

TMD factorisation (a phenomenological introduction)

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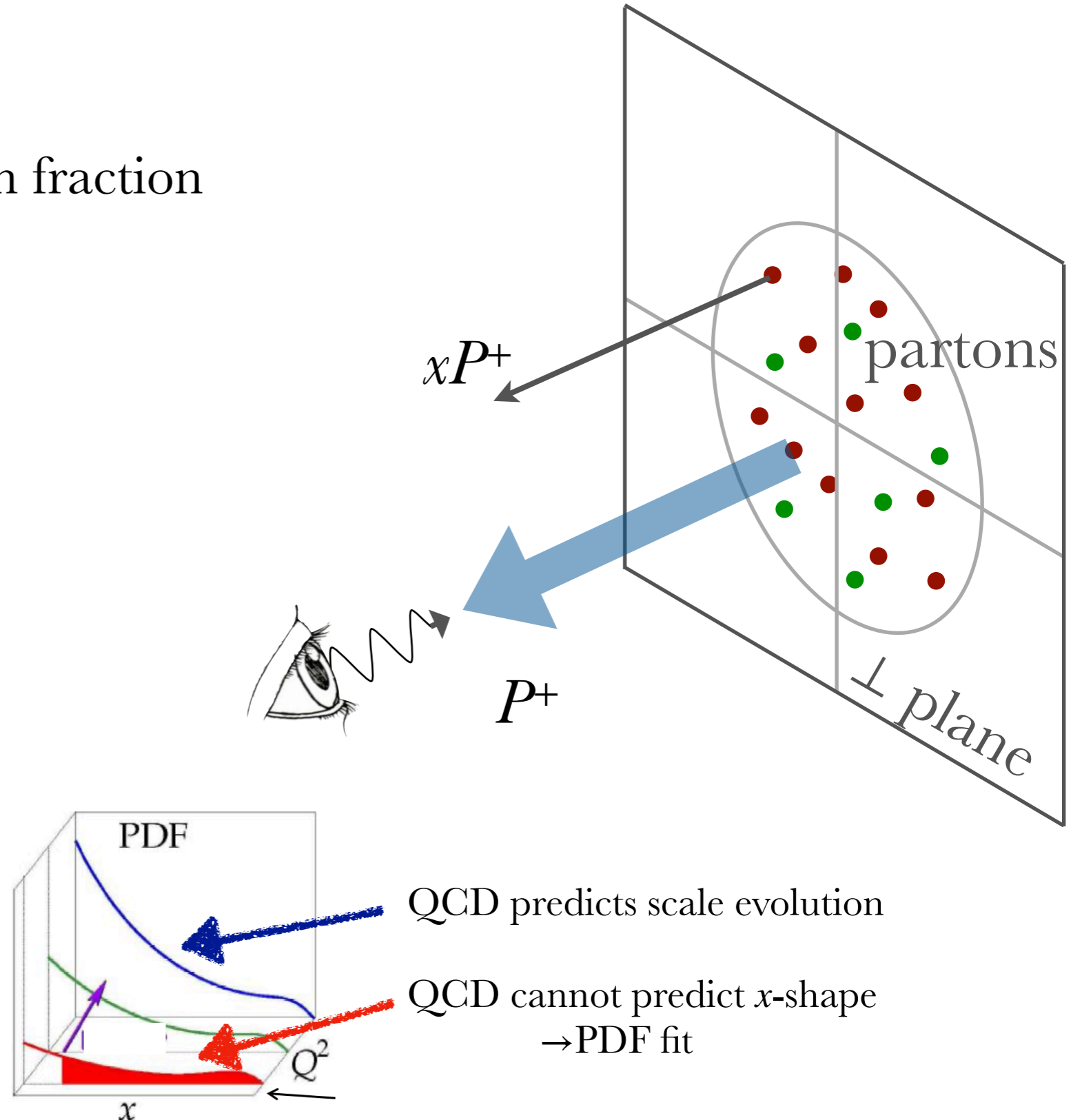
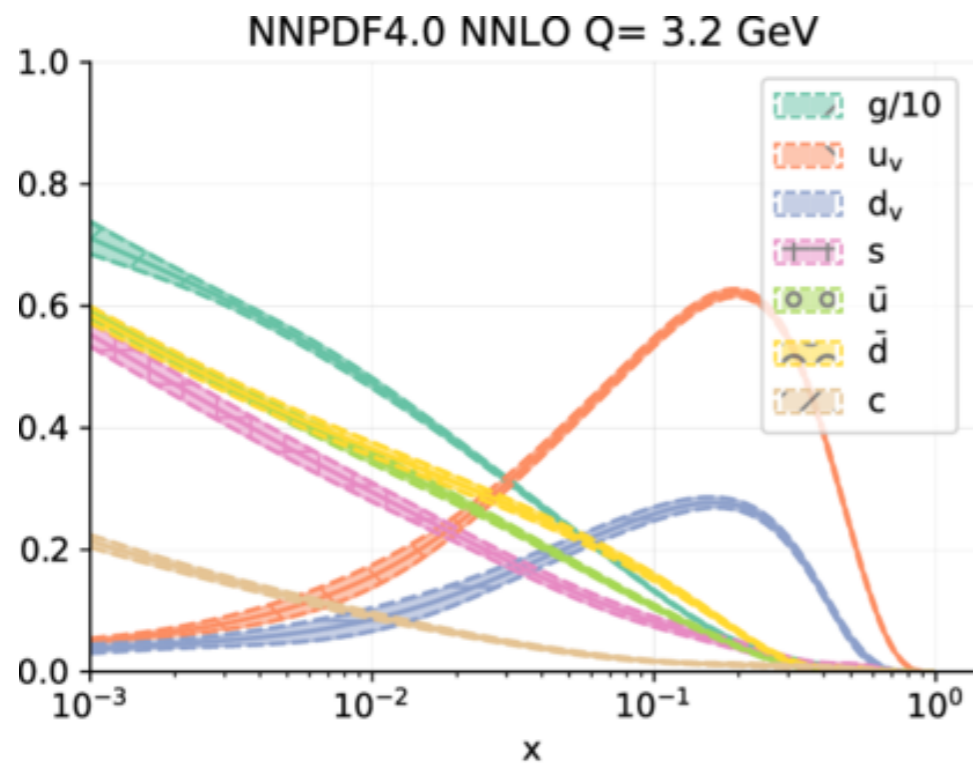
Collinear PDF (FF)

Collinear PDF

$f(x)$ depend on:

x = longitudinal-momentum fraction

1-dim imaging



TMD PDF (FF)

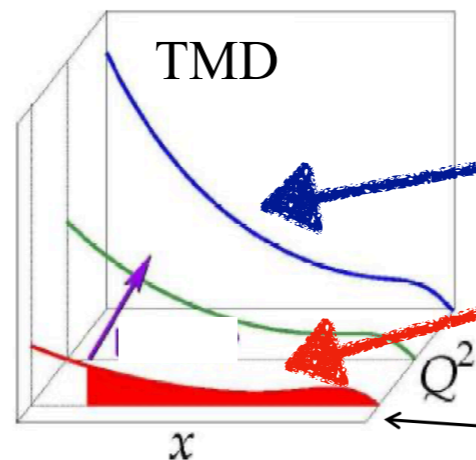
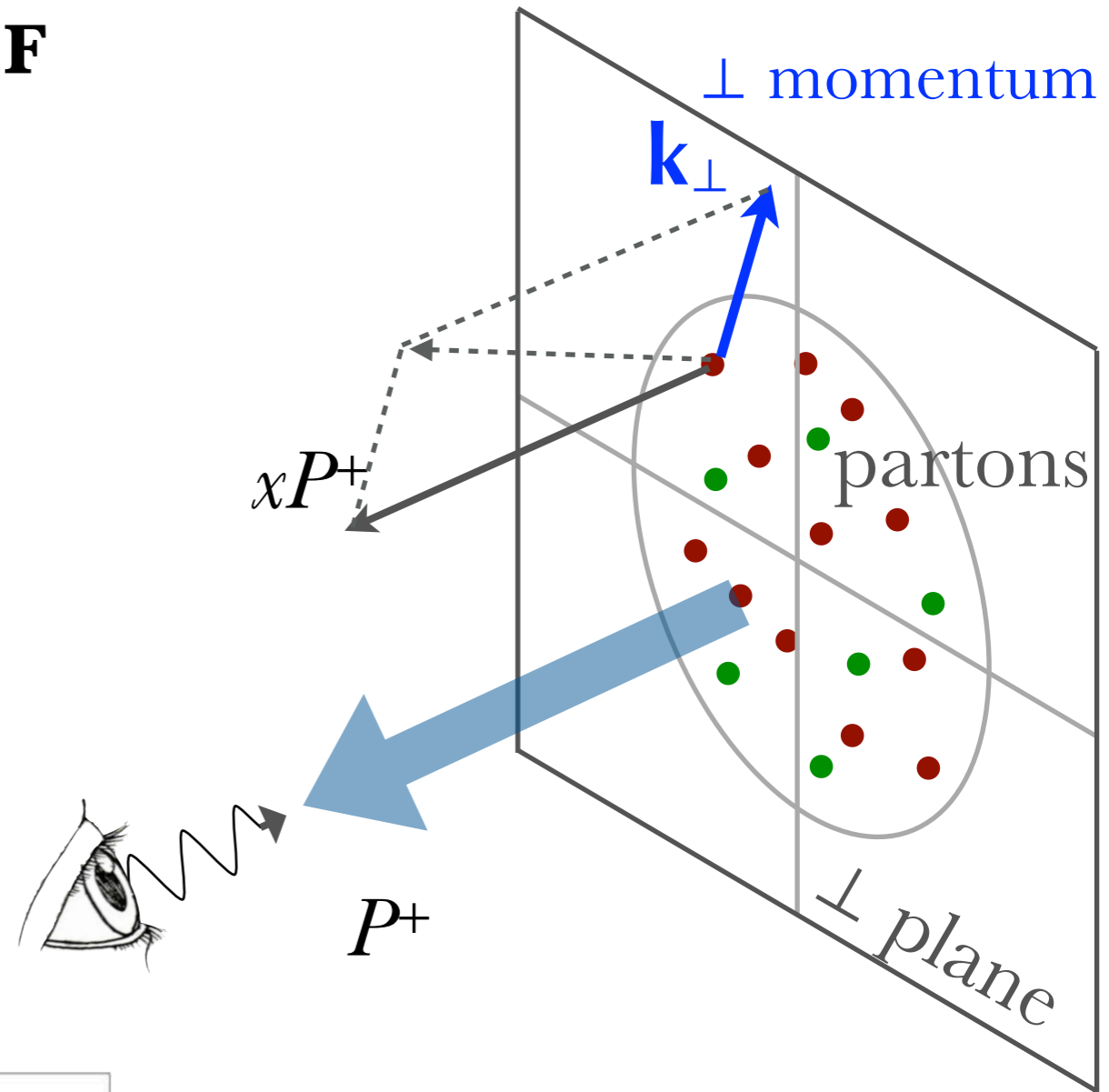
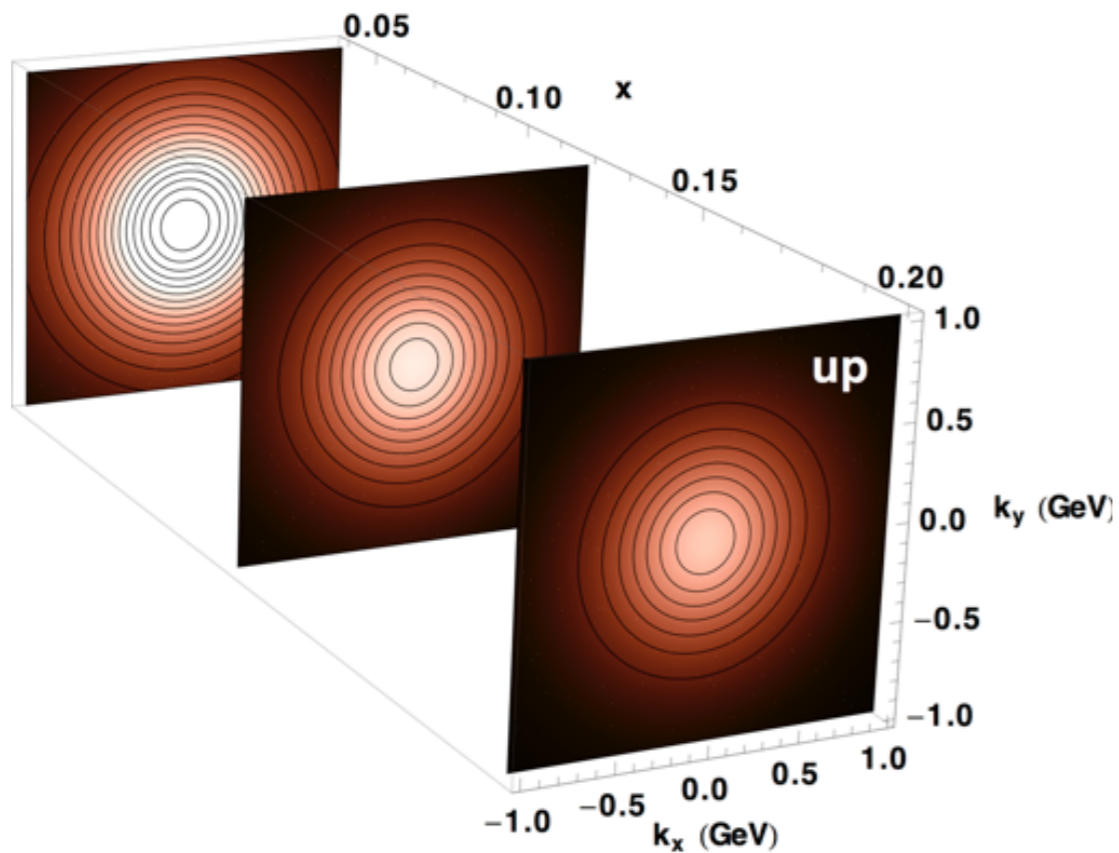
Transverse **M**omentum **D**ependent **P**DF

