NNLO+PS matching

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Outline

- NNLO+PS (QCD):
    - introduction, goals and available methods
    - $\text{MiNNLO}_{\text{PS}}$
    - Geneva
    - similarities / differences

- (selection of) current challenges:
    - NLL showers vs. matching
    - EW corrections
    - F +1 jet @ NNLO+PS

- Focus on pp colliders
- [UNNLOPS currently less developed, see backup]

Introduction
(from NLO+PS to NNLO+PS)

FO vs PS

hard scattering

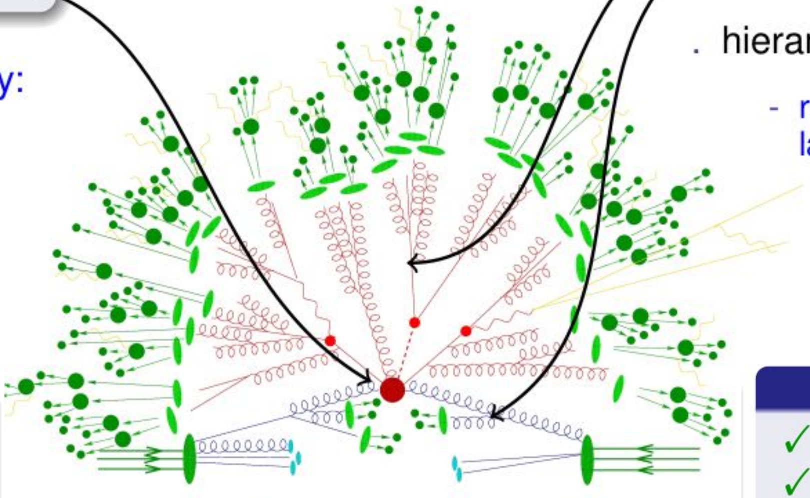
$\Lambda_{\text{QCD}} \ll \mu \approx Q$

- perturbation theory:

$$d\sigma = d\sigma_{\text{LO}} + \alpha_S \, d\sigma_{\text{NLO}} + \alpha_S^2 \, d\sigma_{\text{NNLO}} + \ldots$$

hard process

- ✓ high precision (N^kLO)
- ✓ nowadays NNLO is the standard
- ✗ no “realistic” event
- ✗ (fail when resummation needed)



hadronisation

$\mu \approx \Lambda_{\text{QCD}}$

parton shower

$\Lambda_{\text{QCD}} < \mu < Q$

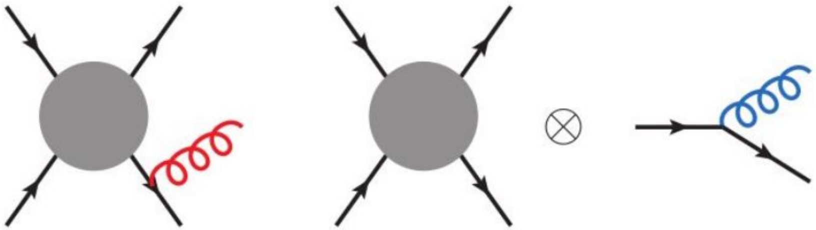
- hierarchy of scales
  - resummation of large logarithms

parton showers

- ✓ realistic + flexible tools
- ✓ widely used by experimental coll’s
- ✗ limited precision (LO)
- ✗ (fail when multiple hard jets)

NLO+PS

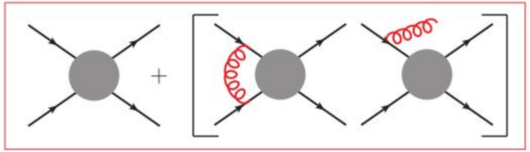
- Problem: overlapping regions



- NLO+PS is well understood, general solutions applicable to virtually any process: MC@NLO and POWHEG [Frixione-Webber '03, Nason '04]
- Other approaches exist, e.g.
  KrkNLO, Vincia [Jadach et al., Skands et al.]
  Geneva, U(N)NLOPS, MAcNLOPS [Alioli et al., Prestel et al./Plätzer, Nason,Salam]

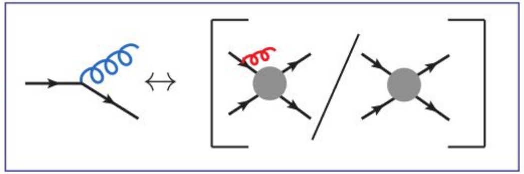
Example: POWHEG

$$B(\Phi_n) \Rightarrow \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$



$$d\sigma_{\mathrm{POW}} = d\Phi_n \quad \bar{B}(\Phi_n) \quad \left\{\Delta(\Phi_n; k_{\mathrm{T}}^{\min}) + \Delta(\Phi_n; k_{\mathrm{T}})\frac{\alpha_s}{2\pi}\frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\, d\Phi_r\right\}$$

[+ p_{T}*-vetoing subsequent emissions*, to avoid double-counting]



$$\Delta(t_{\mathrm{m}}, t) \Rightarrow \Delta(\Phi_n; k_{\mathrm{T}}) = \exp\left\{-\frac{\alpha_s}{2\pi}\int \frac{R(\Phi_n, \Phi_r')}{B(\Phi_n)}\theta(k_{\mathrm{T}}' - k_{\mathrm{T}})\, d\Phi_r'\right\}$$

NLO+PS: tools and accuracy

hard process

- ✓ high precision (N^{k}LO)
- ✓ nowadays NNLO is the standard
- ✗ no "realistic" event
- ✗ (fail when resummation needed)

parton showers

- ✓ realistic + flexible tools
- ✓ widely used by experimental coll's
- ✗ limited precision (LO)
- ✗ (fail when multiple hard jets)

- Available NLO+PS tools: **POWHEG-BOX**, **MG5_aMC@NLO**, **Sherpa** (→MC@NLO), **Herwig7** (MatchBox), Vincia, KrKNLO
  - NLO for inclusive observables (ggH: Higgs rapidity)
  - (N)LL/LO for 1st emission (ggH: $p_{T,H}$ at small/large values)
  - LL for extra emissions (PS)
- Born process can contain jets
- NLO+PS merging (different multiplicities) well understood

NNLO+PS

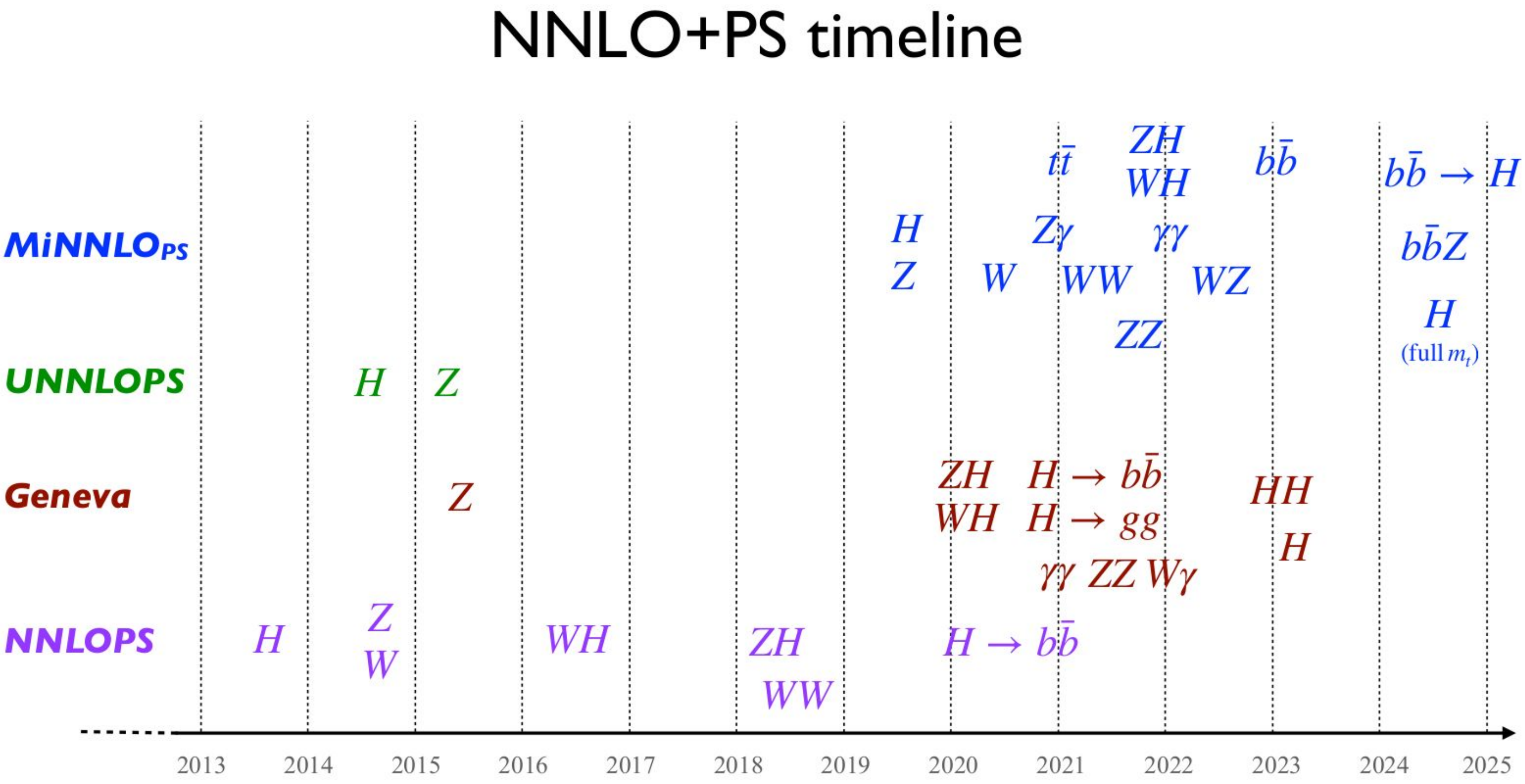
- Consider $F + X$ production (F=massive color singlet)
- NNLO accuracy for observables inclusive on radiation. $[d\sigma/dy_F]$
- NLO(LO) accuracy for $F + 1(2)$ jet observables (in the hard region). $[d\sigma/dp_{T,j_1}]$
  - appropriate scale choice for each kinematics regime
- Sudakov resummation from the Parton Shower (PS) $[\sigma(p_{T,j} < p_{T,\text{veto}})]$
- preserve the PS accuracy (leading log - LL)
  - possibly, no merging scale required.

- methods: reweighted MiNLO′ ("NNLOPS") [Hamilton,et al. '12,'13,...], UNNLOPS [Höche,Li,Prestel '14,...], Geneva [Alioli,Bauer,et al. '13,'15,'16,...], **MiNNLO$_{\text{PS}}$** [Monni,Nason,ER,Wiesemann,Zanderighi '19,...], Vincia+sector showers [Campbell et al, '21]

[Notation: From this point, $X = \sum_k \left(\frac{\alpha_S}{2\pi}\right)^k [X]^{(k)}$]

NNLO+PS: recent progress

[slide from M. Wiesemann]



NNLO+PS: main concepts and notation

- General idea: need to have (N)NLO accuracy across different jet multiplicities

	F (inclusive)	$F{+}j$ (inclusive)	$F{+}2j$ (inclusive)
F-FJ @ NLOPS	NLO	NLO	LO
F @ NNLOPS	NNLO	NLO	LO

$\text{NNLO}^{(F)}, \text{NNLO}_{>0}$ $\text{NLO}^{(FJ)}, \text{NLO}_{>1}$ $\text{LO}^{(F)}, \text{LO}_{>2}$

- Further emissions: parton shower
- (N)NLO calculation recast in MC language (radiation ordered in resolution variable)
  - resolution variables to measure 1st, 2nd,... emission
  - log dependence on resolution parameters → resummation (analytic / Sudakov FF)
  - resummation needs to be accurate enough
  - matching to NNLO ← resummation properties of resolution variable @ NNLL'

$\text{MiNNLO}_{\text{PS}}$: multiplicative-like matching / Geneva: additive-like matching

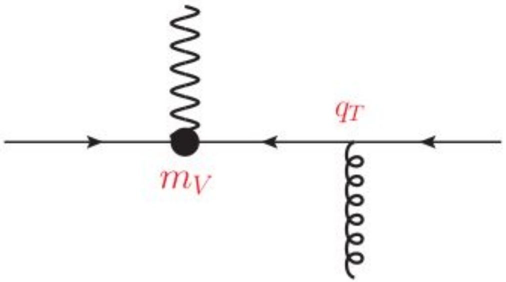
MiNNLO$_{PS}$

The MiNLO' method

Multiscale Improved NLO: a-priori choose scales in multijet NLO computation

[Hamilton,Nason,Zanderighi '12]

$$\bar{B}_{\rm NLO}^{\rm (FJ)} = \frac{\alpha_{\rm S}(\mu_R)}{2\pi}\left[B^{\rm (FJ)} + \frac{\alpha_{\rm S}}{2\pi}V^{\rm (FJ)}(\mu_R) + \frac{\alpha_{\rm S}}{2\pi}\int d\Phi_{\rm r} R^{\rm (FJ)}\right]$$



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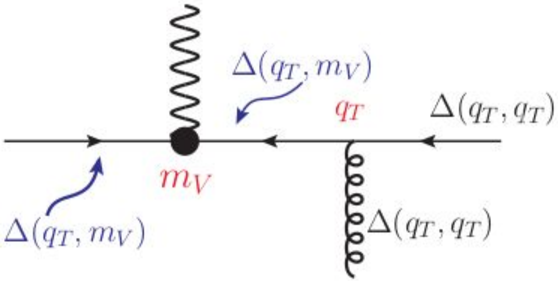
$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} = \frac{\alpha_{\mathrm{S}}(q_{\mathrm{T}})}{2\pi}\left[\Delta_{\mathrm{f}}^2(q_{\mathrm{T}})\left[B^{(\mathrm{FJ})}\left(1 + \frac{\alpha_{\mathrm{S}}}{2\pi}\tilde{S}_{\mathrm{f}}^{(1)}(q_{\mathrm{T}})\right) + \frac{\alpha_{\mathrm{S}}}{2\pi}V^{(\mathrm{FJ})}(\bar{\mu}_R)\right] + \frac{\alpha_{\mathrm{S}}}{2\pi}\int d\Phi_{\mathrm{r}}\Delta_{\mathrm{f}}^2(q_{\mathrm{T}})R^{(\mathrm{FJ})}\right]$$



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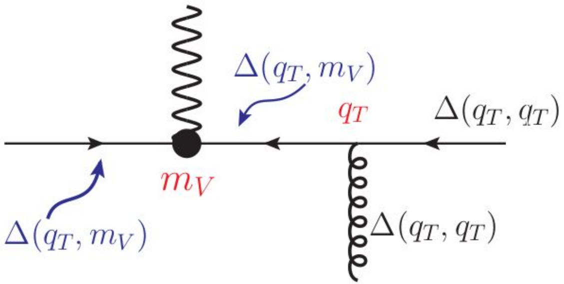
$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} = \frac{\alpha_{\mathrm{S}}(q_{\mathrm{T}})}{2\pi}\left[\Delta^2_{\mathrm{f}}(q_{\mathrm{T}})\left[B^{(\mathrm{FJ})}\left(1 + \frac{\alpha_{\mathrm{S}}}{2\pi}\tilde{S}^{(1)}_{\mathrm{f}}(q_{\mathrm{T}})\right) + \frac{\alpha_{\mathrm{S}}}{2\pi}V^{(\mathrm{FJ})}(\bar{\mu}_R)\right] + \frac{\alpha_{\mathrm{S}}}{2\pi}\int d\Phi_{\mathrm{r}}\Delta^2_{\mathrm{f}}(q_{\mathrm{T}})R^{(\mathrm{FJ})}\right]$$



- $\bar{\mu}_R = q_{\mathrm{T}}$
- $\Delta^2_{\mathrm{f}}(q_{\mathrm{T}}) = \exp(-\tilde{S}_{\mathrm{f}}(q_{\mathrm{T}}))$
- $\tilde{S}_{\mathrm{f}}(q_{\mathrm{T}}) = \int_{q_{\mathrm{T}}^2}^{m_{\mathrm{F}}^2} \frac{dq^2}{q^2}\left[A_{\mathrm{f}}(\alpha_{\mathrm{S}}(q^2))\log\frac{m_{\mathrm{F}}^2}{q^2} + B_{\mathrm{f}}(\alpha_{\mathrm{S}}(q^2))\right]$
- $\frac{\alpha_{\mathrm{S}}}{2\pi}\tilde{S}^{(1)}_{\mathrm{f}}(q_{\mathrm{T}}) = \frac{\alpha_{\mathrm{S}}}{2\pi}\left[\frac{1}{2}A_{1,\mathrm{f}}\log^2\frac{m_{\mathrm{F}}^2}{q_{\mathrm{T}}^2} + B_{1,\mathrm{f}}\log\frac{m_{\mathrm{F}}^2}{q_{\mathrm{T}}^2}\right]$
- $\mu_F = q_{\mathrm{T}}$

