Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani Piano Nazionale di Ripresa e Resilienza
NQSTI National Quantum Science and Technology Institute

Dynamical evolution of quarkonium in strongly interacting matter via open quantum system approach

Gabriele Coci

UNIVERSITÀ degli STUDI di CATANIA

Dipartimento Fisica Astronomia "Ettore Majorana"

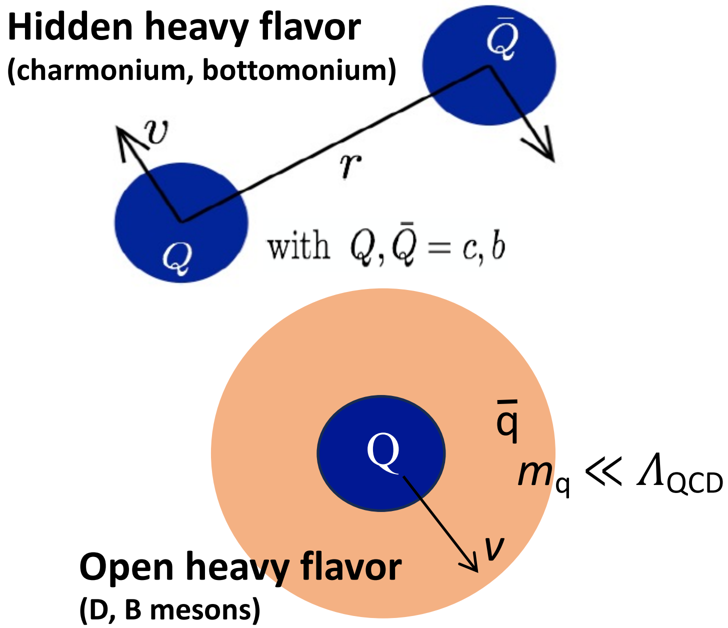
NQSTI - Spoke 1 - A1.4

Responsabile scientifico progetto: Prof. G. Falci

Responsabile scientifico sotto-progetto: Prof. S. Plumari

Meeting SIM e PRIN2022 – 10-11 Settembre 2024

Quarkonia in vacuum



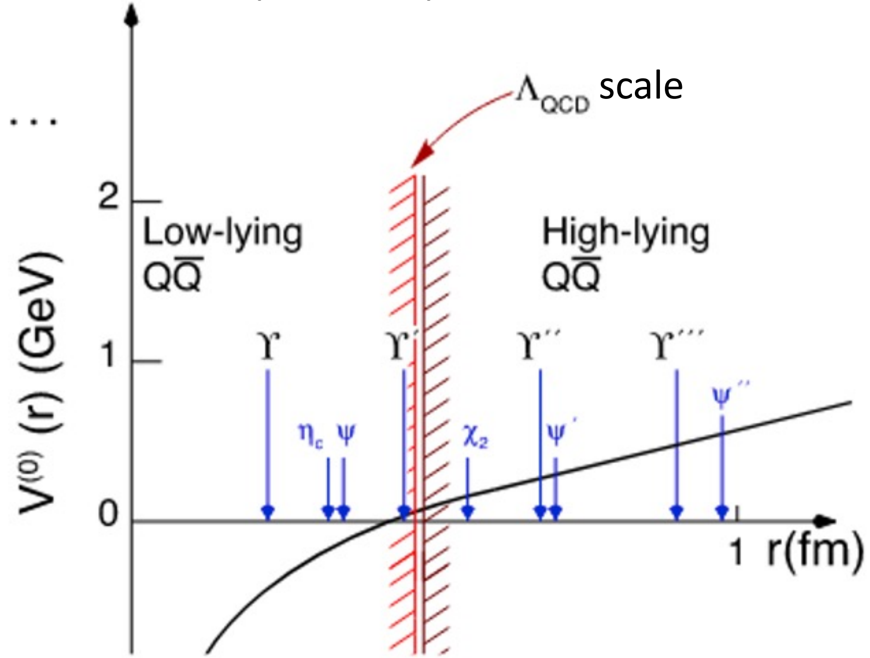
Quarkonia are the quantum bound states of heavy quark with heavy anti-quark (QQbar).

$$M_Q \gg M_Q v \gg M_Q v^2 \simeq E_B \,,\, \Lambda_{QCD}$$

Non-relativistic potential models, based on Schrödinger equation with central potential, give good description of main properties (radius <r>, binding energy E_B).

$$\left[-\frac{1}{M_Q}\frac{\partial}{r^2 \partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{\hat{L}^2}{M_Q r^2} + V_C(r)\right]\psi = E_B \psi \qquad V_C(r) = -\frac{4\alpha_s}{3r} + \sigma r + \ldots$$

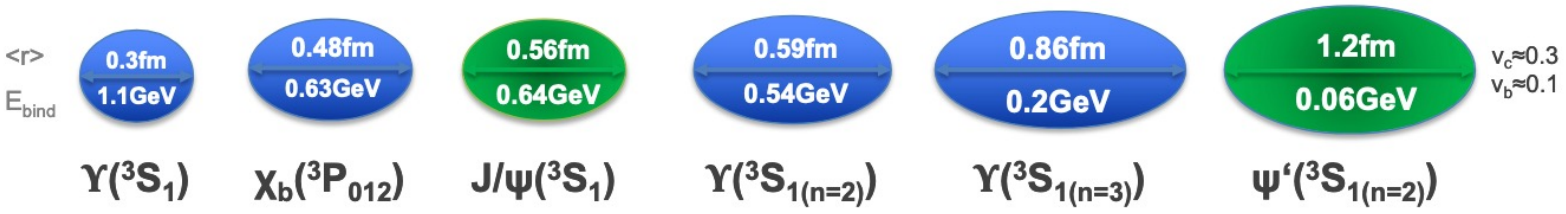
Cornell potential



[S. Godfrey, N. Isgur PRD 82 (1985)]

The different quarkonium radii provide different measures of the transition from a Coulomb-like bound state to a linear-confined bound state.

→ **Quarkonia as probe of QCD confinement**

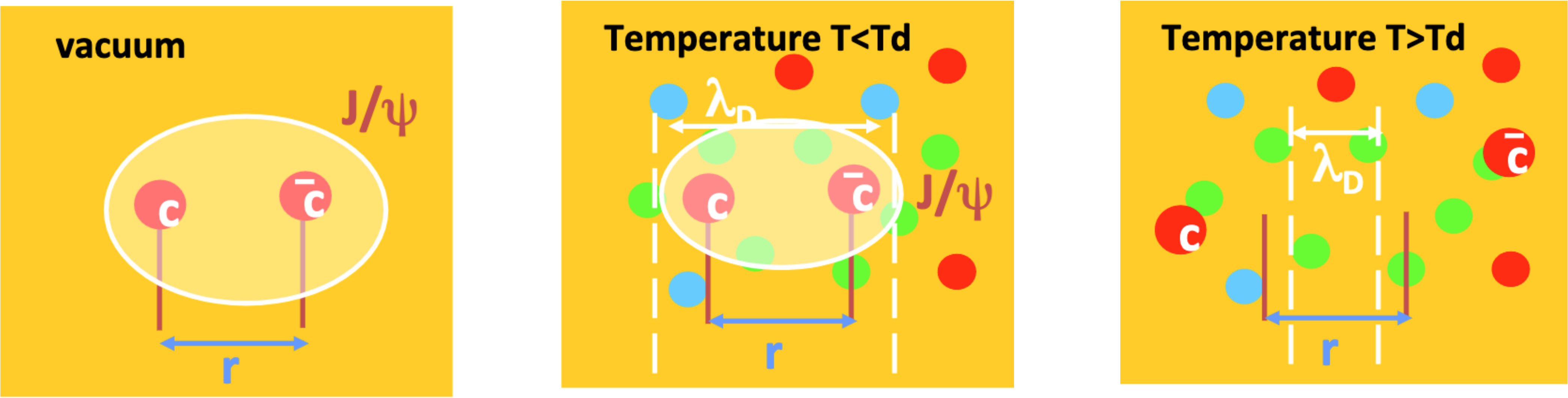


1986 Matsui and Satz idea → **Debye screening**

$$V(r) \propto -\frac{\alpha_s}{r} \rightarrow V(r, T) \propto -\frac{\alpha_s}{r} e^{-r/\lambda_D}$$

Quarkonium suppression clear signature of QGP phase!

[T. Matsui, H. Satz PLB 178 (1986)]

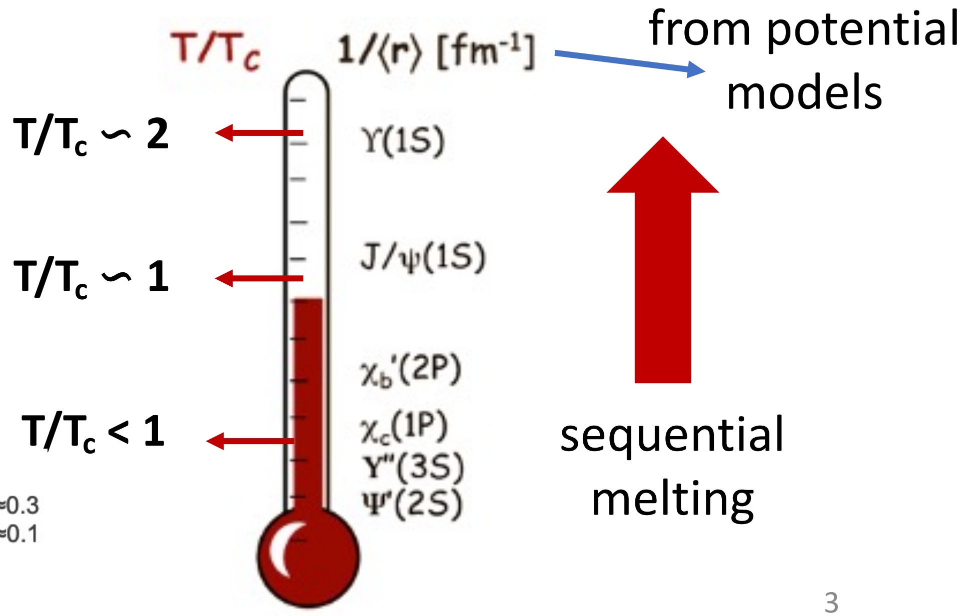


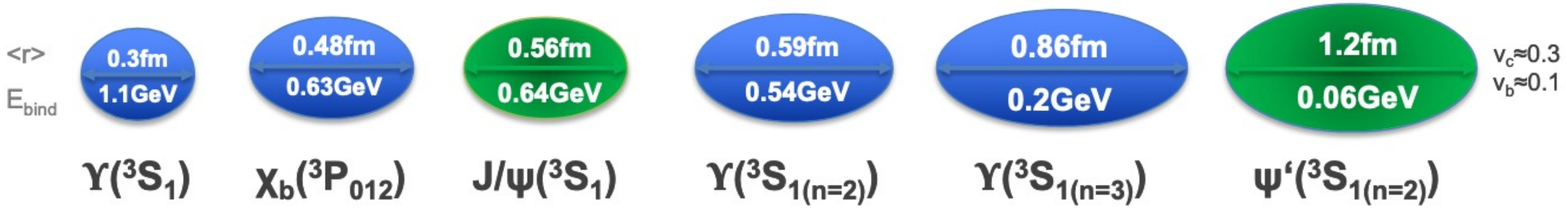
$\lambda_D^{-1} = m_D(T) \backsim (4\pi\alpha_s)^{1/2}\, T$

Can quarkonia be used as thermometer of QGP ?

