Quantum simulation of strongly-correlated vortex phases with atoms in optical lattices

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M. Di Liberto and N. Goldman, Phys. Rev. Research 5, 023064 (2023)

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Quantum Simulation with atoms

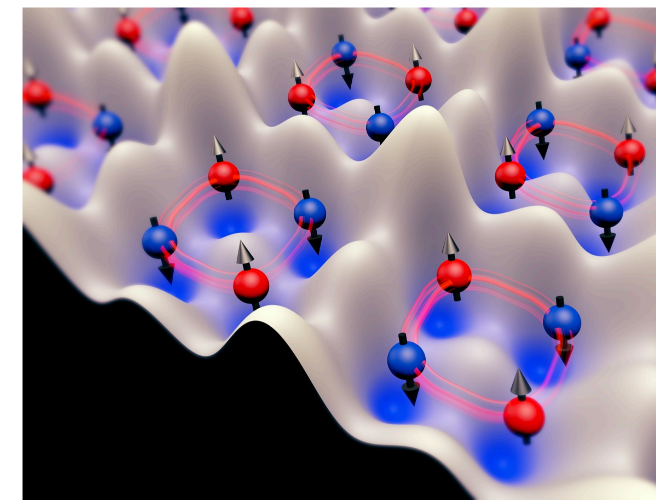
Engineered quantum systems (ultra cold atoms, Rydberg atoms, trapped ions) → Simulate quantum many-body models and phases

Feynman 1982; Lloyd Science 1996

Analog quantum simulation
Encode Hilbert space of $\hat{H}$
on atomic degrees of freedom

Fermi-Hubbard model

$$\hat{H} = -J \sum_{\langle ij \rangle, \sigma} \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$



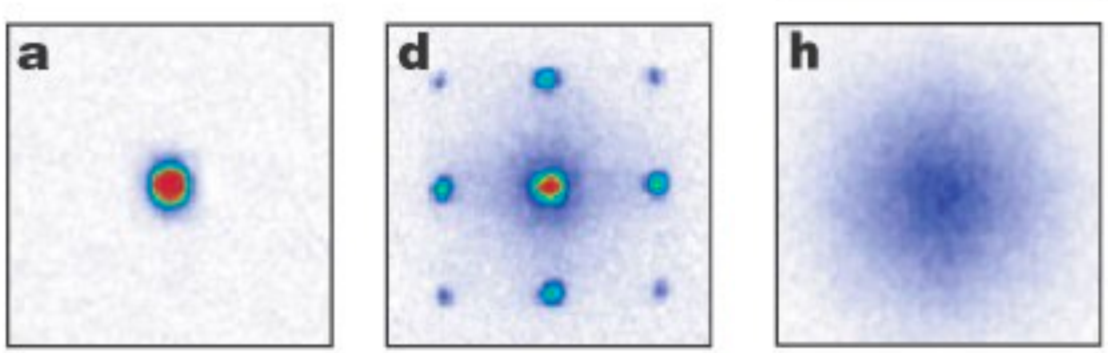
Bose-Hubbard model

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{b}^{\dagger}_{i} \hat{b}_{j} + \frac{U}{2} \sum_{i} \hat{n}_{i}(\hat{n}_{i} - 1)$$

Quantum spin model

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{\sigma}^{z}_{i} \hat{\sigma}^{z}_{j} + h \sum_{i} \hat{\sigma}^{x}_{i}$$

Jaksch PRL 1999
Greiner Nature 2002



Superfluid

$J \gg U$

Mott insulator

$U \gg J$

Achievements

- Strongly-correlated dynamics
- SU(N) models
- Topological phases
- Lattice gauge theories
- Synthetic dimensions
- Quantum magnetism
- Dipolar quantum phases

....

Breaking time-reversal symmetry $\mathscr{T}$

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Breaking $\mathscr{T}$ opens the way to states of matter like Integer and Fractional **Quantum Hall** states for **charged particles** → Atoms are **neutral** (no charge). How to simulate the effect of coupling to gauge fields?

