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Spin physics in relativistic heavy ion collisions

OUTLINE

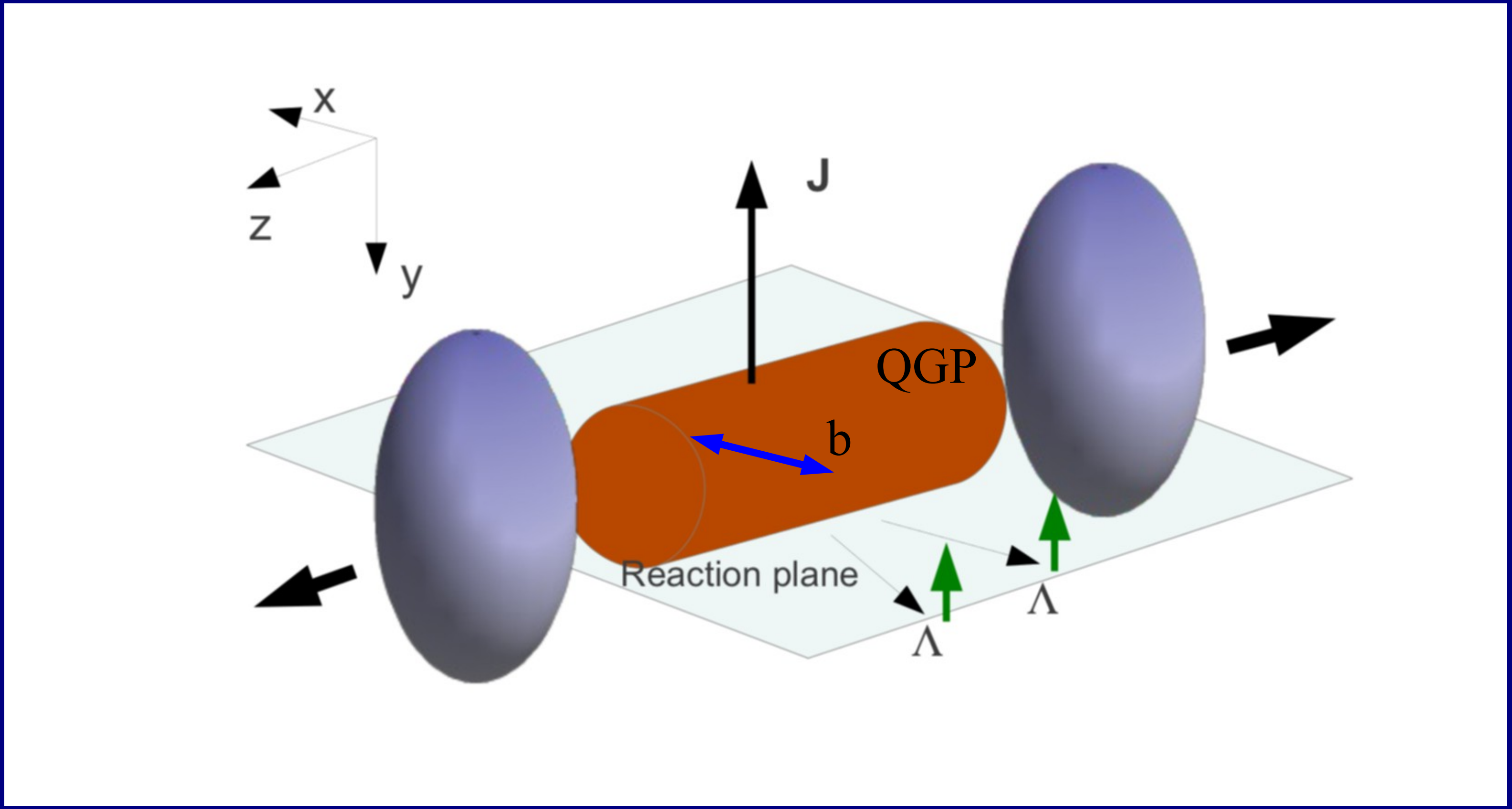
- Introduction
- Global and local spin polarization of Λ hyperons
- Spin polarization as a probe of Quark Gluon Plasma

High Lumi workshop Frascati October 3 2024

Global polarization in relativistic nuclear collisions

Peripheral collisions ➡ Angular momentum ➡ Global polarization w.r.t reaction plane

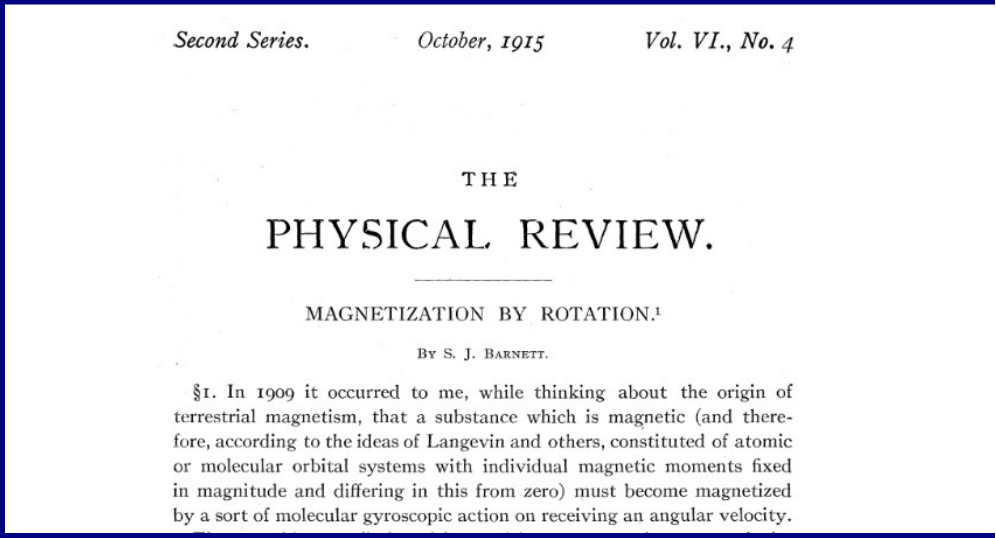
By parton spin-orbit coupling: Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301
By local equilibration: F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906



Barnett effect

S. J. Barnett, *Magnetization by Rotation,*

Phys. Rev. 6, 239–270 (1915).


Second Series. October, 1915 Vol. VI., No. 4

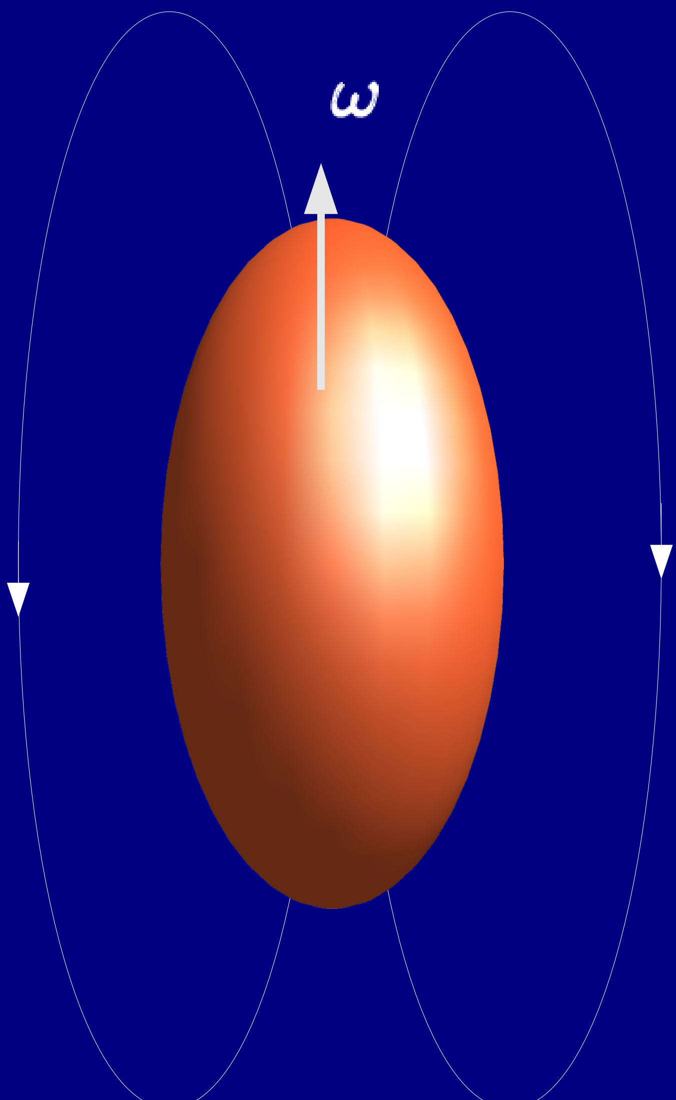
THE

PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

By S. J. Barnett.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.



Spontaneous magnetization of an uncharged body when spun around its axis

$$P \simeq \frac{S+1}{3}\frac{\hbar\omega}{KT} \Rightarrow M = \frac{\chi}{g}\omega$$

It can be seen as a dissipative transformation of the orbital angular momentum into spin of the constituents. The angular velocity decreases and a small magnetic field appears; this phenomenon is accompanied by a heating of the sample. Requires a spin-orbit coupling.

Polarization and vorticity

Local equilibrium at the freeze-out implies a connection between spin polarization and (thermal) vorticity

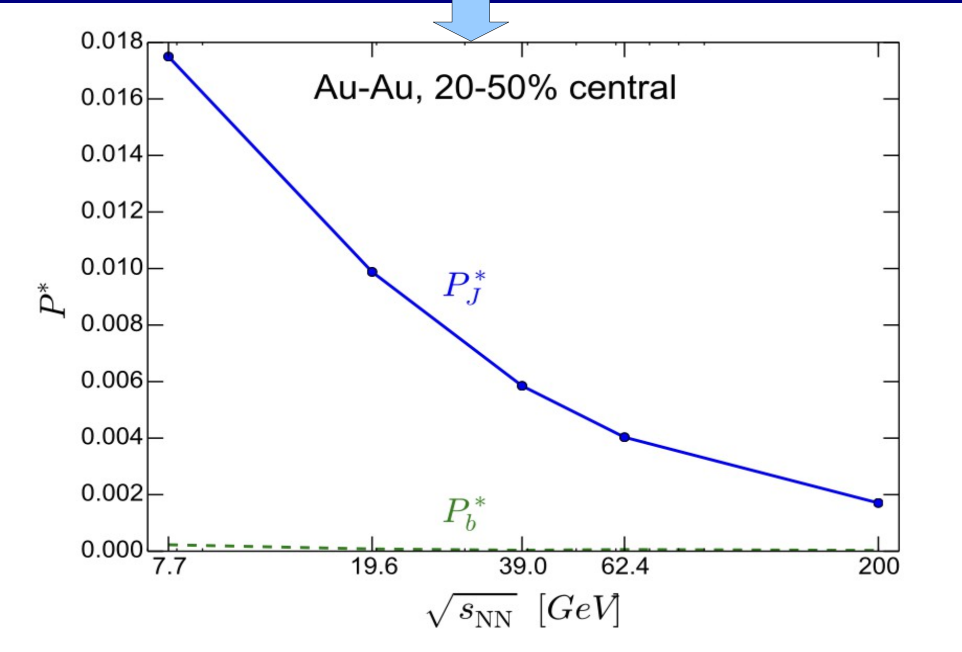
F.B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)

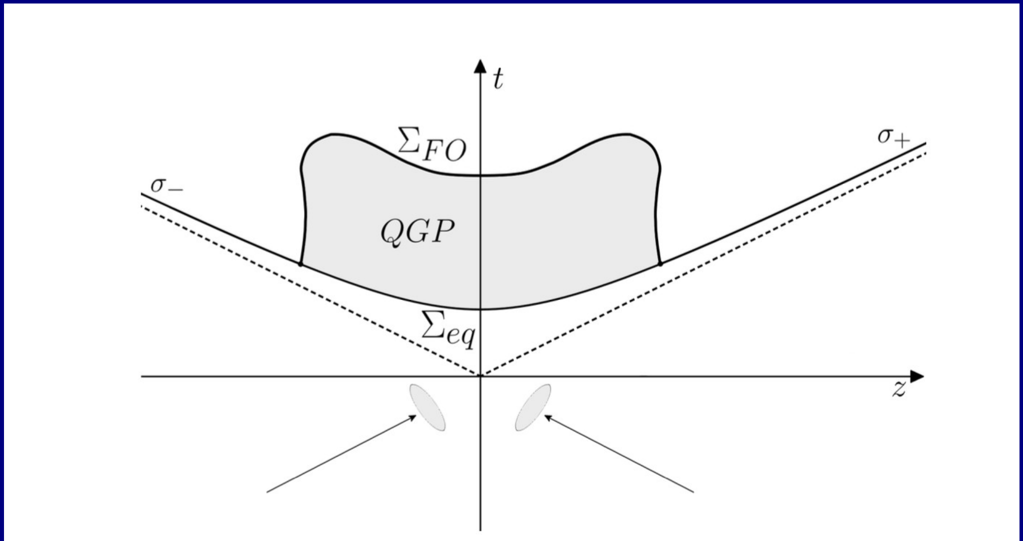
$$n_F = (\mathrm{e}^{\beta\cdot p - \xi} + 1)^{-1}$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1-n_F)\partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

$$\beta = \frac{1}{T}u$$

Quantitative prediction of 3+1D hydrodynamic model of QGP production and evolution