- **<r> from potential models → what happens to V at T≠0 ?**

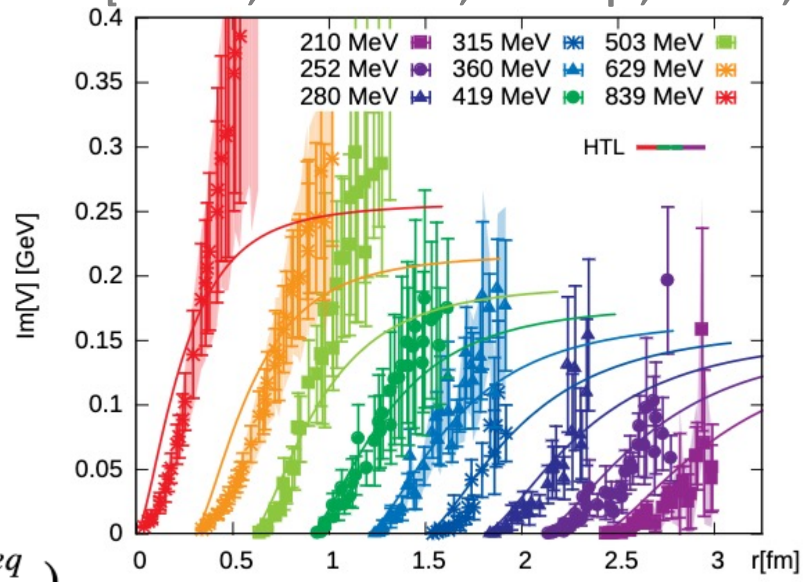
[O. Kaczmarek, F. Zantow PRD 71 (2005)]





Quarkonium description in-medium

[Burnier, Kaczmarek, Rothkopf, PR 114, (2015)]



Quarkonium dissociation not due only to color Debye-screening.

- lQCD provides information of in-medium spectral functions

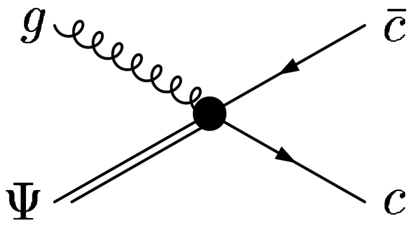
→ **Potential V(r,T) develops an imaginary part** [Laine et al. (2007)]

- **Dynamical dissociation** induced by collisions

$$\frac{dn_{Q\bar{Q}}}{d\tau} = -\Gamma_{Q\bar{Q}}(n_{Q\bar{Q}} - n^{eq}_{Q\bar{Q}})$$

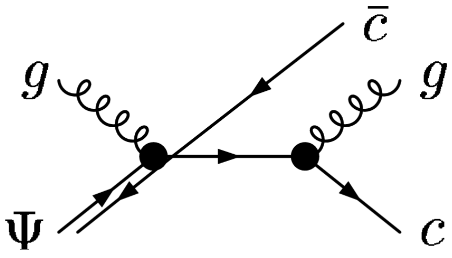
$$\Gamma_{Q\bar{Q}} = \sum_{i=g,q} \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_i(\omega_k, T)\, v_{rel}\, \sigma_{Q\bar{Q}\,i}$$

[L. Grandchamp, R. Rapp PLB 523 (2001), PRL 92 (2004)]



gluon dissociation

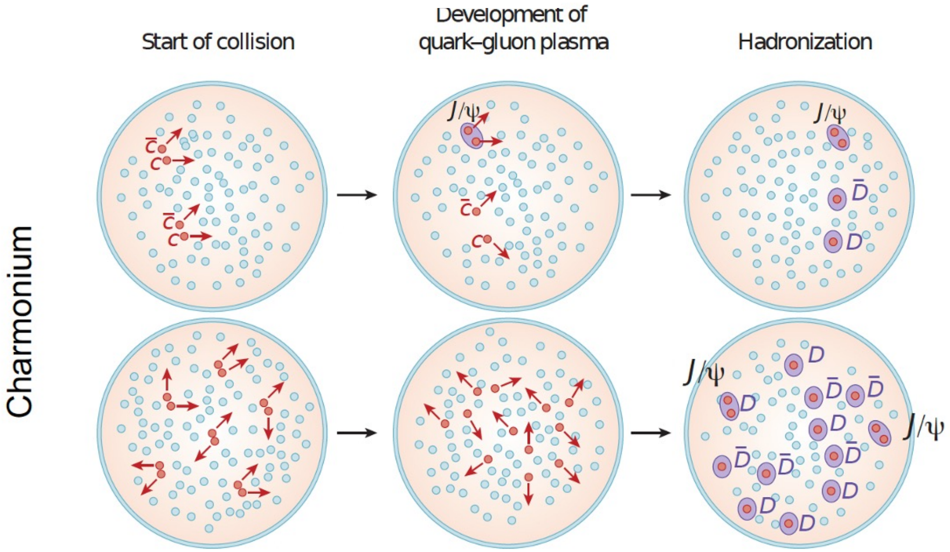
[Peskin NPB 156 (1979)]



quasi-free dissociation

- Large dissociation implies also high formation rate

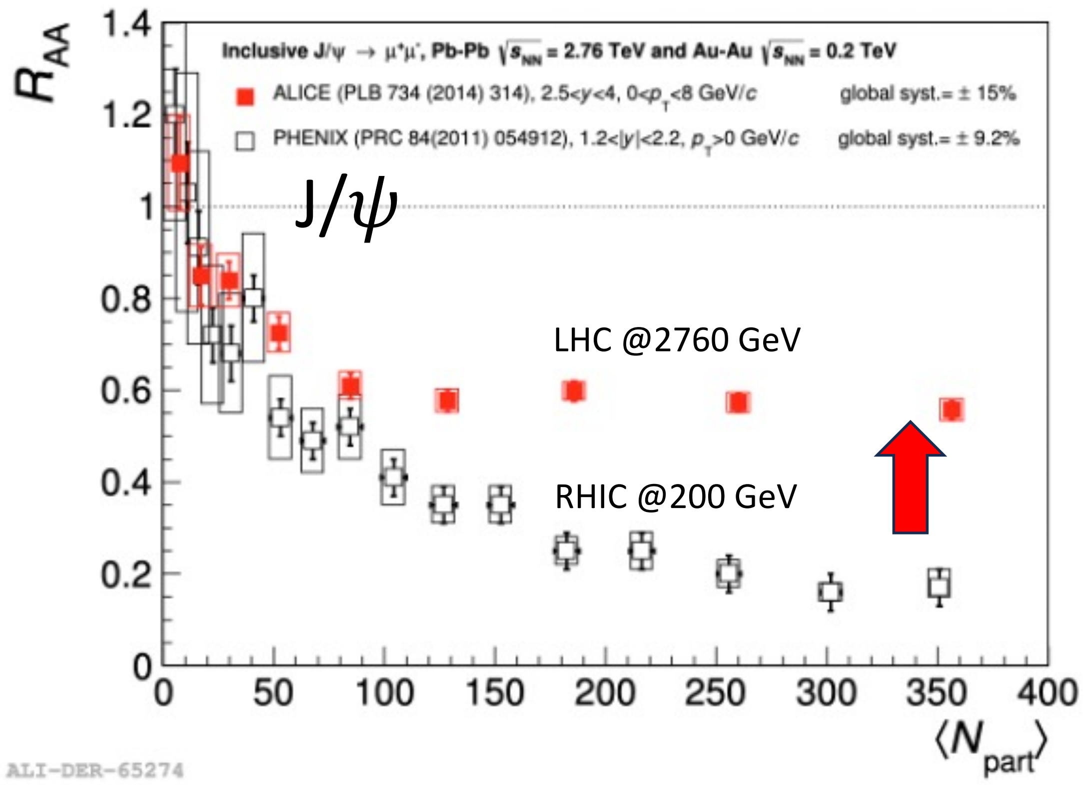
→ **Regeneration** in presence of many charm quarks

