

Status of the Muon g-2 Prediction

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Outline

- **Lepton magnetic moments: the basics**
- **The muon $g-2$ discrepancy: an update**
- **Comment on the muon $g-2$ and the bounds on M_{Higgs}**

The present experimental values:

$$a_e = 1159652180.73 (28) \times 10^{-12}$$

0.24 parts per billion !! Hanneke et al., PRL100 (2008) 120801

$$a_\mu = 116592089 (63) \times 10^{-11}$$

0.5 parts per million !! E821 – Final Report: PRD73 (2006) 072003

$$a_\tau = -0.018 (17)$$

DELPHI - EPJC35 (2004) 159 [$a_\tau^{\text{SM}} = 117721(5) \times 10^{-8}$, Eidelman & MP '07]

Lepton magnetic moments: the basics

The beginning: $g=2$

- Uhlenbeck and Goudsmit in 1926 proposed:

$$\vec{\mu} = g \frac{e}{2mc} \vec{s}$$
$$g = \underline{2} \quad (\text{not } 1!)$$

- Dirac 1928:

$$(i\partial_\mu - eA_\mu) \gamma^\mu \psi = m\psi$$

- A Pauli term in Dirac's eq would give a deviation...

$$a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \quad \rightarrow \quad g = 2(1 + a)$$

...but there was no need for it! $g=2$ stood for ~20 yrs.

Theory of the g-2: Quantum Field Theory

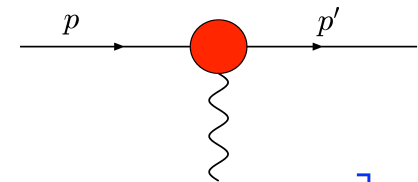
- Kusch and Foley 1948:

$$\mu_e^{\text{exp}} = \frac{e\hbar}{2mc} (1.00119 \pm 0.00005)$$

- Schwinger 1948 (triumph of QED!):

$$\mu_e^{\text{th}} = \frac{e\hbar}{2mc} \left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- Keep studying the lepton- γ vertex:



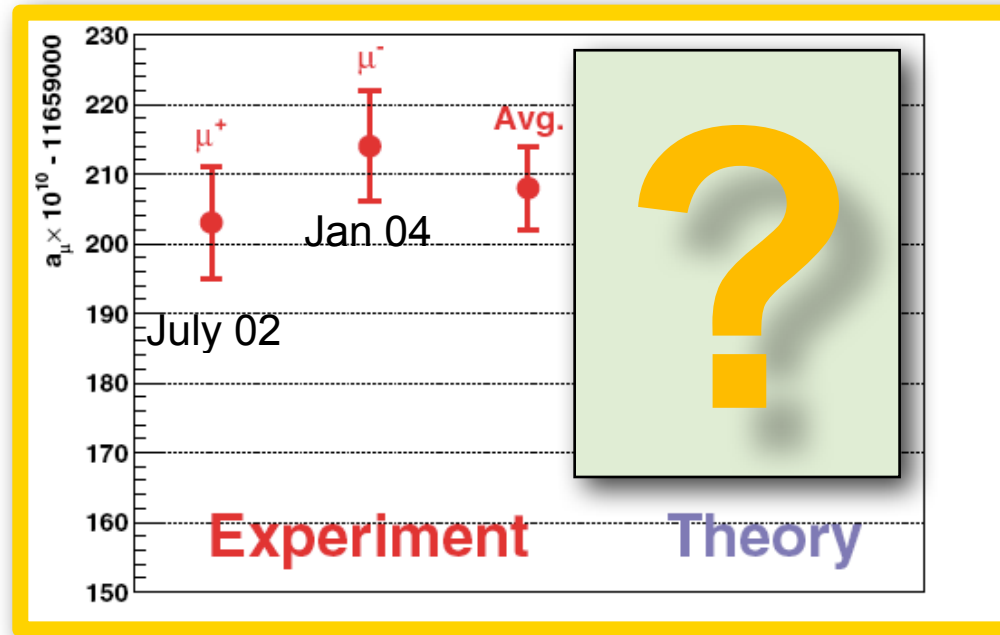
$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \dots \right] u(p)$$

$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure “quantum correction” effect!

The muon $g-2$ discrepancy: an update

The muon g-2: the experimental result



● Today: $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$ [0.5ppm].

● Future: new muon g-2 experiments proposed at:

● Fermilab (P989), aiming at 0.14ppm →

See Venanzoni's talk

● J-PARC aiming at 0.1 ppm

[D. Hetzog & N. Saito, U.Paris, Feb 2010; B. Lee Roberts & T. Mibe, Tau2010]

● Are theorists ready for this (amazing) precision? No(t yet)

a_μ^{SM} : the QED contribution

$$a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857408 (27) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$$+ 24.05050959 (42) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell '08

$$+ 130.805 (8) (\alpha/\pi)^4$$



Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, June & Dec 2007

$$+ 663 (20) (\alpha/\pi)^5 \quad \text{In progress...}$$

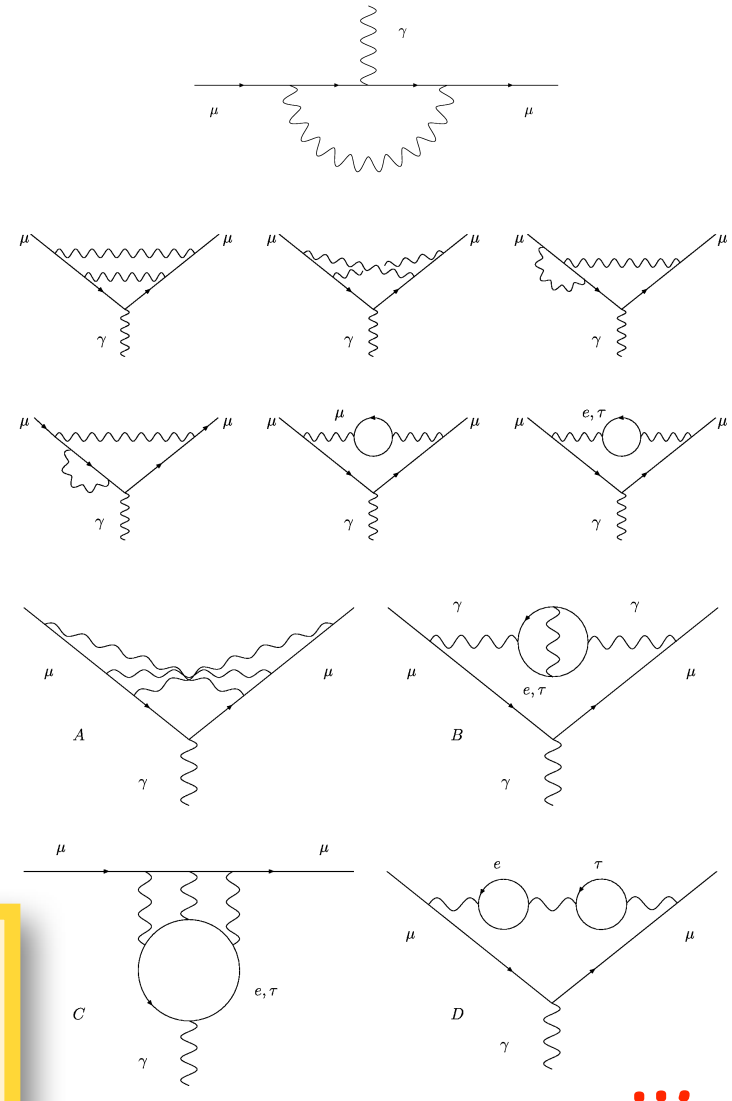
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim, ..., Kataev, Kinoshita & Nio '06, Kinoshita et al. 2011

