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Istituto Nazionale
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Towards the evaluation of NNLO (QCD + QCD-EW + EW) corrections at HL-LHC and future colliders

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Frascati, October 2nd 2024

based on: R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV: 2201.01754, 2205.03345, 2405.00612

Outline

Precision physics at the LHC and future colliders

Recent developments in the prediction of standard candle processes: fermion-pair production

Prospects towards the completion of full NNLO (QCD + QCDxEW + EW) corrections

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Precision physics at the LHC and future colliders

Recent developments in the prediction of standard candle processes: fermion-pair production

Prospects towards the completion of full NNLO (QCD + QCDxEW + EW) corrections

The inclusive production of a fermion pair is a standard candle process both

at LHC (Drell-Yan) $\sigma(pp \to \mu^+\mu^- + X)$

and

at FCC-ee $\sigma(e^+e^- \to \mu^+\mu^- + X)$

the lowest order process, at partonic level, is in both cases $f\bar{f} \to \mu^+\mu^-$: they share very similar computational challenges

The evaluation of NNLO-EW corrections is needed not only at FCC-ee, but already at the LHC or high-intensity facilities !

Motivations

Motivation: statistical precision from small to large fermion-pair invariant masses

Statistical errors

FCC-ee $\sigma(e^+e^- \to \mu^+\mu^- + X)$
arXiv:2206.08326

sqrt(S) (GeV)	luminosity (ab^{-1})	σ (fb)	% error
91	150	2.17595 10^6	0.0002
240	5	1870.84 ± 0.612	0.03
365	1,5	787.74 ± 0.725	0.09

LHC and HL-LHC $\sigma(pp \to \mu^+\mu^- + X)$
arXiv:2106.11953

bin range (GeV)	% error 140 fb^{-1}	% error 3 ab^{-1}
91-92	0.03	6 10^{-3}
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

Theoretical systematics

EW input parameters
large QED corrections
increasingly large EW corrections

proton PDFs
increasingly large QCD, QCD-EW and EW corrections

Are we able to reach (at least) 0.1% precision throughout the whole invariant mass range?

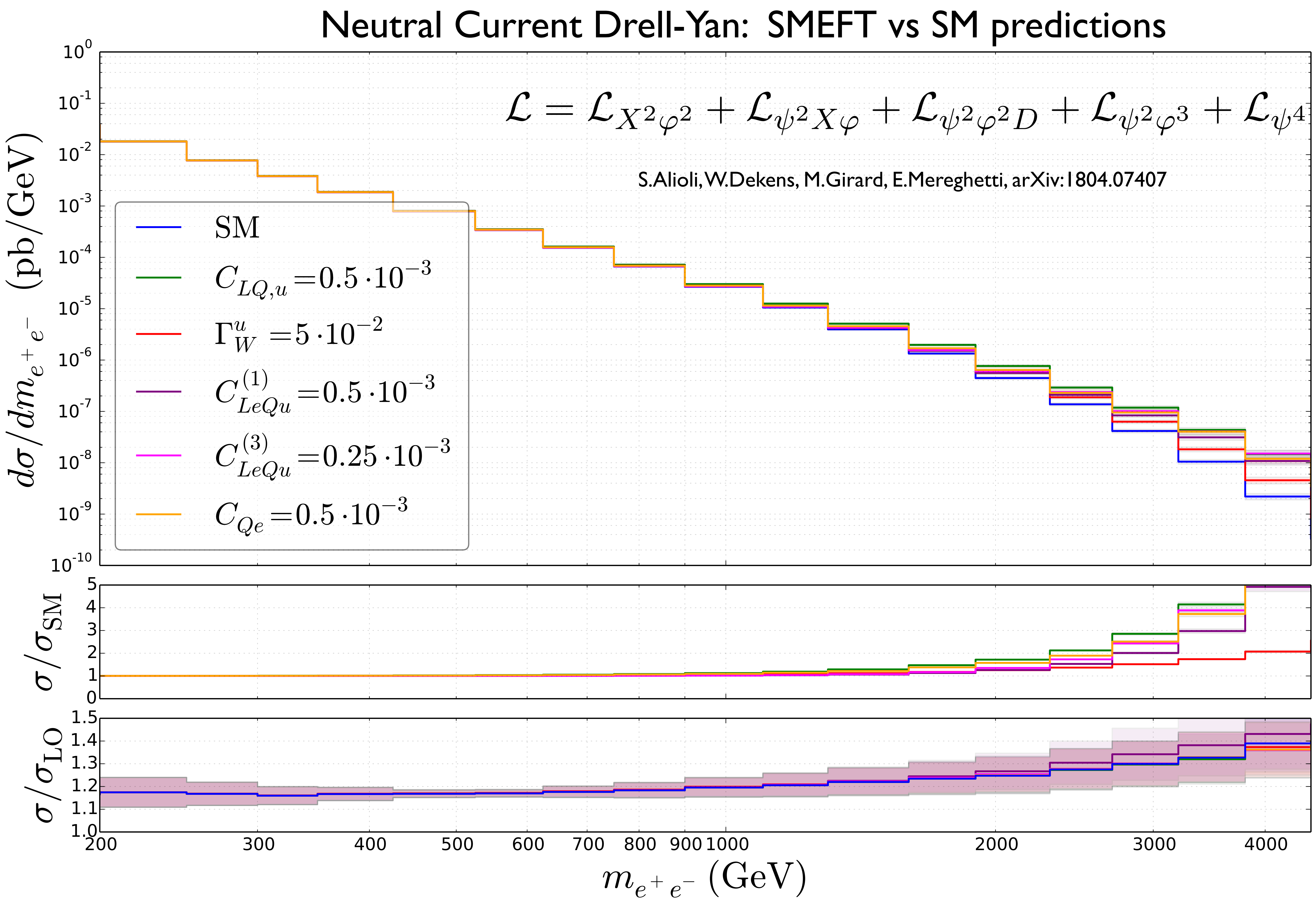
The Drell-Yan case poses the same challenges relevant for FCC-ee

Motivation: impact of higher dimension operators, as a function of the invariant mass

The parameterisation of BSM physics in the SMEFT language can be probed by studying the impact of higher dimension operators as a function of energy.

Deviations from the SM prediction require to answer the question "What is the SM?"

→ SM predictions have to be at the same precision level of the data i.e. (sub) per mille level



Motivation: interplay of precision measurements at Z resonance, low-, and high-energy

The very high precision determination of EW parameters at the Z resonance is a cornerstone of the whole precision program but there is more…

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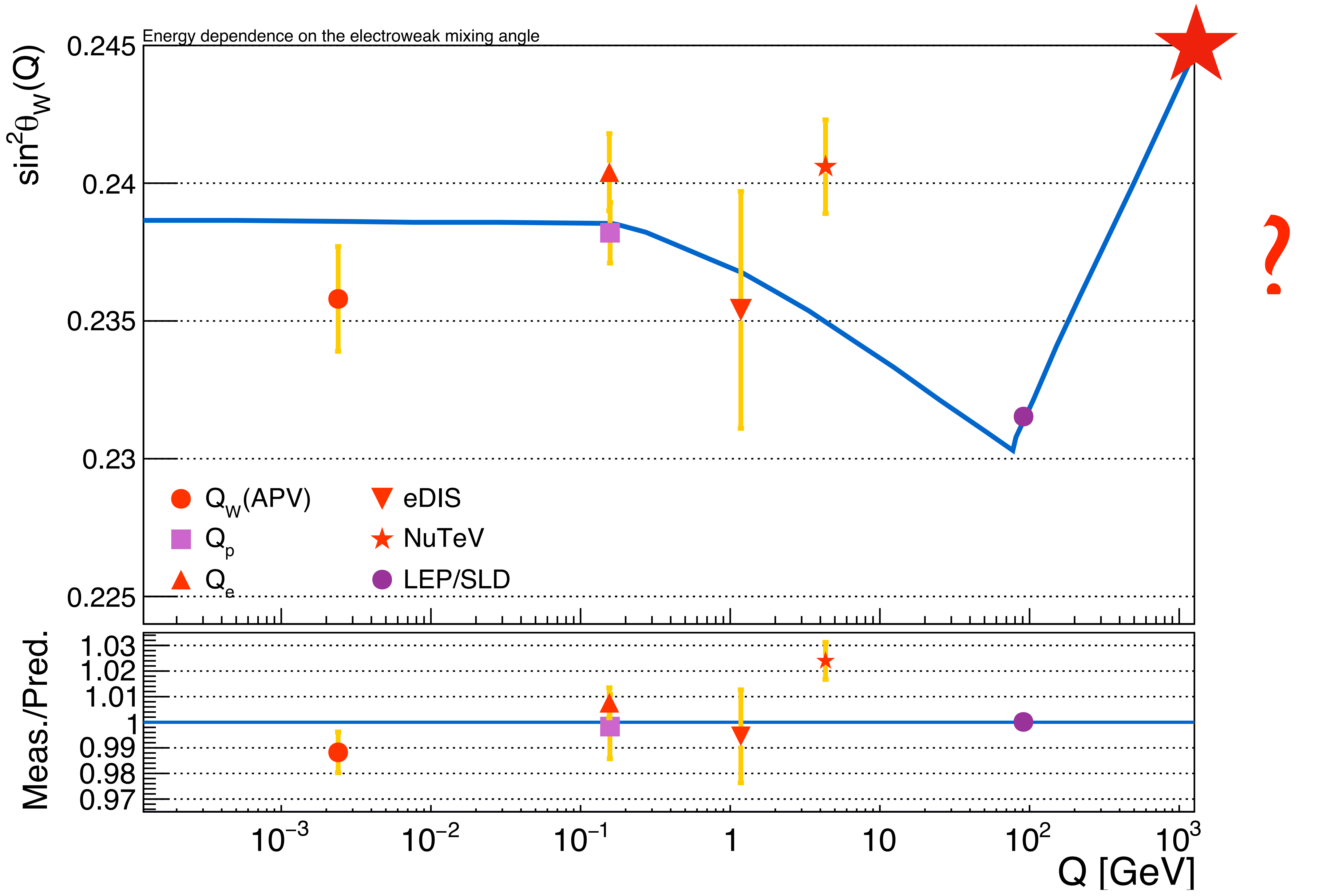
The SM predicts the running of its parameters, like e.g. $\sin^2\hat{\theta}(\mu_R^2)$, with non-trivial features and in turn complementary sensitivity to BSM physics

low-energy (sub-GeV) determinations (P2 in Mainz, Møller at JLab)
high-energy (TeV) determinations (CMS, ATLAS)
offer a stringent test of the SM
complementary to the results at the Z resonance

The running of an MSbar parameter is completely assigned
once boundary and matching conditions are specified

The absence of higher-order SM corrections
could fake a BSM signal

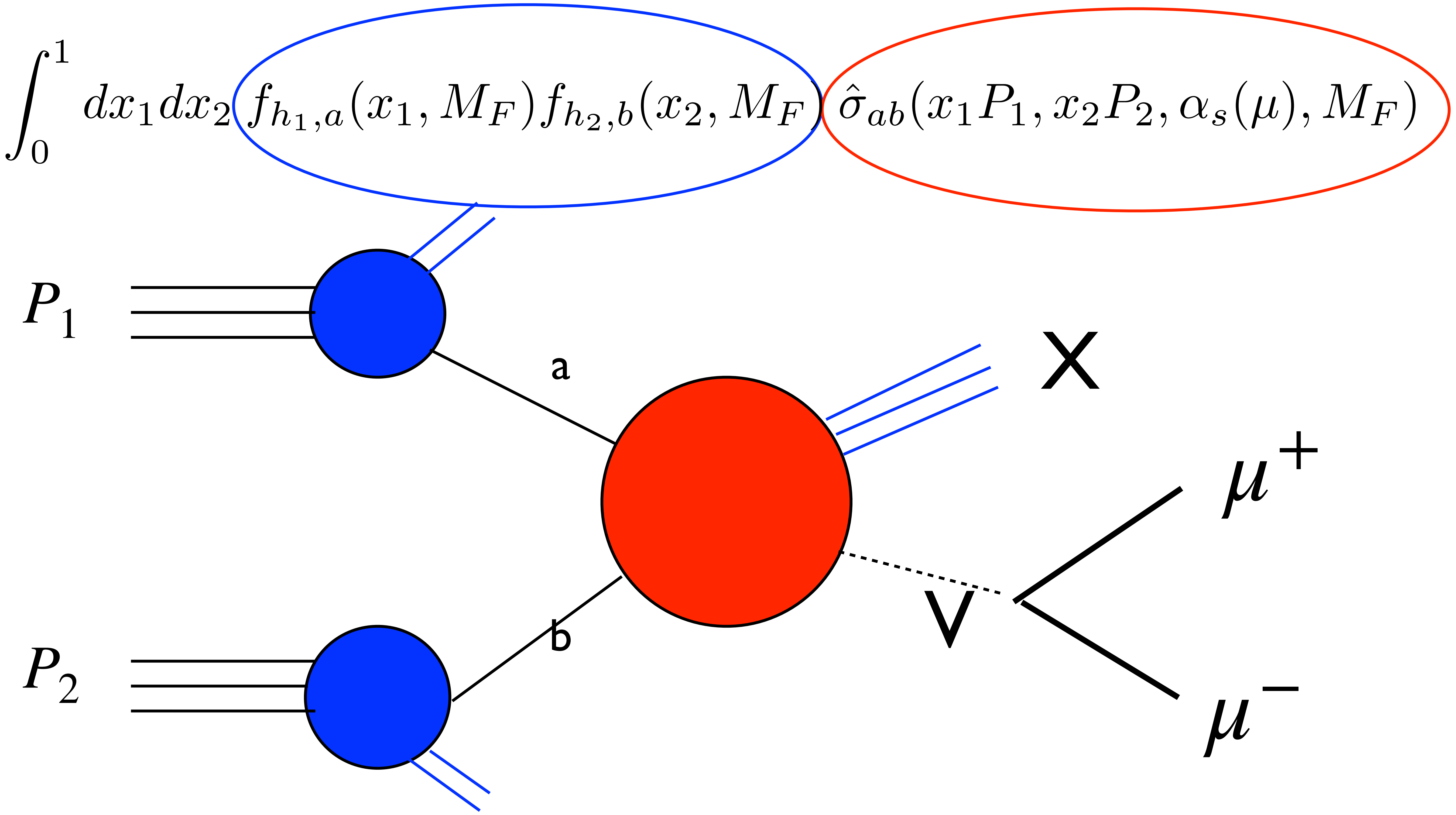


Erler, J., Schott, M., Progr. in Part. and Nucl. Phys. 106 (2019) 68-119

Computational framework

Factorisation theorems and the cross section in the partonic formalism

$$\sigma(P_1,P_2;m_V) = \sum_{a,b}\int_0^1 dx_1 dx_2\, f_{h_1,a}(x_1,M_F) f_{h_2,b}(x_2,M_F)\, \hat\sigma_{ab}(x_1P_1,x_2P_2,\alpha_s(\mu),M_F)$$



Particles $P_{1,2}$ can be protons (→ Drell-Yan @ LHC) or leptons (→ FCC-ee, muon collider)

The partonic content of the scattering particles can be expressed in terms of PDFs
proton PDFs: ABM, CT18, MSHT, NNPDF,… lepton PDFs: Frixione et al. arXiv:1911.12040

The partonic scattering can be computed in perturbation theory, in the full QCD+EW theory,
exploiting the theoretical progress in QCD, in the understanding of its IR structure

Factorisation theorems guarantee the validity of the above picture up to power correction effects

The Drell-Yan cross section in a fixed-order expansion

The Drell-Yan cross section in a fixed-order expansion

The resummation of QCD and QED corrections is central crucial topic (one slide later)

Assuming that the corrections are under control better than O(0.1%) level
we can discuss the evaluation of the hard partonic cross section (and its matching in the resummation formalism)

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Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \\ \alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \\ \alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \\ \alpha_s^3\,\sigma^{(3,0)} + \ldots$$

Altarelli, Ellis, Martinelli (1978)

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

C.Duhr, B.Mistlberger, arXiv:2111.10379

still missing
Sudakov high-energy approximations

Neutral Current

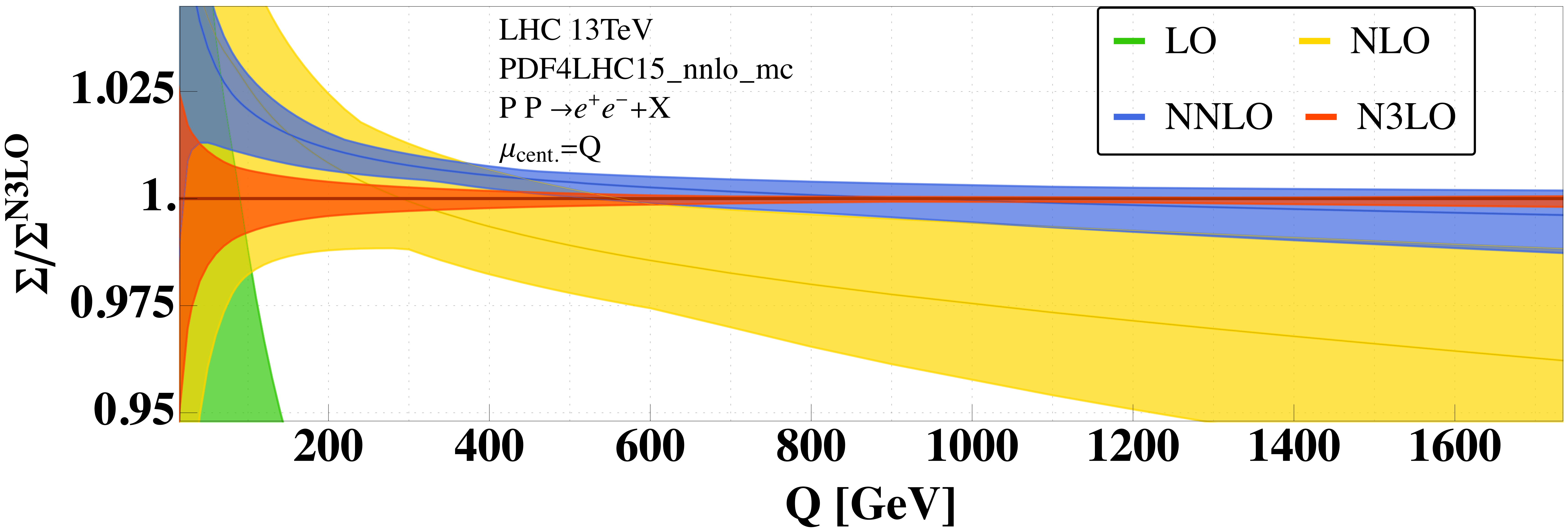
R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)

New!!! Charged-current 2-loop amplitude

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2024)

QCD results: lepton-pair invariant mass

C.Duhr, B.Mistlberger, arXiv:2111.10379



Thanks to the N3LO-QCD results for the Drell-Yan cross section, scale variation band at the few per mille level at any Q

