Estimation of the beta-decay rates of ^{85}Kr, ^{93}Zr, ^{87}Rb and ^{176}Lu

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3rd PANDORA Meeting - 08/10/2024

Introduction

Two theoretical-computational methods for the analysis of nuclear decays in different scenarios

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The Dirac-Hartree-Fock method

- Radial basis functions

The variational method

- Multidimensional Gaussian basis functions

The Dirac-Hartree-Fock method

Our approach is based on the calculation of the total Hamiltonian

$$H = H_{nucl} + H_{e-e} + H_{weak}$$

where

- H_{nucl} contains the interactions between nucleons in the initial and final nuclear states
- H_{e-e} is the electron-electron Coulomb correlation
- H_{weak} is the weak interaction Hamiltonian

The Dirac-Hartree-Fock method

The weak Hamiltonian, which satisfies the Lorentz-invariance,

$$H_{weak} = \frac{G_F}{\sqrt{2}} H_\mu L^\mu + h.c.$$

is defined as the product of leptonic

$$L^\mu = \bar{u}_e \gamma^\mu (1 - \gamma^5) v_\nu$$

and hadronic currents

$$H_\mu = \bar{u}_p \gamma_\mu (1 - x\gamma^5) v_n$$

The Dirac-Hartree-Fock method

The leptonic part

- The leptonic current is factorized into the independent product of the electron and neutrino wavefunctions
  - The neutrino wavefunction $\longrightarrow$ free-particle Dirac equation
  - Electrons interact via a mean-field
  - The electron wavefunction $\longrightarrow$ Dirac-Hartree-Fock equation in a central potential, whose numerical solution was calculated by using a radial basis function Runge-Kutta method

$$\begin{pmatrix} mc^2 + W_V + W_S + \mathbf{A}_P \cdot \sigma - E & -c\sigma \cdot i\nabla - \sigma \cdot \mathbf{A} + W_{PS} \\ -c\sigma \cdot i\nabla - \sigma \cdot \mathbf{A} + W_{PS} & -mc^2 + W_V + \mathbf{A}_P \cdot \sigma - W_S - E \end{pmatrix} \begin{pmatrix} \Psi_L \\ \Psi_S \end{pmatrix} = 0$$

The Dirac-Hartree-Fock method

The hadronic part

- The hadronic current is separable into neutron and proton field operators
    - The decaying neutron $\longrightarrow$ an independent particle correlated only geometrically to the *core* of the remaining nucleons
    - Protons and neutrons $\longrightarrow$ semi-empirical scalar and vector relativistic Wood-Saxon spherical symmetric potential

