



Università
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Heavy hadron production with a coalescence plus fragmentation approach from AA to pp collisions

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In collaboration with:

S. Plumari, M.L. Sambaturo, Y.Sun, V.Greco

Outline

Hadronization:

- Fragmentation
- Coalescence model

Heavy hadrons in AA collisions:

- Λ_c , D spectra and ratio: RHIC and LHC

S. Plumari, et al. Eur. Phys. J. C 78, no.4, 348 (2018)

Heavy hadrons in small systems (pp @ 5.02 TeV):

- Λ_c/D^0 , Ξ_c/D^0 , Ω_c/D^0 *V. Minissale, et al., Phys. Lett. B 821, 136622 (2021)*
- Λ_b/B^0 , Ξ_b/B^0 , Ω_b/B^0 *(new) V. Minissale et al, arXiv:2405.19244 [hep-ph]*

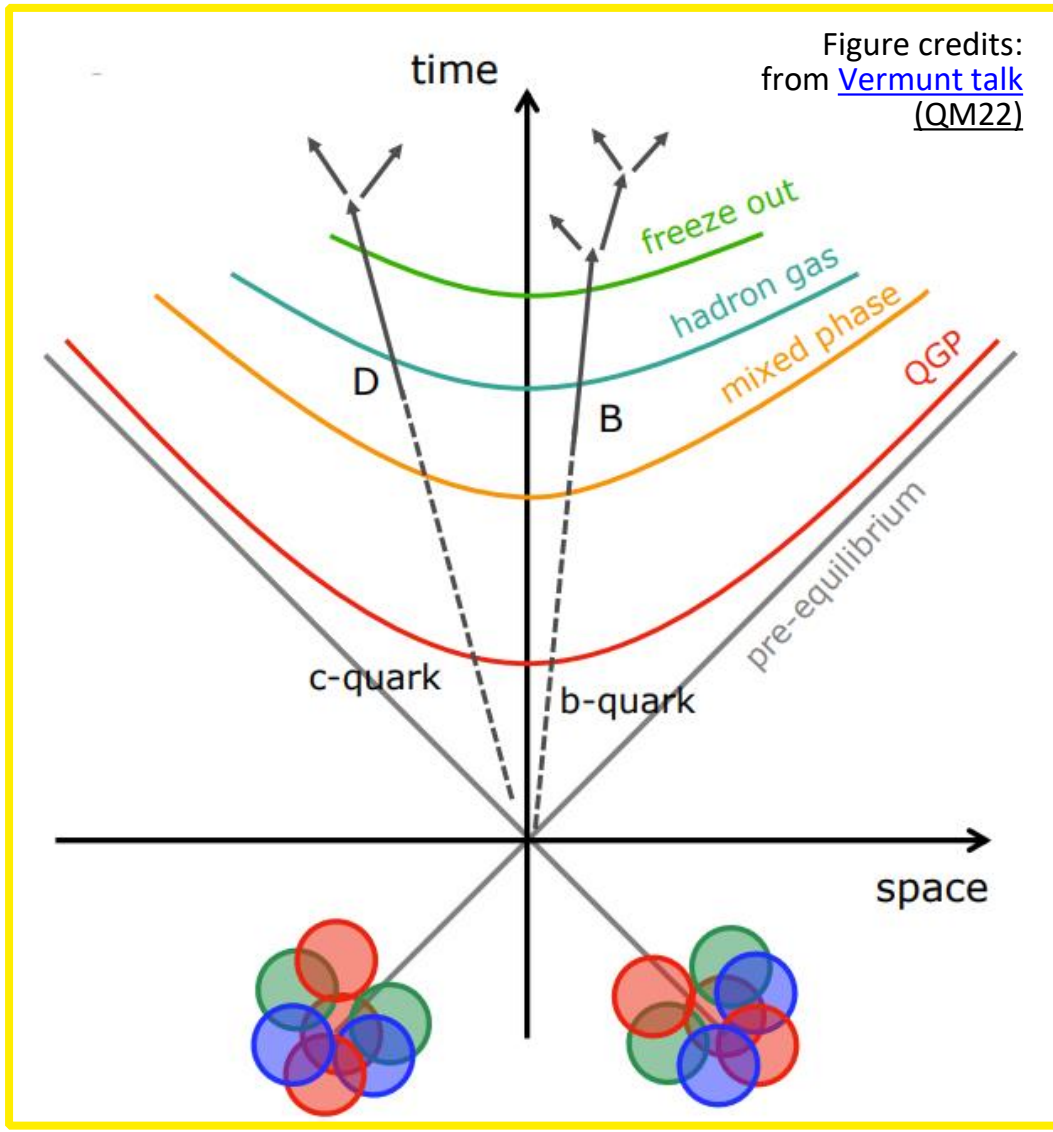
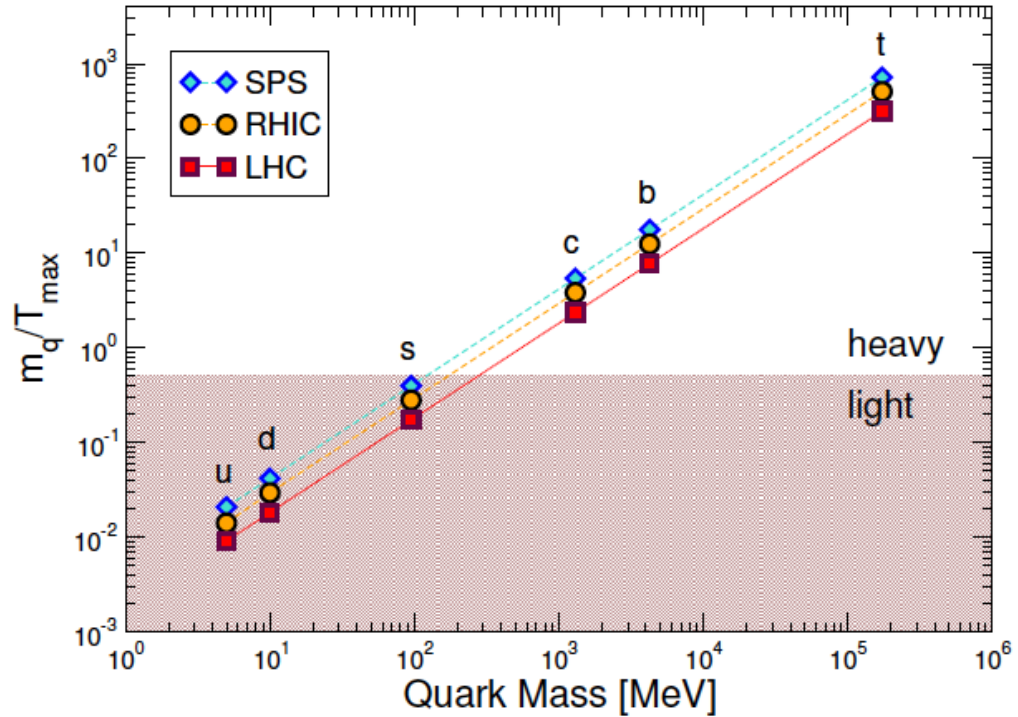
Multicharm production *(new) V. Minissale, et al., Eur. Phys. J. C 84, no.3, 228 (2024)*

Specific of Heavy Quarks

- $m_{c,b} \gg \Lambda_{\text{QCD}}$
produced by pQCD process (out of equilibrium)
 - $m_{c,b} \gg T_0$
negligible thermal production
 - $\tau_0 \ll \tau_{\text{QGP}}$
 - $\tau_{\text{therm.}} \approx \tau_{\text{QGP}} \gg \tau_{g,q}$
- HQs experience the full QGP evolution
- Carry informations about initial stages, more than light quarks

Recent reviews:

- 1) [X.Dong, V. Greco Prog. Part. Nucl. Phys. 104 \(2019\)](#)
- 2) [A.Andronic Eur.Phys.J.C 76 \(2016\) 3, 107](#)
- 3) [F.Prino, R.Rapp, J.Phys.G 43 \(2016\) 9, 093002](#)



Bulk evolution

$$\underbrace{p^\mu \partial_\mu f_{q,g}(x,p)}_{\text{free-streaming}} + \underbrace{M(x) \partial_\mu^x M(x) \partial_p^\mu f_{q,g}(x,p)}_{\text{field interaction } \varepsilon-3p \neq 0} = \underbrace{C_{22}[f_{q,g}]}_{\text{collisions } \eta \neq 0}$$

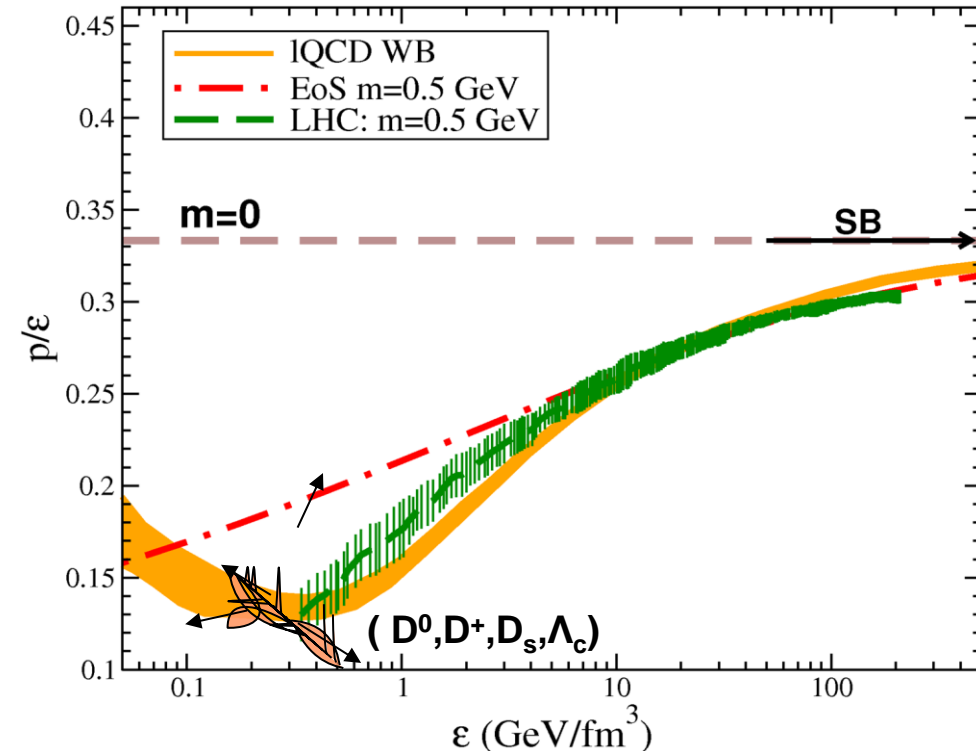
Heavy quark evolution

$$p^\mu \partial_\mu f_Q(x,p) = C[f_q, f_g, f_Q]$$

Describes the evolution of the one body distribution function $f(x,p)$

It is valid to study the evolution of both bulk and Heavy quarks

Possible to include $f(x,p)$ out of equilibrium



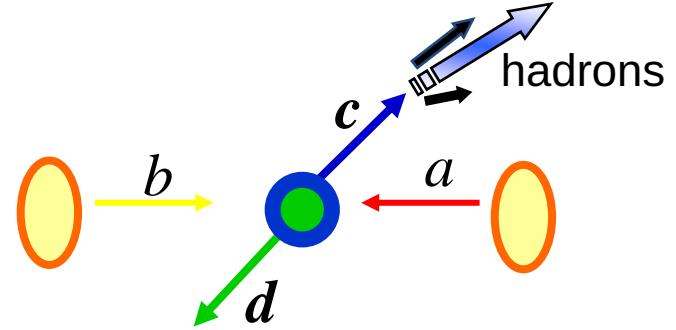
Heavy flavour Hadronization

Fragmentation:

production from hard-scattering processes (PDF+pQCD).
 Fragmentation functions: data parametrization, assumed “universal”

$$\sigma_{pp \rightarrow h} = PDF(x_a, Q^2) PDF(x_b, Q^2) \otimes \sigma_{ab \rightarrow q\bar{q}} \otimes D_{q \rightarrow h}(z, Q^2)$$

Microscopic



Parton shower: String fragmentation(Lund model – PYTHIA)
 +colour reconnection(interaction from different scattering)
 Cluster decay (HERWIG)

Coalescence: recombination of partons in QGP close in phase space

$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta\left(p_T - \sum_i p_{Ti}\right)$$

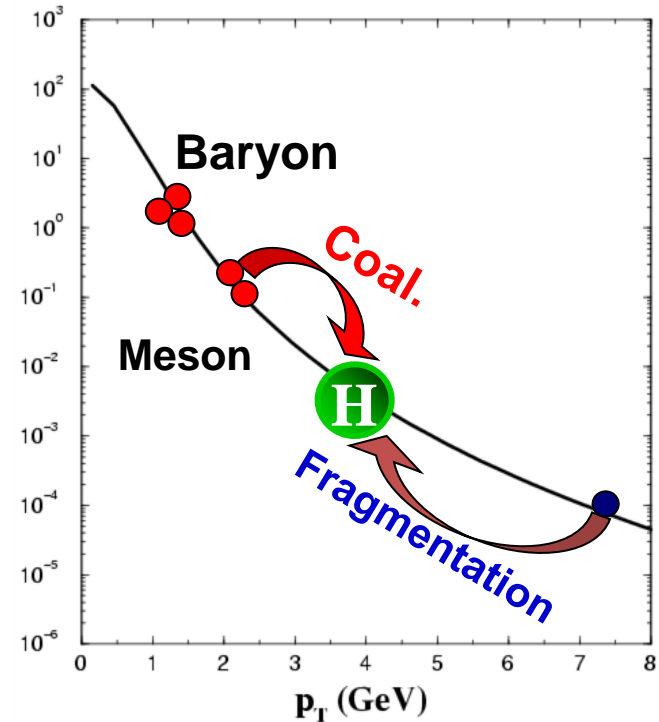
Have described first AA observations in light sector for the enhanced baryon/meson ratio and elliptic flow splitting

Macroscopic

Statistical hadronization:

Equilibrium + hadron-resonance gas + freeze-out temperature.
 Production depends on hadron masses and degeneracy, and on system properties.

*pQCD Charm production + total yield from charm cross section (not Temp.)
 charm hadrons according to thermal weights*



Catania Model: Coalescence + Fragmentation

Statistical factor colour-spin-isospin

Parton Distribution function

Hadron Wigner function

$$C_H = \mathcal{N} f_H$$

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Wigner function – Wave function

Wigner function width fixed by root-mean-square charge radius from quark model

$$\sigma_{ri} = 1/\sqrt{\mu_i \omega}$$

[C.-W. Hwang, EPJ C23, 585 \(2002\)](#)

[C. Albertus et al., NPA 740, 333 \(2004\)](#)

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \phi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \phi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$

$\phi_M(\mathbf{r})$ meson wave function

Assuming gaussian wave function

$$f_H(\dots) = \prod_{i=1}^{N_q-1} 8 \exp\left(\frac{-x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

| Meson | $\langle r^2 \rangle_{ch}$ | σ_{p1} | σ_{p2} |
|-----------------------|----------------------------|---------------|---------------|
| $D^+ = [c\bar{d}]$ | 0.184 | 0.282 | — |
| $D_s^+ = [\bar{s}c]$ | 0.083 | 0.404 | — |
| Baryon | $\langle r^2 \rangle_{ch}$ | σ_{p1} | σ_{p2} |
| $\Lambda_c^+ = [udc]$ | 0.15 | 0.251 | 0.424 |
| $\Xi_c^+ = [usc]$ | 0.2 | 0.242 | 0.406 |
| $\Omega_c^0 = [ssc]$ | -0.12 | 0.337 | 0.53 |

$$\langle r^2 \rangle_{ch} = \frac{3}{2} \left(\frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2 + \frac{m_3^2 (Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2 \right)$$

only one width coming from $\phi_M(\mathbf{r})$,
constraint $\sigma_r \sigma_p = 1$

| Meson | $\langle r^2 \rangle_{ch}$ | σ_{p1} | Baryon | $\langle r^2 \rangle_{ch}$ | σ_{p1} | σ_{p2} |
|--------------------------|----------------------------|---------------|---------------------|----------------------------|---------------|---------------|
| $B^- [b\bar{u}]$ | -0.378 | 0.302 | $\Lambda_b^0 [udb]$ | 0.13 | 0.264 | 0.5 |
| $\bar{B}^0 [b\bar{d}]$ | 0.187 | 0.303 | $\Xi_b^0 [usb]$ | 0.16 | 0.279 | 0.527 |
| $\bar{B}_s^0 [b\bar{s}]$ | 0.119 | 0.374 | $\Xi_b^- [dsb]$ | -0.21 | 0.295 | 0.557 |
| $B_c^- [b\bar{c}]$ | -0.043 | 0.74 | $\Omega_b^- [ssb]$ | -0.18 | 0.318 | 0.592 |

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Normalization \mathcal{N} of $C_H(\dots)$ requiring that $P_{coal}=1$ at $p_T=0$
 The charm that does not coalesce undergo fragmentation

only one width coming from $\phi_M(\mathbf{r})$, constraint $\sigma_r \sigma_p = 1$

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LIGHT

CHARM

Thermal+flow for **u,d,s** ($p_T < 3$ GeV)

In AA collisions charm distribution from the studies of R_{AA} and v_2 of **D-meson** to determine the Space Diffusion coefficient:
parton simulations solving relativistic Boltzmann transport equation

In pp collisions the charm distribution are the FONLL distribution

$$\frac{dN_{q,\bar{q}}}{d^2p_T} \sim \exp\left(-\frac{\gamma_T - p_T \cdot \beta_T \mp \mu_q}{T}\right)$$

$$\beta(r) = \frac{r}{R} \beta_{max}$$

$$V = \pi R^2 \tau \cosh(y_z), R(\tau_f) = R_0(1 + \beta_{max} \tau_f)$$

$$\text{PbPb@5TeV(0-10\%): } \tau_f = 8,5 \frac{fm}{c} \rightarrow V_{|y|<0,5} = 4500 fm^3$$

+quenched minijets for **u,d,s** ($p_T > 3$ GeV)

Coalescence simulation in a fireball with radial flow for light quarks → dimension set by experimental constraints