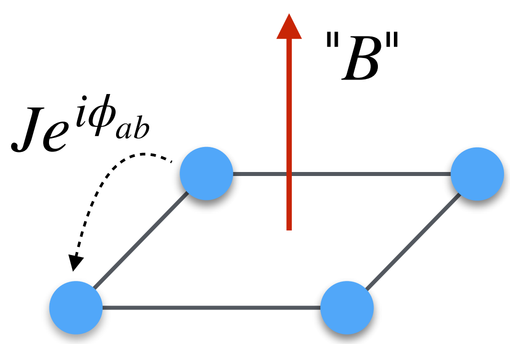
Jaksch Zoller NJP 2003 Celi PRL2014

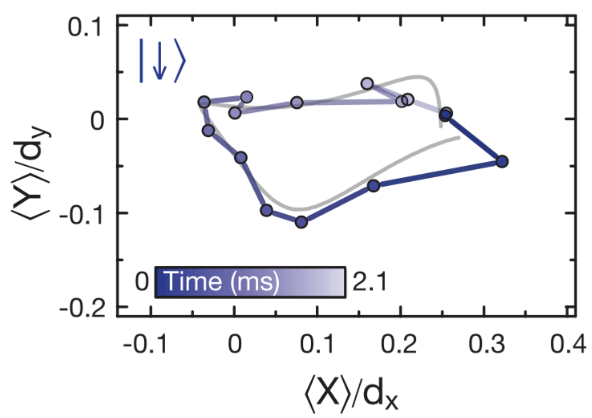
- Synthetic magnetic field

  Complex hopping ↔ Aharonov-Bohm effect

$$\phi_{ab} = \frac{q}{\hbar} \int_a^b \mathbf{A} \cdot d\mathbf{r}$$

Aidelsburger PRL 2013
Fallani Science 2015

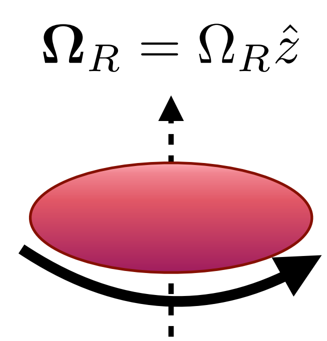


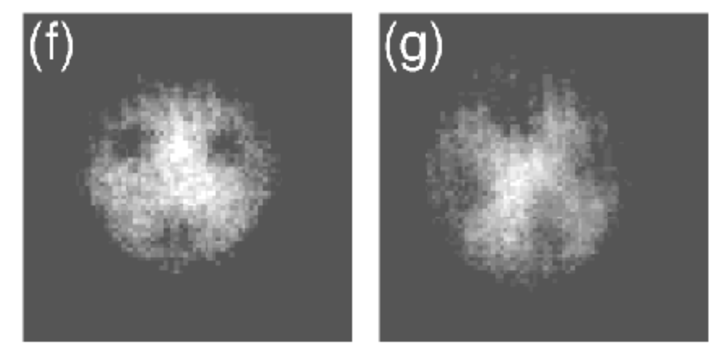


- Rotation

  Coriolis ↔ Lorentz force

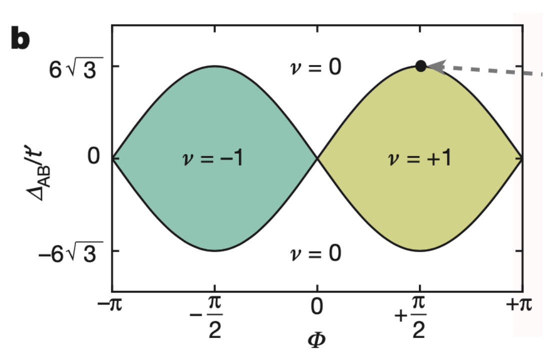
Dalibard PRL 2000

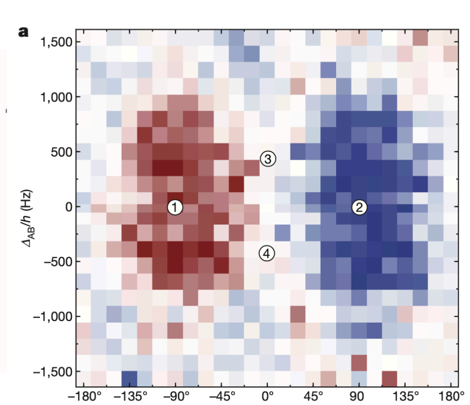




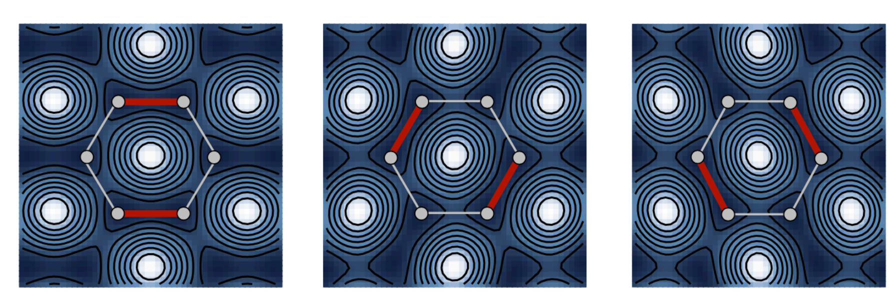
- Circular/chiral lattice shaking

  Haldane model
  Esslinger Nature 2014





Anomalous Floquet topological insulator
Aidelsburger Nat. Phys. 2020



Interactions and $\mathscr{T}$ breaking

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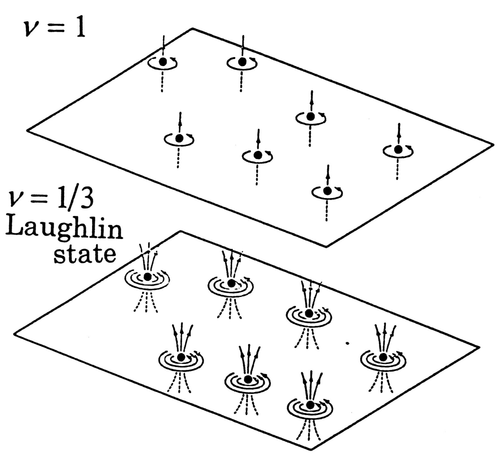
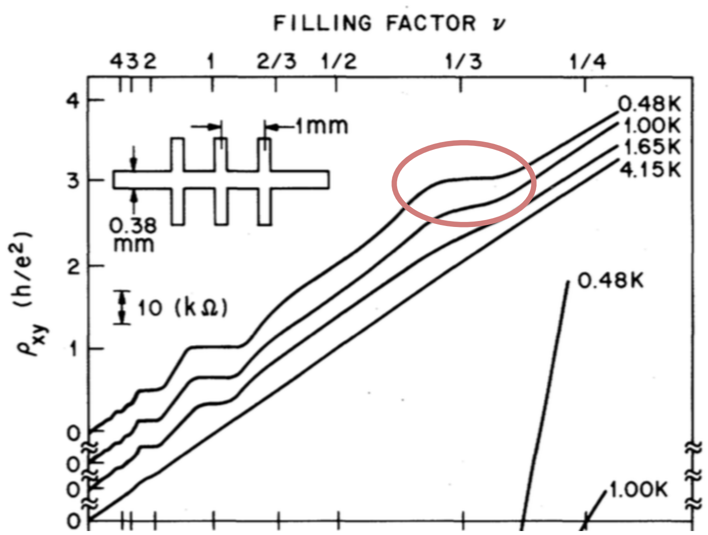
- Fractional Quantum Hall effect (with applied $\vec{B}$)

Störmer, Tsui, Gossard 1982
Laughlin PRL 1983

Anyons excitations with fractional statistics → **Braiding** for quantum computing