$$V_{WS}(r) = -\frac{V_0}{1 + e^{\frac{r-R_N}{a}}}$$

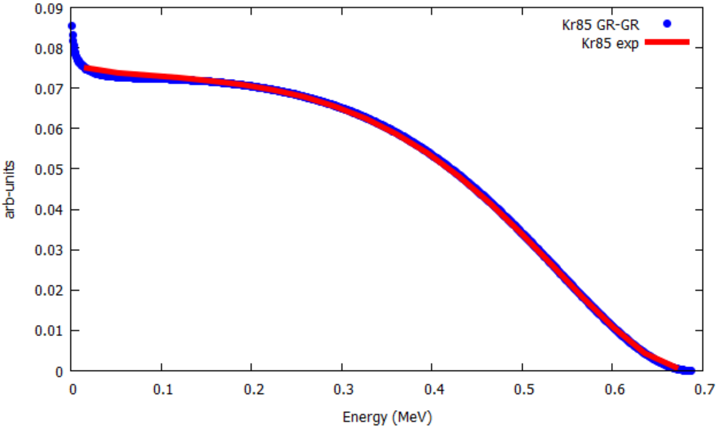
The Dirac-Hartree-Fock method

The main purpose is to compute the transition probability

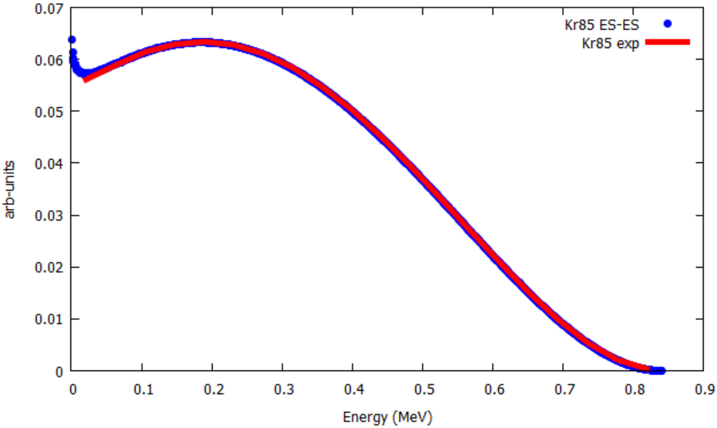
$$N_{i\to f} = 2\pi\, Tr(\hat{\rho}_i H_{weak} P_f H_{weak})\delta(E_i - E_f) + h.c.$$

- $\hat{\rho}_i = p_i \left|i\right\rangle \left\langle i\right|$ is a statistical mixture of initial states $\left|i\right\rangle$
- $P_f = \sum_f \left|f\right\rangle \left\langle f\right|$ is the projector onto the final states $\left|f\right\rangle$