Catania Model: Coalescence + Fragmentation

Statistical factor colour-spin-isospin

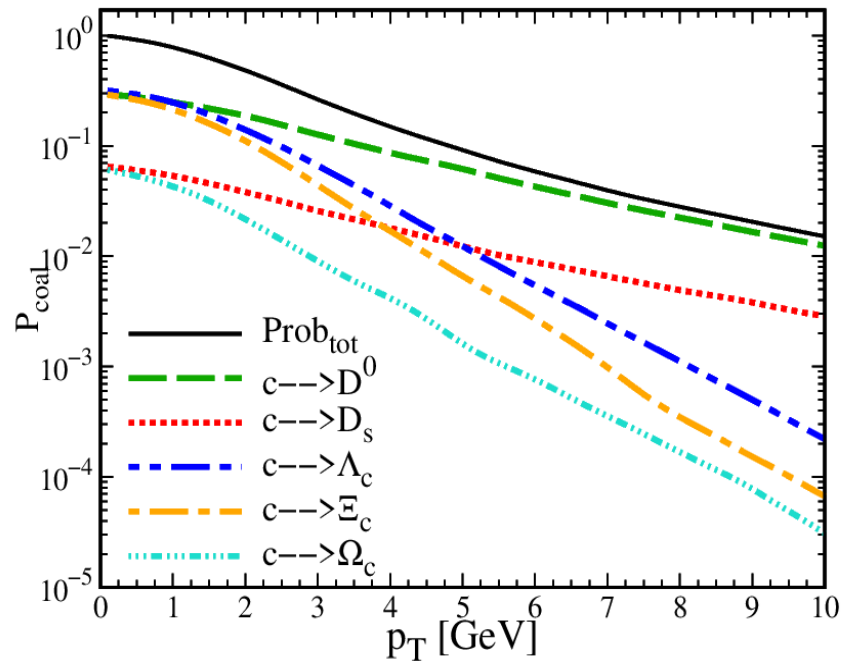
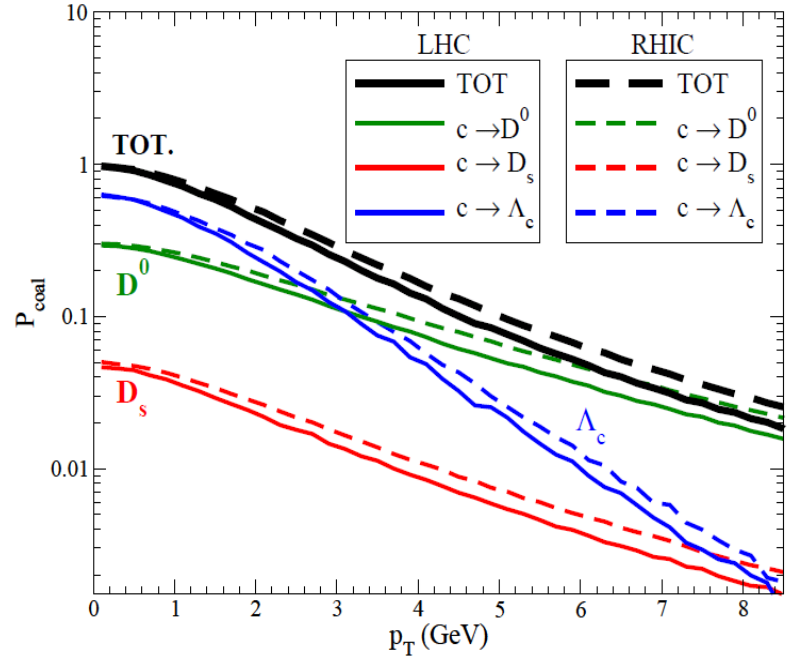
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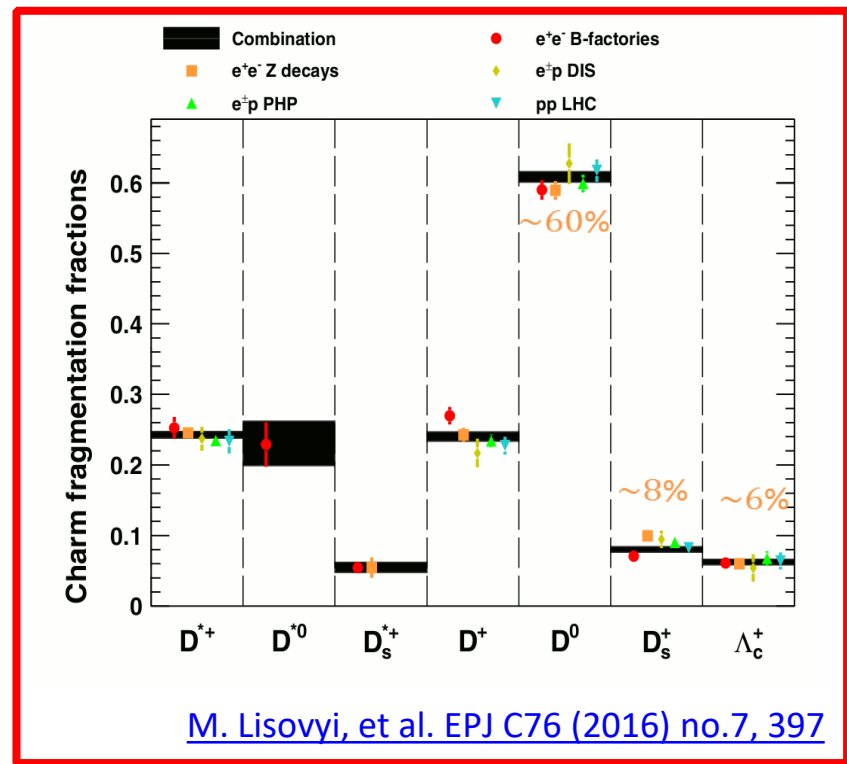
$$\frac{dN_h}{d^2p_h} = \sum_f \int dz \frac{dN_f}{d^2p_f} D_{f \rightarrow h}(z)$$

Parton Distribution Function

The distribution function is evaluated at the Fixed-Order plus Next-to-Leading-Log (FONLL)

[M. Cacciari, P. Nason, R. Vogt, PRL 95 \(2005\) 122001](#)

In AA: bulk+charm evolution with Relativistic Transport Boltzmann Equation



We use the **Peterson fragmentation function**

[C. Peterson, D. Schaller, I. Schmitt, P.M. Zerwas PRD 27 \(1983\) 105](#)

$$D_{f \rightarrow h}(z) \propto \frac{1}{z \left[1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right]^2}$$

Slightly modified to reproduce tail of the Λ_c/D^0

Charm Fragmentation Fraction ($c \rightarrow h$)
 Measurement in $e^\pm p$, $e^+ e^-$ and old pp data

$$\left(\frac{\Lambda_c^+}{D^0} \right)_{e^+ e^-} \simeq 0.1 \quad \left(\frac{D_s^+}{D^0} \right)_{e^+ e^-} \simeq 0.13$$

Heavy flavour: Resonance decay

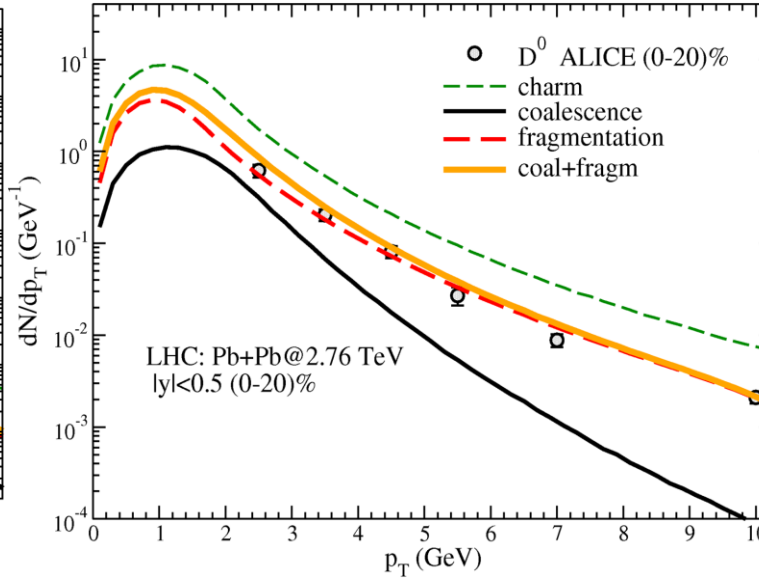
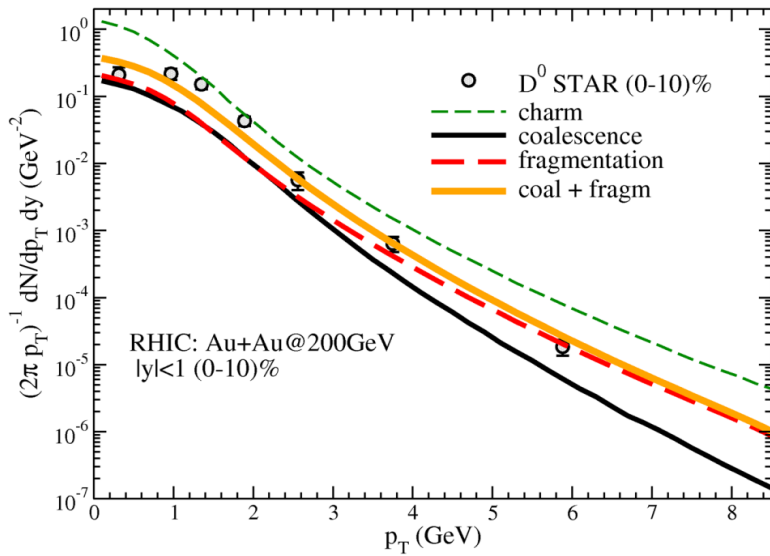
| Meson | Mass(MeV) | I (J) | Decay modes | B.R. |
|---------------------|-----------|-----------------------------|---------------------------|---------|
| $D^+ = \bar{d}c$ | 1869 | $\frac{1}{2} (0)$ | | |
| $D^0 = \bar{u}c$ | 1865 | $\frac{1}{2} (0)$ | | |
| $D_s^+ = \bar{s}c$ | 2011 | $0 (0)$ | | |
| Resonances | | | | |
| D^{*+} | 2010 | $\frac{1}{2} (1)$ | $D^0 \pi^+; D^+ X$ | 68%,32% |
| D^{*0} | 2007 | $\frac{1}{2} (1)$ | $D^0 \pi^0; D^0 \gamma$ | 62%,38% |
| D_s^{*+} | 2112 | $0 (1)$ | $D_s^+ X$ | 100% |
| Baryon | | | | |
| $\Lambda_c^+ = udc$ | 2286 | $0 (\frac{1}{2})$ | | |
| $\Xi_c^+ = usc$ | 2467 | $\frac{1}{2} (\frac{1}{2})$ | | |
| $\Xi_c^0 = dsc$ | 2470 | $\frac{1}{2} (\frac{1}{2})$ | | |
| $\Omega_c^0 = ssc$ | 2695 | $0 (\frac{1}{2})$ | | |
| Resonances | | | | |
| Λ_c^+ | 2595 | $0 (\frac{1}{2})$ | $\Lambda_c^+ \pi^+ \pi^-$ | 100% |
| Λ_c^+ | 2625 | $0 (\frac{3}{2})$ | $\Lambda_c^+ \pi^+ \pi^-$ | 100% |
| Σ_c^+ | 2455 | $1 (\frac{1}{2})$ | $\Lambda_c^+ \pi$ | 100% |
| Σ_c^+ | 2520 | $1 (\frac{3}{2})$ | $\Lambda_c^+ \pi$ | 100% |
| $\Xi_c^{'+,0}$ | 2578 | $\frac{1}{2} (\frac{1}{2})$ | $\Xi_c^{+,0} \gamma$ | 100% |
| Ξ_c^+ | 2645 | $\frac{1}{2} (\frac{3}{2})$ | $\Xi_c^+ \pi^-$, | 100% |
| Ξ_c^+ | 2790 | $\frac{1}{2} (\frac{1}{2})$ | $\Xi_c'^+ \pi$, | 100% |
| Ξ_c^+ | 2815 | $\frac{1}{2} (\frac{3}{2})$ | $\Xi_c'^+ \pi$, | 100% |
| Ω_c^0 | 2770 | $0 (\frac{3}{2})$ | $\Omega_c^0 \gamma$, | 100% |

In our calculations we take into account hadronic channels including the ground states + first excited states

Statistical factor suppression for resonances

$$\frac{|(2J + 1)(2I + 1)|_{H^*}}{|(2J + 1)(2I + 1)|_H} \left(\frac{m_{H^*}}{m_H}\right)^{3/2} e^{-(m_{H^*} - m_H)/T}$$

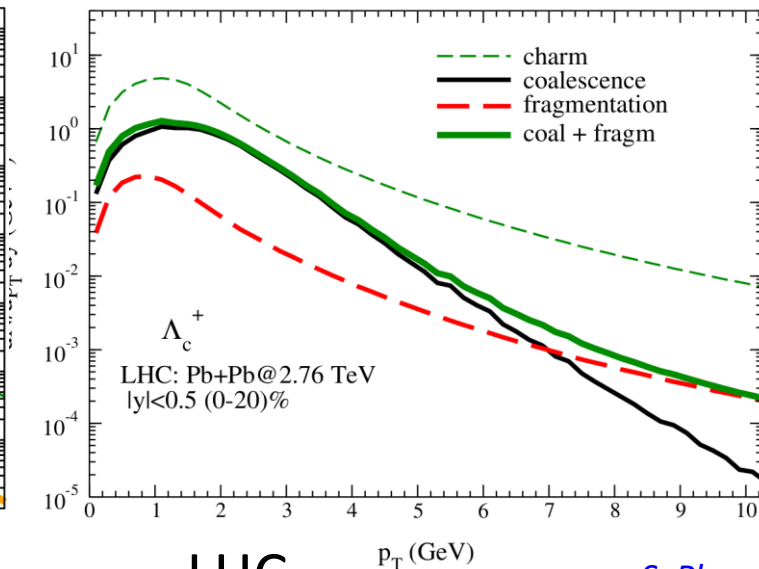
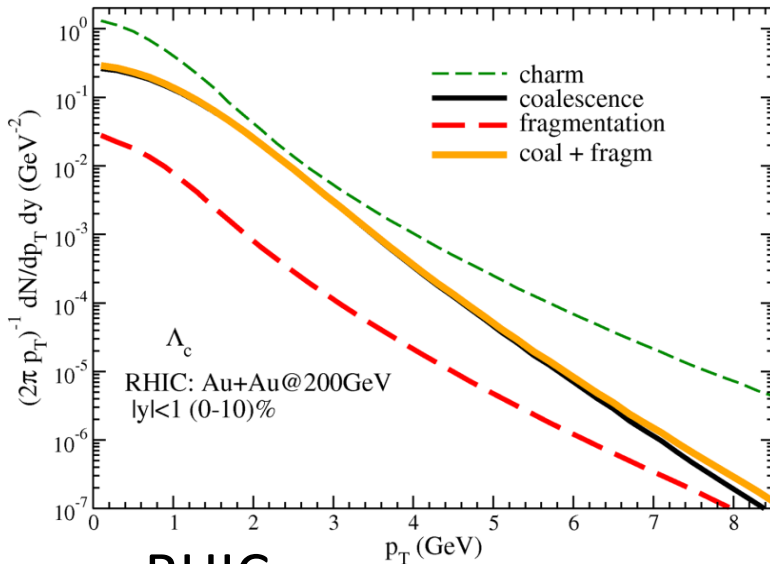
Data from: [STAR Coll. PRL 113, 142301 \(2014\)](#), [ALICE Coll. JHEP 09 \(2012\) 112](#)



D^0

Coalescence lower at LHC than at RHIC

main contribution:
Fragmentation

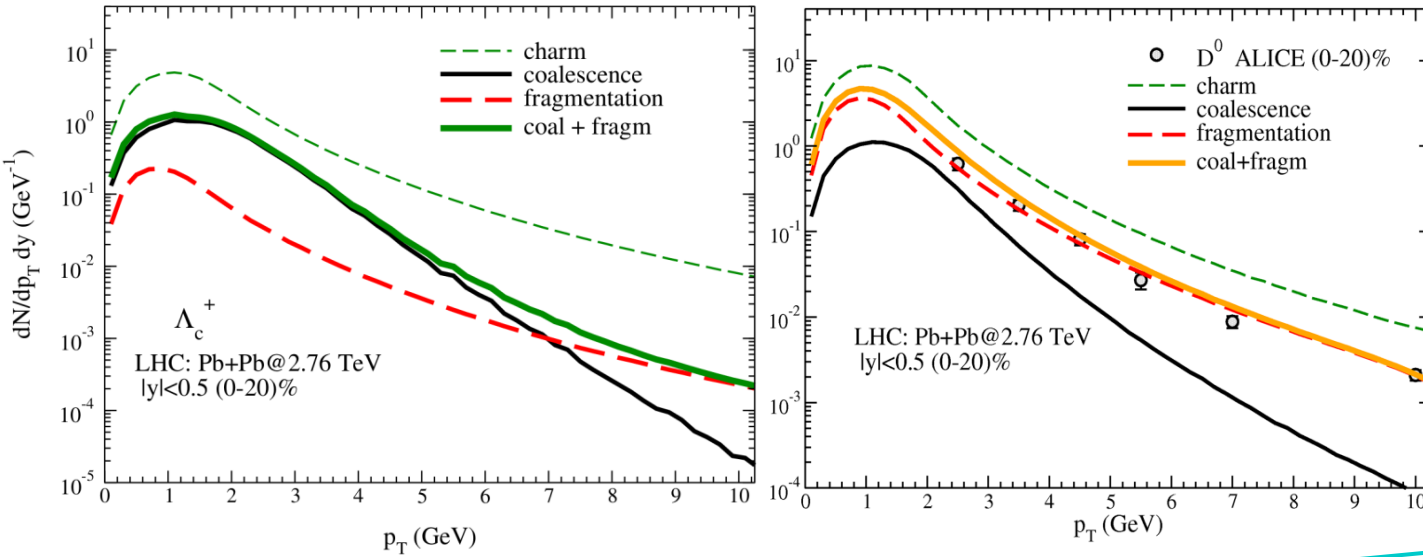


Λ_c

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Data from [ALICE Coll. JHEP 09 \(2012\) 112](#)



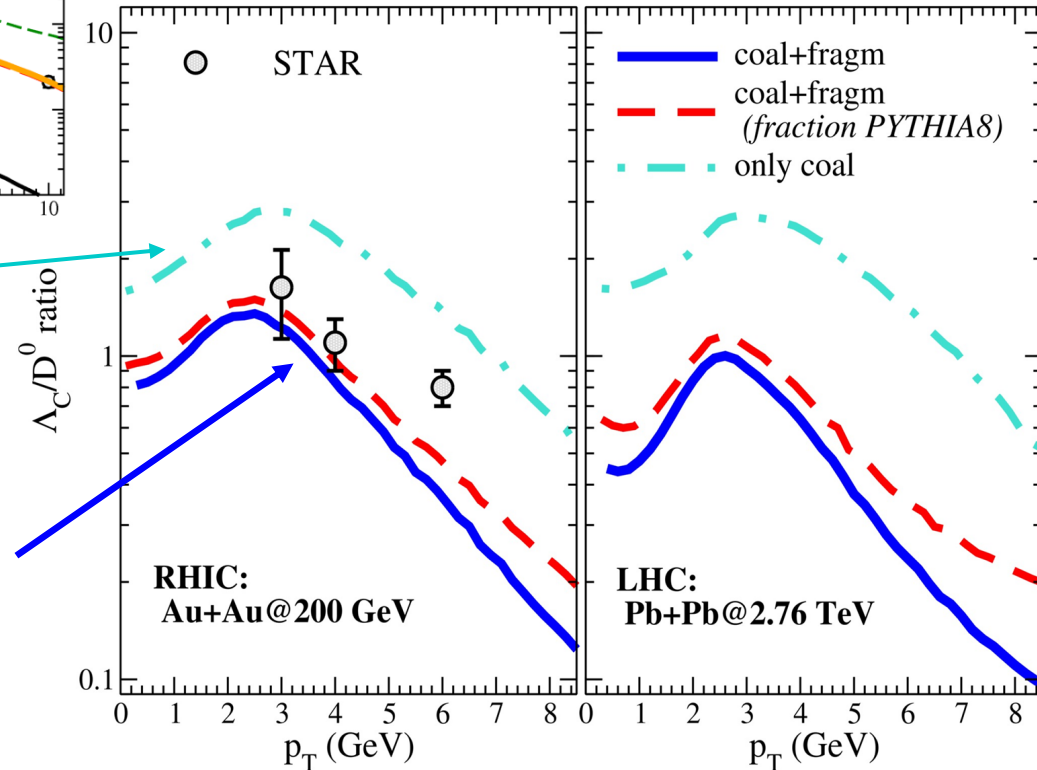
Coalescence lower at LHC than at RHIC

Only Coalescence ratio is similar at both energies.

