Hierarchies from landscape probability gradients and critical boundaries

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TUM

based on 2311.10139

Roma Tre, 2024

Introduction

Gauge Hierarchy problem:

$$\delta m_h^2 \propto \Lambda^2 \ \leftarrow$$ any physics that Higgs interacts with

e.g. $$\frac{m_P^2}{m_h^2} \sim 10^{34}$$

Introduction

Single vacuum* approaches:

$$\delta m_h^2 = {\color{red}0}\,\Lambda^2 + \mathcal{O}(100GeV)$$

↑

supersymmetry

compositeness

extra dimensions

Introduction

Landscape/dynamical approaches:

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$

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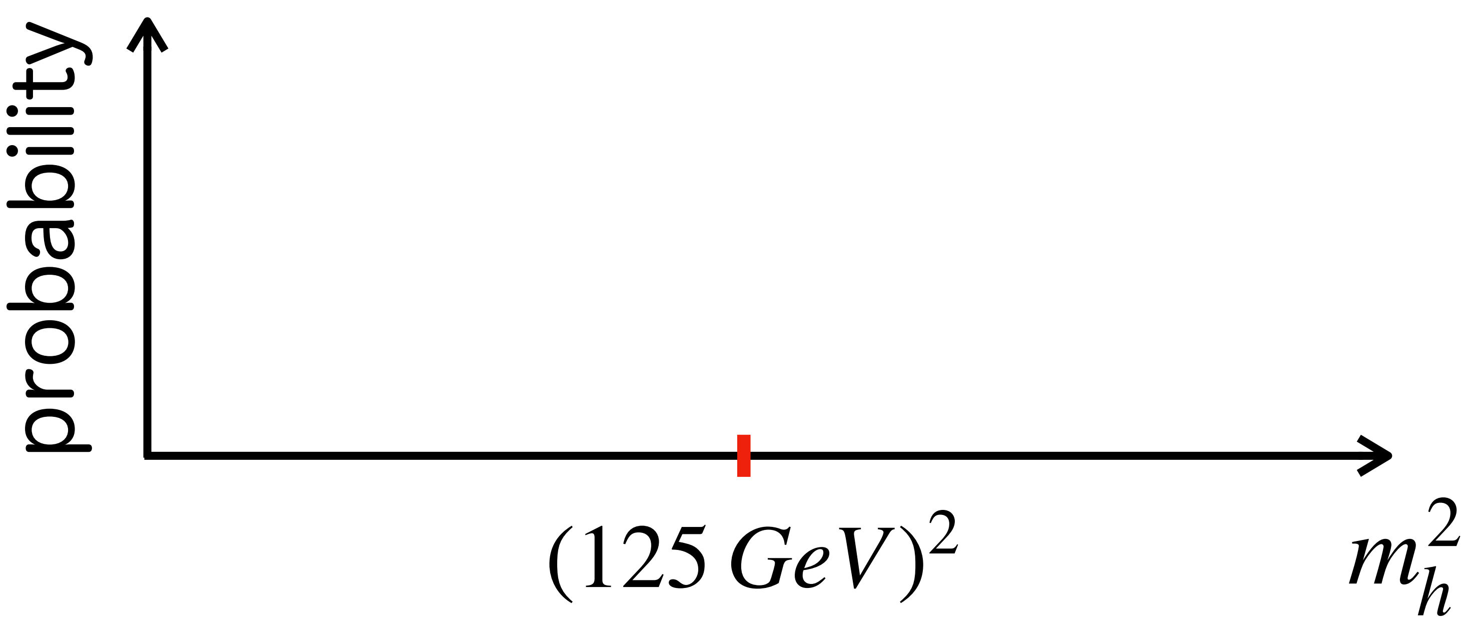
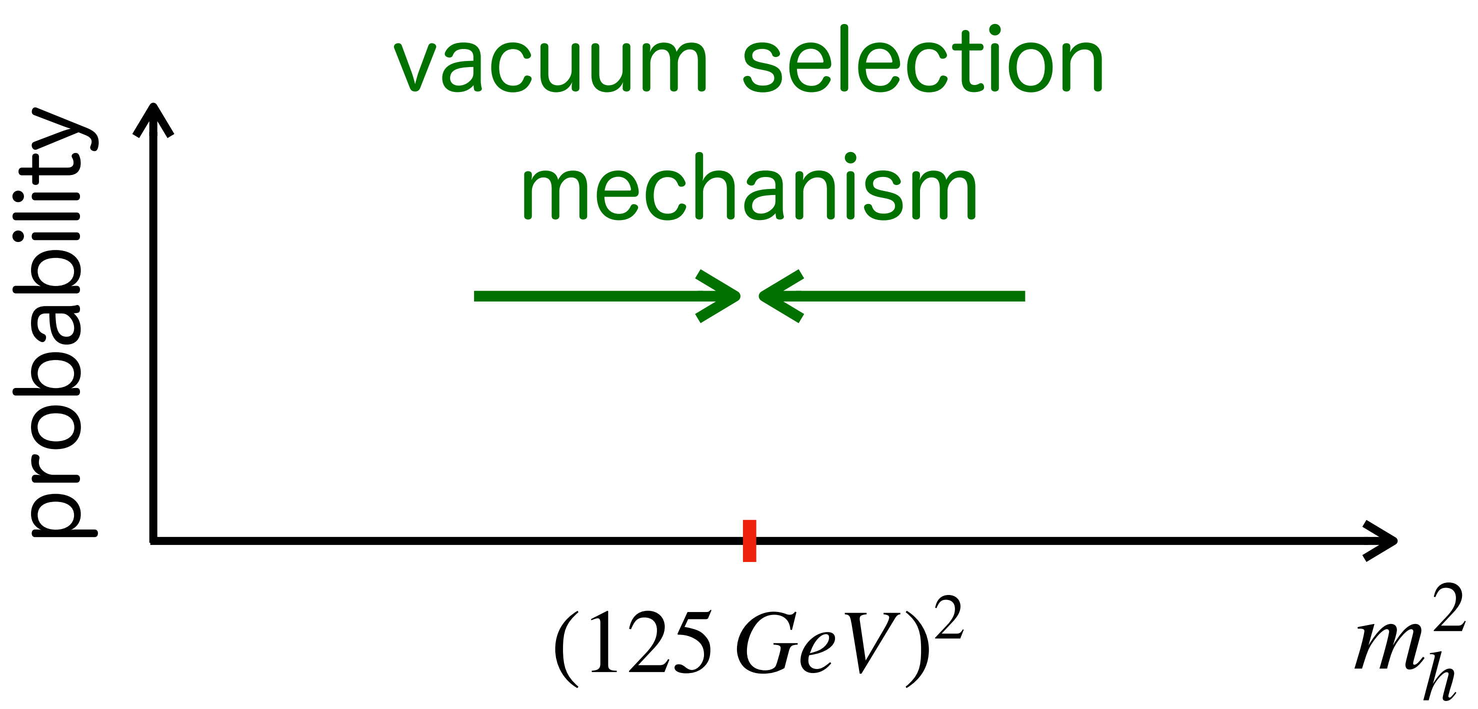


Introduction

Landscape/dynamical approaches:

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$



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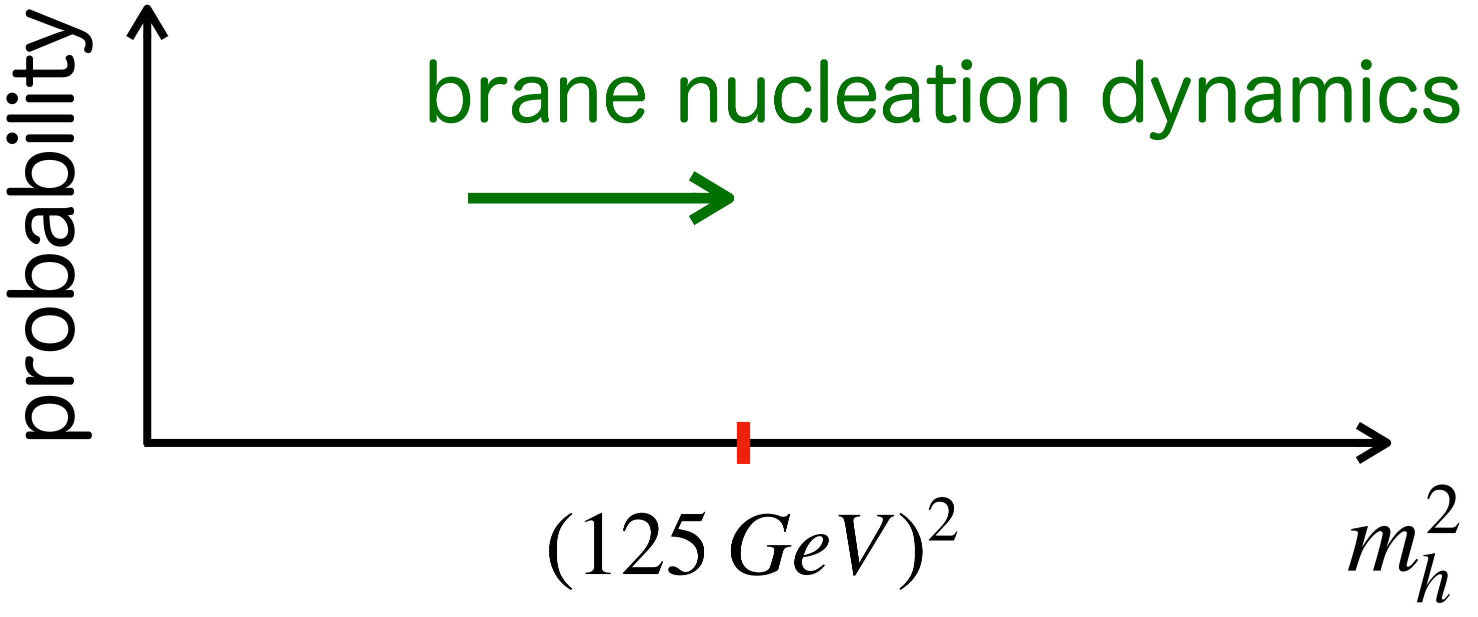
Landscape/dynamical approaches:

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$

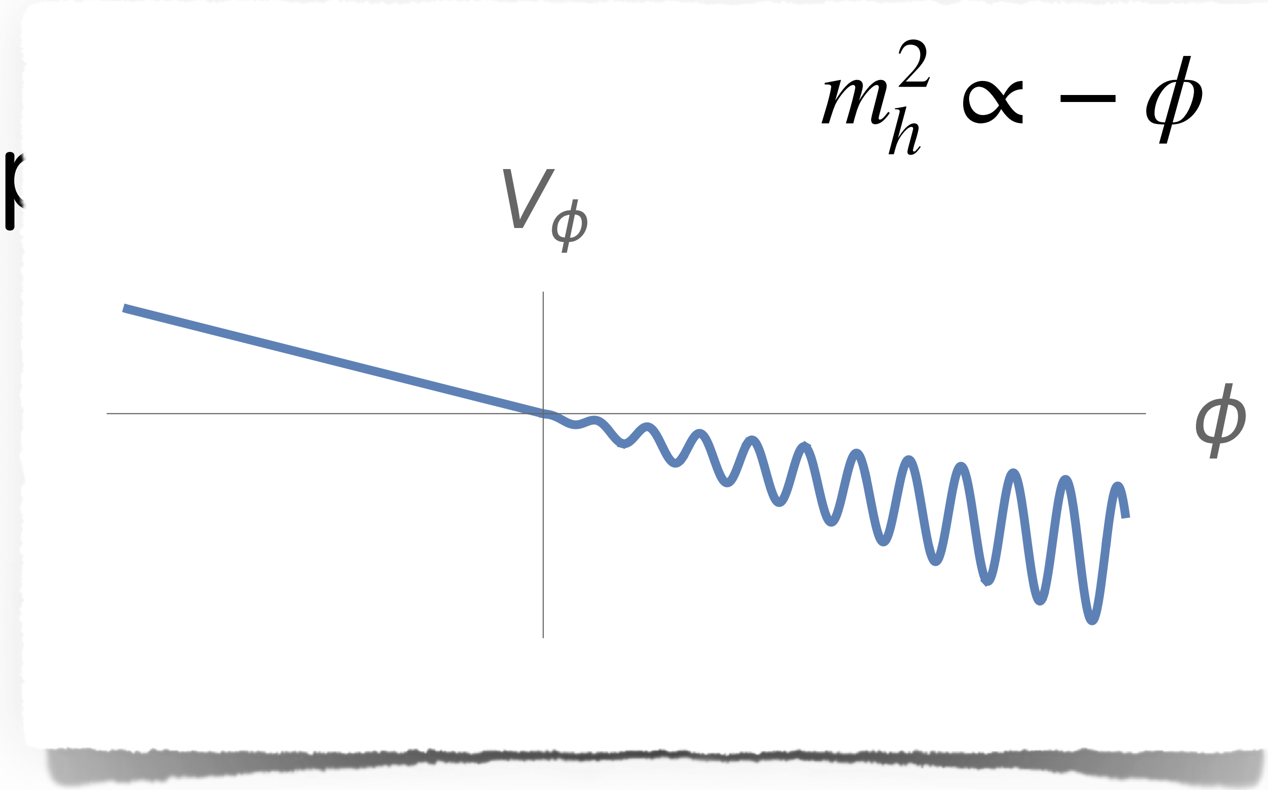
Dvali,Vilenkin 0304043,0410286



Introduction

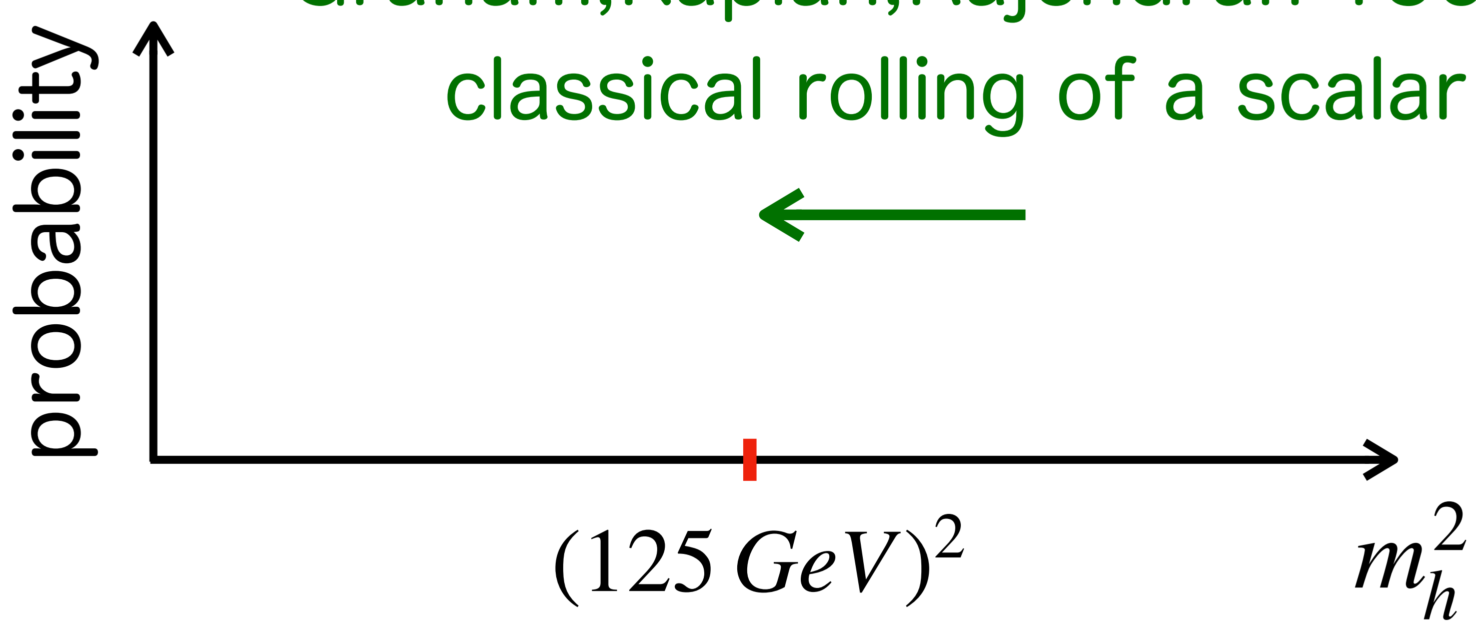
Landscape/dynamical app

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$



Graham,Kaplan,Rajendran 1504.07551

classical rolling of a scalar



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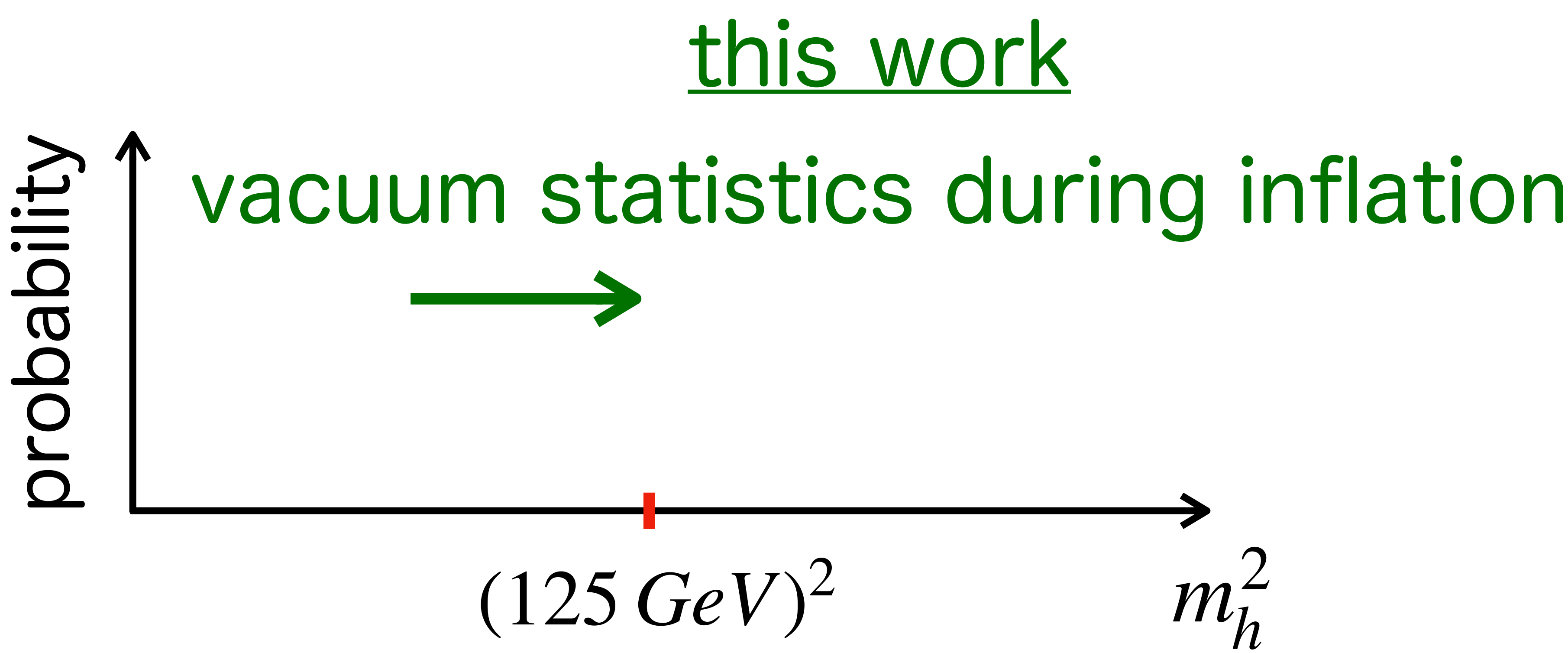


Introduction

Preview of the final mechanism

Landscapes for both mH and CC. Why?

Introduction

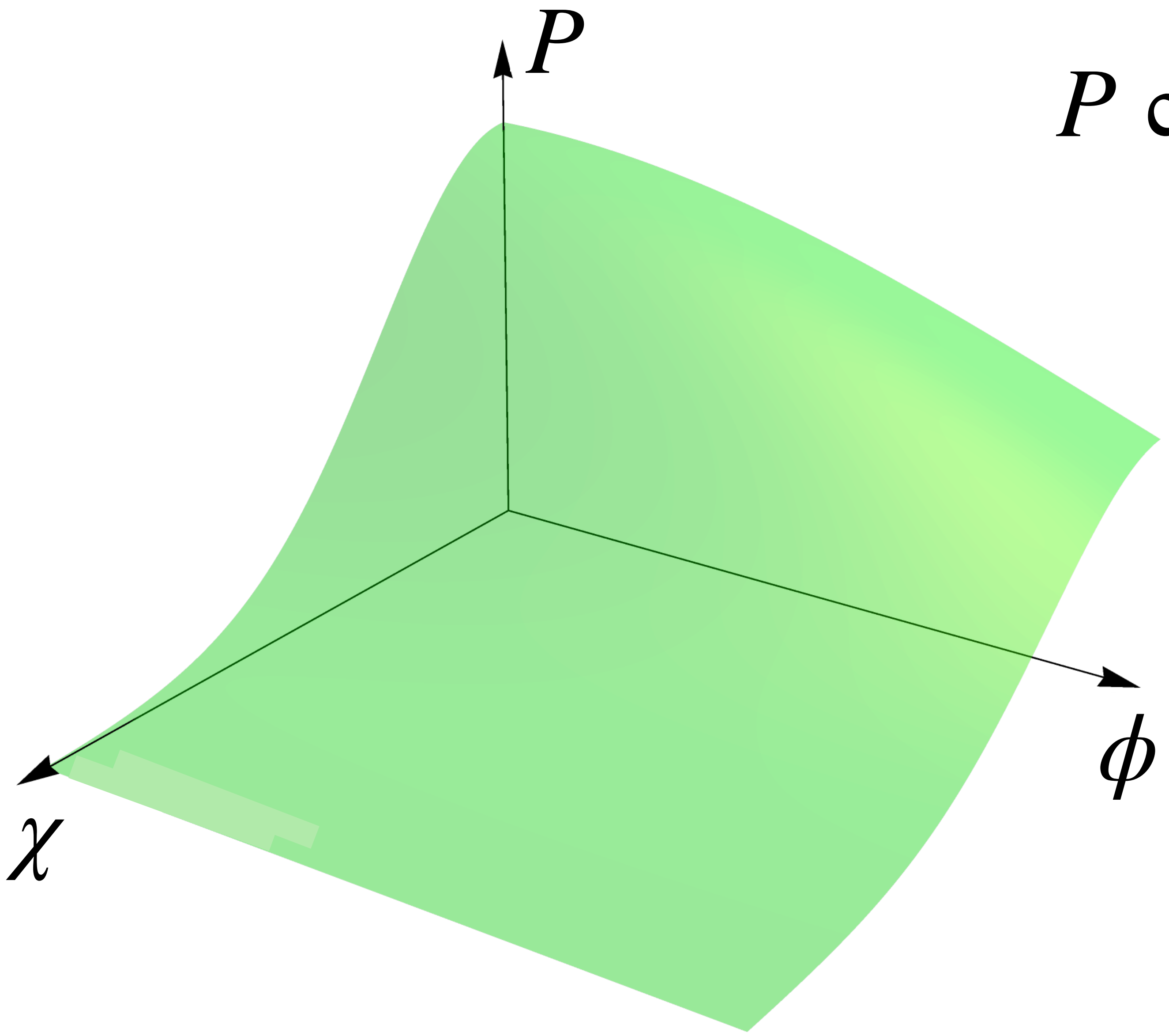
Preview of the final mechanism

Landscapes for both mH and CC. Why?

