SIM 2024

Marco Ruggieri
Dipartimento di Fisica e Astronomia, Catania University,
and INFN, Sezione di Catania
marco.ruggieri@dfa.unict.it

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- *Marco Ruggieri (PA, 100%), Fabio Siringo (PA, 100%)*
- *Paolo Castorina (INFN, 50%)*
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- *Giorgio Comitini (Assegnista, 100%)*
- *Fabrizio Murgana (to rejoin)*
- *Binu (would-be-PhD, to join)*

Research

- *Early stage of HICs (Marco, Parisi, Binu)*
- *Heavy quarks in the early stage of HICs (Marco, Parisi, Lucia)*
- *Superdense QCD, walls, axions and η' (Marco, Fabrizio, Ana&David)*
- *FRG applied to the QCD phase diagram (Marco, Fabrizio)*
- *Stochastic processes with memory (Marco, Santosh et al.)*
- *Non-perturbative QCD at finite T and μ (Fabio, Giorgio)*
- *Supersolids, HICs (Paolo)*
- *Hadronization in HICs (Vincenzo)*

Plan of the talk

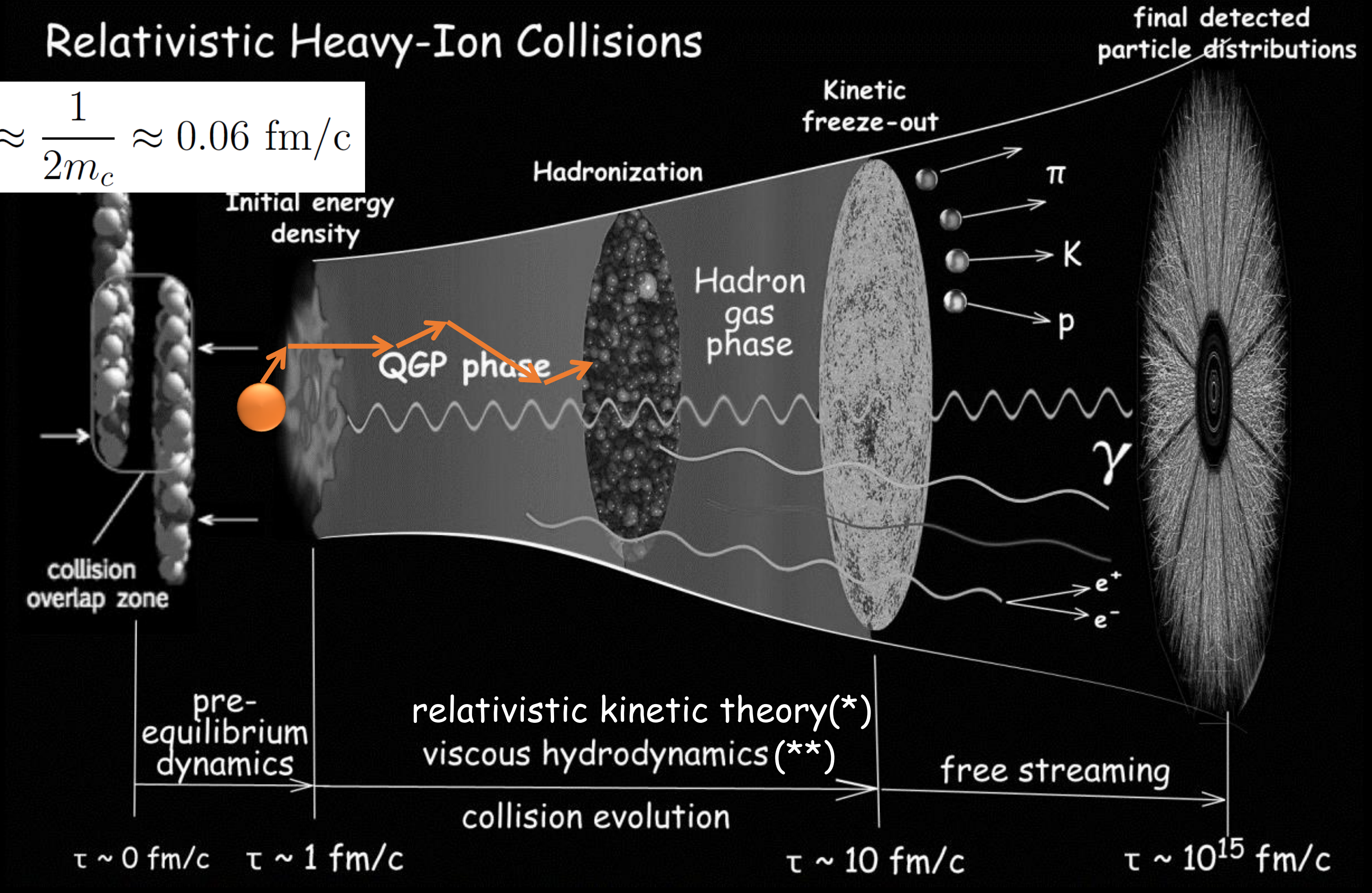
- *Heavy quarks and quarkonia in Glasma (pA and AA)*
- *Topological susceptibility in superdense QCD*
- *Outlook*

Heavy quarks and quarkonia in Glasma

Heavy quarks, c and b, in the pre-equilibrium stage of HICs

$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06\ \text{fm/c}$$

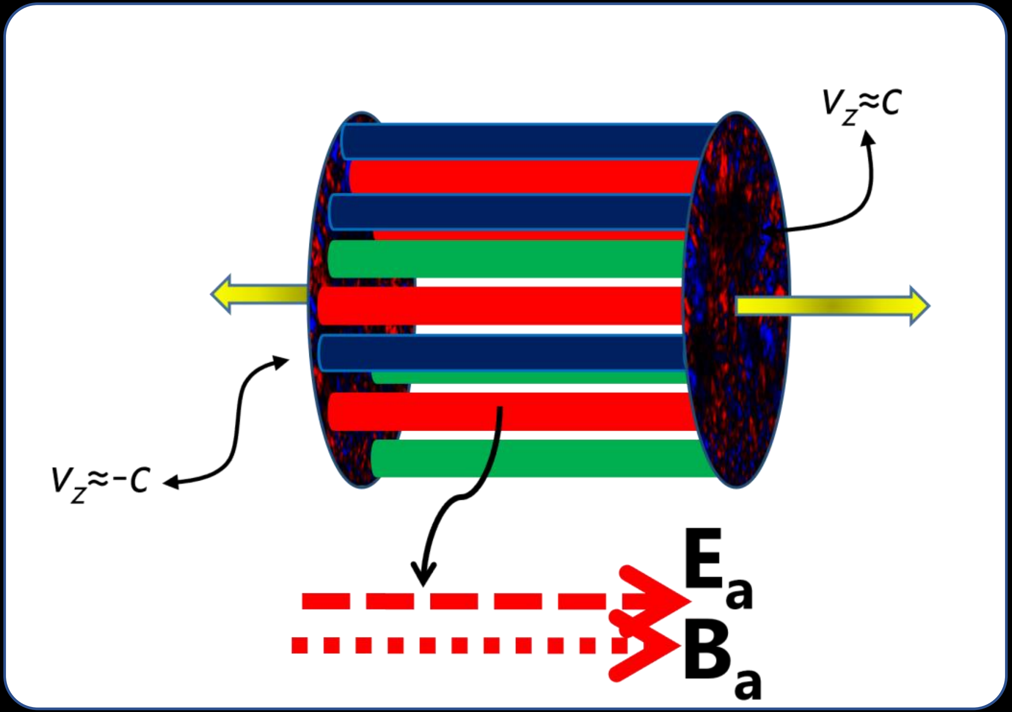
HQs can probe the entire evolution of the medium, from the early stage up to hadronization



The pre-equilibrium stage: Glasma as the initial condition

Many gluons in the early stage

Useful, easy description in terms of ***classical, intense fields***, rather than in terms of one-particle states

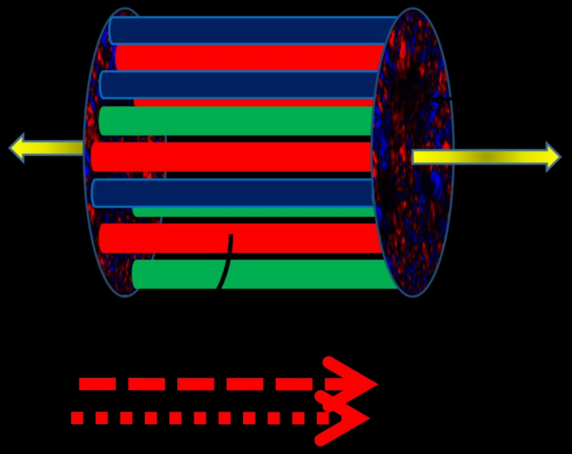


Glasma (*)

Glasma: *initial condition* for the medium produced in RHICs.

(*)Lappi and McLerran (2006)

The pre-equilibrium stage: Glasma as the initial condition



Strength of initial fields: $O(Q_s^2)$

Initial energy density: $O(Q_s^4)$

Q_s: saturation scale

Qs is the only energy scale in this model

$$\boldsymbol{Q_s \approx 1 - 3\ \mathrm{GeV}}$$

The pre-equilibrium stage: evolving the Glasma via CYM equatinos

Due to the large density the gluon field behaves like a classical field:
Dynamics is governed by classical EoMs, namely the classical Yang-Mills (CYM) equations.

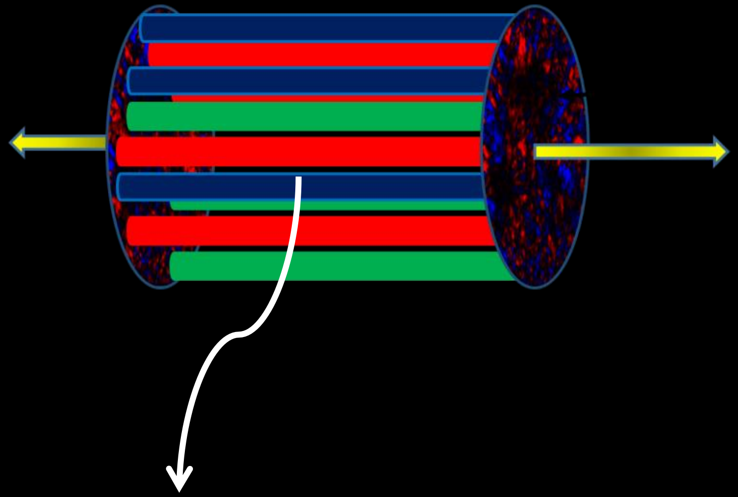
$$\left(D^\mu F_{\mu\nu}\right)^a = 0$$

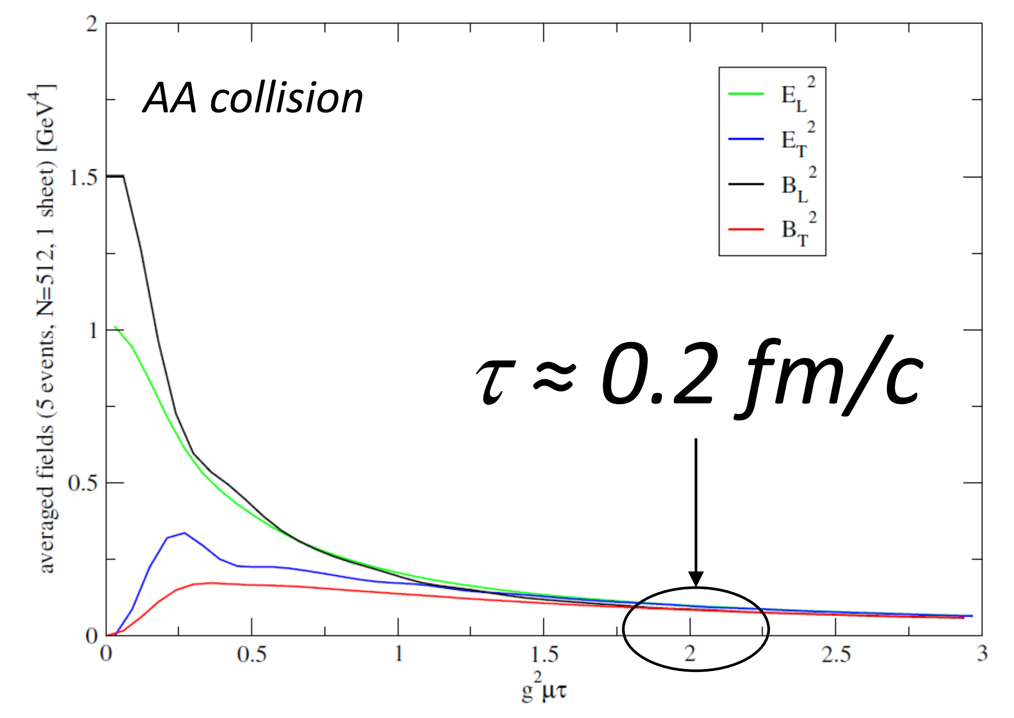
$$\begin{aligned} \tau &= \sqrt{t^2 - z^2} \\ \eta &= \frac{1}{2}\log\left(\frac{t+z}{t-z}\right) \end{aligned}$$

$$\begin{aligned} \partial_\tau E_i &= \tfrac{1}{\tau}\mathcal{D}_\eta F_{\eta i} + \tau \mathcal{D}_j F_{ji}, \\ \partial_\tau E_\eta &= \tfrac{1}{\tau}\mathcal{D}_j F_{j\eta}, \end{aligned} \qquad \begin{aligned} E_i &= \tau \partial_\tau A_i, \\ E_\eta &= \frac{1}{\tau}\partial_\tau A_\eta. \end{aligned}$$

Evolution of the system is studied assuming the Glasma initial condition, and evolving this condition by virtue of the CYM equations.

The pre-equilibrium stage: evolving fields in AA collisions





$$\varepsilon = \mathrm{Tr}\left[E_L^2 + E_T^2 + B_L^2 + B_T^2\right]$$

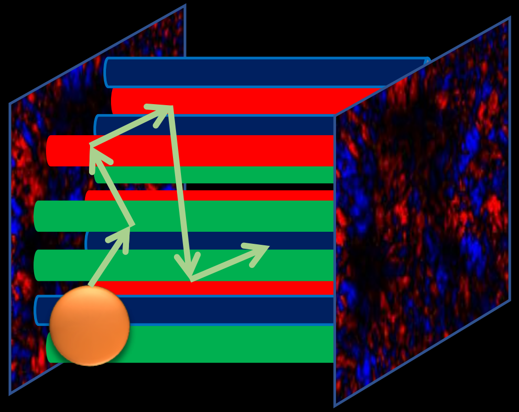
$$\left.\frac{dE_a^x}{dt}\right|_{t=0^+} = \partial_y B_z^a + f_{abc} A_y^b B_z^c$$

Formation time of transverse fields:

$Q_s\tau \approx 1$ namely $\tau \approx 0.1$ fm/c

For pA collisions the evolution of the fields is similar (G. Parisi et al. in preparation).

The pre-equilibrium stage: heavy quarks



Charm quarks

$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06 \text{ fm/c}$$

Relativistic kinetic theory of HQs in Glasma

$$\frac{dx_i}{dt} = \frac{p_i}{E} \qquad E = \sqrt{\boldsymbol{p}^2 + m^2}$$

$$E\frac{dp_i}{dt} = gQ_a F^a_{i\nu} p^\nu$$

$$E\frac{dQ_a}{dt} = -gQ_c \varepsilon^{cba} \boldsymbol{A}_b \cdot \boldsymbol{p}$$

$\boldsymbol{v} \equiv \frac{\boldsymbol{p}}{E}$ (Relativistic) Velocity

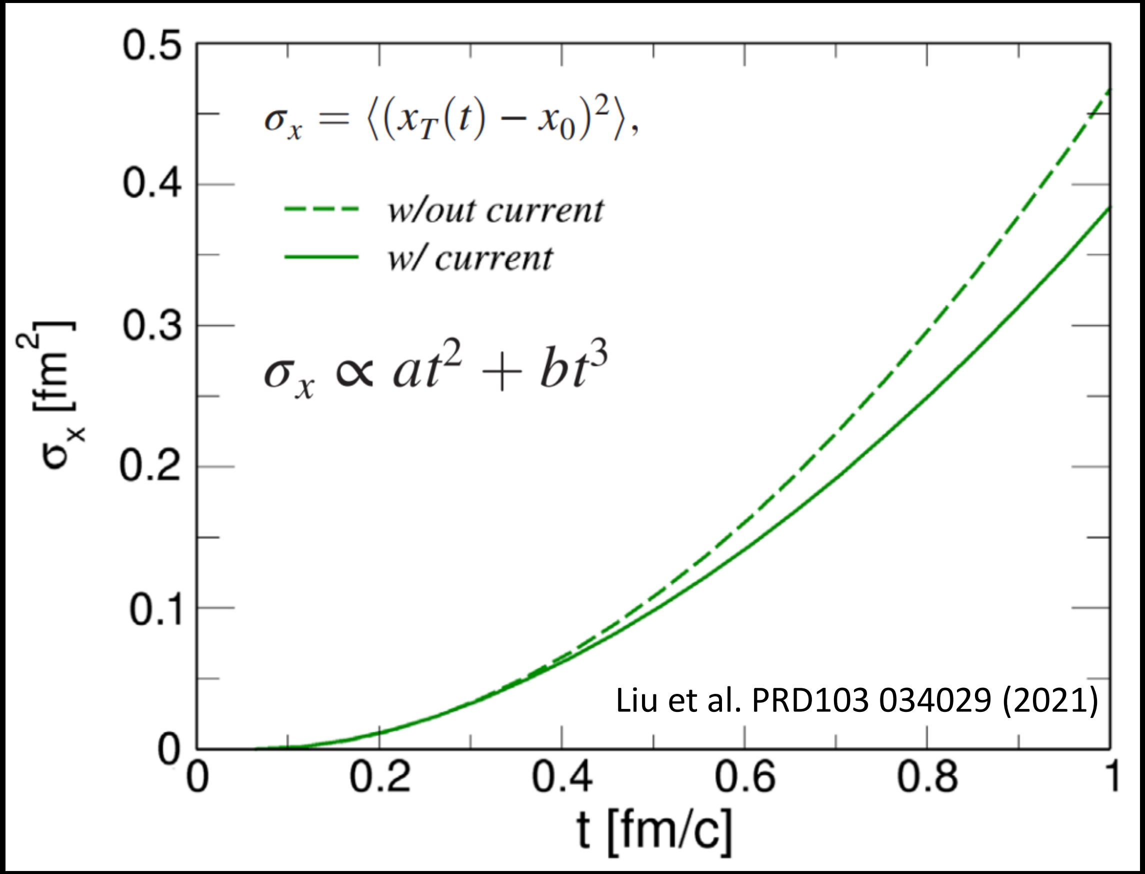
$\frac{d\boldsymbol{p}}{dt} = q\boldsymbol{E} + q\left(\boldsymbol{v} \times \boldsymbol{B}\right)$ Lorentz force

$D_\mu J^\mu_a = 0$ Gauge-invariant conservation of the color current carried by charm quaks + gluons

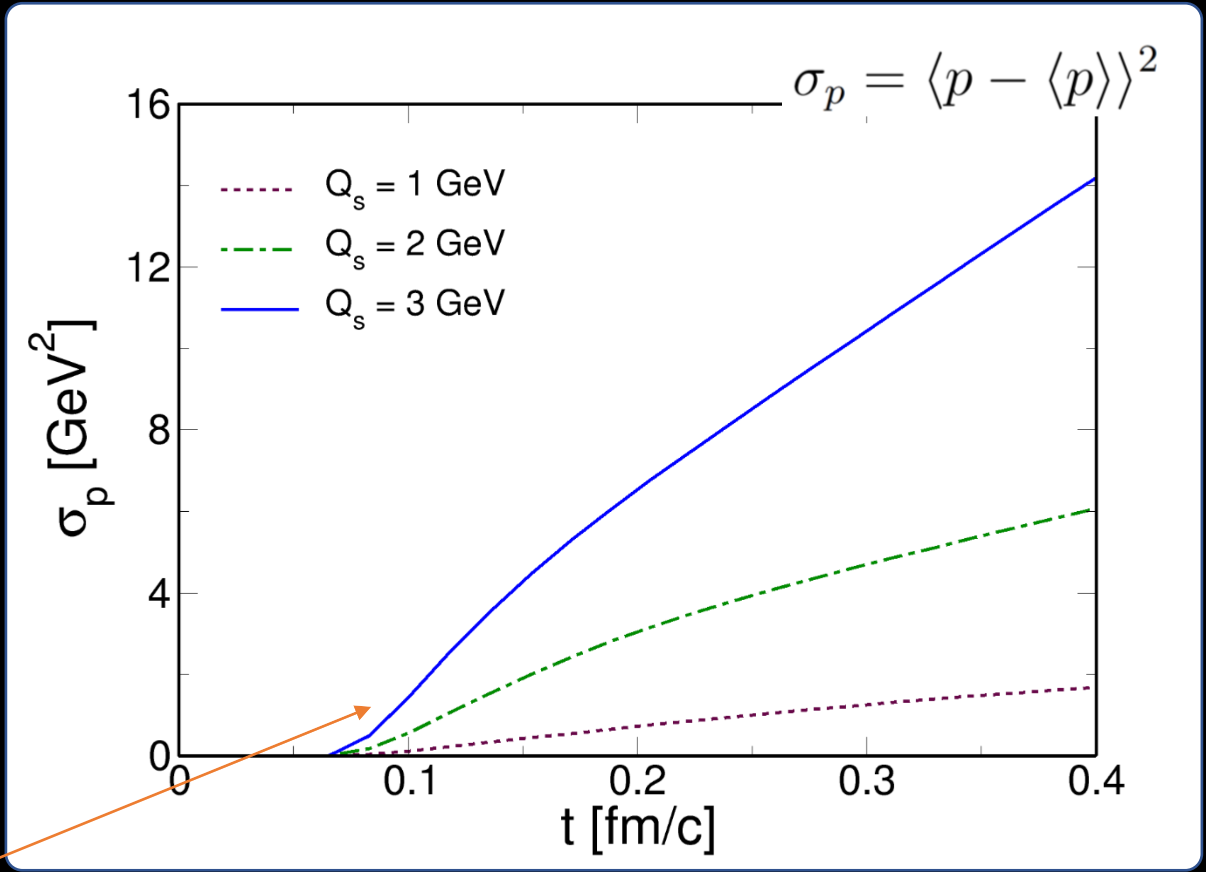
$J^\mu_a = \bar{c}\gamma^\mu T_a c$

Equations of motion of heavy quarks are solved in the background given by the evolving Glasma fields

The pre-equilibrium stage: diffusion of heavy quarks in the color filaments



Liu et al. PRD103 034029 (2021)



Diffusion in the color filament

Standard Brownian motion

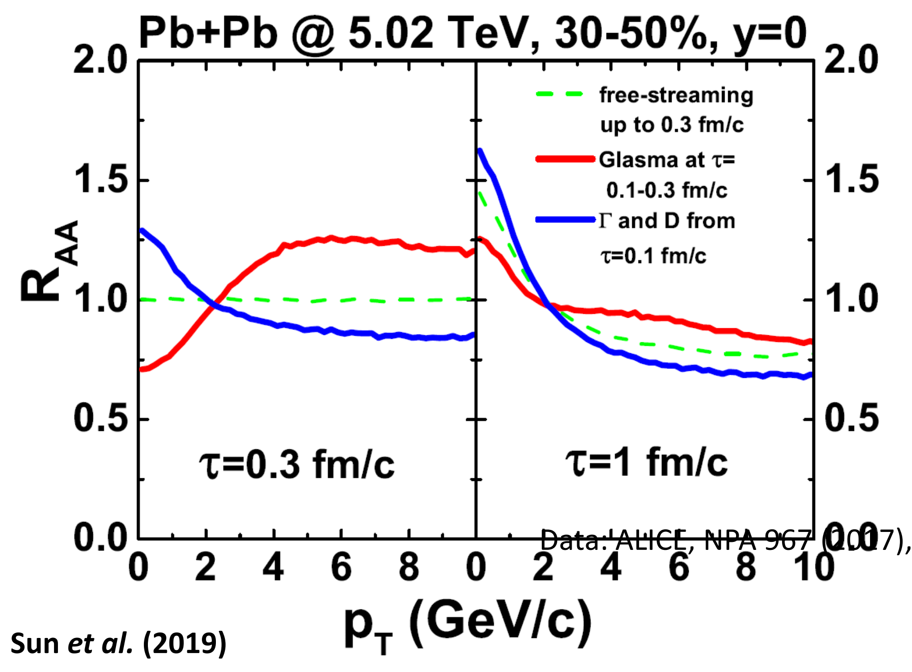
- *Early time:* $\sigma_p \approx Q_s D t^2$
- *Later time:* $\sigma_p \approx 2Dt$

D: momentum diffusion coefficient

See also D. Avramescu *et al.* (2023)

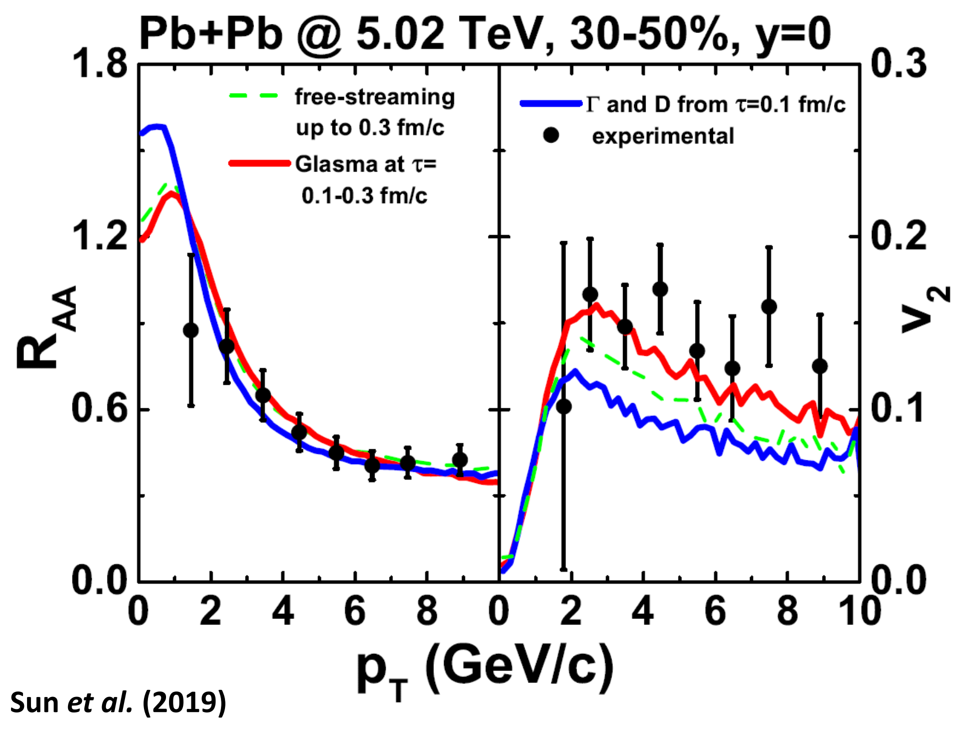
The pre-equilibrium stage: diffusion of heavy quarks in the color filaments

- *Diffusion in the early stage*
- *Evolution in the QGP*



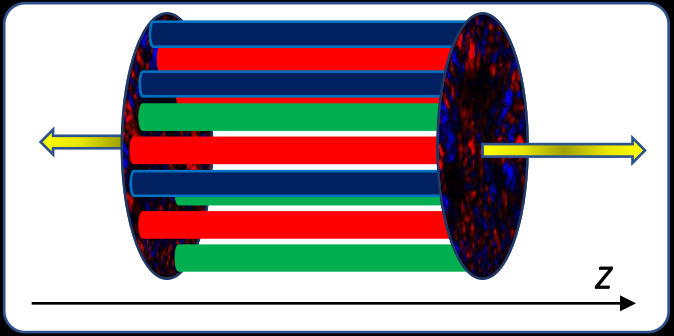
Data: ALICE, NPA 967 (2017),

Sun *et al.* (2019)



Sun *et al.* (2019)

Diffusion in the early stage helps to describe simultaneously the RAA and the v2.



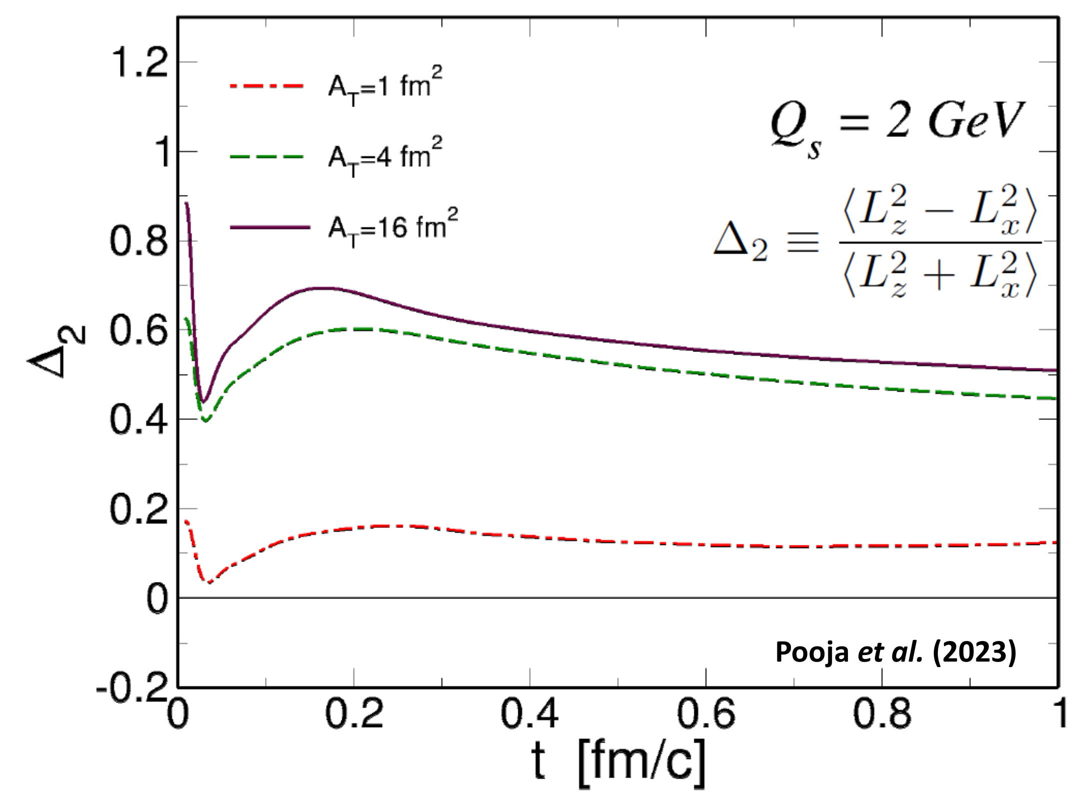
The pre-equilibrium stage: anisotropic angular momentum diffusion

Anisotropic distribution of the momentum
[Pooja et al. (2023), Ipp et al. (2020), Avramescu et al. (2023)]
as well as of angular momentum [Pooja et al. (2023)]

- Static box, «AA collisions»
- Infinite quark mass limit
- Two-colors QCD

Local polarization of c and b,
along the longitudinal direction

Naively: glasma induces vortex-like motion of c and b around color filaments in the transverse plane



Pooja *et al.* (2023)