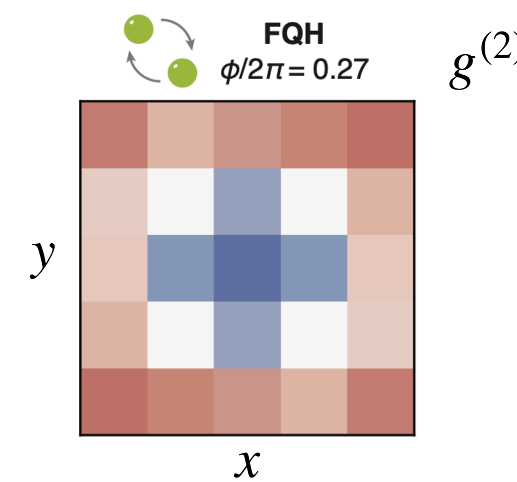




Simon Nature 2020
Greiner Nature 2023

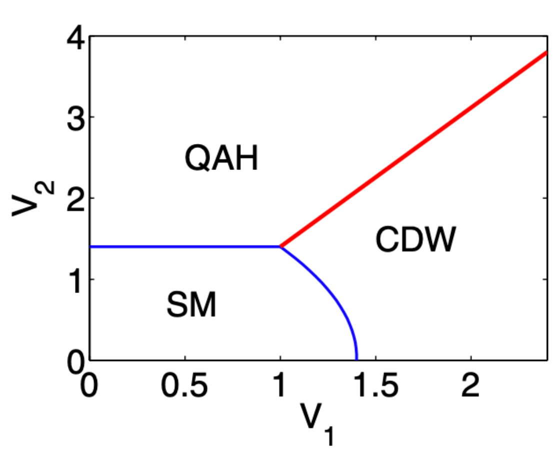
Laughlin states of few particles

Measured two-particle vortex structure
↓
Particle bound to vortex

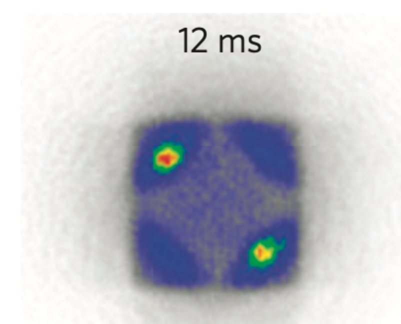


- Spontaneous $\mathscr{T}$ breaking through interactions

Topological Mott insulator
Raghu PRL 2008
Rachel Rep. Prog. Phys. 2018

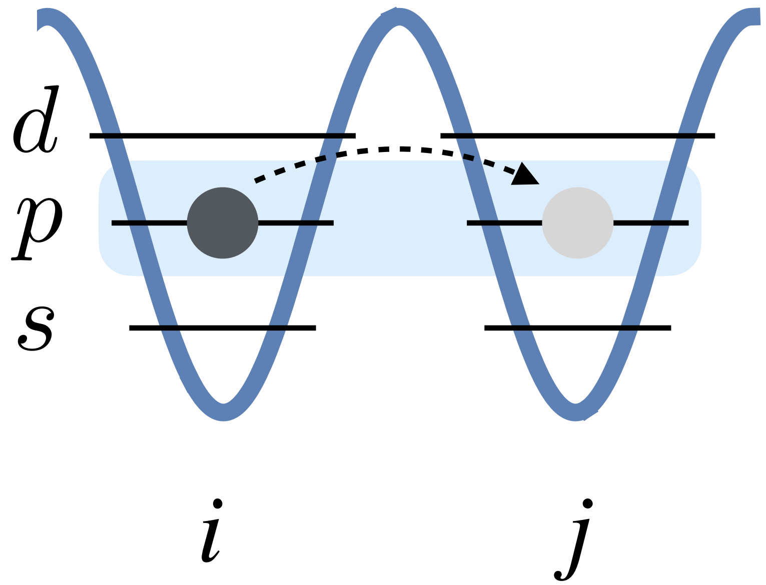


Higher bands
Li and Wu PRA 2006
Hemmerich Nat. Phys. 2012



$p_x + ip_y$ BEC

Intro: bosons in P bands



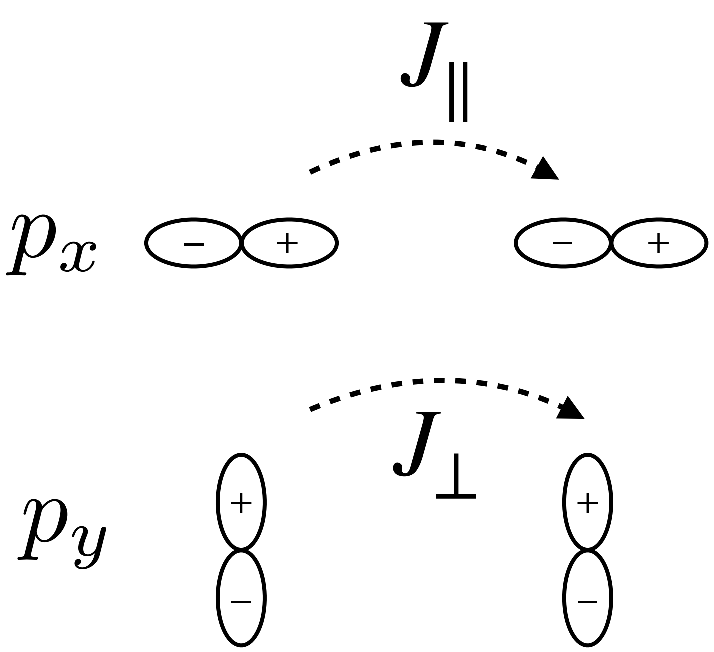
P orbitals (in 2D)

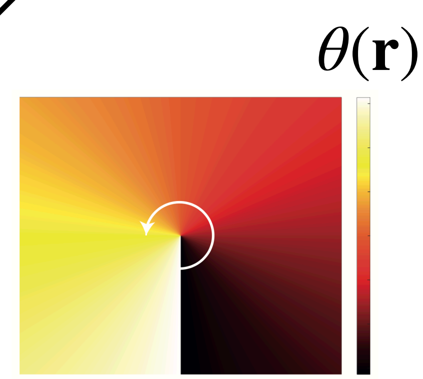
$p_x\,,\; p_y$

- Orbital degeneracy
- Wavefunctions have nodes

Tight-binding model

$$H = H_{\text{hop}} + H_{\text{int}}$$





$$\Psi(\mathbf{r}) = p_x(\mathbf{r}) \pm i p_y(\mathbf{r})$$
$$\approx x \pm iy$$

Vortex

$$H_{\text{int}} = \frac{U}{2} \sum_i \left(n_i^2 - \frac{L_{z,i}^2}{3} \right)$$