$F(x, k_{\perp})$ depend on:

x = longitudinal-momentum fraction

k_{\perp} = (*intrinsic*) transverse-momentum

3-dim imaging



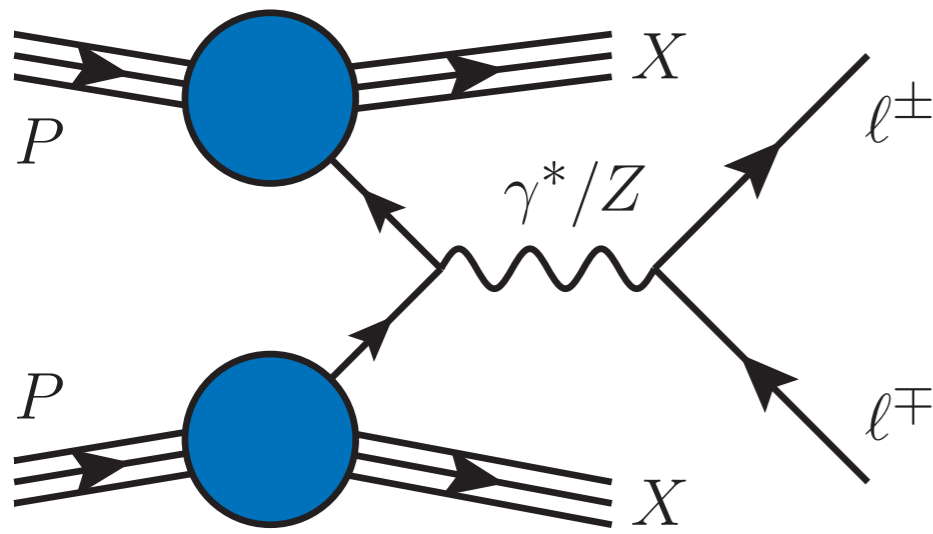
QCD predicts scale evolution

QCD cannot predict x, k_{\perp} -shape
 \rightarrow TMD fit

Factorising processes

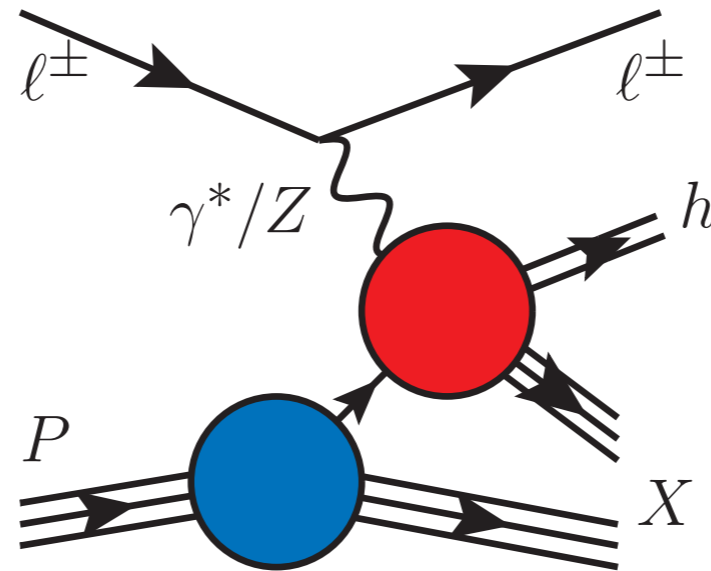
Processes for which TMD factorisation has been **proven**:

Drell-Yan



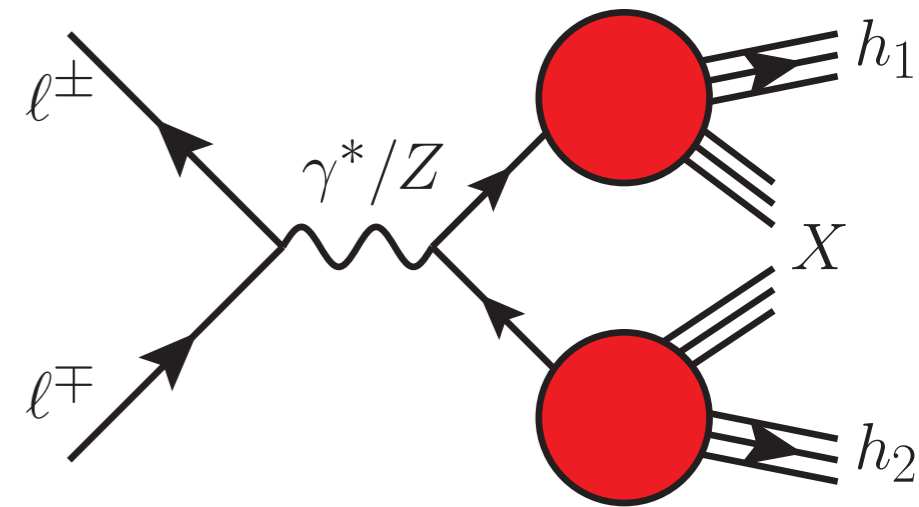
$$PP \longrightarrow l^\pm l^\mp X$$

Semi-inclusive DIS



$$Pl^\pm \longrightarrow l^\pm h X$$

e^+e^- annihilation



$$l^\pm l^\mp \longrightarrow h_1 h_2 X$$

Two TMD **PDFs**

One TMD **PDF** one **FF**

Two TMD **FFs**

Lots of data:

many precise data points:

DIA process from:

low-energy: FNAL

HERMES at DESY

BELLE at KEK

mid-energy: RHIC

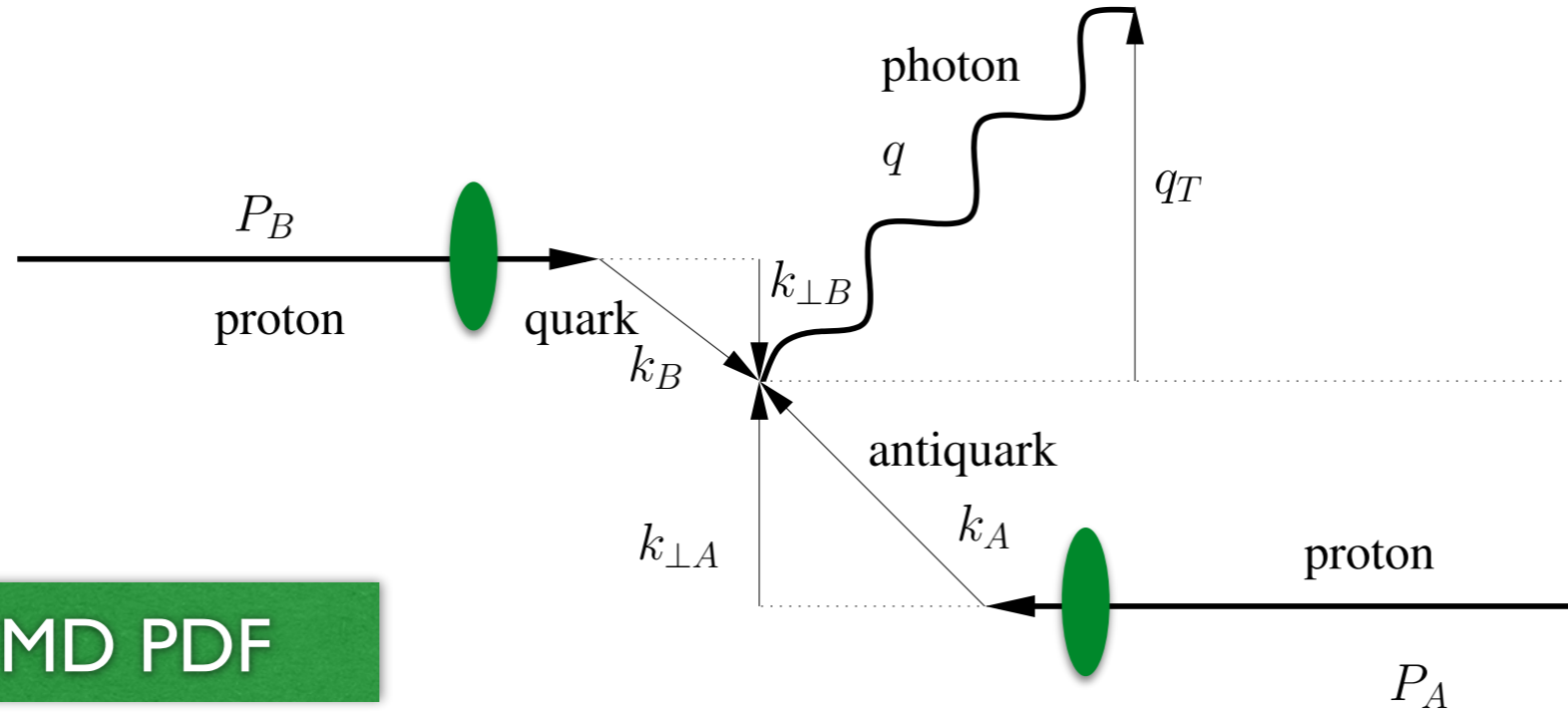
COMPASS at CERN

BABAR at SLAC

high-energy: Tevatron, LHC

missing!

TMD factorisation for DY



quark TMD PDF

$$\frac{d\sigma}{dq_T dy dQ} \propto x_A x_B H^{DY}(Q, \mu) \sum_q c_q(Q^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} \boxed{F^{\bar{q}}(x_A, \mathbf{k}_{\perp A}^2; \mu, \zeta_A)} \boxed{F^q(x_B, \mathbf{k}_{\perp B}^2; \mu, \zeta_B)} \delta^{(2)}(\mathbf{k}_{\perp A} + \mathbf{k}_{\perp B} - \mathbf{q}_T)$$

$$= x_A x_B H^{DY}(Q, \mu) \sum_q c_q(Q^2) \int \frac{db_T}{2\pi} b_T J_0(b_T q_T) \boxed{\hat{F}^{\bar{q}}(x_A, b_T^2; \mu, \zeta_A)} \boxed{\hat{F}^q(x_B, b_T^2; \mu, \zeta_B)}$$

TMD factorisation

- TMD factorisation introduces two independent *artificial* scales:
 - the **renormalisation scale** μ , originating from UV renormalisation
 - the **rapidity scale** ζ , originating from the cancellation of rapidity divergencies between collinear and soft emissions
- The respective **evolution equations** are:

$$\frac{\partial \ln F}{\partial \ln \sqrt{\zeta}} = K(\mu) \quad \text{with:} \quad \frac{\partial K}{\partial \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

$$\frac{\partial \ln F}{\partial \ln \mu} = \gamma_F(\alpha_s(\mu)) - \gamma_K(\alpha_s(\mu)) \ln \frac{\sqrt{\zeta}}{\mu}$$

TMD structure

$$\begin{aligned} F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\ &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\ &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C \end{aligned}$$

TMD structure

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 \end{aligned}$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{\text{QCD}}$
- **perturbative**

TMD structure

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 \end{aligned}$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{\text{QCD}}$
- **perturbative**

- CS and RGE evolution to large b_T
- **perturbative**

Perturbative accuracy

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N ³ LL	2	3	4	NNLO

$$\text{NLL} \quad C^0 \quad \alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$$

$$\text{NLL}' \quad \left(C^0 + \alpha_S C^1 \right) \quad \alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$$

same logarithmic accuracy (difference = NNLL)

TMD structure

$$\begin{aligned}
 F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\
 &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\
 &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C
 \end{aligned}$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{\text{QCD}}$
- **perturbative**

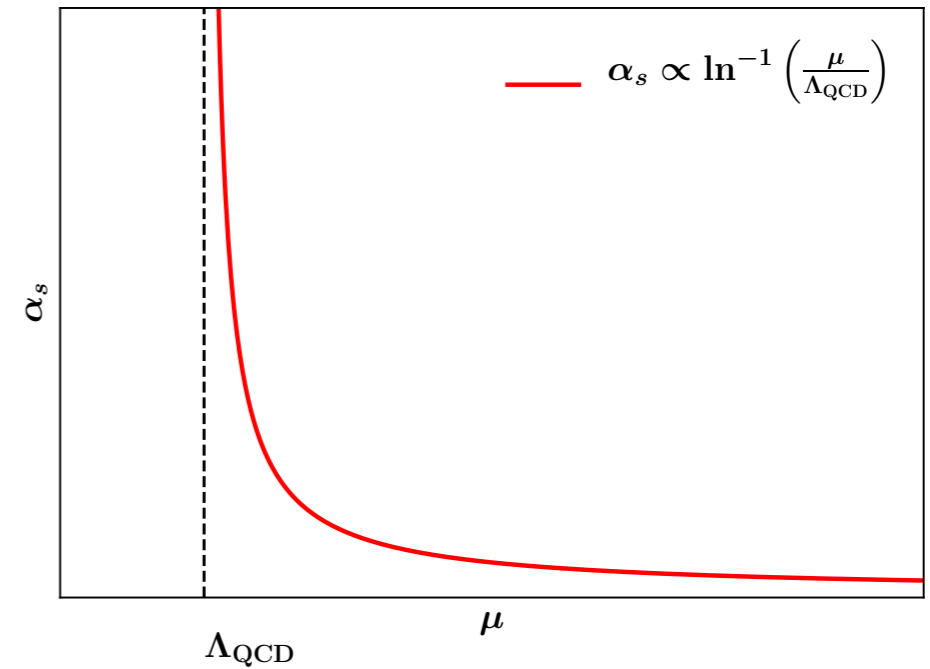
$$(\mu_b = 2e^{-\gamma_E} / b_*)$$

- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole

Non-perturbative: b^* and f_{NP}

$$\alpha_s(\mu_b) = \alpha \left(\frac{2e^{-\gamma_E}}{b} \right) \gg 1 \quad \text{for large } b \text{ values}$$



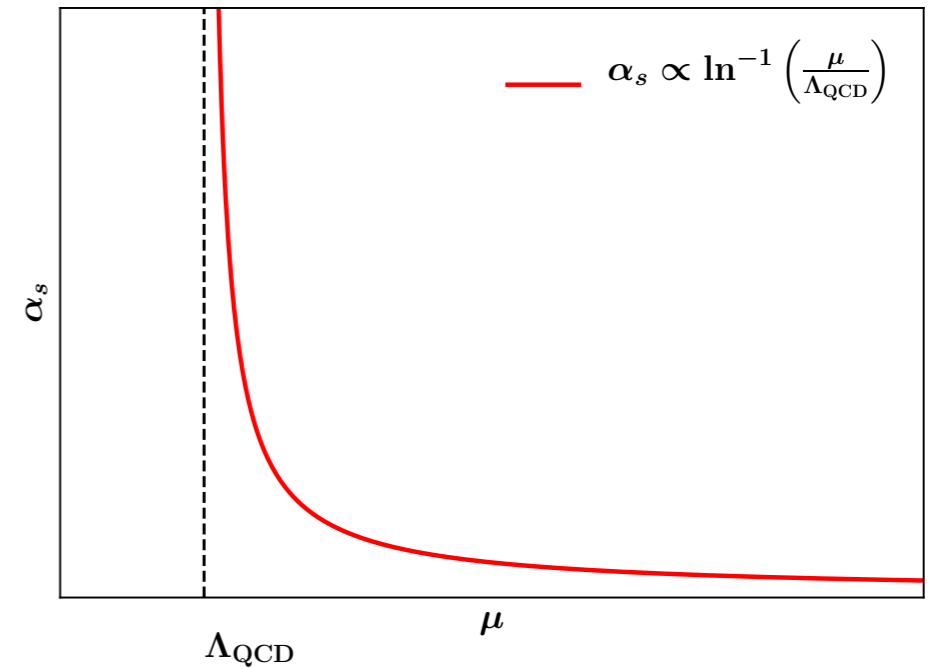
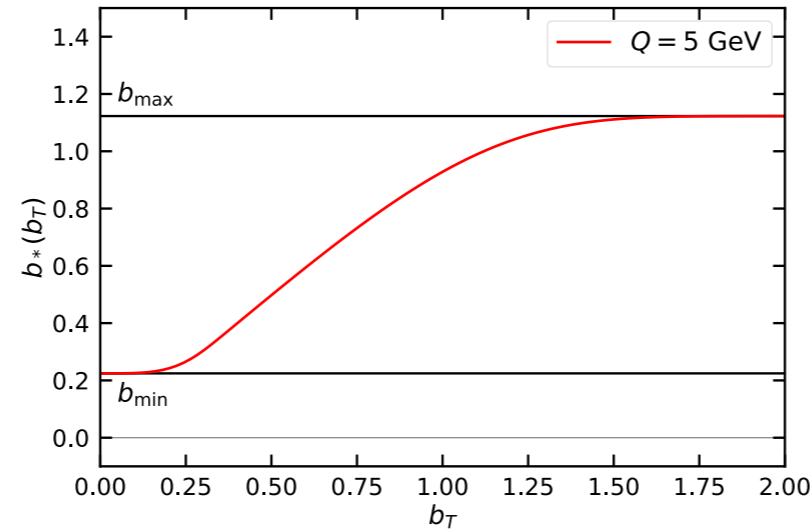
Non-perturbative: b^* and f_{NP}

$$\alpha_s(\mu_b) = \alpha \left(\frac{2e^{-\gamma_E}}{b} \right) \gg 1 \quad \text{for large } b \text{ values}$$

$$b_*(b) = b_{\max} \left(\frac{1 - \exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1 - \exp\left(-\frac{b^4}{b_{\min}^4}\right)} \right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$