- $$\frac{m_P^4}{\Lambda_{cc}(obs)} \sim 10^{120}$$
- most straightforward approach to the smallness of CC is landscape + anthropics
- dynamics of the two landscapes generically interfere hence it is natural to consider them together

Introduction

Preview of the final mechanism



$$P \propto \exp[-\#\phi] \times \exp[-\#\chi]$$

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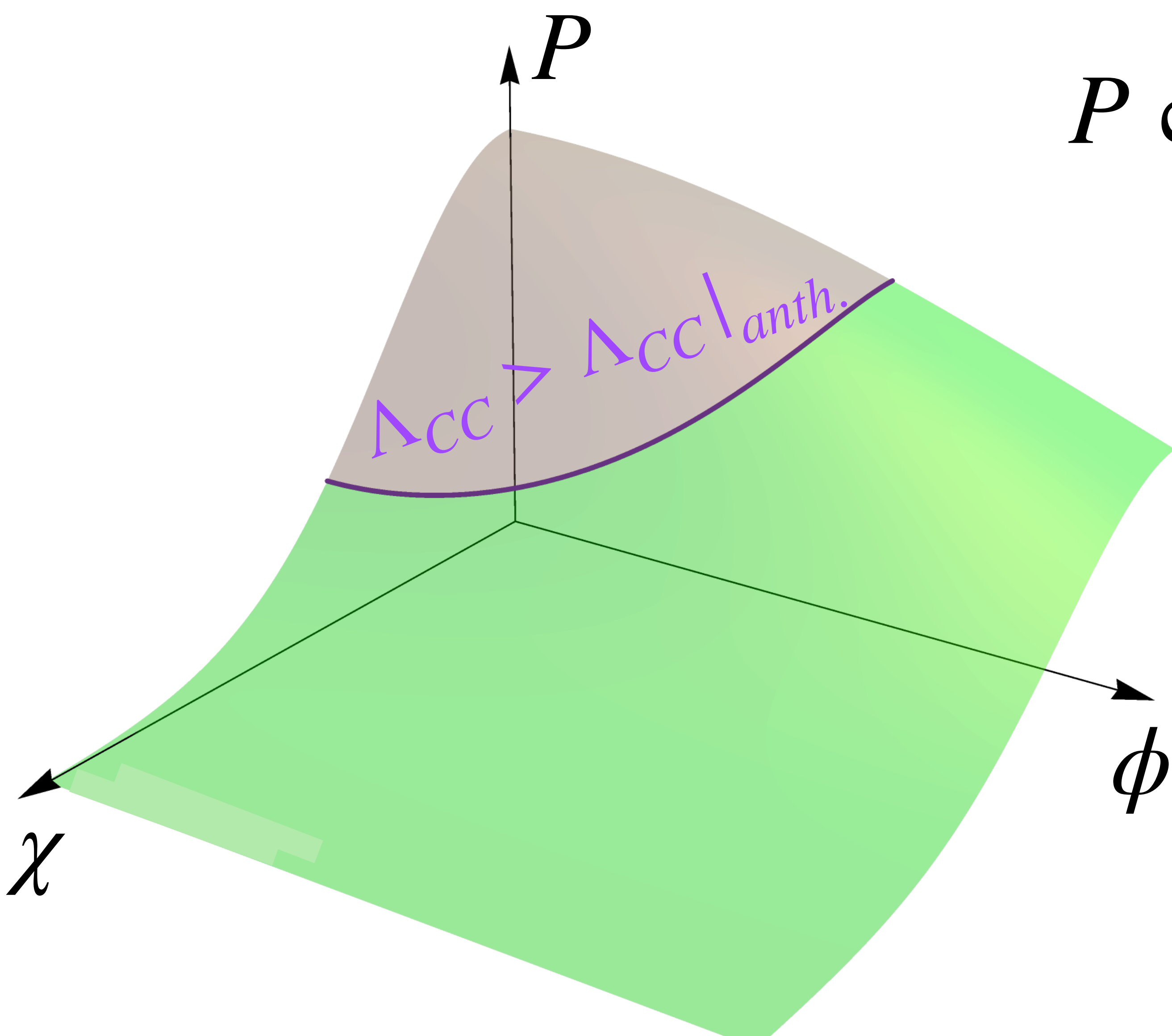
Preview of the final mechanism



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$$\Lambda_{cc} \propto \phi + \chi$$

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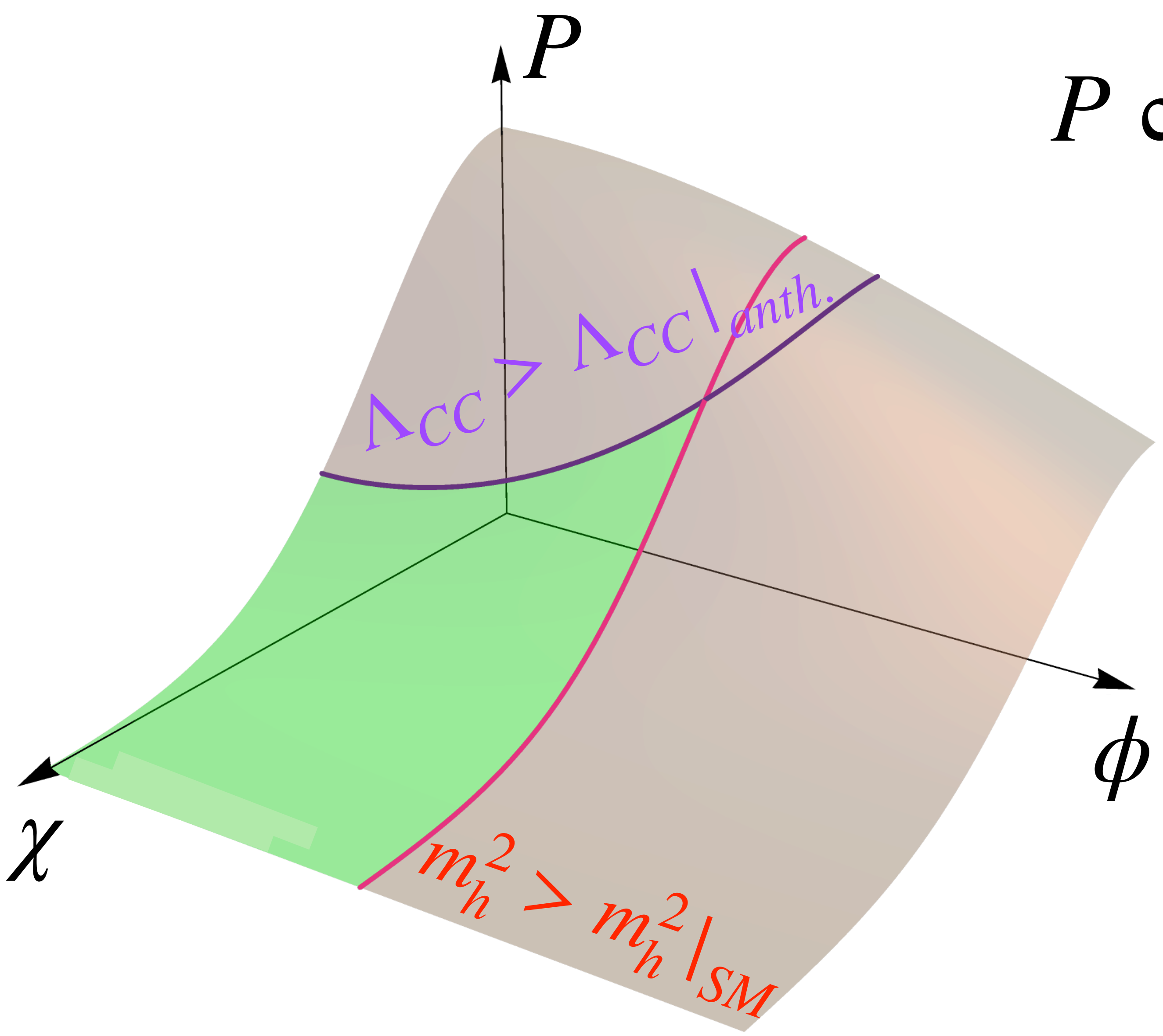

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$$m_h^2 \propto \phi$$

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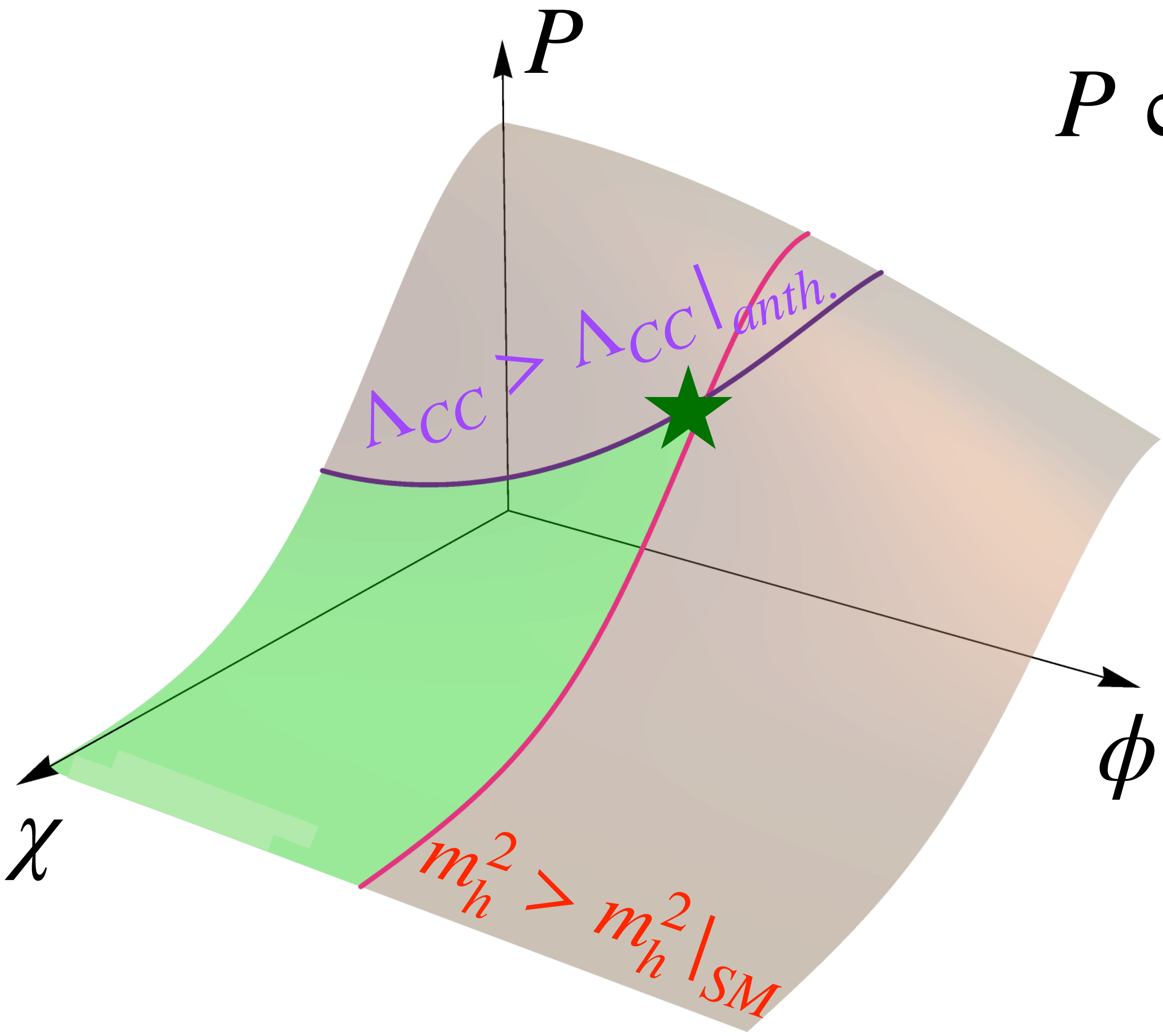

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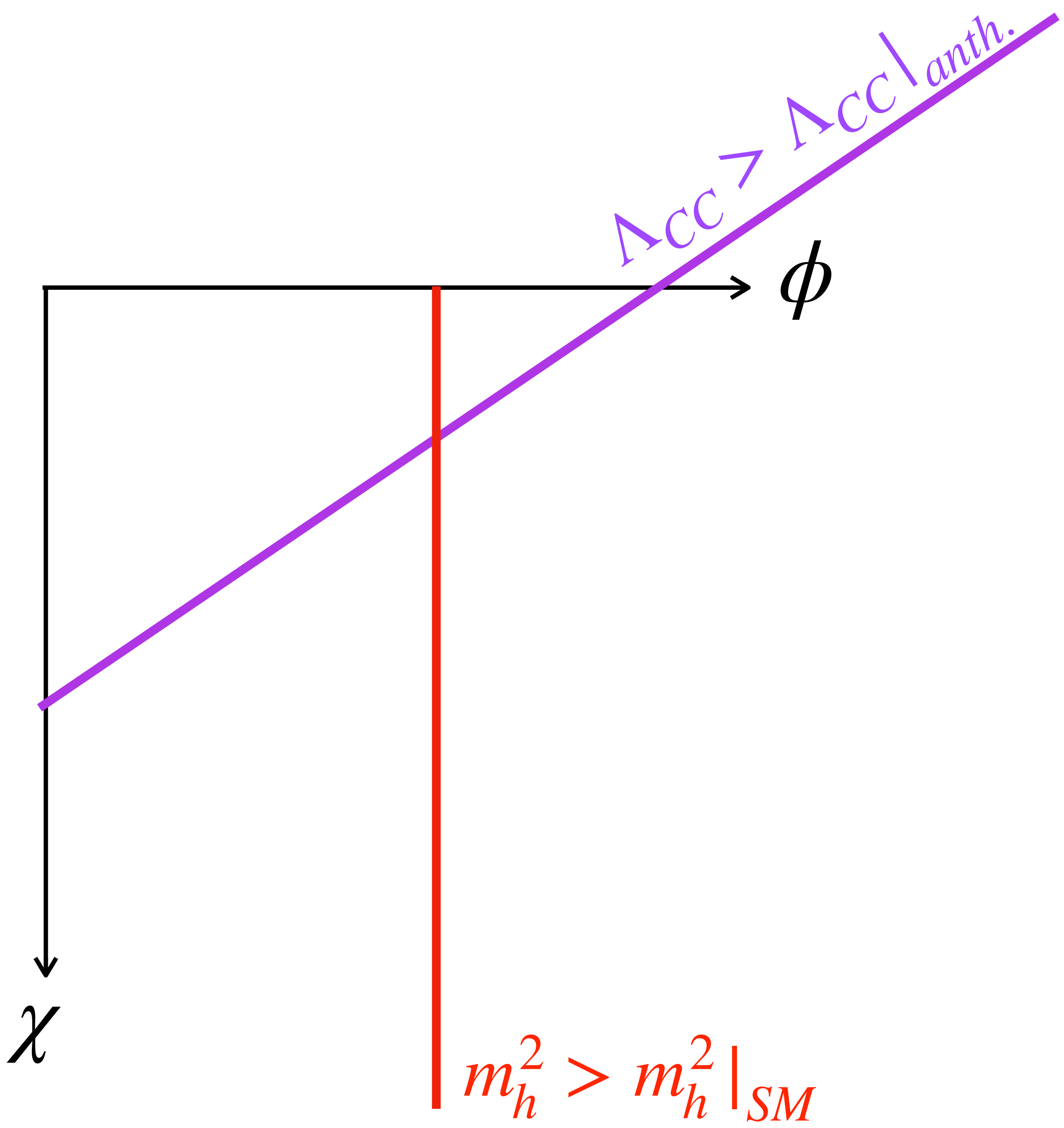

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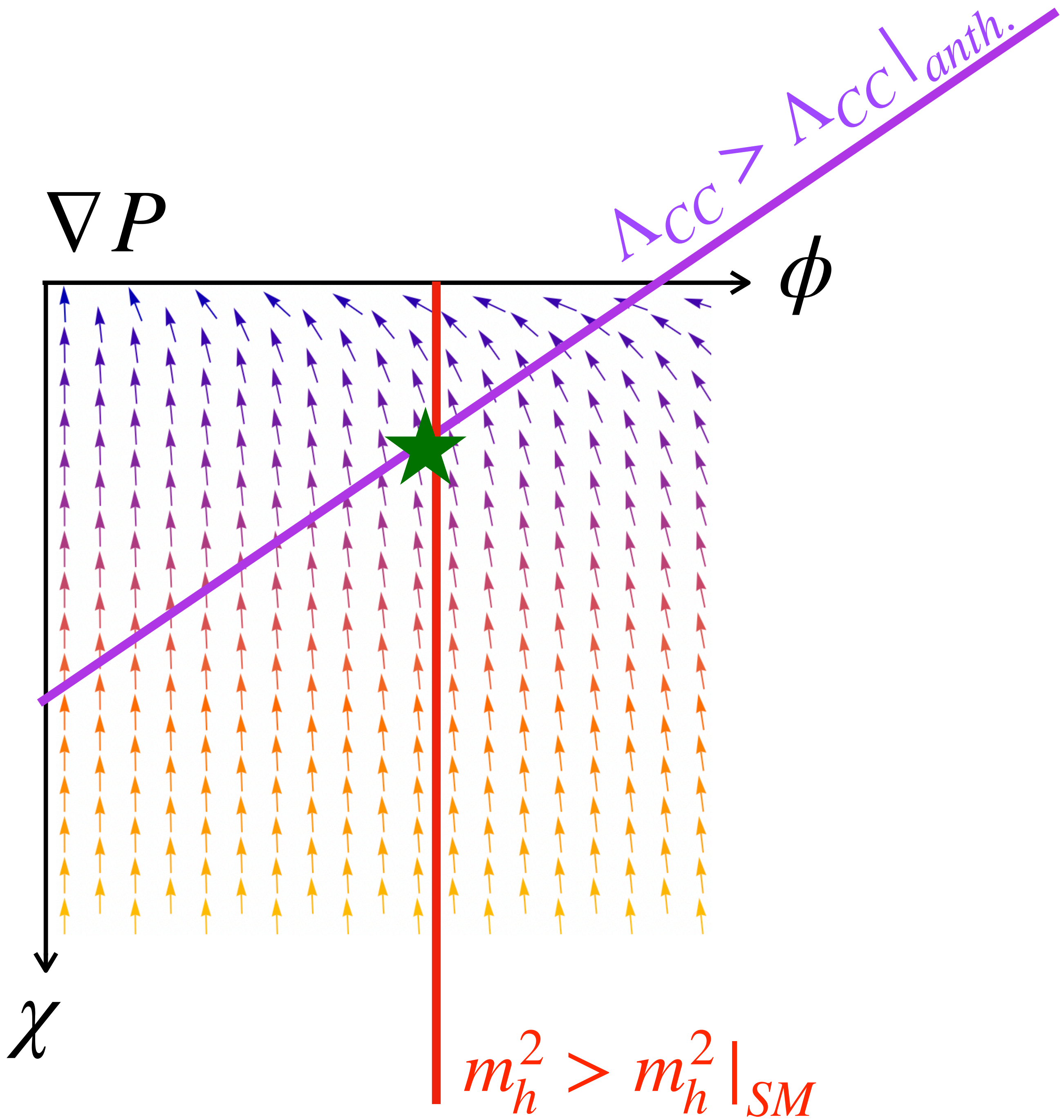


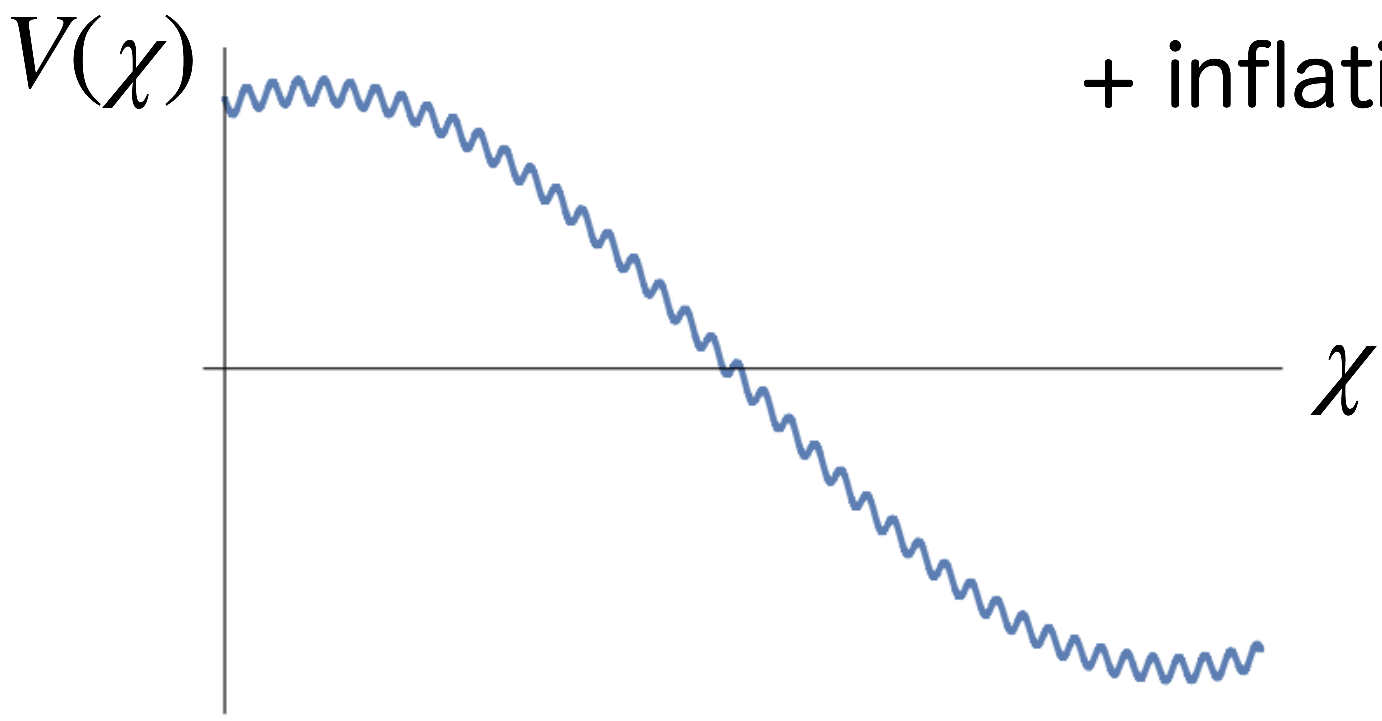
$$\Lambda_{cc} \propto \phi + \chi$$

$$m_h^2 \propto \phi$$

Probability measures

Probability measures

What are the probabilities to observe different vacua?



+ inflation with $V \gg V(\chi)$

Abbott '84

$\chi \propto$ some fundamental parameter

e.g. m_H^2

Probability measures

What are the probabilities to
observe different vacua?

1. standard volume-weighted measure

A. D. Linde, Phys. Lett. B **175**, 395 (1986).
A. D. Linde, D. A. Linde, and A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994), gr-qc/9306035.
A. D. Linde and A. Mezhlumian, Phys. Lett. B **307**, 25 (1993), gr-qc/9304015.

2. local measures

R. Bousso, Phys. Rev. Lett. **97**, 191302 (2006), hep-th/0605263.
L. Susskind (2007), 0710.1129.
Y. Nomura, Astron. Rev. **7**, 36 (2012), 1205.2675.

