Approximate inverse measurement channel for shallow shadows

Lorenzo Piroli
University of Bologna

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Quantum Computing
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based on

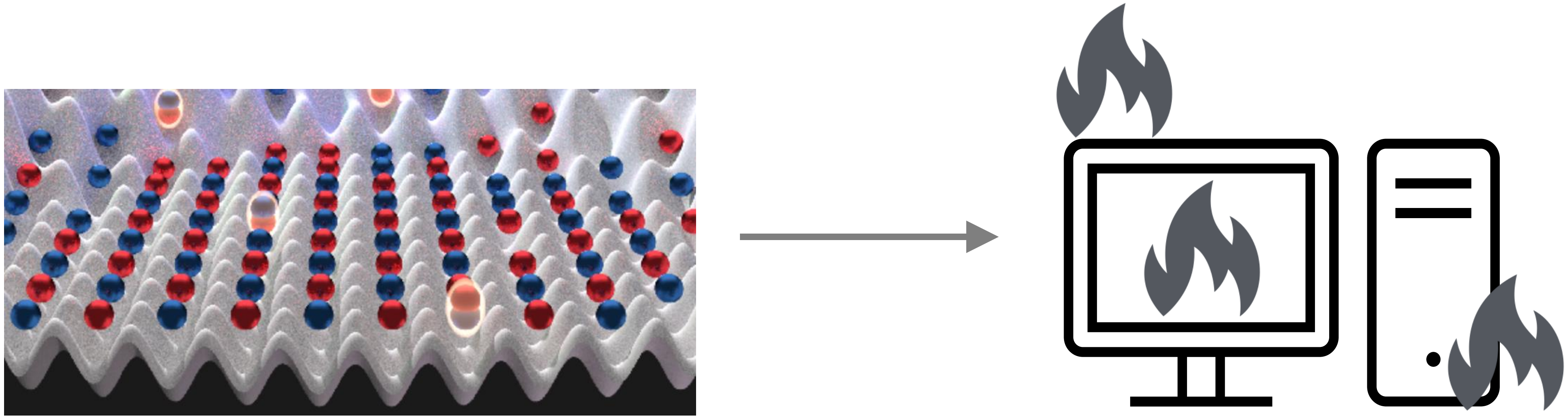
R. Cioli, E. Ercolessi, M. Ippoliti, X. Turkeshi, and **LP**,
Approximate Inverse Measurement Channel for Shallow Shadows,
arXiv:2407.11813.

Riccardo Cioli,
UNIBO

Matteo Ippoliti,
University of Texas

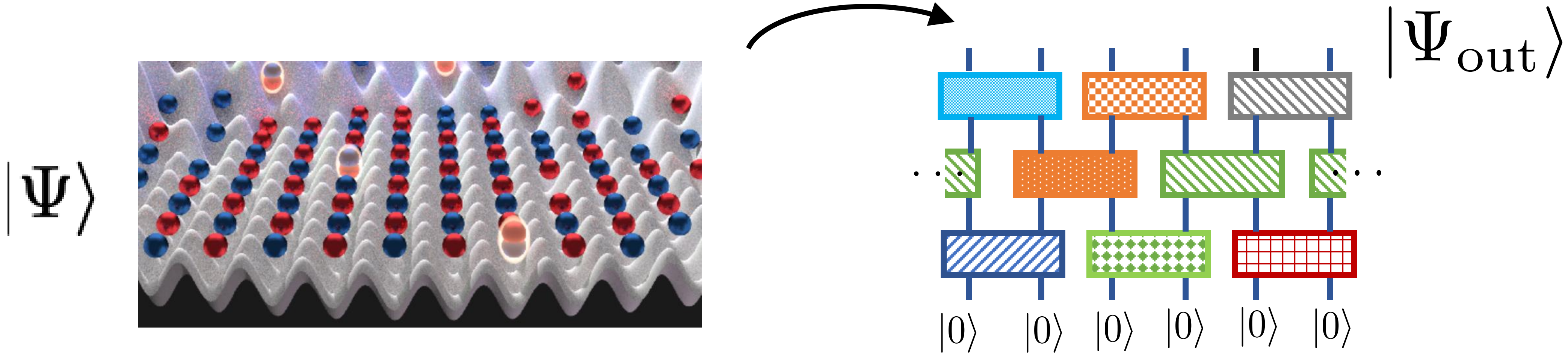
Xhek Turkeshi,
University of Cologne

- Many-body quantum systems are hard to simulate on classical computers



- Large-scale quantum computers would be immediately useful for several tasks, e.g.:
    - Real-time digital quantum simulation
    - Many-body quantum-state preparation