TMD structure

$$\begin{aligned}
 F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\
 &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\
 &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} f_{NP} && : C
 \end{aligned}$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{\text{QCD}}$
- **perturbative**

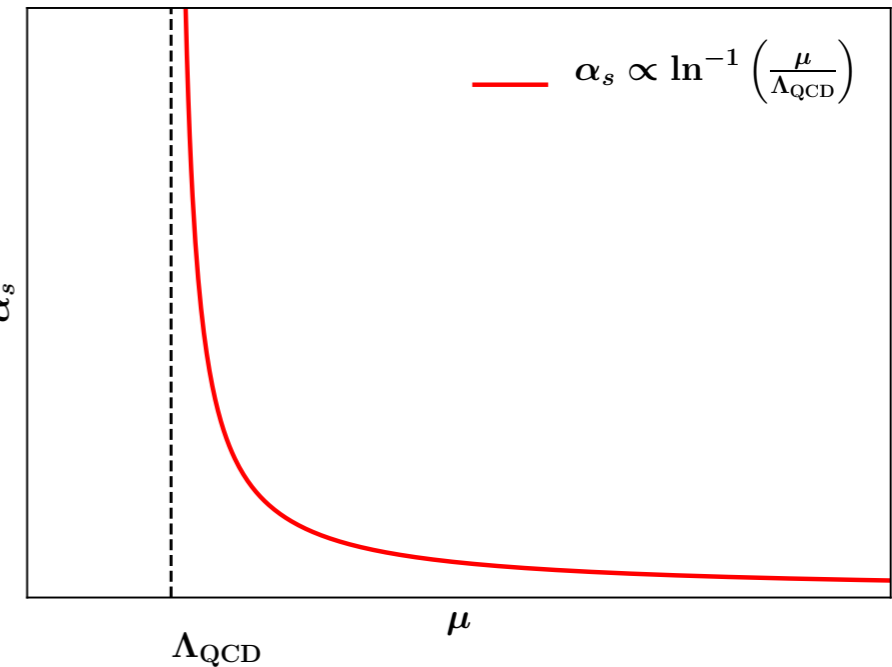
$$(\mu_b = 2e^{-\gamma_E} / b_*)$$

- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole
- f_{NP} “parametrises” the **non-perturbative** transverse modes
- **fit** f_{NP} to data

Non-perturbative: b^* and f_{NP}

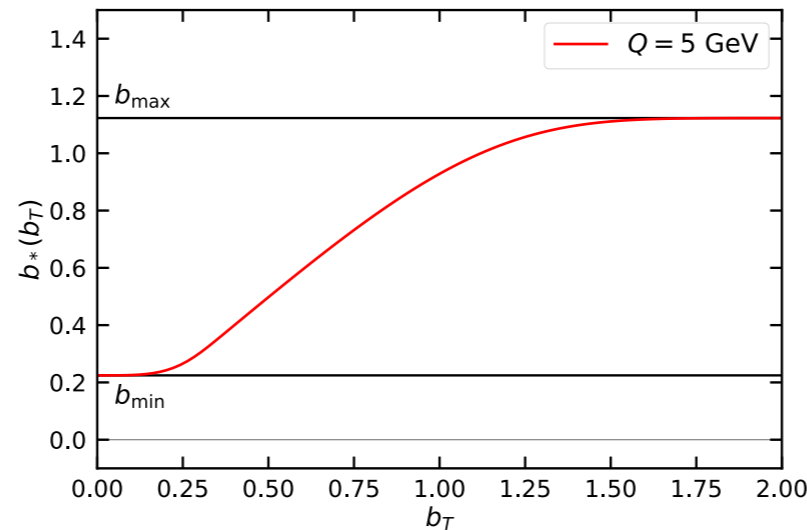
$$\alpha_s(\mu_b) = \alpha \left(\frac{2e^{-\gamma_E}}{b} \right) \gg 1 \quad \text{for large } b \text{ values}$$



$$b_*(b) = b_{\text{max}} \left(\frac{1 - \exp \left(-\frac{b^4}{b_{\text{max}}^4} \right)}{1 - \exp \left(-\frac{b^4}{b_{\text{min}}^4} \right)} \right)^{\frac{1}{4}}$$

$$b_{\text{max}} = 2e^{-\gamma_E}$$

$$b_{\text{min}} = 2e^{-\gamma_E} / Q$$

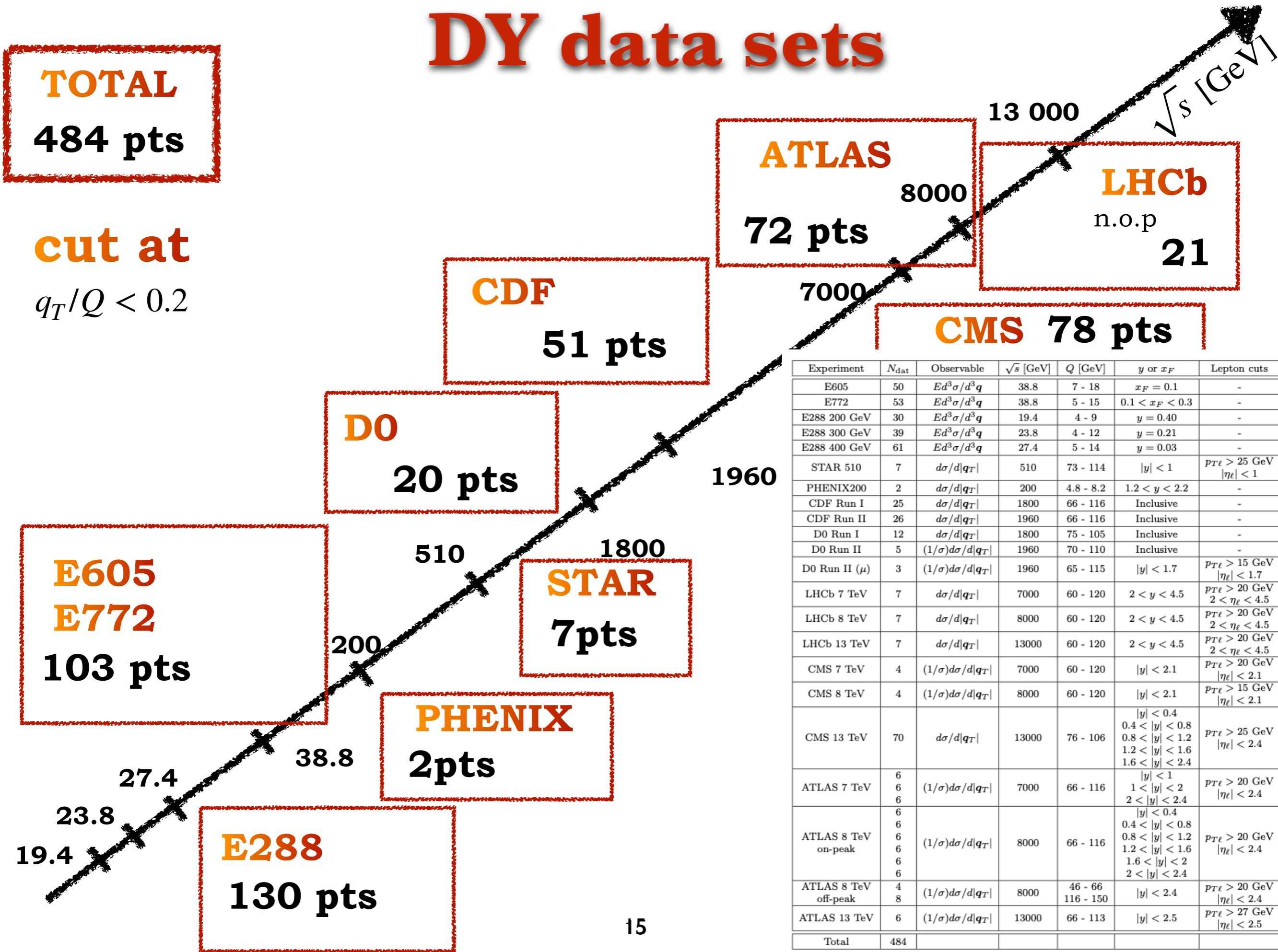


$$F(x, b; \mu, \zeta) = \overset{f_{\text{NP}}}{\left[\frac{F(x, b; \mu, \zeta)}{F(x, b_*(b); \mu, \zeta)} \right]} F(x, b_*(b); \mu, \zeta)$$

- ▶ NP is unavoidable: intrinsically tied to regularisation procedure
- ▶ There is not a universal form factor:
 - ▶ depends on details of b^* and collinear PDFs
 - ▶ requires definition of a functional form
 - ▶ determined through a fit to experimental data

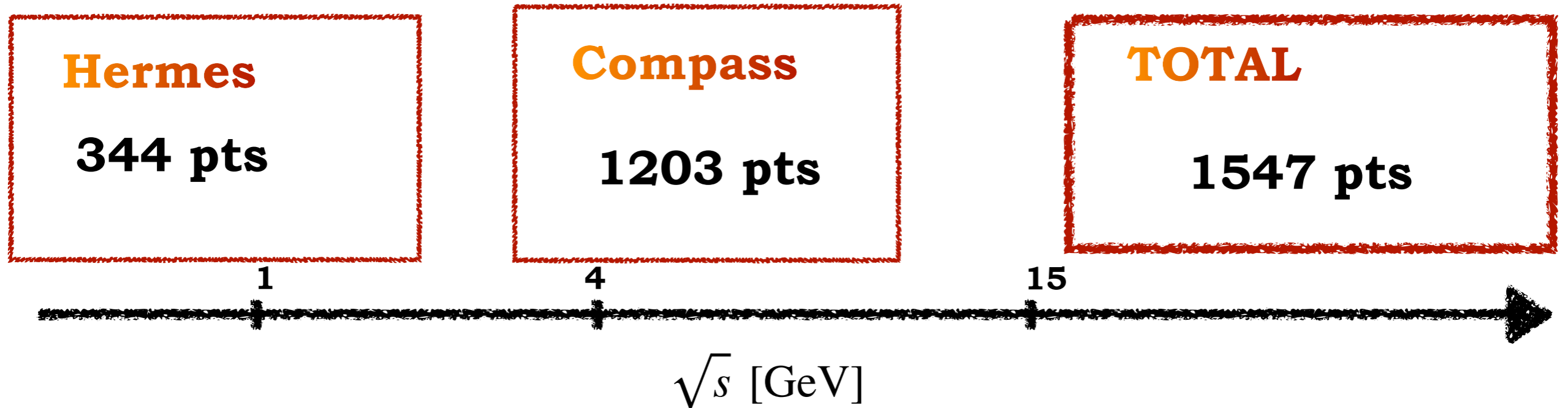
The extraction of TMD PDFs and FFs from low- p_T data

DY data sets



Experiment	N_{dat}	Observable	\sqrt{s} [GeV]	Q [GeV]	y or x_F	Lepton cuts
E605	50	$Ed^3\sigma/d^3q$	38.8	7 - 18	$x_F = 0.1$	-
E772	53	$Ed^3\sigma/d^3q$	38.8	5 - 15	$0.1 < x_F < 0.3$	-
E288 200 GeV	30	$Ed^3\sigma/d^3q$	19.4	4 - 9	$y = 0.40$	-
E288 300 GeV	39	$Ed^3\sigma/d^3q$	23.8	4 - 12	$y = 0.21$	-
E288 400 GeV	61	$Ed^3\sigma/d^3q$	27.4	5 - 14	$y = 0.03$	-
STAR 510	7	$d\sigma/d q_T $	510	73 - 114	$ y < 1$	$p_{T\ell} > 25$ GeV $ \eta_\ell < 1$
PHENIX200	2	$d\sigma/d q_T $	200	4.8 - 8.2	$1.2 < y < 2.2$	-
CDF Run I	25	$d\sigma/d q_T $	1800	66 - 116	Inclusive	-
CDF Run II	26	$d\sigma/d q_T $	1960	66 - 116	Inclusive	-
D0 Run I	12	$d\sigma/d q_T $	1800	75 - 105	Inclusive	-
D0 Run II	5	$(1/\sigma)d\sigma/d q_T $	1960	70 - 110	Inclusive	-
D0 Run II (μ)	3	$(1/\sigma)d\sigma/d q_T $	1960	65 - 115	$ y < 1.7$	$p_{T\ell} > 15$ GeV $ \eta_\ell < 1.7$
LHCb 7 TeV	7	$d\sigma/d q_T $	7000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
LHCb 8 TeV	7	$d\sigma/d q_T $	8000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
LHCb 13 TeV	7	$d\sigma/d q_T $	13000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
CMS 7 TeV	4	$(1/\sigma)d\sigma/d q_T $	7000	60 - 120	$ y < 2.1$	$p_{T\ell} > 20$ GeV $ \eta_\ell < 2.1$
CMS 8 TeV	4	$(1/\sigma)d\sigma/d q_T $	8000	60 - 120	$ y < 2.1$	$p_{T\ell} > 15$ GeV $ \eta_\ell < 2.1$
CMS 13 TeV	70	$d\sigma/d q_T $	13000	76 - 106	$ y < 0.4$ $0.4 < y < 0.8$ $0.8 < y < 1.2$ $1.2 < y < 1.6$ $1.6 < y < 2.4$	$p_{T\ell} > 25$ GeV $ \eta_\ell < 2.4$
ATLAS 7 TeV	6 6 6	$(1/\sigma)d\sigma/d q_T $	7000	66 - 116	$ y < 1$ $1 < y < 2$ $2 < y < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell < 2.4$
ATLAS 8 TeV on-peak	6 6 6 6 6	$(1/\sigma)d\sigma/d q_T $	8000	66 - 116	$ y < 0.4$ $0.4 < y < 0.8$ $0.8 < y < 1.2$ $1.2 < y < 1.6$ $1.6 < y < 2$ $2 < y < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell < 2.4$
ATLAS 8 TeV off-peak	4 8	$(1/\sigma)d\sigma/d q_T $	8000	46 - 66 116 - 150	$ y < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell < 2.4$
ATLAS 13 TeV	6	$(1/\sigma)d\sigma/d q_T $	13000	66 - 113	$ y < 2.5$	$p_{T\ell} > 27$ GeV $ \eta_\ell < 2.5$
Total	484					