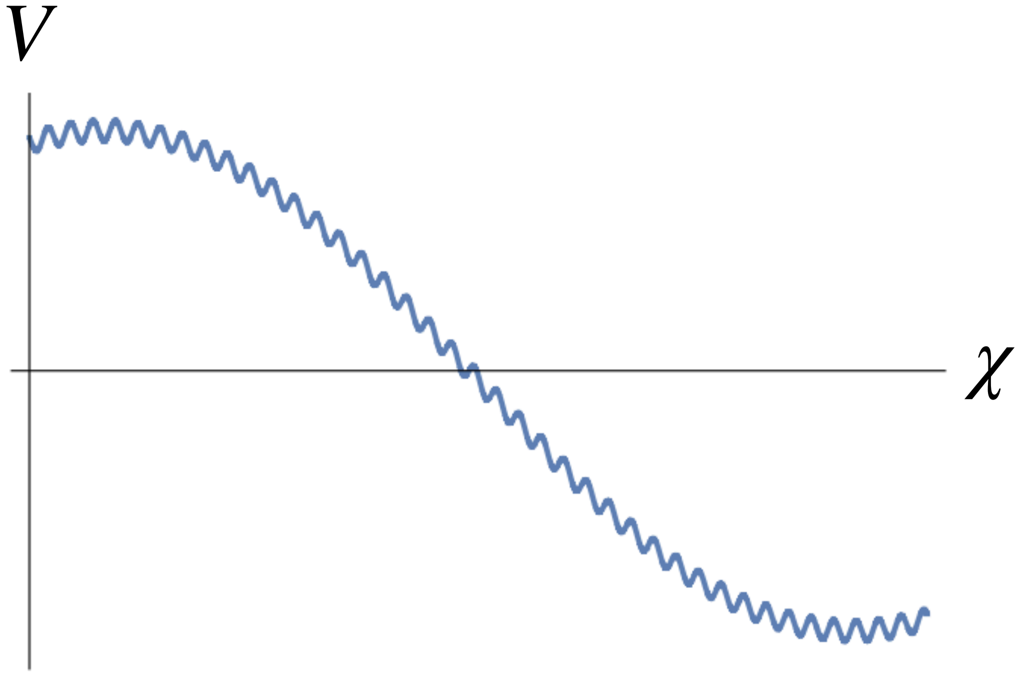
Volume-weighted measures

Probability to observe some type of vacuum (labeled e.g. by the Higgs mass) $\propto$ overall volume of this vacuum at some proper time t

*Youngness paradox: assumed to be solved by a version of the stationary measure prescription

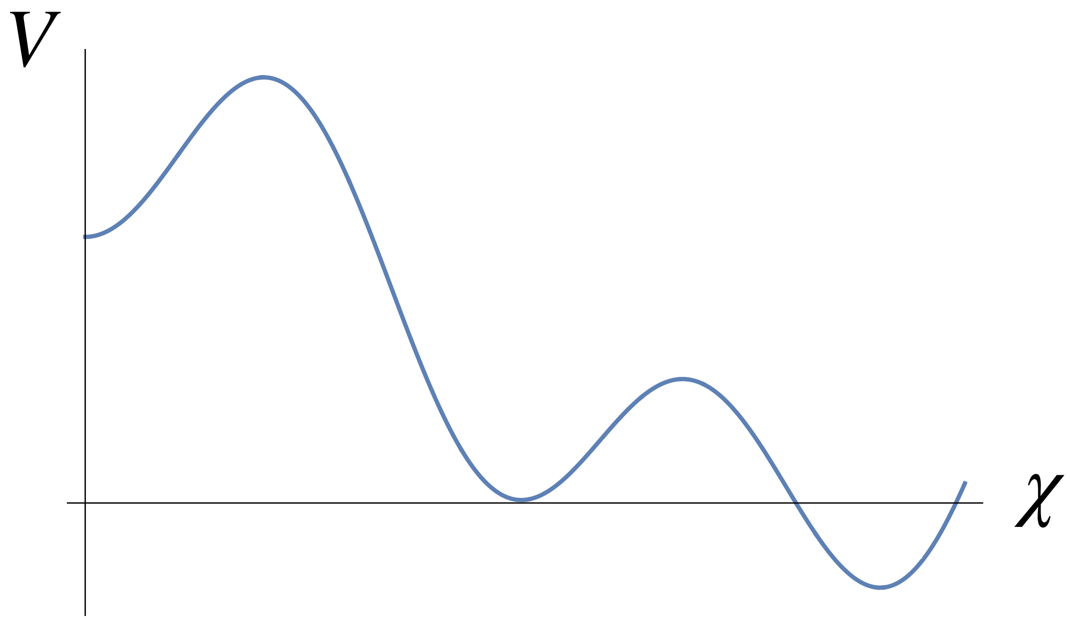
Volume-weighted measures

Probability gradients



Volume-weighted measures

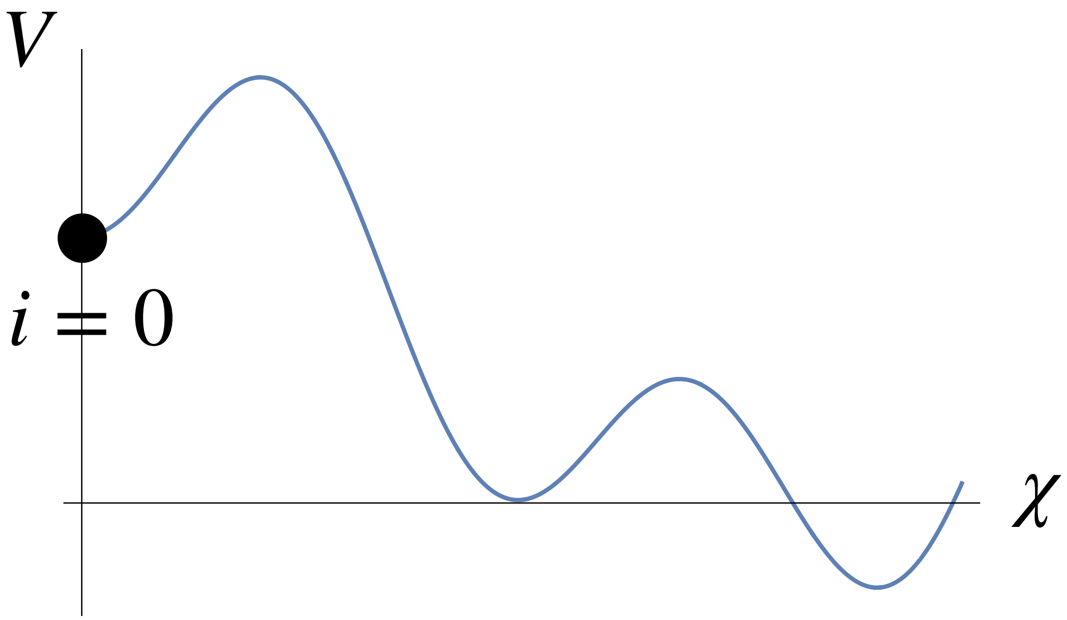
Probability gradients



$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i \to j} + \sum_{j \neq i} P_j \Gamma_{j \to i} + 3H_i P_i$$

Volume-weighted measures

Probability gradients



$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i\to j} + \sum_{j\neq i} P_j \Gamma_{j \to i} + 3H_i P_i$$

- Highest “parent” minimum

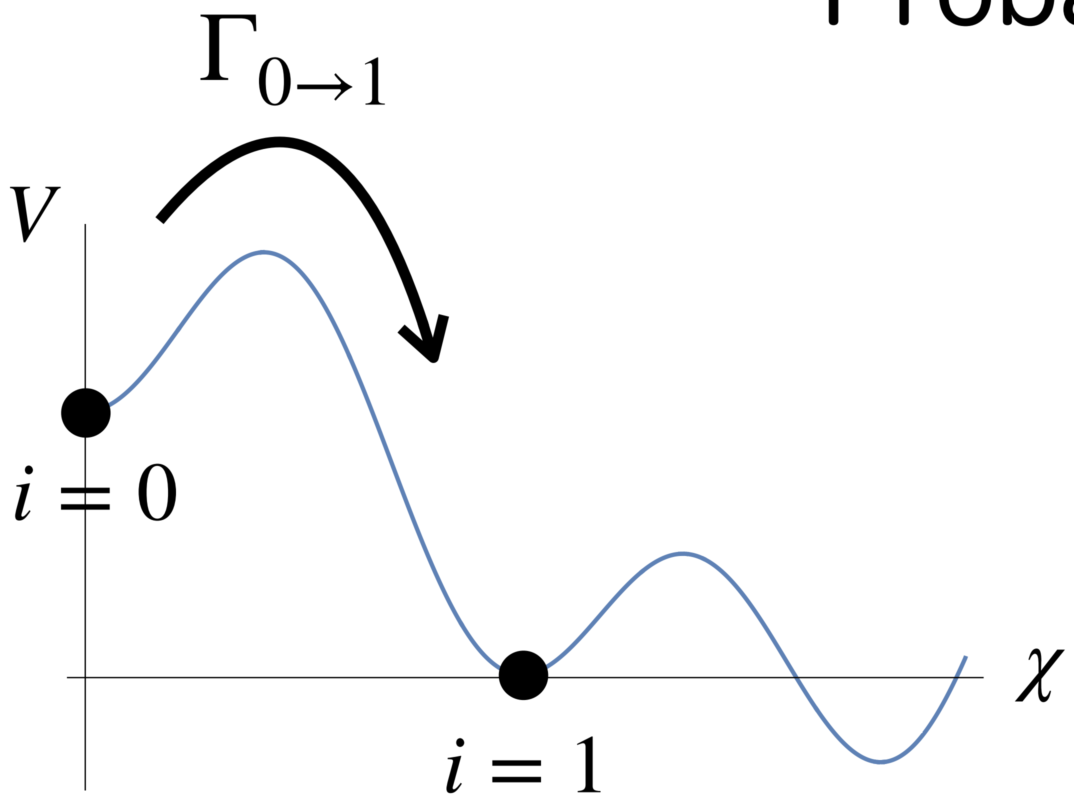
$$\dot{P}_0 \simeq 3H_0 P_0$$

eternal ‘stationary’ inflation:

$$P_0 = C_0\, e^{3H_0 t}$$

Volume-weighted measures

Probability gradients



$j=0$

$$\dot{P}_i = -P_i \sum_{j\neq i} \Gamma_{i\to j} + \sum_{j\neq i} P_j \Gamma_{j\to i} + 3H_i P_i$$

Volume-weighted measures

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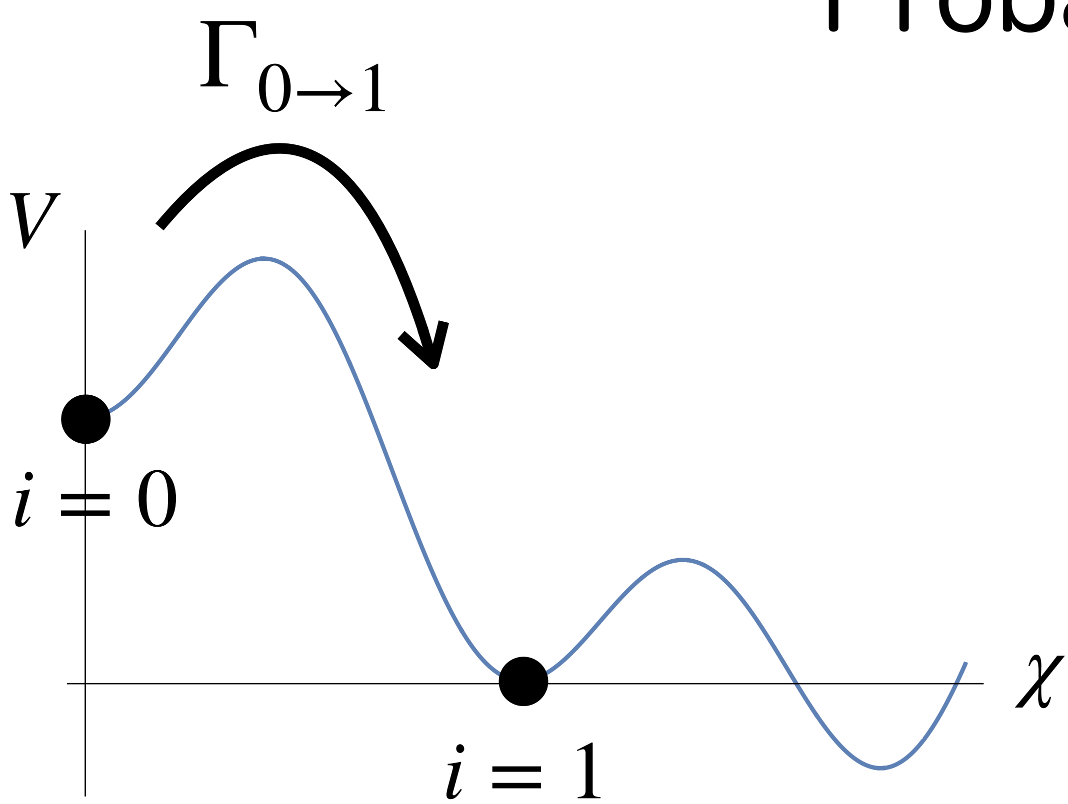


$j = 0$

$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i\to j} + \sum_{j\neq i} P_j \Gamma_{j\to i} + 3H_i P_i$$

- Lower vacuum:

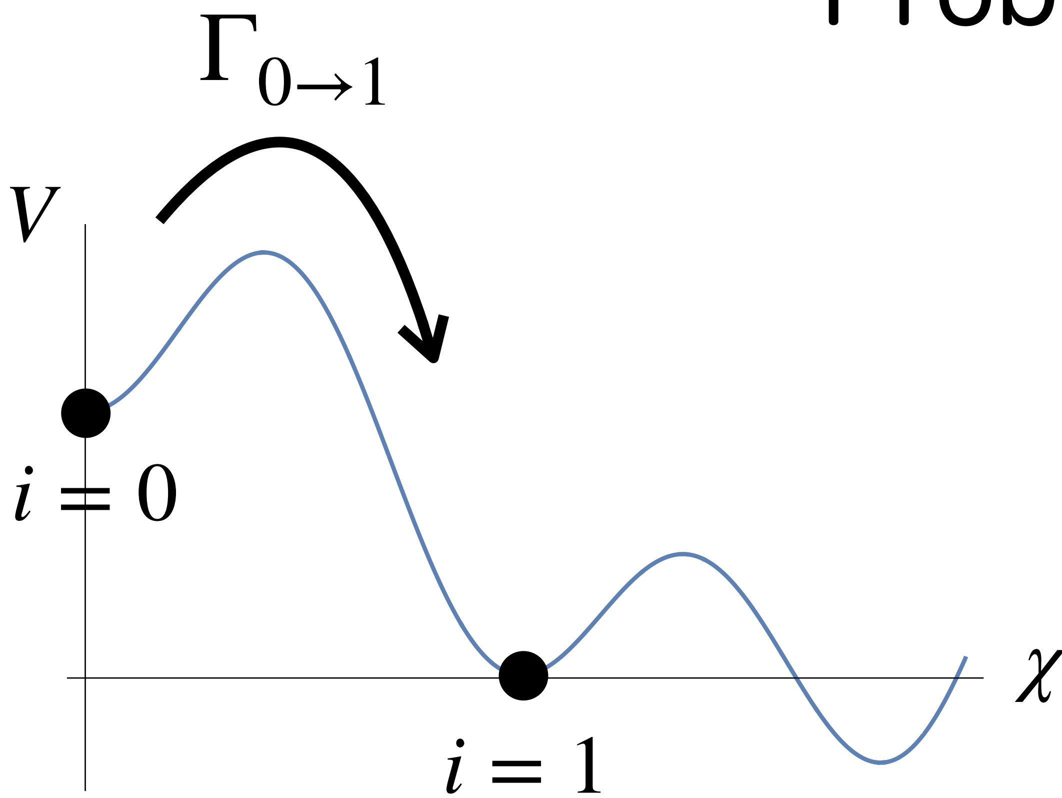
$$\dot{P}_1 \simeq 3H_1 P_1 + P_0 \Gamma_{0\to 1} \longrightarrow$$

eternal 'stationary' inflation:

$$P_1 = C_1 e^{3H_0 t}$$

Volume-weighted measures

Probability gradients



$$3H_0 P_1 = \dot{P}_1 \simeq 3H_1 P_1 + \underbrace{P_0 \Gamma_{0\to 1}}$$

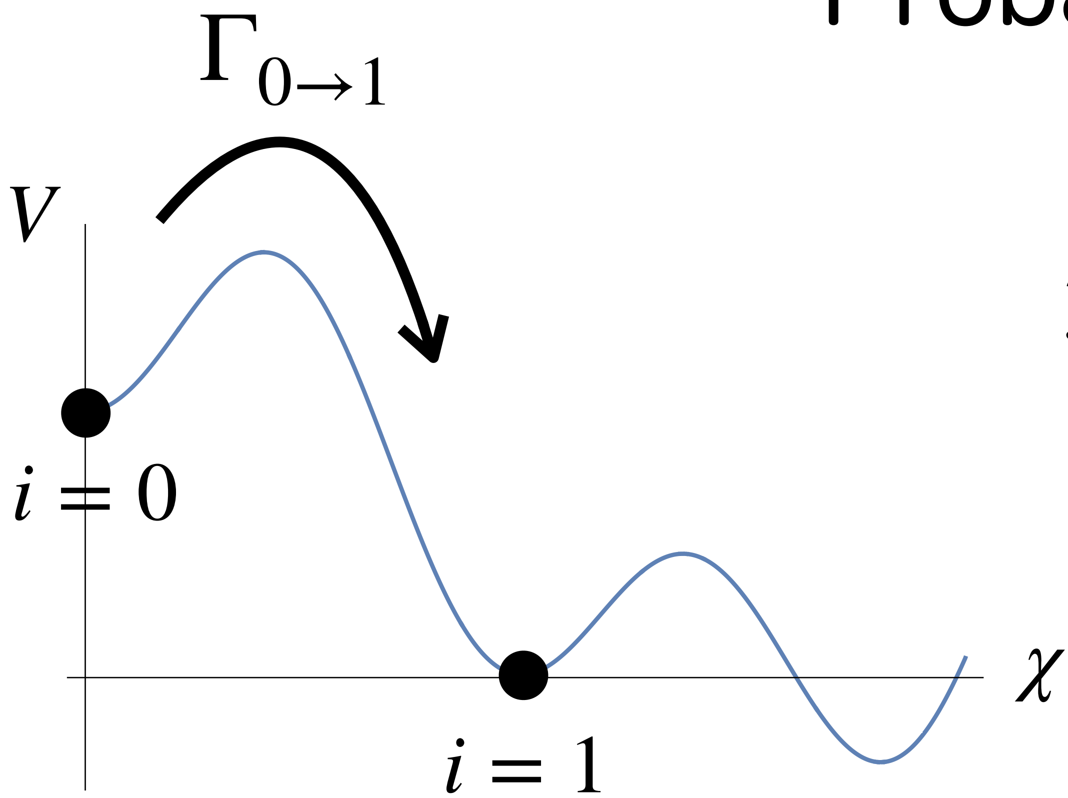
compensates "missing" expansion

$$\Rightarrow \quad P_1 = C_1 e^{3H_0 t}$$

$$\Rightarrow \quad C_1 = \frac{\Gamma_{0\to 1}}{3(H_0 - H_1)} C_0$$

Volume-weighted measures

Probability gradients



$$3H_0 P_1 = \dot{P}_1 \simeq 3H_1 P_1 + \underbrace{P_0 \Gamma_{0\to 1}}_{\text{compensates "missing" expansion}}$$