Adding up, we get:

$$a_\mu^{\text{QED}} = 116584718.08 (14)(04) \times 10^{-11}$$

from coeffs, mainly from 5-loop unc   from $\delta\alpha('08)$

with $\alpha = 1/137.035999084(51) [0.37 \text{ ppb}]$



[A parenthesis on the electron g-2...

a_e^{SM}

$$= (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,89(60) (\alpha/\pi)^2$$

Schwinger 1948 Sommerfeld; Petermann; Suura & Wichmann '57; Elend '66; MP '06

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,78(26) \times 10^{-7}$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,62(60) \times 10^{-9}$$

$$+ 1.181\,234\,016\,827(19) (\alpha/\pi)^3$$

Kinoshita, Barbieri, Laporta, Remiddi, ... , Li, Samuel; Mohr, Taylor & Newell '08, MP '06

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,73(27) \times 10^{-6}$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5819(19) \times 10^{-8}$$

$$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,45(62) \times 10^{-13}$$

$$- 1.9144(35) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio, 2007

$$+ 0.0(4.6) (\alpha/\pi)^5 \quad \text{In progress (12672 mass ind. diagrams!)}$$

Aoyama, Hayakawa, Kinoshita, Nio '07; Aoyama, Hayakawa, Kinoshita, Nio, Jan 2011

$$+ 1.676(20) \times 10^{-12} \quad \text{Hadronic}$$

Krause 1997, Jegerlehner & Nyffeler 2009

$$+ 0.02973(52) \times 10^{-12} \quad \text{Electroweak}$$

Mohr, Taylor & Newell, '08; Czarnecki, Krause, Marciano '96

... and the best determination of alpha]

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{exp}} = 1159652180.73 (28) \times 10^{-12} \quad \text{Hanneke et al, PRL100 (2008) 120801}$$

vs. old (factor of 15 improvement, 1.8σ difference):

$$a_e^{\text{exp}} = 1159652188.3 (4.2) \times 10^{-12} \quad \text{Van Dyck et al, PRL59 (1987) 26}$$

- Equating $a_e^{\text{SM}}(\alpha) = a_e^{\text{exp}} \rightarrow$ best determination of alpha to date:

$$\alpha^{-1} = 137.035\,999\,084\,(12)(37)(2)(33) [0.37\text{ppb}] \quad \text{Hanneke et al, '08}$$

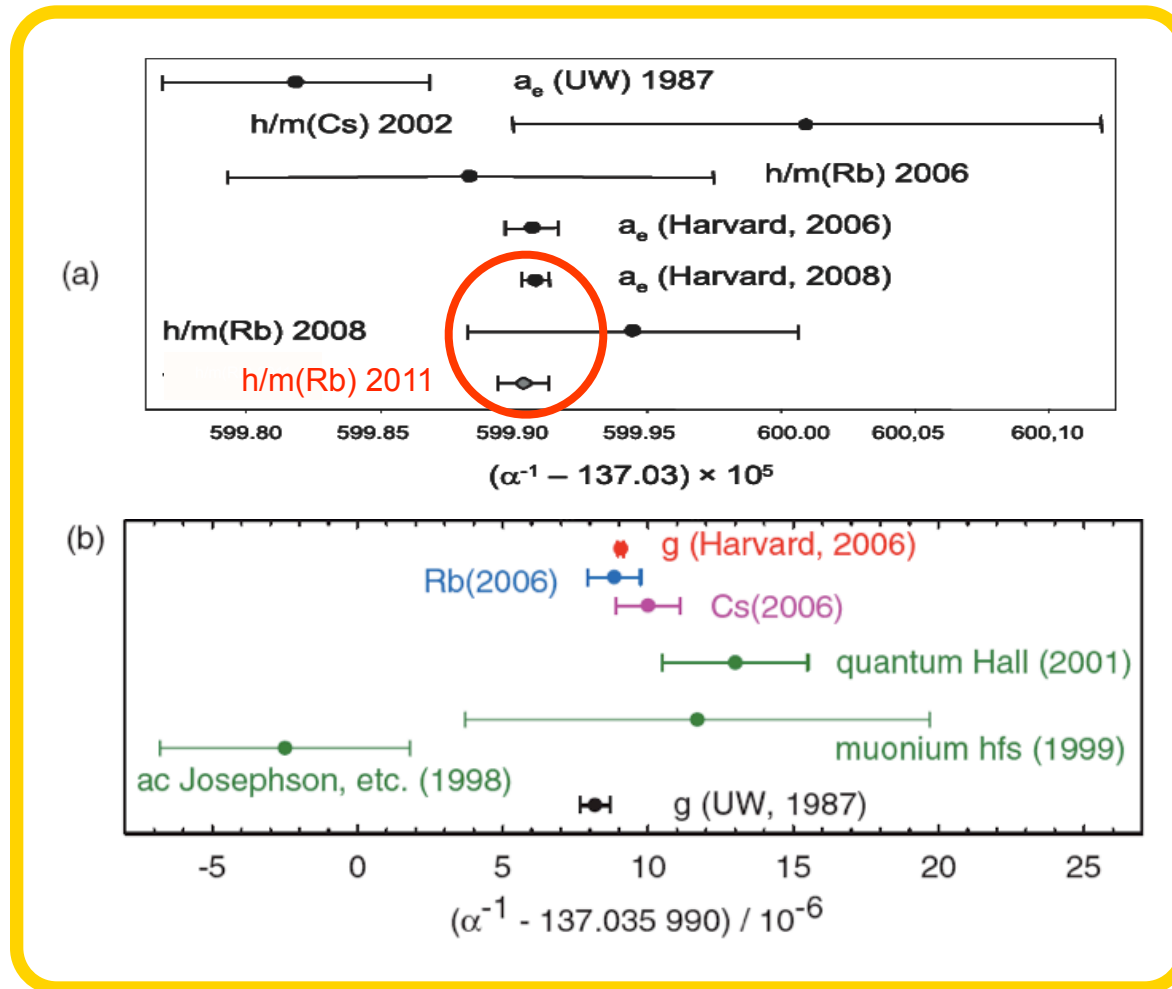
δC_4^{qed} δC_5^{qed} δa_e^{had} δa_e^{exp} (smaller than th!)

