Perturbative methods in non-perturbative QCD

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PIANO NAZIONALE DI RIPRESA E RESILIENZA

Istituto Nazionale di Fisica Nucleare

Meeting SIM e PRIN2022
10 settembre 2024

Introduction

Research topics

- PRIN2022 research grant "Approcci analitici alla QCD non-perturbativa e proprietà del Quark-Gluon Plasma" @ UniCT within NGEU funding (p.c. 2022SM5YAS)
- Working on pert. methods for non-pert. QCD since 2017 w/ F. Siringo and more recently (2021 - ongoing) w/ D. Dudal (KU Leuven, Kortrijk, BE). Joint PhD @ UniCT & KU Leuven
- Two massive reformulations of ordinary PT:
  - the **screened massive expansion** – simple first-principles change of QCD's tree level to account for gluon mass and chiral symmetry breaking (UniCT)
  - the **dynamical model** – explaining gluon mass generation via the formation of a gauge-invariant gluon condensate of dimension 2 (KU Leuven)

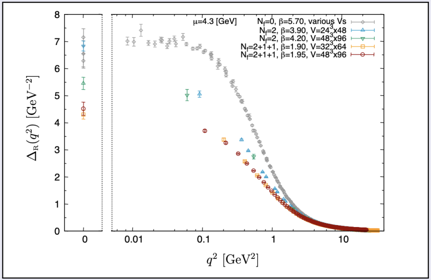
Today's talk

- Main focus: perturbative methods for **finite-temperature** non-perturbative QCD
- Show what can and cannot be obtained by a one-loop (non-ordinary) perturbative expansion
- Won't talk about dynamical model: no finite-T extension yet. Plenty to talk about finite-T screened massive expansion
- Take a step back and talk about motivation, general principles and vacuum theory

The mass of the gluon

Gluons on the lattice

- Relatively recent ('00s - '10s) **lattice calculations** have shown that the gluons **dynamically acquire** a mass $\neq 0$ in the IR



Ayala, *et al.*, Phys. Rev. D86 (2012)

- The gluon propagator **saturates** at $p = 0$ **instead of diverging** as if massless
- Seen in various gauges and for different N_f's

The mass of the gluon

Gluons by other methods

- Anticipated by both **theoretical** and **phenomenological** investigations, e.g. Cornwall, PRD 26 (1984); Field, PRD 66 (2002)

TABLE XV. Estimates of the value of the gluon mass from the literature. For Donnachie and Landshoff, the inverse of the correlation length a is quoted.

Author	Reference	Estimation method	Gluon mass
Parisi and Petronzio	[12]	$J/\psi \to \gamma X$	800 MeV
Cornwall	[8]	Various	500 ± 200 MeV
Donnachie and Landshoff	[59]	Pomeron parameters	687–985 MeV
Hancock and Ross	[61]	Pomeron slope	800 MeV
Nikolaev *et al.*	[62]	Pomeron parameters	750 MeV
Spiridonov and Chetyrkin	[63]	$\Pi^{\mathrm{e.m.}}_{\mu\nu}$, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	750 MeV
Lavelle	[64]	$qq \to qq$, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	640 $\mathrm{MeV}^2/Q(\mathrm{MeV})$
Kogan and Kovner	[67]	QCD vacuum energy, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	1.46 GeV
Field	[68]	PQCD at low scales (various)	$1.5^{+1.2}_{-0.6}$ GeV
Liu and Wetzel	[39]	$\Pi^{\mathrm{e.m.}}_{\mu\nu}$, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	570 MeV
		Glueball current, $\langle \mathrm{Tr}\, G^2_{\mu\nu} \rangle$	470 MeV
Ynduráin	[66]	QCD potential	10^{-10}-20 MeV
Leinweber *et al.*	[69]	Lattice gauge	1.02 ± 0.10 GeV
Field	This paper	$J/\psi \to \gamma X$	$0.721^{+0.016}_{-0.068}$ GeV
		$\Upsilon \to \gamma X$	$1.18^{+0.09}_{-0.29}$ GeV

Field, Phys. Rev. D66 (2002)

- **Confirmed** by the **Dyson-Schwinger** equations approach

The mass of the gluon

$m \neq 0$ forbidden by gauge invariance

QCD Lagrangian: $\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^2 + \overline{\psi}(i\not{D} - M)\psi$

The gluon mass term, $\mathcal{L}_m = \frac{1}{2} m^2 A^2$, is **not** gauge invariant

However

- Masses can receive **quantum corrections** due to the interactions
- In the absence of spontaneous symmetry breaking, this is **perturbatively forbidden** for gauge bosons
- And **non-perturbatively**?

Massive gluons in perturbation theory

- Treat the (transverse) gluons as **massive** at **tree level...**

$$\mathcal{L}_{\text{gluon, kin}} = -\frac{1}{2} A^a_\mu \, [\Delta_m^{-1}]^{\mu\nu}_{ab} \, A^b_\nu \quad ; \qquad \Delta_m(p)^{ab}_{\mu\nu} = -i\delta^{ab} \left[\frac{t_{\mu\nu}(p)}{p^2 - m^2} + \xi \, \frac{\ell_{\mu\nu}(p)}{p^2} \right]$$

- ... **without changing** the overall QCD Lagrangian

$$\mathcal{L}_{\text{QCD, kin}} = \mathcal{L}^{(\text{standard})}_{\text{QCD, kin}} + \delta\mathcal{L}_m \quad ; \qquad \mathcal{L}_{\text{QCD, int}} = \mathcal{L}^{(\text{standard})}_{\text{QCD, int}} - \delta\mathcal{L}_m$$

$$\delta\mathcal{L}_m = \frac{m^2}{2} A^a_\mu \, t^{\mu\nu} A^a_\nu$$

- That is, **add** a gluon mass term to the **kinetic** Lagrangian and **subtract** it back from the **interactions**, so that $\mathcal{L}_{\text{QCD}} \to \mathcal{L}_{\text{QCD}}$

Modified Feynman rules for PT

- The transverse gluons propagate as **massive** (by construction)

μ, a  ν, b

$$= \delta^{ab}\left[\frac{-it_{\mu\nu}(p)}{p^2 - m^2} + \frac{-i\xi\,\ell_{\mu\nu}(p)}{p^2}\right]$$

- A **new** two-gluon interaction **vertex** arises from the shift

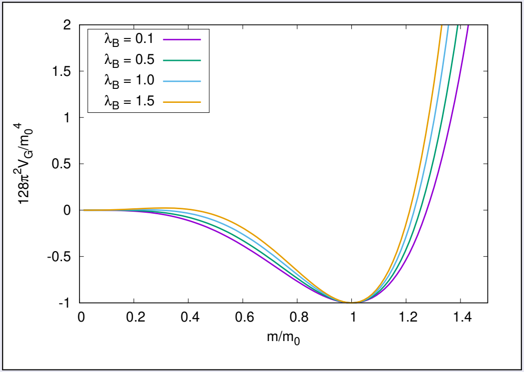
μ, a  ν, b

$$= -im^2\, t^{\mu\nu}(p)\,\delta_{ab}$$

Computational techniques

- Use the **ordinary perturbative** techniques...
- ... but include the **right number** of gluon mass vertices

Gaussian effective potential



Pure Yang-Mills GEP as a function of the gluon mass parameter for different values of the coupling constant

G. C. and F. Siringo, Phys. Rev. D 97, 056013 (2018)

Gluon propagator

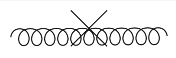
- Definition

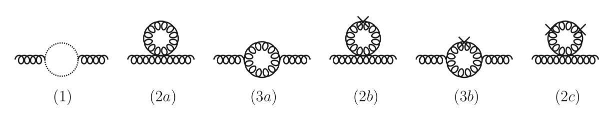
$$\Delta^{ab}_{\mu\nu}(p) = \int d^4x \, e^{ip\cdot x} \, \left\langle T\left\{A^a_\mu(x)A^b_\nu(0)\right\}\right\rangle$$

- General expression

$$\Delta(p^2) = \frac{-i}{Z_A\, p^2 - m^2 - \Pi(p^2)}$$

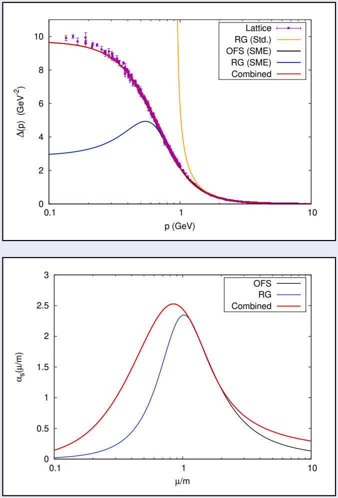
- Polarization diagrams





The Screened Massive Expansion

Pure Yang-Mills theory in the vacuum



- Mass is generated by the gluon self-interaction (i.e. in **loops**)
- **Finite** and ≈ **small** running coupling – **no Landau poles**
- **Excellent agreement** with the lattice at **low** (fixed-scale), **intermediate** (both) and **high** (RG-improved) energy scales, $m \approx 650$ MeV
- One pair of **complex-conjugate** poles in p^2 (four poles in complex-frequency space)

The Screened Massive Expansion

Full QCD in the vacuum

- Like in the gluon sector, add a **non-perturbative mass** for the **quarks...**

$$\mathcal{L}_{q,\text{kin}} = \overline{\psi}(i\partial\!\!\!/ - M)\psi \ \ ; \qquad S(p) = \frac{i}{p\!\!\!/ - M}$$

- ... **without changing** the overall quark Lagrangian

$$\mathcal{L}_{q,\text{kin}} = \mathcal{L}^{(\text{standard})}_{q,\text{kin}} + \delta\mathcal{L}_M \ \ ; \qquad \mathcal{L}_{q,\text{int}} = \mathcal{L}^{(\text{standard})}_{q,\text{int}} - \delta\mathcal{L}_M$$

$$\delta\mathcal{L}_M = (M_R - M)\,\overline{\psi}\psi$$

- The “chiral” mass M is an **independent parameter**, i.e. it is assumed **not** to be of the form $M = M_R(1 + \kappa_1\alpha_s + \cdots)$, so that it can act as a scale for **quark DMG** due to **chiral symmetry violation**

Full QCD: quark propagator

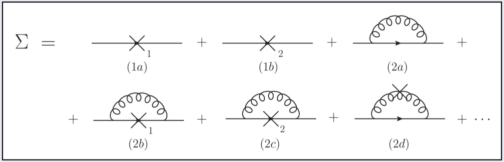
- Definition

$$S(p) = \int d^4x\, e^{ip\cdot x} \,\left\langle T\left\{\psi(x)\overline{\psi}(0)\right\}\right\rangle$$

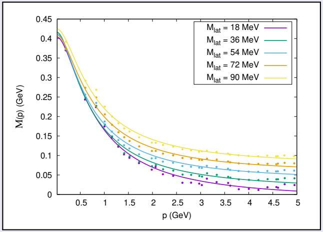
- General expression

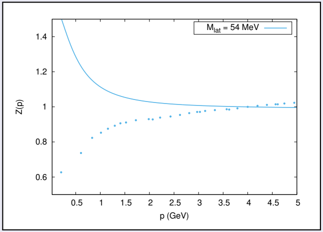
$$S(p) = \frac{i}{\not{p} - M - \Sigma(p)}$$

- Self-energy diagrams



Full QCD in the vacuum



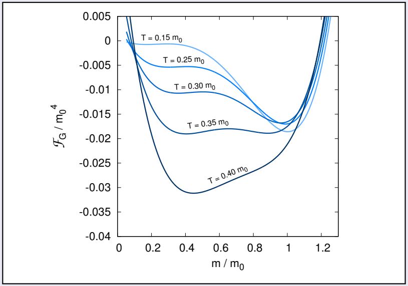


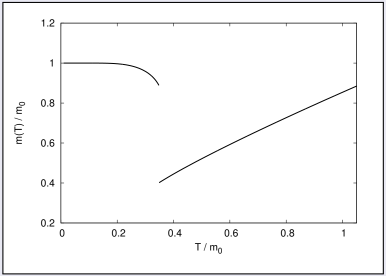
- **Quark mass function** in **excellent agreement** with the lattice for $M \approx 200 - 400$ MeV, depending on M_{lat} and the resummation scheme
- **Wrong behavior** of 1-loop **quark** Z **function** – needs higher orders, as confirmed by using the c.c. scheme. May change at finite T
- Unfortunately, quite a few free parameters (chiral mass, current mass, 2 renorm. constants, gluon mass if not already fixed)

The Screened Massive Expansion

Finite temperature: GEP

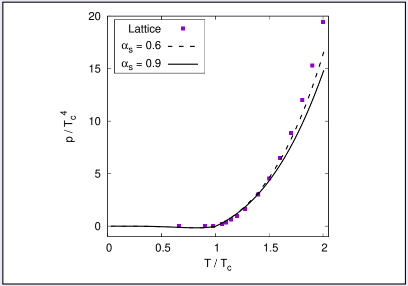
$$\mathcal{F}_G(m,T) = -\frac{1}{\beta\mathcal{V}_3}\ln\int\mathcal{D}\mathscr{F}\,e^{-S_m} + \frac{1}{\beta\mathcal{V}_3}\,\langle S'_{\text{int.}}\rangle_m\ , \quad \mathcal{F}_G(m,T)\geq\mathcal{F}(T)$$

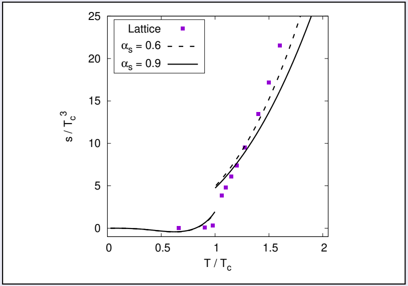




Finite temperature: GEP

$$p = -\left[\mathcal{F}_G(T, m(T)) - \mathcal{F}_G(0, m_0)\right] \ , \qquad s = -\frac{d}{dT}\mathcal{F}_G(T, m(T))$$





$$T_c \approx 0.35 m_0 \approx 230 \text{ MeV}$$

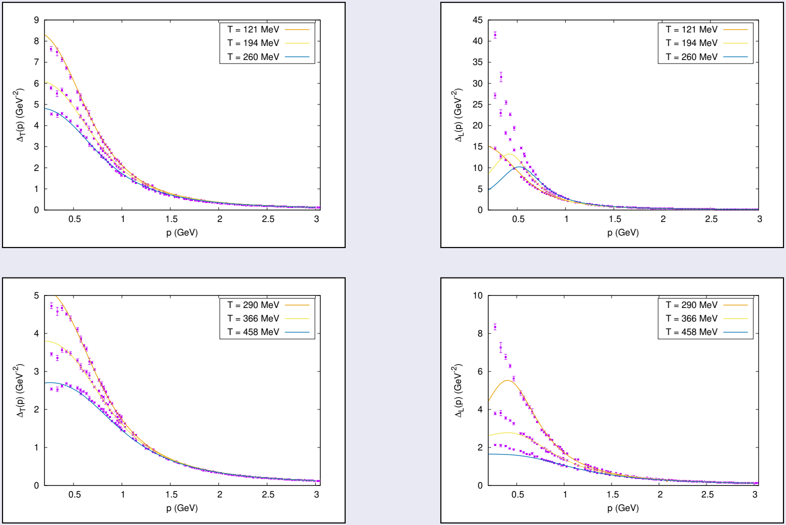
Finite temperature: gluon propagator

$$\Pi^{(a-c)}_{L,T}(p) = \left[\frac{3p^4}{2m^4} - 1\right] I^{00}_{L,T}(p) + \left[4 + \frac{3p^4 + 8m^2p^2 + 4m^4}{2m^4}\right] I^{mm}_{L,T}(p) +$$
$$- \left[\frac{3p^4 + 4m^2p^2 + m^4}{m^4}\right] I^{m0}_{L,T}(p) + \frac{2p^2(p^2 + 2m^2)}{m^2}\, I^{mm}(p) +$$
$$- \frac{2p^2(p^2 + m^2)}{m^2}\, I^{0m}(p) - \left[\frac{2p^2 + 3m^2}{m^2}\right] J_m + \left[\frac{2p^2 + m^2}{m^2}\right] J_0 +$$
$$- \left[8m^2 + \frac{(p^2 + 2m^2)^2}{m^2}\right] \partial I^{mm}_{L,T}(p) + \frac{(p^2 + m^2)^2}{m^2}\, \partial I^{m0}_{L,T}(p) +$$
$$- 2p^2(p^2 + 4m^2)\, \partial I^{mm}(p) + (p^2 + m^2)^2\, \partial I^{m0}(p) + (p^2 + 3m^2)\, \partial J_m .$$