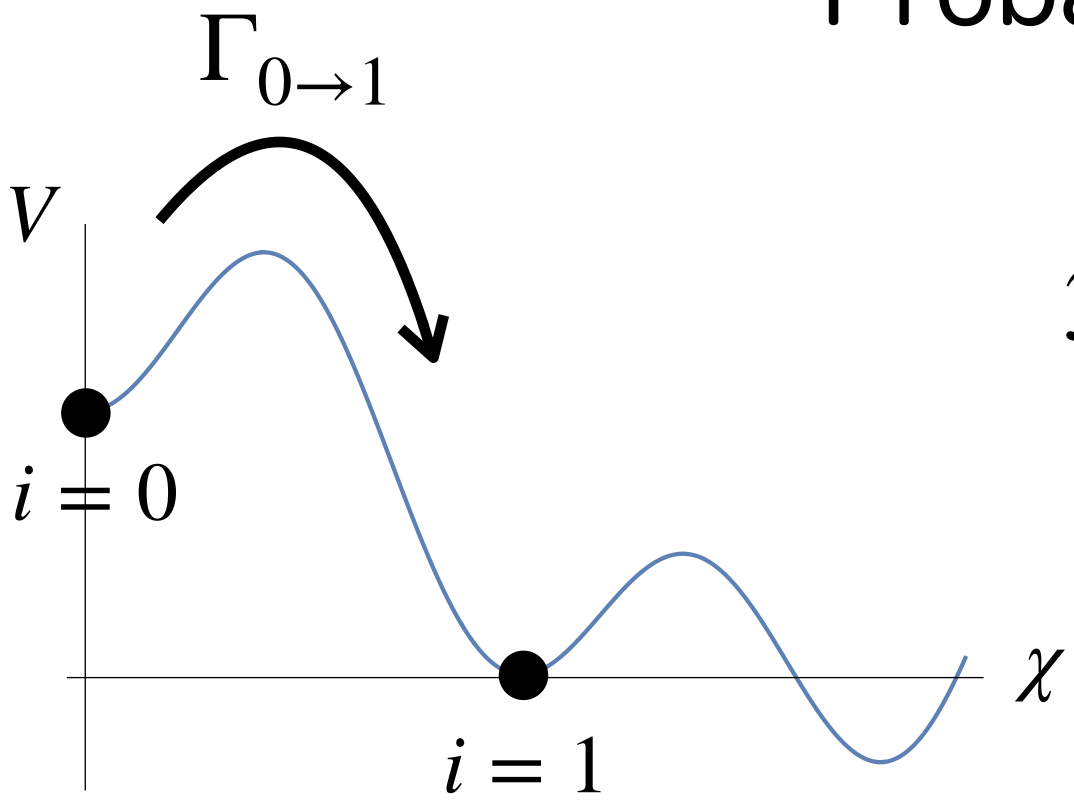
compensates “missing” expansion

$$\Rightarrow \quad P_i = C_i e^{3H_0 t}$$

$$\Rightarrow \quad C_i = \frac{\Gamma_{(i-1)\to i}}{3(H_0 - H_i)} C_{(i-1)}$$

Volume-weighted measures

Probability gradients



$$3H_0P_1 = \dot{P}_1 \simeq 3H_1 P_1 + \underbrace{P_0\Gamma_{0\to1}}$$

compensates "missing" expansion

$$\Rightarrow \quad P_i = C_i\, e^{3H_0 t}$$

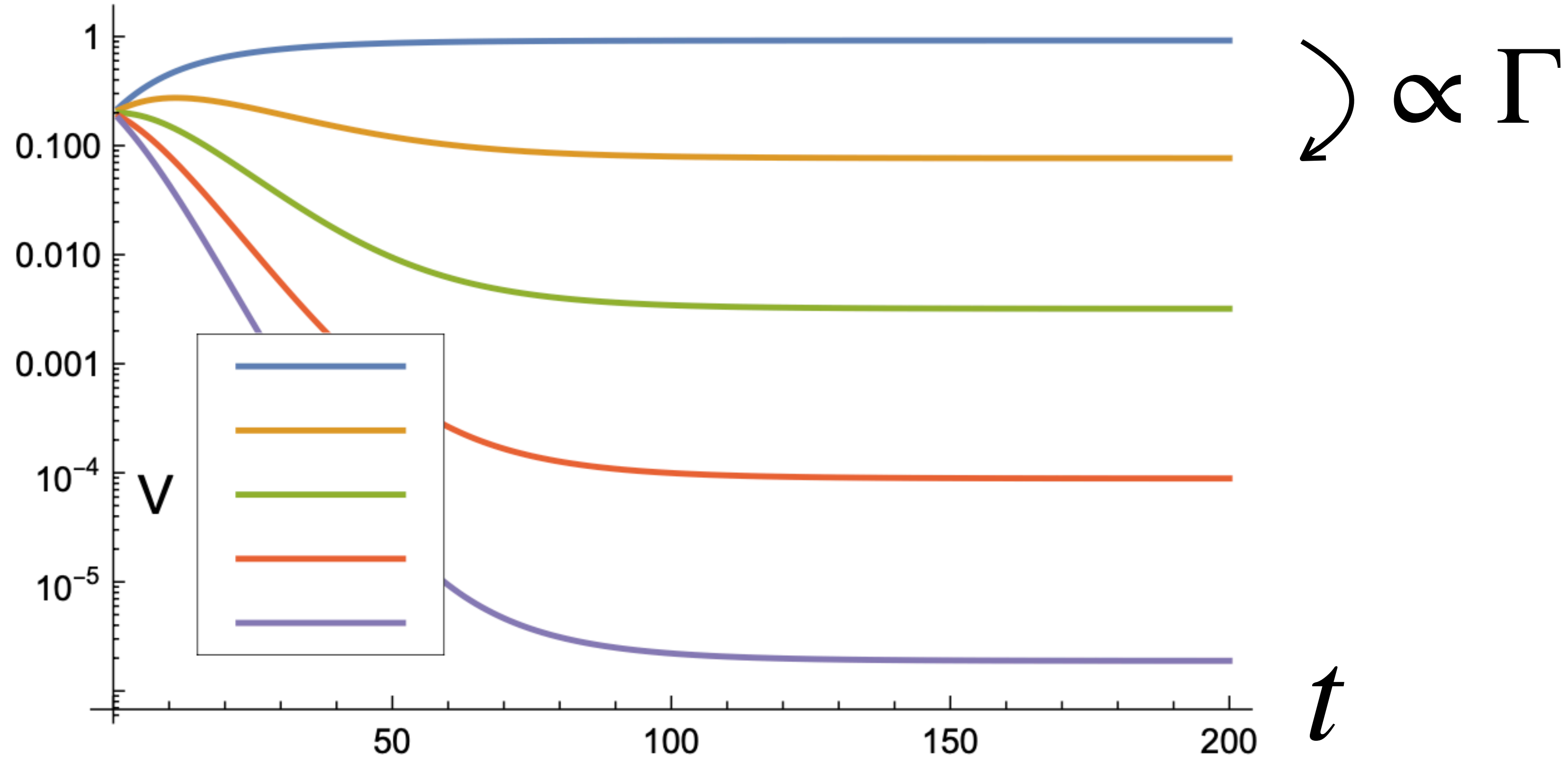
$$\Rightarrow \quad C_i = \left[\prod_{k=0}^{i} \frac{\Gamma_{(k-1)\to k}}{3(H_0 - H_k)}\right] C_0$$

Volume-weighted measures

Probability gradients

numerically:

$P/e^{3H_0 t}$



HM tunneling (|m|<H): $\Gamma \propto \exp\left[-\frac{8\pi^2}{3}\frac{\Delta V_B}{H_j^4}\right]$

Volume-weighted measures

Probability gradients

numerically:

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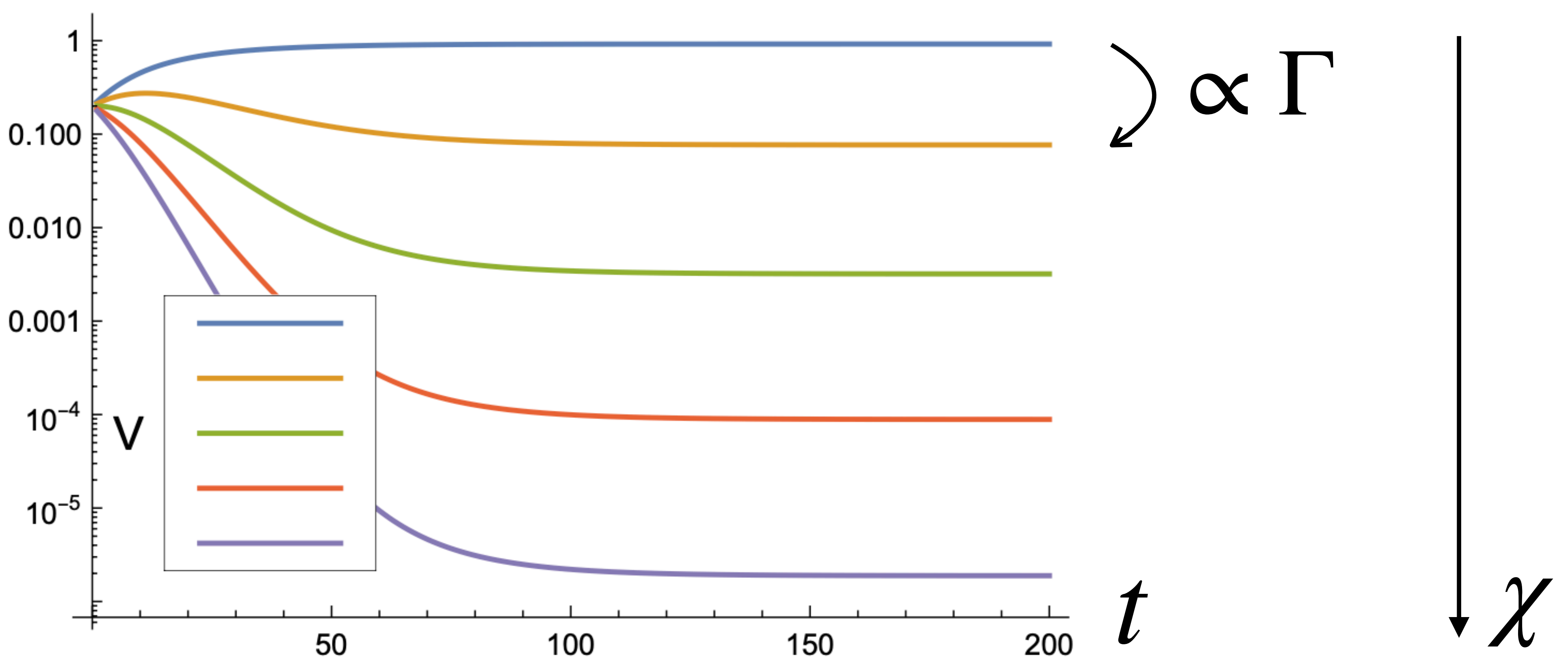

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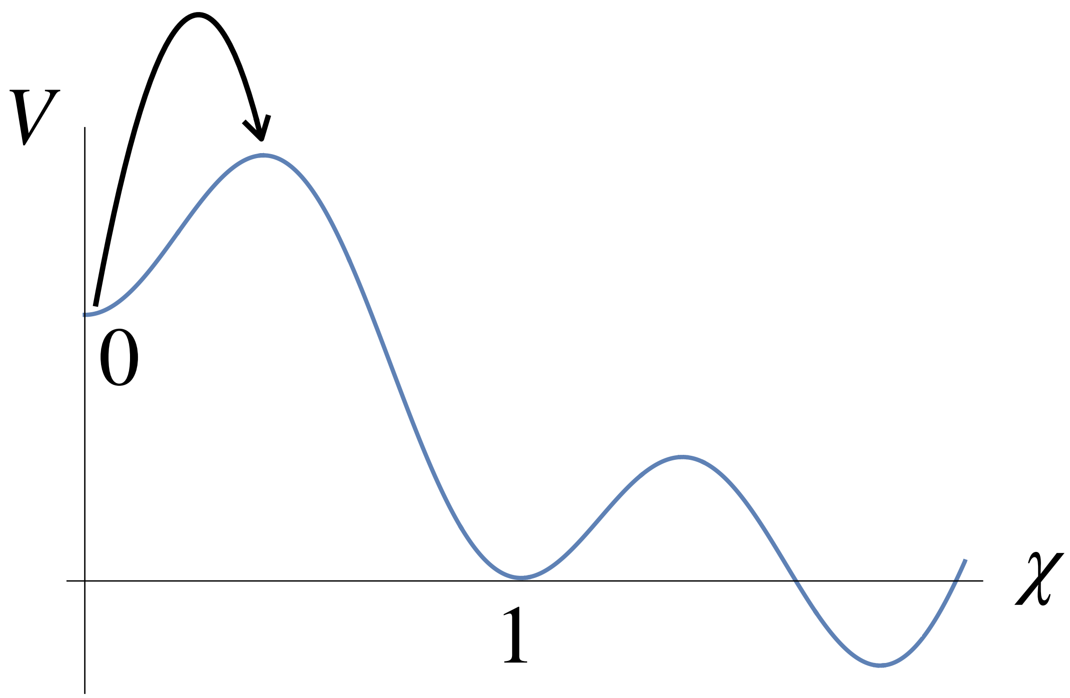
Volume-weighted measures

Stochastic approach

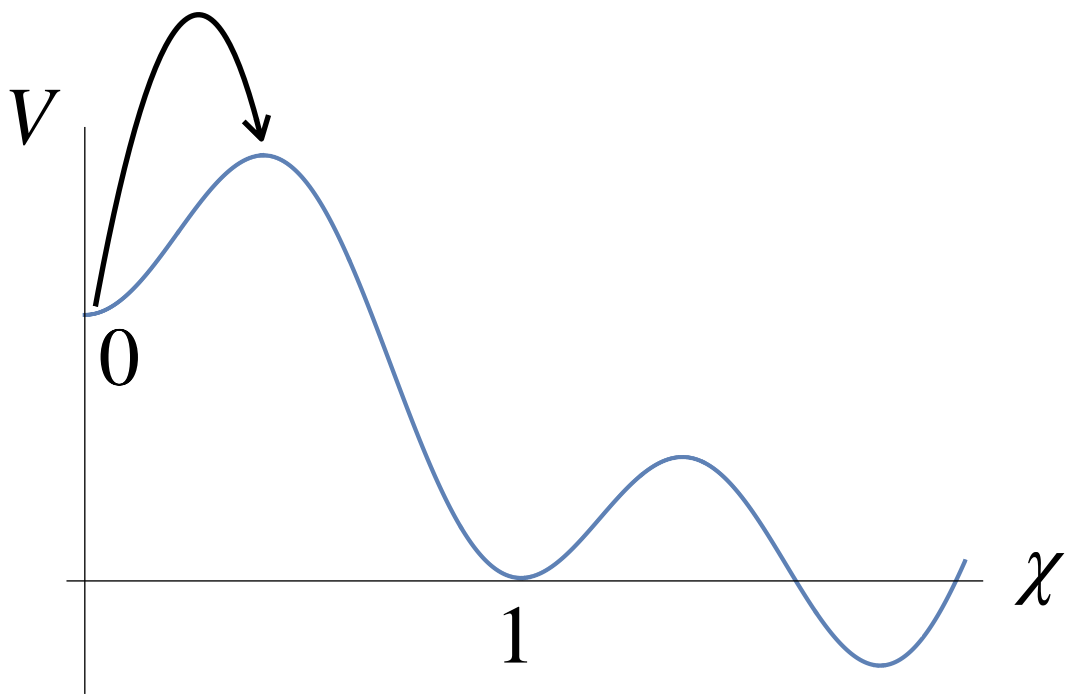


HM tunneling

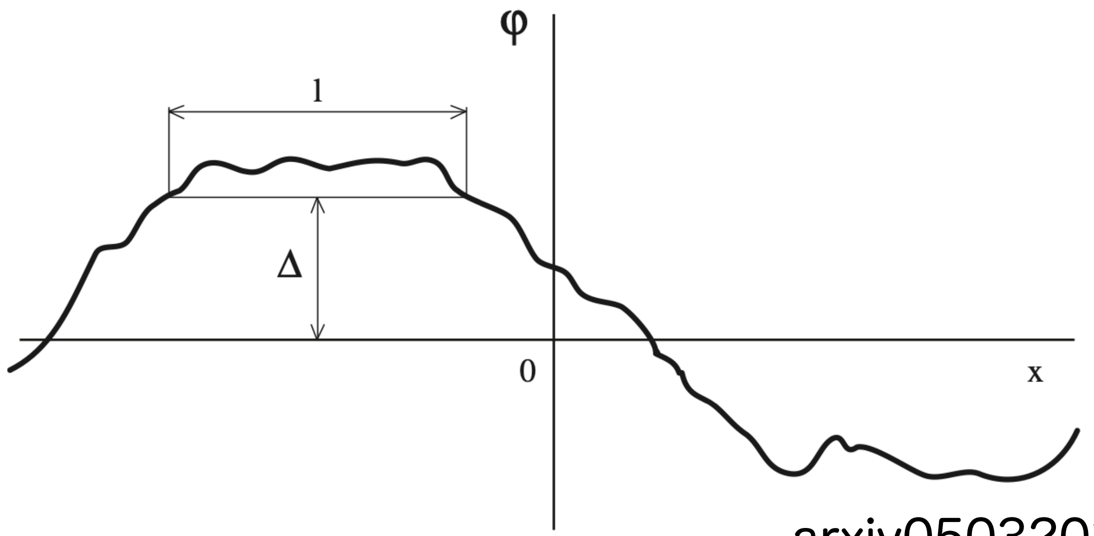
$$\Gamma_{j\to i} \sim H_j \exp\left[-\frac{8\pi^2}{3}\frac{\Delta V_B}{H_j^4}\right]$$

Volume-weighted measures

Stochastic approach



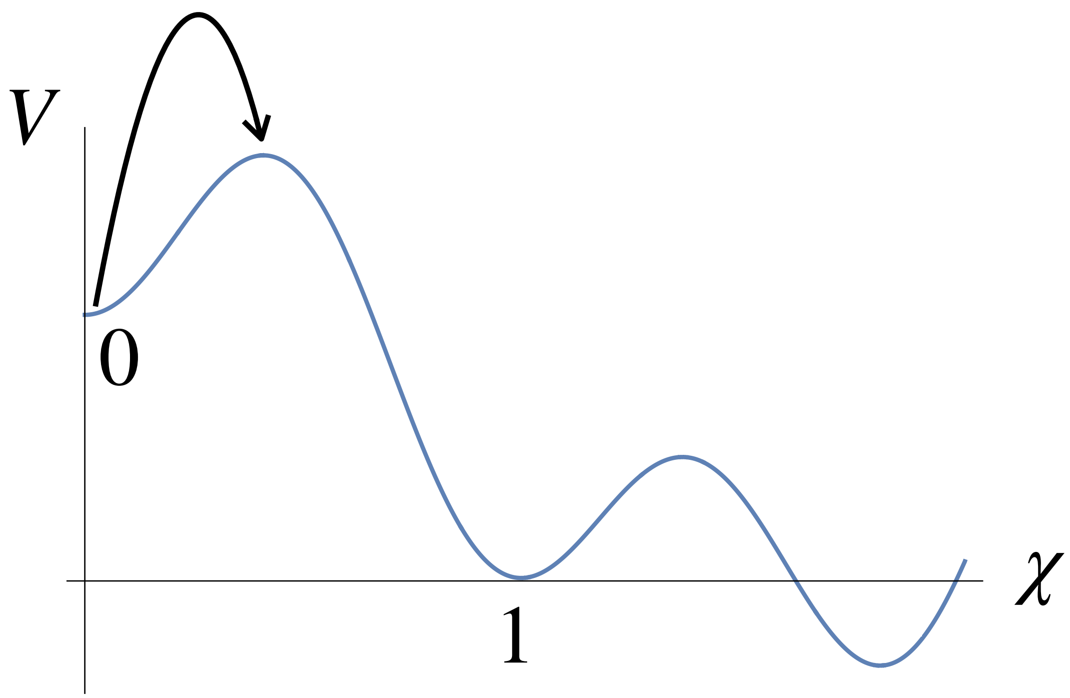
quantum spread



arxiv0503203

Volume-weighted measures

Stochastic approach



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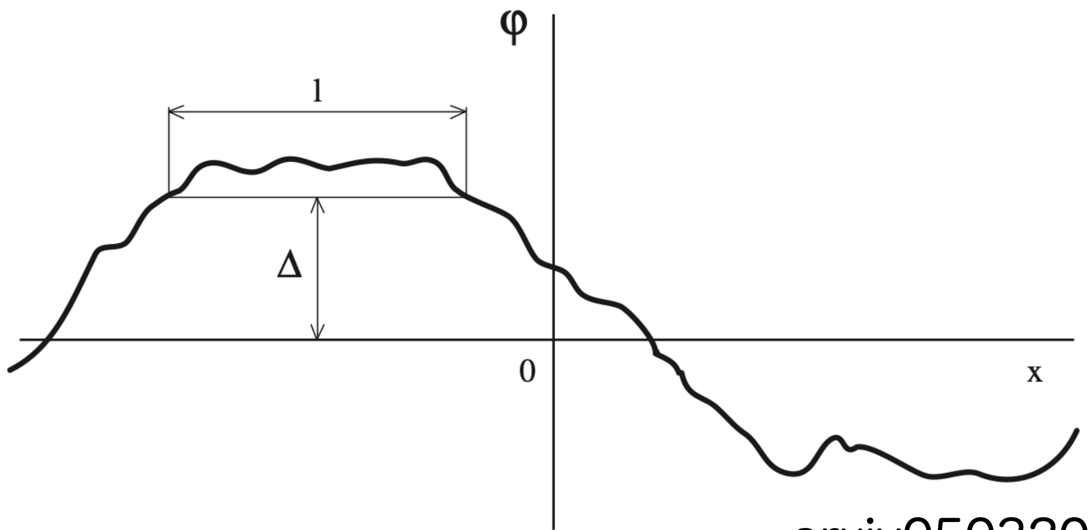


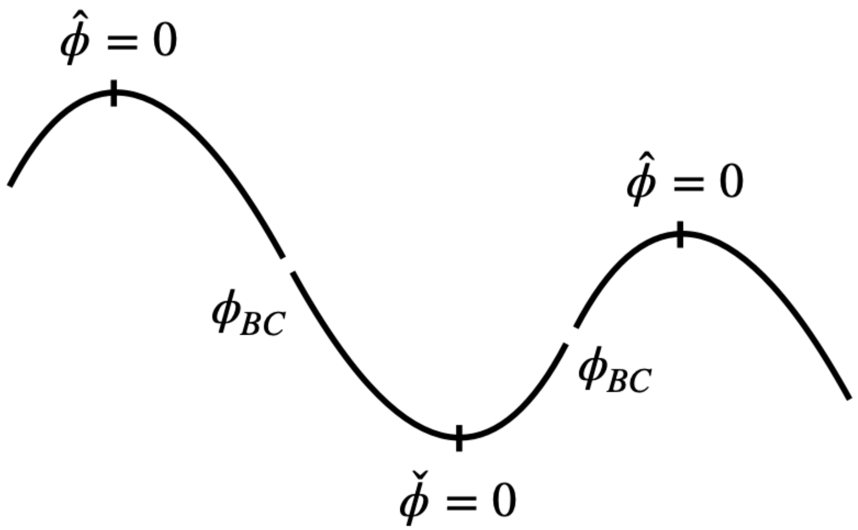
arxiv0503203

$$P(\chi_i) \to P(\chi)$$

$$\dot{P} = \frac{\partial}{\partial\phi}\left(\frac{H^{3(1-\beta)}}{8\pi^2}\frac{\partial}{\partial\phi}(H^{3\beta}P)\right) + \frac{\partial}{\partial\phi}\left(\frac{V'}{3H}P\right) + 3HP$$

Volume-weighted measures

Stochastic approach



$$V = \Lambda + \frac{1}{2} m^2 \phi^2$$

general solution:

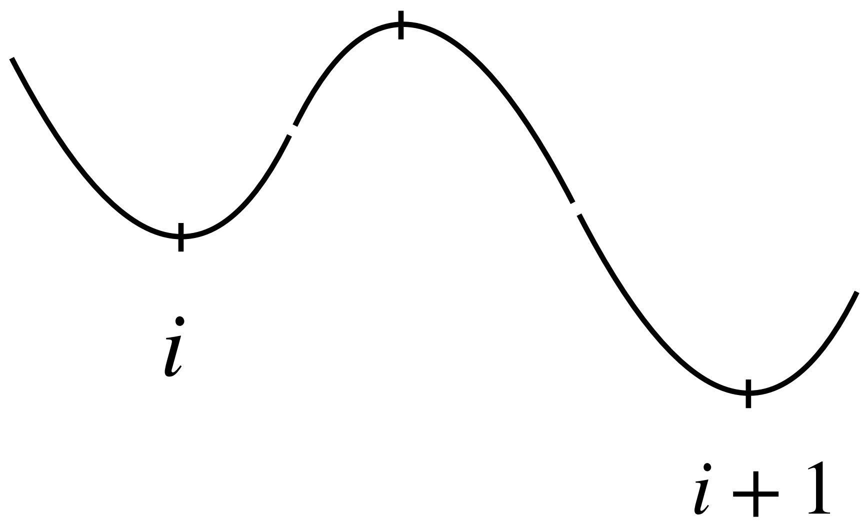
eigenmodes of $\nu \propto -H_s^2 + \dots$

Giudice,McCullough,You, 2105.08617

$$P_\nu = \exp\left[-A\phi^2\right] \left\{ \mathbf{c}_+ D_\nu \left[B\phi\right] + \mathbf{c}_- D_\nu \left[-B\phi\right] \right\} e^{3H_s t}$$

Volume-weighted measures

Matching



P drop between 2 minima:

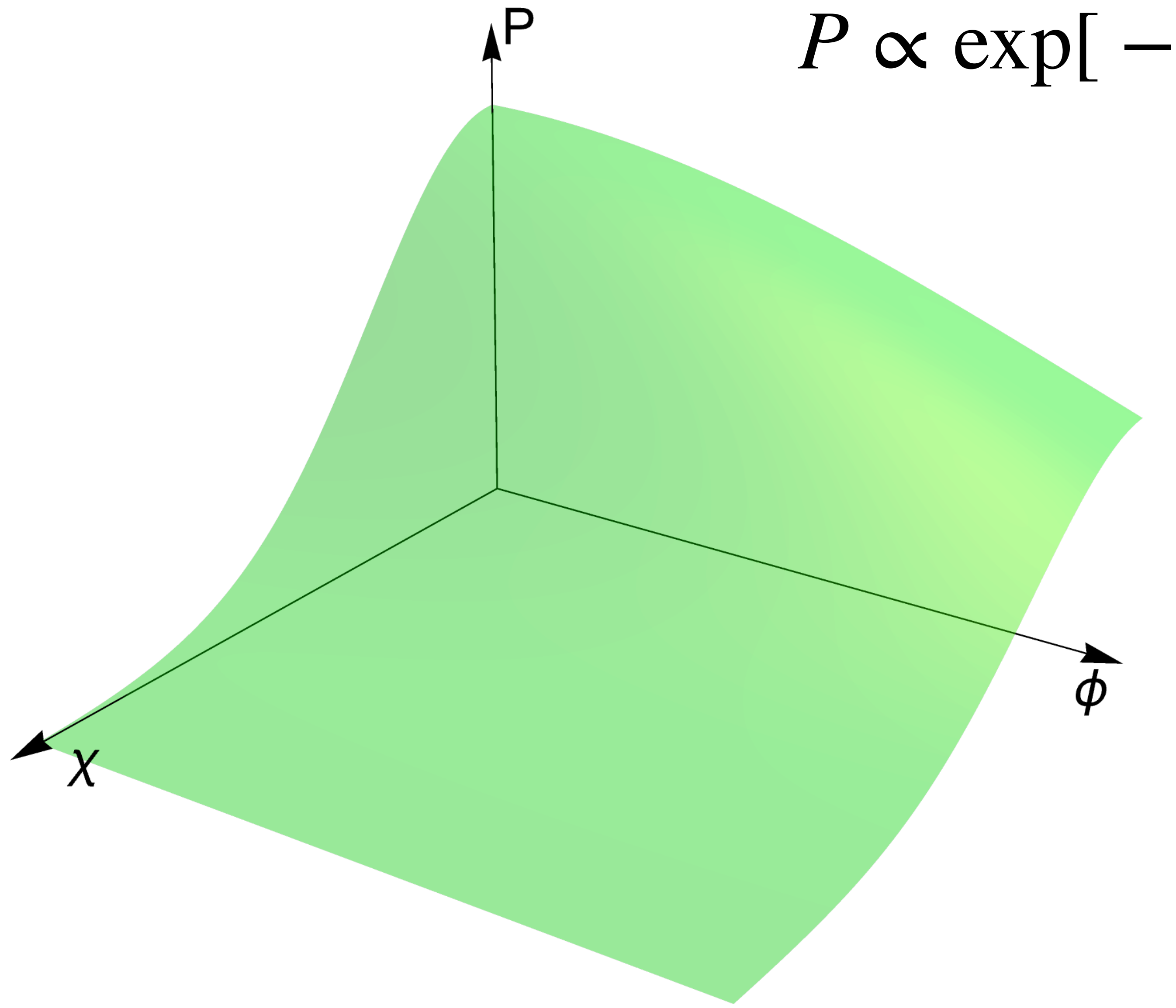
$$\frac{\check{P}_{i+1}(0)}{\check{P}_i(0)} \simeq \frac{\Gamma[-\hat{\nu}_i]\Gamma[-\check{\nu}_{i+1}]}{2\pi}|B\phi_{\rm BC}|^{2(\check{\nu}_i+\hat{\nu}_i+1)} e^{-\frac{8\pi^2}{3}\frac{\Delta V_B}{H^4}} + \mathcal{O}(\epsilon^2)$$

$$\left(\epsilon \sim \frac{H^4}{m_p^2 m^2}\right)$$

Volume-weighted measures

We got the gradients

$$P \propto \exp[-\#\phi] \times \exp[-\#\chi]$$



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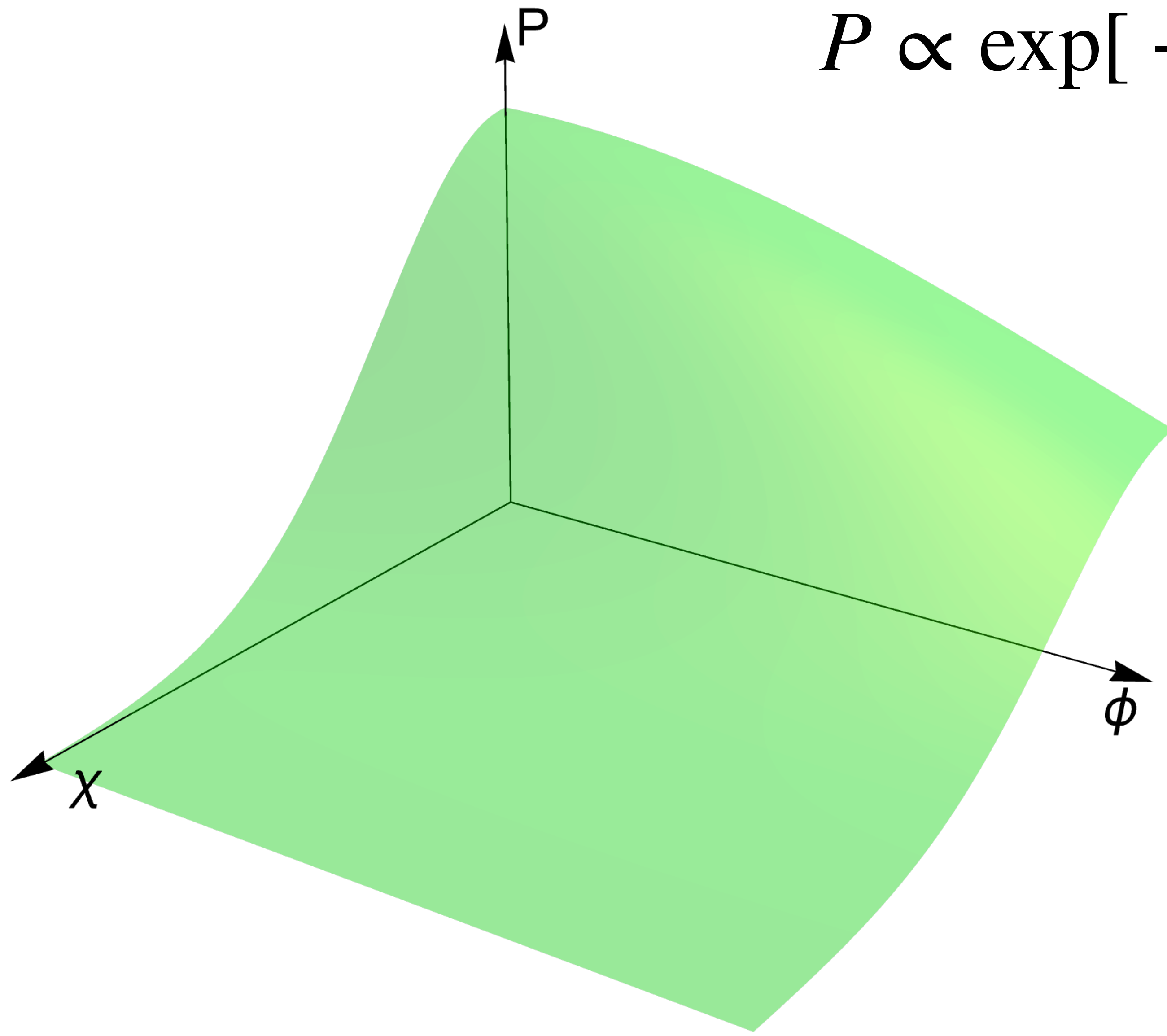
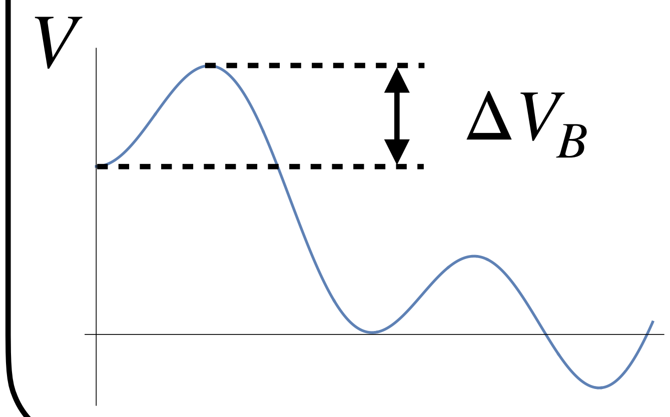


$$P \propto \exp[-\#\phi] \times \exp[-\#\chi]$$

$$\propto \Delta V_B(\phi, \chi)$$



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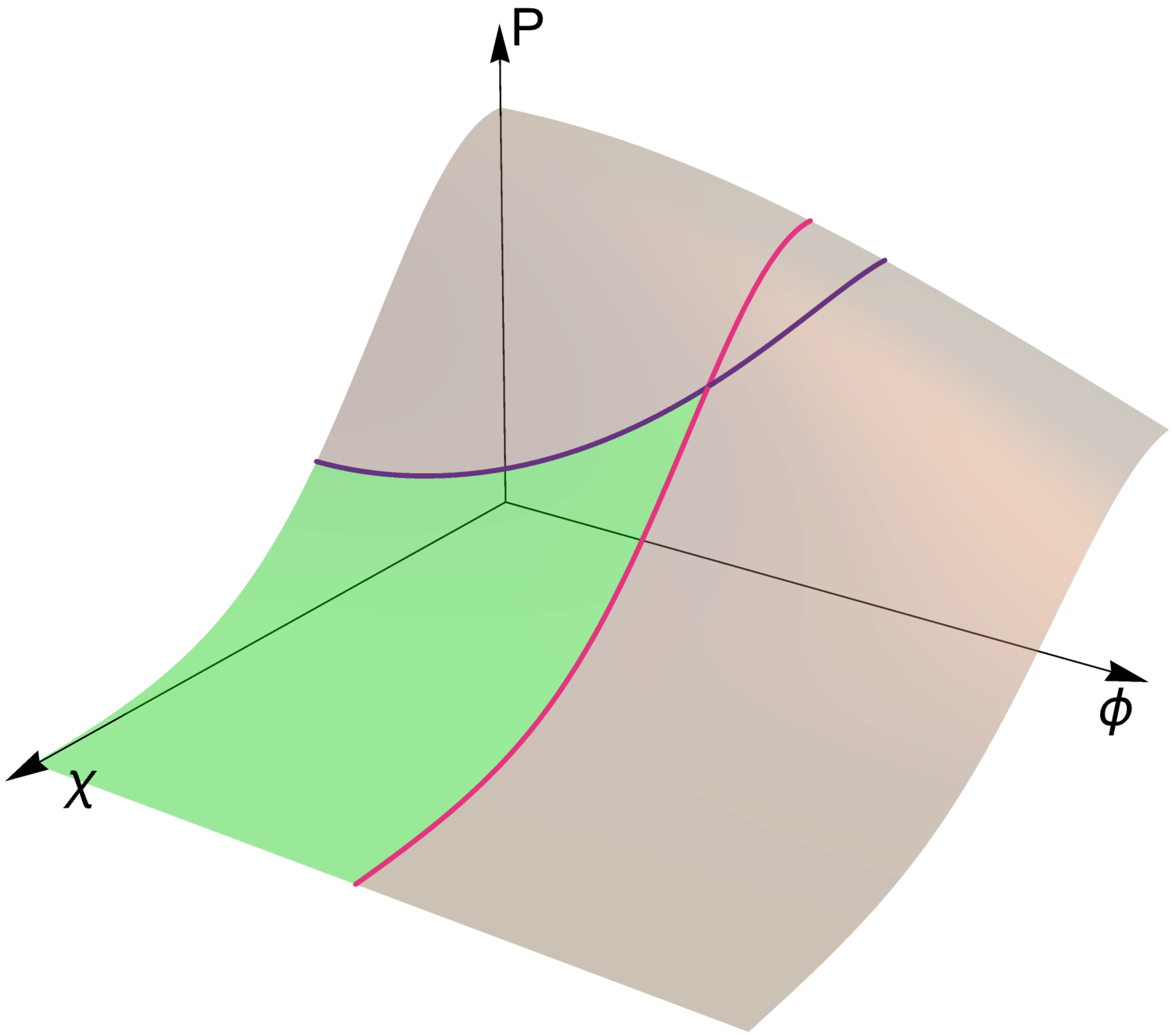
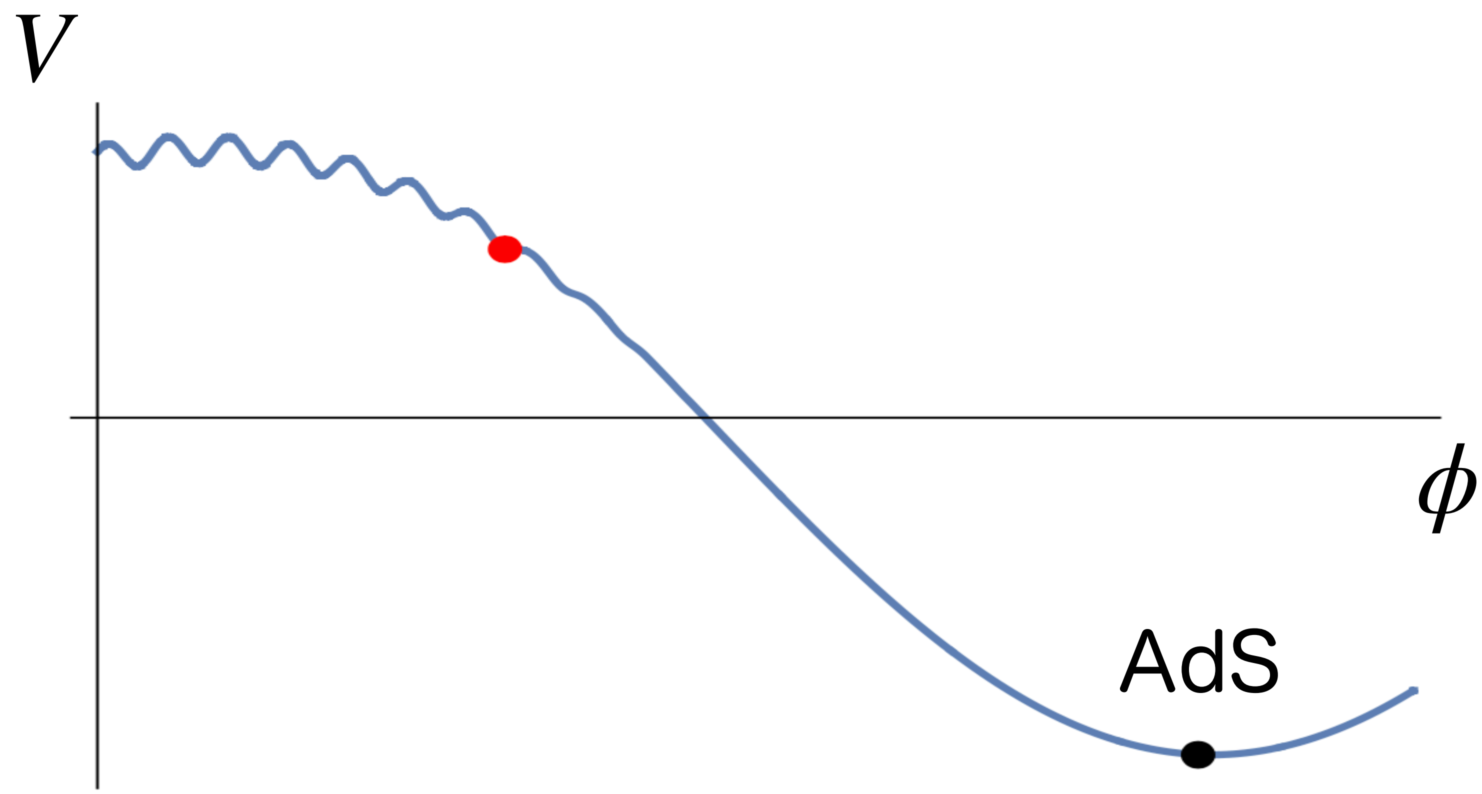


We need to scan mH and introduce the boundaries

mH and CC from gradients & boundaries

Higgs-VEV dependent critical boundary



'inverted'
GKR relaxion
1504.07551

$$V(\phi, h) \supset \mu_\phi^2 h^2 \cos(\phi/f) + M^2 h^2 \cos(\phi/F)$$

mH and CC from gradients & boundaries

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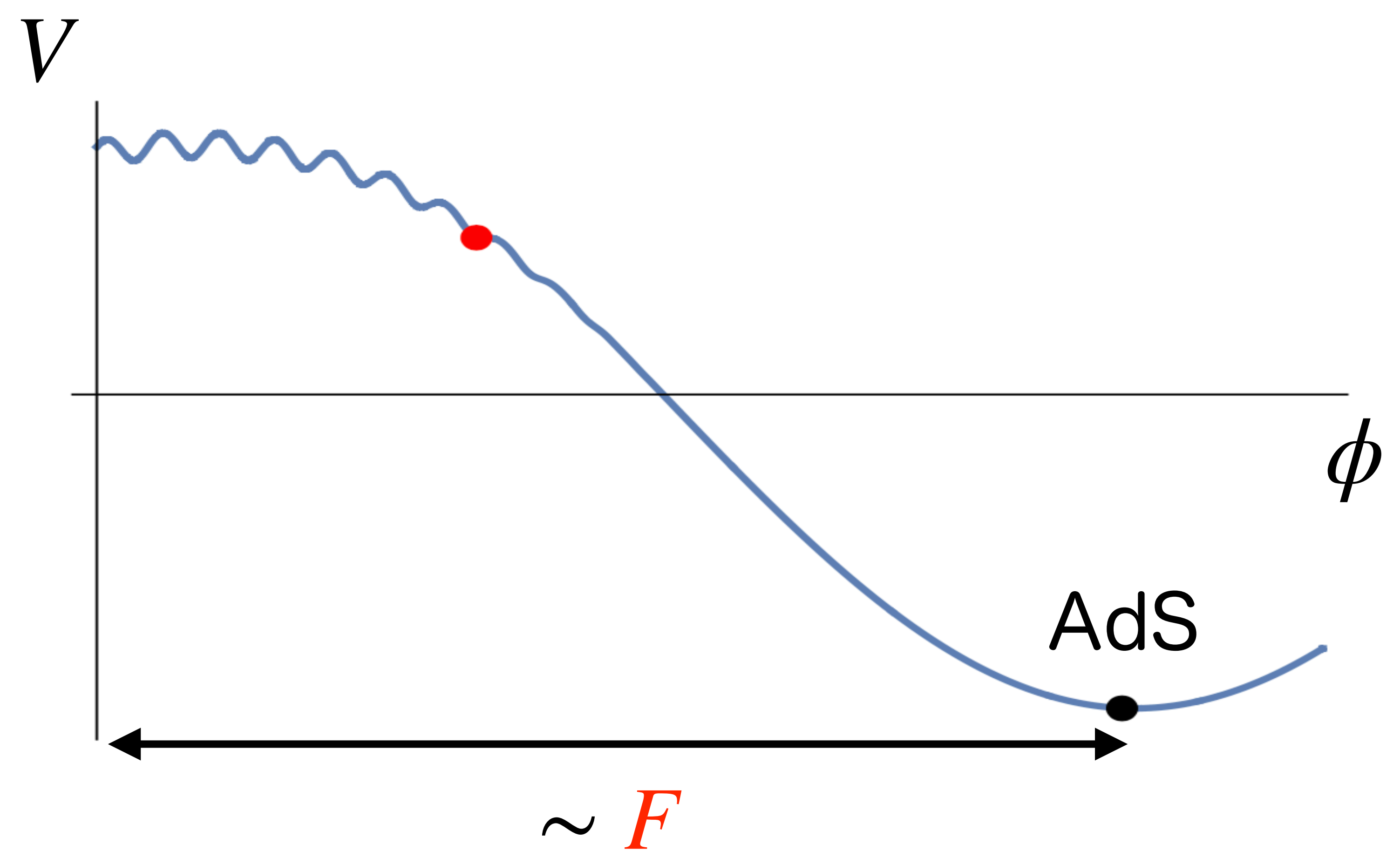

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$$V(\phi, h) \supset \mu_\phi^2 h^2 \cos(\phi/f) + M^2 h^2 \cos(\phi/F)$$

$$m_h^2 = M^2 \cos(\phi/F) + \cdots$$

mH and CC from gradients & boundaries

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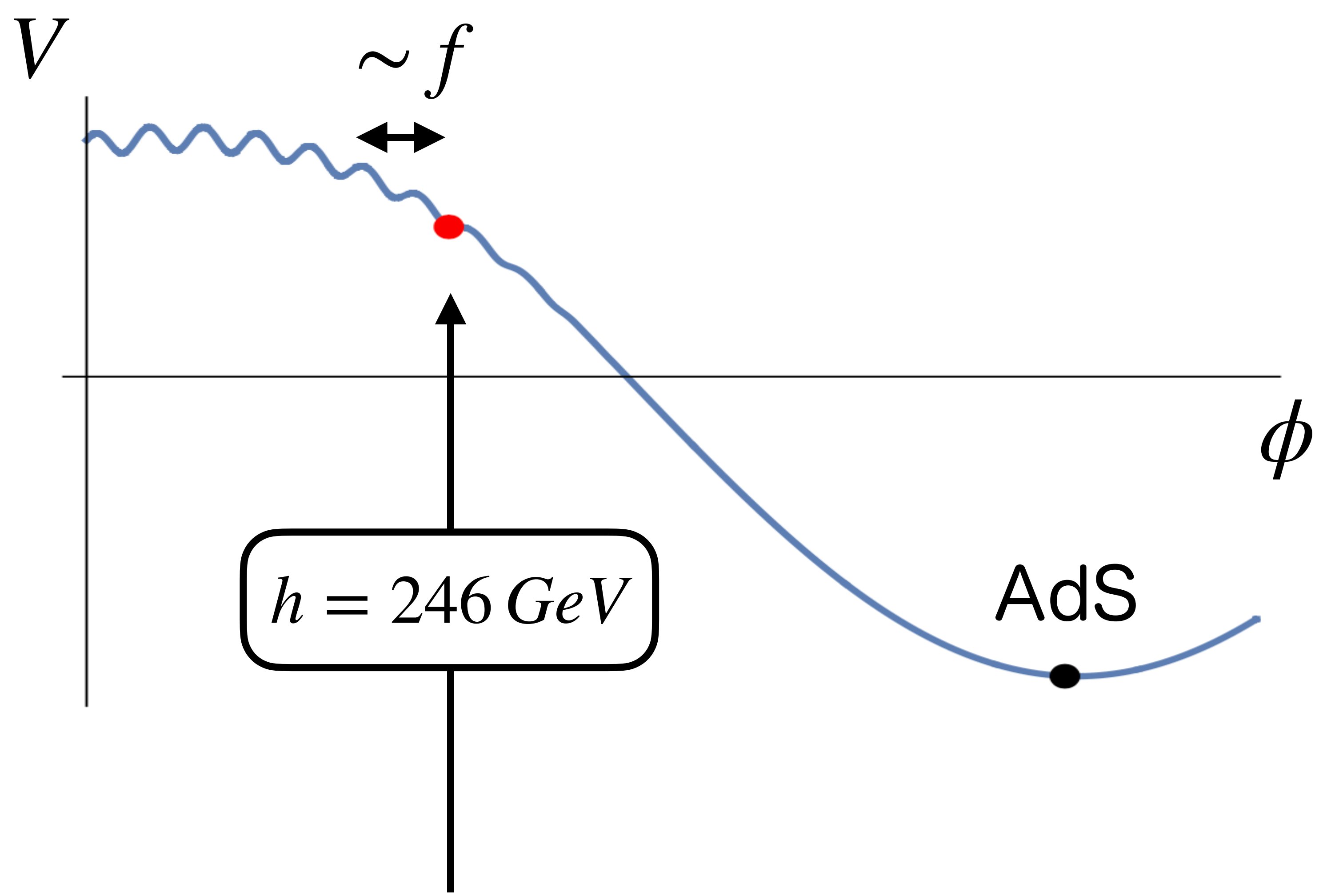