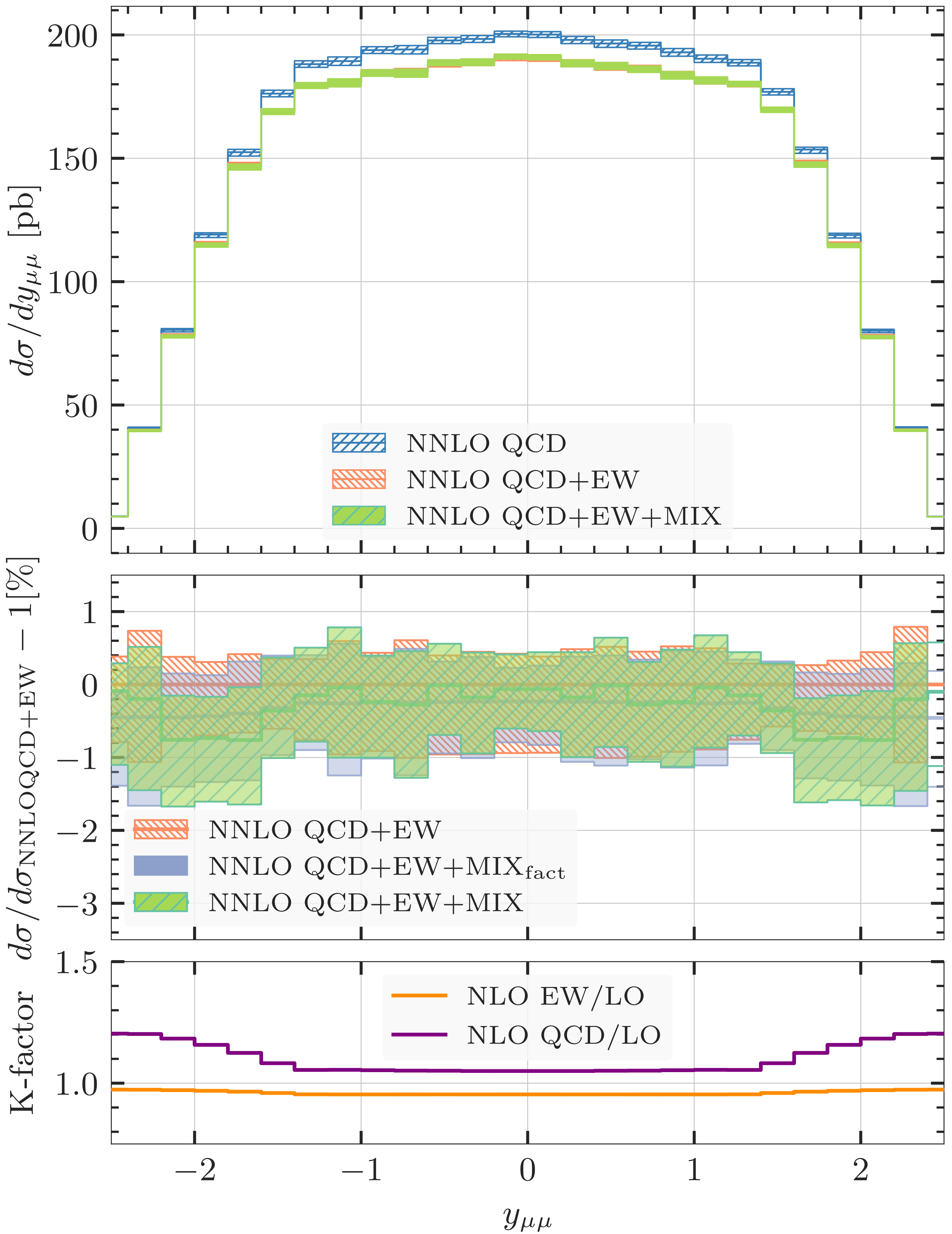
The PDFs are not yet at N3LO

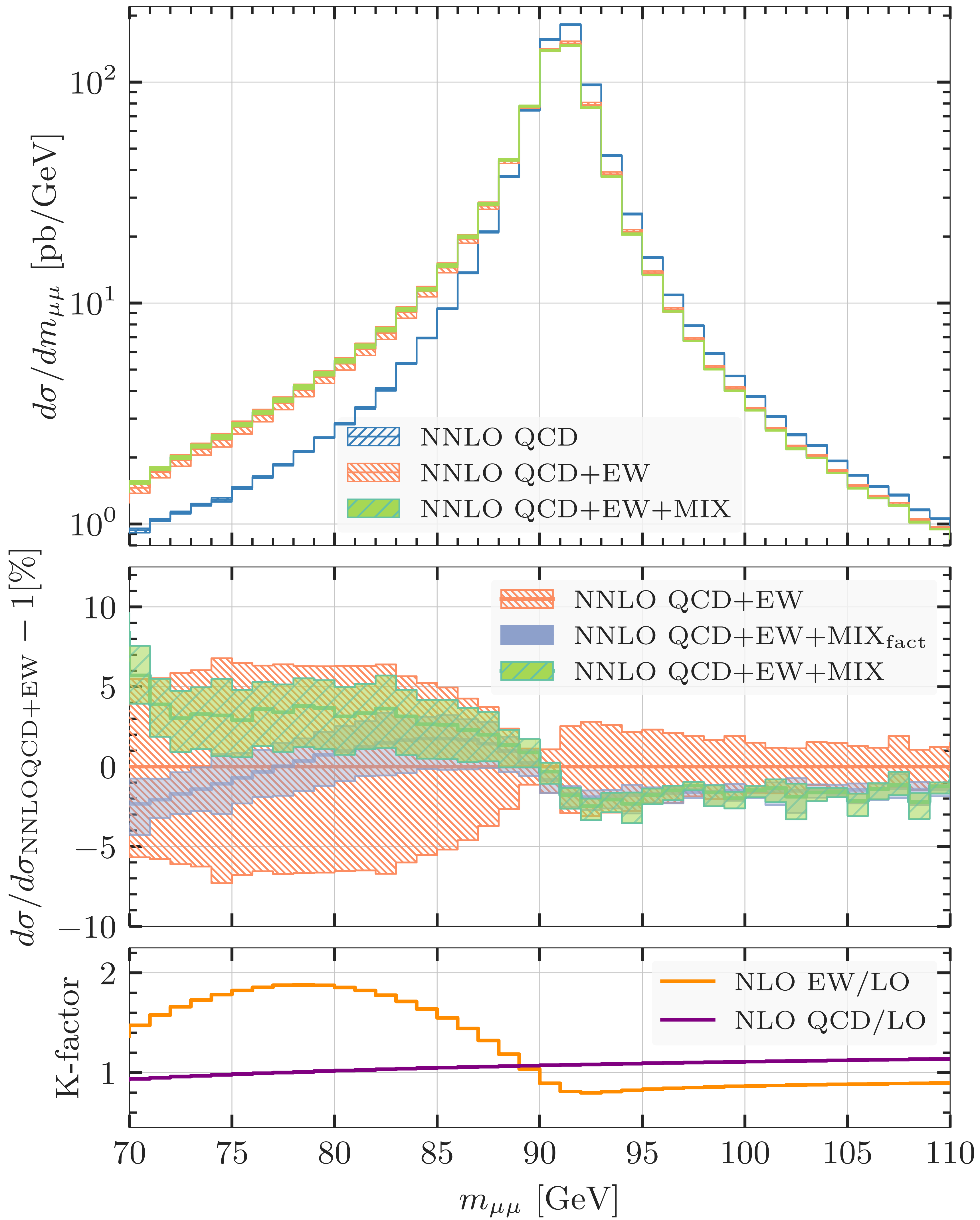
This is promising, in view of the program of searches for deviation from the SM in the TeV range

What about NNLO QCD-EW and NNLO-EW corrections ?

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953 , Phys.Rev.Lett. 128 (2022) 1, 012002 and work in preparation





Non-trivial distortion of the rapidity distribution (absent in the naive factorised approximation)

Large effects below the Z resonance (the factorised approximation fails) → impact on the $\sin^2\theta_{eff}$ determination

O(-1.5%) effects above the resonance → ongoing precision studies in the CERN EW WG

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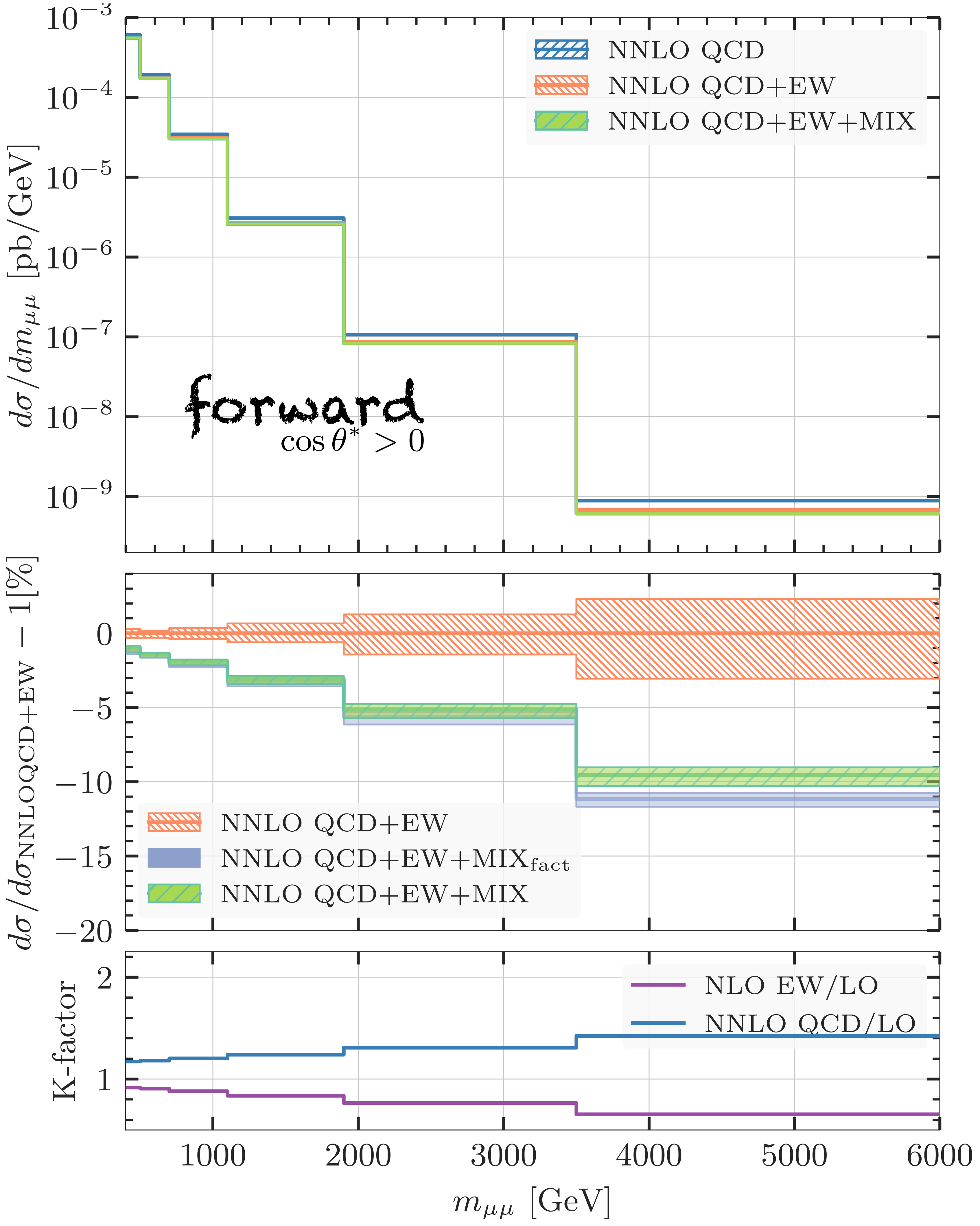

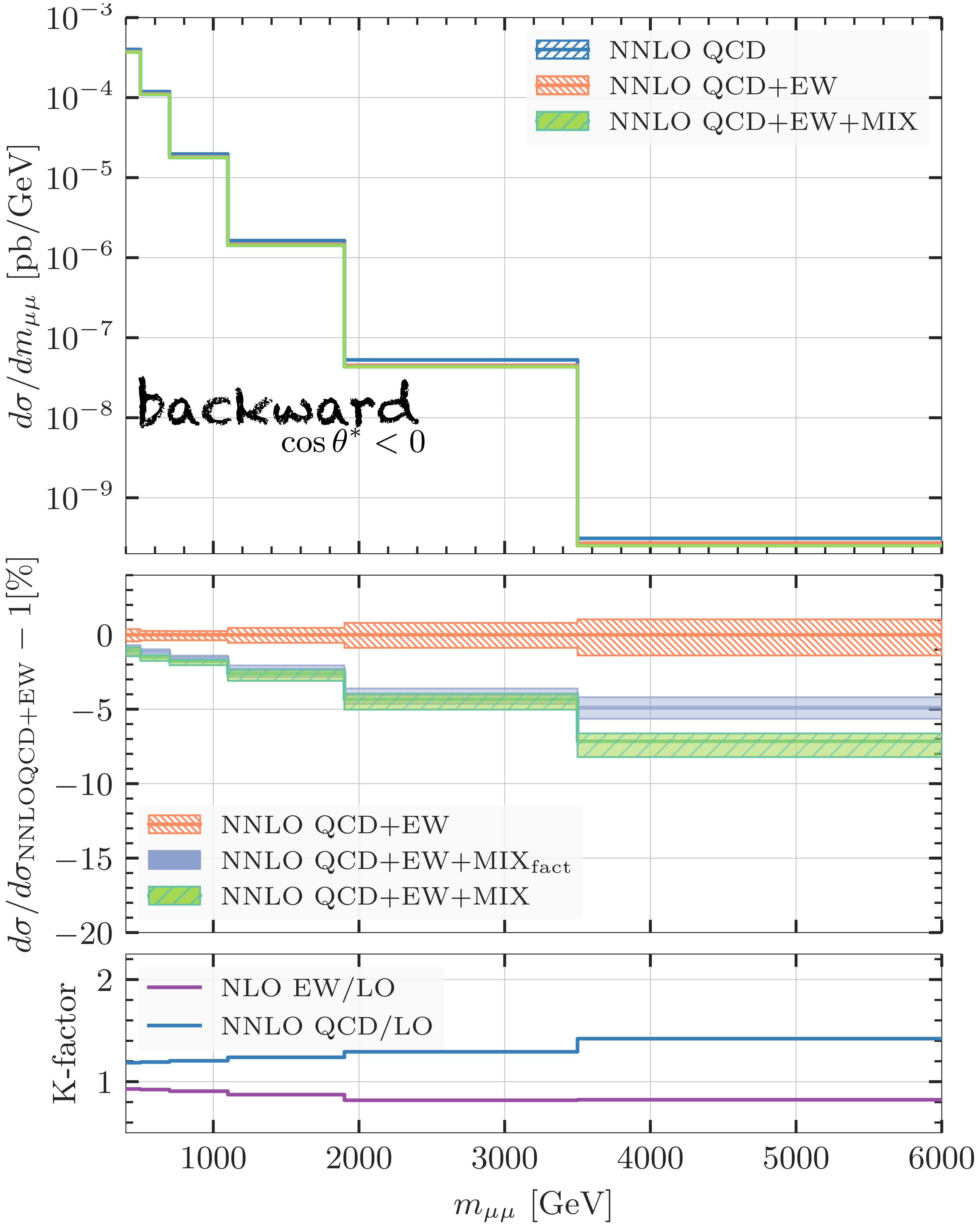

Negative mixed NNLO QCD-EW effects (-3% or more) at large invariant masses,

absent in any additive combination   → impact on the searches for new physics

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

- The exact NNLO QCD-EW corrections yield large effects at large transverse/invariant masses → BSM searches
- m_W determination

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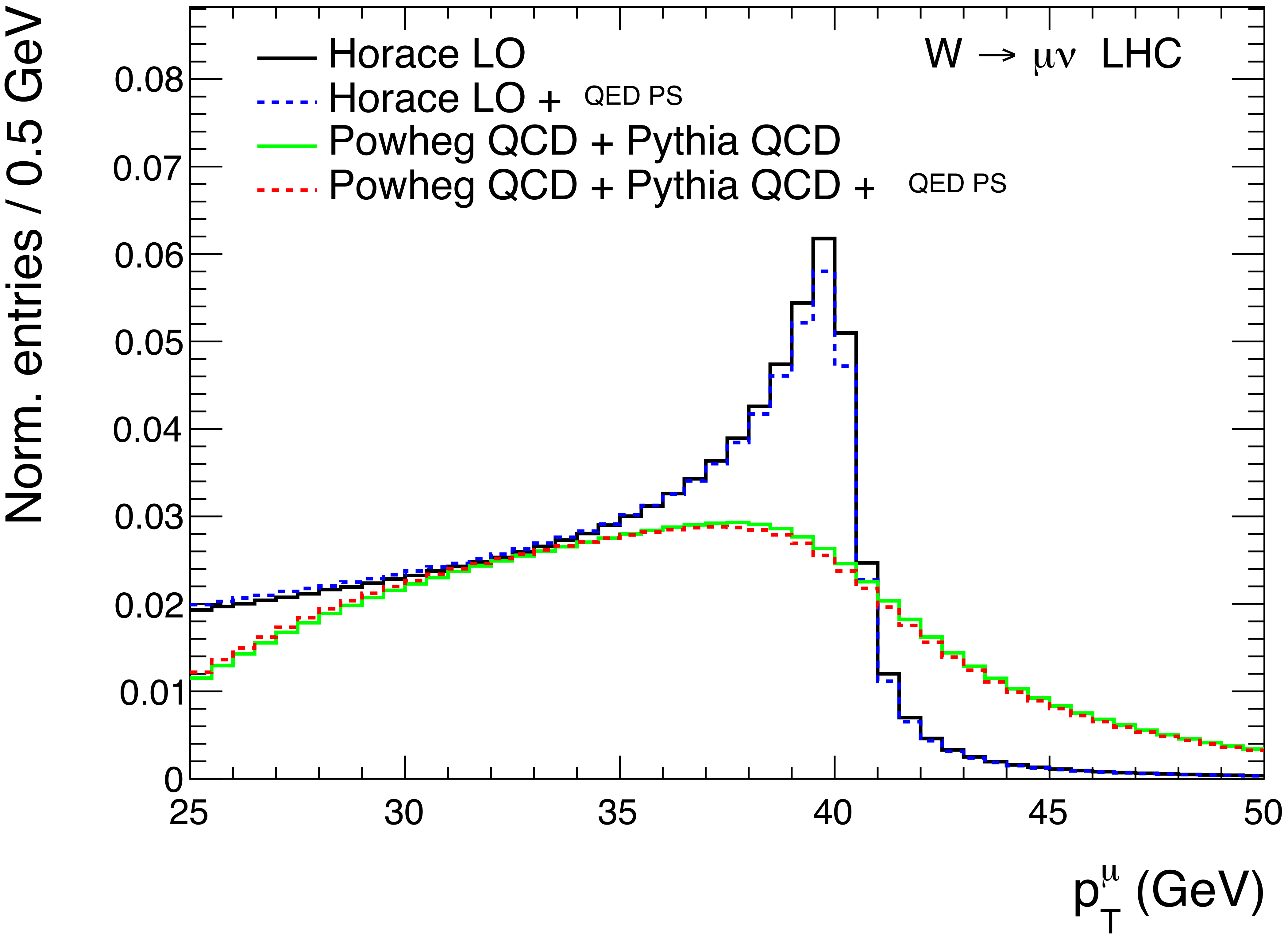
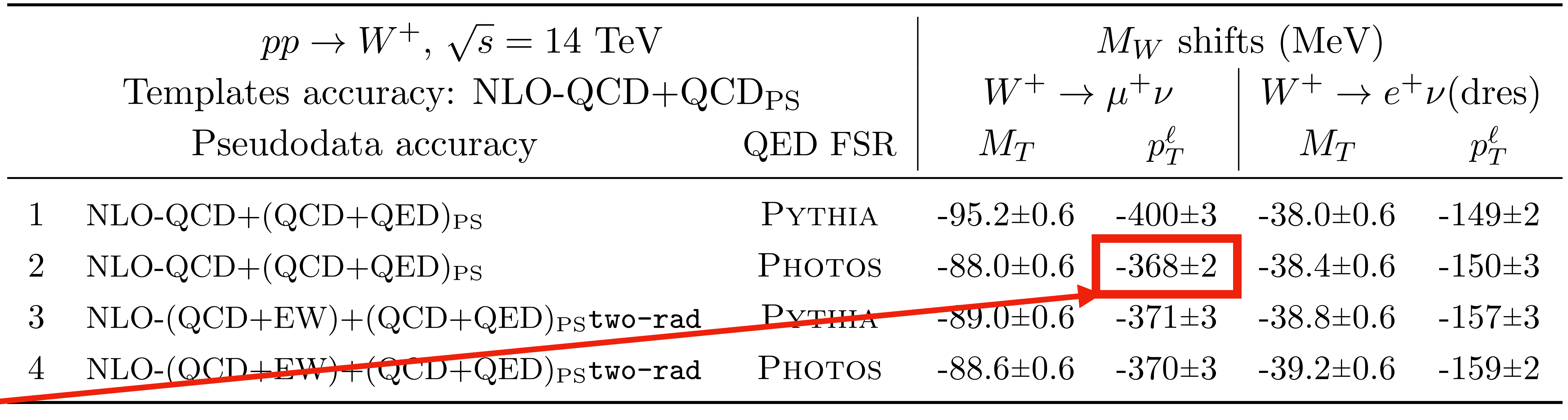


POWHEG simulation NLO QCD+EW +QCDPS + QEDPS



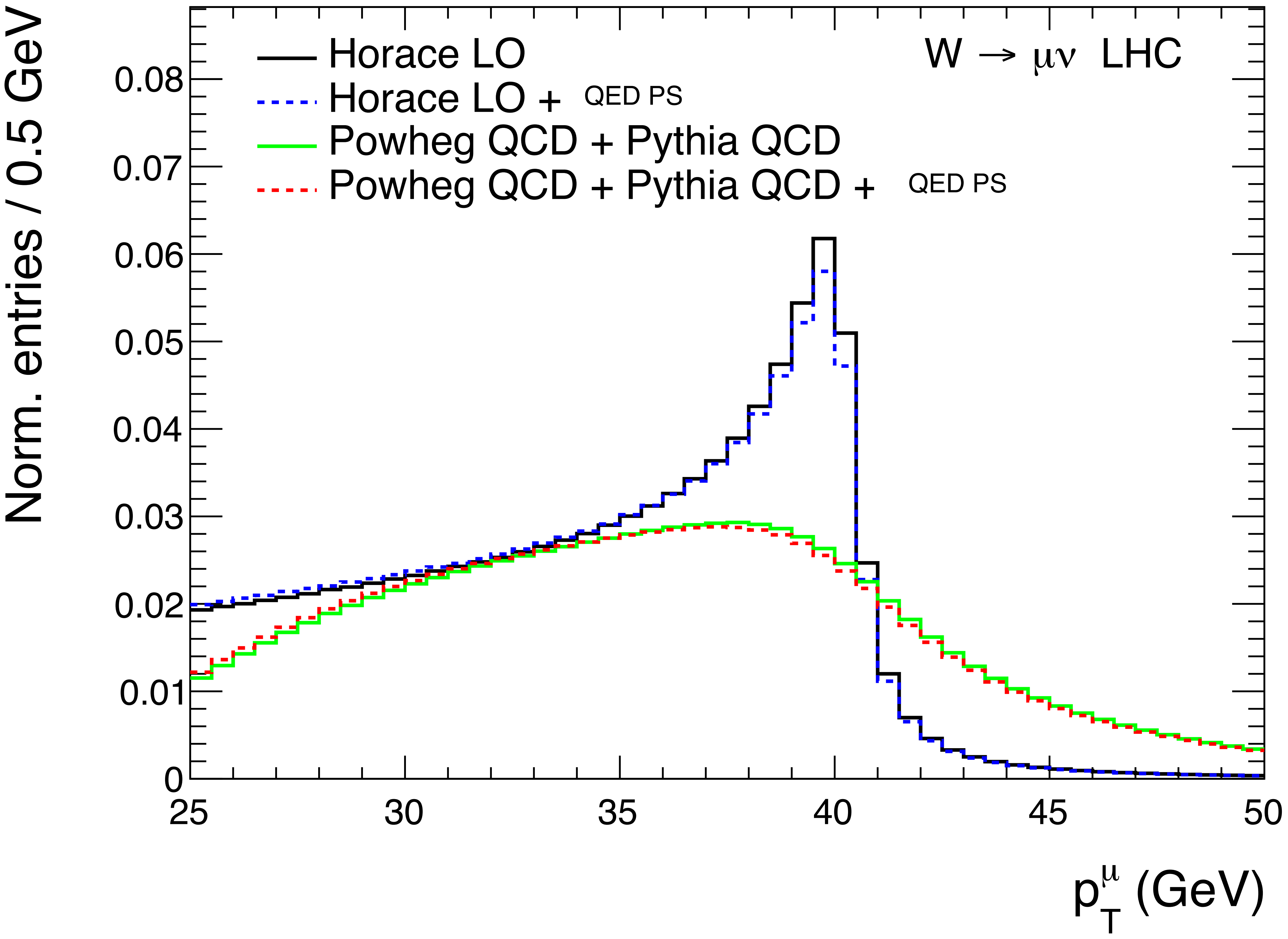
	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{\text{PS}}$ Pseudodata accuracy	QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{\text{PS}}$	Pythia	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{\text{PS}}$	Photos	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{\text{PS}}$`two-rad`	Pythia	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{\text{PS}}$`two-rad`	Photos	-88.6±0.6	-370±3	-39.2±0.6	-159±2

Huge impact of QED and mixed QCD-QED corrections in the m_W determination

What is the theoretical uncertainty on this estimated shift ? e.g. what would be the difference using POWHEG vs MC@NLO ?