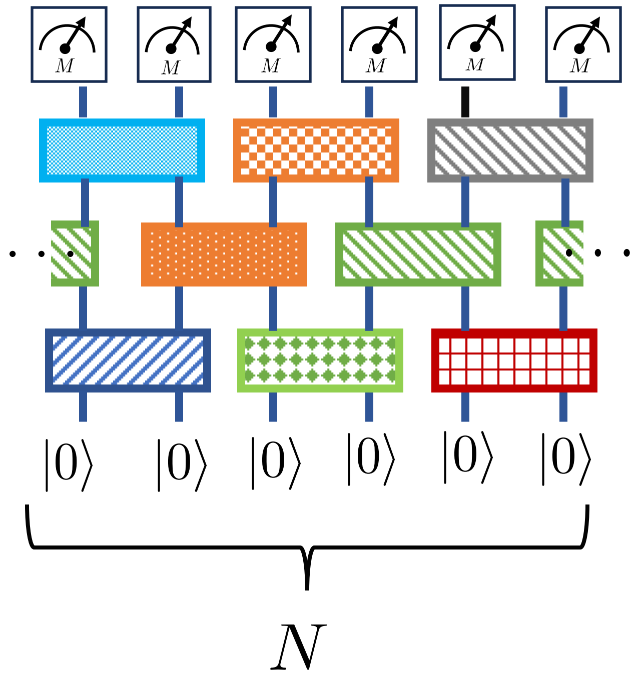
- Example: quantum-state preparation
- Even when wavefunction is known, correlations could be hard to compute



- Target wavefunction encoded in output qubits of quantum computation

Cirac and Zoller, Nature Phys. (2012)

- Some tasks remain hard, even with access to ideal quantum device
- Ex: full quantum-state tomography



$|\Psi_{\text{out}}\rangle$ requires $O(e^N)$ measurements

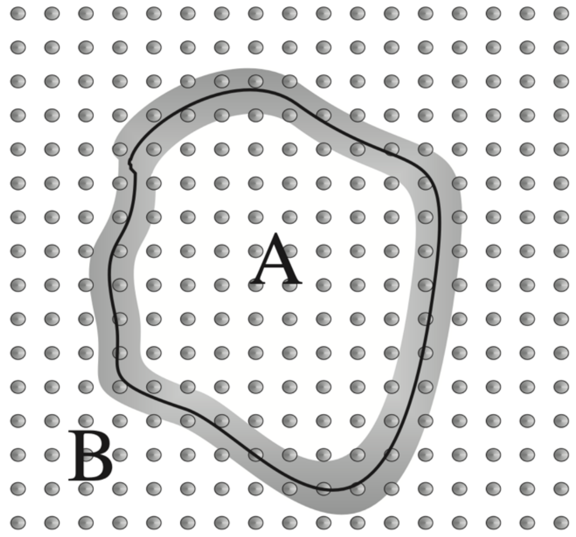
- Full tomography not necessary to probe observables, but in many-body physics we are often interested in more complicated quantities

- Typical quantities of interest beyond simple observables:
  - overlaps and fidelities

$$\mathcal{F} = |\langle \Psi_{\text{target}} | \Psi_{\text{out}} \rangle|^2$$

  - bipartite entanglement entropy of extended systems
  - [...]

