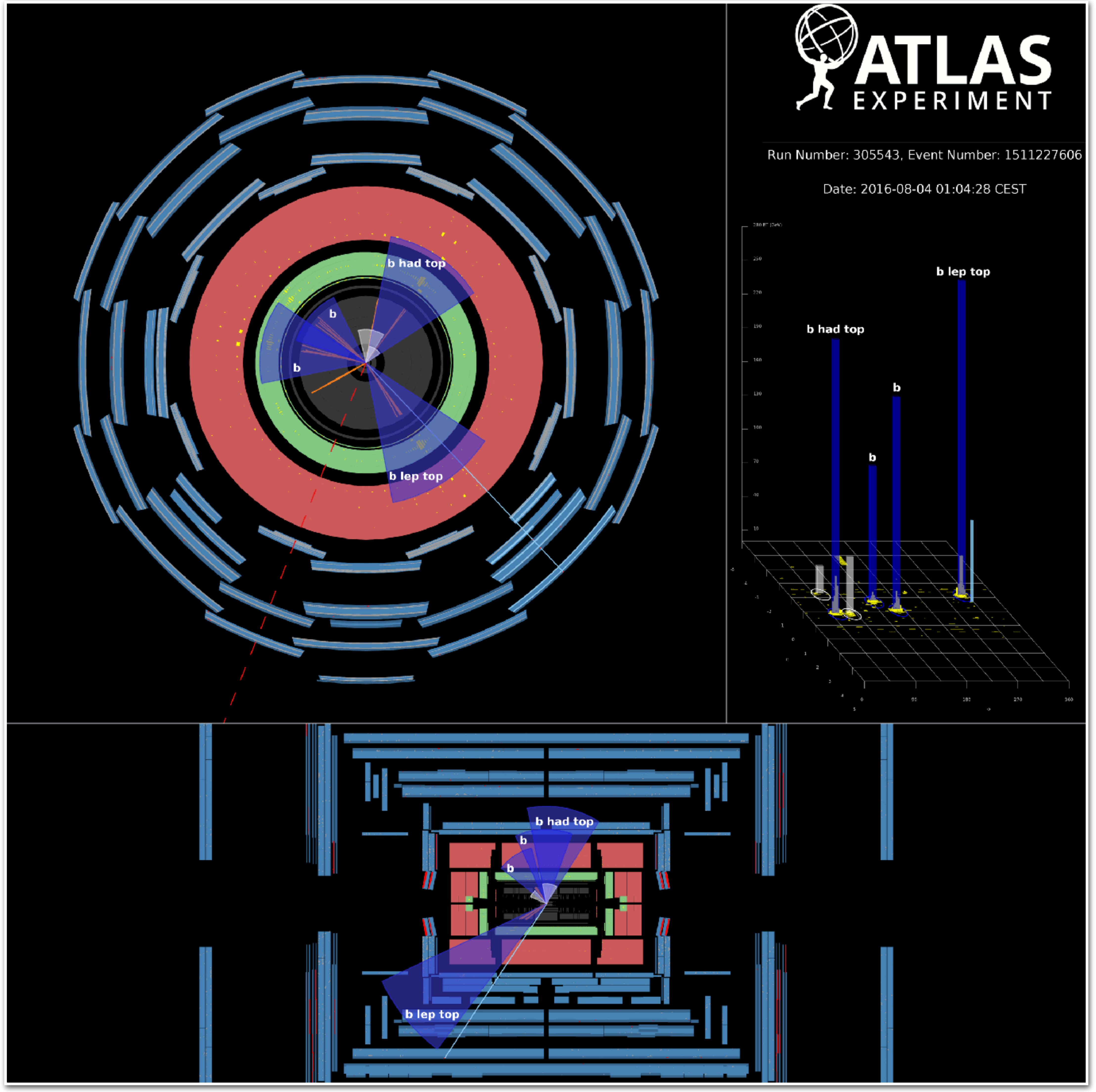
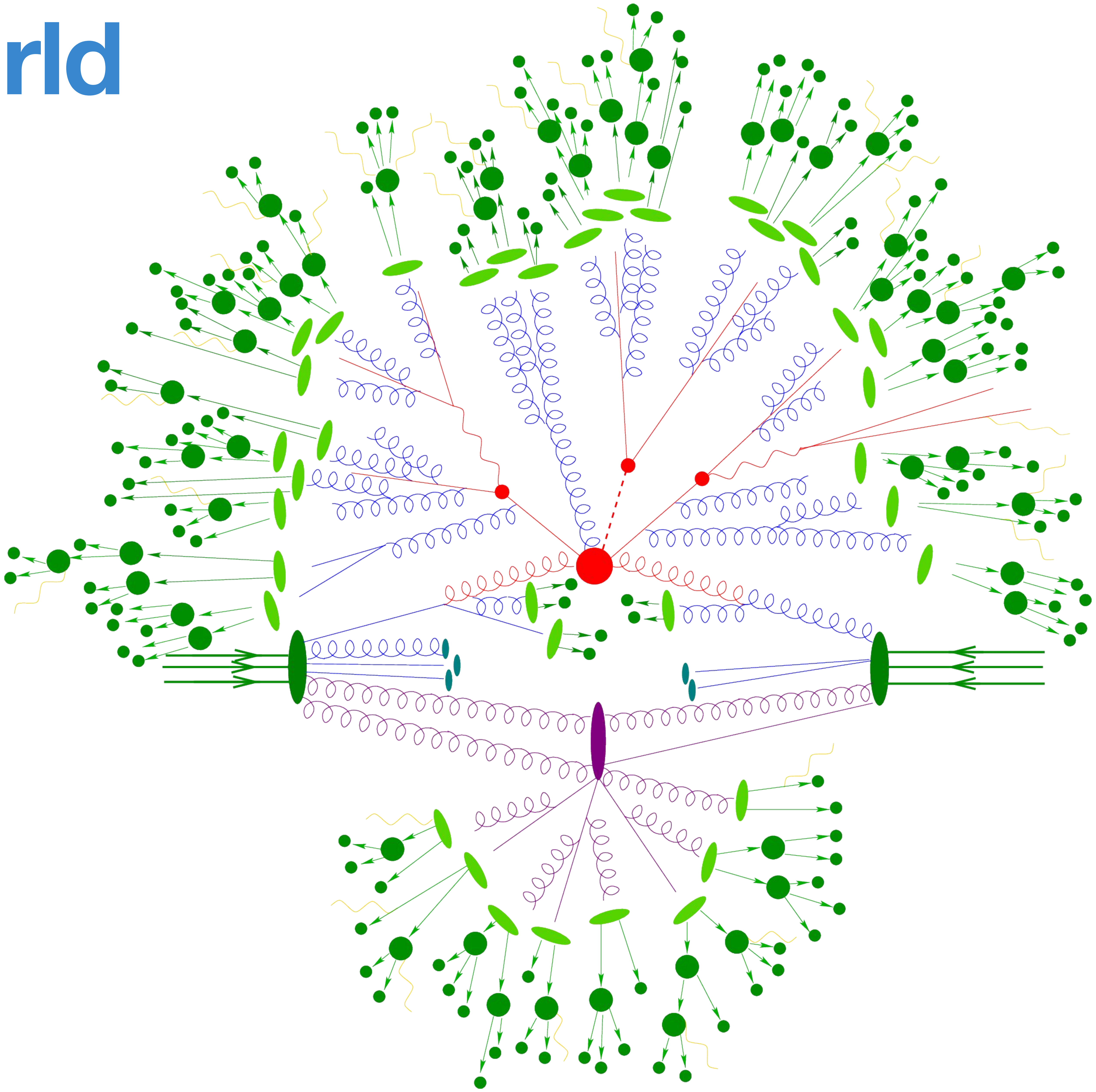
Parton shower accuracy (NLL and beyond)

Daniel Reichelt, 2 October 2024

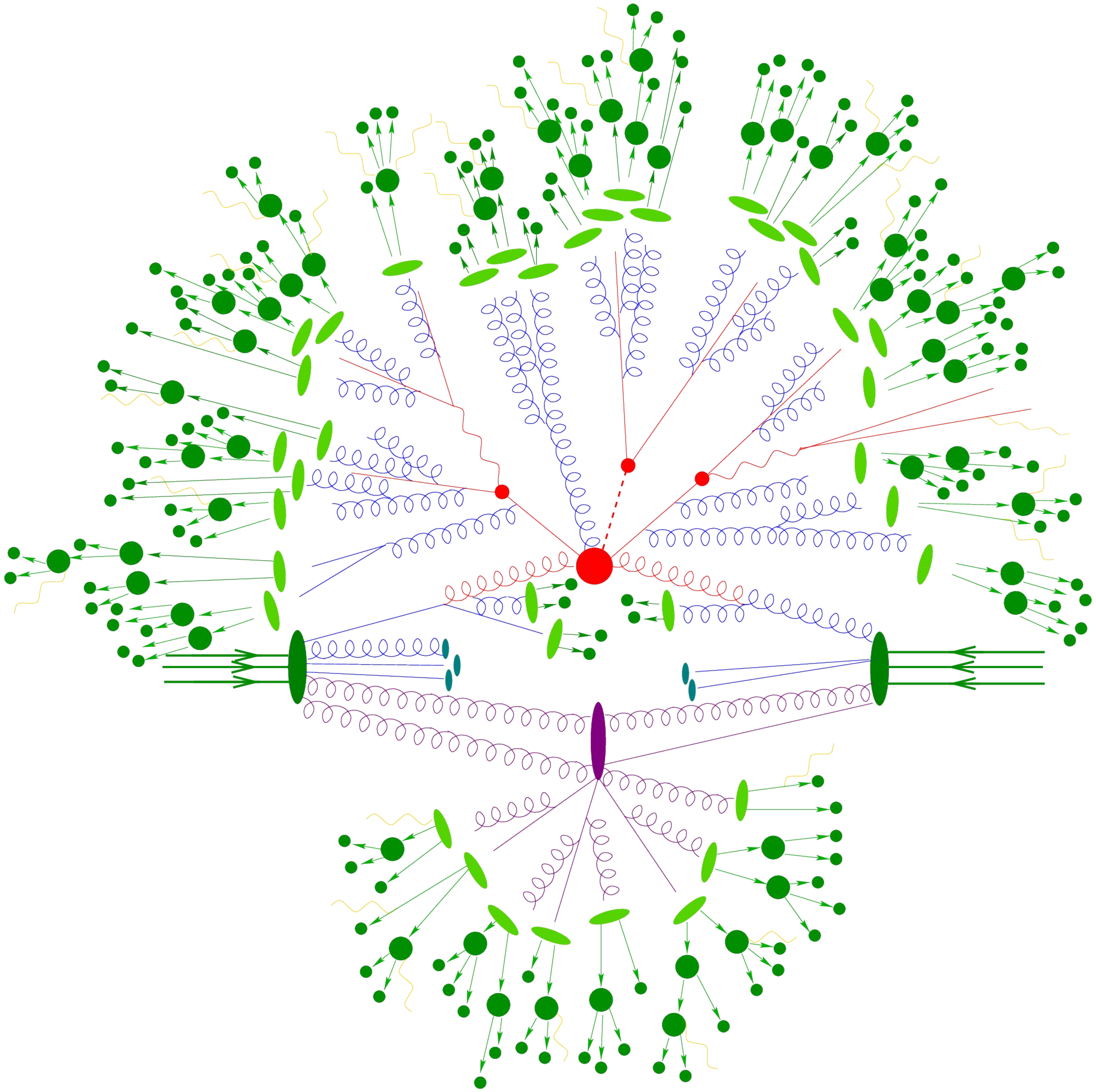
Colliders in the real world





Colliders for theorists

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - PDF/Underlying event
  - Hadronisation
  - QED radiation
  - Hadron Decays