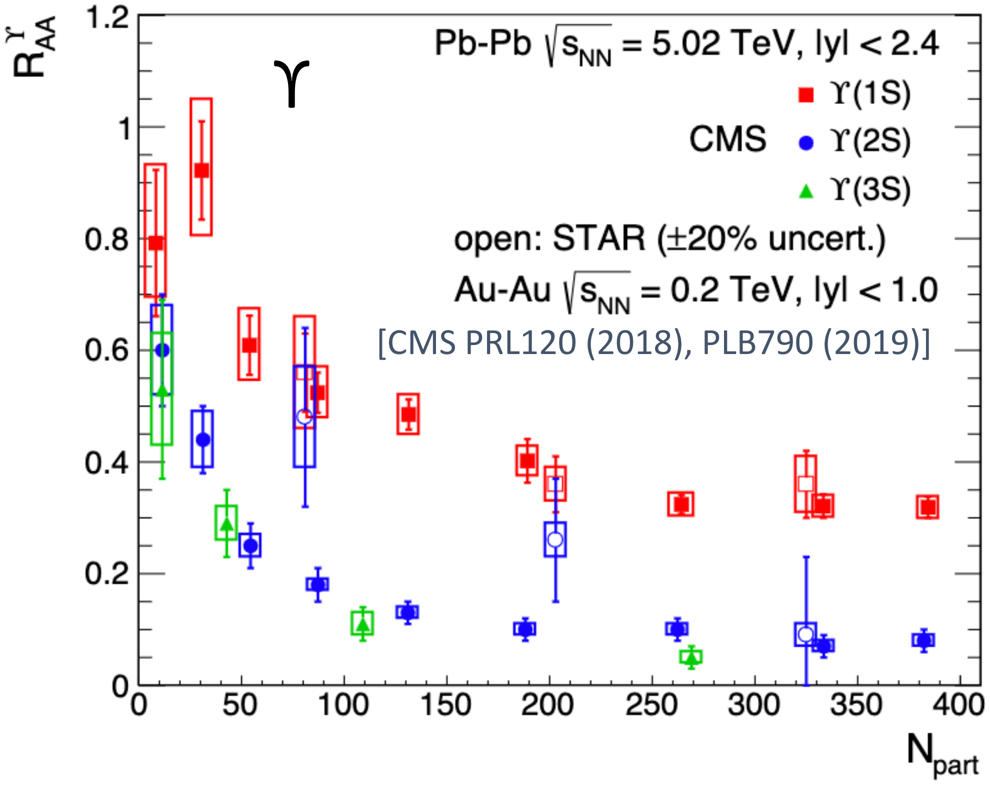


RHIC $N_{ccbar} \backsim 10$

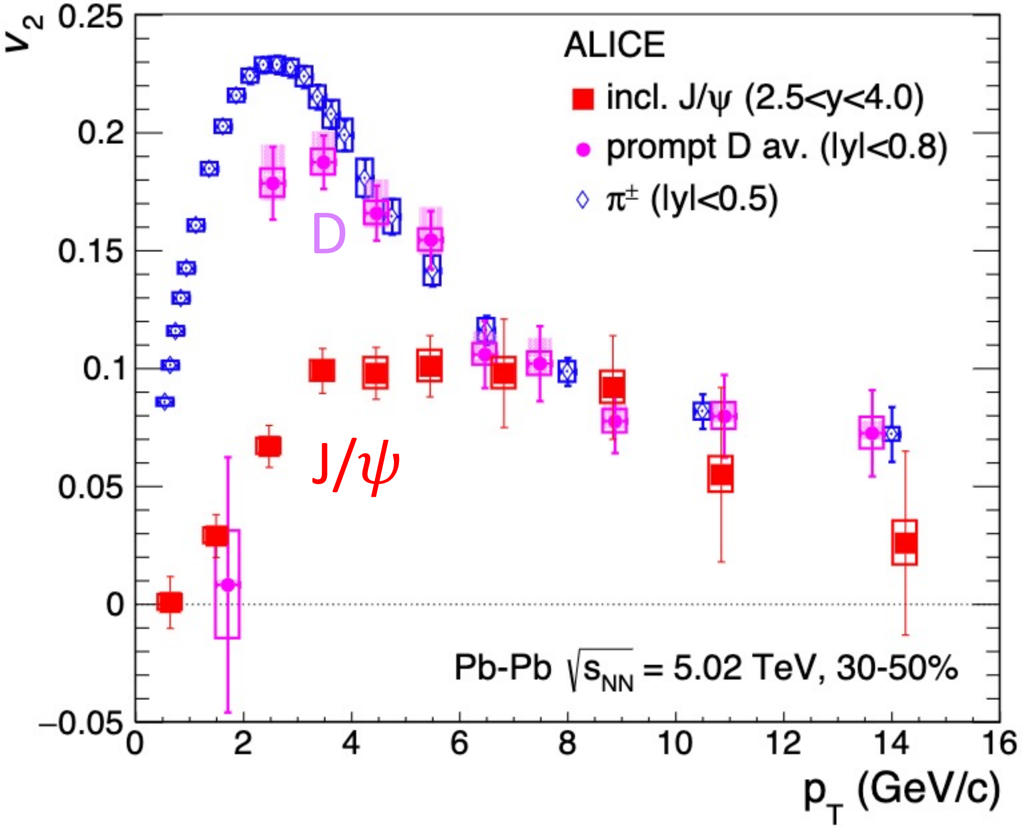
LHC $N_{ccbar} \backsim 100$

All these mechanisms should stay in a consistent framework

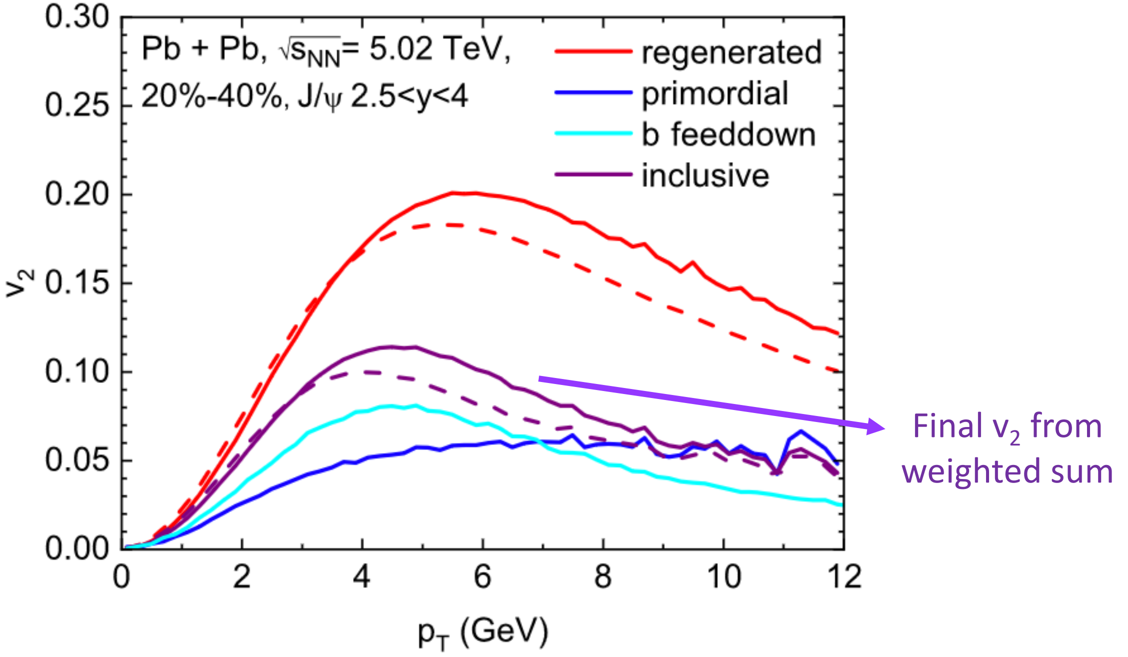




- R_{AA}(RHIC) ≈ 0.3 → significant **suppression** of J/ψ in the QGP phase
- R_{AA}(LHC) > R_{AA}(RHIC) → charmonia **regeneration becomes dominant**
- R_{AA}(Y2S) < R_{AA}(Y1S) < 1 → sequential melting of Y states (regeneration is negligible)



Regenerated J/ψ with Resonance Recombination Model (RRM)
[M. He, B. Wu, R. Rapp PRL 128 (2022)]



- At LHC $v_2(J/\psi) \approx v_2(D)$ → carry information on **thermalization** of charm quark
- At low p_T $v_2(J/\psi)$ from **recombination** process → important observable for **charm hadronization models**
- Small elliptic flow of ϒ at forward and backward rapidity → **thermalization of bottom quark**

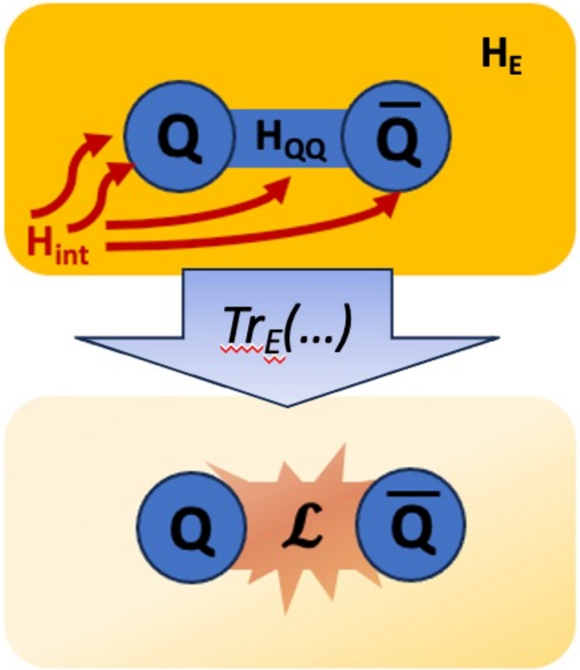
Open Quantum Dynamics for quarkonium evolution

- High-energy physics can draw inspiration from the **theory of Open Quantum Systems (OQS)** which is highly developed in the field of condensed matter physics.

- We can imagine the overall system (**QQbar** + **QGP environment "E"**) as a closed quantum system with hermitian Hamiltonian *H* and density matrix ρ_{tot} which evolves unitarily.

$$H = H_{Q\bar{Q}} \otimes I_E + I_{Q\bar{Q}} \otimes H_E + H_{int} \qquad \frac{d\rho_{tot}}{dt} = -i\,[H, \rho_{tot}]$$

(intial conditions) $\rho_{tot}(0) = \rho_S(0) \otimes \rho_E$



- If we are interested only in the dynamics of the QQbar pair, we can trace out the medium d.o.f. and study the evolution of the QQbar **reduced density matrix**.

$$\rho_{Q\bar{Q}} = Tr_E(\rho_{tot}) \quad \Rightarrow \quad \frac{d\rho_{Q\bar{Q}}}{dt} = -i\,Tr_E([H, \rho_{tot}])$$

[H.-P. Breuer, F. Petruccione: "*The theory of OQS*" (Oxford)]

- In general, the open quantum system time evolution is not unitary.
- In the regime of **Markovian dynamics** the master equation can be written in terms of the generator of a dynamical (semi-group) map, the **Liouvillian super-operator** which is not known from first principles

$$\frac{d\rho_{Q\bar{Q}}}{dt} = \mathcal{L}\rho_{Q\bar{Q}}$$

Markovian quantum process

- Markovian quantum master equation implies trace preserving condition $Tr(\rho_{Q\bar{Q}}(t)) = Tr(\rho_{Q\bar{Q}}(0)) = 1$

but still difficult to solve… need **specific approximation** to get the famous **Lindblad master equation**

$$\frac{d\rho_{Q\bar{Q}}}{dt} = -i[H, \rho_{Q\bar{Q}}] + \sum_n \gamma_n \left(L_n \rho_{Q\bar{Q}} L_n^\dagger - \frac{1}{2}\left\{ L_n^\dagger L_n, \rho_{Q\bar{Q}} \right\} \right)$$

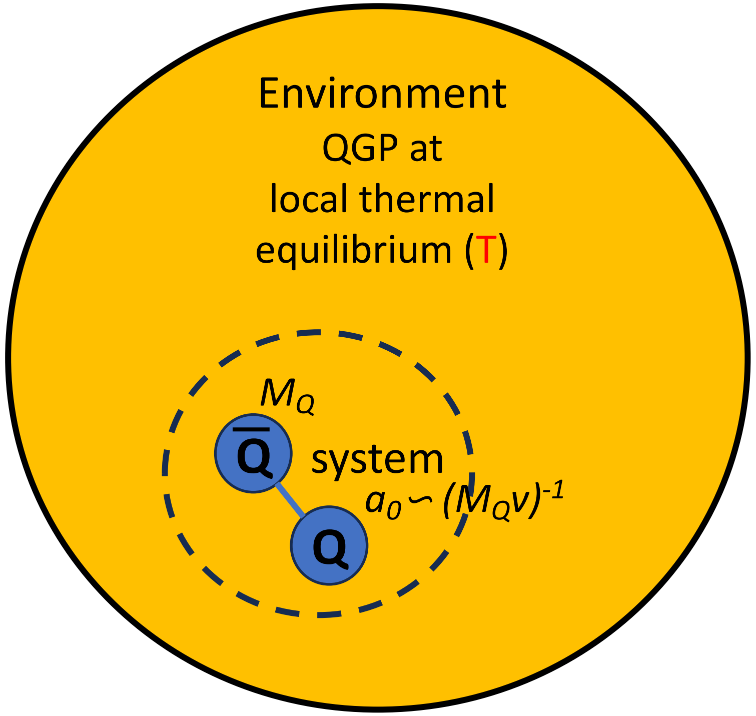
[G. Lindblad (1976) ,Comm. in Math. Phys. 48(2), 119-130]
[Gorini, Kossakowski, Sudarshan (1976) . J. Math. Phys. 17]

- *H* is a Hermitian operator (“**Hamiltonian”**) acting on the system state space, describing the unitary dynamics
- L_n are non-Hermitian **Lindblad operators** which encode the **dissipative part of the dynamics.**
- γ_n are non-negative relaxation rates for the various decay modes → $C_n = \sqrt{\gamma_n} L_n,$ **collapse operators**

Quarkonium suppression in QGP: Open Quantum System approach

<u>Time scales</u>

- Intrinsic time of the system: $\boldsymbol{\tau}_S \backsim 1/E_B \backsim M_Q v^2$
- Correlation time of the environment: $\boldsymbol{\tau}_E \backsim 1/\pi T$
- Relaxation time of the system: $\boldsymbol{\tau}_R \backsim 1 / \text{self-energy} \backsim 1 / a_0^2 (\pi T)^3 \backsim (M_Q v)^2 / (\pi T)^3$



For quarkonium evolution in strong interacting QGP the following hierarchy is valid:

$$M_Q \gg M_Q v \gg \pi T \simeq m_D \left(\sqrt{4\pi\alpha_s} T\right) \gg M_Q v^2 \simeq E_B \,,\, \Lambda_{QCD}$$

$\boldsymbol{\tau}_R >> \boldsymbol{\tau}_E$ the evolution is Markovian

$\boldsymbol{\tau}_S >> \boldsymbol{\tau}_E$ Quantum Brownian Motion (QBM)

- **The dynamics of quantum brownian motion is described by Lindblad equation**

<u>Caldeira-Leggett model (1983):</u>
Quantum random walk of massive particle moving in a potential V and losing energy which is absorbed by "heat bath" environment leading to dissipation.