Entanglement in many-body physics



$$|\psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

- Define reduced state

$$\rho_A = \mathrm{tr}_B \left[|\psi\rangle\langle\psi| \right]$$

- Rényi and von Neumann entanglement entropies

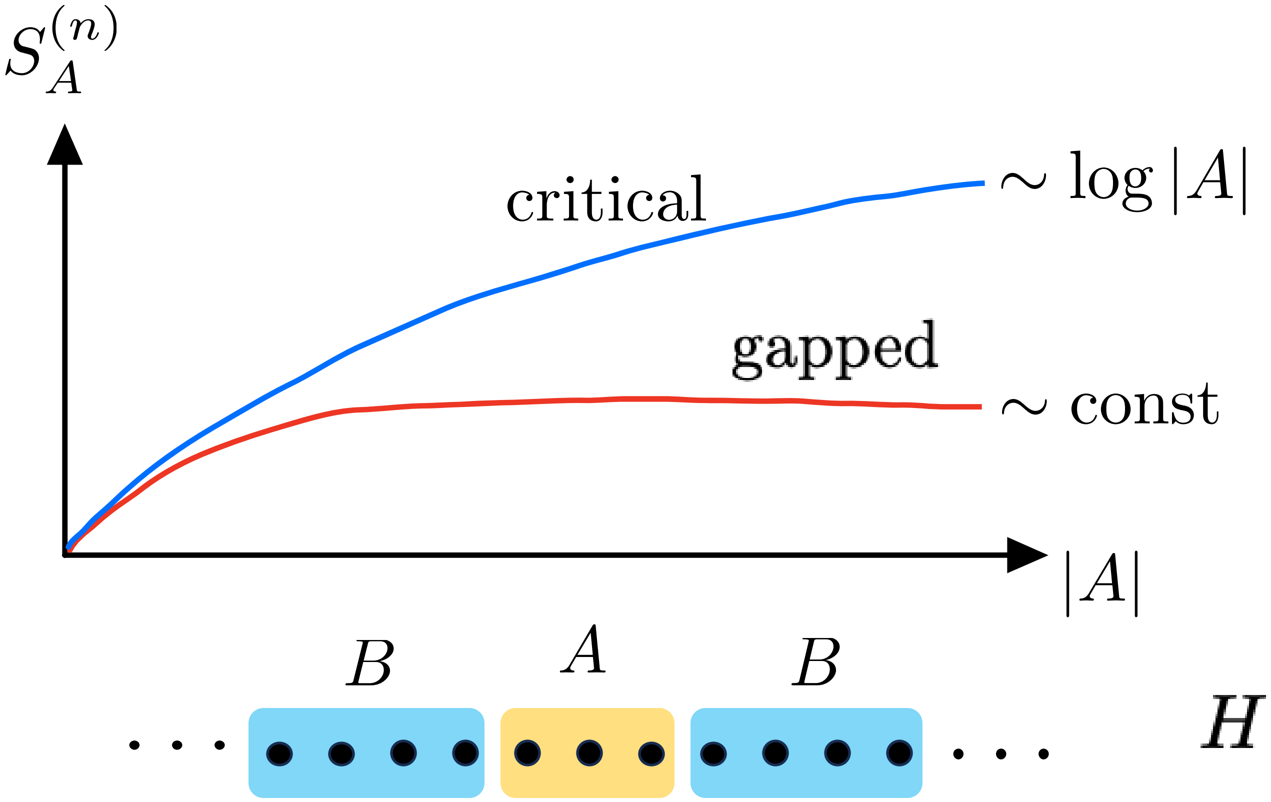
$$S_A^{(n)}[\rho] = \frac{1}{1-n} \log[\mathrm{tr}(\rho^n)] \qquad S_A[\rho] = -\mathrm{tr}\left(\rho \log[\rho]\right)$$

- They answer the question:

  "how far is $|\psi\rangle$ from a (classical) product state"?