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This Talk:

- Well established tools, used for decades to model collider physics
- Also connection to fixed order via matching/merging techniques well established, at least up to NLO
- But: basic shower picture based on leading-log approximation, some simple adjustments to get “at least most of” NLL

Factorisation and Logarithms

- QCD matrix elements factorise in the soft limit as ('Eikonal')

$$d\sigma_{n+1} = d\sigma_n \otimes d\Phi_{+1} \frac{\alpha_s}{2\pi} \sum_{k,i} \mathbb{T}_k \mathbb{T}_i \frac{p_k p_i}{(p_k q)(p_i q)}$$

single emission phase space

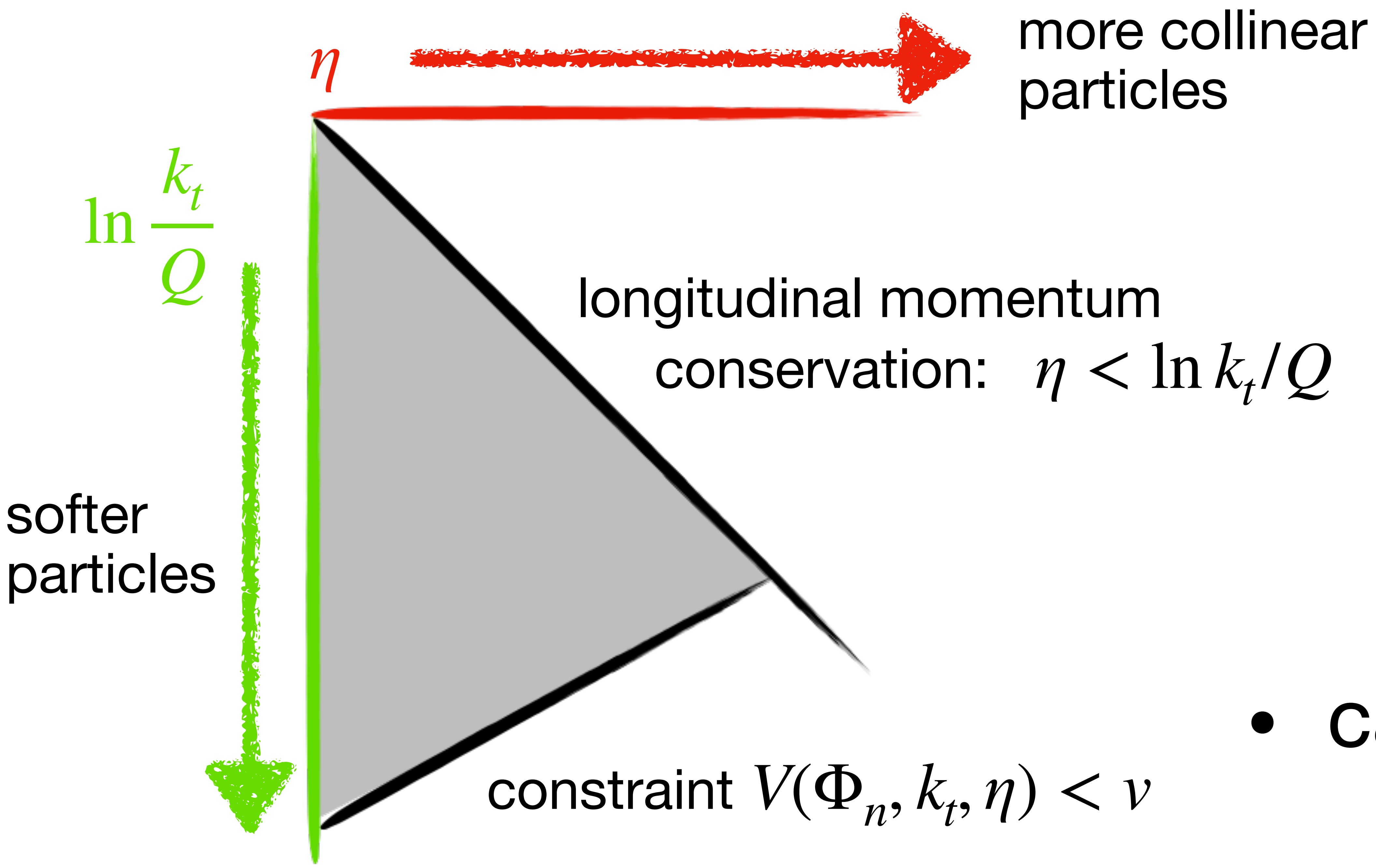
transverse momentum

$$\frac{p_k p_i}{(p_k q)(p_i q)} \sim 1/k_t^2$$

rapidity

azimuthal angle

$$d\Phi_{+1} \sim dk_t^2 \, d\eta \, d\phi$$



- calculate cross section, cut on $V(k_t, \eta) = k_t/Q > v$

$$\to \frac{\alpha_s}{2\pi} \int_{vQ}^{Q} \frac{dk_t}{k_t} \int_0^{\ln k_t/Q} d\eta \sim \frac{\alpha_s}{2\pi} \ln^2 1/v = \frac{\alpha_s}{2\pi} L^2$$

- "probability" for soft gluon emission above v

Parton branchings

- In toy case of constant probability for one emission between two scales

$$P = \int_{t_c}^{t_0} dt' \lambda = \lambda \Delta t$$

- “No emission” probability given by unitarity
  - $\Delta(t_0, t_c) = \exp[-\lambda \Delta t]$
- Poisson-type distribution familiar from radioactive decay
- In reality not constant (see last slide), but Monte-Carlo methods available to generate emissions to corresponding ”no-emission” factor (Veto-algorithm)

Missing ingredients for real (NLL) showers

- Precise choice of scale "ordering" variable $t \to$ I will mostly talk about $t \sim k_t^2$ ordered showers
- More accurate shower kernels
  - match to collinear part of Altarelli-Parisi splitting kernels
  - include CMW scheme (maybe not the Pythia default, but no conceptual question)
  - including additional effects on color, spin, generic higher order splitting kernels
- prescription to construct $n+1$ parton final state (aka recoil scheme)

Parton showers - Cliff notes version

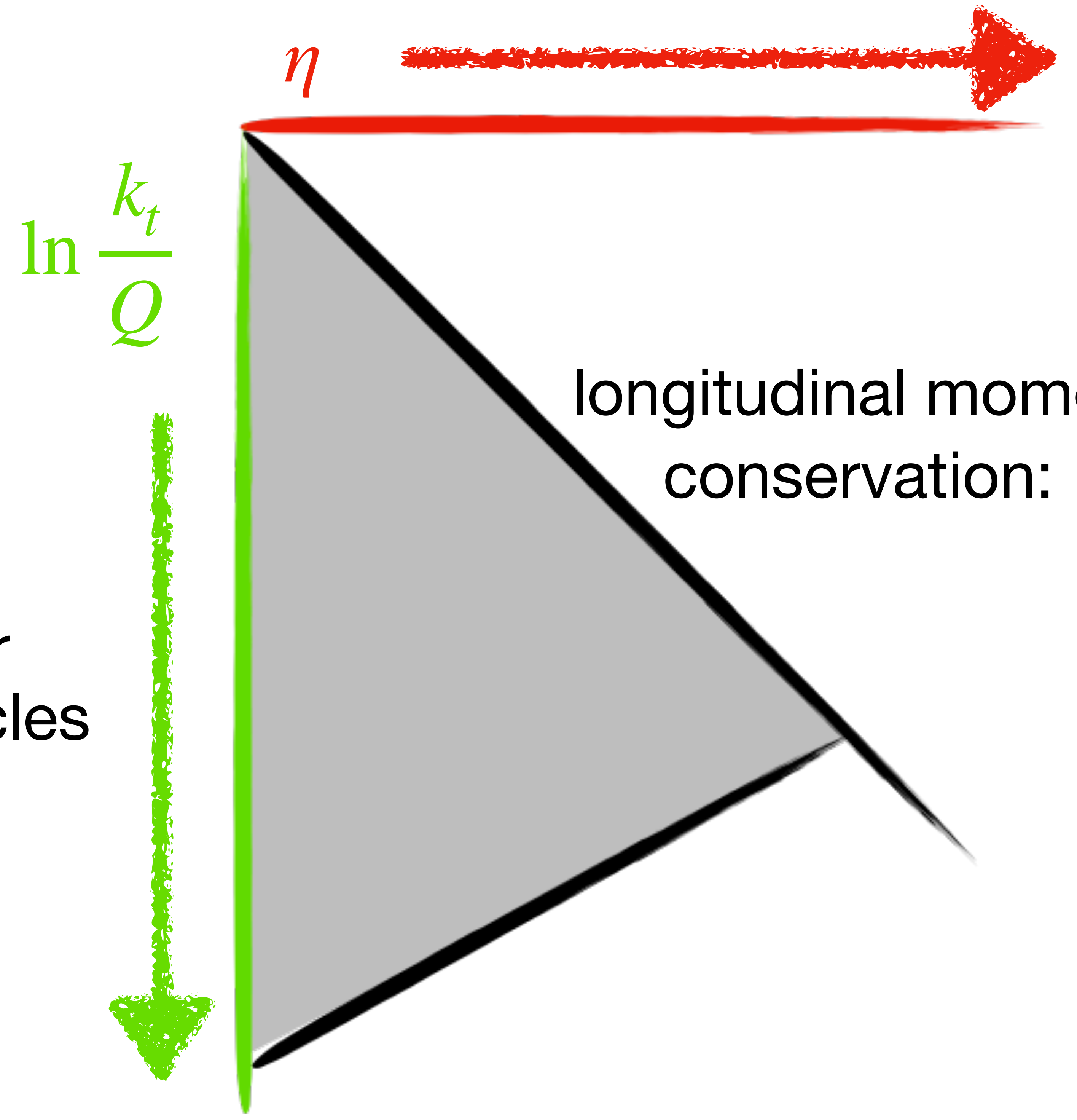
- no-emission probability (sudakov factor) $\sim \exp\left[-\int_{t_0}^{t_1} \frac{dk_t}{k_t} dz \frac{\alpha_S}{2\pi} P(z)\right]$