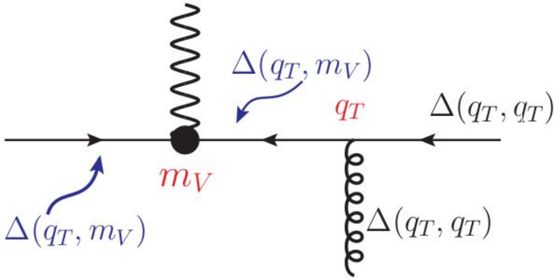
The MiNLO' method

Multiscale Improved NLO: a-priori choose scales in multijet NLO computation

[Hamilton,Nason,Zanderighi '12]

$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{NLO}} = \frac{\alpha_{\mathrm{S}}(\mu_R)}{2\pi}\left[B^{(\mathrm{FJ})} + \frac{\alpha_{\mathrm{S}}}{2\pi} V^{(\mathrm{FJ})}(\mu_R) + \frac{\alpha_{\mathrm{S}}}{2\pi}\int d\Phi_{\mathrm{r}} R^{(\mathrm{FJ})}\right]$$

$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} = \frac{\alpha_{\mathrm{S}}(q_{\mathrm{T}})}{2\pi}\left[\Delta^2_{\mathrm{f}}(q_{\mathrm{T}})\left[B^{(\mathrm{FJ})}\left(1 + \frac{\alpha_{\mathrm{S}}}{2\pi}\tilde{S}^{(1)}_{\mathrm{f}}(q_{\mathrm{T}})\right) + \frac{\alpha_{\mathrm{S}}}{2\pi} V^{(\mathrm{FJ})}(\bar{\mu}_R)\right] + \frac{\alpha_{\mathrm{S}}}{2\pi}\int d\Phi_{\mathrm{r}} \Delta^2_{\mathrm{f}}(q_{\mathrm{T}}) R^{(\mathrm{FJ})}\right]$$



resol. variable: p_T

☞ Sudakov FF included on F+j Born kinematics

- MiNLO-improved FJ yields finite results also when 1st jet is unresolved ($q_{\mathrm{T}} \to 0$)
- $\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}}$ allows to extend the validity of FJ−POWHEG [called "FJ-MiNLO" hereafter]

The MiNLO' method

- formal accuracy of FJ−MiNLO for inclusive observables carefully investigated.

[Hamilton et al. 1212.4504]

- possible to improve FJ−MiNLO such that inclusive NLO is recovered (NLO$^{(\mathrm{F})}$), without spoiling NLO accuracy of F+j (NLO$^{(\mathrm{FJ})}$):

MiNLO′ : NLO+PS merging of F and F+j, without merging scale

- accurate control of subleading small-p_{T} logarithms is needed:
  - include B_2 (NNLL) coefficient in MiNLO-Sudakov.
  - set scales in R, V and subtraction terms equal to q_{T}.
  - without the above requirements, spurious $\alpha_{\mathrm{S}}^{3/2}$ terms show up in $\sigma_{\mathrm{NLO}}^{(\mathrm{F})}$ after integration over q_{T}.

- MiNNLO$_{\mathrm{PS}}$: rather than upgrading the above method through reweighting, add analytic ingredients to get to NNLO

MiNNLO PS (I)

[Monni,Nason,ER,Wiesemann,Zanderighi '19-'20]

- from p_T resummation, differential cross section for $F+X$ production can be written as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_{\mathrm{T}}\mathrm{d}\Phi_{\mathrm{F}}} = \frac{\mathrm{d}}{\mathrm{d}p_{\mathrm{T}}}\left\{\mathcal{L}(\Phi_{\mathrm{F}}, p_{\mathrm{T}})\exp(-\tilde{S}(p_{\mathrm{T}}))\right\} + R_{\mathrm{finite}}(p_{\mathrm{T}})$$

$$\mathcal{L}(\Phi_{\mathrm{F}}, p_{\mathrm{T}}) \ni \{H^{(1)}, H^{(2)}, C^{(1)}, C^{(2)}, (G^{(1)}\cdot G^{(1)})\} \qquad R_{\mathrm{finite}}(p_{\mathrm{T}}) = \frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} - \frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}}$$

- recast it, to match the POWHEG $\bar{B}^{(\mathrm{FJ})}(\Phi_{\mathrm{FJ}})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{D(p_{\mathrm{T}}) + \frac{R_{\mathrm{finite}}(p_{\mathrm{T}})}{\exp[-\tilde{S}(p_{\mathrm{T}})]}\right\}$$

$$D(p_{\mathrm{T}}) \equiv -\frac{\mathrm{d}\tilde{S}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{\mathrm{d}\mathcal{L}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}} \qquad \tilde{S}(p_{\mathrm{T}}) = \int_{p_{\mathrm{T}}}^{Q}\frac{dq^2}{q^2}\left[A_{\mathrm{f}}(\alpha_{\mathrm{S}}(q))\log\frac{Q^2}{q^2} + B_{\mathrm{f}}(\alpha_{\mathrm{S}}(q))\right]$$

- expand the above integrand in power of $\alpha_{\mathrm{S}}(p_{\mathrm{T}})$, keep the terms that are needed to get NLO$^{(\mathrm{F})}$ & NNLO$^{(\mathrm{F})}$ accuracy, when integrating over p_{T}

MiNNLO PS (II)

1→ 0 resolution

$$\frac{\mathrm{d}\bar{B}(\Phi_{\mathrm{FJ}})}{\mathrm{d}\Phi_{\mathrm{FJ}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(1)}\left(1+\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}[\tilde{S}(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(2)} + [D(p_{\mathrm{T}})]^{(\geq 3)} F^{\mathrm{corr}}_{\ell}(\Phi_{\mathrm{FJ}})\right\}$$

- $[D(p_{\mathrm{T}})]^{(\geq 3)} = \underbrace{-\frac{\mathrm{d}\tilde{S}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{\mathrm{d}\mathcal{L}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}}_{D} - \frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}[D(p_{\mathrm{T}})]^{(1)} - \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^2[D(p_{\mathrm{T}})]^{(2)}$
- $F^{\mathrm{corr}}_{\ell}(\Phi_{\mathrm{FJ}})$: projection → recover $[D(p_{\mathrm{T}})]^{(\geq 3)}$ when integrating over Φ_{FJ} at fixed $(\Phi_{\mathrm{F}}, p_{\mathrm{T}})$

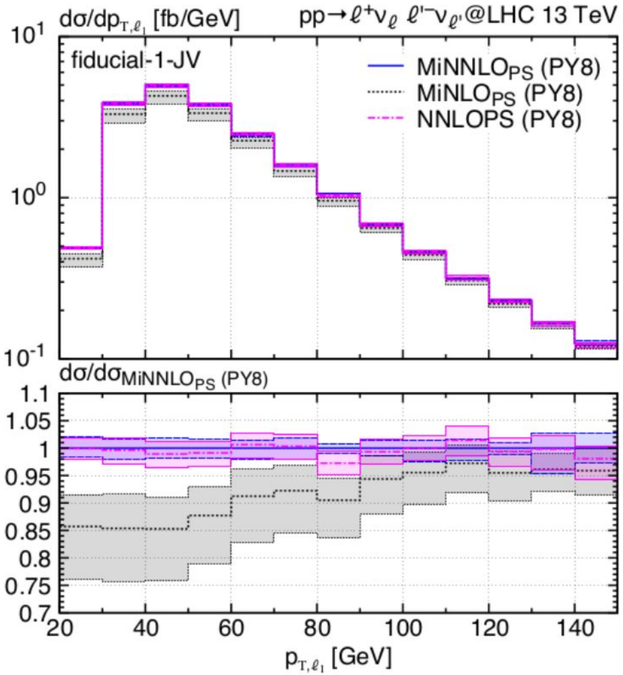
. The second radiation is generated by the usual POWHEG mechanism.

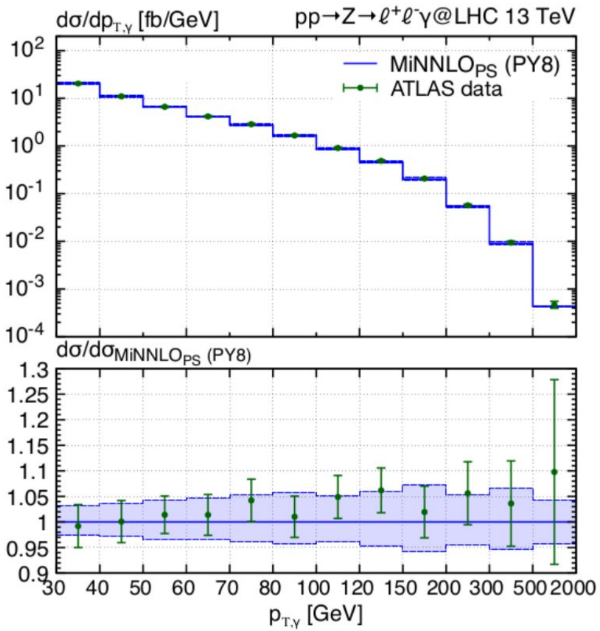
$$\mathrm{d}\sigma = \bar{B}(\Phi_{\mathrm{FJ}})\,\mathrm{d}\Phi_{\mathrm{FJ}}\left\{\Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \mathrm{d}\Phi_{\mathrm{rad}}\Delta_{\mathrm{pwg}}(p_{\mathrm{T,rad}})\frac{R(\Phi_{\mathrm{FJ}},\Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})}\right\}$$

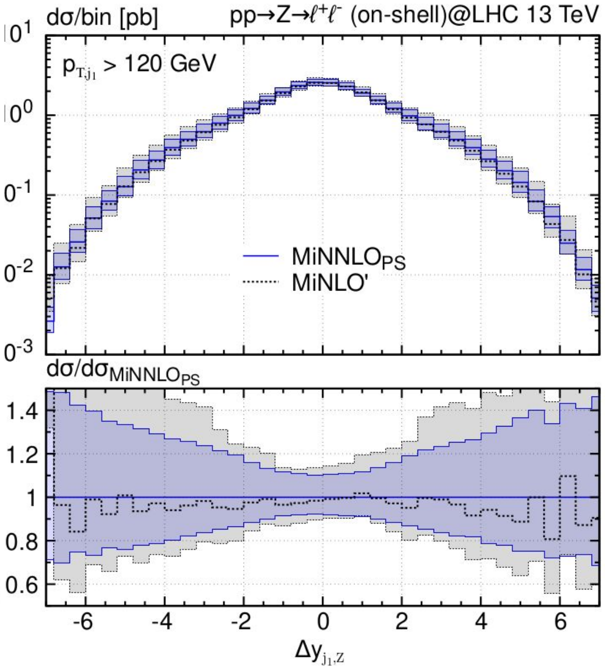
2→ 1 resolution

. if emissions are strongly ordered, same emission probabilities as in k_t-ordered shower
→ LL shower accuracy preserved

Results (I): color singlet







- diboson processes
  [Lombardi,Wiesemann,Zanderighi+{Buonocore,Koole,Rottoli} +{Lindert,Zanoli} '20-'22]

- DY@NNLOPS:
  → $NLO^{(FJ)}$ accuracy retained

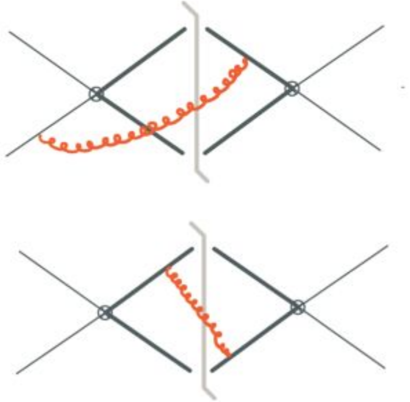
MiNNLO for $t\bar{t}$

- Starting point: resummation formula for $t\bar{t}$ transverse momentum. [Catani,Grazzini,Torre '14]
  Very schematically:

$$d\sigma^F_{\rm res} \sim \frac{d}{dp_T}\left\{e^{-S}\,\mathrm{Tr}(\mathbf{H}\mathbf{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int \frac{dq^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

$$\mathrm{Tr}(\mathbf{H}\mathbf{\Delta}) = \langle M|\,\mathbf{\Delta}\,|M\rangle, \quad \mathbf{\Delta} = \mathbf{V}^\dagger\,\mathbf{D}\,\mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int\frac{dq^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\mathbf{\Gamma}^{(1)}_{\mathbf{t}}+\frac{\alpha_s^2(q)}{(2\pi)^2}\mathbf{\Gamma}^{(2)}_{\mathbf{t}}\right]\right\}$$



- With some approximations (respecting our goal), terms due to soft interference can be rearranged so that the "resummation" can be eventually recasted as:

$$d\sigma^F_{\rm res} \sim \frac{d}{dp_T}\left\{\sum_{i\in\text{colours}} e^{-\bar{S}_i}\,\underbrace{c_i\,\overline{H}\,\overline{(C\otimes f)}\,\overline{(C\otimes f)}}_{\equiv\,\overline{\mathscr{L}}_i}\right\} + \mathcal{O}(\alpha_S^5)$$

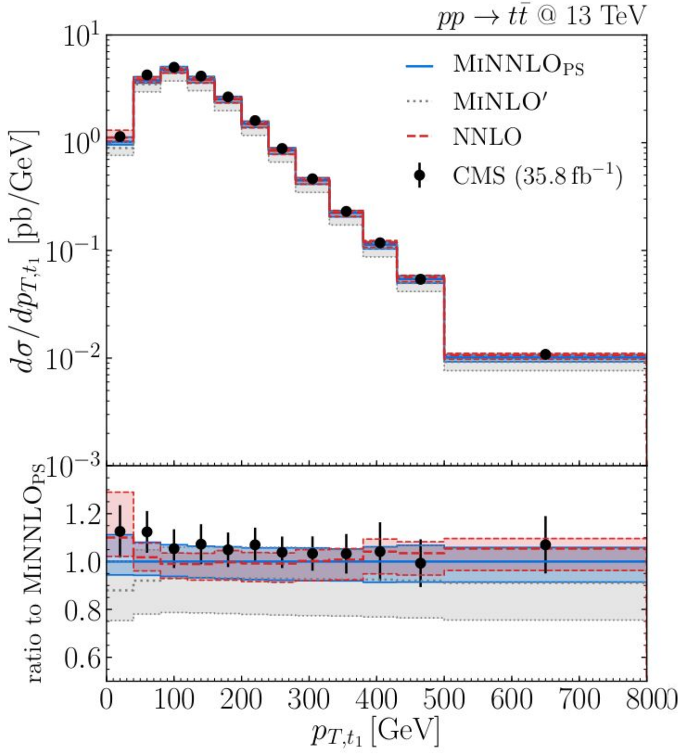
inputs from [Catani, Devoto, Grazzini, Kallweit, Mazzitelli + Sargsyan '19]

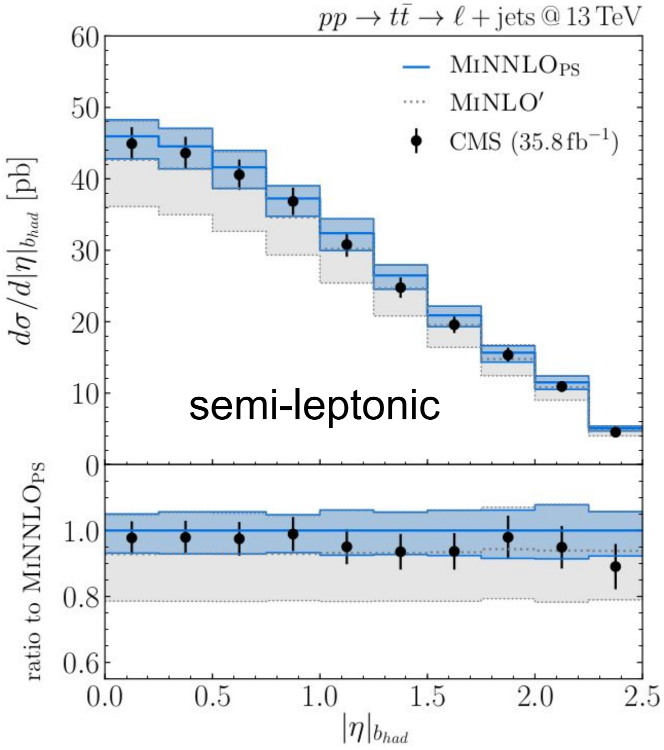
paper: [Catani, Devoto, Grazzini, Mazzitelli '23]

- Each term has the "same structure" as in the color-singlet case!