Entanglement in many-body physics

- Entanglement entropy important both in and out of equilibrium
- Famous example: diagnose criticality in ground-states of local Hamiltonians



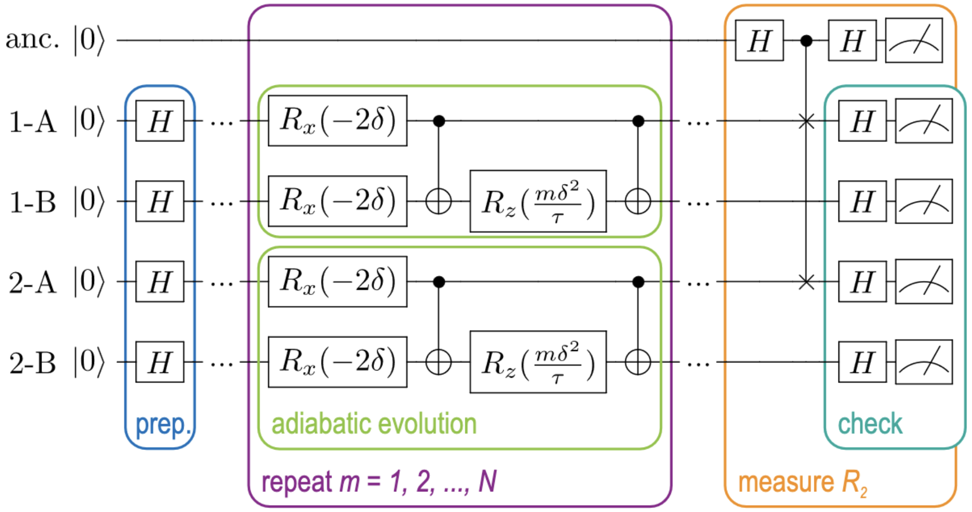
$$H = \sum_j h_{j,j+1}$$

Calabrese and Cardy, J. Stat. Mech. (2004)

- Entanglement entropies are famously hard do measure. Two strategies:

1. Full-state tomography → reconstruct ρ_A & compute $S_A^{(n)} = \frac{1}{1-n}\log[\mathrm{Tr}(\rho_A^n)]$

2. Quantum-interference experiments



Direct measure of purity

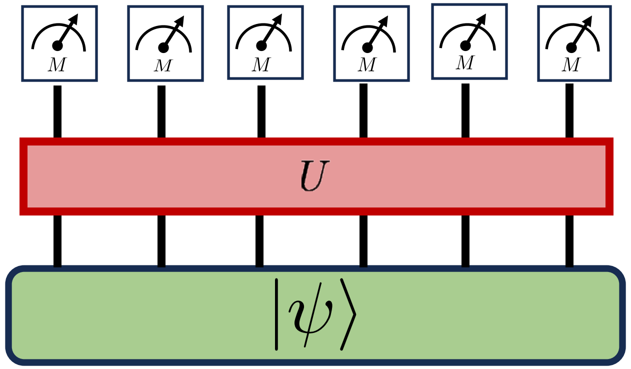
$$\mathrm{Tr}[\rho^2] = P_A(0) - P_A(1)$$

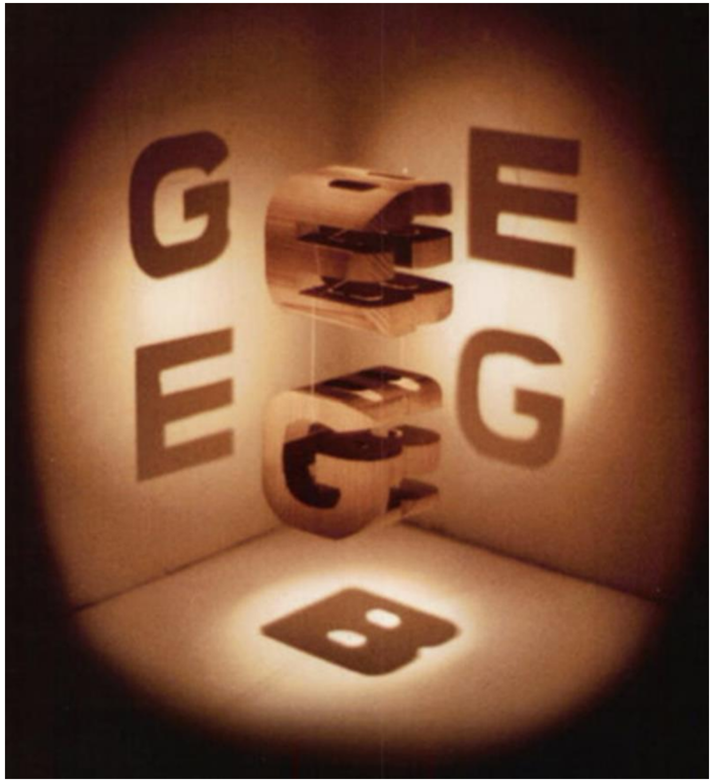
Islam et al. Nature (2015); Kaufman et al. Science (2016)
Linke et al. PRA (2018)