- Main ingredients to a shower:

1. splitting kernels $P(z)$ captures soft and collinear limits of matrix elements

2. fill phase space ordered in evolution variable $(k_t, \theta, q^2, \ldots) \Rightarrow$ here k_t ordered shower

3. generate new final state after emission according to recoil scheme



Colliders for theorists

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  - Q
  - Ha

- See also large amount of effort dedicated to colour accuracy specifically, e.g. [Forshaw, Holguin, Plätzer ’19,’20,’21], [De Angelis, Forshaw, Plätzer ’20], [Nagy, Soper ’19]

What I will not (so much) talk about:

- Issues with colour assignment:
  - inherited from mismatch between PS evolution and resummed observable (different identification of “hardest” emission) [Dasgupta,Dreyer,Hamilton,Monni,Salam ’18], [Hamilton, Medves, Salam, Scyboz, Soyez ’20]
  - for rest of the talk: assume suppression of effect with N_c is sufficient (whether you agree or not, we only have 30 min)

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What I will not (so much) talk about:

- Spin correlations:
  - effective solution known in principle ([Collins '88], [Knowles '88,'90]), with application to angular-ordered and dipole showers [Richardson, Webster '18]
  - see PanScales studies on implications for resummation properties for specific observables [Karlberg, Salam, Scyboz, Verheyen '21], [Hamilton, Karlberg, Salam, Scyboz, Verheyen '21]

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What I will not (so much) talk about:

- Fixed-order inputs:
  - See Emanuelle Re's talk yesterday about NNLOPS methods
  - interplay with log accuracy issues in some points, in particular if NLO emission is performed separately a la Powheg-Box [Hamilton, Karlberg, Salam, Scyboz, Verheyen '21]

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What I will (try to) talk about:

- NLL accurate parton showers
  - PanScales studies on recoil schemes and solutions
  - Pheno with NLL parton showers
- Towards NNLL

Treatment of multiple emissions e.g. in CAESAR

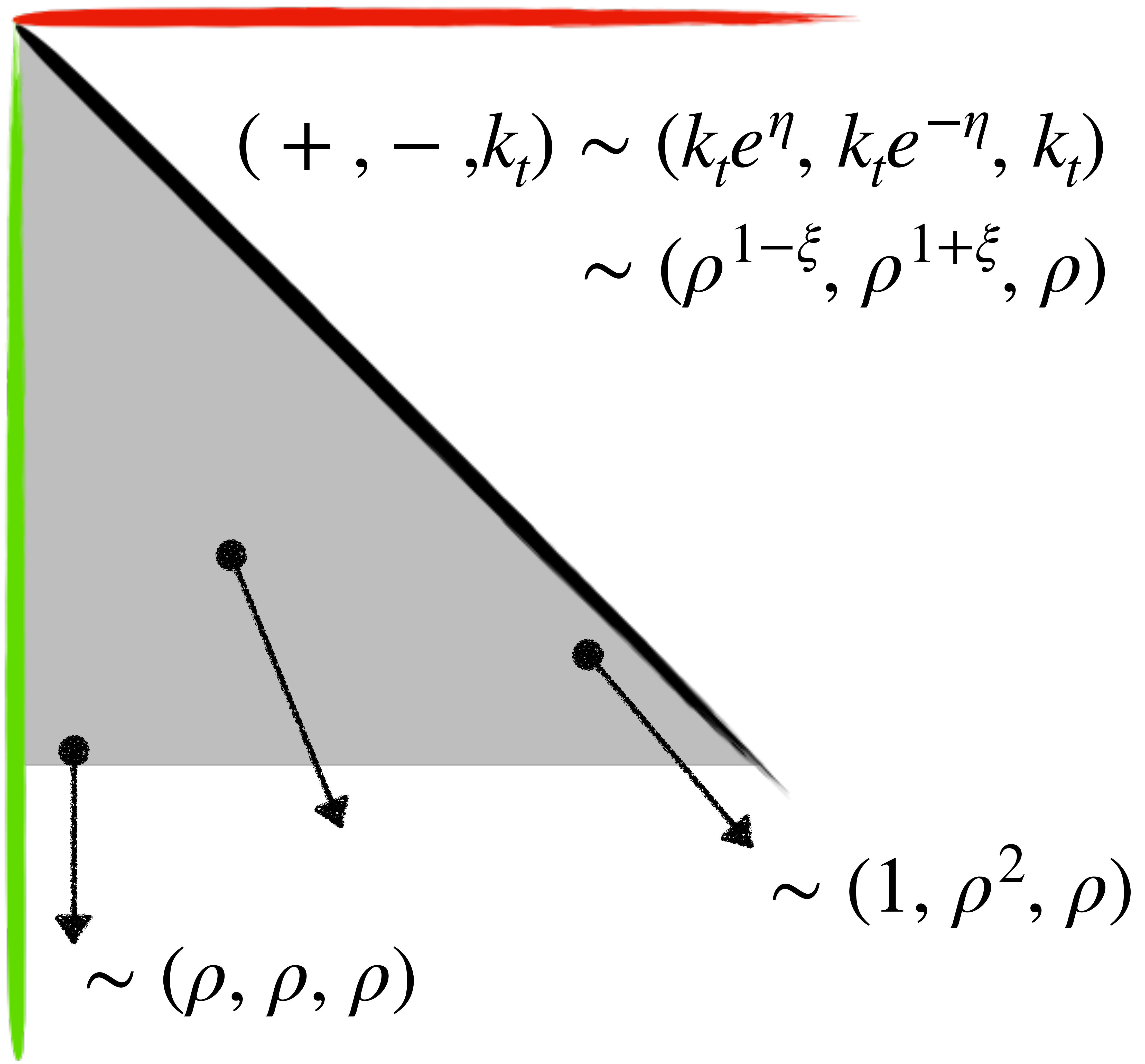
- factorisation of matrix elements in soft collinear limit well known (see last slide)
- how to extract NLL observable independent (i.e. without additional information)?
- method from [Banfi, Salam, Zanderighi '05]: need explicit implementation of soft-collinear limit*:

$$k_t^\rho = k_t \rho \qquad \xi = \frac{\eta}{\eta_{\max}}$$

$$\eta^\rho = \eta - \xi \ln \rho$$

and assume $\rightarrow$ numerically evaluate phase space integrals in this limit

$$V(k_i^\rho) = \rho V(k_i)$$



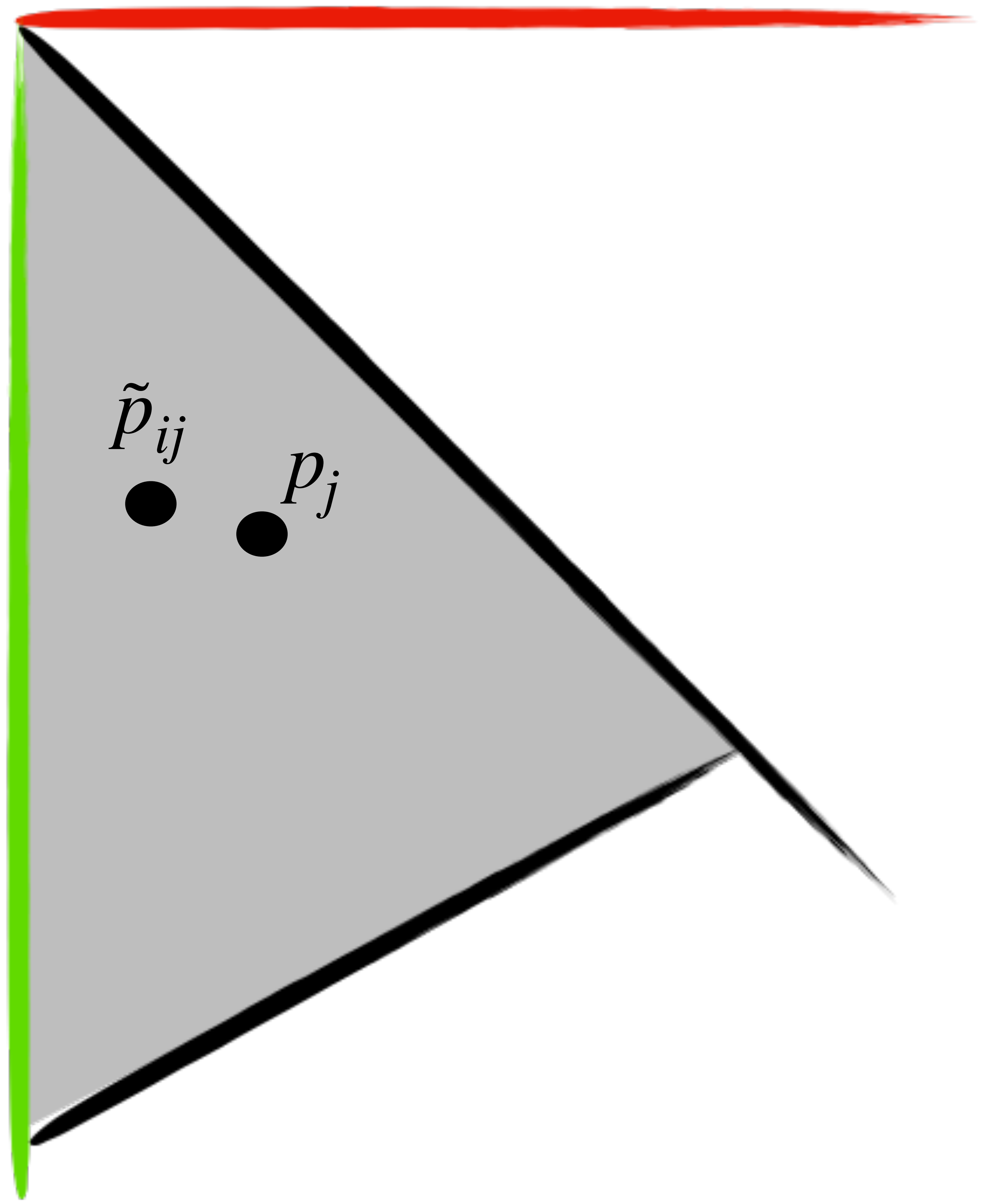
* example assuming $V(k_t, \eta) \sim k_t/Q$ for brevity