Fragmentation ~ 0.1 at both energies.

the **combined ratio is different** because the coalescence over fragmentation ratio at LHC is smaller than at RHIC

Therefore at LHC the larger contribution in particle production from fragmentation leads to a final ratio that is smaller than at RHIC.



[STAR Coll., Phys.Rev.Lett. 124 \(2020\) 17, 172301](#)

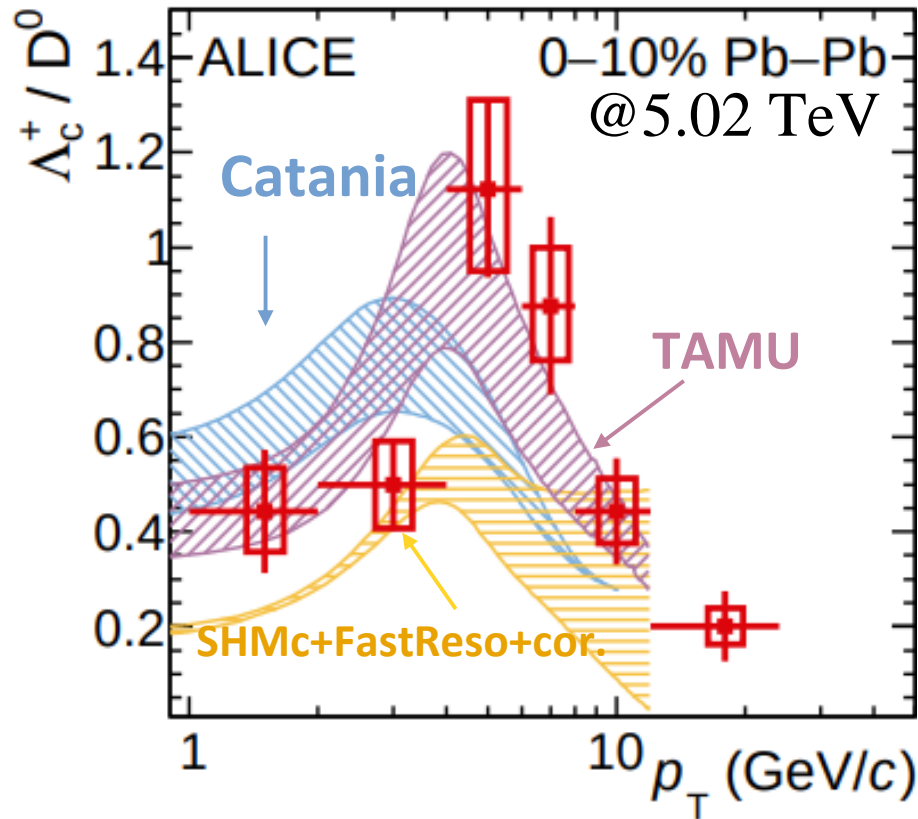
[S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, \(2018\) 348](#)

Results for 0-10% in PbPb @5.02TeV:

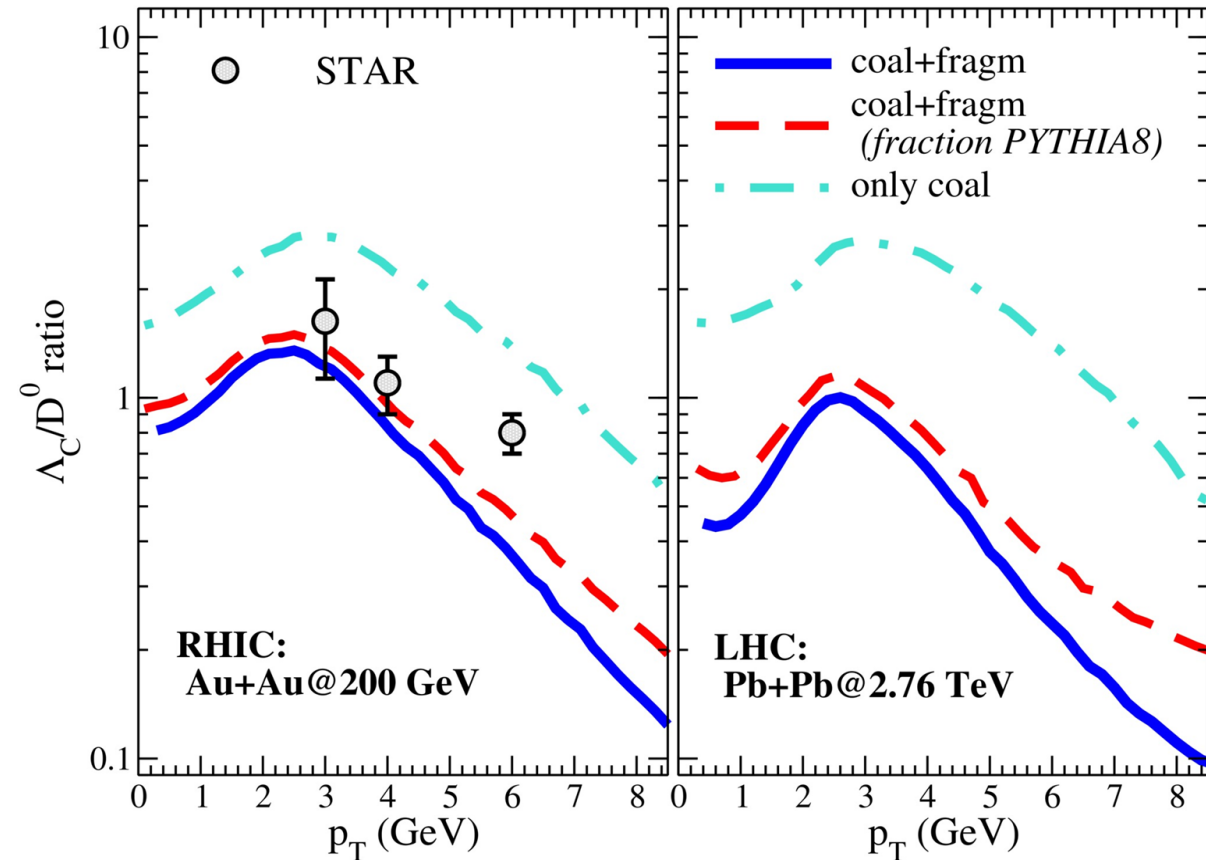
Consistent with the trend shown at RHIC and LHC @2.76TeV

Available data at low $p_T \rightarrow$ differences recombination vs SHM

[STAR Coll., Phys.Rev.Lett. 124 \(2020\) 17, 172301](#)



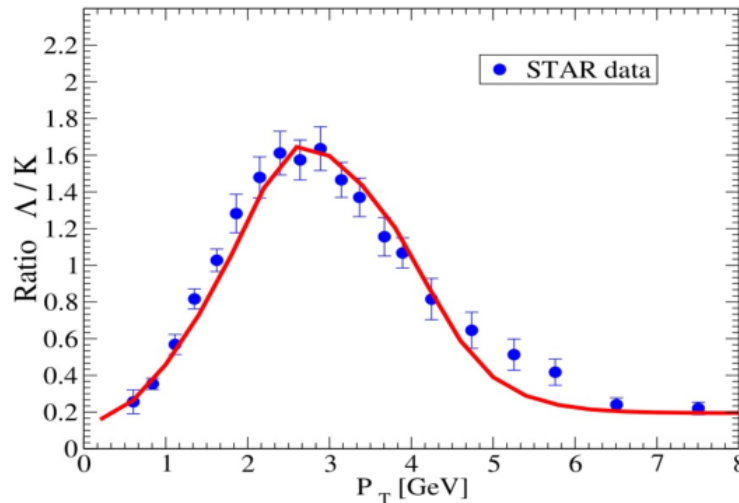
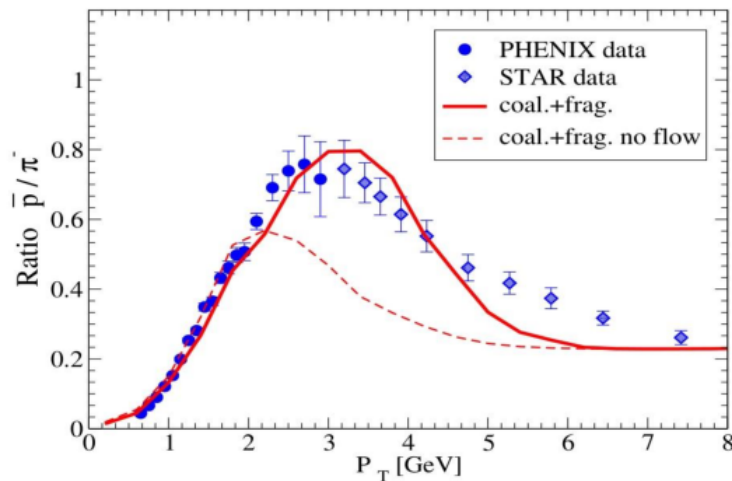
[ALICE Coll. Phys. Lett. B 839 \(2023\) 137796](#)



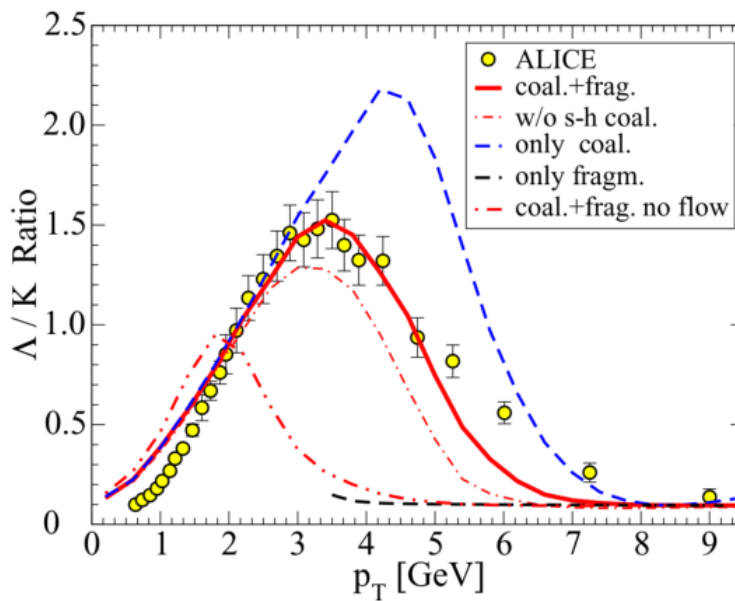
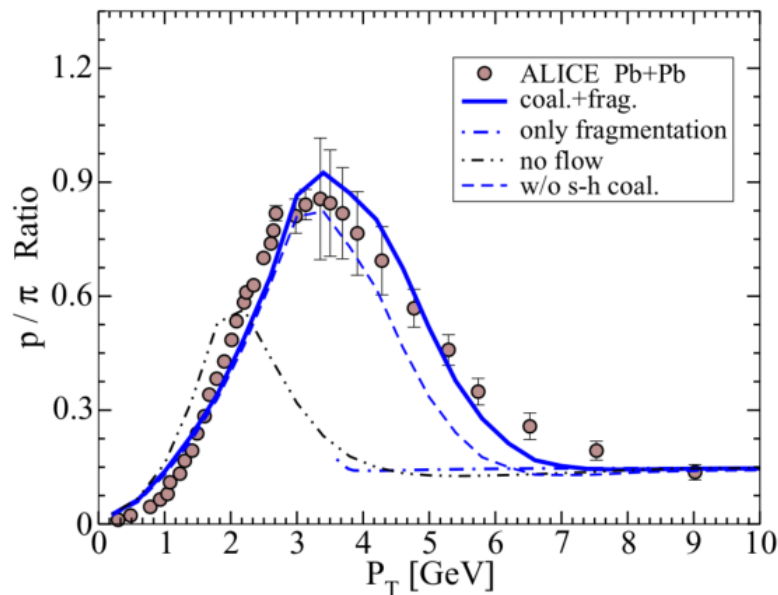
[S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, \(2018\) 348](#)

Baryon to meson ratio at RHIC & LHC

Minissale, Scardina, Greco, Phys.Rev. C 92 (2015) 5,054904



RHIC



LHC

- coalescence naturally predict a baryon/meson enhancement in the region $p_T \approx 2-4\text{GeV}$ with respect to pp collisions
- Lack of baryon yield in the region $p_T \approx 5-7\text{GeV}$

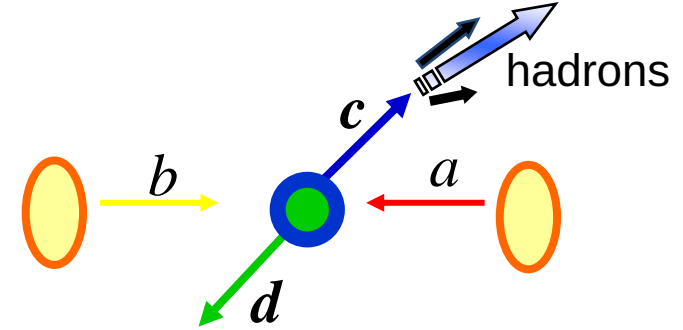
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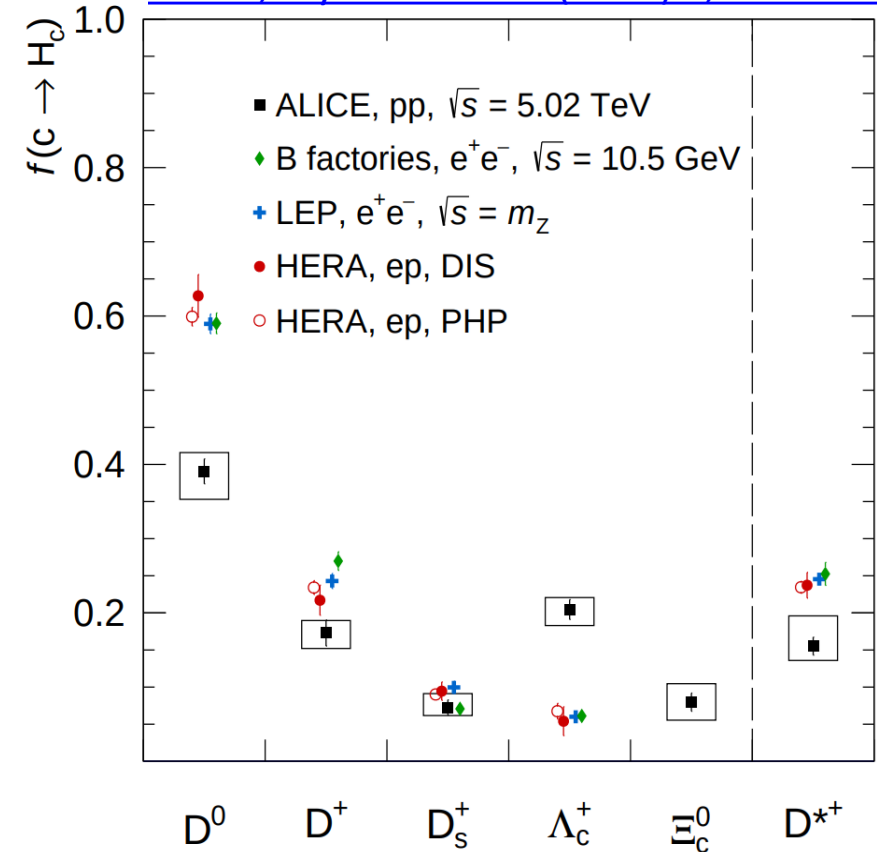


Things get more complicated after experimental evidence in pp@5TeV:

Fragmentation fractions ($c \rightarrow h$) depends on collision system...and QGP presence?

No more Universality?