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

- The exact NNLO QCD-EW corrections yield large effects at large transverse/invariant masses → BSM searches
- m_W determination



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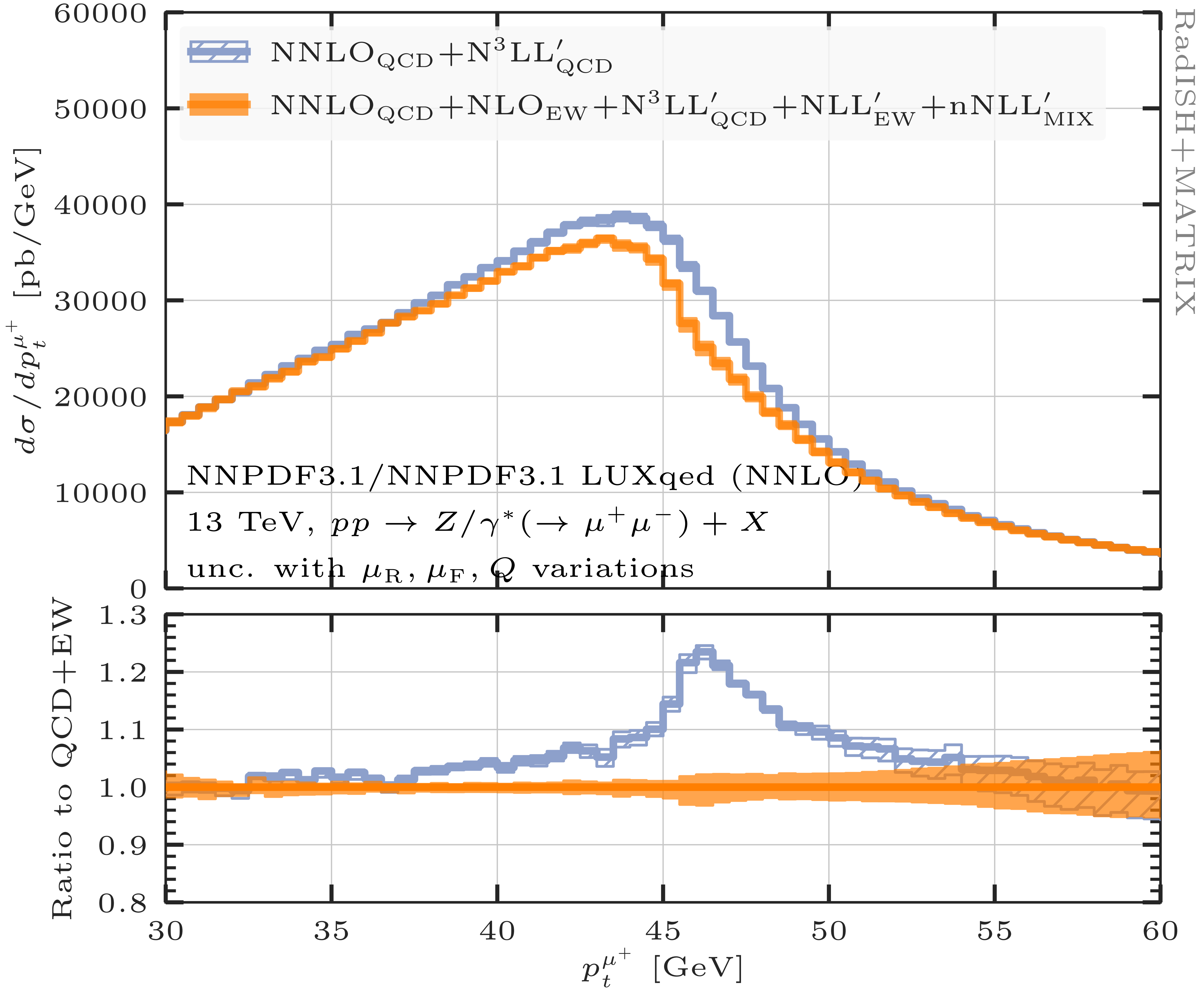
What is the theoretical uncertainty on this estimated shift ? e.g. what would be the difference using POWHEG vs MC@NLO ?

with NNLO QCD-EW results we can fix the dominant source of ambiguity

Towards a tool matching QCD+QED resummation with NNLO QCD-EW fixed order

L.Buonocore, L.Rottoli, P.Torrielli, arXiv:2404.15112



Joined QCD-QED resummation in the RadIsh formulation at N3LL'-QCD + NLL'-EW + nNLL'-mixed accuracy including QED effects from all charged legs

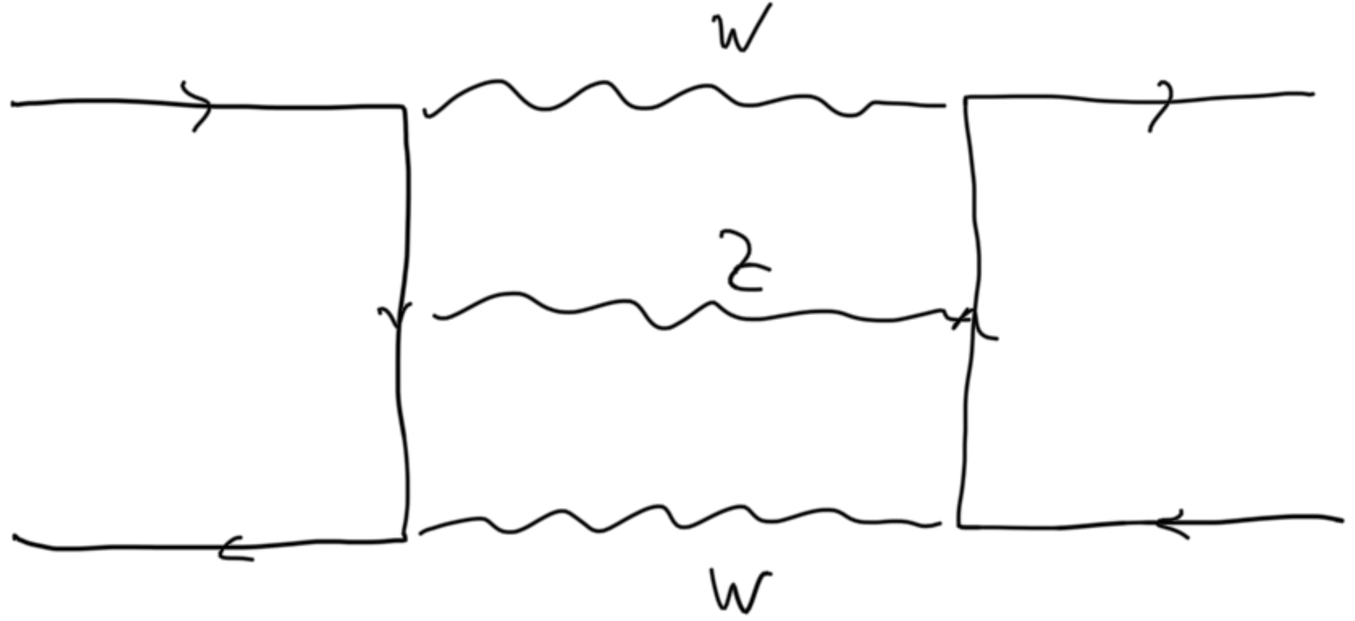
Non-trivial interplay of QCD and EW corrections

Missing final step : Matching with the exact $\mathcal{O}(\alpha\alpha_s)$ corrections needed to reach full NNLL-mixed

→ Reliable estimate of the reduced residual theoretical uncertainties

Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent level

The NNLO-EW corrections to scattering processes are still today one of the frontiers in QFT



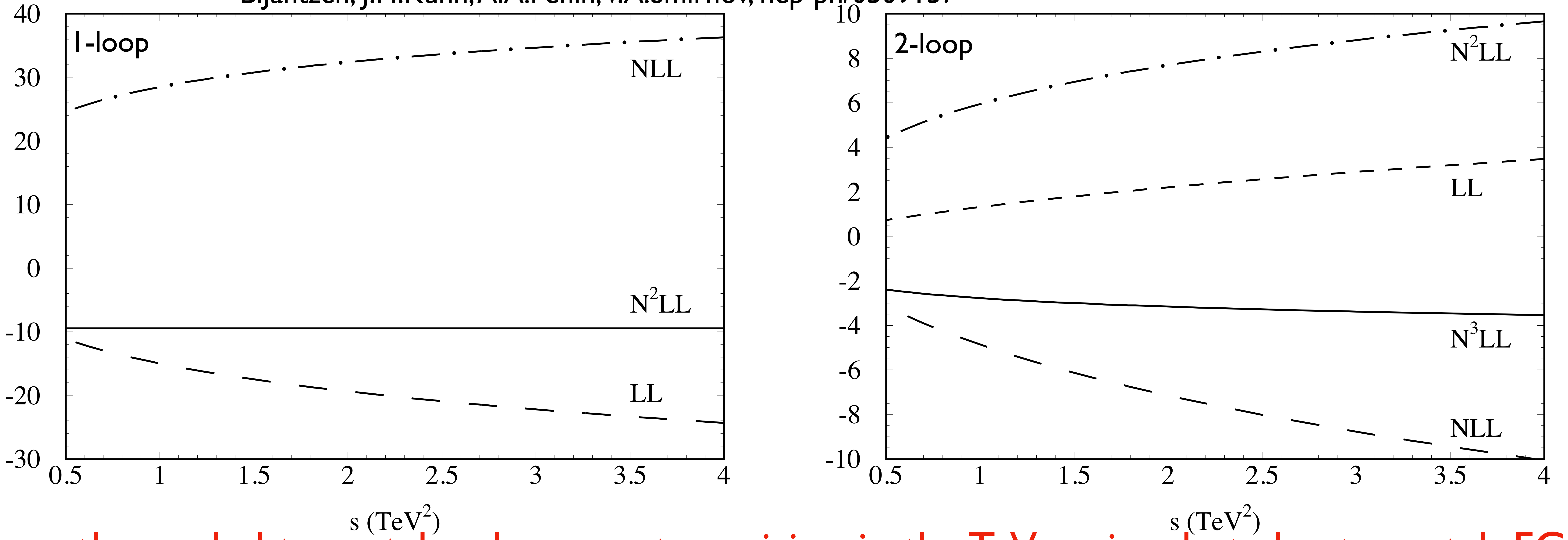
The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions

Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections

At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$

The size of the constant term is not trivial

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157



corrections to $e^+e^- \to q\bar{q}$ due to EW Sudakov logs

urgently needed to match sub-percent precision in the TeV region, but also to match FCC-ee precision

Evaluation of the exact NNLO QCD-EW corrections to NC and CC DY

The Neutral Current Drell-Yan cross section in the SM: perturbative expansion

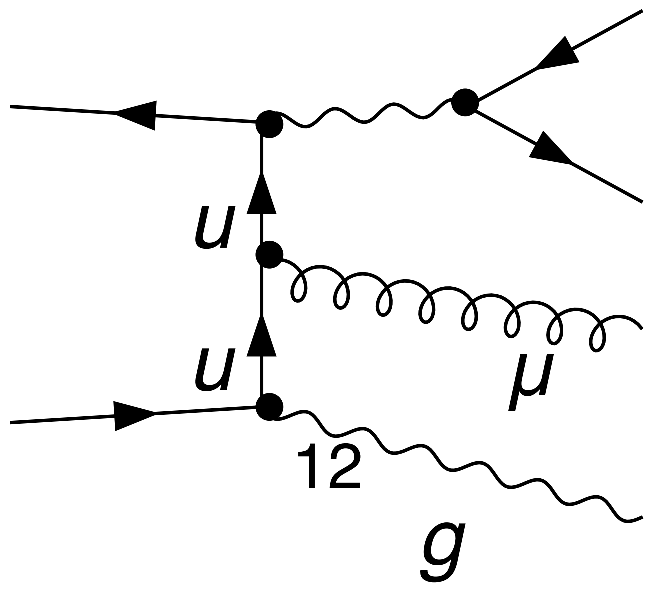
$$\begin{aligned}\sigma(h_1h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} &+ \\ &\alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \\ &\alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \\ &\alpha_s^3\,\sigma^{(3,0)} + \ldots\end{aligned}$$

$$\sigma(h_1h_2 \to l\bar{l} + X) = \sum_{i,j=q\bar{q},g,\gamma} \int dx_1\, dx_2\, f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F)\, \hat{\sigma}(ij \to l\bar{l} + X)$$

$\sigma^{(1,1)}$ requires the evaluation of the xsecs of the following processes, including photon-induced

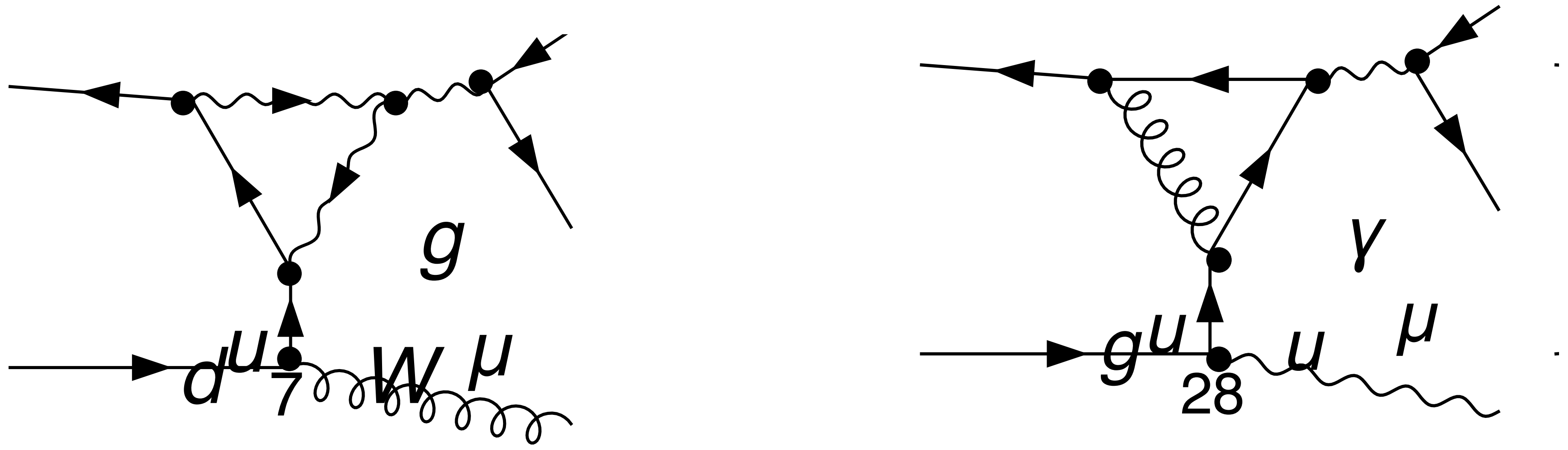
0 additional partons	$q\bar{q} \to l\bar{l},\ \gamma\gamma \to l\bar{l}$	including virtual corrections of $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha), \mathcal{O}(\alpha\alpha_s)$
1 additional parton	$q\bar{q} \to l\bar{l}g,\ qg \to l\bar{l}q$	including virtual corrections of $\mathcal{O}(\alpha)$
	$q\bar{q} \to l\bar{l}\gamma,\ q\gamma \to l\bar{l}q$	including virtual corrections of $\mathcal{O}(\alpha_s)$
2 additional partons	$q\bar{q} \to l\bar{l}g\gamma, qg \to l\bar{l}q\gamma, q\gamma \to l\bar{l}qg, g\gamma \to l\bar{l}q\bar{q}$	
	$q\bar{q} \to l\bar{l}q\bar{q},\ q\bar{q} \to l\bar{l}q'\bar{q}',\ qq' \to l\bar{l}qq',\ q\bar{q}' \to l\bar{l}q\bar{q}',\ qq \to l\bar{l}qq$	at tree level

Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems



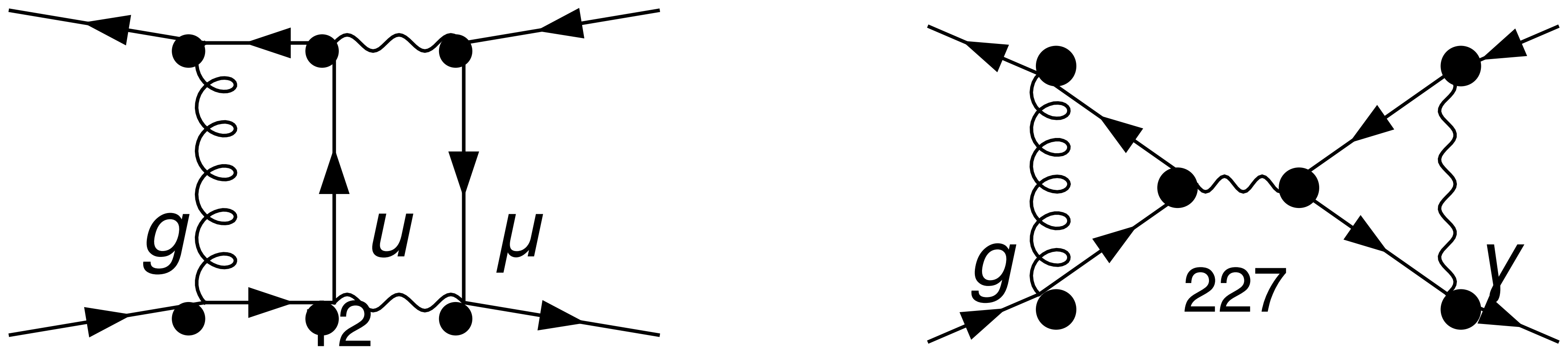
double-real contributions

amplitudes are easily generated with OpenLoops
IR subtraction
care about the numerical convergence when aiming at 0.1% precision



real-virtual contributions

amplitudes are easily generated with OpenLoops or Recola
1-loop UV renormalisation and IR subtraction
care about the numerical convergence when aiming at 0.1% precision



double-virtual contributions

generation of the amplitudes
γ_5 treatment
2-loop UV renormalization
solution and evaluation of the Master Integrals
subtraction of the IR divergences
numerical evaluation of the squared matrix element

Structure of the double virtual amplitude

$$2\mathrm{R}e\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{I}_i(s,t,m;\varepsilon)$$

Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{I}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε

Their size can rapidly "explode" in the GB range

→ careful work to identify the patterns of recurring subexpressions, keeping the total size in the O(1-10 MB) range

Abiss Mathematica package

Structure of the double virtual amplitude

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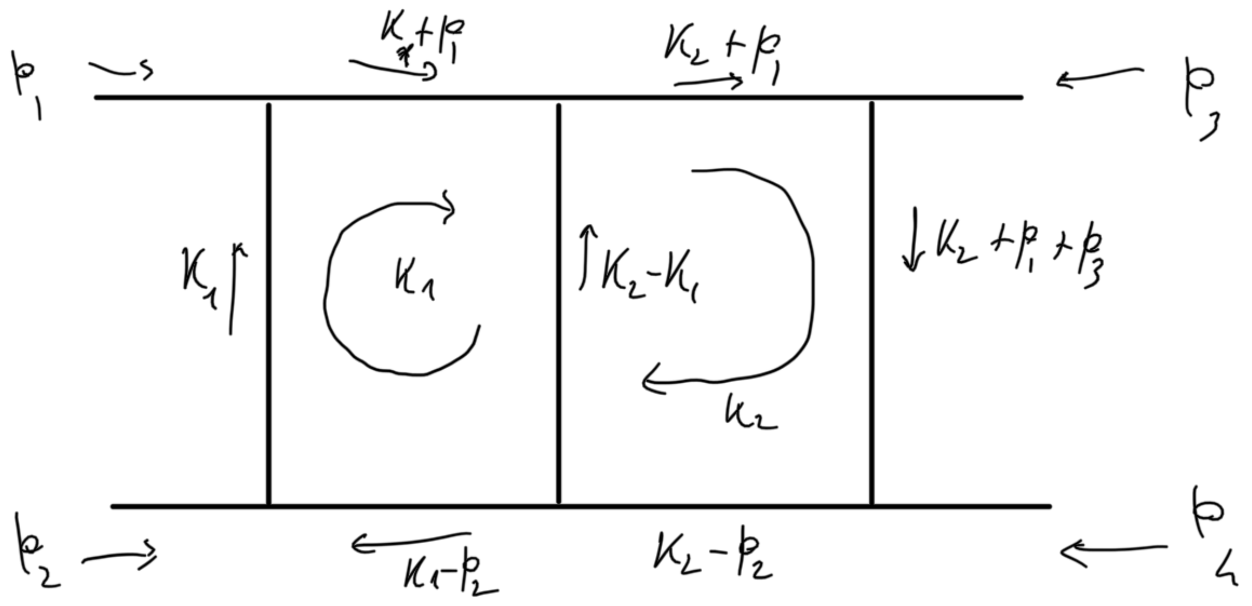
Abiss Mathematica package

The Feynman Integrals $\mathcal{I}_i$ are one of the major challenges in the evaluation of the virtual corrections

$$\mathcal{I}(p_i\cdot p_j;\vec{m}) = \int\frac{d^nk_1}{(2\pi)^n}\int\frac{d^nk_2}{(2\pi)^n}\frac{1}{[k_1^2-m_0^2]^{\alpha_0}\,[(k_1+p_1)^2-m_1^2]^{\alpha_1}\ \ldots\ [(k_1+k_2+p_j)^2-m_j^2]^{\alpha_j}\ \ldots\ [(k_2+p_l)^2-m_l^2]^{\alpha_l}}$$



The complexity of the solution grows with the number of energy scales (masses and invariants) upon which it depends