SIDIS data sets



Experiment	N_{dat}	Observable	Channels	Q [GeV]	x	z	Phase space cuts
HERMES	344	$M(x, z, \mathbf{P}_{hT} , Q)$	$p \rightarrow \pi^+$ $p \rightarrow \pi^-$ $p \rightarrow K^+$ $p \rightarrow K^-$ $d \rightarrow \pi^+$ $d \rightarrow \pi^-$ $d \rightarrow K^+$ $d \rightarrow K^-$	$1 - \sqrt{15}$	$0.023 < x < 0.6$ (6 bins)	$0.1 < z < 1.1$ (8 bins)	$W^2 > 10 \text{ GeV}^2$ $0.1 < y < 0.85$
COMPASS	1203	$M(x, z, \mathbf{P}_{hT}^2, Q)$	$d \rightarrow h^+$ $d \rightarrow h^-$	1 - 9 (5 bins)	$0.003 < x < 0.4$ (8 bins)	$0.2 < z < 0.8$ (4 bins)	$W^2 > 25 \text{ GeV}^2$ $0.1 < y < 0.9$
Total	1547						

cut at

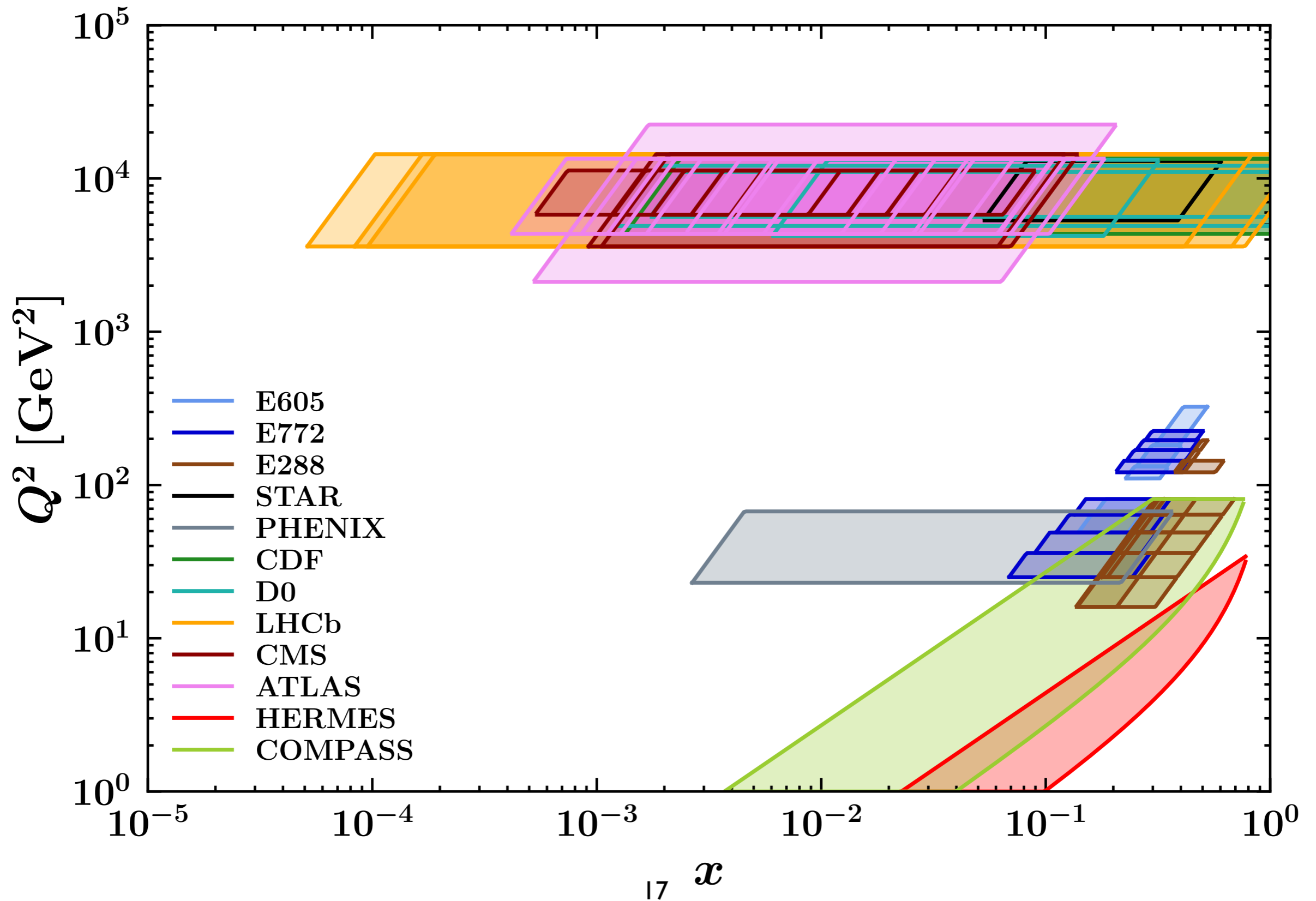
$Q > 1.4 \text{ GeV}$
(coll. factorisation)

$0.2 < z < 0.7$
(no exclusive processes)

$$P_{hT}|_{\text{max}} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

$(q_T/Q < 0.2)$

(x, Q^2) coverage



TMD global fits

	Accuracy	HERMES	COMPASS	DY fixed target	DY collider	N. of points	χ^2/N
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N3LL ⁻	✓	✓	✓	✓	1039	1.06
MAP 2022 arXiv:2206.07598	N3LL ⁻	✓	✓	✓	✓	2031	1.06
MAP 2024 arXiv:2405.13833	N3LL	✓	✓	✓	✓	2031	1.08

Global extractions: quick facts

PV17 (NLL)

SV19 (N3LL)

MAP22 (N3LL)

MAP24 (N3LL)

11 parameters

11 parameters

21 parameters

20(96) parameters

JHEP 06 (2017) 081

JHEP 06 (2020) 137

JHEP 10 (2022) 127

JHEP 08 (2024) 28

Functional forms

$$f_{NP}(x, b) = \exp\left(-\frac{\lambda_1(1-x) + \lambda_2x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3x\lambda_4b^2}}b^2\right)$$

SV

$$f_{1NP}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}}\right)$$

MAP

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

$$\mathcal{D}(\mu, b) = \mathcal{D}_{\text{resum}}(\mu, b^*(b)) + c_0 b b^*(b),$$

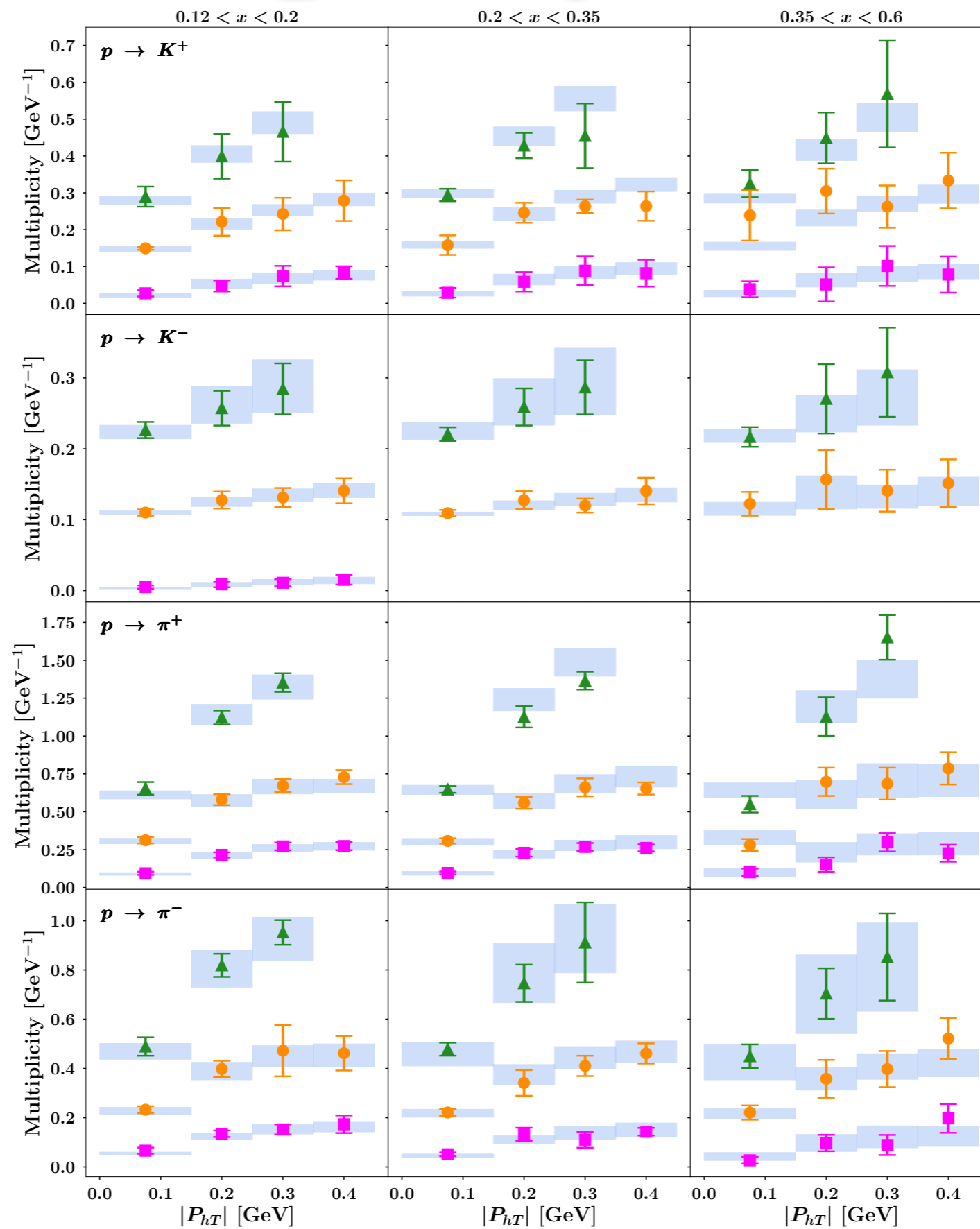
$$b^*(b) = \frac{b}{\sqrt{1 + b^2/B_{\text{NP}}^2}}$$

Collins-Soper kernel

$$f_{NP}(x, b_T^2) \exp\left\{g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right\}$$

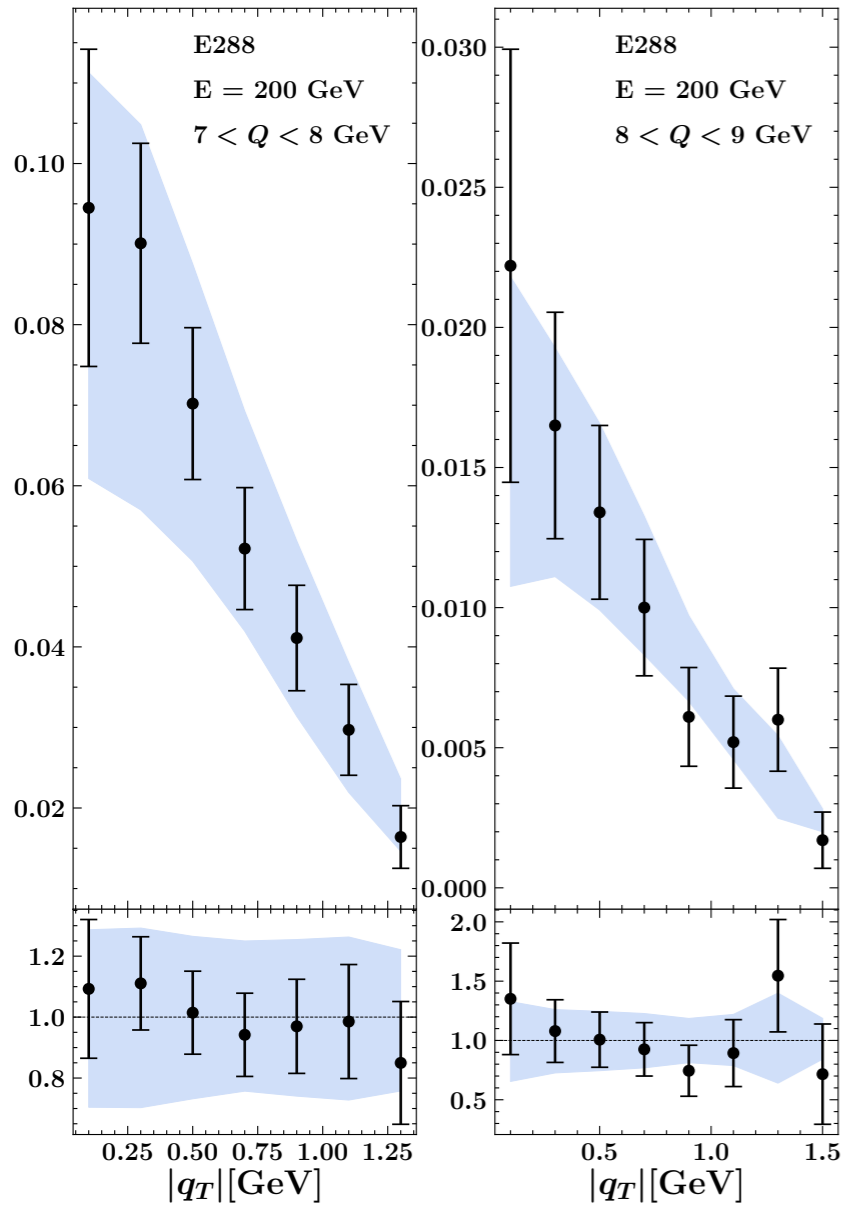
$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