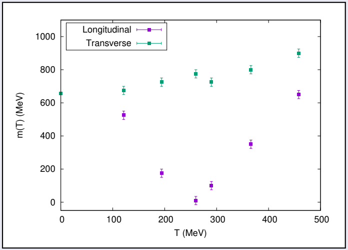
$$I^{\alpha\beta}(y,\omega) = \int_0^\infty \frac{x\mathrm{d}x}{8\pi^2 y} \left\{ \frac{n(\epsilon_{x,\alpha})}{\epsilon_{x,\alpha}}\ \mathrm{Re}\ L_\beta(\omega + i\epsilon_{x,\alpha};\, y, x) + \ \alpha \leftrightarrow \beta \right\}$$

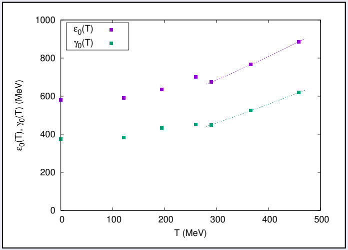
$$L_\alpha(z; y, x) = \log \frac{z^2 + \epsilon^2_{x+y,\alpha}}{z^2 + \epsilon^2_{x-y,\alpha}}$$

Finite temperature: gluon propagator



Finite temperature: gluon propagator

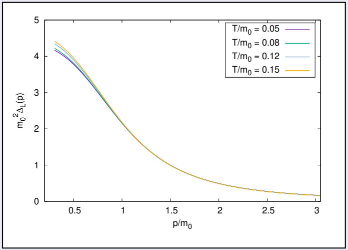


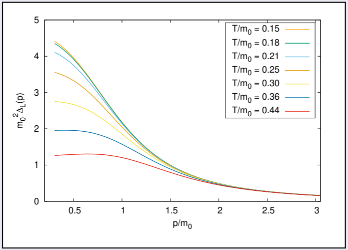


- Need to separately fit the mass parameter for the two components
- Gluon mass parameter and $p = 0$ poles as a function of T. Poles from 3D-transverse propagator
- For $T > T_c$, real part of the poles (mass) $\approx$ HTL + condensate ($\alpha_s \approx 0.42$). Im. part (damping?) not compatible with usual $O(g^3)$ value and topology

The Screened Massive Expansion

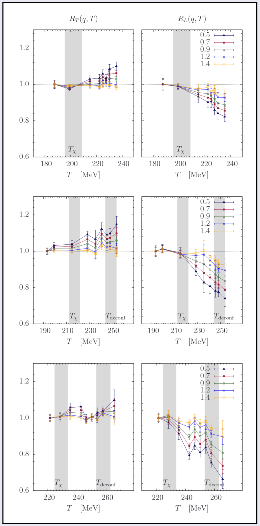
Finite temperature: long. gluon propagator and phase diagram





- Non-monotonic behavior of longitudinal gluon propagator is a stable feature: does not depend on T-dependence of parameters (T_c does)
- Maximum in T of long. propagator @ fixed p marks phase transition
- Can we use this fact to qualitatively predict the phase diagram of **full QCD**?

Finite temperature: long. gluon propagator and phase diagram



- Even at $\mu = 0$ not a lot of lattice data for the gluon propagator 1. below and across the phase transition, 2. for SU(3), 3. with full momentum dependence
- $\Longleftarrow$ basically the best available: Aouane *et al.*, Phys. Rev. D 87, 114502 (2013), arXiv:1212.1102. $T_{\rm min}$ too large, too few data points below T_c (also, uncertain qualitative consistency?)

Finite temperature: long. gluon propagator and phase diagram

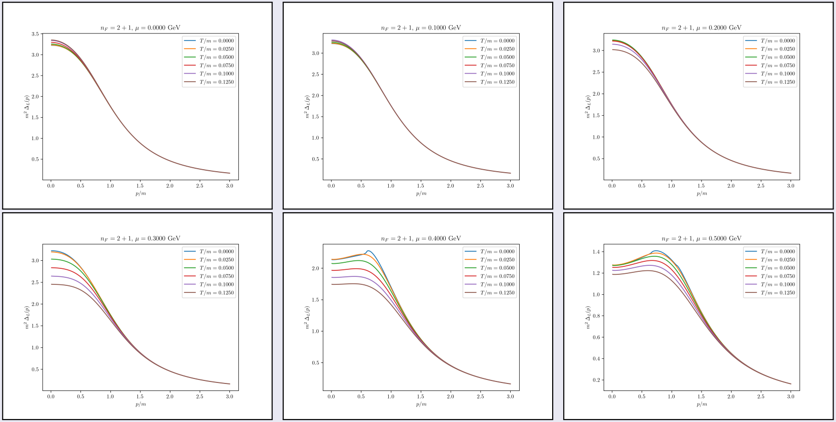
Strategy

- Use a simple model for quark dynamics: what counts for the gluon propagator is the quark's effective IR mass

$$S(p) = \frac{i}{\not{p} - M} \qquad \text{with} \qquad M \sim 300\text{-}400 \text{ MeV}$$

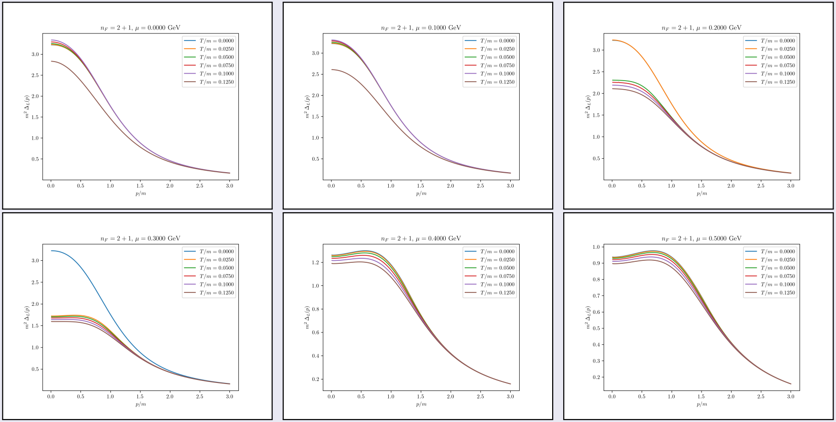
- Introduce a physically meaningful number of quarks – $N_f = 2 + 1$ or $N_f = 2 + 1 + 1$ – with uniform (i.e. baryonic) chemical potential
- Fix parameters at $(T, \mu) = (0, 0)$ and study maximum of $\Delta_L(p, T, \mu)$ with respect to T at $p \approx 0$ and fixed μ

Finite temperature: long. gluon propagator and phase diagram



$$(m_{u,d}, m_s) = (350, 450)\ \text{MeV}$$

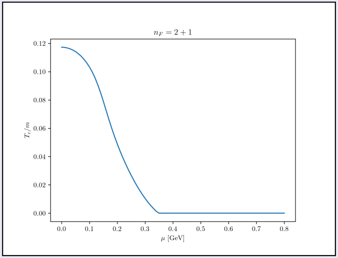
Finite temperature: long. gluon propagator and phase diagram

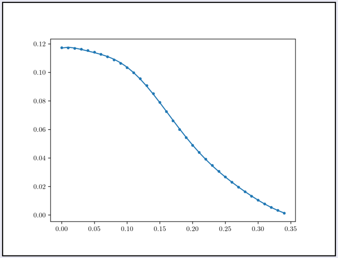


$$(m_{u,d}, m_s) = (350, 450)\ \text{MeV @ } T < T_c\,,\ (3, 90)\ \text{MeV @ } T > T_c$$

The Screened Massive Expansion

Finite temperature: long. gluon propagator and phase diagram





- Expected shape of phase diagram, independent of (constant) parameters
- $\mu_c \approx M_{\text{min}}$, the **chiral** mass of the lightest quarks: simple interpretation. Heavier quarks essentially play no role (qualitatively)
- Downsides: no quantitative prediction for T_c or κ, no distinction between types of transition
- Change in concavity hints to a change in type of transition?

Next steps

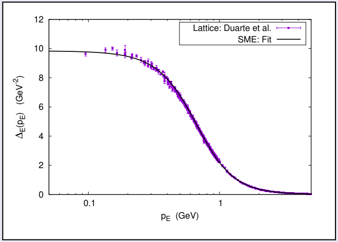
- The quark propagator at finite T and μ is half computed.
- We need to make contact with the phenomenology. A good way to make use of propagators could be to provide input for quasi-particle models.
- For $T > T_c$ and $\mu = 0$ *some* (not many, but still *some*) lattice data for the unquenched gluon propagator and the quenched quark propagator are available. Use them to fit momentum-dependent masses for quasi-particles?

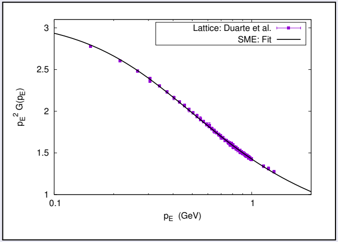
Thank you

Backup Slides

The Screened Massive Expansion

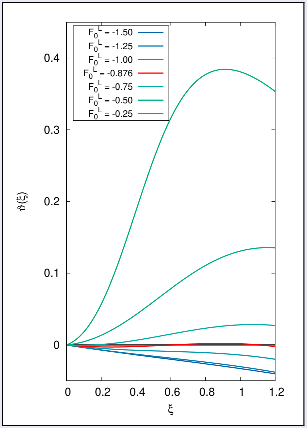
Pure Yang-Mills theory in the vacuum: fixed scale, full fit





- **Gluon DMG** in the IR by a **non-trivial mechanism**: the gluon mass cancels at tree level
- **Excellent agreement** with the **lattice** for $m = 0.654$ GeV (full fit of the data)
- Need to fit a **spurious** free parameter, e.g. F_0
- **Complex-conjugate poles** in the gluon propagator, pointing to **confinement**

Pure Yang-Mills theory in the vacuum: fixed scale, optimized



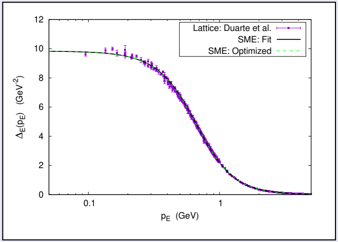
- Spurious free parameter **fixed** by **minimizing the gauge-dependence** of the **phases** of the **residues** of the **gluon poles**
- Phase gauge-parameter independent to within **3 parts in 1000**

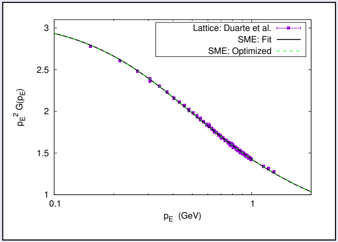
$$|\theta(\xi) - \theta(0)| < 2.7 \cdot 10^{-3} \quad \text{at}$$

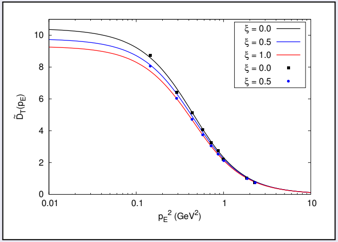
$$F_0 = -0.876$$

Pure Yang-Mills theory in the vacuum: fixed scale, optimized

- Optimized results **indistinguishable** from a full fit for $m = 0.656$ GeV
- **Predictivity** of the method **restored**: first-principles SME

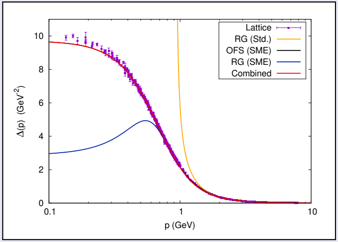


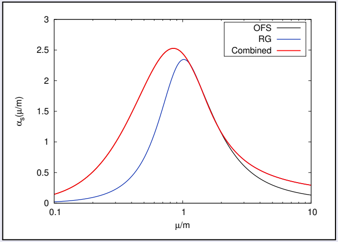




The Screened Massive Expansion

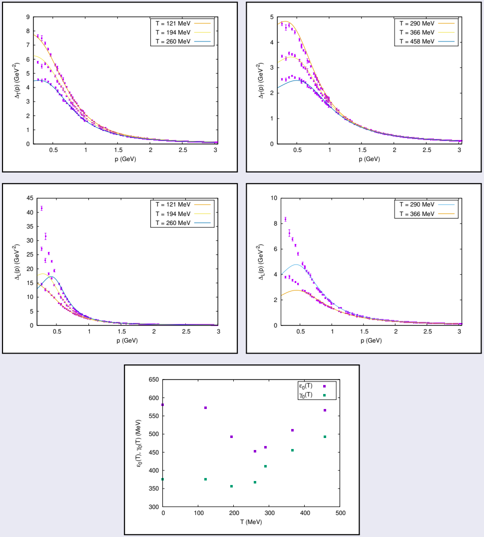
Pure Yang-Mills theory in the vacuum: RG improvement





- Renormalization in the **MOM-Taylor scheme**
- **Finite** and **moderately small** running coupling – **no Landau poles**
- **Excellent agreement** with the lattice at **intermediate** to **high** energies
- Coupling **too large** at $p \approx m$: needs **2 loops** at low energies
- **Optimization** by **matching** RG and fixed scale results

Pure Yang-Mills theory at finite temperature



- **Semi-quantitative** predictions
- **Excellent agreement** with the lattice in the **3D-transverse** sector
- **Good agreement** with the lattice in the **3D-longitudinal** sector at **low T** and **large p**
- **Sub-optimal** in the **3D-longitudinal** sector at **high T** and **small p**
- Lattice-aided predictions on the gluon **dispersion relations** and **pole position** as a function of T, displaying a **confining pattern**

Trying to explain gluon DMG

- **Condensates** – i.e. VEVs of the form $\langle \mathcal{O}(0) \rangle$ – are known to play a **major role** in dynamical mass generation
- A **quadratic** condensate of the form $\langle A^2 \rangle$ has the right dimensions to produce a gluon mass...
- ... but its introduction in the QCD action spoils **gauge symmetry** $\Longrightarrow$ look for a gauge-invariant analogue

$$\boxed{\left\langle (A^h)^2 \right\rangle}$$

where $\quad A^U = U^\dagger \left(A_\mu + \frac{i}{g}\,\partial \right) U, \quad h : \min_{h \in SU(3)} \int d^4x\, (A^h)^2$

equivalently, $\quad \partial \cdot (A^h) = 0$

The Dynamical Model

Trying to explain gluon DMG

- The **effect** of a non-zero $\langle (A^h)^2 \rangle$ on the IR gluodynamics can be investigated by making use of the **LCO formalism**
- A^h is a **non-local** operator: perturbatively,

$$A^h_\mu = \left(\delta^\nu_\mu - \frac{\partial_\mu \partial^\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{\partial \cdot A}{\partial^2}, A_\nu\right] + \frac{ig}{2}\left[\frac{\partial \cdot A}{\partial^2}, \partial_\nu \frac{\partial \cdot A}{\partial^2}\right] + \cdots\right)$$

