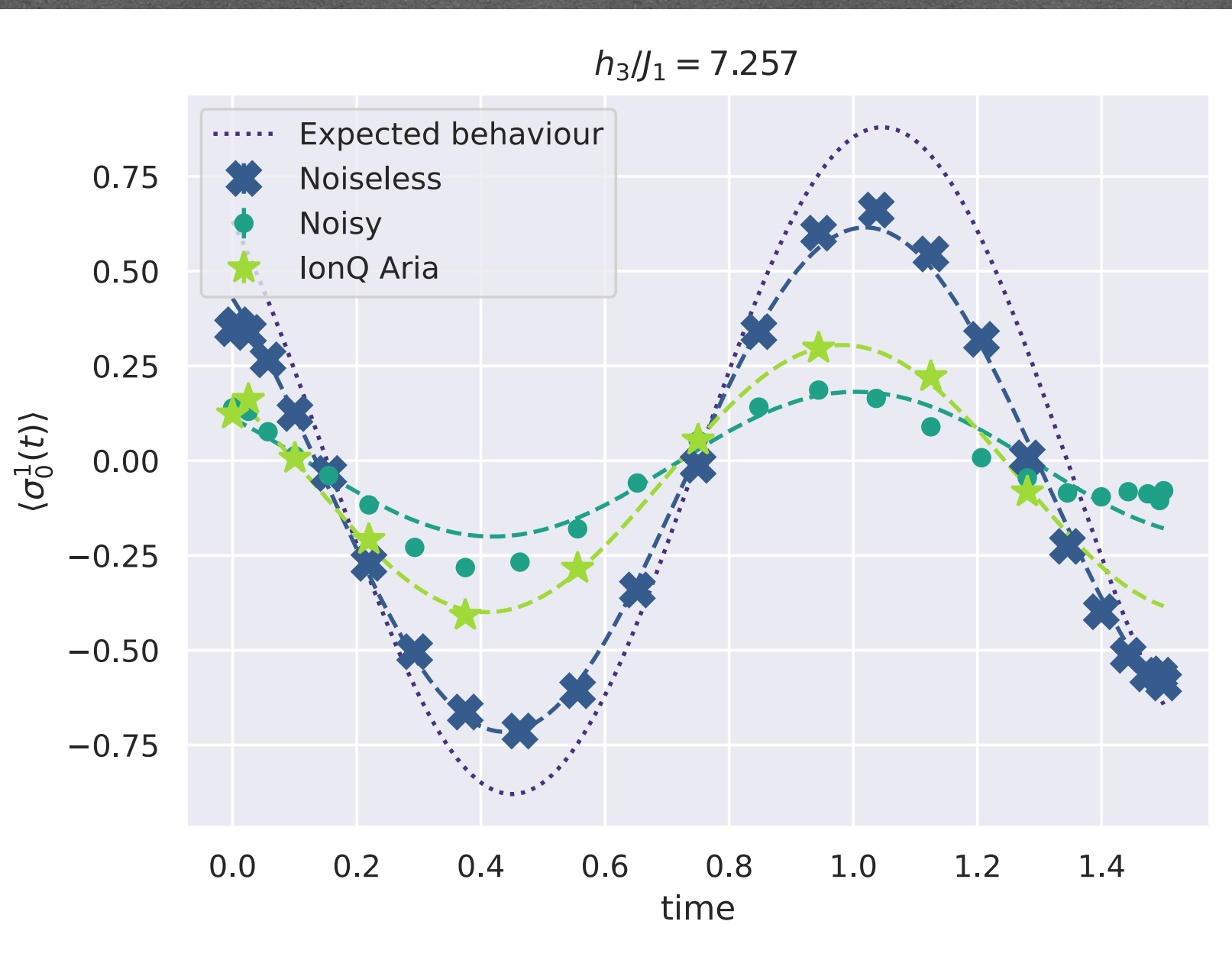
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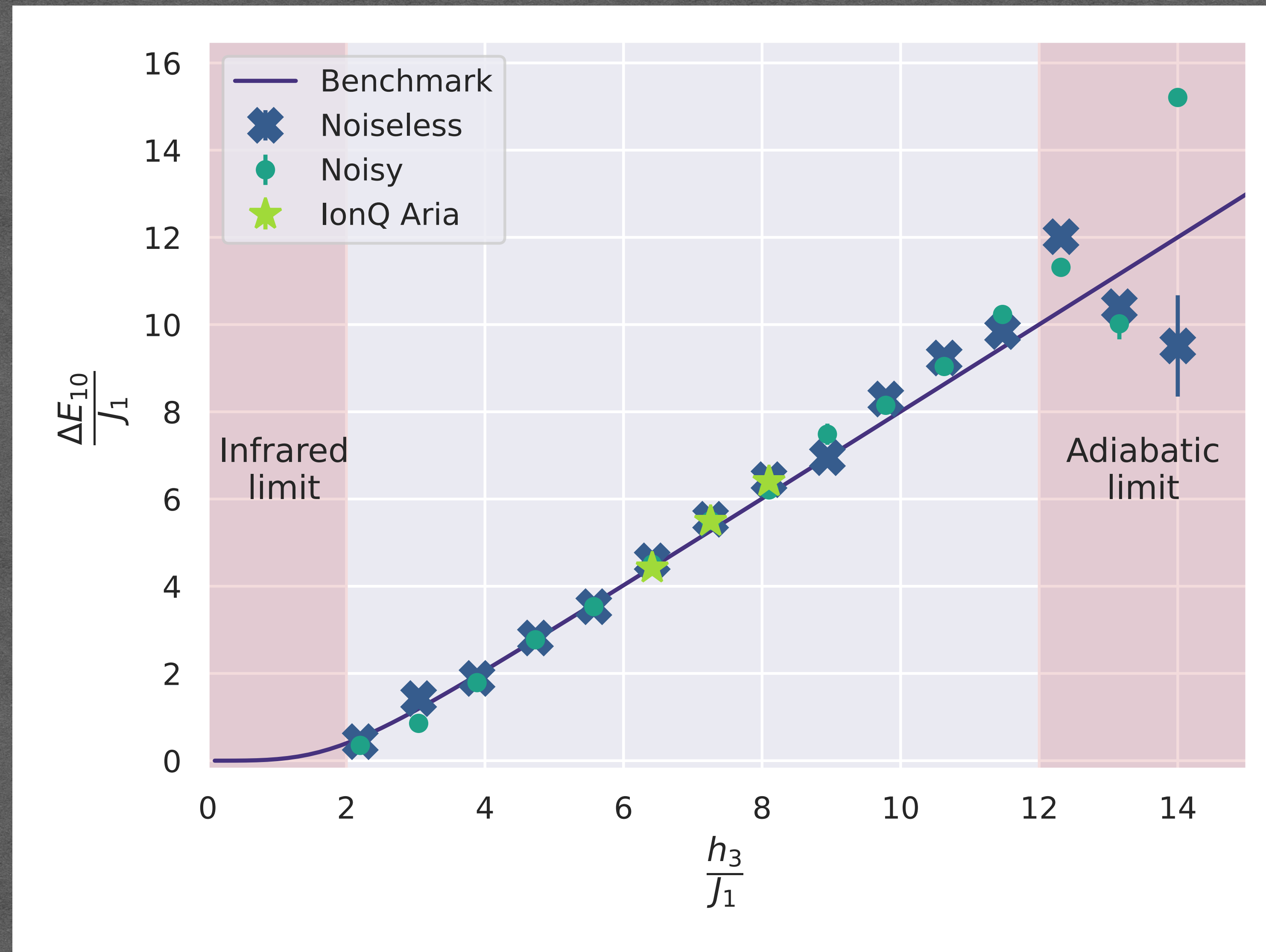
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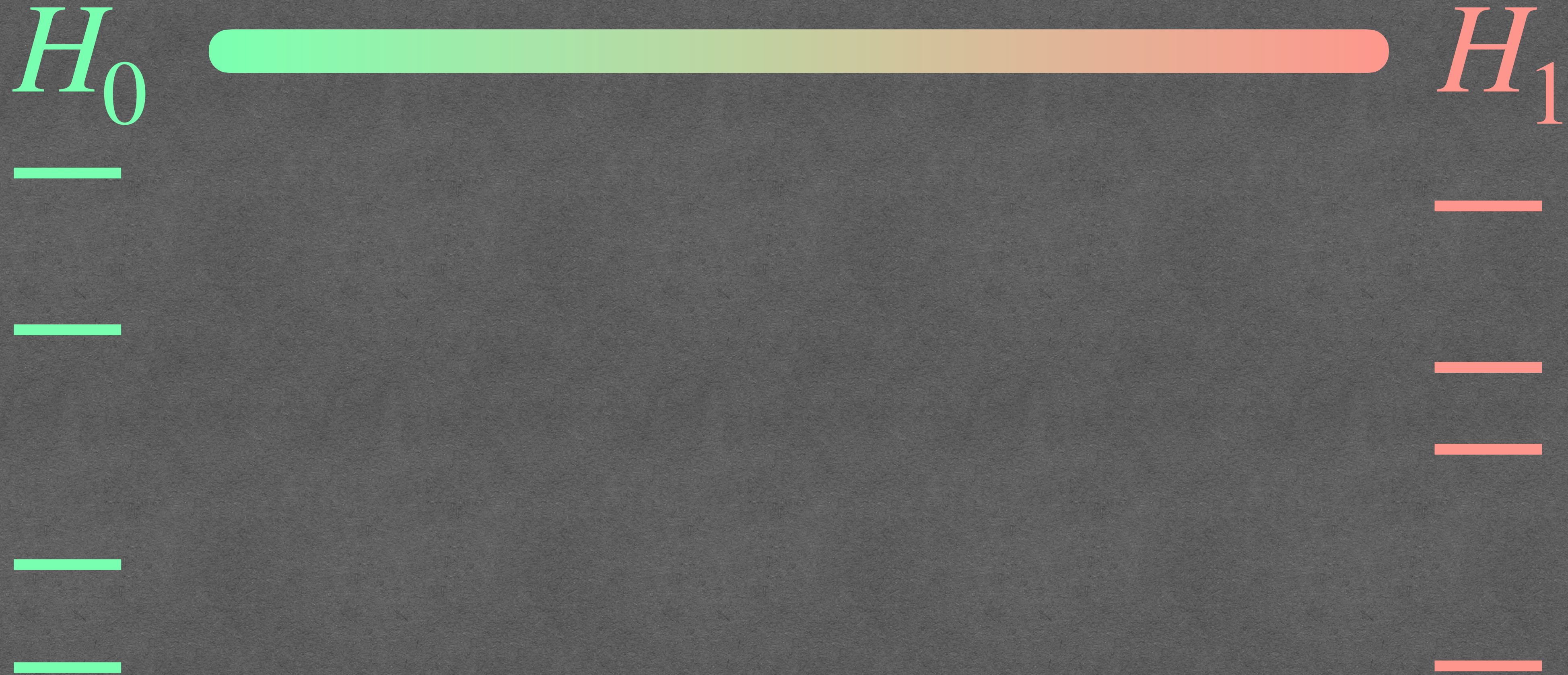