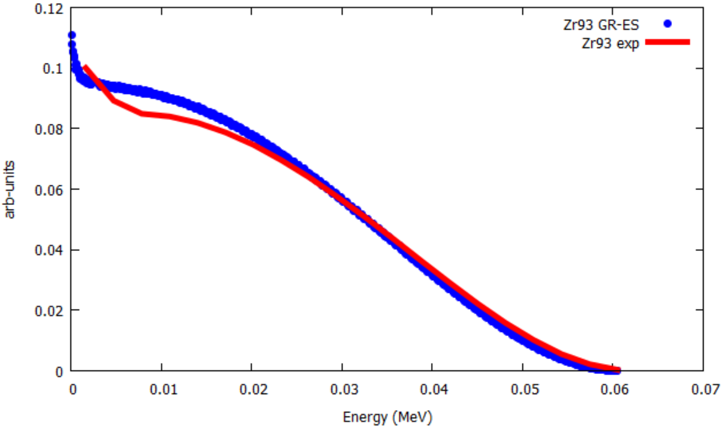
The β-decay spectrum of ^{85}Kr



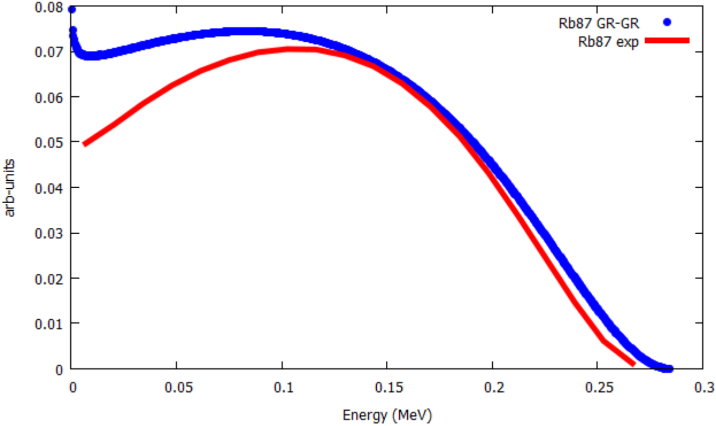
The β-decay spectrum of ^{85}Kr



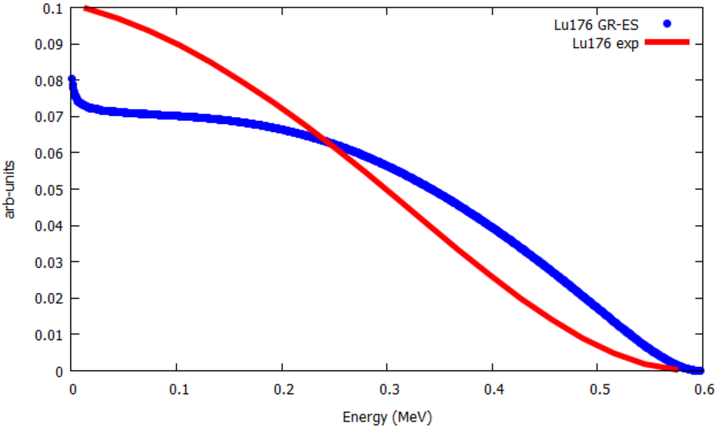
The β-decay spectrum of ^{93}Zr



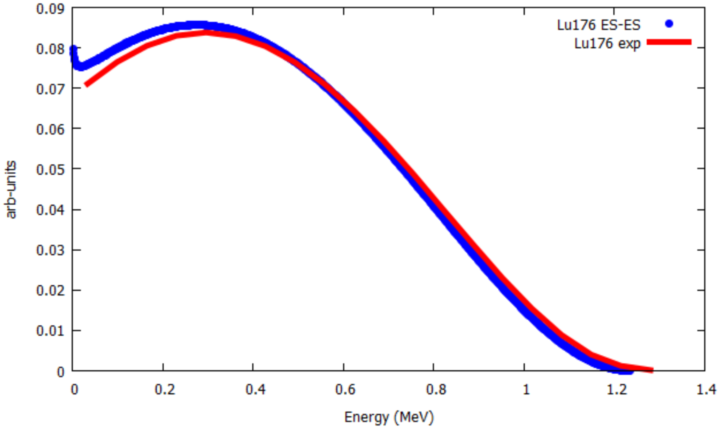
The β-decay spectrum of ^{87}Rb



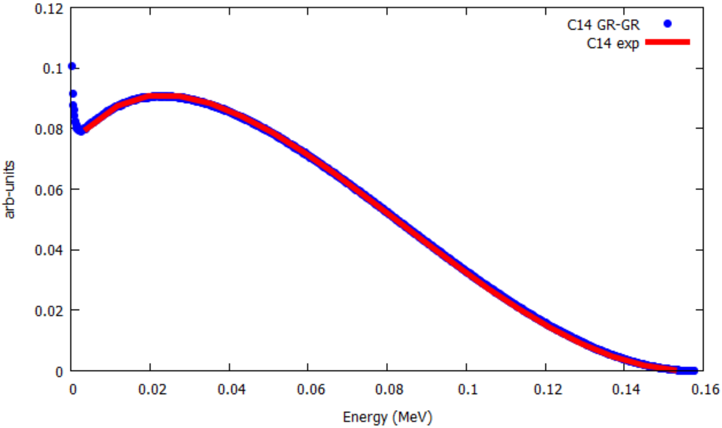
The β-decay spectrum of ^{176}Lu



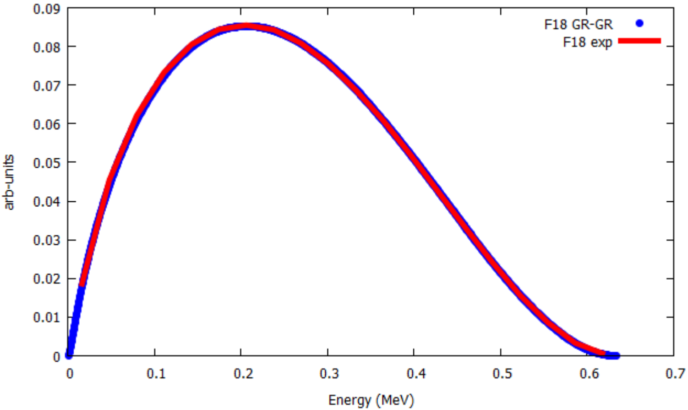
The β-decay spectrum of ^{176}Lu