Classical shadows

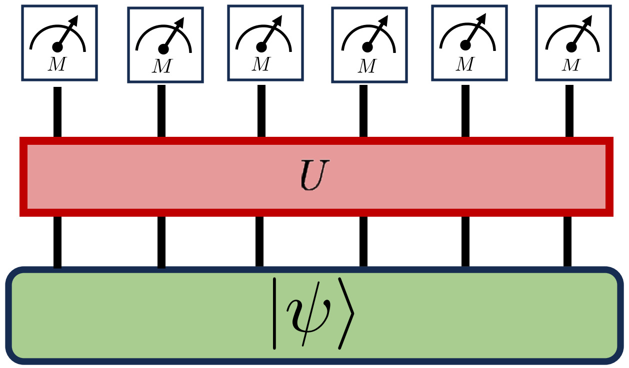
- Collect classical "snapshots" (shadows) from which to reconstruct properties of the quantum state
- "Measure first, ask questions later"
- Naturally tailored for digital devices
- Suitable to probe both observables, entanglement-related quantities & more

The randomized measurement toolbox, Elben *et al.* Nature Rev. Phys. (2022)





The recipe



1. Apply random unitary U
2. Perform on-site projective measurements with outcomes
3. Repeat runs $\mathrm{r} = 1, \ldots, M$, and classically store $\{b_j^{(r)}\},\ U^{(r)}$
4. Define *classical shadows*

$$\rho^{(r)} = (2^N + 1) U^{(r)} \left(\otimes_j |b_j^{(r)}\rangle\langle b_j^{(r)}| \right) U^{(r)\dagger} - \mathbf{1}$$

- Claim:

$$\lim_{M\to\infty} \frac{1}{M} \sum_r \rho^{(r)} = \rho$$

- From M measurements (shadows), construct desired estimators

Fidelity → $$\mathcal{F}^{(e)} = \frac{1}{M} \sum_{r=1}^{M} \langle \Psi_{\text{target}} | \rho^{(r)} | \Psi_{\text{target}} \rangle$$

Purity → $$\mathcal{P}^{(e)} = \frac{1}{M(M-1)} \sum_{r \neq r'} \mathrm{Tr}\left(\rho^{(r)} \rho^{(r')}\right)$$

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- The efficiency of the method controlled by the estimator variances

$$\mathrm{Var}\left[\mathcal{F}^{(e)}\right] \sim O(1) \qquad \mathrm{Var}\left[\mathcal{P}^{(e)}\right] \sim \frac{2^{2N}}{(M-1)^2}$$