$$L_z = i(p_x^\dagger p_y - p_y^\dagger p_x)$$

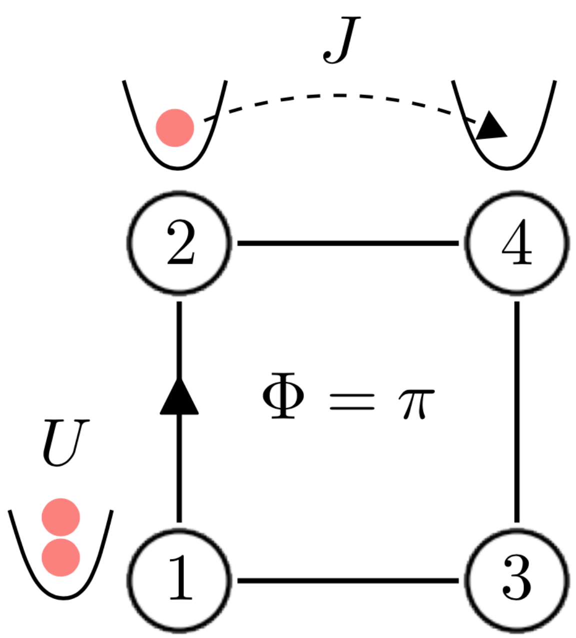
Ground state:
maximize angular momentum

Isacsson and Girvin PRA 2006
Liu and Wu PRA 2006
Li and Liu 2015

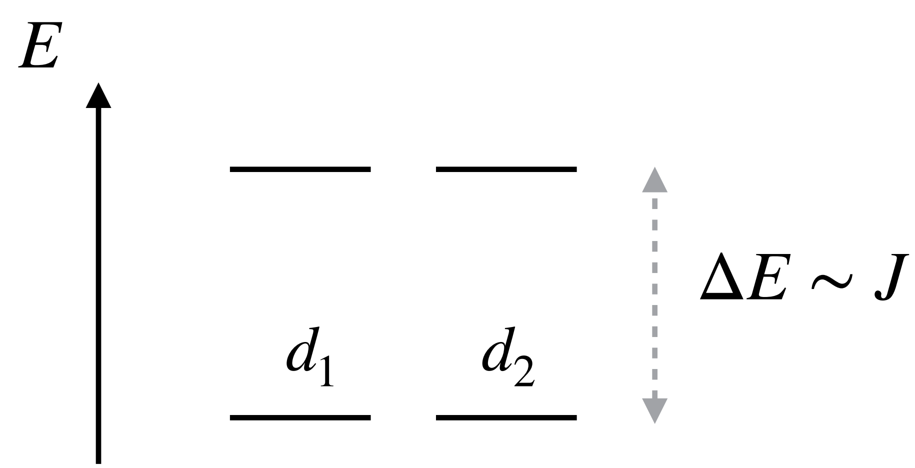
π-flux plaquette: the building block

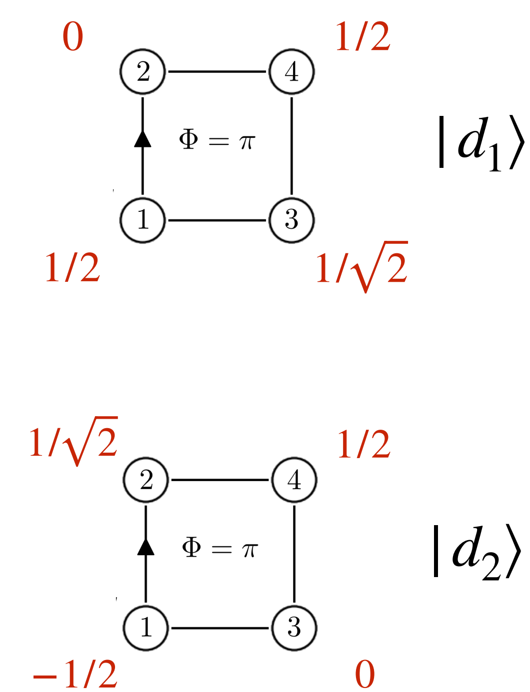
MDL and N. Goldman, Phys. Rev. Research 5, 023064 (2023)

Goal: π-flux plaquettes as building blocks to simulate p-band physics



Two-fold degeneracy
of the single-particle spectrum





We project the **Bose-Hubbard** model onto the lowest two states d_1, d_2

$$\hat{H}_{\text{eff}} \approx \hat{P}\hat{H}\hat{P} \quad (U \ll J)$$

$$\hat{H}_{\text{eff}} = \frac{3U}{16}\hat{n}^2 - \frac{U}{8}\hat{n} - \frac{U}{16}\hat{L}_z^2$$

Exps: Mukherjee (**MDL**) PRL 2018; Mittal Nature 2019; Kremer Nat. Comm. 2020; Jörg Light:Sci.App 2020; Caceres PRL 2022, Houck arXiv 2023

Many-body spectrum

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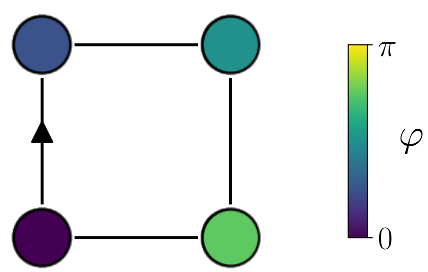
$$\hat{H}_{\text{eff}} = \frac{3U}{16}\hat{n}^2 - \frac{U}{8}\hat{n} - \frac{U}{16}\hat{L}_z^2$$

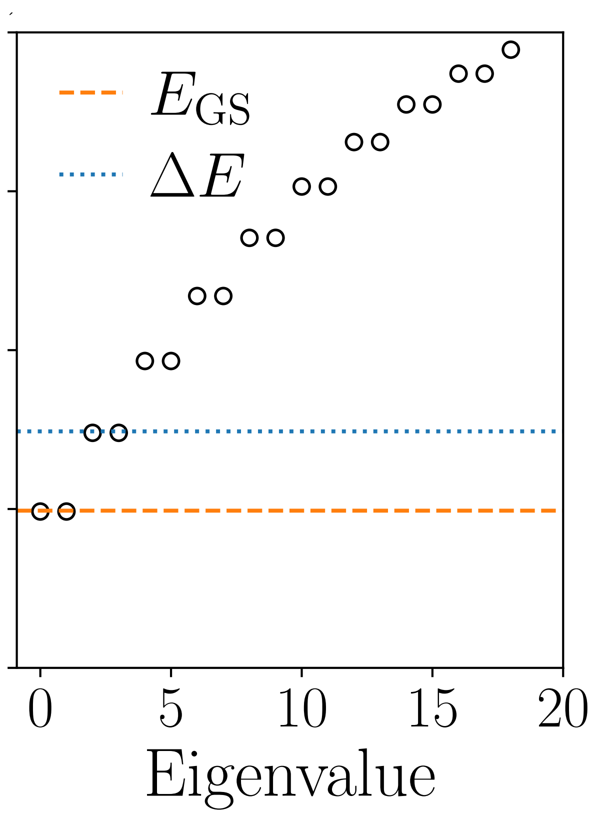
$$[\hat{H}_{\text{eff}}, \hat{L}_z] = 0 \qquad \hat{d}_{\pm} \sim \hat{d}_1 \pm i\hat{d}_2$$

BEC phase winding



$$|n_+, n_-\rangle = \frac{1}{\sqrt{n_+!\,n_-!}}(\hat{d}_+^\dagger)^{n_+}(\hat{d}_-^\dagger)^{n_-}|0\rangle$$