Results (II): $t\bar{t}$

[$\mathbf{t\bar{t}}$: Mazzitelli,Monni,Nason,ER,Wiesemann,Zanderighi '20-'21]
[$\mathbf{b\bar{b}}$: Mazzitelli,Ratti,Wiesemann,Zanderighi '23]

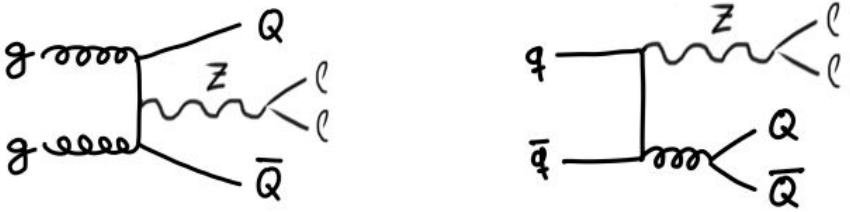


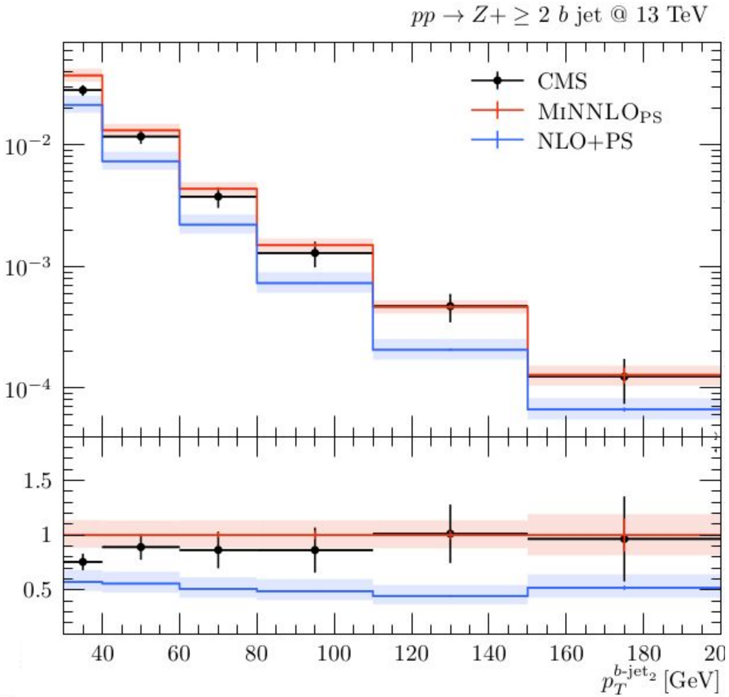


- nice agreement with NNLO (and with data - both ATLAS and CMS). $\mu_{core} = H_T/4$
- implemented top-quark decays @ tree level + approximated off-shell effects

Results (III): Zbb̄ (4FS) NNLO+PS

[Mazzitelli,Sotnikov,Wiesemann '24]





- 4FS/5FS: known at NLO+PS (also with combination)
- differences 4FS/5FS, tension 4FS and data
- 4FS: large pert. uncertainties

- NNLO correction large (50%), no overlap with NLO, still large pert. uncertainty
  - 2-loop amplitude:

$$2\mathrm{Re}\langle R^{(0)} | R^{(2)} \rangle = \sum_{i=1}^{4} \kappa_i \log^i(m_b/\mu_R) + 2\mathrm{Re}\langle R_0^{(0)} | R_0^{(2)} \rangle + \mathcal{O}(m_b/\mu)$$

massive amplitude — coefficients of massification (κ_i) — massless amplitude [Abreu, Cordero, Ita, Klinkert, Page, Sotnikov '21] — power corrections

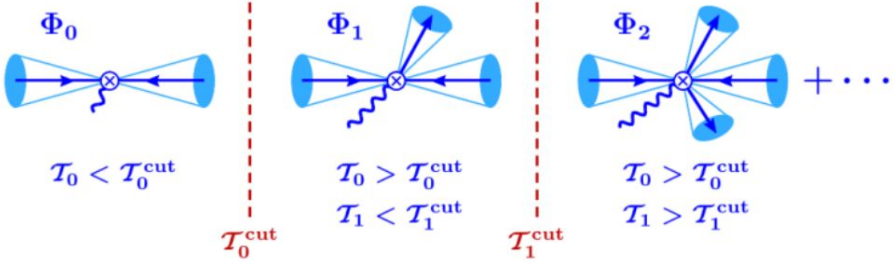
- MiNNLO$_{PS}$: tension with data lifted (+ good agreement with NLO+PS 5FS where expected)

Geneva

Geneva: main idea

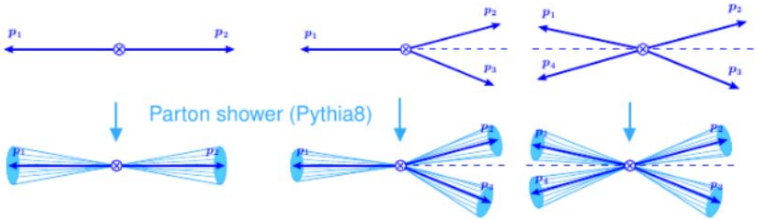
- Main idea: construct IR-finite events using a resolution parameter τ_N, whose resummation properties are accurately known
  - slice phase space into jet-bins: τ_N^{cut} translate an M-parton event to a N-jet event $(N \leq M)$, fully differential in Φ_N.

$$\mathcal{T}_N = \sum_{i=1}^{n} \min_{q_1, \dots, q_N} (q_a \cdot \hat{p}_i, q_b \cdot \hat{p}_i, q_1 \cdot \hat{p}_i, \dots, q_N \cdot \hat{p}_i)$$



$\tau_N \to 0$ *N*-jet event where extra emissions are soft or collinear to resolved jets

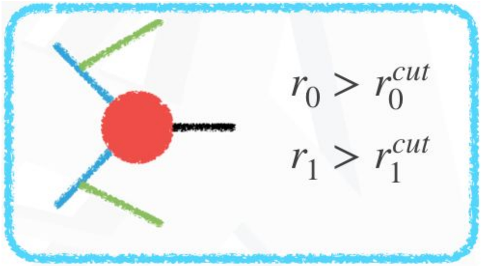
- Parton Shower: add radiation to higher multiplicities bins, fill 0- and 1-jet bins
  - constraints on τ_N^{cut}: PS not allowed to affect the accuracy of the cross section reached at partonic level



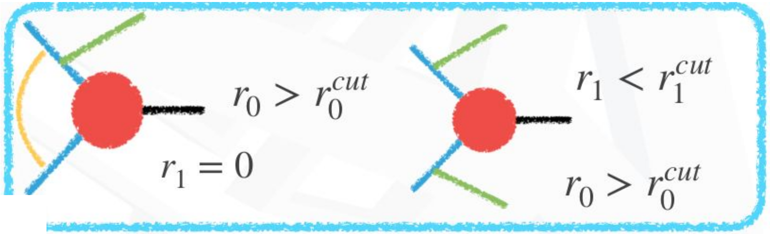
1st papers:
0- and 1-jettiness as res. variable

Geneva: details

$$\tau_i^{cut} \equiv r_i^{cut}$$

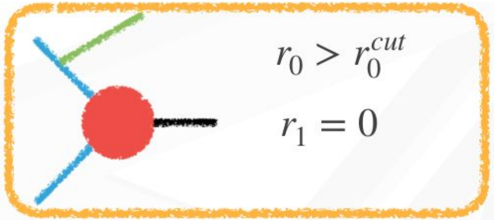


- 2-jet inclusive: 2 resolved emissions.
  (final) events must have integrated $LO_{>2}$ accuracy
- event "weight": full LO matrix element + resummation
  (terms from "complement" to other jet bins)



- 1-jet exclusive: 1 hard + 1 unresolved.
  (final) events must have integrated NLO_1 accuracy
  → local subtraction (~ q_T/jettiness subtraction)

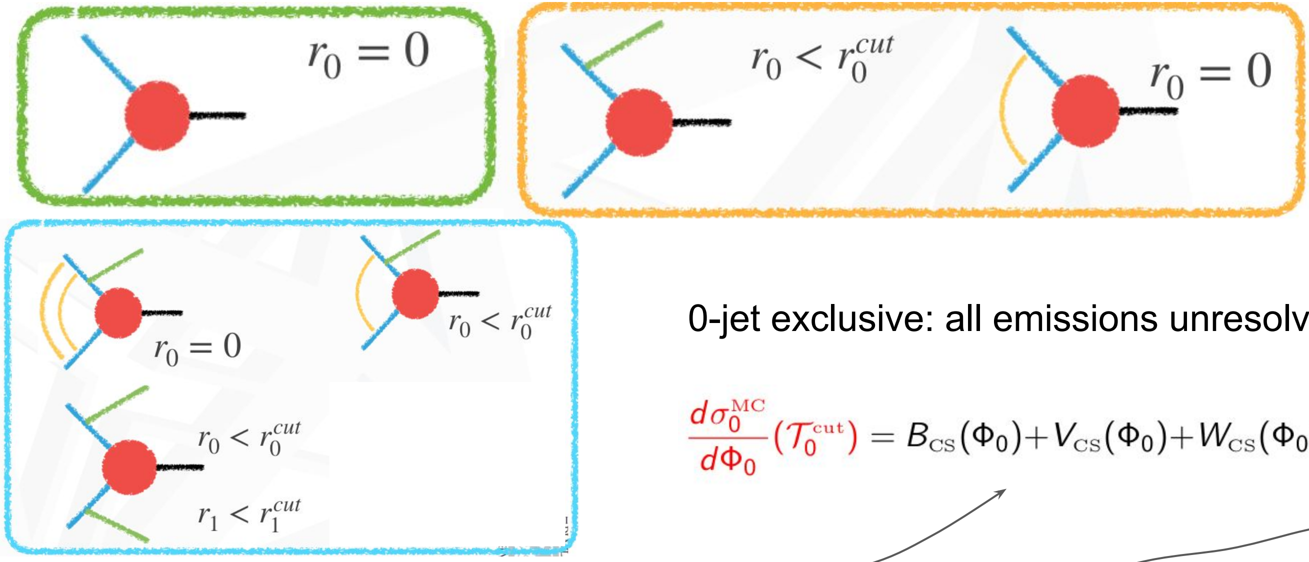
$$\frac{d\sigma^{\rm MC}_{\geq 1}}{d\Phi_1} = \left[\left(\frac{d\sigma^{\rm NNLL'}}{d\Phi_0\, d\mathcal{T}_0} - \frac{d\sigma^{\rm NNLL'}}{d\Phi_0\, d\mathcal{T}_0}\bigg|_{\rm NLO_1}\right) P_{0\to 1}(\Phi_1) + \frac{d\sigma^{\rm NLO_1}_{\rm CS+jet}}{d\Phi_1}\right] \theta(\mathcal{T}_0 - \mathcal{T}_0^{\rm cut})$$



- $P_{0\to 1}$ needed (resummation not expressed in full Φ_1)
- 1 unresolved: τ_1^{cut} must be resummed
  →gets $U_1(\Phi_1, \mathcal{T}_1^{\rm cut})$ weight in final expression
- τ_0 preserved by maps close to singular region

Geneva: details

$\tau_i^{cut} \equiv r_i^{cut}$



0-jet exclusive: all emissions unresolved

$$\frac{d\sigma_0^{\rm MC}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}) = B_{\rm CS}(\Phi_0) + V_{\rm CS}(\Phi_0) + W_{\rm CS}(\Phi_0) + \frac{d\sigma^{\rm NNLL'}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}) + \frac{d\sigma_{\rm CS}^{\rm nonsing}}{d\Phi_0}(\mathcal{T}_0^{\rm cut})$$

- contains: hard function + resummation below τ_0^{cut} + $NLO_{>1}$ below cut (subtracted through expansion of τ_0 NNLL' resummation)

- no shower emissions above τ_0^{cut}

Geneva: final partonic formula

→ τ_0 resummation

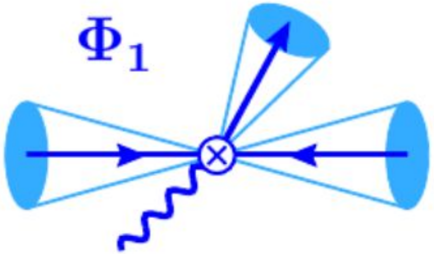
Φ_0

$$\frac{\mathrm{d}\sigma^{\mathrm{MC}_0}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_0^{\mathrm{nons}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})$$

$$\frac{\mathrm{d}\sigma_0^{\mathrm{nons}}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_0^{\mathrm{NNLO}_0}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{\mathrm{d}\sigma^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{\mathrm{NNLO}_0}$$

→ τ_1 resummation

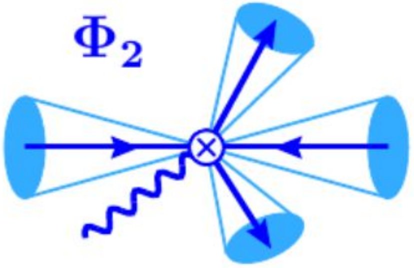
Φ_1



$$\frac{\mathrm{d}\sigma^{\mathrm{MC}_1}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq 1}^{C}}{\mathrm{d}\Phi_1} U_1(\Phi_1, \mathcal{T}_1^{\mathrm{cut}})\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_1^{\mathrm{match}}}{\mathrm{d}\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}})$$

$$\frac{d\sigma_{\geq 1}^{\mathrm{MC}}}{d\Phi_1} = \left[\left(\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0\, d\mathcal{T}_0} - \frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0\, d\mathcal{T}_0}\bigg|_{\mathrm{NLO}_1}\right) P_{0\to 1}(\Phi_1) + \frac{d\sigma_{\mathrm{CS+jet}}^{\mathrm{NLO}_1}}{d\Phi_1}\right] \theta(\mathcal{T}_0 - \mathcal{T}_0^{\mathrm{cut}})$$

Φ_2



$$\frac{\mathrm{d}\sigma^{\mathrm{MC}_{\geq 2}}}{\mathrm{d}\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_{\geq 1}^{C}}{\mathrm{d}\Phi_1} U_1'(\Phi_1, \mathcal{T}_1)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\Big|_{\Phi_1 = \Phi_1^{\mathcal{T}}(\Phi_2)} \times \mathcal{P}(\Phi_2)\, \theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) + \frac{\mathrm{d}\sigma_{\geq 2}^{\mathrm{match}}}{\mathrm{d}\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$

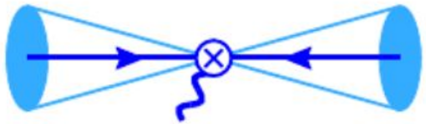
023

Geneva: final partonic formula

→ τ_0 resummation

→ τ_1 resummation

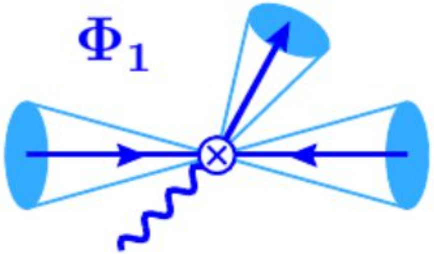
Φ_0



$$\frac{d\sigma^{MC_0}}{d\Phi_0}(\mathcal{T}_0^{cut}) = \frac{d\sigma^{NNLL'}}{d\Phi_0}(\mathcal{T}_0^{cut}) + \frac{d\sigma_0^{nons}}{d\Phi_0}(\mathcal{T}_0^{cut})$$

$$\frac{d\sigma_0^{nons}}{d\Phi_0}(\mathcal{T}_0^{cut}) = \frac{d\sigma_0^{NNLO_0}}{d\Phi_0}(\mathcal{T}_0^{cut}) - \left[\frac{d\sigma^{NNLL'}}{d\Phi_0}(\mathcal{T}_0^{cut})\right]_{NNLO_0}$$