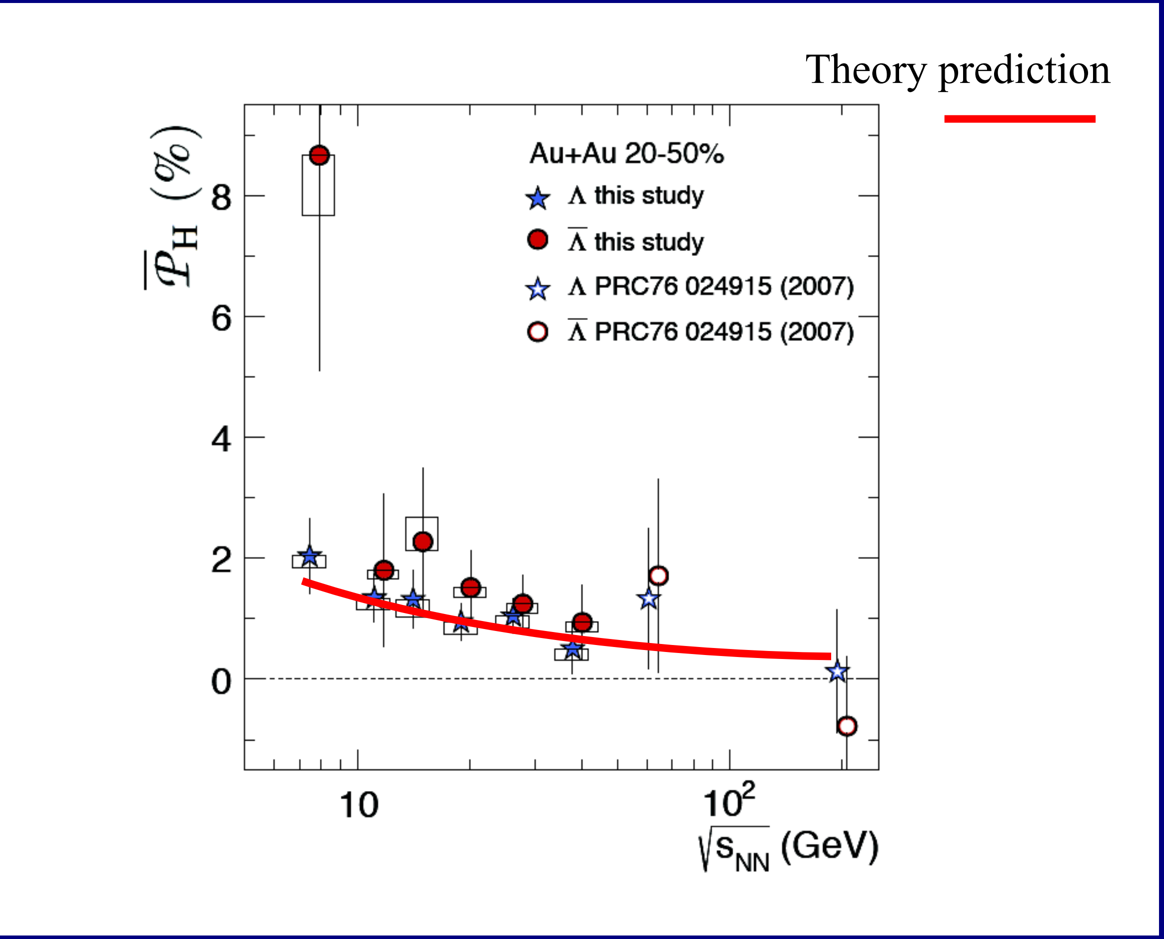


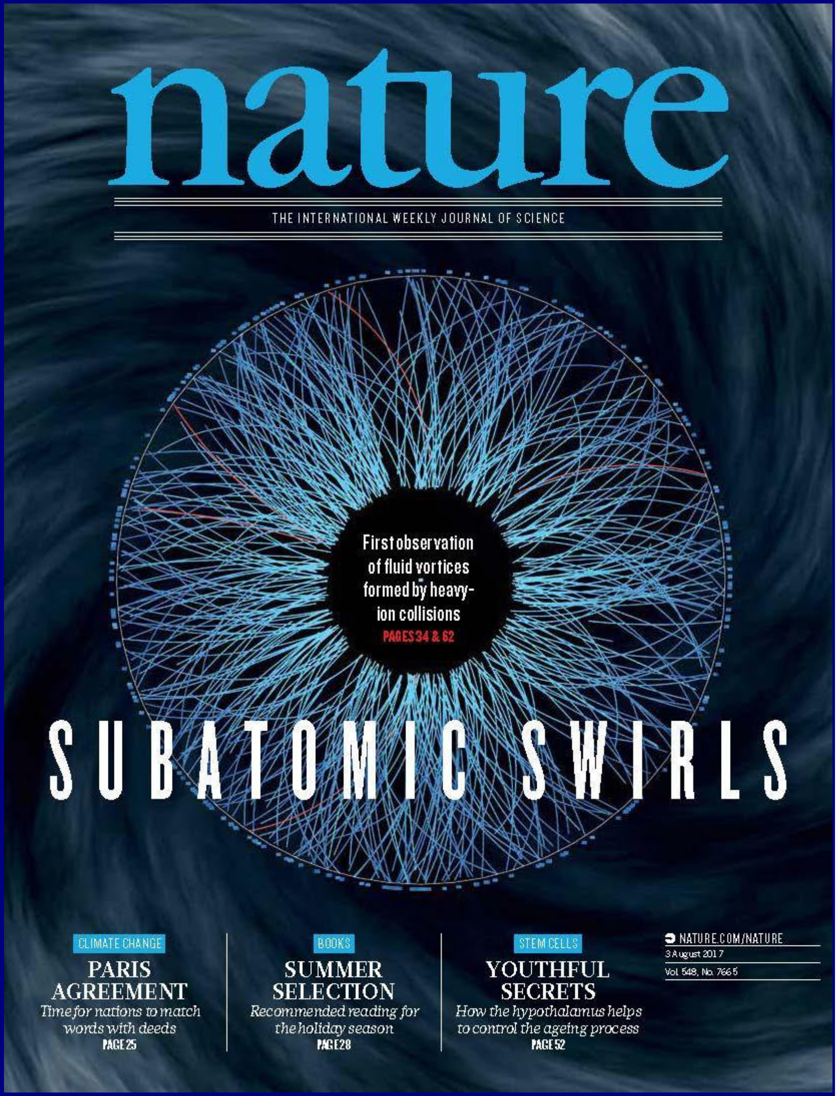


I. Karpenko, F.B., Eur. Phys. J. C 77 (2017) 213

Discovery of polarization in heavy ion collisions

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017

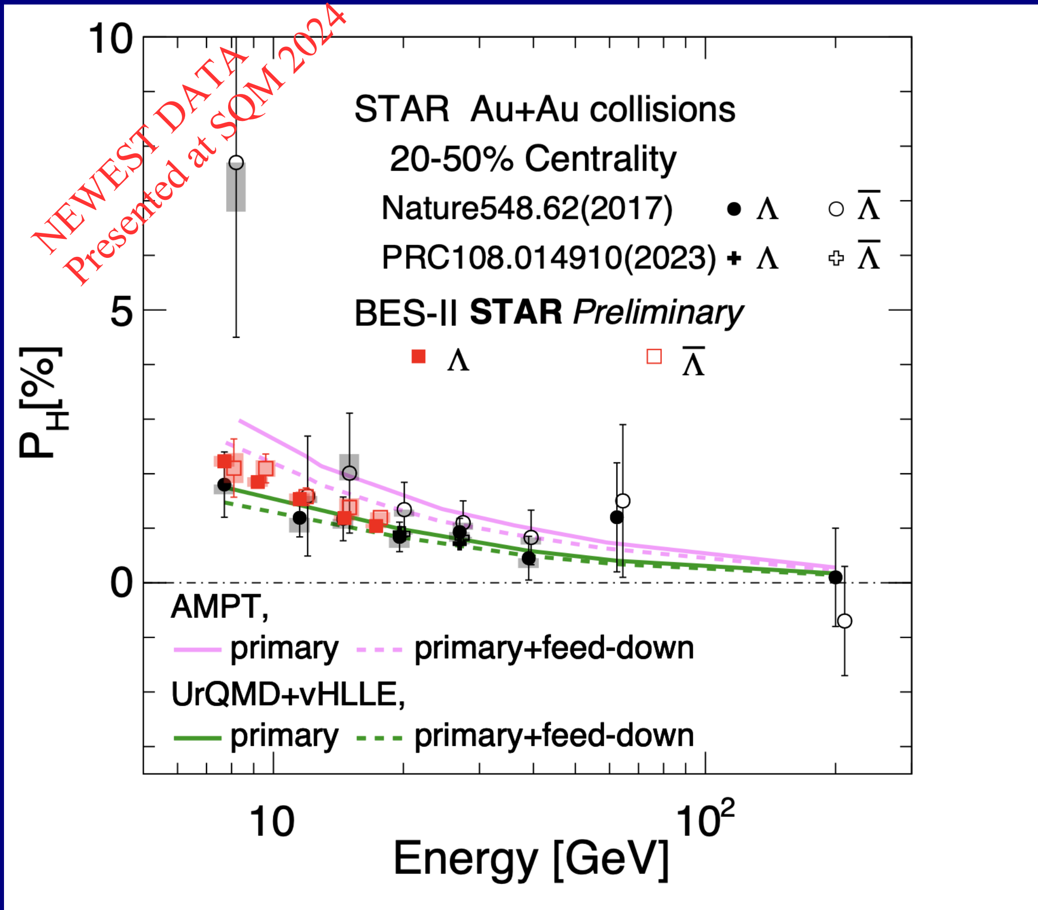


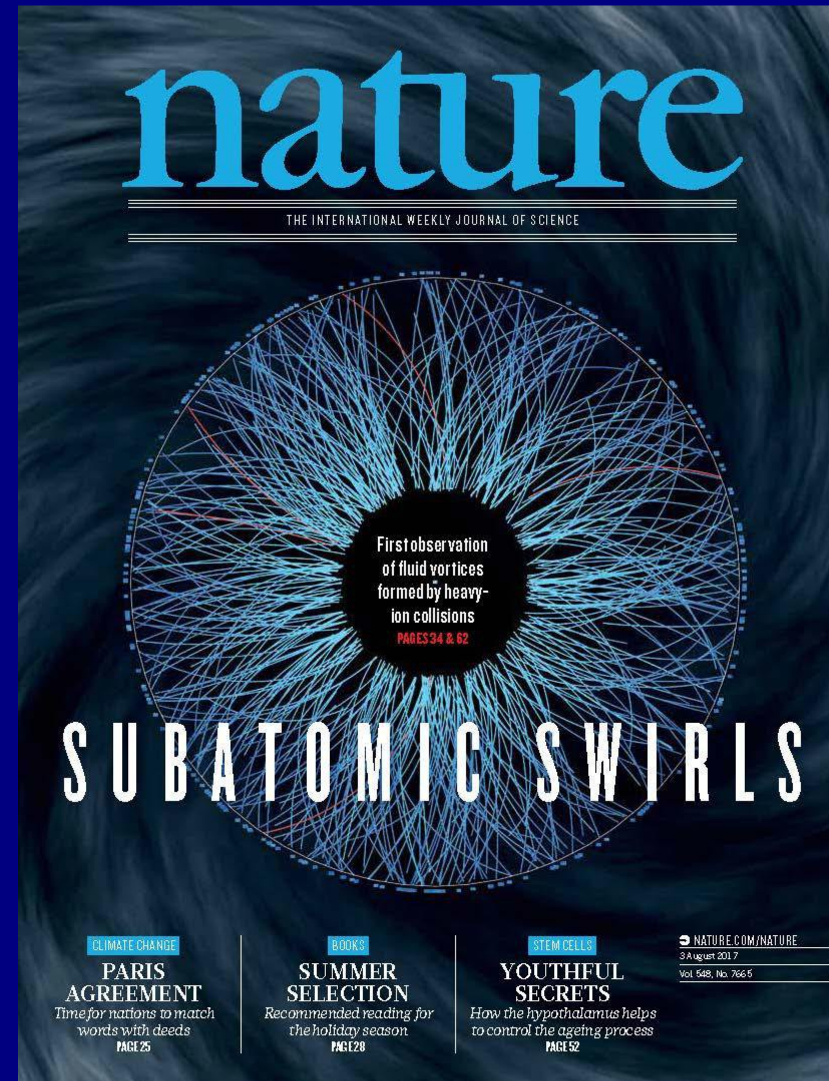


Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven by a mean field (such as EM) whose coupling is *C-odd.*
In agreement with the predictions based on spin-vorticity formula