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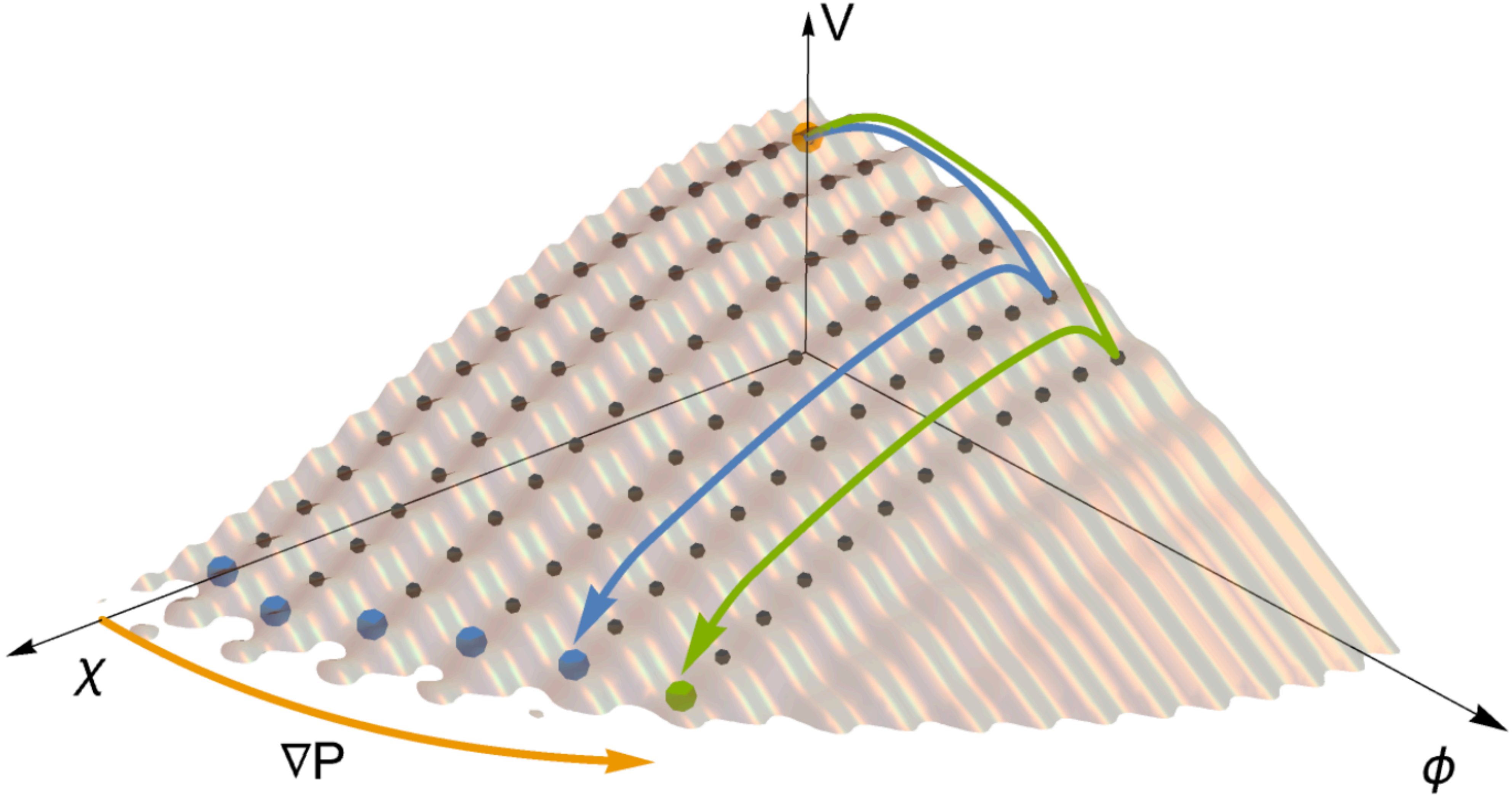
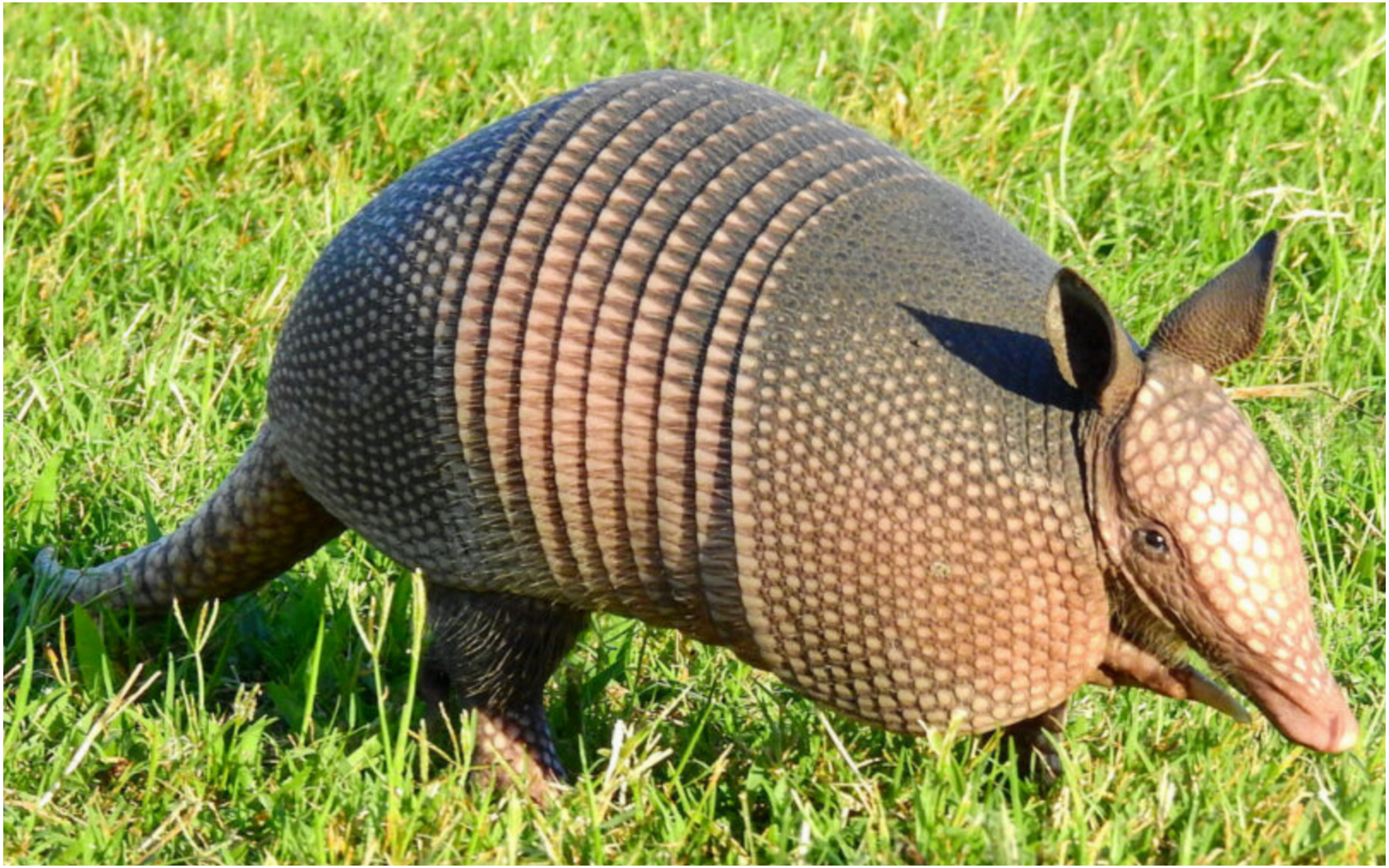


factorization:

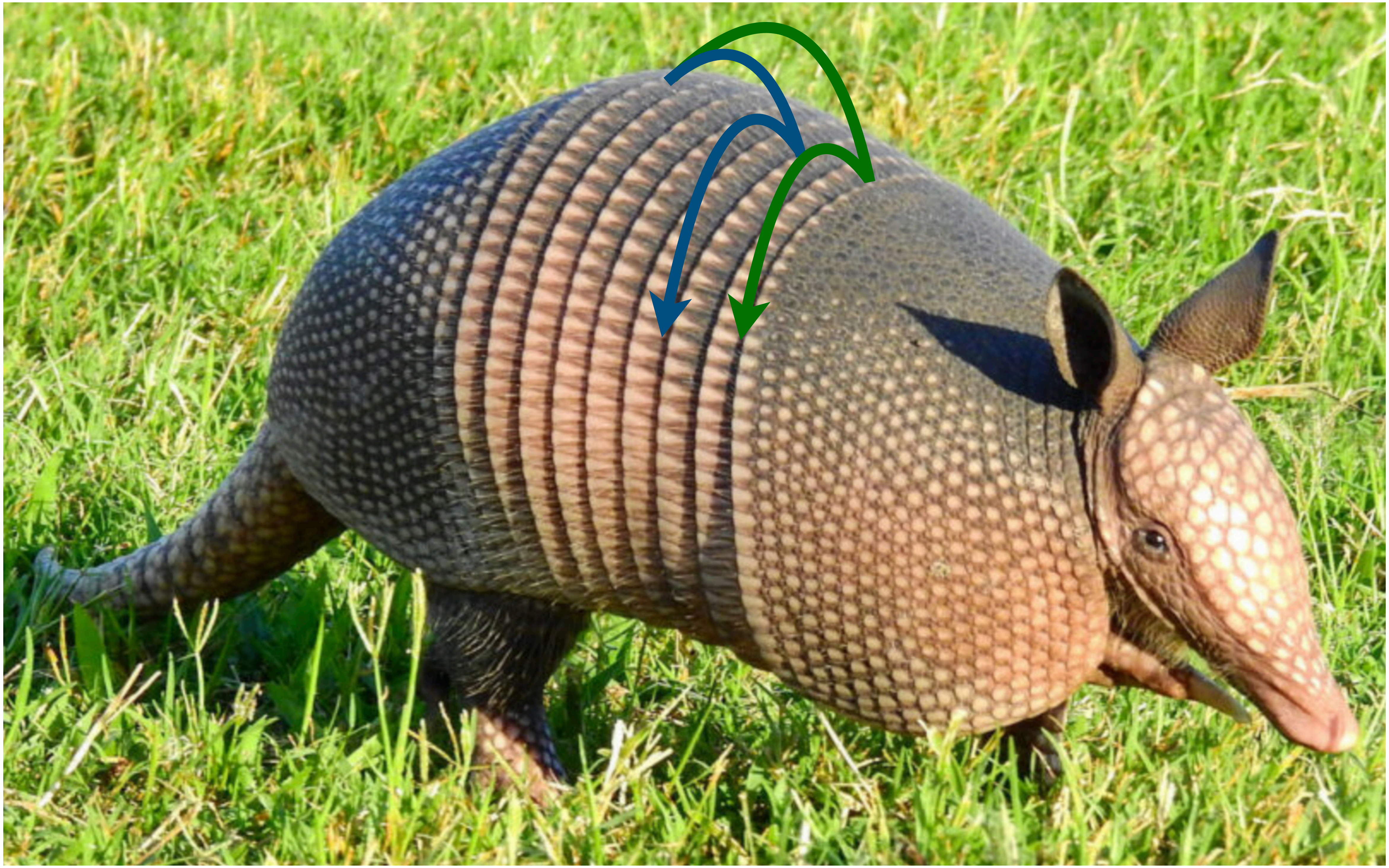
$$P(\phi,\chi) \simeq P(\phi)\,P(\chi)$$

$$\rightarrow \quad \frac{P_{\text{green}}}{P_{\text{blue}}} \sim \frac{\Gamma_\phi}{\Gamma_\chi} \gg 1$$

Armadillo

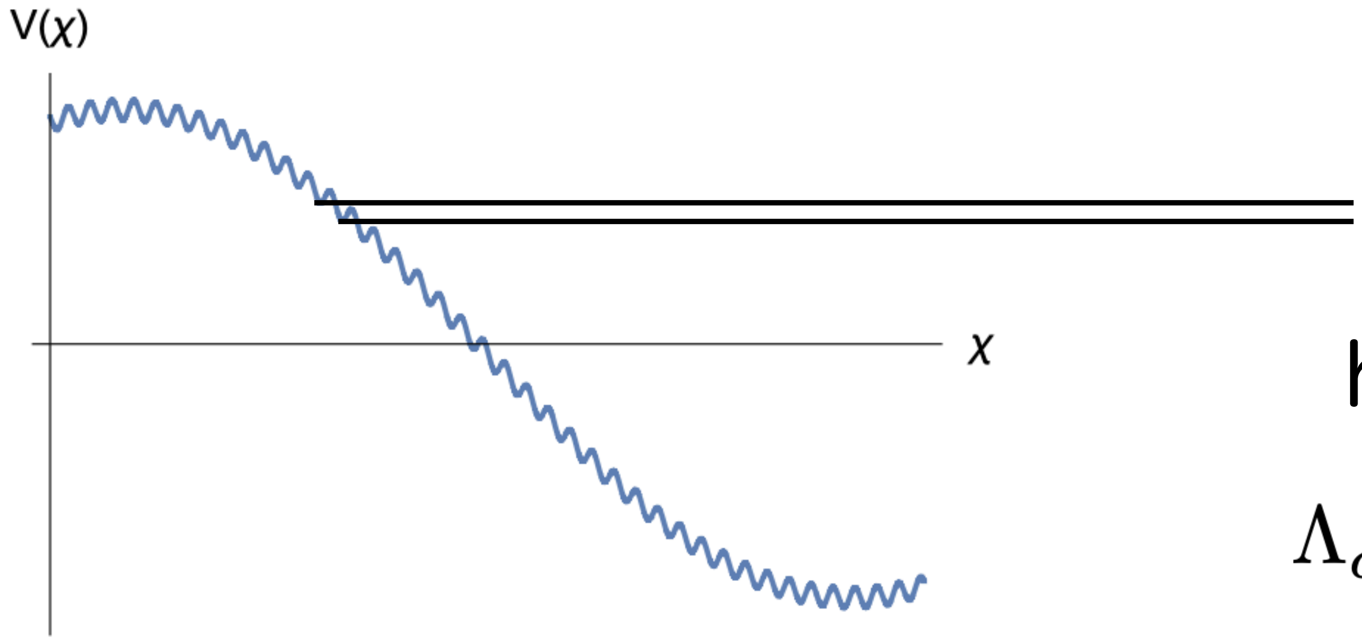


Armadillo



mH and CC from gradients & boundaries

CC solution?



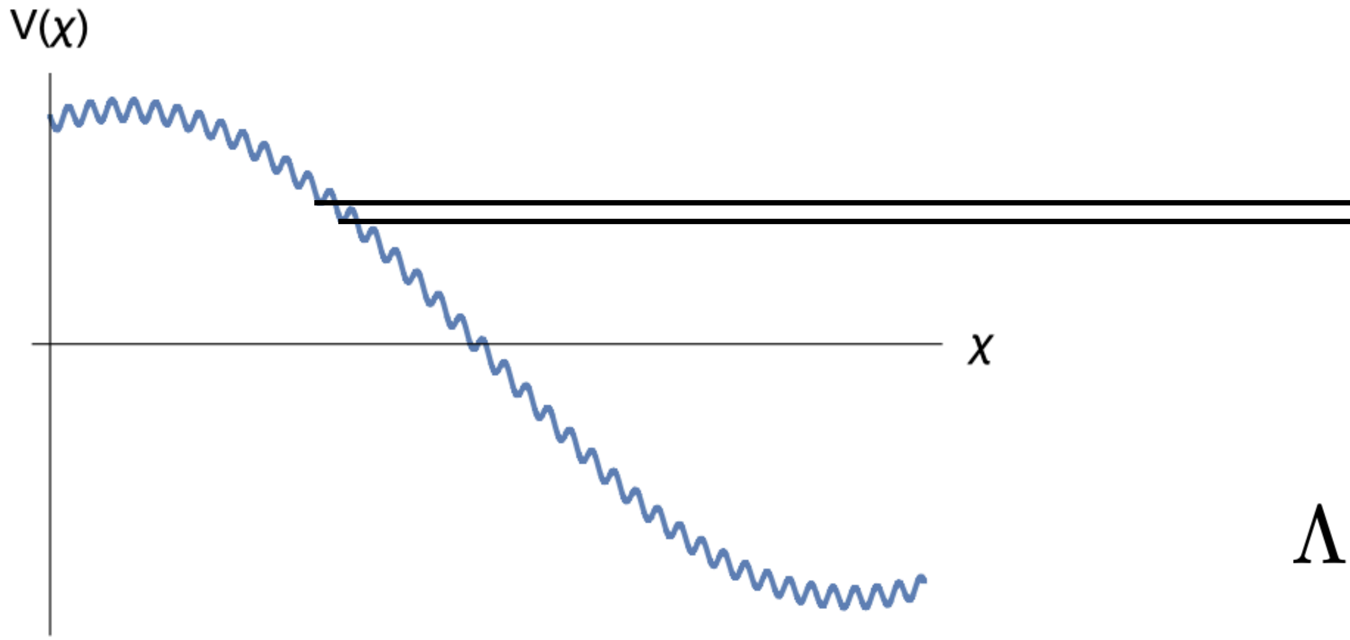
$$\Delta\Lambda_{cc\,\chi} \simeq M_\chi^4/N_\chi$$

has to be within

$$\Lambda_{cc(obs.)} \simeq 10^{-47}\mathrm{GeV}^4 \quad (1)$$

mH and CC from gradients & boundaries

CC solution?



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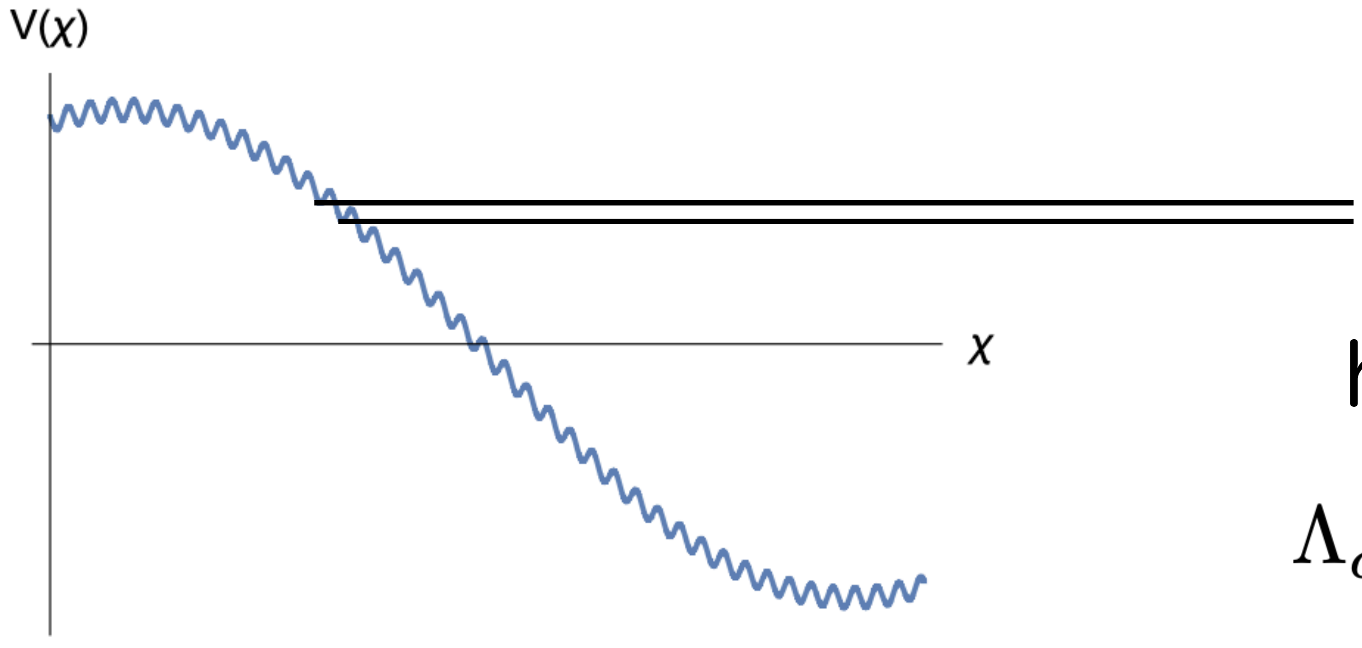
In addition, $P(\chi)$ prefers less tunnelings, hence higher Λ, close to the upper anthropic bound $\sim\ 10^3\Lambda_{cc(obs.)}$

$\Rightarrow$ one needs a sufficiently mild grad $P(\chi)$ (2)

mH and CC from gradients & boundaries

CC solution?