- However, its LCO action can be **localized** in any gauge...

$$S_{\text{LCO}} = S_{\text{FP}} + \int d^4x \left(\tau^a \partial \cdot A^{h,a} + \overline{\eta}^a \partial \cdot D(A^h)\eta^a\right) + \\ + \int d^4x \left\{\frac{1}{2\zeta}(\delta\sigma)^2 - \frac{1}{2\zeta}(\sigma + \delta\sigma)(A^h)^2 + \frac{1}{8\zeta}[(A^h)^2]^2\right\}$$

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- ... and contains a **gluon mass term** $m^2 = -\frac{\sigma}{\zeta} = -\frac{\langle (A^h)^2 \rangle}{2\zeta}$

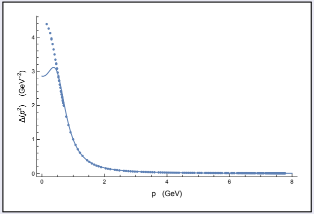
Computational techniques

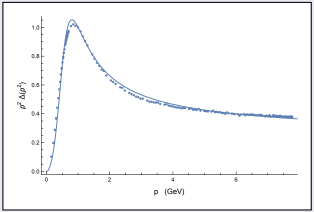
- Find the **value** of the condensate by minimizing its **effective potential** $\Gamma[\sigma]$; this introduces a renormalization constant ζ
- Use the **reduction of couplings** to express ζ as a power series in the coupling g, $\zeta = \frac{1}{g^2}\sum_{n=0}^{+\infty} \zeta_n\, g^{2n} \quad \Longrightarrow$

$$\zeta = \frac{\zeta_0}{g^2} + \zeta_1 + \cdots \ , \qquad \zeta_0 = \frac{9N_A}{13N}\,,\ \zeta_1 = \frac{161N_A}{52\cdot 16\pi^2}$$

- Apply the **ordinary** perturbative methods to the condensate's LCO action – by which the gluon is **massive** $(\sigma \neq 0)$ – ...
- ... to compute e.g. the **gluon propagator** $\Longleftarrow$ done in the **Landau gauge**, where the calculations are the simplest

Pure Yang-Mills theory results in the vacuum





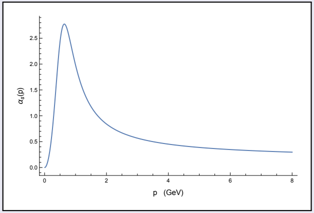
- **Renormalized** in the Dynamically Infrared-Safe (**DIS**) scheme

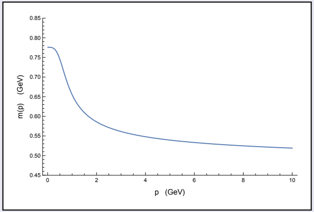
$$Z_{g^2} Z_A Z_c^2 = 1 \ , \quad Z_2 Z_c = 1 + \frac{5}{8} \frac{N g^2}{16\pi^2}$$

- Uses a **single** free parameter: the coupling α_s at the initial scale μ_0
- The gluon mass parameter m^2 is **computed** via the **gap equation**
- **RG-improved** propagators in **excellent agreement** with the lattice data, except for the deep IR $p \lesssim 0.5$ GeV (expected at 1 loop)

The Dynamical Model

Pure Yang-Mills theory results in the vacuum





- As in the SME, the **DM-DIS** running coupling is **finite** (no Landau poles) and **moderately small**
- The running gluon mass displays a typical **saturating** behavior...

$$m(\mu_0 = 1 \text{ GeV}) = 0.655 \text{ GeV}$$
$$m(0) \approx 0.78 \text{ GeV}$$

- ... and slowly **decreases to zero** in the UV

$$m^2(\mu) \sim [\ln \mu^2]^{-\frac{9}{44}} \quad (\mu \to \infty)$$

The QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - M)\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^a_{bc}\, A^b_\mu A^c_\nu\ , \qquad D_\mu = \partial_\mu - igA^a_\mu T_a$$

$$\begin{aligned}\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\,\partial_\mu A^a_\nu\ (\partial^\mu A^{a\,\nu} - \partial^\nu A^{a\,\mu}) + \overline{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \\ - g\, f^a_{bc}\,\partial_\mu A^a_\nu\, A^{b\,\mu} A^{c\,\nu} - \frac{1}{4}\, g^2 f^a_{bc} f^a_{de}\, A^b_\mu A^c_\nu A^{d\,\mu} A^{e\,\nu} + \\ + g\,\overline{\psi}\gamma^\mu T_a \psi A^a_\mu\end{aligned}$$

Gauge symmetry

$$A_\mu^U = U\left(A_\mu + \frac{i}{g}\,\partial_\mu\right)U^\dagger\ , \qquad \psi^U = U\psi$$

$$\mathcal{L}_{\mathrm{QCD}}[A^U,\psi^U] = \mathcal{L}_{\mathrm{QCD}}[A,\psi]$$
$$S_{\mathrm{QCD}}[A^U,\psi^U] = S_{\mathrm{QCD}}[A,\psi]$$

$$J_a^\mu = \overline{\psi}\gamma^\mu T_a\psi + f_{abc}\,F^{b\,\mu\nu}A_\nu^c\ , \qquad \partial_\mu J_a^\mu = 0\ .$$

The Faddeev-Popov Lagrangian

$$\mathcal{L}_{\mathrm{FP}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - M)\psi - \frac{1}{2\xi}\,(\partial \cdot A)^2 + \partial^\mu \overline{c}^a D_\mu c^a$$

$$D_\mu c^a = \partial_\mu c^a + g f^a_{bc}\, A^b_\mu c^c$$

$$\begin{aligned}\mathcal{L}_{\mathrm{FP}} = &-\frac{1}{2}\,\partial_\mu A^a_\nu\,\left(\partial^\mu A^{a\,\nu} - \partial^\nu A^{a\,\mu}\right) - \frac{1}{2\xi}\,\partial^\mu A^a_\mu \partial^\nu A^a_\nu + \\ &+ \partial^\mu \overline{c}^a \partial_\mu c^a + \overline{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \\ &- g f^a_{bc}\,\partial_\mu A^a_\nu\, A^{b\,\mu} A^{c\,\nu} - \frac{1}{4}\, g^2 f^a_{bc} f^a_{de}\, A^b_\mu A^c_\nu A^{d\,\mu} A^{e\,\nu} + \\ &+ g f^a_{bc}\,\partial^\mu \overline{c}^a A^b_\mu c^c + g\,\overline{\psi}\gamma^\mu T_a \psi A^a_\mu\end{aligned}$$

BRST symmetry

$$\delta A^a_\mu = \epsilon D_\mu c^a \;, \qquad \delta\psi = i\epsilon g\, c^a T_a \psi \;,$$

$$\delta c^a = -\frac{1}{2}\epsilon g f^a_{bc}\, c^b c^c \;, \qquad \delta\overline{c}^a = \epsilon B^a \;,$$

$$\delta B^a = 0$$

$$\mathcal{S}_{\mathsf{FP}}[A+\delta A, \psi+\delta\psi, c+\delta c, \overline{c}+\delta\overline{c}, B+\delta B] = S_{\mathsf{FP}}[A, \psi, c, \overline{c}, B]$$

$$j^\mu_B = -F^{a\,\mu\nu} D_\nu c^a + B^a D^\mu c^a - g\,\overline{\psi}\gamma^\mu T_a \psi\, c^a + \frac{g}{2} f^a_{bc}\, \partial^\mu \overline{c}^a c^b c^c$$

$$\partial_\mu j^\mu_B = 0$$

BRST symmetry

$$\delta = \epsilon s \quad \Longrightarrow \quad s^2 = 0$$

$$Q_B = \int d^3x \, j^0_B \quad \Longrightarrow \quad Q_B^2 = 0$$

$$[Q_B, F]_{\mp} = -isF$$

$$Q_a = \int d^3x \, j^0_a \quad \Longrightarrow \quad Q^a = \frac{1}{g} \oint d^2x_i \, F^{0i} + \frac{1}{g} \int d^3x \, \{Q_B, D_0 \overline{c}^a\}$$

BRST cohomology

$$Q_c = \int d^3x\, j_c^0 \quad : \quad (c, \overline{c}) \to (e^{-\lambda} c, e^{\lambda} \overline{c})$$

$$\mathcal{H}_{\text{phys}} = (\text{Ker}\{Q_B\} \cap \text{Ker}\{Q_c\}) / \text{Im}\{Q_B\}$$

$$Q_B\, |\text{phys}\rangle = 0\ , \qquad\qquad Q_c\, |\text{phys}\rangle = 0$$

Slavnov-Taylor identities

$$Q_B \left|0\right\rangle = 0 \quad \Longrightarrow \quad \left\langle 0\right| [Q_B, \mathcal{O}]_{\mp} \left|0\right\rangle = -i \left\langle s\mathcal{O}\right\rangle = 0$$

- Examples

$$0 = \left\langle T\left\{ s\left(B^a(x)\overline{c}^b(y) \right) \right\} \right\rangle = \left\langle T\left\{ B^a(x)B^b(y) \right\} \right\rangle$$

$$0 = \left\langle T\left\{ s\left(A^a_\mu(x)\overline{c}^b(y) \right) \right\} \right\rangle = \left\langle T\left\{ D_\mu c^a(x)\overline{c}^b(y) + A^a_\mu(x)B^b(y) \right\} \right\rangle$$

$$\begin{aligned} 0 = \left\langle T\left\{ s(A^a_\mu(x)A^b_\nu(y)\overline{c}^c(z)) \right\} \right\rangle &= \langle T\{D_\mu c^a(x)A^b_\nu(y)\overline{c}^c(z)\}\rangle + \\ &+ \langle T\{A^a_\mu(x)D^b_\nu c(y)\overline{c}^c(z))\}\rangle + \\ &+ \langle T\{A^a_\mu(x)A^b_\nu(y)B^c(z)\}\rangle \end{aligned}$$

$$\begin{aligned} 0 = \langle T\{s(\psi(x)\overline{\psi}(y)\overline{c}^b(z))\}\rangle &= ig\langle T\{T_a\psi(x)\overline{\psi}(y)\, c^a(x)\overline{c}^b(z)\}\rangle + \\ &- ig\langle T\{\psi(x)\overline{\psi}(y)T_a\, c^a(y)\overline{c}^b(z)\}\rangle + \\ &+ \langle T\{\psi(x)\overline{\psi}(y)B^b(z)\}\rangle \end{aligned}$$

Slavnov-Taylor identities

- Consequences

$$\left\langle T\left\{B^a(x)B^b(0)\right\}\right\rangle = 0$$

$$\left\langle T\left\{D_\mu c^a(x)\bar{c}^b(0)\right\}\right\rangle = -\left\langle T\left\{A^a_\mu(x)B^b(0)\right\}\right\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot x}\, \frac{p_\mu}{p^2}\, \delta^{ab}$$

$$\left\langle T\left\{A^a_\mu(x)A^b_\nu(0)\right\}\right\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot x}\, \left[\Delta^{ab}_T(p)\, t_{\mu\nu}(p) + \frac{-i\xi}{p^2}\, \delta^{ab}\, \ell_{\mu\nu}(p)\right]$$

etc. ...

Nielsen identities

$$\frac{\partial}{\partial\xi}\langle\mathcal{O}\rangle = \frac{i}{2}\int d^4x\ \langle T\left\{\mathcal{O}B^a(x)B^a(x)\right\}\rangle =$$
$$= \mp\frac{i}{2}\int d^4x\ \langle T\left\{(s\mathcal{O})\ B^a(x)\overline{c}^a(x)\right\}\rangle$$

- Gluon propagator:

$$\frac{\partial}{\partial\xi}\left\langle T\left\{A^a_\mu(x)A^b_\nu(y)\right\}\right\rangle = -\frac{i}{2}\int d^4z\left[\left\langle T\left\{D_\mu c^a(x)A^b_\nu(y)B^c(z)\overline{c}^c(z)\right\}\right\rangle + \right.$$
$$\left. + \left\langle T\left\{A^a_\mu(x)D_\nu c^b(y)B^c(z)\overline{c}^c(z)\right\}\right\rangle\right]$$

Nielsen identities

- Equivalently (transverse):

$$\frac{\partial}{\partial\xi}\,\Delta_T^{-1}(p,\xi) = F_T(p,\xi)\,\Delta_T^{-1}(p,\xi)$$

$$\mathcal{F}^{ab}_{\mu\nu}(p) = \left[F_T(p)\,\Delta_T(p)\;t_{\mu\nu}(p) + \frac{i}{p^2}\;\ell_{\mu\nu}(p)\right]\,\delta^{ab}$$

$$\mathcal{F}^{ab}_{\mu\nu}(x-y) = -\frac{i}{2\xi}\int d^4z\,\left\{\,\left\langle T\left\{D_\mu c^a(x)A^b_\nu(y)\partial\cdot A^c(z)\overline{c}^c(z)\right\}\right\rangle + \right.$$
$$\left. +(x\leftrightarrow y, \mu\leftrightarrow\nu, a\leftrightarrow b)\right\}$$

Nielsen identities

- Consequences

$$\Delta_T^{-1}(p_0(\xi),\xi) = 0 \qquad \Longrightarrow$$

$$\begin{aligned}
0 &= \frac{d}{d\xi}\,\Delta_T^{-1}(p_0(\xi),\xi) = \\
&= \frac{\partial}{\partial\xi}\,\Delta_T^{-1}(p_0(\xi),\xi) + \frac{\partial}{\partial p}\Delta_T^{-1}(p_0(\xi),\xi)\,\frac{dp_0}{d\xi}(\xi) = \\
&= F_T(p_0(\xi),\xi)\,\Delta_T^{-1}(p_0(\xi),\xi) + \frac{\partial}{\partial p}\Delta_T^{-1}(p_0(\xi),\xi)\,\frac{dp_0}{d\xi}(\xi) = \\
&= \frac{\partial}{\partial p}\Delta_T^{-1}(p_0(\xi),\xi)\,\frac{dp_0}{d\xi}(\xi) \qquad \Longrightarrow \qquad \frac{dp_0}{d\xi} = 0
\end{aligned}$$

Standard perturbation theory: set up

$$S_{\mathrm{FP}} = S_0 + S_{\mathrm{int.}}$$

$$S_0 = \lim_{g\to 0} S_{\mathrm{FP}}\ , \qquad S_{\mathrm{int.}} = S_{\mathrm{FP}} - S_0$$

$$S_0 = \int d^4x \left\{ -\frac{1}{2}\,\partial_\mu A^a_\nu(\partial^\mu A^{a\,\nu} - \partial^\nu A^{a\,\mu}) - \frac{1}{2\xi}\,\partial^\mu A^a_\mu \partial^\nu A^a_\nu + \right.$$

$$\left. + \overline{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \partial^\mu \overline{c}^a \partial_\mu c^a \right\}$$

$$S_{\mathrm{int}} = \int d^4x \left\{ -g f^a_{bc}\,\partial_\mu A^a_\nu\, A^{b\,\mu} A^{c\,\nu} - \frac{1}{4}\, g^2 f^a_{bc} f^a_{de}\, A^b_\mu A^c_\nu A^{d\,\mu} A^{e\,\nu} + \right.$$

$$\left. + g\,\overline{\psi}\gamma^\mu T_a \psi A^a_\mu + g f^a_{bc}\,\partial^\mu \overline{c}^a A^b_\mu c^c \right\}$$