Discovery of polarization in heavy ion collisions

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017





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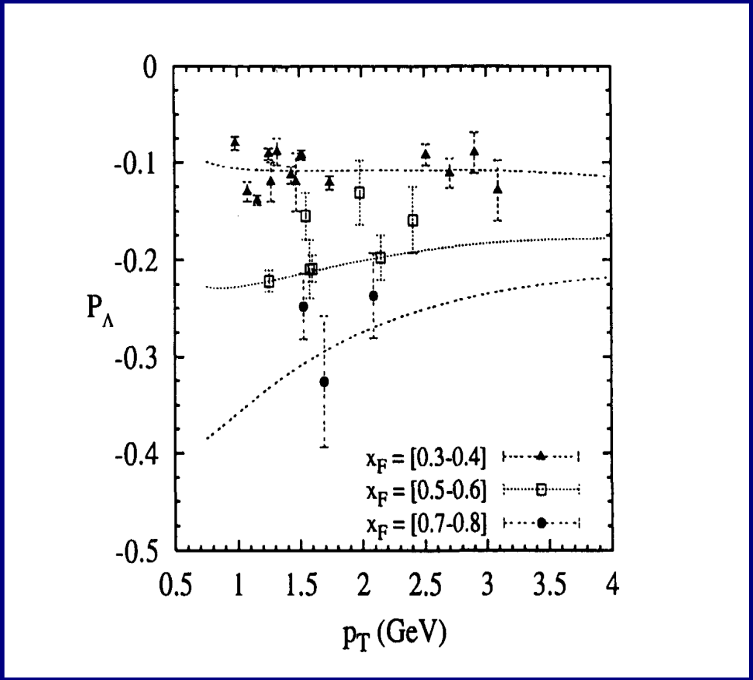
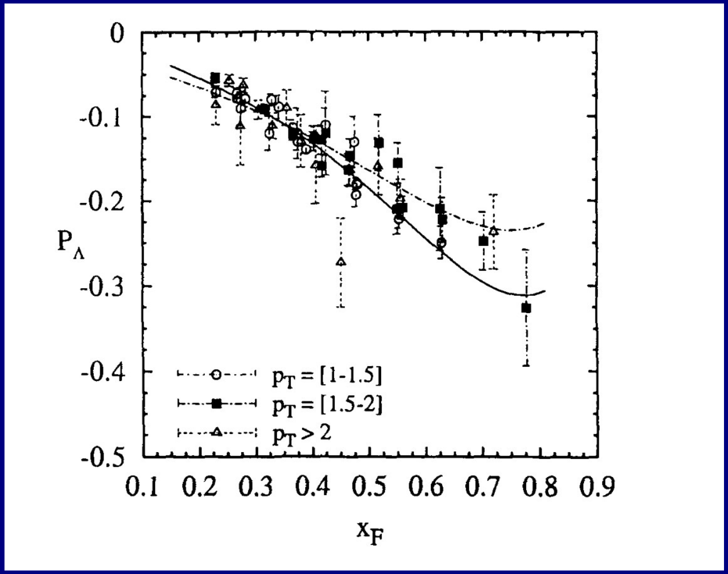
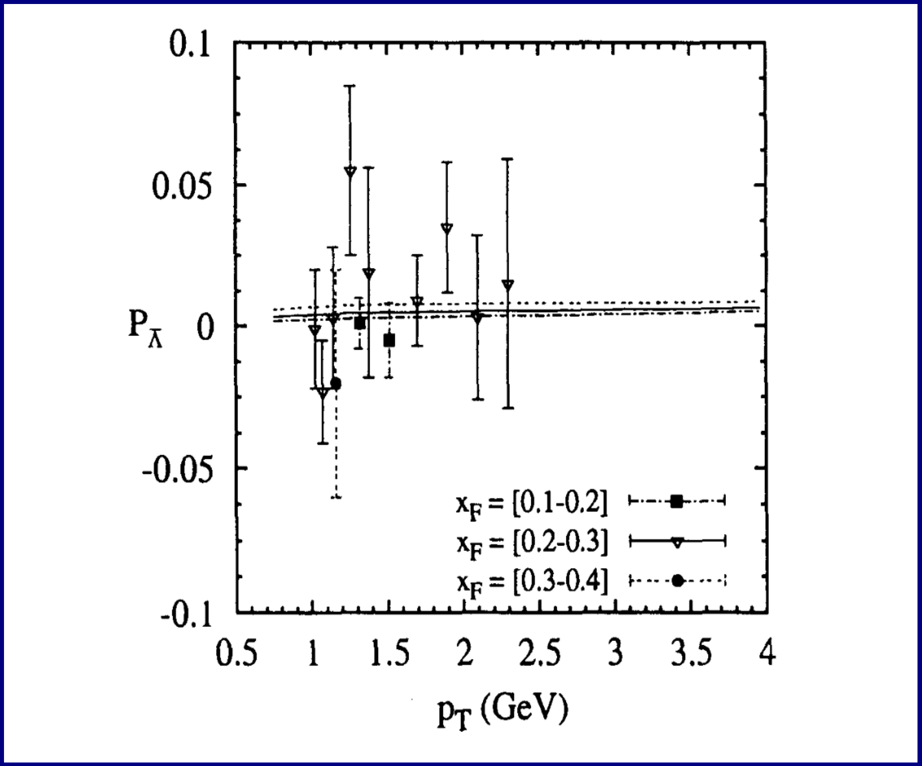
Comparison with NN collisions





Λ is polarized perpendicular to the production plane (no global polarization)

$$x_F = \frac{p_z}{|p_{zMAX}|}$$

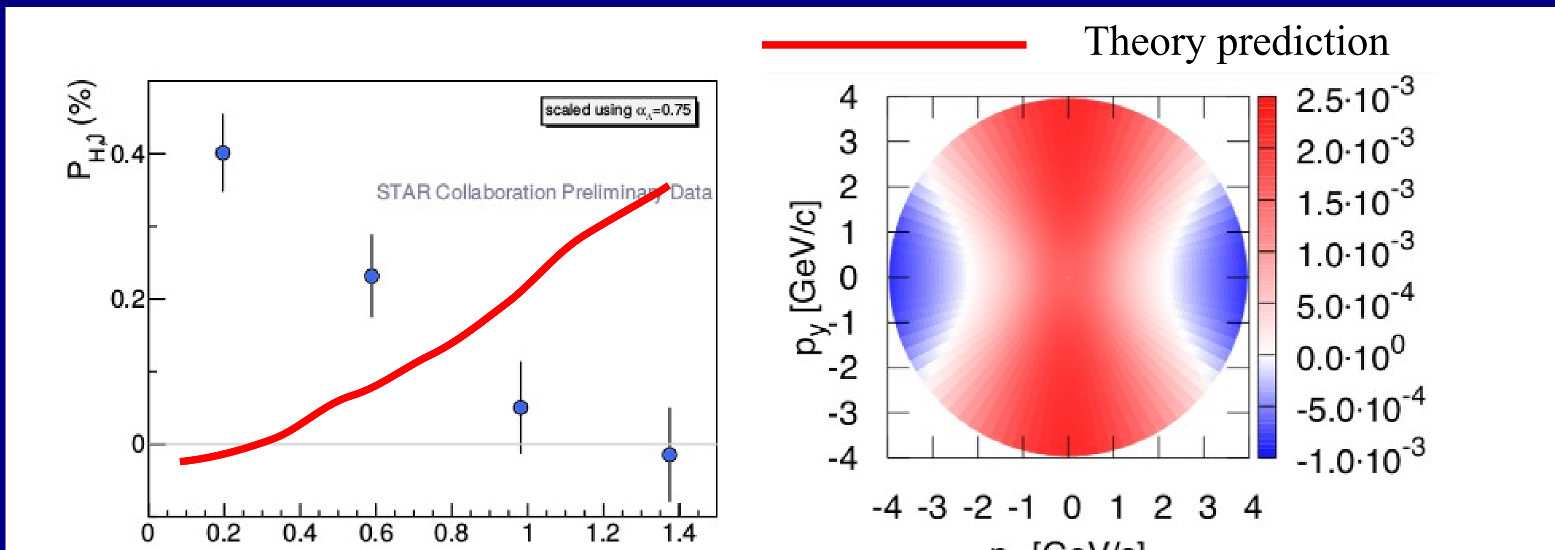


Polarization of anti-Λ almost vanishing compared to Λ

M. Anselmino et al., Czech. J. Phys. 51 (2001) Suppl. A, compilation of data from pBe collisions

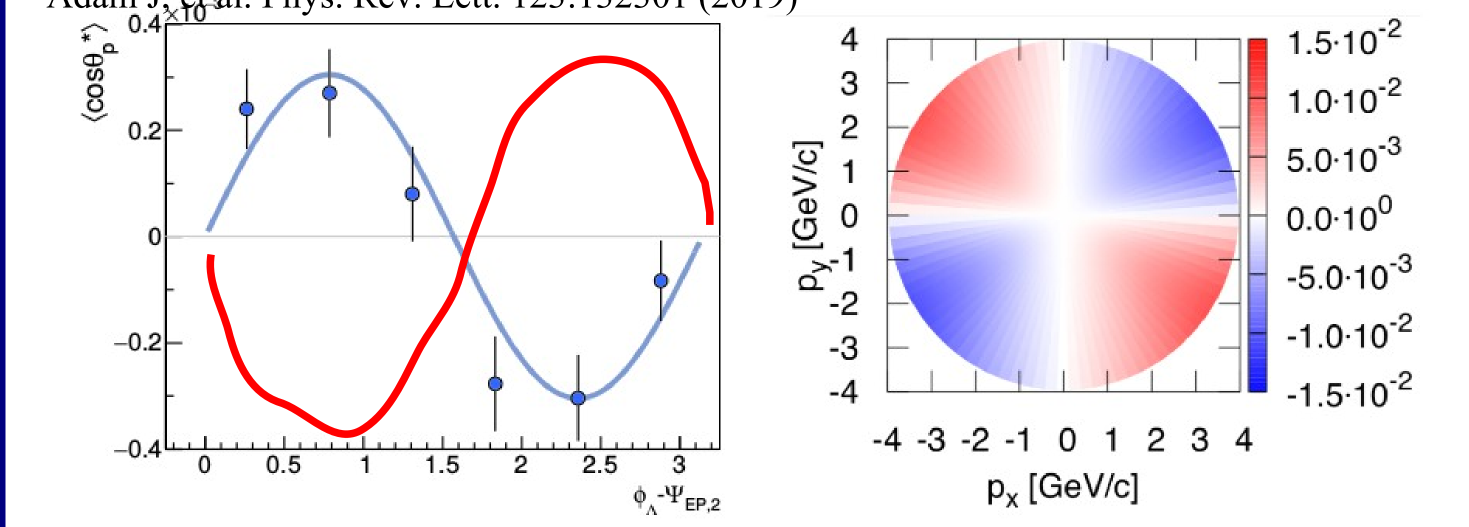
(old) Puzzle: momentum dependence of polarization

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \ n_F}$$



Niida T. Nucl. Phys. A982:511 (2019)

Spin component along J at p_z=0



Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)

Spin component along beam line at p_z=0

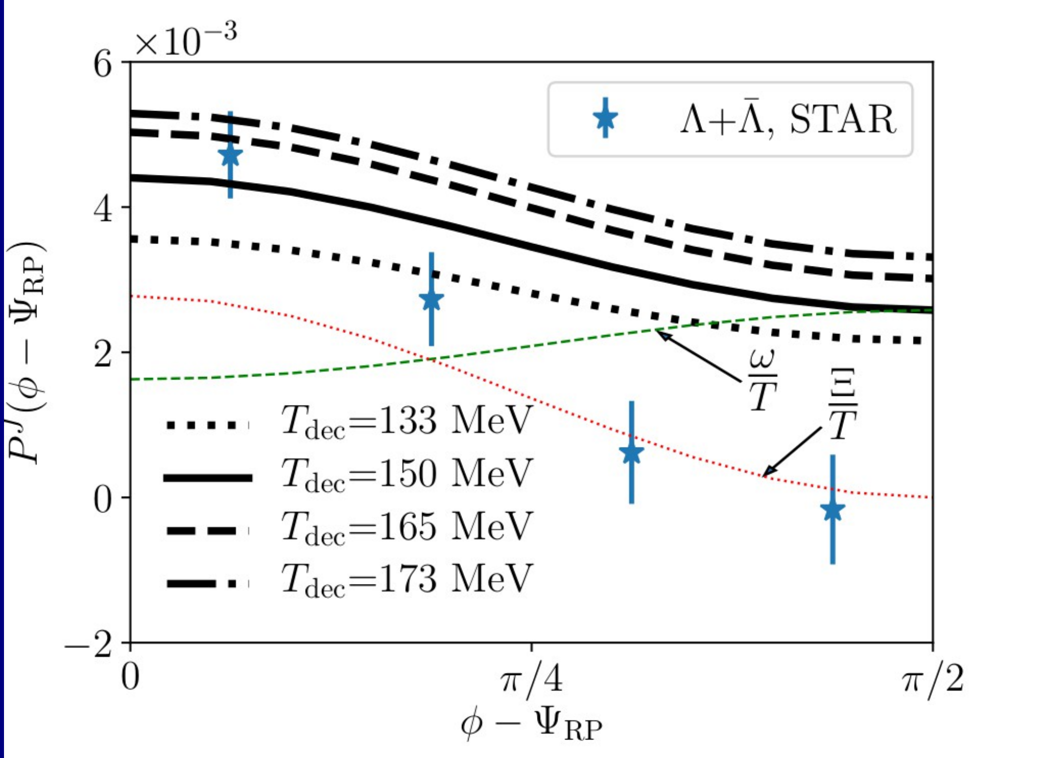
New term found: spin-thermal shear coupling