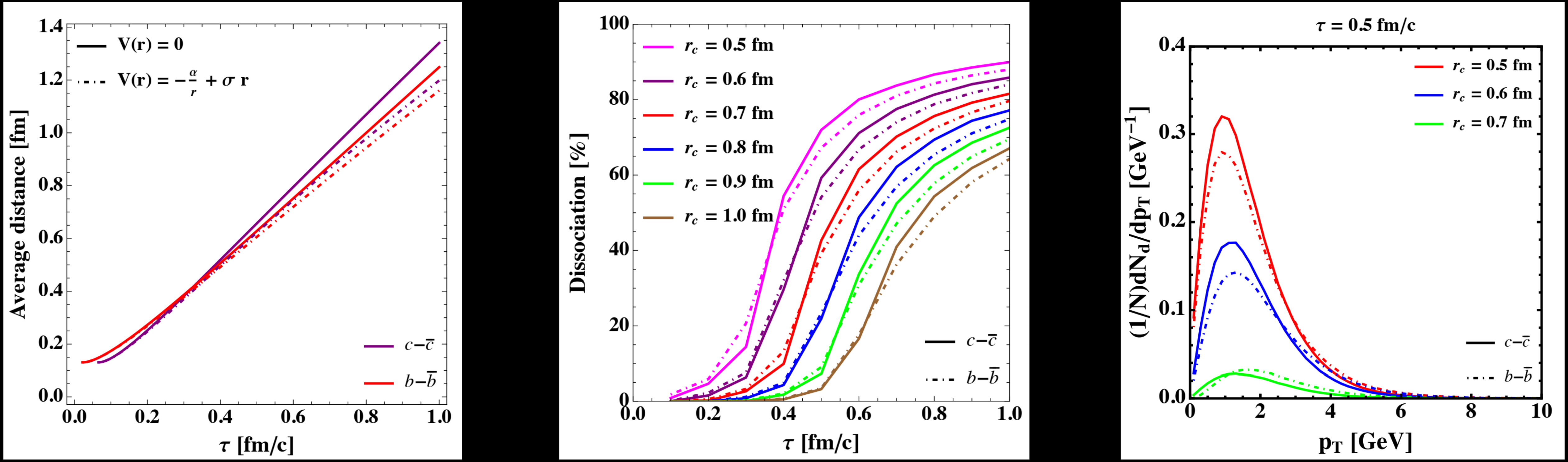
Quarkonya vs Glasma

$$\frac{dx^i}{dt} = \frac{p^i}{E},$$
$$\frac{dp^i}{dt} = gQ_a F_a^{i\nu} \frac{p_\nu}{E} - \frac{\partial V}{\partial x_i},$$
$$E\frac{dQ_a}{dt} = g\varepsilon_{abc} A_b^\mu p_\mu Q_c,$$

color-singlet potential

$$V(r) = -\frac{3\alpha_s}{4r} + \sigma r$$

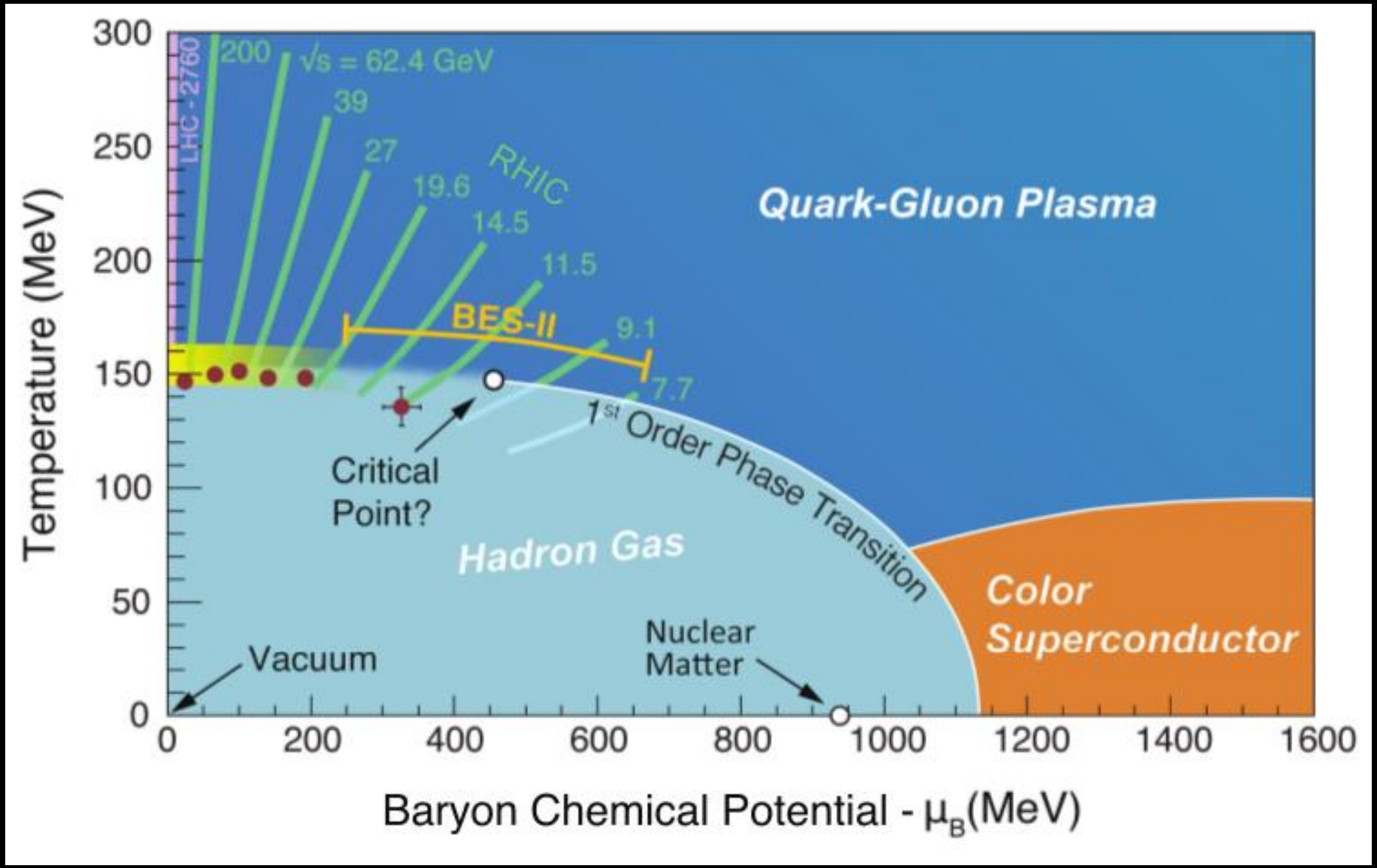
Dissociation of q-qbar pairs in Glasma



Pooja et al. (2404.05315)

Topological susceptibility in superdense QCD

The QCD phase diagram



→

Direction of increasing density

Two-flavor superconductive quark matter

2SC ansatz

$$\langle q^T_{\alpha i} C i \gamma_5 \mathcal{P}_L q_{\beta j} \varepsilon_{\alpha\beta 3} \varepsilon_{ij3} \rangle = -h_L, \quad \langle q^T_{\alpha i} C i \gamma_5 \mathcal{P}_R q_{\beta j} \varepsilon_{\alpha\beta 3} \varepsilon_{ij3} \rangle = h_R$$

Ruester et al. (2005)
Blaschke et al. (2005)
Rapp et al. (1998, 2000)
Alford et al. (1998)

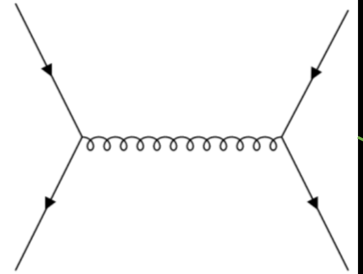
$$\Delta_L = 2G_D h_L, \quad \Delta_R = 2G_D h_R$$

L and R condensates

$$\Delta_{\mathrm{S}} = \Delta_R - \Delta_L, \quad \Delta_{\mathrm{PS}} = \Delta_R + \Delta_L$$

scalar *pseudoscalar*

Superconductive quark matter at finite θ



$U(1)_A$ preserving

$$\mathcal{L}_{\rm int} = G_D(q^TCi\gamma_5\varepsilon\varepsilon q)(\bar{q}Ci\gamma_5\varepsilon\varepsilon\bar{q}^T) + G_D(q^TC\varepsilon\varepsilon q)(\bar{q}C\varepsilon\varepsilon\bar{q}^T)$$
$$+\zeta G_De^{i\theta}(q^TCi\gamma_5\mathcal{P}_L\varepsilon\varepsilon q)(\bar{q}Ci\gamma_5\mathcal{P}_L\varepsilon\varepsilon\bar{q}^T)$$
$$+\zeta G_De^{-i\theta}(q^TCi\gamma_5\mathcal{P}_R\varepsilon\varepsilon q)(\bar{q}Ci\gamma_5\mathcal{P}_R\varepsilon\varepsilon\bar{q}^T)$$

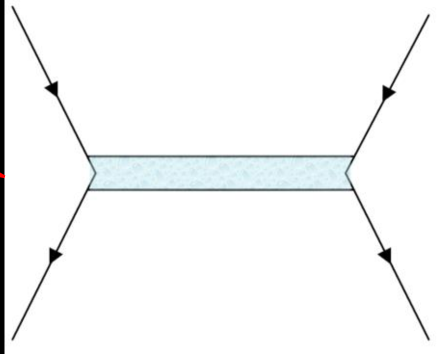
$U(1)_A$ breaking (t'Hooft-like)

$$\zeta G_D = \int d\rho\; n_0(\rho)\left(\frac{4}{3}\pi^2\rho^3\right)^2$$

$$n_0(\rho) \propto \frac{1}{\alpha_s^{2N_c}}\rho^{-5}\exp\left(-1/\alpha_s\right)e^{-N_f\mu^2\rho^2}$$

ρ: instanton size

$$\ell_{\rm ave} \equiv \langle\rho\rangle \propto 1/\mu$$



Topological susceptibility in 2SC at T=0

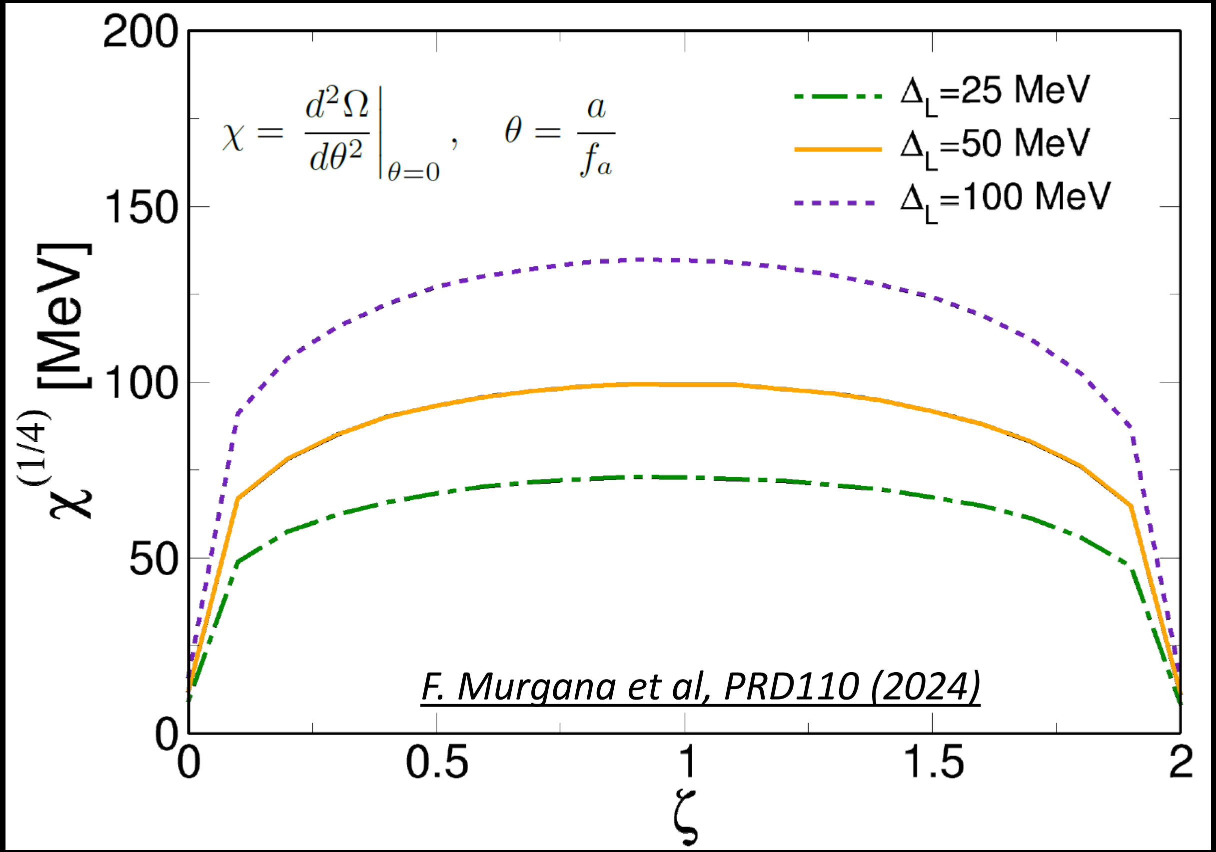
Analytical result for χ:

$$\chi = \frac{\Delta_L^2}{2G_D}\zeta\frac{2-\zeta}{2+\zeta},$$

Analogous to the Di Vecchia-Veneziano formula for the QCD vacuum

$$\chi = |\langle\bar{q}q\rangle|\frac{m_u m_d}{m_u+m_d}$$

Veneziano (1979), Di Vecchia-Veneziano (1980), Leutwyler and Smilga (1992), Crewther (1977)



F. Murgana et al, PRD110 (2024)

$$\chi = \left.\frac{d^2\Omega}{d\theta^2}\right|_{\theta=0}, \quad \theta = \frac{a}{f_a}$$

$$m_a^2 = \frac{1}{f_a^2}\left.\frac{d^2\Omega}{d\theta^2}\right|_{\theta=0}$$

$$m_a^2 = \frac{\Delta_L^2}{2G_D f_a^2}\frac{2-\zeta}{2+\zeta}\zeta,$$

Axion mass in 2SC(*)

(*)A similar result holds for the CFL phase (Murgana et al. In preparation)

Conclusions

- *Glasma as the initial condition in high energy nuclear collisions (pA and AA)*
- *Heavy quarks (c and b) can probe the pre-equilibrium stage, gluon-dominated, stage*
- *Interaction of HQs with evolving Glasma fields potentially affects observables (spectra, v2 and possibly more)*
- *Quarkonia melting in the early stage: rough model, which however gives indications on the amount of quark-antiquark pairs that can melt before the QGP forms*

- *Superconductive QCD at very high density: topological susceptibility for the 2SC and the CFL phases, potentially interesting for the QCD-axion and the η' physics (walls, spectral functions)*

Outlook

Outlook 0: (3+1)D Glasma and HQs in pA collisions

Wong equations for HQs in Glasma, augmented with:

- *η-dependent initial state fluctuations on top of the Glasma fields*

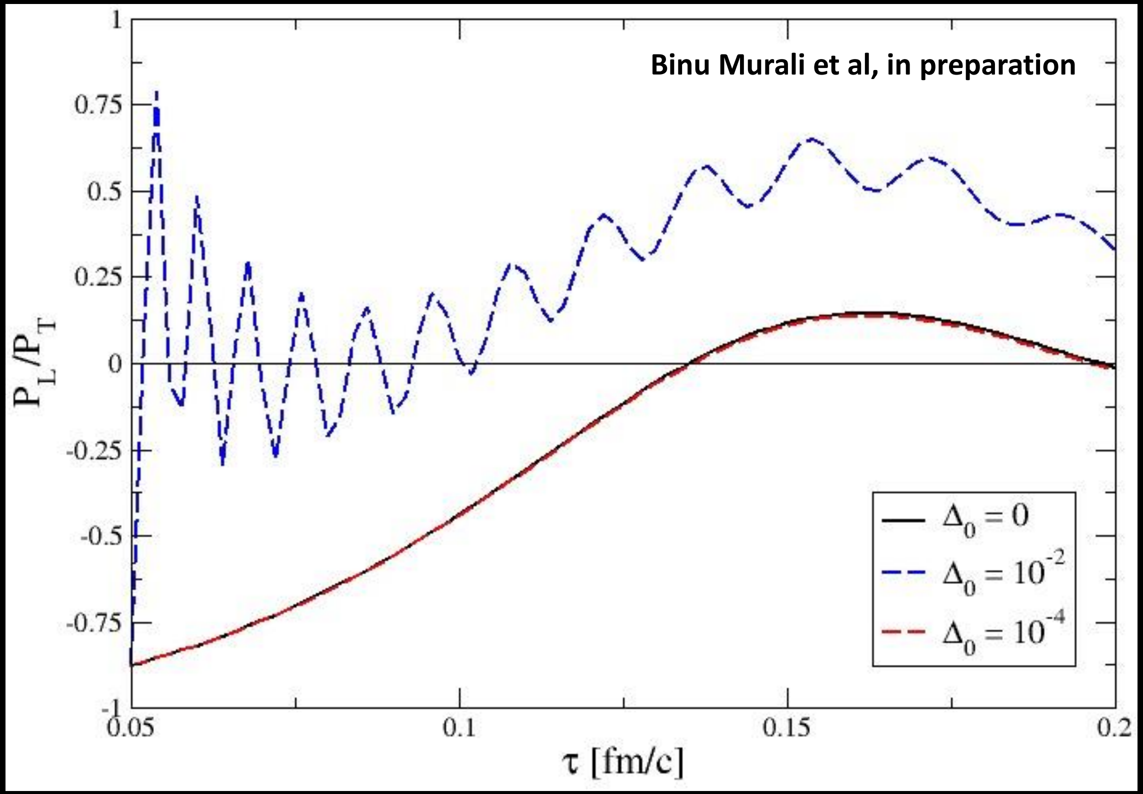
$$\delta E^i(\mathbf{x}_\perp, \eta) = -\partial_\eta F(\eta) \xi_i(\mathbf{x}_\perp),$$
$$\delta E^\eta(\mathbf{x}_\perp, \eta) = F(\eta) \sum_{i=x,y} D_i \xi_i(\mathbf{x}_\perp).$$

$$\langle \xi_i(\mathbf{x}_\perp) \xi_j(\mathbf{y}_\perp) \rangle = \delta_{ij} \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp),$$

$$F(\eta) \propto \Delta_0 \sum_n \left(\frac{2\pi\eta}{N_\eta} n \right)$$

Modes added on top of the Glasma

pA collisions, with η-dependent fluctuations



Binu Murali et al, in preparation

Quick partial isotropization in pA collisions

Outlook 1: quarkonia in Glasma, AA and pA collisions

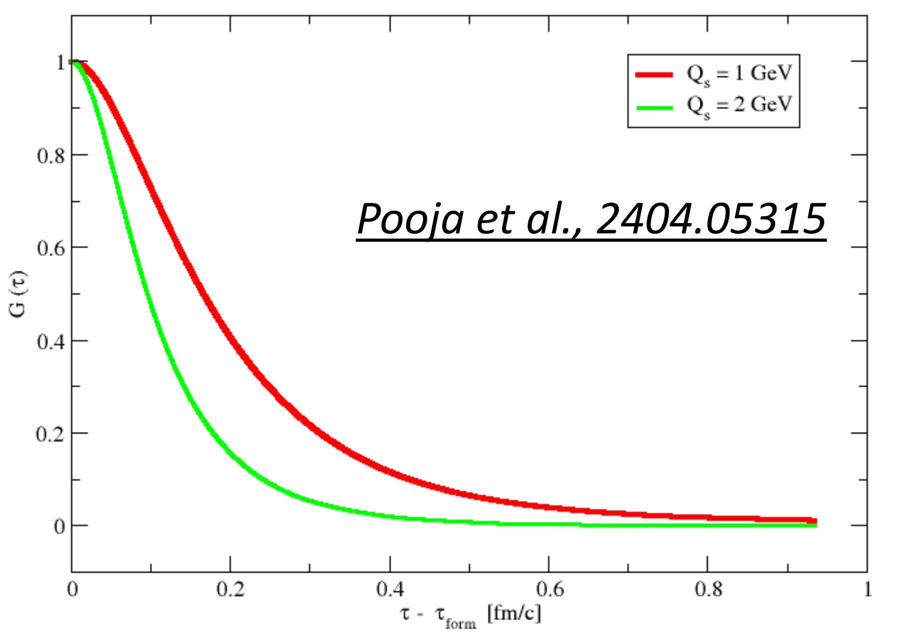
Wong equations for quarkonia in Glasma, augmented with:

- *singlet-to-octet and octet-to-singlet transitions implemented stochastically, with probabilities related to the color content of the pair:*

$$Q_a\bar{Q}_b = c_0\delta_{ab} + \sum_{n=1}^{N_c^2-1} c_n f_{nab} \qquad \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{1}+\mathbf{8}$$

- *alternatively, a simpler implementation can be obtained relating probabilities to the equal-time, gauge-invariant correlator*

$$G(\tau) = -\sum_{a=1}^{N_c^2-1} \langle Q_a(\tau)\bar{Q}_a(\tau)\rangle$$



Pooja et al., 2404.05315

Outlook 1: quarkonia in Glasma, AA and pA collisions

Wong equations for quarkonia in Glasma, augmented with:

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$$Q_a\bar{Q}_b = c_0\delta_{ab} + \sum_{n=1}^{N_c^2-1} c_n f_{nab} \qquad \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{1}+\mathbf{8}$$

- *survival or melting of each pair can be determined from the overlap in phase space by assigning a probability,* $P_{\text{hadronization}}$*, to the formation of a bound state(*), then*

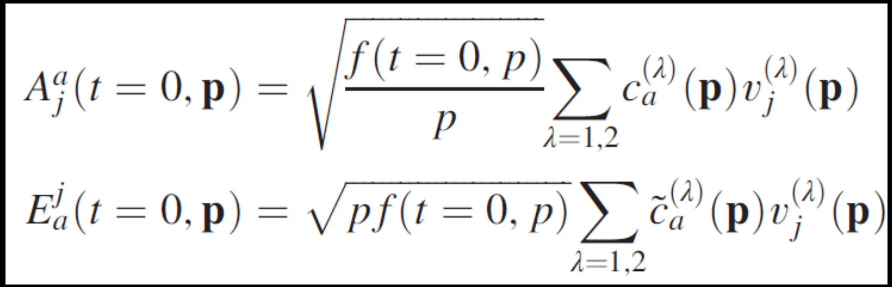
$$\mathcal{P}_{\text{melting}} = 1 - \mathcal{P}_{\text{hadronization}}$$

(*)Similarly to what is done in coalescence models with the Wigner wave function.

Outlook 3: paradigm shift for the initial condition

Replace Glasma with a gluon-saturated “distribution function”, f, for the initial condition.

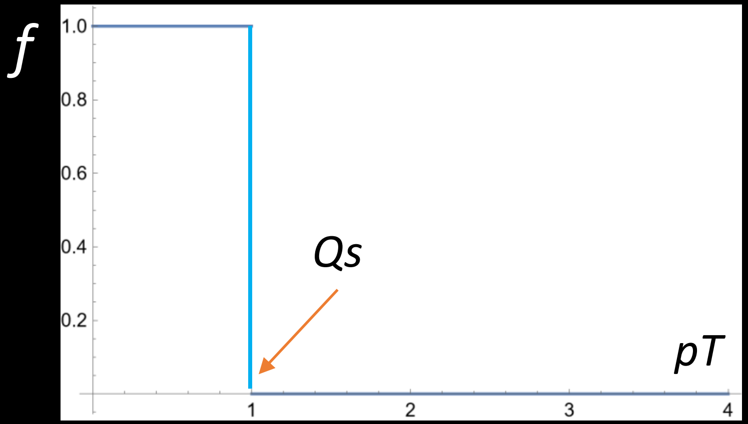
f roughly models the production of a lot of gluons below Qs and only a few gluons above Qs



$$A_j^a(t=0,\mathbf{p}) = \sqrt{\frac{f(t=0,p)}{p}}\sum_{\lambda=1,2} c_a^{(\lambda)}(\mathbf{p})v_j^{(\lambda)}(\mathbf{p})$$

$$E_a^j(t=0,\mathbf{p}) = \sqrt{pf(t=0,p)}\sum_{\lambda=1,2} \tilde{c}_a^{(\lambda)}(\mathbf{p})v_j^{(\lambda)}(\mathbf{p})$$

Bogulavski et al, PRD98, 014006 (2018)



Q_s in the transverse plane is built up in the same way we make it for the Glasma.

f is then converted into fields and these are evolved by CYM equations.

- *Pro: takes into account very early-time evolution which is probably untreatable, particularly when there are strong fluctuations (*)*
- *Contro: it is not the result of a first-principle calculation(*)*

(*)Strong fluctuations and non-perturbative dynamics could lead to a less anisotropic system with respect to the Glasma in a very short time, but treating this process with CYM equations might not be enough, hence change the initial condition.

$$\chi = \frac{\Delta_L^2}{2G_D}\zeta\frac{2-\zeta}{2+\zeta},$$

F. Murgana et al, PRD110 (2024)

Outlook 5: topological susceptibility in superdense QCD, via CJT-HDET

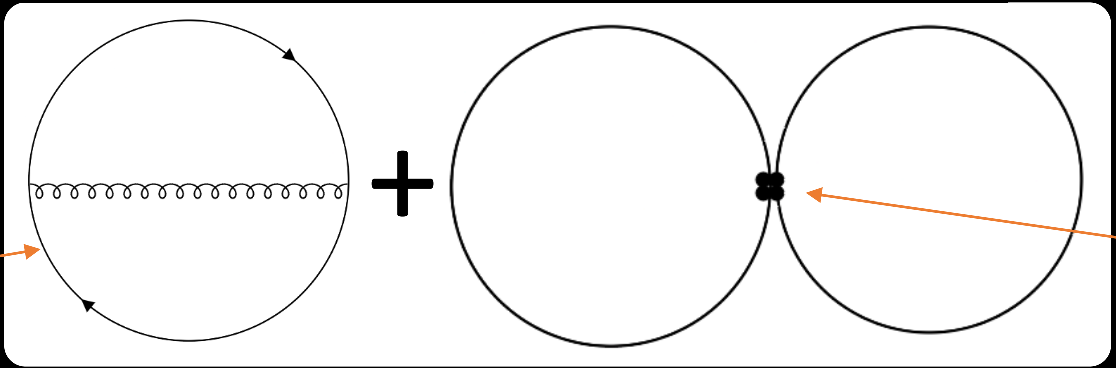
$$\Omega = \overbrace{-i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\log S_0^{-1}(p)S(p) - S_0^{-1}(p)S(p) + 1\right]}^{\text{quarks}} \underbrace{+\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\log D_0^{-1}(p)D(p) - D_0^{-1}(p)D(p) + 1\right]}_{\text{gluons}} + V_2(S,D)$$

Full HDET quark propagator

Dressed gluon propagator

$$V_2(S,D) = -\sum(\text{2PI vacuum bubbles})$$

$=$



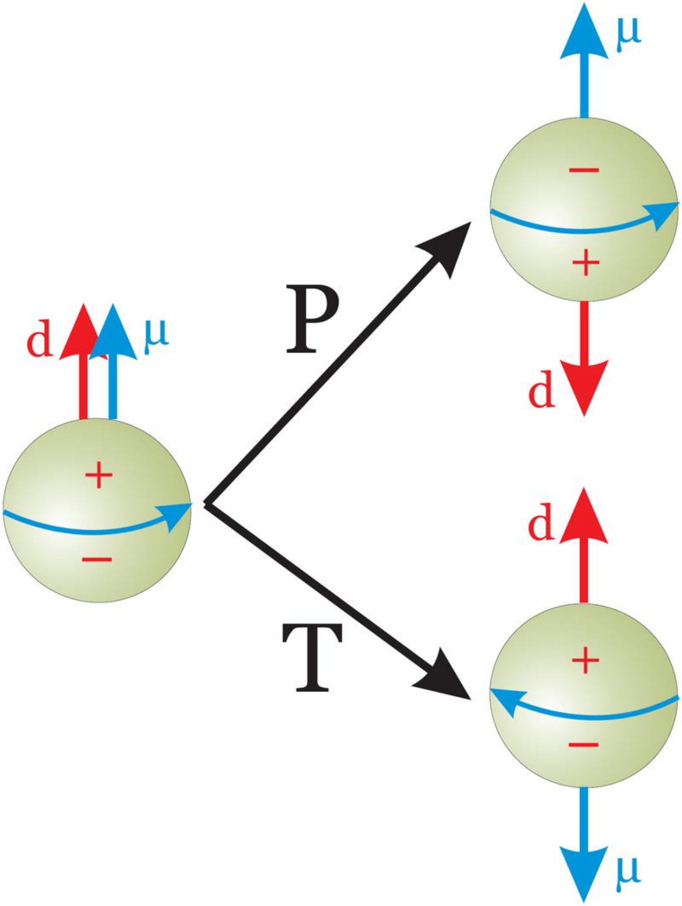
$U(1)_A$ preserving one-gluon exchange

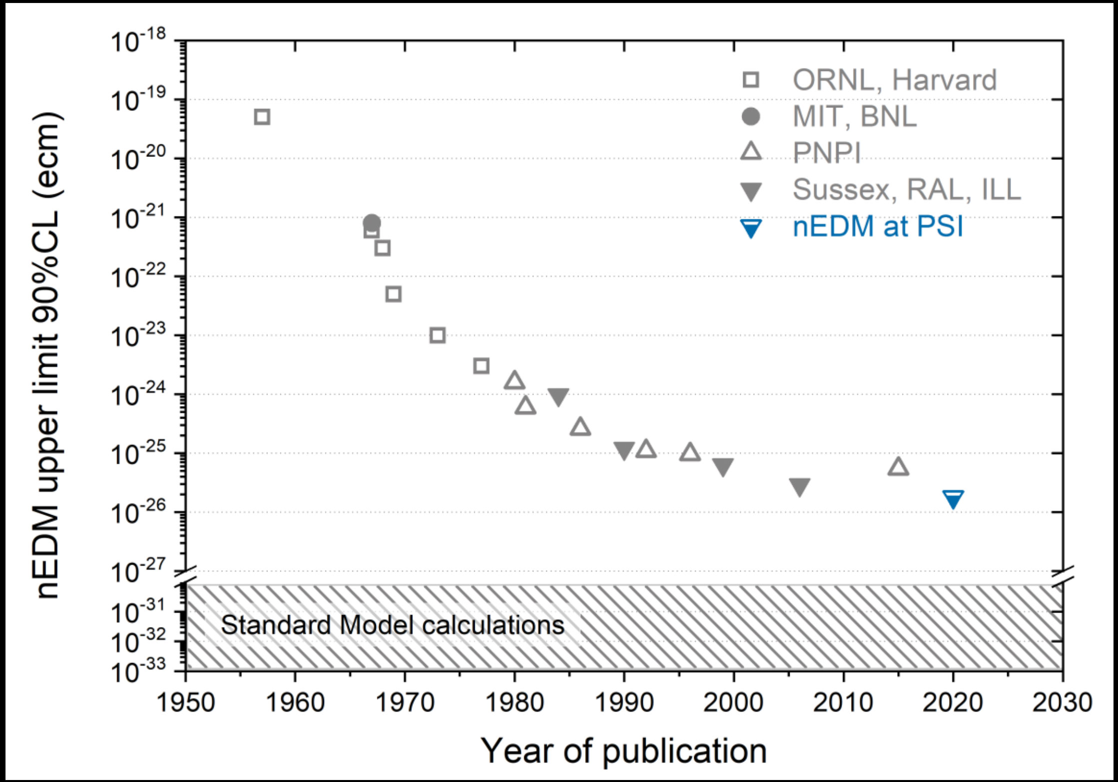
$U(1)_A$ breaking θ-dependent one-instanton exchange

Appendix

$$\mathcal{L}_{\mathrm{odd}} \propto \theta \tilde{F} \cdot F$$

$$\tilde{F} \equiv \varepsilon_{\mu\nu\rho\sigma} F_a^{\rho\sigma}$$





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$$\mathcal{L}_{\rm odd} \propto \theta \tilde{F} \cdot F$$

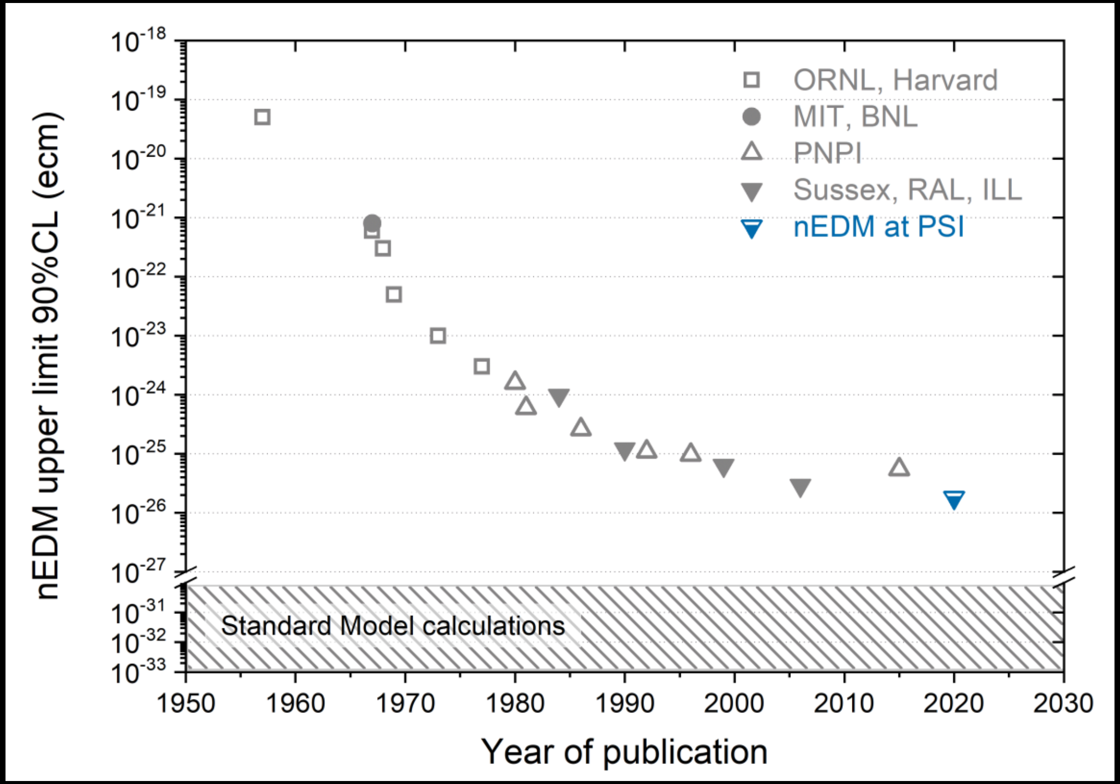
$$\tilde{F} \equiv \varepsilon_{\mu\nu\rho\sigma} F_a^{\rho\sigma}$$

Naturalness

θ=O(1)

Experimental limit

$\theta \lesssim 10^{-10}$ radians



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A vanilla introduction to the QCD axion

$$\mathcal{L}_{\rm odd} \propto \theta \tilde{F} \cdot F \qquad \tilde{F} \equiv \varepsilon_{\mu\nu\rho\sigma} F_a^{\rho\sigma}$$

$$\mathcal{L}_{\rm axion} \propto \frac{A}{f_a} \tilde{F} \cdot F$$

f_a: axion decay constant

$$\mathcal{L}_{\rm odd} + \mathcal{L}_{\rm axion} \propto \left(\theta + \frac{A}{f_a}\right) \tilde{F} \cdot F$$

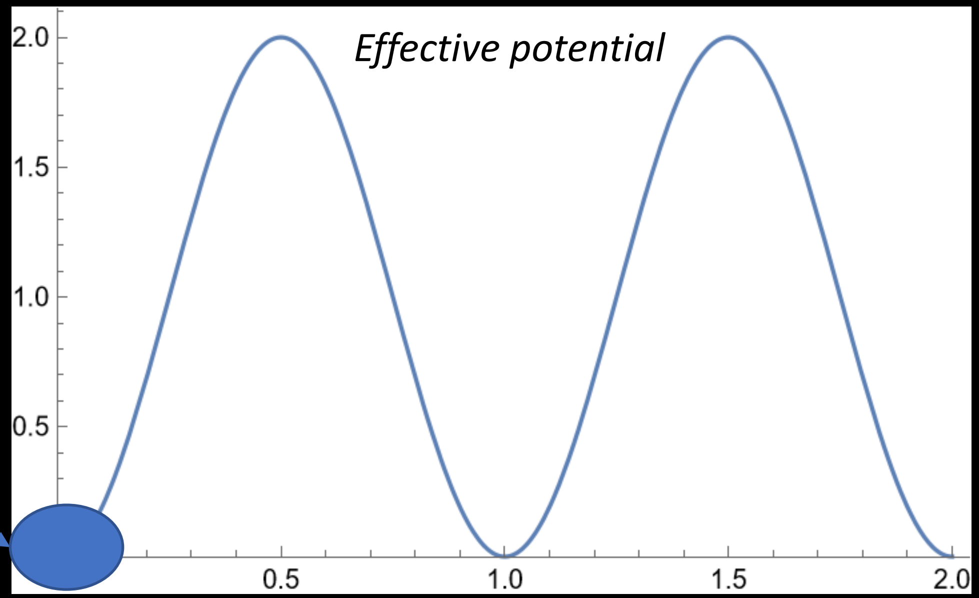
Peccei and Quinn (1977); Shifman, Vainshtein and Zakharov (1979); Dine, Fischler and Srednicki (1981); Wilczek (1977); Weinberg (1977)

The θ-term is cancelled by the expectation value of A(*)

$$\mathcal{L}_{\text{odd}} + \mathcal{L}_{\text{axion}} \propto \left(\theta + \frac{A}{f_a}\right) \tilde{F} \cdot F$$

Minimum

$$\theta + \frac{\langle A \rangle}{f_a} = 0$$



(*)At the tree level. Potential P-odd observables can take nonzero contributions from loops. These however are suppressed by 1/f_a at least.

