



Esperimento PVLAS per la misura della birifrangenza magnetica del vuoto

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per la collaborazione PVLAS



"Hands holding the void"
Alberto Giacometti



PVLAS collaboration

University and INFN - Ferrara

- G. Di Domenico
- G. Messineo
- L. Piemontese
- G. Zavattini

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- F. Della Valle
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- R. Pengo
- G. Ruoso



Summary

- Introduction:
 - Aim of the PVLAS experiment
 - Experimental technique
- PVLAS
 - Overview of published results
- Development phases
 - Improvements with respect to PVLAS-LNL
 - Ferrara Test apparatus
 - Final experiment



Classical Electromagnetism in vacuum

Classical vacuum has no structure.

$$\begin{aligned} \text{div } \vec{\mathbf{D}} &= 0; & \text{rot } \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} \\ \text{div } \vec{\mathbf{B}} &= 0; & \text{rot } \vec{\mathbf{H}} &= \frac{\partial \vec{\mathbf{D}}}{\partial t} \end{aligned}$$

$$L_{EM} = \frac{1}{2\mu_0} \left(\frac{\mathbf{E}^2}{c^2} - \mathbf{B}^2 \right)$$

The superposition principle is valid
with $c = 1/\sqrt{\epsilon_0\mu_0}$

$$\begin{aligned} \vec{\mathbf{D}} &= \frac{\partial L_{EM}}{\partial \vec{\mathbf{E}}} \\ \vec{\mathbf{H}} &= -\frac{\partial L_{EM}}{\partial \vec{\mathbf{B}}} \end{aligned}$$



↓

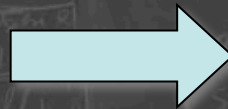
$$\vec{\mathbf{D}} = \epsilon_0 \vec{\mathbf{E}}; \quad \mu_0 \vec{\mathbf{H}} = \vec{\mathbf{B}}$$



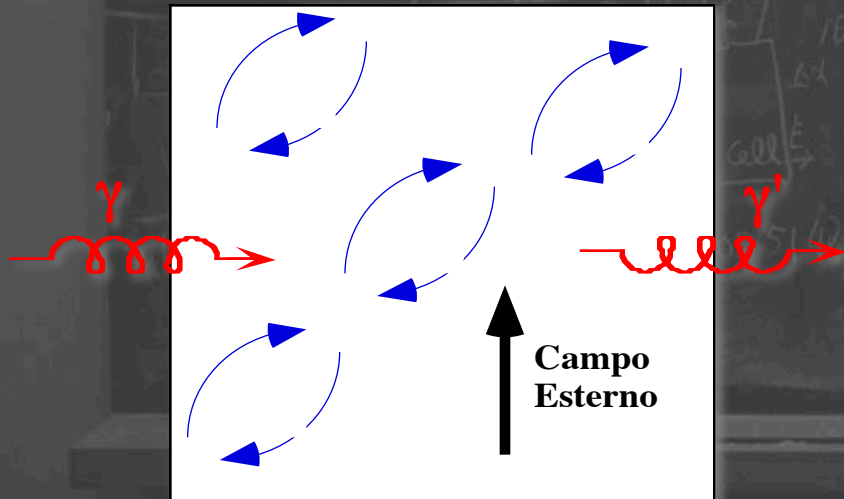
Heisenberg's Uncertainty Principle

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Vacuum is the minimum energy state and can **fluctuate into anything** compatible with vacuum



Vacuum has a **structure** which can be observed by perturbing it and probing it.



- QED tests in bound systems - Lamb shift
- QED tests in charged particles - (g-2)
- **QED tests with photons is missing**
- Macroscopically observable (small) non linear effects have been predicted since 1936 but have never been directly observed yet.