Spectrum is made of eigenstates of angular momentum L_z



$$|\text{GS}\rangle \rightarrow |\overset{n_+}{N}, \overset{n_-}{0}\rangle$$

All particles in one angular momentum state

$$|\text{1p exc}\rangle \rightarrow |\overset{n_+}{N-1}, \overset{n_-}{1}\rangle$$

Excitations change angular momentum

$$\Delta E = \frac{g}{4} - \frac{U}{4}$$

Mean-field

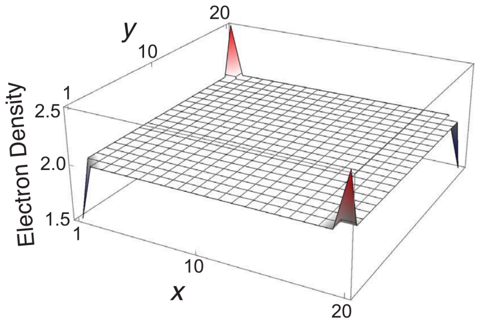
Particle number fluctuations

π-flux plaquettes as building blocks

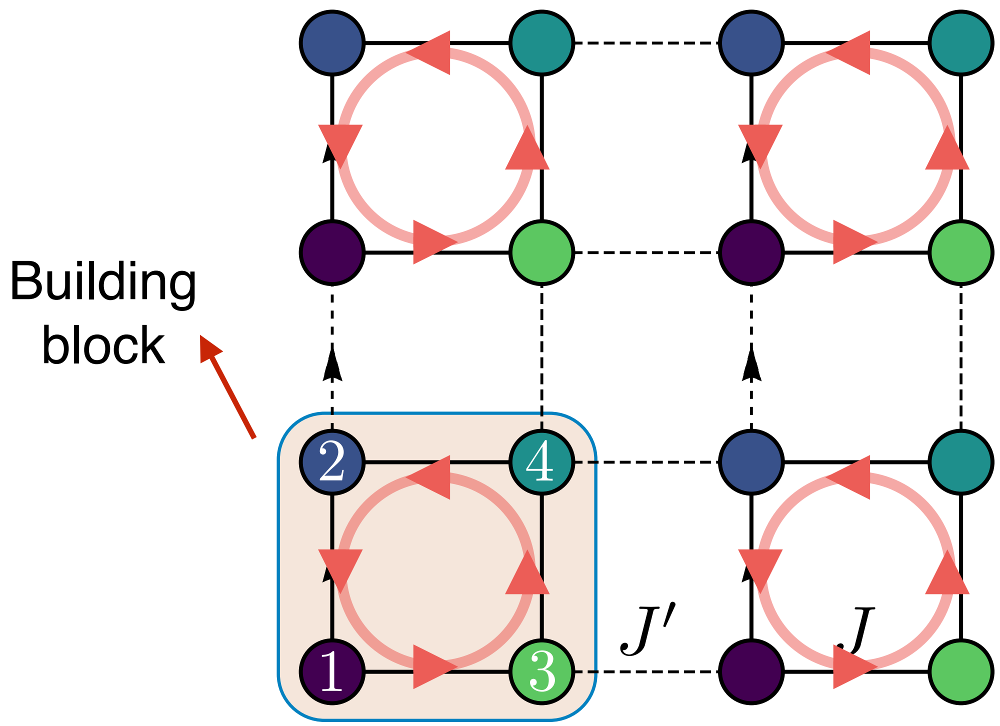
- Complex lattices with chiral order by assembling **dimerized** ($J' \ll J$) π-flux plaquettes

<u>Example</u>:
The BBH model
(higher-order TI)

Benalcazar, Bernevig, Hughes Science 2017

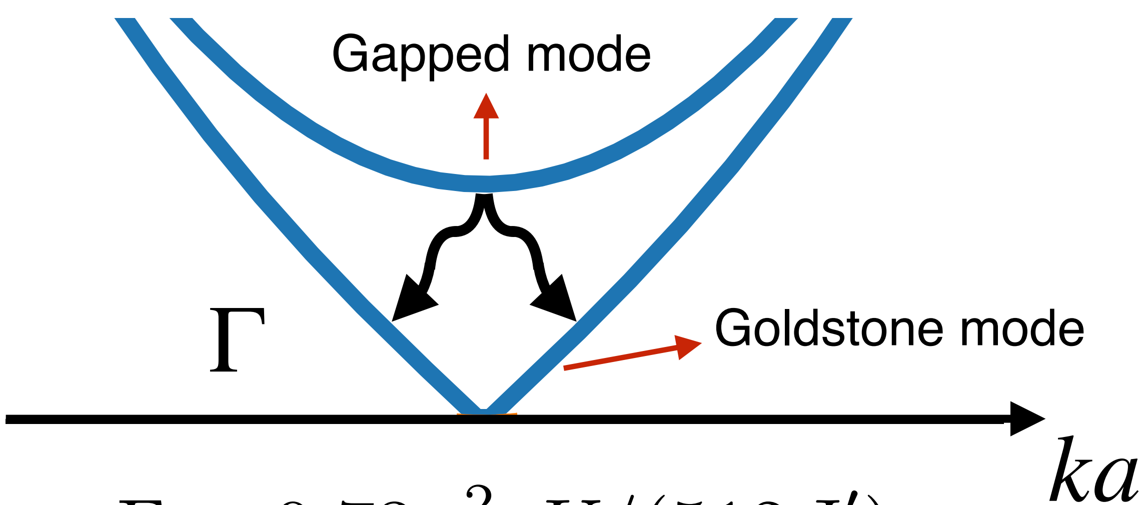


$$\hat{H}_{\text{eff}}^{(inter)} \propto -J' \sum_{\langle i,j \rangle} (\hat{d}^{\dagger}_{1,i}\hat{d}_{1,j} + \hat{d}^{\dagger}_{2,i}\hat{d}_{2,j})$$



Chiral Superfluid - Vortices array

- Bogoliubov dispersion ($ka \ll \pi$)



$$\Gamma = 0.73\pi^2 gU/(512J')$$

Requires $\Gamma < \omega_0$ (i.e. $U < 18J'$)