Eigenstates superposition: Ising model

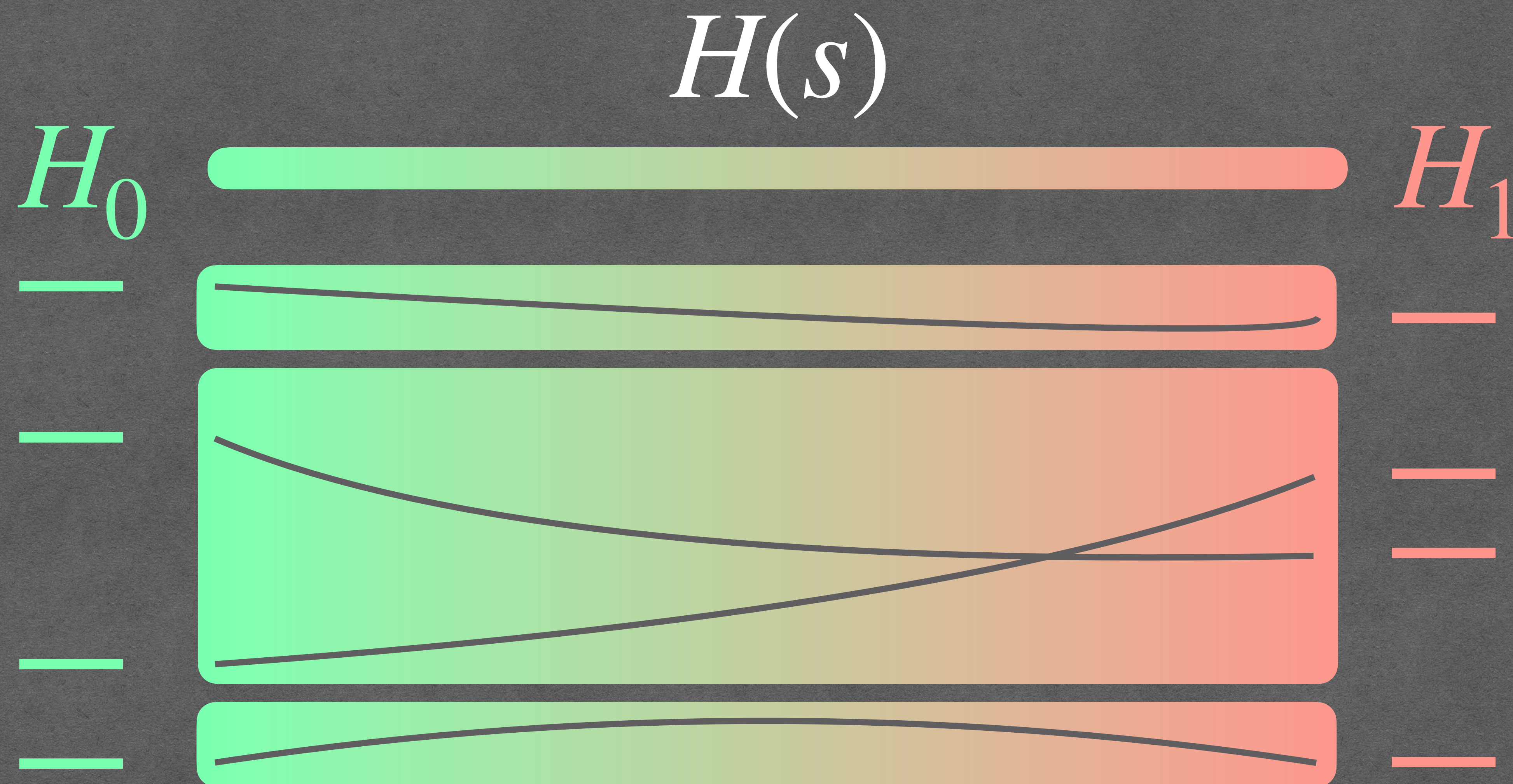


Adiabatic Preparation AP

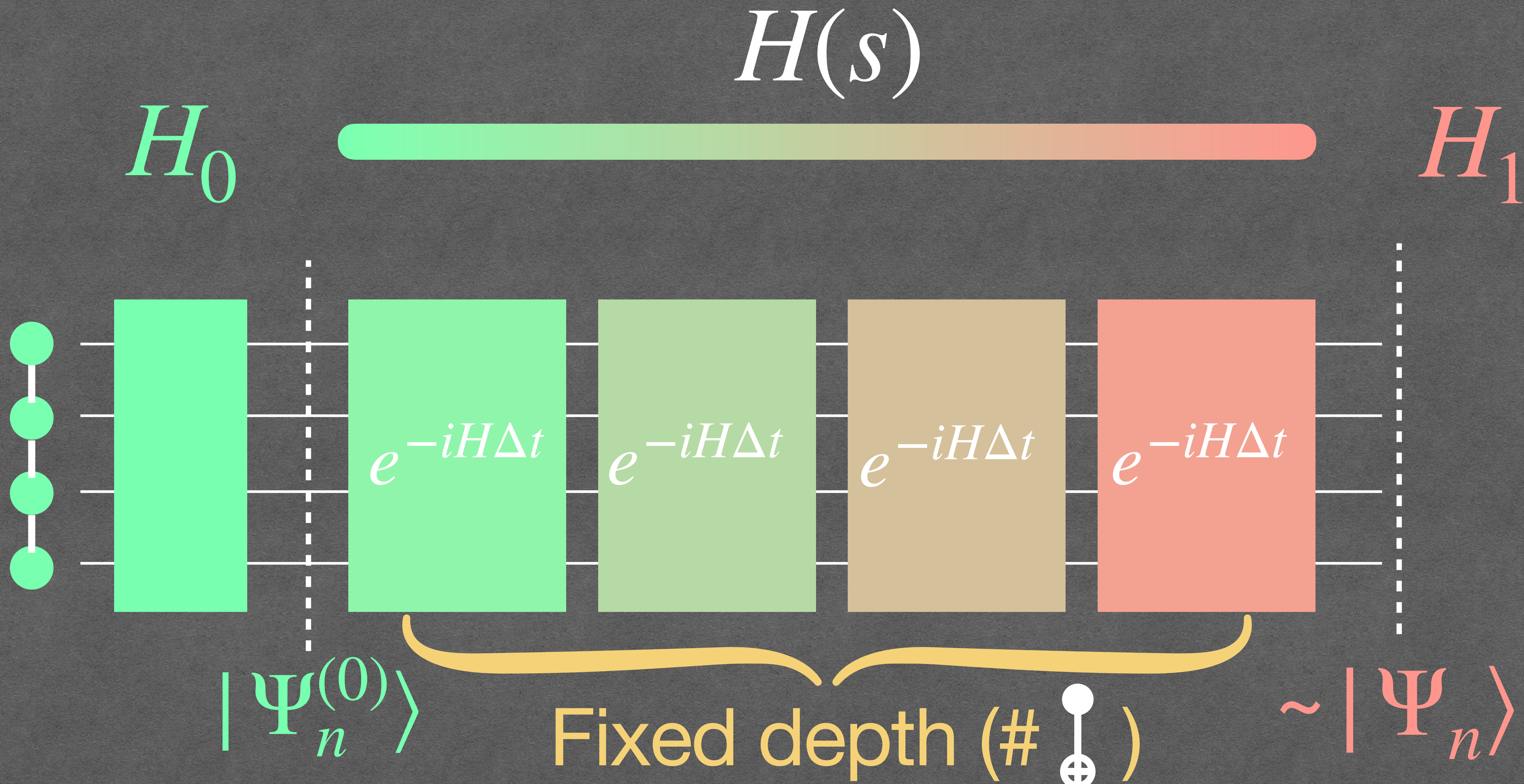
$$H(s) = H_0 + f(s)[H_1 - H_0]$$



Adiabatic Preparation AP



AP on quantum computers



Losses bound for AP

Linear Adiabatic Theory. Exponential Estimates

G. Nenciu

$\exists C, \tilde{g} > 0$ such that

$$\left| \left| 1 - |\Psi_n\rangle\langle\Psi_n| \right| \right| \leq C \exp\left[-\tau/\tilde{g}\right]$$

τ : Duration of the adiabatic process

Losses bound for AP

Linear Adiabatic Theory. Exponential Estimates

G. Nenciu

$\exists C, \tilde{g} > 0$ such that

$$\text{🔥} \leq C \exp \left[-(\# \text{⊕}) / \tilde{g} \right]$$

Fixed by the hardware

CAN WE REDUCE \tilde{g} ?

Exponential optimization of quantum state preparation via adiabatic thermalization

Davide Cugini,¹ Davide Nigro,¹ Mattia Bruno,² and Dario Gerace¹

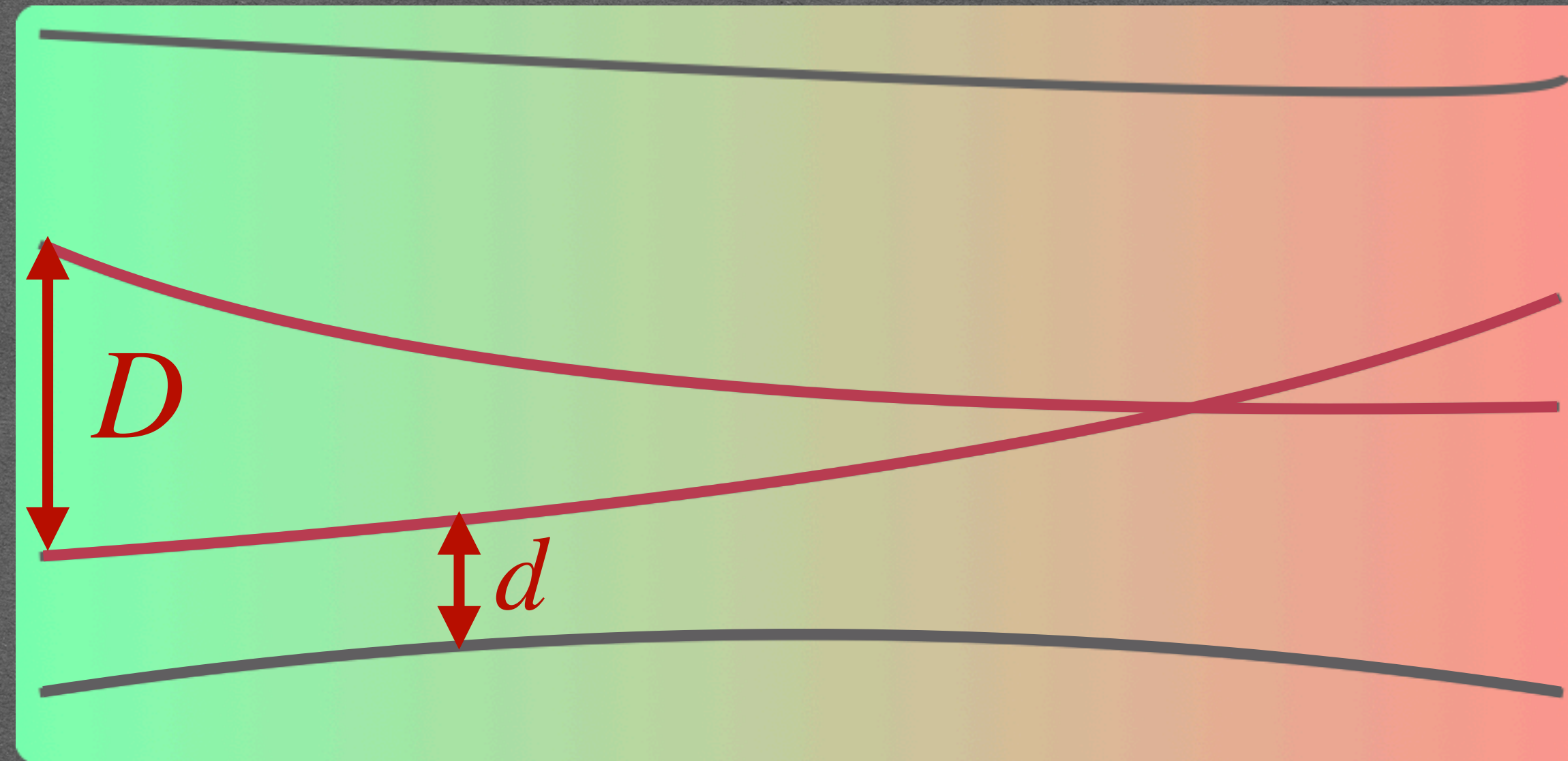
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²*Dipartimento di Fisica, Università di Milano-Bicocca, and INFN, sezione di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy*

$$\tilde{g} = \tilde{g}(\|H_1 - H_0\|, D, d)$$

Increasing function in terms of

$$\|\Delta\| = \|H_1 - H_0\|, \quad D, \quad d^{-1}$$



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$$\tilde{g} = 4 \frac{W(D, d)}{\rho(\|\Delta\|, d)}$$

$$W(d, D) = \left[2^8 \left(1 + 2 \frac{D}{\pi d} \right)^2 \left(\frac{\pi^2}{3} + \frac{4}{d} \right) \right]^{7/3}$$