Propagation of light

Photon propagation in vacuum as depicted with Feynman diagrams

Without external field



Real photon

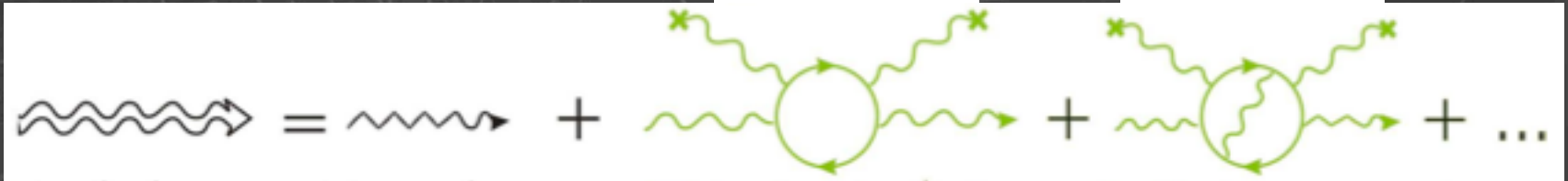
Bare photon

Virtual pairs

\vec{B}, \vec{E}

\vec{B}, \vec{E}

With external field



Real photon

Bare photon

Virtual pairs

Radiative corrections

c depends on polarization and external field!



Euler-Heisenberg Effective Lagrangian

For fields much smaller than the critical field
($B \ll 4.4 \cdot 10^9$ T; $E \ll 1.3 \cdot 10^{18}$ V/m) one can write

W Heisenberg and H Euler, *Z. Phys.* **98**, 714 (1936)
H Euler, *Ann. Phys.* **26**, 398 (1936)

$$L = L_{em} + L_{HE} = \frac{1}{2\mu_0} \left(\frac{E^2}{c^2} - B^2 \right) + \frac{A_e}{\mu_0} \left[\left(\frac{E^2}{c^2} - B^2 \right)^2 + 7 \left(\frac{\vec{E}}{c} \cdot \vec{B} \right)^2 \right] + \dots$$

$$A_e = \frac{2}{45\mu_0} \left(\frac{\alpha^2 \hbar^3}{m_e c^2} \right) = 1.32 \cdot 10^{-24} \text{ T}^{-2}$$

CPT and Lorentz invariant \Rightarrow Coefficients determined by theory

Are neglected:

- α^3 and higher order terms
- virtual pairs with particles different from $e^+ e^-$



Induced Magnetic Birefringence of Vacuum

• By applying the constitutive relations to L_{EH} one finds

$$\vec{D} = \frac{\partial L_{EH}}{\partial \vec{E}}$$

$$\vec{H} = -\frac{\partial L_{EH}}{\partial \vec{B}}$$



$$\vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 A_e \left[4 \left(\frac{E^2}{c^2} - B^2 \right) \vec{E} + 14 (\vec{E} \cdot \vec{B}) \vec{B} \right]$$

$$\mu_0 \vec{H} = \vec{B} + A_e \left[4 \left(\frac{E^2}{c^2} - B^2 \right) \vec{B} - 14 \left(\frac{\vec{E} \cdot \vec{B}}{c^2} \right) \vec{E} \right]$$

• Light propagation is still described by Maxwell's equations in media but they no longer are linear due to E-H correction.

Index of refraction

Linearly polarized light passing through a transverse external magnetic field perpendicular to \vec{k} .

$$\begin{cases} \varepsilon_{\parallel} = 1 + 10 A_e \mathbf{B}_{Ext}^2 \\ \mu_{\parallel} = 1 + 4 A_e \mathbf{B}_{Ext}^2 \\ n_{\parallel} = 1 + 7 A_e \mathbf{B}_{Ext}^2 \end{cases} \quad \begin{cases} \varepsilon_{\perp} = 1 - 4 A_e \mathbf{B}_{Ext}^2 \\ \mu_{\perp} = 1 + 12 A_e \mathbf{B}_{Ext}^2 \\ n_{\perp} = 1 + 4 A_e \mathbf{B}_{Ext}^2 \end{cases}$$



- $v \neq c$
- anisotropy

A_e can be determined by measuring the magnetic birefringence of vacuum.

$$\Delta n_{(\alpha^2)} = 3A_e B^2$$

$$\Delta n_{(\alpha^3)} = 3A_e B^2 \left(1 + \frac{25}{4\pi} \alpha \right) = \frac{2}{15} \frac{\alpha^2 \hbar^3}{m_e^4 c^5} \left(1 + \frac{25}{4\pi} \alpha \right) \frac{B^2}{\mu_0}$$

$$\Delta n = (4.031699 \pm 0.000002) \cdot 10^{-24} \left(\frac{B}{1T} \right)^2$$