Fit quality: SIDIS

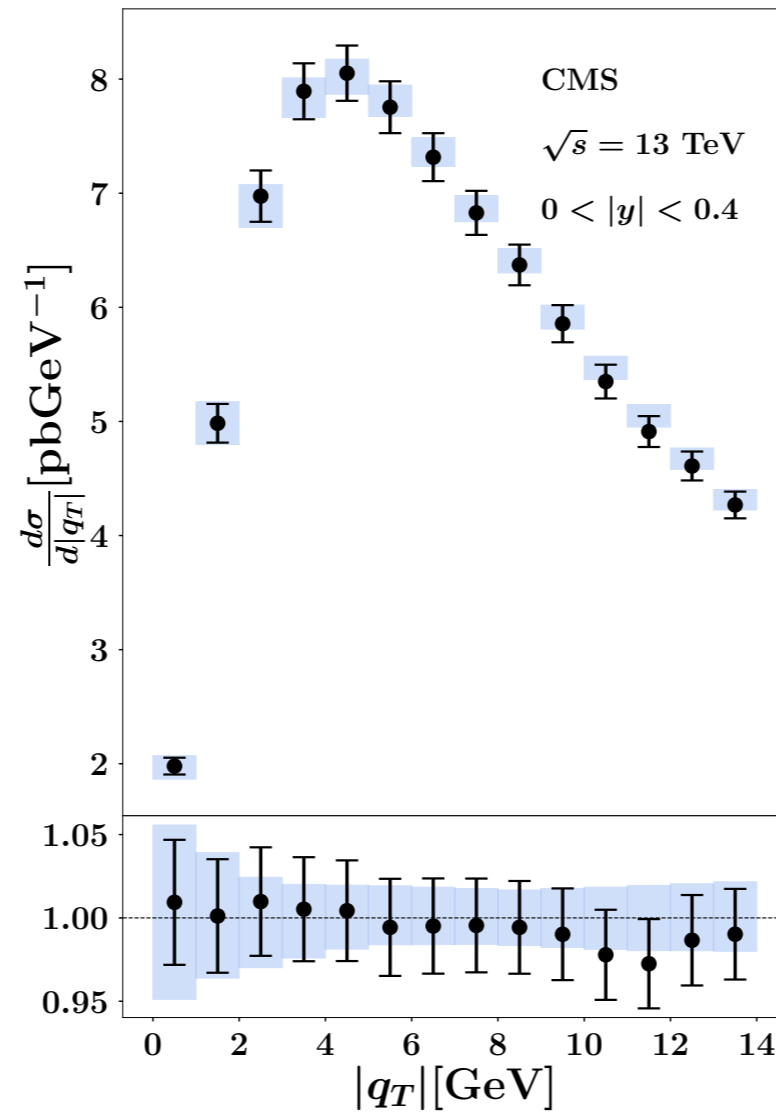


Fit quality: Drell-Yan

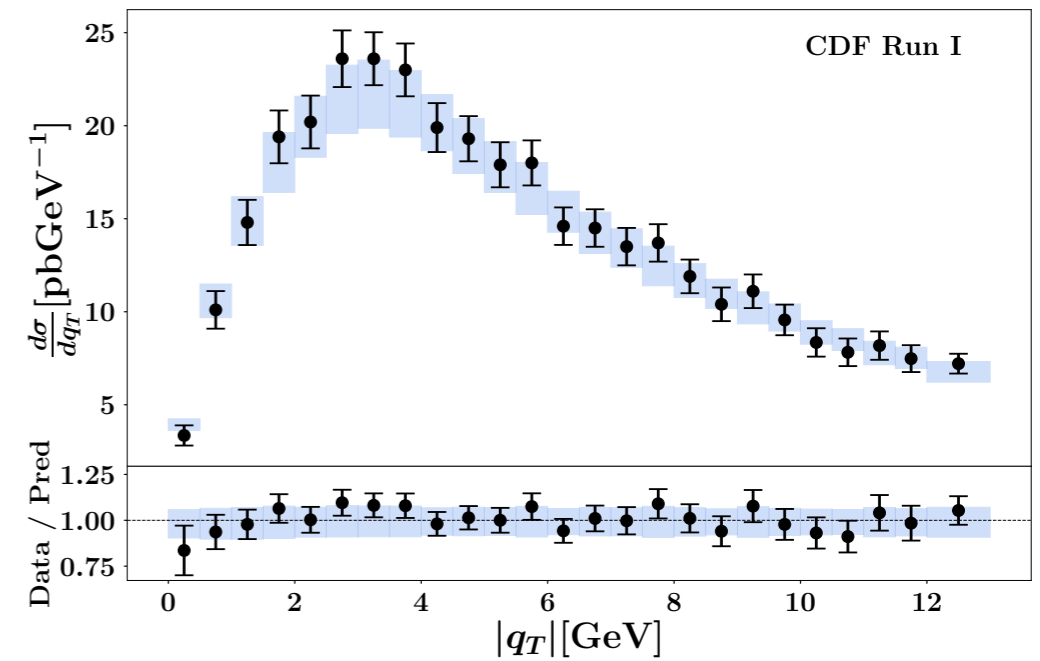
E288



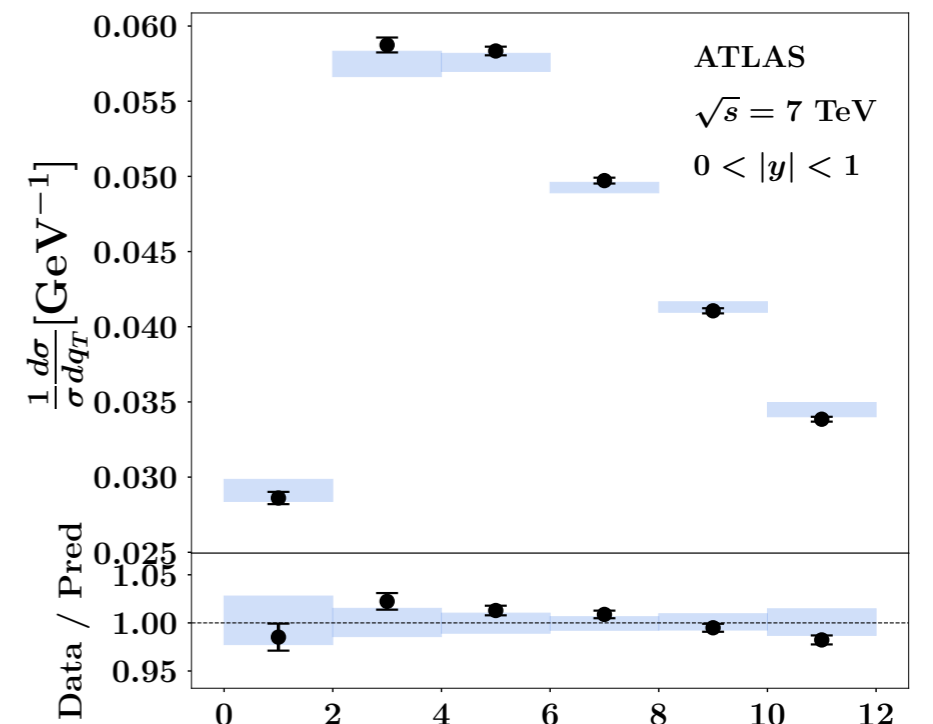
CMS



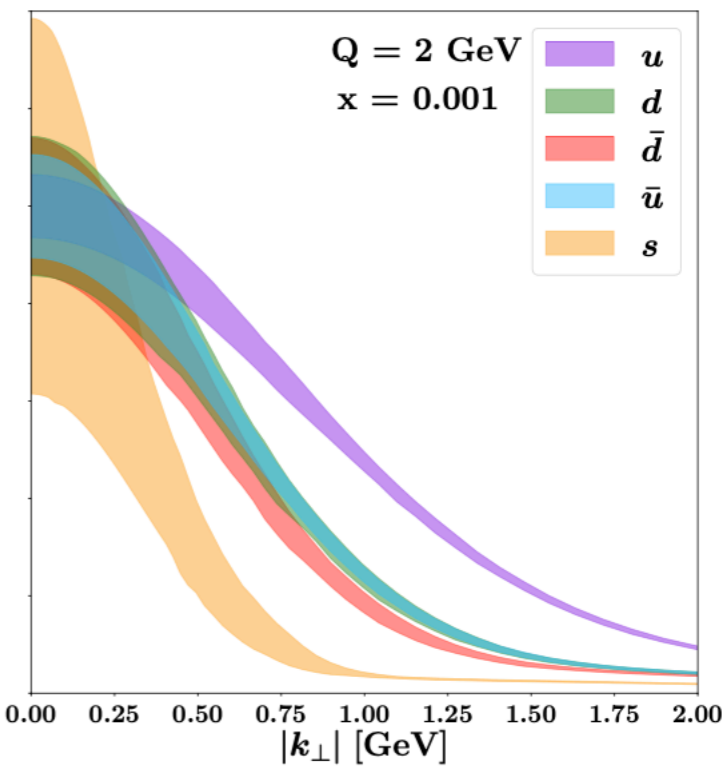
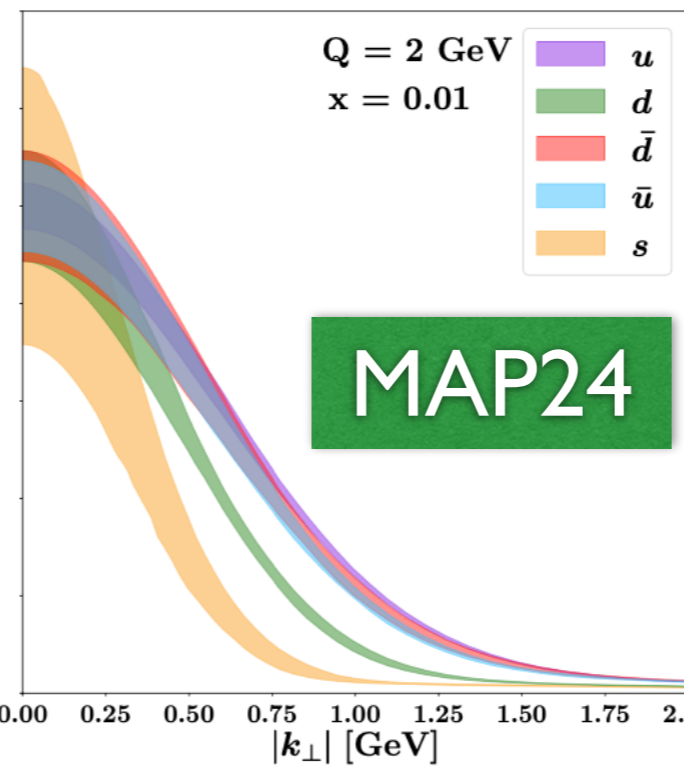
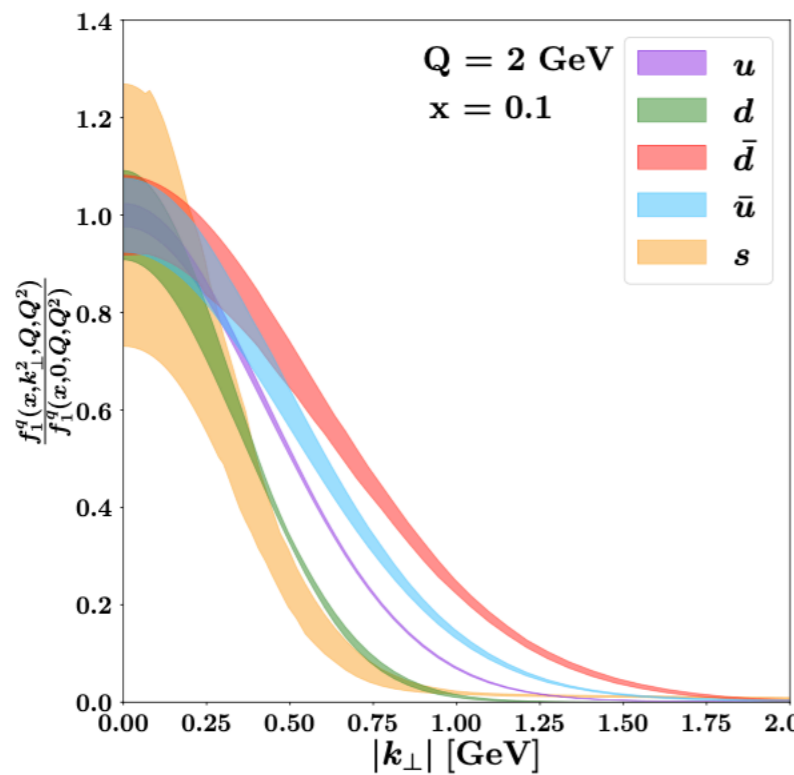
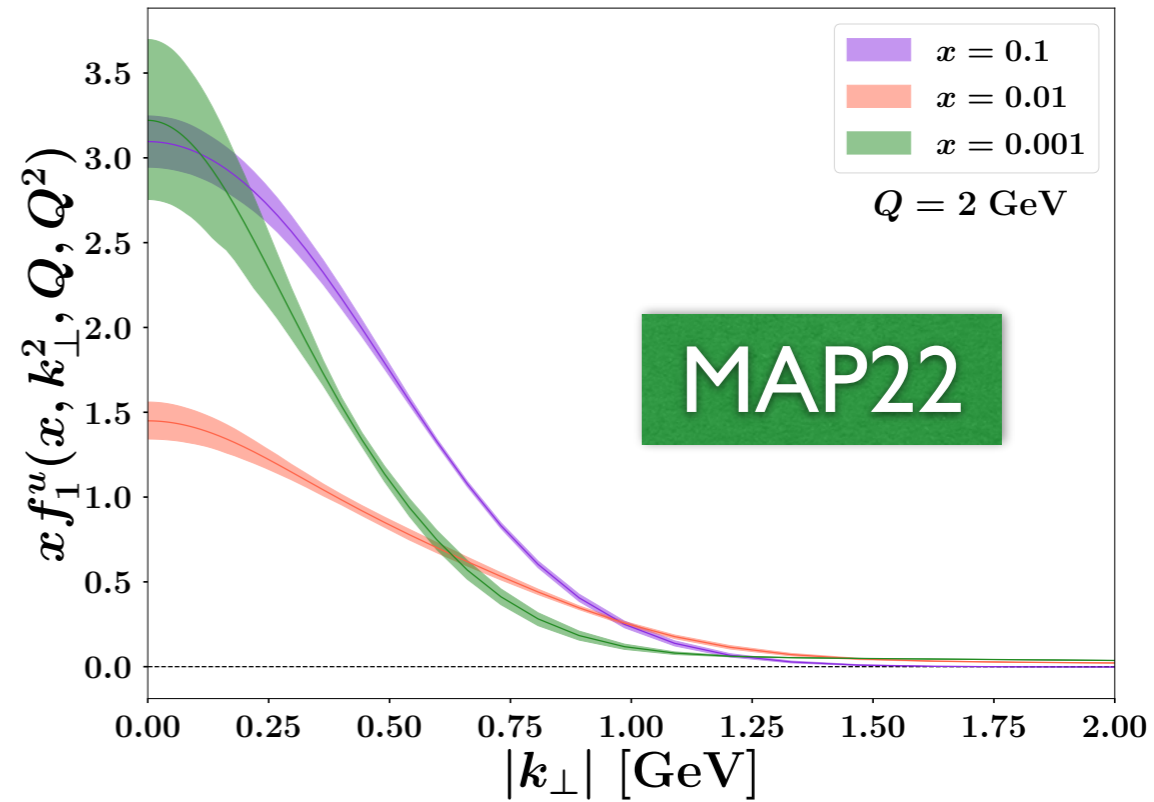
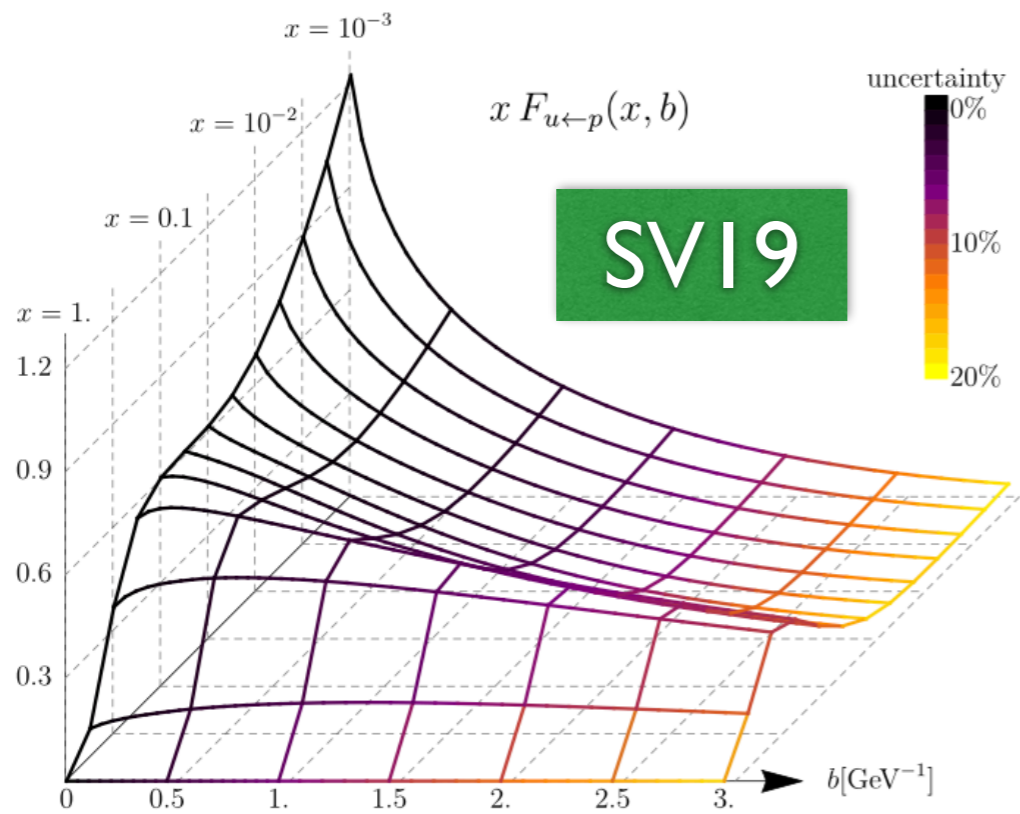
CDF