- Exponential improvement over traditional tomographic methods

- Unfortunately, hard to realize random global unitaries in current NISQ era

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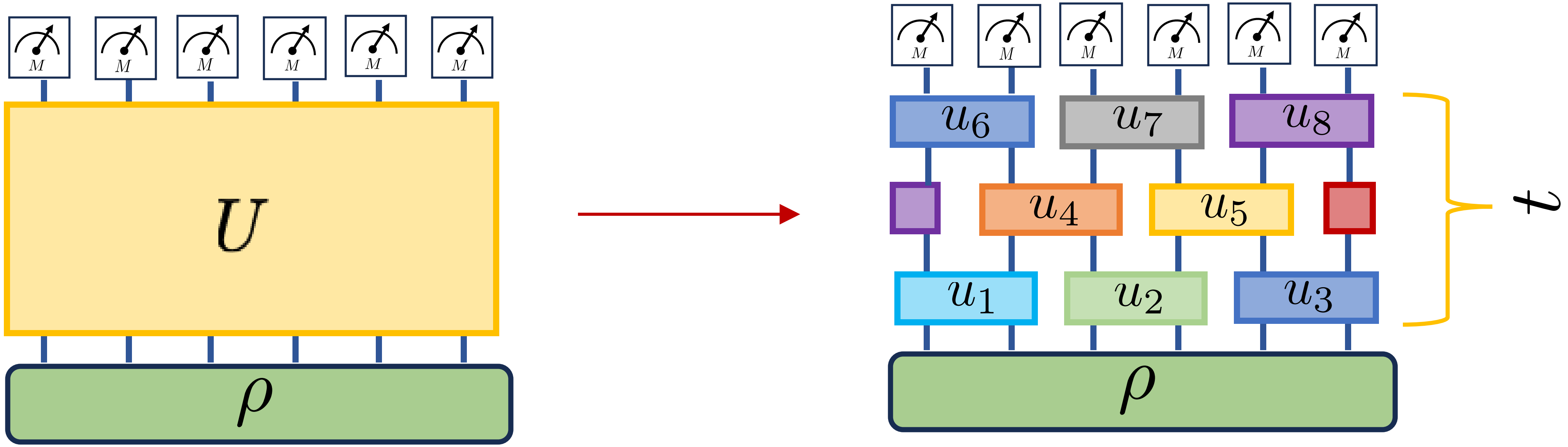
- Our work: replace global unitary with shallow random quantum circuits



Bertoni et al. PRL 2024; Akhtar et al. Quantum 2023
Arienzo et al. QIC 2023

- Challenges:
  - bound depth of the circuit (quantum computations should be fast)
  - bound the variance of observables (avoid too many measurements)
  - bound postprocessing computational cost

- We propose simple postprocessing scheme of shadows to solve the problem

- Showing that the suggested protocol works requires answering questions on the behavior of random quantum circuits