[ALICE, Phys.Rev.D 105 \(2022\) 1, L011103](#)

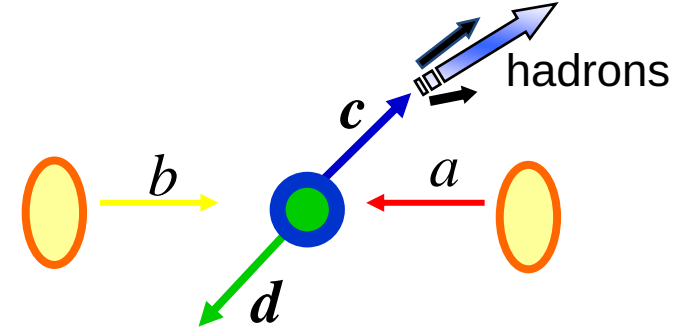


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[ALICE, PRL 127 202301 \(2021\)](#)
[ALICE, PRC 104 054905 \(2021\)](#)

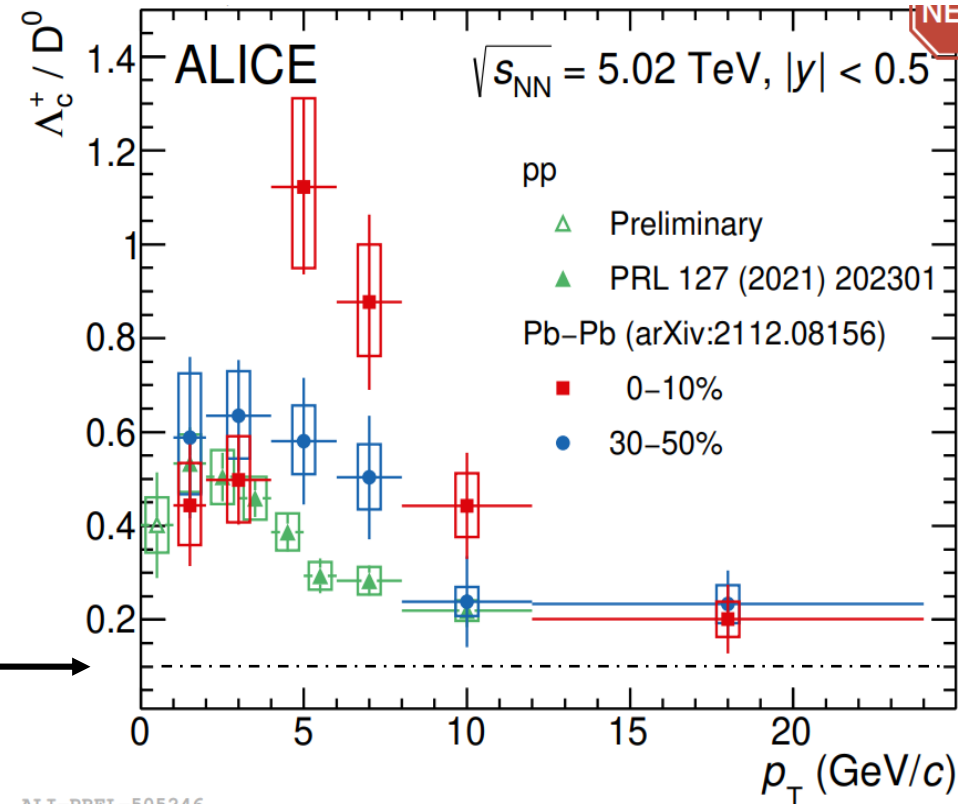
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No more Universality?

Baryon/meson ratio is underestimated, and no p_T dependence

$$\left(\frac{\Lambda_c^+}{D^0} \right)_{e^+e^-} \approx 0.1 \longrightarrow$$



Small systems: Coalescence in pp?

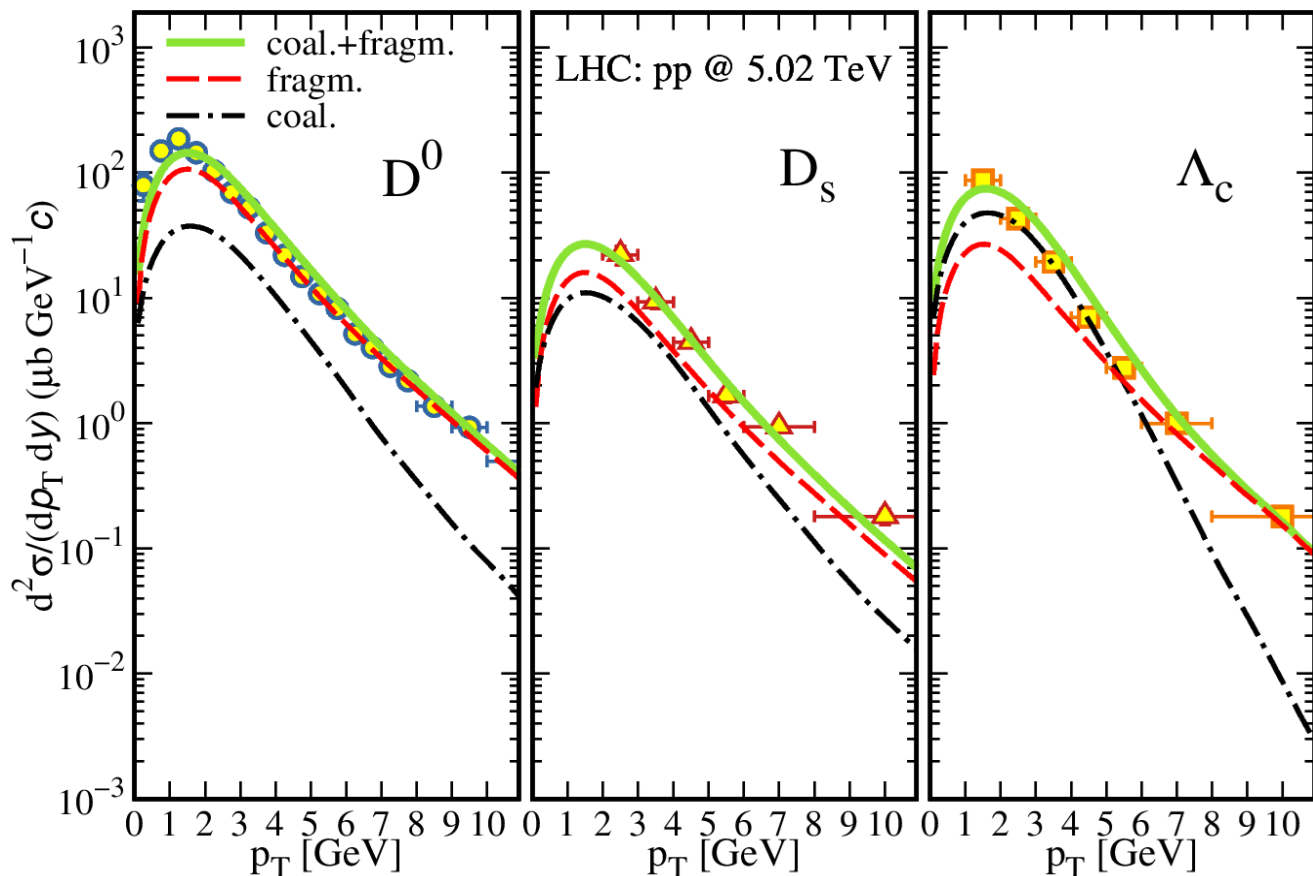
Common consensus of possible presence of QGP in smaller system.

If we assume in p+p @ 5 TeV a medium similar to the one simulated in hydro:

What if:

- Assuming QGP formation also in pp?
- What coalescence+fragmentation predicts in this case?

Data from:
[S. Acharya et al. \(ALICE\), Eur. Phys. J. C 79, 388 \(2019\)](#)
[ALICE Coll., Phys.Rev.Lett. 127 \(2021\) 20, 202301 - Phys.Rev.C 104 \(2021\) 5, 054905](#)



p+p @ 5 TeV

- $\tau_{pp} = 2 \text{ fm}/c$
- $\beta_0 = 0.4$
- $R = 2.5 \text{ fm}$
- $V \sim 30 \text{ fm}^3$

LIGHT

• Thermal Distribution ($p_T < 2 \text{ GeV}$)

$$\frac{dN_q}{d^2r_T d^2p_T} = \frac{g_q \tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T \cdot \beta_T)}{T}\right)$$

• Minijet Distribution ($p_T > 2 \text{ GeV}$)
 NO QUENCHING

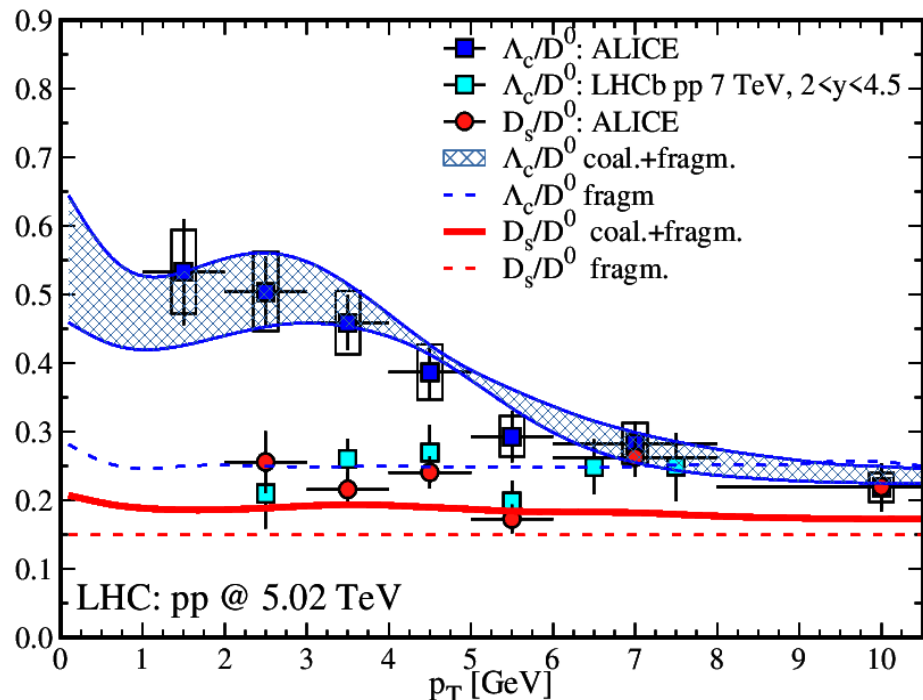
CHARM

FONLL Distribution

wave function widths σ_p of baryon and mesons kept the same from AA to pp

Small systems: Coalescence in pp?

V. Minissale, S. Plumari, V. Greco, *Physics Letters B* 821 (2021) 136622



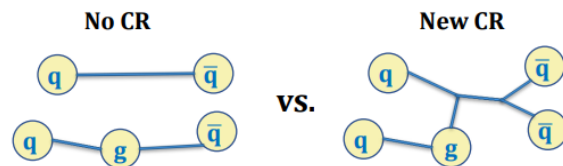
Error band correspond to $\langle r^2 \rangle$ uncertainty in quark model

Other models:

[He-Rapp, Phys.Lett.B 795 \(2019\) 117-121](#): Increase ≈ 2 to Λ_c production: SHM with resonance not present in PDG

PYTHIA8 + color reconnection

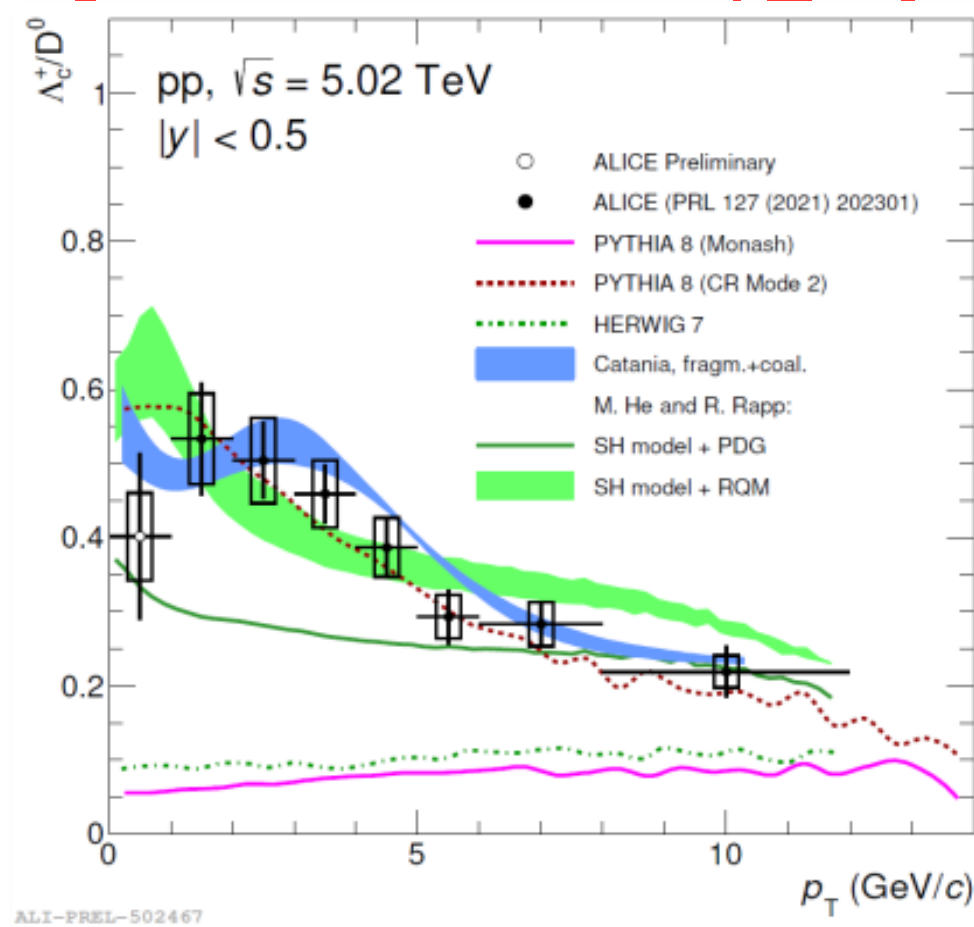
CR with SU(3) weights and string length minimization



Reduction of rise-and-fall behaviour in Λ_c / D^0 ratio:

- Confronting with AA: Coal. contribution smaller w.r.t. Fragm.
- FONLL distribution flatter w/o evolution trough QGP
- Volume size effect

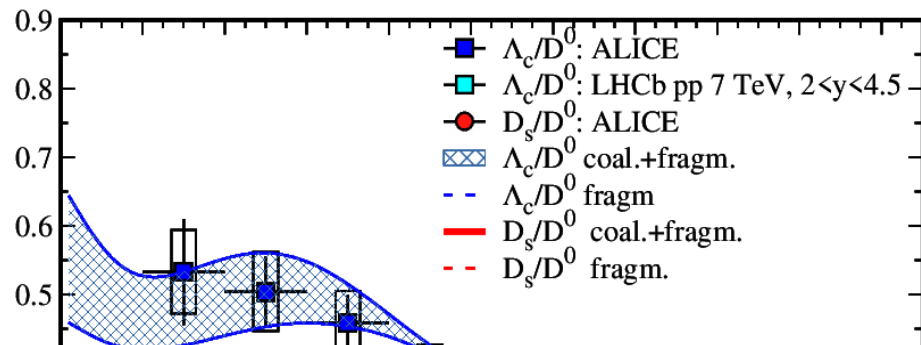
The increase of Λ_c production in pp have effect on R_{AA} of Λ_c



[ALICE, Phys.Rev.Lett. 127 \(2021\) 20, 202301](#)

Small systems: Coalescence in pp?

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622



Reduction of rise-and-fall behaviour in Λ_c / D^0 ratio:

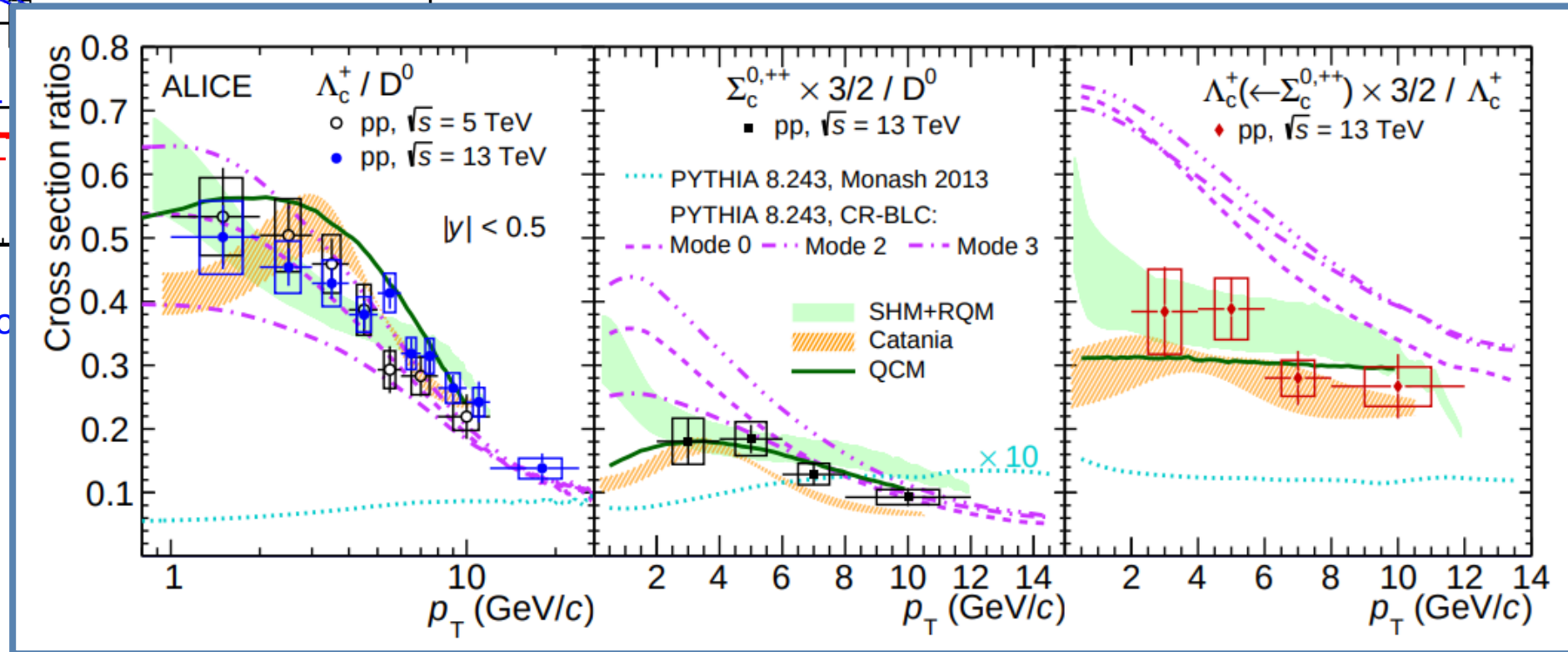
-Confronting with AA: Coal. contribution smaller w.r.t. Fragm.