The double virtual amplitude: the Master Integrals

The complexity of the MIs depends on the number of energy scales

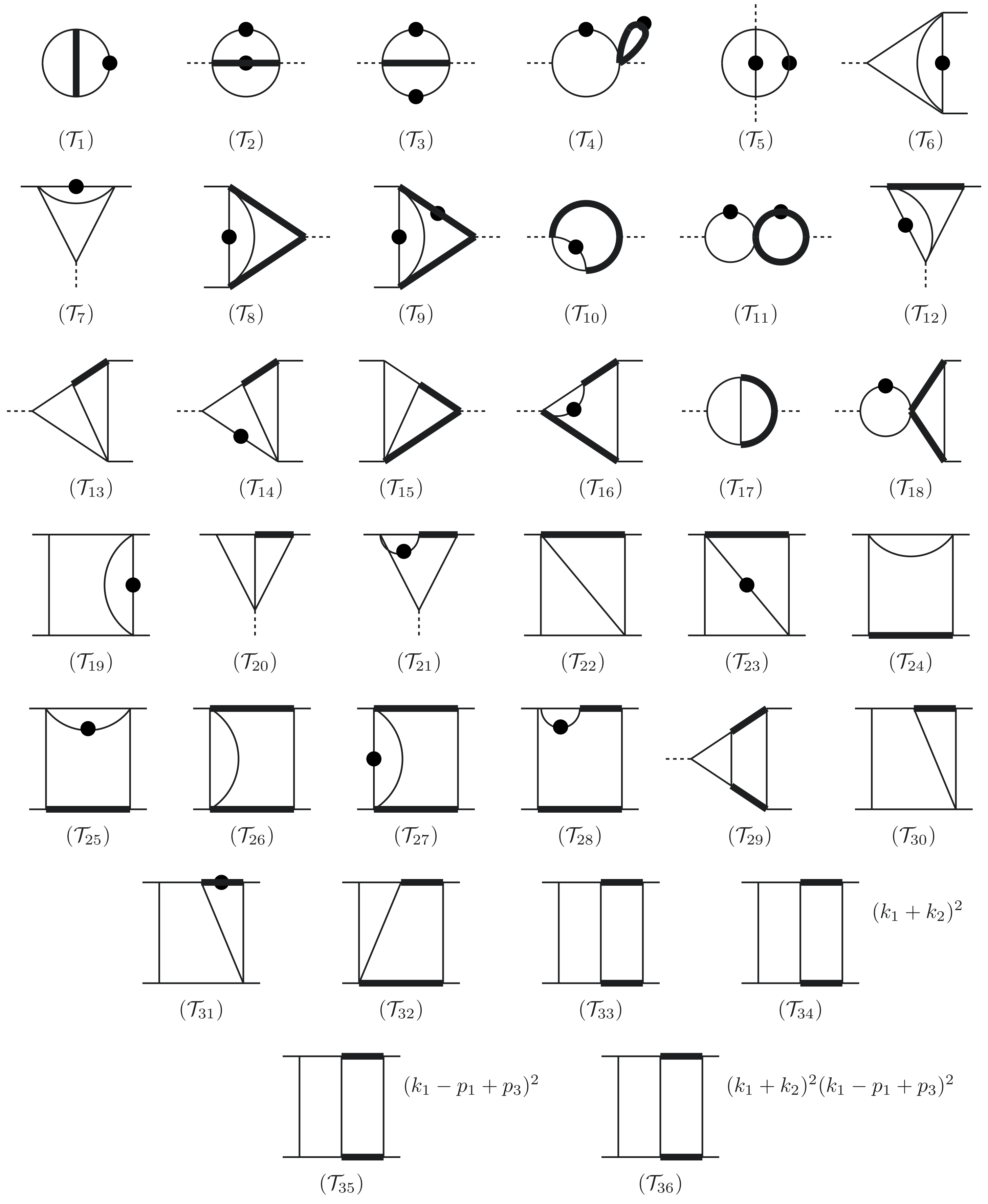
NNLO QCD-EW corrections to NC and CC Drell-Yan feature 0, 1, or 2 internal massive lines
dependence on 2 kinematical invariants (s,t)

NNLO EW corrections to NC Drell-Yan feature up to 5 internal massive lines (2 distinct masses, external fermions massless)

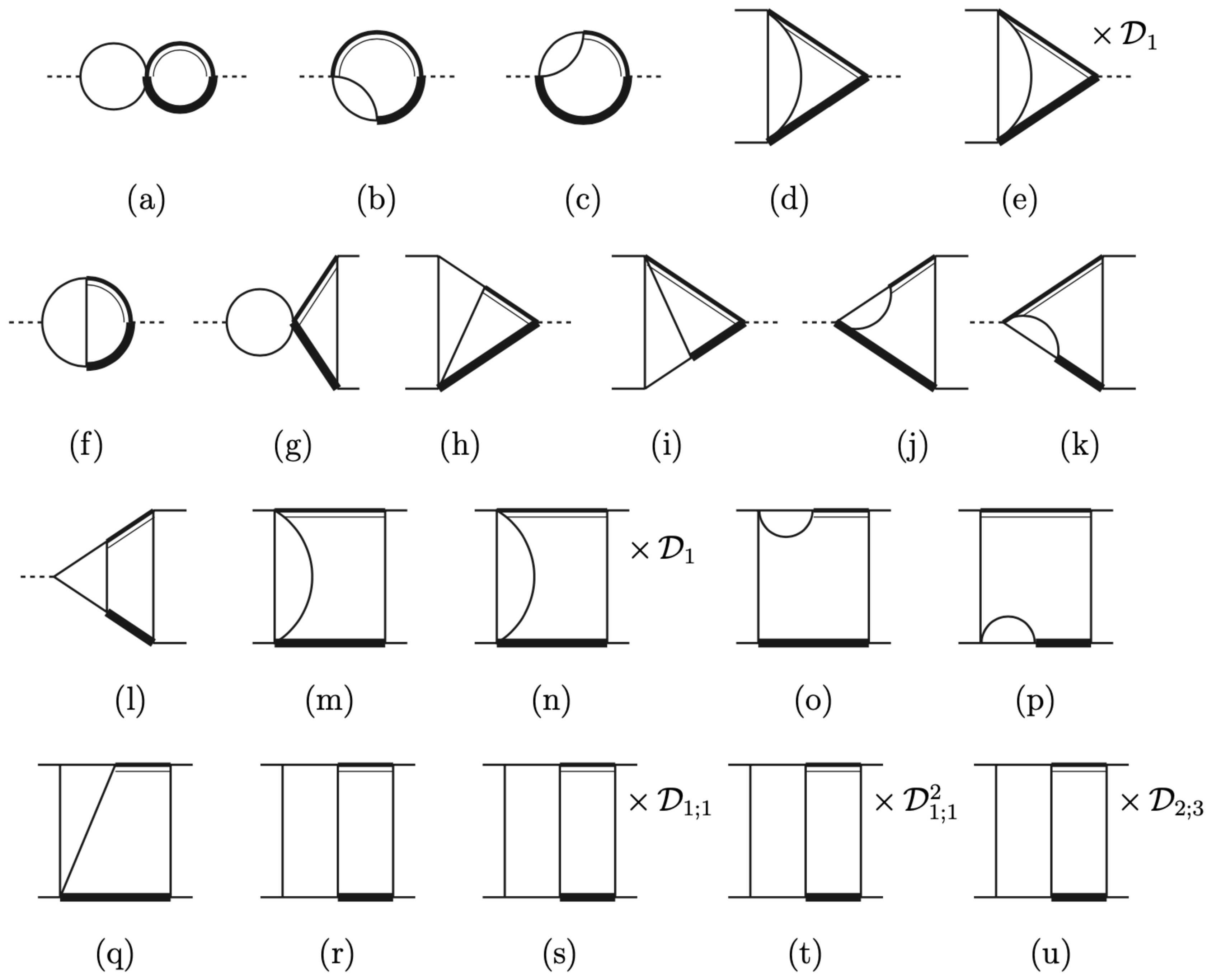
2-loop virtual QCD-EW corrections to NC and CC DY: Master Integrals with 2 massive lines

Neutral-Current DY

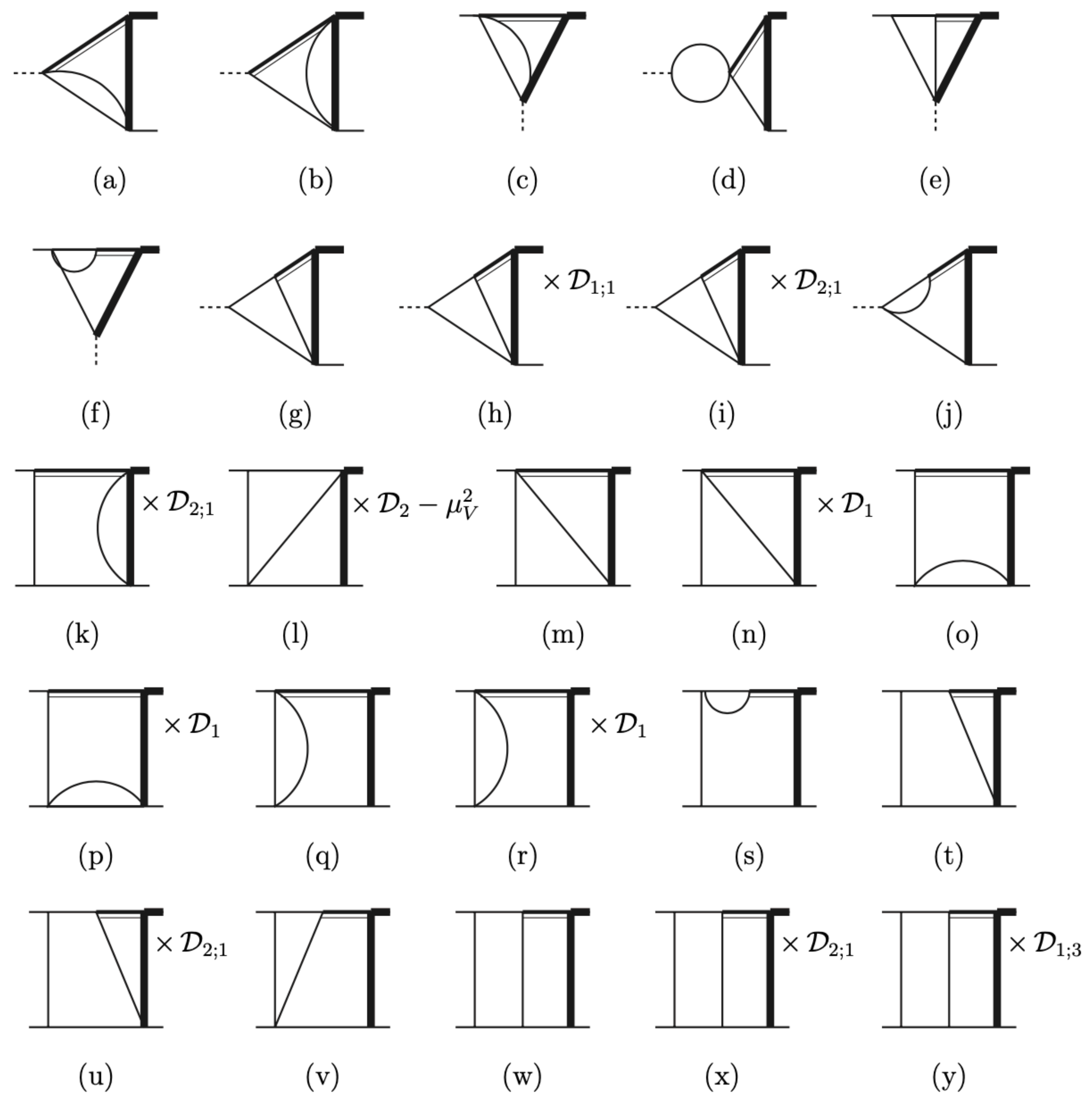
Charged-Current DY



Master Integrals with two equal mass internal lines



Master Integrals with two different internal masses



Master Integrals with one W and one internal massive lepton lines

2-loop virtual QCD-EW corrections to NC and CC DY: Master Integrals with 2 massive lines

subset	# Master Integrals	# Integral families	#MIs in the largest integral family
NC DY @ NNLO QCD-EW	401	16	36
CC DY @ NNLO QCD-EW	274	11	53
NC DY @ NNLO EW	3245	56	148

the number of Master Integrals in one family sets the computational complexity (potentially coupled quantities)

The double virtual amplitude: the Master Integrals

The complexity of the MIs depends on the number of energy scales

NNLO QCD-EW corrections to NC and CC Drell-Yan feature 0, 1, or 2 internal massive lines
dependence on 2 kinematical invariants (s,t)

NNLO EW corrections to NC Drell-Yan feature up to 5 internal massive lines (2 distinct masses, external fermions massless)

The solution of the MIs can be obtained with different approaches (depending on the problem complexity)

Solution of the MIs differential equations:

solution in closed form in terms of special functions GINAC
GPLs, elliptic polylogs, elliptic functions
→ availability of the relevant power series
→ arbitrary precision

solution via power expansions DiffExp
→ full analytical control but no functional relations SeaSyde
AMFlow

MIs direct numerical integration:

sector decomposition allows to reorganise the integration PySecDec
→ great flexibility but reduced numerical precision compared to the other approaches
→ sufficient for gauge cancellations?

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations (cfr `DiffExp` by M.Hidding, arXiv:2006.05510).
The MIs are replaced by formal series with unknown coefficients → algebraic eqs for the unknown coefficients of the series.

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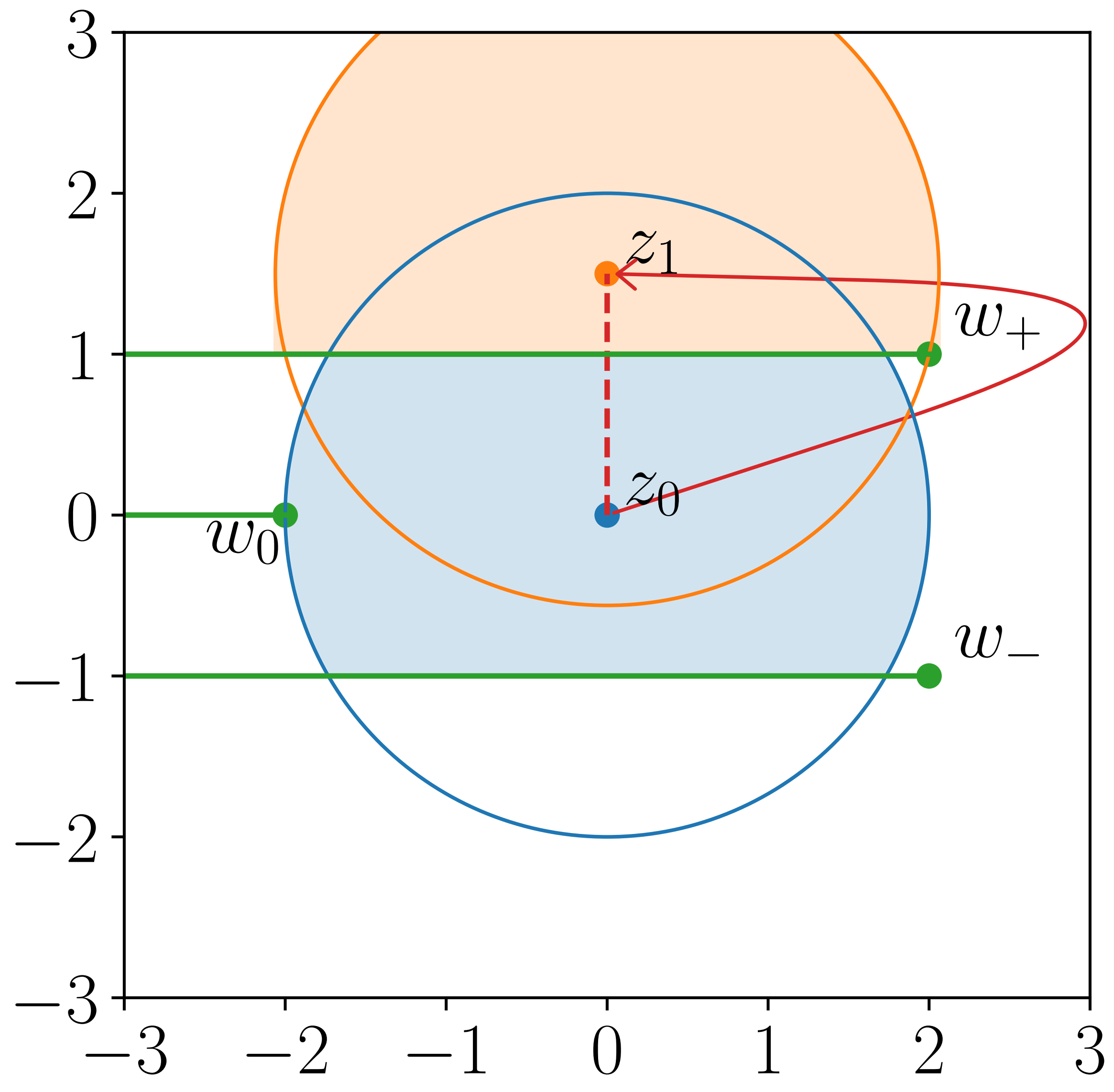
We need complex-valued masses of W and Z bosons (unstable particles) → `SeaSyde`

Complete knowledge about the singular structure of the MI
can be read directly from the differential equation matrix

The analytic continuation is unambiguously under control,
working in the complex plane of each kinematical variable, one variable at a time



Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations (cfr `DiffExp` by M.Hidding, arXiv:2006.05510).
The MIs are replaced by formal series with unknown coefficients → algebraic eqs for the unknown coefficients of the series.

We need complex-valued masses of W and Z bosons (unstable particles) → `SeaSyde`

Complete knowledge about the singular structure of the MI
can be read directly from the differential equation matrix

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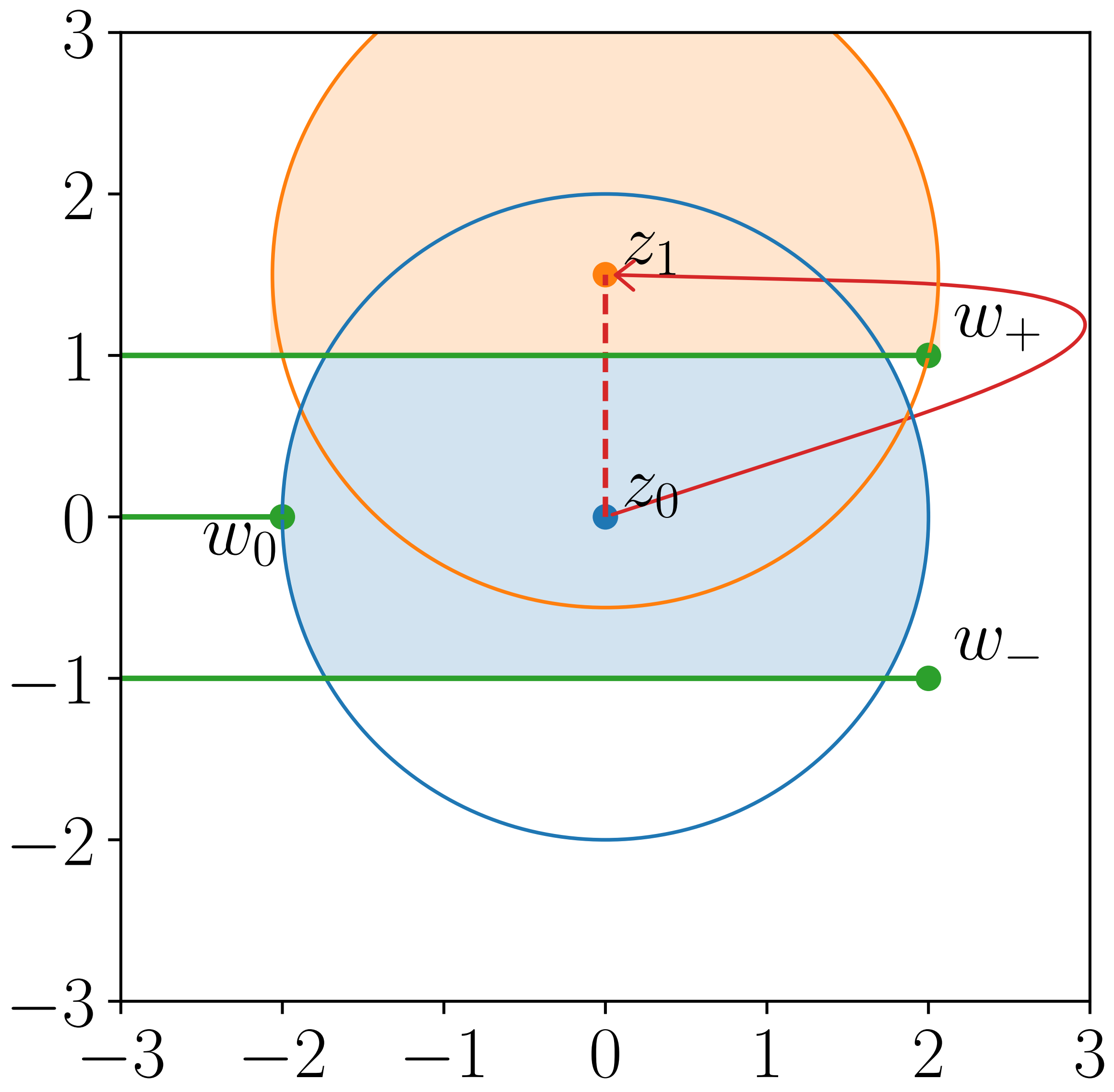
The solution can be computed with an arbitrary number of significant digits,
but not in closed form → semi-analytical