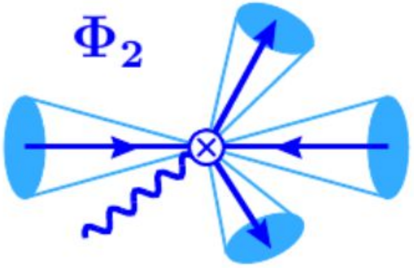
Φ_1



$$\frac{d\sigma^{MC_1}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{cut}; \mathcal{T}_1^{cut}) = \frac{d\sigma^{C}_{\geq 1}}{d\Phi_1} U_1(\Phi_1, \mathcal{T}_1^{cut})\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{cut}) + \frac{d\sigma_1^{match}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{cut}; \mathcal{T}_1^{cut})$$

$$\frac{d\sigma^{MC}_{\geq 1}}{d\Phi_1} = \left[\left(\frac{d\sigma^{NNLL'}}{d\Phi_0\, d\mathcal{T}_0} - \frac{d\sigma^{NNLL'}}{d\Phi_0\, d\mathcal{T}_0}\bigg|_{NLO_1}\right) P_{0\to 1}(\Phi_1) + \frac{d\sigma^{NLO_1}_{CS+jet}}{d\Phi_1}\right] \theta(\mathcal{T}_0 - \mathcal{T}_0^{cut})$$

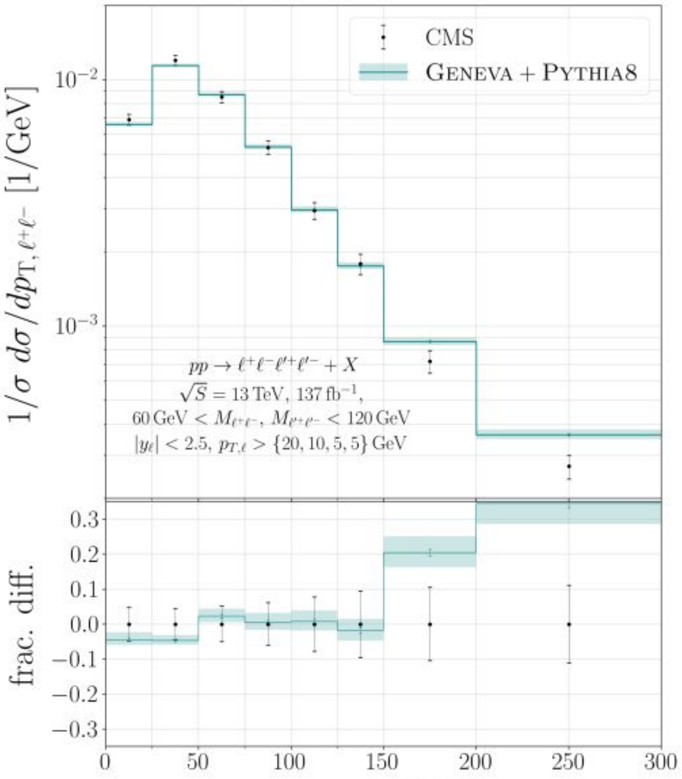
Φ_2



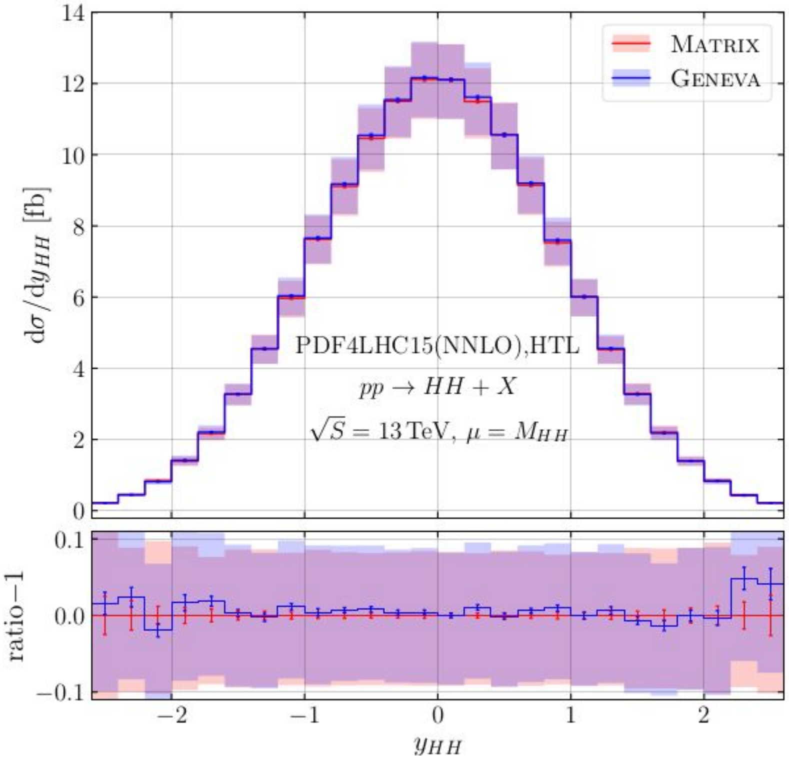
$$\frac{d\sigma^{MC_{\geq 2}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{cut}, \mathcal{T}_1 > \mathcal{T}_1^{cut}) = \frac{d\sigma^{C}_{\geq 1}}{d\Phi_1} U_1'(\Phi_1, \mathcal{T}_1)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{cut})\Big|_{\Phi_1 = \Phi_1^{\mathcal{T}}(\Phi_2)} \times \mathcal{P}(\Phi_2)\, \theta(\mathcal{T}_1 > \mathcal{T}_1^{cut}) + \frac{d\sigma^{match}_{\geq 2}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{cut}, \mathcal{T}_1 > \mathcal{T}_1^{cut})$$

023

Results (I): colour singlet



- diboson processes
  → large p_T → EW Sudakov effects (?)



- pp → HH

Subtraction in Geneva

- Formulated in full generality → 0-jettiness can be changed with $\{q_T, p_T^j\}$

- Jettiness subtraction is non local

→ missing power corrections below τ_0^{cut} (and τ_1^{cut})

→ small a-posteriori reweighting

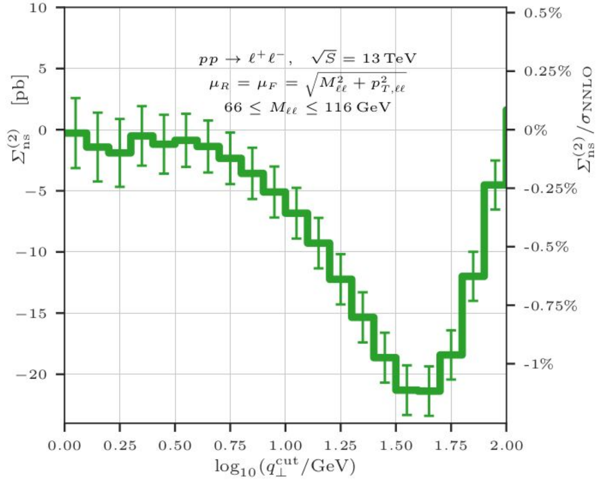
→ can be ameliorated using other resolution parameters

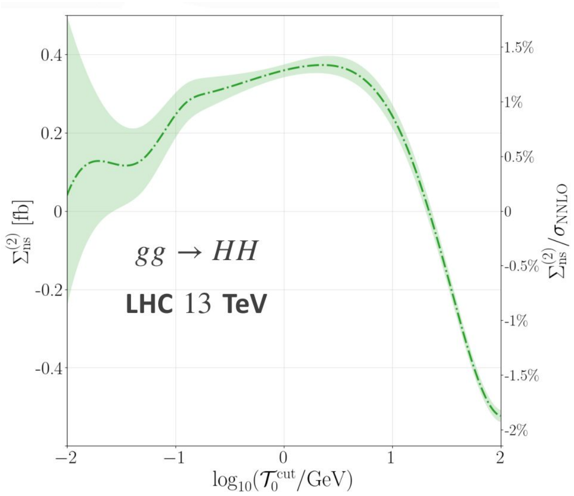
(+ smart subtraction in 0-jet bin, using only LO_1)

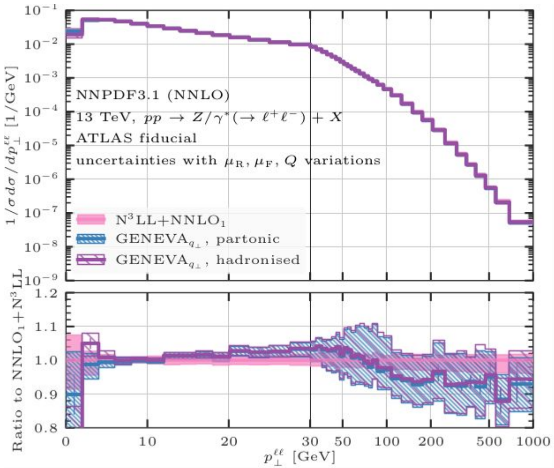
[Geneva DY '21, Geneva WW '23]

different resol. parameters

change shower interface



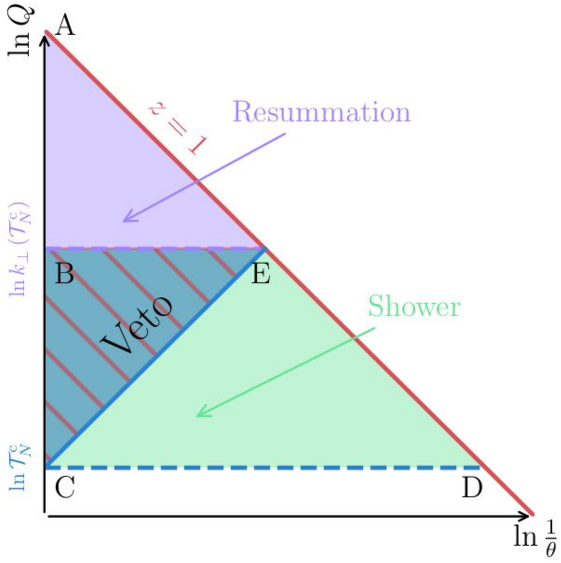




- $MiNNLO_{PS}$: Sudakov form factor suppresses $p_T \to 0$ limit

Results (II): different showers

[adapted from slide by S. Alioli]



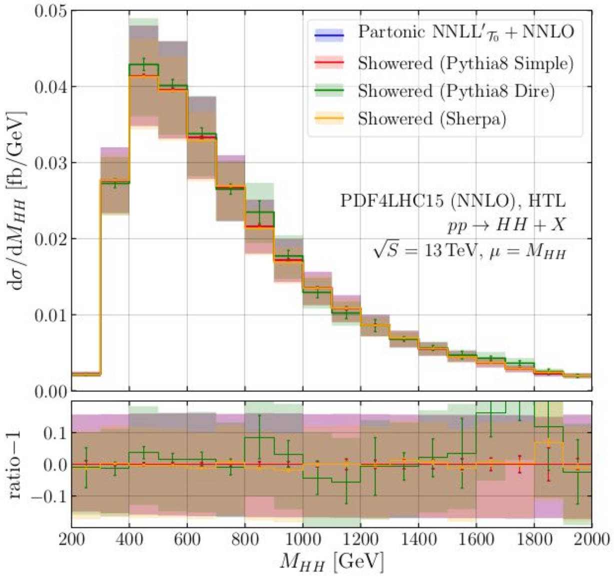
$\mathcal{T}_N(\Phi_{N+1})$ measures the hardness of the N+1-th emission

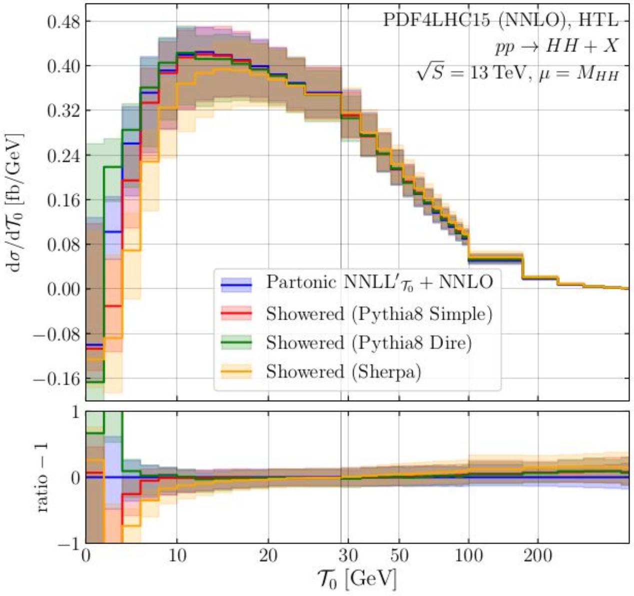
- If shower ordered in k_T, start from largest value allowed by N-jettiness
- Let the shower evolve unconstrained.
- At the end veto an event if after $M \geq 1$ shower emissions $\mathcal{T}_N(\Phi_{N+M}) > \mathcal{T}_N(\Phi_N + 1)$ and **retry** the whole shower.

$$\mathcal{T}_{N+M-1}(\Phi_{N+M}) \leq \mathcal{T}_{N+M-2}(\Phi_{N+M}) \leq \ldots \leq \mathcal{T}_N(\Phi_{N+M})$$

- 2-jet bin: avoid spoiling resummation accuracy of τ   (0/1 jet bin: start at resol. cut)
- shower accuracy for other observables more subtle

Results (II): different showers





Pythia8 vs. Dire vs. Sherpa

Results (III): ptj as resolution variables

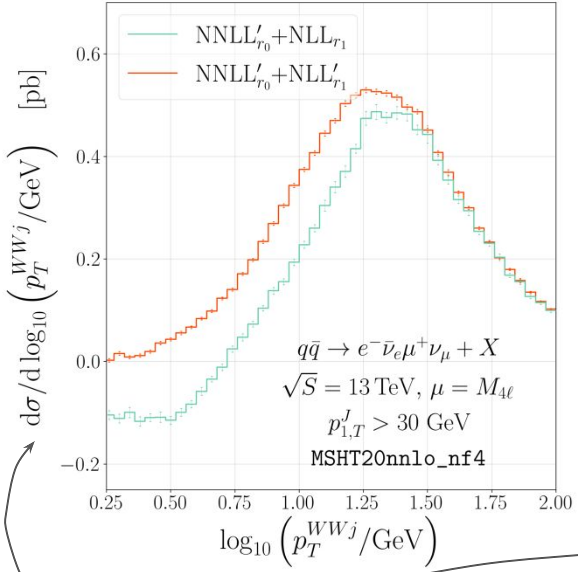
[Gavardi,Lim,Alioli,Tackmann '23]

r_o^{cut} : pt^{j1} @ NNLL'

r_1^{cut} : pt^{j2} @ NLL'

$$\frac{d\sigma_1^{MC}}{d\Phi_1}(r_1^{cut}) = \Bigg\{ \left[\frac{d\sigma^{NNLL' r_0}}{d\Phi_0\, dr_0} - \frac{d\sigma^{NNLL' r_0}}{d\Phi_0\, dr_0}\bigg|_{NLO_1} \right] \mathcal{P}_{0\to 1}(\Phi_1)\, U_1(\Phi_1, r_1^{cut})$$
$$+ \frac{d\sigma^{NLO_1}}{d\Phi_1}(r_1^{cut}) + \frac{d\sigma^{NLL' r_1}}{d\Phi_1}(r_1^{cut}) - \frac{d\sigma^{NLL' r_1}}{d\Phi_1}(r_1^{cut})\bigg|_{NLO_1} \Bigg\} \theta(r_0 > r_0^{cut})$$
$$+ \frac{d\sigma^{LO_1}_{nonproj}}{d\Phi_1}\, \theta(r_0 < r_0^{cut})$$

$$\frac{d\sigma_2^{MC}}{d\Phi_2} = \Bigg\{ \left[\frac{d\sigma^{NNLL' r_0}}{d\Phi_0\, dr_0} - \frac{d\sigma^{NNLL' r_0}}{d\Phi_0\, dr_0}\bigg|_{NLO_1} \right] \mathcal{P}_{0\to 1}(\Phi_1)\, U_1'(\Phi_1, r_1)\, \mathcal{P}_{1\to 2}(\Phi_2)$$
$$+ \frac{d\sigma^{LO_2}}{d\Phi_2} + \left[\frac{d\sigma^{NLL' r_1}}{d\Phi_1\, dr_1} - \frac{d\sigma^{NLL' r_1}}{d\Phi_1\, dr_1}\bigg|_{LO_2} \right] \mathcal{P}_{1\to 2}(\Phi_2) \Bigg\} \theta(r_1 > r_1^{cut})\, \theta(r_0 > r_0^{cut})$$
$$+ \frac{d\sigma^{LO_2}_{nonproj}}{d\Phi_2}\, \theta(r_1 < r_1^{cut})\, \theta(r_0 > r_0^{cut})\,.$$