- Compare it with other determinations (independent of a_e):

$\alpha^{-1} = 137.036\,000\,00\,(110)\,[7.7\text{ppb}]$	PRA73 (2006) 032504 (Cs)
$\alpha^{-1} = 137.035\,999\,45\,(62)\,[4.6\text{ppb}]$	PRL101 (2008) 230801 (Rb)
$\alpha^{-1} = 137.035\,999\,037\,(91)\,[0.7\text{ppb}]$	PRL106 (2011) 080801 (Rb)

Excellent agreement \rightarrow beautiful test of QED at 4-loop level!

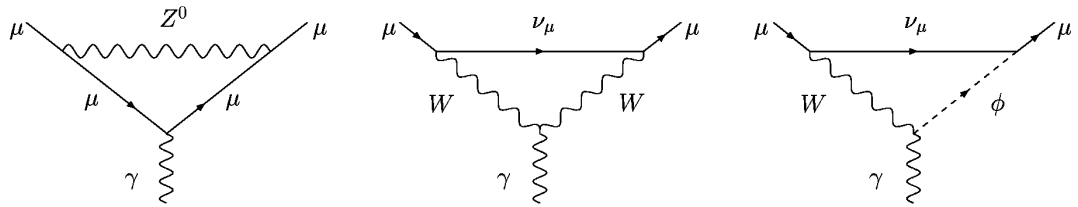
Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
 Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801
 Bouchendira et al, PRL106 (2011) 080801

a_μ^{SM} : the Electroweak contribution

One-loop term:



$$a_\mu^{\text{EW}}(1\text{-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2} \left[1 + \frac{1}{5} (1 - 4\sin^2\theta_W)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

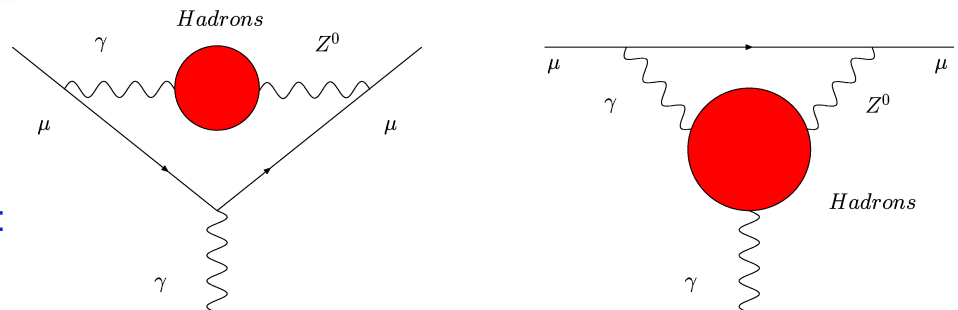
One-loop plus higher-order terms:

$$a_\mu^{\text{EW}} = 154 (2) (1) \times 10^{-11}$$

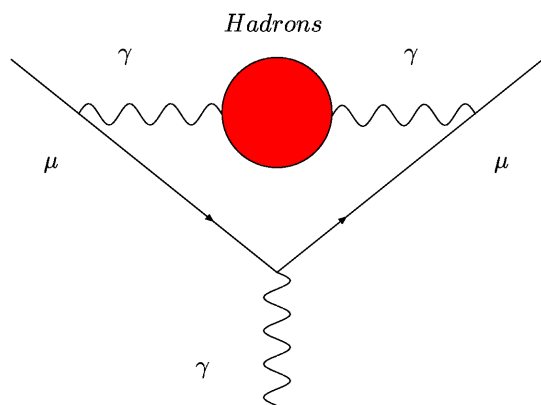
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano, Vainshtein '02; Degrossi, Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk, Czarnecki '05; Vainshtein '03.

Higgs mass variation, M_{top} error, 3-loop nonleading logs

Hadronic loop uncertainties:



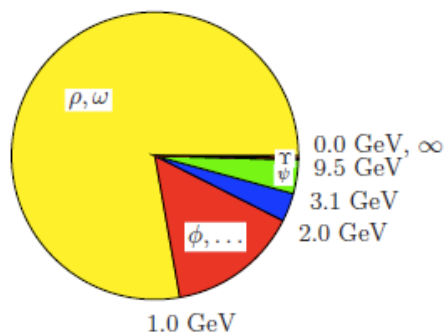
a_μ^{SM} : the hadronic leading-order (HLO) contribution



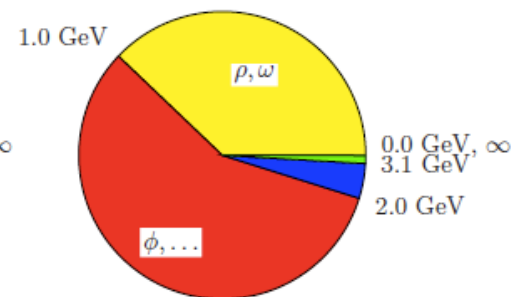
$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

Central values



Errors²



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$a_\mu^{\text{HLO}} = 6903 (53)_{\text{tot}} \times 10^{-11}$$

F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$$= 6923 (42)_{\text{tot}} \times 10^{-11}$$

Davier et al, arXiv:1010.4180 (incl. BaBar & KLOE10 2 π)

$$= 6949 (37)_{\text{exp}} (21)_{\text{rad}} \times 10^{-11}$$

Hagiwara et al. (HLMNT11), arXiv:1105.3149



a_μ^{SM} : the HLO contribution from e^+e^- data

- The overall agreement of all e^+e^- data looks reasonably good, but discrepancies exist.
- **Energy scan**: Agreement between the **CMD2 & SND** $\pi^+\pi^-$ data at VEPP-2M. First preliminary results presented in Sep 2011 by the **CMD3 & SND** exp. at the new VEPP-2000 collider in Novosibirsk.
- **Initial State Radiation Method (ISR)**: Collider operates at fixed energy but s_π can vary continuously. Important independent method made possible by strong th & exp interplay.
- **KLOE**: The “small angle” (2008) and “large angle” (2010) $\pi^+\pi^-$ ISR analyses agree. July 2011 (EPS): new preliminary measure by the $\pi\pi\gamma/\mu\mu\gamma$ ratio presented. It confirms their earlier results.