A vanilla introduction to the QCD axion

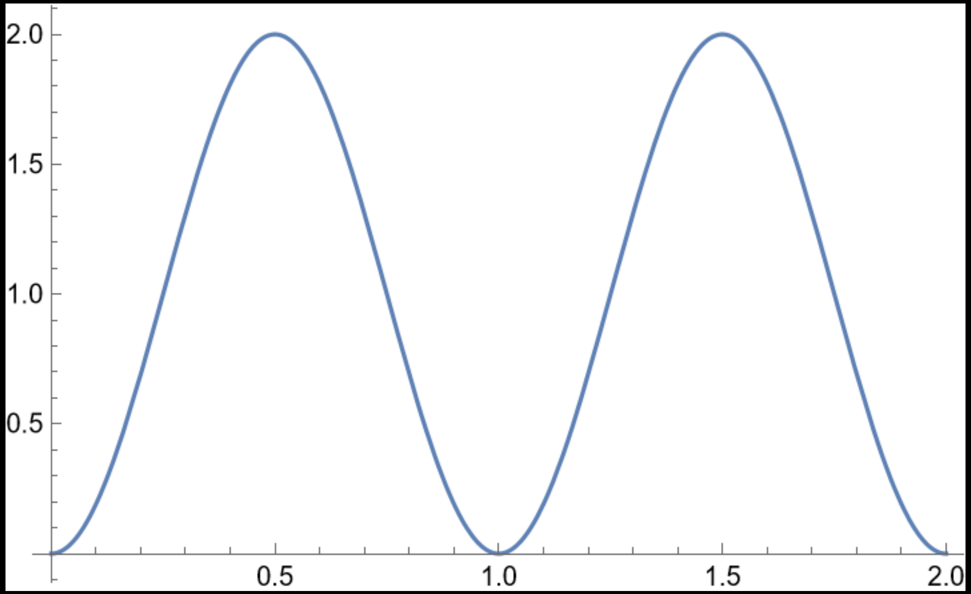
$$\mathcal{L}_{\rm odd} \propto \theta \tilde{F} \cdot F$$

$$\tilde{F} \equiv \varepsilon_{\mu\nu\rho\sigma} F_a^{\rho\sigma}$$

$$\mathcal{L}_{\rm odd} + \mathcal{L}_{\rm axion} \propto \left(\theta + \frac{A}{f_a}\right) \tilde{F} \cdot F$$

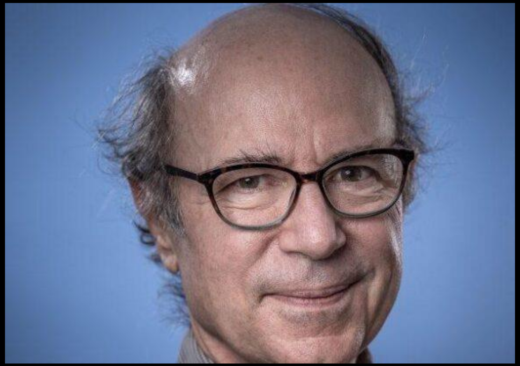
Effective potential

$$V\left(\theta + \frac{A}{f_a}\right) \propto 1 - \cos\left(\theta + \frac{A}{f_a}\right)$$



The θ-term is cancelled by the expectation value of A

$$\mathcal{L}_{\rm odd} + \mathcal{L}_{\rm axion} \propto \left(\theta + \frac{A}{f_a}\right) \tilde{F} \cdot F$$



Wilczek (1977)

By The logo may be obtained from Axion (brand)., Fair use, https://en.wikipedia.org/w/index.php?curid=63685011

The model

Lagrangian density

$$\mathcal{L} = \bar{q}\left(i\not{\partial} + \hat{\mu}\gamma_0 - m_0\right)q + \bar{e}\left(i\not{\partial} + \mu_e\gamma_0\right)e + \mathcal{L}_{\text{int}}$$

Chemical potential matrix

$$\hat{\mu} = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix} \otimes \mathbf{1}_c$$

$$\mu_u = \mu - \frac{2}{3}\mu_e, \quad \mu_d = \mu + \frac{1}{3}\mu_e$$

$$\mu_d = \mu_u + \mu_e$$

Strong interaction

$$\frac{a}{f_a}\tilde{F}\cdot F$$

$$\mathcal{L}_{\text{int}} = G_1\left[(\bar{q}\tau_a q)(\bar{q}\tau_a q) + (\bar{q}\tau_a i\gamma_5 q)(\bar{q}\tau_a i\gamma_5 q)\right]$$

U(1)$_A$-preserving

$$+8G_2\left[e^{i\frac{a}{f_a}}\det(\bar{q}_R q_L) + e^{-i\frac{a}{f_a}}\det(\bar{q}_L q_R)\right]$$

U(1)$_A$-breaking

Coupling of a to quarks

A little remark

$$\mathcal{L} \propto \theta \tilde{F} \cdot F$$

QCD at finite θ

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} = G_1 \left[(\bar{q}\tau_a q)(\bar{q}\tau_a q) + (\bar{q}\tau_a i\gamma_5 q)(\bar{q}\tau_a i\gamma_5 q)\right] \\ +8G_2 \left[e^{i\theta}\det(\bar{q}_R q_L) + e^{-i\theta}\det(\bar{q}_L q_R)\right]\end{aligned}$$

The 1-loop thermodynamic potential

$$\sigma = \langle \bar{q}q \rangle ,\; \eta = \langle \bar{\eta} i \gamma_5 \eta \rangle$$

Chiral condensate (scalar)

η-condensate (pseudoscalar)

The 1-loop thermodynamic potential

$$\Omega = \Omega_{\mathrm{mf}} + \Omega_{1-\mathrm{loop}} + \Omega_e$$

$$\Omega_{\mathrm{mf}} = -G_2(\eta^2 - \sigma^2)\cos(a/f_a) + G_1(\eta^2 + \sigma^2) - 2G_2\sigma\eta\sin(a/f_a)$$

$$\sigma = \langle \bar{q}q \rangle,\ \eta = \langle \bar{\eta} i \gamma_5 \eta \rangle$$

$$\Omega_{1-\mathrm{loop}} = -4N_c \sum_{f=u,d} \int \frac{d^3p}{(2\pi)^3} \left[\frac{E_p}{2} + \frac{1}{2\beta} \log(1 + e^{-\beta(E_p - \mu_f)})(1 + e^{-\beta(E_p + \mu_f)}) \right]$$

$$\alpha_0 = -2\left[G_1 + G_2\cos(a/f_a)\right]\sigma + 2G_2\eta\sin(a/f_a)$$

$$E_p = \sqrt{p^2 + \Delta^2}, \quad \Delta^2 = (m_0 + \alpha_0)^2 + \beta_0^2$$

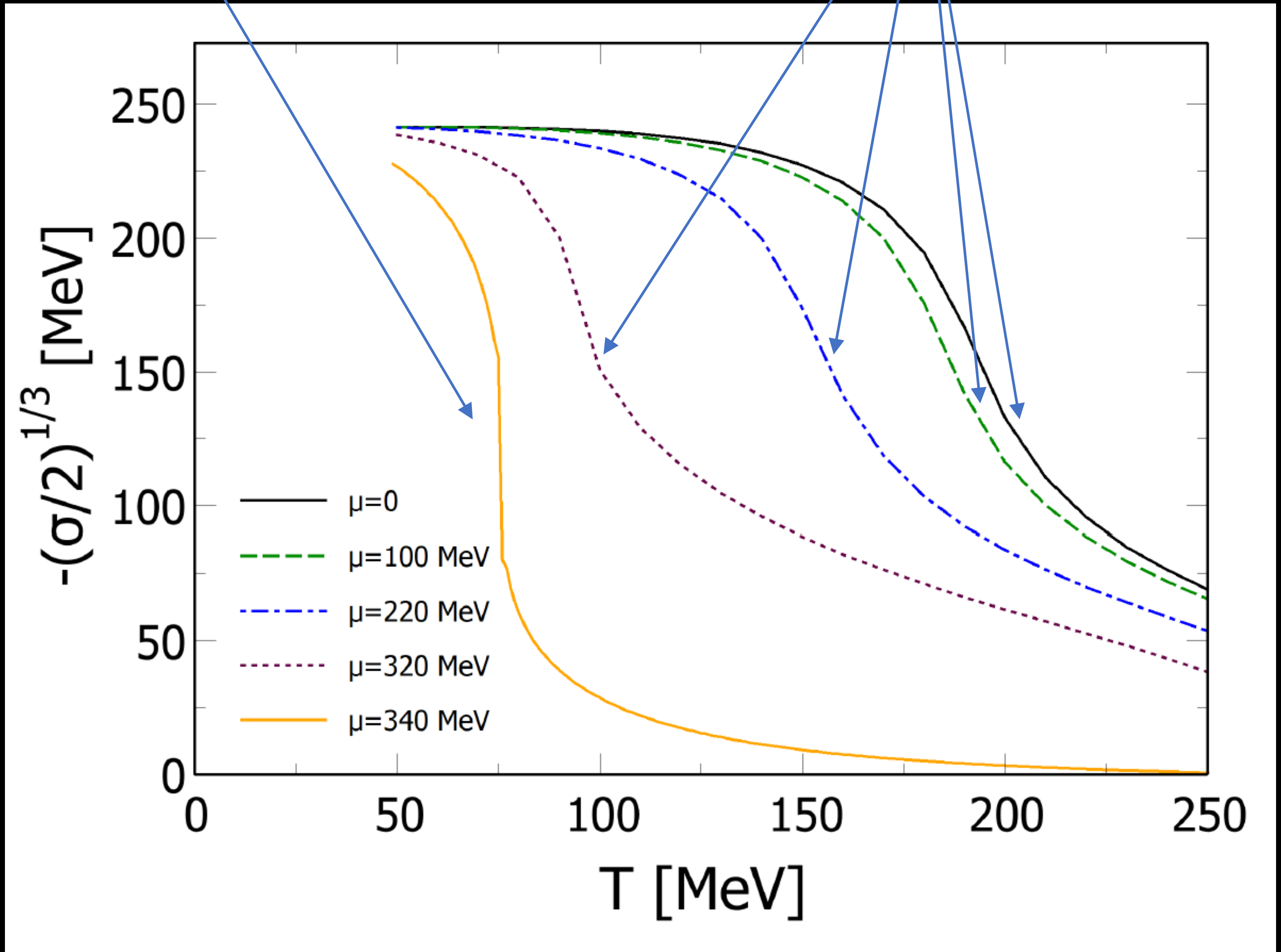
$$\beta_0 = -2\left[G_1 - G_2\sin(a/f_a)\right]\eta + 2G_2\sigma\sin(a/f_a).$$

$$\Omega_e = -2T\frac{4\pi}{8\pi^3}\left(\frac{7\pi^4}{180}T^3 + \frac{\pi^2\mu_e^2 T}{6} + \frac{\mu_e^4}{12T}\right)$$

Chiral condensate

crossover

1st order



Z.Bonan *et al.* (2023)

Gap equations

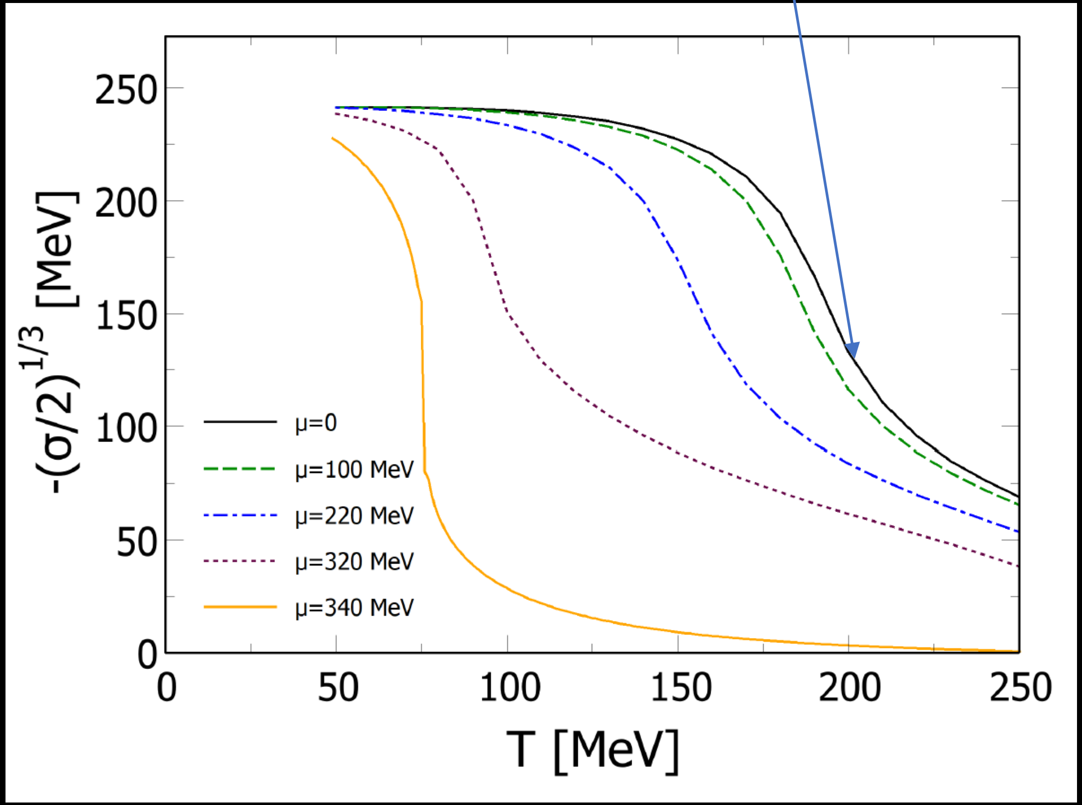
$$\frac{\partial \Omega}{\partial \sigma} = 0, \qquad \frac{\partial \Omega}{\partial \eta} = 0$$

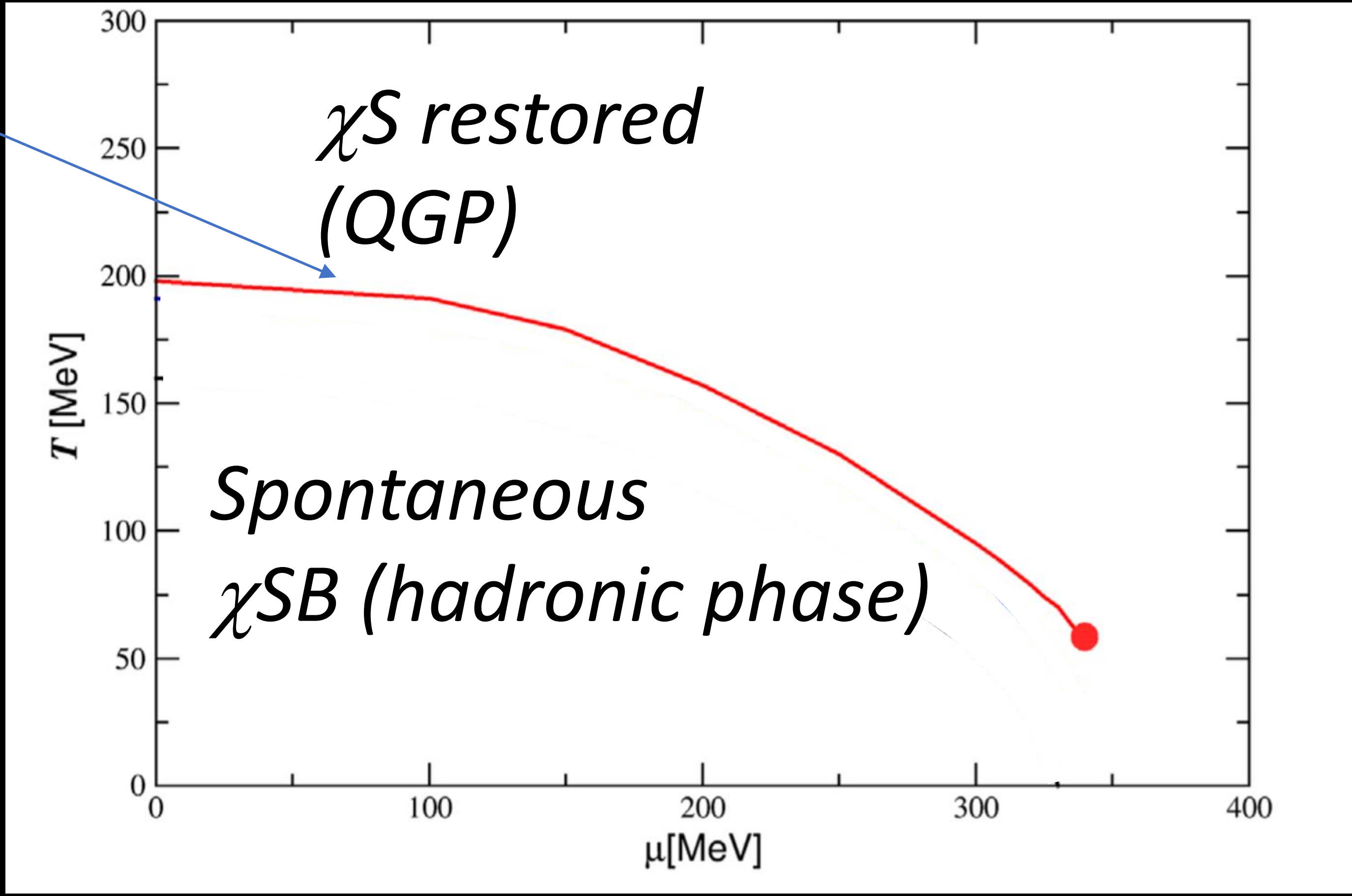
Electrical neutrality

$$\frac{\partial \Omega}{\partial \mu_e} = 0$$

Phase diagram

crossover





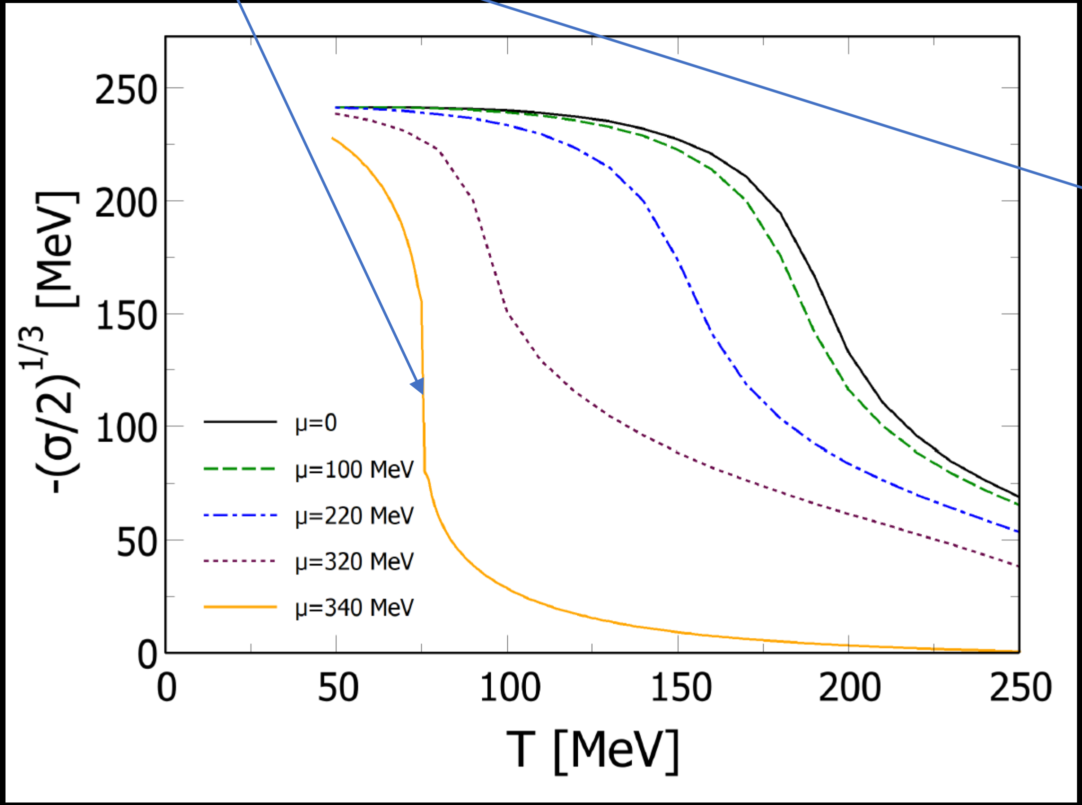
χS restored (QGP)

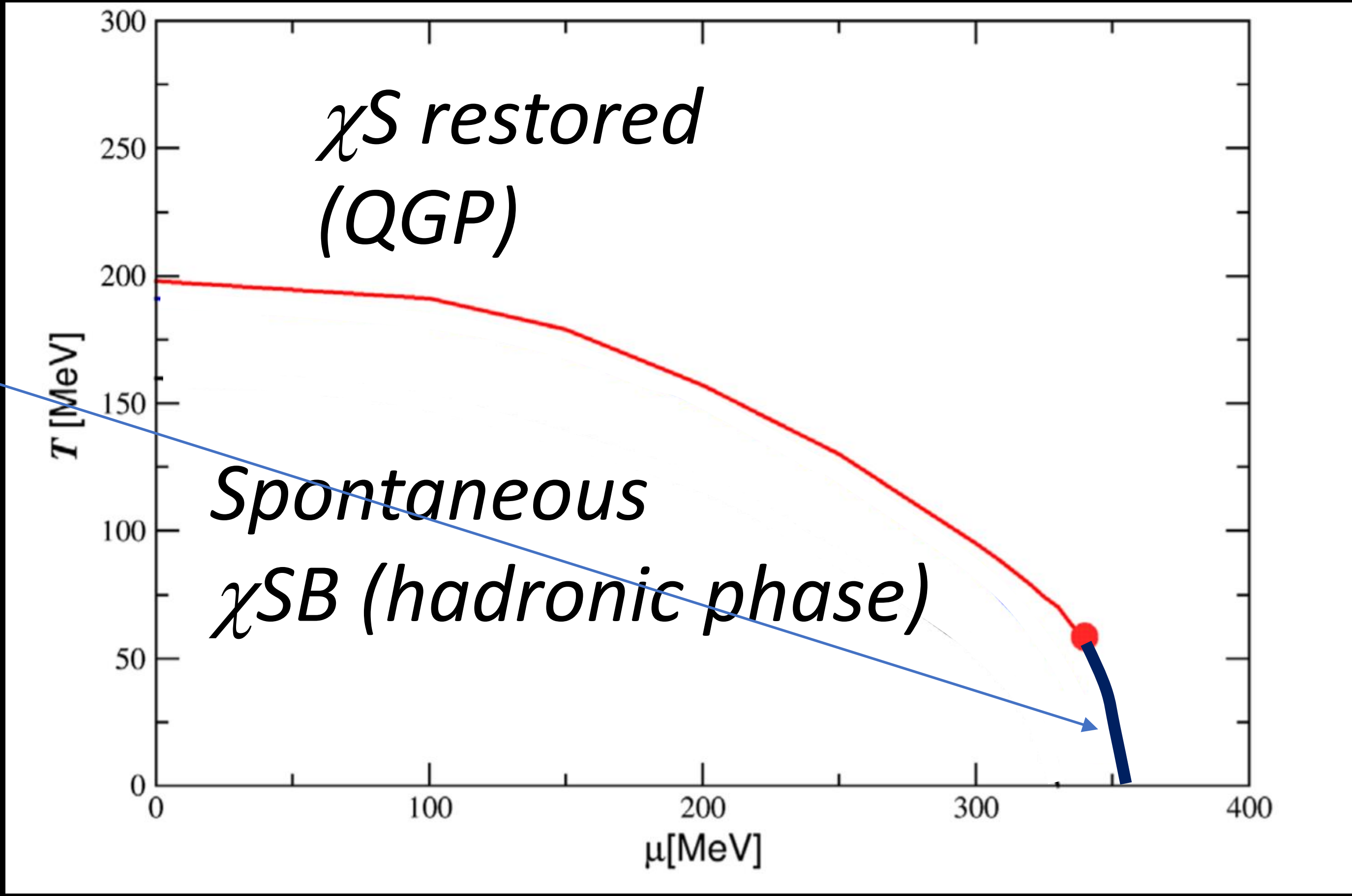
Spontaneous χSB (hadronic phase)

M.R. *et al.* (2021)

Phase diagram

1st order





χS restored (QGP)

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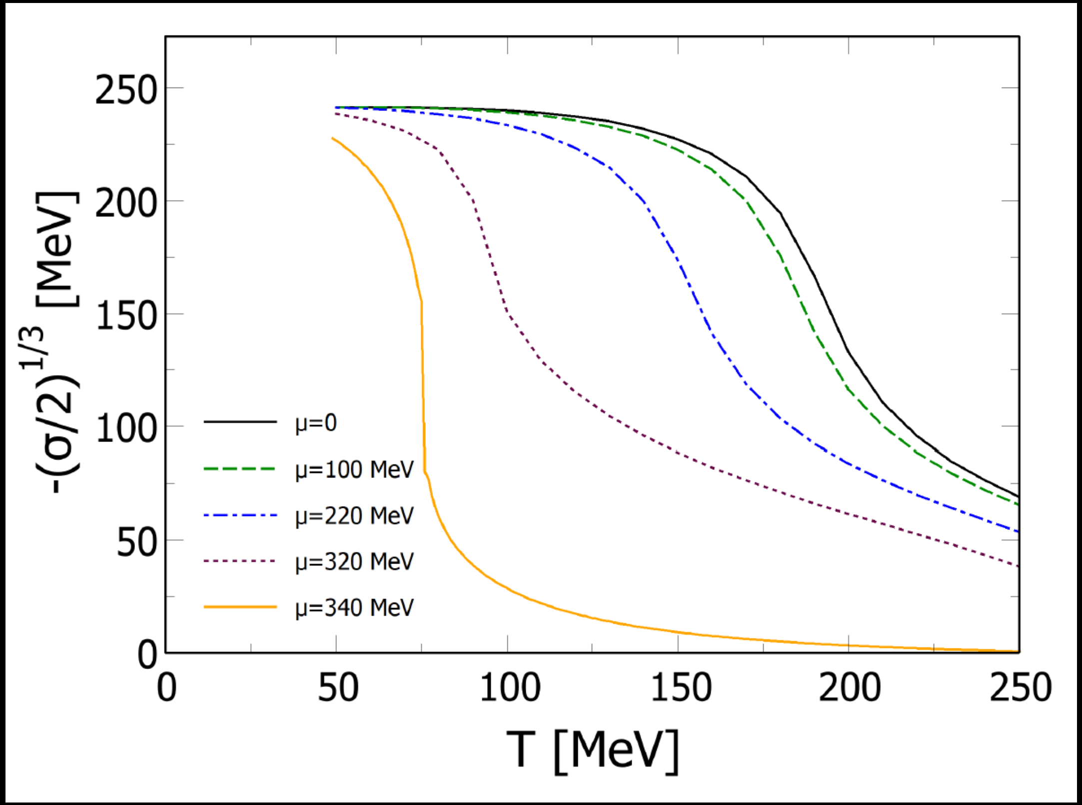
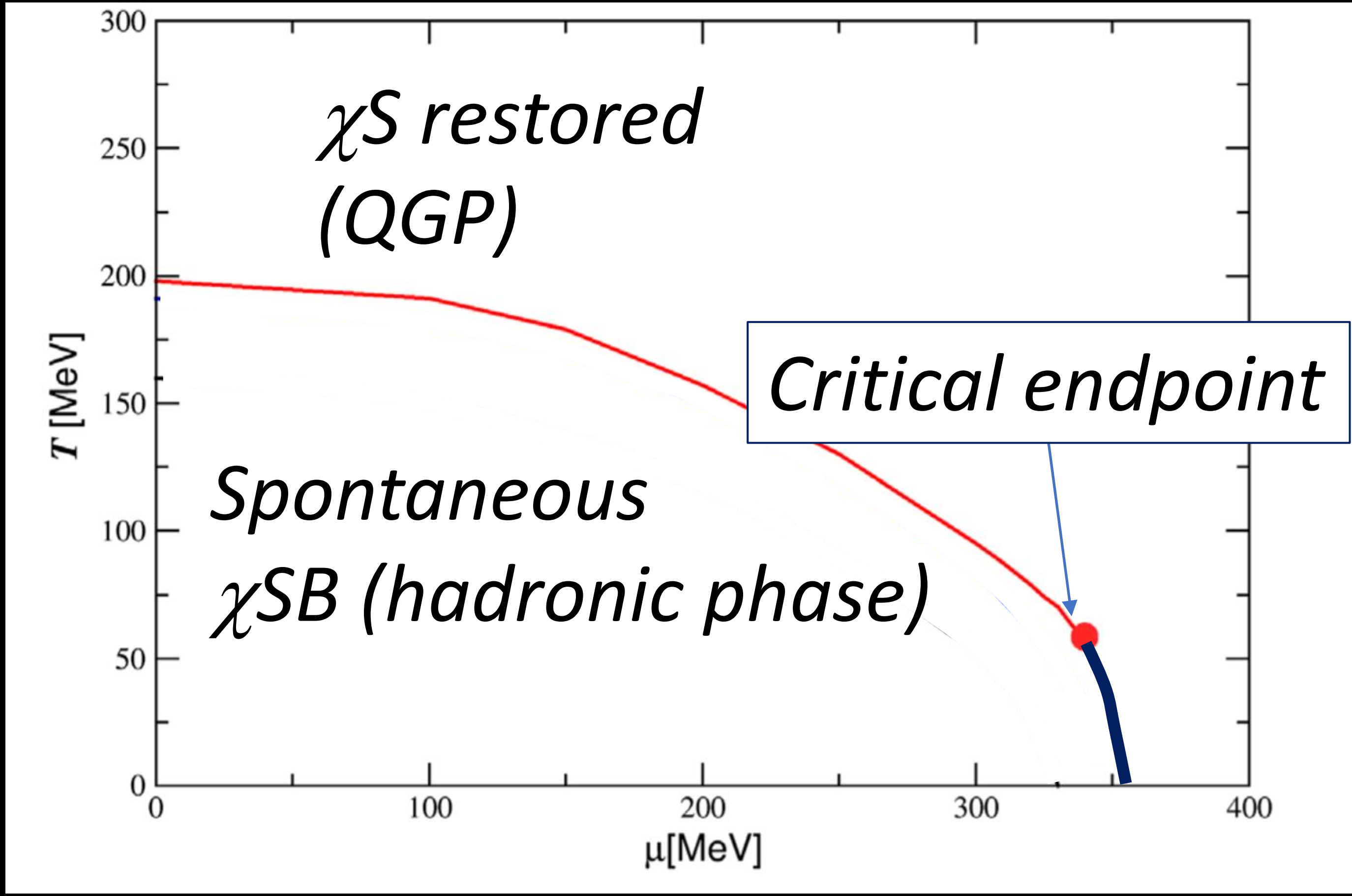
Phase diagram





χS restored (QGP)

Spontaneous χSB (hadronic phase)

Critical endpoint

M.R. *et al.* (2021)

Real second order phase transition, with divergent correlation length (critical behavior)

Topological susceptibility

Put

$$\theta = a/f_a$$

then

$$\chi = \left.\frac{\partial^2\Omega}{\partial\theta^2}\right|_{\theta=0}$$

Qualitative understanding

$$\chi = |\langle\bar{q}q\rangle|\frac{m_u m_d}{m_u + m_d}$$

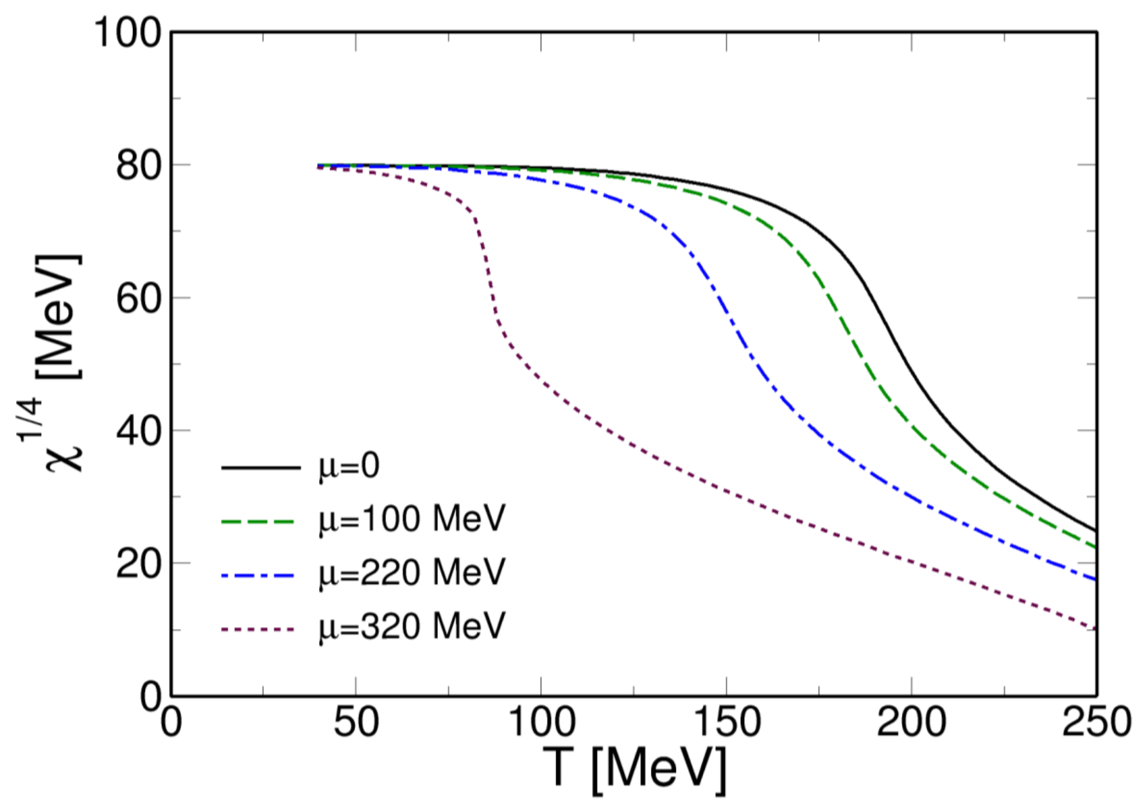
For vacuum-QCD:

Veneziano (1979), Di Vecchia-Veneziano (1980), Leutwyler and Smilga (1992), Crewther (1977)

It works faily well also around the QCD crossover

M.R. and Gatto (2011)

Z.Bonan *et al.* (2023)



Measures fluctuations of the topological charge

Is the QCD axion sensitive to the QCD phase transitions?