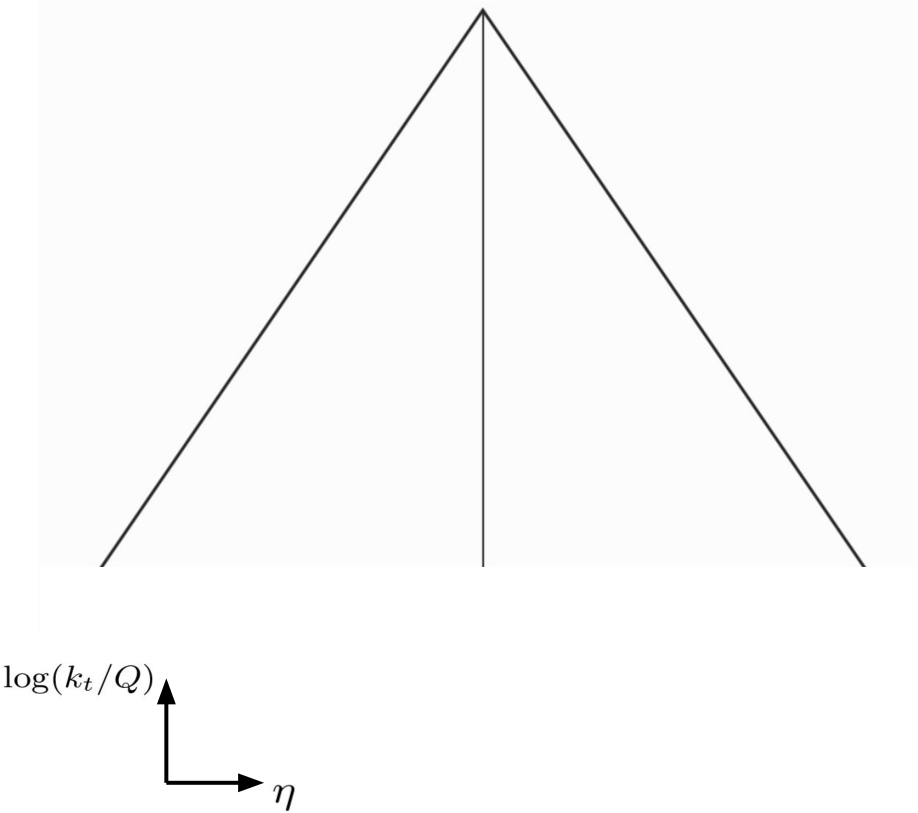


- better physical behaviour (if $pt^{j2} << pt^{j1}$)

- easier shower matching than with τ_0

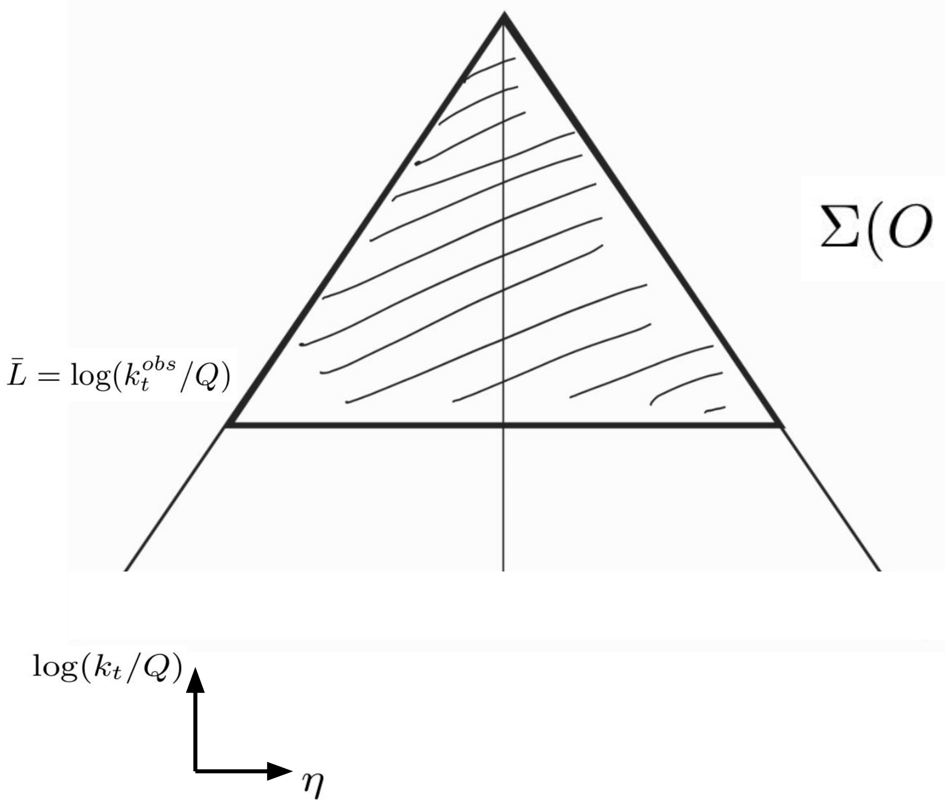
what’s next

Matching vs. shower accuracy



Resol. parameter: choice not crucial for NNLO accuracy

Matching vs. shower accuracy



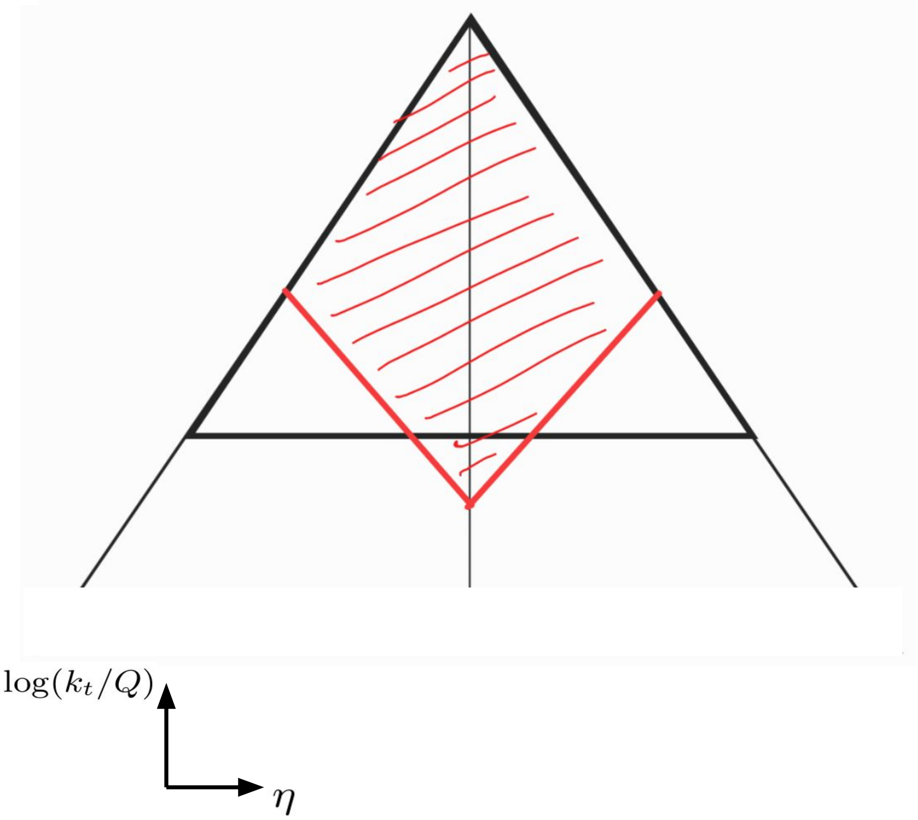
Resol. parameter: interplay with shower crucial for log. accuracy

$$\Sigma(O < e^{\bar{L}})$$

$$O \sim \frac{k_t}{Q}$$

→ shaded area: correct veto

Matching vs. shower accuracy



Resol. parameter: interplay with shower crucial for log. accuracy

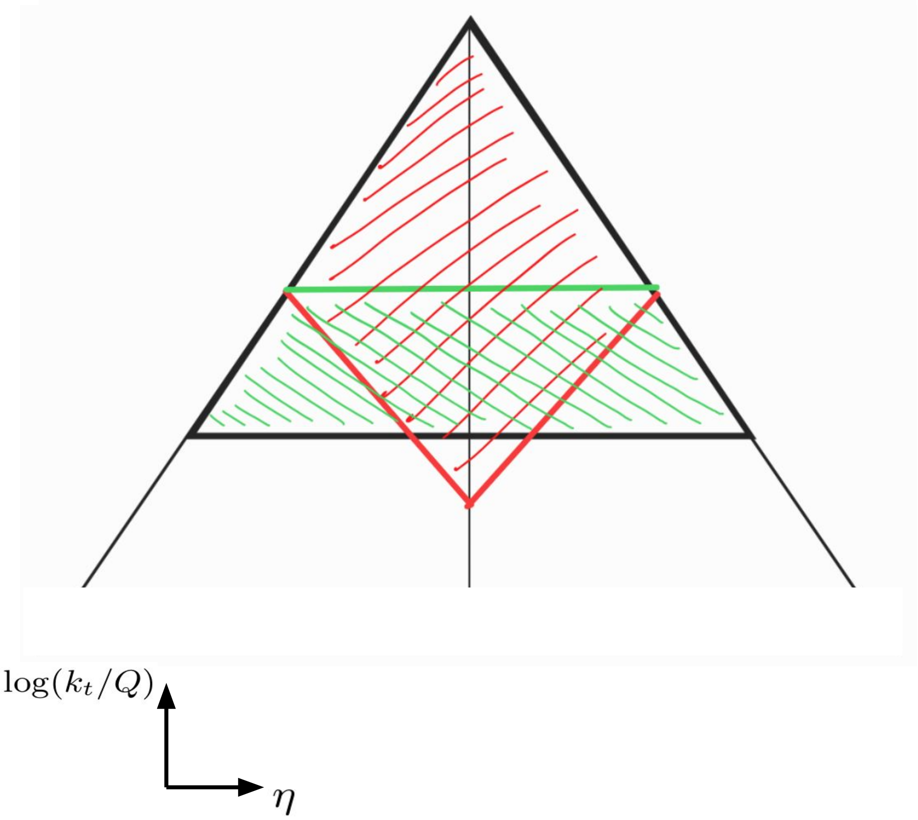
$$O \sim \frac{k_t}{Q}$$

scaling of gen. resolution

$$O_{\text{gen}} \sim \frac{k_t}{Q} e^{-\beta_{\text{gen}}|\eta|}$$

→ generator vetoes red area

Matching vs. shower accuracy



Resol. parameter: interplay with shower crucial for log. accuracy

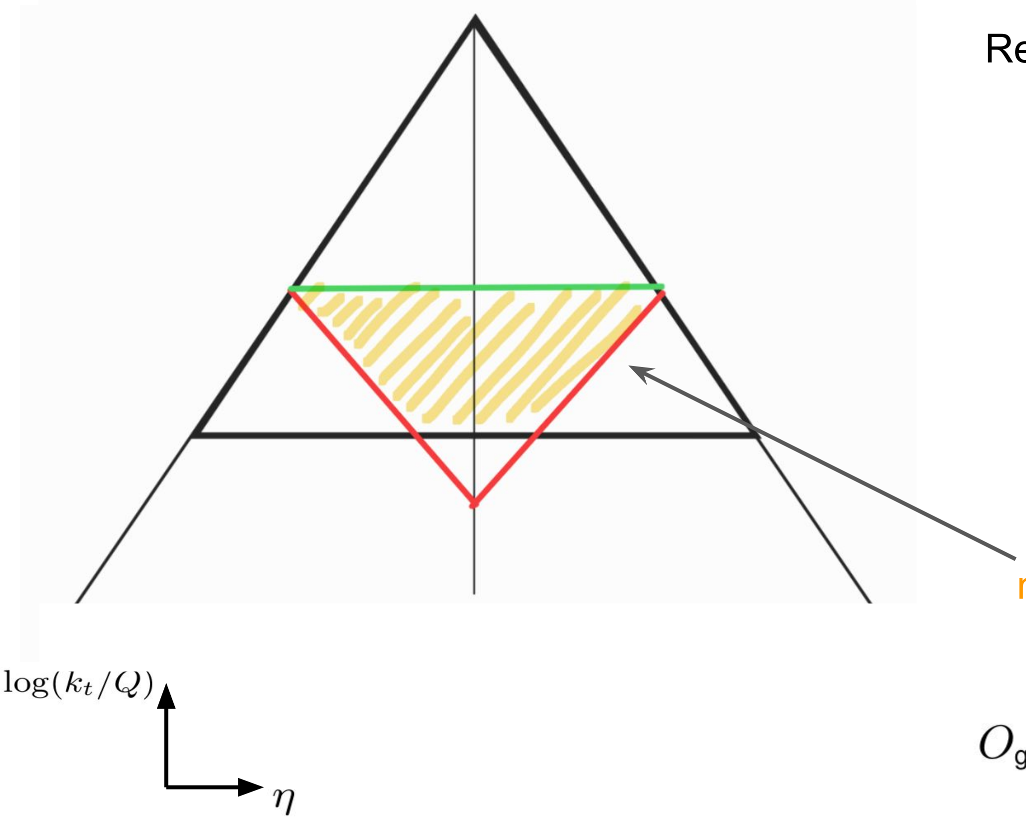
$$O \sim \frac{k_t}{Q}$$

scaling of PS resolution

$$O_{\text{PS}} \sim \frac{k_t}{Q} e^{-\beta_{\text{PS}}|\eta|}$$

→ PS: vetoes green area (here β_{PS}=0)

Matching vs. shower accuracy



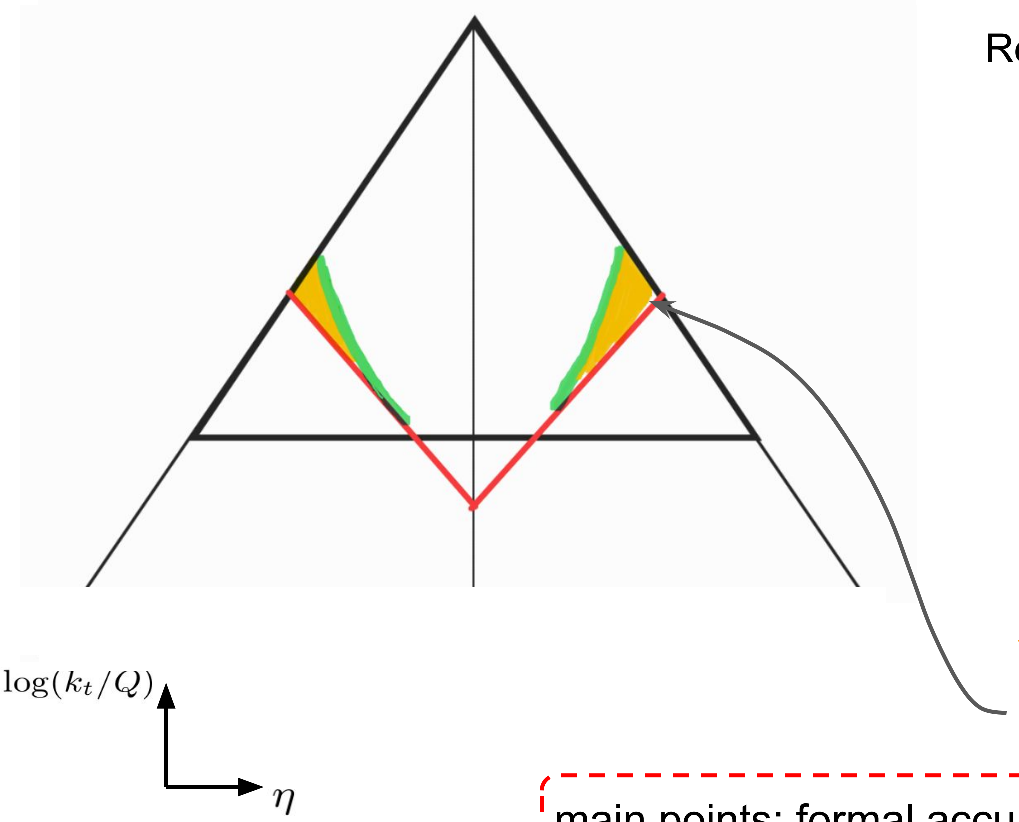
Resol. parameter: interplay with shower crucial for log. accuracy

$$O \sim \frac{k_t}{Q}$$

mismatch of ordering → double counting → breaking of LL accuracy

$$O_{\text{gen}} \sim \frac{k_t}{Q} e^{-\beta_{\text{gen}}|\eta|} \qquad O_{\text{PS}} \sim \frac{k_t}{Q} e^{-\beta_{\text{PS}}|\eta|}$$

Matching vs. shower accuracy



Resol. parameter: interplay with shower crucial for log. accuracy

$$O \sim \frac{k_t}{Q}$$

NLL accuracy: no contour mismatch in single-log region.