See Mandaglio's talk

- Reasonable agreement between **KLOE** and **CMD2-SND** (especially below the ρ). The contributions to a_μ^{HLO} agree.
- **BaBar**: ISR $\pi^+\pi^-$ result from 0.3 to 3 GeV! (2009). **Discrepancy** between the results of BaBar and KLOE.

a_μ^{SM} : the HLO contribution from tau-decay data

- The **isospin symmetry** is the symm. of the Lagrangian of strong int. under rotation in flavor space of u & d quarks.
- This global symm. implies the existence of 3 conserved currents: V_μ^\pm and V_μ^3 . The current $V_\mu^3 = J_\mu^{\text{em}, I=1}$ enters the evaluation of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$, while $V_\mu^- = \bar{d}\gamma_\mu u$ enters that of the decays $\tau^- \rightarrow \nu_\tau + \pi^0 + \pi^-$. **In the limit of exact isospin symmetry**, at leading order we have:

$$\sigma(e^+e^- \rightarrow \pi^+\pi^-) = \frac{4\pi\alpha^2}{s} v_0(s)$$

$$\frac{d\Gamma}{ds} (\tau^- \rightarrow \pi^-\pi^0\nu_\tau) = |V_{ud}|^2 \frac{G_F^2 M_\tau^3}{32\pi^3} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) v_-(s)$$

with $v_0(s) = v_-(s)$. But the isospin symmetry is **not exact...**

a_μ^{SM} : the HLO contribution from tau-decay data

- The τ data of ALEPH, CLEO & OPAL are higher than the CMD2/SND/KLOE ones, particularly above the ρ .
- The 2008 $a_\mu^{\pi\pi}$ τ result of Belle agrees with Aleph-Cleo-Opal (some deviations from Aleph's spectral functions).
- Revisited analysis: (Davier et al, EPJ C71 (2011) 1515)

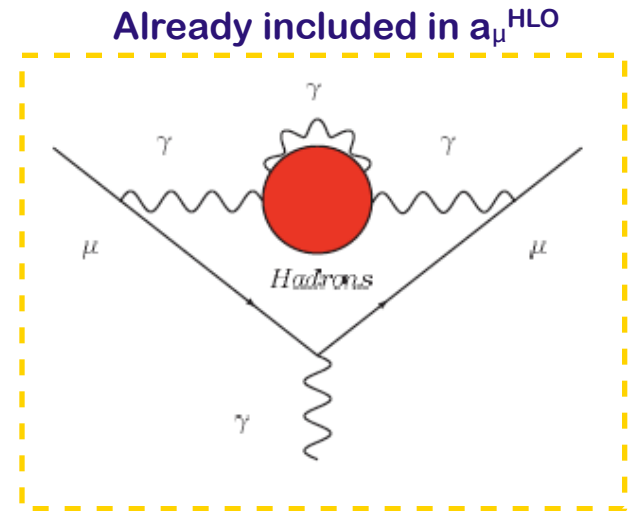
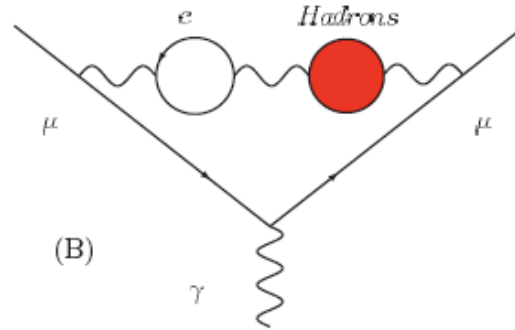
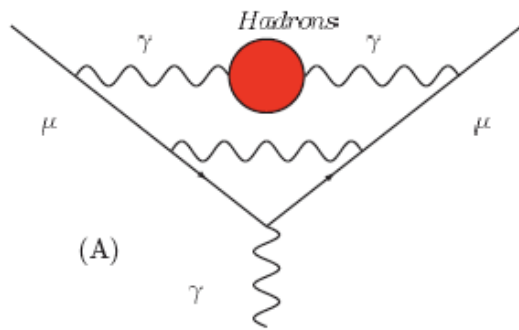
$$a_\mu^{\text{HLO}} = 7015 (47) \times 10^{-11}$$

Belle's data included + IB corrections revisited & updated

(Marciano & Sirlin '88; Cirigliano, Ecker, Neufeld '01-'02, Flores-Baez et al. '06 & '07)

- The discrepancy with e^+e^- data has decreased.
Inconsistencies in e^+e^- or τ data? All possible isospin-breaking (IB) effects taken into account?
- Claims exist that e^+e^- and τ data are actually consistent after IB effects & vector meson mixings considered (Benayoun et al., 2007, 2009 and [arXiv:1106.1315](#), Jegerlehner & Szafron 2011).
- Lattice calculation? See Jansen's talk

● HHO: Vacuum Polarization



$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{\text{HHO}}(\text{vp}) = -98 (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

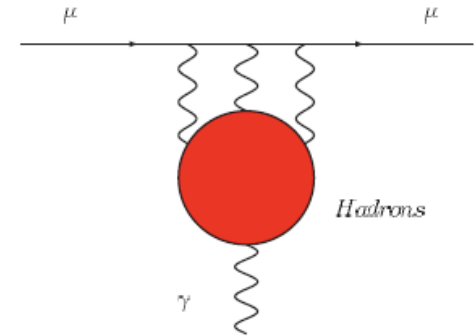
Only tiny shifts if τ data are used instead of the e^+e^- ones

Davier & Marciano '04.

● HHO: Light-by-light contribution

🔊 Unlike the HLO term, for the hadronic l-b-l term we must rely on theoretical approaches.

🔊 This term had a troubled life! Recent values:



$a_\mu^{\text{HHO}}(b) = + 80 (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{\text{HHO}}(b) = +136 (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{\text{HHO}}(b) = +105 (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_\mu^{\text{HHO}}(b) = +116 (39) \times 10^{-11}$	Jegerlehner & Nyffeler '09

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

🔊 **“Bound”** $a_\mu^{\text{HHO}}(|b|) < \sim 160 \times 10^{-11}$ Erler&Sanchez '06, Pivovarov '02 (Boughezal&Melnikov'11)

🔊 **Recent large result: $217 (91) \times 10^{-11}$** Fischer, Goecke, Williams, PRD83 (2011) 094006

🔊 **Had l-b-l is likely to become the ultimate limitation of the SM prediction**

🔊 **Lattice? Very hard, but in progress!**

See Jansen's and Moricciani's talks

The muon g-2: Standard Model vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$$a_{\mu}^{\text{EXP}} = 116592089 (63) \times 10^{-11}$$

E821 – Final Report: PRD73 (2006) 072
with latest value of $\lambda=\mu_{\mu}/\mu_p$ (CODATA'06)

	$a_{\mu}^{\text{SM}} \times 10^{11}$	$(\Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}}) \times 10^{11}$	σ
[1]	116 591 782 (59)	307 (86)	3.6
[2]	116 591 802 (49)	287 (80)	3.6
[3]	116 591 828 (50)	261 (80)	3.2
[4]	116 591 894 (54)	195 (83)	2.4

with $a_{\mu}^{\text{HHO}}(|b|) = 105 (26) \times 10^{-11}$

- [1] F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1
- [2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar and KLOE10 2π)
- [3] HLMNT11: Hagiwara et al, JPG38 (2011) 085003 (incl BaBar and KLOE10 2π)
- [4] Davier et al, Eur.PJ C71 (2011) 1515, τ data.