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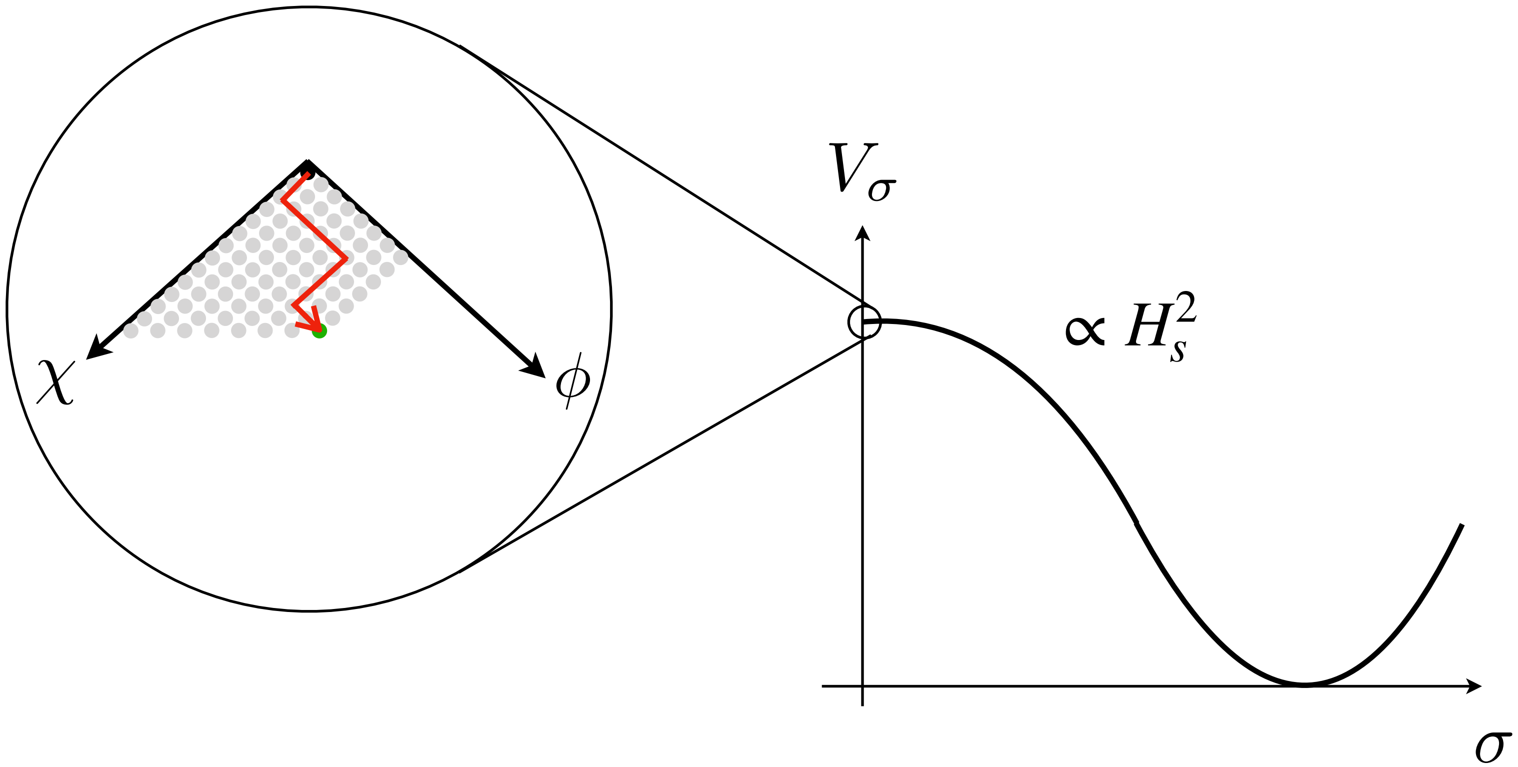
$\Rightarrow$ one needs a sufficiently mild grad $P(\chi)$ (2)

We evade (1), (2) by assuming some additional fine-scanning sector.

mH and CC from gradients & boundaries

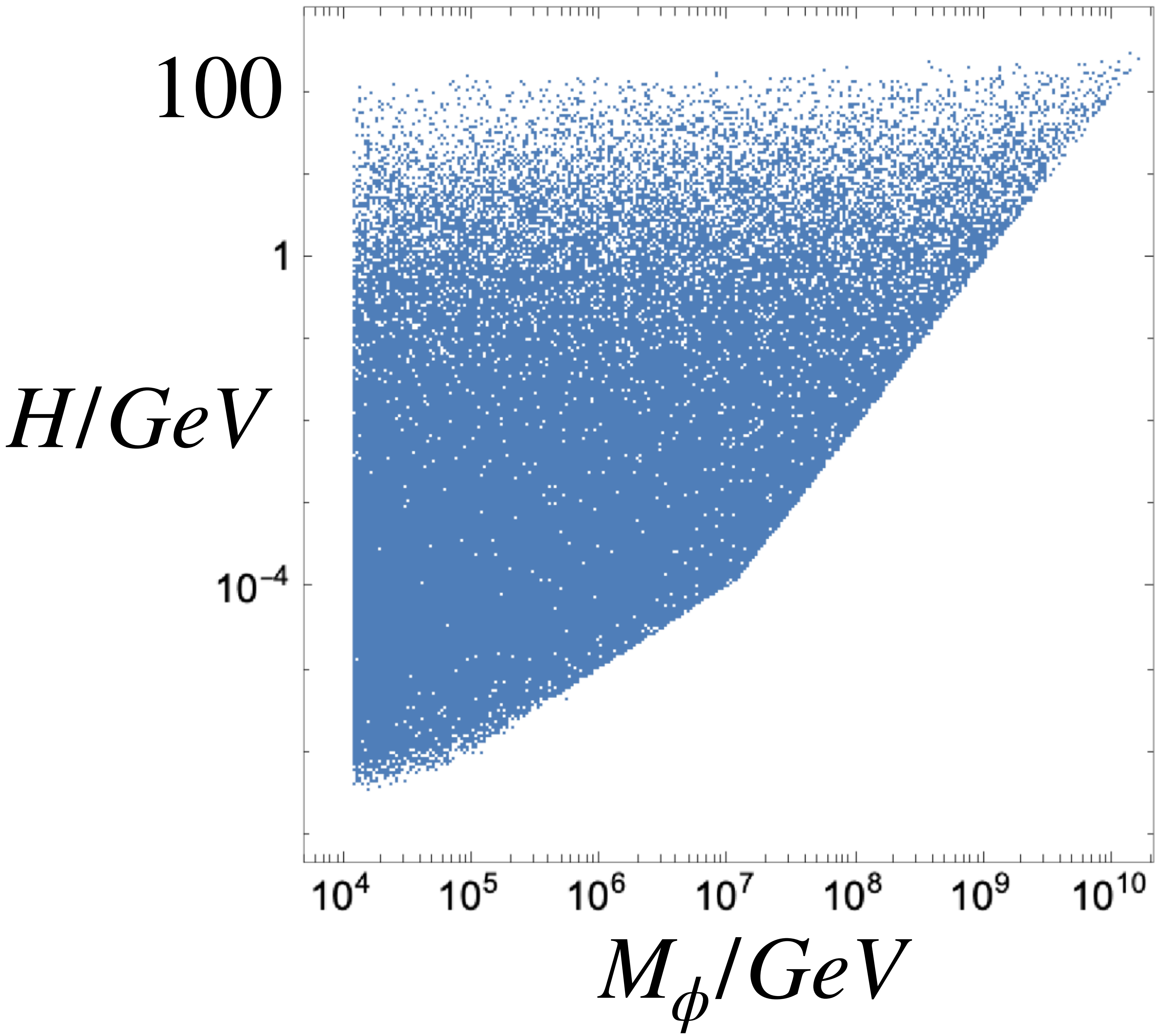
Slow-roll inflation



We assume some slow-roll inflation in the background, responsible for eternal inflation at a scale H_s

mH and CC from gradients & boundaries

Parameter space



~where new EW physics is

mH and CC from gradients & boundaries

Parameter space

- Hierarchical suppression over ϕ landscape requires

$$\Gamma_\phi \sim \exp\left[-\frac{8\pi^2}{3}\frac{\Delta V_B}{H^4}\right] \ll 1 \qquad \Rightarrow \qquad H \lesssim \Delta V_B{}^{1/4} \sim \sqrt{\mu_\phi v_{\rm SM}} \lesssim v_{sm}$$

I'm too restrictive here!

- Landscape energy contribution is subdominant in H_s

$$M_\phi \lesssim \sqrt{m_P H} \lesssim \sqrt{m_P v_{sm}}$$

mH and CC from gradients & boundaries

Parameter space

$$m_\phi \simeq 10^{-20} eV \dots 1 GeV$$

mH and CC from gradients & boundaries

Parameter space

$$m_\phi \simeq 10^{-20} eV \ldots 1 GeV$$

$$\begin{aligned} m_\phi^2 &\simeq \partial_h^2 \left[\mu^2 h^2 \cos \phi / f\right] \\ &\simeq \mu^2 h^2 / f^2 \\ &\lesssim v_{SM}^4 / f^2 \end{aligned}$$

Local measures

Local measures

Motivation

Extrapolation of black hole complementarity to inflationary space.

The physically meaningful description of the universe should be confined to a region of space accessible to some hypothetical observer.

R. Bousso, Phys. Rev. Lett. **97**, 191302 (2006), hep-th/0605263.
L. Susskind (2007), 0710.1129.
Y. Nomura, Astron. Rev. **7**, 36 (2012), 1205.2675.

Local measures

What is P(vac)?

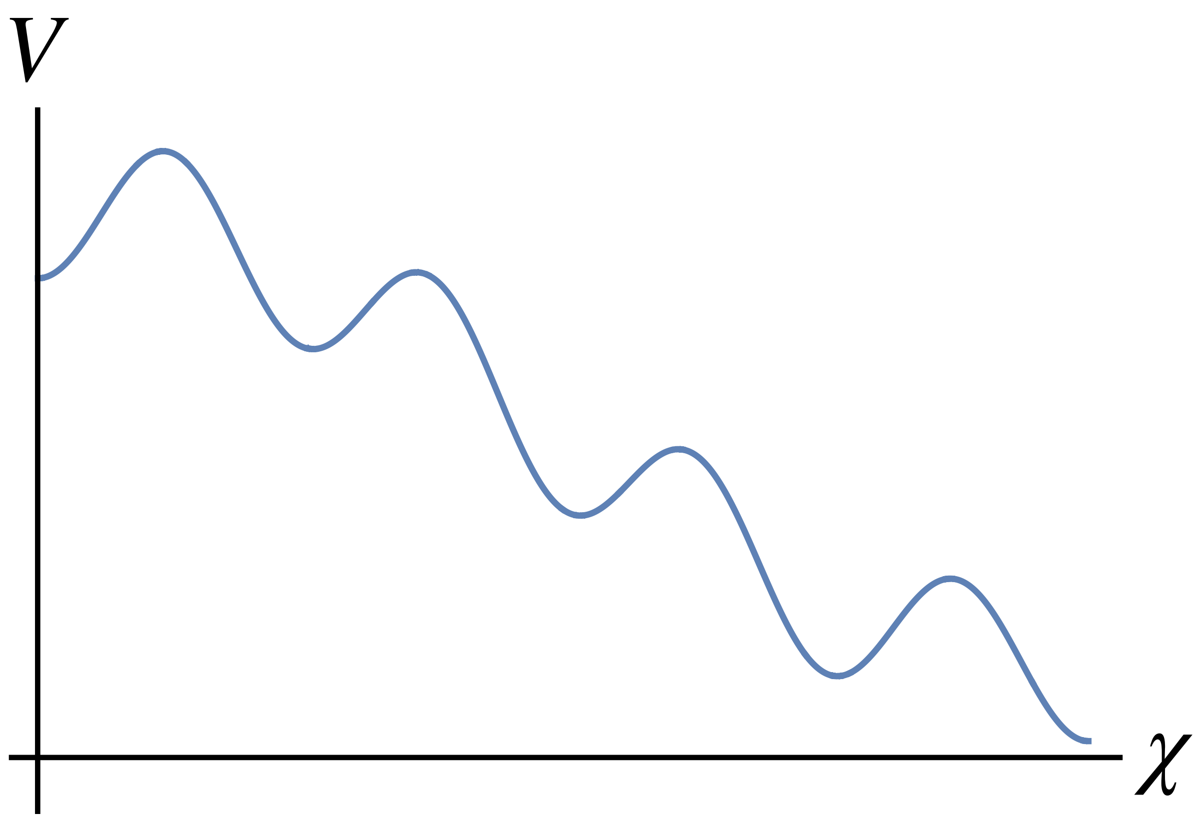
Time that a worldline spends (or a number of times it enters) in a given vacuum on its way to AdS

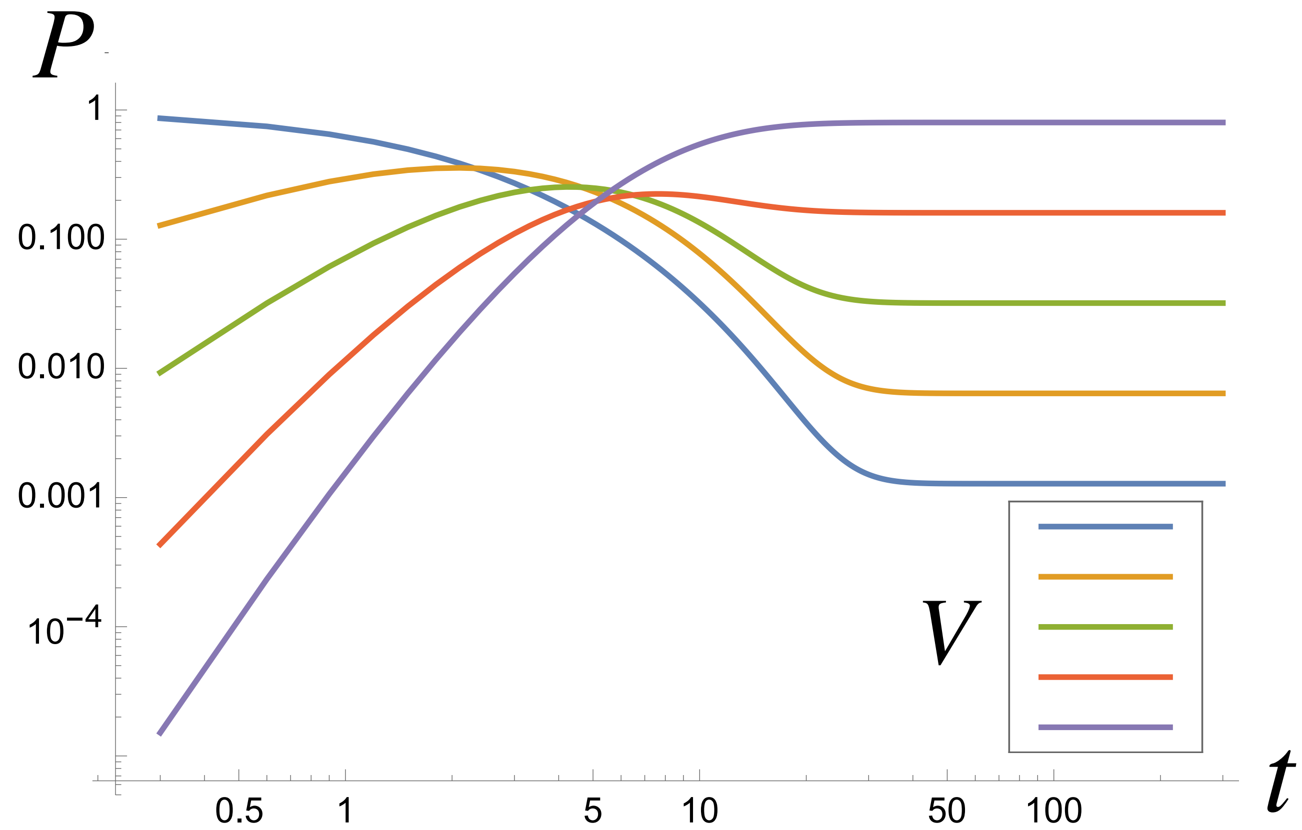
$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i \to j} + \sum_{j \neq i} P_j \Gamma_{j \to i}$$

Linde, 0611043

Local measures

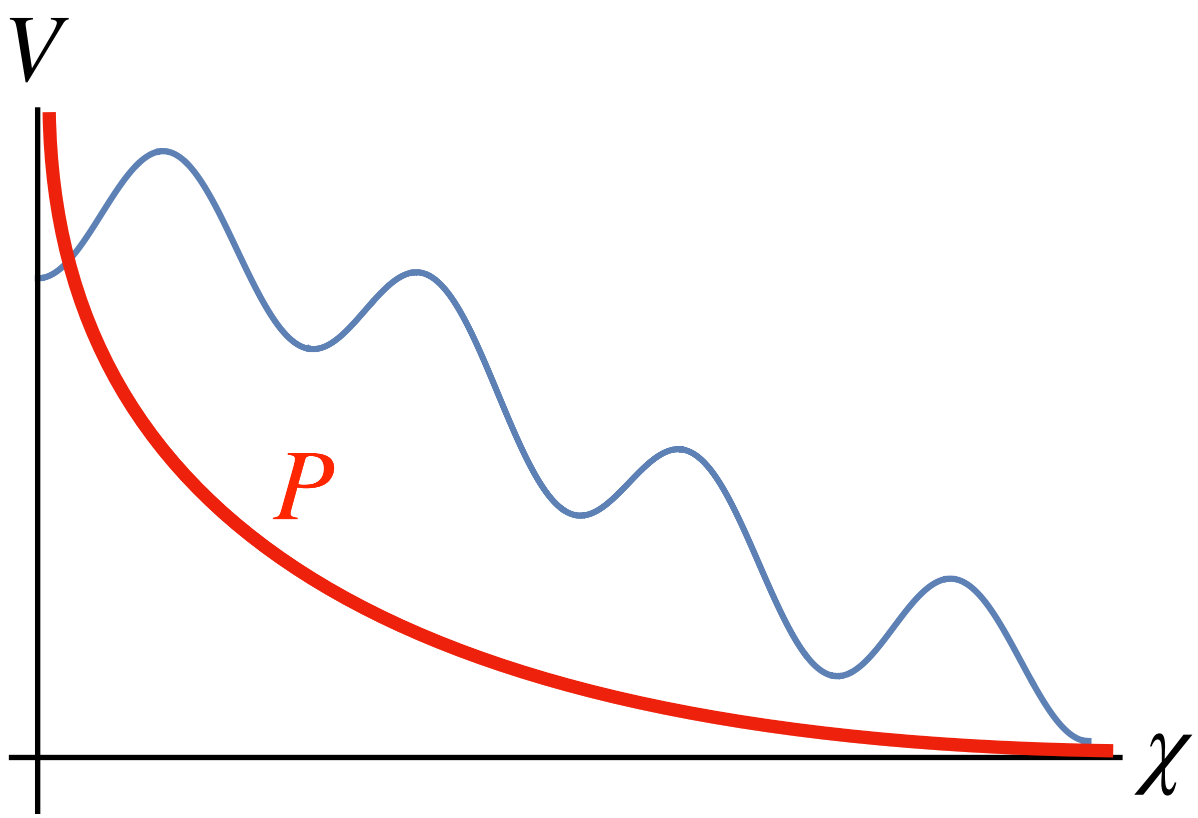
Probability gradients

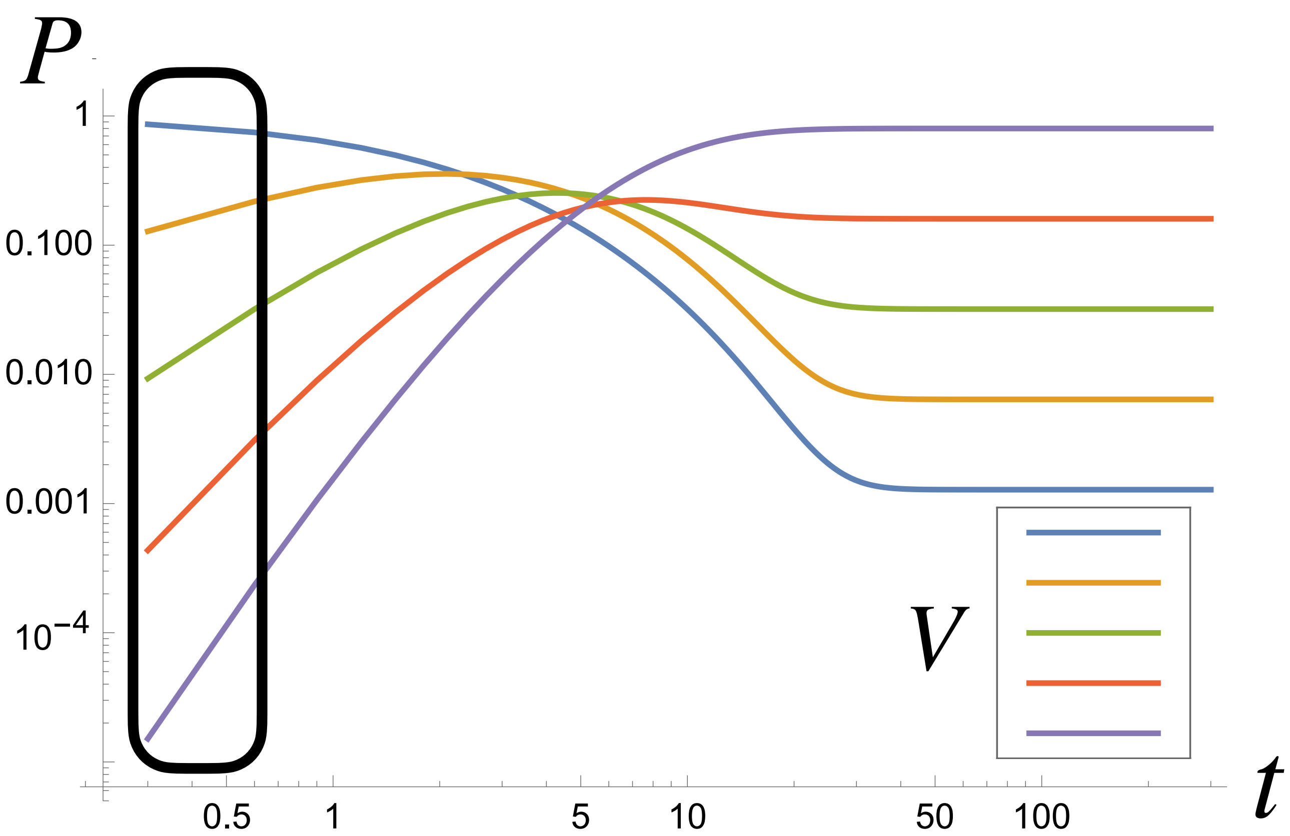




Local measures

Probability gradients





1. Dominated by initial conditions

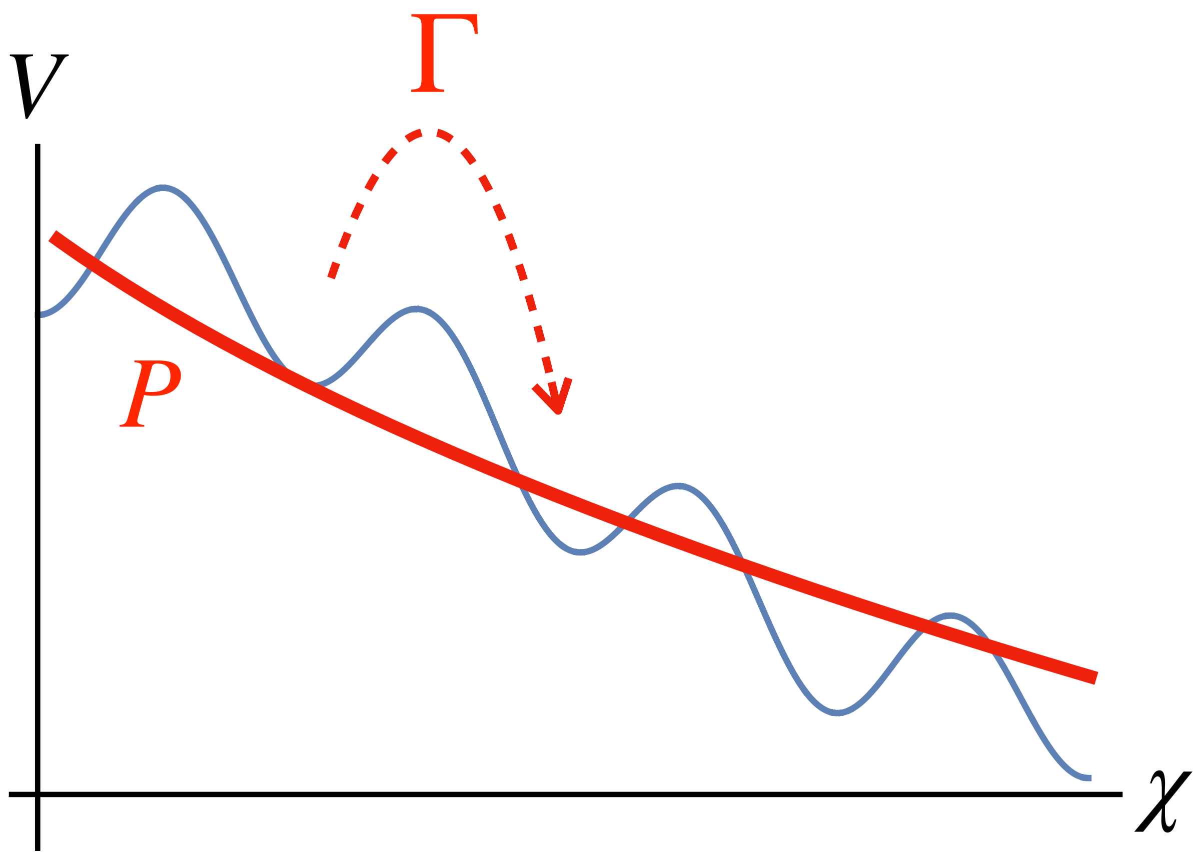
e.g. "quantum creation of the universe"