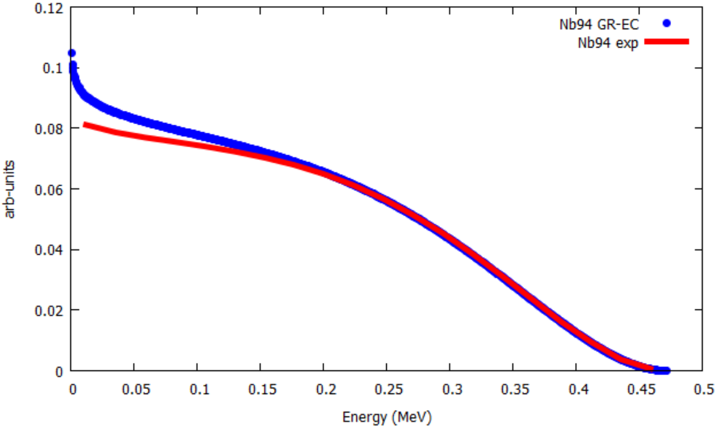
The β-decay spectrum of ^{14}C



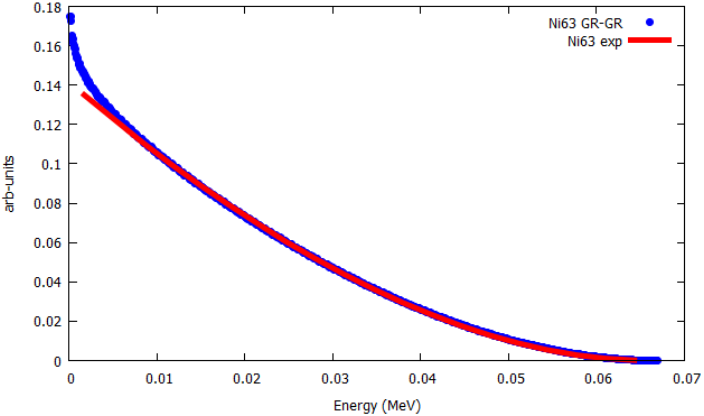
The β-decay spectrum of ^{18}F



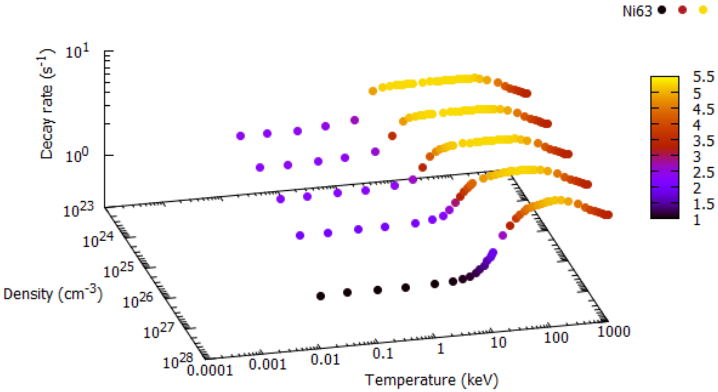
The β-decay spectrum of ^{94}Nb



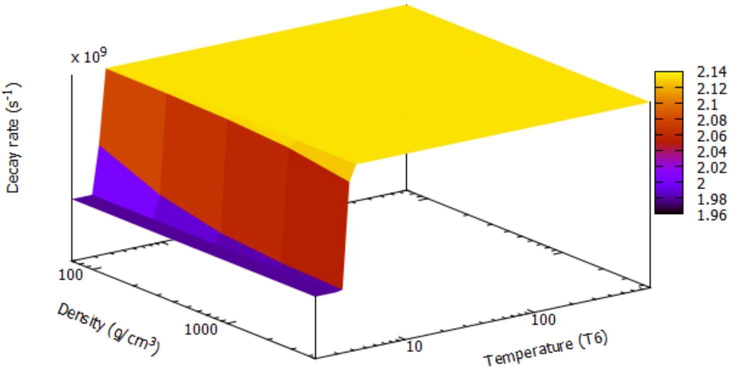
The β-decay spectrum of ^{63}Ni



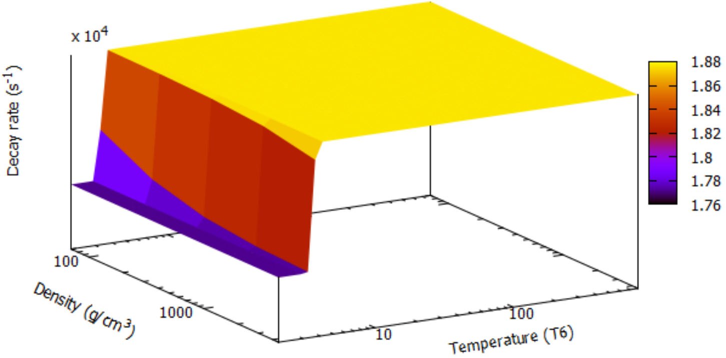
Decay rate of ^{63}Ni as a function of T and n_p



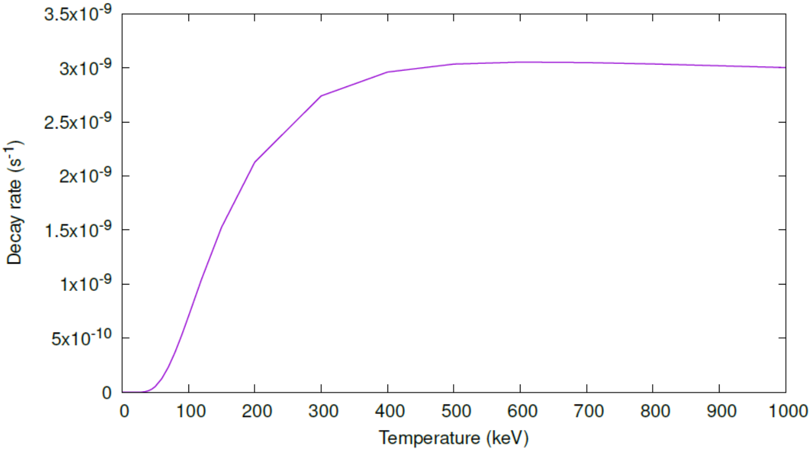