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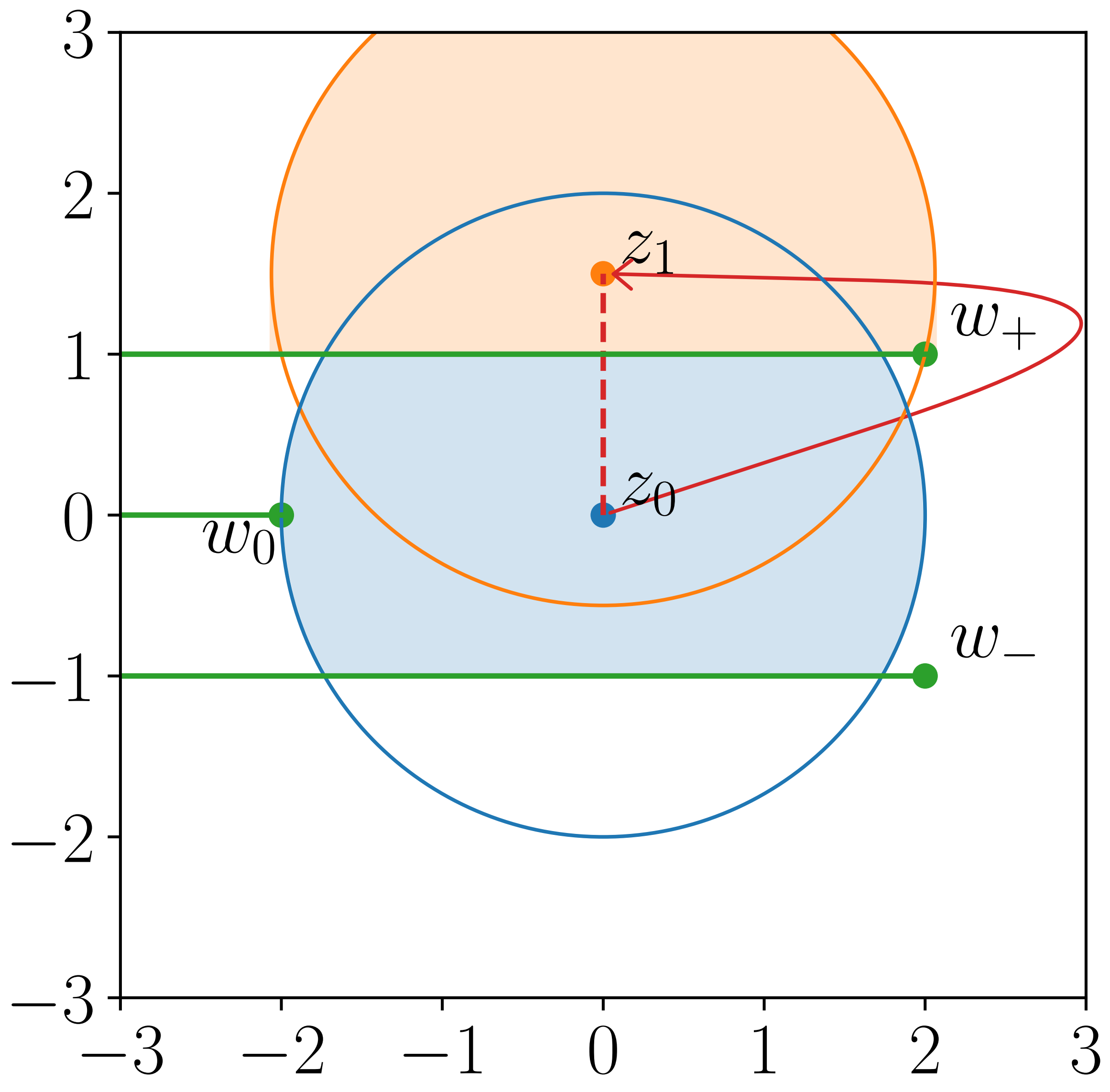

Pros: fully general approach valid for an arbitrary loop integral

Issues with increasing number of MIs:
- writing the differential equations
- evaluation time (size of the system + length of each matrix element)

The double virtual amplitude: the Master Integrals

The complexity of the MIs depends on the number of energy scales

NNLO QCD-EW corrections to NC and CC Drell-Yan feature 0, 1, or 2 internal massive lines
dependence on 2 kinematical invariants (s,t)

NNLO EW corrections to NC Drell-Yan feature up to 5 internal massive lines (2 distinct masses, external fermions massless)

The solution of the MIs can be obtained with different approaches (depending on the problem complexity)

Solution of the MIs differential equations:

solution in closed form in terms of special functions GINAC
GPLs, elliptic polylogs, elliptic functions
→ availability of the relevant power series
→ arbitrary precision

solution via power expansions DiffExp
→ full analytical control but no functional relations SeaSyde
AMFlow

MIs direct numerical integration:

sector decomposition allows to reorganise the integration PySecDec
→ great flexibility but reduced numerical precision compared to the other approaches
→ sufficient for gauge cancellations?

The evaluation time of the problem grows with the number of energy scales

the number of coupled MIs, each with increasing complexity, determines a longer evaluation time

Evaluation timings

The interference term $2\mathrm{Re}\langle \mathscr{M}^{(1,1),fin} | \mathscr{M}^{(0,0)} \rangle$ contributes to the hard function $H^{(1,1)}$

After the subtraction of all the universal IR divergences, it is a finite correction

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Single phase-space point evaluation

from O(15 s) for NC DY @ NNLO QCD+EW (optimised diff.eqs. systems)
to O(600 s) for CC DY @ NNLO QCD+EW (non optimised choice of MIs → generic diff.eqs. systems)
to O(#### s) for NC DY @ NNLO EW

The evaluation "on-the-fly" is not affordable in MC simulations → numerical grid + interpolation

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Numerical grids

for DY @ NNLO QCD-EW, sampling on NLO results, a 3250-points grid in (s,t) is sufficient for

- interpolation with excellent accuracy
- negligible evaluation time in MC simulations

Larger phase-space (e.g. $t\bar{t}H$ production) have more kinematical variables (extra factor to the total evaluation time)

Licensing (Wolfram) is becoming an issue for massive distributed evaluations

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Precision phenomenology

In CC DY, one grid requires O(3 weeks) to be prepared: too long! e.g. if we need O(100) templates for MW studies
→ exploit analytical properties

Fast numerical evaluation with arbitrary W-mass values

Compute once, for a given value $\overline{m}_W$ of the W boson mass, the numerical grid $\mathscr{M}^{(1,1)} = \mathscr{M}^{(1,1)}(\overline{m}_W)$

To determine $\mathscr{M}^{(1,1)} = \mathscr{M}^{(1,1)}(m_W)$

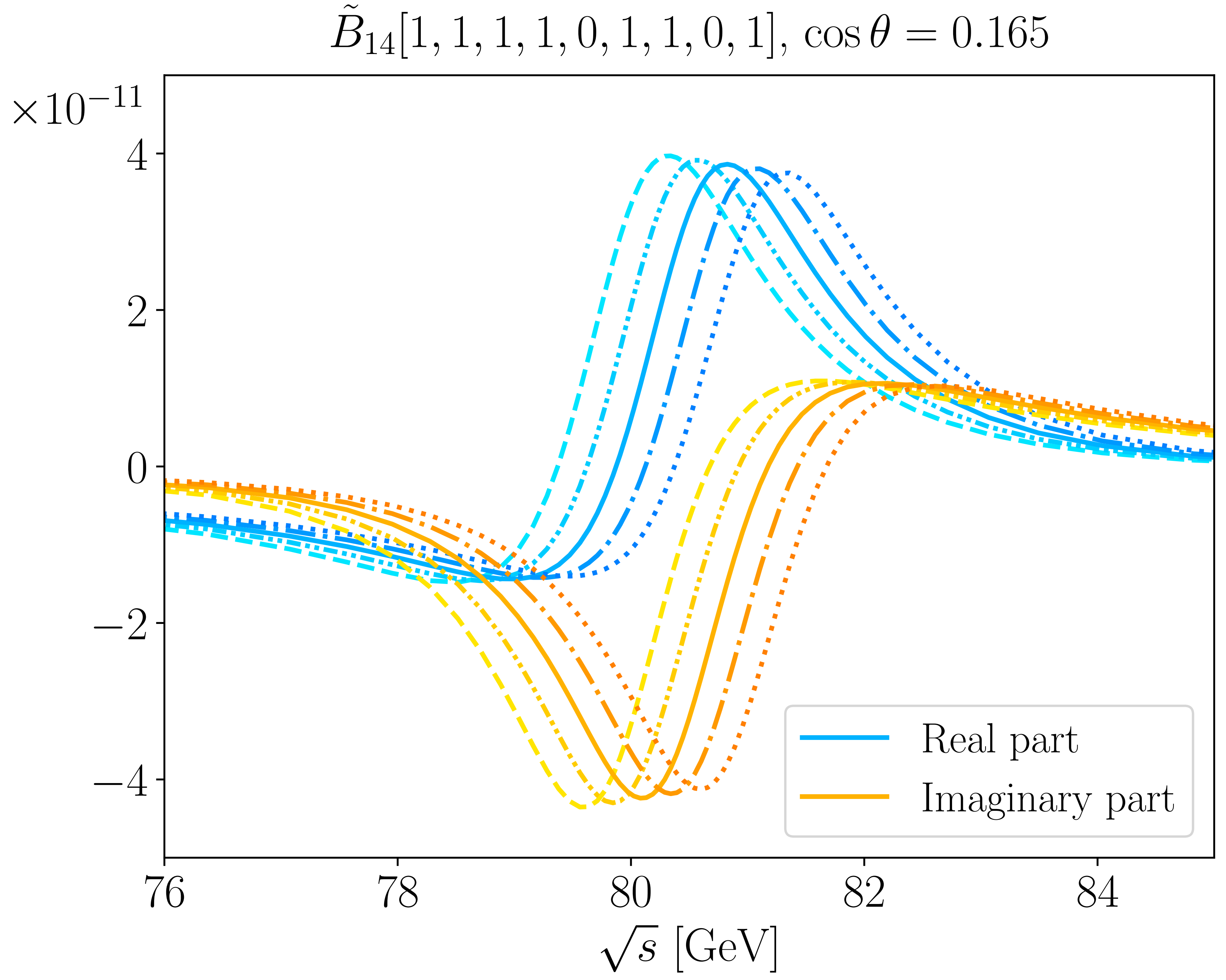
solving the differential equations w.r.t. m_W ,

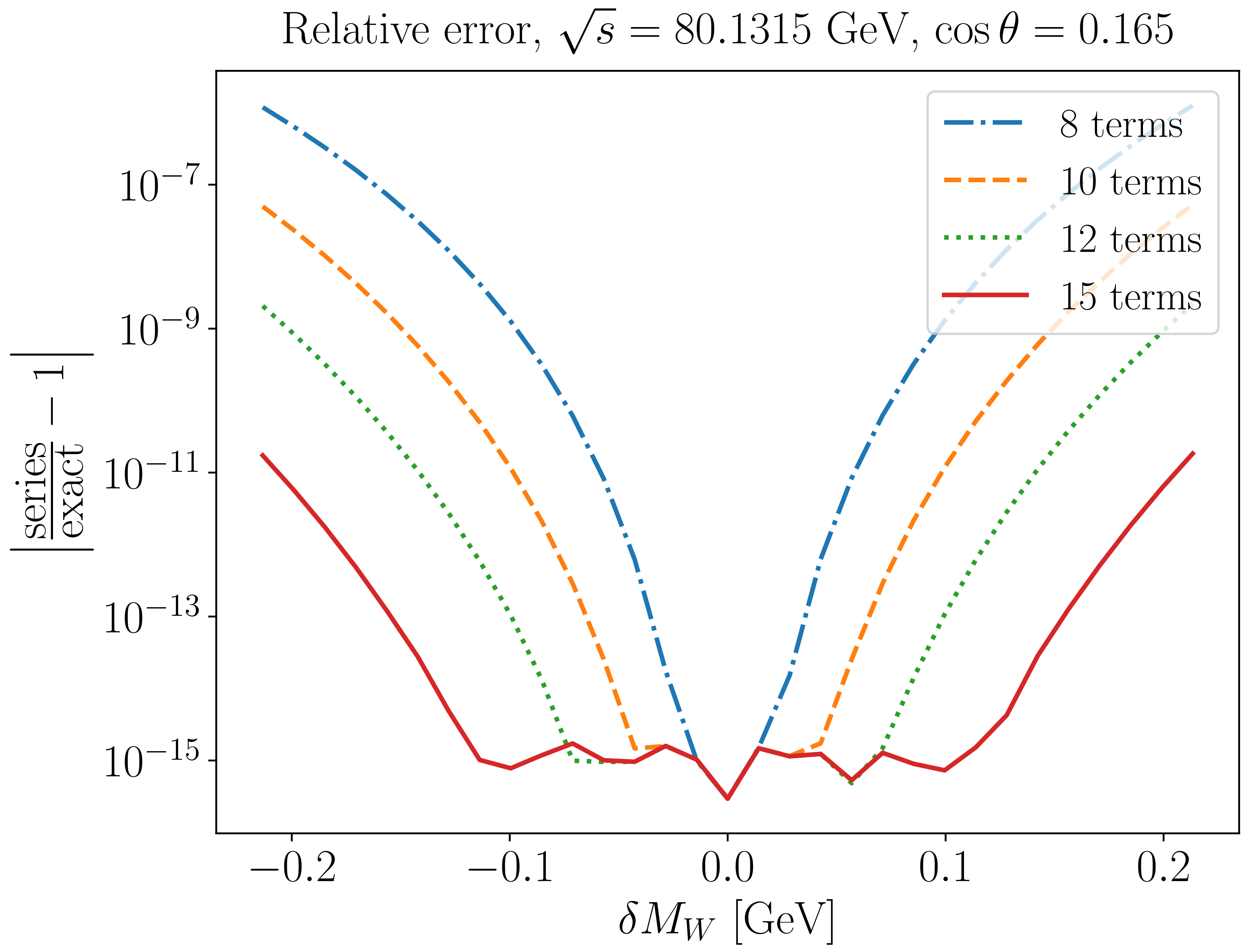
the first grid $\mathscr{M}^{(1,1)} = \mathscr{M}^{(1,1)}(\overline{m}_W)$ is the boundary condition

The solution is cast as a "symbolic grid" with 3250 power series in $\delta m_W = m_W - \overline{m}_W$,

For a generic m_W choice,

the actual numerical grid is evaluated in negligible time and available for simulation

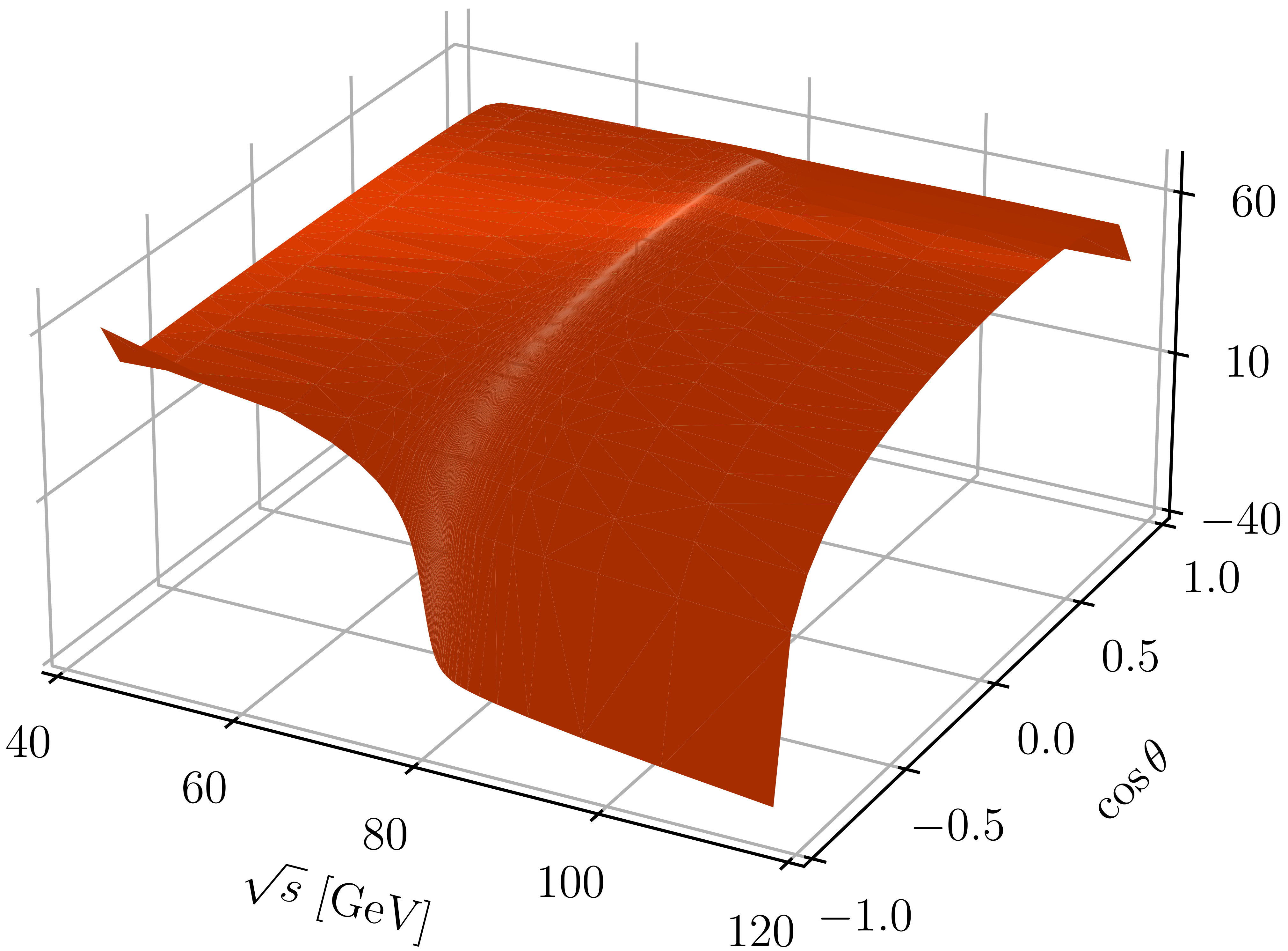


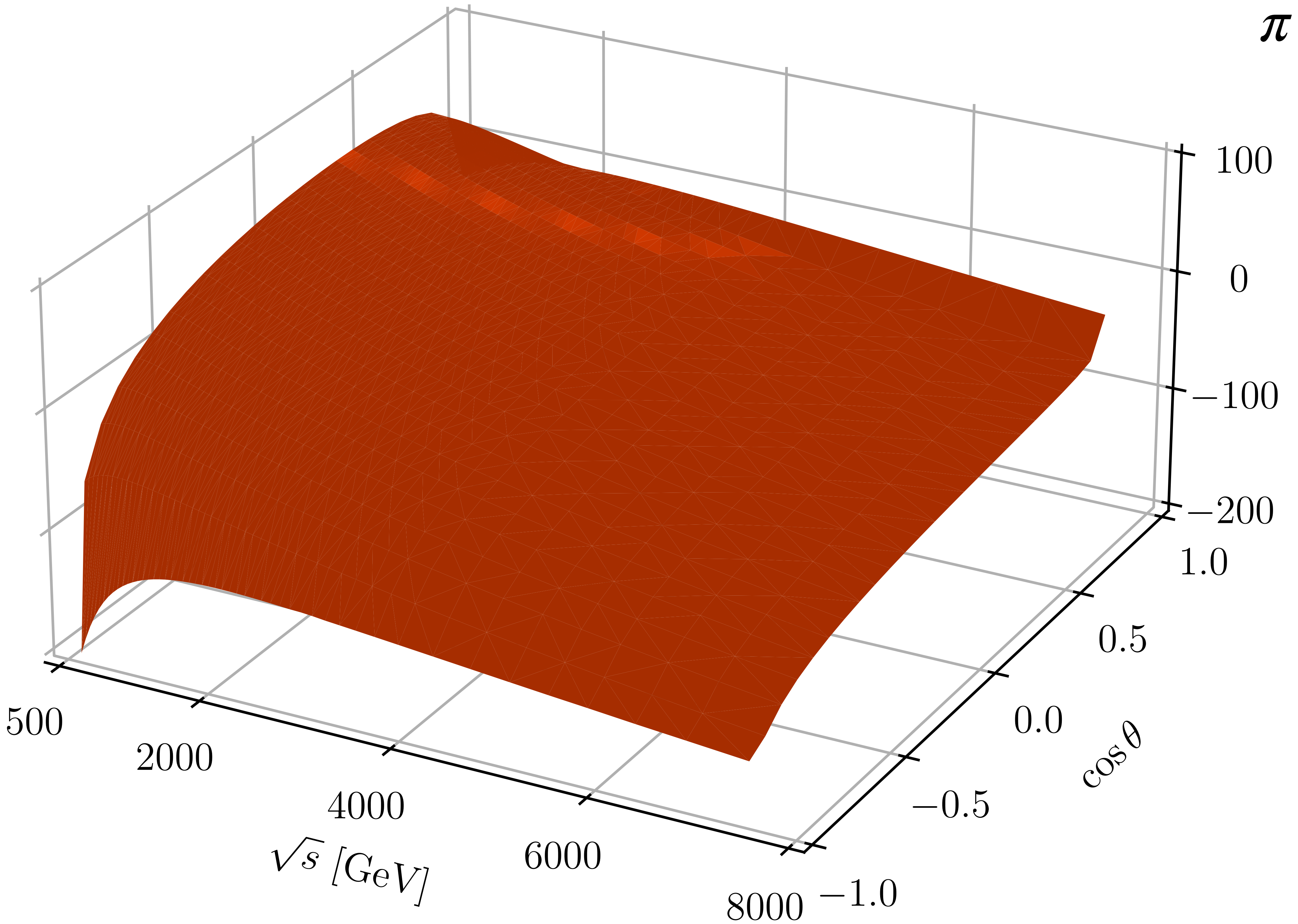


Finite 2-loop exact QCD-EW virtual corrections to Charged-Current Drell-Yan

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, arXiv:2405.00612

in units $\frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\sigma_0$





- Ready for MC simulation

- Expected large effects at large transverse masses, analogously to the NC DY case
- Improved theoretical stability in PDFs determination at (sub)percent level
- Relevance in the discussion of the *W* resonance region, when matching fixed-order and QCD-QED resummation → m_W fit

Conclusions

Precision

- The NNLO (QCD + QCDxEW + EW) corrections are needed to match the final HL-LHC precision

Steady progress is pushing the frontier of NNLO calculations from QCD-EW to full EW

These results will be the core of the calculations needed at the FCC-ee
to describe fermion-pair production in the whole energy range

The Standard Model benchmark

- The availability of these corrections will establish the SM benchmark with precision comparable to the data

→ increase the significance of an observed deviation, as a function of energy → relevant to SMEFT studies

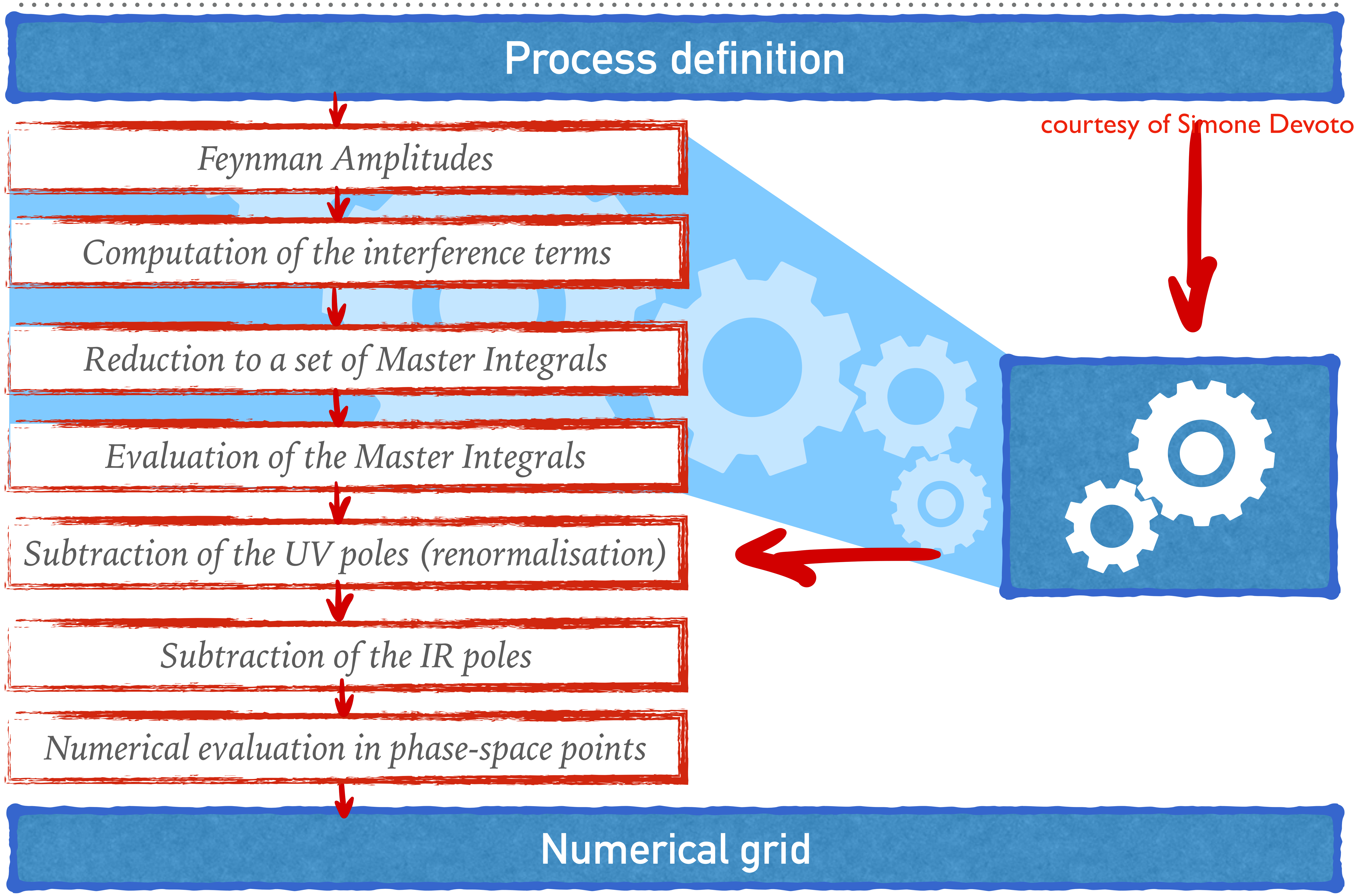
The computational burden

- Precision phenomenology requires:
  - significant computational resources to achieve the necessary precision level
  - renewed mathematical effort to simplify the representation of the problem
  - efficient multidimensional interpolation techniques (is ML at this precision level?)