Note that the th. error is now about the same as the exp. one

The muon $g-2$ and the bounds on the Higgs boson mass

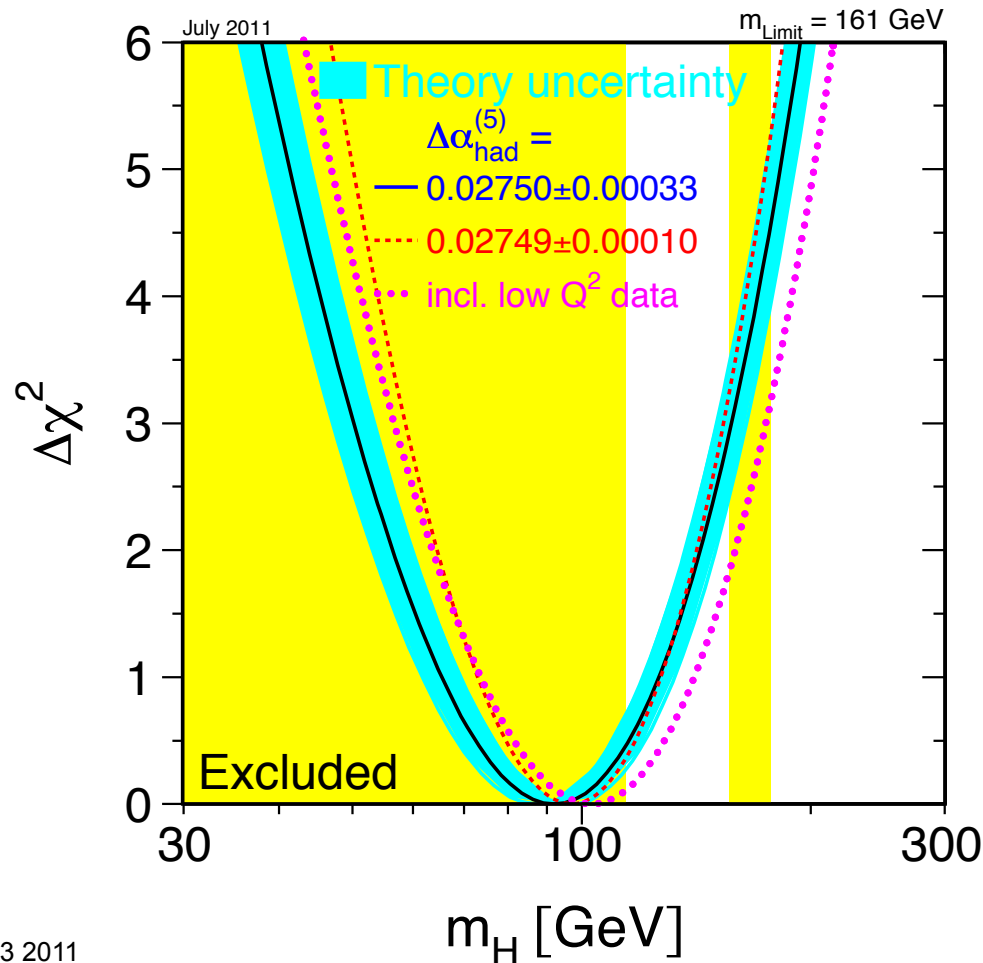
W.J. Marciano, A. Sirlin, MP

PRD78 (2008) 013009

[updated in Chin. Phys. C34 (2010) 735]

The EW Bounds on the SM Higgs mass

The Higgs is the last missing particle of the SM: comparing SM predictions (that depend on its mass M_H via loops) with precise experiment we get indirect information on M_H : clear preference for a light Higgs (below ~ 160 GeV)



The Hadronic Contribution to $\alpha(M_Z)$

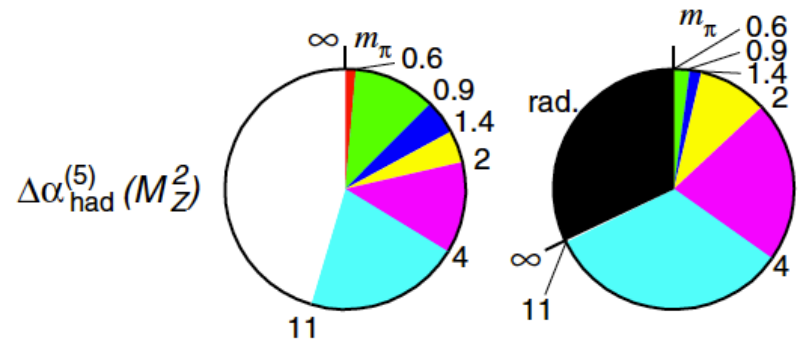
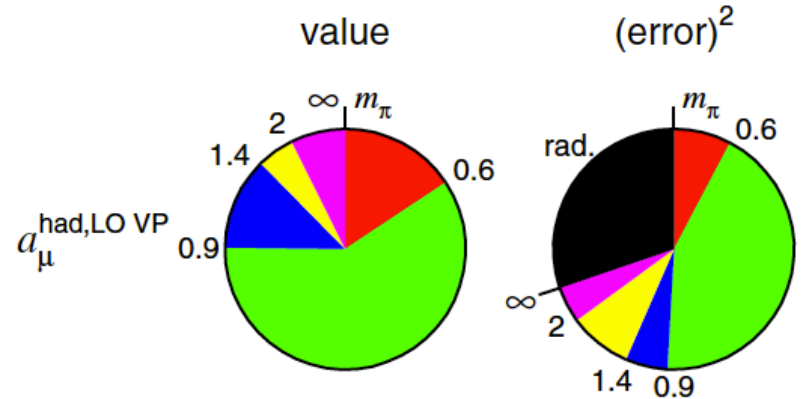
The effective fine-structure constant at the scale M_Z is given by:

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha(M_Z)} \quad \text{with} \quad \Delta\alpha = \Delta\alpha_{\text{lep}} + \Delta\alpha_{\text{had}}^{(5)} + \Delta\alpha_{\text{top}}$$

The light-quark part is given by:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = \frac{M_Z^2}{4\alpha\pi^2} P \int_{4m_\pi^2}^{\infty} ds \frac{\sigma(s)}{M_Z^2 - s}$$

Progress due to significant data improvement (in particular BES):



Hagiwara et al, JPG38 (2011) 085003

$\Delta\alpha_{\text{had}}^{(5)}(M_Z) =$

0.02800 (70)	Eidelman, Jegerlehner'95
0.02775 (17)	Kuhn, Steinhauser 1998
0.02749 (12)	Troconiz, Yndurain 2005
0.02758 (35)	Burkhardt, Pietrzyk 2005
0.02761 (23)	F. Jegerlehner 2008
0.02763 (14)	Hagiwara et al, JPG38 (2011) 085003
0.02749 (10)	Davier et al, EPJ C71 (2011) 1515
0.02750 (33)	Burkhardt, Pietrzyk arXiv:1106.2991

The EW Bounds on the SM Higgs mass

- The dependence of SM predictions on the Higgs mass, via loops, provides a powerful tool to set bounds on its value.