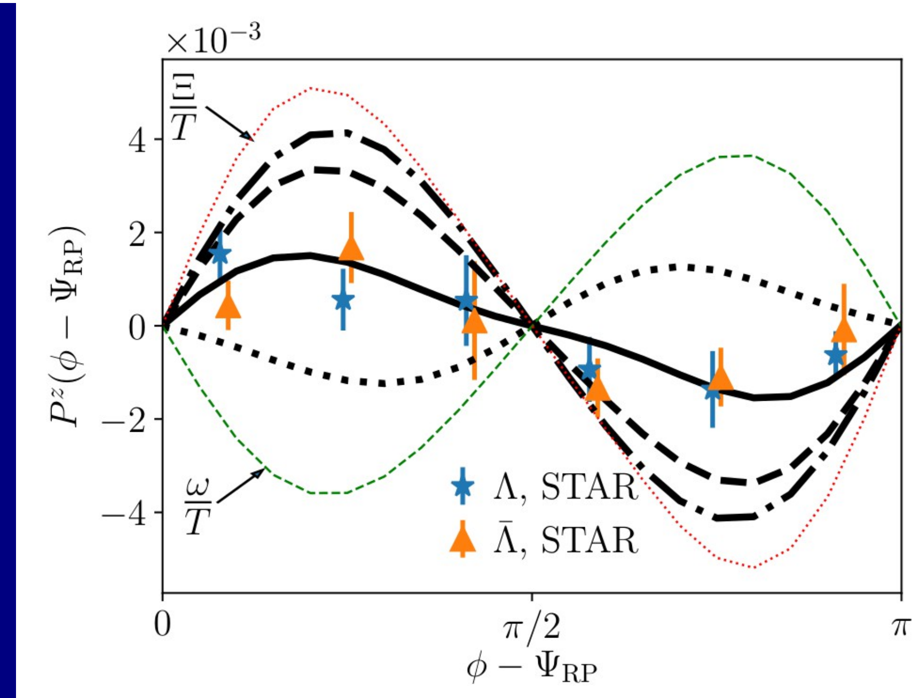
$$S^{\mu}_{\xi}(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F},$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519
S. Liu, Y. Yin, JHEP 07 (2021) 188
Confirmed by C. Yi, S. Pu, D. L. Yang, Phys.Rev.C 104 (2021) 6, 064901
Y. C. Liu, X. G. Huang, Sci.China Phys.Mech.Astron. 65 (2022) 7, 272011

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right).$$

$$S^{\mu}_{\rm ILE}(p) = \quad (\; - \epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon}\Xi_{\lambda\sigma}\right]}{8mT_{\rm dec}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

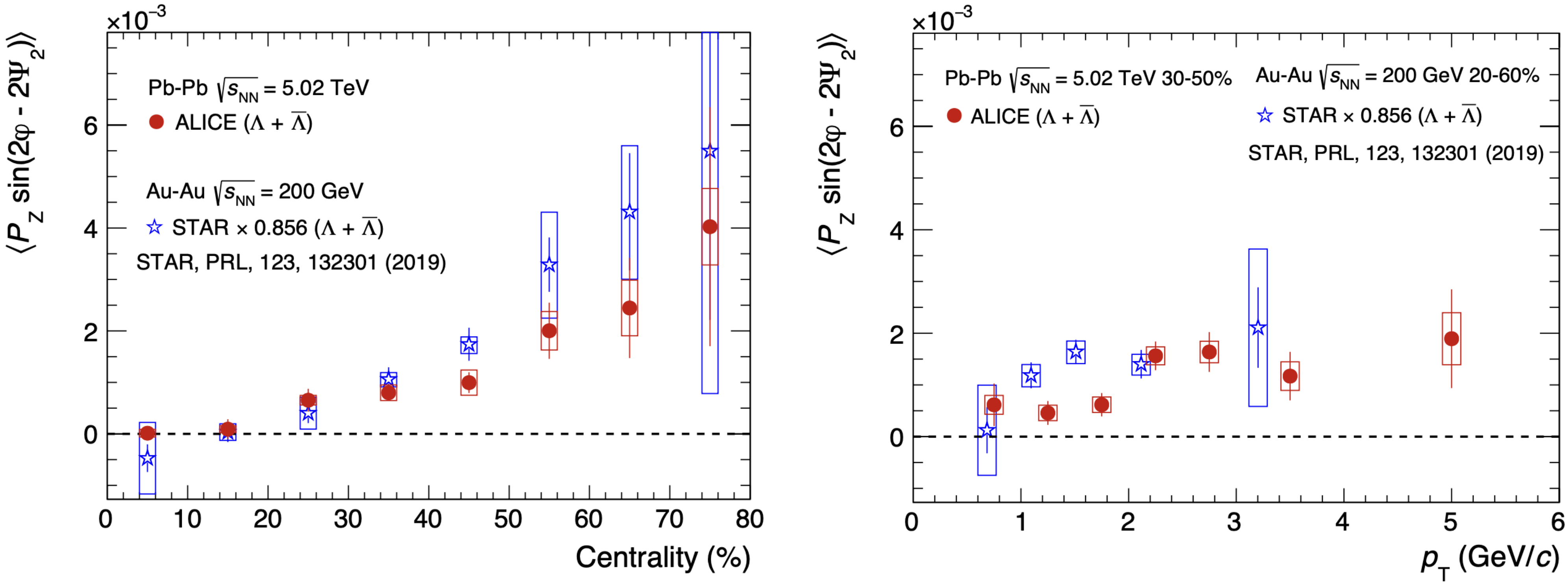




F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko,
Phys. Rev. Lett. 127 (2021) 272302

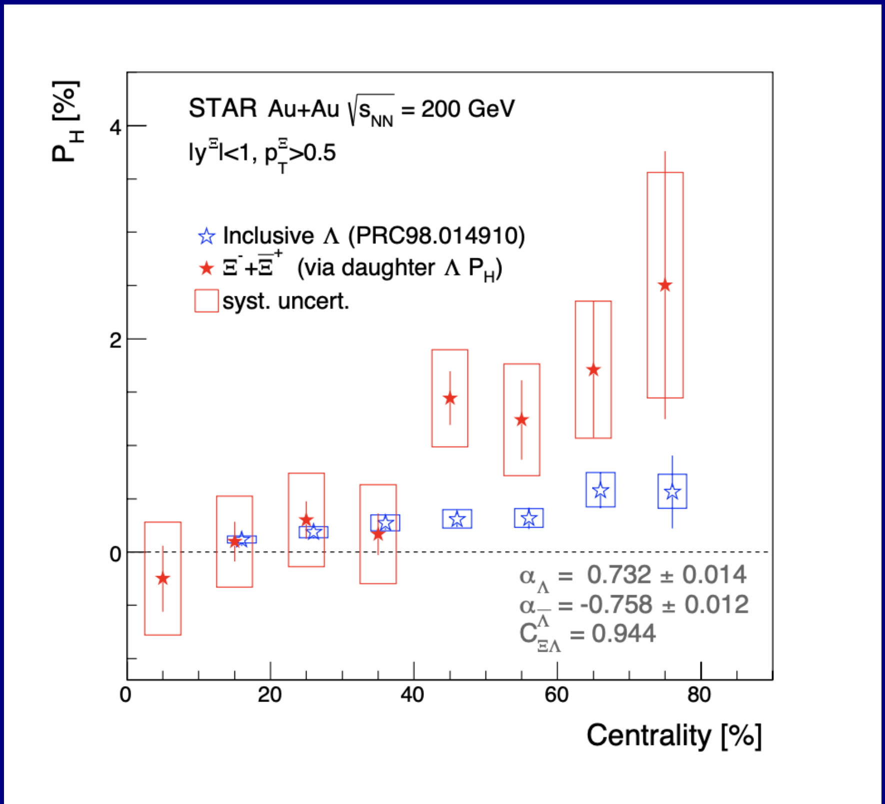
Measurement at the LHC energy

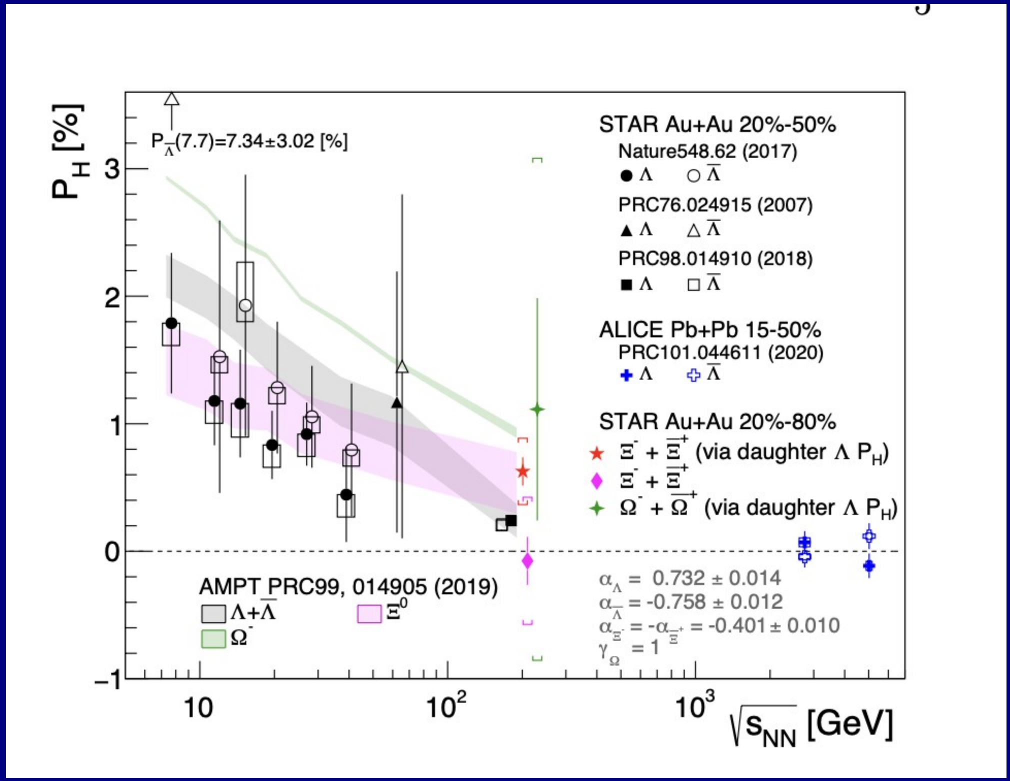
ALICE, Phys. Rev. Lett. 128, 172005. (2022)



Heavier hyperon polarization

STAR Collaboration, Phys. Rev. Lett. 126 (2021) 16, 162301



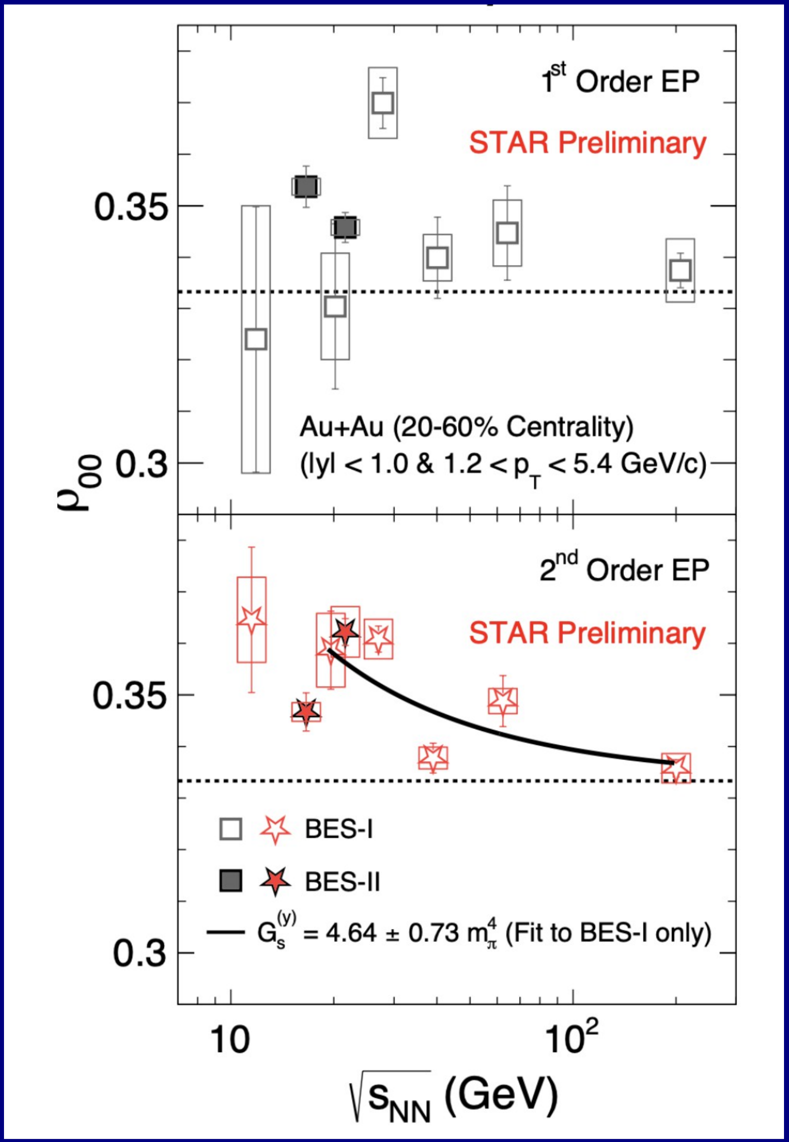


Polarization consistent with S+1 scaling, though with a large statistical error

Will become an important probe with high statistics

Vector meson spin alignment

$$\phi \longrightarrow K^+ \, K^-$$



G. Wilks (STAR coll.) SQM 2024

Spin density matrix:

$$\Theta(\boldsymbol{k}) = \frac{1}{3}\mathbb{1} + \frac{1}{2}\sum_{i=1}^{3} P^i(\boldsymbol{k}) S^i + \frac{1}{\sqrt{6}}\sum_{i,j=1}^{3} \mathfrak{T}^{ij}(\boldsymbol{k})(S^i S^j + S^j S^i),$$