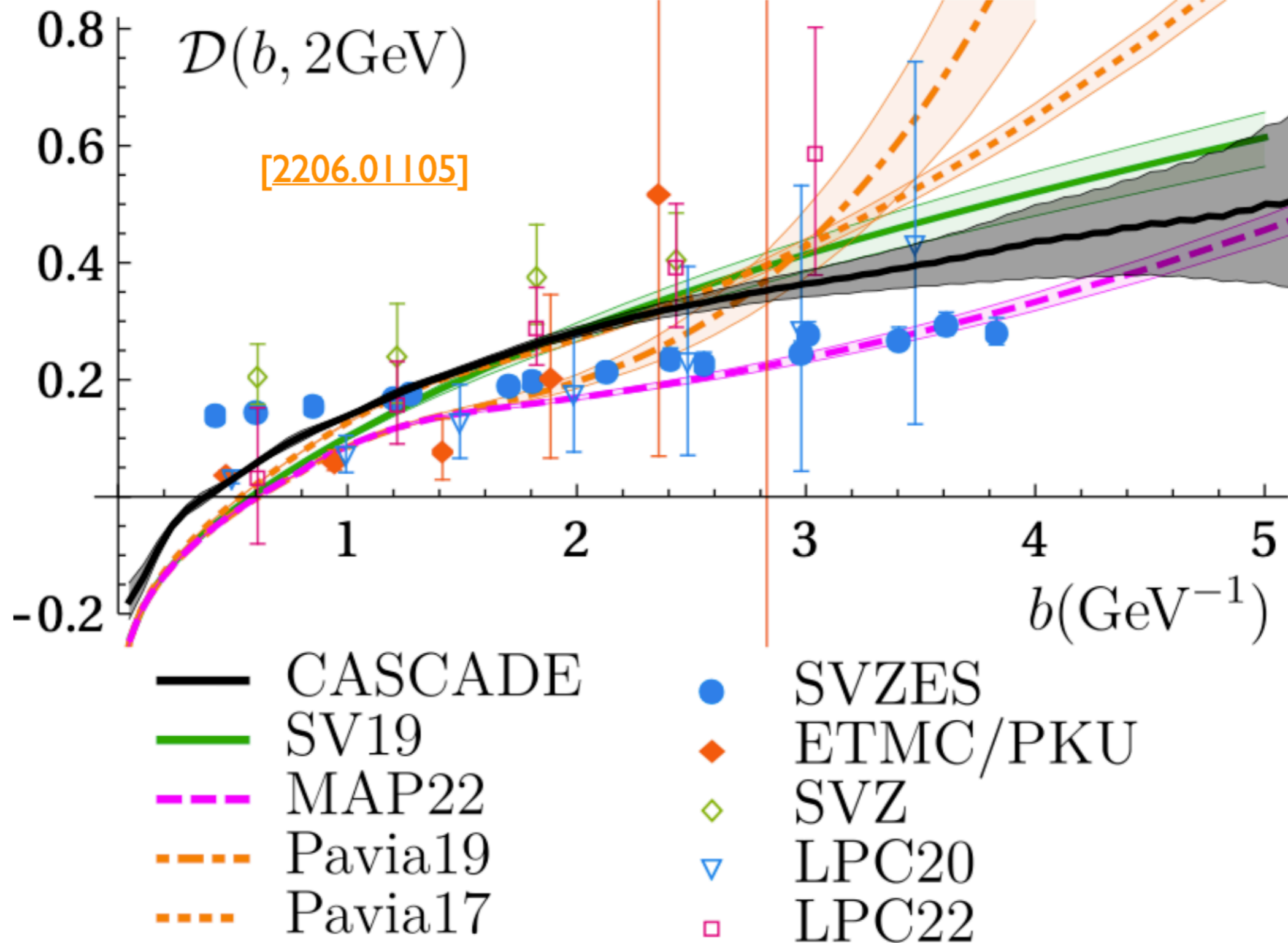
ATLAS



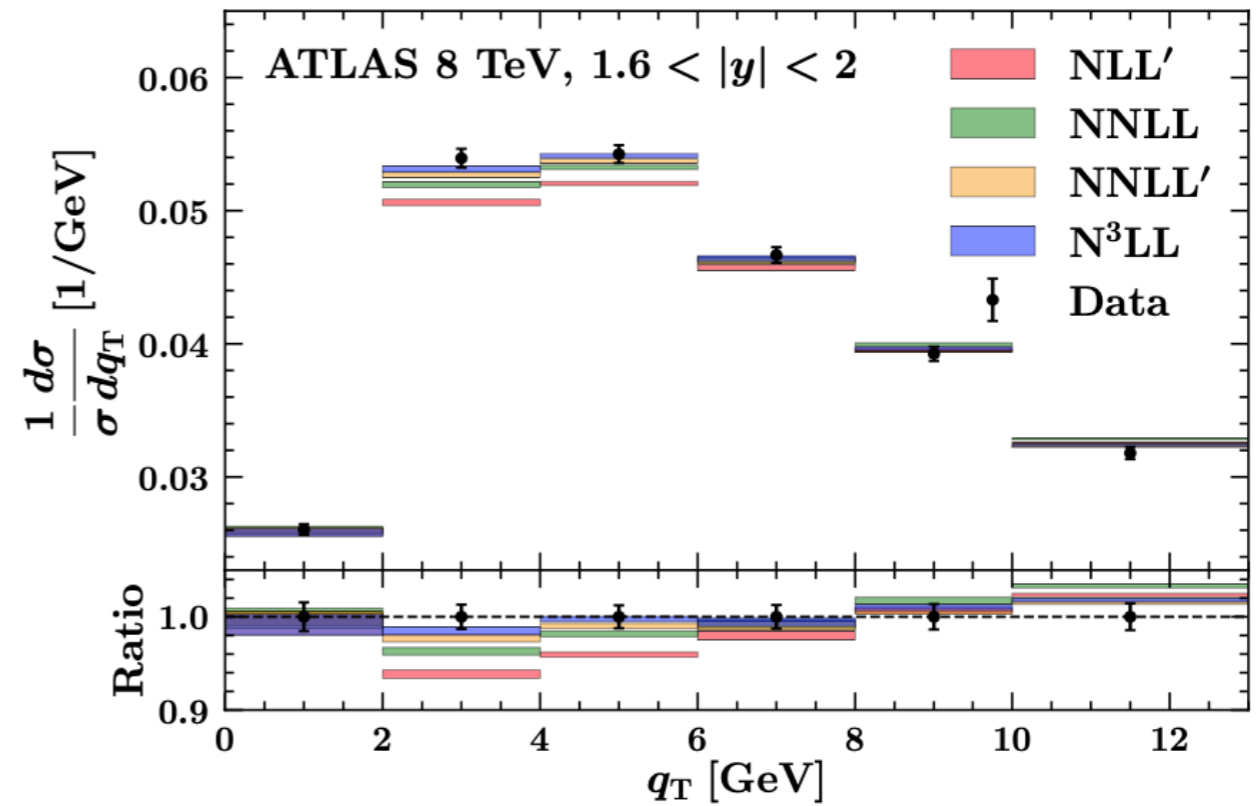
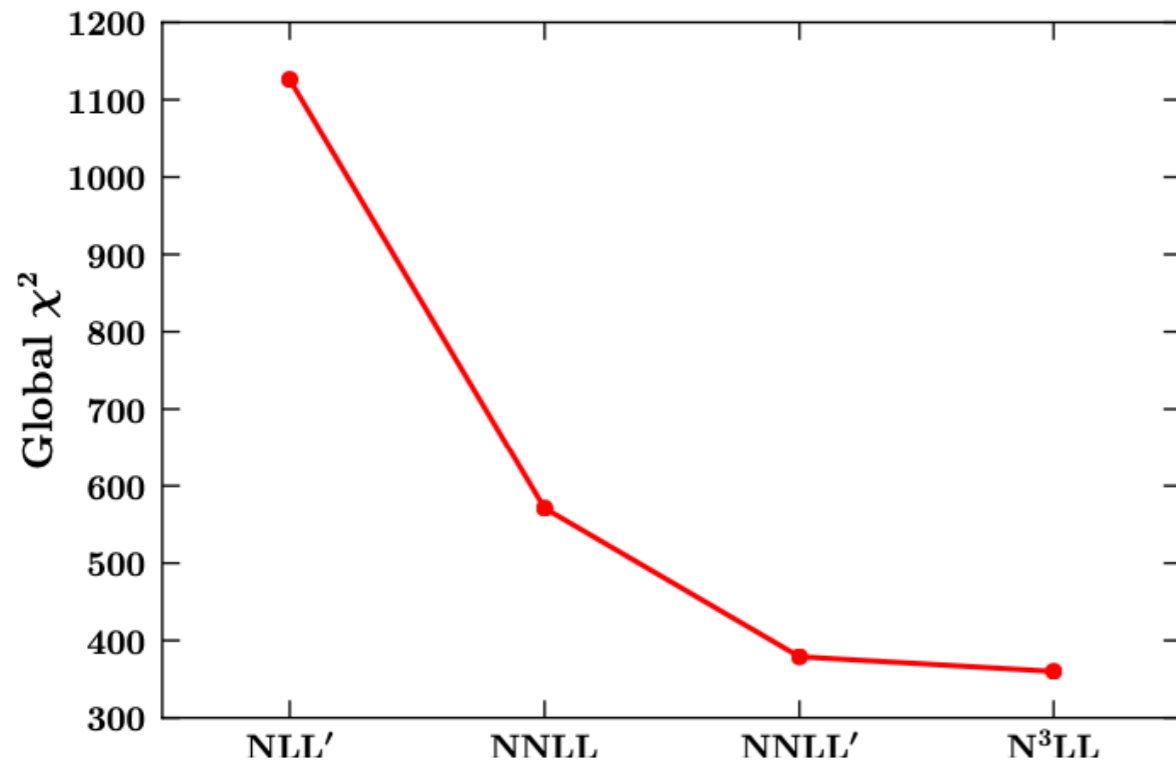
TMD PDFs



Collins-Soper kernel



Perturbative convergence



Order	NLL'	NNLL	NNLL'	N ³ LL
χ^2 /d.o.f.	3.19	1.62	1.07	1.02

Importance of x -dependence

Test: x -independent fit at N³LL with Davies, Webber, Stirling (1985) NP parameterisation:

$$f_{\text{NP}}^{\text{DWS}}(b_T, \zeta) = \exp \left[-\frac{1}{2} \left(g_1 + g_2 \ln \left(\frac{\zeta}{2Q_0^2} \right) \right) b_T^2 \right]$$

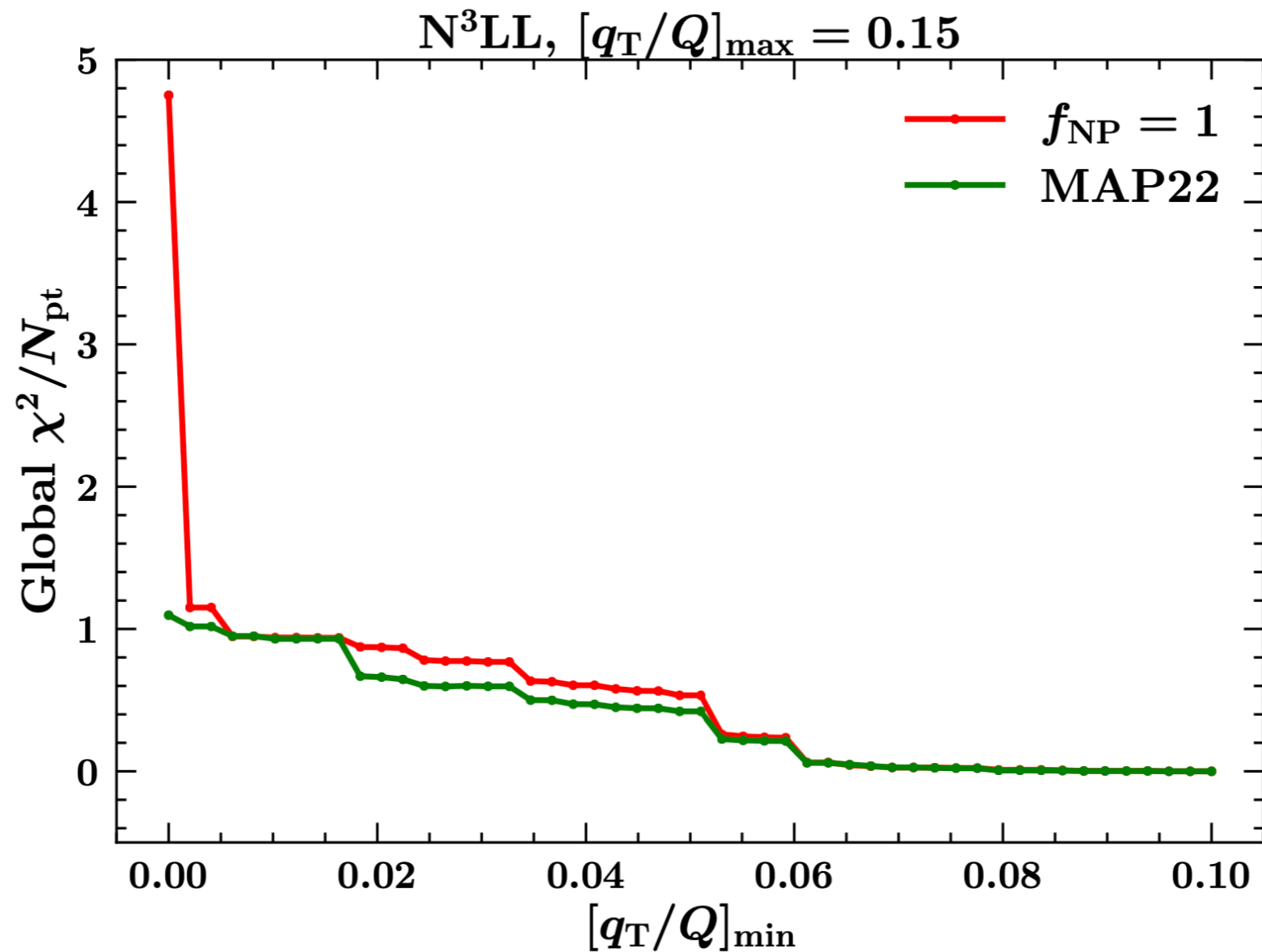
with and without ATLAS data

	Full dataset	No y -differential data
Global χ^2/N_{dat}	1.339	0.895
g_1	0.304	0.207
g_2	0.028	0.093