- **Comparing the theoretical predictions of M_W and $\sin^2 \theta_{\text{eff}}^{\text{lept}}$**

[convenient formulae in terms of M_H , M_{top} , $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ and $\alpha_s(M_Z)$ by Degrandi, Gambino, MP, Sirlin '98; Degrandi, Gambino '00; Ferroglia, Ossola, MP, Sirlin '02; Awramik, Czakon, Freitas, Weiglein '04 & '06]

with $M_W = 80.399 (23) \text{ GeV}$ [LEP+Tevatron]

$\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23153 (16)$ [LEP+SLC]

and

$\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02760 (15)$ [HLMNT09, now 0.02763(14)]

$M_{\text{top}} = 173.1 (1.3) \text{ GeV}$ [CDF-D0 '09, now 173.2(9)]

$\alpha_s(M_Z) = 0.118 (2)$ [PDG 2008, now 0.1184(7)]

we get

$$M_H = 96^{+32}_{-25} \text{ GeV} \quad \& \quad M_H < 153 \text{ GeV} \quad 95\% \text{CL}$$

- The value of $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ is a key input of these EW fits...

How do we explain Δa_μ ?

- Δa_μ can be explained in many ways: errors in HHO-LBL, QED, EW, HHO-VP, g-2 EXP, **HLO**; or **New Physics**.
- Can Δa_μ be due to **hypothetical mistakes in the hadronic $\sigma(s)$** ?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$.
- **Consider:**

$$\begin{aligned} a_\mu^{\text{HLO}} &\rightarrow a = \int_{4m_\pi^2}^{s_u} ds f(s) \sigma(s), & f(s) &= \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2, \\ \Delta\alpha_{\text{had}}^{(5)} &\rightarrow b = \int_{4m_\pi^2}^{s_u} ds g(s) \sigma(s), & g(s) &= \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\epsilon > 0$), in the range:

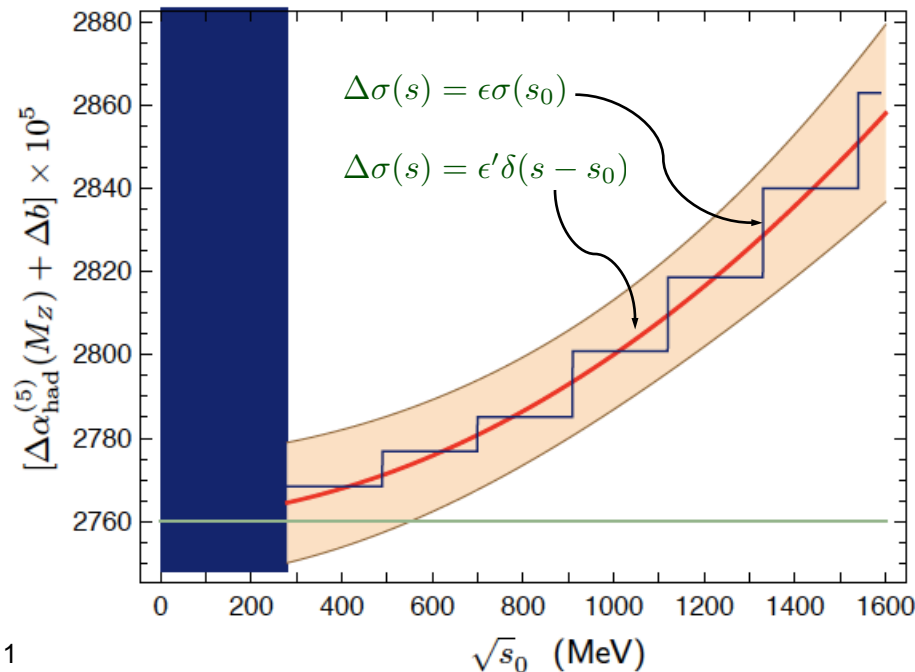
$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \quad \longrightarrow$$

Shifts of a_μ^{HLO} and $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$

- If this shift $\Delta\sigma(s)$ in $[\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$ is adjusted to bridge the g-2 discrepancy, the value of $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ increases by:

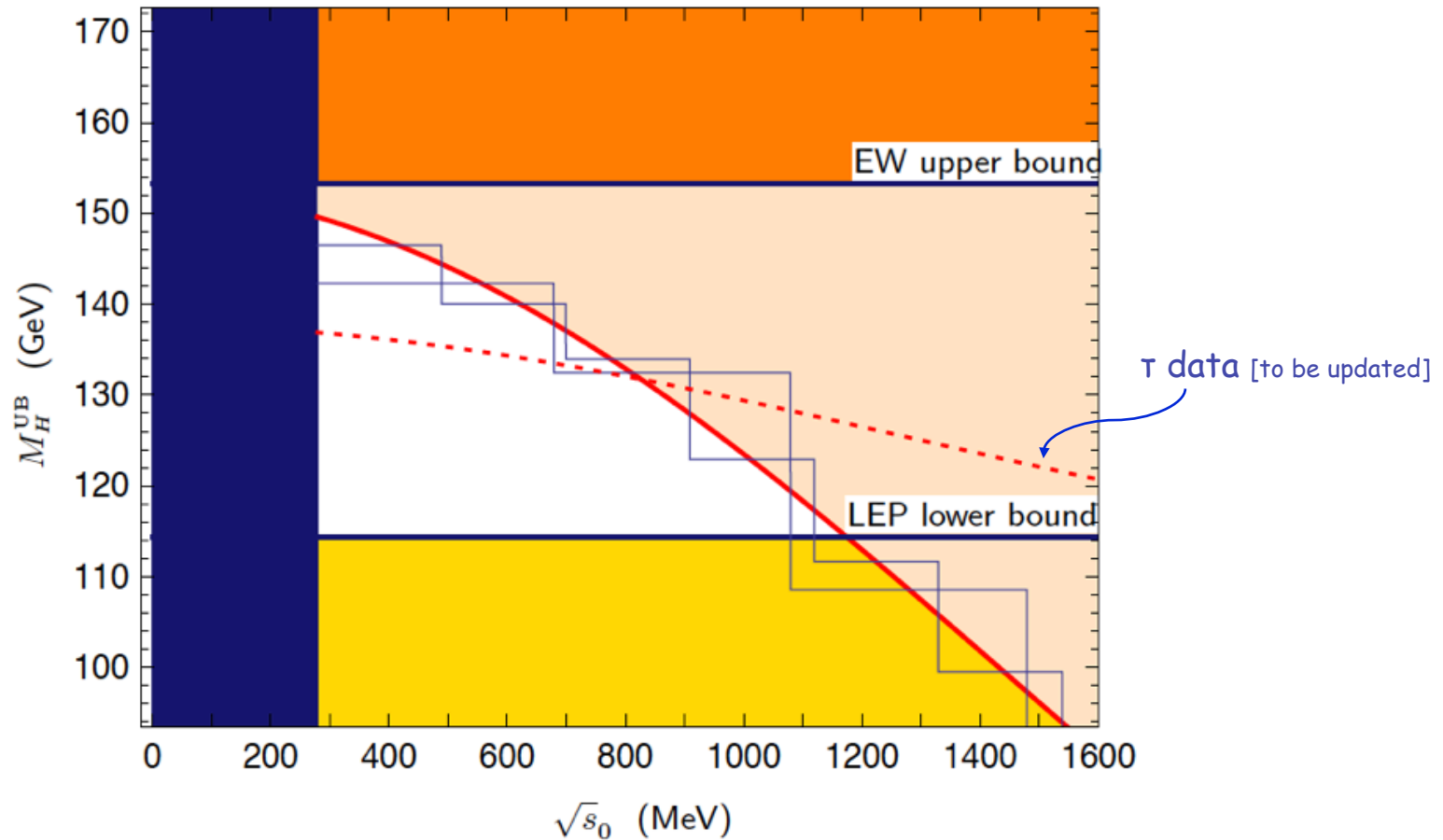
$$\Delta b(\sqrt{s_0}, \delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} g(t^2) \sigma(t^2) t dt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} f(t^2) \sigma(t^2) t dt}$$