-FONLL distribution flatter w/o evolution trough QGP

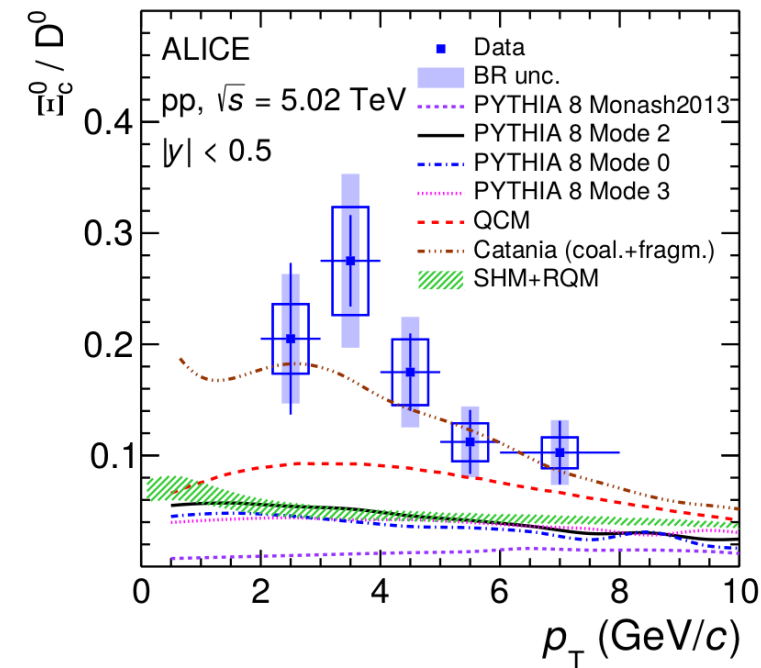
-Volume size effect

The increase of Λ_c production in pp have effect on R_{AA} of Λ_c

Error band correspond to



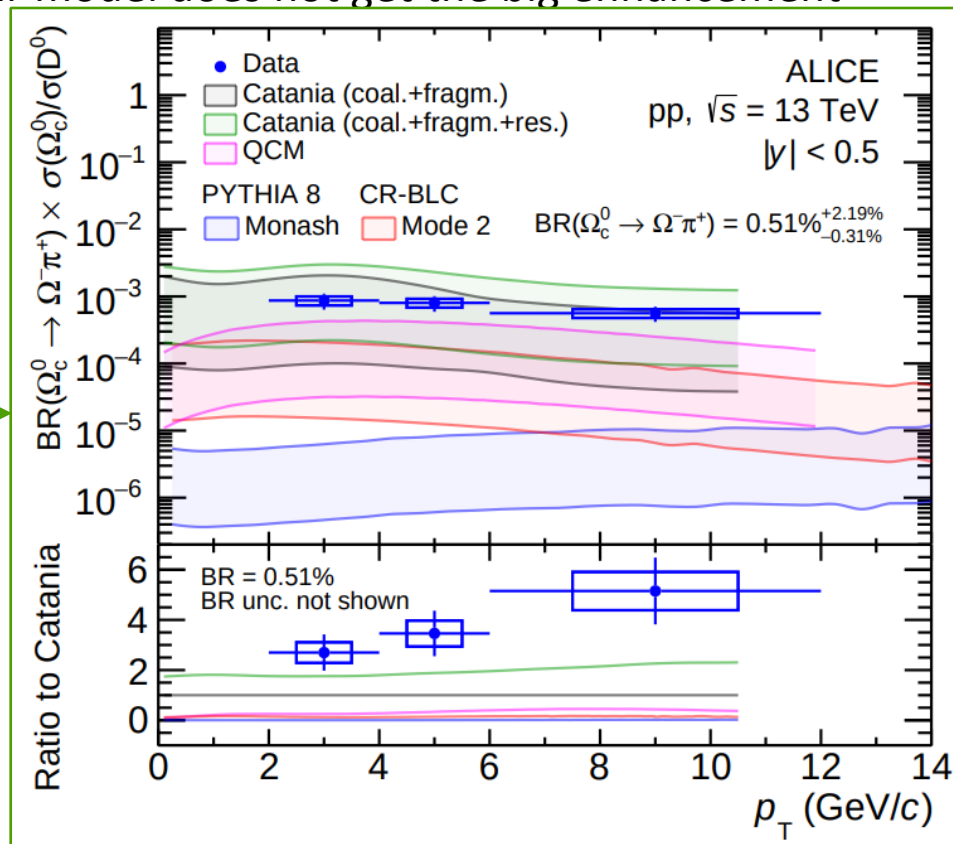
Small systems: Coalescence in pp?



New measurements of heavy hadrons at ALICE:

- Ξ_c/D^0 ratio, same order of Λ_c/D^0 : coalescence gives enhancement
- very large Ω_c/D^0 ratio, our model does not get the big enhancement

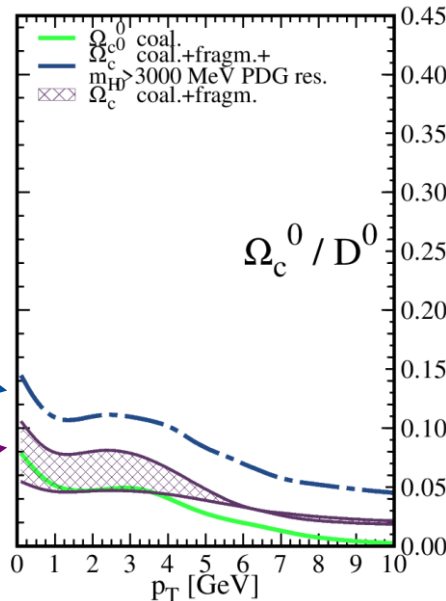
Uncertainties bands coming from the Branching Ratio error



Assuming additional PDG resonances with $J=3/2$ and decay to Ω_c additional to $\Omega_c^0(2770)$

$\Omega_c^0(3000), \Omega_c^0(3005), \Omega_c^0(3065), \Omega_c^0(3090), \Omega_c^0(3120)$
supply an idea of how these states may affect the ratio

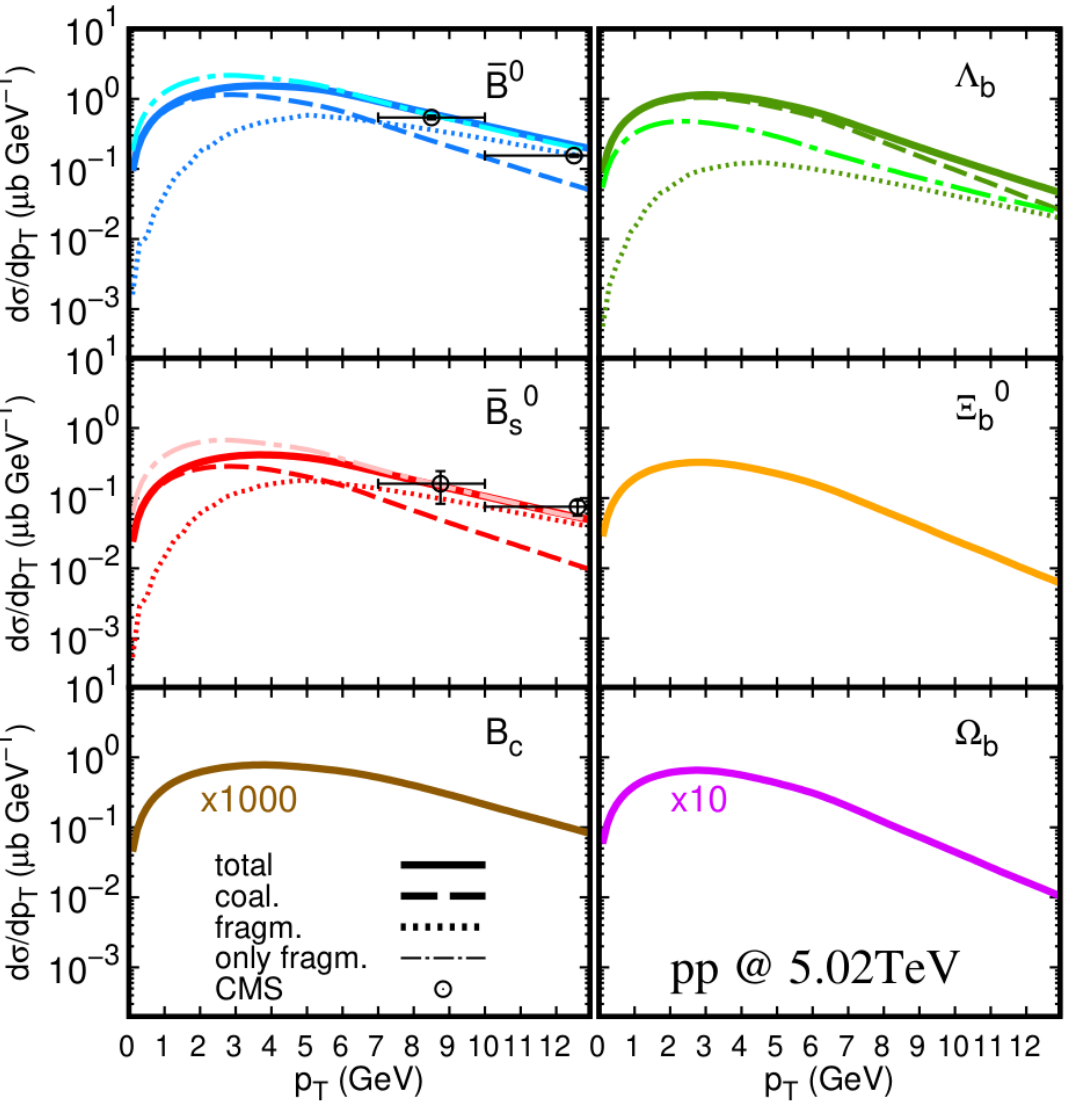
Error band correspond to $\langle r^2 \rangle$ uncertainty in quark model



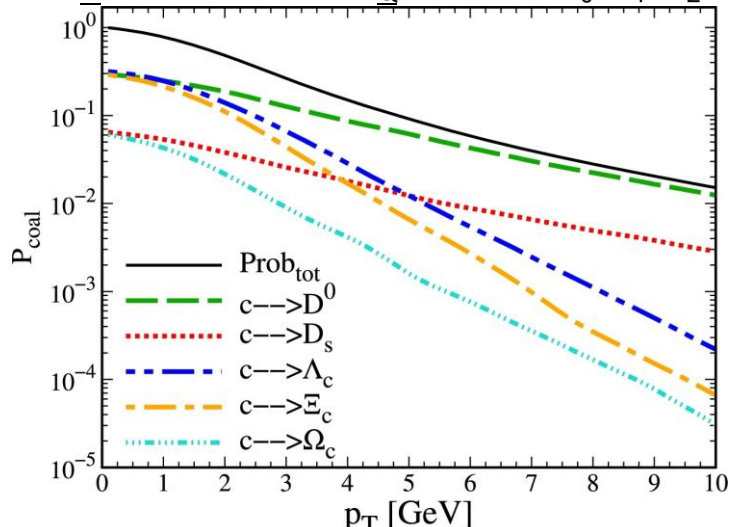
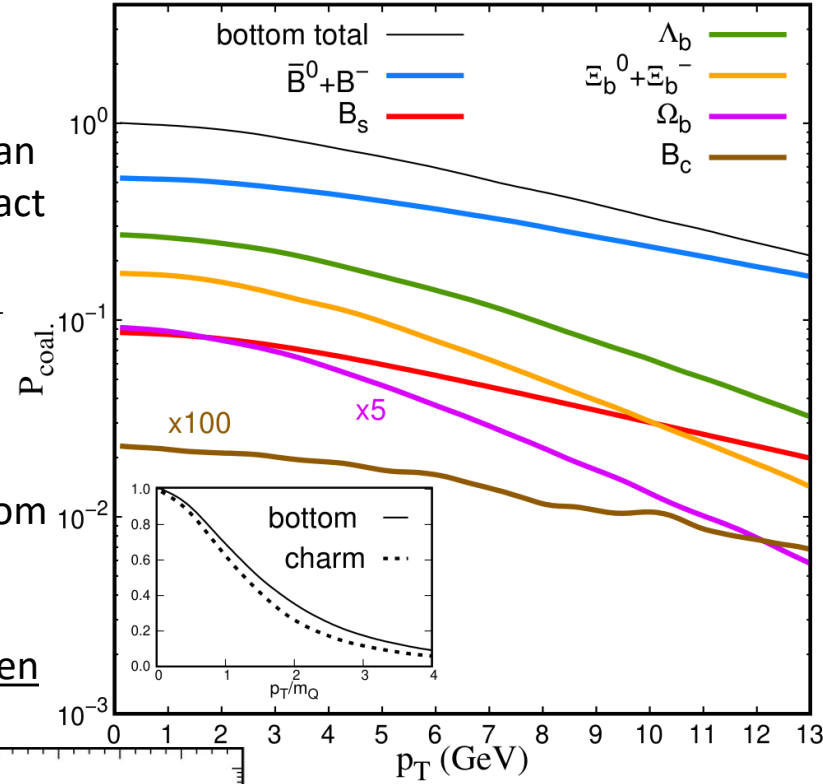
[ALICE Coll. JHEP 10 \(2021\) 159](#)
[ALICE Coll. arXiv:2205.13993](#)
[V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 \(2021\) 136622](#)

Small systems: Coalescence in pp for bottom hadron?

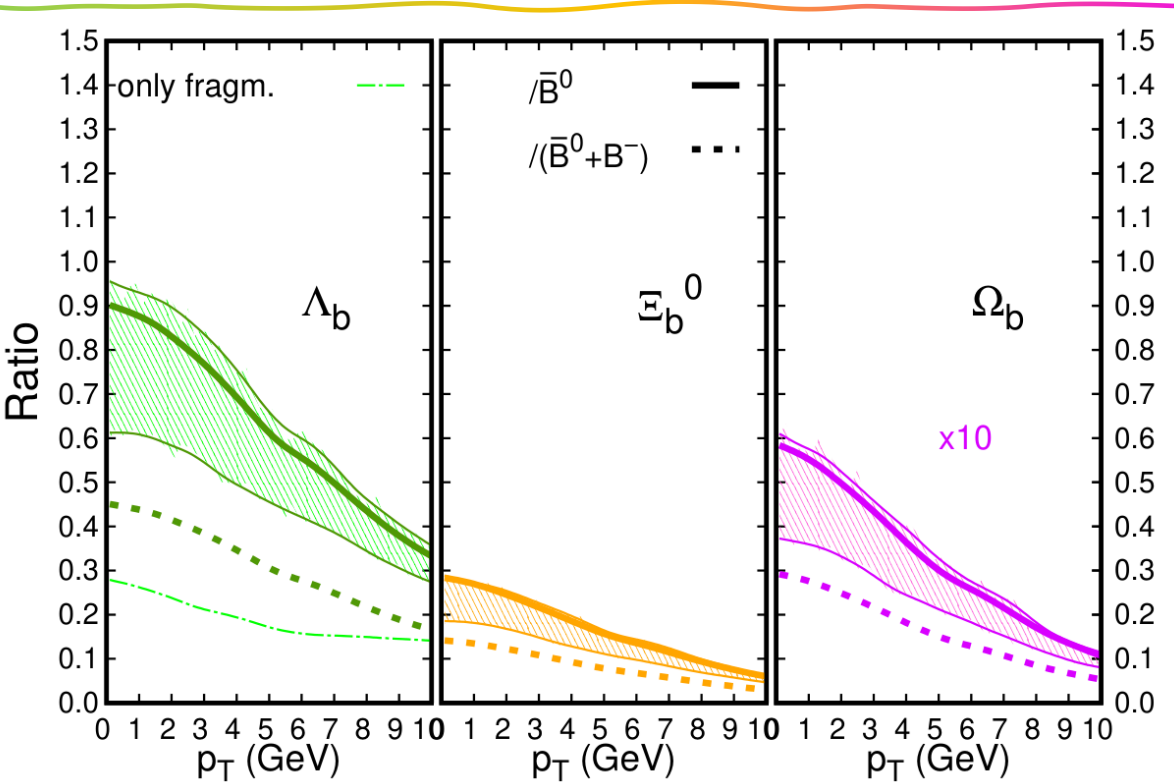
Data from: [A. M. Sirunyan et al. \(CMS\), PRL 119, 152301 \(2017\).](#)



- Bottom P_{coal} is flatter than charm \rightarrow Coal. big impact on bottom hadron production in a wider p_T region
- B meson at $p_T < 5$ GeV mainly from Coal
- Λ_b production mainly from Coal. for $p_T < 10$ GeV
- Similar probabilities when p_T is scaled with m_Q



Small systems: Coalescence in pp for bottom hadron?