e.g. hard-collinear region

main points: formal accuracy + assessment of PS uncertainty

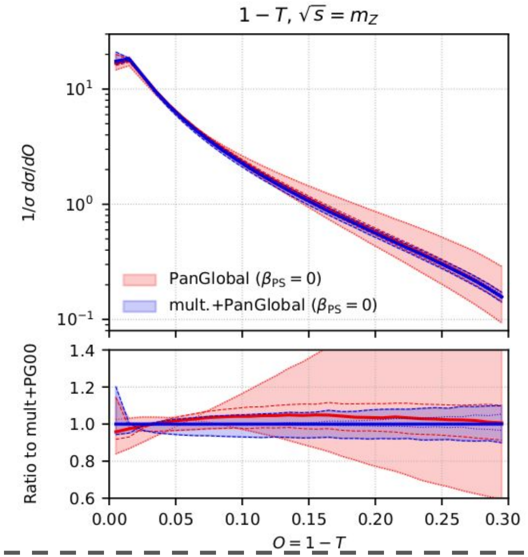
Matching vs. shower accuracy

- Interplay with shower already delicate at NLO+PS (if aiming at NLL)

  - thrust in e^+e^-: NLO+PS multiplicative matching + NLL shower [Hamilton et al. 2301.09645]

  - dots: modified splitting function in hard region
  - dashes: μ_R scale variation (also in hard matrix elements)
  - if wrong matching, shower breaks

  - here matching fulfils NNDL accuracy $\alpha_s^n L^{2n-2}$ (i.e. the same accuracy of NLL+NLO)



- At NNLO+PS: more complex → 1st and 2nd emission from generator

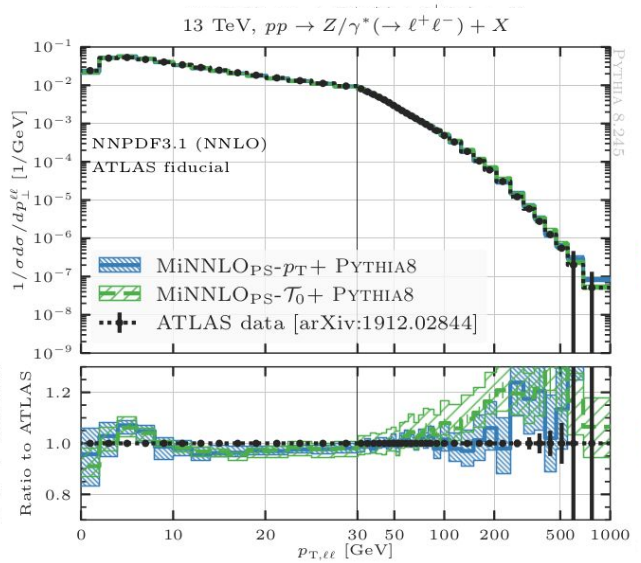
- MiNNLO-q_T / Geneva-p_T^{j1}: LL matching to kt-ordered showers straightforward
- Geneva-τ_0 / Geneva-q_T: truncated-vetoed showers to match with kt-ordered showers
- MiNNLO-τ_0 requires changing POWHEG mappings…

MiNNLO(τ_0) / Geneva(q_T)

- MiNNLOPS (0-jettiness):

→ $[D(p_T)]^{(\geq 3)}$ / Sudakov fact. changed accordingly

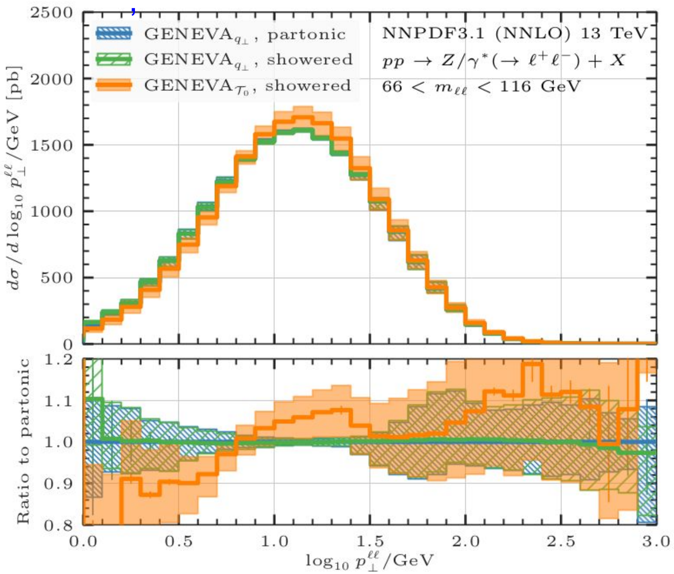
[Ebert,Rottoli,Wiesemann,Zanderighi,Zanoli '24]



- Geneva (q_T):

→ delicate interplay with shower

[Geneva DY '21: Alioli,Bauer,Broggio, Gavardi,Kallweit,Lim,Nagar,Napoletano,Rottoli '21]



- matching with parton shower not fully accurate here (mappings not suited yet → j1-region spoiled)

- some differences in DY pT spectrum when using τ_0

NLOQCD + NLOEW + PS

- NLO_{EW}+PS not conceptually solved in full generality
  - bottleneck: processes with "QCD/EW interference" at LO
  - possible for some processes, e.g. DY, dibosons

POWHEG: exact matching of EW corrections for *n*- and *n+1*-body contributions

1st papers: [Barze et al. '12,'13, Carloni et al. '16]

- Use the `POWHEG BOX RES` framework [Jezo, Nason '15]

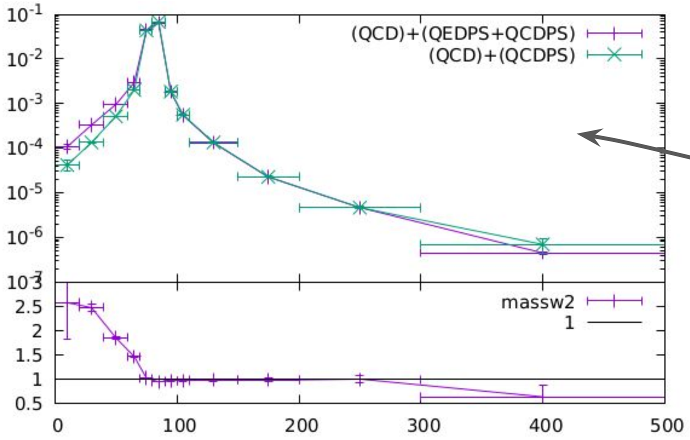
$$\bar{B}(\Phi_B) = B(\Phi_B) + [V_{\text{QCD}}(\Phi_B) + V_{\text{EW}}(\Phi_B)] + \int d\Phi_{\text{rad}} [R_{\text{QCD}}(\Phi_B, \Phi_{\text{rad}}) + R_{\text{EW}}(\Phi_B, \Phi_{\text{rad}})]$$

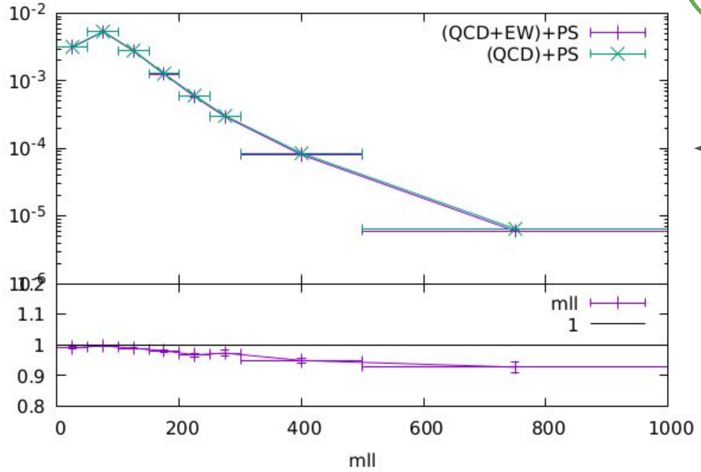
$$\Delta_{p_T}(\Phi_B) = \Delta_{p_T}^{\text{QCD}}(\Phi_B) \times \Delta_{p_T}^{\text{EW}}(\Phi_B)$$

- generate one radiation from each resonance
- requires dedicated interface to Parton Shower
- additive scheme + factorizable & mixed $\alpha_S^n \alpha_{\text{EW}}^m$ terms, only in collinear limit

Other approaches exist (e.g. Sherpa)

NLOQCD + NLOEW + PS: diboson production





- possible to have control on few percent effects
- $\text{NLO}_{\alpha_S+\alpha} + \text{PS}_{\alpha_S,\alpha}$ / $\text{NLO}_{\alpha_S} + \text{PS}_{\alpha_S,\alpha}$:

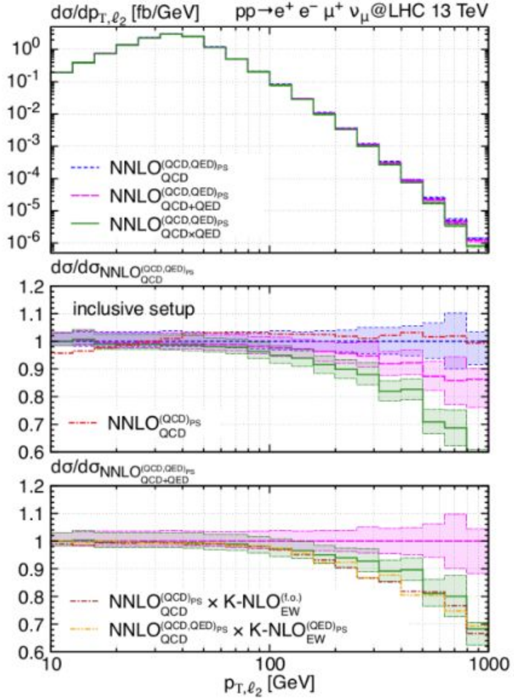
  - NLO weak, non-log QED $\mathcal{O}(\alpha)$, mixed
- $\text{NLO}_{\alpha_S+\alpha} + \text{PS}_{\alpha_S,\alpha}$ / $\text{NLO}_{\alpha_S} + \text{PS}_{\alpha_S}$:

  - NLO weak, QED $\mathcal{O}(\alpha)$, leading-log QED $\mathcal{O}(\alpha^n)$ $(n > 2)$, mixed

$\text{NNLO}_{\text{QCD}}+\text{NLO}_{\text{EW}}$, various approx.

$$\text{NNLO}_{\text{QCD}}^{(\text{QCD,QED})_{\text{PS}}} \times \text{K-NLO}_{\text{EW}}^{(\text{QCD,QED})_{\text{PS}}}$$

$$\text{NNLO}_{\text{QCD}}^{(\text{QCD,QED})_{\text{PS}}} + \delta\text{NLO}_{\text{EW}}^{(\text{QCD,QED})_{\text{PS}}}$$



F+jet @ NNLO+PS

- Geneva with 1-jettiness [Alioli,Bell,Billis,Broggio,Dehnadi,Lim,Marinelli,Nagar,Napoletano,Rahn ‘23]

$$\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_\kappa \exp\Big\{4(C_a+C_b)K_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + 4C_c K_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) - 2(C_a+C_b+C_c)K_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H)$$

$$- 2C_c L_J\, \eta_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) - 2(C_a L_B + C_b L'_B)\eta_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + K_{\gamma_{\mathrm{tot}}}$$

$$+ \left[C_a \ln\left(\frac{Q_a^2 u}{st}\right) + C_b \ln\left(\frac{Q_b^2 t}{su}\right) + C_{\kappa_j}\ln\left(\frac{Q_J^2 s}{tu}\right) + (C_a+C_b+C_c)L_S\right]\eta_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H)$$

$$+ \sum_{R=F,A}\Big[8\,(D_{aR}+D_{bR})K_{g^R}(\mu_B,\mu_H) + 8\,D_{cR}K_{g^R}(\mu_J,\mu_H)$$

$$- 4\,(D_{aR}+D_{bR}+D_{cR})K_{g^R}(\mu_S,\mu_H) - 4D_{cR}L_J\eta_{g^R}(\mu_J,\mu_H) - 4\,(D_{aR}L_B + D_{bR}L'_B)\eta_{g^R}(\cdots$$

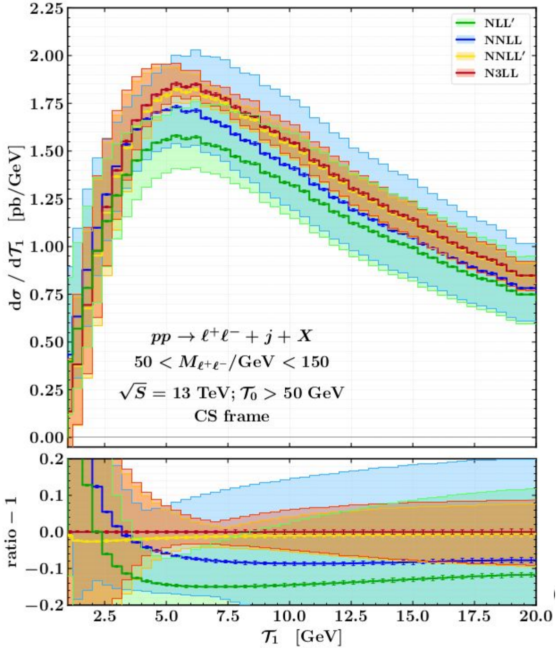
$$+ 2\left[D_{aR}\ln\left(\frac{Q_a^2 u}{st}\right) + D_{bR}\ln\left(\frac{Q_b^2 t}{su}\right) + D_{cR}\ln\left(\frac{Q_J^2 s}{tu}\right) + (D_{aR}+D_{bR}+D_{cR})L_S\right]\eta_{g^R}(\mu\cdots$$

$$\times H_\kappa(\Phi_1,\mu_H)\tilde{S}^\kappa(\partial_{\eta_S}+L_S,\mu_S)\tilde{B}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B)\tilde{B}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B)\,\tilde{J}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_\cdots$$

$$\times \frac{Q^{-\eta_{\mathrm{tot}}}}{\mathcal{T}_1^{\,1-\eta_{\mathrm{tot}}}}\,\frac{\eta_{\mathrm{tot}}\; e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(1+\eta_{\mathrm{tot}})}$$

Up to NNLL’

New N3LL ingredients



F+jet

- Geneva with 1-jettiness [Alioli,Bell,Billis,Broggio,Dehnadi,Lim,Marinelli,Nagar,Napoletano,Rahn '23]

$$\frac{d\sigma^{\mathrm{N^3LL}}}{d\Phi_1 d\mathcal{T}_1} = \sum_\kappa \exp\Big\{4(C_a+C_b)K_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + 4C_c K_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) - 2(C_a+C_b+C_c)K_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H)$$

$$- 2C_c L_J\, \eta_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) - 2(C_a L_B + C_b L'_B)\eta_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + K_{\gamma_{\mathrm{tot}}}$$

$$+ \left[C_a \ln\left(\frac{Q_a^2 u}{st}\right) + C_b \ln\left(\frac{Q_b^2 t}{su}\right) + C_{\kappa_j} \ln\left(\frac{Q_J^2 s}{tu}\right) + (C_a+C_b+C_c)L_S\right]\eta_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H)$$

$$+ \sum_{R=F,A}\Big[8\,(D_{aR}+D_{bR})K_{g^R}(\mu_B,\mu_H) + 8\,D_{cR}K_{g^R}(\mu_J,\mu_H)$$

$$- 4\,(D_{aR}+D_{bR}+D_{cR})K_{g^R}(\mu_S,\mu_H) - 4D_{cR}L_J\eta_{g^R}(\mu_J,\mu_H) - 4\,(D_{aR}L_B + D_{bR}L'_B)\eta_{g^R}($$

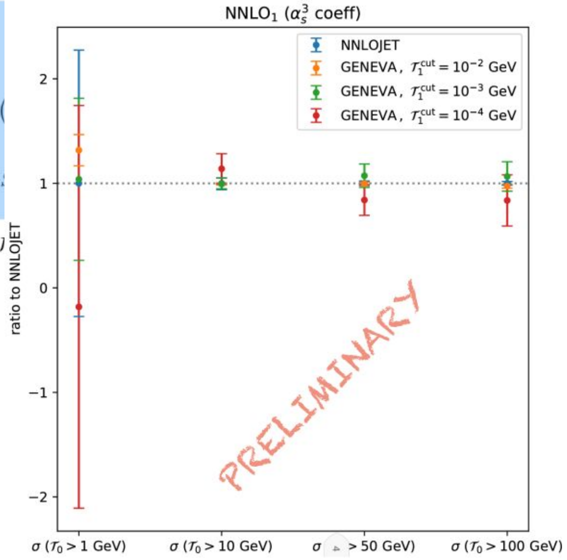
$$+ 2\left[D_{aR}\ln\left(\frac{Q_a^2 u}{st}\right) + D_{bR}\ln\left(\frac{Q_b^2 t}{su}\right) + D_{cR}\ln\left(\frac{Q_J^2 s}{tu}\right) + (D_{aR}+D_{bR}+D_{cR})L_S\right]\eta_{g^R}(\mu$$

$$\times H_\kappa(\Phi_1,\mu_H)\tilde{S}^\kappa(\partial_{\eta_S}+L_S,\mu_S)\tilde{B}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B)\tilde{B}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B)\,\tilde{J}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J$$

$$\times \frac{Q^{-\eta_{\mathrm{tot}}}}{\mathcal{T}_1^{1-\eta_{\mathrm{tot}}}}\frac{\eta_{\mathrm{tot}}\, e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(1+\eta_{\mathrm{tot}})}$$