$O(\alpha^4), O(\alpha^5)$? Also a theoretical challenge

$$\Delta n = 2.5 \cdot 10^{-23} \text{ for } B = 2.5 \text{ T}$$



Cotton-Mouton Effect

Vacuum behaves like a gas: Cotton-Mouton effect

Gas	CM constant (atm Tesla ⁻²)	vacuum equiv. pressure (mbar)
N ₂	-2.45·10 ⁻¹³	1.6·10 ⁻⁸
Ar	6.8·10 ⁻¹⁵	5.8·10 ⁻⁷
Kr	9.9·10 ⁻¹⁵	4·10 ⁻⁷
Ne	2.8·10 ⁻¹⁶	1.4·10 ⁻⁵
He	1.8·10 ⁻¹⁶	2.2·10 ⁻⁵
H ₂	8.5·10 ⁻¹⁵	4.7·10 ⁻⁷

$$\Delta n_{CM} = CM \frac{P}{P_{atm}} B_0^2$$

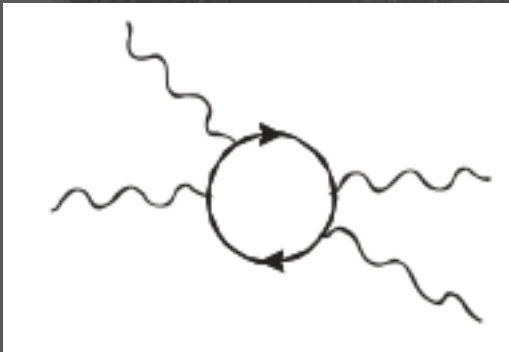


For He:

Vacuum is 'equivalent' to
5.8·10¹¹ He atoms/cm³



Light-Light scattering



Very low energy photon-photon scattering is proportional to A_e^2 .

For non polarized light:

$$\sigma_{\gamma\gamma}^{[*]} = \frac{973\mu_0^2}{20\pi} \frac{E_\gamma^6}{\hbar^4 c^4} A_e^2$$

From Euler-Heisenberg Lagrangian (S.I. units)

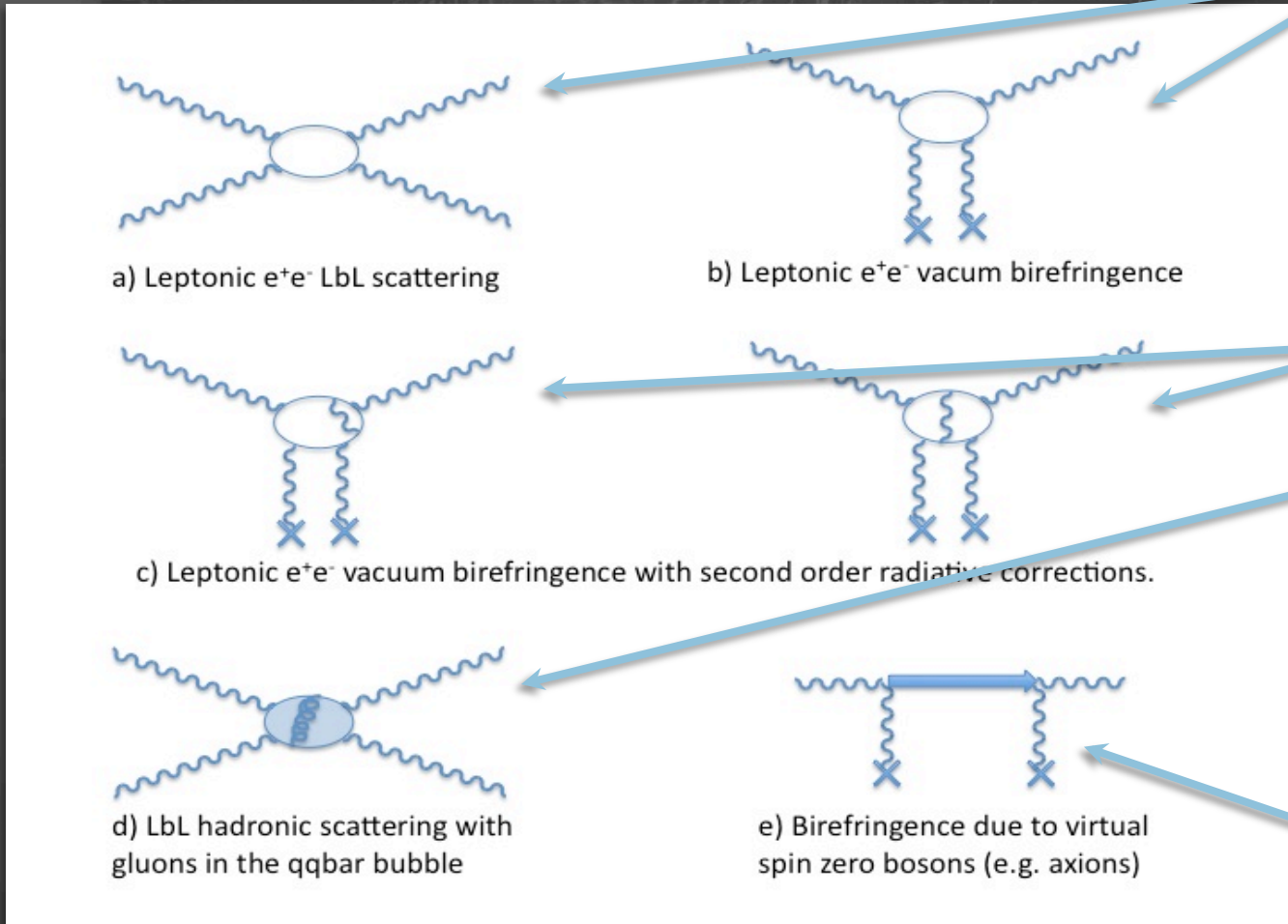
- For light at 1064 nm this predicts a value of $\sigma_{\gamma\gamma} = 1.8 \cdot 10^{-65} \text{ cm}^2$
- Experimentally Bernard et al.^[**] have published $\sigma_{\gamma\gamma} < 1.5 \cdot 10^{-48} \text{ cm}^2$