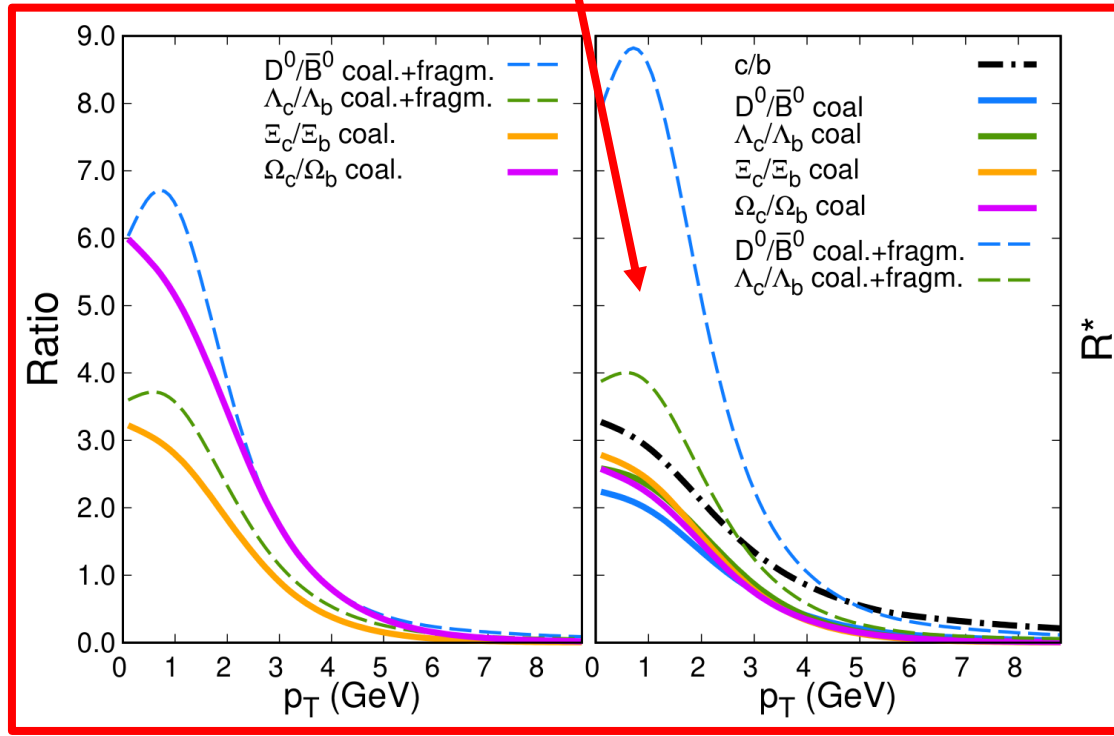
Error band correspond to $\langle r^2 \rangle$ uncertainty in quark model
 Coal gives enhancement of Baryon/meson ratio

D/B, Λ_c/Λ_b , Ξ_c/Ξ_b , Ω_c/Ω_b provide information about hadronization and $f(c)/f(b)$
 Scaling when only coal. is assumed, considering only the g.s.

$$g_{H^*} = \frac{|(2J+1)(2I+1)|_{H^*}}{|(2J+1)(2I+1)|_H} \left(\frac{m_{H^*}}{m_H}\right)^{3/2} e^{-(m_{H^*}-m_H)/T}$$

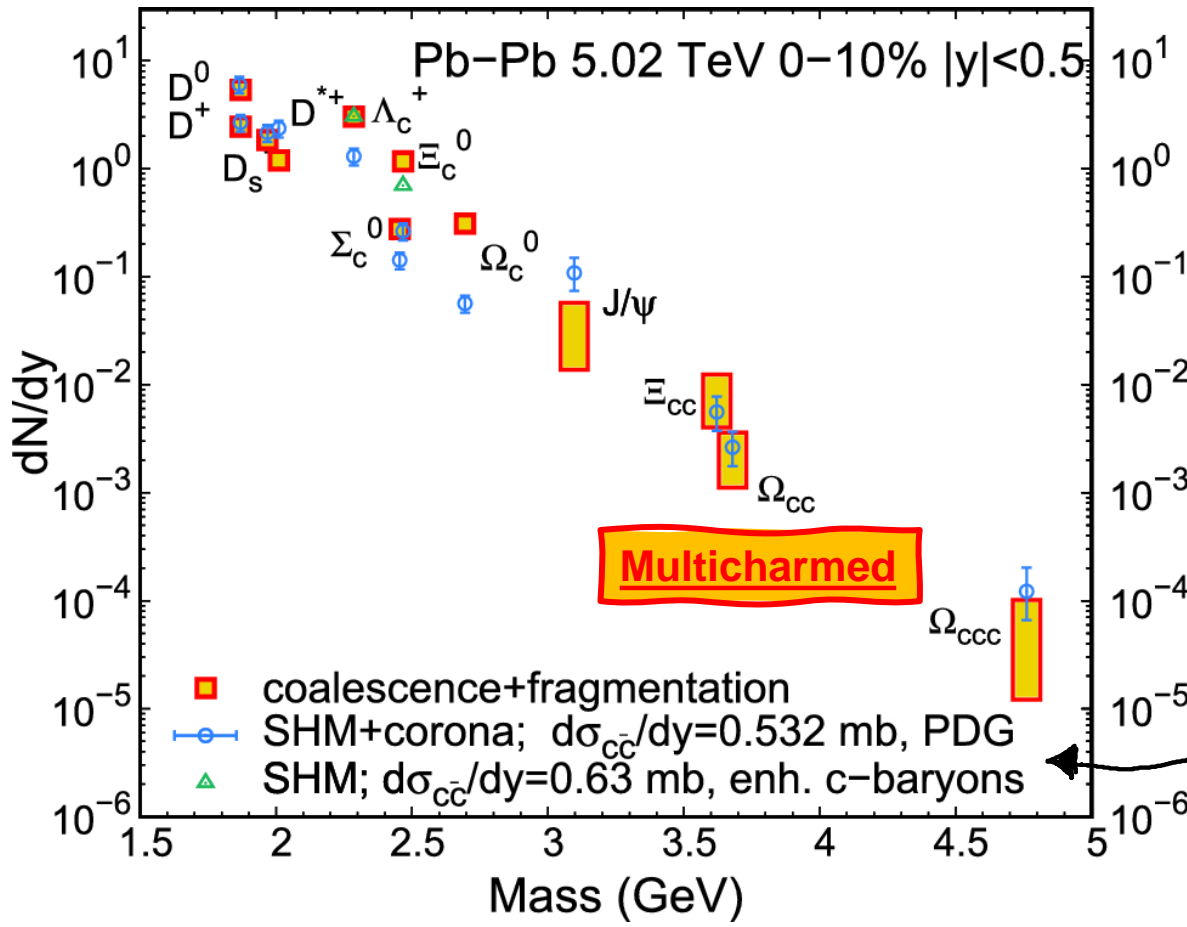
$$\mathcal{F}_H^{c,b} = 1 + \sum_{Res} g_{H^*}^{c,b}$$

$$R_{H_{c,b}}^* = \left(\frac{\mathcal{F}_H^c}{\mathcal{F}_H^b}\right)^{-1} \frac{d\sigma^{H_c}/dp_T}{d\sigma^{H_b}/dp_T}$$

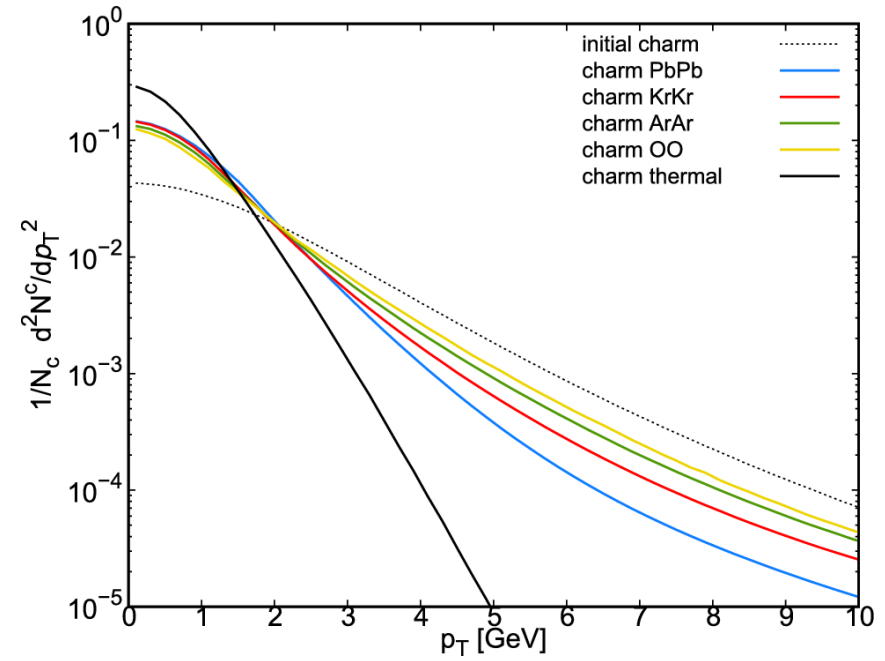


Yields in PbPb from coalescence vs SHM

V. Minissale, S. Plumari, Y. Sun and V. Greco, *Eur. Phys. J. C* 84, no.3, 228 (2024)



→ upper limit: charm thermal distribution
 → lower limit: PbPb distribution with widths rescaled as standard Harm. Oscill. (ω from Ω_c^0)



We employ same volume in SHM
 A. Andronic et al., [JHEP 07 \(2021\) 035](#)

| | OO | ArAr | KrKr | PbPb |
|----------------------|------|------|------|------|
| $R_0 (fm)$ | 2.76 | 3.75 | 4.9 | 6.5 |
| $R_{max} (fm)$ | 5.2 | 7.65 | 10.1 | 14.1 |
| $\tau (fm)$ | 4 | 5 | 6.2 | 8 |
| β_{max} | 0.55 | 0.6 | 0.64 | 0.7 |
| $V_{ y <0.5} (fm^3)$ | 345 | 920 | 2000 | 5000 |

$\Sigma_c^0, \Xi_c^0, \Omega_c^0$, widths from quark model
 Ξ_{cc}, Ω_{cc} widths obtained rescaling with harm. oscillator

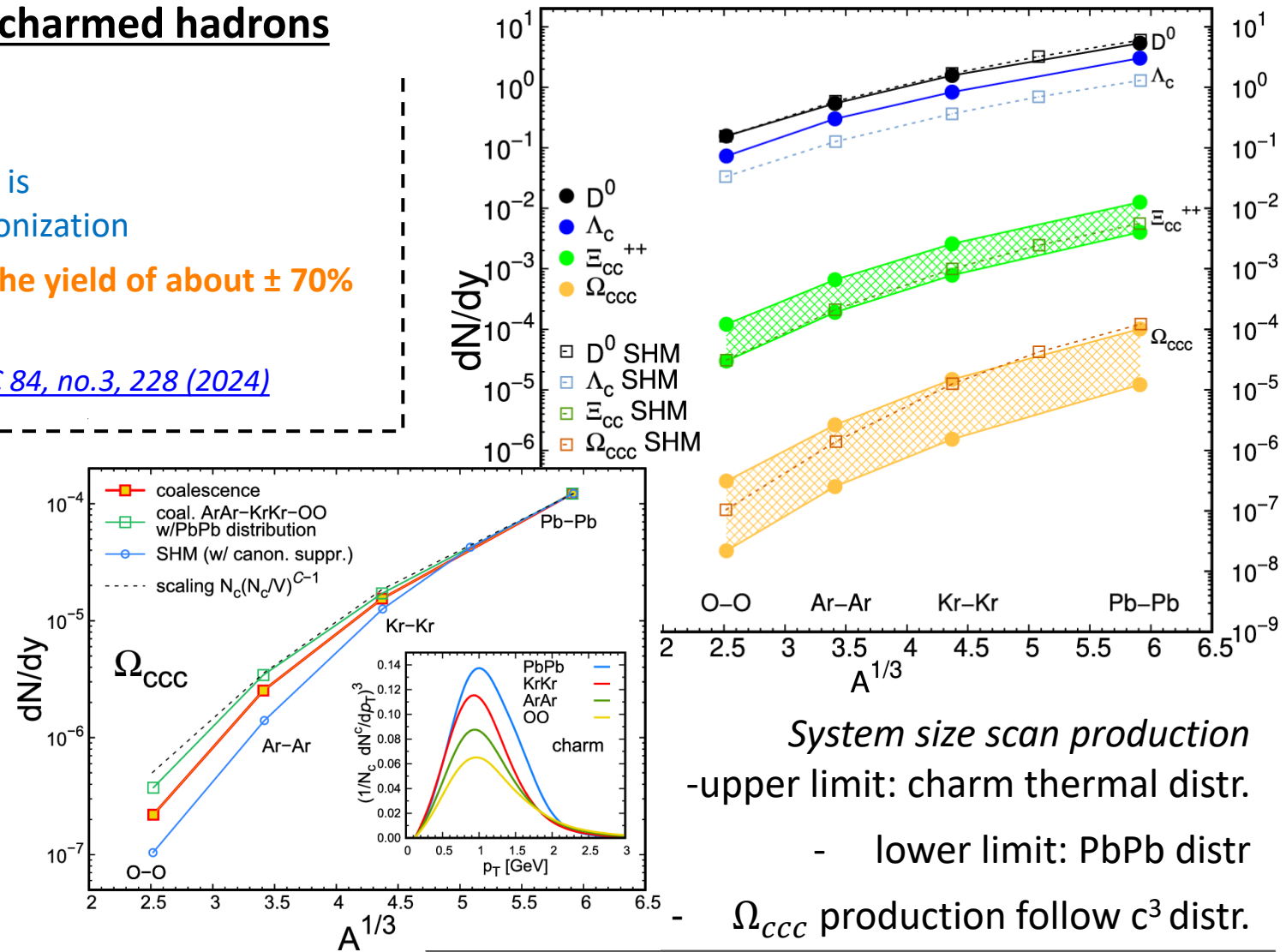
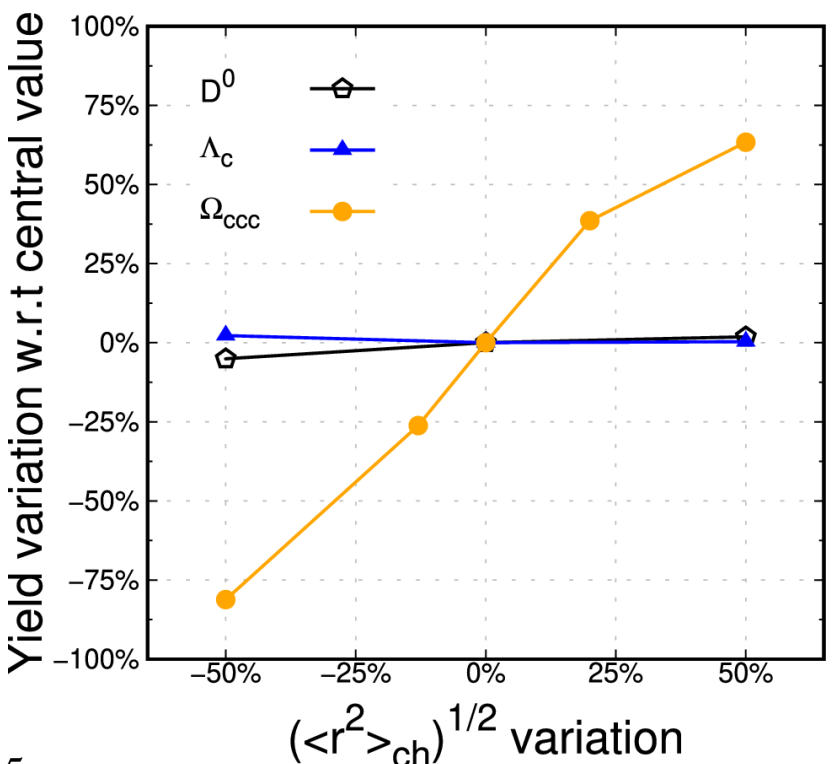
Widths and system size effects on multicharmed hadrons

Widths effects on production:

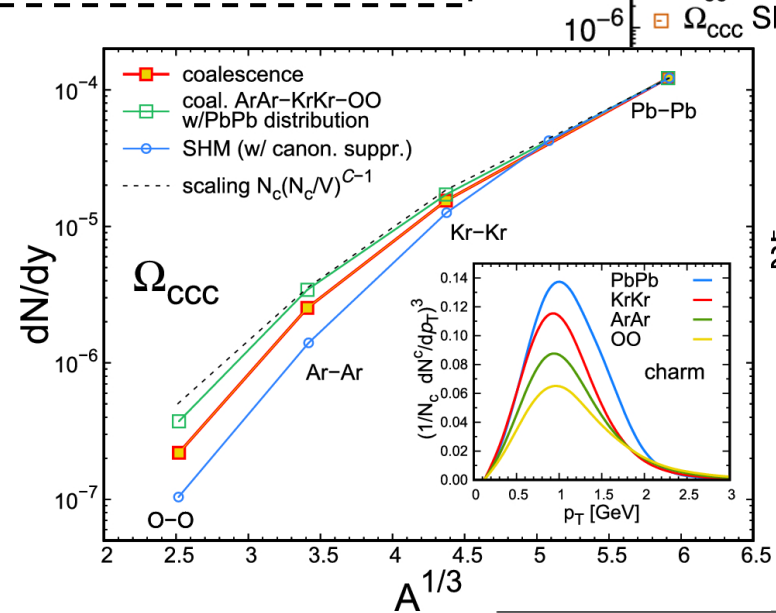
D^0 and Λ_c determine the yield, the radius variation is compensated by the constraint on the charm hadronization

$A \pm 50\%$ in the radius of Ω_{ccc} induces a change in the yield of about $\pm 70\%$

[V. Minissale, S. Plumari, Y. Sun and V. Greco, Eur. Phys. J. C 84, no.3, 228 \(2024\)](#)



System size scan production
 - upper limit: charm thermal distr.
 - lower limit: PbPb distr
 - Ω_{ccc} production follow c^3 distr.