Tensor component

Spin alignment much larger than expected from local equilibrium calculations at the leading order in the gradient expansion

Dissipative contribution calculation in:

S. Y. F. Liu, Feng-Li, arXiv: 2206.11890

D. Wagner, N. Weickgennant, E. Speranza, Phys.Rev.Res. 5 (2023) 1, 013187

Alternative model based proposed by several authors

Qun Wang, Xin-Li Sheng, L. Oliva and others

What can polarization tell us about QGP?

$$S^\mu(p) = -\frac{\epsilon^{\mu\rho\sigma\tau} p_\tau}{8m \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F} \int_\Sigma \mathrm{d}\Sigma \cdot p \times n_F (1 - n_F) \left[\varpi_{\rho\sigma} + 2\hat{t}_\rho \frac{p^\lambda}{E_p} \xi_{\lambda\sigma} - \frac{\hat{t}_\rho \partial_\sigma \zeta}{2E_p} \right]$$

$$n_F = \frac{1}{\exp[\beta \cdot p - \mu q] + 1},$$

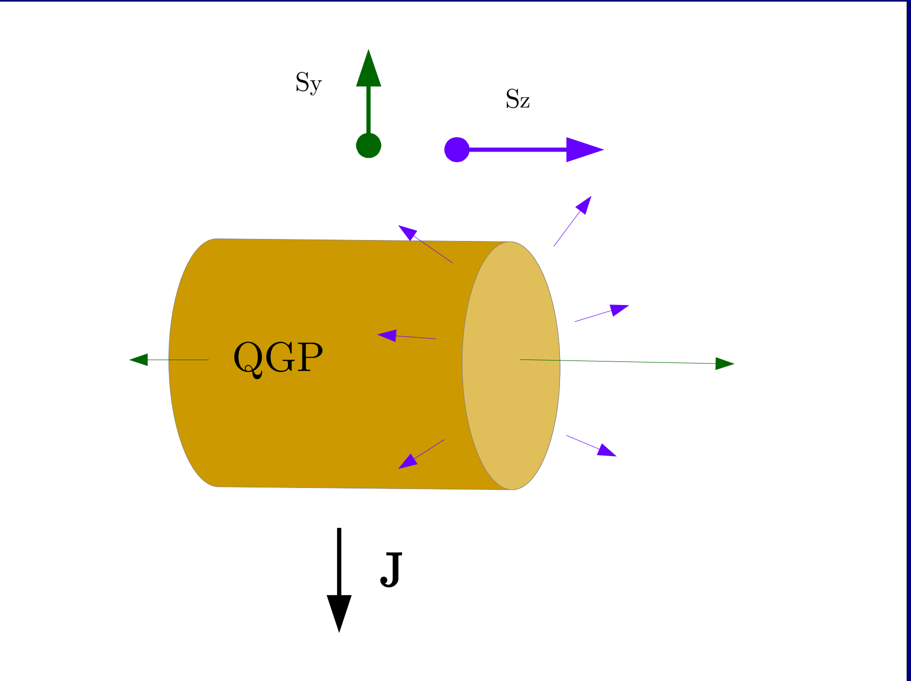
$$\varpi_{\mu\nu} = -\frac{1}{2} \left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu \right).$$

$$\xi_{\mu\nu} = \frac{1}{2} \left(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu \right).$$

$$\beta = \frac{1}{T} u$$

$$\zeta = \frac{\mu}{T}$$

Spin polarization, unlike any other observable, at the leading order depends on hydrodynamic GRADIENTS, therefore it is a very sensitive probe of hydrodynamic motion



S_y sensitive to longitudinal expansion

S_z sensitive to radial expansion

Sensitivity to initial conditions and viscosity

A. Palermo, F.B., E. Grossi, I. Karpenko, Eur. Phys. J. 84 (2024) 9, 920

Recent hydro calculations of Λ polarization in relativistic heavy ion collisions

S. Alzhrani, S. Ryu, and C. Shen, Phys. Rev. C **106**, 014905 (2022), arXiv:2203.15718 [nucl-th].
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Phys. Rev. Lett. **127**, 272302 (2021), arXiv:2103.14621 [nucl-th].
B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, Phys. Rev. Lett. **127**, 142301 (2021), arXiv:2103.10403 [hep-ph].
X.-Y. Wu, C. Yi, G.-Y. Qin, and S. Pu, Phys. Rev. C **105**, 064909 (2022), arXiv:2204.02218 [hep-ph].
Z.-F. Jiang, X.-Y. Wu, H.-Q. Yu, S.-S. Cao, and B.-W. Zhang, Acta Phys. Sin. **72**, 072504 (2023).
Z.-F. Jiang, X.-Y. Wu, S. Cao, and B.-W. Zhang, Phys. Rev. C **108**, 064904 (2023), arXiv:2307.04257 [nucl-th].
V. H. Ribeiro, D. Dobrigkeit Chinellato, M. A. Lisa, W. Matioli Serenone, C. Shen, J. Takahashi, and G. Torrieri, Phys. Rev. C **109**, 014905 (2024), arXiv:2305.02428 [hep-ph].

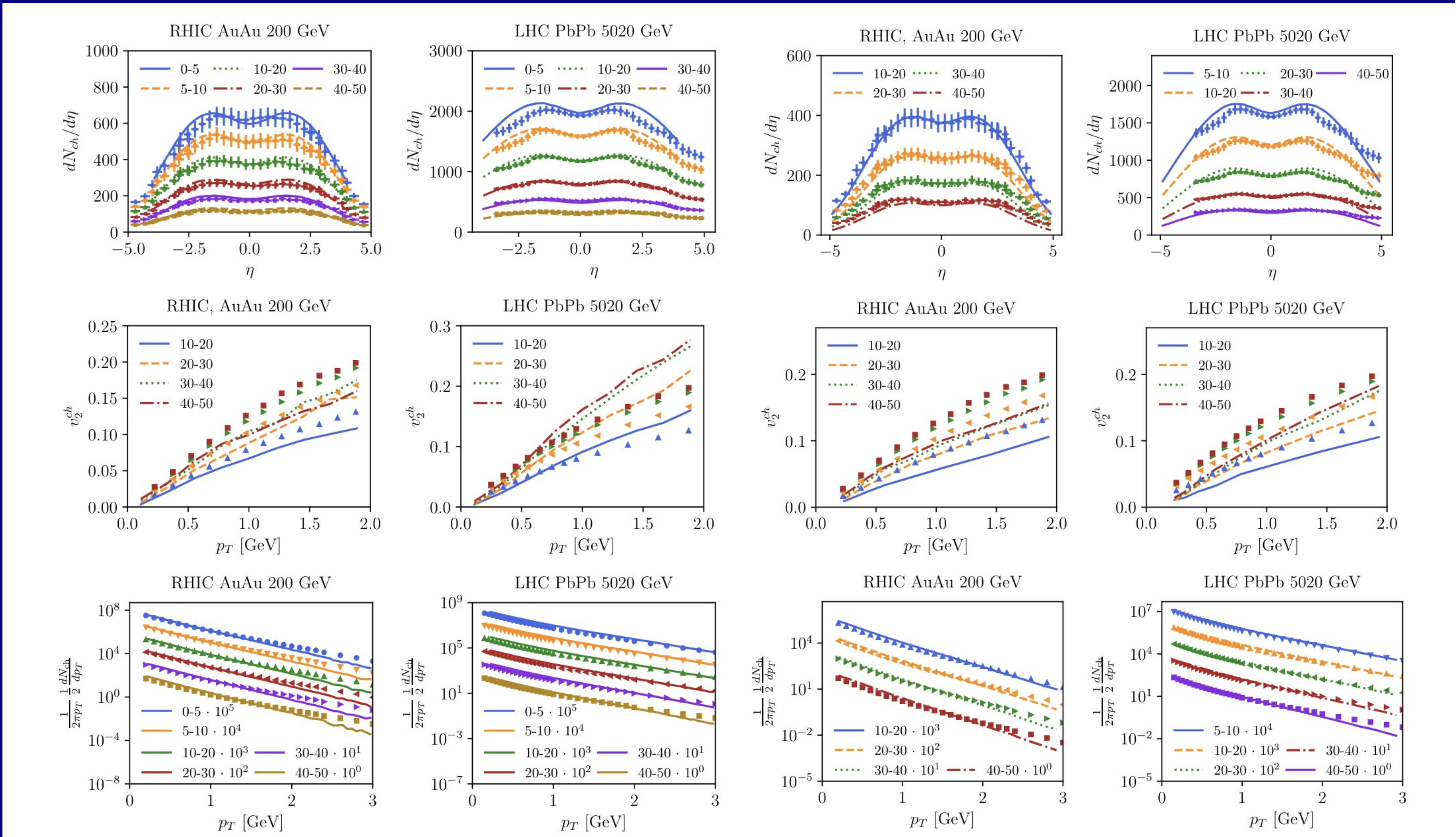
Numerical implementation of 3+1 D causal viscous hydrodynamics (VHLLE) with statistical hadronization and particle rescattering (afterburner SMASH)

Initial state model: SUPERMC (C. Shen et al.), GLISSANDO (Monte-Carlo Glauber)

Polarization transferred to Λ in secondary decays of Σ^0 and Σ^* taken into account