Conservation of initial Q (Qbar) number: $Tr(\rho_{Q\bar{Q}}(t)) = 1$

QBM applied to quarkonia:

[Akamatsu et al. PRD 85 (2012), PRD 87 (2013) , PRD 91 (2015)]
[Brambilla et al. PRD 96 (2017), PRD 100 (2019)]
[D. De Boni JHEP 08 (2017) 064]
[Blaizot, Escobedo JHEP 06 (2018) 034]
[Gossiaux et al. Annals Phys. 368 (2016) 267-295, JHEP 06 (2024)]

$$\frac{d\rho_{Q\bar{Q}}}{dt} = -i[H, \rho_{Q\bar{Q}}] + \sum_n \left(C_n \rho_{Q\bar{Q}} C_n^\dagger - \frac{1}{2}\left\{ C_n^\dagger C_n, \rho_{Q\bar{Q}} \right\} \right) \quad \rho_{Q\bar{Q}} = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

QTRAJ 1.0

[N. Brambilla, M. Strickland et al. JHEP 05 (2021)]
[M. Strickland http://personal.kent.edu/~mstrick6]
[H. Omar, M. Strickland et al. Comp. Phys. Comm. (2021)]

- Block density matrix: $\rho_s(t)$ and $\rho_o(t)$ are the populations of singlets (bound) and octets (unbound) states.

- Hamiltonian and collapse operators derived within the potential non-relativistic QCD (**pNRQCD**) EFT.

$$h_s = \frac{p^2}{M_Q^2} \underbrace{- \frac{N_c^2 - 1}{2N_c} \frac{\alpha_s}{r}}_{V_s(r)}, \quad h_o = \frac{p^2}{M_Q^2} \underbrace{+ \frac{1}{2N_c} \frac{\alpha_s}{r}}_{V_o(r)}$$

vacuum “Coulomb” QQbar

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2 - 2}{2(N_c^2 - 1)} \end{pmatrix}$$

in-medium QQbar

[N. Brambilla et al. NPB 566 (2000)]
[N. Brambilla et al. PRD 96 (2017)]

- Six collapse operators induce S→O , O→O , O→S and dipole transitions among ground and excited states.

$$C_i^S = \sqrt{\frac{\kappa(t)}{N_c^2 - 1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix}$$

$$C_i^O = \sqrt{\frac{(N_c^2 - 4)\,\kappa(t)}{2(N_c^2 - 1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

i=x, y, z

$$\Gamma_S = \sum_{i=1}^{3} C_i^{S\dagger} C_i^S = \kappa(t)\, r^2 \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{N_c^2 - 1} \end{pmatrix}$$

$$\Gamma_O = \sum_{i=1}^{3} C_i^{O\dagger} C_i^O = \kappa(t)\, r^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

partial decay widths

$$\Gamma = \Gamma_S + \Gamma_O = \kappa(t)\, r^2 \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2 - 2}{2(N_c^2 - 1)} \end{pmatrix}$$

total decay width

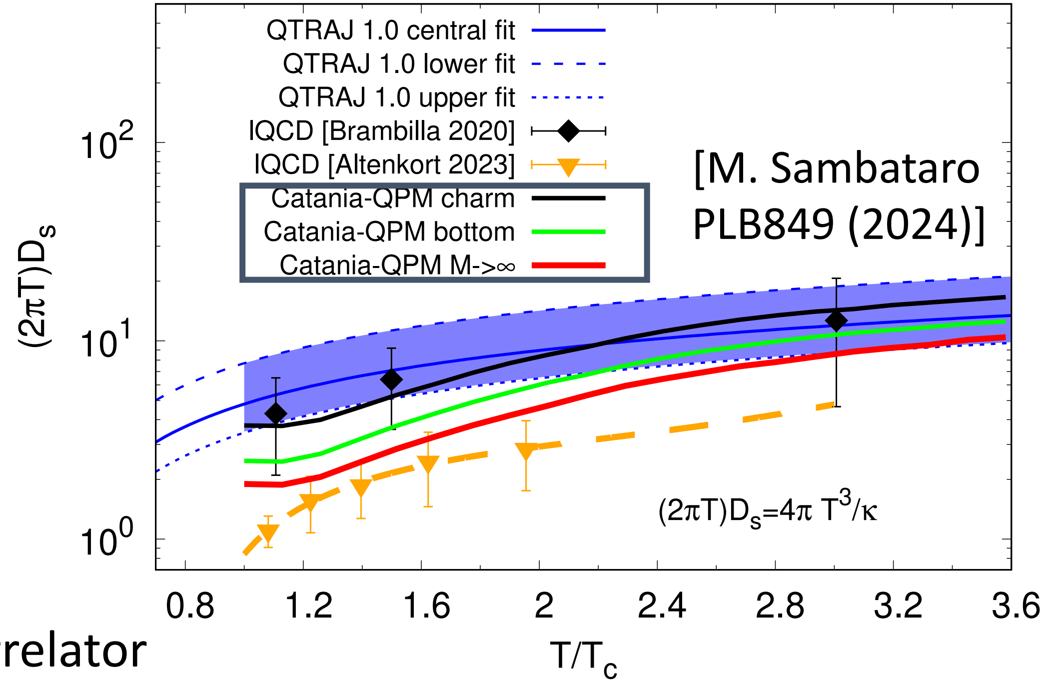
In OQS/pNRQCD approach: Quarkonium evolution = Quantum Brownian motion

- In-medium H and collapse operators depend on two HQ transport coefficients: $\gamma(t)$, $\kappa(t)$

- **κ(T) is the HQ momentum diffusion coefficient**

$$(2\pi T)D_s = \frac{2\pi T^2}{M_Q}\tau_{th} = \frac{4\pi T^3}{\kappa(T)}$$

- **Default parametrization in QTRAJ nis not in agreement with latest D_S(T) from lQCD data and transport estimates…**



- γ(T) imaginary part of the integral of the chromo-electric correlator

→ $\gamma \neq 0$ to match pNRQCD with lQCD

→ non general agreement on the value of γ: **dispersion effects?**

- **HQ transport coefficients affect both the unitary and the dissipative evolution in Lindblad eq.**

Solution of the Lindblad equation for QQbar evolution

- Solving the Lindblad equation is numerically challenging:
  for a Hilbert space of dim. N, the density matrix has N^2 entries → need an efficient algorithm and parallelization.

- The dimension of QQbar density matrix increases with # of lattice size, # of quarkonium states (S-wave, P-wave,…).

❑ Master equation unraveling techniques are numerical methods used to simulate the dynamics of OQS.

→ QTRAJ 1.0 in based on the Monte Carlo Wave Function (MCWF) / quantum trajectory method.

$$\frac{d\rho_{Q\bar{Q}}}{dt} = -i[H, \rho_{Q\bar{Q}}] + \sum_n \left(C_n \rho_{Q\bar{Q}} C_n^\dagger - \frac{1}{2}\left\{ C_n^\dagger C_n, \rho_{Q\bar{Q}} \right\} \right)$$

$$\Gamma_n \equiv C_n^\dagger C_n \,, \; \Gamma = \sum_n \Gamma_n \rightarrow H_{\text{eff}} = H - \frac{i}{2}\Gamma$$

Assume a pure state at time t

$$\rho_{Q\bar{Q}}(t) = |\psi(t)\rangle\langle\psi(t)|$$

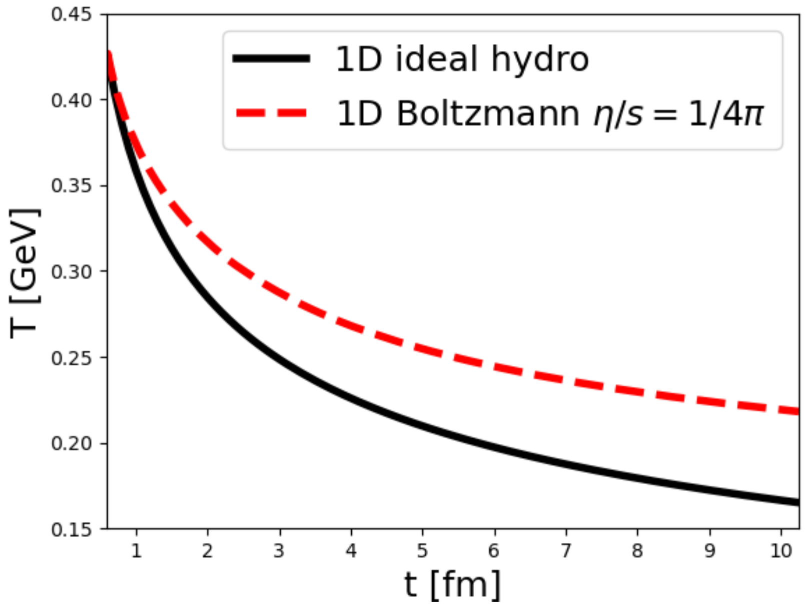
$$\rho_{Q\bar{Q}}(t+\Delta t) \approx (1 - \langle\psi(t)|\,\Gamma\,|\psi(t)\rangle) \frac{(1 - iH_{\text{eff}}\Delta t)\,|\psi(t)\rangle\langle\psi(t)|\,(1 + iH_{\text{eff}}^\dagger \Delta t)}{1 - \langle\psi(t)|\,\Gamma\,|\psi(t)\rangle} + \sum_n \langle\psi(t)|\,\Gamma_n\,|\psi(t)\rangle \frac{C_n\,|\psi(t)\rangle\langle\psi(t)|\,C_n^\dagger}{\langle\psi(t)|\,\Gamma_n\,|\psi(t)\rangle}$$

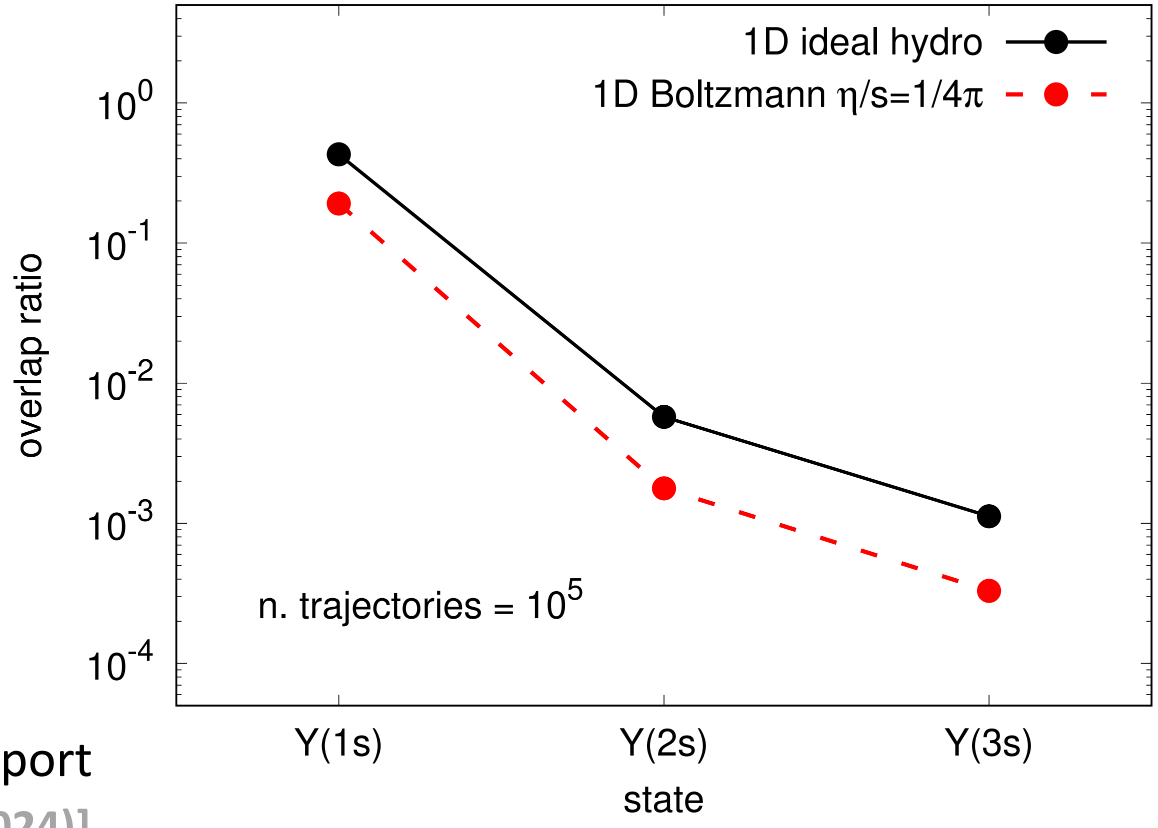
1. With survival probability the system wavefunction evolves to the state $(1 - iH_{\text{eff}}\Delta t)\,|\psi(t)\rangle$