- Main result:
  to any error δ, efficient measurement scheme is achieved by

$$D \sim \log(N/\delta) \quad \text{(fidelity)}$$

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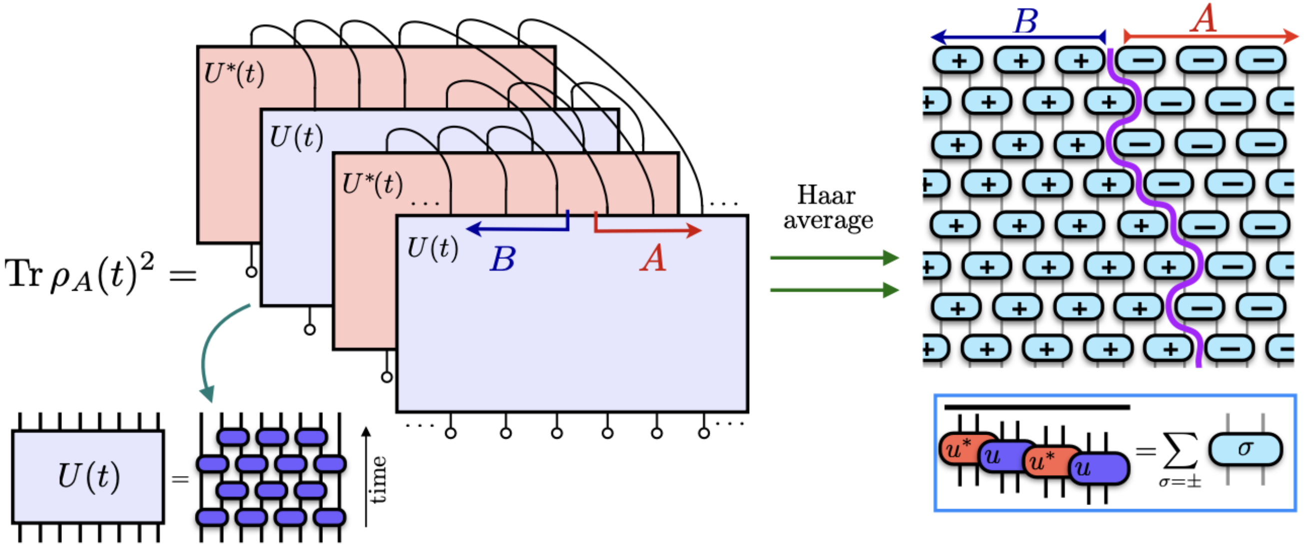
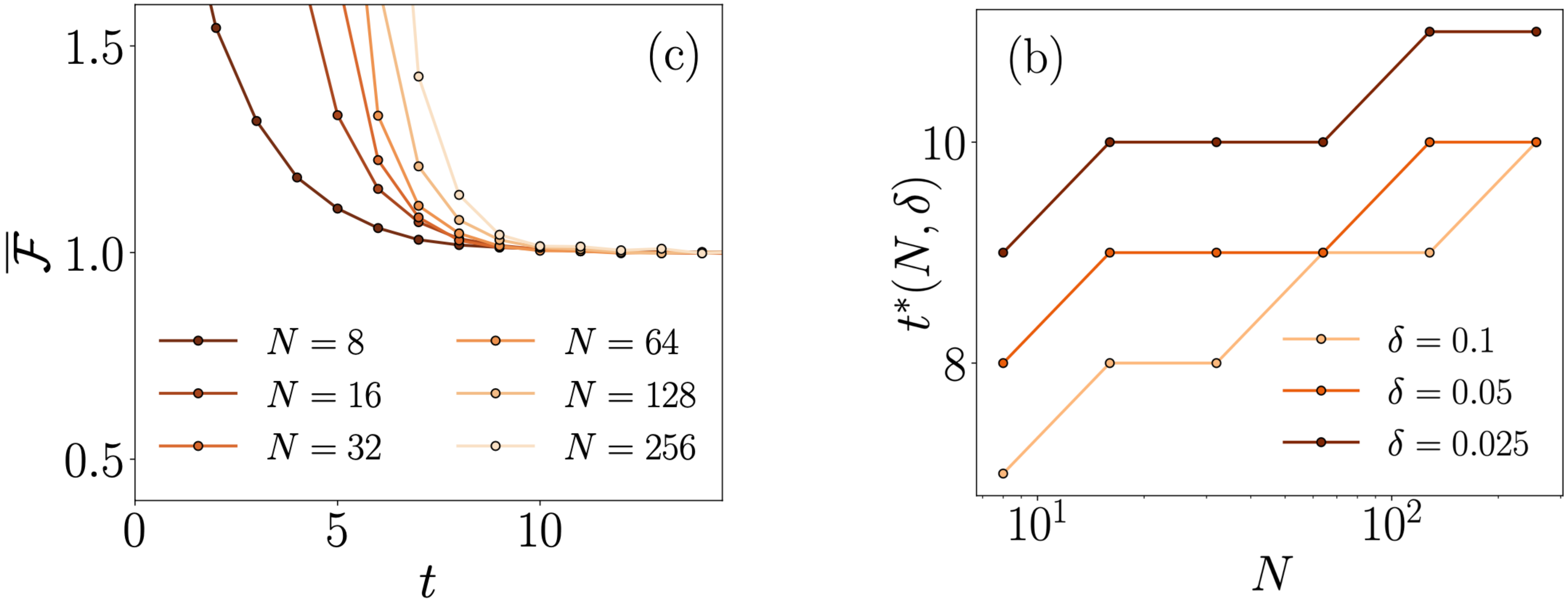
- Results derived studying " statistical mechanics" of random quantum circuits