Linear case

$$\rho \left(\frac{d}{\|\Delta\|} \right) = \inf_{s \in [0, 1]} \sup_{\rho \in C} \left[|\rho| : |f(s + \rho) - f(s)| < \frac{d}{4\|\Delta\|} \right] \quad \rho \left(\frac{d}{\|\Delta\|} \right) = \frac{d}{4\|\Delta\|}$$

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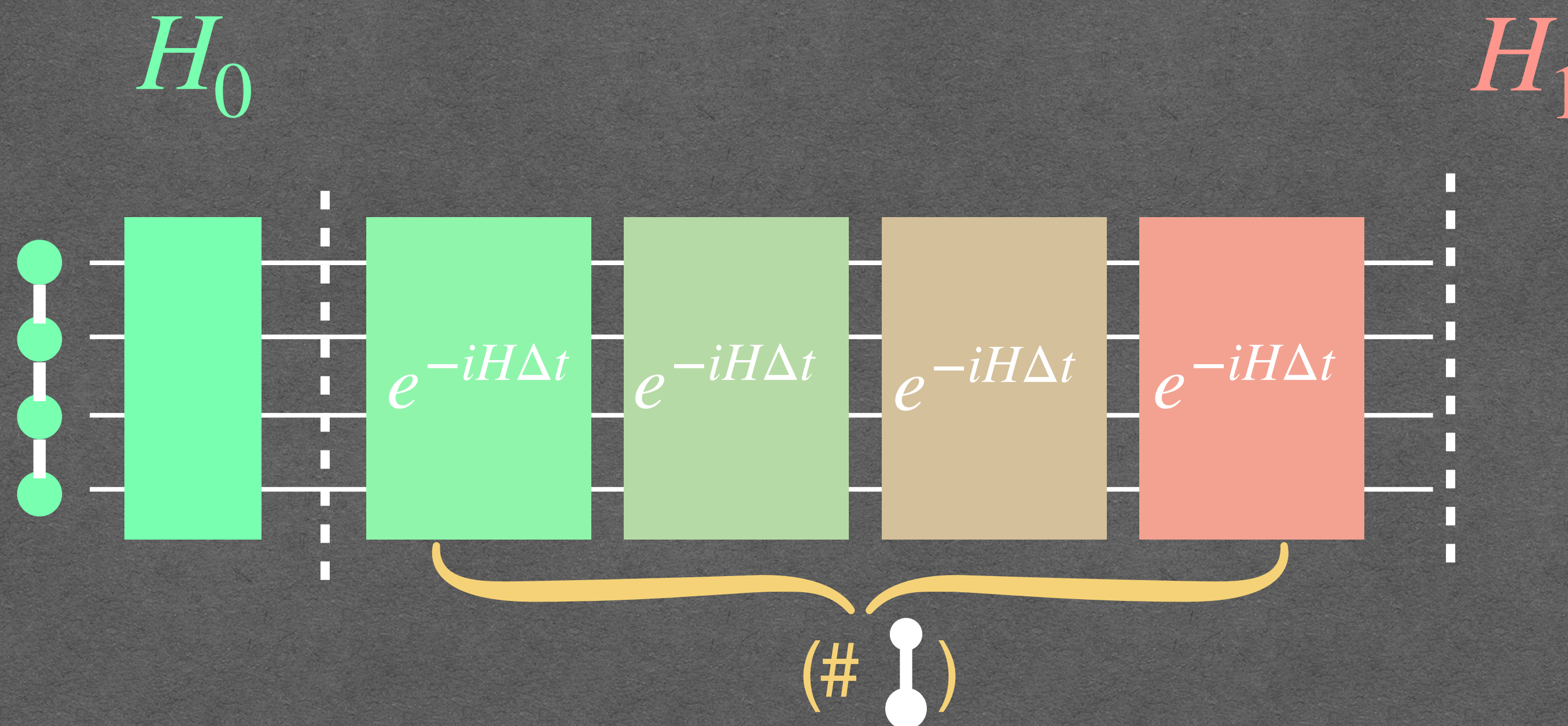
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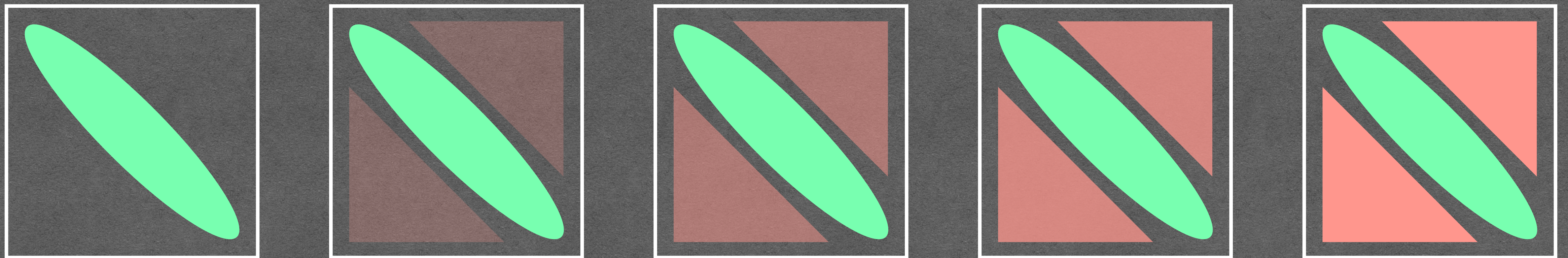
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Exponential optimisation of AP

$$H_0 = \text{diag}(H_1)$$

H_1

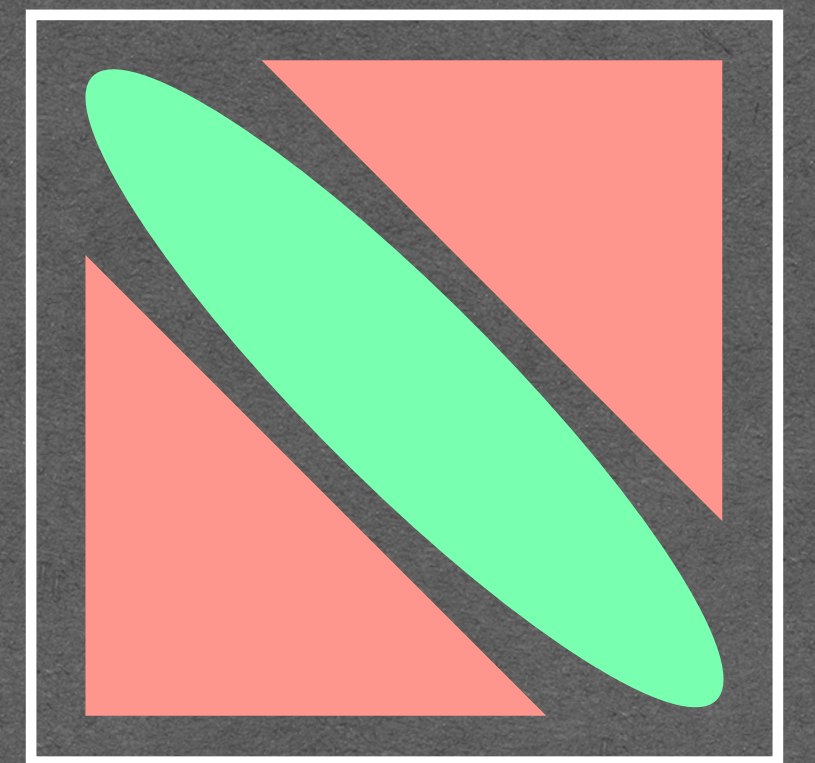
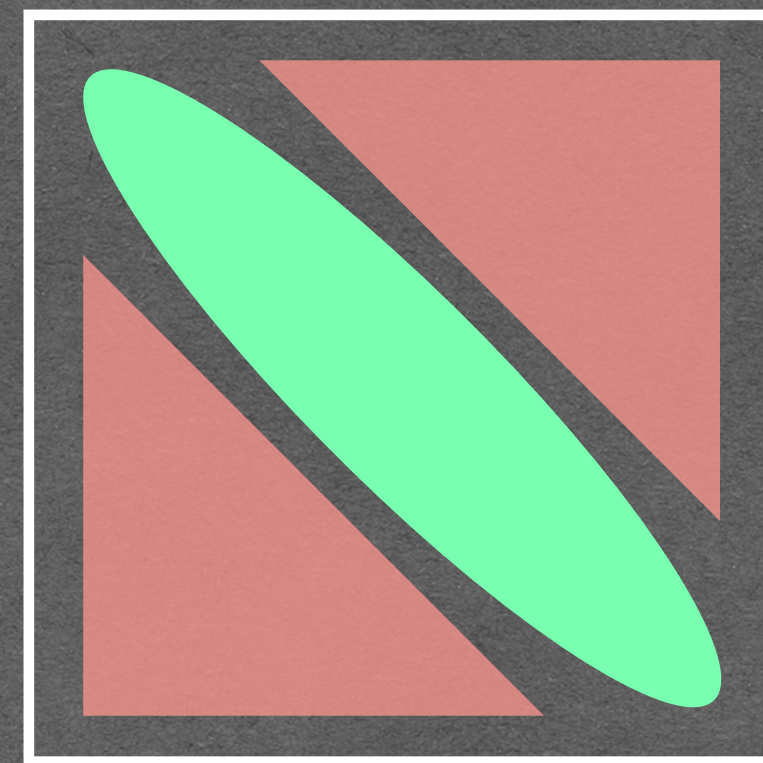
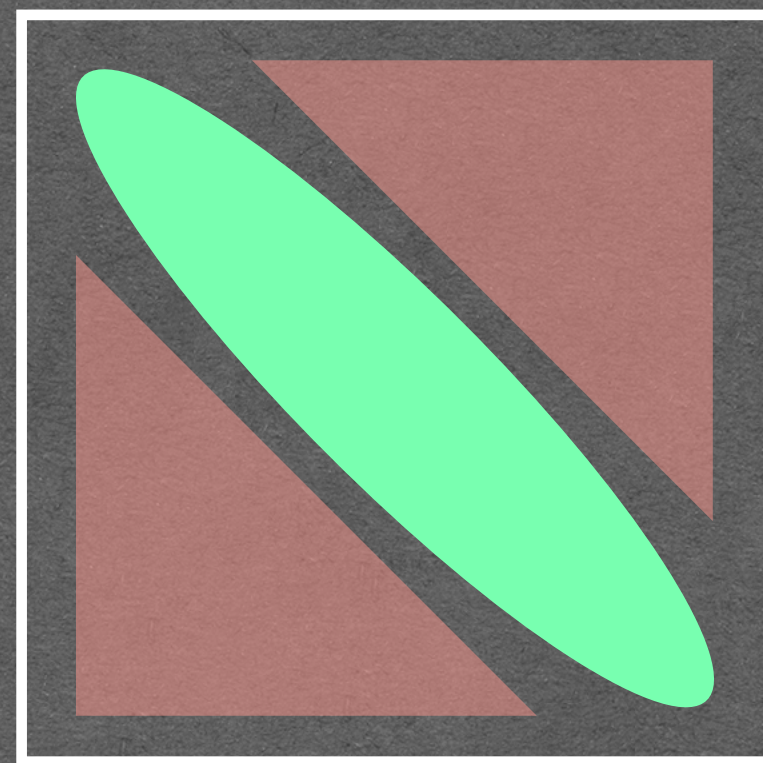
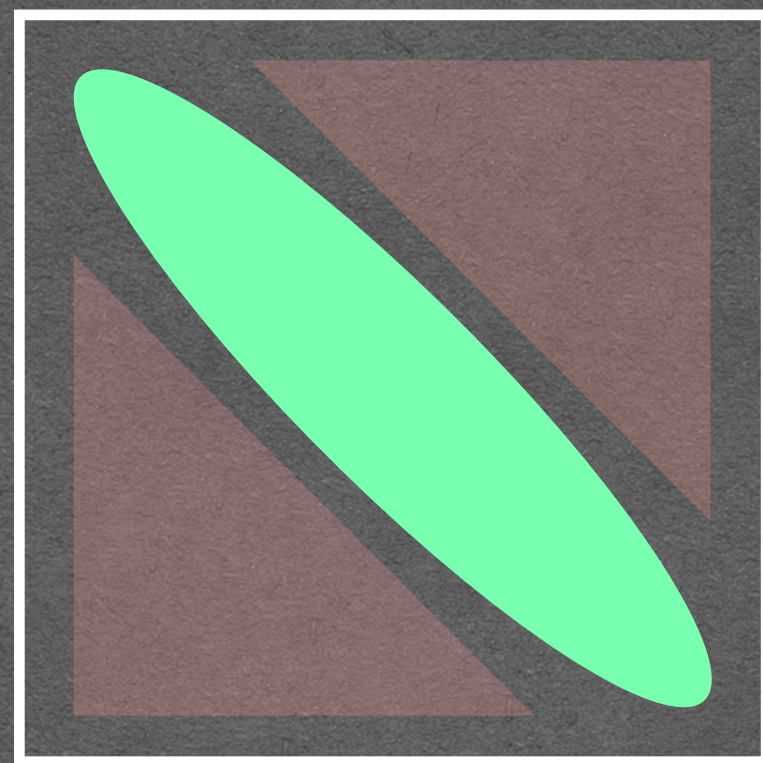
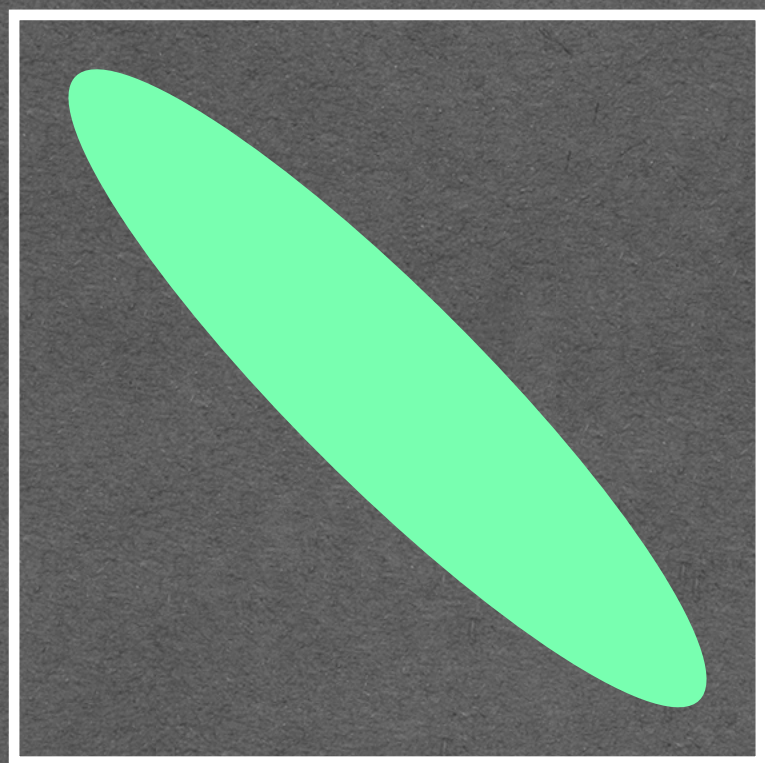