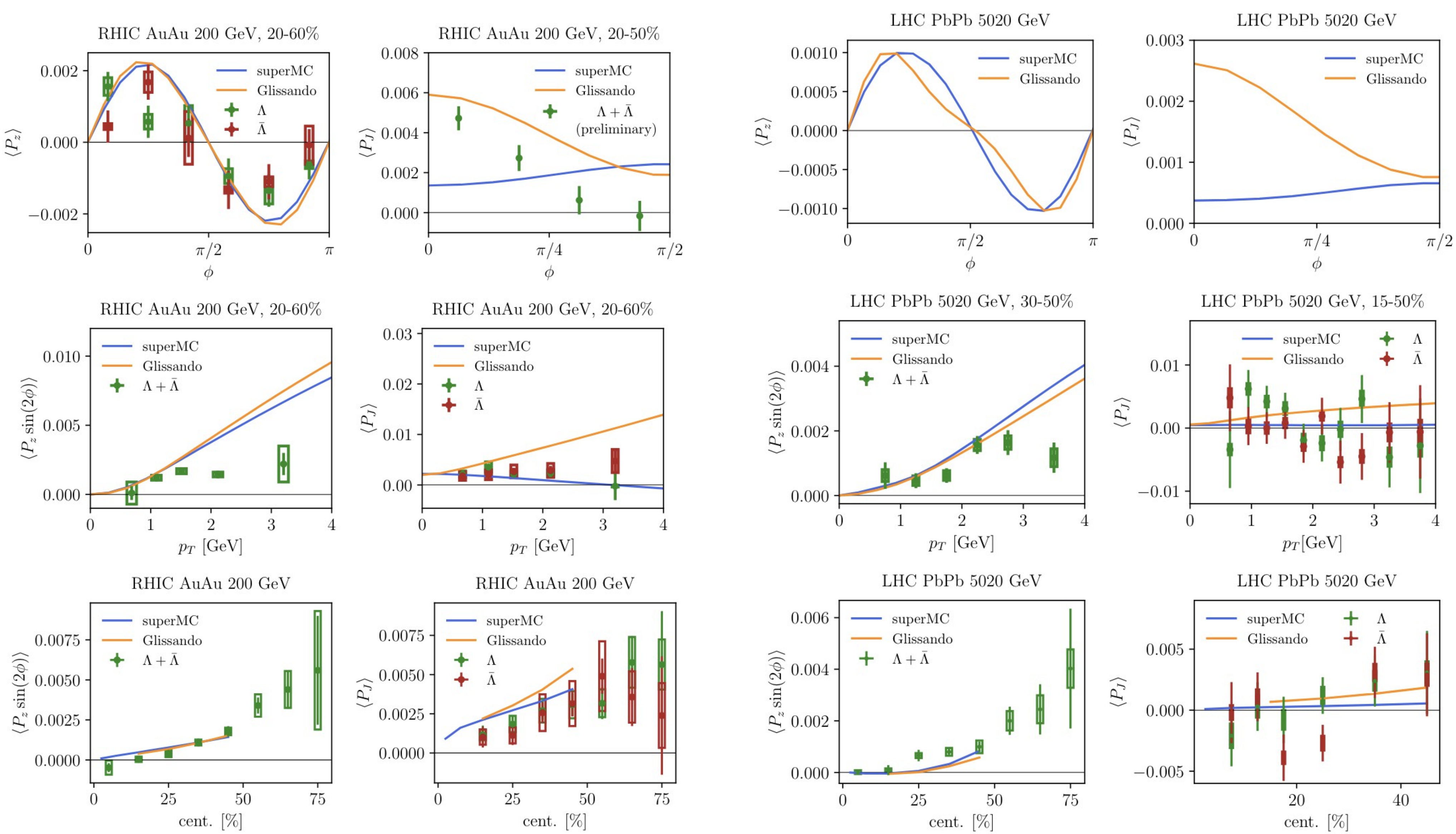
Qualification of the code

Benchmark distributions



SUPERMC GLISSANDO

RESULTS

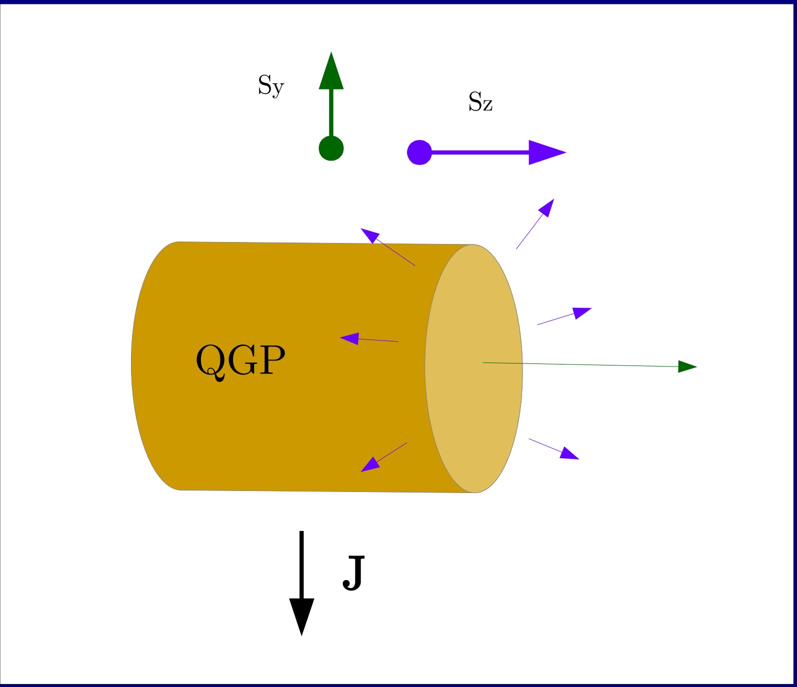


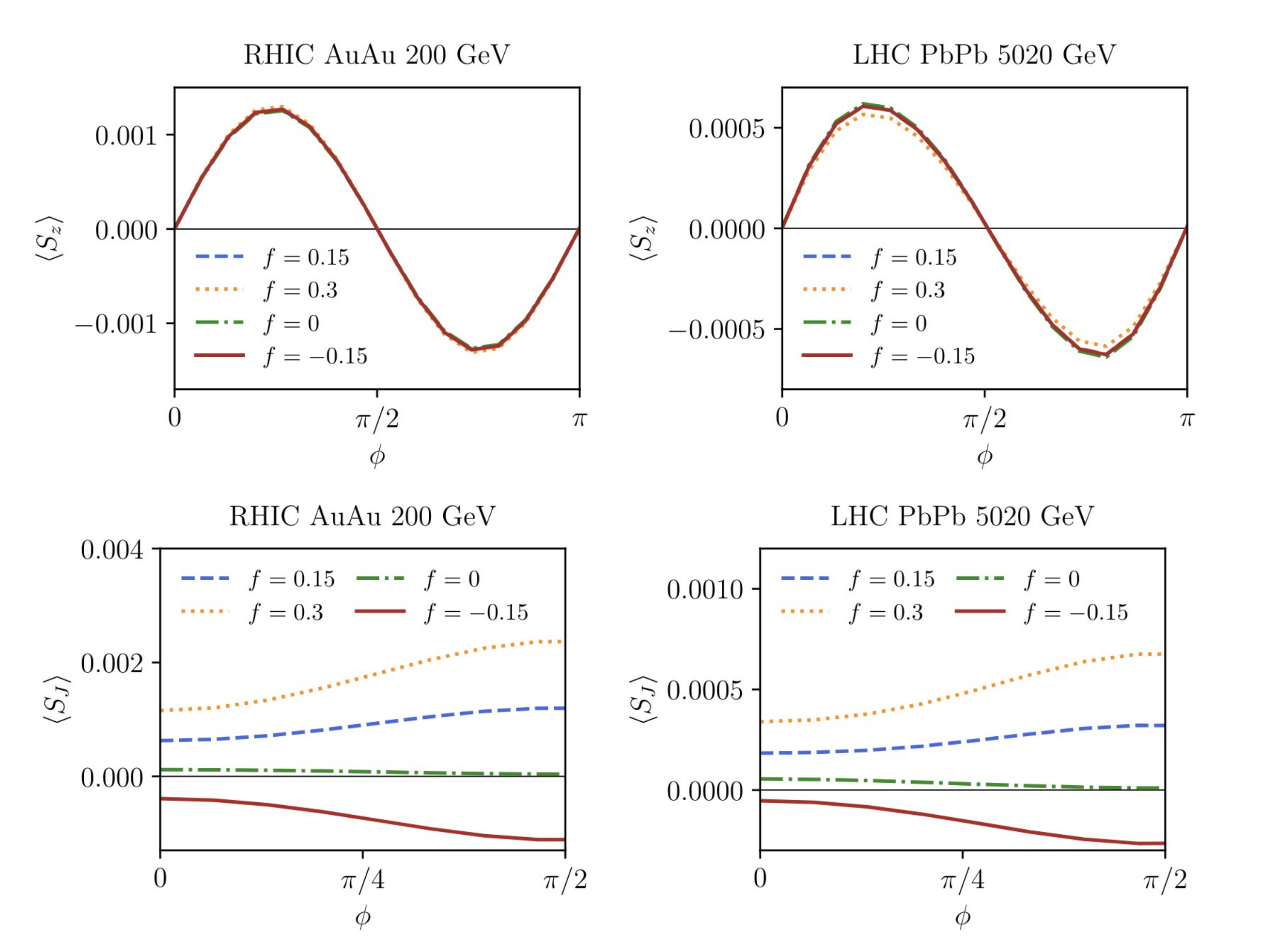
RHIC AuAu 200 GeV, 20-60%
RHIC AuAu 200 GeV, 20-50%
LHC PbPb 5020 GeV
LHC PbPb 5020 GeV
RHIC AuAu 200 GeV, 20-60%
RHIC AuAu 200 GeV, 20-60%
LHC PbPb 5020 GeV, 30-50%
LHC PbPb 5020 GeV, 15-50%
RHIC AuAu 200 GeV
RHIC AuAu 200 GeV
LHC PbPb 5020 GeV
LHC PbPb 5020 GeV
superMC
Glissando
Λ
$\bar{\Lambda}$
$\Lambda + \bar{\Lambda}$
(preliminary)
$\langle P_z \rangle$
$\langle P_J \rangle$
$\langle P_z \sin(2\phi) \rangle$
ϕ
p_T [GeV]
cent. [%]

Sensitivity to initial longitudinal flow

Variation of SUPERMC flow parameter

$$T^{\tau\tau} = \rho \cosh(f\, y_{CM})$$
$$T^{\tau\eta} = \frac{\rho}{\tau} \sinh(f\, y_{CM})$$





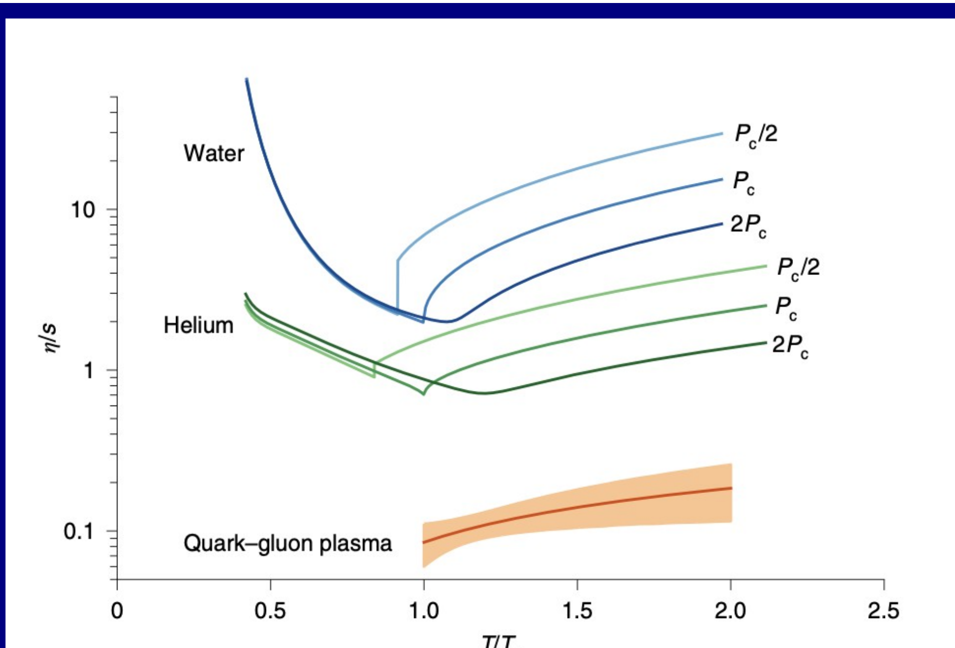
Shear and bulk viscosity of the QGP

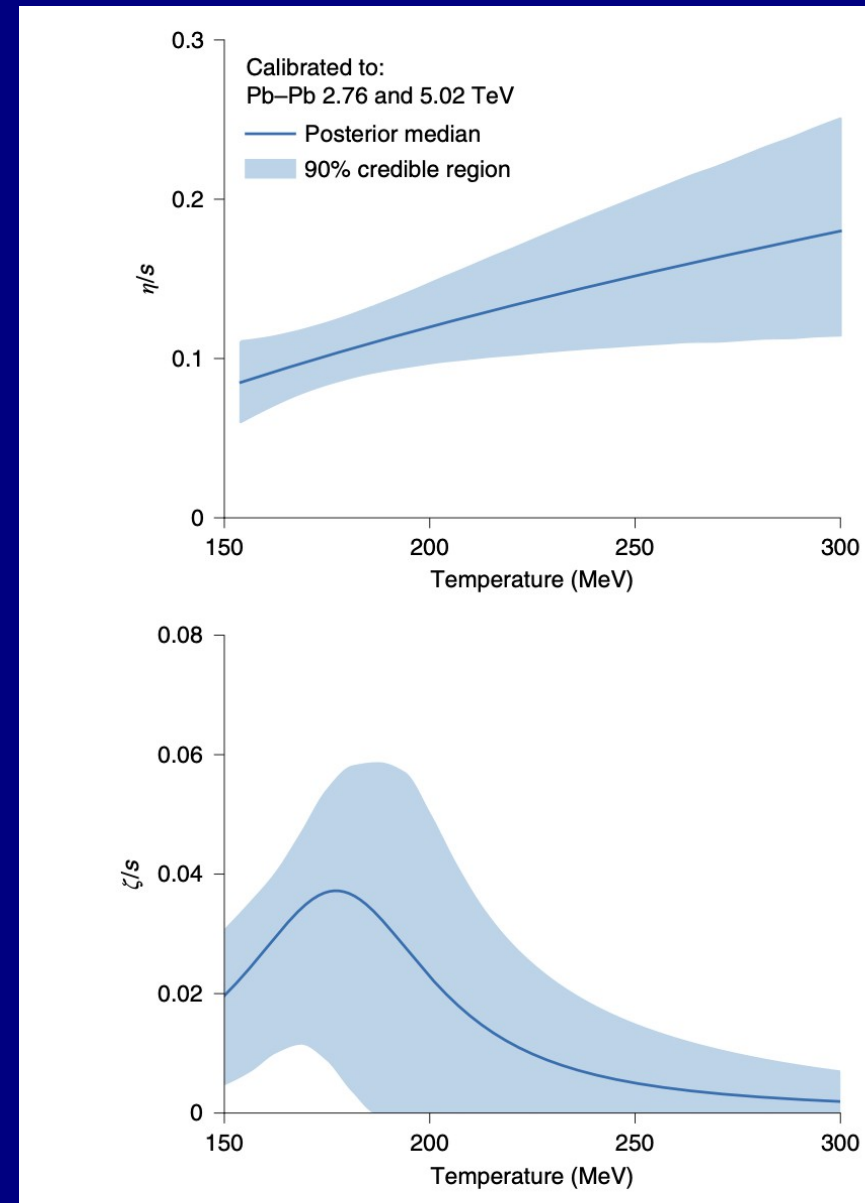
Measuring the shear and bulk viscosity of the Quark Gluon Plasma is one of the most important objectives