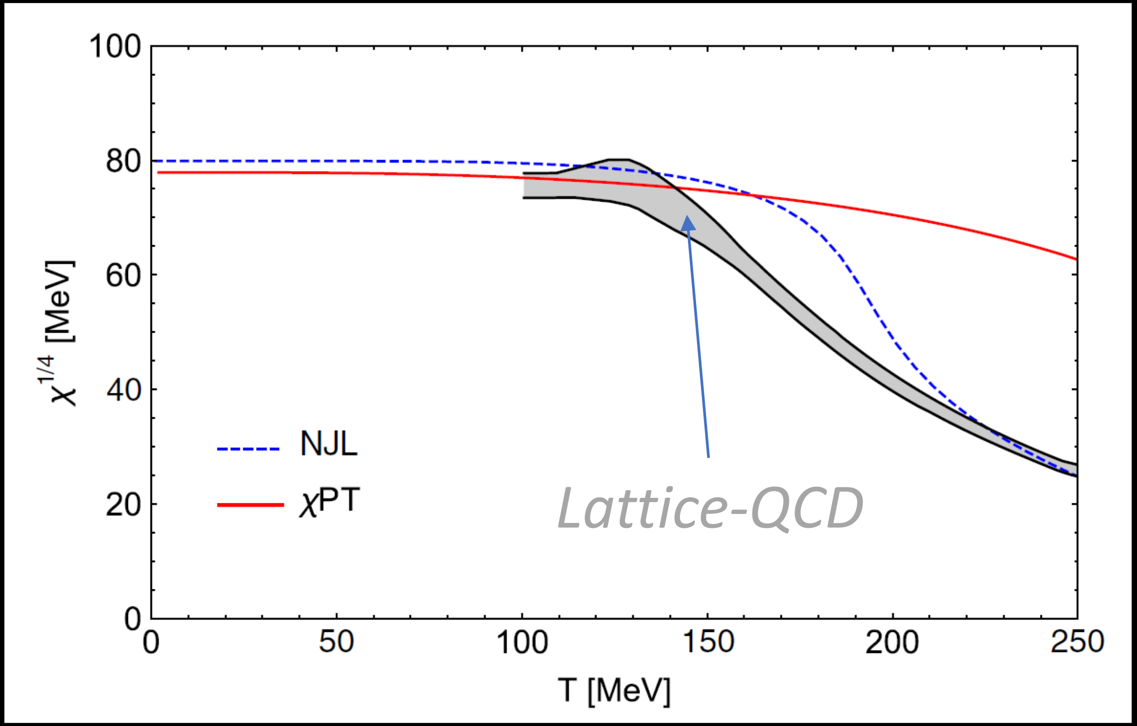
Spoiler alert: YES IT IS

$\theta = a/f_a$

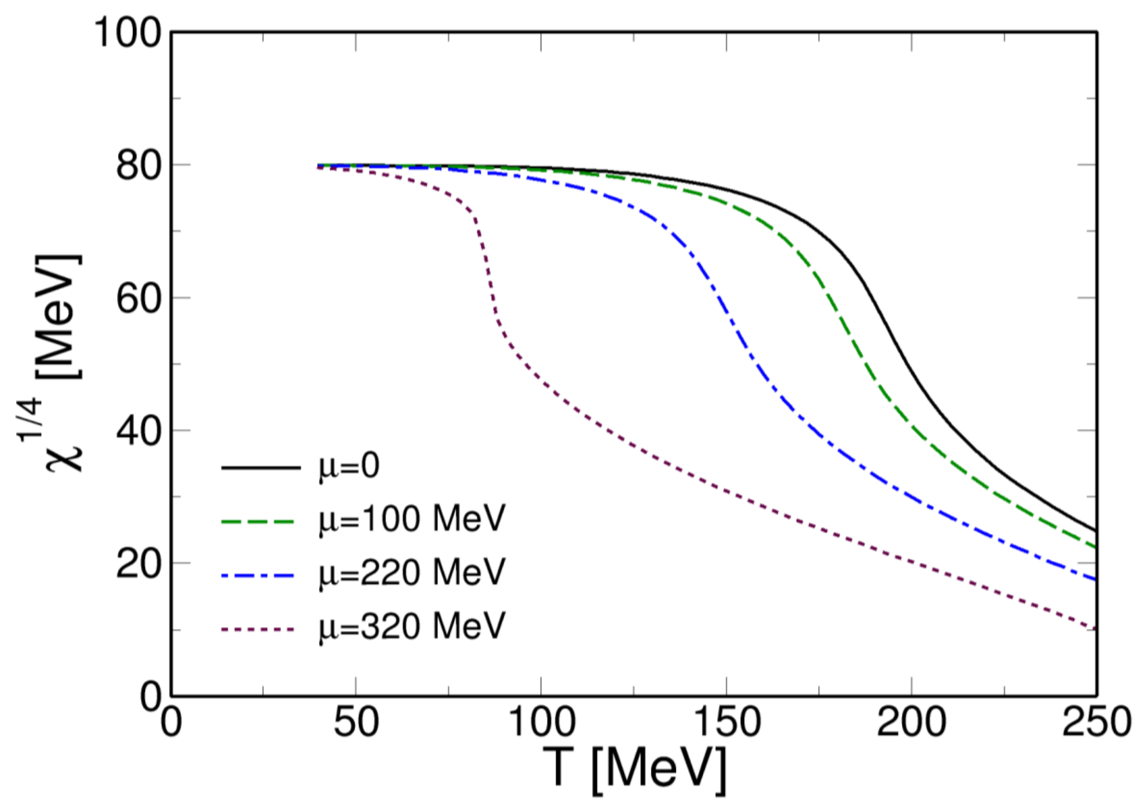
Topological susceptibility

$$\chi = \frac{\partial^2\Omega}{\partial\theta^2}\bigg|_{\theta=0}$$

Z.Bonan *et al.* (2023)



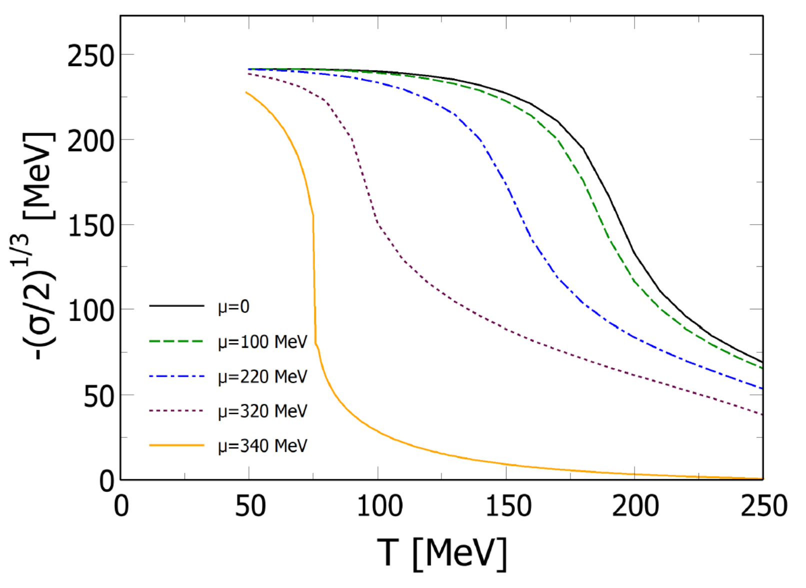
Lu *et al.* (2019)
Lattice data from Borsanyi *et al.* (2016)
χPT result from Grilli di Cortona *et al.* (2016)

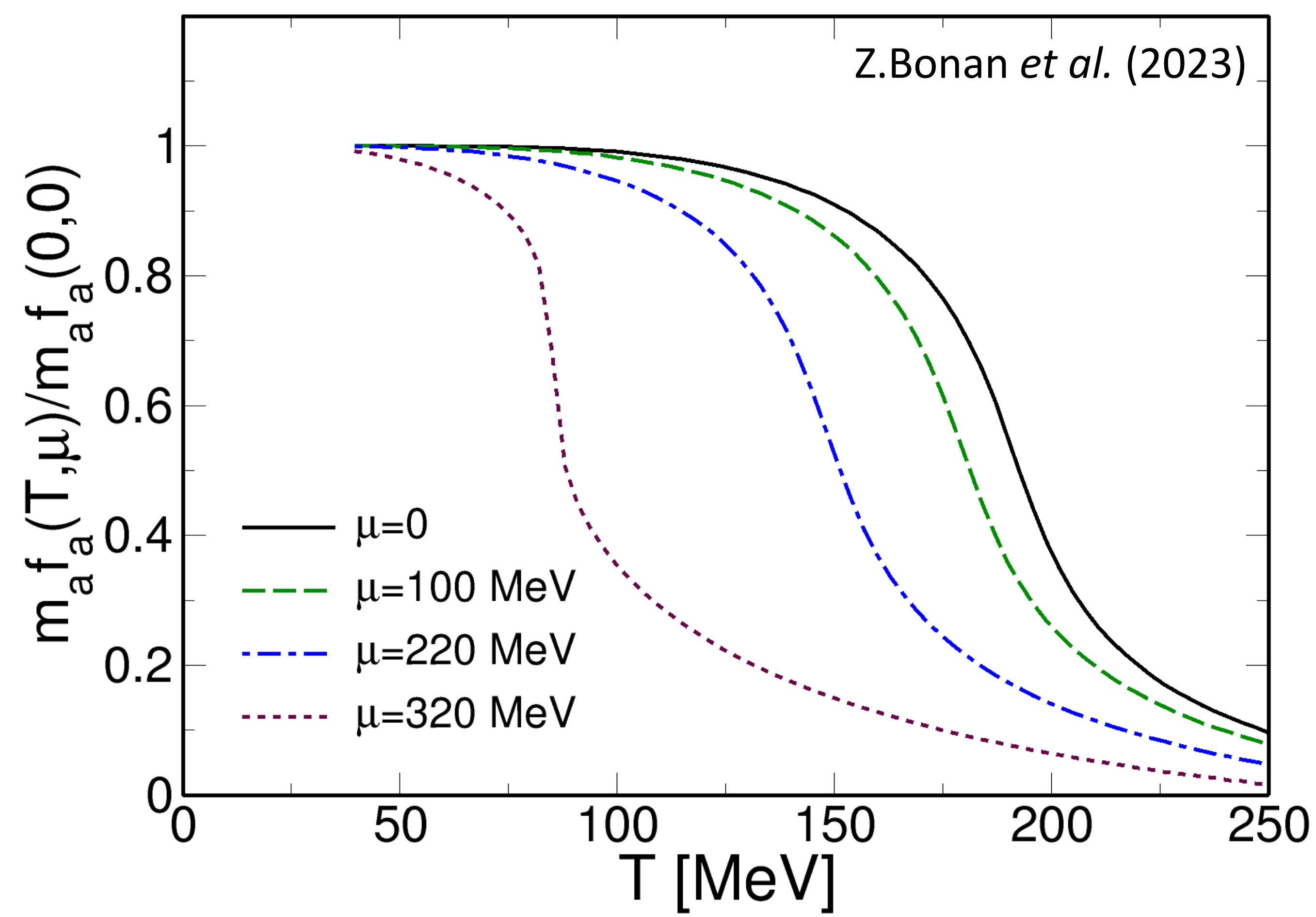


Measures fluctuations
of the topological charge

The axion mass

$$\Omega \approx \Omega(a=0) + \frac{m_a^2}{2}a^2 + \frac{\lambda_a}{4!}a^4$$




Z.Bonan *et al.* (2023)

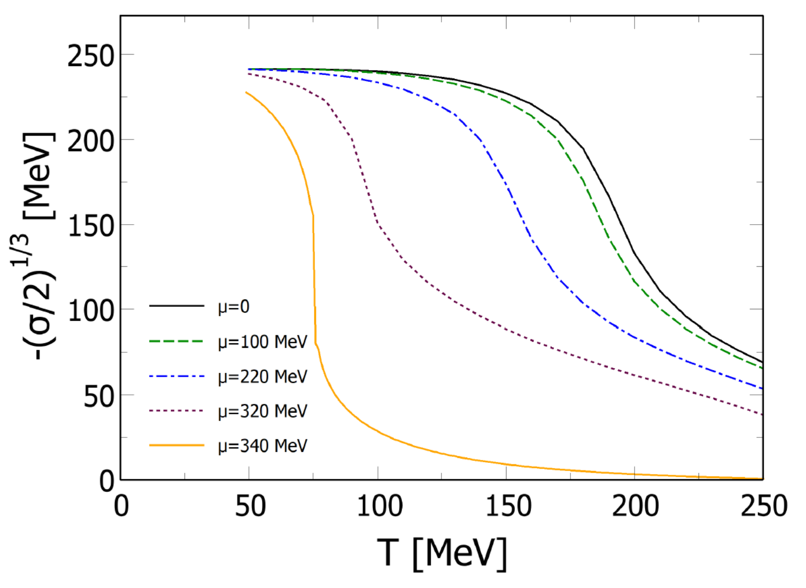
$$T = \mu = 0$$

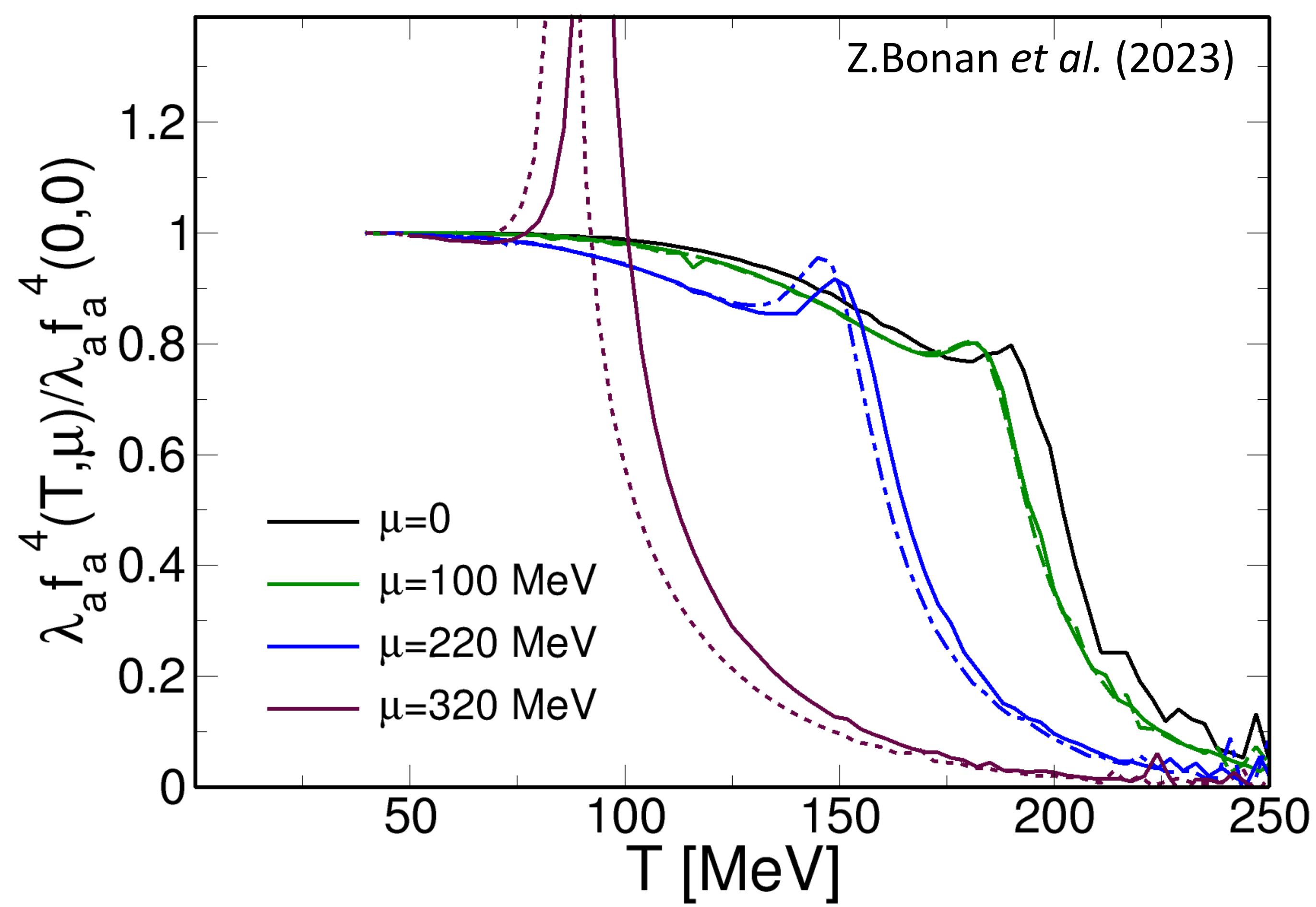
$$m_a f_a = 6.38 \times 10^3 \text{ MeV}^2$$

Axion mass is very sensitive to the phase transition.

The axion self-coupling

$$\Omega \approx \Omega(a=0) + \frac{m_a^2}{2}a^2 + \frac{\lambda_a}{4!}a^4$$





In agreement with Abhishek *et al.* (2021), Bandyopadhyay et al. (2019)

$$T = \mu = 0$$

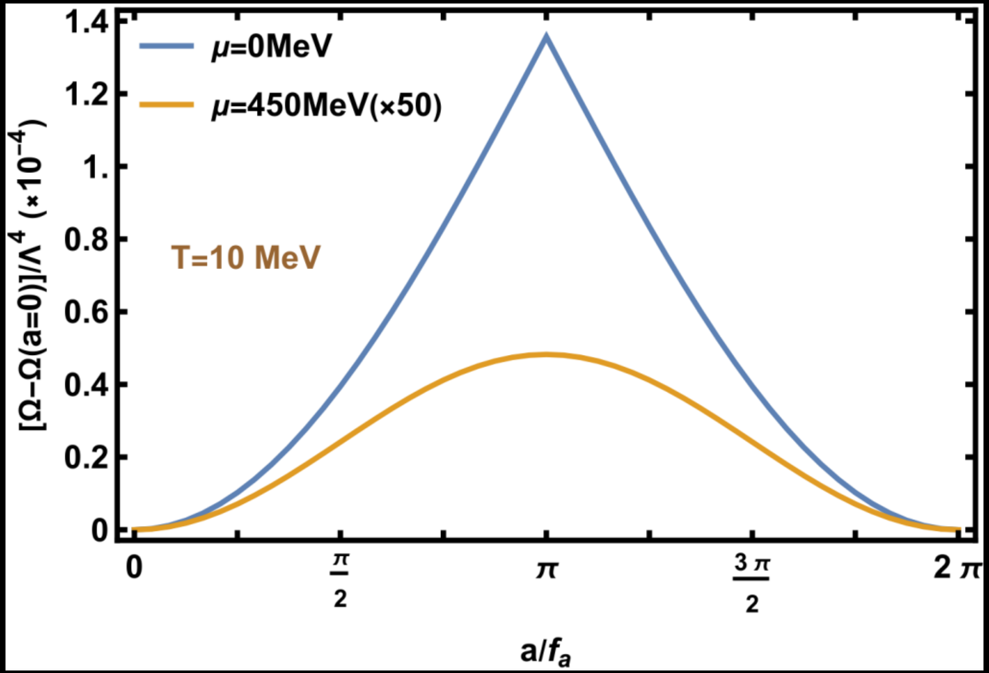
$$\lambda_a f_a^4 = -(55.63 \text{ MeV})^4$$

Axion coupling gets enhanced in the critical region

The axion walls

$$\mathcal{L} = \frac{1}{2}\partial^\mu a \partial_\mu a - V(a/f_a)$$

$$V(\theta) = \Omega(\theta) - \Omega(0)$$



In general

$$(m_a f_a) \int_\pi^{\theta(m_a x)} \frac{d\theta}{\sqrt{V(\theta)}} = \pm\sqrt{2} m_a x$$

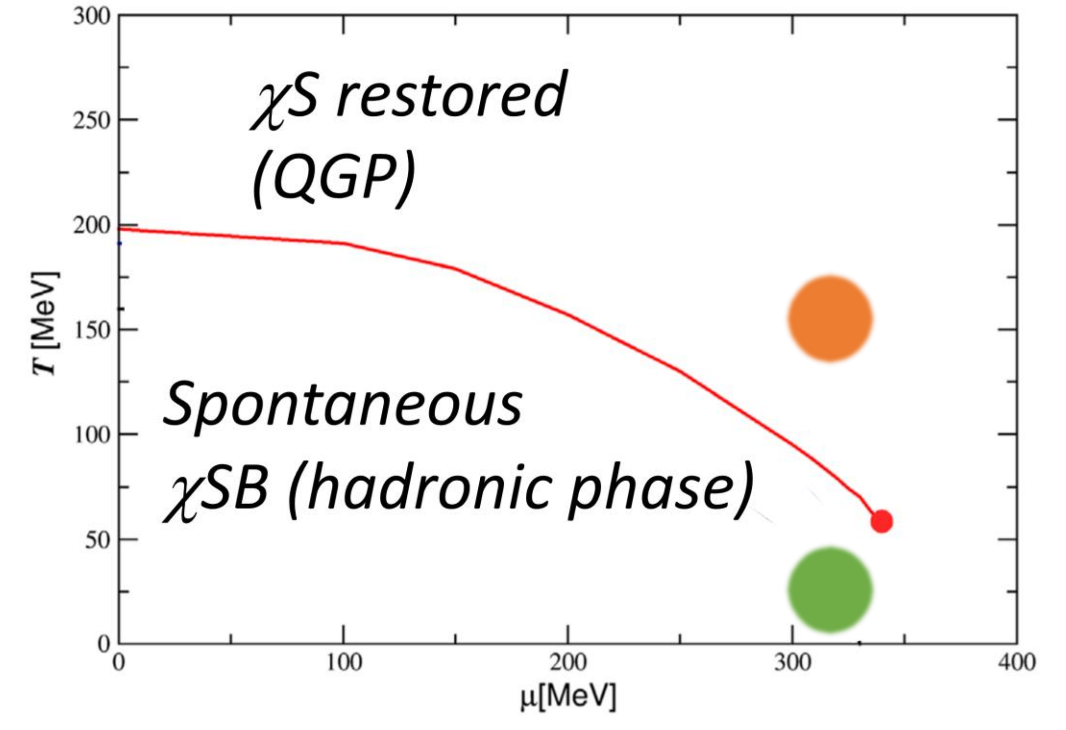
In the chiral restored phase

$$V(\theta) = V_0(1-\cos\theta) = m_a^2 f_a^2 (1-\cos\theta)$$

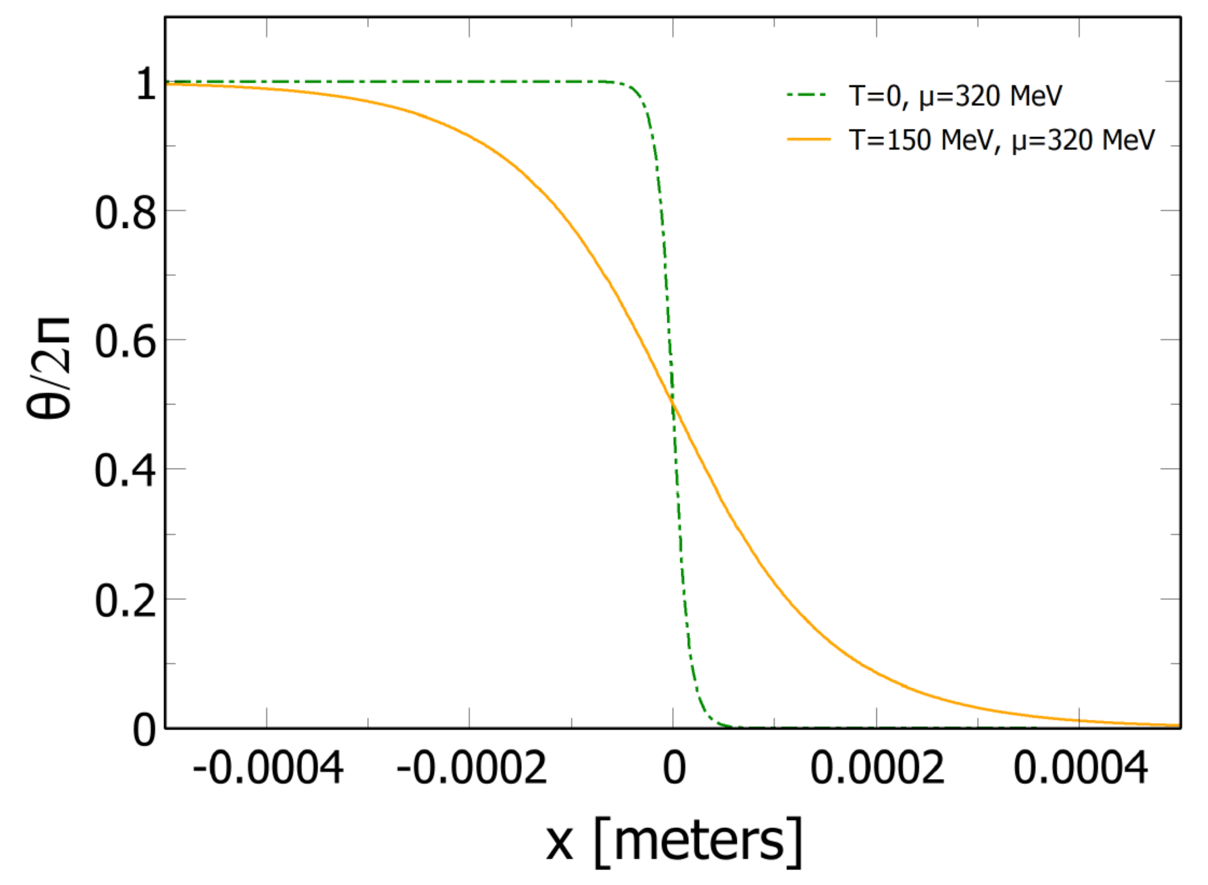
$$\theta_\pm(x) = 4\arctan\exp\left(\pm m_a x\right)$$

Sine-Gordon soliton

The axion walls



Z.Bonan *et al.* (2023)



The previous description of QCD at high density might be incomplete, because it ignores the possibility of the formation of new condensates.

$$\langle qq \rangle \propto \Delta$$

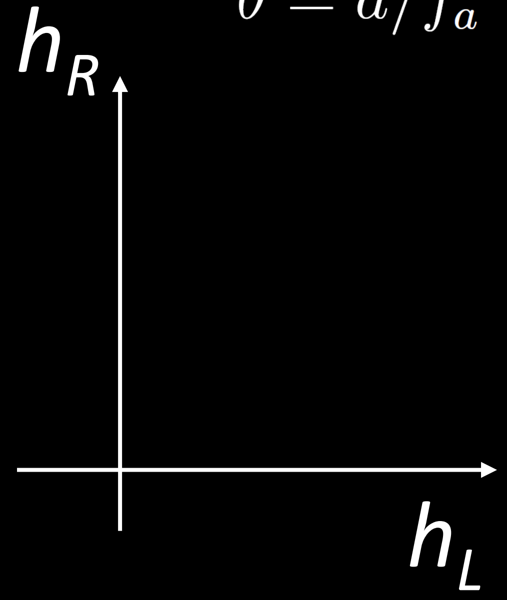
superconductive gap

quark-quark condensate

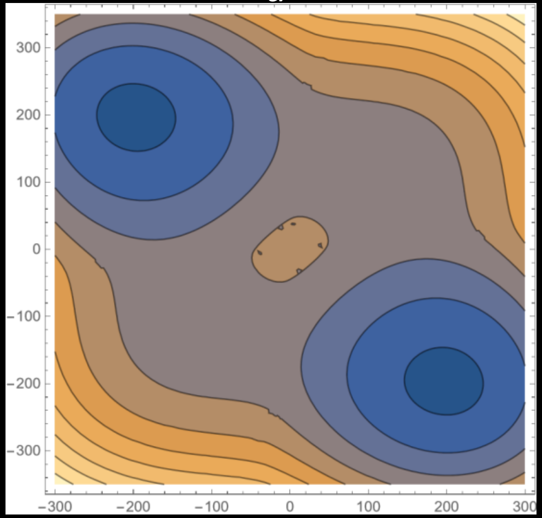
Condensation can happen both in the scalar and pseudoscalar channels.