Exponential optimisation of AP

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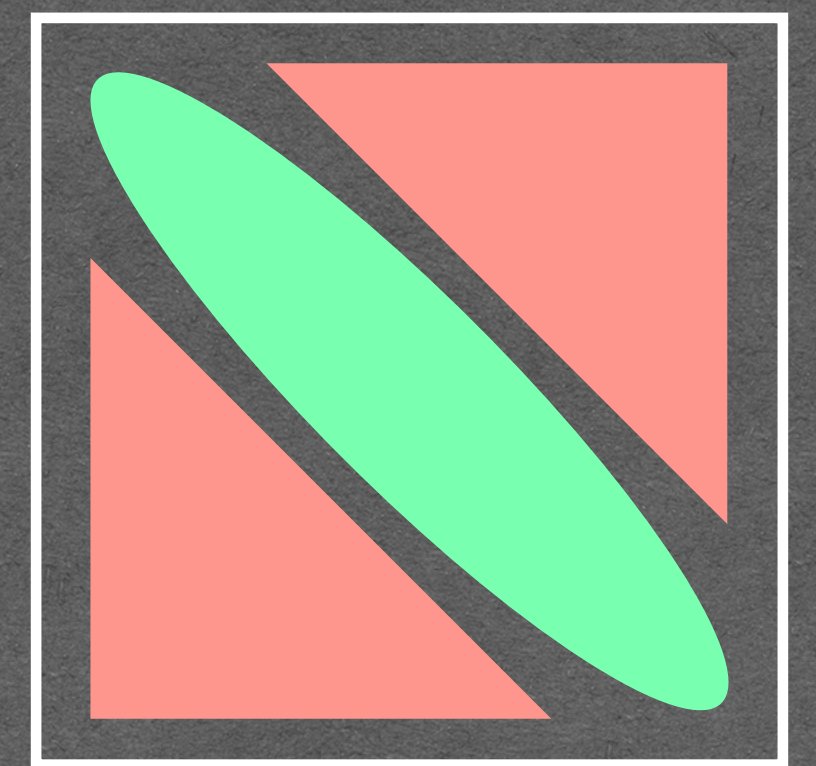
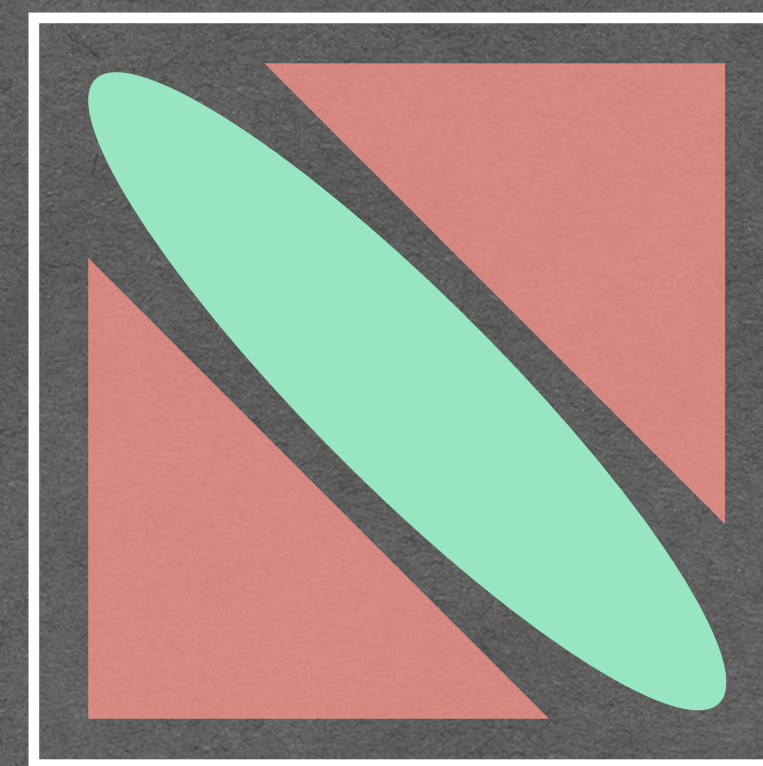
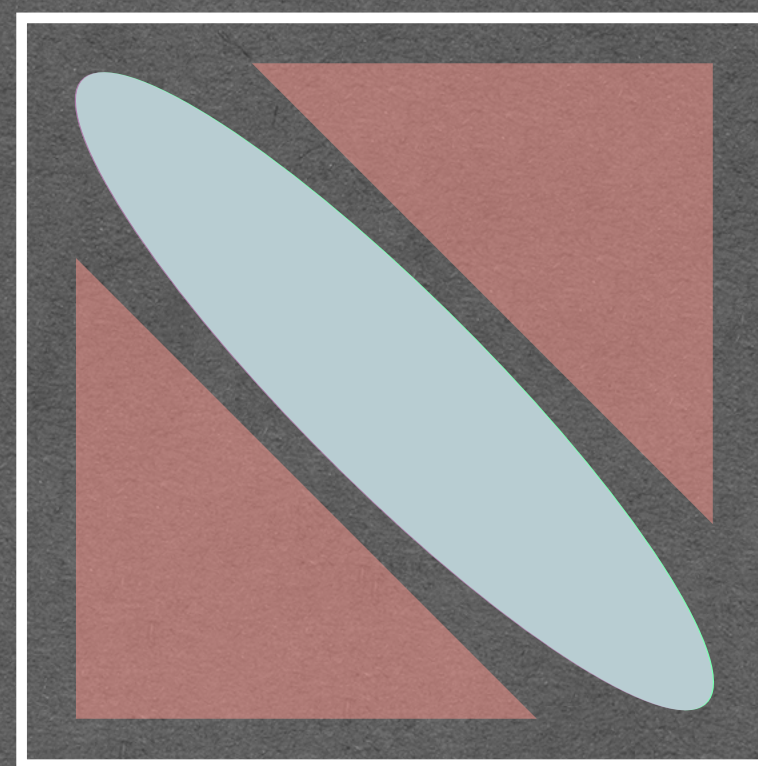
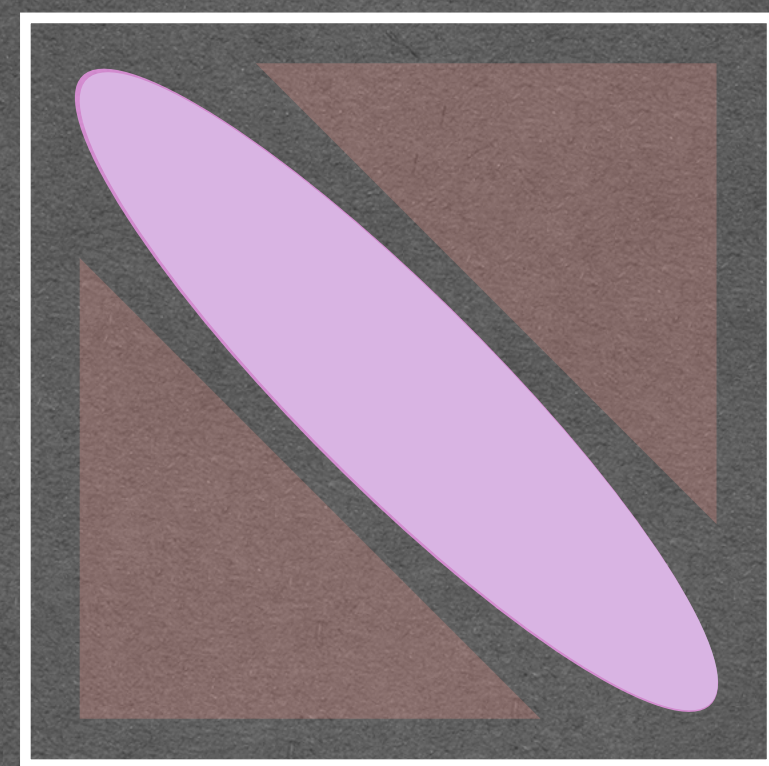
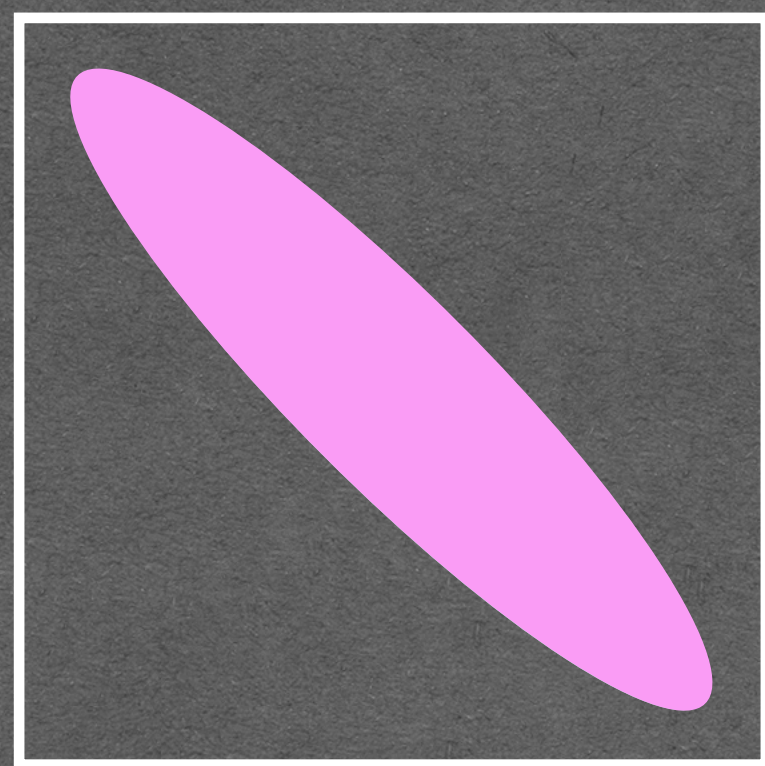