Effect of recoil on accuracy

- question: do recoil effects indeed vanish in soft limit (i.e. $\rho \to 0$)?*
  [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]

$$p_i = z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\perp$$
$$p_j = (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\perp$$
$$p_k = (1-y)\tilde{p}_k \ .$$

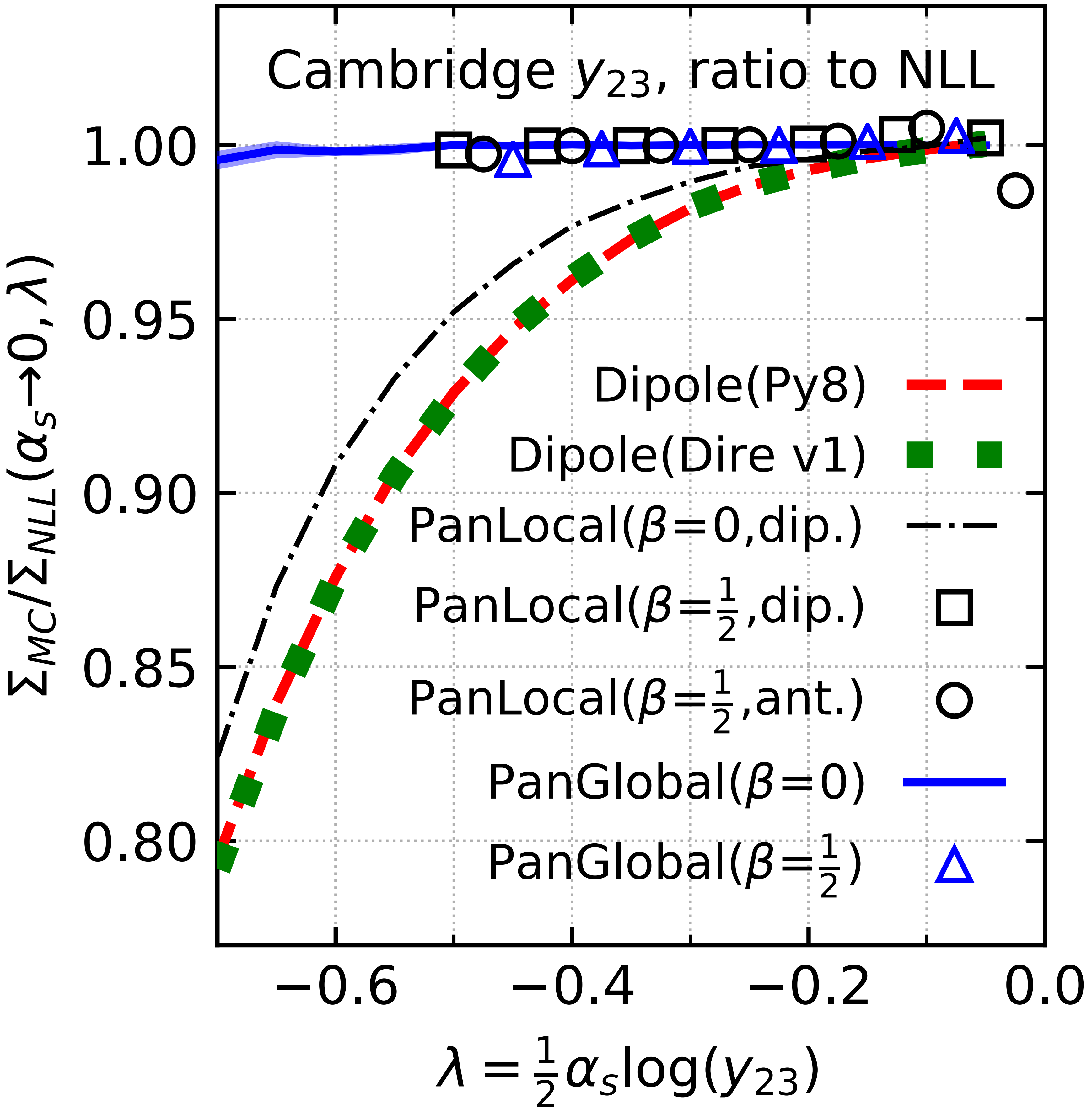
- consider situation where we first emit $\tilde{p}_{ij}$ from p_a, p_b, then emit p_j, $\tilde{p}_{ij} \to p_i, p_j$



- transverse momentum of p_i will be $k_t^i \sim k_t^{ij} + k_t^j \to k_t^{ij}$ as $\dfrac{k_t^j}{k_t^i} \to 0$
- but, relevant limit is $\dfrac{\Delta k_t^i}{k_t^i} \to \dfrac{\rho k_t^j}{\rho k_t^i} = \mathcal{O}(1)$

New Parton Showers - NLL accuracy

- typical claim based on accuracy of splitting functions etc.
  - parton showers ~ NLL accurate if CMW scheme for strong coupling is used
- observation in [Dasgupta, Dreyer, Hamilton, Monni, Salam ’18] [Dasgupta, Dreyer, Hamilton, Monni, Salam ’20] (PanScales collaboration):
  - subtleties arise in distribution of recoil for subsequent emissions ⇒ phase space where accuracy is spoiled if soft gluon absorbs recoil
  - apparently restricts k_t ordered showers to global recoil schemes

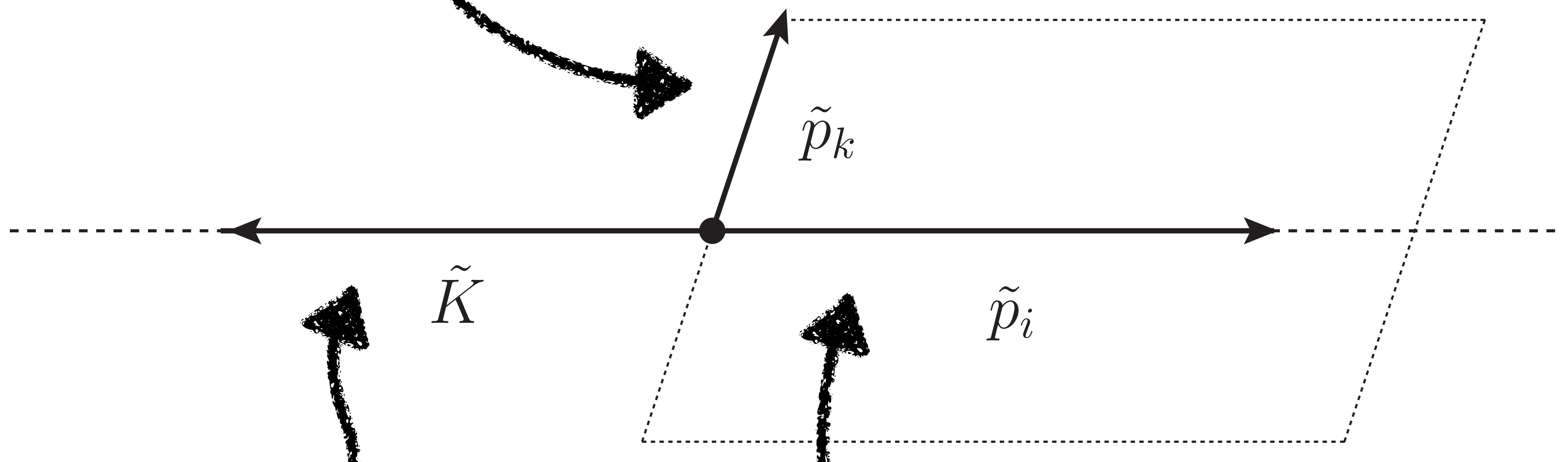


Kinematics - global recoil scheme (Alaric example)

[Herren, Höche, Krauss, DR, Schönherr,’22]

- Before splitting:

colour spectator

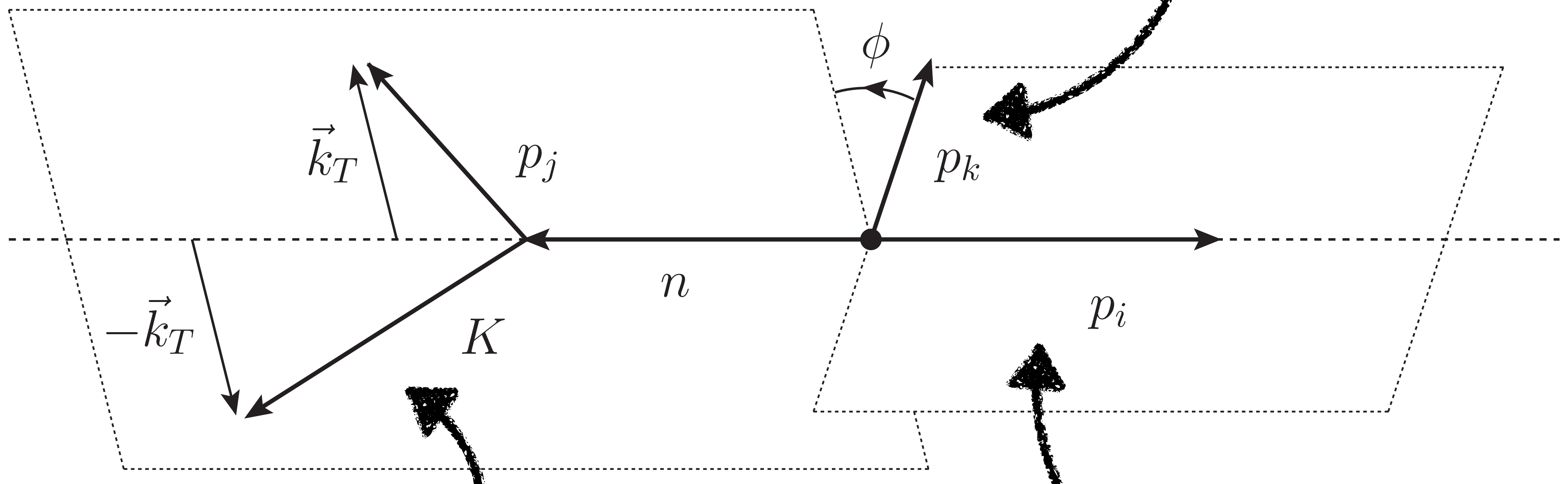


$\sum$ other momenta k_i

splitter

- After splitting:

$p_k = \tilde{p}_k$



$p_i = z\tilde{p}_i$

$K^2 = \tilde{K}^2$

$\tilde{K} + \tilde{p}_i = K + p_i + p_j$

$k_i^\mu \rightarrow \Lambda^\mu{}_\nu k_i^\nu$

[Catani, Seymour ’97]

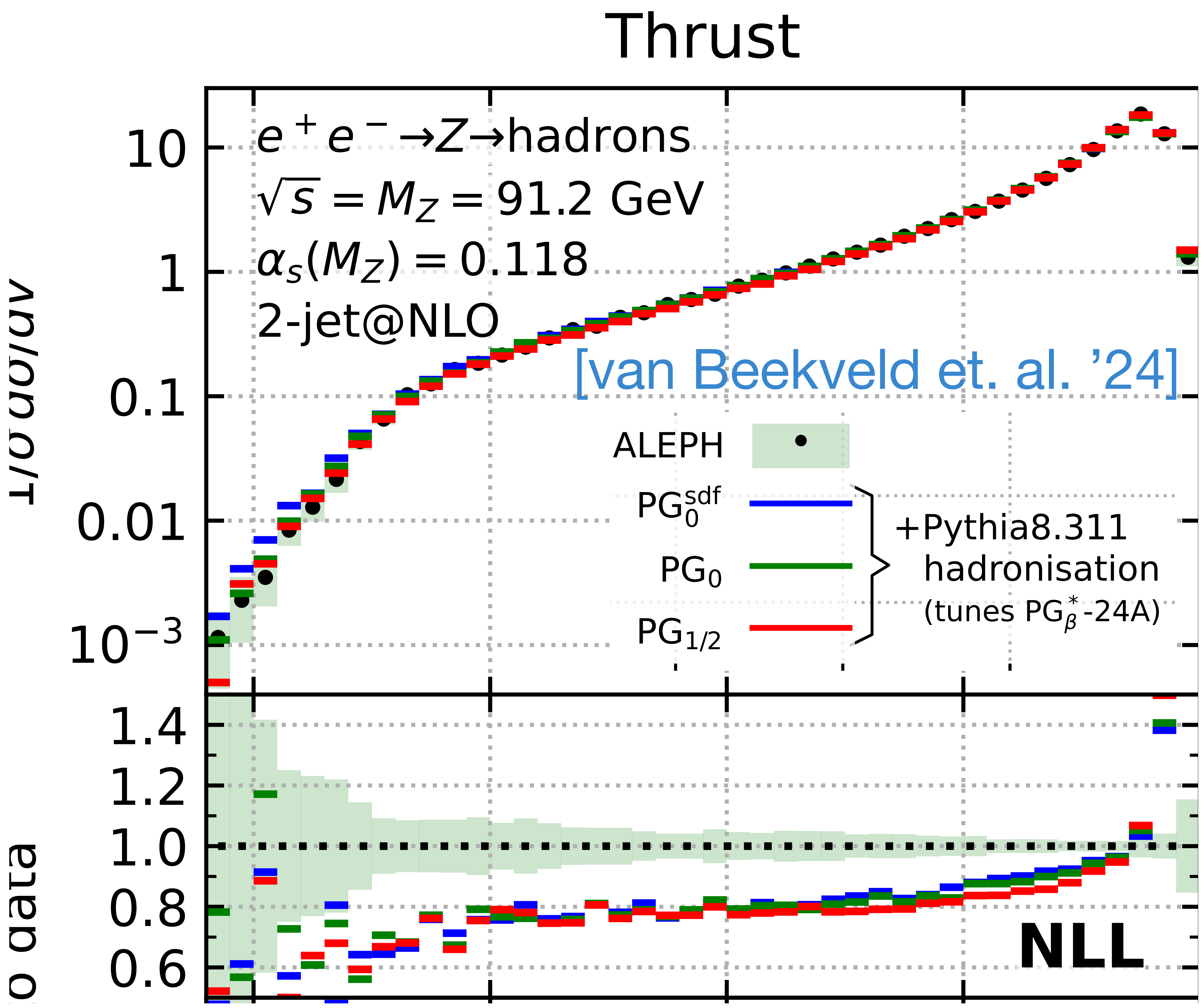
$$\Lambda^\mu{}_\nu = g^\mu{}_\nu - \frac{(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{K\cdot\tilde{K}+\tilde{K}^2} + 2\frac{K^\mu \tilde{K}_\nu}{\tilde{K}^2} \;\rightarrow\; \Lambda^\mu{}_\nu \tilde{K}^\nu = K^\mu$$

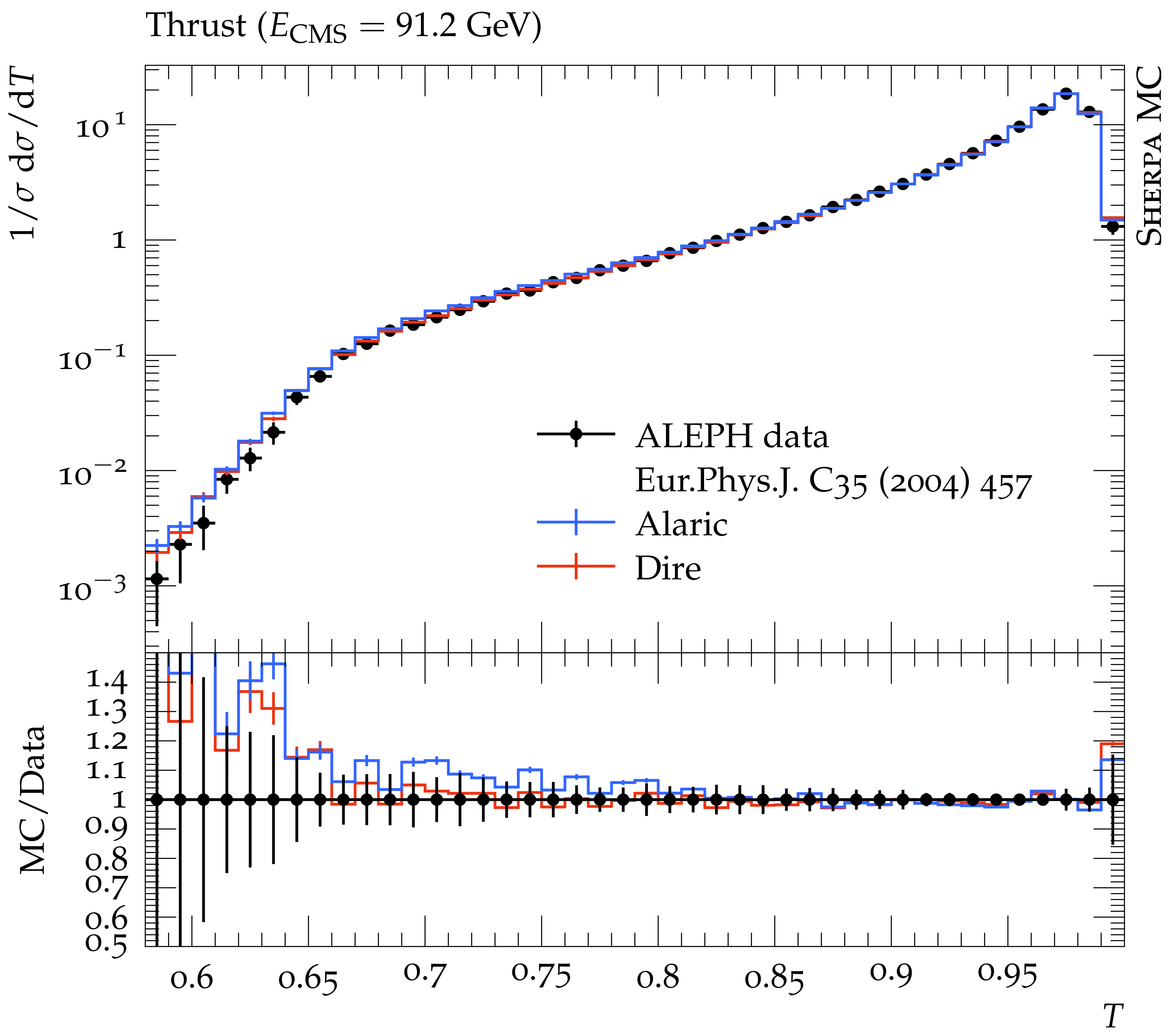
Lorentz transformation distributes recoil to hardest particles!