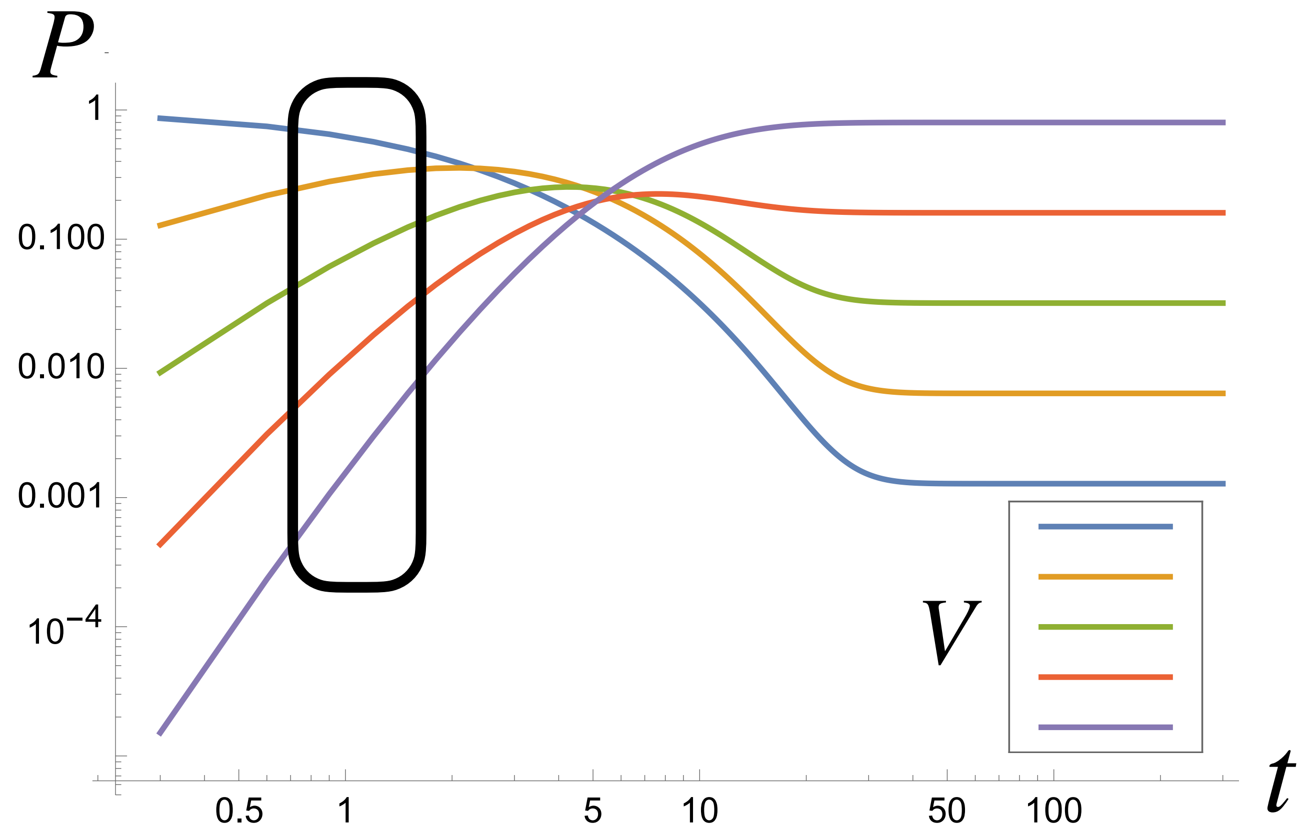
$$P(t=0) \propto \exp\left[-\frac{3}{8}\frac{m_P^4}{V(\chi)}\right] \propto \exp\left[\frac{8\pi^2}{3}\frac{V(\chi)}{H^4}\right]$$

A. D. Linde, Lett. Nuovo Cim. **39**, 401 (1984).
A. Vilenkin, Phys. Rev. D **30**, 509 (1984).

Local measures

Probability gradients



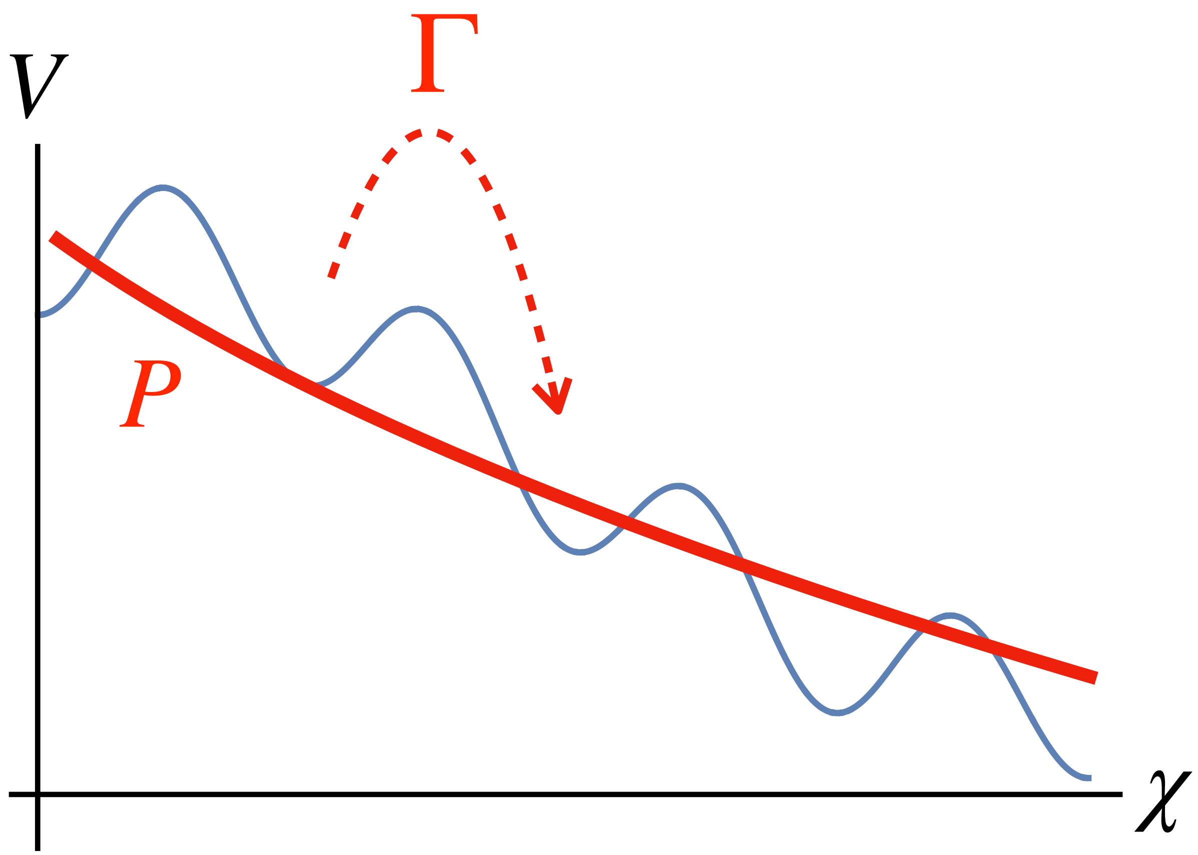


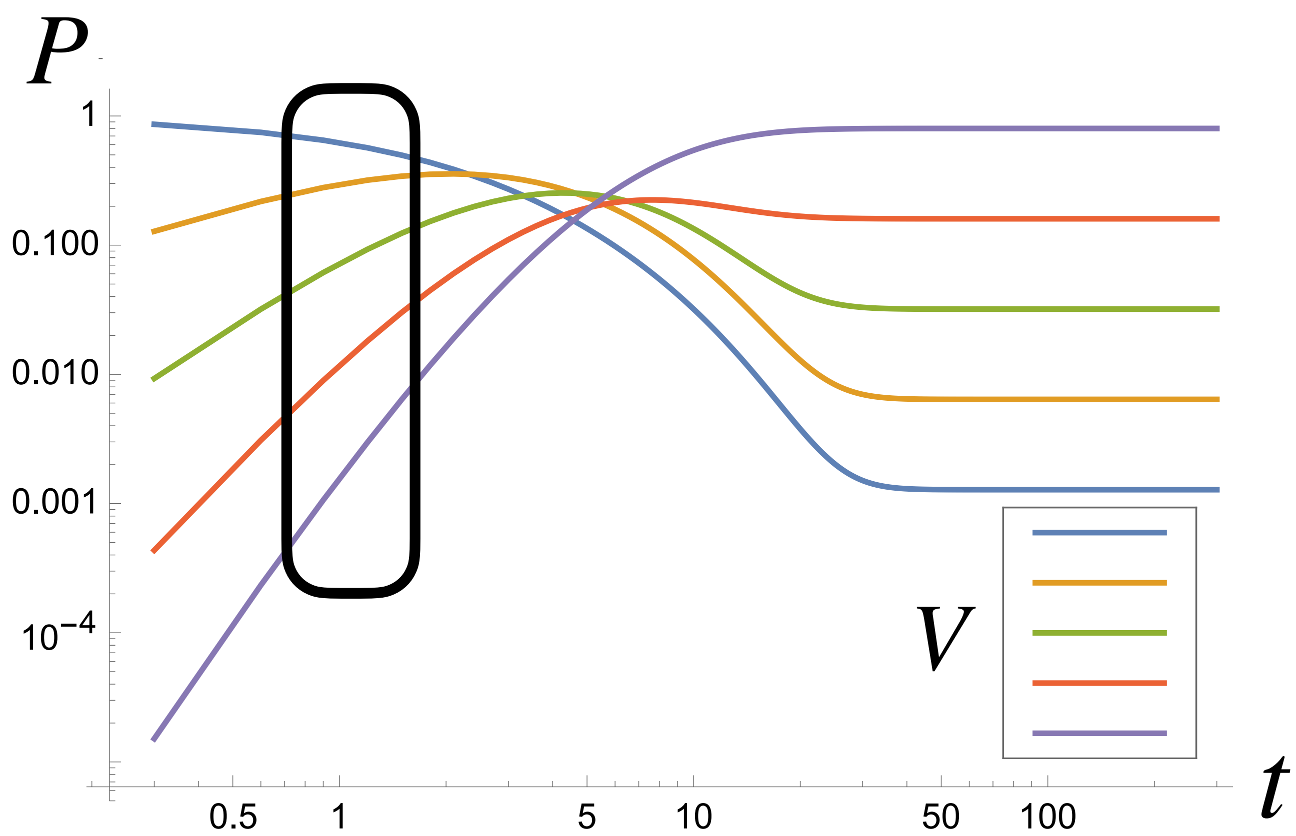
2. I.C. + Dynamics

$$P = \exp[\kappa t]P_{t=0}, \text{ with } \kappa_{ij} = \Gamma_{j\to i} - \delta_{ij}\sum_k \Gamma_{j\to k}$$

Local measures

Probability gradients



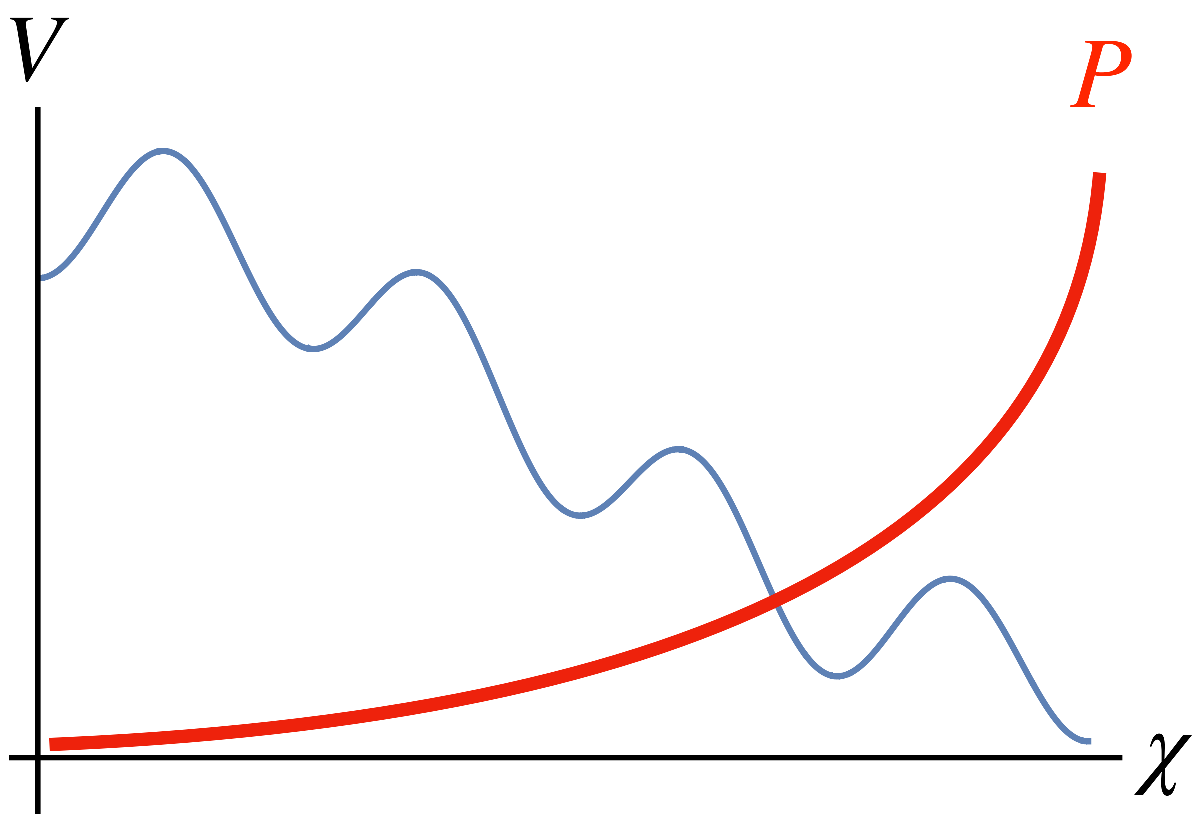


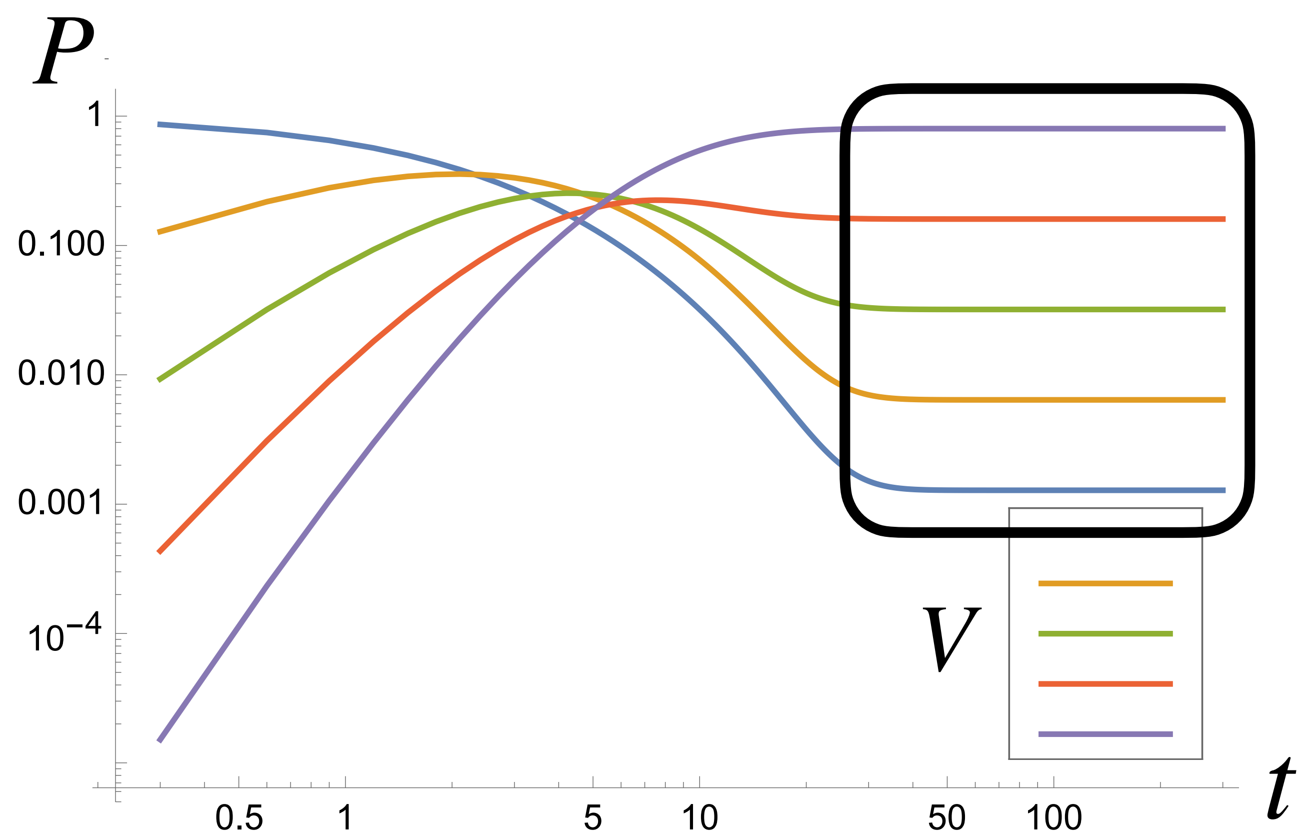
2. I.C. + Dynamics

$$P_i \simeq \frac{1}{i!}(\kappa t)^i P_{t=0} \simeq \frac{1}{i!}(\Gamma\, t)^i$$

Local measures

Probability gradients





3. Equilibrium independent of I.C.

(if no sinks)

$$P_i \propto \exp\left[\frac{3}{8}\frac{m_P^4}{V(\chi_i)}\right] \propto \exp\left[-\frac{8\pi^2}{3}\frac{V(\chi_i)}{H^4}\right]$$

Local measures

Probability gradients

3 regimes, end of slow-roll picks the time of sampling.

Regime 2 has probability defined by Γ similarly to the V-weighted case.

Experimental tests

Experimental tests

All the pheno associated with the relaxion.

(although param. space is somewhat different)

Experimental tests

Come from the "trigger"

$$V(\phi, h) \supset \mu_\phi^2 h^2 \cos(\phi/f)$$

$$\downarrow$$

$$\sin\theta_{h\phi} \sim \mu^2 v / m_h^2 f$$

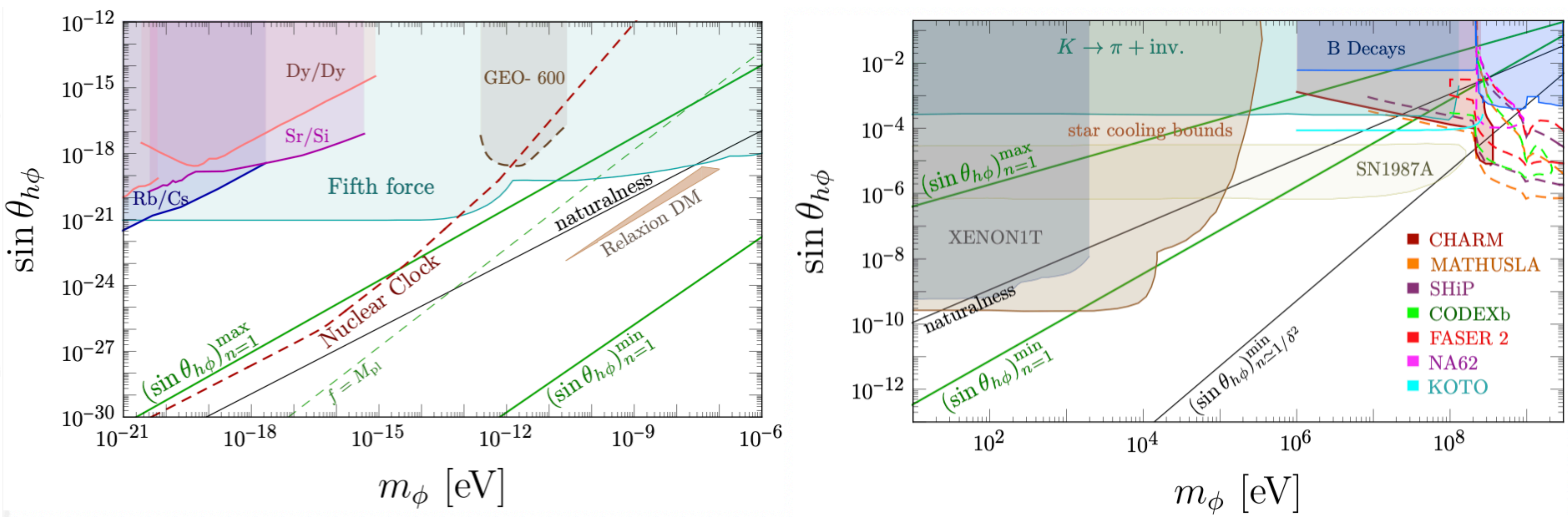
$$\downarrow$$

$$\phi \times [SM][SM]$$

other triggers discussed e.g. in Arkani-Hamed, D'Agnolo, Kim 2012.04652

Experimental tests

green lines delimit relaxion parameter space



Banerjee,OM,Kim,Perez 2004.02899

Experimental tests

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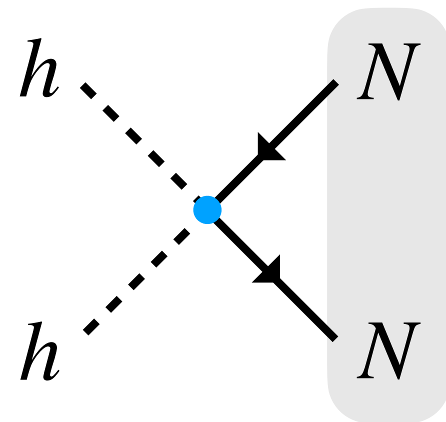
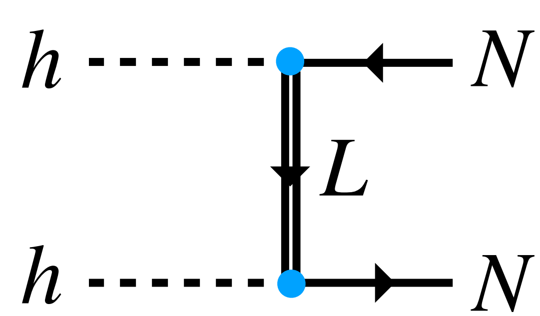


EW singlet fermion N



EW doublet fermion L

$$m_L \lesssim 4\pi v_{SM}$$

Graham,Kaplan,Rajendran 1504.07551

Experimental tests

- These were only 'local' probes:

$$V(\phi, h) \simeq \frac{1}{2} V''_{\phi} \phi^2 + V''_{\phi h} \phi h + \dots$$

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- Can one probe the global landscape structure?
  e.g. ϕ displacement by density effects:

  - Balkin,Serra,Springmann,Stelzl,Weiler 2106.11320
  - Hook,Huang 1904.00020

Conclusions

Dynamical solution for the Higgs mass in the presence of the CC landscape for two “orthogonal” measures.

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> A.Guth, 0002188

Predictions are uncertain, which doesn’t mean that they are not physically significant.

Conclusions

Dynamical solution for the Higgs mass in the presence of the CC landscape for two “orthogonal” measures.

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A.Guth, 0002188

Predictions are uncertain, which doesn’t mean that they are not physically significant.

Landscapes & anthropics $\neq$ giving up on exp testablity: potential probes from astrophysics to colliders

Thank you!