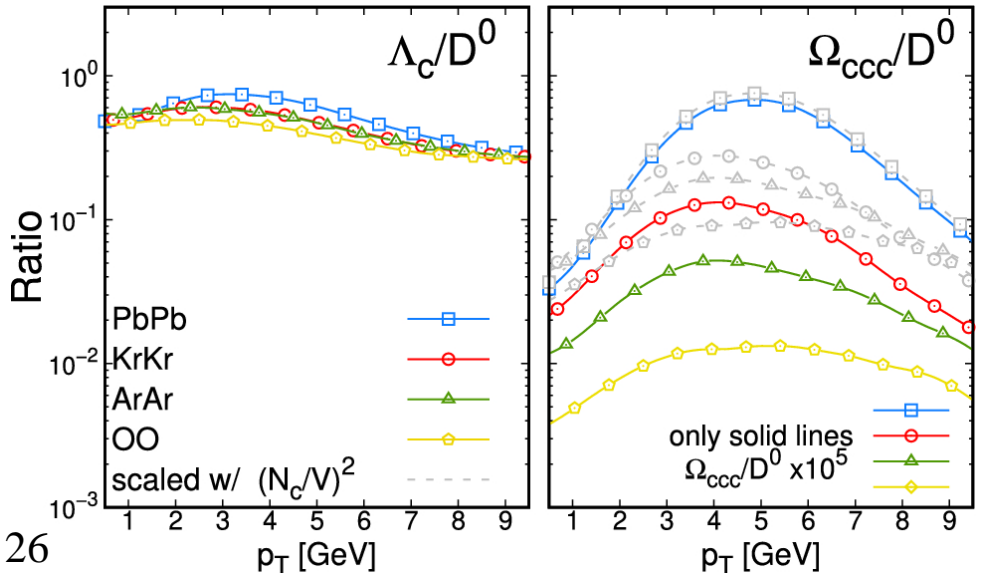
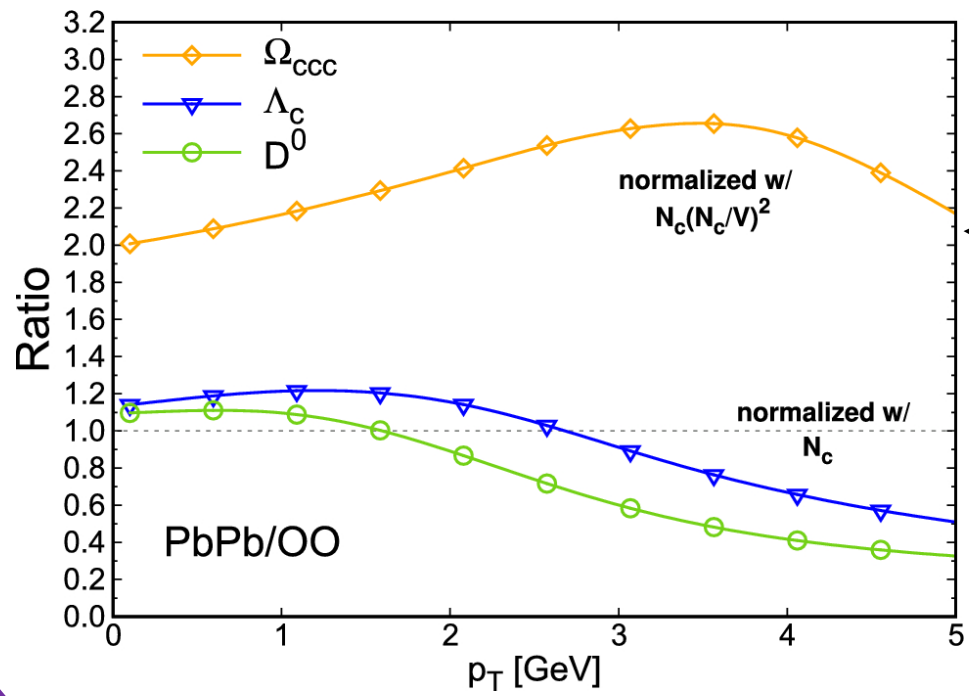
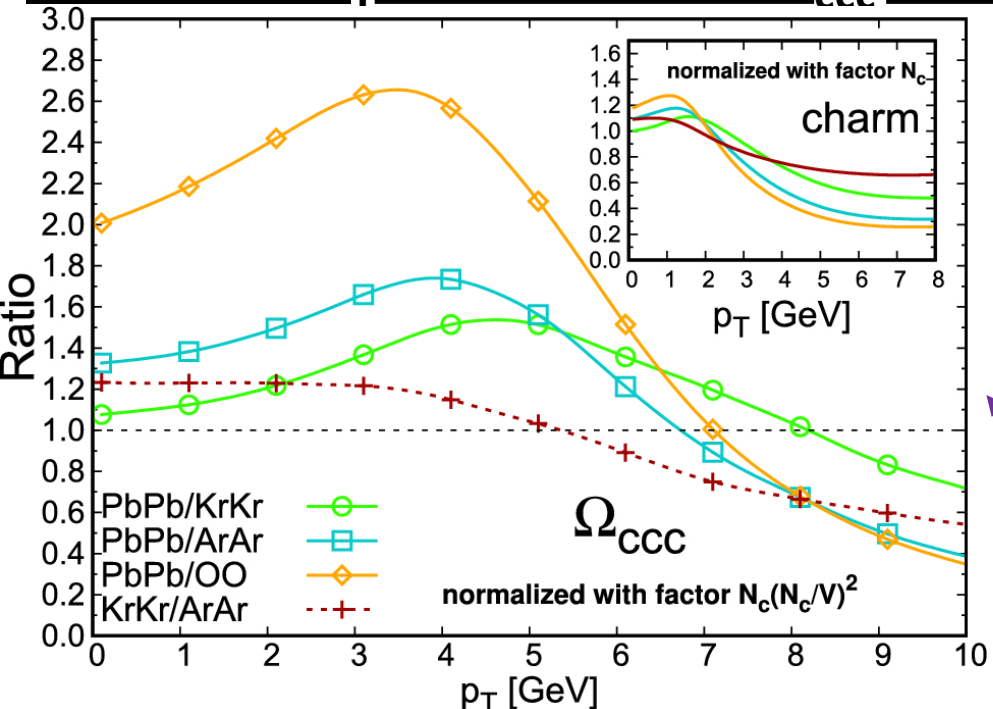


Follow the $N_c \left(\frac{N_c}{V}\right)^{C-1}$ scaling at fixed distribution

| | D^0 | Λ_c | $\Xi_{cc}^{+,++}$ | Ω_{ccc} |
|------|-------|-------------|--------------------------|---------------------------|
| OO | 0.156 | 0.0732 | $3-12.1 \cdot 10^{-5}$ | $2.2-29.2 \cdot 10^{-8}$ |
| ArAr | 0.543 | 0.301 | $1.9-6.6 \cdot 10^{-4}$ | $2.5-26.3 \cdot 10^{-7}$ |
| KrKr | 1.564 | 0.835 | $0.78-2.6 \cdot 10^{-3}$ | $1.5-14.9 \cdot 10^{-6}$ |
| PbPb | 5.343 | 3.0123 | $4-12.5 \cdot 10^{-3}$ | $0.12-1.01 \cdot 10^{-4}$ |

Ratios of p_T distribution Ω_{ccc} in PbPb/KrKr/ArAr/OO

V. Minissale, S. Plumari, Y. Sun and V. Greco, *Eur. Phys. J. C* 84, no.3, 228 (2024)



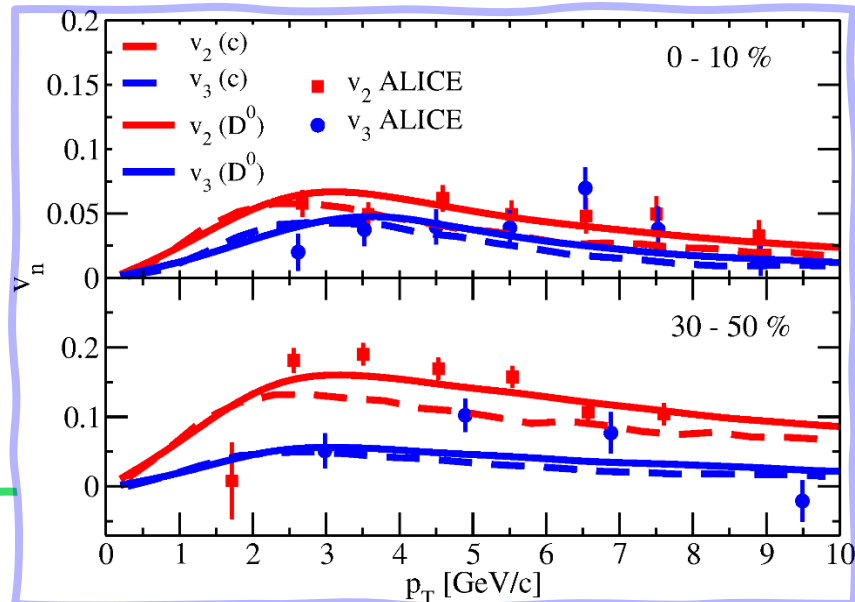
It can be a meter of non-equilibrium.
 Translation of features of charm spectra at low p_T to hadron high momentum region.

More sensitive multicharm Ω_{ccc}/D^0 respect to Λ_c/D^0

More sensitive to system change w.r.t. $D^0 \Lambda_c$ (fragm. Effect)

Implications and developments:

The large Λ_c, Ξ_c production has effects on the R_{AA} of D^0 , because of the charm conservation

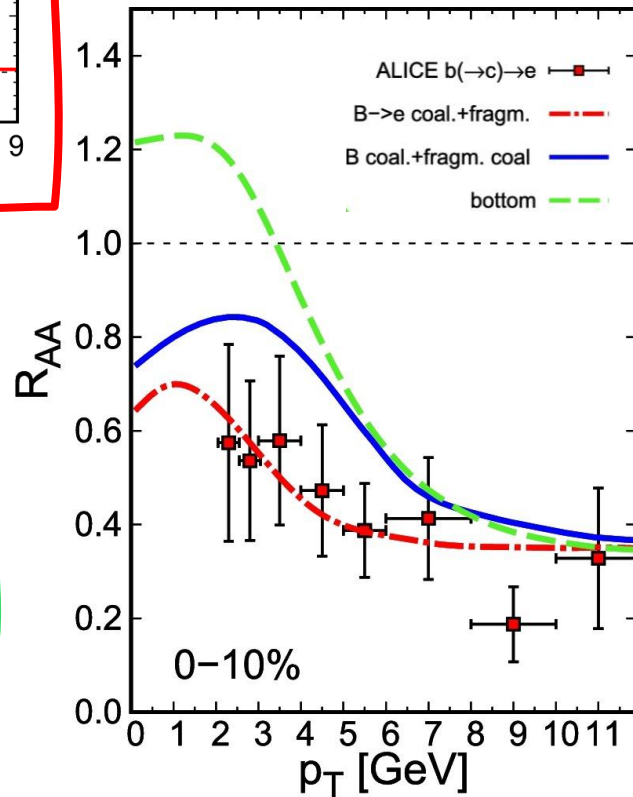


Coalescence give an enhancement to the $v_n(p_T)$ of final hadrons compared to the charm $v_n(p_T)$.

[Sambataro, Sun, Minissale, Plumari, Greco, Eur.Phys.J.C 82 \(2022\) 9, 833](#)

Electrons from semileptonic B meson decay with a coal + fragm model for B meson production

[Sambataro, Minissale, Plumari, Greco Phys.Lett.B 849 \(2024\) 138480](#)



Conclusions

- **Charm hadronization in AA:**

Coalescence+fragmentation gives enhancement of Λ_c production at intermediate momentum region:
 $\Lambda_c/D^0 \sim 1$ for $p_T \sim 3 \text{ GeV}$

- ***In p+p assuming a medium:***

Charm:

Coal.+fragm. good description of heavy baryon/meson ratio (closer to Λ_c/D^0 , Ξ_c/D^0 , Ω_c/D^0 data)

Bottom:

- ✓ B, B_s good agreement with exp. data.
- ✓ Coal.+fragm. Enhancement of Λ_b
- ✓ Predictions for Λ_b/B^0 , Ξ_b/B^0 , Ω_b/B^0
- ✓ D/B, Λ_c/Λ_b , Ξ_c/Ξ_b provide information about hadronization and $f(c)/f(b)$

- **Multicharm hadrons:** - role of non-equilibrium distribution function
-in accord with SHM predictions

Backup Slides

Quark Gluon Plasma in Ultra-Relativistic Heavy-Ion Collisions

Initial Stage

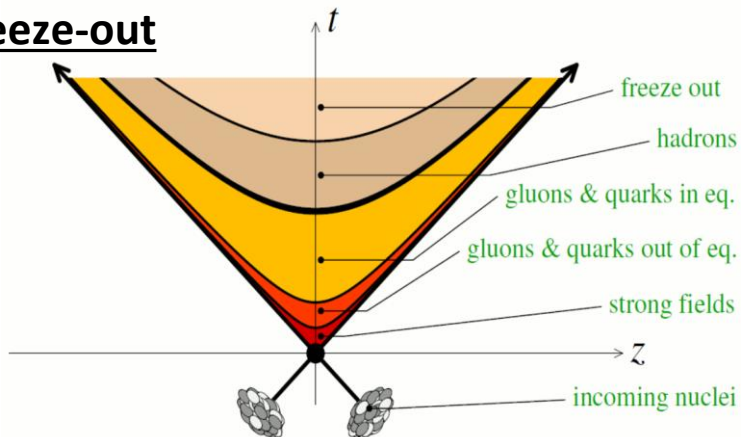
Pre-equilibrium stage

Expansion

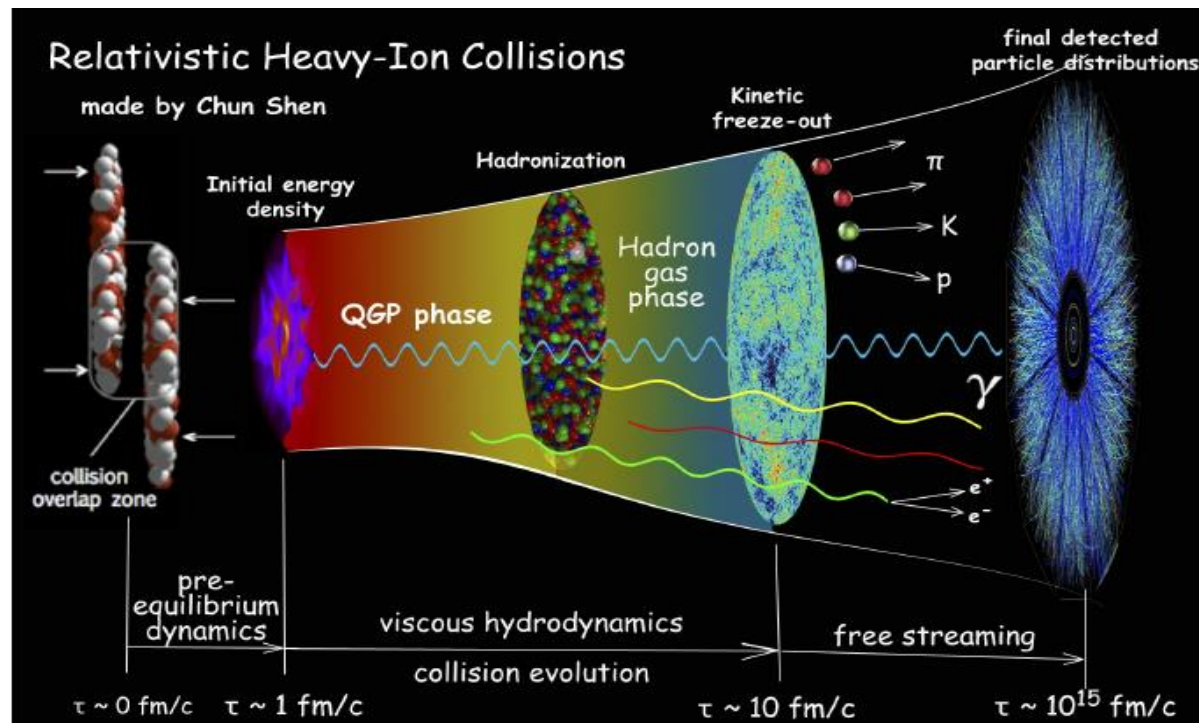
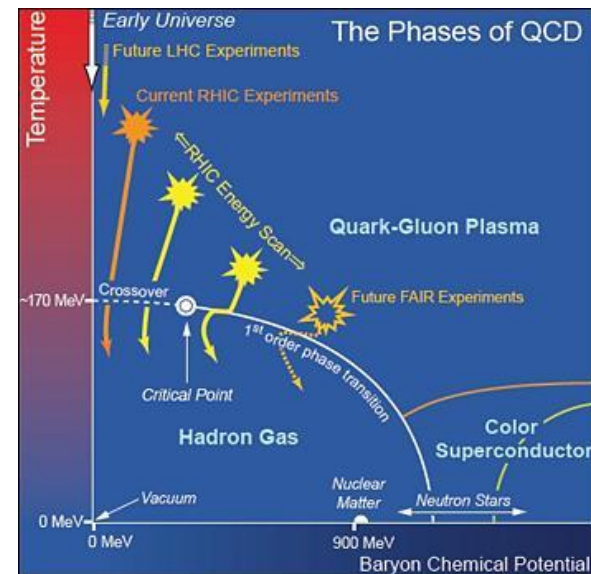
QGP

Hadronization

Chemical and kinetic freeze-out

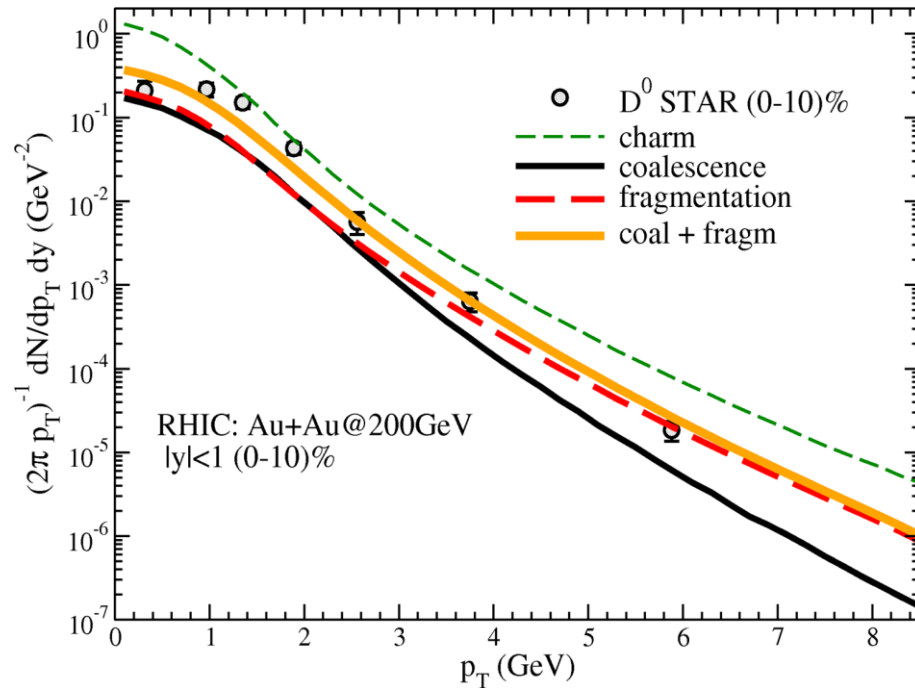


- Nuclear matter: Critical Energy and Temperature in the transition between confined and deconfined phase
- If $T > T_c$ colour charges are deconfined in a Quark Gluon Plasma (QGP)
- Different value of T and ρ for deconfinement \rightarrow Phase Diagram