Standard perturbation theory: Feynman rules

- Zero-order gluon propagator

μ, a  $\nu, b \;=\; -\frac{i}{p^2}\,\delta^{ab}\,\left[t_{\mu\nu}(p) + \xi\,\ell_{\mu\nu}(p)\right]$

- Zero-order ghost propagator

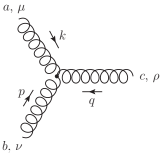
a  $b \;=\; \frac{i}{p^2}\,\delta^{ab}$

- Zero-order quark propagator

 $\;=\; \frac{i}{\not{p} - M}\,\mathbb{1}_{3\times 3}$

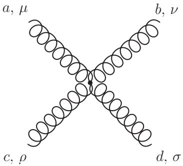
Standard perturbation theory: Feynman rules

- 3-gluon vertex



$$= g\, f^{abc} \left[\eta^{\mu\nu} (k-p)^\rho + \eta^{\nu\rho} (p-q)^\mu + \eta^{\rho\mu} (q-k)^\nu \right]$$

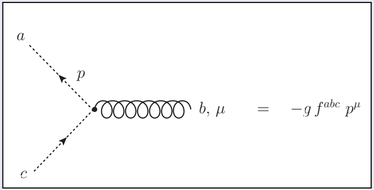
- 4-gluon vertex



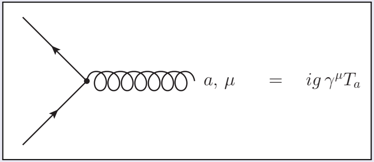
$$\begin{aligned} = \quad & -ig^2 \left[f^{abe} f^{cde} \left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} \right) + \right. \\ & + f^{ace} f^{bde} \left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} \right) + \\ & \left. + f^{ade} f^{bce} \left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} \right) \right] \end{aligned}$$

Standard perturbation theory: Feynman rules

- ghost-gluon vertex



- quark-gluon vertex



Renormalization & the renormalization group

$$A^a_{B\,\mu} = \mu^{-\frac{\epsilon}{2}}\, Z_A^{1/2}\, A^a_{R\,\mu}\ , \qquad \psi_B = \mu^{-\frac{\epsilon}{2}} Z_\psi^{1/2}\, \psi_R\ ,$$

$$c^a_B = \mu^{-\frac{\epsilon}{2}} Z_c^{1/2}\, c^a_R\ , \qquad \overline{c}^a_B = \mu^{-\frac{\epsilon}{2}} Z_c^{1/2}\, \overline{c}^a_R\ ,$$

$$g_B = \mu^{-\frac{\epsilon}{2}} Z_g\, g_R\ , \qquad \xi_B = Z_\xi\, \xi_R\ , \qquad M_B = Z_M\, M_R$$

$$\gamma_A = \frac{\mu}{Z_A}\frac{dZ_A}{d\mu}\ , \qquad \gamma_c = \frac{\mu}{Z_c}\frac{dZ_c}{d\mu}\ , \qquad \gamma_\psi = \frac{\mu}{Z_\psi}\frac{dZ_\psi}{d\mu}\ ,$$

$$\beta_g = \mu\frac{dg_R}{d\mu}\ , \qquad \gamma_\xi = \frac{\mu}{\xi_R}\frac{d\xi_R}{d\mu}\ , \qquad \gamma_M = \frac{\mu}{M_R}\frac{dM_R}{d\mu}$$

Renormalization & the renormalization group

- Gluon propagator

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_g\,\frac{\partial}{\partial g_R}+\gamma_M\,M_R\,\frac{\partial}{\partial M_R}+\gamma_\xi\,\xi_R\,\frac{\partial}{\partial \xi_R}+\gamma_A\right)\,\Delta^{ab}_{\mu\nu}(p;\mu)=0$$

$$\Delta_T\left(p^2;g_R(\mu),M_R(\mu),\xi_R(\mu),\mu\right)=$$
$$=e^{-\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\,\gamma_A(\mu')}\,\Delta_T\left(p^2;g_R(\mu_0),M_R(\mu_0),\xi_R(\mu_0),\mu_0\right)$$

- Ghost propagator

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_g\,\frac{\partial}{\partial g_R}+\gamma_M\,M_R\,\frac{\partial}{\partial M_R}+\gamma_\xi\,\xi_R\,\frac{\partial}{\partial \xi_R}+\gamma_c\right)\,\mathcal{G}^{ab}(p;\mu)=0$$

$$\mathcal{G}\left(p^2;g_R(\mu),M_R(\mu),\xi_R(\mu),\mu\right)=$$
$$=e^{-\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\,\gamma_c(\mu')}\,\mathcal{G}\left(p^2;g_R(\mu_0),M_R(\mu_0),\xi_R(\mu_0),\mu_0\right)$$

IR Landau pole

$$\mu \frac{dg}{d\mu} = \beta_g = -\frac{\beta_0 g^3}{16\pi^2}\,, \qquad \beta_0 = \frac{11}{3}N - \frac{2}{3}n_f$$

$$g^2(\mu) = \frac{g^2(\mu_0)}{1 + \frac{\beta_0 g^2(\mu_0)}{16\pi^2}\ln(\mu^2/\mu_0^2)}$$

$$\Lambda = \mu_0 \exp\left(-\frac{8\pi^2}{\beta_0 g^2(\mu_0)}\right) \quad \Longrightarrow \quad \lim_{\mu\to\Lambda} g(\mu) = \infty$$

Lattice QCD: set-up

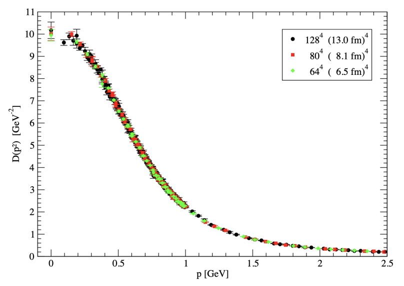
$$S_{\mathrm{W}} = \frac{6}{g_0^2} \sum_{x,\mu<\nu} \left(1 - \frac{1}{3} \mathrm{Tr} \left\{ U_{\mu\nu}(x) \right\} \right)$$

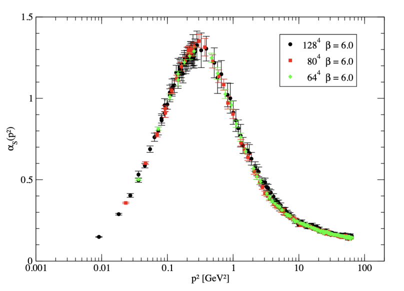
$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat{e}_\mu) U_\mu^\dagger(x + a\hat{e}_\nu) U_\nu^\dagger(x)$$

$$A_\mu(x + a\hat{e}_\mu/2) = \frac{1}{2ig_0 a} \left(U_\mu(x) - U_\mu^\dagger(x) \right) + O(a^2)$$

$$U_{\mu\nu} = \mathbb{1} + ig_0 a^2 F_{\mu\nu} - \frac{g_0^2 a^4}{2} F_{\mu\nu} F_{\mu\nu} + O(a^6) \ ,$$

Lattice QCD: results





Gluon propagator & Taylor coupling

Duarte *et al.*, PRD 94, 014502 (2016)

OPE: set-up and results

$$\mathcal{O}_1(x)\mathcal{O}_2(0) \to \sum_n C_{12}^n(x)\,\mathcal{O}_n(0) \qquad (x \to 0)$$

- Gluon propagator (Landau gauge)

$$\Delta(p) \to -\frac{iJ(p^2)}{p^2} + c_{A^2}\,\frac{\langle A^2\rangle}{(p^2)^2} \approx -\frac{iJ(p^2)}{p^2 - ic_{A^2}\,\langle A^2\rangle}$$

$$c_{A^2} = -i\,\frac{Ng^2}{4N_A} \qquad \Longrightarrow \qquad m^2 = \frac{Ng^2}{4N_A}\,\langle A^2\rangle$$

- Quark propagator (Landau gauge)

$$S(p) \to \frac{iZ(p^2)}{\not p} + c_{\overline{\psi}\psi}\,\frac{\langle\overline{\psi}\psi\rangle}{(p^2)^2} \approx \frac{iZ(p^2)}{\not p + ic_{\overline{\psi}\psi}\,\langle\overline{\psi}\psi\rangle/p^2}$$

$$c_{\overline{\psi}\psi} = -i\frac{3N_Ag^2}{8N^2} \qquad \Longrightarrow \qquad \mathcal{M}(p^2) = -\frac{3N_Ag^2}{8N^2}\,\frac{\langle\overline{\psi}\psi\rangle}{p^2}$$

The Gribov-Zwanziger approach: set-up and results

$$\int \mathcal{D}\mathscr{F}\ e^{-S_{GZ}}\Big|_{\langle H[A]\rangle_\gamma = 4\mathcal{V}_4 N_A} = \int \mathcal{D}\mathscr{F}\ e^{-S_{\mathrm{FP}}}\Big|_{\xi=0}\, \Theta\left(-\partial^\mu D_\mu\right)$$

$$S_{\mathrm{GZ}} = S_{\mathrm{FP}}\Big|_{\xi=0} + \int d^4x \left(\overline{\phi}^{ac}_\mu\, K^{ab}\,\phi^{bc\,\mu} - \overline{\omega}^{ac}_\mu K^{ab}\omega^{bc\,\mu} + \right.$$
$$\left. + \gamma^2 g f_{abc}\, A^{a\,\mu}(\phi^{bc}_\mu + \overline{\phi}^{bc}_\mu)\right) - 4\gamma^2\mathcal{V}_4 N_A$$

$$H[A] = g^2 \int d^4x\, f_{abc} f_{dec}\, A^b_\mu\, [K^{-1}(A)]^{ad} A^{e\,\mu}\ , \qquad K = -\partial\cdot D$$

equiv., $dW/d\gamma = 0$, $S_{\mathrm{GZ}} = S_{\mathrm{FP}}\big|_{\xi=0} + \gamma^4\left(H[A] - 4\mathcal{V}_4 N_A\right)$

The Gribov-Zwanziger approach: set-up and results

$$\Delta_{\mathrm{GZ}}^{-1}(p^2) = p^2 + \frac{2Ng^2\gamma^4}{p^2} \,, \qquad \Delta_{\mathrm{GZ}}(p^2) = \frac{p^2}{p^4 + 2Ng^2\gamma^4}$$

- With condensates $\langle \overline{\phi}_\mu^{ab} \, \phi^{ab\,\mu} - \overline{\omega}_\mu^{ab} \omega^{ab\,\mu} \rangle$, $\langle A^2 \rangle$:

$$\Delta_{\mathrm{RGZ}}(p^2) = \frac{p^2 + M^2}{p^4 + M^2 p^2 + 2Ng^2\gamma^4} \,,$$

$$\Delta_{\mathrm{RGZ}}^{(\langle A^2 \rangle)}(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + 2Ng^2\gamma^4 + M^2 m^2}$$

The Curci-Ferrari model: set-up

$$S_{\mathrm{CF}} = S_{\mathrm{YM}} + \int d^4x \left(iB^a \partial \cdot A^a + \overline{c}^a \partial^\mu D_\mu c^a + \frac{1}{2} m^2 A^a_\mu A^{a\mu} \right)$$

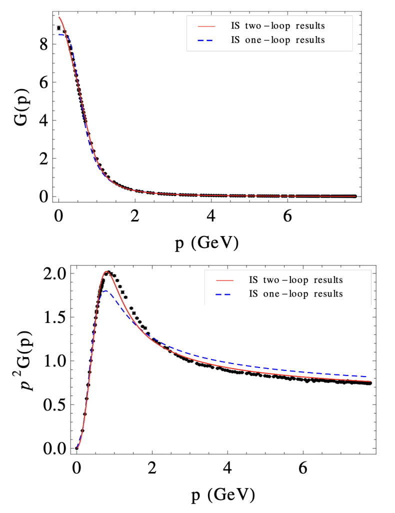
$$s_{m^2} A^a_\mu = -D_\mu c^a \ , \quad s_{m^2} c^a = \frac{g}{2} f^a_{bc}\, c^b c^c \ , \quad s_{m^2} \overline{c}^a = iB^a \ , \quad s_{m^2} B^a = im^2 c^a$$

$$s^2_{m^2} \propto m^2 \neq 0$$

$$Z_g Z_A^{1/2} Z_c = 1 \ , \qquad Z_{m^2} Z_A Z_c = 1$$

$$\Delta(p^2) = \frac{1}{Z_A\, p^2 + Z_A Z_{m^2}\, m^2 + \Pi_T^{(\mathrm{CF})}(p^2)}$$

The Curci-Ferrari model: results



Gluon propagator & dressing function, infrared-safe scheme

Gracey *et al.*, PRD 100, 034023 (2019)

Set-up

$$S_{\text{FP}} = S_m + S'_{\text{int.}}$$

$$S_m = i \int \frac{d^4p}{(2\pi)^4} \left\{ \frac{1}{2} A^a_\mu(-p)\, [\Delta_m^{-1}(p)]^{\mu\nu}_{ab}\, A^b_\nu(p) + \overline{c}^a(p)\, [\mathcal{G}_0^{-1}(p)]_{ab}\, c^b(p) \right\}$$

$$\Delta^{ab}_{m\,\mu\nu}(p) = \delta^{ab} \left(\frac{-it_{\mu\nu}(p)}{p^2 - m^2} + \xi\, \frac{-i\ell_{\mu\nu}(p)}{p^2} \right)$$

$$S'_{\text{int.}} = S_{\text{int.}} - \delta S\,, \qquad -\delta S = -i \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{2} A^a_\mu(-p)\, \Gamma^{\mu\nu}_{ab}(p)\, A^b_\nu(p)$$

$$\Gamma^{\mu\nu}_{ab}(p) = -im^2\, t^{\mu\nu}(p)\, \delta_{ab}$$

3. The Screened Massive Expansion

Feynman rules

- Zero-order gluon propagator

μ, a  $\nu, b \quad = \quad \delta^{ab}\left[\frac{-i t_{\mu\nu}(p)}{p^2 - m^2} + \frac{-i\xi\, \ell_{\mu\nu}(p)}{p^2}\right]$

- Gluon mass counterterm

μ, a  $\nu, b \quad = \quad -i m^2\, t^{\mu\nu}(p)\, \delta_{ab}$

- + standard QCD rules

Gaussian effective potential

$$V_G = \frac{i}{\mathcal{V}_4} \ln \int \mathcal{D}\mathscr{F}\, e^{iS_m} - \frac{i}{\mathcal{V}_4} \left\langle S'_{\text{int.}} \right\rangle_m \,, \qquad V_G \geq \mathcal{E}$$