Decay rate of ^{85}Kr as a function of T and n_p



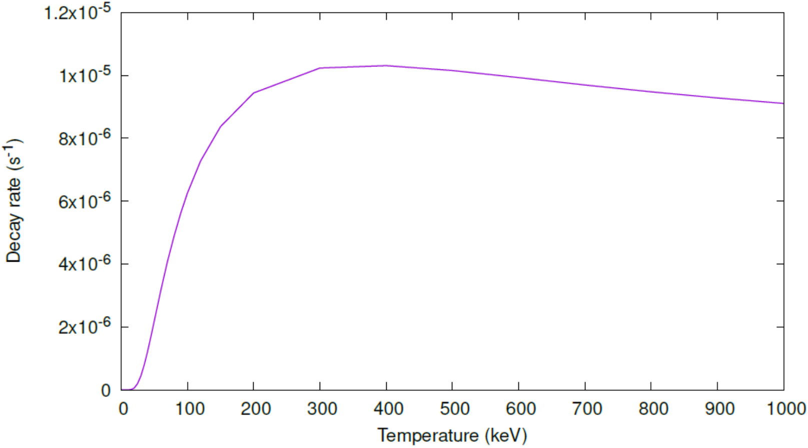
Decay rate of ^{85}Kr as a function of T and n_p



Decay rate of ^{93}Zr as a function of T



Decay rate of ^{176}Lu as a function of T



The variational method

- We define a Gaussian function $n \times n_d - dimensional$

$$g\left(A, \mathbf{b}; \mathbf{r}\right) = \exp\left[-\left(\mathbf{r}-\mathbf{b}\right)^t A \left(\mathbf{r}-\mathbf{b}\right)\right]$$