Exponential optimisation of AP

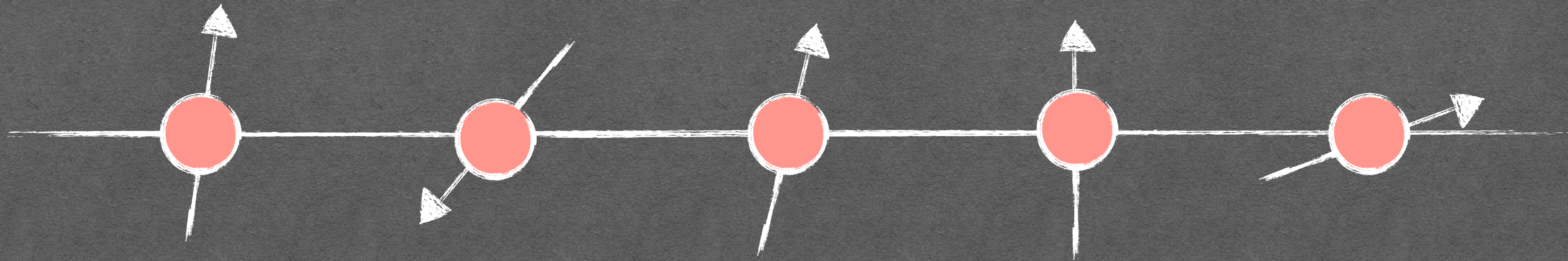
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XZ Heisenberg model

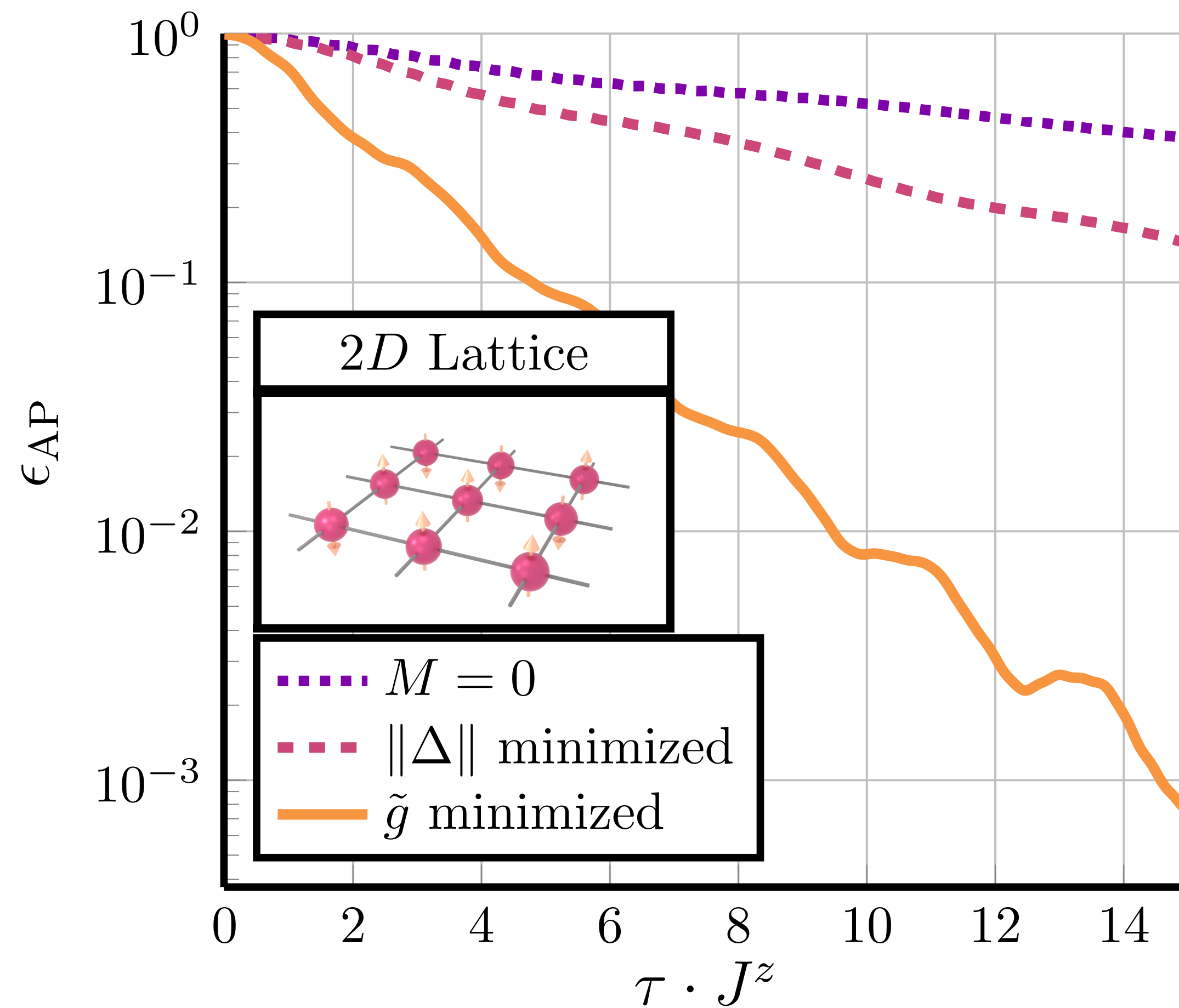
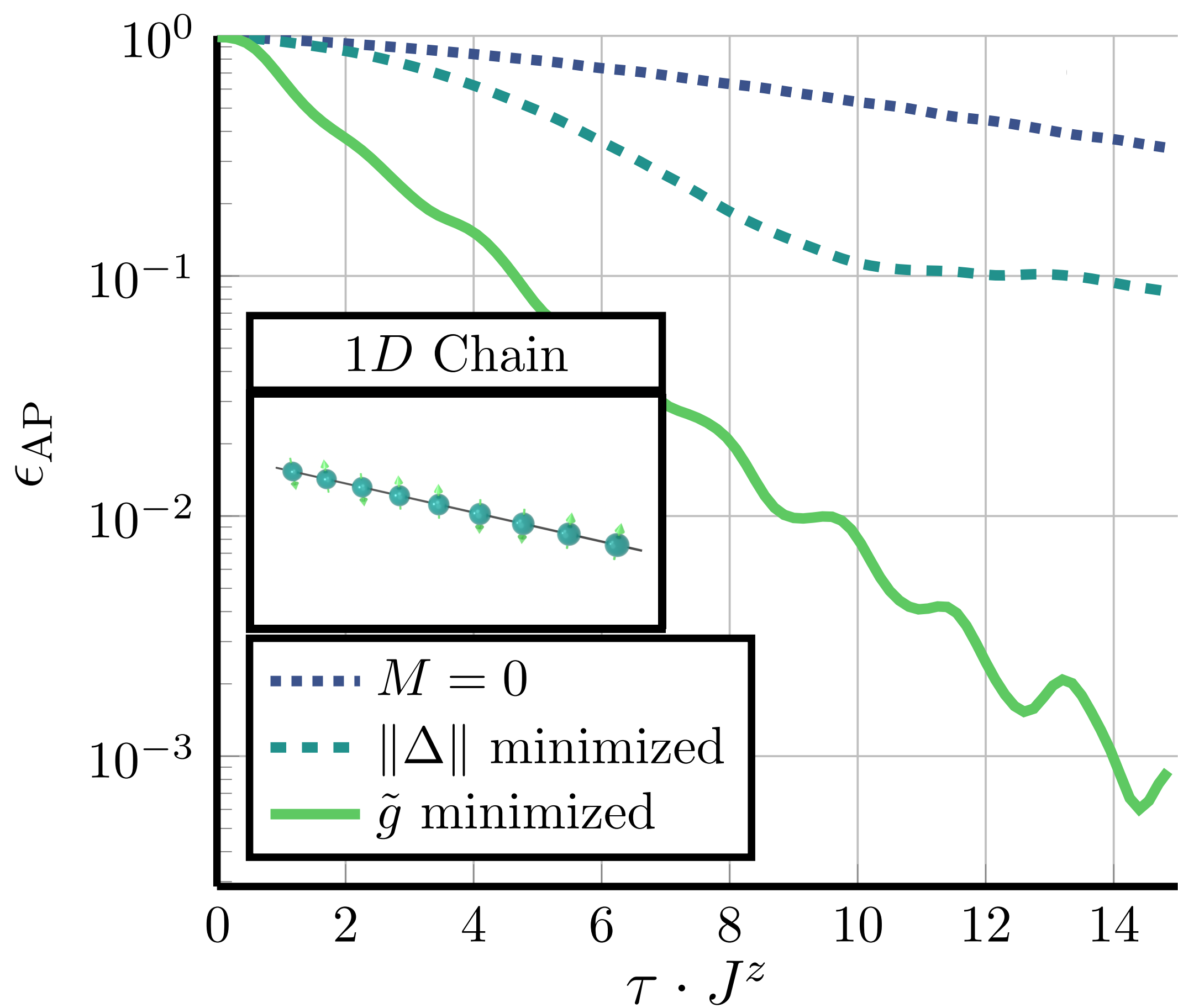