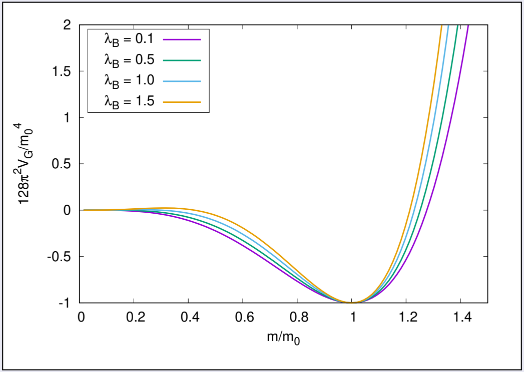
$$V_G(m^2) = \frac{3N_A m^4}{128\pi^2} \left(\alpha\, \ln^2 \frac{m^2}{m_0^2} + 2\, \ln \frac{m^2}{m_0^2} - 1 \right)$$

$$\alpha = \frac{9N\alpha_s}{8\pi}\,, \qquad \alpha_s = \frac{g^2}{4\pi}$$

$$V'_G(m^2) = 0 \quad \Longleftrightarrow \quad m^2 = m_0^2 \neq 0$$

$$V_G(m_0^2) = -\frac{3N_A m_0^4}{128\pi^2} < 0 = V_G(m^2 = 0)$$

Gaussian effective potential



Pure Yang-Mills GEP as a function of the gluon mass parameter for different values of the coupling constant

G. C. and F. Siringo, Phys. Rev. D 97, 056013 (2018)

Ghost propagator

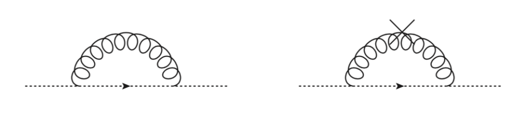
- Definition

$$\mathcal{G}^{ab}(p) = \int d^4x\, e^{ip\cdot x}\ \left\langle T\left\{c^a(x)\overline{c}^b(0)\right\}\right\rangle$$

- General expression

$$\mathcal{G}(p^2) = \frac{i}{Z_c\, p^2 - \Sigma(p^2)}$$

- Self-energy diagrams



Ghost propagator

- Self-energy

$$\Sigma(p^2) = \frac{\alpha}{4}\,p^2\left(1-\frac{\xi}{3}\right)\left(\frac{2}{\epsilon}-\ln\frac{m^2}{\overline{\mu}^2}\right) - \alpha\,p^2\left(G(s)-\frac{2}{3}-\frac{\xi}{12}\ln s\right)$$

$$G(s) = \frac{1}{12}\left[\frac{(1+s)^2(2s-1)}{s^2}\ln(1+s) - 2s\ln s + \frac{1}{s} + 2\right]$$

- Field-strength renormalization

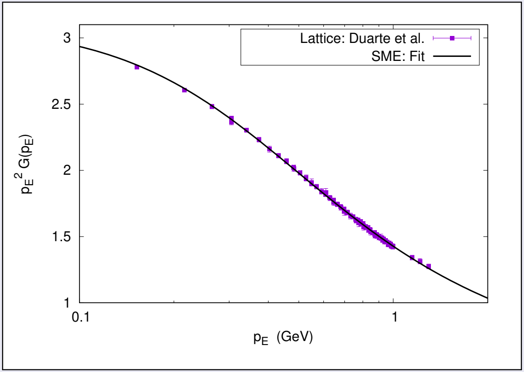
$$Z_c = 1 + \frac{\alpha}{4}\left[\left(1-\frac{\xi}{3}\right)\left(\frac{2}{\epsilon}-\ln\frac{m^2}{\overline{\mu}^2}\right) + \frac{8}{3} + 4g_0\right]$$

- Renormalized propagator

$$\mathcal{G}(p^2) = \frac{iZ_{\mathcal{G}}}{p^2\left(G(s)-\xi\ln s/12+G_0\right)}$$

Ghost propagator

- Lattice fit (Landau gauge)



$$Z_{\mathcal{G}} = 1.0994,\ G_0 = 0.1464 \quad @\ m = 0.654\ \text{GeV}$$

3. The Screened Massive Expansion

Gluon propagator

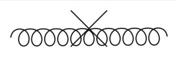
- Definition

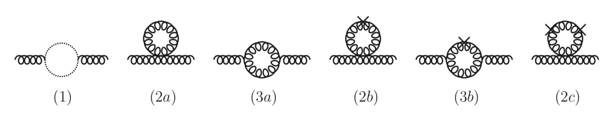
$$\Delta^{ab}_{\mu\nu}(p) = \int d^4x\, e^{ip\cdot x}\, \left\langle T\left\{A^a_\mu(x) A^b_\nu(0)\right\}\right\rangle$$

- General expression

$$\Delta(p^2) = \frac{-i}{Z_A\, p^2 - m^2 - \Pi(p^2)}$$

- Polarization diagrams





Gluon propagator

- Loop polarization

$$\Pi(p^2) = -m^2 + \Pi_{\text{loop}}(p^2) \quad \Longrightarrow \quad \Delta(p^2) = \frac{-i}{Z_A\, p^2 - \Pi_{\text{loop}}(p^2)}$$

$$\Pi_{\text{loop}}(p^2) = \frac{\alpha}{3}\left(\frac{13}{6} - \frac{\xi}{2}\right) p^2 \left(\frac{2}{\epsilon} - \ln\frac{m^2}{\overline{\mu}^2}\right) - \alpha\, p^2\,(F(s) + \xi F_\xi(s) + \mathcal{C})$$

$$F(s) = \frac{5}{8s} + \frac{1}{72}\,[L_a(s) + L_b(s) + L_c(s) + R_a(s) + R_b(s) + R_c(s)]\;,$$

$$F_\xi(s) = \frac{1}{4s} - \frac{1}{12}\left[2s\ln s - \frac{2(1-s)(1-s^3)}{s^3}\ln(1+s) + \frac{3s^2-3s+2}{s^2}\right]$$

Gluon propagator

- Loop polarization

$$L_a(s) = \frac{3s^3 - 34s^2 - 28s - 24}{s}\sqrt{\frac{4+s}{s}}\ \ln\left(\frac{\sqrt{4+s}-\sqrt{s}}{\sqrt{4+s}+\sqrt{s}}\right),$$

$$L_b(s) = \frac{2(1+s)^2}{s^3}\left(3s^3 - 20s^2 + 11s - 2\right)\ \ln(1+s),$$

$$L_c(s) = \left(2 - 3s^2\right)\ \ln s$$

$$R_a(s) = -\frac{4+s}{s}\left(s^2 - 20s + 12\right),$$

$$R_b(s) = \frac{2(1+s)^2}{s^2}\left(s^2 - 10s + 1\right),$$

$$R_c(s) = \frac{2}{s^2} + 2 - s^2$$

Gluon propagator

- Field-strength renormalization

$$Z_A = 1 + \frac{\alpha}{3}\left(\frac{13}{6} - \frac{\xi}{2}\right)\left(\frac{2}{\epsilon} - \ln\frac{m^2}{\overline{\mu}^2}\right) + \alpha(f_0 - \mathcal{C})$$

- Renormalized propagator

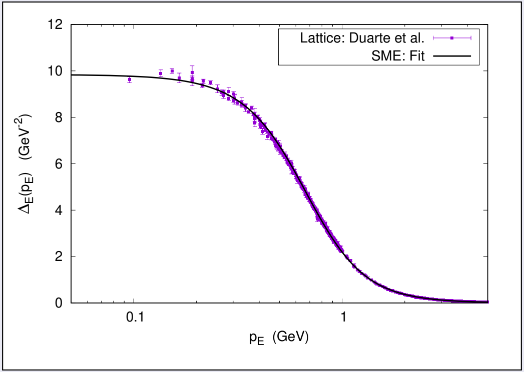
$$\Delta(p^2) = \frac{-iZ_\Delta}{p^2(F(s) + \xi\, F_\xi(s) + F_0)}$$

- Zero-momentum limit

$$\Delta(p^2 = 0) = \frac{-iZ_\Delta}{-\frac{5m^2}{8}\left(1 + \frac{2\xi}{5}\right)}$$

Gluon propagator

- Lattice fit (Landau gauge)



$$Z_\Delta = 2.6308,\ F_0 = -0.8872,\ m = 0.6541\ \text{GeV}$$

Optimization: gluon sector

$$\Delta^{-1}(p^2,\xi) = iZ_\Delta^{-1}\, p^2\, J^{-1}(-p^2/m^2,\xi)$$

$$\Delta^{-1}(p_0^2,\xi) = 0 \quad \Longleftrightarrow \quad J^{-1}(-p^2/m^2,\xi) = 0$$

$$J^{-1}(s,\xi) = F(s) + \xi\, F_\xi(s) + F_0$$

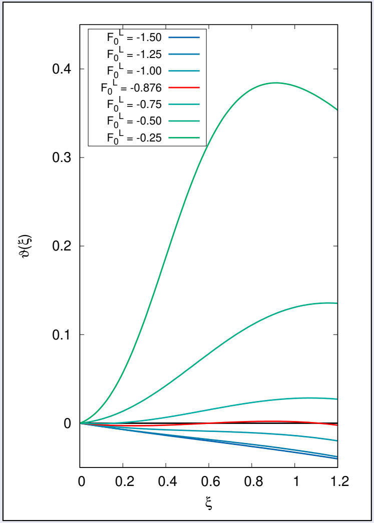
$$\mathrm{Re}\left\{F(-p_0^2/m^2(\xi)) + \xi\, F_\xi(-p_0^2/m^2(\xi))\right\} + F_0(\xi) = 0$$

$$\mathrm{Im}\left\{F(-p_0^2/m^2(\xi)) + \xi\, F_\xi(-p_0^2/m^2(\xi))\right\} = 0$$

$$R(\xi) = \lim_{p_E^2\to -p_0^2} (p_E^2 + p_0^2)\Delta_E(p_E^2,\xi) = \left(\left.\frac{\partial \Delta_E^{-1}}{\partial p_E^2}\right|_{p_E^2=-p_0^2}\right)^{-1} =$$

$$= |R(\xi)|e^{i\theta(\xi)}$$

Optimization: gluon sector



Phase difference $\theta(\xi) - \theta(0)$ of the residue of the gluon propagator as a function of the gauge ξ

Optimization: gluon sector

$$F_0(0) = -0.876$$

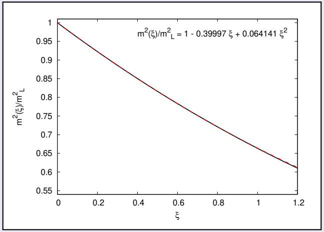
$$p_0^2 = (0.4575 \pm 1.0130\, i)\, m^2(0)$$

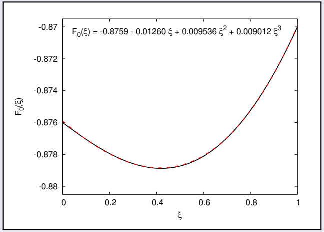
$$m^2(\xi) \approx \left(1 - 0.39997\,\xi + 0.064141\,\xi^2\right)\, m^2(0)$$

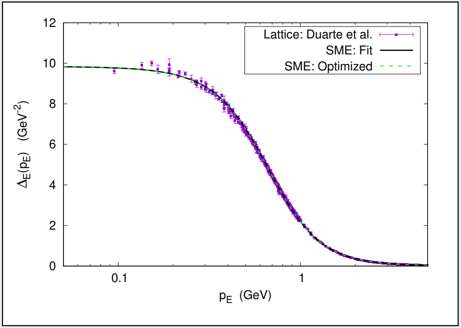
$$F_0(\xi) \approx -0.8759 - 0.01260\,\xi + 0.009536\,\xi^2 + 0.009012\,\xi^3$$

$$\theta(\xi) = 1.262^{+0.22\%}_{-0.22\%}$$

Optimization: gluon sector





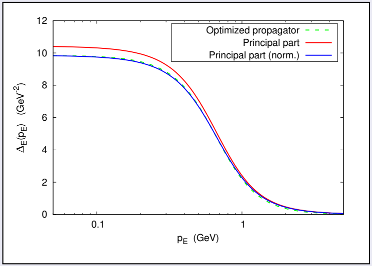


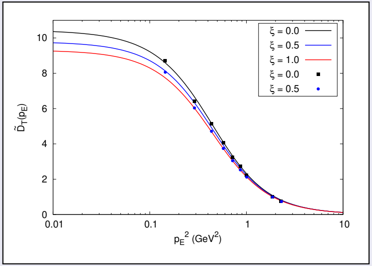
$$F_0(0) = -0.876$$

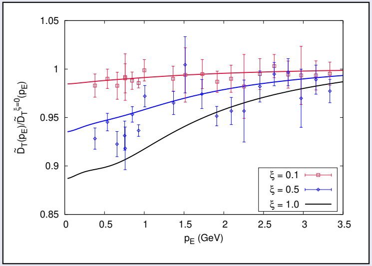
$$Z_\Delta = 2.6481,\ m(0) = 0.6557\ \text{GeV}$$

$$p_0 = (\pm 0.5810 \pm 0.3751\ i)\ \text{GeV}$$

Optimization: gluon sector







Optimization: ghost sector

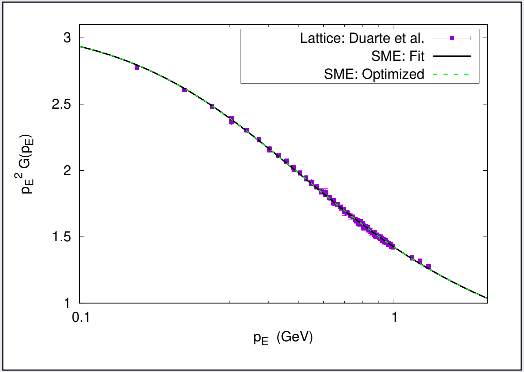
$$G_0 = \frac{1}{\alpha(\mu)} \left(\mu^2\, \mathcal{G}_E(\mu^2)\right)^{-1} - G(\mu^2/m^2) + \frac{\xi}{12}\, \ln(\mu^2/m^2)\ ,$$

$$F_0 = \frac{1}{\alpha(\mu)} \left(\mu^2\, \Delta_E(\mu^2)\right)^{-1} - F(\mu^2/m^2) - \xi\, F_\xi(\mu^2/m^2)$$

$$G_0 = \left(1 + \frac{m^2}{\mu^2}\right)^{-1} \left(F(\mu^2/m^2) + \xi\, F_\xi(\mu^2/m^2) + F_0\right) + \\ - G(\mu^2/m^2) + \frac{\xi}{12}\, \ln(\mu^2/m^2)$$

$$\frac{\partial G_0}{\partial \mu} = 0 \quad \Longrightarrow \quad G_0 = 0.1452$$

Optimization: ghost sector



$$Z_{\mathcal{G}} = 1.0959 \quad @\ G_0 = 0.1452,\ m = 0.6557 \text{ GeV}$$

Renormalization Group improvement

- Renormalization conditions (MOM-Taylor scheme)

$$\Delta(\mu^2) = \mathcal{G}(\mu^2) = \frac{1}{\mu^2}\ , \qquad Z_g = Z_A^{-1/2} Z_c^{-1}$$

- Renormalization counterterms

$$Z_A = 1 + \frac{\alpha}{3}\left(\frac{13}{6} - \frac{\xi}{2}\right)\left(\frac{2}{\epsilon} - \ln\frac{m^2}{\overline{\mu}^2}\right) - \alpha(F(\mu^2/m^2) + \mathcal{C})\ ,$$

$$Z_c = 1 + \frac{\alpha}{4}\left[\left(1 - \frac{\xi}{3}\right)\left(\frac{2}{\epsilon} - \ln\frac{m^2}{\overline{\mu}^2}\right) + \frac{8}{3}\right] - \alpha\, G(\mu^2/m^2)$$

Renormalization Group improvement

- Anomalous dimensions

$$\gamma_A = -2\alpha \frac{\mu^2}{m^2} F'(\mu^2/m^2) , \qquad \gamma_c = -2\alpha \frac{\mu^2}{m^2} G'(\mu^2/m^2)$$