nature physics
LETTERS
https://doi.org/10.1038/s41567-019-0611-8

Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma

Jonah E. Bernhard*, J. Scott Moreland and Steffen A. Bass

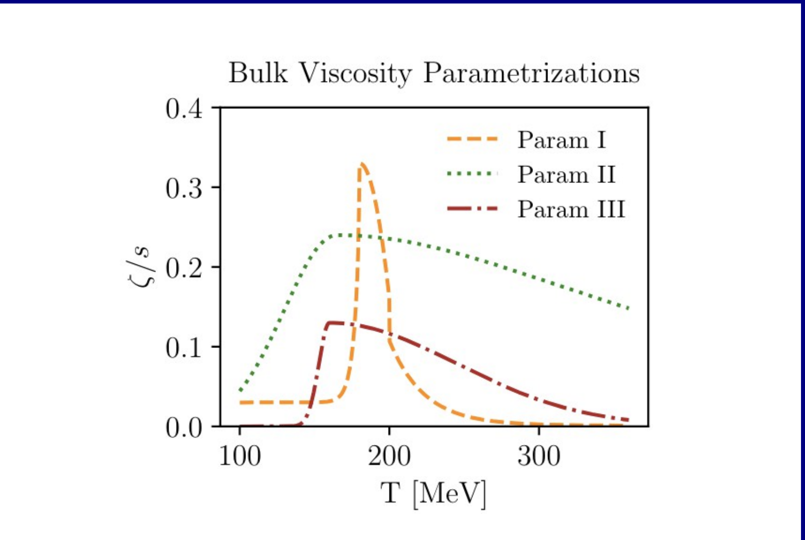


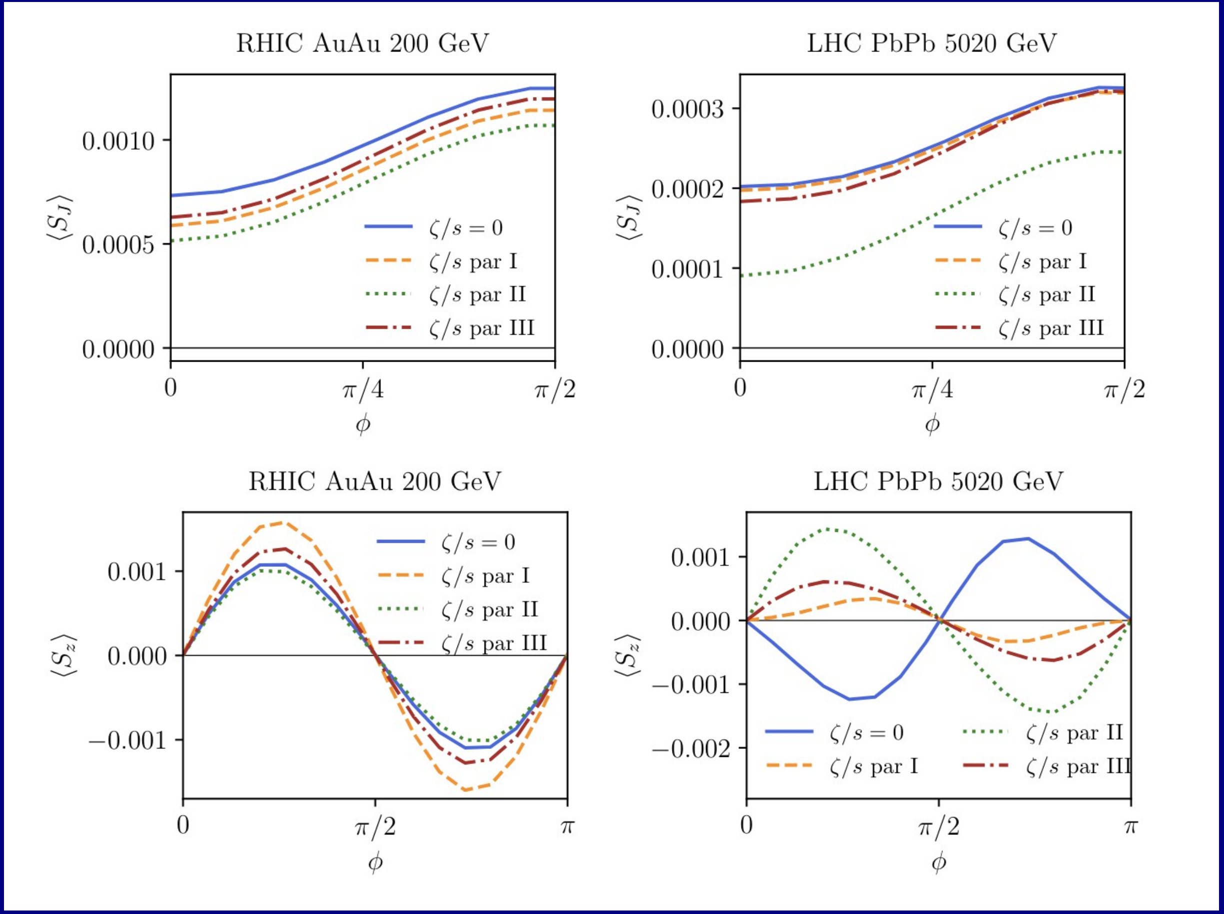


Fit by using momentum-related observables

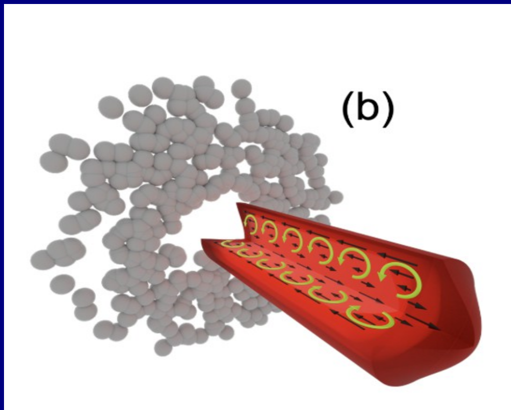
Sensitivity of polarization to bulk viscosity

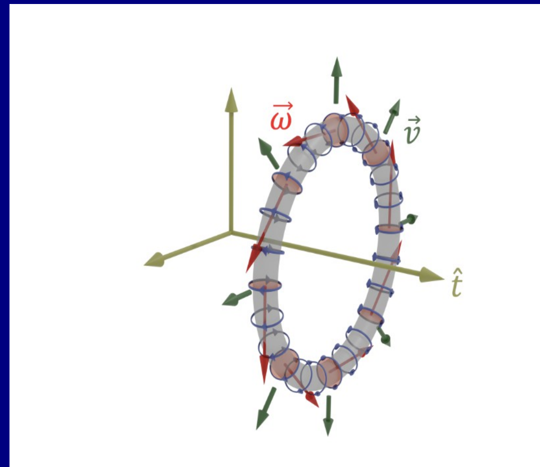
While polarization seems not to depend much on shear viscosity, it turns out to be very sensitive to bulk viscosity at the highest LHC energy





Polarization as a probe of jets and critical point





$$\mathcal{R}_{\Lambda}^{\hat{t}} \equiv \frac{\epsilon^{\mu\nu\rho\sigma} S_{\mu} n_{\nu} \hat{t}_{\rho} p_{\sigma}}{|S||\epsilon^{\mu\nu\rho\sigma} n_{\nu} \hat{t}_{\rho} p_{\sigma}|} \quad .$$

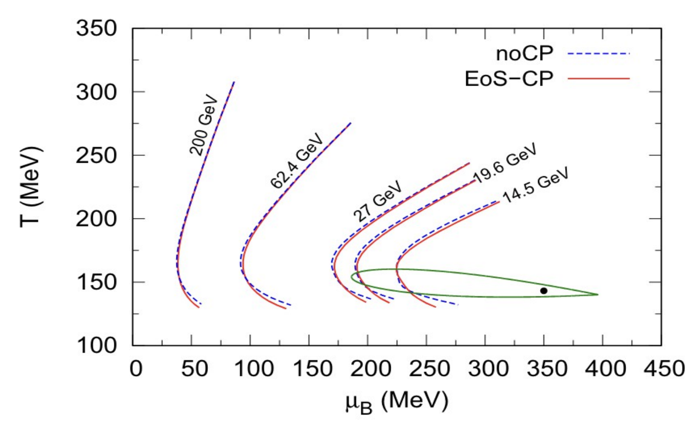
Shooting a proton or a jet through a heavy nucleus is expected to produce vortex rings, which can possibly be detected through spin polarization

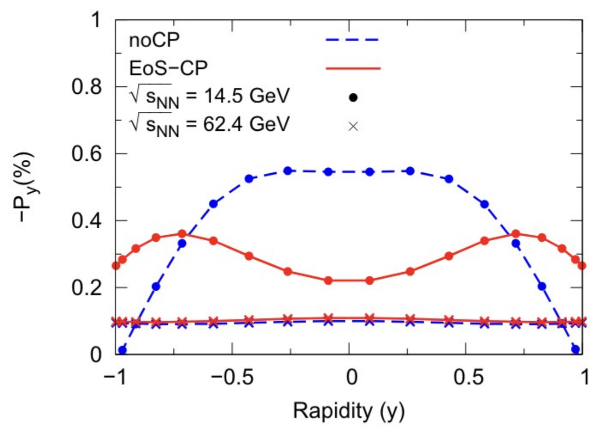
V. H. Ribeiro et al., Phys.Rev.C 109 (2024) 1, 014905; M. Lisa et al., Phys.Rev.C 104 (2021) 1, 011901

Polarization as a probe of the QCD critical point

Critical behaviour of viscous coefficients

$$\zeta = \zeta_0 \left(\frac{\xi}{\xi_0}\right)^3 , \ \eta = \eta_0 \left(\frac{\xi}{\xi_0}\right)^{0.05}$$





S. K. Singh, Jan e-Alam, Eur. Phys. J. C 83 (2023) 585

Summary and outlook

- Spin polarization is a new powerful probe of Quark Gluon Plasma; it is a probe of the *gradients* in the fluid.
- Local equilibrium+hydrodynamic model reproduces the measured Λ polarization
- Vector mesons spin alignment larger than expected: a dissipative correction to local equilibrium or an indication of other mechanisms?
- Full potential is currently limited by statistics. High luminosity will make it one of the most, if not the most, effective probes of QGP formation and evolution at high energy

Why?

Analysis of the different gradient components of the polarization

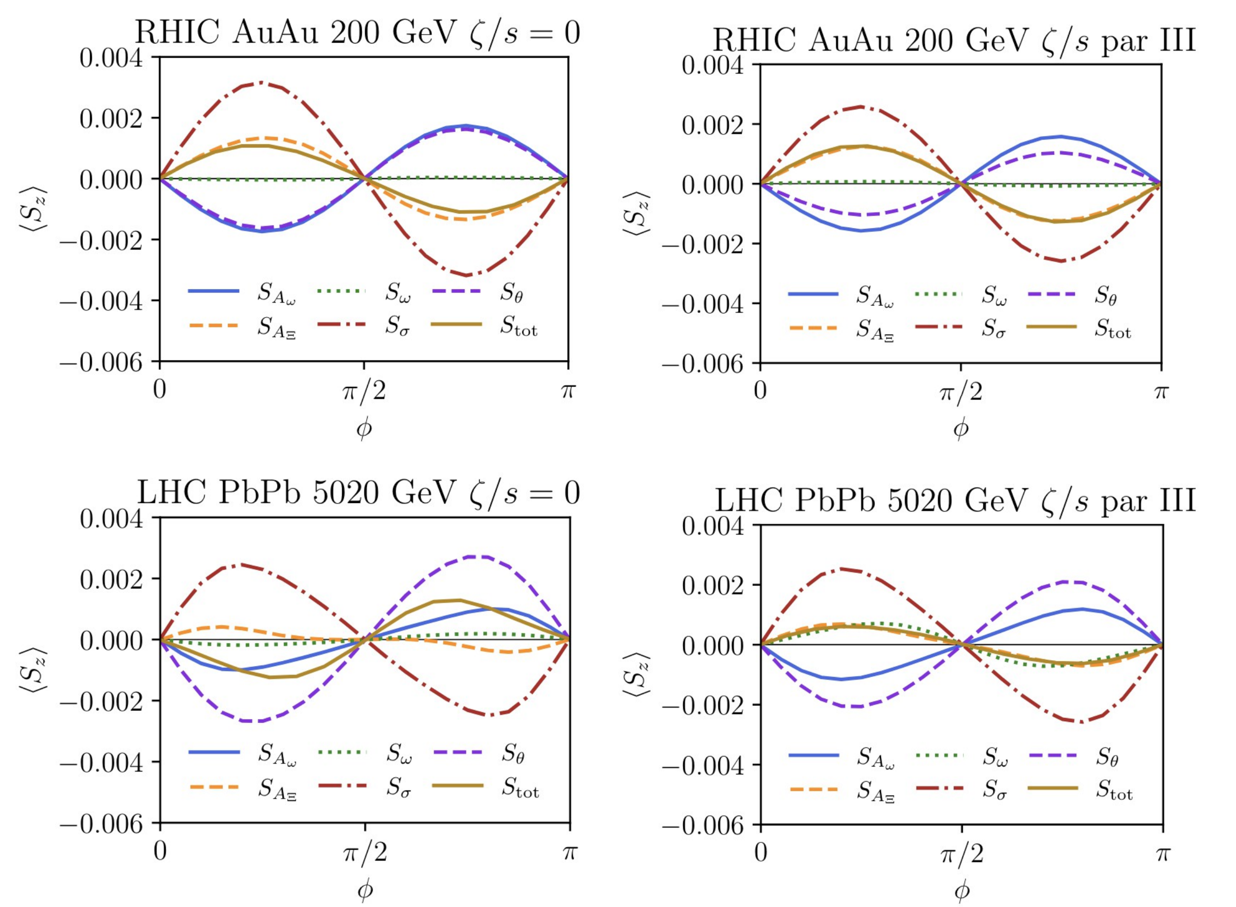
$$S^\mu_{A_\omega} = -\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\; A_\nu u_\rho}{8mT_H \int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F},$$

$$S^\mu_{\omega} = \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\; [\omega^\mu u\cdot p - u^\nu \omega\cdot p]}{4mT_H \int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F},$$

$$S^\mu_{A_\Xi} = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho \frac{p_\tau}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\,[u_\sigma A\cdot p + A_\sigma u\cdot p]}{8mT_H \int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F},$$

$$S^\mu_{\sigma} = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho p_\tau \frac{p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\sigma_{\lambda\sigma}}{4mT_H \int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F},$$

$$S^\mu_{\theta} = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho p_\tau \frac{p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\theta\Delta_{\lambda\sigma}}{12mT_H \int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}.$$



Why do we have a dependence on Σ ?