2. With jump probability the system wavefunction makes a stochastic quantum jump to the new state $C_n\,|\psi(t)\rangle$

Preliminary Results

We focus on how different bulk/environment evolution affects Υ suppression



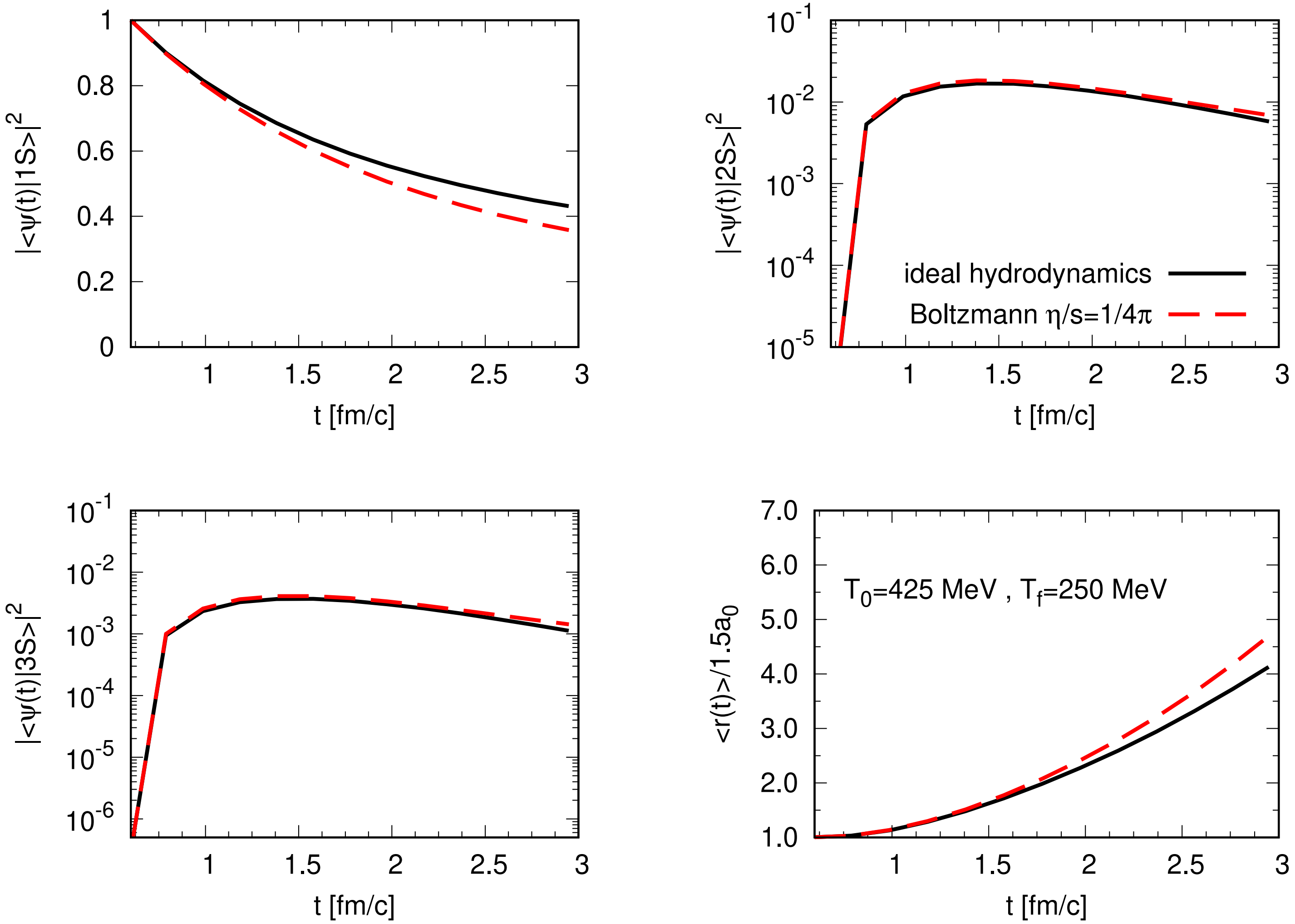


- 1D ideal hydro results reproduce QTRAJ 1.0 benchmark
- T(t) of 1D Boltzmann bulk at fixed η/s from Catania transport

[V. Nugara et al. EPJ C 84 (2024)]

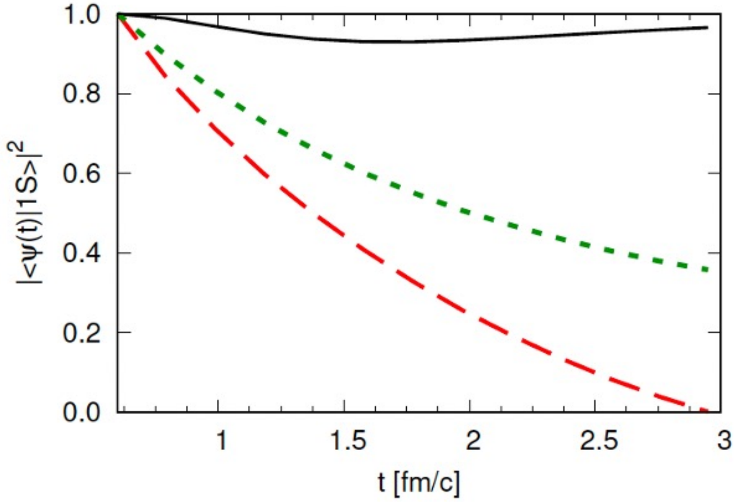
- Enhancement of suppression of Υ S-states due to slower bulk evolution

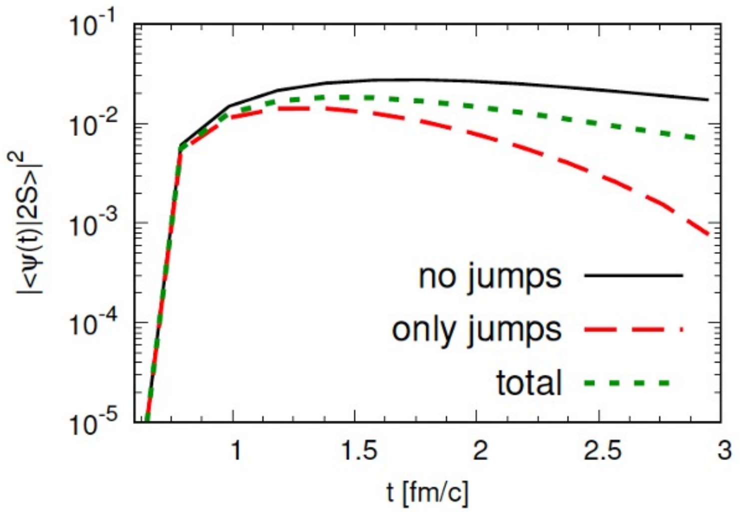
Preliminary Results

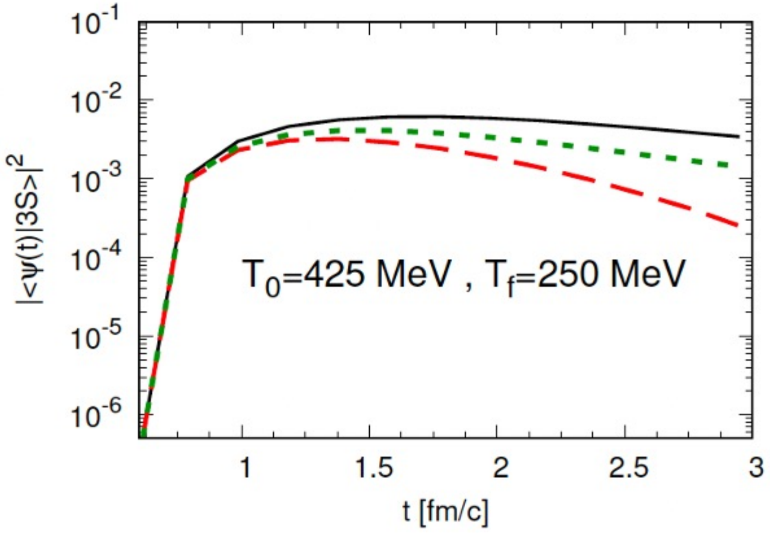


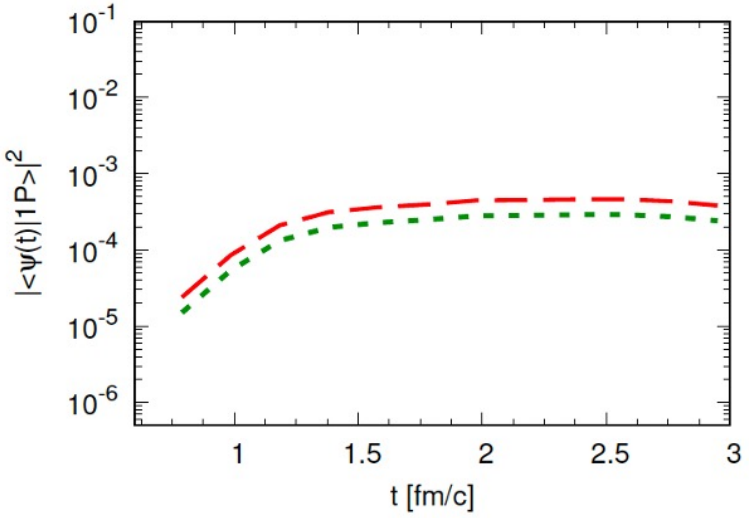
- Final state overlap depends on bulk/environment lifetime
- Time evolution of probability amplitude provide more information!
- Need to extract the distribution of quantum jumps as function of time!