New Parton Showers - NLL accuracy

- Several solutions/re-evaluations of parton shower concepts:
- [Dasgupta, Dreyer, Hamilton,Monni, Salam, Soyez ’20], [vanBeekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso,Soyez ’22] …
  - partitioning of splitting functions and appropriate choice of evolution variable can lead to NLL accurate shower for local and global recoil strategies
- [Forshaw, Holguin, Plätzer ’20]
  - Connections between angular ordered and dipole showers
- [Nagy, Soper ’11]
  - local transverse, global longitudinal recoil
- [Herren, Höche, Krauss, DR, Schönherr,’22], [Höche, Asse ’23], [Höche, Krauss, DR ‘24]
  - global recoil, enables analytic comparison to resummation and proof of NLL accuracy
- [Preuss ’24]
  - global recoil in antenna shower Vinca

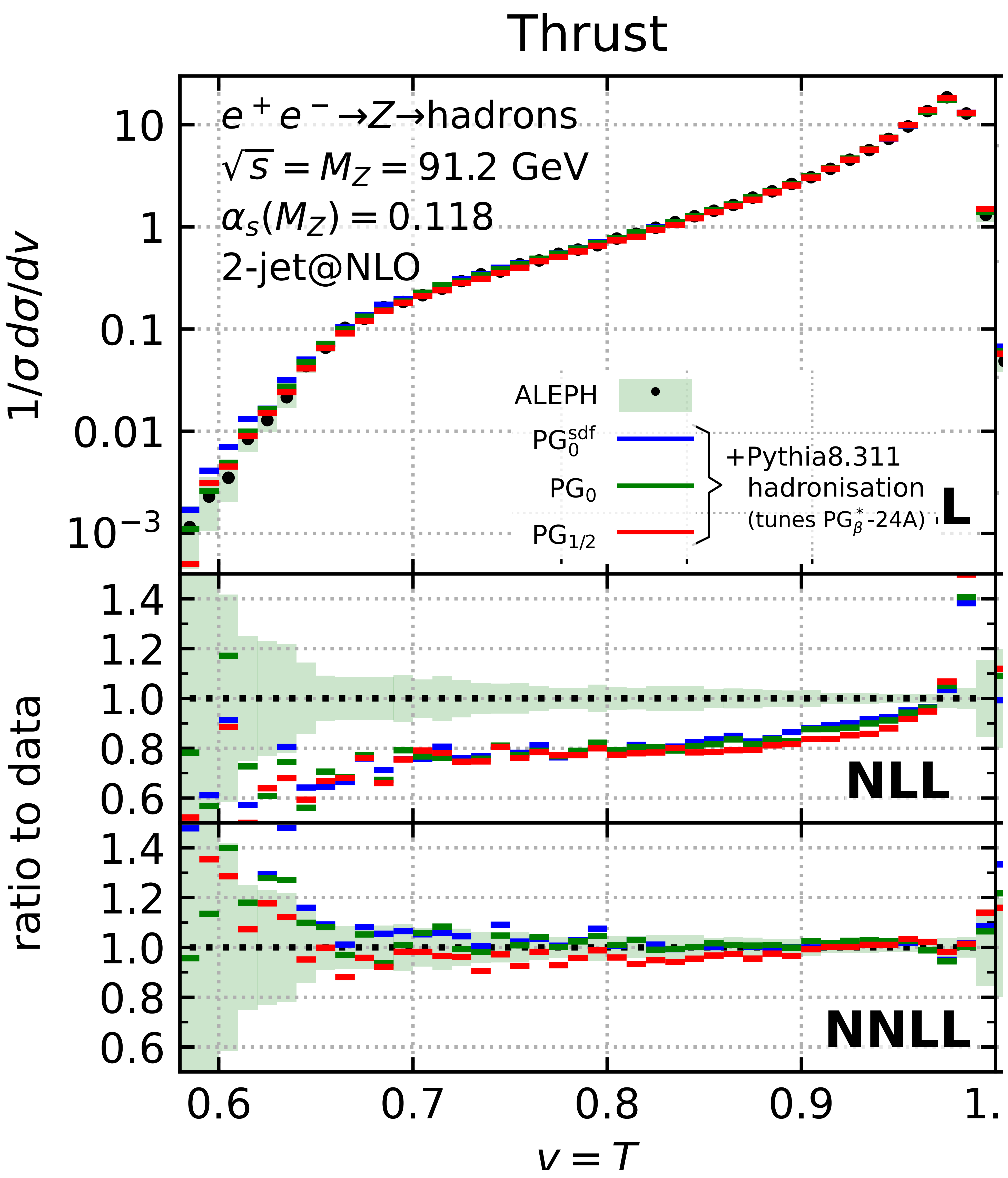
Pheno with NLL showers





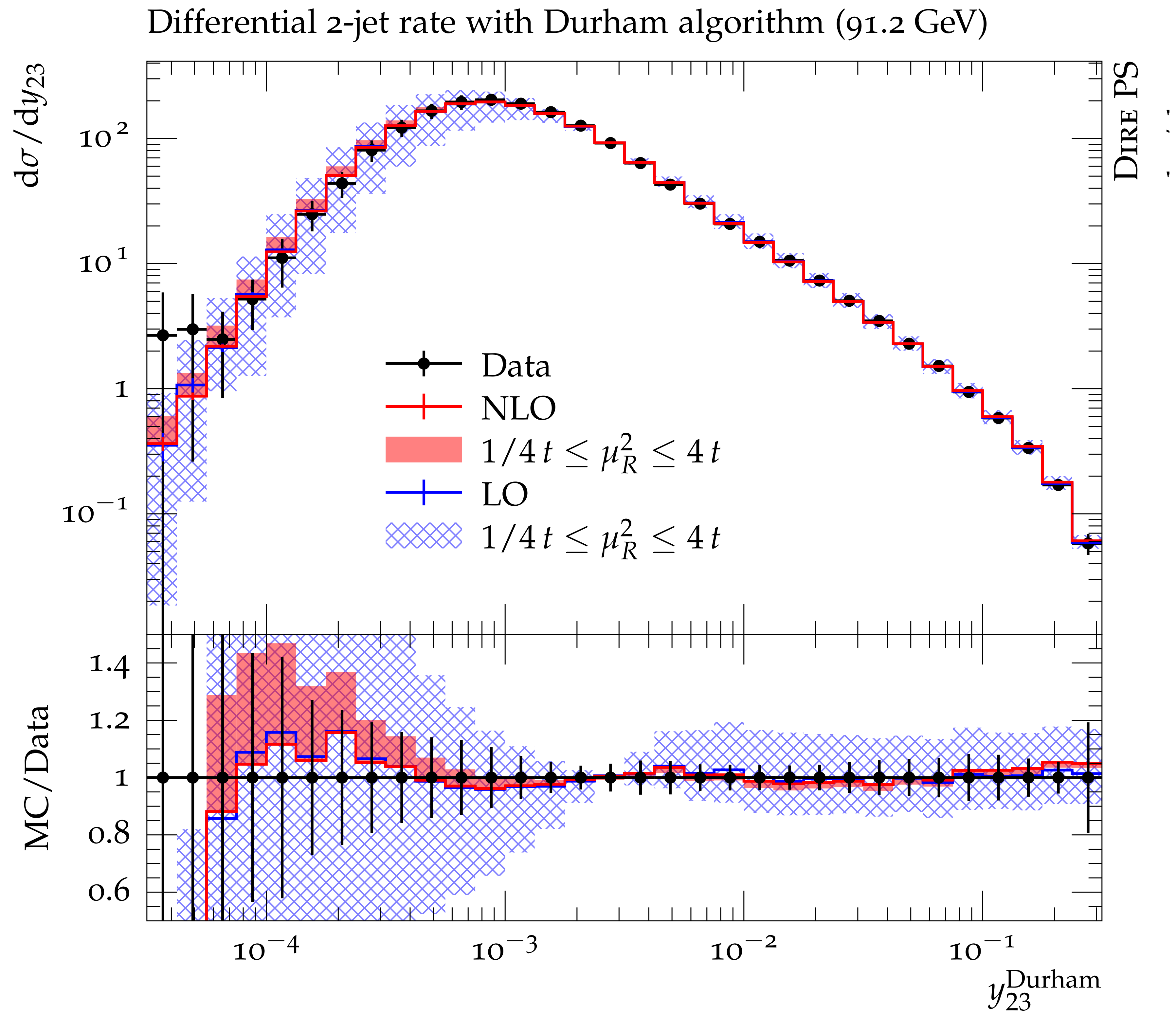
- PanScales shower and Alaric @ NLL accuracy
  - significantly different conclusion about ability of NLL shower to describe data (similar level of tuning, $\alpha_S = 0.118$ is fixed, string fragmentation parameters in Pythia 8 tuned to LEP data)

Towards NNLL



[van Beekveld et. al. ’24]

- Conclusion from PanScales studies: NNLL needed to describe even simple observables
- Achieved by multiplicative matching of NLO splitting kernels via + correction terms capturing effect of inclusive gluon emissions