Up to NNLL'

New N3LL ingredients



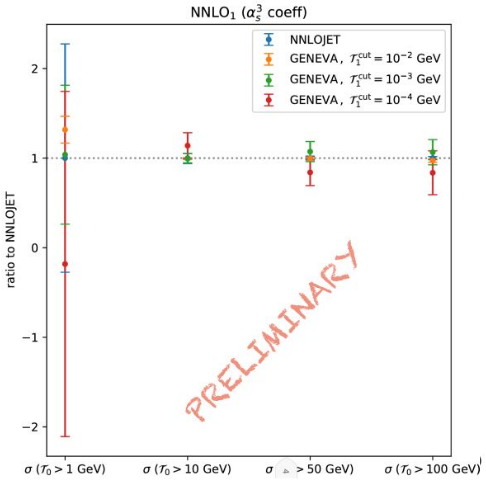
F+jet

- Geneva with 1-jettiness [Alioli,Bell,Billis,Broggio,Dehnadi,Lim,Marinelli,Nagar,Napoletano,Rahn ‘23]

$$
\begin{aligned}
\frac{d\sigma^{\mathrm{N^3LL}}}{d\Phi_1 d\mathcal{T}_1} = \sum_\kappa \exp\Big\{ & 4(C_a+C_b)K_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + 4C_c K_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) - 2(C_a+C_b+C_c)K_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H) \\
& - 2C_c L_J\, \eta_{\Gamma_{\mathrm{cusp}}}(\mu_J,\mu_H) - 2(C_a L_B + C_b L'_B)\eta_{\Gamma_{\mathrm{cusp}}}(\mu_B,\mu_H) + K_{\gamma_{\mathrm{tot}}} \\
& + \left[C_a \ln\left(\frac{Q_a^2 u}{st}\right) + C_b \ln\left(\frac{Q_b^2 t}{su}\right) + C_{\kappa_j}\ln\left(\frac{Q_J^2 s}{tu}\right) + (C_a+C_b+C_c)L_S\right]\eta_{\Gamma_{\mathrm{cusp}}}(\mu_S,\mu_H) \\
& + \sum_{R=F,A}\Big[8\,(D_{aR}+D_{bR})K_{g^R}(\mu_B,\mu_H) + 8\,D_{cR}K_{g^R}(\mu_J,\mu_H) \\
& - 4\,(D_{aR}+D_{bR}+D_{cR})K_{g^R}(\mu_S,\mu_H) - 4D_{cR}L_J\eta_{g^R}(\mu_J,\mu_H) - 4\,(D_{aR}L_B + D_{bR}L'_B)\eta_{g^R}(\mu_B,\mu_H) \\
& + 2\left[D_{aR}\ln\left(\frac{Q_a^2 u}{st}\right) + D_{bR}\ln\left(\frac{Q_b^2 t}{su}\right) + D_{cR}\ln\left(\frac{Q_J^2 s}{tu}\right) + (D_{aR}+D_{bR}+D_{cR})L_S\right]\eta_{g^R}(\mu_S,\mu_H)\Big]\Big\} \\
& \times H_\kappa(\Phi_1,\mu_H)\tilde{S}^\kappa(\partial_{\eta_S}+L_S,\mu_S)\tilde{B}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B)\tilde{B}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B)\,\tilde{J}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J) \\
& \times \frac{Q^{-\eta_{\mathrm{tot}}}}{\mathcal{T}_1^{1-\eta_{\mathrm{tot}}}}\frac{\eta_{\mathrm{tot}}\, e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(1+\eta_{\mathrm{tot}})}
\end{aligned}
$$

Up to NNLL’

New N3LL ingredients



- MiNNLO$_{PS}$ with 1-jettiness formulated [Ebert,Rottoli,Wiesemann,Zanderighi,Zanoli ‘24]

$$\frac{d\sigma^{\mathrm{sing}}(\mathcal{T}^{\mathrm{cut}})}{d\Phi_{\mathrm{FJ}}} = \sum_\kappa \tilde{\mathcal{L}}_\kappa(\mathcal{T}^{\mathrm{cut}})e^{-\mathcal{S}_\kappa(\mathcal{T}^{\mathrm{cut}})}$$

$\longrightarrow$ mappings, shower interface,...

Conclusions

- NNLO+PS matching with $\mathrm{MiNNLO_{PS}}$ and Geneva:
  - many results, for color singlet and heavy-quarks (+color singlet)

- F+1 jet @ NNLO+PS is work in progress

- NLL showers → details of matching matter if one wants to keep NLL shower accuracy
  [talk by D. Reichelt]

- QCD+EW corrections: still room for improvement

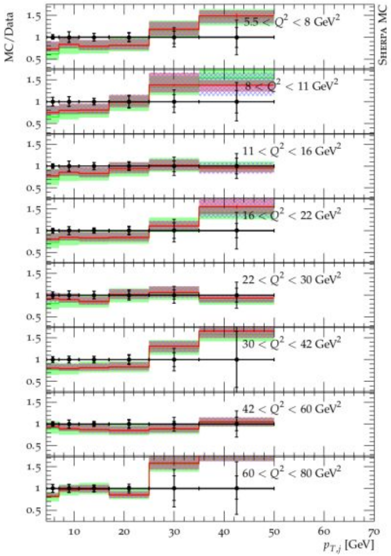
Backup slides

UNNLOPS

UNNLOPS

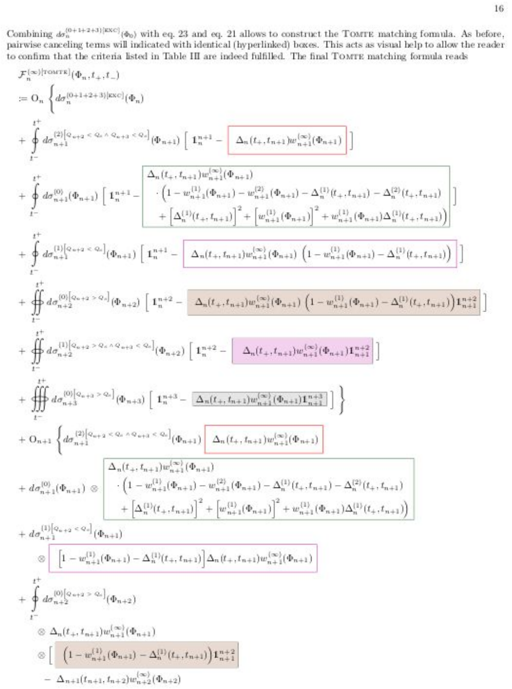
- Main idea: Promote to NLO accuracy an "unitarised" CKKW approach, by carefully adding higher order contributions, and removing the pre-existing approximate α_S terms.
- Supplement results with missing NNLO ingredients.

- DIS @ NNLO+PS [Höche,Kuttimalai,Li '18]



- plot: DIS 1-jet inclusive
- red/blue: UNNLOPS, green: NLO result

- towards N3LO+PS [Prestel '21]



16

Combining $d\sigma_n^{(0+1+2+3)|\text{EXC}}(\Phi_0)$ with eq. 23 and eq. 21 allows to construct the TOMTE matching formula. As before, pairwise canceling terms will indicated with identical (hyperlinked) boxes. This acts as visual help to allow the reader to confirm that the criteria listed in Table III are indeed fulfilled. The final TOMTE matching formula reads

$$
\begin{aligned}
&\mathcal{F}_n^{(\infty)|\text{TOMTE}}(\Phi_n, t_+, t_-) \\
&:= \mathrm{O}_n \Big\{ d\sigma_n^{(0+1+2+3)|\text{EXC}}(\Phi_n) \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+1}^{(2)[Q_{n+2}<Q_n \wedge Q_{n+3}<Q_n]}(\Phi_{n+1}) \Big[\mathbf{1}_n^{n+1} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})} \Big] \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+1}^{(0)}(\Phi_{n+1}) \Big[\mathbf{1}_n^{n+1} - \boxed{\begin{aligned} &\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \\ &\cdot \Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - w_{n+1}^{(2)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1}) - \Delta_n^{(2)}(t_+,t_{n+1}) \\ &+ \big[\Delta_n^{(1)}(t_+,t_{n+1})\big]^2 + \big[w_{n+1}^{(1)}(\Phi_{n+1})\big]^2 + w_{n+1}^{(1)}(\Phi_{n+1})\Delta_n^{(1)}(t_+,t_{n+1})\Big) \end{aligned}} \Big] \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+1}^{(1)[Q_{n+2}<Q_n]}(\Phi_{n+1}) \Big[\mathbf{1}_n^{n+1} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1})\Big)} \Big] \\
&+ \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(0)[Q_{n+2}>Q_n]}(\Phi_{n+2}) \Big[\mathbf{1}_n^{n+2} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1})\Big) \mathbf{1}_{n+1}^{n+2}} \Big] \\
&+ \oiint_{t^-}^{t^+} d\sigma_{n+2}^{(1)[Q_{n+2}>Q_n \wedge Q_{n+3}<Q_n]}(\Phi_{n+2}) \Big[\mathbf{1}_n^{n+2} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbf{1}_{n+1}^{n+2}} \Big] \\
&+ \oiiint_{t^-}^{t^+} d\sigma_{n+3}^{(0)[Q_{n+3}>Q_n]}(\Phi_{n+3}) \Big[\mathbf{1}_n^{n+3} - \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \mathbf{1}_{n+1}^{n+3}} \Big] \Big\} \\
&+ \mathrm{O}_{n+1} \Big\{ d\sigma_{n+1}^{(2)[Q_{n+2}<Q_n \wedge Q_{n+3}<Q_n]}(\Phi_{n+1}) \boxed{\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})} \\
&+ d\sigma_{n+1}^{(0)}(\Phi_{n+1}) \otimes \boxed{\begin{aligned} &\Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \\ &\cdot \Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - w_{n+1}^{(2)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1}) - \Delta_n^{(2)}(t_+,t_{n+1}) \\ &+ \big[\Delta_n^{(1)}(t_+,t_{n+1})\big]^2 + \big[w_{n+1}^{(1)}(\Phi_{n+1})\big]^2 + w_{n+1}^{(1)}(\Phi_{n+1})\Delta_n^{(1)}(t_+,t_{n+1})\Big) \end{aligned}} \\
&+ d\sigma_{n+1}^{(1)[Q_{n+2}<Q_n]}(\Phi_{n+1}) \\
&\quad \otimes \boxed{\Big[1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1})\Big] \Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1})} \\
&+ \oint_{t^-}^{t^+} d\sigma_{n+2}^{(0)[Q_{n+2}>Q_n]}(\Phi_{n+2}) \\
&\quad \otimes \Delta_n(t_+,t_{n+1}) w_{n+1}^{(\infty)}(\Phi_{n+1}) \\
&\quad \otimes \Big[\boxed{\Big(1 - w_{n+1}^{(1)}(\Phi_{n+1}) - \Delta_n^{(1)}(t_+,t_{n+1})\Big) \mathbf{1}_{n+1}^{n+2}} \\
&\quad - \Delta_{n+1}(t_{n+1},t_{n+2}) w_{n+2}^{(\infty)}(\Phi_{n+2})
\end{aligned}
$$

top pair-production @ NNLO+PS: MiNNLO for $t\bar{t}$

$$\frac{d\sigma}{d\vec{q}_\perp d\Phi_F} \sim \sum_f |M^{(0)}_{f\bar{f}\to t\bar{t}}|^2 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\cdot\vec{q}_\perp} e^{-R_f(b)} \mathrm{Tr}\,(\mathbf{H}_f \mathbf{\Delta}_{\mathrm{soft}}) \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{\bar{f}j} \otimes h^{[j]})$$

$$\mathrm{Tr}\,(\mathbf{H}_f \mathbf{\Delta}_{\mathrm{soft}}) = \frac{\langle M_{f\bar{f}} | \mathbf{\Delta} | M_{f\bar{f}} \rangle}{|M^{(0)}_{f\bar{f}}|^2}, \qquad \mathbf{\Delta} = \mathbf{V}^\dagger \mathbf{D} \mathbf{V}$$

$$\mathbf{V} = \mathcal{P} \exp \left\{ - \int_{\frac{b_0^2}{b^2}}^{M^2_{t\bar{t}}} \frac{dq^2}{q^2} \mathbf{\Gamma}_t(\Phi_{t\bar{t}}, \alpha_s(q)) \right\}$$

- LL+NNLO accuracy: azimuthally averaged distribution becomes

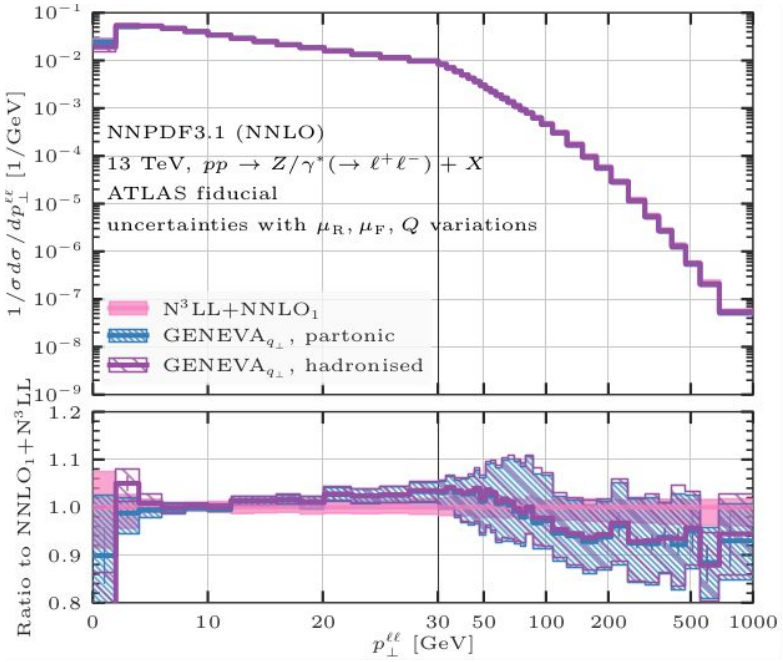
$$\left[\frac{d\sigma}{d\vec{q}_\perp d\Phi_F}\right]_\phi \sim \frac{d}{dq_\perp} \left[\sum_f e^{-S_f(q_\perp)} \langle M^{(0)}_{f\bar{f}} | (\mathbf{V}_{\mathrm{NLL}})^\dagger \mathbf{V}_{\mathrm{NLL}} | M^{(0)}_{f\bar{f}} \rangle [\mathrm{Tr}\,(\mathbf{H}_f \mathbf{D}_{\mathrm{soft}}) \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{\bar{f}j} \otimes h^{[j]})]_\phi \right]$$

$$B(\alpha_s) = \frac{\alpha_s}{2\pi} B^{(1)} + \frac{\alpha_s^2}{(2\pi)^2} B^{(2)}$$

$$\sim \frac{\langle M^{(0)}_{f\bar{f}} | \mathbf{D}^{(1)} | M^{(0)}_{f\bar{f}} \rangle}{|M^{(0)}_{f\bar{f}}|^2} \left((C^{(1)}_{fi} \otimes f_i)(f_{\bar{f}}) \right)$$

- diagonalization of $\mathbf{V}_{\mathrm{NLL}}$ → recast as sum of "colour-singlet-like" terms

Geneva(qT) vs. resummation+FO



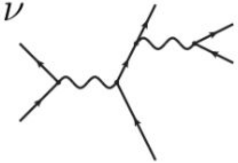
- using p_T as resolution parameter
  → large p_T NNLO effects missing
  (as in all NNLOPS generators so far)