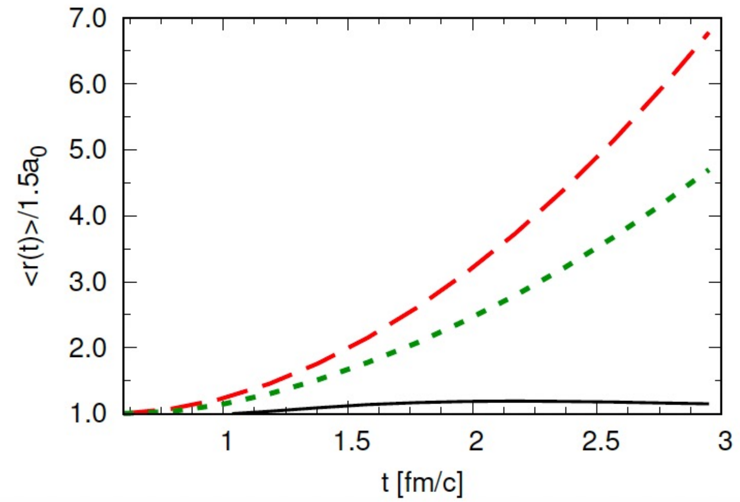
Preliminary Results







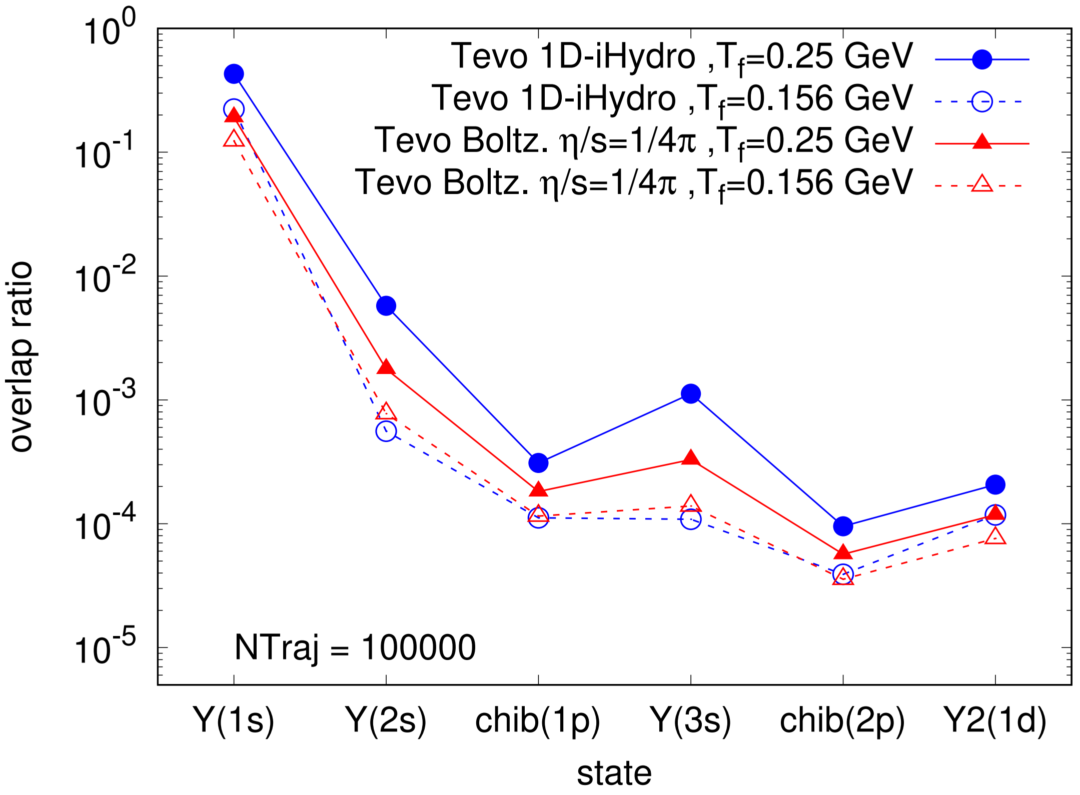




1D Boltzmann bulk evo at fixed $\eta/s=1/4\pi$

- Quantum jumps accelerate ground state suppression
- non-zero angular momentum states occupied only with quantum jumps

Preliminary Results

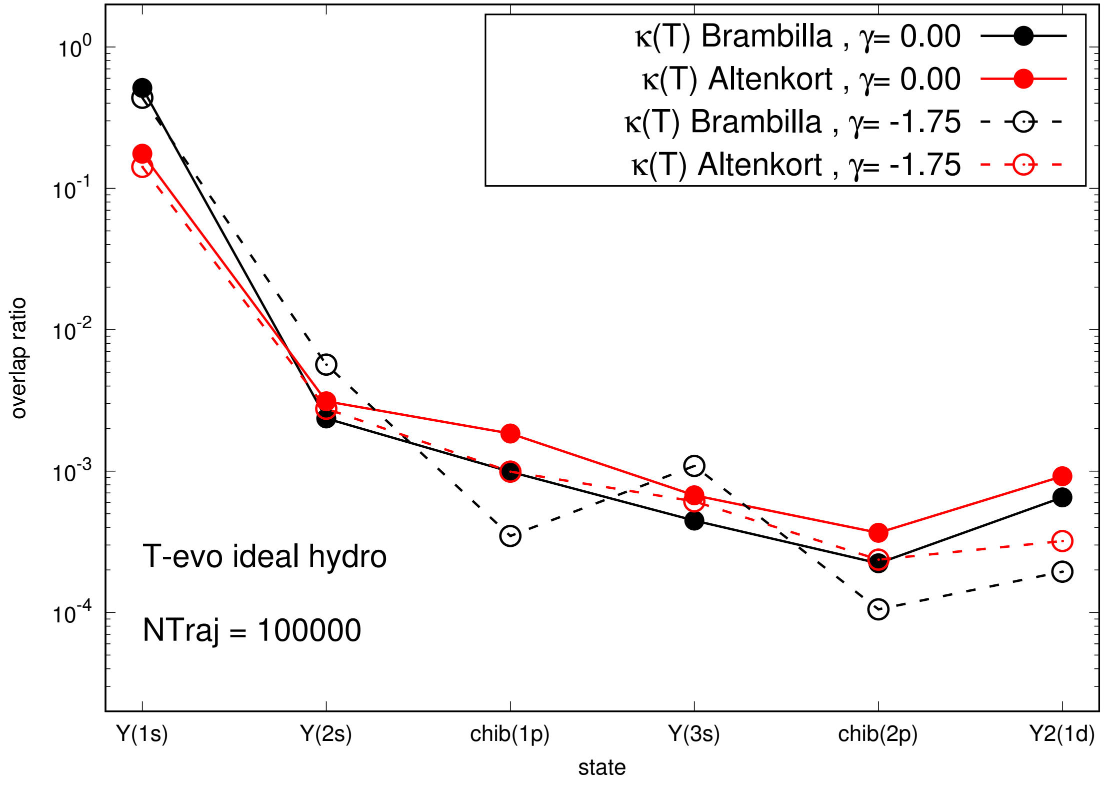


- Effect of different bulk evolution decreases for 1D simulations pushed to **T_f∽156 MeV**
- **Theory (collapse operators from NLO E_B/T expansion) and code version**

[N. Brambilla et al. JHEP 08 (2022)]
[N. Brambilla et al. PRD 108 (2023)]
[N. Brambilla et al. PRD 109 (2024)]

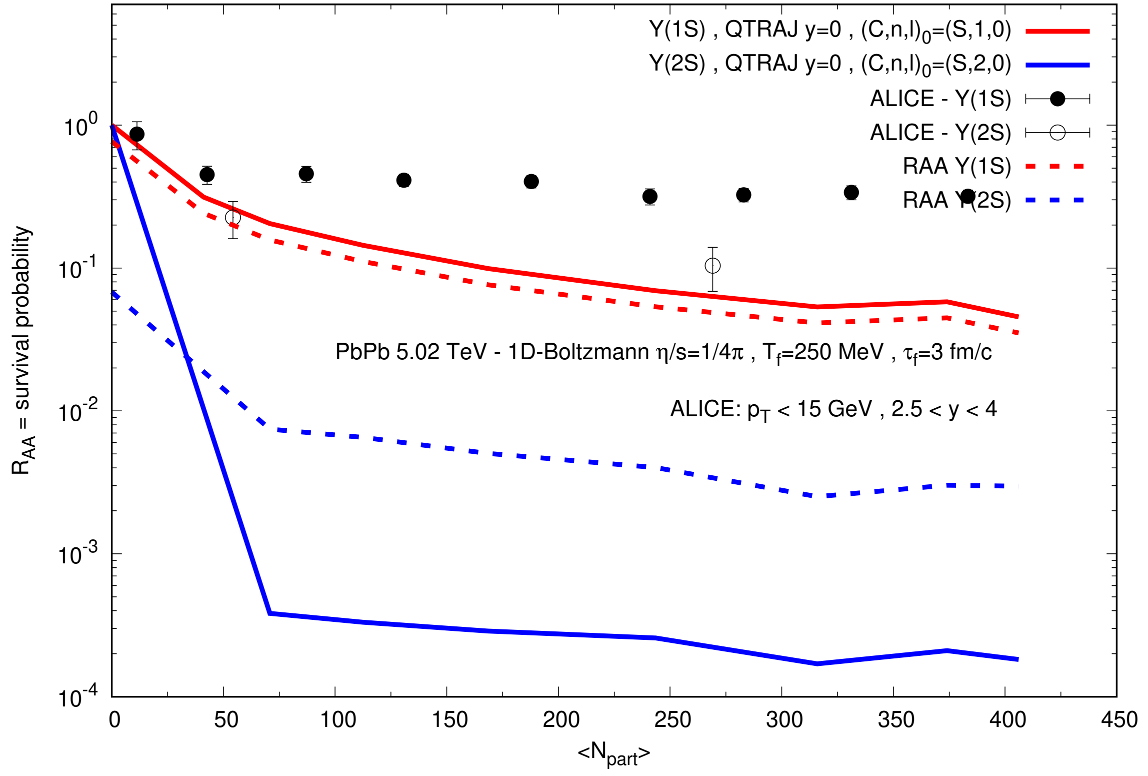
Preliminary Results

We focus also on the impact of HQ transport coeff. on Y suppression



- Lower HQ D_s → higher Y(1S) suppression ???
- Investigate different temperature dependence

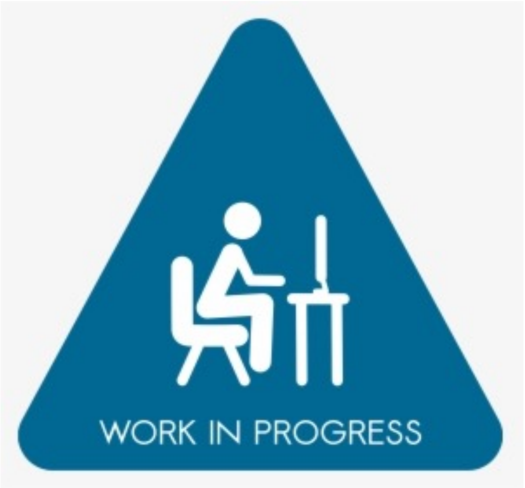
Very preliminary results !!!



- Effect of simulating 1D Boltzmann bulk evolution at fixed eta/s=1/4pi
  → **need full 3D realistic expansion** + **phase-space evolution of QQbar system through the QGP**

- Feed-down to compare with exp. data: Υ(1S) small corrections, Υ(2S) increase R_{AA} by factor 10.

Conclusions



- The suppression of quarkonium in HICs provide important insights on the properties of strong interacting matter. The challenge is to understand what are the dominant dynamical mechanisms leading to quarkonium dissolution (and recombination).

- Open Quantum Systems approaches allow to study quarkonium dissolution and recombination in one consistent framework.