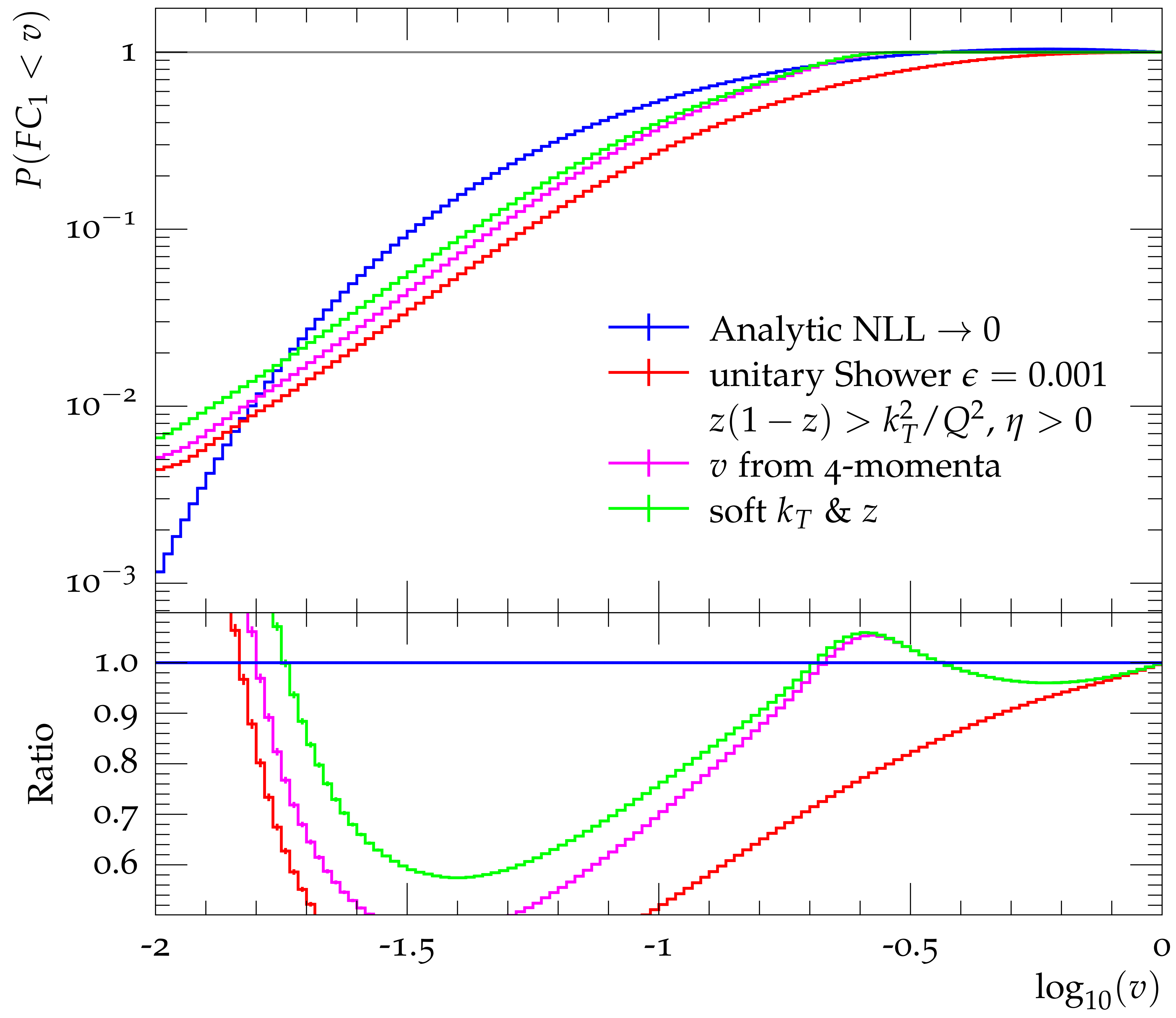


[Höche, Prestel ’17]

- Appears to be in contrast with small effects found so far in implementing higher order splitting functions (though not in complete NNLL framework yet) [Höche, Prestel ’17], [Dulat, Höche, Prestel ‘18], [Gellersen, Höche, Prestel]

Beyond logarithmic accuracy

- Observations
  - LL and NLL accurate showers can be very similar (e.g. failing of NLL accuracy numerically undetectable for Dire in prominent observables like Thrust)
  - NLL accurate showers can differ significantly from NLL result away from strict limit
  - ⇒ subleading effect play a significant role in phenomenological successful parton showers, more systematic understanding desirable, see also [Höche, Siegert, DR ’17]



Alaric beyond NLL - subleading effects

assume Sudakov decompose like

$$p_i^\mu = z_i \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_i\, 2p_{ij}\bar{n}}\,\bar{n}^\mu + k_t^\mu\ ,$$

$$p_j^\mu = z_j \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_j\, 2p_{ij}\bar{n}}\,\bar{n}^\mu - k_t^\mu$$

derivation of splitting functions leads to:

$$P^{(\mathrm{F})}_{qq\,\|}(p_i,p_j,\bar{n}) = C_F\,(1-\varepsilon)(1-z_i)$$

$$P^{(\mathrm{F})}_{gg\,\|}(p_i,p_j,\bar{n}) = 2C_A\, z_i z_j\ ,$$

$$P^{(\mathrm{F})}_{gq\,\|}(p_i,p_j,\bar{n}) = T_R\left[\,1-\frac{2\,z_i z_j}{1-\varepsilon}\,\right]\ .$$

actual shower kinematics:

$$p_i = z\,\tilde{p}_i\ ,$$

$$p_j = (1-z)\,\tilde{p}_i + v\big(\tilde{K}-(1-z+2\kappa)\,\tilde{p}_i\big) - k_\perp\ ,$$

$$K = \tilde{K} - v\big(\tilde{K}-(1-z+2\kappa)\,\tilde{p}_i\big) + k_\perp\ ,$$

$$p_i = \frac{z}{1-v(1-z+\kappa)}\,\hat{p}_{ij} + \frac{z}{1-v(1-z+\kappa)}\,k_\perp + \mathcal{O}\left(\frac{k_\perp^2}{2\tilde{p}_i\tilde{K}}\right)\ ,$$

$$p_j = \frac{(1-z)(1-v)-v\kappa}{1-v(1-z+\kappa)}\,\hat{p}_{ij} - \frac{z}{1-v(1-z+\kappa)}\,k_\perp + \mathcal{O}\left(\frac{k_\perp^2}{2\tilde{p}_i\tilde{K}}\right)$$

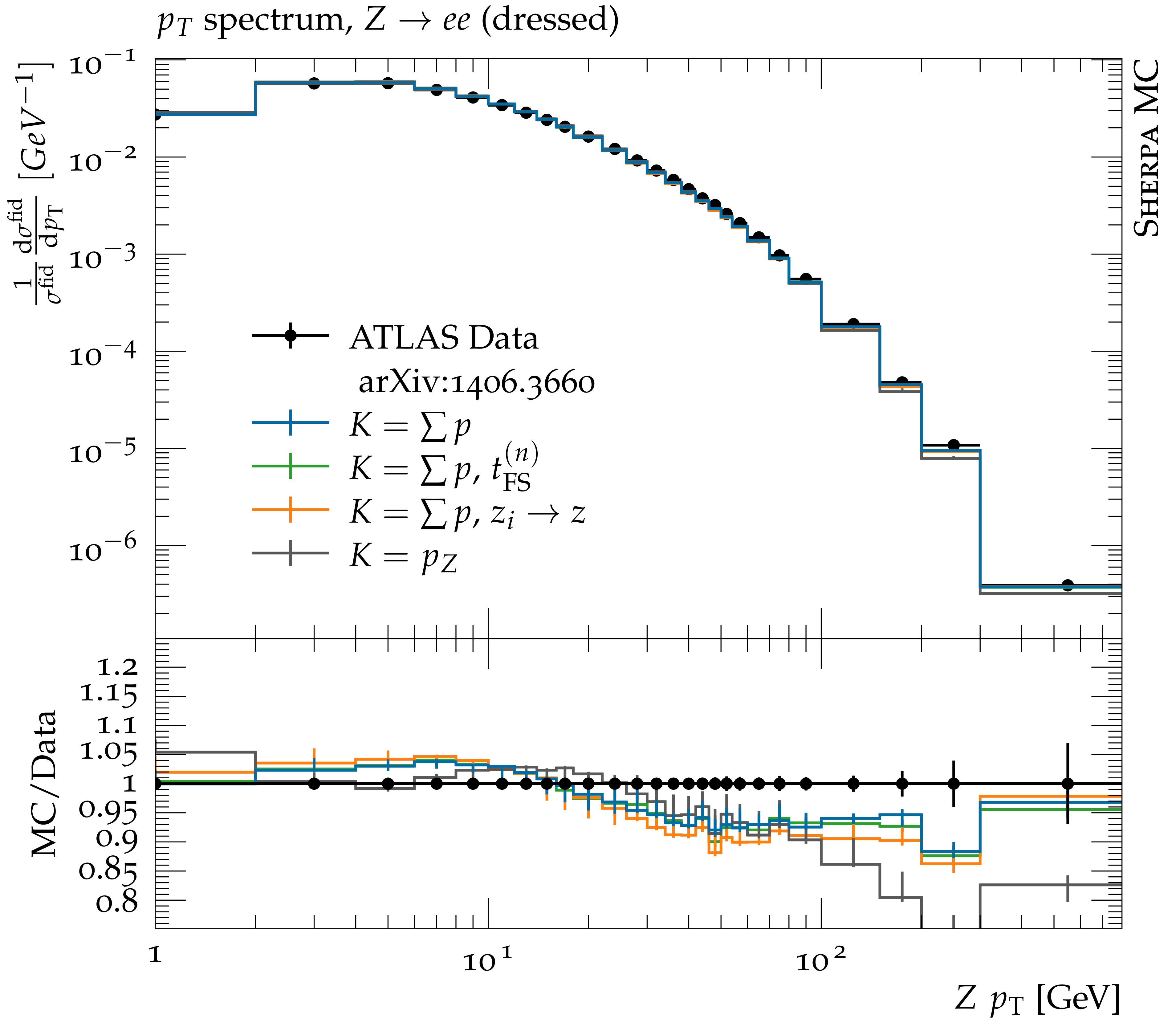
ultimately, “proper” splitting variables:

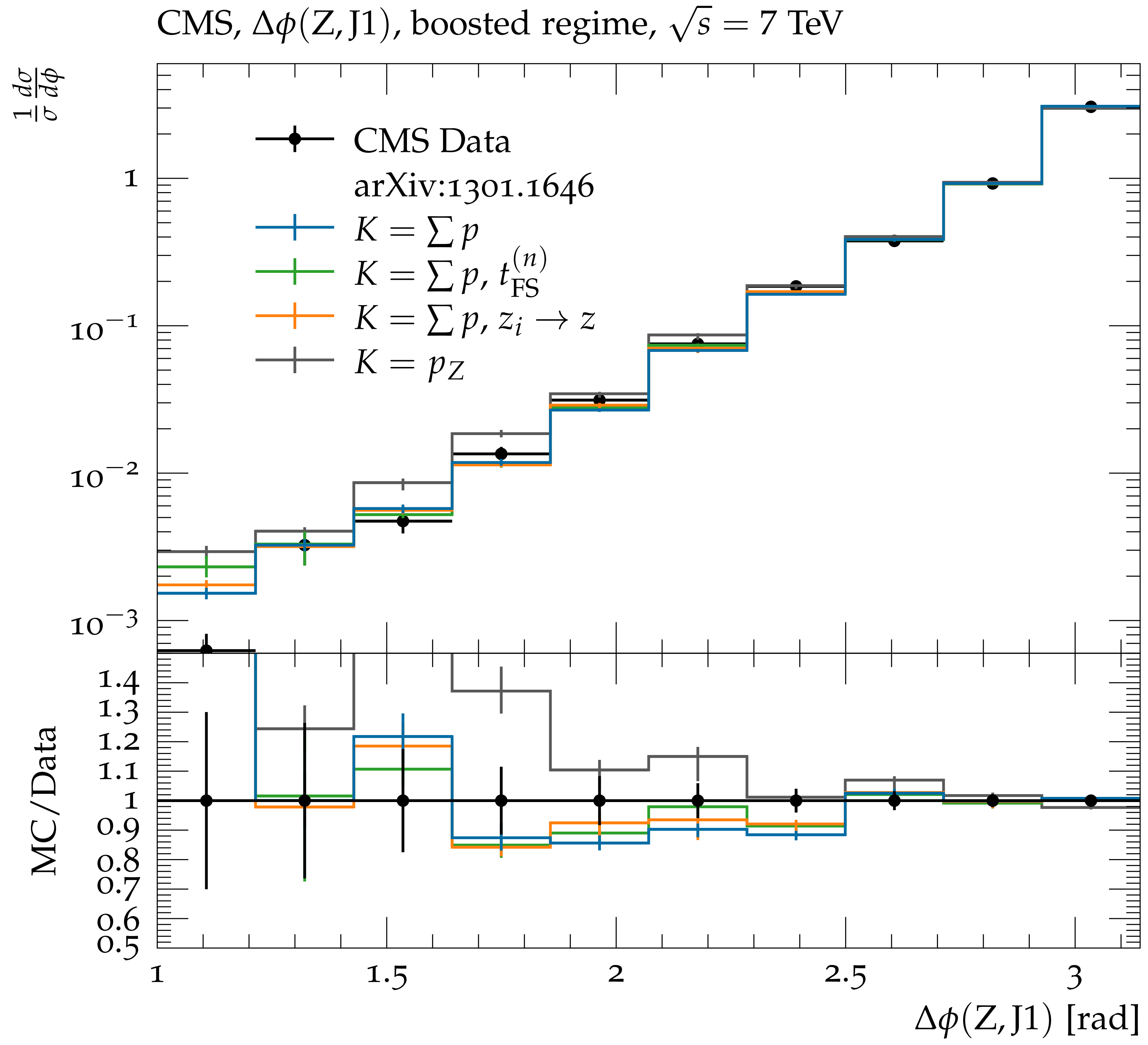
$$z_i = \frac{z}{1-v(1-z+\kappa)}\ ,$$

$$z_j = 1 - \frac{z}{1-v(1-z+\kappa)}$$

Alaric — subleading effects in Z+jets

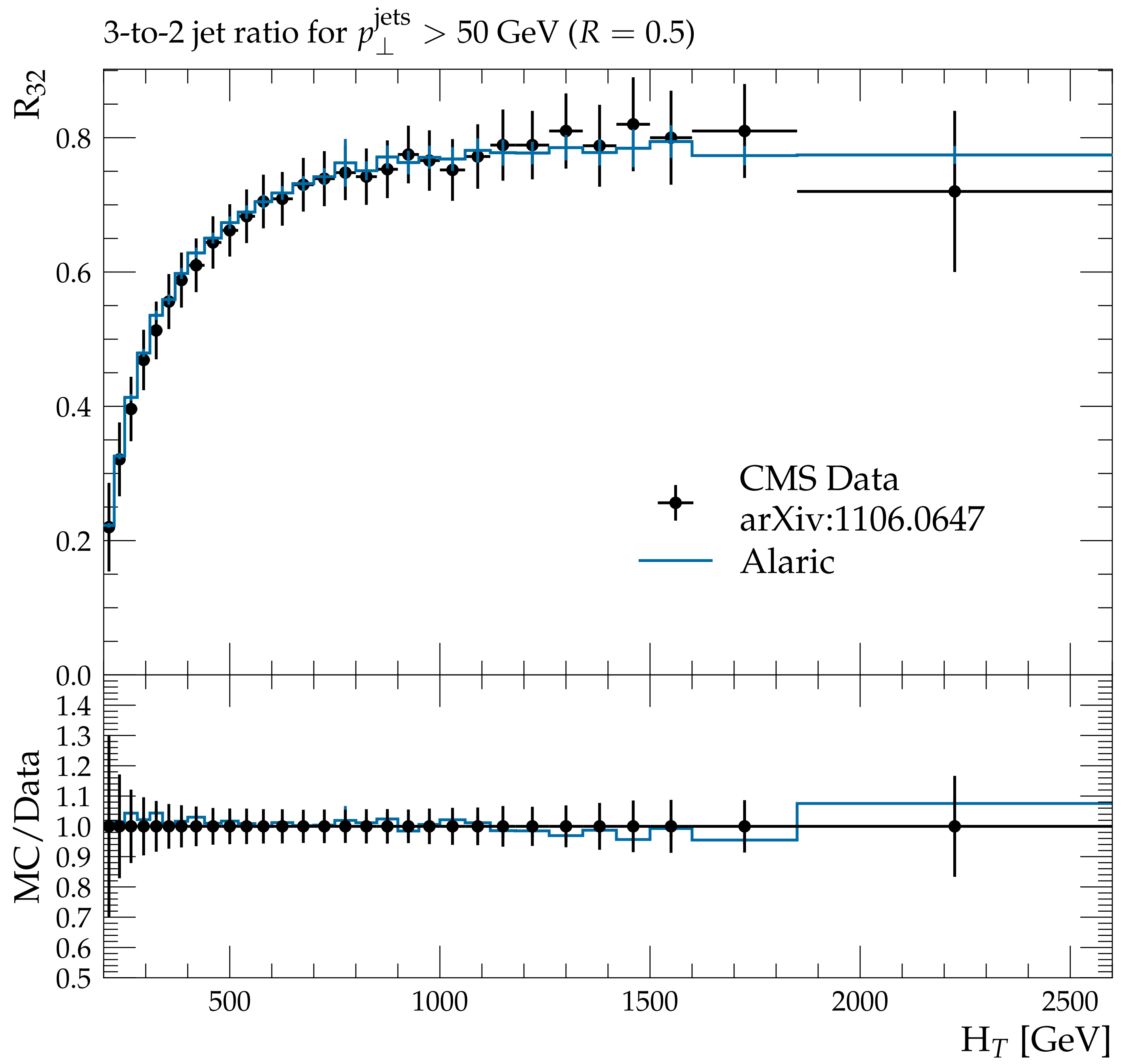
- effects/choices beyond NLL accuracy:
  - choice of evolution variable (up to factors of $z \sim 1$)
  - identify PS parameter z with z_i, z_j
  - choice of recoil momentum K (NLL accuracy needs "hard" K)

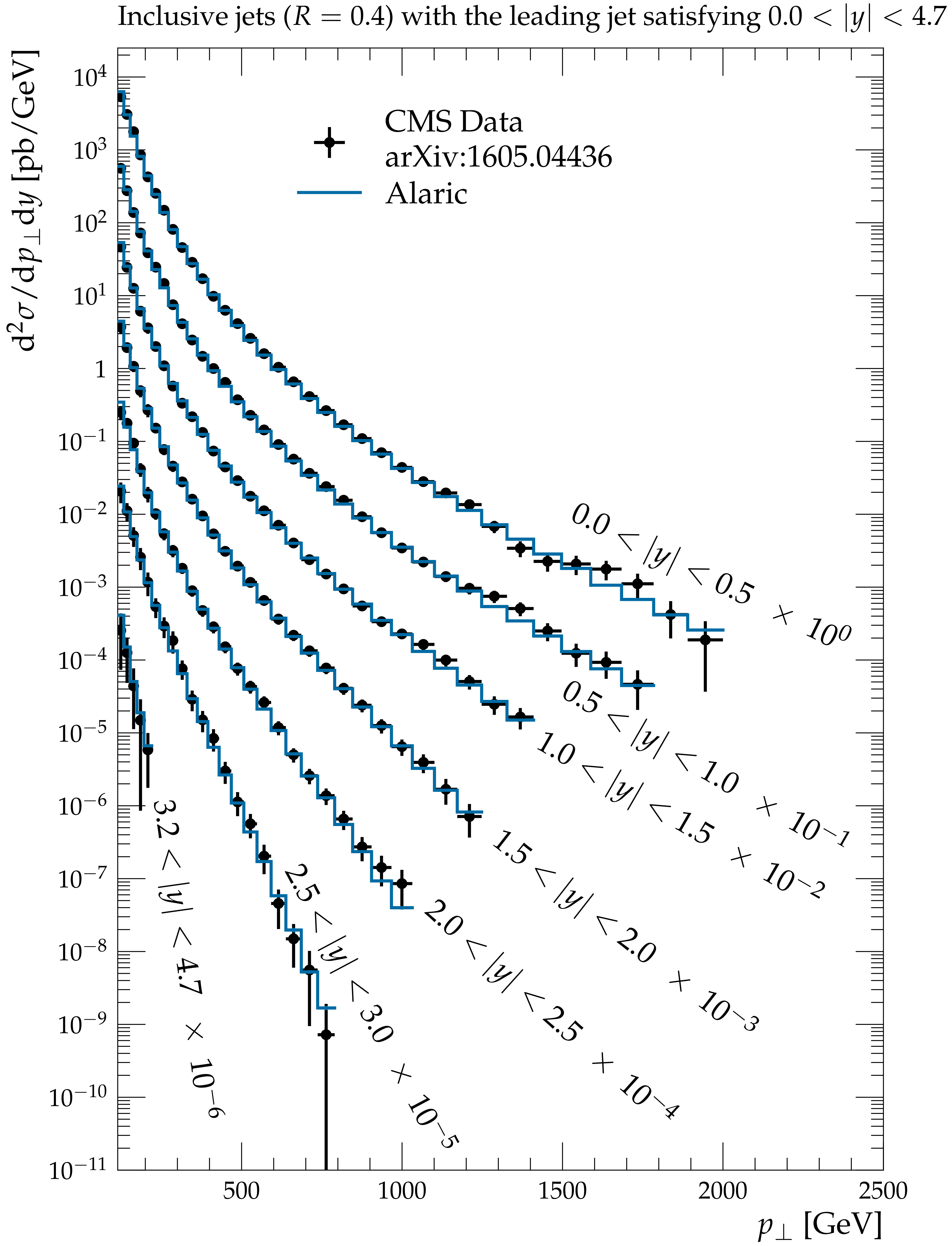




Alaric at the LHC — jets

- [Höche, Krauss, DR '24] extend Alaric method to IS evolution
- satisfactory description of inclusive and dijet events
- transverse momentum spectrum of leading jet and ratio 3-to-2 jet rate





Conclusion

- Progress on logarithmic accuracy of parton showers (as compared to resummed calculations)
- Effect on "general-purpose" nature to be seen
  - reminder to Paolo Nason's talk yesterday,
    " 'best' theory framework [has] not always [been] successful in SMC land "
- Outlook:
  - Probably NNLL PS matched with NNLO fixed order in near future (at least on time scale of future collider)
  - Non-perturbative corrections/soft physics effect might become limiting factors