Thank you

Towards the NNLO-EW corrections to $\sigma(f\bar{f} \to \mu^+\mu^- + X)$

STRUCTURE OF A LOOP COMPUTATION



Process definition

Feynman Amplitudes

Computation of the interference terms

Reduction to a set of Master Integrals

Evaluation of the Master Integrals

Subtraction of the UV poles (renormalisation)

Subtraction of the IR poles

Numerical evaluation in phase-space points

Numerical grid

courtesy of Simone Devoto

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

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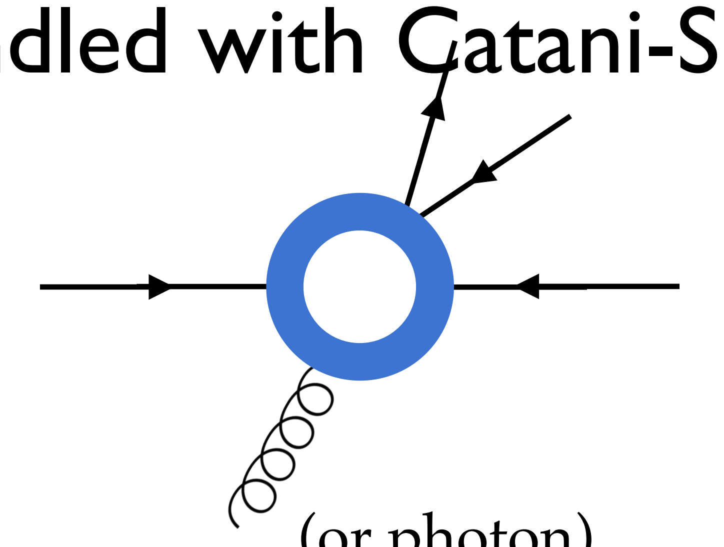
(Catani, Torre, Grazzini, 2014, Buonocore,Grazzini,Tramontano 2019.)

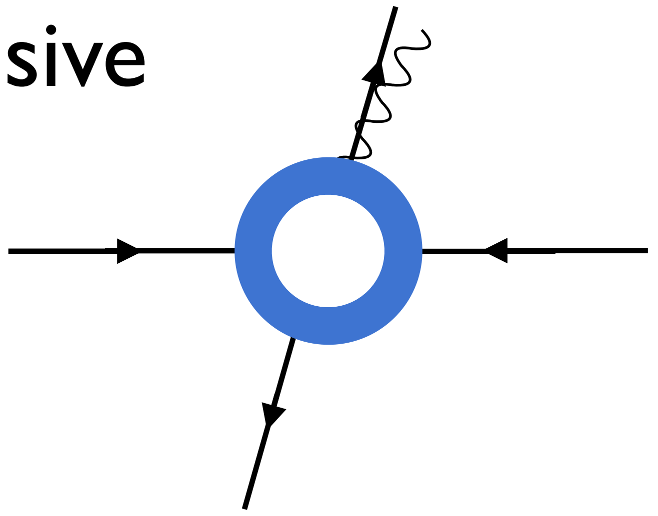
the gauge-boson phase space is split into $q_T = 0$ and $q_T > 0$ regions

$r_{cut} = q_T^{cut}/Q$

for ISR, if $q_T > 0$ the emitted parton is always resolved
and the process under study receives only NLO corrections
which can be handled with Catani-Seymour dipoles



in the FSR case, with $q_T > 0$,
the emitted parton is always resolved
only if the emitter is massive



the final state consists of a pair of massive leptons (treated as bare) to regulate the collinear (mass) singularities

The q_T-subtraction and the residual cut-off dependency

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

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When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

Logarithmic sensitivity on r_{cut} in the double unresolved limit $\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m)$

The counterterm removes the IR sensitivity to the cutoff variable $\int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right) \sim c_0 + \mathcal{O}(r_{cut}^m)$

→ we need small values of the cutoff

→ explicit numerical tests to quantify the bias induced by the cutoff choice (cfr. Buonocore, Kallweit, Rottoli, Wiesemann, arXiv:2111.13661
Camarda, Cieri, Ferrera, arXiv:2111.14509)

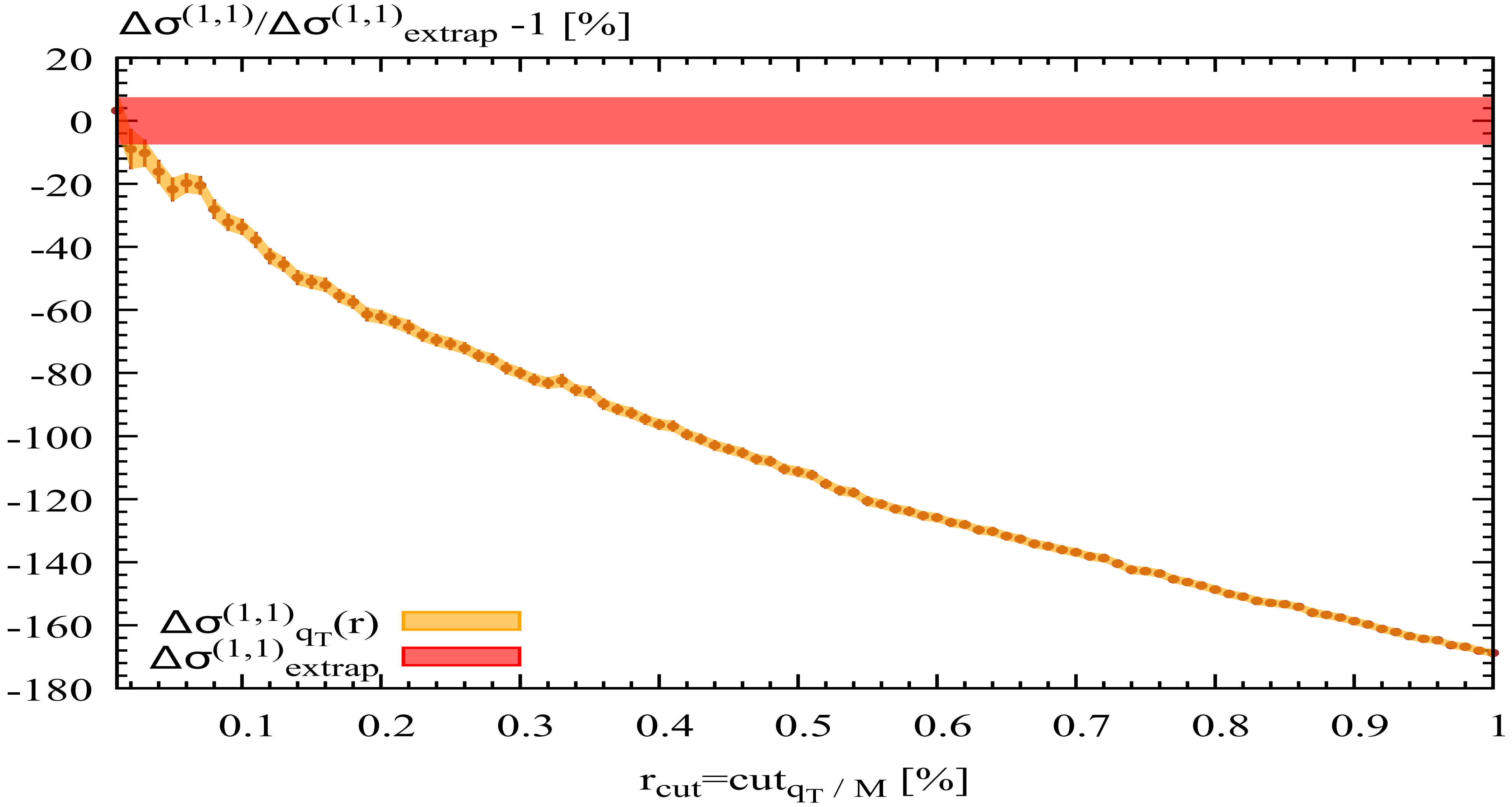
we can fit the r_{cut} dependence and extrapolate in the $r_{cut} \to 0$ limit

Dependence on r_{cut} of the NNLO QCD-EW corrections to NC DY

courtesy of S.Kallweit

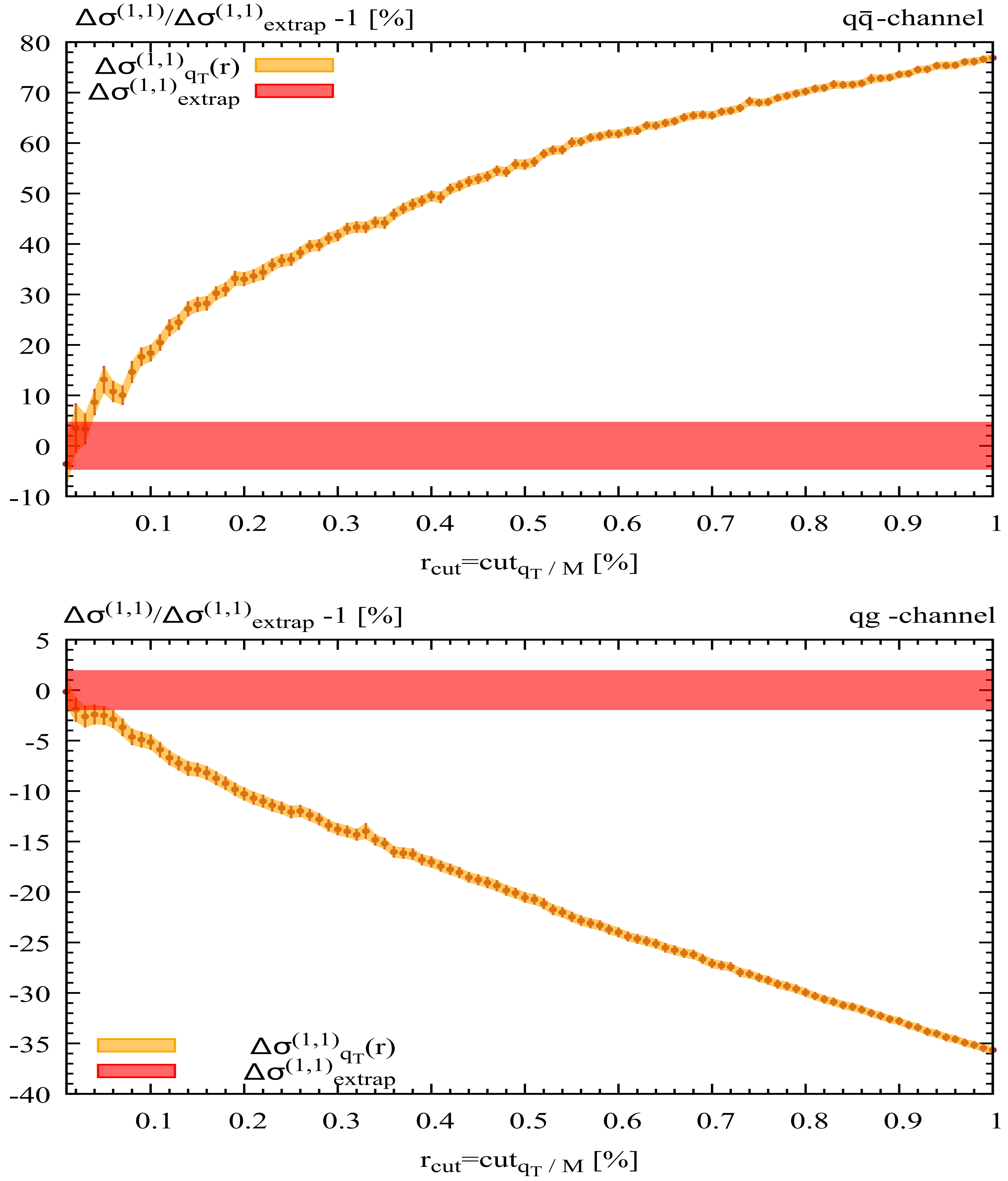
Symmetric-cut scenario

$p_{T,\ell^\pm} > 25$ GeV $\quad y_{\ell^\pm} < 2.5 \quad m_{\ell\ell} > 50$ GeV



- **large power corrections in r_{cut} for mixed corrections**
  - ➥ explained by overall small size of corrections, and in parts also by cancellation between partonic channels
- **by far less dramatic dependence at level of cross sections**
  - ➥ better than permille precision at inclusive level

Splitting into partonic channels



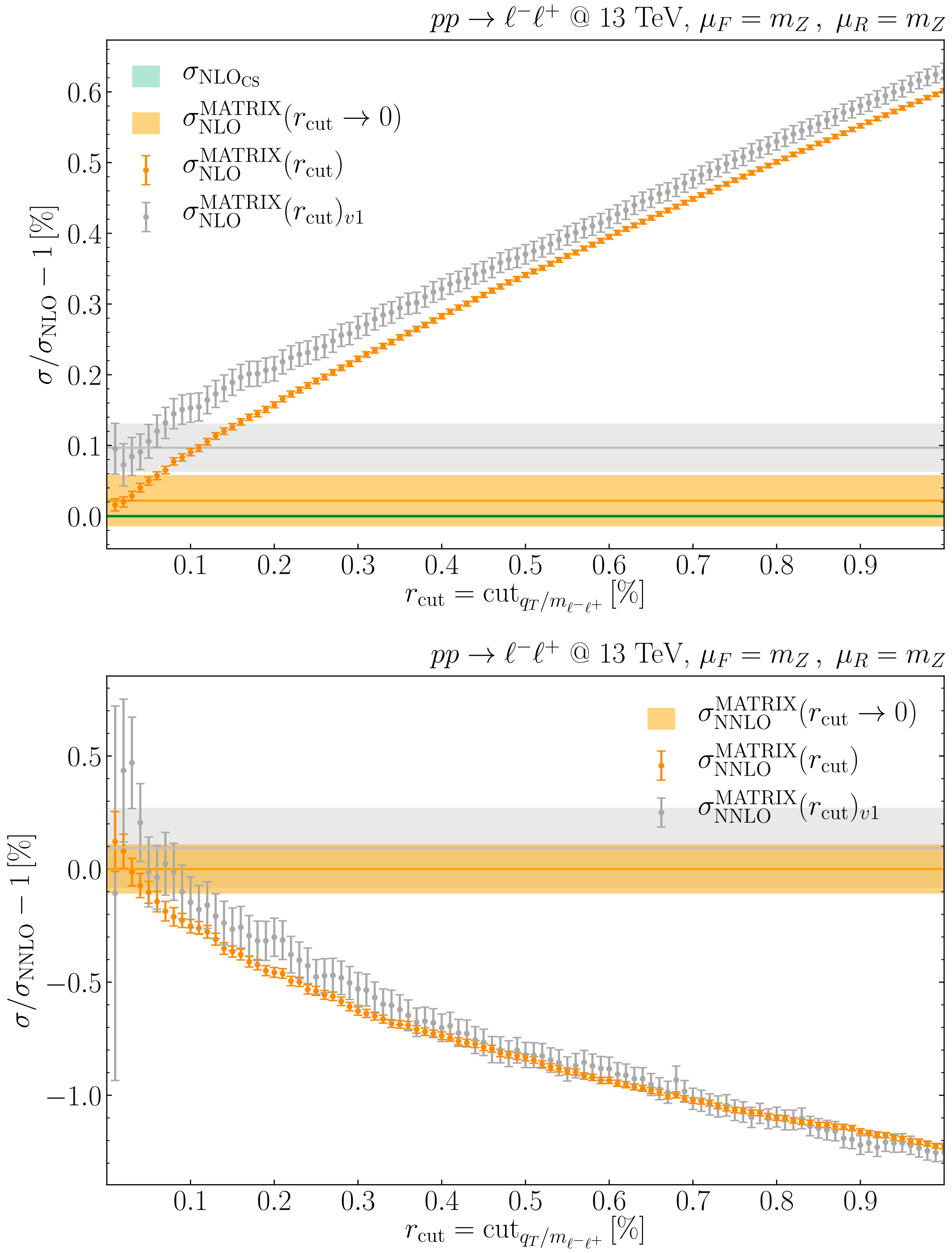
The q_T-subtraction and the residual cut-off dependency in different acceptance setups

courtesy of S.Kallweit

(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, 2111.13661)