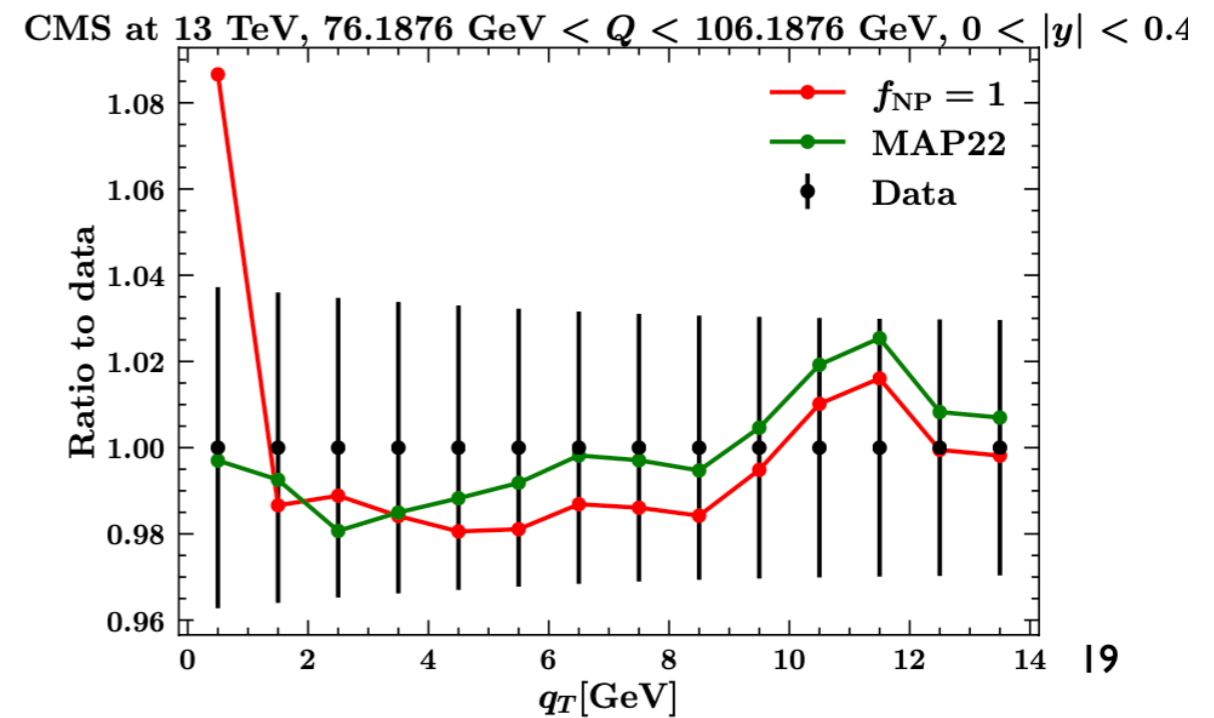
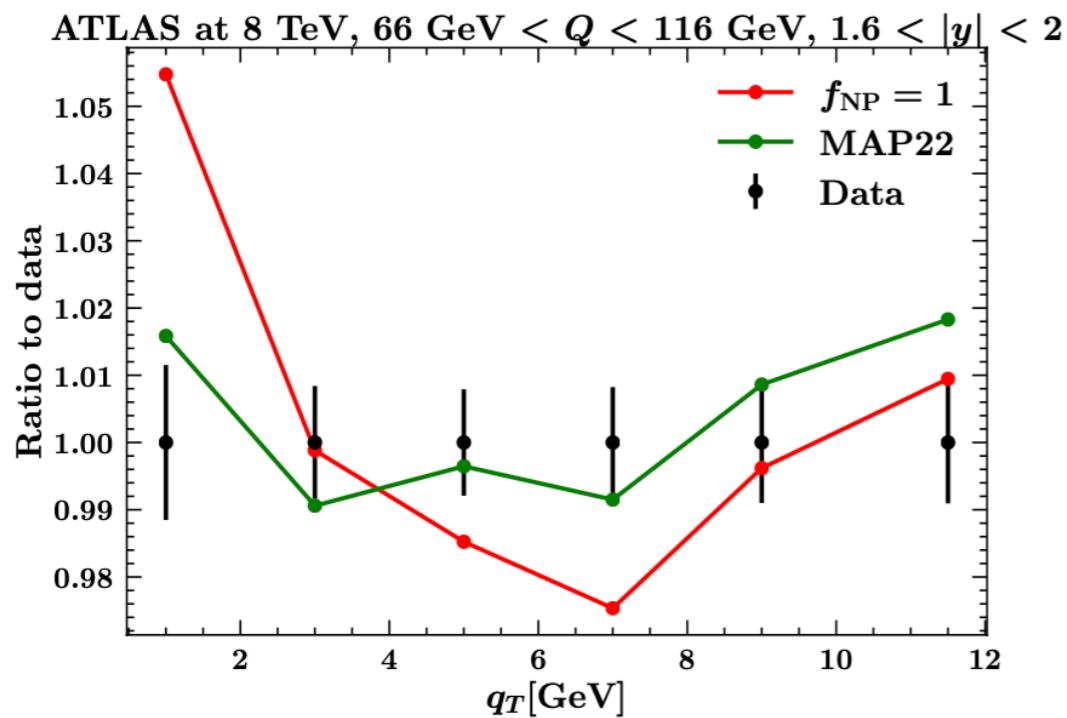
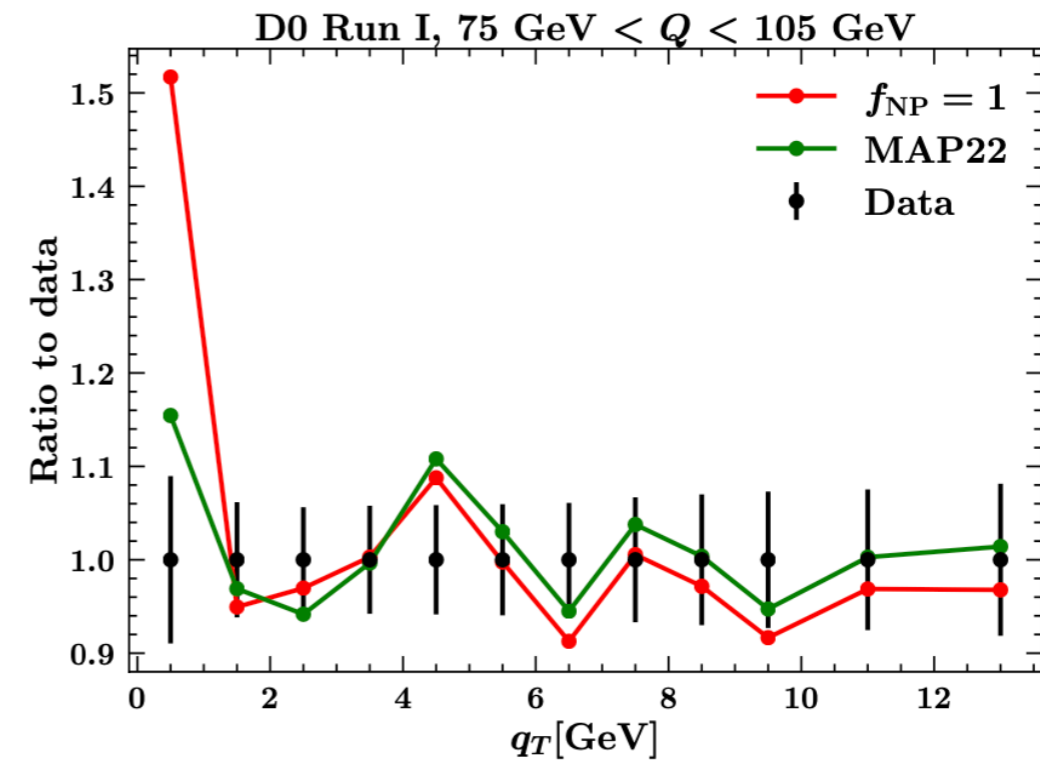
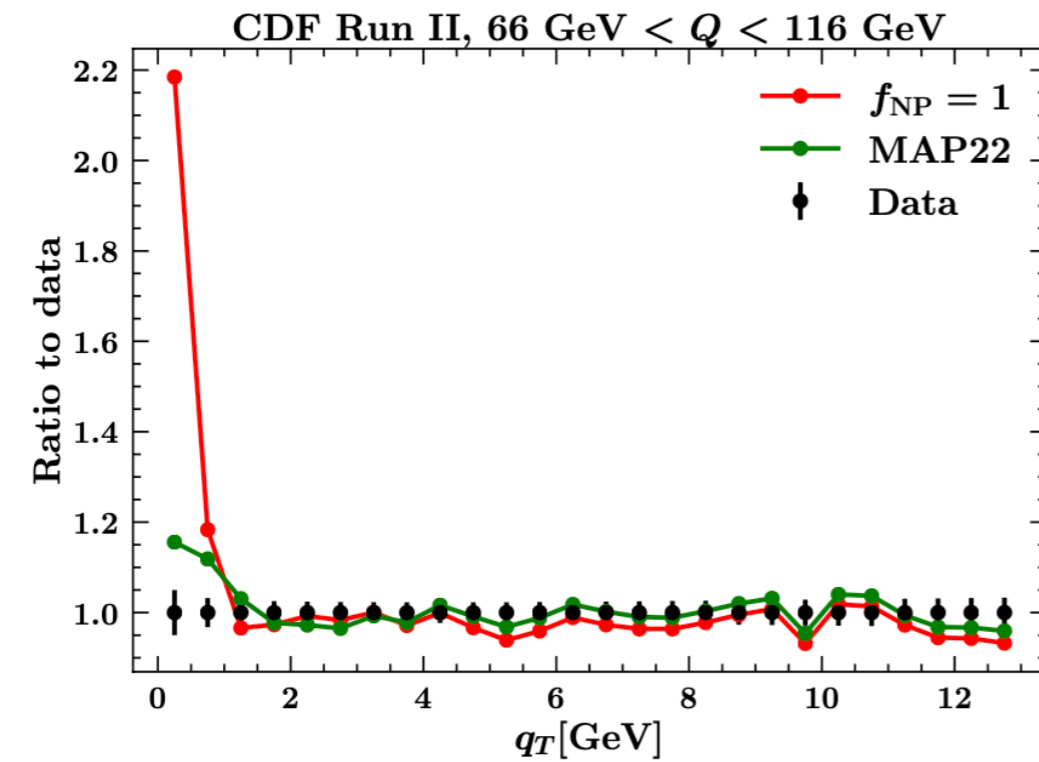
- χ^2 significantly higher for full dataset (1.339 vs. 1.020)
 - ★ x -dependence **required** to describe data
- χ^2 significantly lower without ATLAS data
 - ★ x -dependence at N³LL **driven by ATLAS data**

Relevance of f_{NP} at high Q

- N³LL fit to DY data only with $f_{NP} = 1$ or MAP 22
- different values of $[q_T/Q]_{min}$

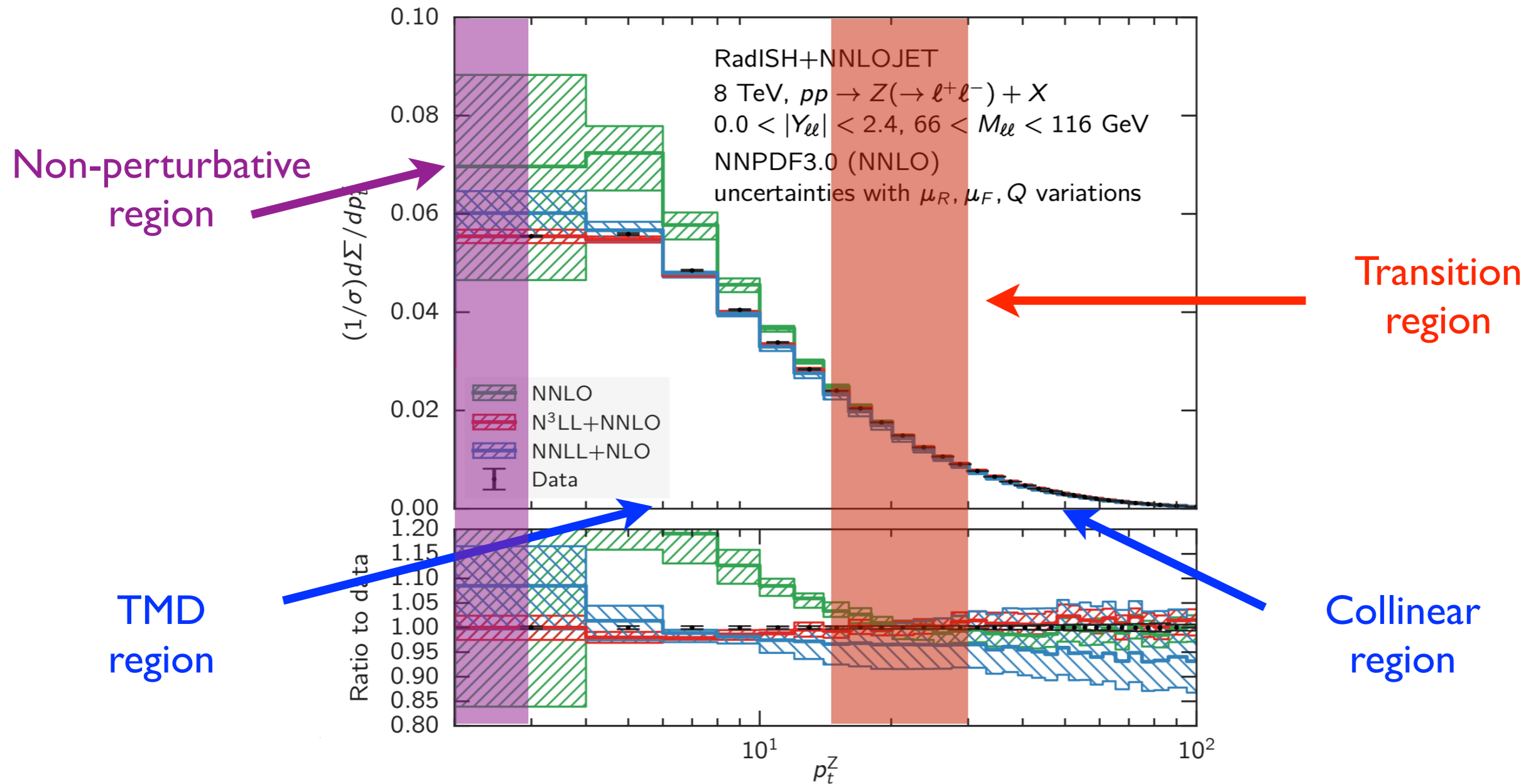


Relevance of f_{NP} at high Q



Future: matching with F.O.

🍏 **Matching** between TMD and collinear factorisations:



🍏 Well-understood procedure at the LHC energies where usually $Q \gg \Lambda_{\text{QCD}}$:

🍏 clear separation of TMD and collinear, non-perturbative confined to very low q_T .