- Adding this shift to $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02760(15)$ [HLMNT09], with $\Delta a_\mu = 316(79) \times 10^{-11}$ [HLMNT09], we obtain:



The muon g-2: connection with the SM Higgs mass

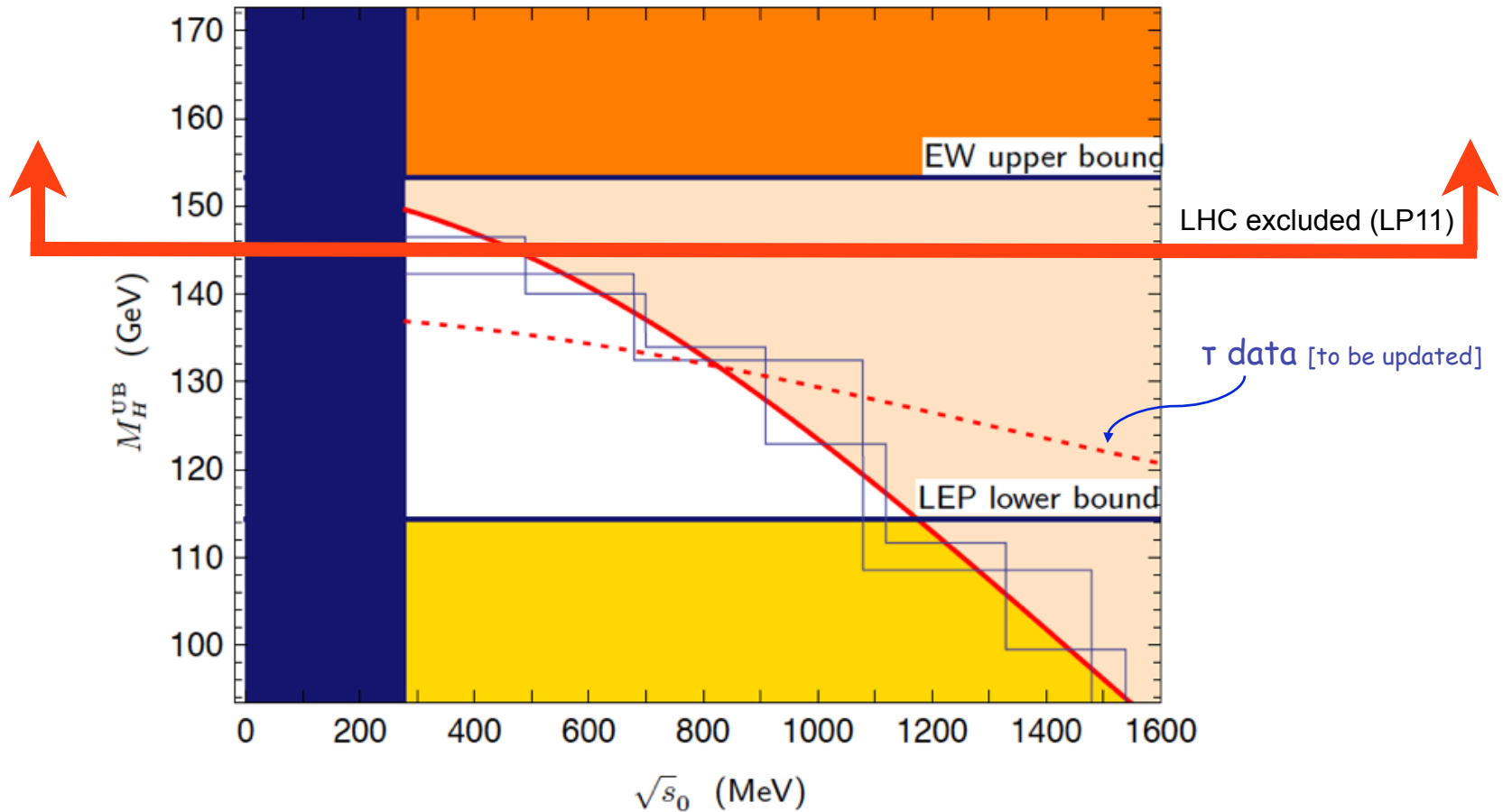
- How much does the M_H upper bound change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ by Δb] to accommodate Δa_μ ?



- $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02760(15)$
- $\Delta a_\mu = 316(79) \times 10^{-11}$

The muon g-2: connection with the SM Higgs mass

- How much does the M_H upper bound change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ by Δb] to accommodate Δa_μ ?