Symmetric cuts

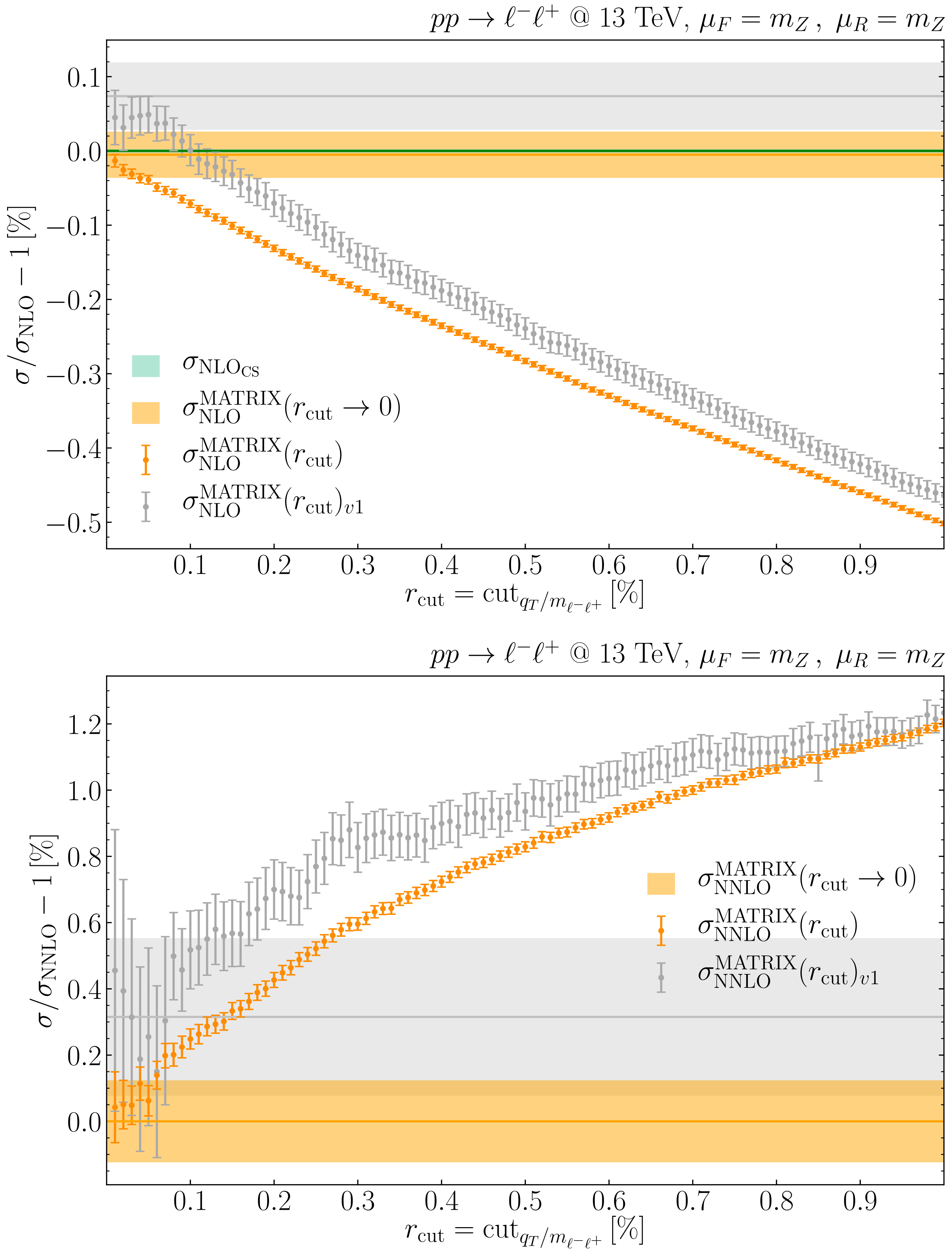
- $p_{\mathrm{T},\ell^\pm} > 25$ GeV



➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ_1 and ℓ_2

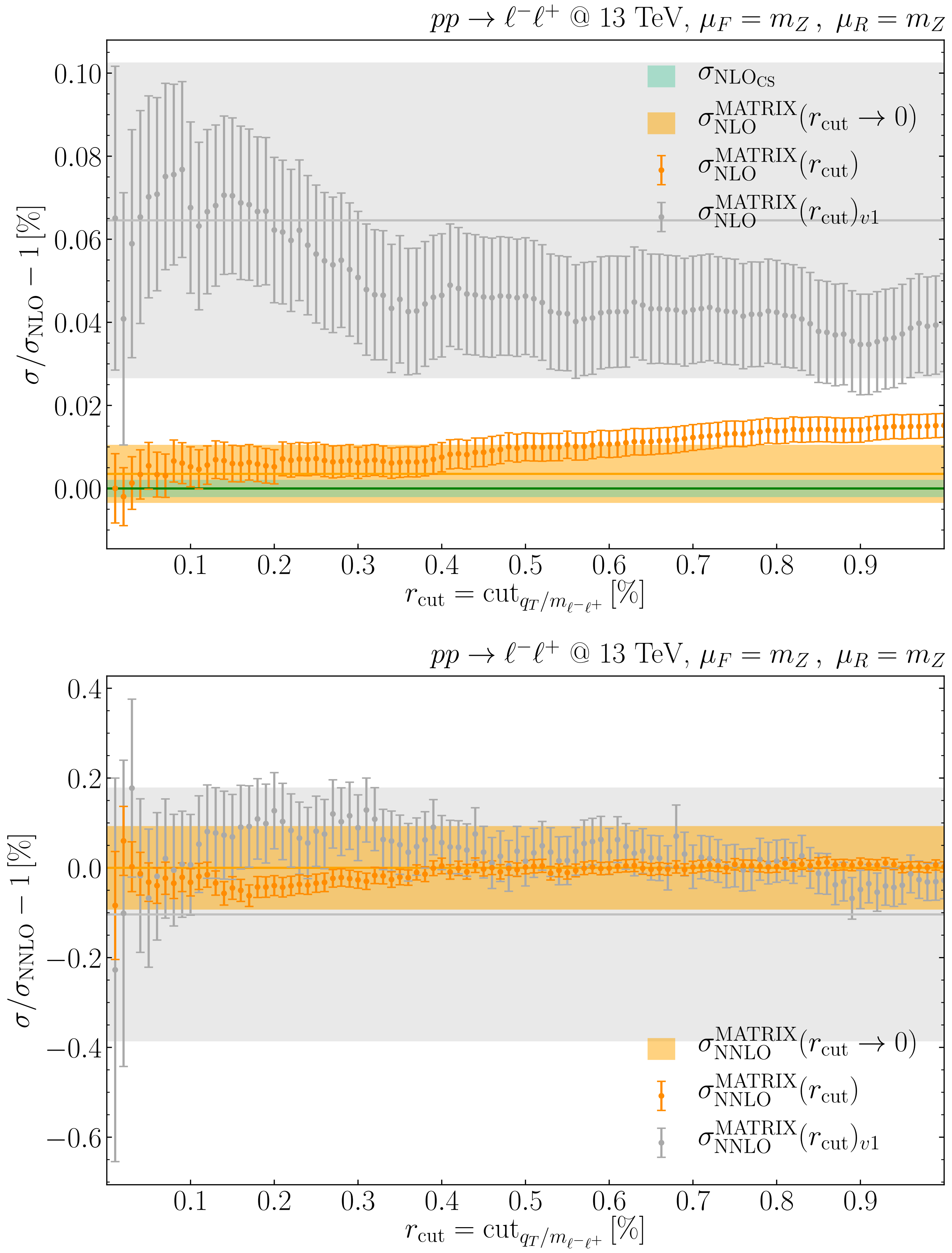
- $p_{\mathrm{T},\ell_1} > 25$ GeV $p_{\mathrm{T},\ell_2} > 20$ GeV



➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ^+ and ℓ^-

- $p_{\mathrm{T},\ell^+} > 25$ GeV $p_{\mathrm{T},\ell^-} > 20$ GeV

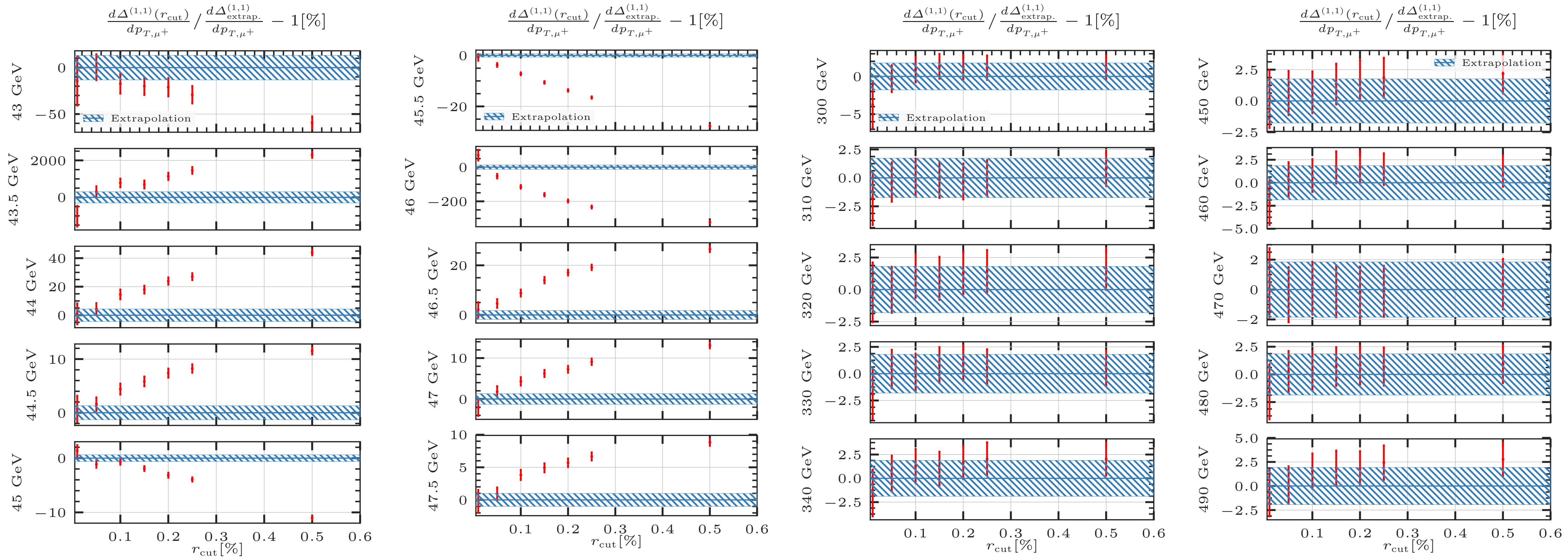


➥ no significant dependence on r_{cut}

Differential sensitivity to r_{cut}

Binwise r_{cut} dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

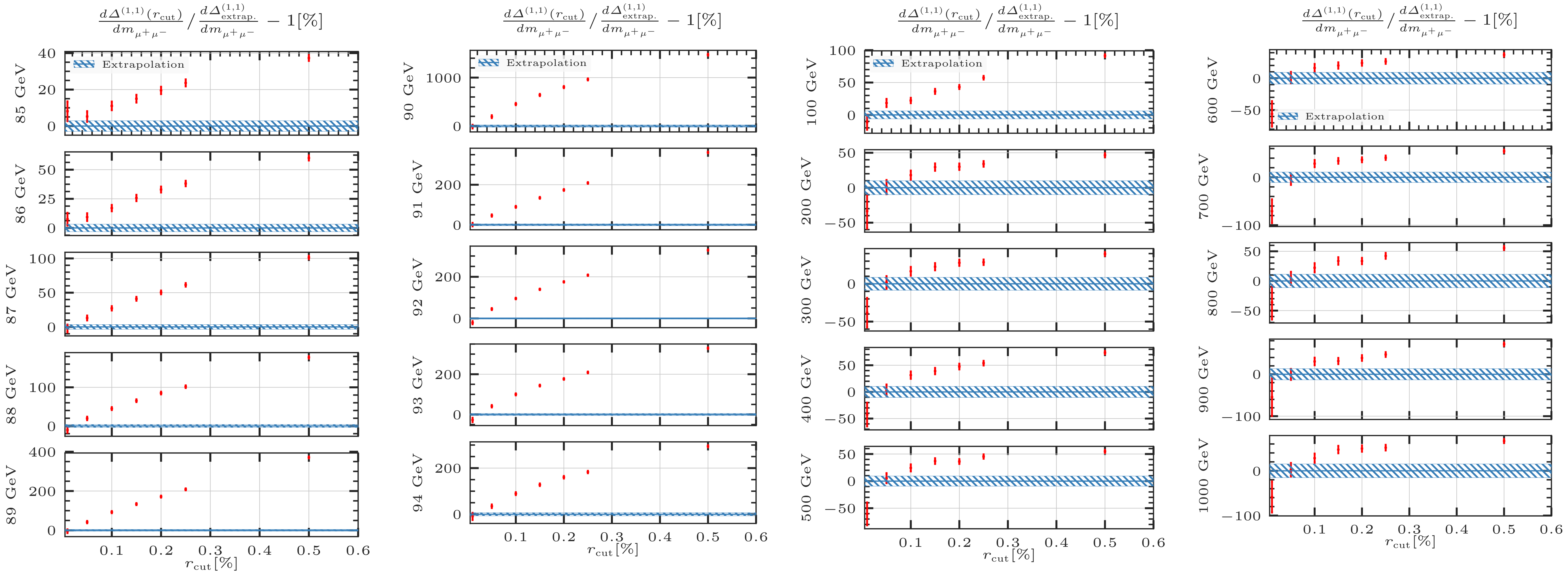
Differential distribution in p_{T,μ^+}: peak (left panels) and tail (right panels) regions



- large r_{cut} dependence in particular around the peak of the distribution, and typically precision of $\lesssim$ 3% on the relative mixed QCD–EW corrections (artificially large where corrections are basically zero)

Binwise r_{cut} dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

Differential distribution in $m_{\mu^+\mu^-}$: peak (left panels) and tail (right panels) regions



- quite large r_{cut} dependence throughout, and lower numerical precision of $\lesssim$ 10% on the relative mixed QCD–EW corrections (but still permille-level precision at the level of cross sections)

Subtraction of the IR divergences from the 2-loop amplitude

we identify QCD-QED (poles up to $1/\varepsilon^4$) and QCD-weak (poles up to $1/\varepsilon^2$ with cumbersome coefficients) diagrams

$$|\mathcal{M}^{(1,0),fin}\rangle = |\mathcal{M}^{(1,0)}\rangle - \mathcal{I}^{(1,0)}|\mathcal{M}^{(0)}\rangle\,,$$

standard NLO-QCD subtraction

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{I}^{(0,1)}|\mathcal{M}^{(0)}\rangle\,.$$

NLO-EW subtraction, with massive leptons

$$|\mathcal{M}^{(1,1),fin}\rangle = |\mathcal{M}^{(1,1)}\rangle - \mathcal{I}^{(1,1)}|\mathcal{M}^{(0)}\rangle - \tilde{\mathcal{I}}^{(0,1)}|\mathcal{M}^{(1,0),fin}\rangle - \tilde{\mathcal{I}}^{(1,0)}|\mathcal{M}^{(0,1),fin}\rangle\,.$$

$$\mathcal{I}^{(1,0)} = \left(\frac{s}{\mu^2}\right)^{-\epsilon} C_F\left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi) + \zeta_2\right)\,,$$

$$\mathcal{I}^{(0,1)} = \left(\frac{s}{\mu^2}\right)^{-\epsilon}\left[\frac{Q_u^2+Q_d^2}{2}\left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi) + \zeta_2\right) + \frac{4}{\epsilon}\Gamma_l^{(0,1)}\right]$$

$$\Gamma_l^{(0,1)} = -\frac{1}{4}\Big[Q_l^2\,(1-i\pi) + Q_l^2\log\left(\frac{m_l^2}{s}\right) + \\ + 2Q_uQ_l\log\left(\frac{(2p_1\cdot p_4)}{s}\right) - 2Q_dQ_l\log\left(\frac{(2p_2\cdot p_4)}{s}\right)\Big]$$

$$\mathcal{I}^{(1,1)} = \left(\frac{s}{\mu^2}\right)^{-2\epsilon} C_F\Big[\frac{Q_u^2+Q_d^2}{2}\left(\frac{4}{\epsilon^4} + \frac{1}{\epsilon^3}(12+8i\pi) + \frac{1}{\epsilon^2}(9-28\zeta_2+12i\pi) + + \frac{1}{\epsilon}\left(-\frac{3}{2} + 6\zeta_2 - 24\zeta_3 - 4i\pi\zeta_2\right)\right) \\ + \left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon}(3+2i\pi) + \zeta_2\right)\,\frac{4}{\epsilon}\,\Gamma_l^{(0,1)}\Big]$$

The analytical check of the cancellation of the IR poles in the QCD-weak sector is one very demanding test of the calculation.

In CC-DY for the first time we achieved a completely numerical check of the cancellation of all the IR poles

The hard-virtual coefficient

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

$$\mathscr{H}^{(1,1)} = H^{(1,1)}\, C_1\, C_2$$

The process independent collinear functions C_1, C_2 are known up to N3LO

The process dependent hard function H is defined upon subtraction of the universal IR contributions

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The process dependent hard function H is defined
upon subtraction of the universal IR contributions

$$2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(1,1)} \rangle \;=\; \sum_{k=-4}^{0} \varepsilon^k f_i(s,t,m)$$

after UV renormalisation the poles are only of IR origin

$$|\mathscr{M}_{fin}\rangle \equiv (1-I)|\mathscr{M}\rangle \qquad H \propto \langle \mathscr{M}_0 | \mathscr{M}_{fin} \rangle$$

$$H^{(1,0)} = \frac{2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(1,0)}_{fin} \rangle}{|\mathscr{M}^{(0,0)}|^2}, \qquad H^{(0,1)} = \frac{2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(0,1)}_{fin} \rangle}{|\mathscr{M}^{(0,0)}|^2}, \qquad H^{(1,1)} = \frac{2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(1,1)}_{fin} \rangle}{|\mathscr{M}^{(0,0)}|^2}$$

NLO-QCD

NLO-EW

NNLO QCD-EW

The double virtual amplitude: UV renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

Complex mass scheme

$$\mu_{W0}^2 = \mu_W^2 + \delta\mu_W^2, \quad \mu_{Z0}^2 = \mu_Z^2 + \delta\mu_Z^2, \quad e_0 = e + \delta e$$

$$\frac{\delta s^2}{s^2} = \frac{c^2}{s^2}\left(\frac{\delta\mu_Z^2}{\mu_Z^2} - \frac{\delta\mu_W^2}{\mu_W^2}\right)$$

the mass counterterms are defined
at the complex pole of the propagator

the weak mixing angle is complex valued $c^2 \equiv \mu_W^2/\mu_Z^2$

BFG EW Ward identity → cancellation of the UV divergences combining vertex and fermion WF corrections

The bare couplings of Z and photon to fermions
in the (G_μ, μ_W, μ_Z) input scheme
are given by

$$\frac{g_0}{c_0} = \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left[1 - \frac{1}{2}\Delta r + \frac{1}{2}\left(2\frac{\delta e}{e} + \frac{s^2-c^2}{c^2}\frac{\delta s^2}{s^2}\right)\right] \equiv \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left(1 + \delta g_Z^{G_\mu}\right)$$

$$g_0 s_0 = \sqrt{4\sqrt{2}G_\mu\mu_W^2 s^2}\left[1 + \tfrac{1}{2}\left(-\Delta r + 2\tfrac{\delta e}{e}\right)\right] \equiv e_{ren}^{G_\mu}\left(1 + \delta g_A^{G_\mu}\right)$$

Gauge boson renormalised propagators

$$\Sigma_{R,T}^{AA}(q^2) = \Sigma_T^{AA}(q^2) + 2\,q^2\,\delta g_A$$
$$\Sigma_{R,T}^{ZZ}(q^2) = \Sigma_T^{ZZ}(q^2) - \delta\mu_Z^2 + 2\,(q^2 - \mu_Z^2)\,\delta g_Z$$

$$\Sigma_{R,T}^{AZ}(q^2) = \Sigma_T^{AZ}(q^2) - q^2\,\frac{\delta s^2}{sc}$$
$$\Sigma_{R,T}^{ZA}(q^2) = \Sigma_T^{ZA}(q^2) - q^2\,\frac{\delta s^2}{sc},$$

After the UV renormalisation, the singular structure is entirely due to IR soft and/or collinear singularities

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$\mathrm{T}r(\gamma_\alpha \dots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \dots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \dots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$Tr(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

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- 't Hooft-Veltman treat γ_5 (anti)commuting in (4) $n - 4$ dimensions preserving the cyclicity of the traces (one counterterm is needed)
- Kreimer treats γ_5 anticommuting in n dimensions, abandoning the cyclicity of the traces (→ need of a starting point)

- Heller, von Manteuffel, Schabinger verified that the IR-subtracted squared matrix element are identical in the two approaches

- we adopted the naive anticommuting prescription (Kreimer); we use $\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$ to compute traces with one γ_5
  - we computed the 2-loop amplitude and, independently, the IR subtraction term; both depend on the prescription chosen
  - the cancellation of all the lowest order poles is checked (and non trivial)
  - absence of fermionic triangles because of colour conservation

Differential equations and IBPs

- Not all the Feynman integrals in one amplitude are independent
  → exploit Integration-by-parts (IBP) and Lorentz identities to reduce to a basis of independent Master Integrals

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_1^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\, [(k_1+p_1)^2 - m_1^2]^{\alpha_1}\, \ldots\, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j}\, \ldots\, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} \;=\; 0$$

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_2^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\, [(k_1+p_1)^2 - m_1^2]^{\alpha_1}\, \ldots\, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j}\, \ldots\, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}} \;=\; 0$$

- Henn's conjecture (2013): if a change of basis exists which leads to $d\vec{J}(\vec{s};\varepsilon) = \varepsilon \tilde{\mathbf{A}}(\vec{s}) \cdot \vec{J}(\vec{s};\varepsilon)$
  then the solution is expressed in terms of iterated integrals (Chen integral representation)
  depending only on the results at previous orders in the ε expansion

Differential equations and IBPs

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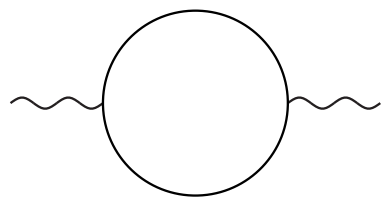
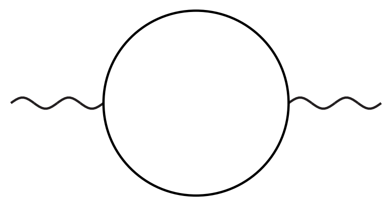
- The independent Master Integrals (MIs) satisfy a system of first-order linear differential equations with respect to each of the kinematical invariants / internal masses

When considering the complete set of MIs, the system can be cast in homogeneous form: $d\vec{I}(\vec{s};\varepsilon) = \mathbf{A}(\vec{s};\varepsilon) \cdot \vec{I}(\vec{s};\varepsilon)$

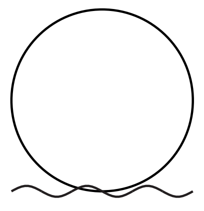
$$\frac{d}{dk^2}$$



$$+ \frac{1}{2}\left[\frac{1}{k^2} - \frac{(D-3)}{(k^2+4m^2)}\right]$$



$$= -\frac{(D-2)}{4m^2}\left[\frac{1}{k^2} - \frac{1}{(k^2+4m^2)}\right]$$



- Henn's conjecture (2013): if a change of basis exists which leads to $d\vec{J}(\vec{s};\varepsilon) = \varepsilon\tilde{\mathbf{A}}(\vec{s}) \cdot \vec{J}(\vec{s};\varepsilon)$
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Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

A Simple Example

$$\begin{cases} f'(x) + \dfrac{1}{x^2-4x+5} f(x) = \dfrac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k+r)\, c_k\, x^{(k+r-1)}$$

$$\begin{cases} rc_0 = 0 \\ \frac{1}{5}c_0 + c_1(r+1) = 0 \\ \frac{4}{25}c_0 + \frac{1}{5}c_1 + c_2(2+r) = 0 \\ \ldots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10}x^2 + \frac{11}{150}x^3 + \ldots$$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \overbrace{\frac{1}{(x'+2)} f_{hom}^{-1}(x')}^{\text{Expanded around } x'=0}$$

$$= \frac{1}{2}x - \frac{7}{40}x^2 + \frac{2}{75}x^3 + \ldots$$

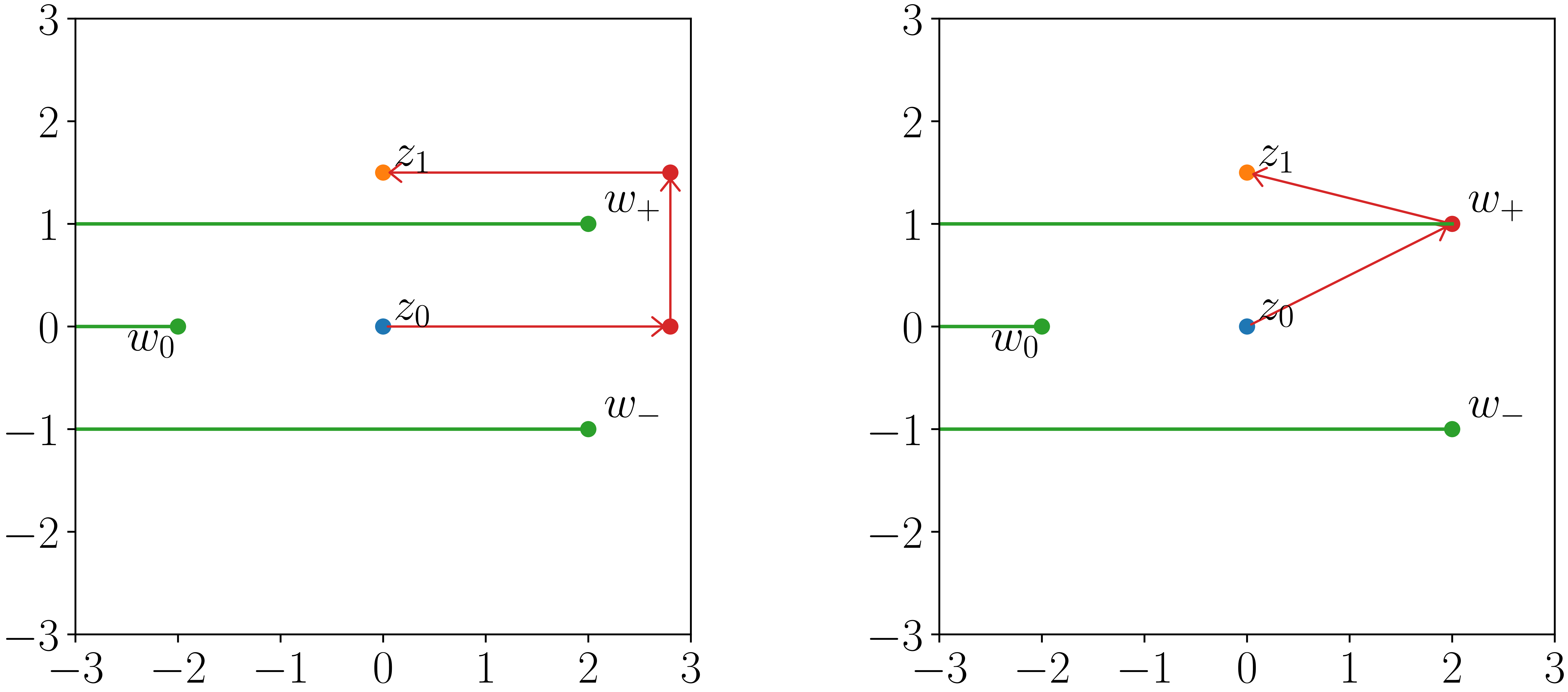
$$f(x) = f_{part}(x) + C\, f_{hom}(x)$$

$$f(0) = 1 \rightarrow C = \frac{1}{5}$$

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

- **Taylor expansion**: **avoids** the singularities;
- **Logarithmic expansion**: uses the singularities as **expansion points**.
- Logarithmic expansion has larger convergence radius but requires longer evaluation time. **We use Taylor expansion as default**.