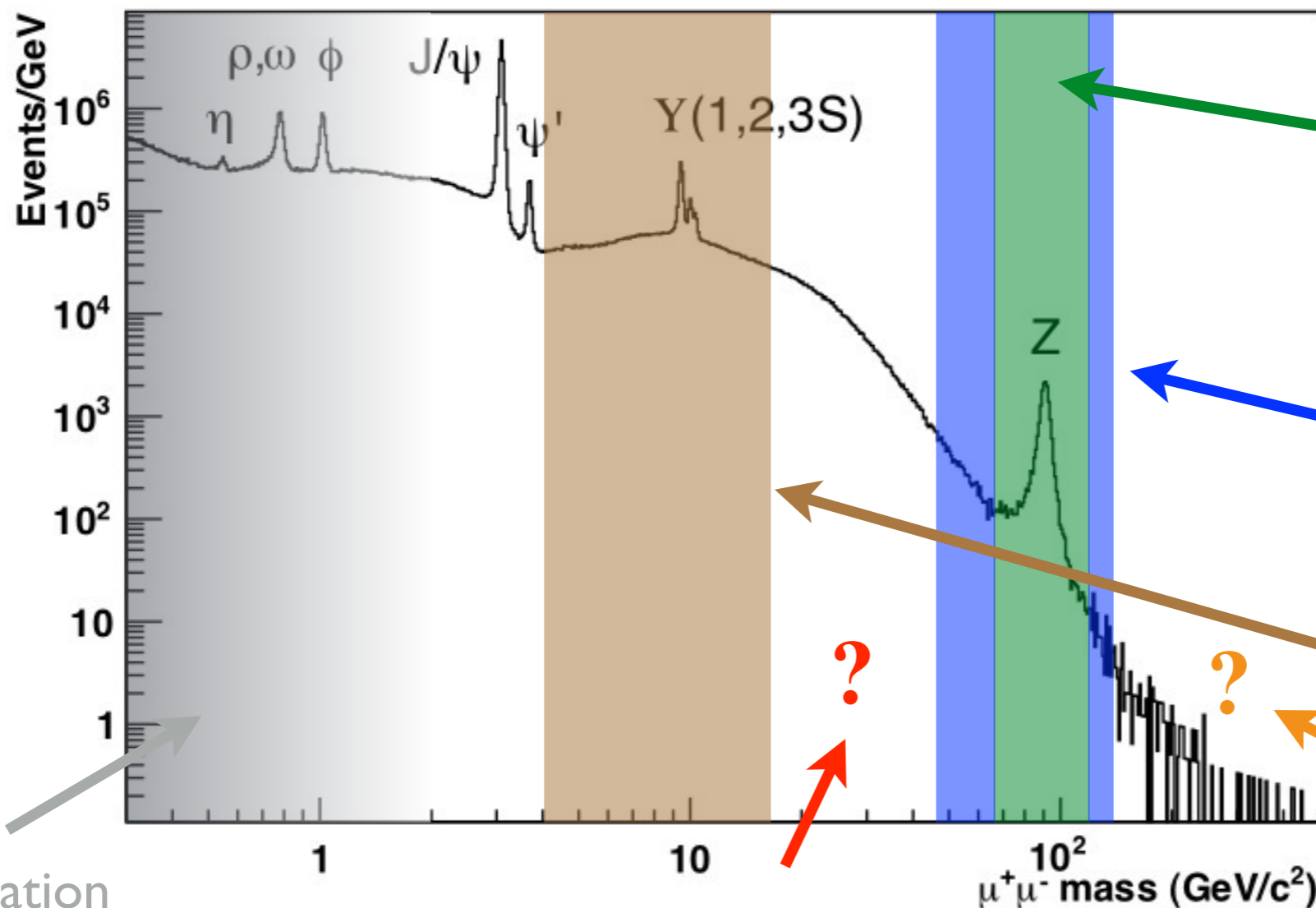
🍏 Not so much so for current (and future) SIDIS data due to smaller Q :

🍏 need to *identify* and *study* the transition region.

Future: Exp. Measurements

- 🍏 TMD factorisation applies for $q_T \ll Q$:
- 🍏 the region $q_T \approx \Lambda_{\text{QCD}}$ is relevant for hadron structure, no matter how large Q
- 🍏 As Q increases the cross section drops and low q_T becomes hard to access.

Need as many (low- q_T + y -binned) data as possible!



Precise data from the LHC and Tevatron

Less precise data from the LHC that extends to low q_T

Some poor old fixed-target data

Low- q_T data in this region absent but would be very welcome!

Is $q_T \approx \Lambda_{\text{QCD}}$ attainable with good precision here? 29

Perturbation theory breaking and resonances

Future: Exp. Measurements

$$\phi_\eta^* = \tan\left(\frac{\pi - \Delta\phi_\ell}{2}\right) \sqrt{1 - \tanh^2\left(\frac{\Delta\eta_\ell}{2}\right)}$$

[Banfi et al., 1009.1580]

Small ϕ^* is mapped onto small q_T , this observable is expected to carry important information on hadron structure.

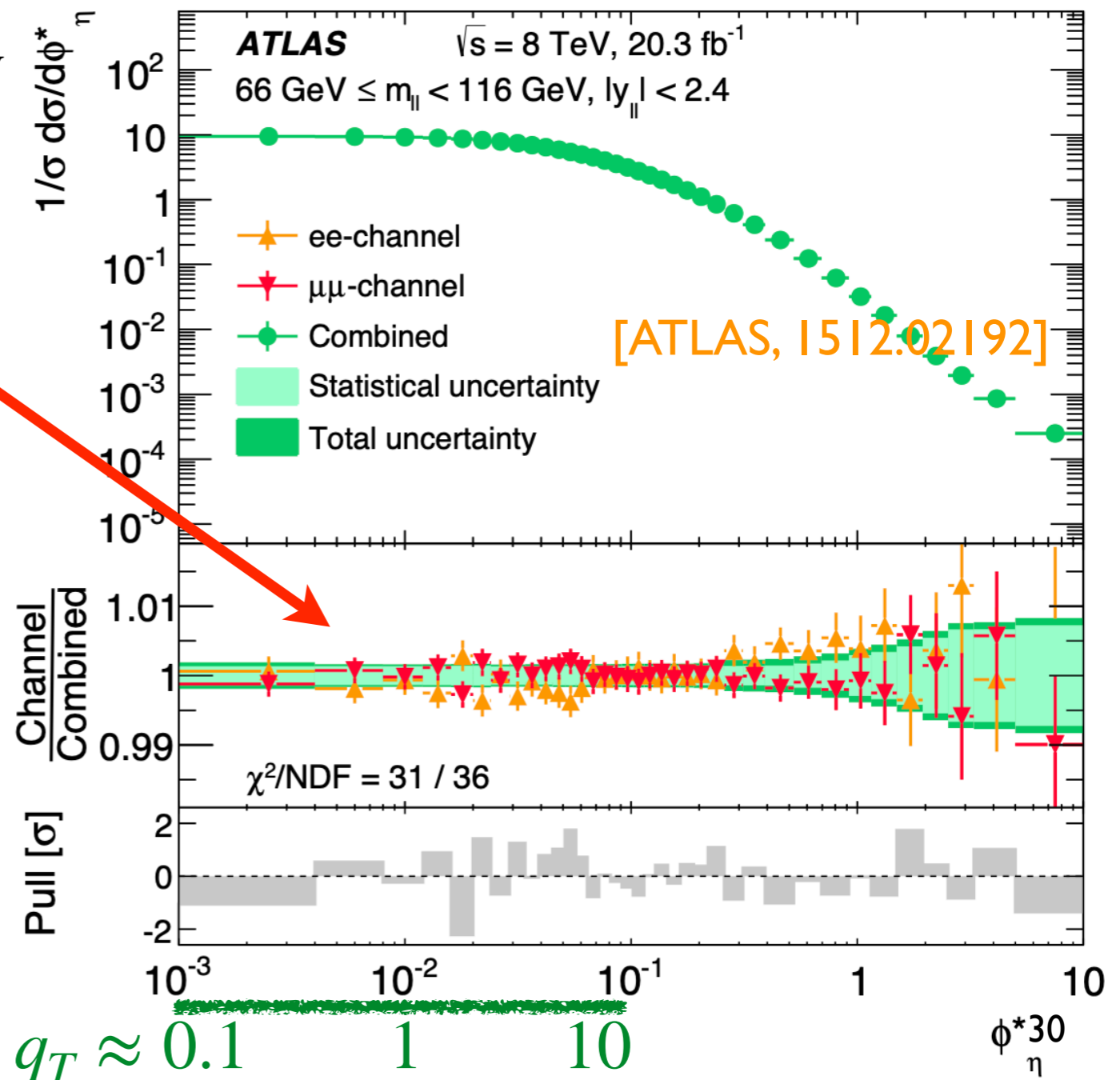
Experimentally very clean because it only involves angles.

Only angles: insanely precise!

$$\phi_\eta^* \approx \frac{q_T}{M}$$

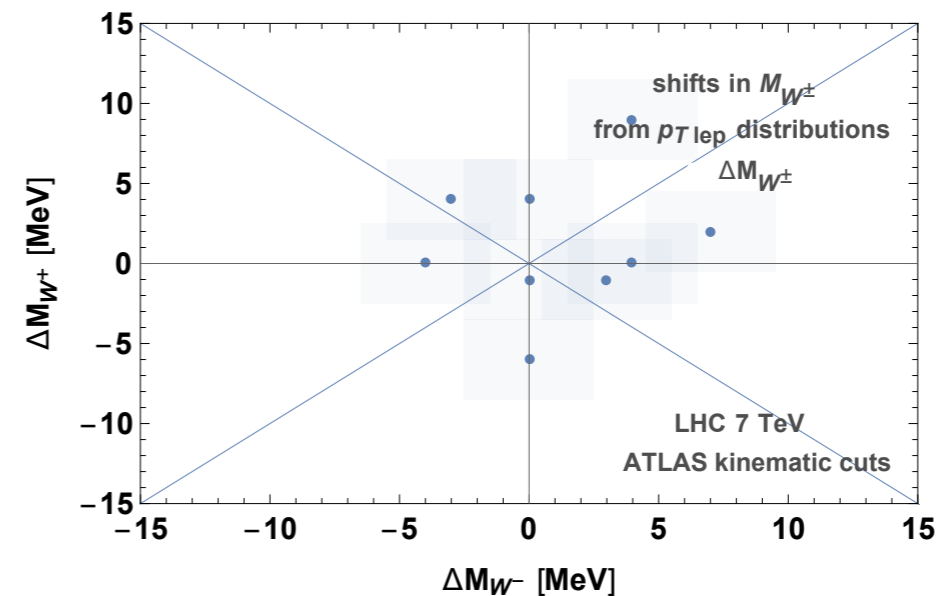
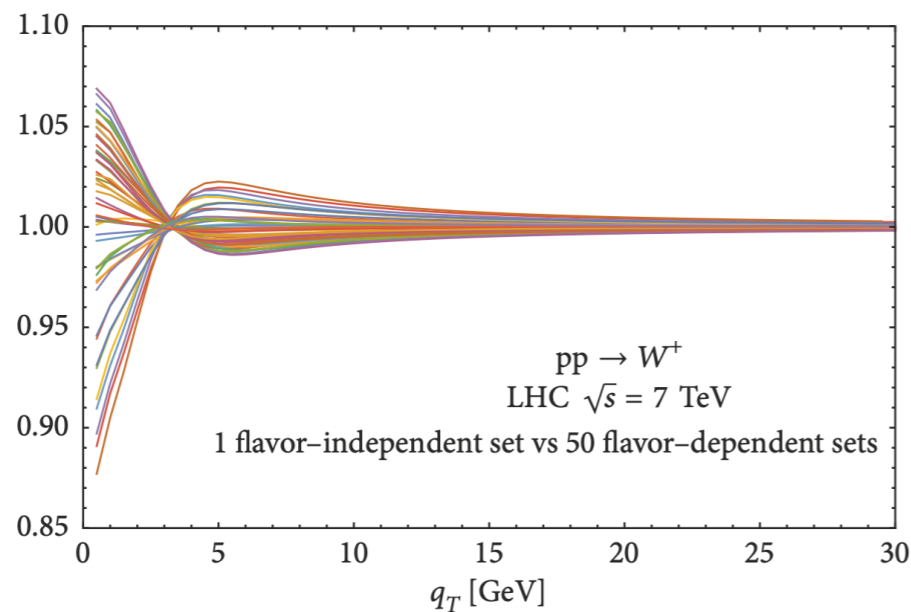
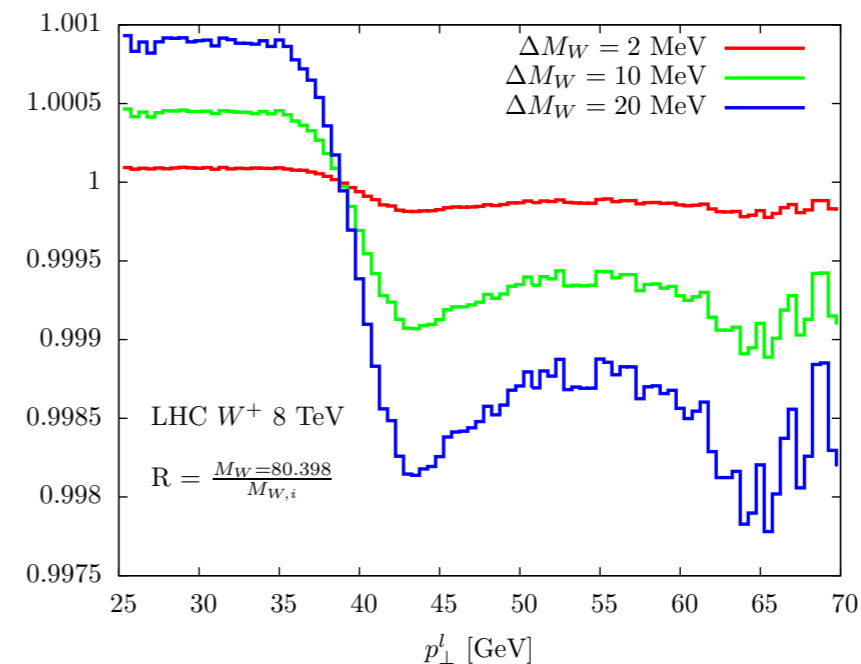
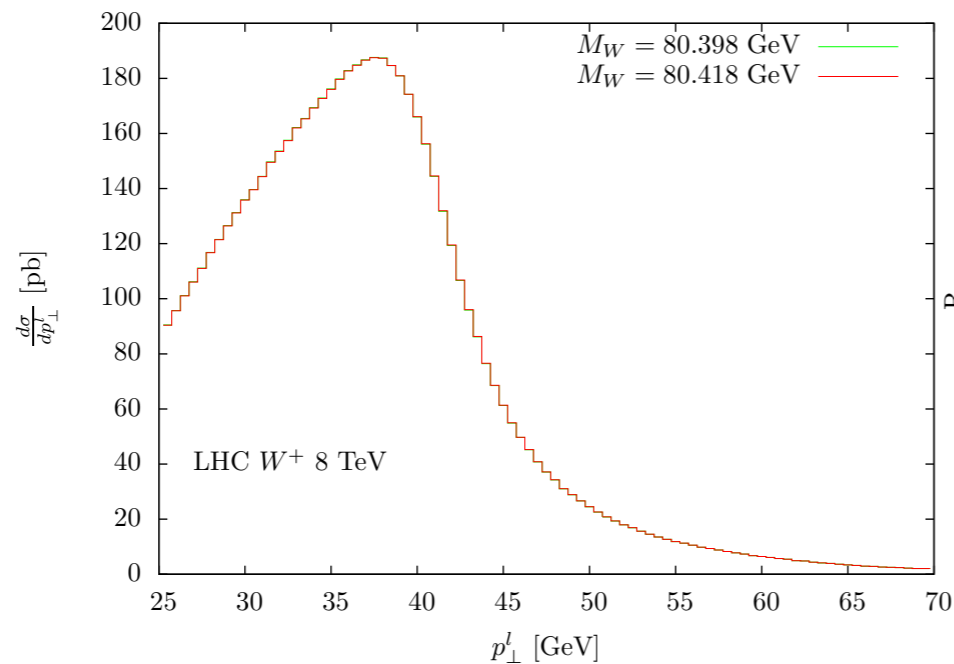
definitely relevant for hadron structure

it might be interesting to check shape variation with rapidity and m_{ll} at low ϕ_η^* (TMD (x, Q^2)-dependence)



Future: W mass measurements

- $p_{Tl} \leftarrow q_{TW} \leftarrow$ resummation + intrinsic- k_T
- All analyses assume flavour-independence
- impact of flavour-dependent intrinsic- k_T comparable to PDF variations



Thank you!