- Beta function

$$\beta_\alpha = \frac{d\alpha}{d\ln\mu^2} = \frac{\alpha}{2}(\gamma_A + 2\gamma_c) = -\alpha^2 \frac{\mu^2}{m^2} H'(\mu^2/m^2)$$

$$H(s) = F(s) + 2G(s)$$

$$\beta_\alpha = \frac{d\alpha}{d\ln\mu^2} = \frac{\alpha}{2}(\gamma_A + 2\gamma_c) = -\alpha^2 \frac{\mu^2}{m^2} H'(\mu^2/m^2)$$

- Solution of the beta function equation

$$\alpha(s) = \frac{\alpha(s_0)}{1 + \alpha(s_0)\,[H(s) - H(s_0)]}$$

Renormalization Group improvement

- Running coupling: UV limit

$$\alpha_s(\mu^2) \to \frac{\alpha_s(\mu_0^2)}{1 + \frac{11N}{3}\frac{\alpha_s(\mu_0^2)}{4\pi}\ln\left(\mu^2/\mu_0^2\right)}$$

- Running coupling: IR limit

$$\alpha_s(\mu^2) \to \frac{32\pi}{15N}\frac{\mu^2}{m^2} \to 0$$

- Landau pole/maximum ($\mu_0 = 6.098m$)

$$\alpha_s(\mu_0) \geq 0.469 \quad \Longrightarrow \quad \text{Landau pole}$$

$$\alpha_s(\mu_0) < 0.469 \quad \Longrightarrow \quad \text{maximum at } \mu_\star = 1.022m$$

Renormalization Group improvement

- RG-improved propagators

$$\Delta\left(p^2;\mu_0^2\right)=\frac{1}{p^2}\exp\left(-\int_{\mu_0^2/m^2}^{p^2/m^2}ds\;\alpha(s)\,F'(s)\right)\ ,$$

$$\mathcal{G}\left(p^2;\mu_0^2\right)=\frac{1}{p^2}\exp\left(-\int_{\mu_0^2/m^2}^{p^2/m^2}ds\;\alpha(s)\,G'(s)\right)$$

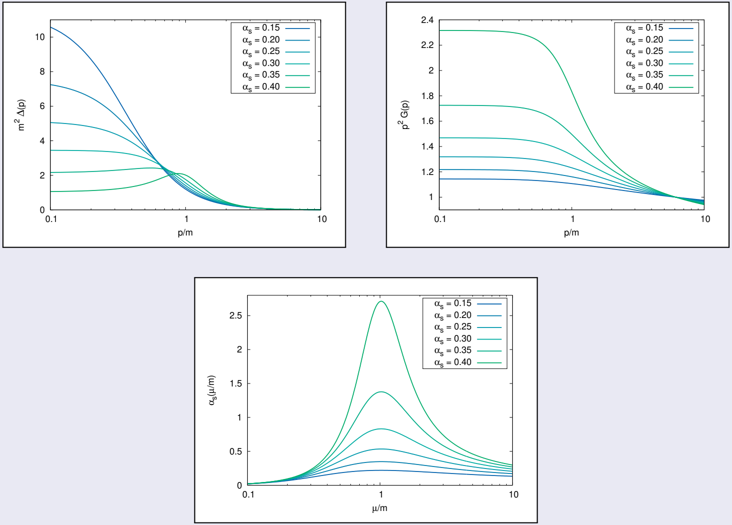
- RG-improved propagators: UV limit

$$\Delta\left(p^2;\mu_0^2\right)\to\frac{1}{p^2}\left[\frac{\alpha_s(p^2)}{\alpha_s(\mu_0^2)}\right]^{\frac{13}{22}}\ ,\qquad \mathcal{G}\left(p^2;\mu_0^2\right)\to\frac{1}{p^2}\left[\frac{\alpha_s(p^2)}{\alpha_s(\mu_0^2)}\right]^{\frac{9}{44}}$$

- RG-improved propagators: IR limit

$$\Delta\left(p^2;\mu_0^2\right)\to\frac{\kappa}{m^2}\ ,\qquad \mathcal{G}\left(p^2;\mu_0^2\right)\to\frac{\kappa'}{p^2}$$

Renormalization Group improvement



RG improvement: intermediate scale matching

- Taylor relation (general scheme)

$$\alpha_s(\mu^2) = \alpha_s(\mu_0^2) \left[\frac{\Delta\left(\mu^2;\mu_0^2\right)}{\Delta\left(\mu^2;\mu^2\right)}\right] \left[\frac{\mathcal{G}\left(\mu^2;\mu_0^2\right)}{\mathcal{G}\left(\mu^2;\mu^2\right)}\right]^2$$

- Taylor relation (optimized fixed scale, MOM)

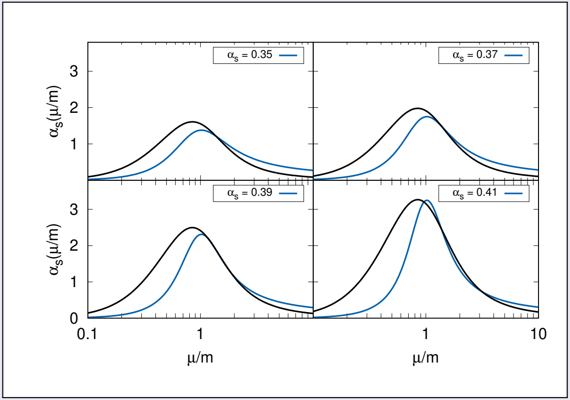
$$\alpha_s^{\text{(OFS)}}(\mu^2) = \kappa\,\left[F(\mu^2/m^2) + F_0\right]^{-1}\left[G(\mu^2/m^2) + G_0\right]^{-2}$$

- Matching condition

$$\alpha_s^{\text{(OFS)}}(\mu^2) = \alpha_s(\mu_1^2) \left[\frac{F(\mu_1^2/m^2) + F_0}{F(\mu^2/m^2) + F_0}\right] \left[\frac{G(\mu_1^2/m^2) + G_0}{G(\mu^2/m^2) + G_0}\right]^2$$

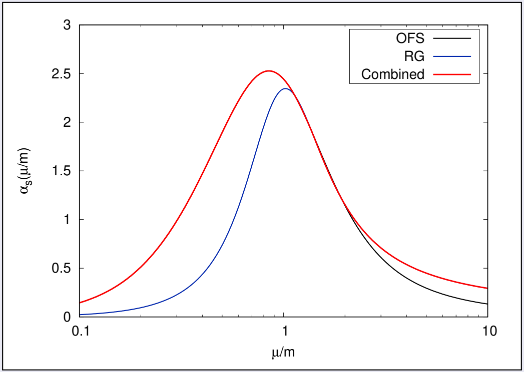
RG improvement: intermediate scale matching

- Matching condition visualized ($\mu_1 = 1.372m$)



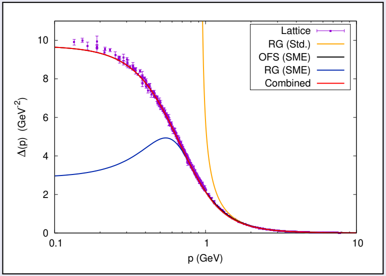
RG improvement: intermediate scale matching

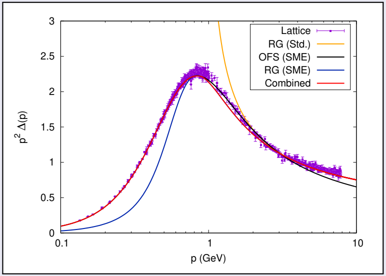
- Best match ($\mu_1 = 1.372m$): $\alpha_s(\mu_0^2) = 0.391$

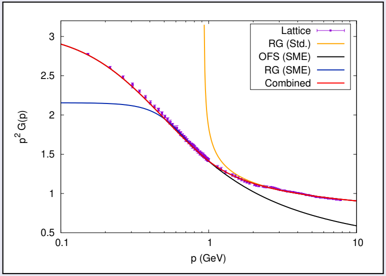


$$\alpha_s^{\text{(OFS)}}(\mu^2) = \alpha_s(\mu^2) \pm 1\% \quad \text{for} \quad \mu \in [1.1m, 2m]$$

RG improvement: intermediate scale matching

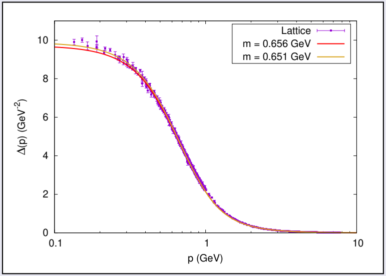


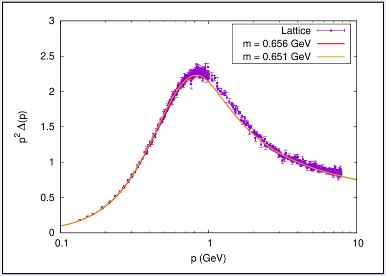




RG improvement: intermediate scale matching

- Combined fit





$$m = 0.651 \text{ GeV}$$

Finite temperature: set-up

$$Z = \int \mathcal{D}\mathscr{F}\ \exp\left(-\int_0^\beta d\tau \int d^3x\, \mathcal{L}_E\right) \qquad \beta = 1/T$$

$$\mathcal{O}(\tau) = e^{\tau H}\mathcal{O}e^{-\tau H}$$

$$G_N(\tau_1,\dots,\tau_N) = Z^{-1}\mathrm{Tr}\left\{e^{-\beta H}\, T_\tau\left\{\mathcal{O}_1(\tau_1)\cdots\mathcal{O}_N(\tau_N)\right\}\right\}$$

$$G_N(\tau_1,\dots,0,\dots,\tau_N) = \pm G_N(\tau_1,\dots,\beta,\dots,\tau_N)$$

Finite temperature: set-up

$$A^a_\mu(\tau=\beta,\vec{x}) = A^a_\mu(\tau=0,\vec{x})$$

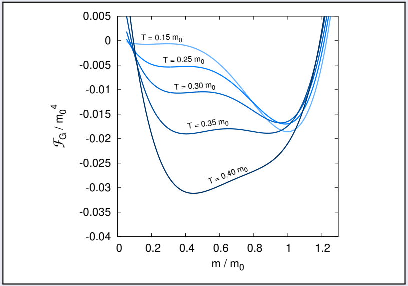
$$c^a(\tau=\beta,\vec{x}) = -c^a(\tau=0,\vec{x})\ , \quad \overline{c}^a(\tau=\beta,\vec{x}) = -\overline{c}^a(\tau=0,\vec{x})$$

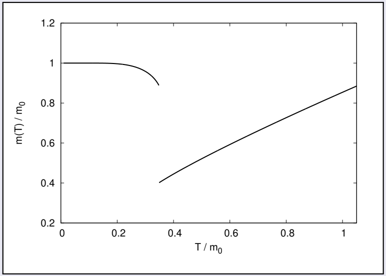
$$A^a_\mu(\tau,\vec{x}) = \sum_n \int \frac{d^3p}{(2\pi)^3}\ e^{i(\omega_n\tau+\vec{p}\cdot\vec{x})}\ A^a_{\mu,n}(\vec{p}) \qquad \omega_n = 2\pi nT$$

$$c^a(\tau,\vec{x}) = \sum_n \int \frac{d^3p}{(2\pi)^3}\ e^{i(\omega_n\tau+\vec{p}\cdot\vec{x})}\ c^a_n(\vec{p}) \qquad \omega_n = (2n+1)\pi T$$

Finite temperature: GEP

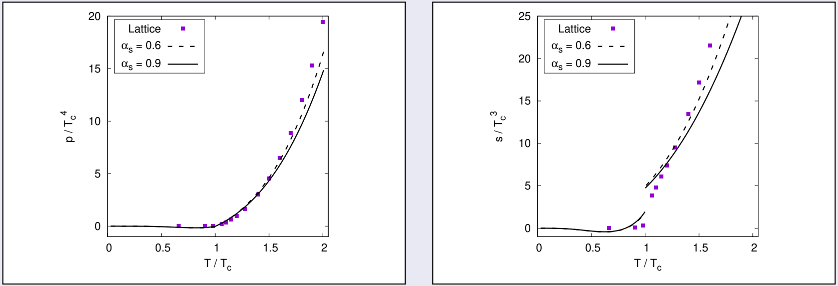
$$\mathcal{F}_G(m,T) = -\frac{1}{\beta\mathcal{V}_3}\ln\int\mathcal{D}\mathscr{F}\ e^{-S_m} + \frac{1}{\beta\mathcal{V}_3}\ \langle S'_{\text{int.}}\rangle_m\ ,\quad \mathcal{F}_G(m,T)\geq\mathcal{F}(T)$$





Finite temperature: GEP

$$p = -\left[\mathcal{F}_G(T,m(T)) - \mathcal{F}_G(0,m_0)\right] \ , \qquad s = -\frac{d}{dT}\mathcal{F}_G(T,m(T))$$



$$T_c \approx 0.35 m_0 \approx 230 \text{ MeV}$$

Finite temperature: gluon propagator

$$\Delta^{ab}_{\mu\nu}(p,T) = \left[\Delta_T(p,T)\,\mathcal{P}^T_{\mu\nu}(p) + \Delta_L(p,T)\,\mathcal{P}^L_{\mu\nu}(p) + \frac{\xi}{p^2}\,\ell_{\mu\nu}(p)\right]\delta^{ab}$$

$$\mathcal{P}^T_{\mu\nu}(p) = (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{|\vec{p}|^2}\right) \ ,$$

$$\mathcal{P}^L_{\mu\nu}(p) = t_{\mu\nu}(p) - \mathcal{P}^T_{\mu\nu}(p)$$

$$[\mathcal{P}^{T,L}(p)]^2 = \mathcal{P}^{T,L}(p)\ , \quad \mathcal{P}^{T,L}(p)\cdot t(p) = t(p)\cdot\mathcal{P}^{T,L}(p) = \mathcal{P}^{T,L}(p)$$

$$\mathcal{P}^{T,L}(p)\cdot\mathcal{P}^{L,T}(p) = \mathcal{P}^{T,L}(p)\cdot\ell(p) = \ell(p)\cdot\mathcal{P}^{T,L}(p) = 0$$

$$\mathrm{Tr}\{\mathcal{P}^T(p)\} = 2\ , \quad \mathrm{Tr}\{\mathcal{P}^L(p)\} = 1\ , \quad \mathcal{P}^T(p) + \mathcal{P}^L(p) + \ell(p) = \mathbb{1}$$

Finite temperature: gluon propagator

$$\Delta_{T,L}(p,T) = \Delta_{T,L}(p^4,|\vec{p}|,T)\ ,$$

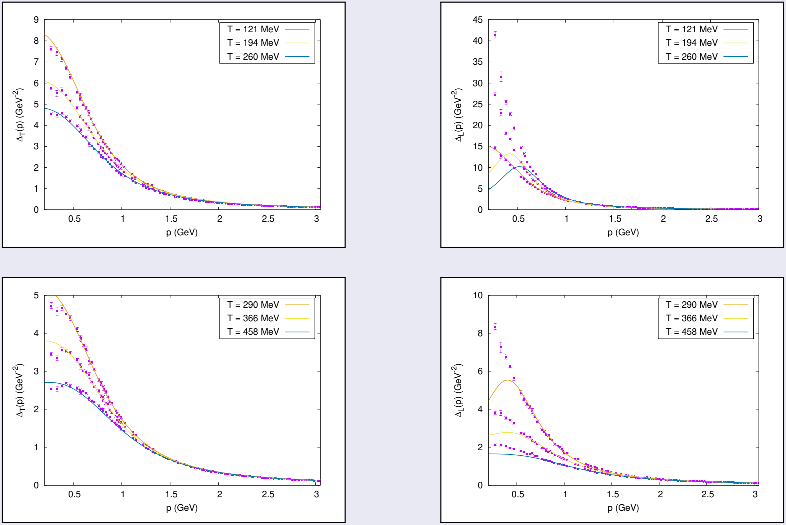
$$\Delta_T(p,T=0) = \Delta_L(p,T=0) = \Delta(p)\ ,$$

$$\Delta_T(p^4,|\vec{p}|=0,T) = \Delta_L(p^4,|\vec{p}|=0,T) \qquad (p^4 \neq 0)$$