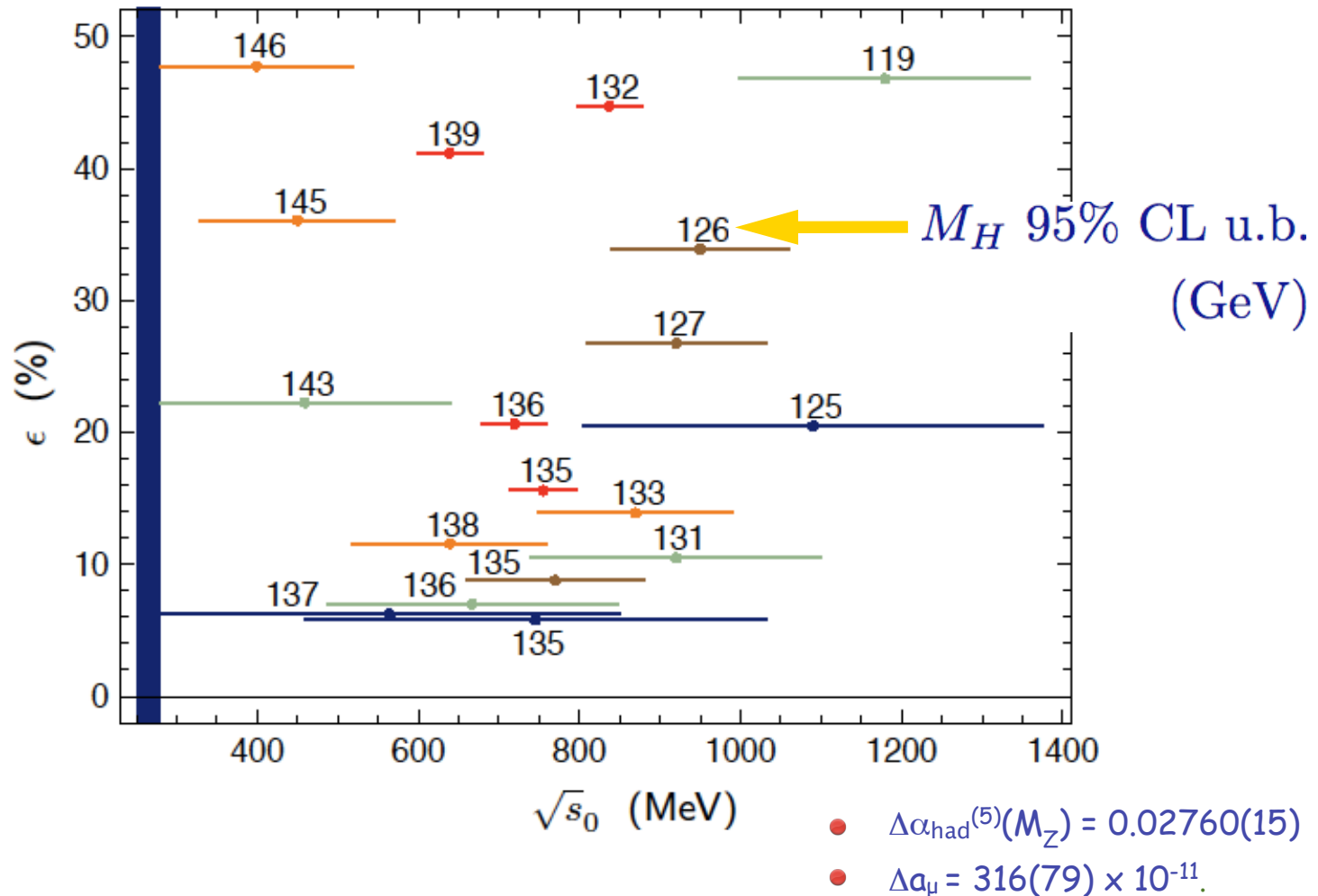
- $\Delta\alpha_{\text{had}}^{(5)}(M_Z) = 0.02760(15)$
- $\Delta a_\mu = 316(79) \times 10^{-11}$

The muon g-2: connection with the SM Higgs mass (II)

- The LEP direct-search lower bound is $M_H^{LB} = 114.4 \text{ GeV}$ (95%CL) and the LHC excludes it between 145 GeV & 446 GeV , with a “hole” between 288 & 296 GeV (LP2011).
- We showed that the hypothetical shifts $\Delta\sigma = \varepsilon\sigma(s)$ that bridge the muon g-2 discrepancy are in conflict with the LEP lower limit when $\sqrt{s_0} > \sim 1.2 \text{ GeV}$
- While the use of τ data in the calculation of a_μ^{HLO} reduces the muon g-2 discrepancy, it increases $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$, lowering the M_H upper bound. Near-conflict with the M_H lower bound if one tries to overcome the full Δa_μ (2.4σ) by further $\Delta\sigma$ increase
- In a scenario where τ data agree with e^+e^- ones below $\sim 1 \text{ GeV}$ after isospin viol. effects & vector meson mixings (Benayoun et al., 2007, 2009 and [arXiv:1106.1315](#), Jegerlehner & Szafron 2011), we could assume that Δa_μ is bridged by hypothetical errors above $\sim 1 \text{ GeV}$. If so, M_H^{UB} falls below M_H^{LB} !!

How realistic are these shifts $\Delta\sigma(s)$?

- How realistic are these shifts $\Delta\sigma(s)$ when compared with the quoted exp. uncertainties? Study the ratio $\epsilon = \Delta\sigma(s)/\sigma(s)$:



How realistic are these shifts $\Delta\sigma(s)$? (II)

- As the quoted exp. uncertainty of $\sigma(s)$ below 1 GeV is \sim a few per cent (or less), with a detailed analysis **we concluded that the possibility to explain the muon $g-2$ with these shifts $\Delta\sigma(s)$ appears to be unlikely**
- If, however, we allow variations of $\sigma(s)$ up to $\sim 6\%$ (7%), M_H^{UB} is reduced to less than ~ 137 GeV (138 GeV). E.g., the $\sim 6\%$ shift in $[0.6, 1.2]$ GeV, required to fix Δa_μ , lowers M_H^{UB} to 133 GeV. Some tension with the $M_H > \sim 120$ GeV “vacuum stability” bound.
- Scenarios where Δa_μ is accommodated without affecting M_H^{UB} are possible, but considerably more unlikely.
- Reminder:** the above M_H upper bounds, like the LEP-EWWG ones, depend on the value of $\sin^2 \theta_{eff}^{lept}$: usual problems. They also depend on M_t & δM_t : we made simple formulae to translate the M_H upper bounds above into new values corresponding to M_t & δM_t inputs different from those employed here.

Conclusions

The muon ($g-2$) provides a beautiful example of interplay between theory and experiment:

g_e probed at $\ll \text{ppt}$! $\rightarrow \alpha$ and strong test of QED's validity (SM too?)

g_μ probed at $\ll \text{ppb}$! tests full SM \rightarrow great to unveil New Physics.

g_τ theory is ready! (Lots of) room for experimental improvement..

The discrepancy Δa_μ is $\sim 3.5\sigma$ (if e^+e^- data are used; with tau data the deviation is 2.4σ). New $g-2$ exp: QED & EW terms ready, but real challenge for HAD contribution. More work & data needed!

Δa_μ can be due to New Physics or to problems in a_μ^{SM} (or a_μ^{EXP} !).

Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?

Unlikely, in view of current experimental error estimates of $\sigma(s)$.

However, if this turns out to be the solution, then the upper

bound on the Higgs mass drops to about $M_H < 135 \text{ GeV}$.

Together with $M_H > 114 \text{ GeV}$ from LEP, this leaves a narrow

window where the LHC may find it very soon!

The End