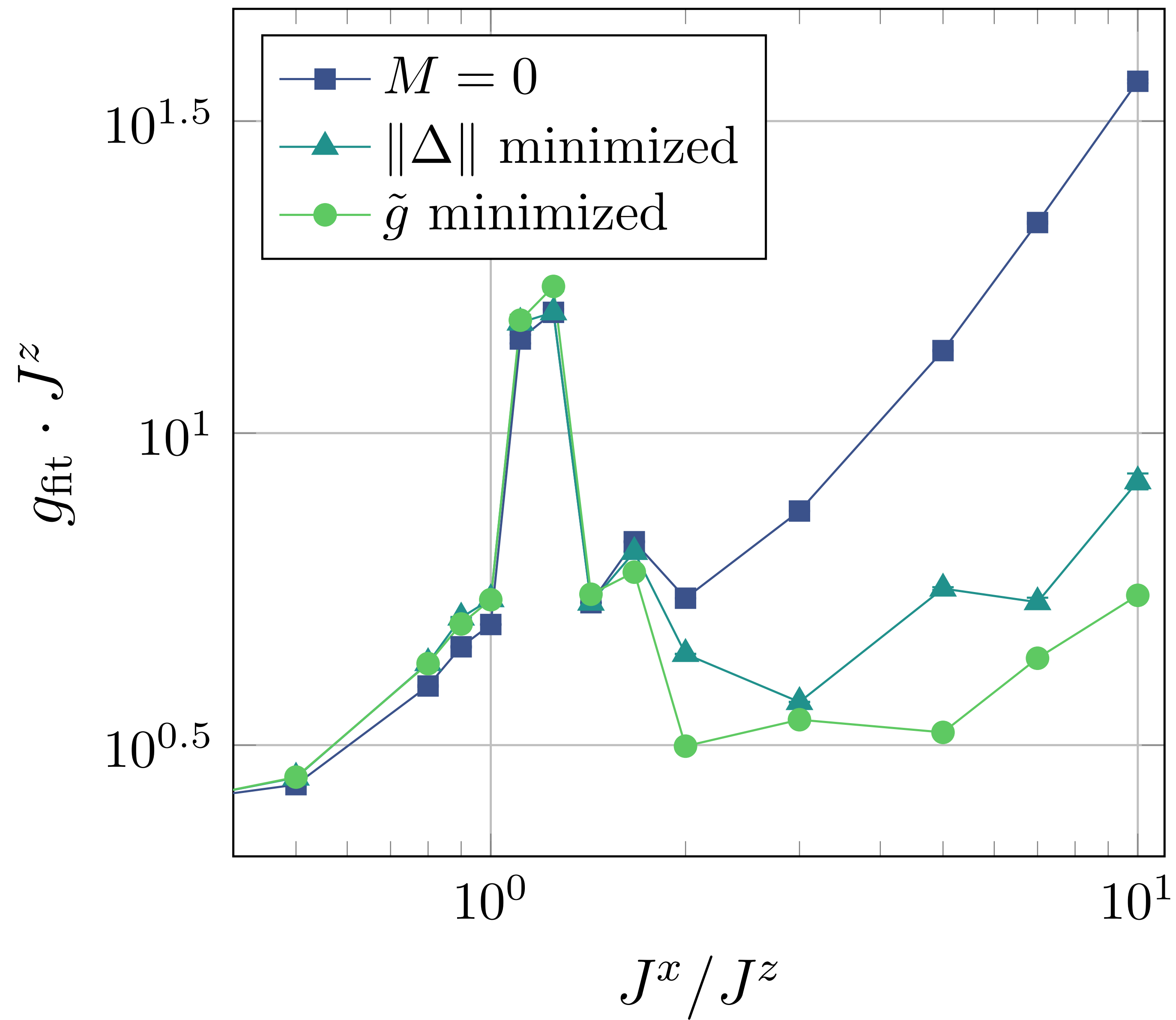
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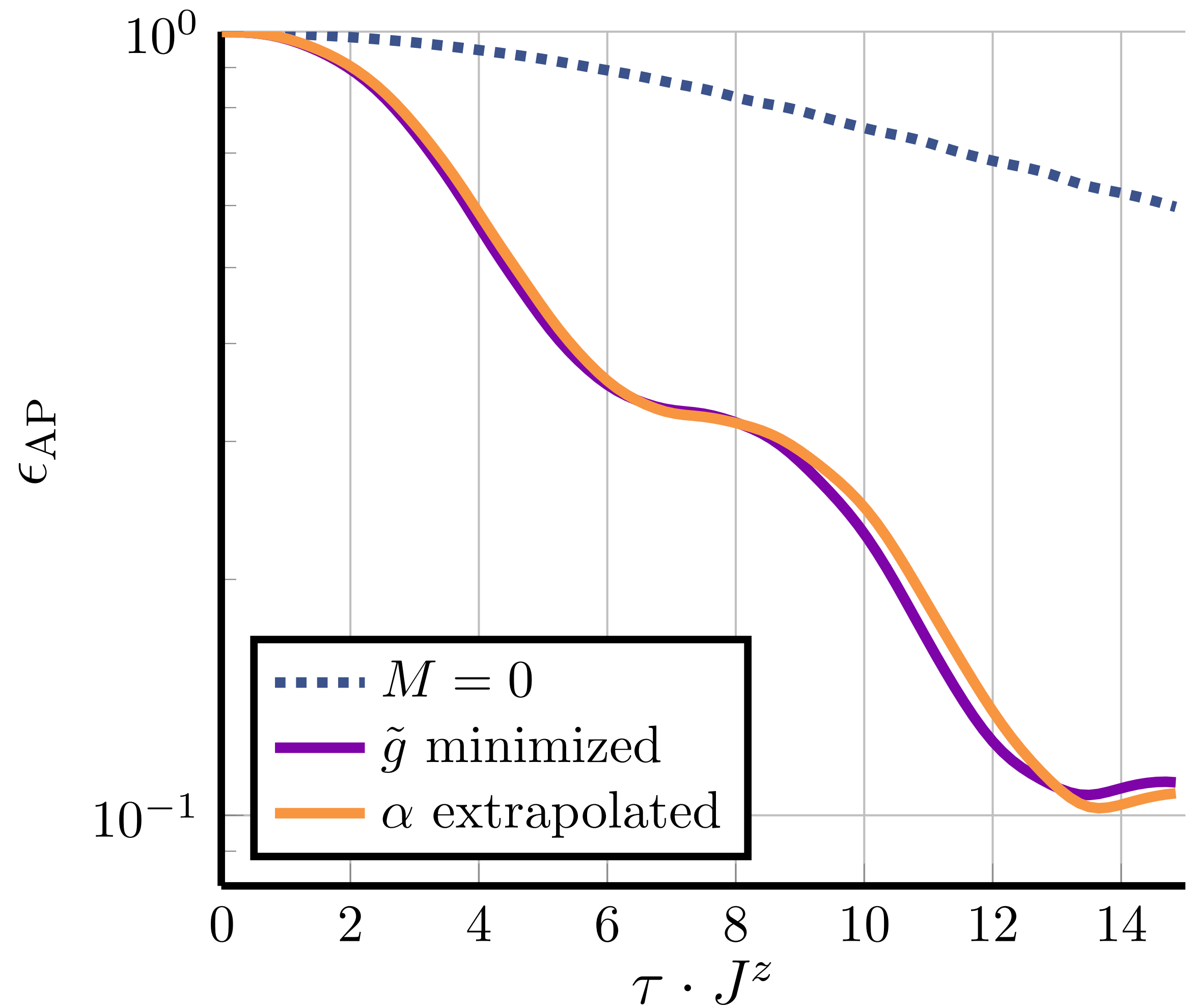
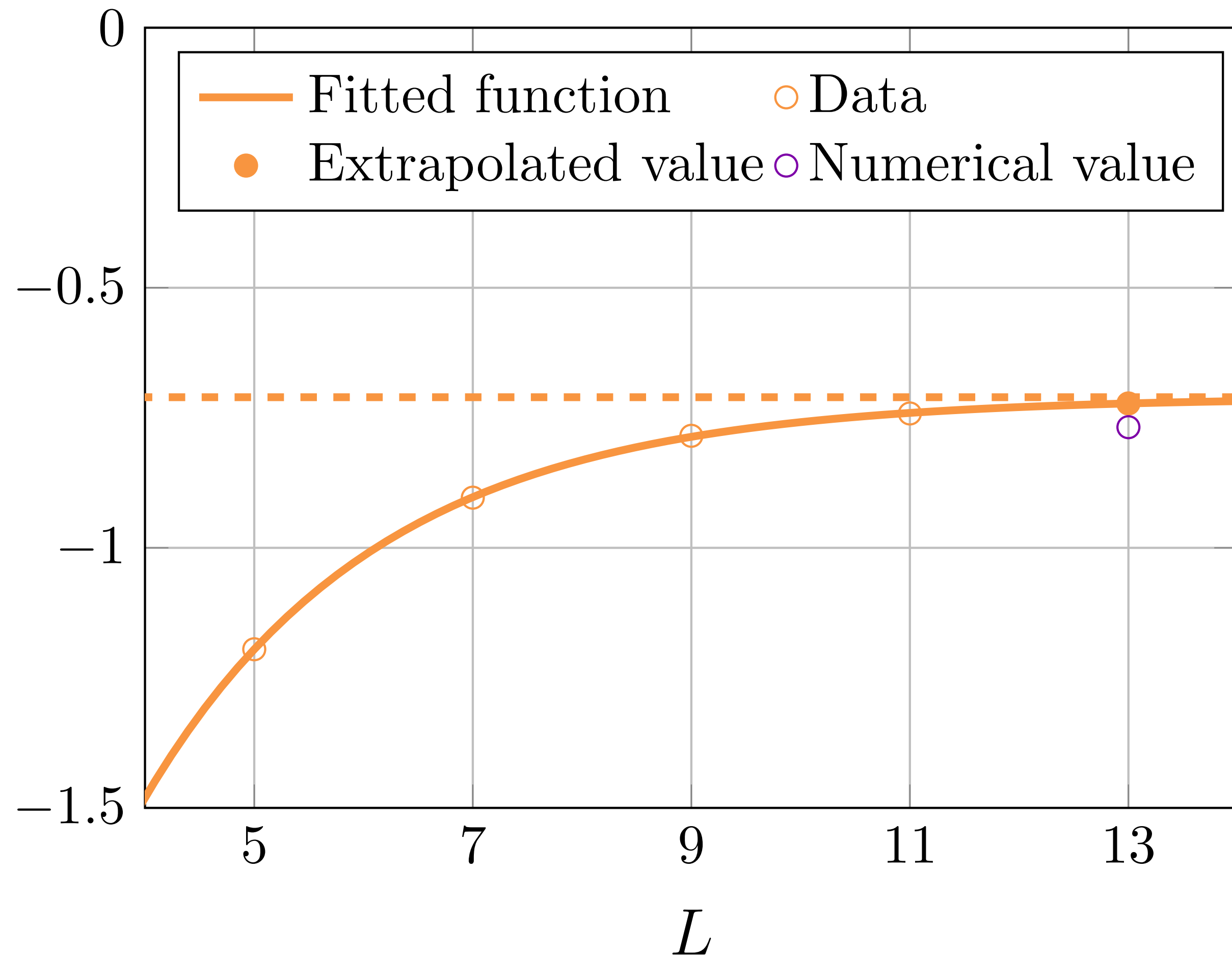
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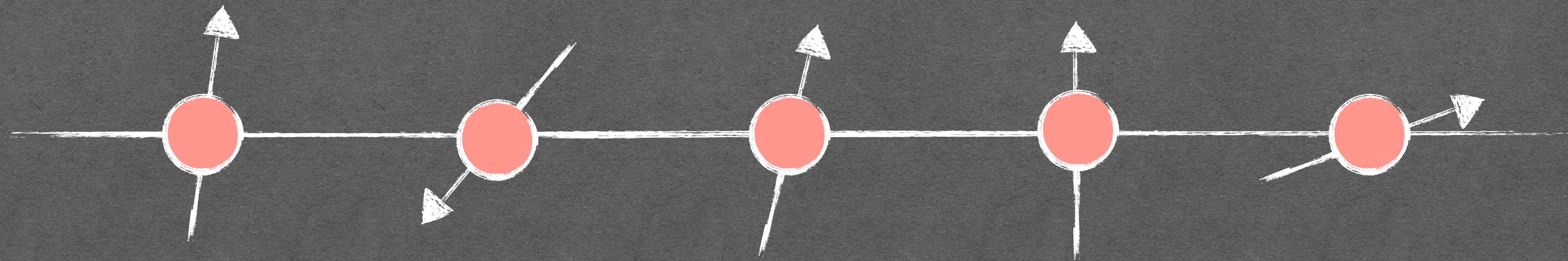