The collective mode is unstable towards decay into phonons

Strongly-correlated regime

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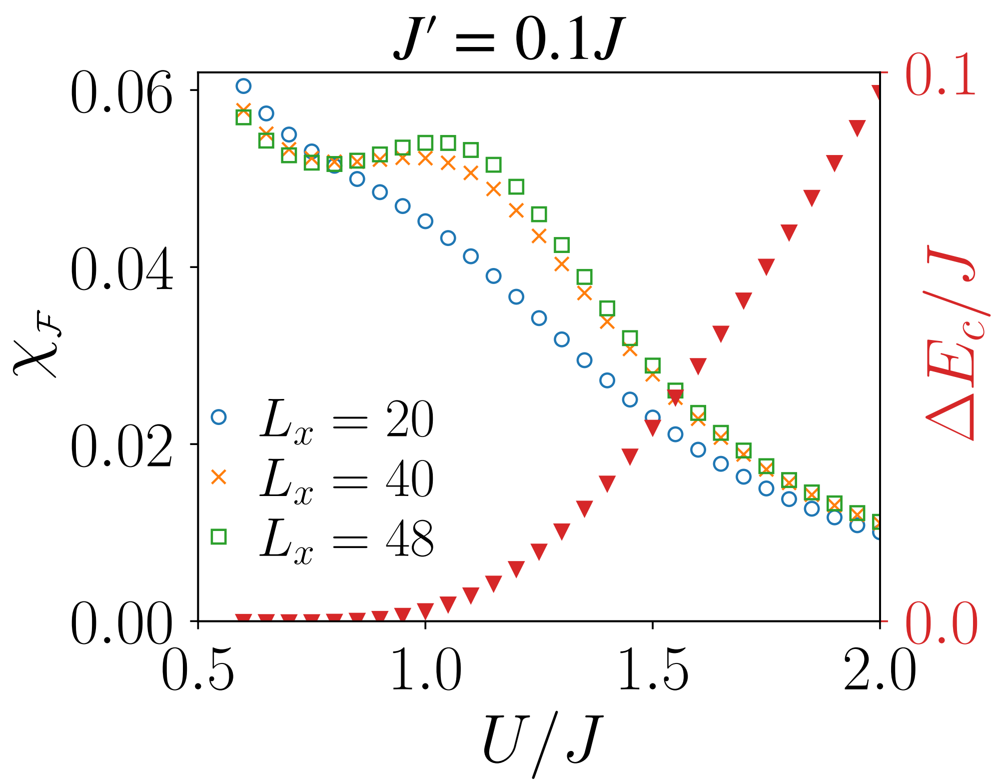
- Strongly-interacting limit $\nu U \sim J'$

Paramekanti PRB 2012, 2013

Phase transitions from superfluid to p-band Mott insulator

Li and Liu Rep. Prog. Phys. 2015

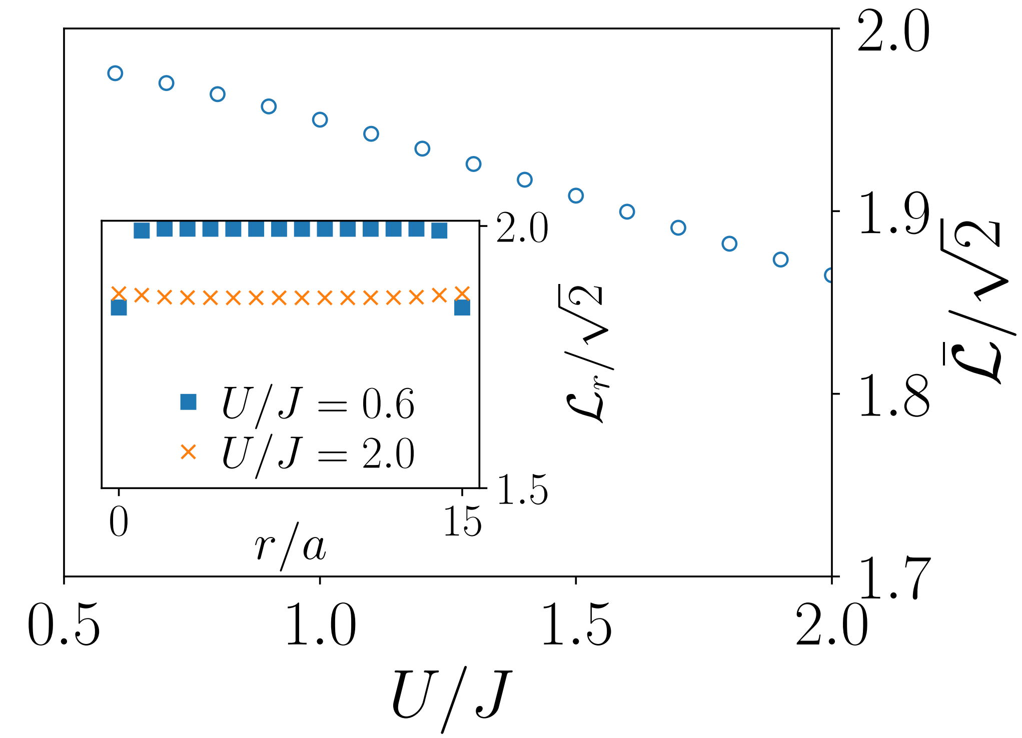


Fidelity susceptibility and charge gap (DMRG) signals transition into a Mott phase

$$\mathcal{F}(\delta U) = |\langle \Psi(U) | \Psi(U + \delta U) \rangle|$$

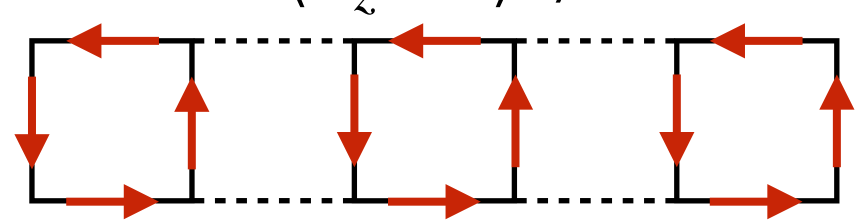
$$\chi_{\mathcal{F}} = -\frac{2}{L_x} \lim_{\delta U \to 0} \frac{\partial^2 \mathcal{F}}{\partial \delta U^2}$$