$\Omega(h_L,h_R)$at T=0

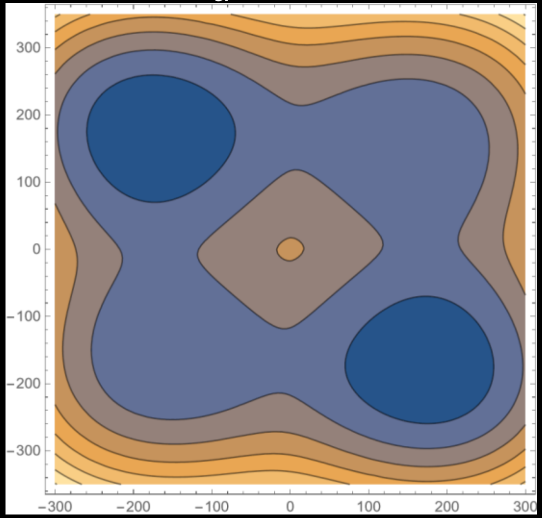
$\theta = a/f_a$



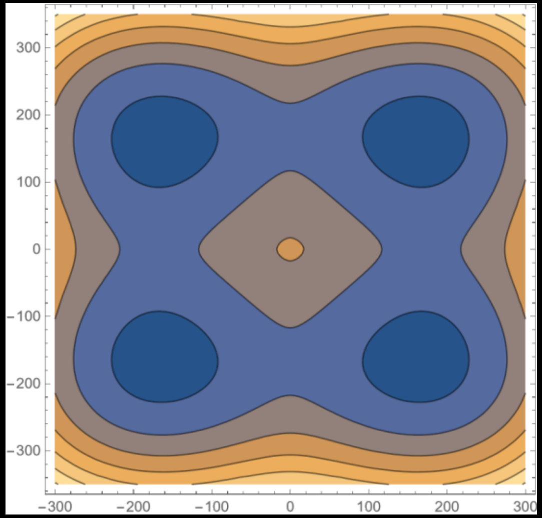
$a/f_a=0$



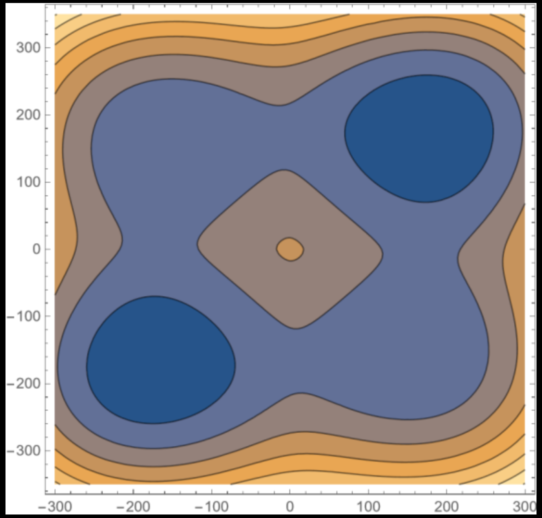
$a/f_a=\pi/2-\varepsilon$



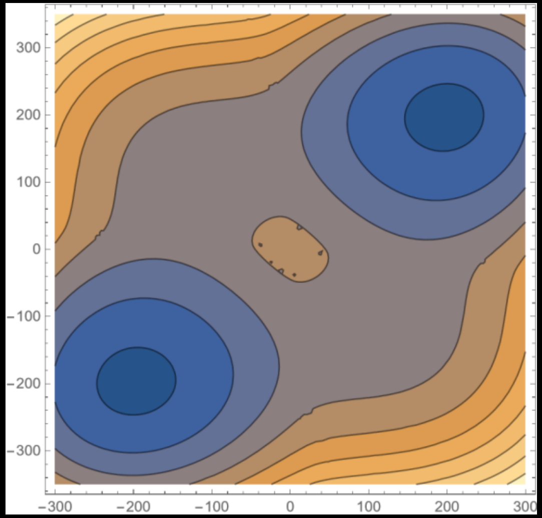
$a/f_a=\pi/2$



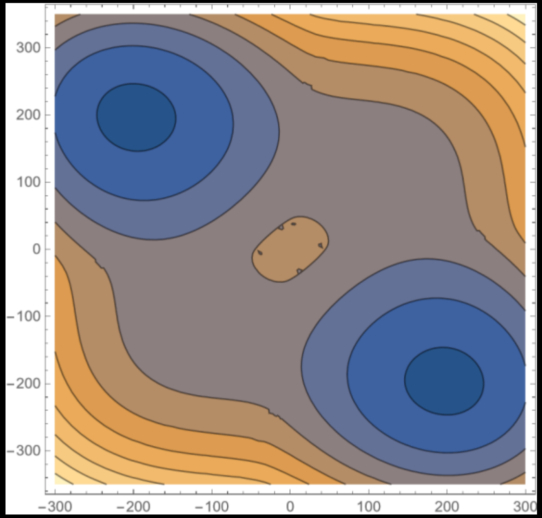
$a/f_a=\pi/2+\varepsilon$



$a/f_a=\pi$

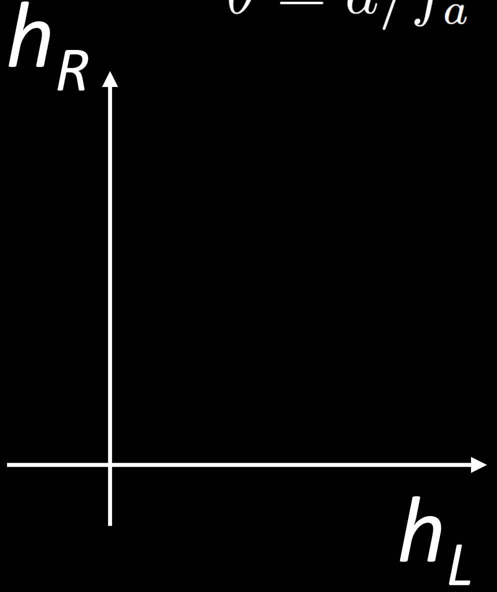


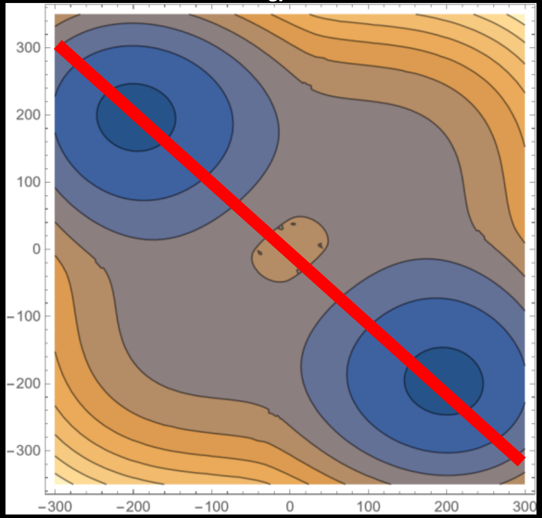
$a/f_a=2\pi$

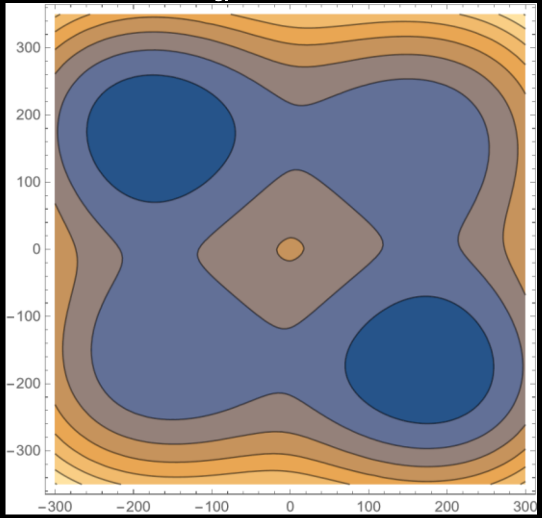


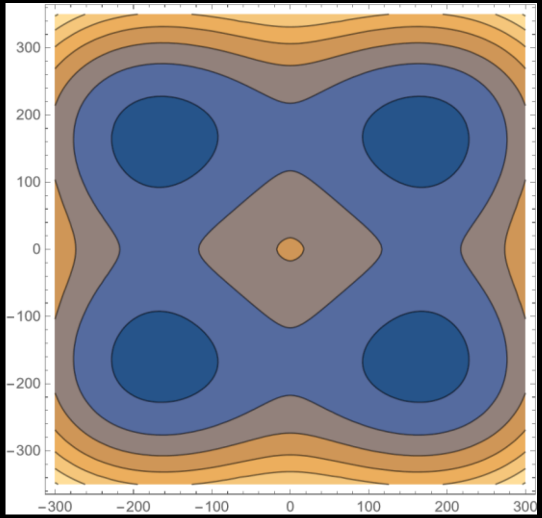
$\theta = a/f_a$

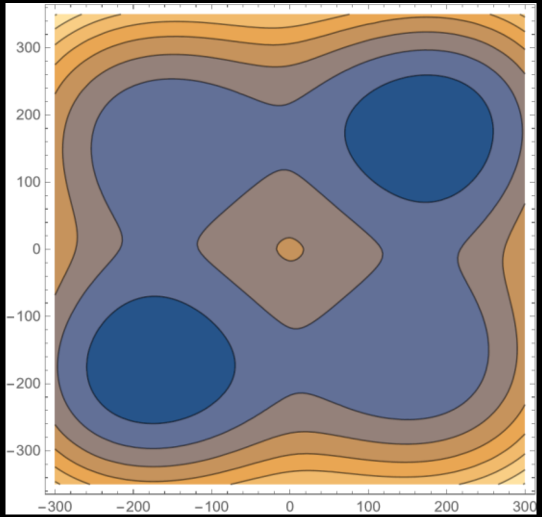
$\Omega(h_L,h_R)$ at $T=0$

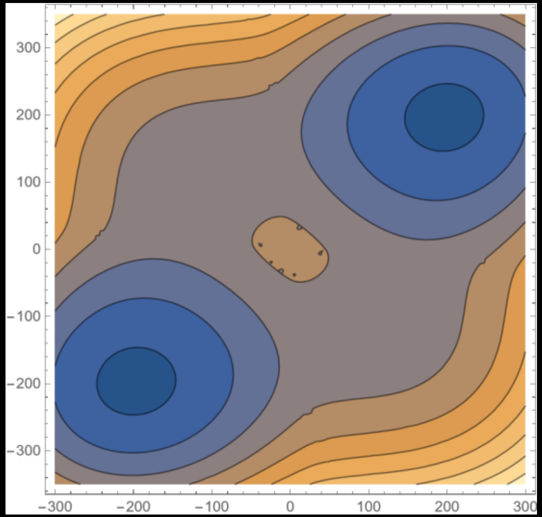


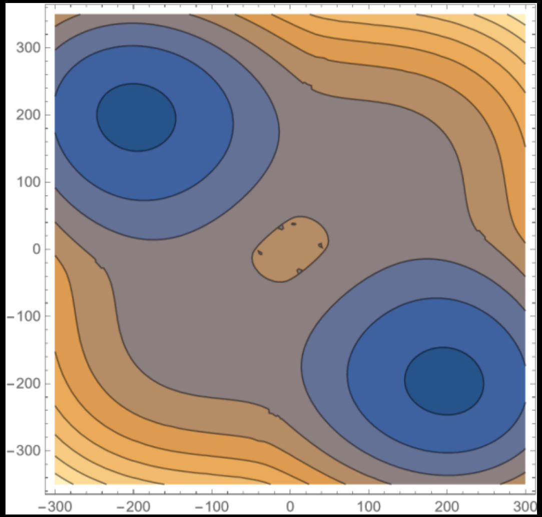

$a/f_a=0$


$a/f_a=\pi/2-\varepsilon$


$a/f_a=\pi/2$


$a/f_a=\pi/2+\varepsilon$


$a/f_a=\pi$

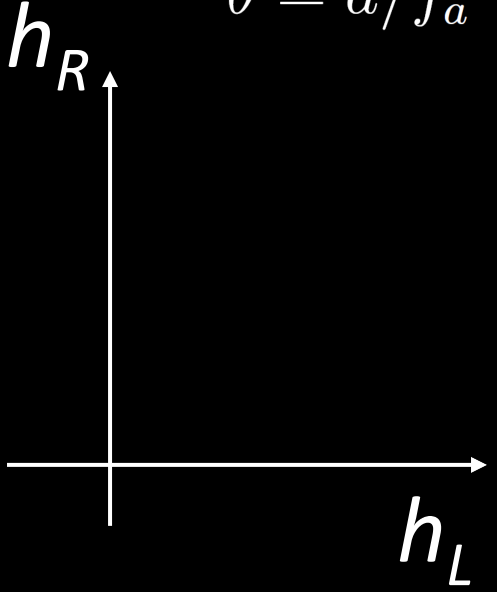

$a/f_a=2\pi$

$h_L=- h_R$

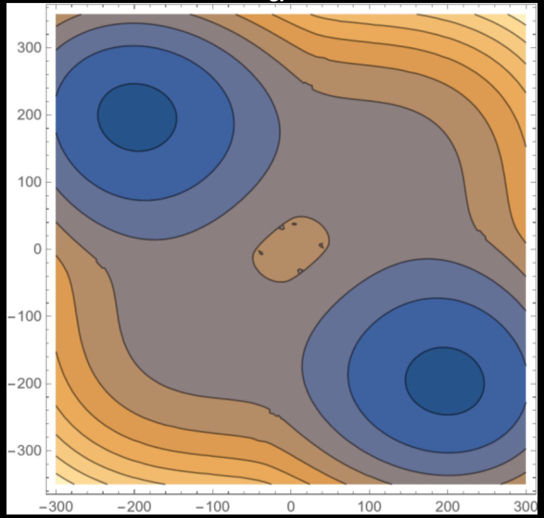
$\Delta_3=\Delta_5$

$\Omega(h_L,h_R)$ at $T=0$

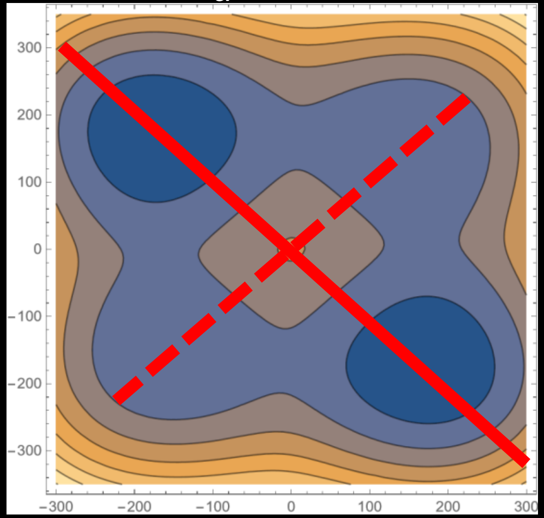
$\theta = a/f_a$



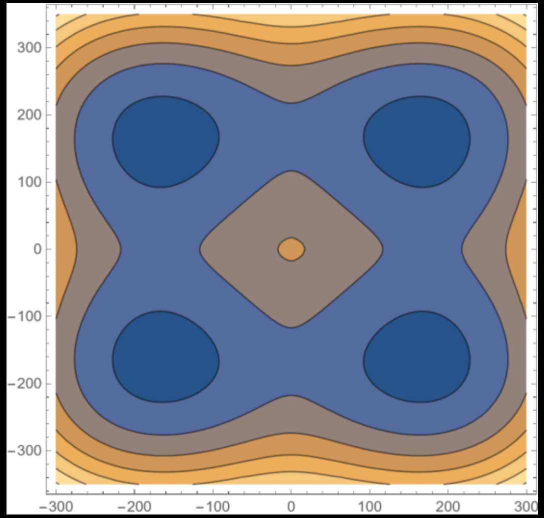
$a/f_a=0$



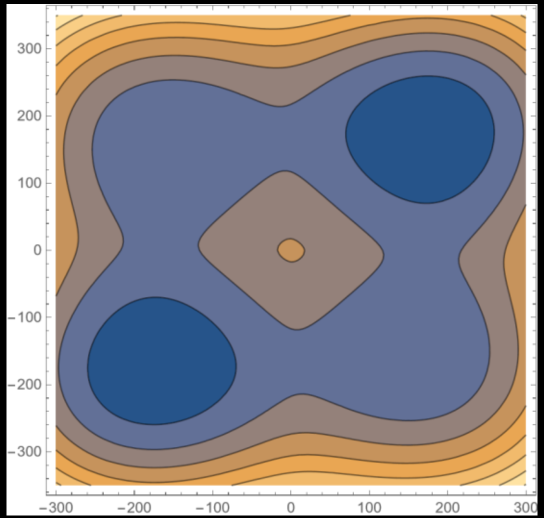
$a/f_a=\pi/2-\varepsilon$



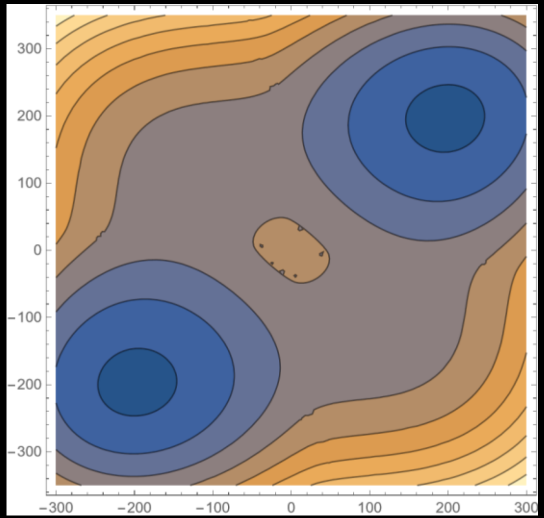
$a/f_a=\pi/2$



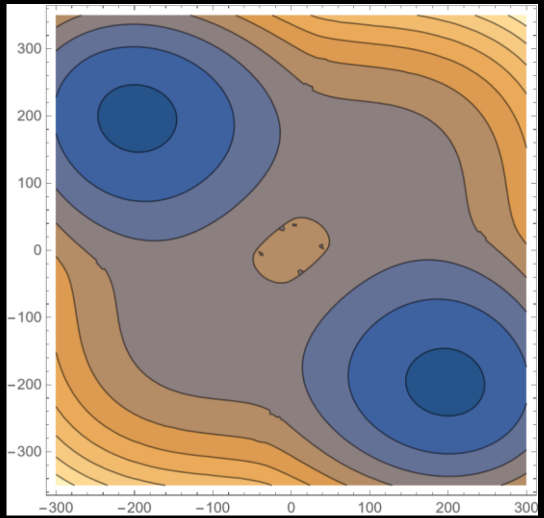
$a/f_a=\pi/2+\varepsilon$



$a/f_a=\pi$



$a/f_a=2\pi$

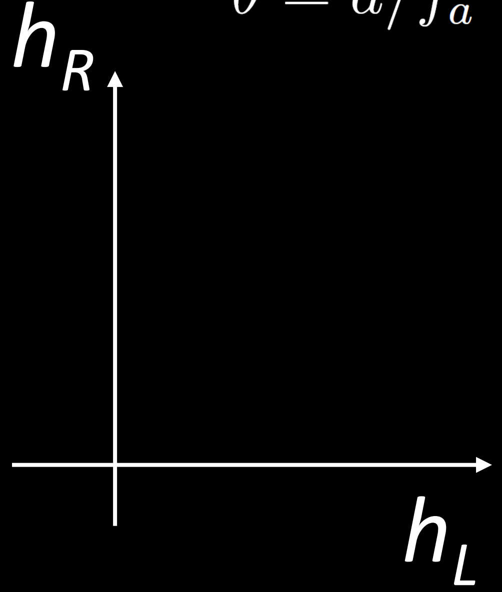


$h_L=-\ h_R$

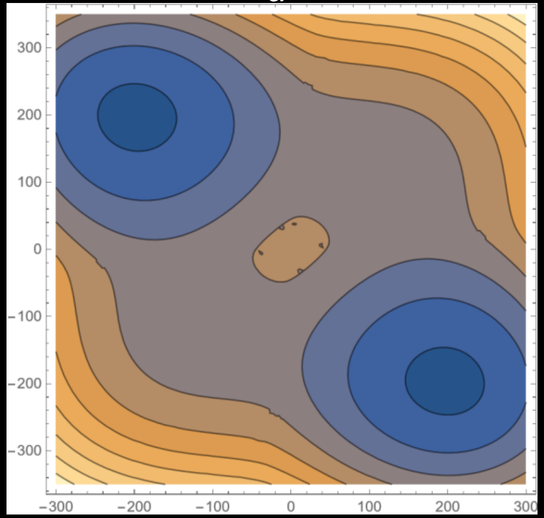
$\Delta_3=\Delta_5$

$\Omega(h_L,h_R)$ at $T=0$

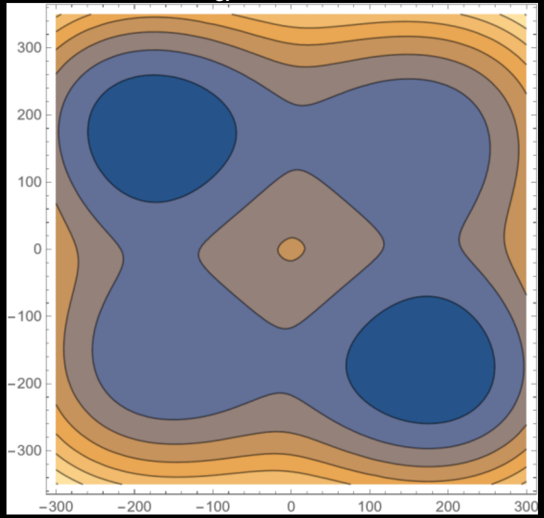
$$\theta = a/f_a$$



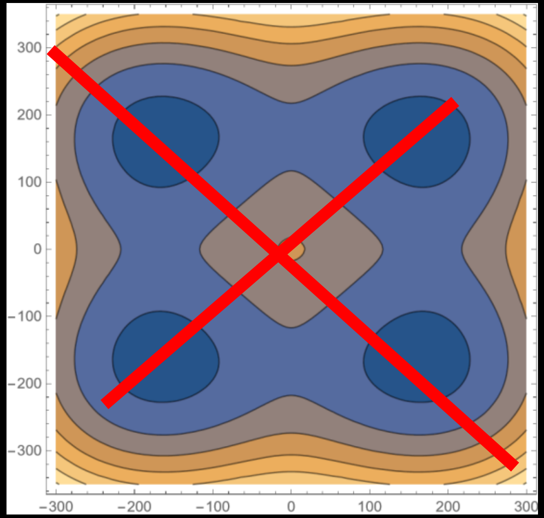
$a/f_a=0$



$a/f_a=\pi/2-\varepsilon$



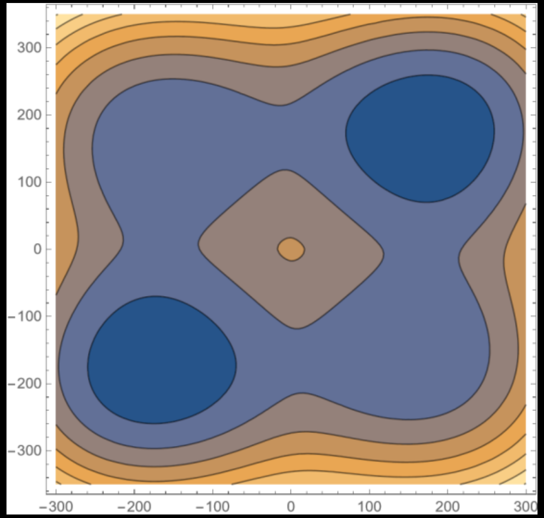
$a/f_a=\pi/2$



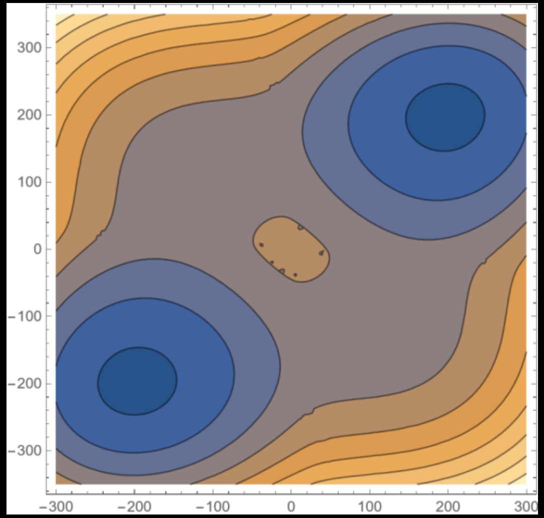
$h_L=\pm h_R$

$\Delta_3=\Delta_5$

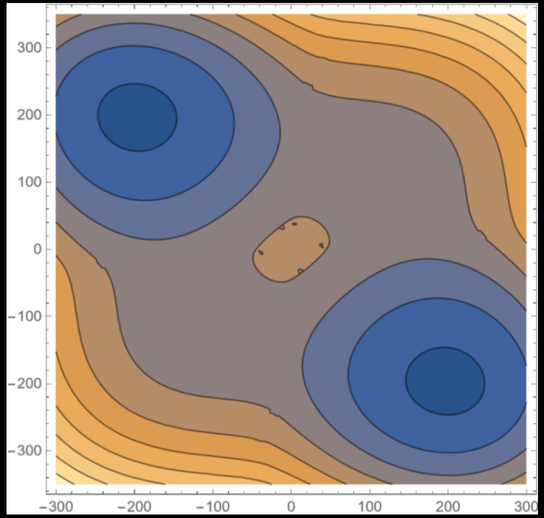
$a/f_a=\pi/2+\varepsilon$



$a/f_a=\pi$

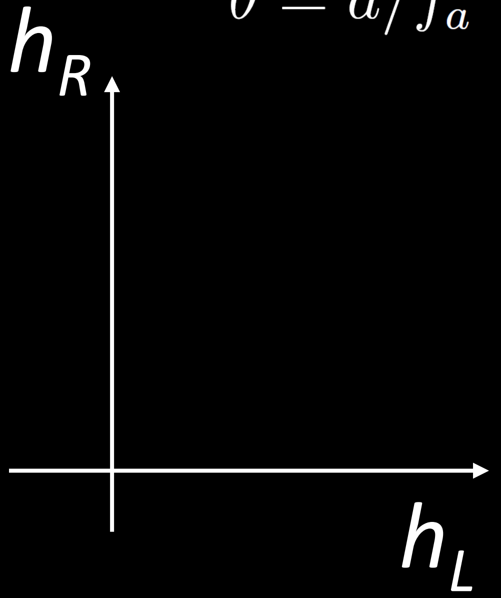


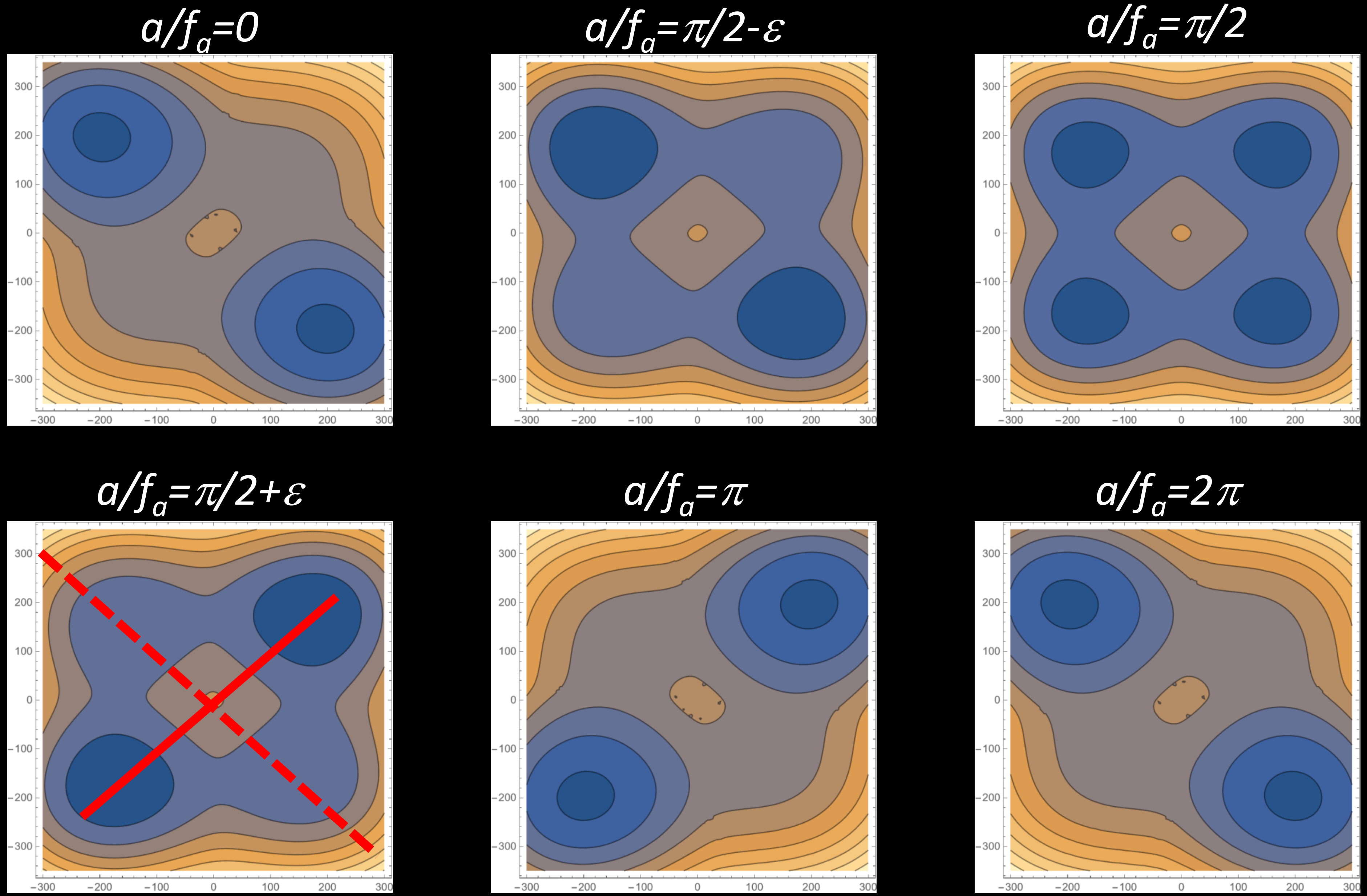
$a/f_a=2\pi$



$\Omega(h_L,h_R)$ at T=0

$\theta = a/f_a$



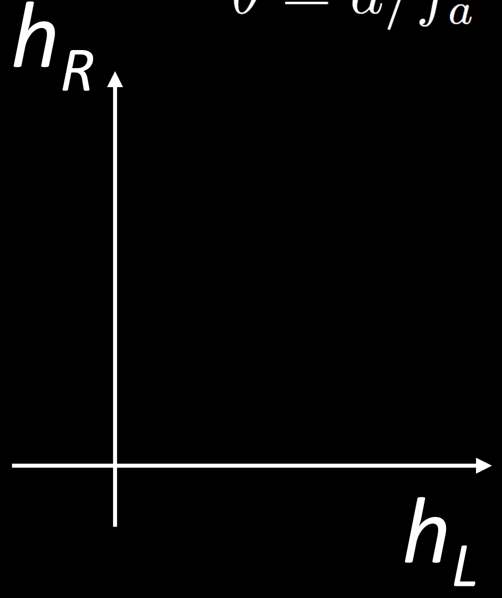


$h_L=+\,h_R$

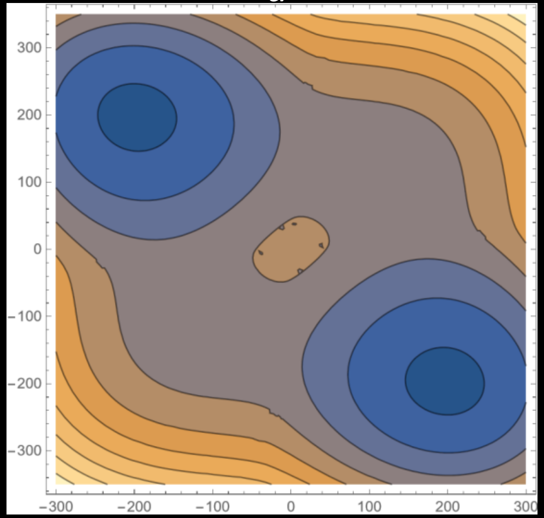
$\Delta_3=\Delta_5$

$\Omega(h_L,h_R)$at T=0

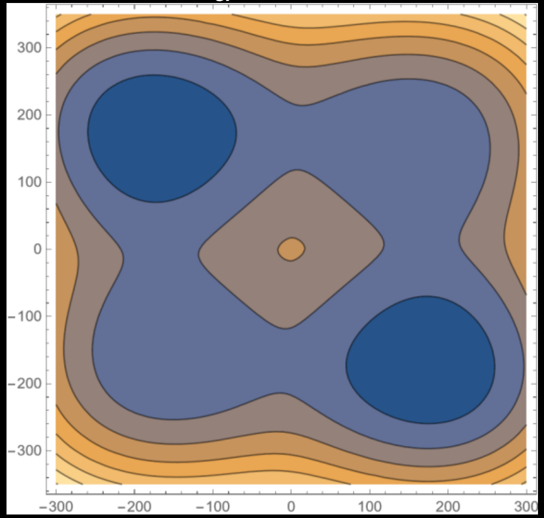
$\theta = a/f_a$