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{I\ K}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{I\ K}$ does not vanish, therefore it does depend on the integration hypersurface and

$$\widehat{\Lambda} \widehat{Q}^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{Q}^{\alpha\beta}$$

(My) Expected theoretical developments

New calculations of dissipative corrections, both for fermion polarization and vector meson alignment

Appraisal of thermal shear-spin coupling formula

Calculation of the contribution of second order derivatives (hopefully small)

COMPUTATION:

Increase code accuracy at low energy, explore the potential of critical point discovery potential

Density operator of quantum relativistic fluid

Needed to calculate the Wigner function!

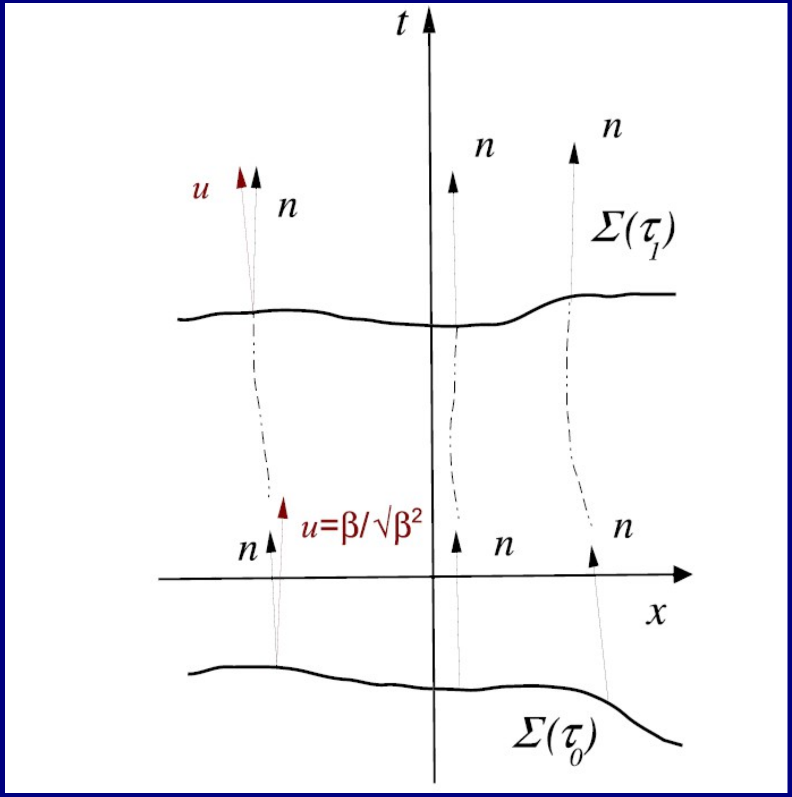
$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

General covariant
Local thermodynamic
Equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right)\right]$$

$$\beta = \frac{1}{T}u$$

$$\zeta = \frac{\mu}{T}$$



The operator is obtained by maximizing the entropy

$$S = -\mathrm{tr}(\widehat{\rho}\log\widehat{\rho})$$

with the constraints of fixed energy-momentum density

Zubarev, 1979, Ch, Van Weert 1982

See also:
F. B., L. Bucciantini, E. Grossi, L. Tinti,
Eur. Phys. J. C 75 (2015) 191 (∩frame)

T. Hayata, Y. Hidaka, T. Noumi, M. Hongo,
Phys. Rev. D 92 (2015) 065008

The actual statistical operator (Zubarev theory)

The above density operatot is "time" dependent, cannot be the actual one!

In the Zubarev's theory, this is the LTE at some initial "time":

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right] .$$

NOTE: T_B stands for the symmetrized Belinfante stress-energy tensor

With the Gauss theorem

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \ \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right] ,$$

Local equilibrium, non-dissipative terms

Dissipative terms

Incidentally: global thermodynamic equilibrium

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Independent of the 3D hypersurface Δ if

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \qquad \partial_\mu \zeta = 0$$

Killing equation

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

The density operator becomes

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

Local thermodynamic equilibrium approximation

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp \left[- \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \, \widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right]$$
$$= \frac{1}{Z} \exp \left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \, \widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right]$$

Corresponding to the ideal fluid: Neglecting dissipative term in the exponent of the density operator

$$W(x,k) \simeq W(x,k)_{\rm LE} = {\rm Tr}(\widehat{\rho}_{\rm LE} \widehat{W}(x,k))$$

What is this new term?

Does it have a non-relativistic limit?
Let us decompose it

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma \left(\frac{1}{T}\right) u_\rho + \frac{1}{2}\partial_\rho \left(\frac{1}{T}\right) u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

All terms are relativistic (they vanish in the infinite *c* limit) EXCEPT grad T terms, which give rise to:

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v} \times \frac{\int \mathrm{d}^3\mathrm{x}\, n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\, n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Application to relativistic heavy ion collisions

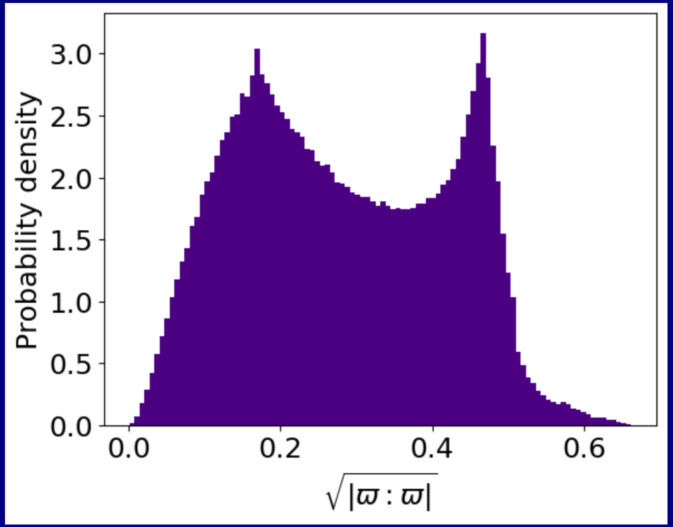
F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko, arXiv:2103.14621

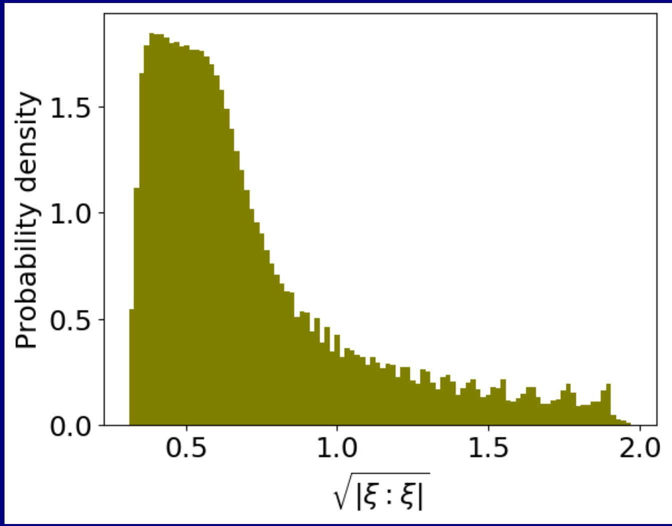
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau} \frac{p_\tau p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\hat{t}_\rho \xi_{\sigma\lambda}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

Is linear response theory adequate?





Isothermal hadronization

At high energy, Δ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T) u^\mu$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} \beta_\nu \right] = \frac{1}{Z} \exp\left[- \frac{1}{T} \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} u_\nu \right]$$

Only NOW u can be expanded!

$$u_\nu(y) = u_\nu(x) + \partial_\lambda u_\nu(x)(y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2T}(\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x)) \widehat{J}^{\mu\nu}_x - \frac{1}{2T}(\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x)) \widehat{Q}^{\mu\nu}_x + \ldots]$$

A short theory summary

F. B., Lecture Notes in Physics 987, 15 (2021) arXiv:2004.04050

Spin polarization vector for spin ½ particles:

$$S^\mu(p) = \frac{1}{2} \frac{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4(\gamma^\mu \gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma \cdot p \; \mathrm{tr}_4 W_+(x,p)}$$

Wigner function:

$$\begin{aligned}\widehat{W}(x,k)_{AB} &= -\frac{1}{(2\pi)^4} \int \mathrm{d}^4y \, \mathrm{e}^{-ik\cdot y} : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) : \\ &= \frac{1}{(2\pi)^4} \int \mathrm{d}^4y \, \mathrm{e}^{-ik\cdot y} : \overline{\Psi}_B(x+y/2)\Psi_A(x-y/2) :\end{aligned}$$

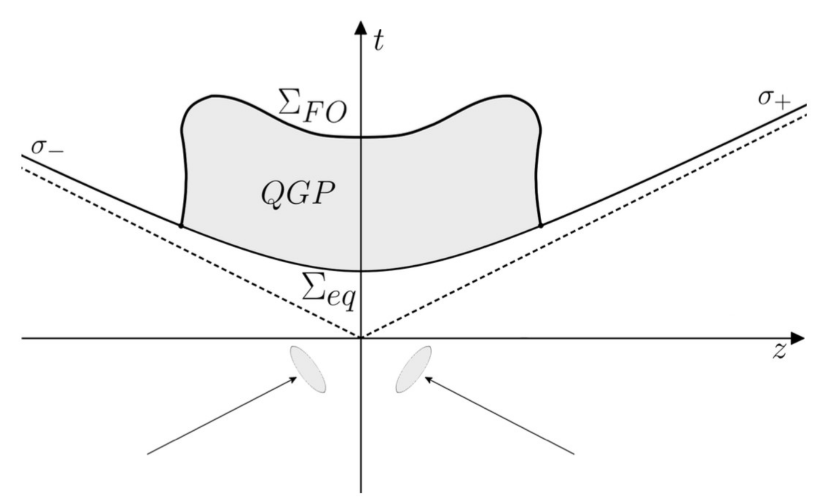
$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

Local equilibrium density operator:

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

$$\beta = \frac{1}{T} u$$

$$\zeta = \frac{\mu}{T}$$



Hydrodynamic limit: Taylor expansion

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}^{\mu\nu}_B(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T^{\mu\nu}_B(y)\beta_\nu(x) = \beta_\nu(x)\int_\Sigma {\rm d}\Sigma_\mu T^{\mu\nu}_B(y) = \beta_\nu(x)\widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp\left[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \ldots\right]$$

$$\widehat{J}^{\mu\nu}_x = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}^{\lambda\nu}_B(y) - (y-x)^\nu\widehat{T}^{\lambda\mu}_B(y)$$

$$\widehat{Q}^{\mu\nu}_x = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}^{\lambda\nu}_B(y) + (y-x)^\nu\widehat{T}^{\lambda\mu}_B(y)$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Linear response theory

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\; \mathrm{e}^{z(\widehat{A}+\widehat{B})}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}} \simeq \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\; \mathrm{e}^{z\widehat{A}}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}}$$

$$\widehat{A} = -\beta_\mu(x)\widehat{P}^\mu$$

$$\widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$W(x,k) \simeq \frac{1}{Z}\mathrm{Tr}(\mathrm{e}^{\widehat{A}+\widehat{B}}\widehat{W}(x,k)) \simeq \ldots$$

CORRELATORS

$$\langle \widehat{Q}^{\mu\nu}_x \widehat{W}(x,p)\rangle \qquad \langle \widehat{J}^{\mu\nu}_x \widehat{W}(x,p)\rangle$$

Spin mean vector at leading order in thermal vorticity

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}_x^{\mu\nu} - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}_x^{\mu\nu} + \ldots]$$

Neglected by "prejudice" until 2021

$$S^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\beta\gamma\delta}p_\delta \frac{\int {\rm d}\Sigma_\lambda p^\lambda {\rm tr}_4(\Sigma_{\beta\gamma}W_+(x,p))}{\int {\rm d}\Sigma_\lambda p^\lambda {\rm tr}_4 W_+(x,p)}$$

+ Linear response theory

$$n_F = ({\rm e}^{\beta\cdot p - \xi} + 1)^{-1}$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma {\rm d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma {\rm d}\Sigma_\tau p^\tau \; n_F}$$

See also

R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904
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