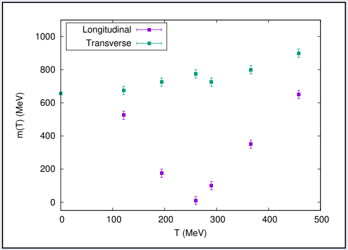
$$\Delta_{T,L}(p,T) = \frac{Z_{T,L}(T)}{p^2[F(s(T)) + F_0^{T,L}(T) + \pi_{T,L}(p,m(T),T)]}$$

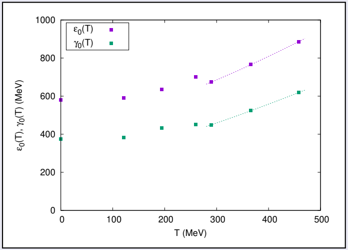
$$s(T) = p^2/m^2(T)\ , \quad \pi_{T,L}(p,m,T=0) = 0$$

Finite temperature: gluon propagator



Finite temperature: gluon propagator





Gluon mass parameter and poles as a function of T. Poles from 3d-transverse propagator

Full QCD: set-up

$$\mathcal{L}_q = \overline{\psi}(i\partial\!\!\!/ - M_R + g\,\gamma^\mu A^a_\mu T_a)\psi$$

$$\mathcal{L}_q = \mathcal{L}_{q,0} + \mathcal{L}_{q,\text{int}}$$

$$\mathcal{L}_{q,0} = \overline{\psi}(i\partial\!\!\!/ - M)\psi\ , \quad \mathcal{L}_{q,\text{int}} = \overline{\psi}(gA\!\!\!/^a T_a + M - M_R)\psi$$

$$M \neq M_R\left(1 + c_1\,\frac{\alpha_s}{4\pi} + \cdots\right)$$

$$S_M(p) = \frac{i}{p\!\!\!/ - M}$$

Full QCD: quark propagator

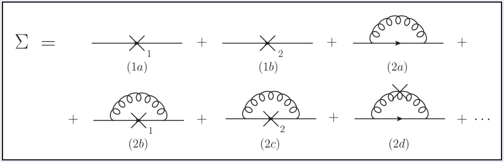
- Definition

$$S(p) = \int d^4x\, e^{ip\cdot x} \left\langle T\left\{\psi(x)\overline{\psi}(0)\right\}\right\rangle$$

- General expression

$$S(p) = \frac{i}{\not{p} - M - \Sigma(p)}$$

- Self-energy diagrams



Full QCD: quark propagator

- Loop self-energy

$$\Sigma(p) = -M + M_R + \Sigma^{(\text{loop})}(p) \quad \Longrightarrow \quad S(p) = \frac{i}{\not{p} - M_R - \Sigma^{(\text{loop})}(p)}$$

$$\Sigma^{(\text{loop})}(p) = \not{p}\, \Sigma_V(p^2) + \Sigma_S(p^2)$$

- Mass and Z-function

$$S(p) = \frac{iZ(p^2)}{\not{p} - \mathcal{M}(p^2)}$$

$$Z(p^2) = 1/A(p^2)\,, \quad \mathcal{M}(p^2) = B(p^2)/A(p^2)$$

$$A(p^2) = 1 - \Sigma_V(p^2)\,, \quad B(p^2) = M_R + \Sigma_S(p^2)$$

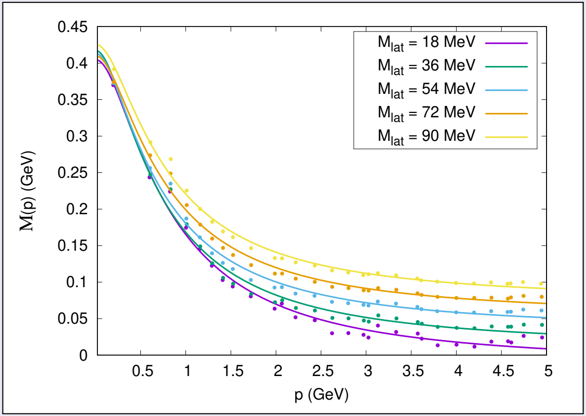
Full QCD: quark propagator

- Zero-momentum limit

$$\mathcal{M}(0) = \frac{M_R + \Sigma_S(0)}{1 - \Sigma_V(0)} \sim M \qquad (M \gg M_R \implies \Sigma_S(0) \sim M)$$

Full QCD: quark propagator

- Lattice fit (Landau gauge) – mass function



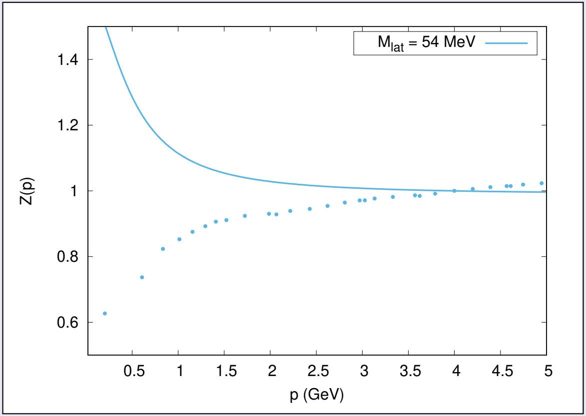
Full QCD: quark propagator

- Lattice fit (Landau gauge) – mass function

M_{lat} (MeV)	M (MeV)	α_s	M_R (MeV)	P_0 (MeV)
18	268.0	2.605	-14.0	$\pm 387.4 \pm 180.9i$
$18^{\star}$	197.6	3.128	6.8	$\pm 349.2 \pm 193.1i$
36	228.7	2.788	10.2	$\pm 371.7 \pm 185.4i$
54	221.4	2.663	33.9	$\pm 375.2 \pm 177.2i$
72	238.4	2.393	53.4	$\pm 392.9 \pm 167.6i$
90	249.0	2.261	73.8	$\pm 410.8 \pm 170.2i$

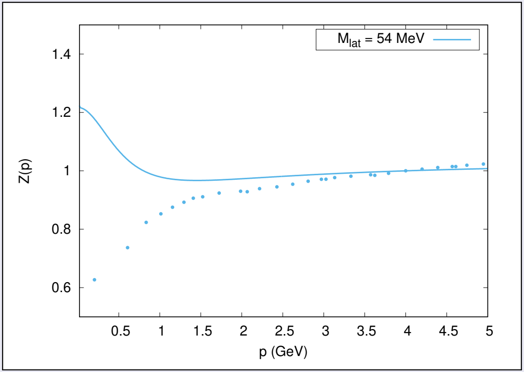
Full QCD: quark propagator

- Lattice fit (Landau gauge) – Z-function



Full QCD: quark propagator

- Lattice fit (Landau gauge) – Z-function, CC scheme



$$\Delta_m(p^2) = \frac{1}{p^2 + m^2} \to \frac{1}{2\text{Re}\{R\}} \left[\frac{R}{p^2 + p_0^2} + \frac{\overline{R}}{p^2 + \overline{p_0^2}} \right]$$

The field A^h: definition and gauge invariance

$$f_A[U] = \mathrm{Tr}\left\{ \int d^4x \, A^U \cdot A^U \right\} \, , \quad A^U_\mu = U^\dagger \left(A_\mu + \frac{i}{g} \, \partial_\mu \right) U$$

$$h \in SU(3) \;\; : \quad f_A[h] = \min_{U \in SU(3)} f_A[U]$$

$$\Longleftrightarrow \quad \partial^\mu A_\mu^{h[A]} = 0$$

$$h[A^U] = U^\dagger h[A] \quad \Longrightarrow \quad A^{h[A]} \to (A^U)^{h[A^U]} = A^{h[A]}$$

The field A^h: perturbative expression

$$h[A] = e^{ig\xi[A]}$$

$$\xi[A] = \frac{\partial\cdot A}{\partial^2} + i\,\frac{g}{\partial^2}\left[\partial\cdot A, \frac{\partial\cdot A}{\partial^2}\right] + i\,\frac{g}{\partial^2}\left[A_\mu, \partial^\mu\,\frac{\partial\cdot A}{\partial^2}\right] + \\ + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial\cdot A}{\partial^2}, \partial\cdot A\right] + \cdots$$

$$A_\mu^h = \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\phi^\nu[A]$$

$$\phi_\mu[A] = A_\mu - ig\left[\frac{\partial\cdot A}{\partial^2}, A_\mu\right] + \frac{ig}{2}\left[\frac{\partial\cdot A}{\partial^2}, \partial_\mu\frac{\partial\cdot A}{\partial^2}\right] + \cdots$$

The quadratic operator $(A^h)^2$: effective action

$$Z[J] = e^{-W[J]} = \int \mathcal{D}\mathscr{F}\; e^{-S^{(1)}[J]}$$

$$S^{(1)}[J] = S_{\mathrm{FP}} + \int d^4x \left[\frac{J}{2}(A^h)^2 - \frac{\zeta}{2} J^2\right]$$

$$\frac{\delta W}{\delta J}[J] = \frac{1}{2}\left\langle (A^h)^2 \right\rangle_J - \zeta J = \sigma[J]$$

$$\Gamma[\sigma] = W[J_\sigma] - \int d^4x\, J_\sigma\, \sigma$$

$$\frac{\delta \Gamma}{\delta \sigma}[\sigma] = 0 \quad \Longleftrightarrow \quad \sigma = \frac{1}{2}\left\langle (A^h)^2 \right\rangle_{J=0}$$

The quadratic operator $(A^h)^2$: effective action, localized

$$1 = \int \mathcal{D}F \; \delta(F) \; , \quad F \to F[\xi] = \partial^\mu A_\mu^{h(\xi)}$$

$$S^{(1)}[J] \to S^{(2)}[J] = S^{(1)}[J] + \int d^4x \; \left(\tau^a \partial^\mu A_\mu^{h,a} + \overline{\eta}^a \partial^\mu D_\mu(A^h) \eta^a \right)$$

$$s\tau^a = s\overline{\eta}^a = s\eta^a = 0 \; , \qquad sh = -igc^a T_a h$$

$$s\xi^a = -c^a + \frac{g}{2} f^a_{bc} \, c^b \xi^c + O(g^2)$$

$$sS^{(2)} = 0 \; , \quad s^2 = 0$$

The quadratic operator $(A^h)^2$: effective action, localized + linearized

$$1 = \mathcal{N} \int \mathcal{D}\sigma \, e^{-\Delta S_2} \,, \quad \Delta S_2 = \frac{1}{2\zeta} \int d^4x \left(\sigma - \frac{1}{2} (A^h)^2 + \zeta J \right)^2$$

$$\begin{aligned} S^{(2)}[J] \to S^{(3)}[J] = S^{(2)}[J] + \Delta S_2 = \\ = S_{\mathrm{FP}} + \int d^4x \left(\tau^a \partial^\mu A_\mu^{h,a} + \overline{\eta}^a \partial^\mu D_\mu(A^h) \eta^a \right) + \\ + \int d^4x \left\{ J\sigma + \frac{1}{2\zeta} \sigma^2 - \frac{1}{2\zeta} \sigma (A^h)^2 + \frac{1}{8\zeta} [(A^h)^2]^2 \right\} \end{aligned}$$

$$s\sigma = 0$$

$$sS^{(3)} = 0 \,, \quad s^2 = 0$$

The quadratic operator $(A^h)^2$: effective action, calculation

$$\sigma \to \sigma + \delta\sigma \quad : \quad \langle \delta\sigma \rangle = 0$$

$$\Gamma[\sigma] = \frac{1}{2\zeta} \int d^4x \, \sigma^2 - \ln \int_{\langle \delta\sigma \rangle = 0} \mathcal{D}\mathscr{F} \; e^{-I}$$

$$I = S_{\mathsf{FP}} + \int d^4x \, \left(\tau^a \partial^\mu A_\mu^{h,a} + \overline{\eta}^a \partial^\mu D_\mu(A^h) \eta^a \right) + \\ + \int d^4x \, \left\{ \frac{1}{2\zeta} (\delta\sigma)^2 - \frac{1}{2\zeta} (\sigma + \delta\sigma)(A^h)^2 + \frac{1}{8\zeta} \left[(A^h)^2 \right]^2 \right\}$$

The quadratic operator $(A^h)^2$: effective action, Feynman rules

$$I = S_{\mathsf{FP}} + \int d^4x \left(\tau^a \partial^\mu A^{h,a}_\mu + \overline{\eta}^a \partial^\mu D_\mu(A^h)\eta^a\right) +$$

$$+ \int d^4x \left\{ \frac{1}{2\zeta}(\delta\sigma)^2 - \frac{1}{2\zeta}(\sigma + \delta\sigma)(A^h)^2 + \frac{1}{8\zeta}[(A^h)^2]^2 \right\}$$

$$D_{(\delta\sigma)}(p^2) = \zeta$$

$$-\frac{\sigma}{2\zeta}(A^h)^2 = -\frac{\sigma}{2\zeta}(A - \partial\xi)^2 + O(g/\zeta) \quad \Longrightarrow \quad m^2 = -\frac{\sigma}{\zeta}$$

+ an infinite number of interactions

to lowest order: a cubic $\delta\sigma A^2$ and a quartic $(A^2)^2$

The quadratic operator $(A^h)^2$: effective action, Landau gauge ($\overline{\text{MS}}$)

$$I_L = S_{\text{FP}}\big|_{\alpha=0} + \int d^4x \left\{ \frac{1}{2\zeta}(\delta\sigma)^2 - \frac{1}{2\zeta}(\sigma + \delta\sigma)\, A^2 + \frac{1}{8\zeta}(A^2)^2 \right\}$$

$$V(\sigma) = \frac{\mu^{2\epsilon}}{2\zeta}\sigma^2 - \frac{3N_A}{64\pi^2\zeta^2}\mu^{2\epsilon}\sigma^2 \left[\ln\left(-\frac{\overline{\mu}^2}{\mu^\epsilon \sigma/\zeta} \right) + \frac{5}{6} \right]$$

$$m^2 = -\frac{\mu^\epsilon \sigma}{\zeta} \quad \Longrightarrow \quad V(m^2) = \zeta\frac{m^4}{2} - \frac{3N_A}{64\pi^2} m^4 \left(\ln\frac{\overline{\mu}^2}{m^2} + \frac{5}{6} \right)$$

Reduction of couplings

- Roughly

$$\zeta = \zeta(g^2) \quad \Longrightarrow \quad \frac{\partial \zeta}{\partial g^2} = \frac{\mu\, d\zeta/d\mu}{\mu\, dg^2/d\mu} = \frac{\gamma_\zeta \zeta}{\beta_{g^2}}$$

$$\gamma_\zeta = \frac{\gamma_0}{\zeta} + \text{h.o.}\ , \quad \beta_{g^2} = -\beta_0\, g^4 + \text{h.o.} \quad \Longrightarrow \quad \frac{\partial \zeta}{\partial g^2} = -\frac{\gamma_0}{\beta_0}\frac{1}{g^4} + \text{h.o.}$$

$$\beta(g^2) = \frac{\gamma_0}{\beta_0}\frac{1}{g^2} + \text{h.o.}$$

- Explicit calculation:

$$\zeta(g^2) = \frac{N_A}{g^2 N}\frac{9}{13} + \frac{161}{52}\frac{N_A}{16\pi^2} + \cdots$$