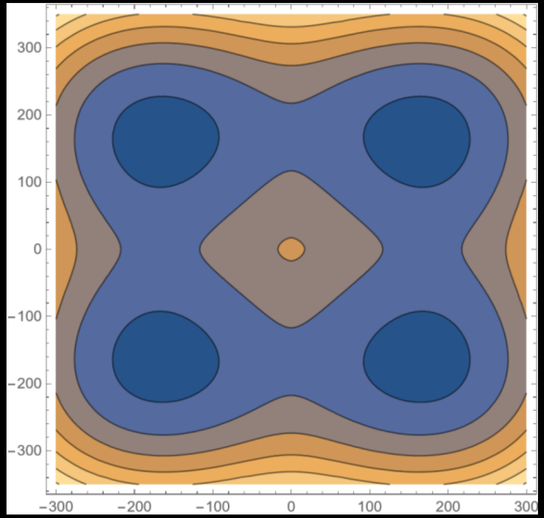
$a/f_a=0$



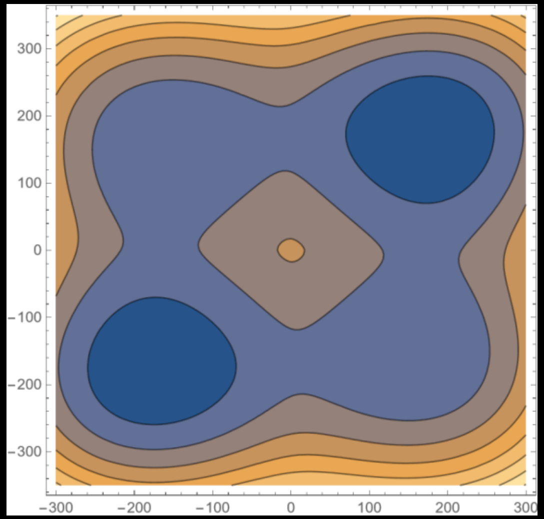
$a/f_a=\pi/2-\varepsilon$



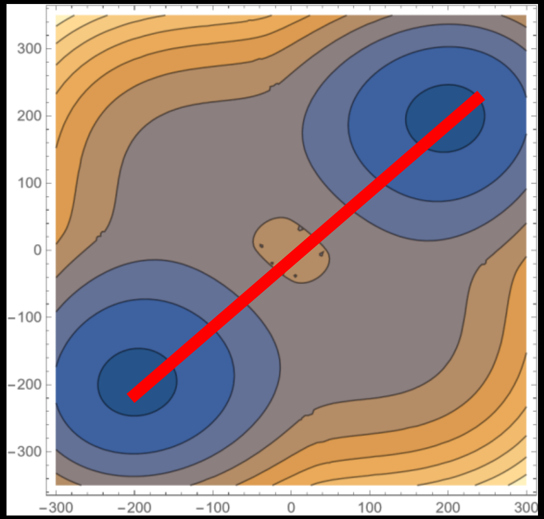
$a/f_a=\pi/2$



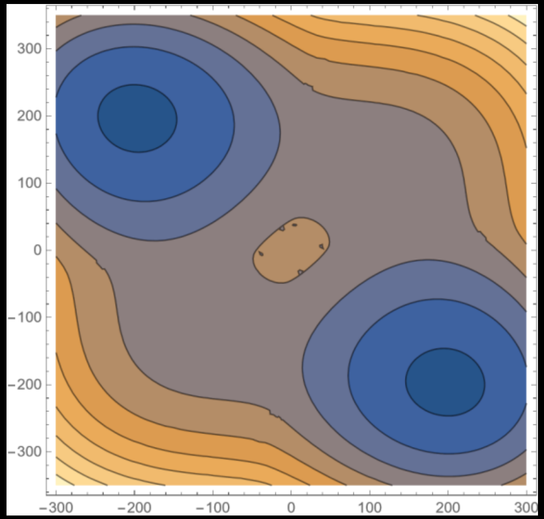
$a/f_a=\pi/2+\varepsilon$



$a/f_a=\pi$



$a/f_a=2\pi$

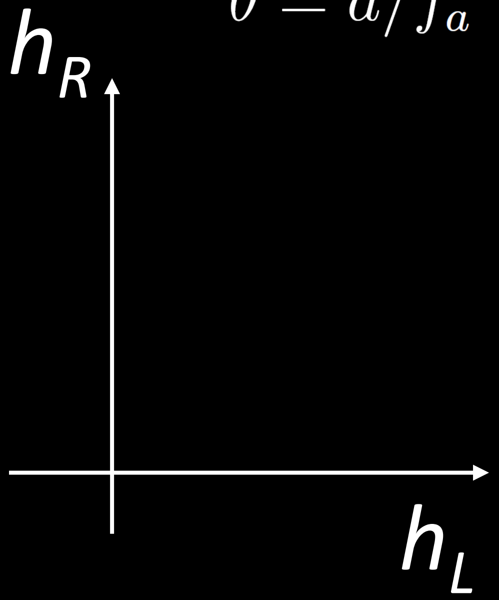


$h_L=+\, h_R$

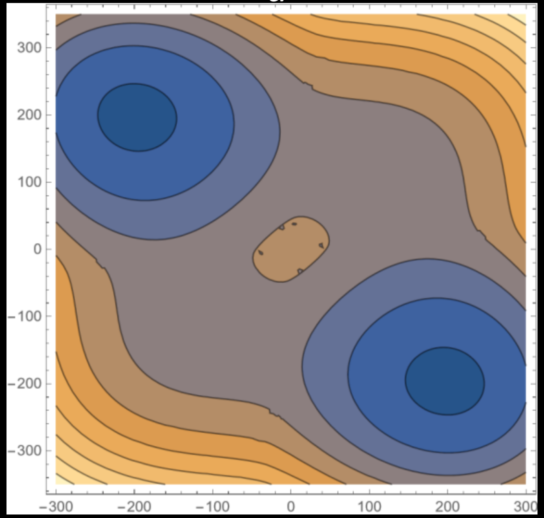
$\Delta_3=\Delta_5$

$\theta = a/f_a$

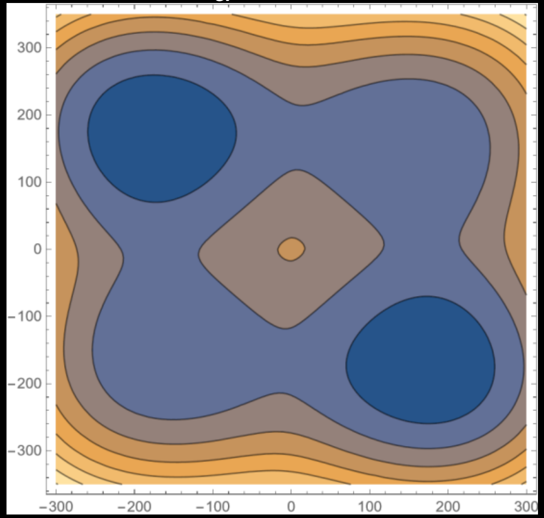
$\Omega(h_L,h_R)$ at $T=0$



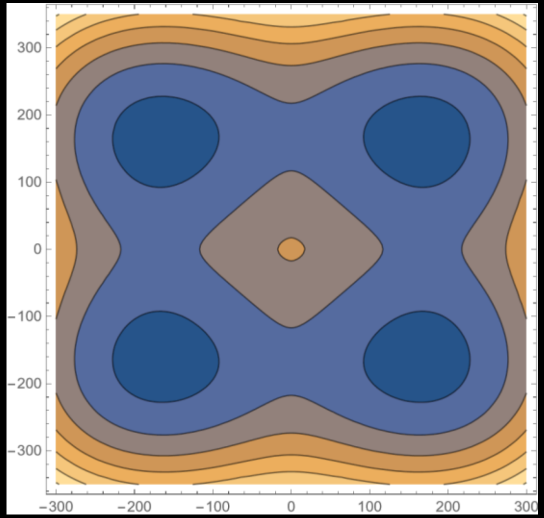
$a/f_a=0$



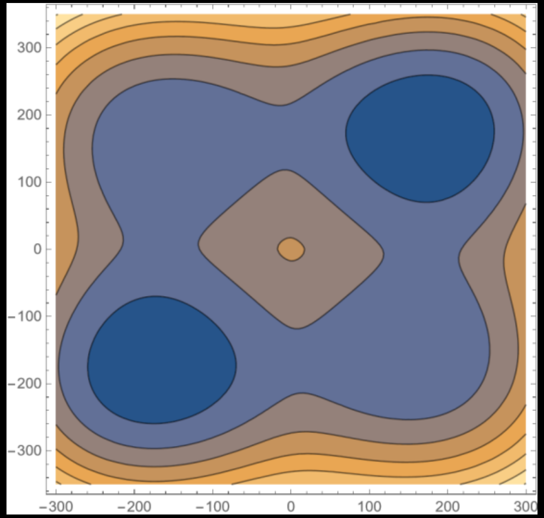
$a/f_a=\pi/2-\varepsilon$



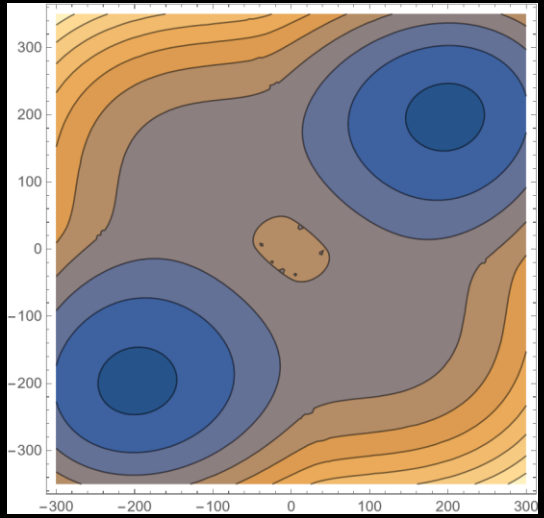
$a/f_a=\pi/2$



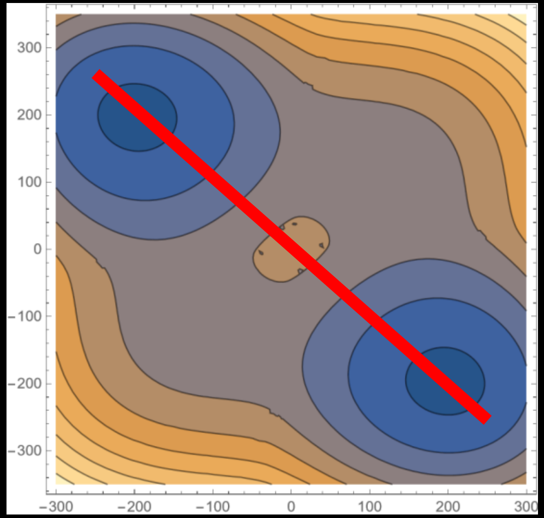
$a/f_a=\pi/2+\varepsilon$



$a/f_a=\pi$



$a/f_a=2\pi$

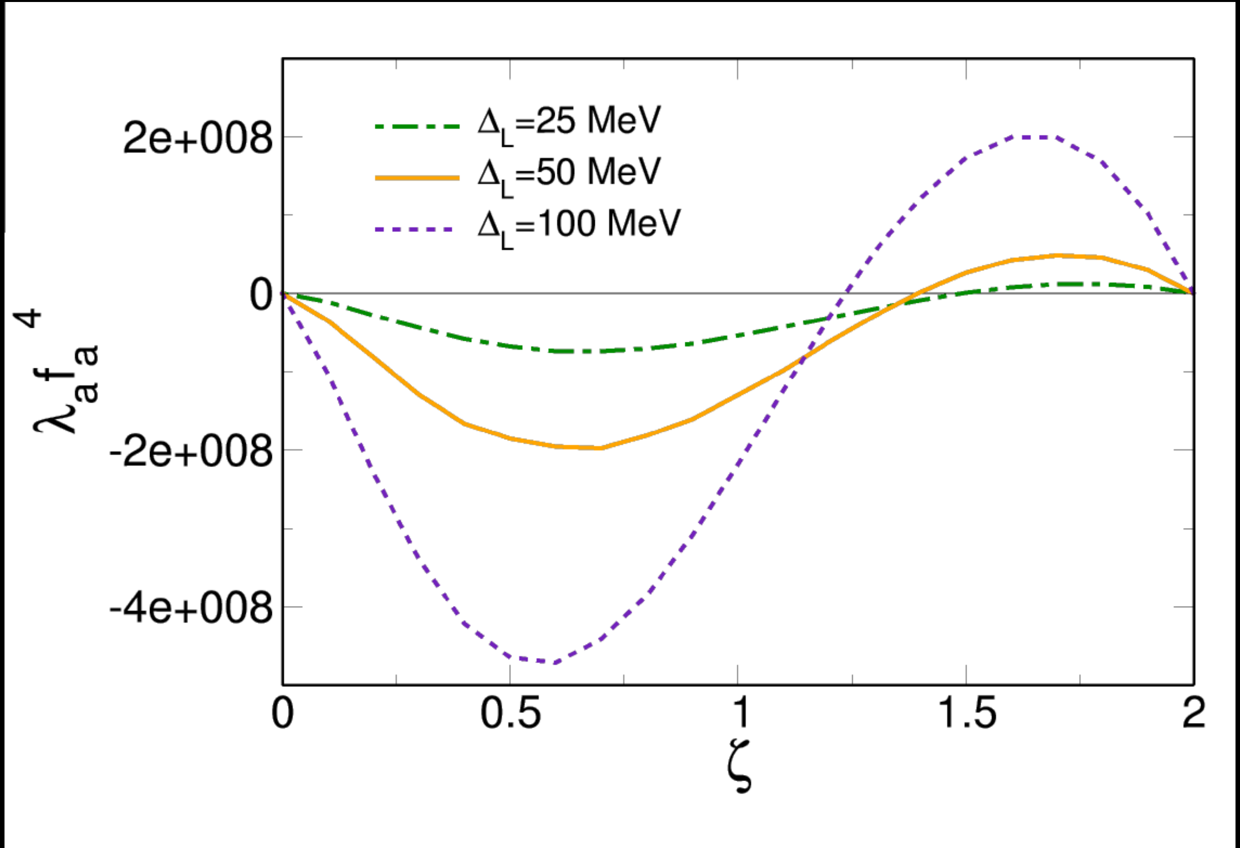


$h_L=- h_R$

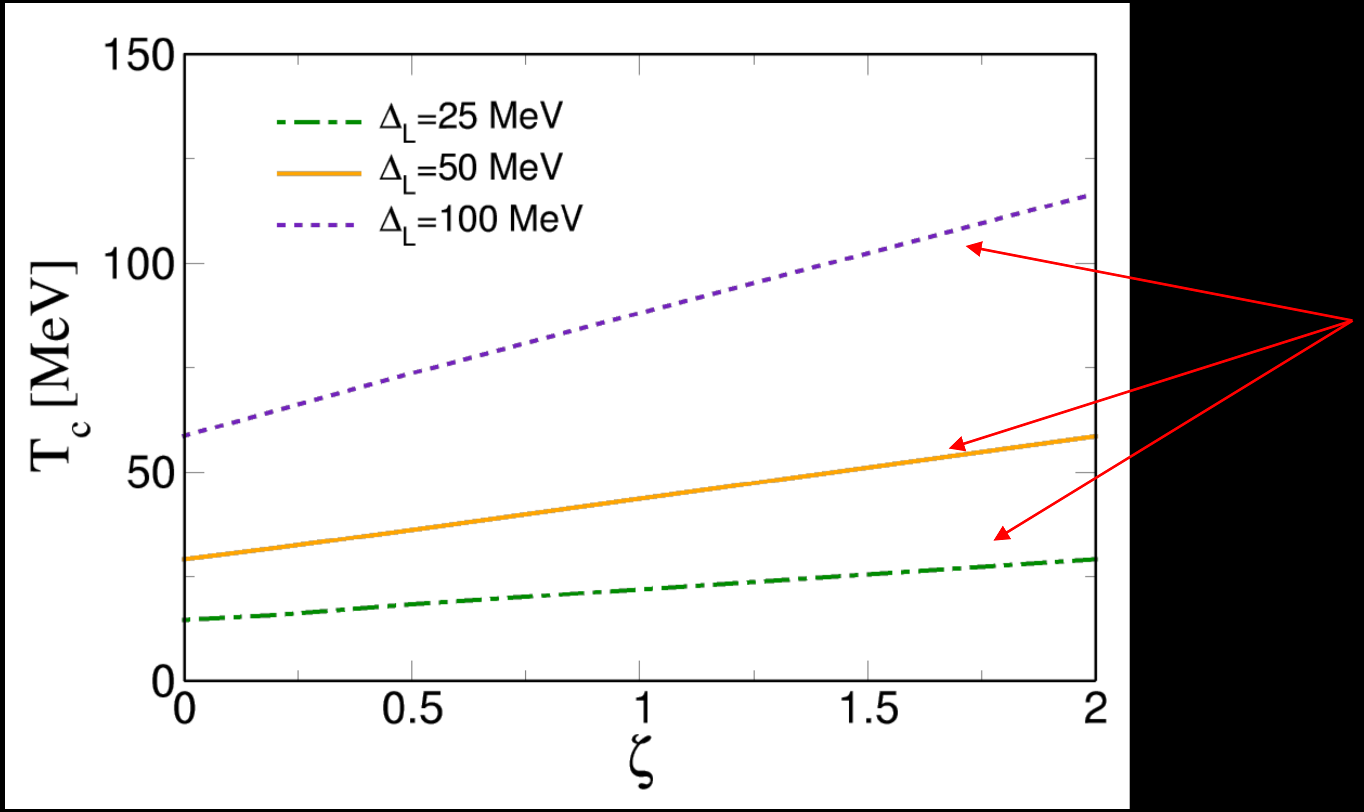
$\Delta_3=\Delta_5$

Axion self-coupling in the 2SC phase

$$\lambda_a = \frac{1}{f_a^4}\,\frac{d^4V(\theta)}{d\theta^4}\bigg|_{\theta=0}$$



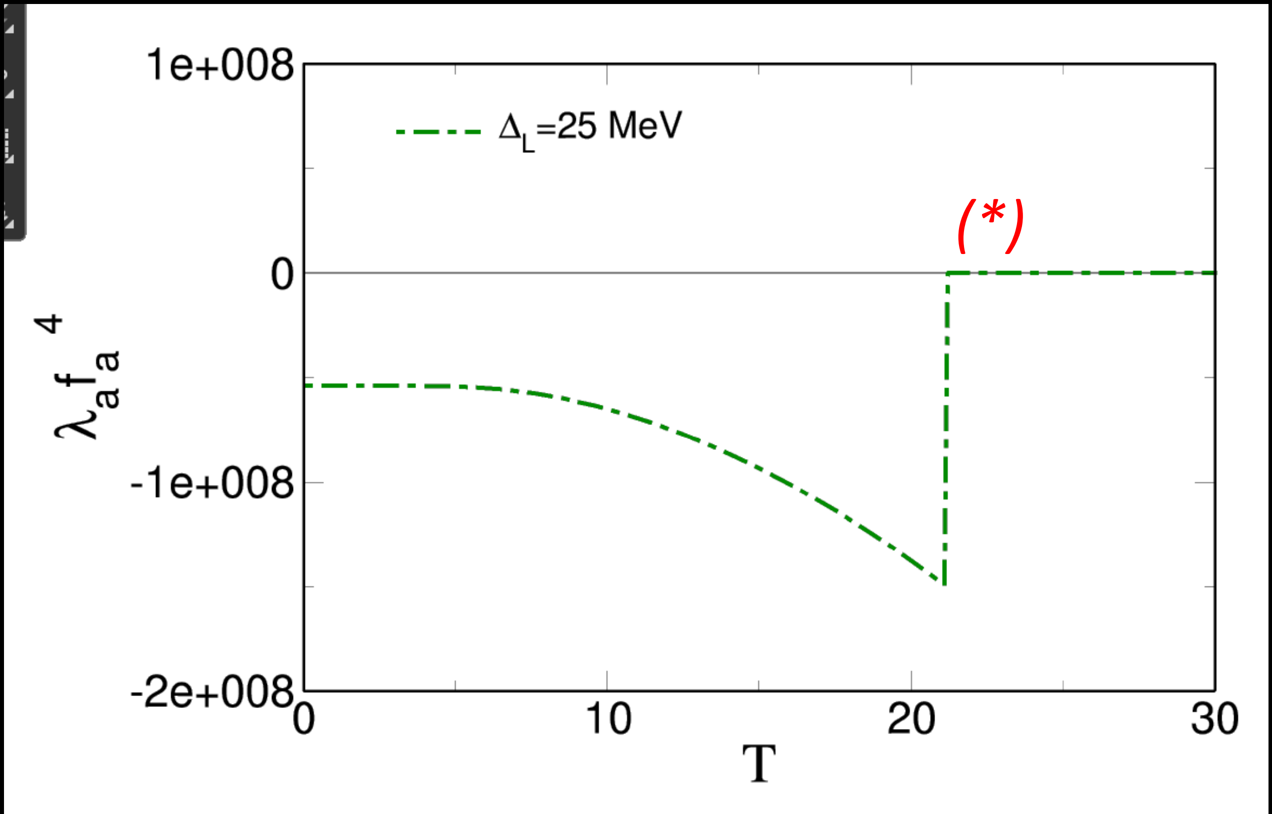
Critical temperature versus ζ

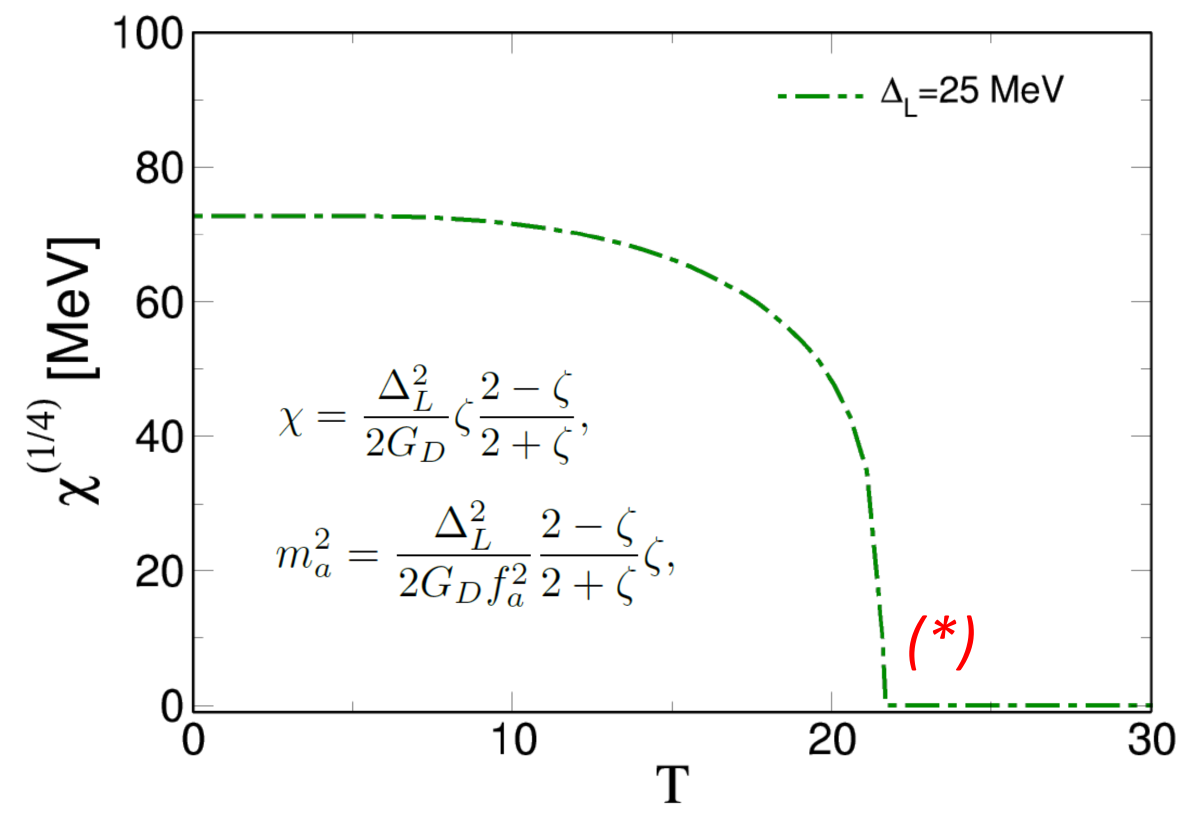


$\Delta_3=0$

Second-order phase transition to normal quark matter

Mass and coupling versus temperature





$$\chi = \frac{\Delta_L^2}{2G_D}\zeta\frac{2-\zeta}{2+\zeta},$$

$$m_a^2 = \frac{\Delta_L^2}{2G_D f_a^2}\frac{2-\zeta}{2+\zeta}\zeta,$$

Second-order phase transition to normal quark matter

(*) λ_a=0 and χ=0 for T>T_c might be a drawback of the mean field approximation.

Superconductive gap in the quark spectrum

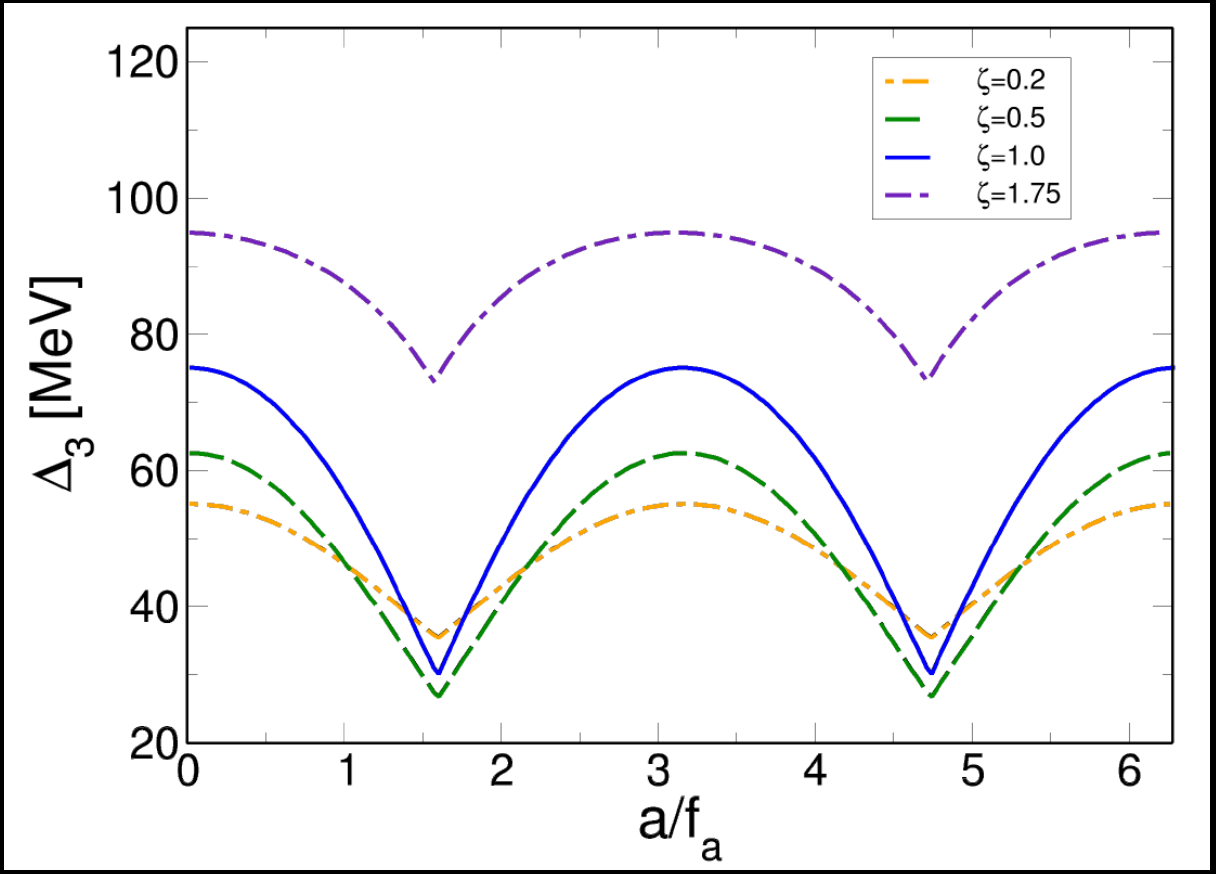
$$\Delta_3^2 = \zeta^2 G_D^2 h_L^2 + 4G_D^2 h_R^2 - 4\zeta G_D^2 h_L h_R \cos(a/fa),$$
$$\Delta_5^2 = \zeta^2 G_D^2 h_R^2 + 4G_D^2 h_L^2 - 4\zeta G_D^2 h_L h_R \cos(a/fa),$$

Δ_3, Δ_5

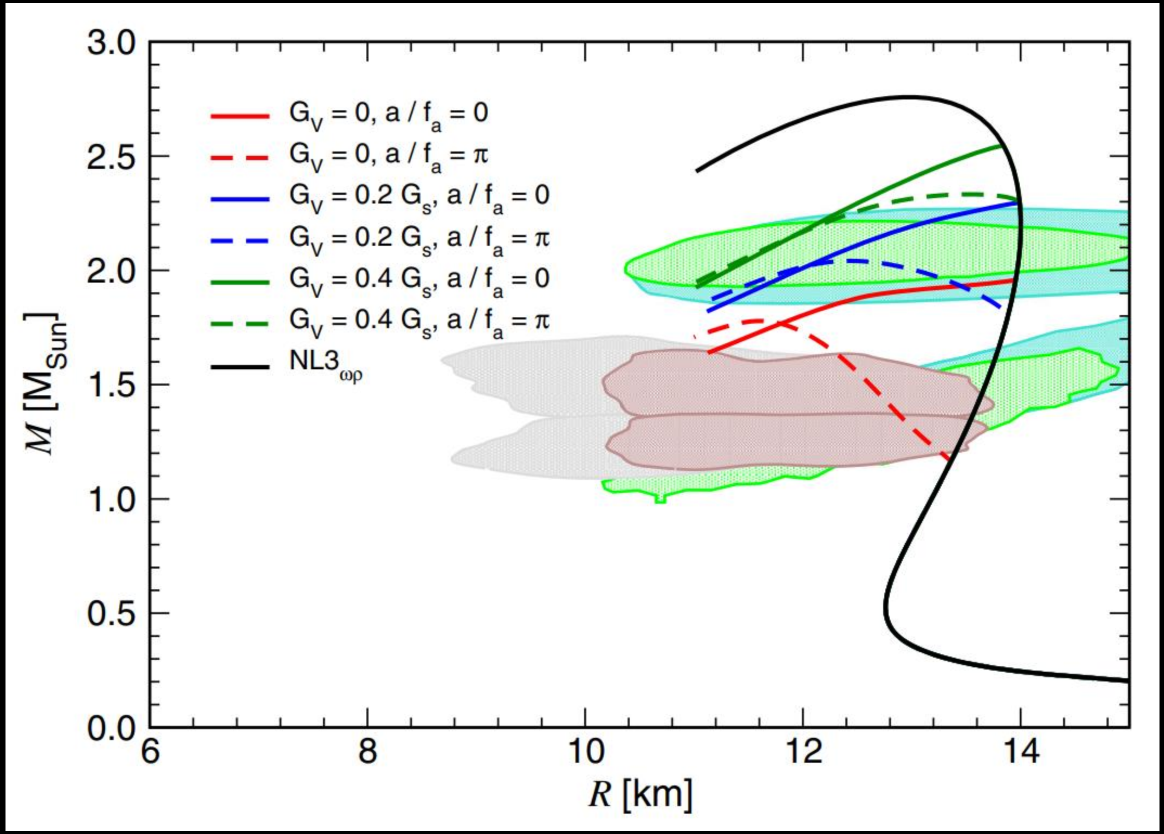
superconductive gaps in the quark spectrum.

Within our model Δ_3=Δ_5.

M.R. et al, in preparation



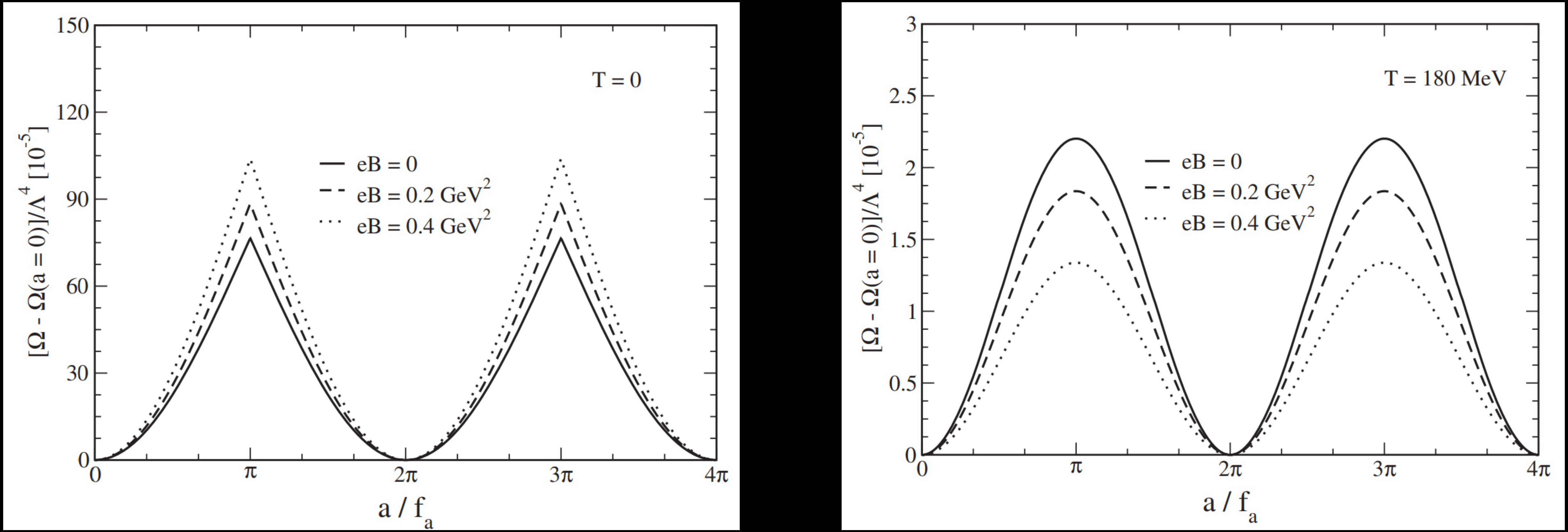
Lopes et al. (2021)



Axion effects on the stability of hybrid stars

Axions stabilize massive neutron stars by weakening the quark-hadron phase transition and bringing it to lower densities.