Filling $\nu = 1/2$ per plaquette



Local angular momentum remains finite across the transition

$$\langle \hat{L}_z^{(\mathrm{plaq})} \rangle \neq 0$$



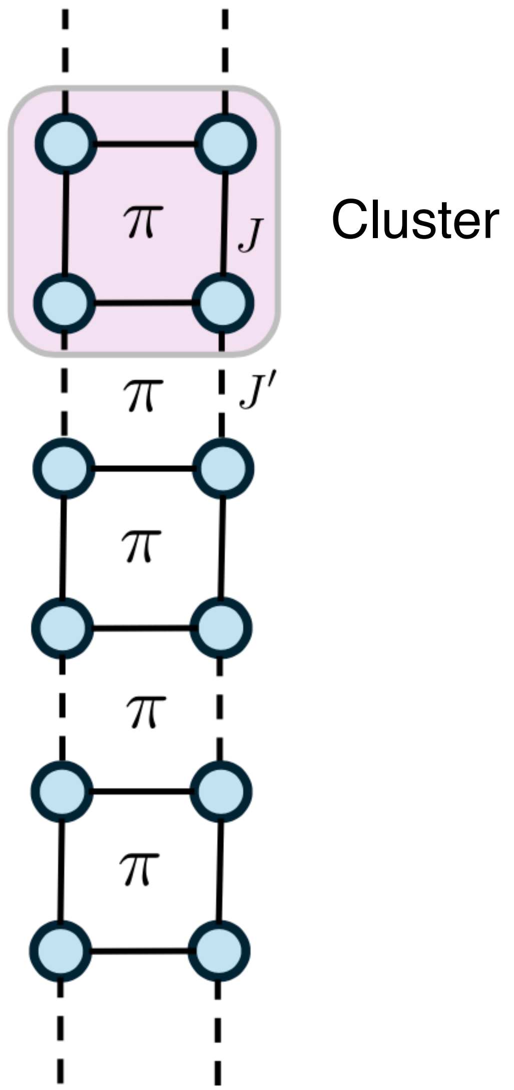
Building the phase diagram

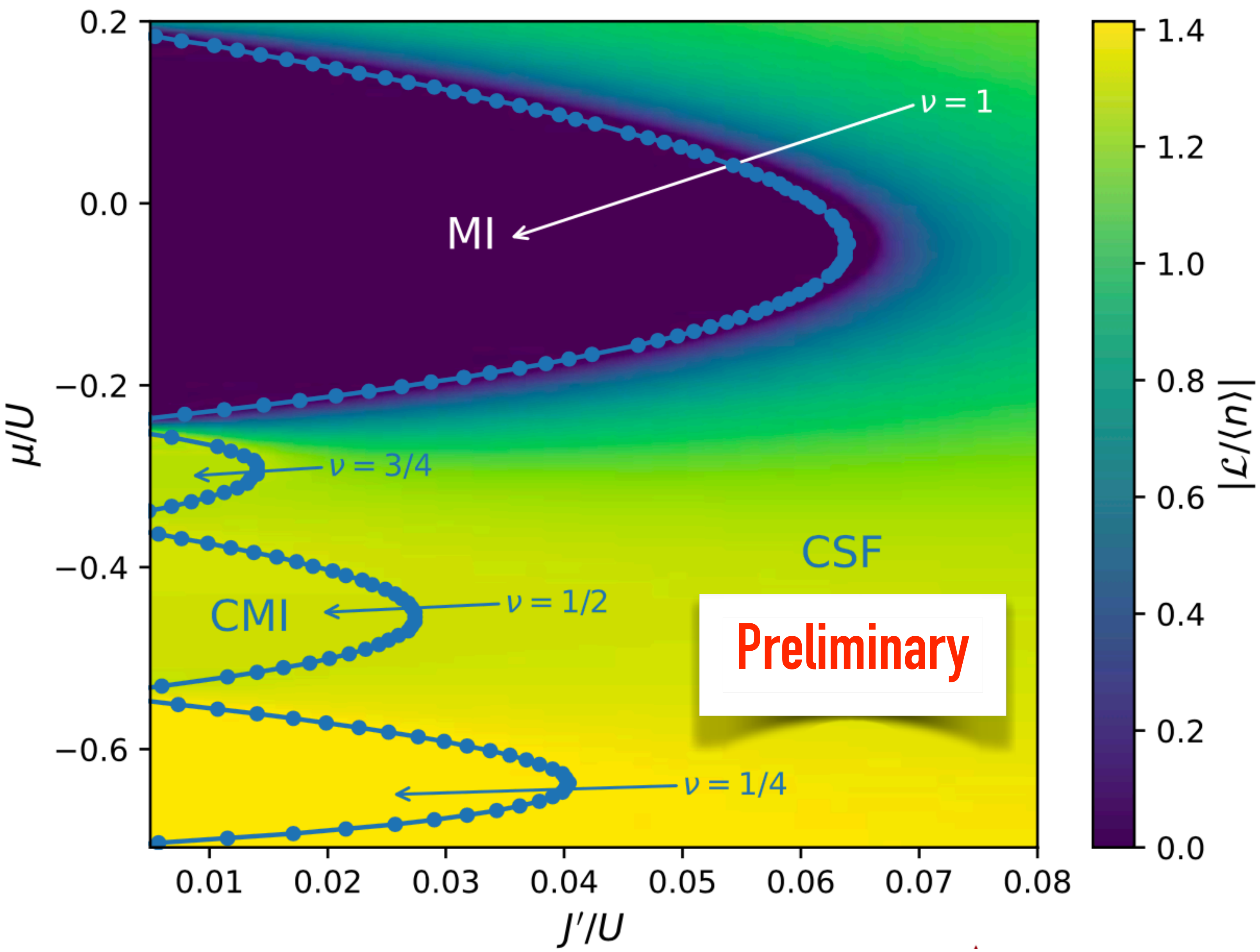
- Cluster Gutzwiller variational ansatz

M. Lanaro et al. (**MDL**), in preparation

$$|\psi\rangle \equiv \otimes_p \left(\sum_{\{\vec{n}\}} A_p(n_1, n_2, n_3, n_4) \,|\, n_1, n_2, n_3, n_4 \rangle_p \right)$$

- Identify quantum phases
- Ansatz for excitations and collective modes (chiral modes)
- Quantum dynamics





State-preparation and measurement

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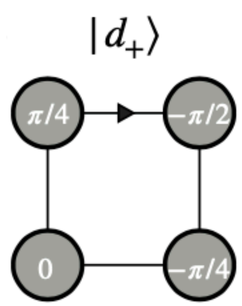
- Loop-current imprint via double well control