The quadratic operator $(A^h)^2$: effective action ($\overline{\text{MS}}$)

$$m^2 = -\frac{\mu^\epsilon \sigma}{(\zeta_0/g^2 + \zeta_1) + O(g^2)} = m_0^2 \left(1 - \frac{\zeta_1}{\zeta_0} g^2 + O(g^4)\right)$$

$$m_0^2 = -\frac{\mu^\epsilon g^2 \sigma}{\zeta_0}$$

$$V(m_0^2) = \frac{9}{13}\frac{N_A}{N}\frac{m_0^4}{2g^2} - \frac{3N_A}{64\pi^2} m_0^4 \left(\ln\frac{\overline{\mu}^2}{m_0^2} + \frac{113}{39}\right) + O(g^2)$$

The quadratic operator $(A^h)^2$: gap equation ($\overline{\text{MS}}$)

$$V'(m_0^2) = \frac{9}{13}\frac{N_A}{N}\frac{m_0^2}{g^2} - \frac{3N_A}{32\pi^2}\, m_0^2 \left(\ln\frac{\overline{\mu}^2}{m_0^2} + \frac{187}{78}\right)$$

$$V'(m_0^2) = 0 \quad \Longleftrightarrow \quad m_0^2 = \overline{\mu}^2\, \exp\left(\frac{187}{78} - \frac{3\cdot 32\pi^2}{13Ng^2}\right)$$

$$V(m_0^2) = -\frac{3N_A m_0^4}{128\pi^2} < 0 = V(m_0^2 = 0)$$

Renormalization in detail (Landau gauge)

$$J_B\, A_B^h \cdot A_B^h = (Z_J\, Z_{A^h})\, J\, A^h \cdot A^h = Z_2\, J\, A^h \cdot A^h\ ,$$
$$\zeta_B\, J_B^2 = Z_\zeta \zeta\, \mu^{-\epsilon}\, J^2$$

$$\Delta S = \int d^4x\, \left(\frac{Z_2}{2}\, J\, A^h \cdot A^h - \mu^{-\epsilon} \frac{Z_\zeta \zeta}{2}\, J^2 \right)$$

$$\sigma[J] = \frac{\delta W}{\delta J}[J] = \frac{Z_2}{2} \left\langle A^h \cdot A^h \right\rangle_J - Z_\zeta \zeta\, \mu^{-\epsilon}\, J$$

$$I_L = S_{\mathsf{FP}}\big|_{\alpha=0} + \int d^dx\, \left[\frac{\mu^\epsilon}{2 Z_\zeta \zeta} (\delta\sigma)^2 - \frac{\mu^\epsilon Z_2}{2 Z_\zeta \zeta} (\sigma + \delta\sigma) A^2 + \frac{\mu^\epsilon Z_2^2}{8 Z_\zeta \zeta} (A^2)^2 \right]$$

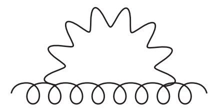
Gluon propagator (Landau gauge)

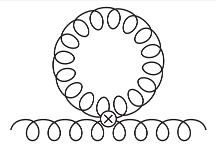
- General expression

$$[\Delta^{-1}(p)]^{ab}_{\mu\nu} = \delta^{ab}\left[Z_A\, p^2\, t_{\mu\nu}(p) + (1+\delta Z_2 - \delta Z_\zeta)\, m^2\, \delta_{\mu\nu} + \Pi_{\mu\nu}(p)\right]$$

- Polarization diagrams

Ordinary (massive) +





Gluon propagator (Landau gauge)

- Polarization (off-shell)

$$\Pi_{\mu\nu}(p) = \Pi^{(\mathrm{CF})}_{\mu\nu}(p) + \frac{(d-1)N_A}{2}\frac{\mu^\epsilon}{\zeta}\delta_{\mu\nu}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2+m^2}$$

- Propagator (off-shell)

$$\Delta^{-1}_{\mu\nu}(p) = Z_A p^2\, t_{\mu\nu}(p) + \delta Z_2\, m^2\, \delta_{\mu\nu} + \Pi^{(\mathrm{CF})}_{\mu\nu}(p) + \\ + \delta_{\mu\nu}\left(Z_\zeta^{-1}\, m^2 + \frac{(d-1)N_A}{2}\frac{\mu^\epsilon}{\zeta}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2+m^2}\right)$$

Gluon propagator (Landau gauge)

- Propagator (on-shell)

$$V'(m^2) = 0 \iff Z_\zeta^{-1}\, m^2 + \frac{(d-1)N_A}{2}\frac{\mu^\epsilon}{\zeta}\int \frac{d^dq}{(2\pi)^d}\frac{1}{q^2+m^2} = 0$$

$$\Delta(p^2) = \frac{1}{Z_A p^2 + \delta Z_2\, m^2 + \Pi_T^{(\mathrm{CF})}(p^2)}$$

$$m^2 = -\frac{\mu^\epsilon\, g^2 Z_2 \left\langle A^2 \right\rangle}{2\zeta_0} \qquad \left(\zeta_0 = \frac{9N_A}{13N}\right)$$

- Divergences

$$[\Pi_T^{(\mathrm{CF})}(p^2)]_{\mathrm{div.}} = -\lambda\left(\frac{13}{6}\,p^2 - \frac{3}{4}\,m^2\right)\frac{2}{\epsilon}\,, \quad \lambda = \frac{N\alpha_s}{4\pi}$$

Ghost propagator (Landau gauge)

- Polarization diagrams

Ordinary (massive gluon)

- Propagator

$$\mathcal{G}(p^2) = \frac{1}{Z_c\, p^2 + \Sigma^{(\mathrm{CF})}(p^2)}$$

- Divergences

$$[\Sigma^{(\mathrm{CF})}(p^2)]_{\mathrm{div}} = -\frac{3\lambda}{4}\, p^2\, \frac{2}{\epsilon}$$

Renormalization (Landau gauge)

- Needed counterterms

$$\delta Z_A = \frac{13\lambda}{6}\,\frac{2}{\epsilon} + \text{fin.}\,, \quad \delta Z_2 = -\frac{3\lambda}{4}\,\frac{2}{\epsilon} + \text{fin.}\,, \quad \delta Z_c = \frac{3\lambda}{4}\,\frac{2}{\epsilon} + \text{fin.}$$

$$([Z_2 Z_c]_{\text{div}} = 1)$$

- Mass renormalization

$$m^2 = -\frac{g^2\mu^\epsilon}{2\zeta_0} Z_2 \left\langle A^2 \right\rangle = -\frac{g_B^2\mu^\epsilon}{2\zeta_0}\,\frac{Z_2}{Z_g^2 Z_A} \left\langle A_B^2 \right\rangle = Z_{m^2}\, m_B^2$$

$$Z_{m^2} = \frac{Z_2 Z_c^2}{Z_{g^2} Z_A Z_c^2}$$

Renormalization (Landau gauge)

- Renormalization conditions: propagators and coupling (MOM-Taylor scheme)

$$\Delta(\mu^2;\mu^2) = \mathcal{G}(\mu^2;\mu^2) = \frac{1}{\mu^2}\,, \qquad Z_{g^2} Z_A Z_c^2 = 1$$

i.e.

$$Z_A = 1 - \delta Z_2 \frac{m^2}{\mu^2} - \frac{\Pi_T^{(\mathrm{CF})}(p^2=\mu^2)}{\mu^2}\,, \quad Z_c = 1 - \frac{\Sigma^{(\mathrm{CF})}(p^2=\mu^2)}{\mu^2}$$

$$(\text{plus } \; Z_{m^2} = Z_2 Z_c^2)$$

Renormalization (Landau gauge)

- DIS scheme: $Z_2 \neq Z_c^{-1}$

  $Z_2 Z_c = 1$ yields a massless gluon propagator $\implies$ choose something else, e.g.

$$\delta Z_2(\mu) = -\delta Z_c(\mu) + \lim_{\mu' \to 0} [\delta Z_c(\mu')]_{\text{fin.}} = -\delta Z_c(\mu) + \frac{5\lambda}{8}$$

RG improvement (Landau gauge)

- Anomalous dimensions

$$\gamma_A = -\mu \frac{d}{d\mu}\left(\frac{\Pi_T^{(\mathrm{CF})}(p^2=\mu^2)}{\mu^2} + \frac{m^2}{\mu^2}\frac{\Sigma^{(\mathrm{CF})}(p^2=\mu^2)}{\mu^2} + \frac{5\lambda m^2}{8\mu^2} \right),$$

$$\gamma_c = -\mu \frac{d}{d\mu}\left(\frac{\Sigma^{(\mathrm{CF})}(p^2=\mu^2)}{\mu^2} \right)$$

$$\mu \frac{dm^2}{d\mu} = \gamma_{m^2}\, m^2 \quad \Longrightarrow \quad \gamma_{m^2} = \gamma_2 + 2\gamma_c = \gamma_c \quad (\gamma_2 = -\gamma_c)$$

- Beta function

$$\beta_\lambda = \lambda(\gamma_A + 2\gamma_c)$$

RG improvement (Landau gauge)

- Identities

$$\gamma_A = \frac{\beta_\lambda}{\lambda} - 2\gamma_{m^2} \,, \qquad \gamma_c = \gamma_{m^2}$$

- RG-improved propagators

$$\Delta(p^2;\mu_0^2) = \frac{1}{p^2}\frac{\lambda(p^2)}{\lambda(\mu_0^2)}\frac{m^4(\mu_0^2)}{m^4(p^2)} \,, \qquad \mathcal{G}(p^2;\mu_0^2) = \frac{1}{p^2}\frac{m^2(p^2)}{m^2(\mu_0^2)}$$

- UV limit

$$\lambda(\mu^2) \sim 1/\ln(\mu^2) \,, \quad m^2(\mu^2) \sim [\lambda(\mu^2)]^{\frac{9}{44}} \sim [\ln\mu^2]^{-\frac{9}{44}} \,,$$

$$p^2\Delta(p^2) \sim [\lambda(p^2)]^{\frac{13}{22}} \sim [\ln p^2]^{-\frac{13}{22}} \,, \quad p^2\mathcal{G}(p^2) \sim [\lambda(p^2)]^{\frac{9}{44}} \sim [\ln p^2]^{-\frac{9}{44}}$$

- IR limit

$$\lambda(\mu^2) \sim \frac{\mu^2}{m^2} \,, \quad m^2(\mu^2) \sim \text{const} \,, \quad \Delta(p^2) \sim m^{-2}(0) \,, \quad p^2\mathcal{G}(p^2) \sim \text{const}$$

RG improvement (Landau gauge): lattice fit

- RG-improved potential ($\overline{\text{MS}}$)

$$V(m^2) = \frac{9}{13}\frac{N_A}{N}\frac{m^4(\mu)}{2g^2(\mu)}\left(1+\beta_0\frac{g^2(\mu)}{16\pi^2}\ln\frac{m^2(\mu)}{\mu^2}\right)^{1+\gamma_0/\beta_0}$$

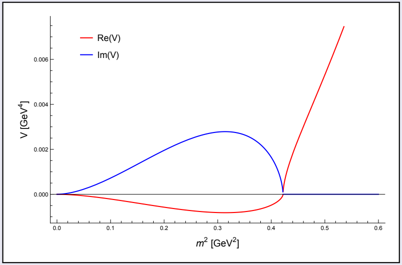
$$\beta_0 = \frac{11N}{3}\,, \qquad \gamma_0 = -\frac{3N}{2}$$

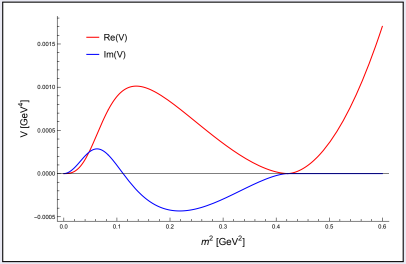
- Scheme conversion ($\overline{\text{MS}} \leftrightarrow$ DIS)

$$m^2_{\text{DIS}} = \frac{Z_{m^2,\overline{\text{MS}}}}{Z_{m^2,\text{DIS}}}\, m^2_{\overline{\text{MS}}}\,, \qquad \lambda_{\text{DIS}} = \frac{Z^2_{g,\overline{\text{MS}}}}{Z^2_{g,\text{DIS}}}\,\lambda_{\overline{\text{MS}}}$$

RG improvement (Landau gauge): lattice fit

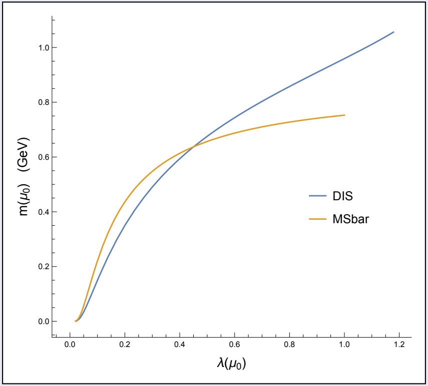
- Next-to-leading log RG-improved potential ($\overline{\text{MS}}$)



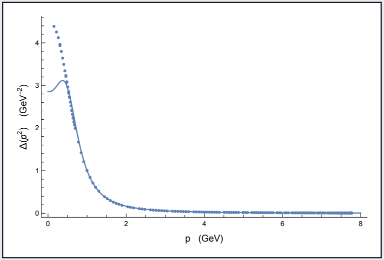


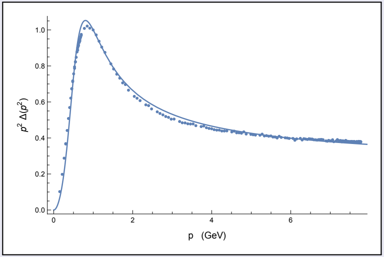
RG improvement (Landau gauge): lattice fit

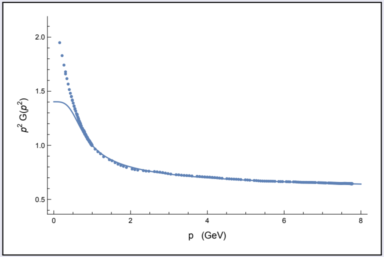
- Solutions of the gap equation ($\mu_0 = 1$ GeV)



RG improvement (Landau gauge): lattice fit





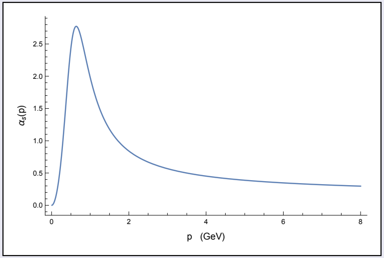


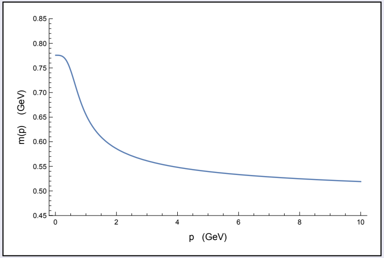
$$\lambda(\mu_0^2) = 0.473 \implies$$

$$\implies m(\mu_0^2) = 0.655 \text{ GeV}$$

$$@ \ \mu_0 = 1 \text{ GeV}$$

RG improvement (Landau gauge): lattice fit





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