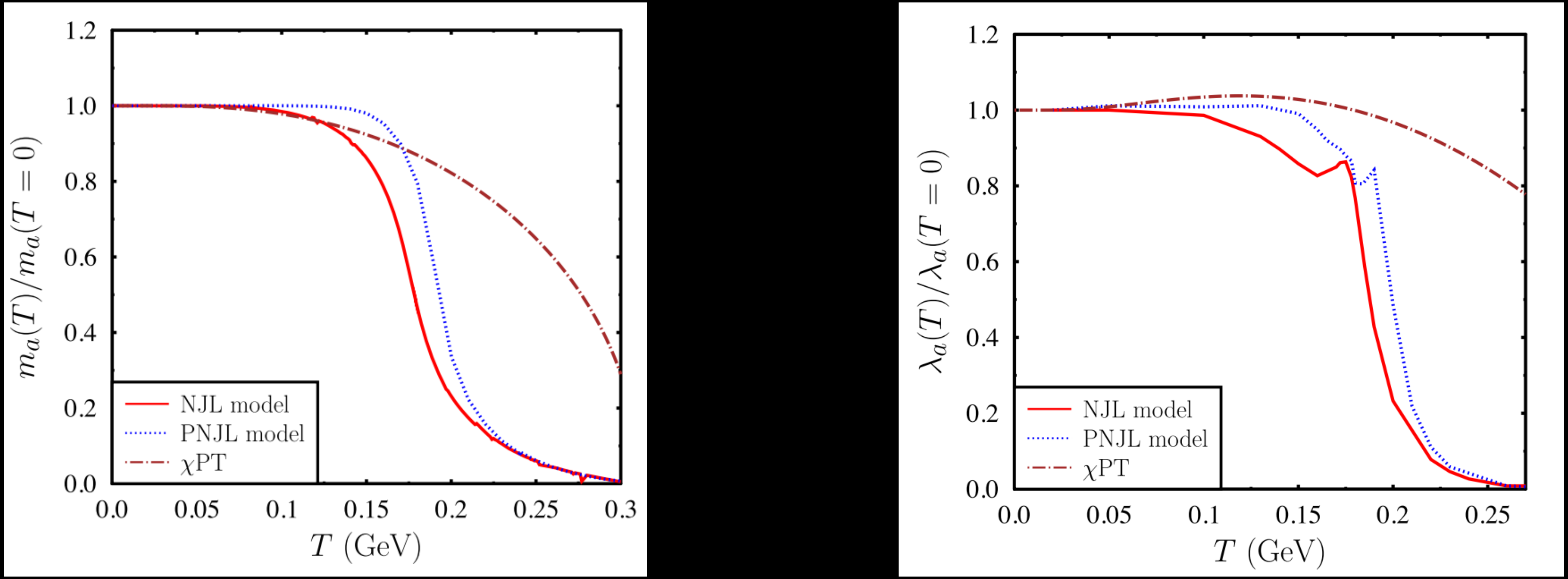
Bandyopadhyay et al. (2019)



QCD axion in a hot and magnetized medium

Magnetic field enhances the axion potential at T=0, while lowers it above T_c (inverse magnetic catalysis).

Abishek et al. (2019)



In-medium QCD axion within PNJL model

Study of the QCD axion using a model that effectively (statistically) contains confinement.

Comparison with perturbative QCD-Langevin

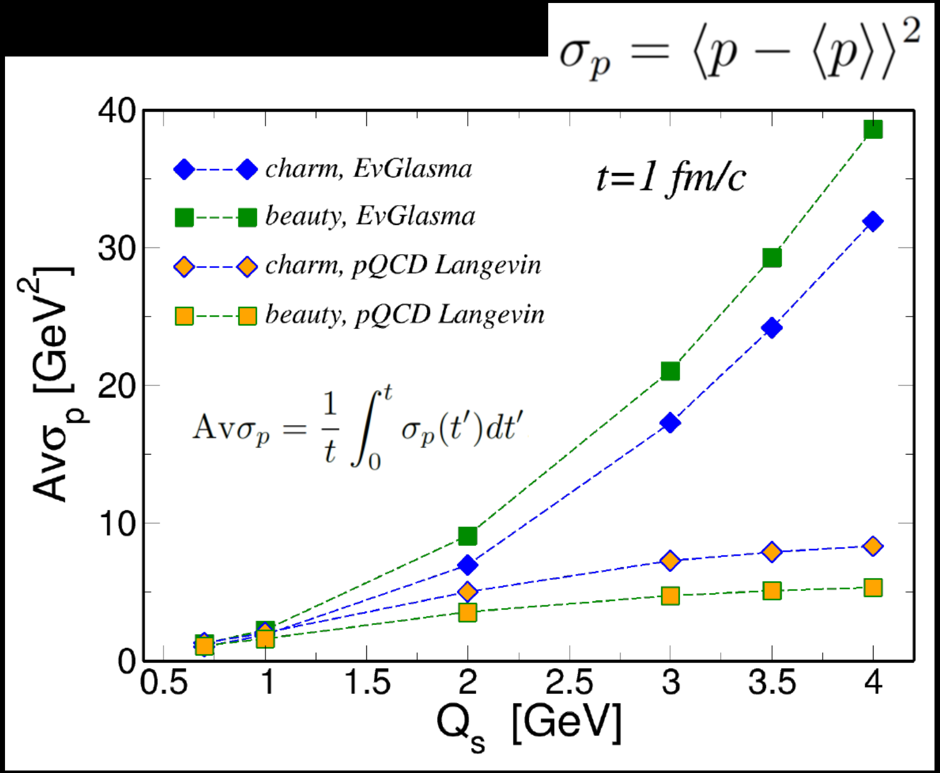
Standard Brownian motion

$$\frac{dp}{dt} = -\gamma p + \xi$$

$$\langle \xi(t_1)\xi(t_2) \rangle = 2D\delta(t_1 - t_2)$$

energy loss

random kicks from the medium
(momentum diffusion)



$$\sigma_p = \langle p - \langle p \rangle \rangle^2$$

Average diffusion coefficient of HQs in Glasma agrees with pQCD for small values of Q_s (diluted Glasma).

Quasi-quark spectrum

Quark dispersion laws

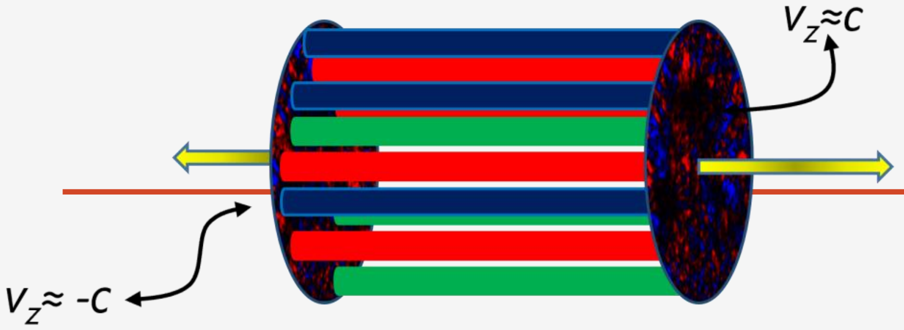
$$\begin{aligned}
\varepsilon_{1,\pm} &= \pm|p-\mu|, \\
\varepsilon_{2,\pm} &= \pm|p+\mu|, \\
\varepsilon_{3,\pm} &= \pm\sqrt{(p-\mu)^2+\Delta_3^2} \\
\varepsilon_{4,\pm} &= \pm\sqrt{(p+\mu)^2+\Delta_3^2} \\
\varepsilon_{5,\pm} &= \pm\sqrt{(p-\mu)^2+\Delta_5^2} \\
\varepsilon_{6,\pm} &= \pm\sqrt{(p+\mu)^2+\Delta_5^2},
\end{aligned}$$

Δ_3, Δ_5

superconductive gaps in the quark spectrum.

$$\begin{aligned}
\Delta_3^2 &= \zeta^2 G_D^2 h_L^2 + 4G_D^2 h_R^2 - 4\zeta G_D^2 h_L h_R \cos(a/fa), \\
\Delta_5^2 &= \zeta^2 G_D^2 h_R^2 + 4G_D^2 h_L^2 - 4\zeta G_D^2 h_L h_R \cos(a/fa),
\end{aligned}$$

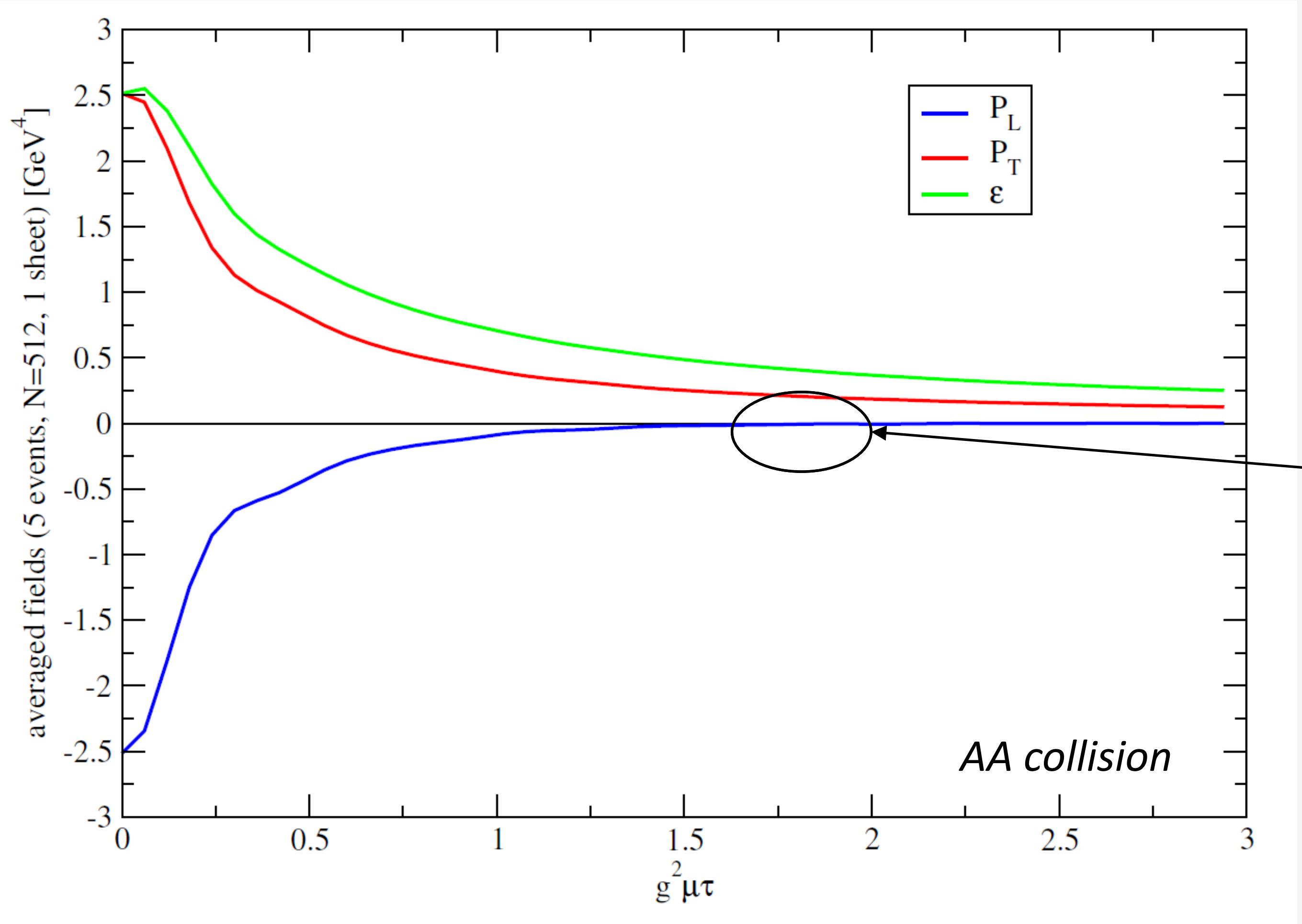
The free streaming regime, AA collisions



$$\varepsilon = \mathrm{Tr}\left[E_L^2 + E_T^2 + B_L^2 + B_T^2\right]$$

$$P_L = \mathrm{Tr}\left[-E_L^2 - B_L^2 + E_T^2 + B_T^2\right]$$

$$P_T = \mathrm{Tr}\left[E_L^2 + B_L^2\right]$$



$\tau \approx 0.2$ fm/c

Longitudinal pressure vanishes(*)

$P_L \neq P_T$: the system is quite anisotropic. This anisotropy affects observables, e.g. those of the heavy quarks.

A vanilla introduction to the QCD axion

QCD-axion

$$\mathcal{L}_{\text{odd}} + \mathcal{L}_{\text{axion}} \propto \frac{a}{f_a}\tilde{F} \cdot F$$

axion decay constant

The axion mass, m_a, and f_a are very uncertain, mostly due to lack of direct observation of this elusive particle.

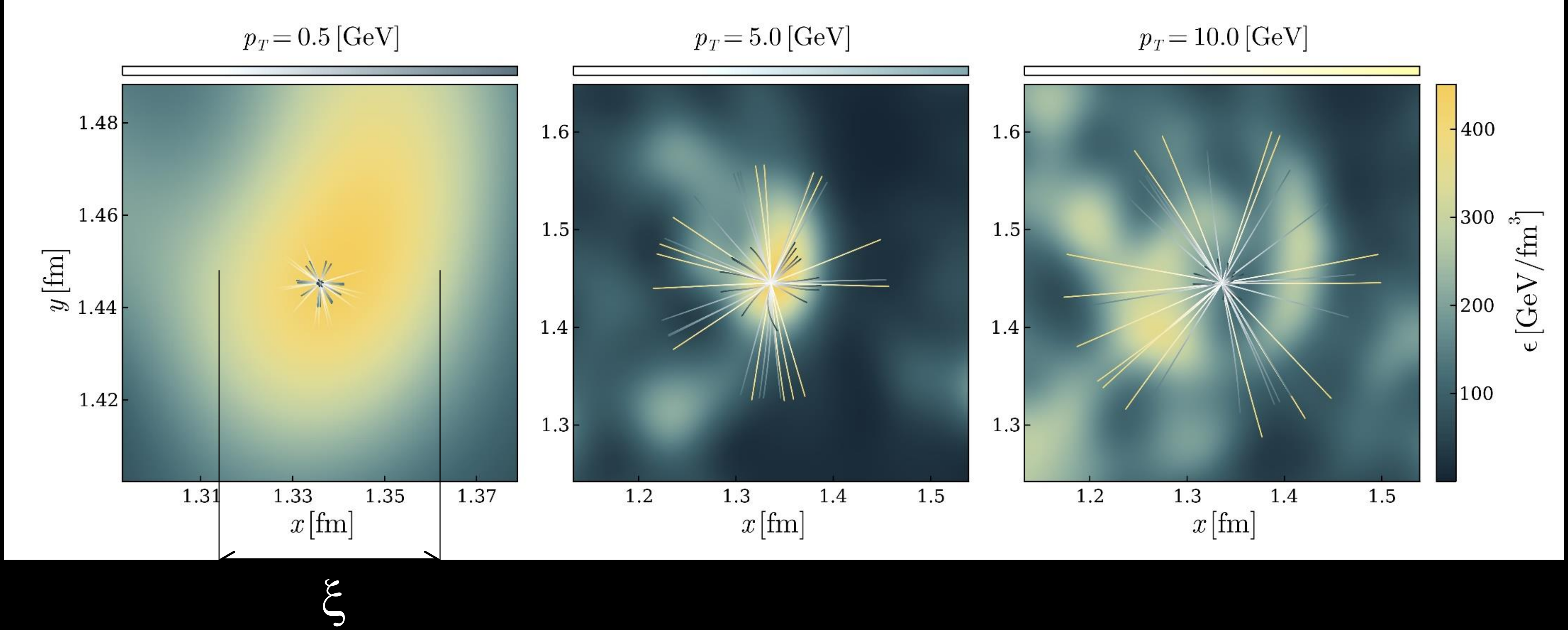
Cosmological models with inflation: $f_a = O(10^{11}$ GeV – 10^{18} GeV)

Constraints from astrophysics: $m_a = O(10^{-11}$ eV – 10 eV)(*)

(*)See for example Giannotti, https://arxiv.org/pdf/2205.06831

The pre-equilibrium stage: diffusion of heavy quarks in the color filaments

Dana Avramescu *et al.* (2023)

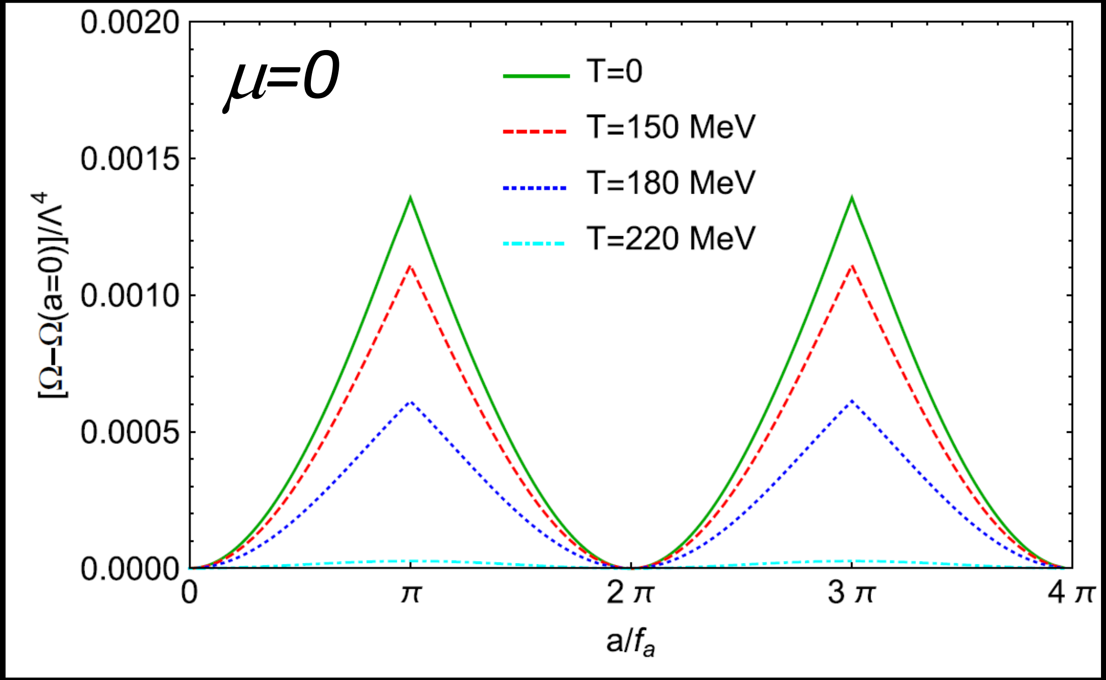


Slow color charges spend some time within one single filament: diffusion in a coherent field, rather than in a random medium.

The force exerted on these charges is time-correlated.

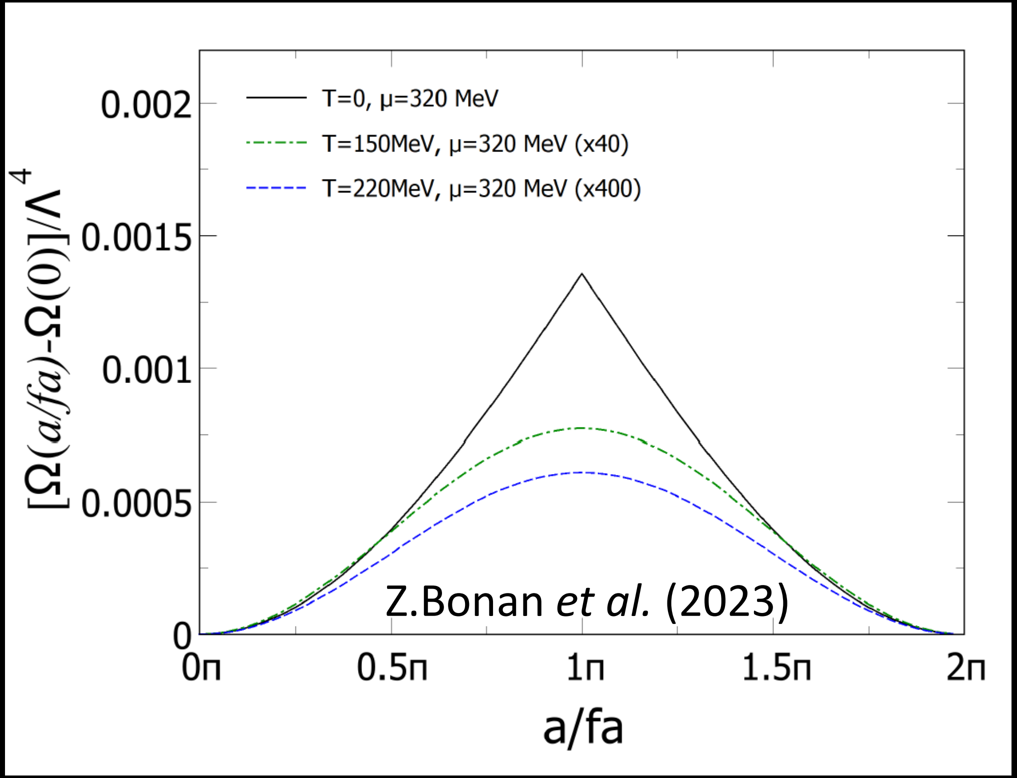
The axion potential

Axion potential in hot QCD



Lu *et al.* (2019)
Bandyopadhyay et al. (2019)

Axion potential in dense QCD



Z.Bonan *et al.* (2023)

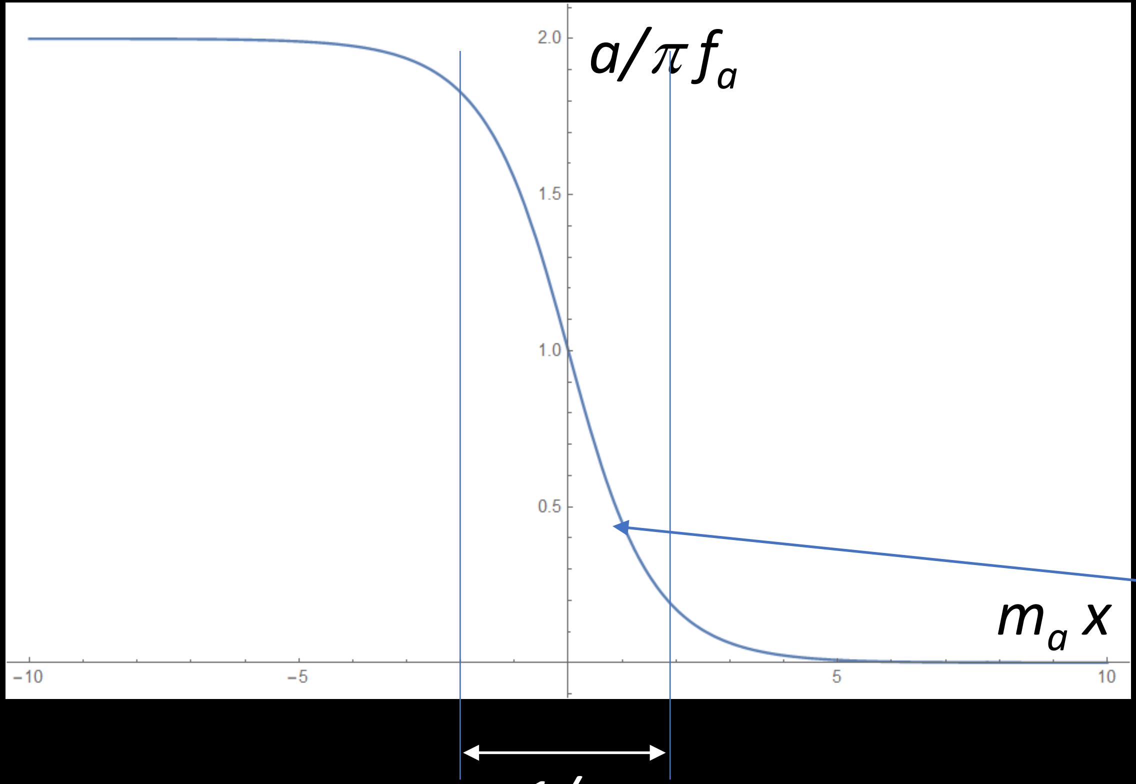
Chiral restoration implies a substantial decrease of the free energy barrier:

Energy cost to form field configurations connecting two adjacent vacua is lowered.

The axion walls

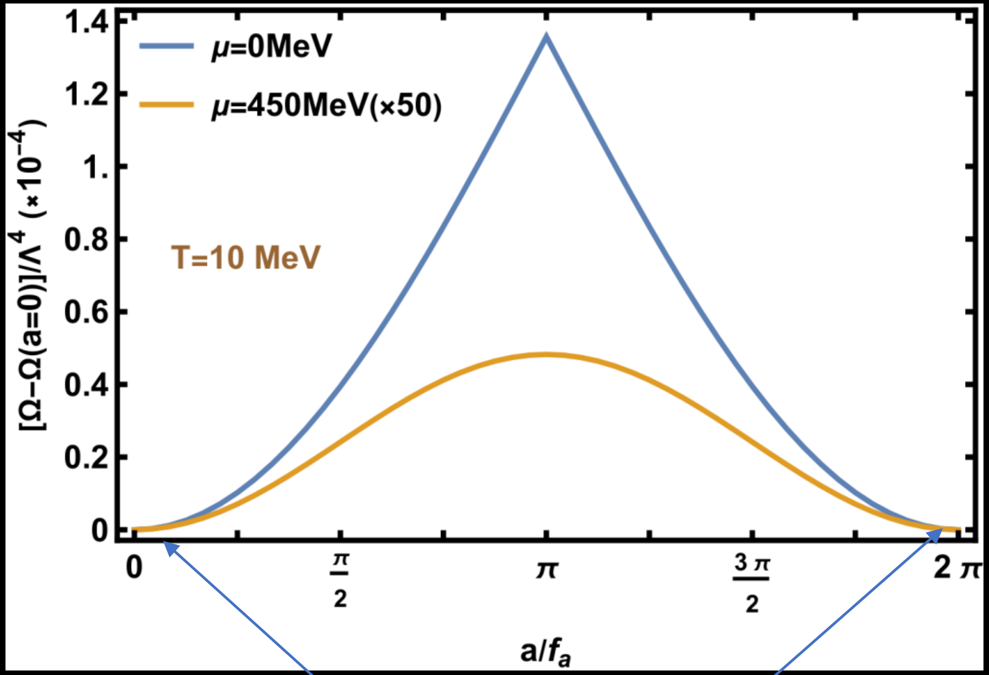
$$\mathcal{L} = \frac{1}{2}\partial^\mu a \partial_\mu a - V(a/f_a)$$

$$V(\theta) = \Omega(\theta) - \Omega(0)$$



≈1/m_a

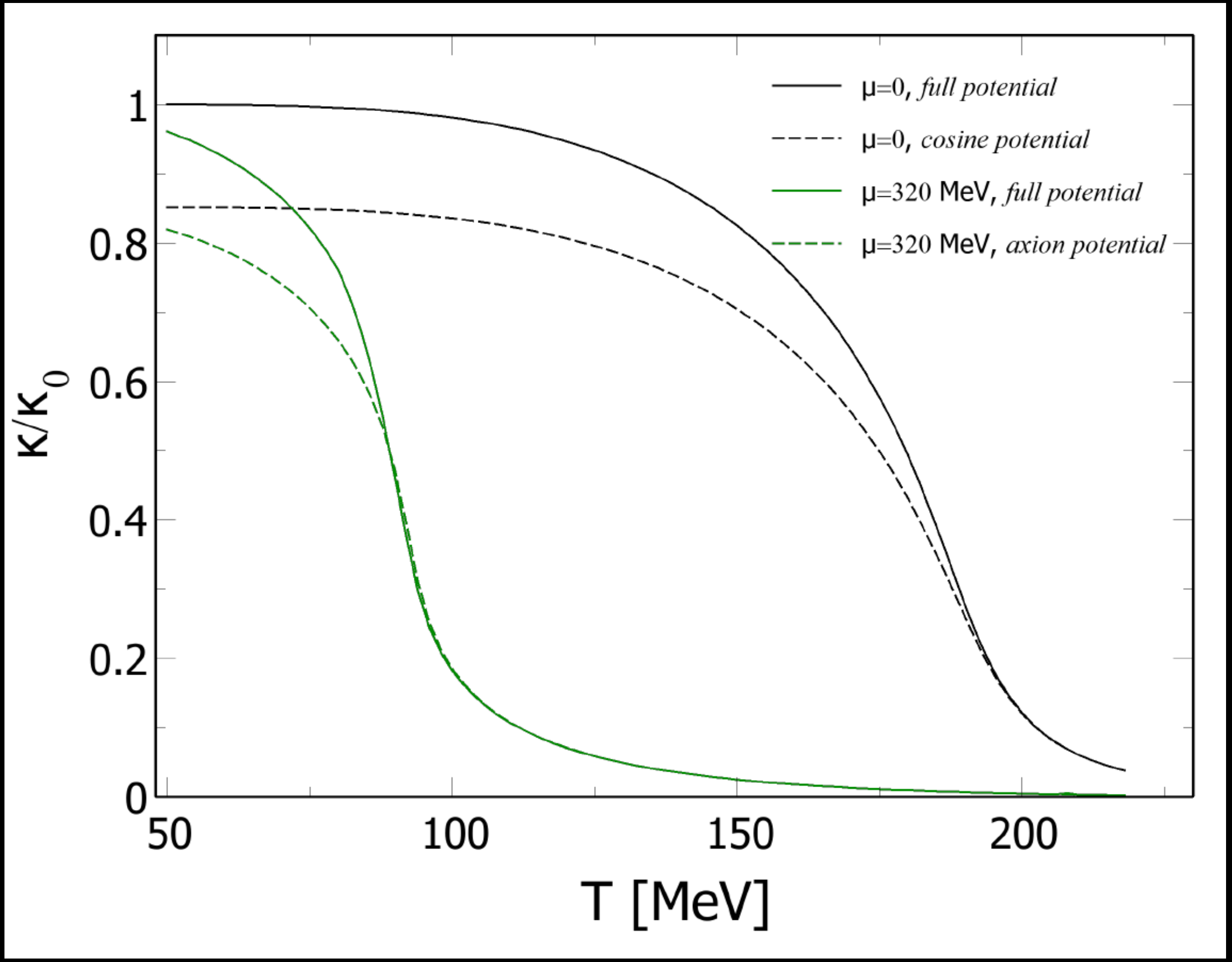
Energy accumulates here



Field configuration that interpolates between two adjacent vacua

The axion wall surface tension

Z.Bonan *et al.* (2023)



Surface tension of the walls

$$\kappa \equiv \frac{E}{L^2} = \int_{-\infty}^{+\infty} dx \left[\frac{1}{2}\left(\frac{da}{dx}\right)^2 + V(a/f_a)\right]$$

Restoration of chiral symmetry lowers the κ *of the walls.*

$$\kappa \equiv \frac{E}{L^2} = \int_{-\infty}^{+\infty} dx \left[\frac{1}{2} \left(\frac{da}{dx} \right)^2 + V(a/f_a) \right]$$

Free energy cost to add one wall $\sim L^2$

Free energy of bulk quark matter $\sim \mu^4 L^3, T^4 L^3, \mu^2 T^2 L^3$

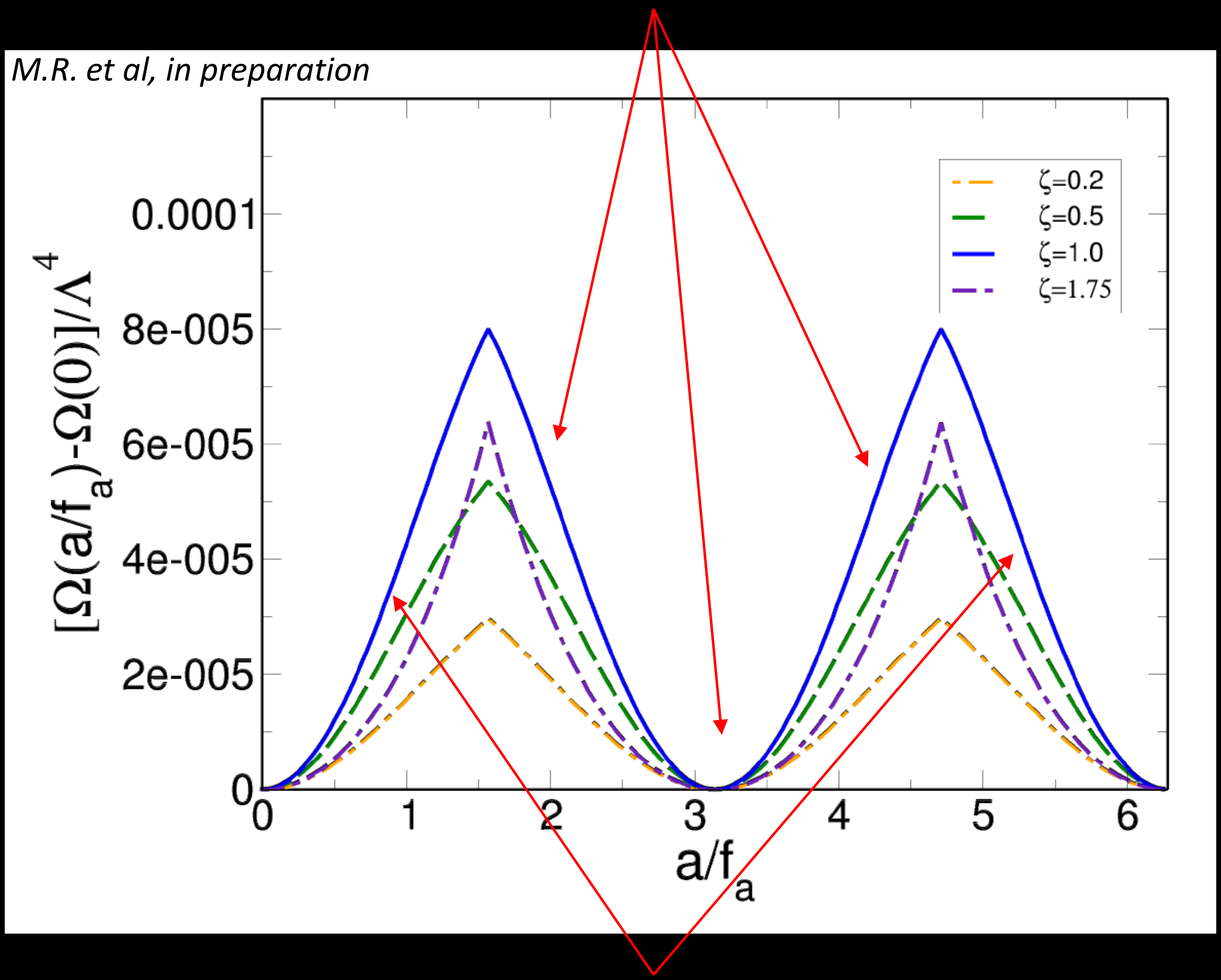
Ratio of the two $\sim 1/L$

In the thermodynamic limit, the free energy cost of adding one wall to the bulk quark matter is zero:

Axion walls might be abundant in quark matter

Axion potential at T=0, for two-flavor CSC

h_L=+ h_R
Pseudoscalar condensate Δ_{PS}



M.R. et al, in preparation

h_L=- h_R
Scalar condensate Δ_S

F. Murgana et al, PRD110 (2024)

One of the novelties of our study
Two adjacent minima have different parity(*)

(*)This is different from what happens in non-superconductive phases of QCD.
The same happens in the CFL phase.