- We expand the many-particle wave-function

$$\Psi = \sum_{i=1}^{n} c_i g_i$$

The variational method

- To solve the Schrödinger equation

$$H\Psi = E\Psi$$

- We apply Rayleigh-Ritz variational method *(generalized eigenvalue problem)*

$$Hc = ENc$$

where

$$H_{ij} = \langle g_i | H | g_j \rangle \quad and \quad N_{ij} = \langle g_i | g_j \rangle \quad (i, j = 1, ..., n)$$

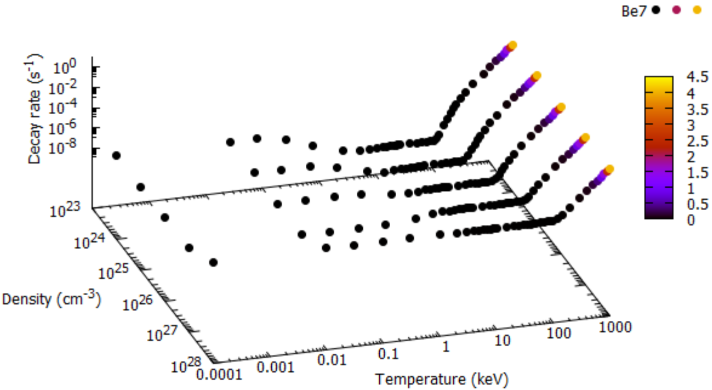
Results

Conf.	Bound DHF (s^{-1})	Bound (MDG)	
		Energy	Rate (s^{-1})
$1s^2 2s^2$	1.4804×10^{-7}	-14.6651	1.4817×10^{-7}
$1s^2 2s3s\ {}^3S$	1.4478×10^{-7}	-14.2803	1.3928×10^{-7}
$1s^2 2s3s\ {}^1S$		-14.2681	1.3406×10^{-7}
$1s2s^2 3s$	0.7788×10^{-7}	/	/
$1s^2 2s$	1.4648×10^{-7}	-14.3232	1.4730×10^{-7}
$1s^2 3s$	1.4258×10^{-7}	-13.8272	1.3601×10^{-7}
$1s2s^2$	0.8014×10^{-7}	/	/
$1s^2$	1.4284×10^{-7}	-13.6544	1.4552×10^{-7}
$1s2s\ {}^3S$	0.7706×10^{-7}	-9.1263	0.7598×10^{-7}
$1s3s\ {}^1S$	0.7252×10^{-7}	-8.4352	0.6449×10^{-7}
$1s$	0.7140×10^{-7}	-8.0	0.8617×10^{-7}
$2s$	0.0893×10^{-7}	-2.0	0.1077×10^{-7}

Results

kT (eV)	n_{el} (cm^{-3})	rate (s^{-1}) (bound MDG)	rate (s^{-1})(total DHF)
2	1.e10	1.455157e-07	1.409283e-07
12	1.e10	5.193027e-13	6.101676e-16
22	1.e10	5.490840e-17	1.263426e-18
32	1.e10	1.422436e-18	9.994987e-20
42	1.e10	1.876488e-19	2.370322e-20
2	1.e11	1.455157e-07	1.409283e-07
12	1.e11	5.192746e-12	6.101676e-15
22	1.e11	5.490840e-16	1.263442e-17
32	1.e11	1.422436e-17	9.994677e-19
42	1.e11	1.876488e-18	2.369779e-19
2	1.e12	1.455157e-07	1.409283e-07
12	1.e12	5.189941e-11	6.101674e-14
22	1.e12	5.490839e-15	1.263442e-16
32	1.e12	1.422436e-16	9.994651e-18
42	1.e12	1.876488e-17	2.369700e-18

Decay rate of ^7Be as a function of T and n_p



- A good agreement with our results and the experimental data
- Temperature and density $\longrightarrow$ nuclear decay rates
- Future goals? Implementation of our model

Thank you for your attention !