A possible approach: extrapolation



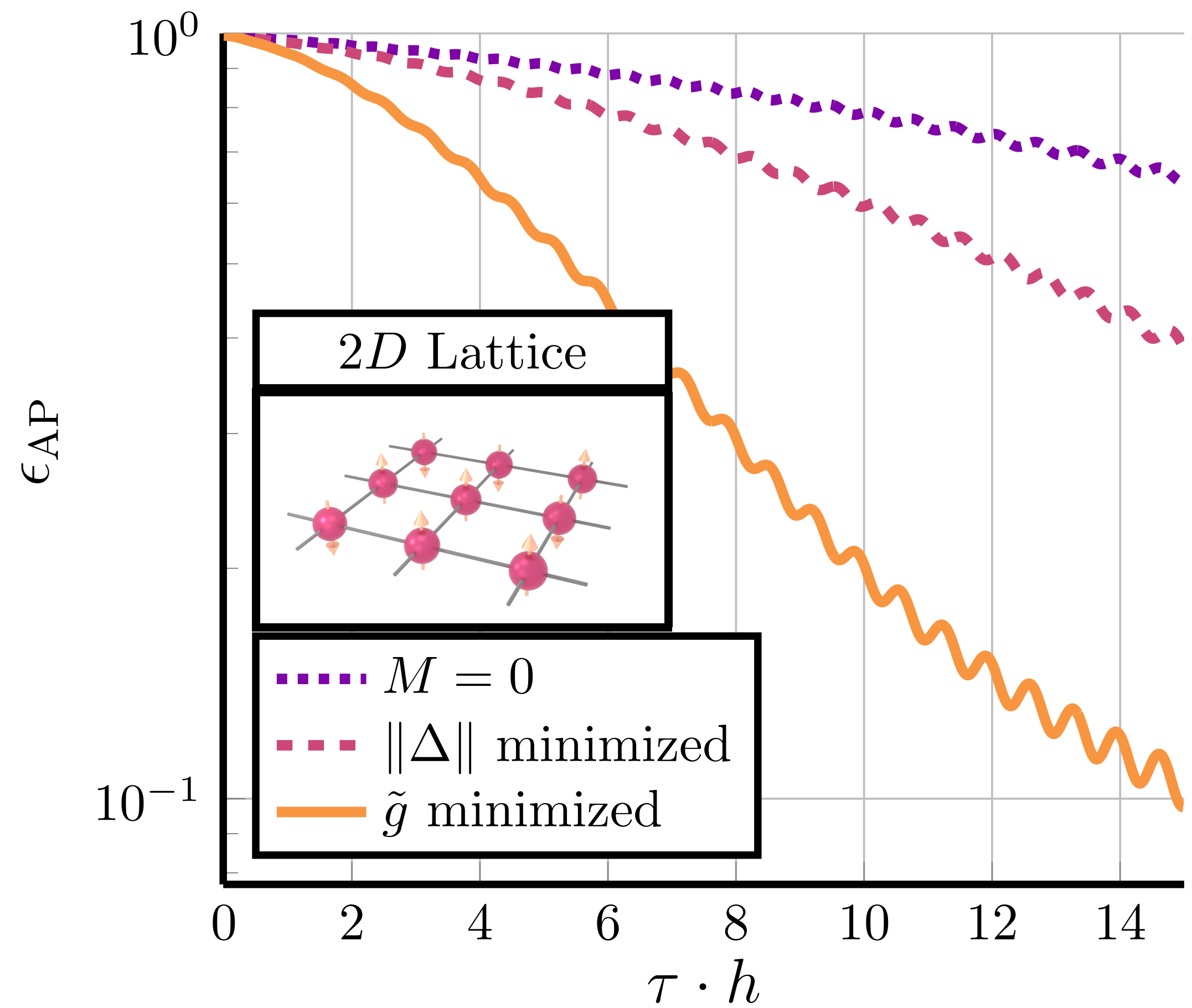
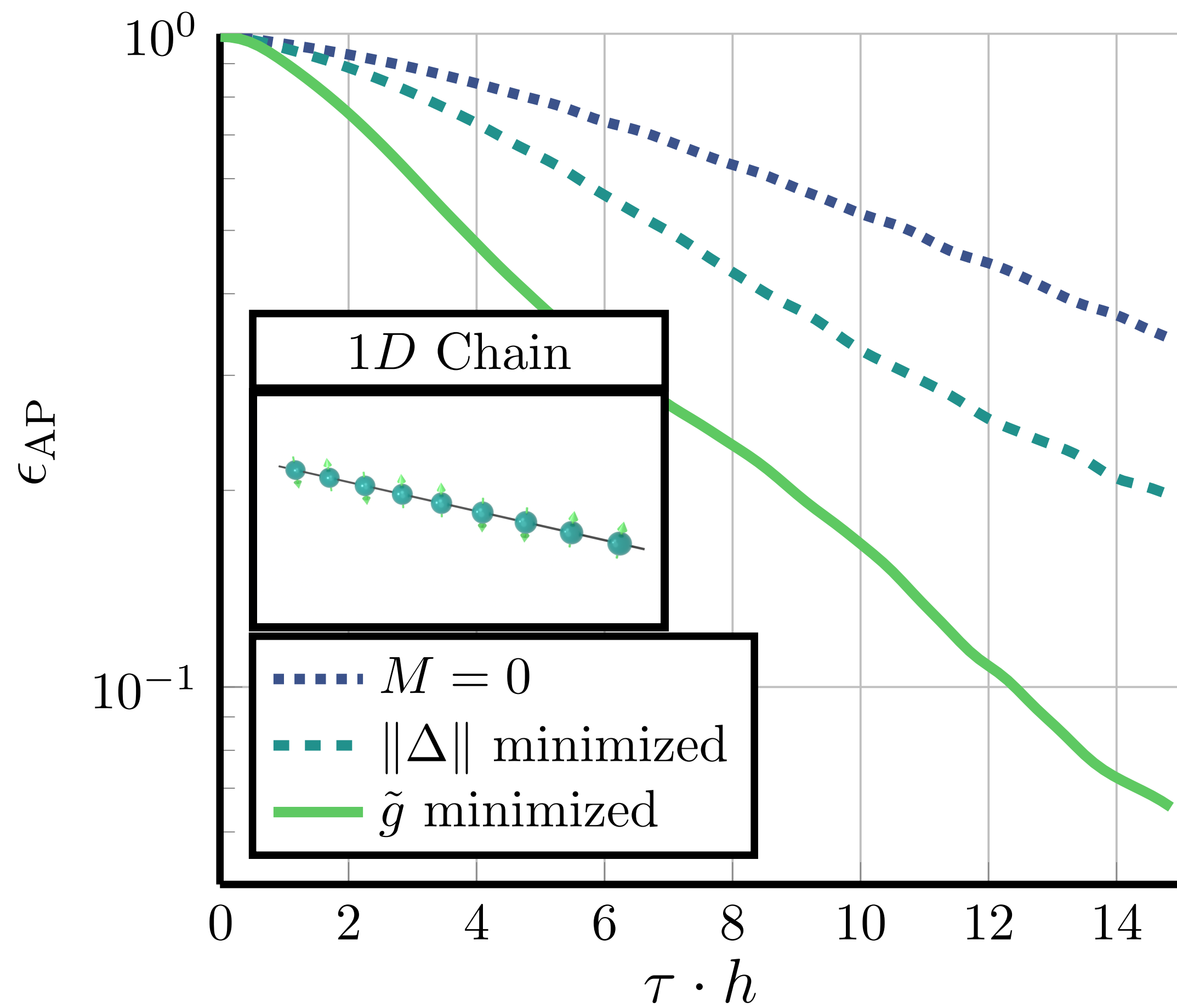
Ising model in a magnetic field

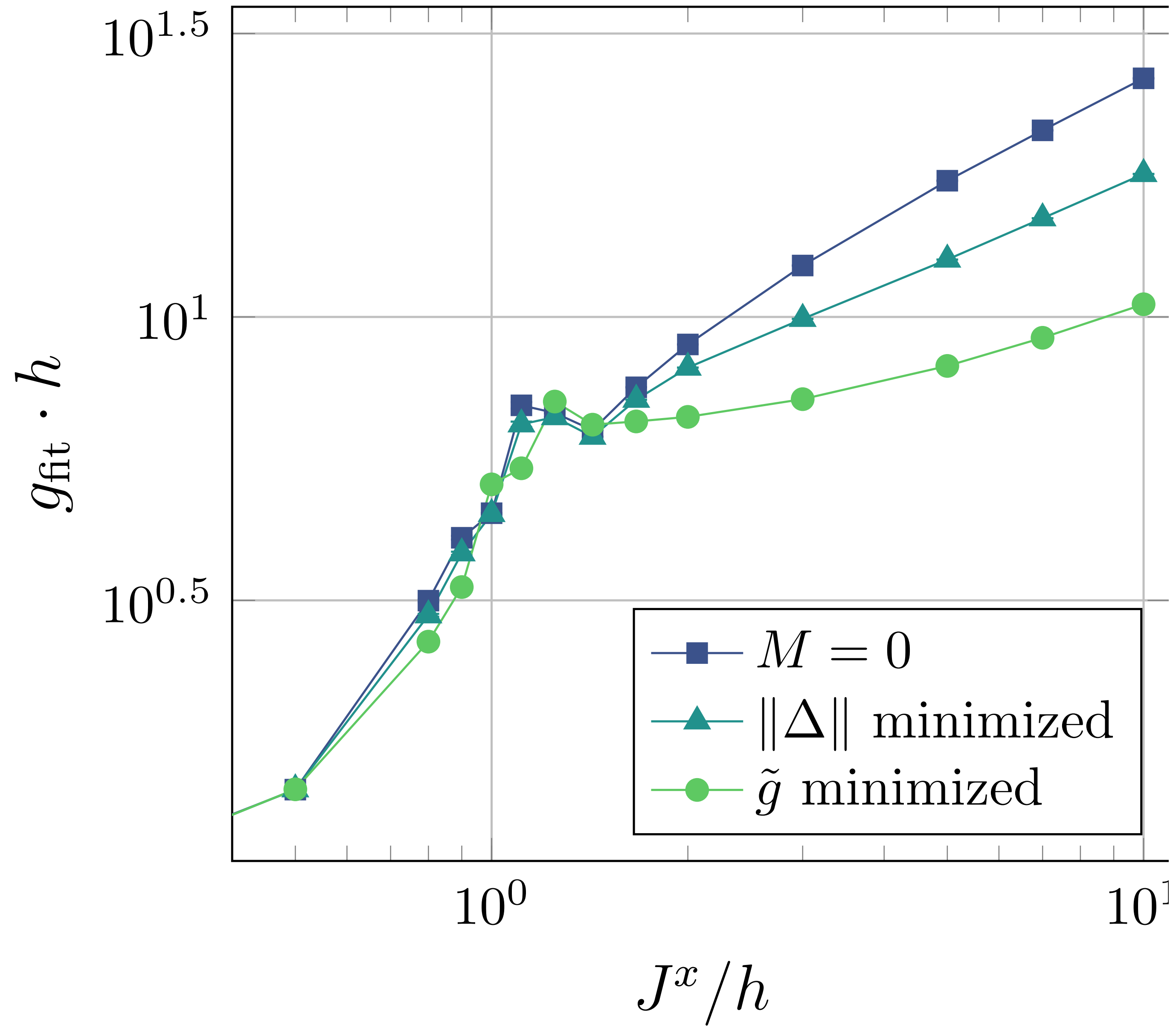


$$H_1 = -\frac{h}{2} \sum_i Z_i - \frac{J^x}{2} \sum_{\langle i,j \rangle} (X_i X_j)$$