A vanilla introduction to the QCD axion

$$\mathcal{L}_{\mathrm{odd}} \propto \theta \tilde{F} \cdot F$$

$$\tilde{F} \equiv \varepsilon_{\mu\nu\rho\sigma} F_a^{\rho\sigma}$$

$$\mathcal{L}_{\mathrm{axion}} \propto \frac{A}{f_a} \tilde{F} \cdot F$$

$$\frac{\langle A \rangle}{f_a} = -\theta$$

$$A = \langle A \rangle + a$$

QCD-axion

$$\mathcal{L}_{\mathrm{odd}} + \mathcal{L}_{\mathrm{axion}} \propto \frac{a}{f_a} \tilde{F} \cdot F$$

axion decay constant

Peccei and Quinn (1977); Shifman, Vainshtein and Zakharov (1979);
Dine, Fischler and Srednicki (1981); Wilczek (1977); Weinberg (1977)

Outlook 2: OQS for HQs in Glasma, AA and pA collisions

- *Extension of the Lindblad equation to processes with memory(*).*
- *Simple problem: one electric charge (say an electron) in a flux tube (quantum diffusion in a coherent field), Markovian case.*
- *Extension to the non-Markovian case*
- *Extension to the case of HQs in Glasma color filaments.*

- *Pro: has the advantage to allow for the description of the dissipation*
- *Contro: non-Markovian extensions of the Lindblad equation are not trivial*

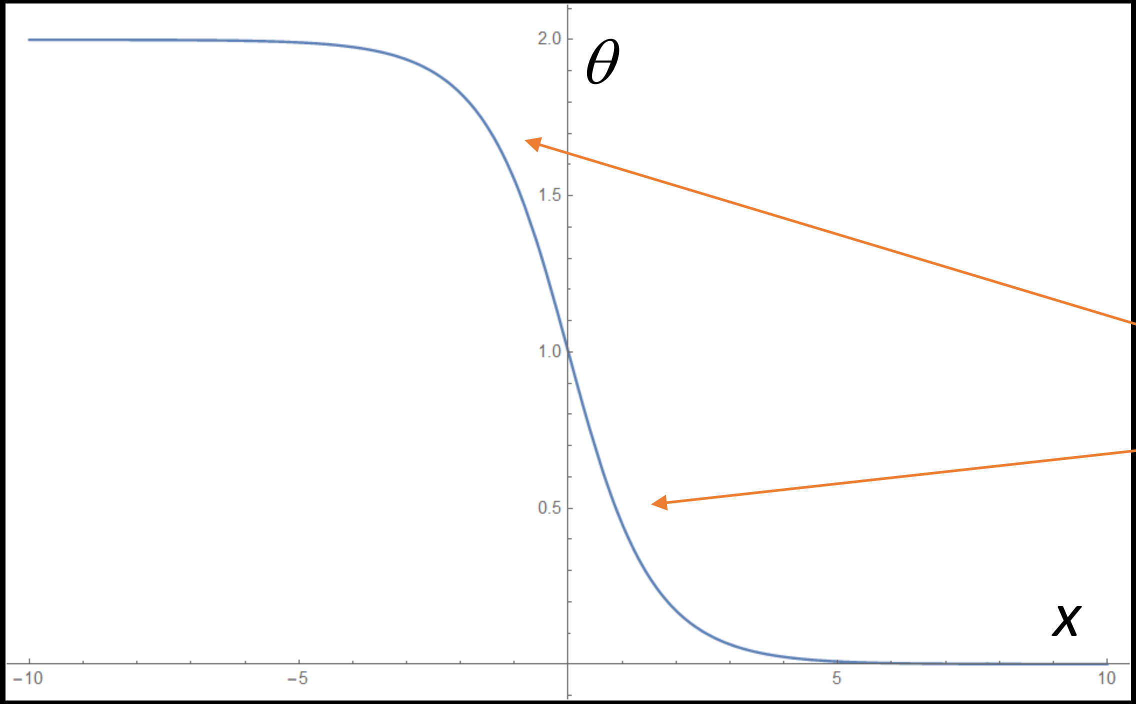
(*)Extension of the Lindblad equation to the coupling of baths with memory is an actual active research topic even in Open Quantum systems, hence the interest for this problem goes well beyond that of HQs in HICs.

Outlook 4: FRG applied to superdense phases of QCD

- *Inhomogeneous phases near the critical endpoint of the phase diagram (QM-like models)*
- *Homogeneous and inhomogeneous color-superconductive phases (HDET)*

See Murgana’s talk

Outlook 6: coupling of quarks to axions and η' walls



$$\theta(\boldsymbol{x},t) = \frac{\eta'}{f_\eta} + \frac{a}{f_a}$$

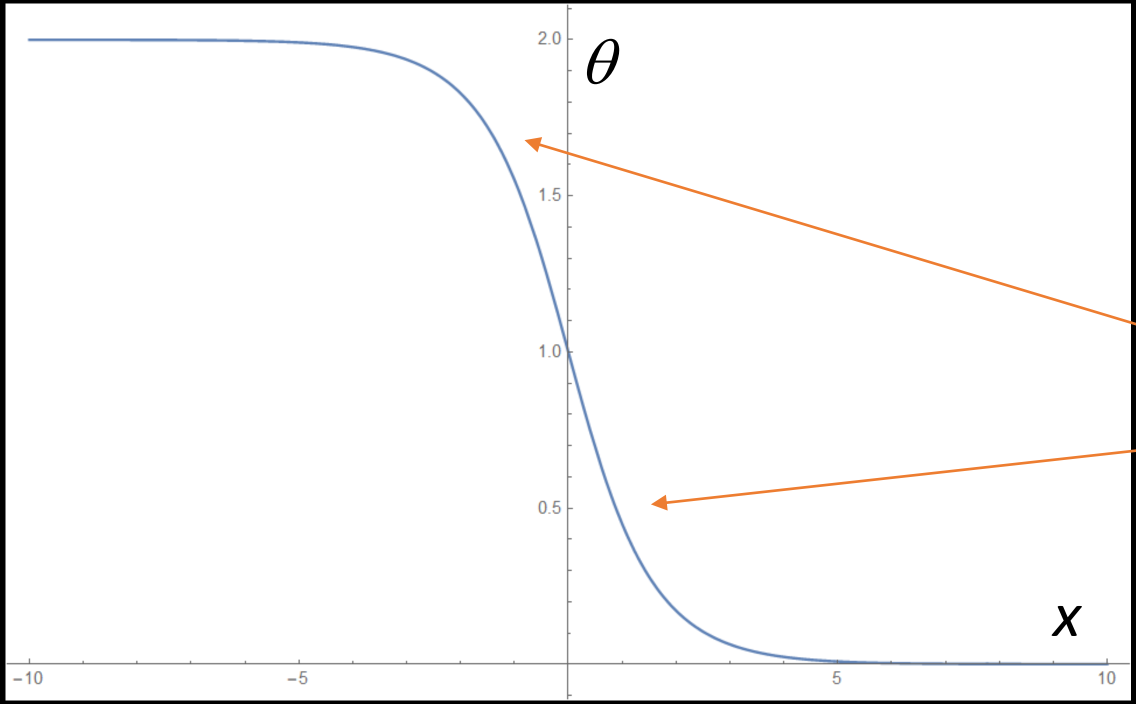
$$\begin{aligned}\mathcal{L}_{\text{coupling}} &= \partial_\mu\theta(\boldsymbol{x},t)\ \bar{\psi}\gamma^\mu\gamma_5\psi \\ &= \partial_0\theta(\boldsymbol{x},t)\ \underbrace{\psi^\dagger\gamma_5\psi} - \boldsymbol{\nabla}\theta(\boldsymbol{x},t)\cdot\bar{\psi}\boldsymbol{\gamma}\gamma_5\psi\end{aligned}$$

$$n_5 = n_R - n_L$$

Chiral density

Difference of densities of R and L quarks

Outlook 6: coupling of quarks to axions and η' walls



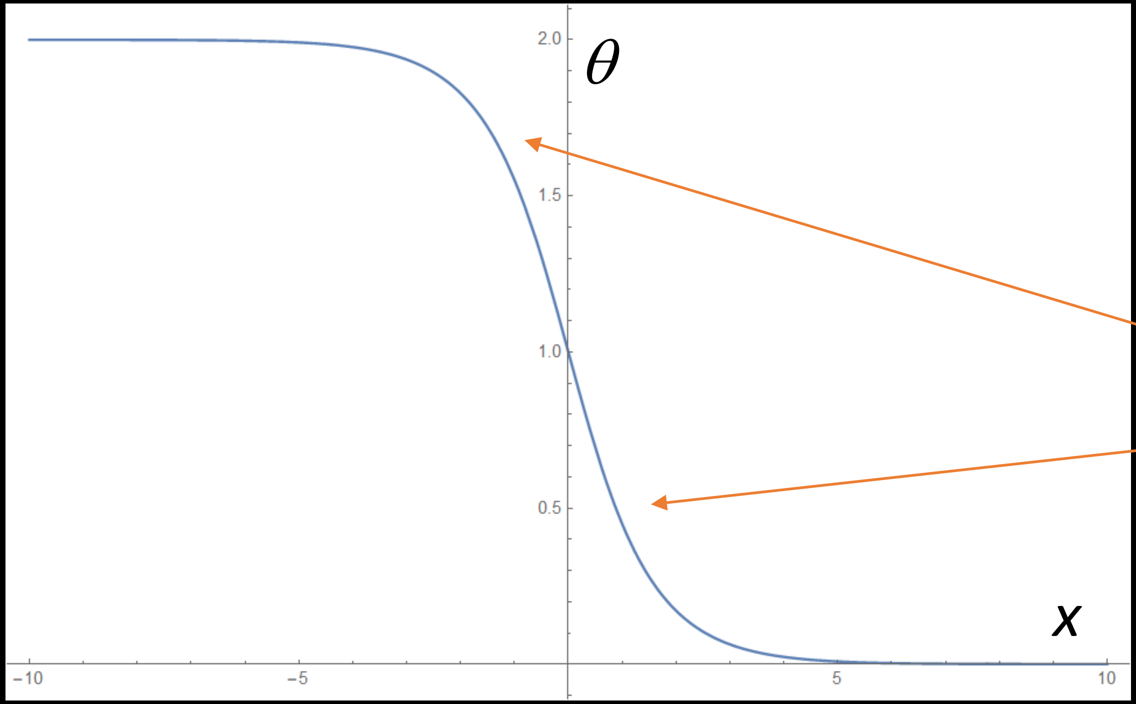
$$\theta(\boldsymbol{x},t) = \frac{\eta'}{f_\eta} + \frac{a}{f_a}$$

$$\begin{aligned}\mathcal{L}_{\text{coupling}} &= \partial_\mu\theta(\boldsymbol{x},t)\ \bar{\psi}\gamma^\mu\gamma_5\psi \\ &= \partial_0\theta(\boldsymbol{x},t)\ \psi^\dagger\gamma_5\psi - \boldsymbol{\nabla}\theta(\boldsymbol{x},t)\cdot\bar{\psi}\boldsymbol{\gamma}\gamma_5\psi \\ &\equiv \mu_5(\boldsymbol{x},t)\ \psi^\dagger\gamma_5\psi - \boldsymbol{\nabla}\theta(\boldsymbol{x},t)\cdot\bar{\psi}\boldsymbol{\gamma}\gamma_5\psi\end{aligned}$$

chiral chemical potential

$$n_5 = n_R - n_L$$

Outlook 6: coupling of quarks to axions and η' walls



$$\theta(\boldsymbol{x},t) = \frac{\eta'}{f_\eta} + \frac{a}{f_a}$$

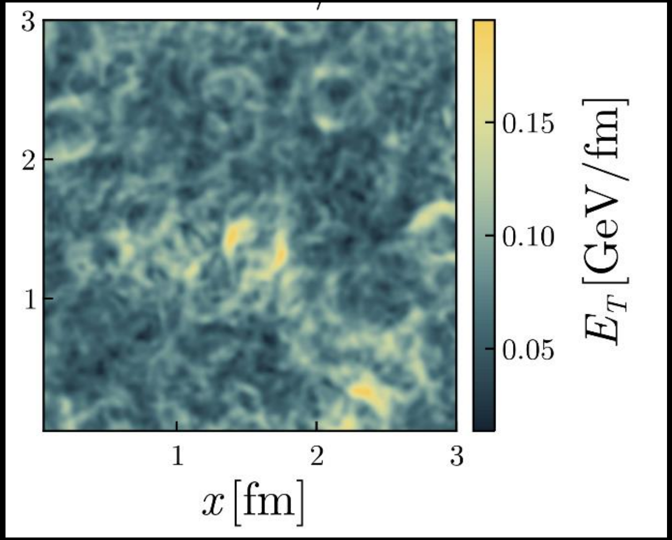
$$\mathcal{L}_{\text{coupling}} \;=\; \mu_5(\boldsymbol{x},t)\;\psi^\dagger\gamma_5\psi - \boldsymbol{\nabla}\theta(\boldsymbol{x},t)\cdot\bar{\psi}\boldsymbol{\gamma}\gamma_5\psi$$

- *Walls will form and deform to (metastable) bubbles due to surface tension*
- *Scattering of quarks on walls leads to local chirality imbalance*
- *Strong magnetic fields in the core coupled to n_5 might lead to chiral magnetic effect in superdense quark matter(*)*

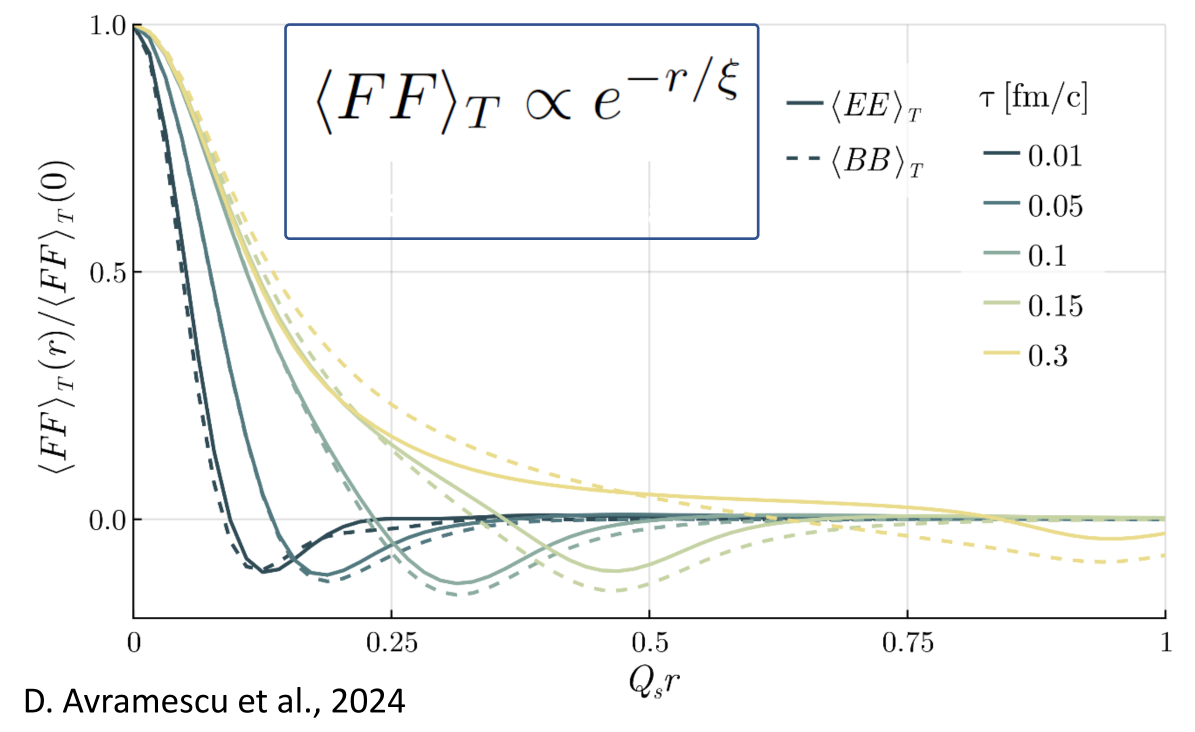
(*)Bonan et al. (2023); Murgana and Ruggieri (in preparation).
Mutatis mutandis, this can be implemented in RHICs as well.

The pre-equilibrium stage: diffusion of heavy quarks in the color filaments

Fields arrange in correlation domains, aka filaments, of transverse area ≈ξ^2:
$\xi^2 = O(1/Q_s^2)$



Field correlators in the transverse plane



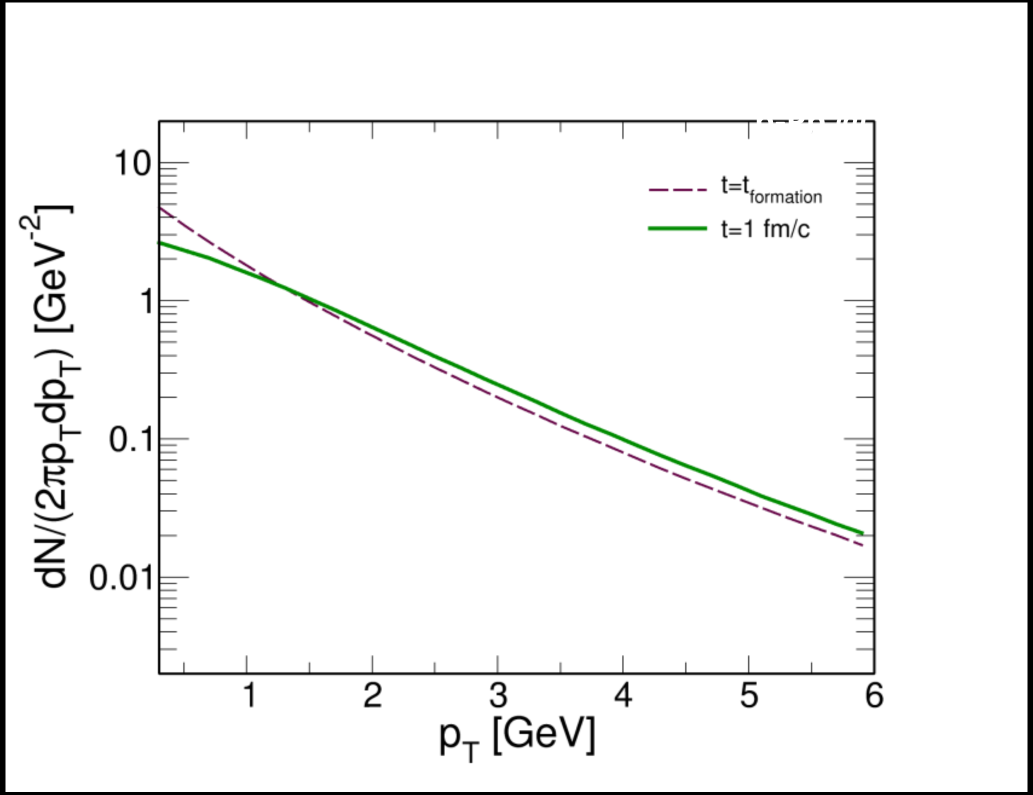
D. Avramescu et al., 2024

The force experienced by HQs in the pre-equilibrium stage is time-correlated: diffusion with memory.

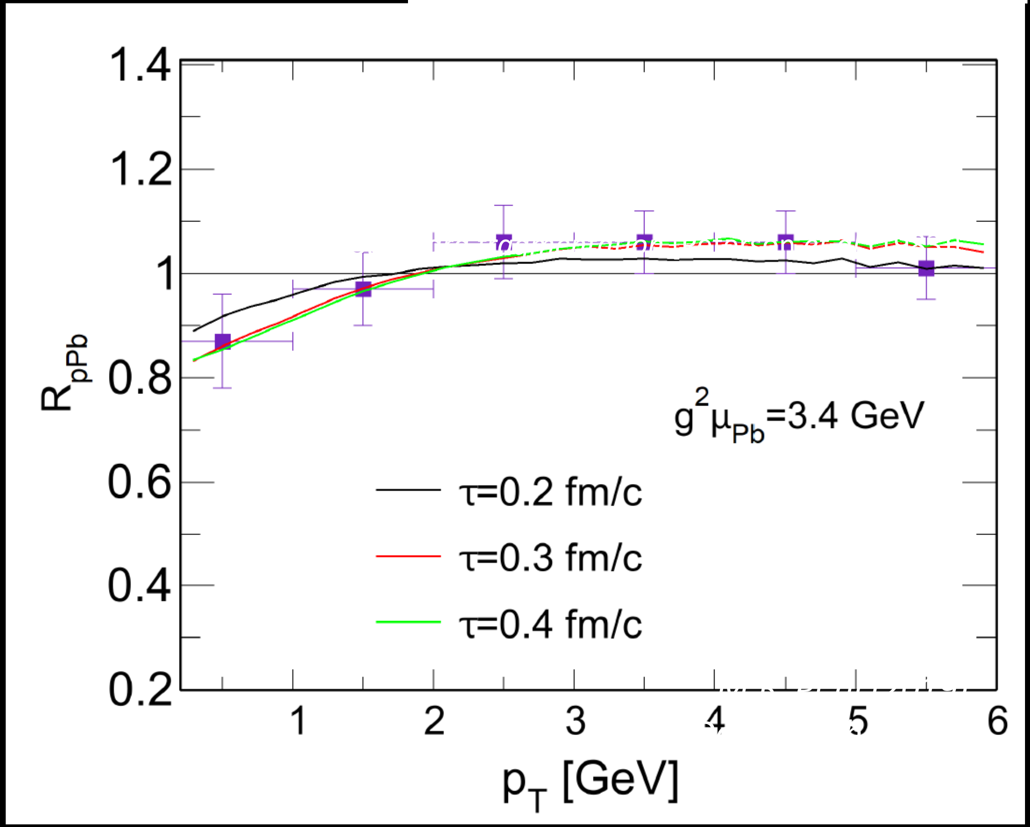
The pre-equilibrium stage: diffusion of heavy quarks in the color filaments

Initial distribution

From perturbative QCD, aka ***FONLL*** [Cacciari *et al.* (2001, 2012)]



$$R_{\mathrm{pPb}} = \frac{(dN/d^2p_T)_{\mathrm{final}}}{(dN/d^2p_T)_{\mathrm{pQCD}}}$$



$R_{pPb} \neq 1$

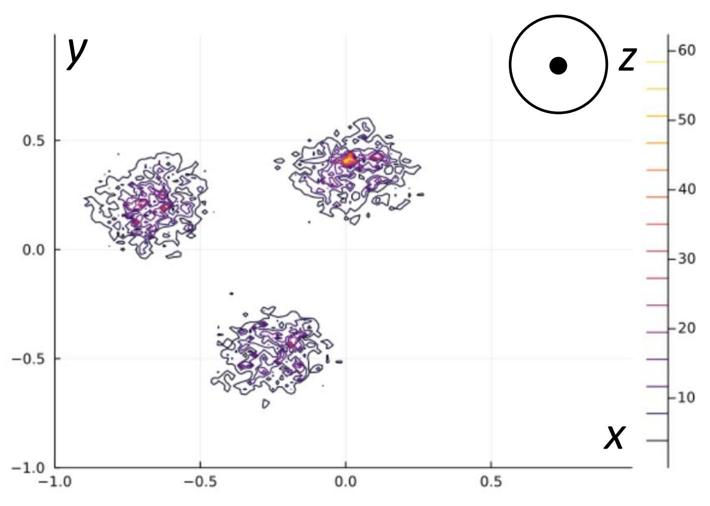
Interaction with the fields created by the collision

See also

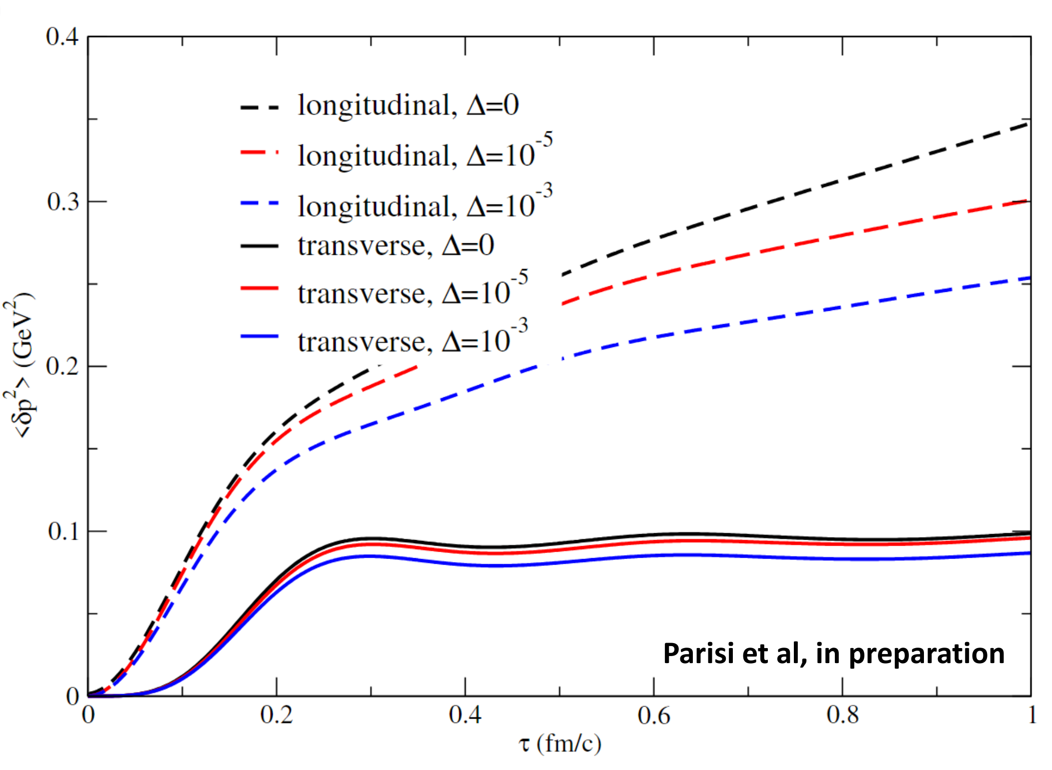
M. Ruggieri and S. K. Das (2018)

Y. Sun et al. (2019) for an estimate of the effect on the elliptic flow.

Outlook 0: (3+1)D Glasma and HQs in pA collisions



pA collisions, with η-dependent fluctuations
HQs momentum spreading

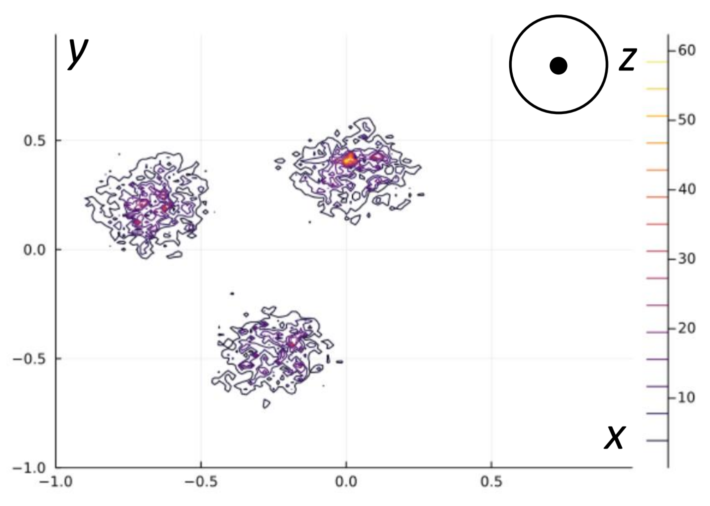


Parisi et al, in preparation

- Realistic pA collisions
- *Infinite quark mass limit*(*)
- Three-colors QCD

(*)The extension to the case of realistic quark masses is under progress.

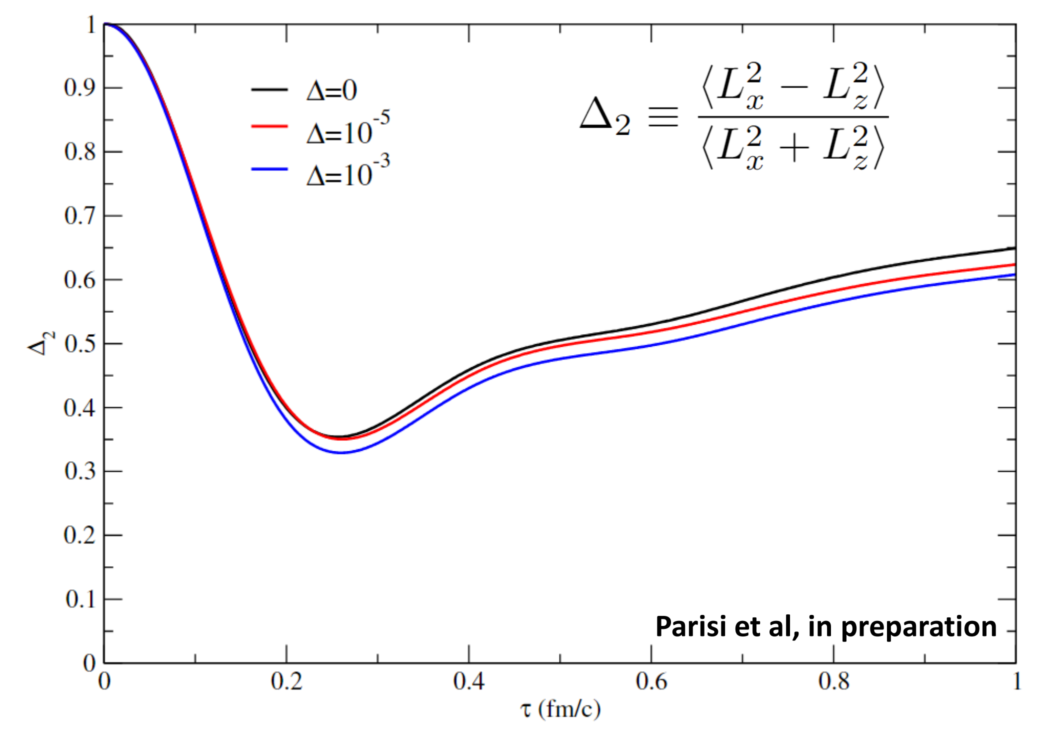
Outlook 0: (3+1)D Glasma and HQs in pA collisions



$$\Delta_2 = \frac{[\langle z^2\rangle - \langle x^2\rangle - \langle y^2\rangle]\langle p_T^2\rangle/2 + \langle y^2\rangle\langle p_z^2\rangle}{[\langle z^2\rangle + \langle x^2\rangle + \langle y^2\rangle]\langle p_T^2\rangle/2 + \langle y^2\rangle\langle p_z^2\rangle}$$

pA collisions, with η-dependent fluctuations

HQs angular momentum spreading



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