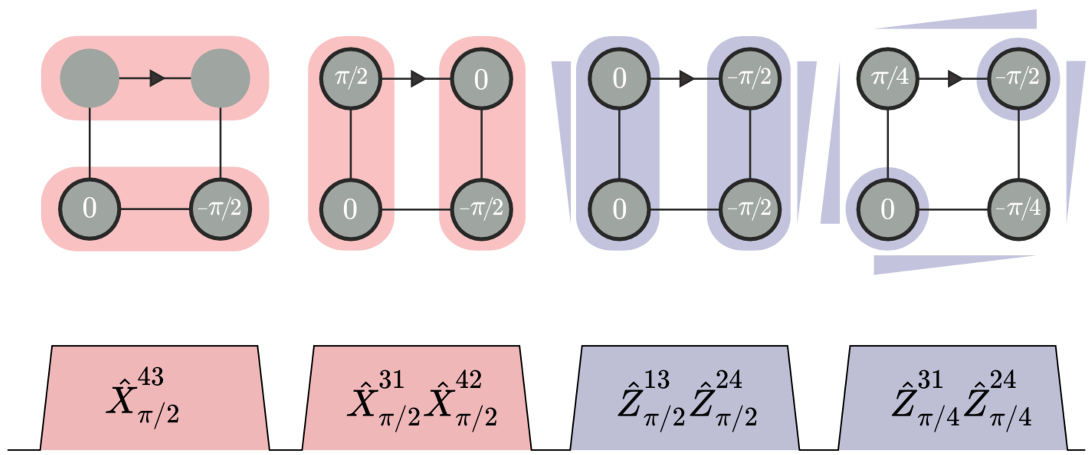
  Impertro et al. (Aidelsburger) PRL 2024

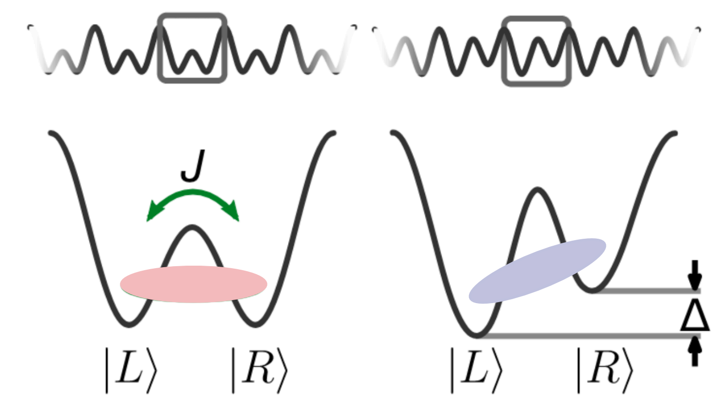
A. Stepanenko and **MDL**, arXiv:2410.06184

$$X_t^{ab} \equiv \exp\left(-i(-J_{ab}b_a^\dagger b_b + \mathrm{h.c.})t/\hbar\right)$$

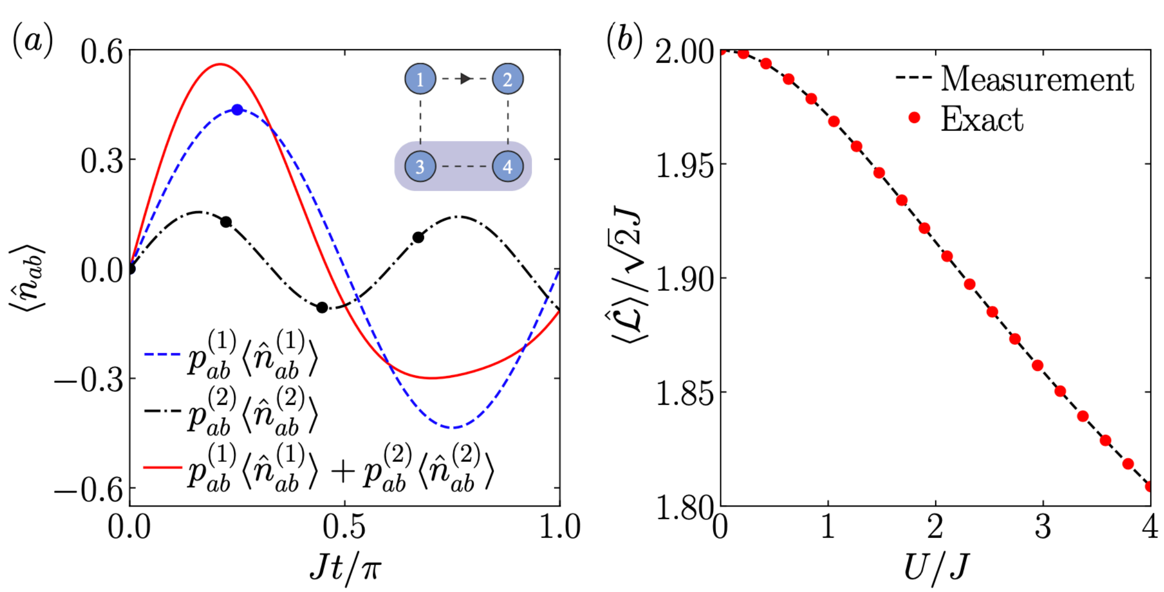
$$Z_t^{ab} \equiv \exp\left(-i\Delta(\hat{n}_a - \hat{n}_b)t/2\hbar\right)$$







- Chirality measurement via double well oscillations (in the presence of interactions)



- Measure density in a double well at different times
- Convert density meas. to current via continuity equation

$$\mathcal{J}_0^{(2)} = -\csc\left(\frac{U\pi}{2\omega}\right)\left[\frac{U}{4}\mathcal{N}_0^{(2)}\cos\left(\frac{U\pi}{2\omega}\right) + \frac{U}{4}\mathcal{N}_2^{(2)} - \frac{\omega}{2}\mathcal{N}_1^{(2)}\sin\left(\frac{3U\pi}{4\omega}\right) - \frac{\omega}{2}\mathcal{N}_3^{(2)}\sin\left(\frac{U\pi}{4\omega}\right)\right]$$

- Weight with respect to occupation

$$\langle \hat{j}_{ab} \rangle(\tau_0) = p_{ab}^{(1)}\mathcal{J}_0^{(1)} + p_{ab}^{(2)}\mathcal{J}_0^{(2)}$$

Conclusions and outlook

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MDL and N. Goldman, Phys. Rev. Research 5, 023064 (2023)
A. Stepanenko and **MDL,** arXiv:2410.06184 (2024)
M. Lanaro et al. (**MDL**) (in preparation)

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- Orbital-like order for bosons in dimerised lattices with π-flux
- Building chiral phases using spontaneous time-reversal symmetry breaking mechanism
- Make state preparation of the gapped phase two particles at a time with atoms and superconducting circuits
- Can there be topology in these systems?
- Exotic models with flux? 3D BBH?