$$H_0 = -\frac{h}{2} \sum_i Z_i + \alpha \sum_i (Z_i)$$

$$J^x = 5h$$






Adiabatic Preparation Error

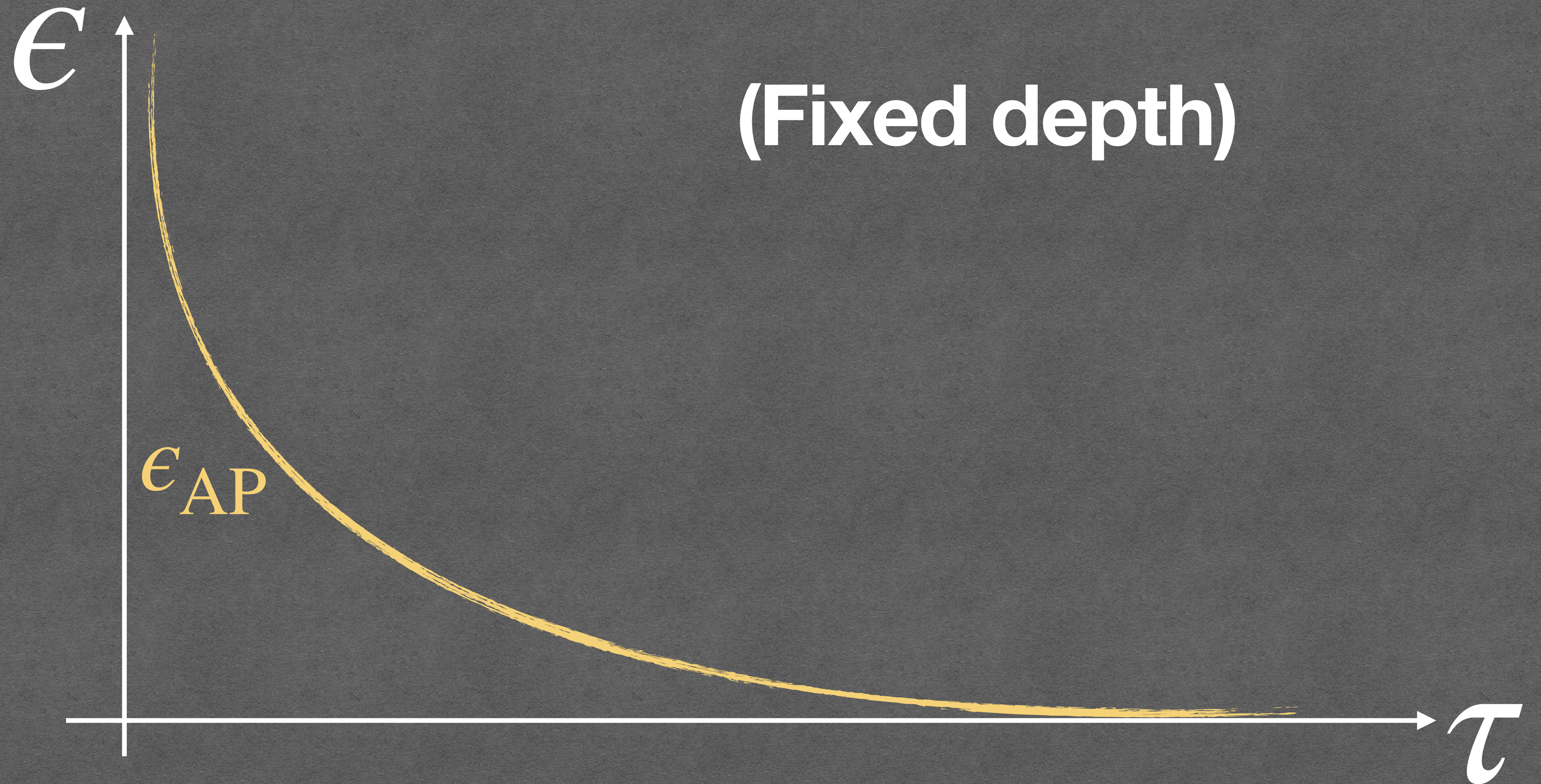
ϵ

(Fixed depth)

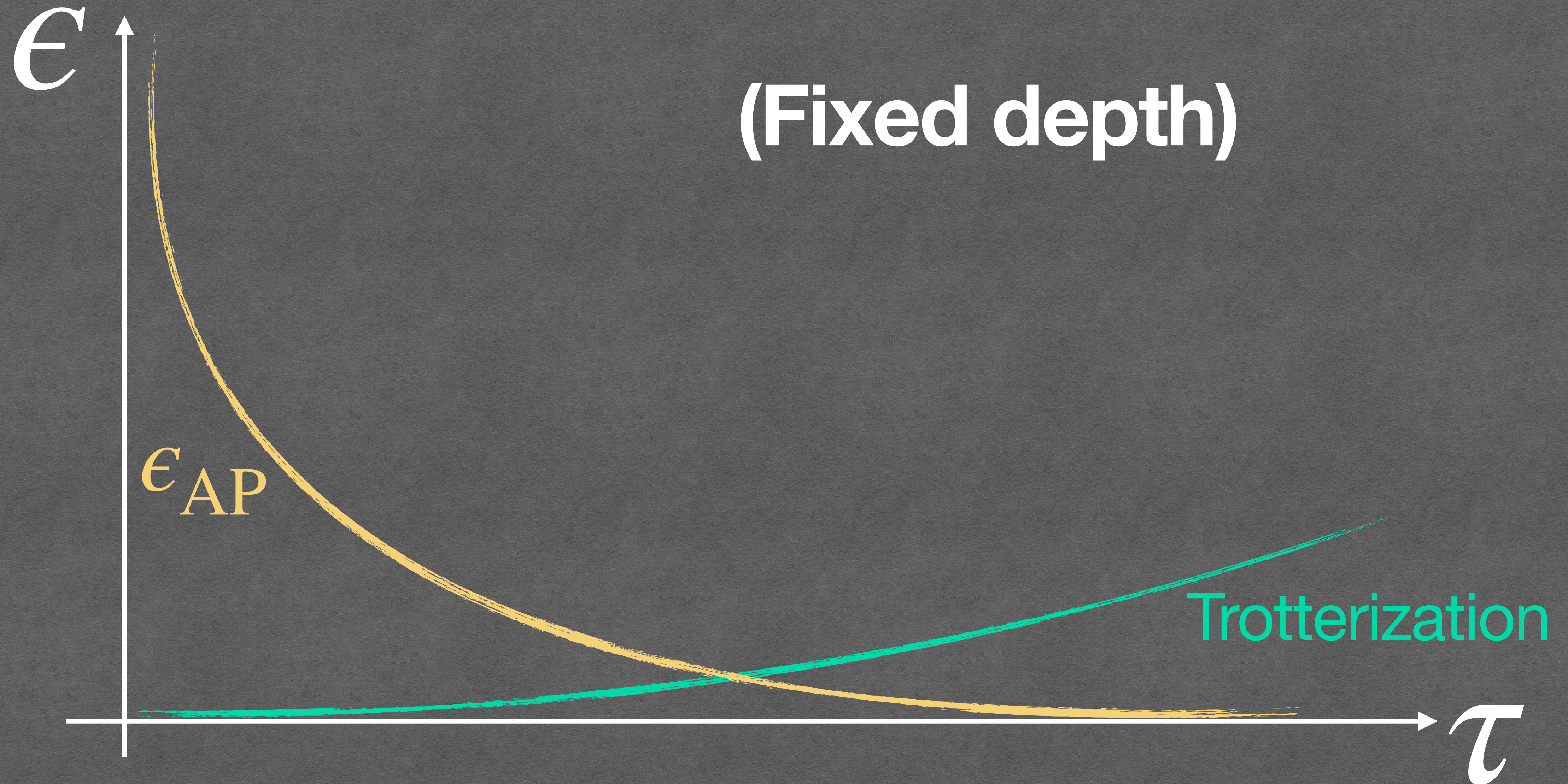
τ



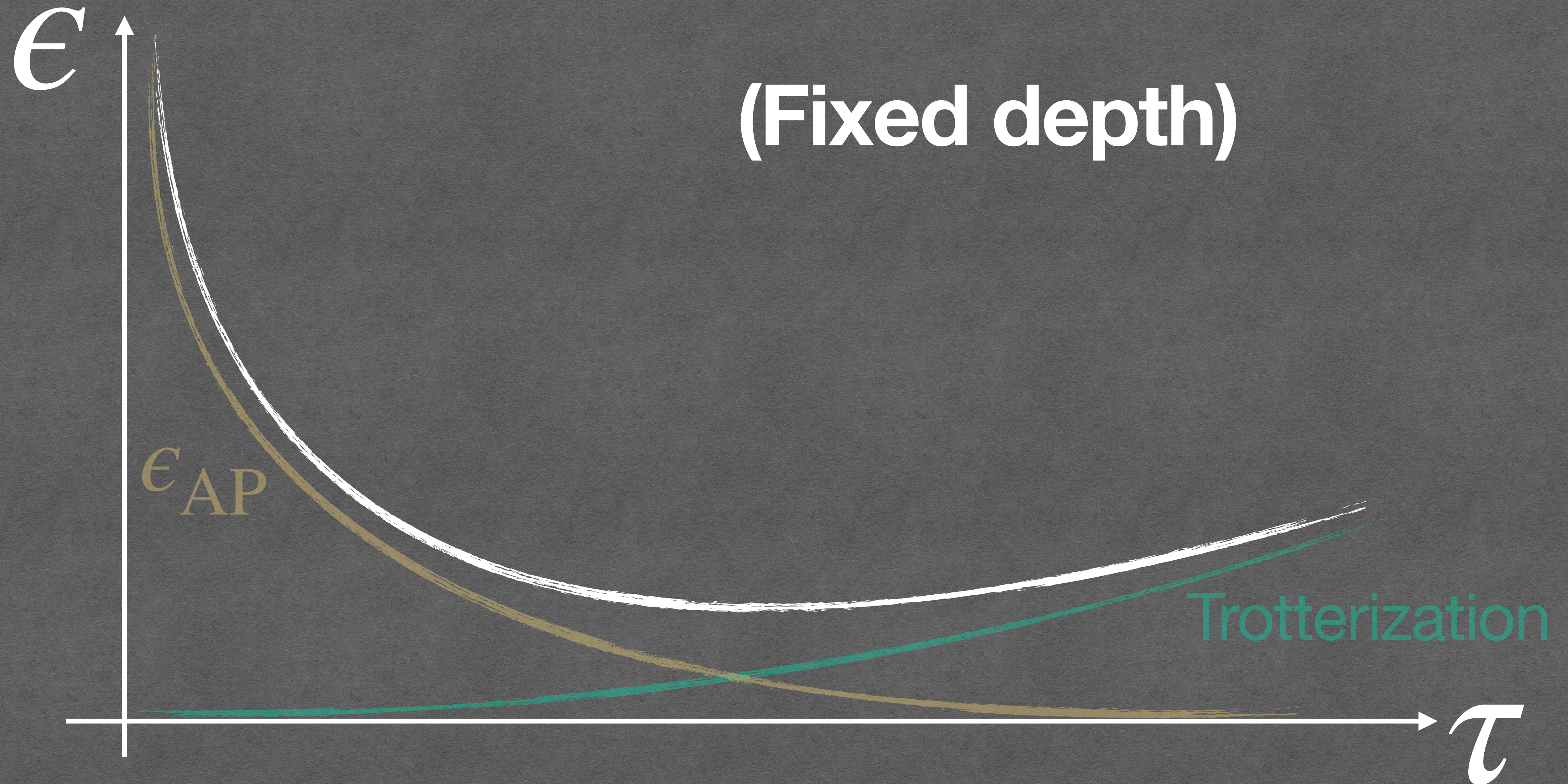
Adiabatic Preparation Error



Adiabatic Preparation Error



Adiabatic Preparation Error



Adiabatic Preparation Error