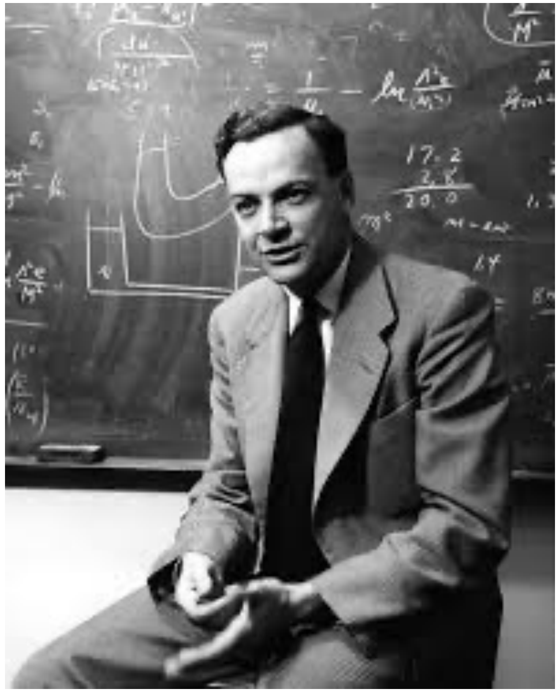
Outlooks

Development	Couple Lindblad's equation (and possible non-Markovian extensions) for quarkonium dynamics with numerical tools to simulate realistic HICs.
Evaluate	Compare OQS approach with semi-classical theoretical results.
Results	Investigate key quarkonia observables (R_{AA} , v_2) with experiments.

Future Aspects

- **<u>Quantum simulations of open quantum systems in HICs</u>**
  - Open quantum system approach offers the possibility to overcome semiclassical approximations in dynamical evolution of natural systems using **quantum algorithms**.
  - Available **quantum resources** (IBM-Q, D-Wave) give the possibility to study natural (quantum) systems and processes in full quantum device.

- Can novel approaches based on **<u>Quantum Information</u>** (entanglement and decoherence) elucidate the long-standing problem of describing hadrons in terms of their fundamental constituents and interactions?



"Nature isn't classical… if you want to do simulations, you'd better to do quantum mechanical… it's a wonderful problem, because it doesn't look so easy." – Richard Feynman (1982)

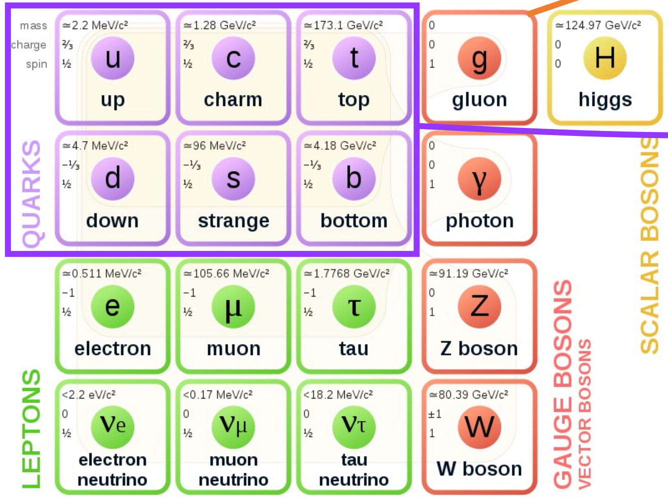
Thank you for your attention!

Backup

Quantum Chromodynamics (QCD)

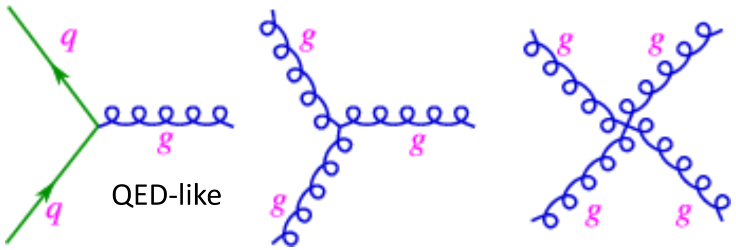
Natural units $\hbar = c = k_B = 1$

Standard Model of elementary particles



$$\mathcal{L}_{QCD} = \sum_f^{N_f} \bar{q}_f(i\gamma^\mu\partial_\mu - m_f - ig\gamma^\mu A_\mu)q_f - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

f = (anti-)quark flavor



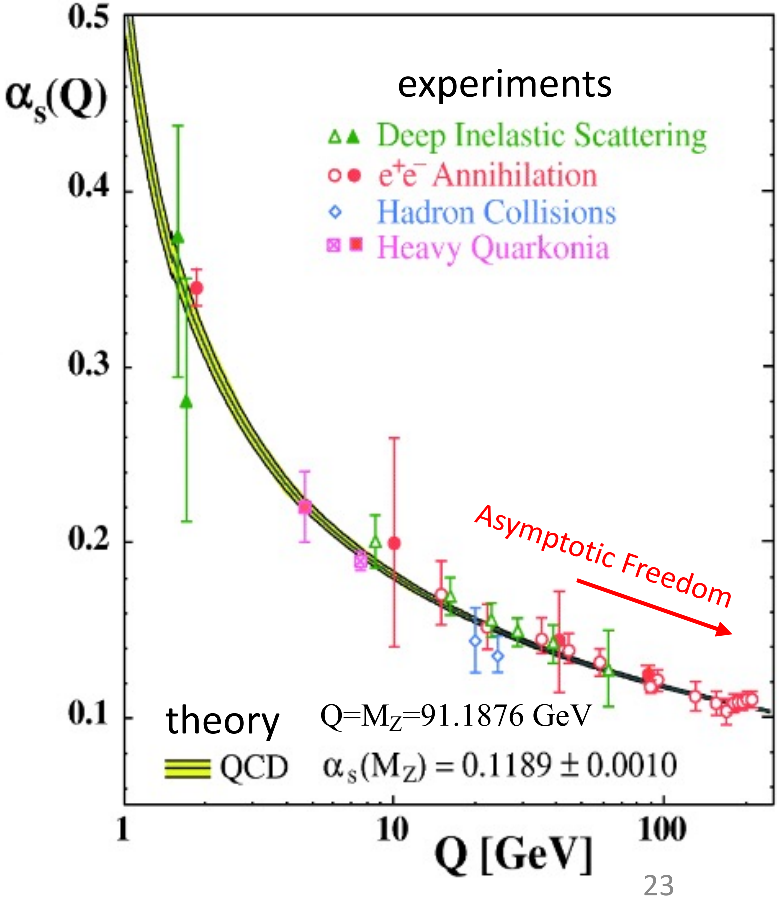
$$A_\mu = T^a A_\mu^a\,,\; F_{\mu\nu} = T^a[\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c]$$

gluon gauge field and strength tensor

$$[T^a, T^b] = i f^{abc} T^c$$ *non-abelian gauge SU(3) symmetry group*

$$q_f(x) \to U(x) q_f(x)\,,\; U(x) = \exp[-i\theta^a(x) T^a]$$

$$T^a A_\mu^a(x) \to U(x)(T^a A_\mu^a(x))U^+(x) - \frac{i}{g}(\partial_\mu U(x))U^+(x)$$



<u>The main properties of strong interaction:</u>

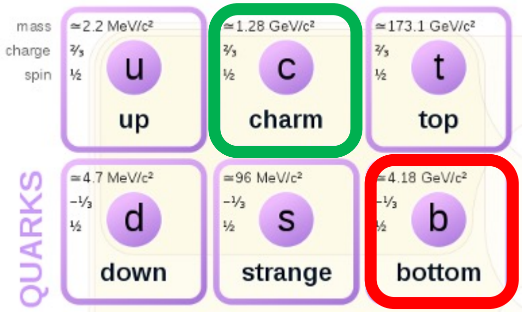
- **Confinement**: quarks and gluons are bound in colorless hadrons.
- **Asymptotic freedom**: the force strength diminishes with increasing energy (temperature) → running of QCD coupling constant.

$$\alpha_s(Q^2) = \frac{4\pi}{\left(11 - \frac{2}{3}N_f\right)\ln\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)} \Rightarrow$$

- Q ≫ Λ_{QCD} perturbative
- Q ≈ Λ_{QCD} pQCD breaks

$\Lambda_{QCD} \backsim 0.2$ GeV $\backsim 1$ fm^{-1}

Properties of heavy quarks



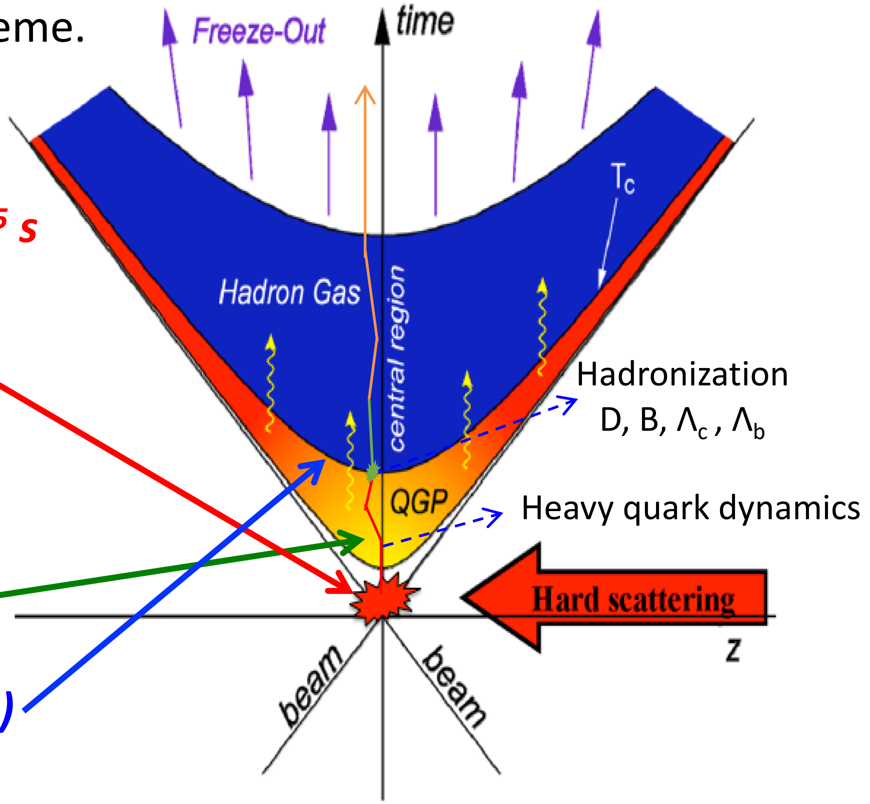
References
[J. Zhao et al. *Prog.Part.Nucl.Phys.* 114 (2020)]
[V. Greco, X. Dong *Prog.Part.Nucl.Phys.* 104 (2019)]
[R.Rapp, F.Prino J.Phys. G43 (2016)]
[R. Rapp, H. van Hees arxiv: 0903.1096 (2009)]