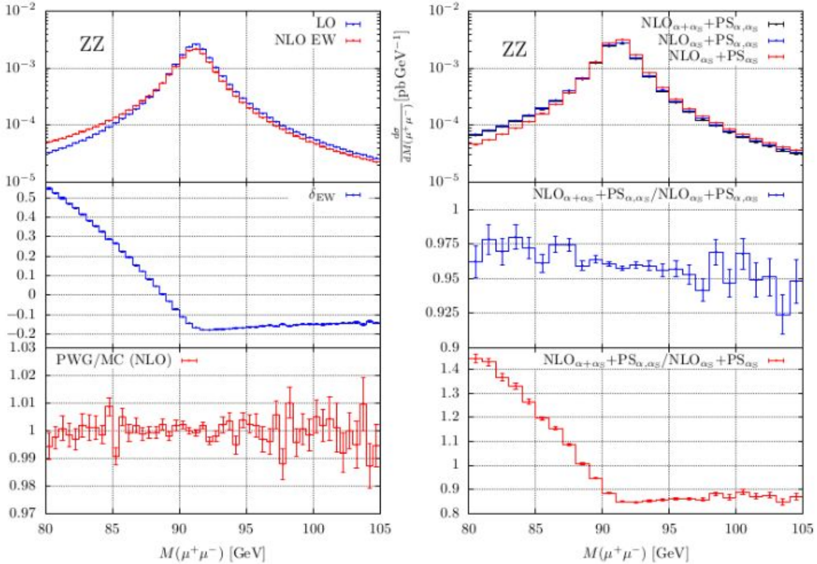
Diboson production in POWHEG-BOX: EW+QCD

[Chiesa,ER,Oleari ‘20]

loop amplitudes from `Recola2`

$$pp \to 4l,\; pp \to 2l2\nu,\; pp \to 3l\nu$$





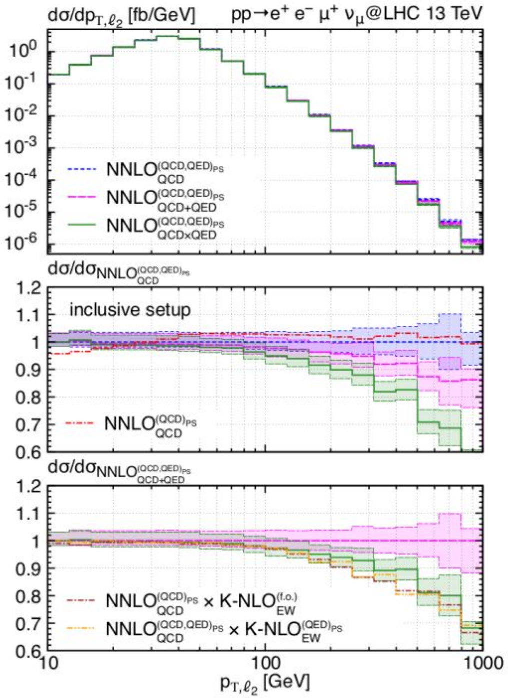
- possible to have control on few percent effects
- $NLO_{\alpha_S+\alpha}$ + $PS_{\alpha_S,\alpha}$ / NLO_{α_S} + $PS_{\alpha_S,\alpha}$:

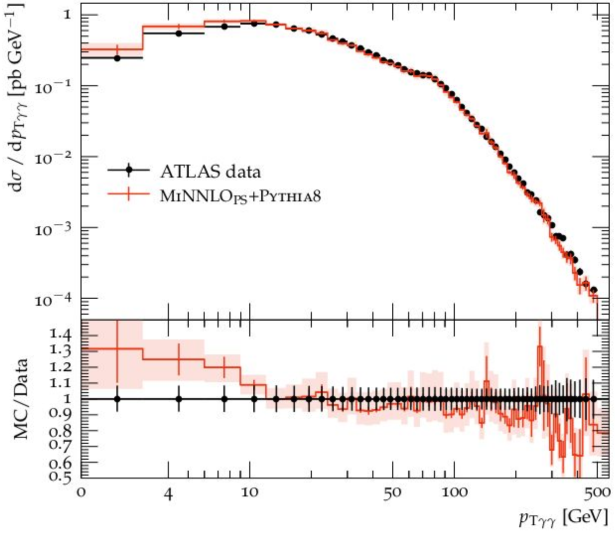
  - NLO weak, non-log QED $\mathcal{O}(\alpha)$, mixed

- $NLO_{\alpha_S+\alpha}$ + $PS_{\alpha_S,\alpha}$ / NLO_{α_S} + PS_{α_S}:

  - NLO weak, QED $\mathcal{O}(\alpha)$, leading-log QED $\mathcal{O}(\alpha^n)$ $(n > 2)$, mixed

Diboson production in POWHEG-BOX: NNLO QCD





Zγ [Lombardi et al. ‘20]

WW [Lombardi et al. ‘20]

ZZ [Buonocore et al. ‘21]

WZ [Lindert et al. ‘22]

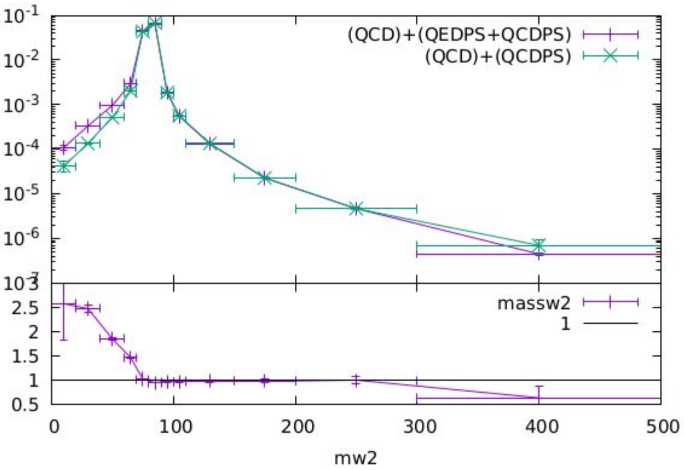
γγ [Gavardi et al. ‘22]

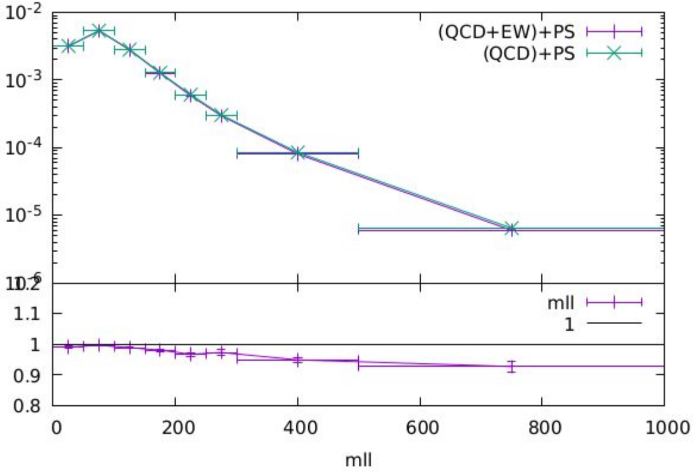
$$\mathrm{NNLO}_{\mathrm{QCD}}^{(\mathrm{QCD,QED})_{\mathrm{PS}}} \times \mathrm{K\text{-}NLO}_{\mathrm{EW}}^{(\mathrm{QCD,QED})_{\mathrm{PS}}}$$

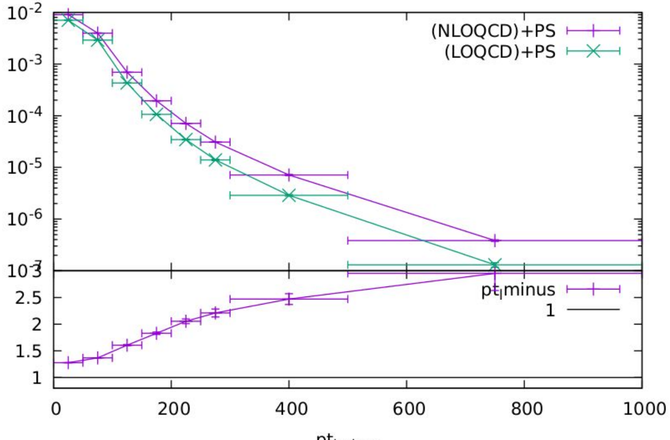
$$\mathrm{NNLO}_{\mathrm{QCD}}^{(\mathrm{QCD,QED})_{\mathrm{PS}}} + \delta\mathrm{NLO}_{\mathrm{EW}}^{(\mathrm{QCD,QED})_{\mathrm{PS}}}$$

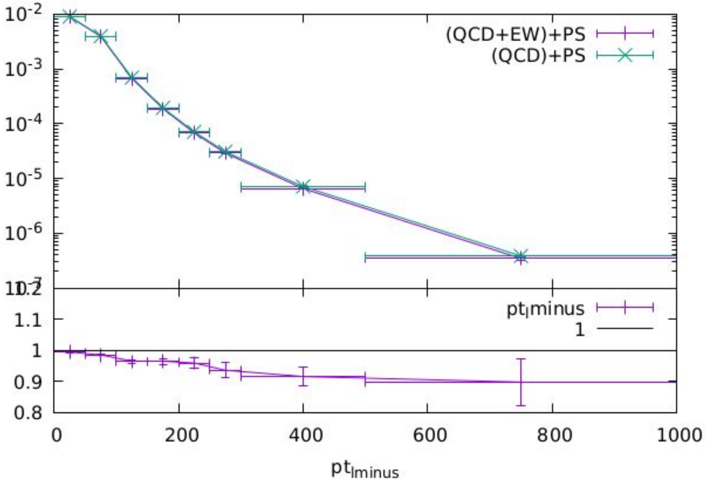
- left: $W^{\pm}Z$. It includes also NLO$_{\mathrm{EW}}$+PS corrections in various approximations
- right: $\gamma\gamma$. It required also some minor modification to the MiNNLO$_{\mathrm{PS}}$ master formula

WW QCD+EW: plots



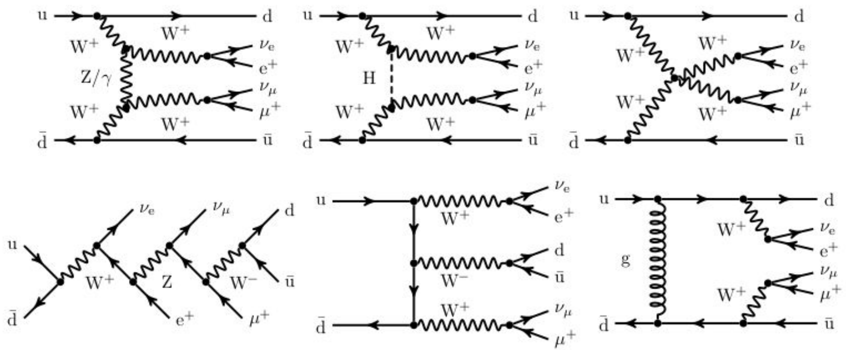






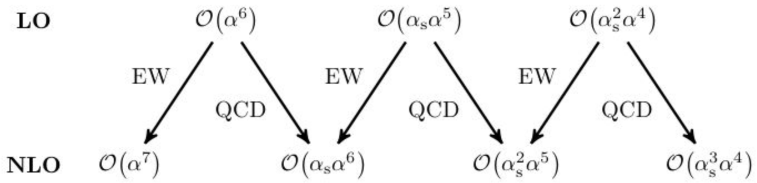
NLOEW+PS: bottlenecks

[slide from M. Chiesa]



$$\mathcal{M} \simeq \quad \mathcal{O}(\alpha^3) \qquad \mathcal{O}(\alpha^3) \qquad \mathcal{O}(\alpha_S\alpha^2)$$

$$\text{LO} \simeq \quad \mathcal{O}(\alpha^6) \qquad \mathcal{O}(\alpha^5\alpha_S) \qquad \mathcal{O}(\alpha_S^2\alpha^4)$$



MiNNLO PS (details)

- from p_T resummation, differential cross section for $F+X$ production can be written as:

$$\frac{d\sigma}{dp_T d\Phi_F} = \frac{d}{dp_T}\left\{\mathcal{L}(\Phi_F, p_T)\exp(-\tilde{S}(p_T))\right\} + R_{\text{finite}}(p_T)$$

$$\mathcal{L}(\Phi_F, p_T) \ni \{H^{(1)}, H^{(2)}, C^{(1)}, C^{(2)}, (G^{(1)}\cdot G^{(1)})\} \qquad R_{\text{finite}}(p_T) = \frac{d\sigma_{FJ}}{d\Phi_F dp_T} - \frac{d\sigma^{\text{sing}}}{d\Phi_F dp_T}$$

- recast it, to match the POWHEG $\bar{B}^{(FJ)}(\Phi_{FJ})$

$$\frac{d\sigma}{d\Phi_F dp_T} = \exp[-\tilde{S}(p_T)]\left\{D(p_T) + \frac{R_{\text{finite}}(p_T)}{\exp[-\tilde{S}(p_T)]}\right\}$$

$$D(p_T) \equiv -\frac{d\tilde{S}(p_T)}{dp_T}\mathcal{L}(p_T) + \frac{d\mathcal{L}(p_T)}{dp_T} \qquad \tilde{S}(p_T) = \int_{p_T}^{Q}\frac{dq^2}{q^2}\left[A_f(\alpha_S(q))\log\frac{Q^2}{q^2} + B_f(\alpha_S(q))\right]$$

- expand the above integrand in power of $\alpha_S(p_T)$, keep the terms that are needed to get NLO$^{(F)}$ & NNLO$^{(F)}$ accuracy, when integrating over p_T
- after expansion, all the terms with explicit logs will be of the type $\alpha_S^m(p_T)L^n$, with $n = 0, 1$.

$$\int^{Q}\frac{dp_T}{p_T}L^n\alpha_S^m(p_T)\exp(-\tilde{S}(p_T)) \sim (\alpha_S(Q))^{m-(n+1)/2} \qquad L = \log Q/p_T$$

MiNNLO PS (details)

$$\frac{d\sigma}{d\Phi_F dp_T} = \exp[-\tilde{S}(p_T)]\Bigg\{ \frac{\alpha_S(p_T)}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(1)} \left(1 + \frac{\alpha_S(p_T)}{2\pi}[\tilde{S}(p_T)]^{(1)}\right) + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^2 \left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(2)} + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^3 [D(p_T)]^{(3)} + \text{regular terms}\Bigg\}$$

- as expected, for NLO$^{(F)}$ accuracy, we recovered `MiNLO'`, exactly
- $[D(p_T)]^{(3)}$ is the $\alpha_S^3(p_T)$ expansion of $D(p_T) = -\frac{d\tilde{S}(p_T)}{dp_T}\mathcal{L}(p_T) + \frac{d\mathcal{L}(p_T)}{dp_T}$
  - higher-order terms $\mathcal{O}(\alpha_S^4(p_T))$ will produce terms beyond accuracy, after integration on p_T
- "regular terms": $[R_{\text{finite}}(p_T)/\exp[-\tilde{S}(p_T)]]^{(3)}$.
  - power suppressed $\rightarrow$ after integration they are of order $\mathcal{O}(\alpha_S^3)$.

MiNNLO PS (details)

$$\frac{d\sigma}{d\Phi_F dp_T} = \exp[-\tilde{S}(p_T)]\Big\{ \frac{\alpha_S(p_T)}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(1)}\left(1+\frac{\alpha_S(p_T)}{2\pi}[\tilde{S}(p_T)]^{(1)}\right) + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(2)} + \left(\frac{\alpha_S(p_T)}{2\pi}\right)^3 [D(p_T)]^{(3)} F_\ell^{corr}(\Phi_{FJ}) \Big\}$$

- as expected, for NLO$^{(F)}$ accuracy, we recovered `MiNLO'`, exactly
- $[D(p_T)]^{(3)}$ is the $\alpha_S^3(p_T)$ expansion of $D(p_T) = -\frac{d\tilde{S}(p_T)}{dp_T}\mathcal{L}(p_T) + \frac{d\mathcal{L}(p_T)}{dp_T}$
  - higher-order terms $\mathcal{O}(\alpha_S^4(p_T))$ will produce terms beyond accuracy, after integration on p_T
- "regular terms": $[R_{\rm finite}(p_T)/\exp[-\tilde{S}(p_T)]]^{(3)}$.
  - power suppressed $\rightarrow$ after integration they are of order $\mathcal{O}(\alpha_S^3)$.
- $[D(p_T)]^{(3)}$: extracted from $p_T \rightarrow 0$ limit, depends on (Φ_F, p_T), not on Φ_{FJ}
  - in practice, we need to integrate over $\Phi_{FJ} \Rightarrow$ smooth mapping to evaluate $[D(p_T)]^{(3)}$
  - $F_\ell^{corr}(\Phi_{FJ})$: projection $\rightarrow$ recover $[D(p_T)]^{(3)}$ when integrating over Φ_{FJ} at fixed (Φ_F, p_T)

Matching and NLL showers

[Hamilton et al. 2301.09645]