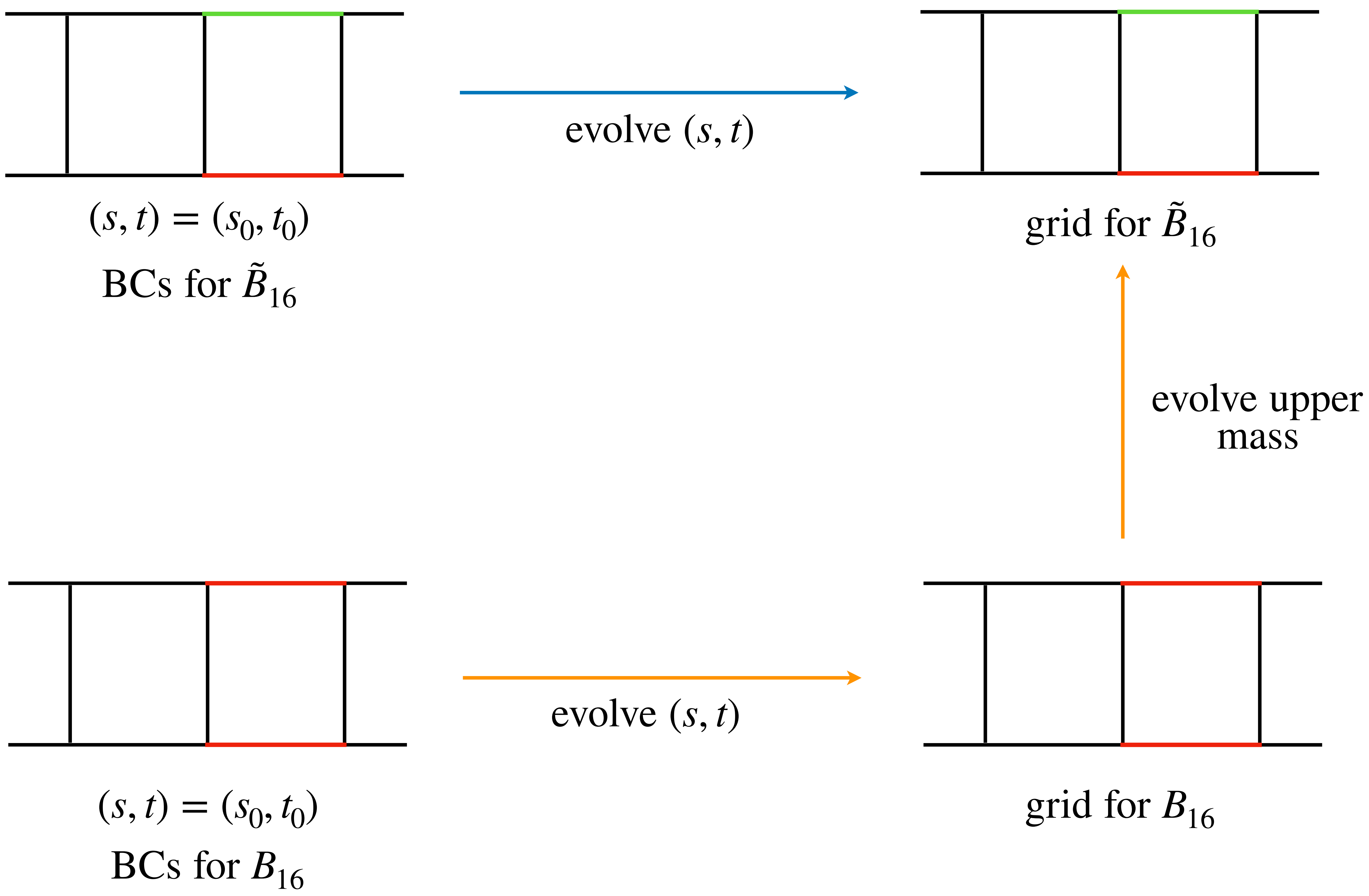


Exploiting the flexibility of the Differential Equations approach

The CC-DY Master Integrals can be evaluated with two different approaches:

- compute the BCs with AMFlow and then
  solve the differential equations in the invariants *s* and *t*

- use the results of the NC DY process as BCs
  (two equal internal masses, arbitrary *s* and *t*)
  then solve the differential equation
  in the mass parameter from (m_Z, m_Z) to (m_W, m_Z)



$(s, t) = (s_0, t_0)$
BCs for $\tilde{B}_{16}$

evolve (s, t)

grid for $\tilde{B}_{16}$

evolve upper mass

$(s, t) = (s_0, t_0)$
BCs for B_{16}

evolve (s, t)

grid for B_{16}

Perfect agreement of the two approaches

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ mathematical and theoretical developments and computation of universal building blocks

- 2-loop virtual Master Integrals with internal masses

  U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193, R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581, M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491, S.Hasan, U.Schubert, arXiv:2004.14908, M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130

- New methods to solve the Master Integrals

  M.Hidding, arXiv:2006,05510, D.X.Liu, Y.-Q. Ma, arXiv:2201.11669, T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2205.03345

- Altarelli-Parisi splitting functions including QCD-QED effects

  D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612

- renormalization

  G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

→ on-shell Z and W production as a first step towards full Drell-Yan

- pole approximation of the NNLO QCD-EW corrections

  S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016, 2401.15682

- analytical total cross section including NNLO QCD-QED and NNLO QED corrections

  D. de Florian, M.Der, I.Fabre, arXiv:1805.12214

- ptZ distribution including QCD-QED analytical transverse momentum resummation

  L. Cieri, G. Ferrera, G. Sborlini, arXiv:1805.11948

- fully differential on-shell Z production including exact NNLO QCD-QED corrections

  M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428

- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections

  R. Bonciani, F. Buccioni, R.Mondini, AV, arXiv:1611.00645, R. Bonciani, F. Buccioni, N.Rana, I.Triscari, AV, arXiv:1911.06200, R. Bonciani, F. Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections

  F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221, A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671,

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ complete Drell-Yan

- neutrino-pair production including NNLO QCD-QED corrections
  L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315
- 2-loop NC and CC amplitudes
  M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918 , T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2201.01754, 2405.00612
- NNLO QCD-EW corrections to charged-current DY (2-loop contributions in pole approximation).
  L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539
- NNLO QCD-EW corrections to neutral-current DY
  R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, N.Rana, F.Tramontano, AV, arXiv:2102.12539, F. Buccioni, F. Caola, H.A.Chawdhry, F.Devoto, M.Heller, A.V.Manteuffel, K.Melnikov, R.Roentsch, C.Signorile-Signorile, arXiv:2203.11237

→ mixed QCD-QED resummation

- initial-state corrections
  L. Cieri,G.Ferrera, G.Sborlini,, arXiv:1805.11948, A.Autieri, L. Cieri,G.Ferrera, G.Sborlini,, arXiv:2302.05403
- initial and final state corrections
  L.Buonocore, L'Rottoli, P.Torrielli, arXiv:2404.15112

Charged Current Drell-Yan: NNLO QCD-EW results with approximated 2-loop virtual corrections

L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539

Exact LO, NLO (QCD+EW), NNLO QCD corrections
are combined with mixed QCD-EW corrections

Partonic subprocesses with 1 and 2 additional partons
are evaluated exactly at NLO and LO respectively

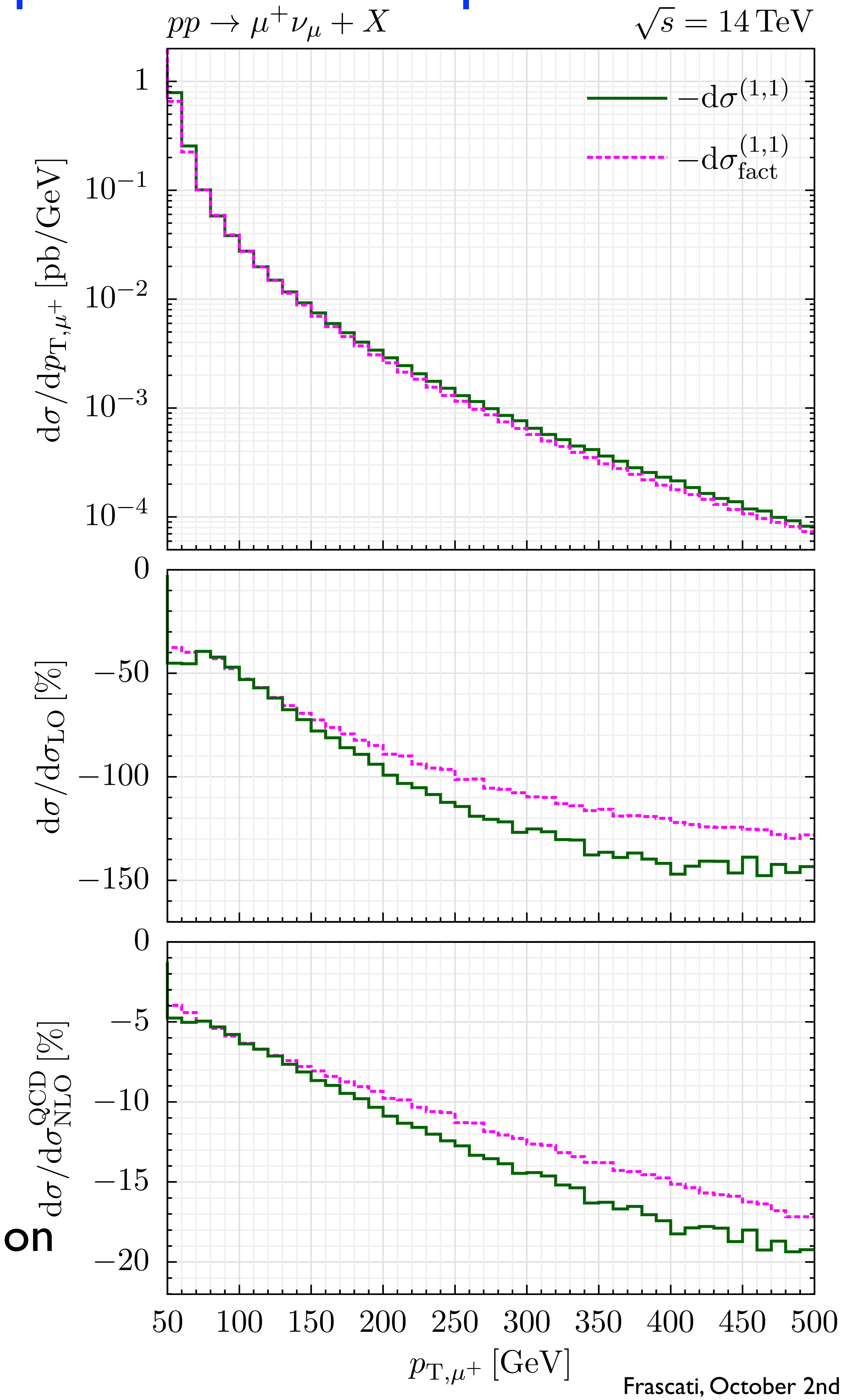
The 2-loop virtual corrections to $q\bar{q}' \to \ell\nu_\ell$ treated in pole approximation

Accurate description of the charged lepton $p_\perp^\ell$ spectrum,
dominated by the (exact) real radiation effects
resonant configurations

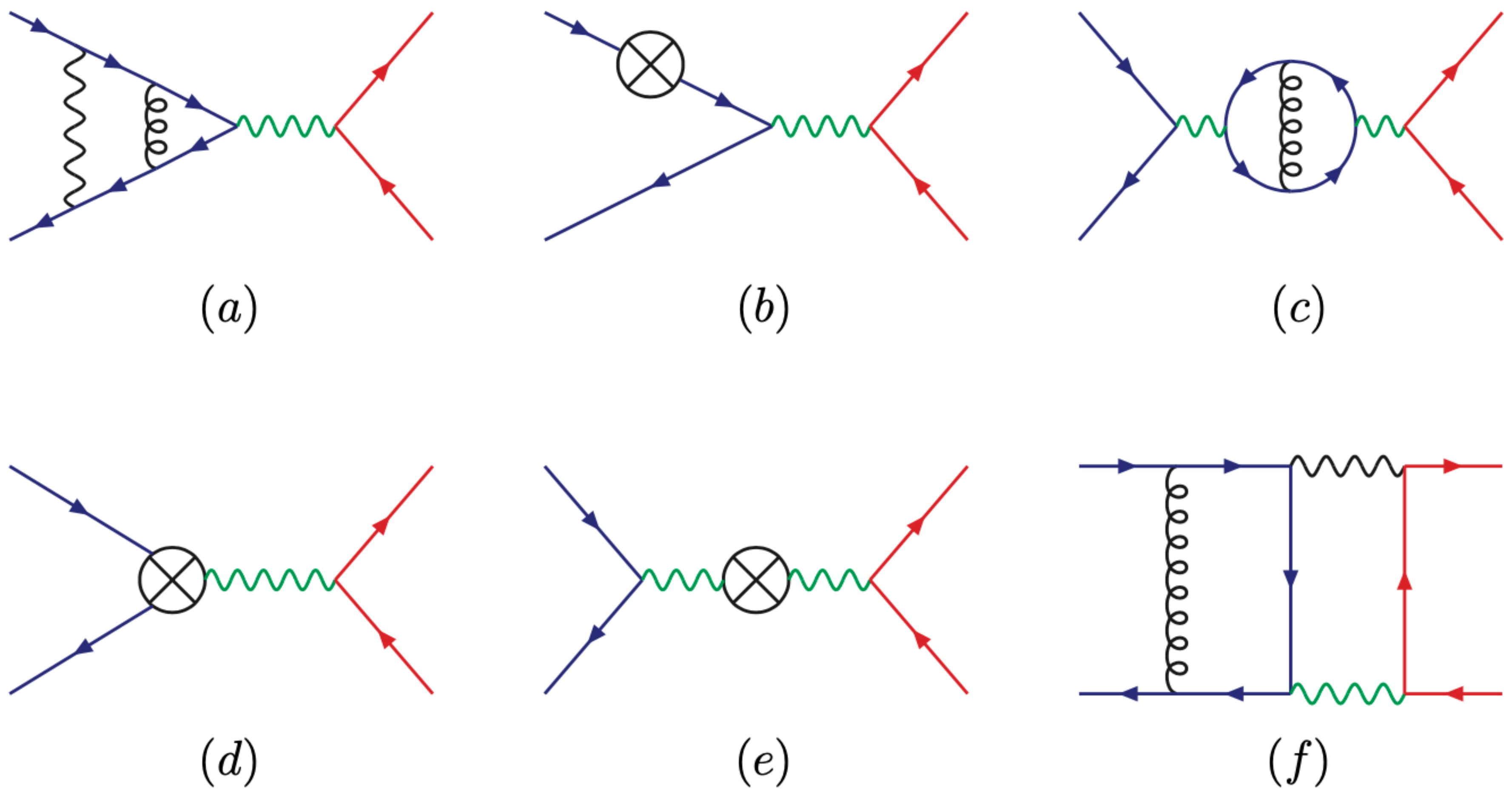
The factorisation of QCD and EW corrections is not accurate at large $p_\perp^\ell$

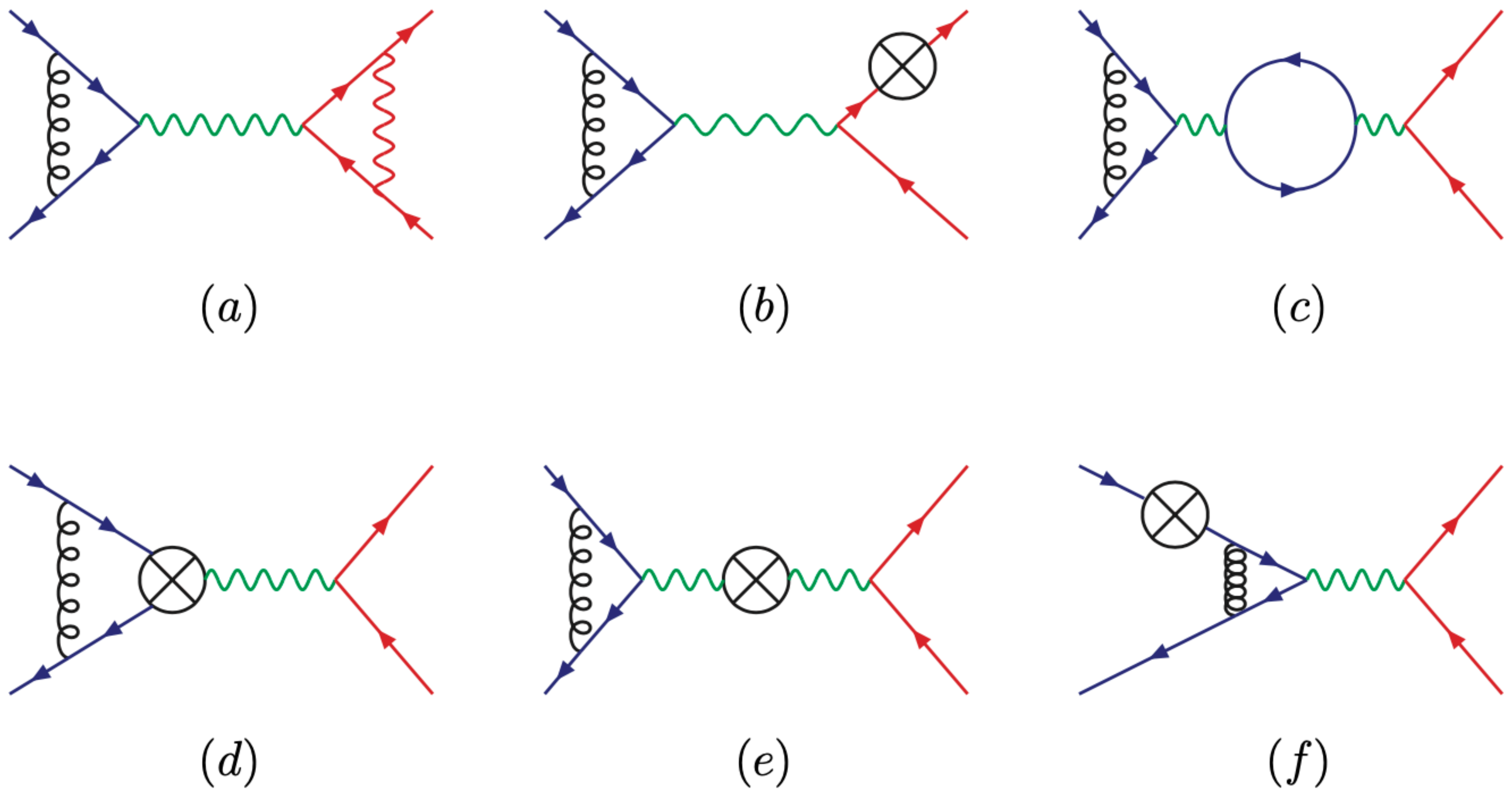
The lepton-pair transverse mass might receive large non-negligible
2-loop virtual corrections at large mass, poorly described in pole approximation

→ new results !



2-loop virtual QCD-EW corrections to the Charged-Current Drell-Yan in the SM





The Charged-Current process is mediated by a *W* exchange

For a general lepton-pair invariant mass,
there is no general gauge invariant separation
of initial- and final-state photonic corrections,
at variance with the NC DY case

We consider a massive final-state lepton,
yielding mass logarithms instead of collinear poles in dim.reg.

The presence of two weak bosons with different masses (*W* and *Z*)
is a new challenge for the solution of the Feynman integrals

Large number of terms → increased automation level