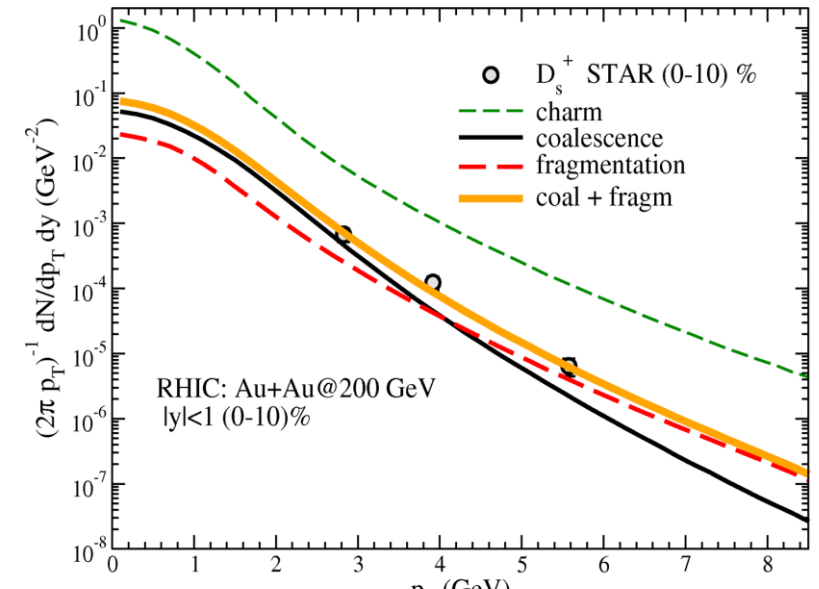
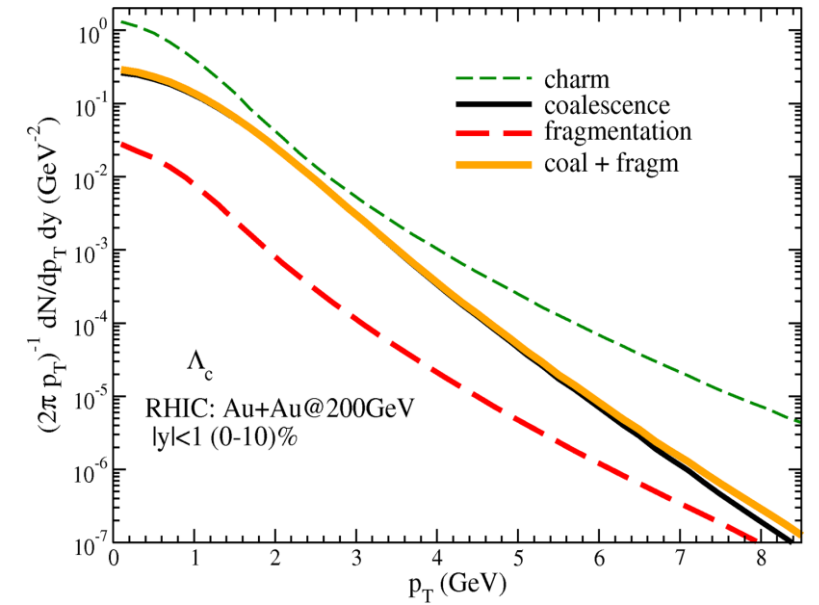


RHIC: results

S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348



Data from STAR Coll. PRL **113** (2014) no.14, 142301



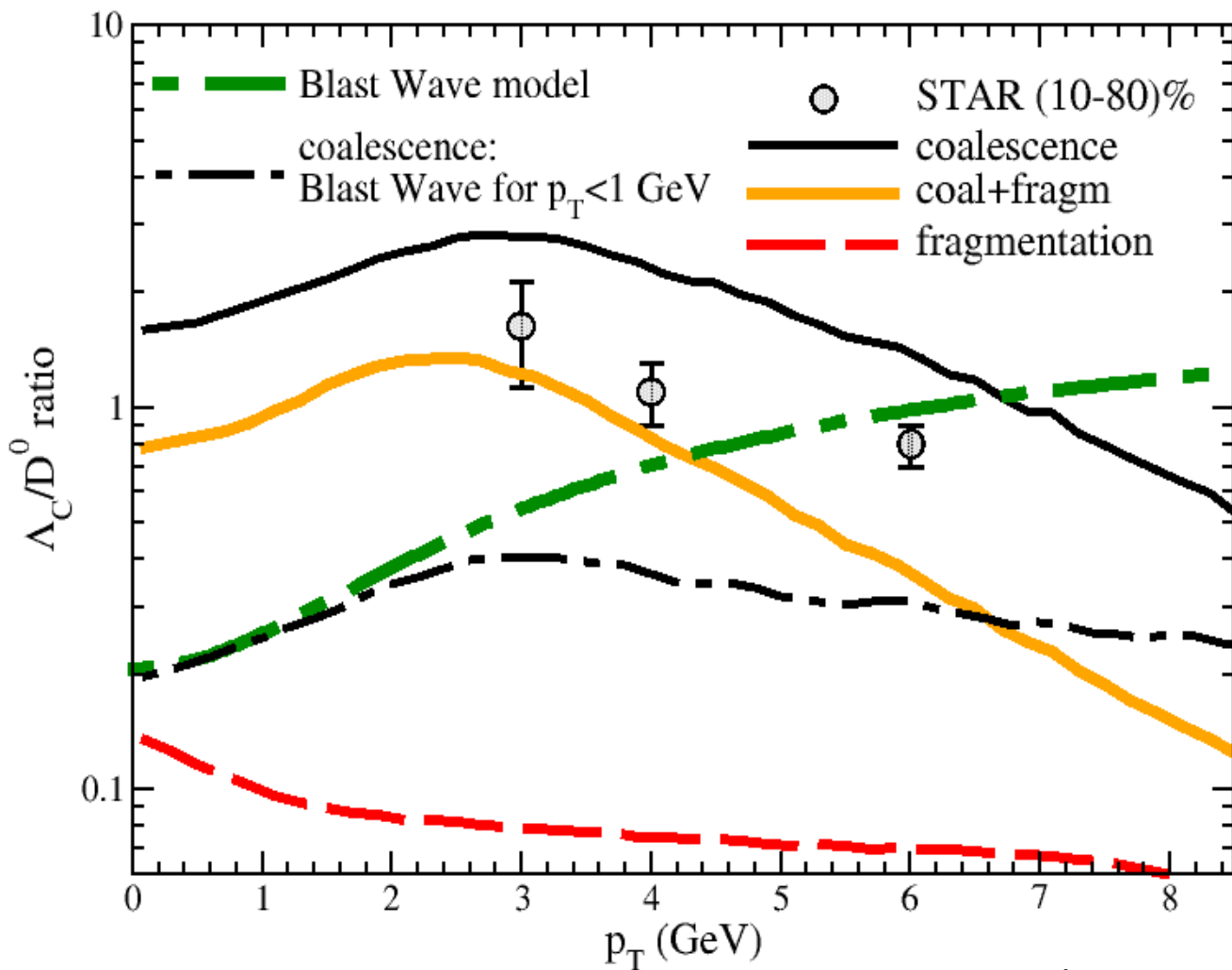
- For D^0 coalescence and fragmentation comparable at 2 GeV

- fragmentation fraction for D_s^+ are small and less than about 8% of produced total heavy hadrons

- Λ_c^+ fragmentation is even more smaller, coalescence gives the dominant contribution

RHIC: Baryon/meson

STAR, Phys.Rev.Lett. 124 (2020) 17,
172301



Compared to light baryon/meson ratio the Λ_c/D^0 ratio has a larger width (flatter)

More flatter \rightarrow should coalescence extend to higher p_T ? Indication also in light sector

V. Minissale, F. Scardina, V. Greco **PRC** **92**,054904 (2015)

Cho, Sun, Ko et al., **PRC** **101** (2020) 2, 024909

Needed data at low p_T

Multicharm production Pb-Pb, Kr-Kr, Ar-Ar, O-O

[V. Minissale, S. Plumari, Y. Sun and V. Greco, *Eur. Phys. J. C* 84, no.3, 228 \(2024\)](#)

| Baryon | | | |
|------------------------------|------|--|-------------------|
| $\Xi_{cc}^{+,++} = dcc, ucc$ | 3621 | $\frac{1}{2} \left(\frac{1}{2}\right)$ | |
| $\Omega_{scc}^+ = scc$ | 3679 | $0 \left(\frac{1}{2}\right)$ | |
| $\Omega_{ccc}^{++} = ccc$ | 4761 | $0 \left(\frac{3}{2}\right)$ | |
| Resonances | | | |
| Ξ_{cc}^* | 3648 | $\frac{1}{2} \left(\frac{3}{2}\right)$ | $1.71 \times g.s$ |
| Ω_{scc}^* | 3765 | $0 \left(\frac{3}{2}\right)$ | $1.23 \times g.s$ |

like S.Cho and S.H. Lee, PRC101 (2020)
from R.A. Briceno et al., PRD 86(2012)

Strengths of the approach:

- Does not rely on distribution in equilibrium for charm
→ useful for small AA down to pp collisions and at $p_T > 3-4$ GeV
- Provide a p_T dependence of spectra and their ratios vs p_T

Widths from harmonic oscillator
rescaling and from $\langle r \rangle$ of
Tsingua approach

| | $\sigma_{p_1} (GeV)$ | $\sigma_{p_2} (GeV)$ | $\sigma_{r_1} (fm)$ | $\sigma_{r_2} (fm)$ |
|---|----------------------|----------------------|---------------------|---------------------|
| Ξ_c | 0.262 | 0.438 | 0.751 | 0.450 |
| Ω_c | 0.345 | 0.557 | 0.572 | 0.354 |
| Ξ_{cc}^ω | 0.317 | 0.573 | 0.622 | 0.344 |
| $\Omega_{ccc}^{\sigma_r, \sigma_p=3/2}$ | 0.522 | 0.522 | 0.566 | 0.566 |

Elliptic Flow – Quark Number Scaling

Fourier expansion of the azimuthal distribution

$$f(\varphi, p_T) = 1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos n\varphi$$

momentum anisotropy in the transverse plane

n=2 Elliptic flow

Assumption

coalescence brings to

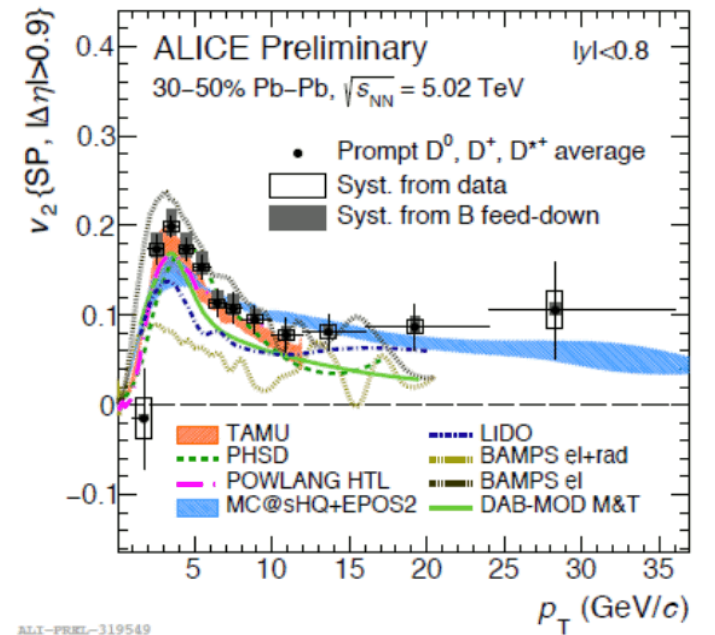
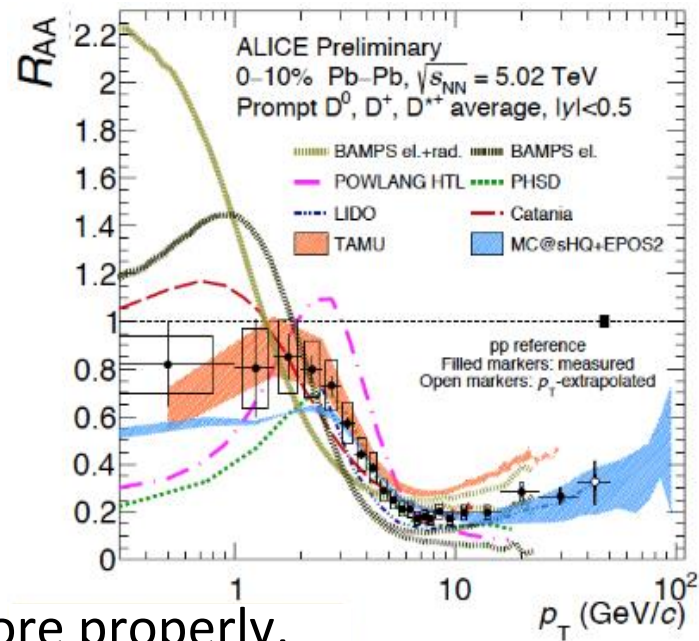
$$v_{2,M}(p_T) \approx 2v_{2,q}(p_T/2)$$
$$v_{2,B}(p_T) \approx 3v_{2,q}(p_T/3)$$

Partonic
elliptic flow

Hadronic
elliptic flow

- one dimensional
- Dirac delta for Wigner function
- isotropic radial flow
- not including resonance effect

Transport approaches



Models not really tested at $p \rightarrow 0$
 The new data \rightarrow determine $D_s(T)$ more properly,
 i.e. $p \rightarrow 0$ where it is defined and computed in IQCD

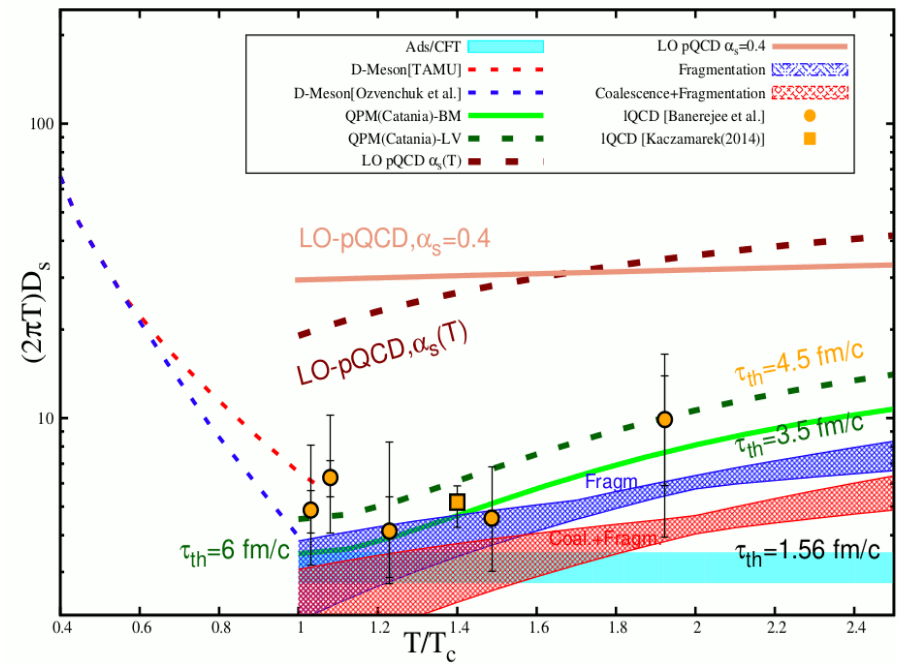
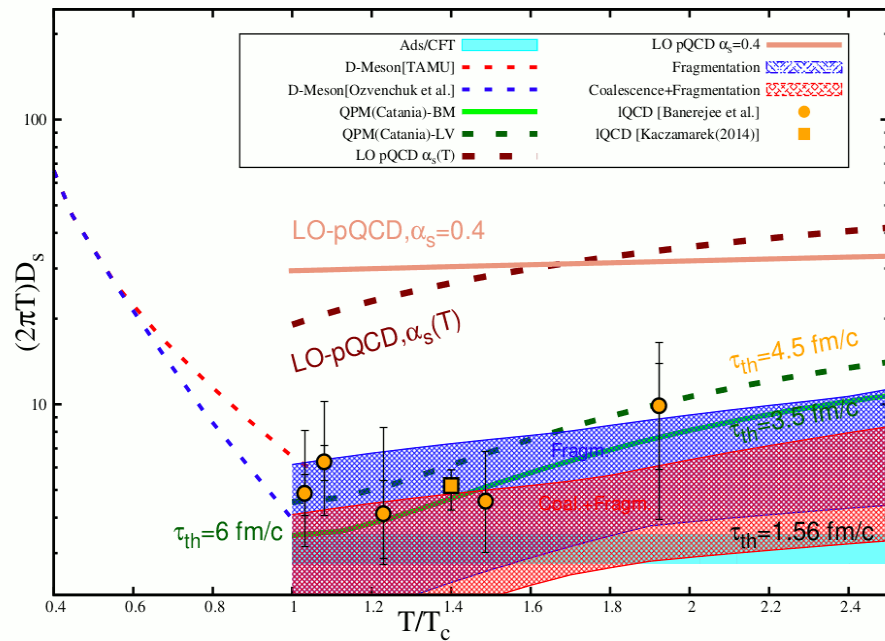
| | Catania | Duke | Frankfurt(PHSD) | LBL | Nantes | TAMU |
|-------------------------|--------------|--------------|------------------------|--------------|----------|--------------------|
| Initial HQ (p) | FONLL | FONLL | pQCD | pQCD | FONLL | |
| Initial HQ (x) | binary coll. | binary coll. | binary coll. | binary coll. | | binary coll. |
| Initial QGP | Glauber | Trento | Lund | | EPOS | |
| QGP partons | Boltzm. mass | Vishnu m=0 | Boltzm. m(T) | Vishnu m=0 | EPOS m=0 | 2d ideal hydro m=0 |
| formation time QGP | 0.3 fm/c | 0.6 fm/c | 0.6 fm/c (early coll.) | 0.6 fm/c | 0.3 fm/c | 0.4 fm/c |
| interactions in between | HQ-glasma | no | HQ-preformed plasma | no | | no |

2018-2019

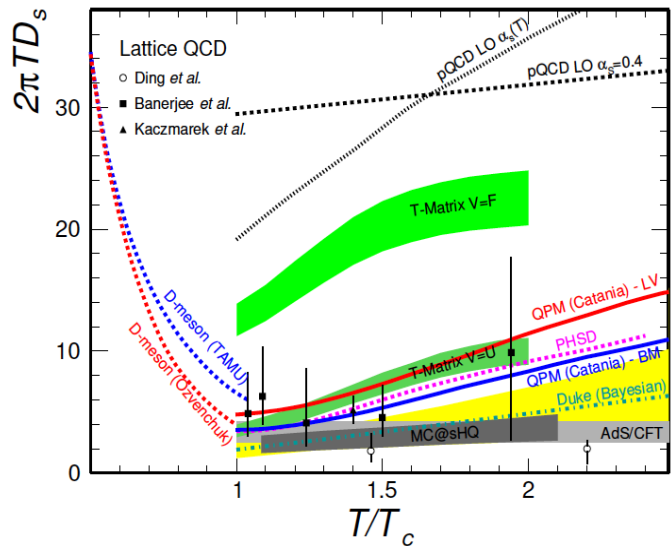
Several Collab. in joint activities:

- EMMI-RRTF:
R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS:
S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)

Transport coefficient



X. Dong and V. Greco, PPNP(2019)



Different hadronization models can affect the extraction of the charm quark diffusion coefficient

2018-2019

Several Collab. in joint activities:

- EMMI-RRTF: R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS: S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)