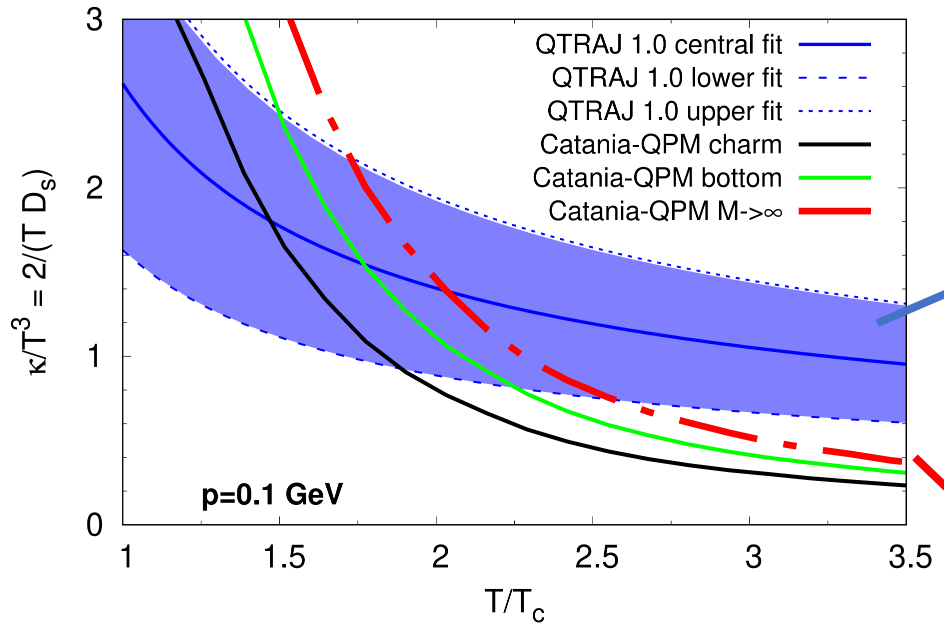
- Heavy quark (charm and bottom) masses: $M_c \backsim 1.3\ GeV$, $M_b \backsim 4.2\ GeV$

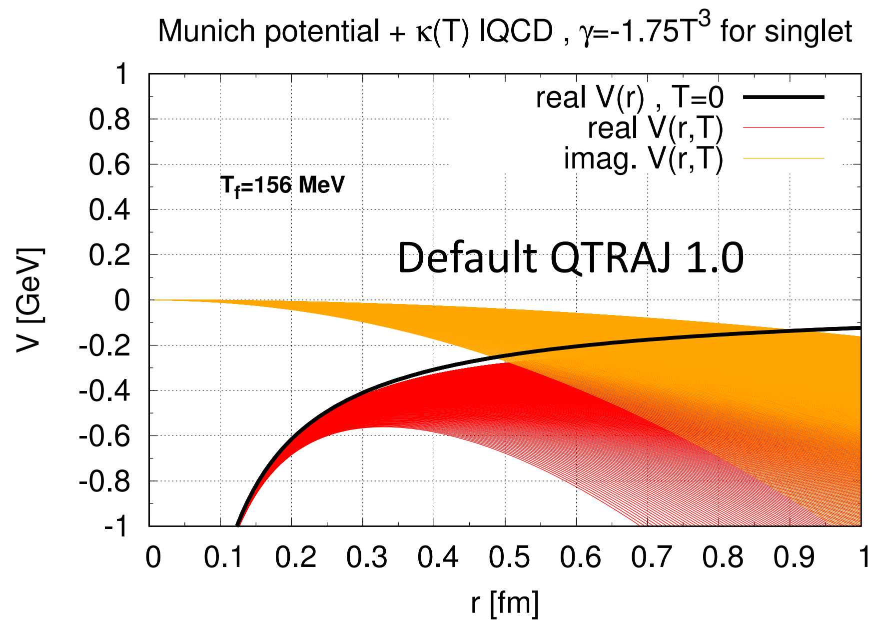
- $M_Q >> \Lambda_{QCD}$ = 0.2 GeV (Particle Physics)
  - Initial QQbar production can be computed within pQCD scheme.
  - Formation time of hard scattering processes:

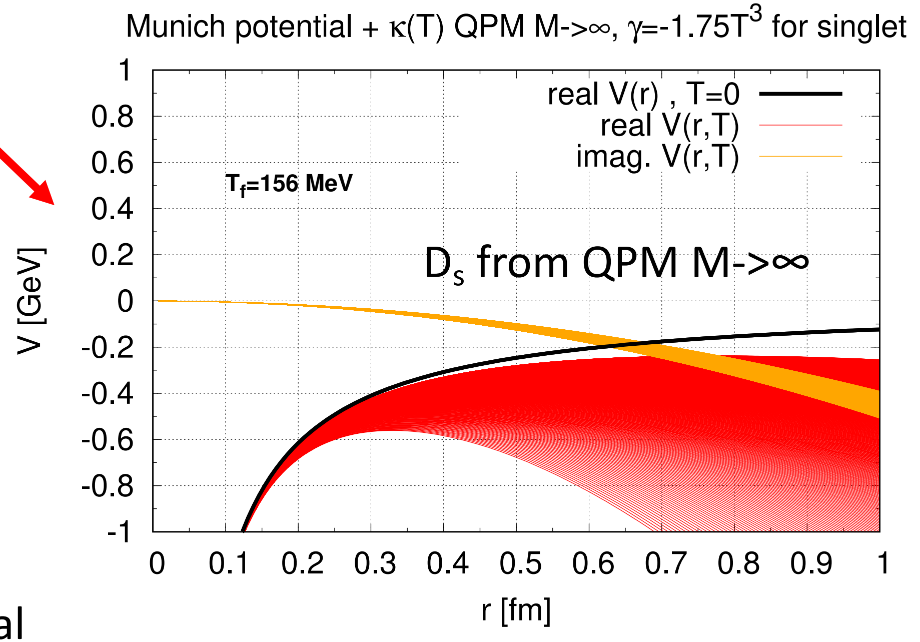
$$\tau_f = 1/(2M_Q) \leq 0.1\ fm \approx 10^{-25}\ s$$

- $M_Q >> T_{QGP} \approx$ 0.3-0.5 GeV (Plasma Physics)
  - Thermal production from the medium is negligible.
  - Created before the QGP formation: $\tau_f << \tau_{QGP} \approx 1\ fm$
  - Thermalization time ≈ QGP lifetime: $\tau_{th}(Q) \approx \tau_{QGP} >> \tau_{th}(q,g)$









$$H_{eff} = \begin{pmatrix} h_S & 0 \\ 0 & h_O \end{pmatrix} + \frac{1}{2} r^2 \begin{pmatrix} \gamma(T) - i\kappa(T) & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)}\gamma(T) - i\kappa(T) \end{pmatrix} = \frac{p^2}{M_Q} + \begin{pmatrix} V_S(r,T) & 0 \\ 0 & V_O(r,T) \end{pmatrix}$$

$$Re\,(V_S(r,T)) = -C_F \frac{\alpha_S}{r} + \frac{1}{2} r^2\, \gamma(T)$$

$$Im\,(V_S(r,T)) = -\frac{1}{2} r^2\, \kappa(T)$$

Fixed γ=-1.75T³ , change κ(T) → effect on S(O) in-medium potential

Discretized grid for the radial coordinate: $r_k = k\,\Delta r$ k=0,1,…,N_r with $\Delta r \ll a_0$ (Bohr radius)
Space dimension D = $N_E * N_r * 2$ (2 from color quantum number C=0 singlet, C=1 octet)

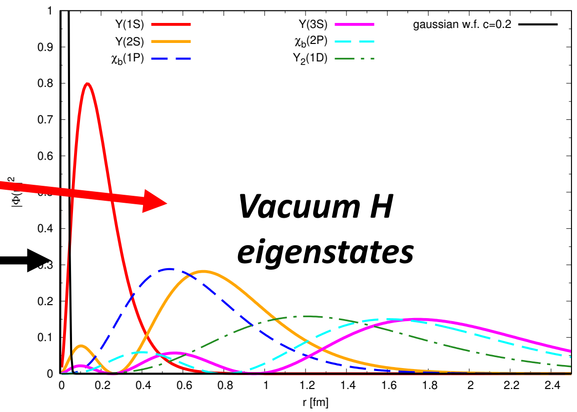
QTRAJ 1.0

Initial Conditions:

$$\rho_{Q\bar{Q}}(0) = \sum_{n,l,C} P_{n,l,C}(0)\,|\phi_{n,l,C}\rangle\langle\phi_{n,l,C}|$$

$$\rho_{Q\bar{Q}}(0) = |\psi_{Ga}\rangle\langle\psi_{Ga}| \quad , \quad \psi_{Ga}(r) \propto e^{-r^2/(ca_0)^2}$$

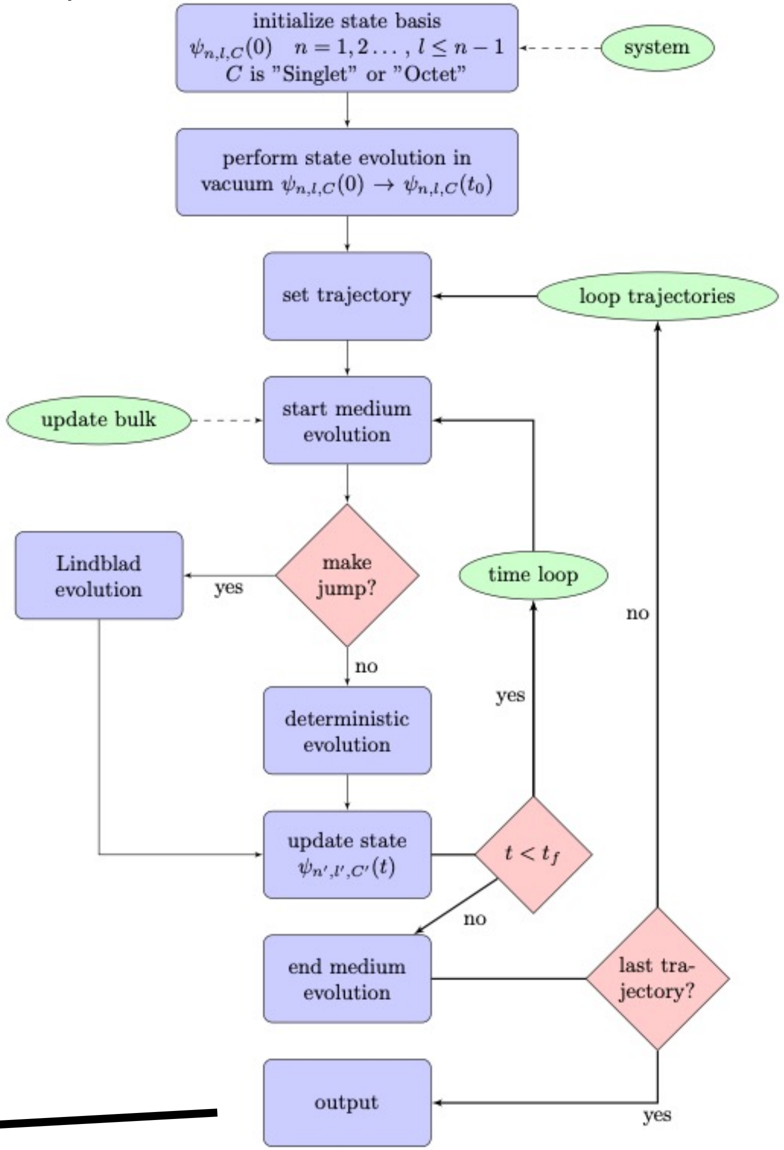
Gaussian function with $c*a_0 \ll a_0$, c=0.2
Qqbar from init. hard scattering production



Vacuum H eigenstates

In-medium evolution for each quantum trajectory wave-function:

- In the first trajectory the wave-function evolves deterministically (no jumps)
- Since H_{eff} is non-Hermitian, the norm of the wave-function decreases during the deterministic evolution → **the jump probability is confronted with the norm.**
- **Quantum jumps can be performed instantaneously using waiting-time approach.**



Output: final state overlaps
$$|\langle\psi(t_f)|\phi_{n,l,0}\rangle|^2 = \frac{1}{N_{traj}}\sum_{k=1}^{N_{traj}} \frac{|\langle\psi_k(t_f)|\phi_{n,l,0}\rangle|^2}{|\langle\psi_k(t_f)|\psi_k(t_f)\rangle|^2}$$