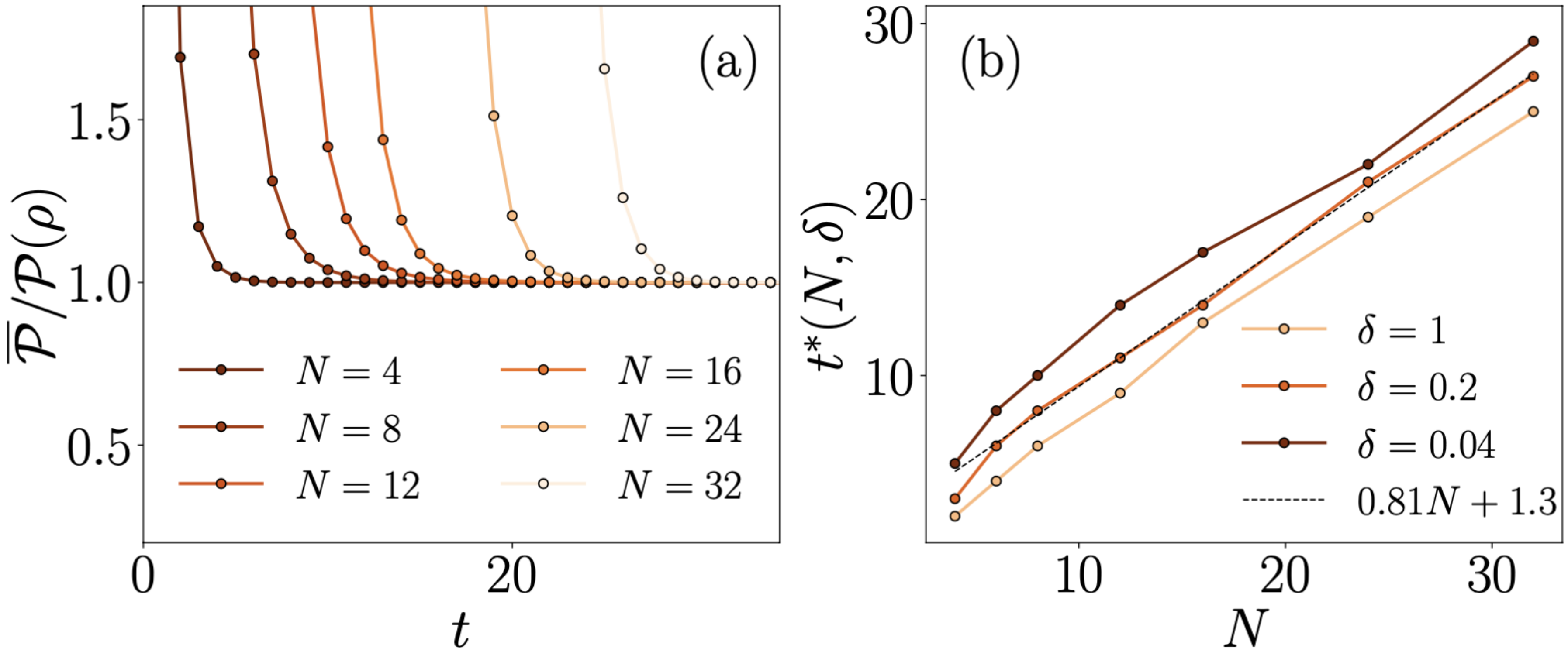


Fisher et al., Ann. Rev. (2023)
Dalzell et al. PRX Quantum (2022)



$$|\Psi_{\text{out}}\rangle = |\Psi_{\text{target}}\rangle = |\text{GHZ}\rangle$$

$$|\text{GHZ}\rangle = (|0\cdots 0\rangle + |1\cdots 1\rangle)/\sqrt{2}$$



$$\rho = \bigotimes_{j=1}^{N} \rho_j \qquad \rho_j = \cos^2(\mu)|0\rangle\langle 0| + \sin^2(\mu)|1\rangle\langle 1|,$$

Outlook

- Our work exploits and contributes to recent developments of statistical mechanics ideas in quantum information/computation

- Much room for improvements: our protocol could be combined with other schemes to speed up entanglement detection in many-body systems, see e.g.

  Vermersch, Ljubotina, Cirac, Zoller, Serbyn, **LP**,
  Many-Body Entropies and Entanglement from Polynomially Many Local Measurements, PRX (2024)

- To do: systematic study of noise and decoherence, and error mitigation strategies

Thank you for your attention!