*Duane et al., Phys Rev. D, vol 57 p. 2443 (1998)

**Bernard D. et al., The European Physical Journal D, vol 10, p. 141 (1999)



Summary of possible 4 photon processes



- Described by the Euler-Heisenberg Lagrangian. **Should be there.** Also includes MCPs
- Radiative correction 1.45%
- Hadronic contribution. Difficult to extract from indirect measurements. $g-2$ open problem.
- Contribution from hypothetical new particles coupling to two photons.



Aim of PVLAS

- We want to study the speed of light in the perturbed vacuum and therefore study changes in the refractive index

$$n_{\text{vacuum}} = 1 + (\delta n_r - i\kappa)_{\text{field}}$$

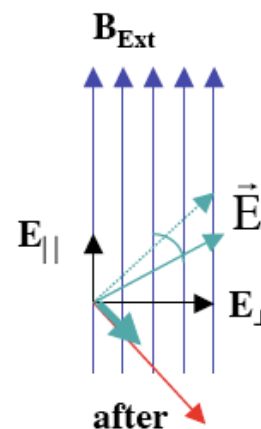
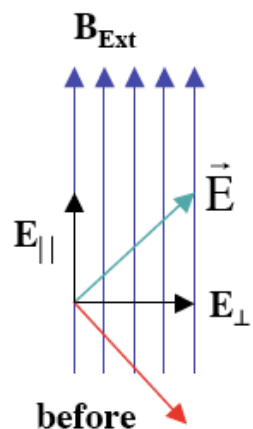
- Absolute changes of n_{vacuum} are too difficult to measure so we study anisotropies due to the perturbing field.
- Linear birefringence and linear dichroism
- Linear dichroism (photon splitting) results to be exceedingly small



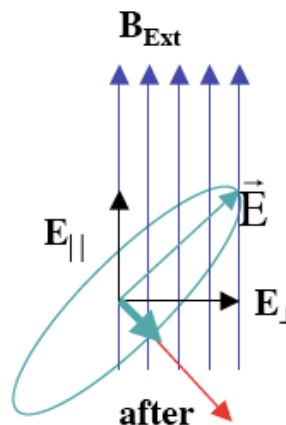
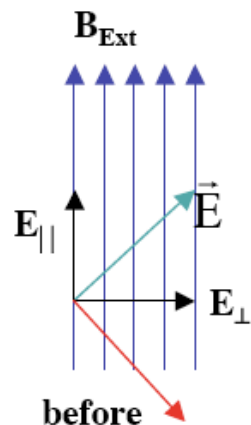
Propagation of the photon in an external field

Dichroism ΔK

- (Photon splitting)
- Real particle production



apparent rotation ε



ellipticity ψ

Both Δn and ΔK are defined with sign



Summing up

Experimental study of the quantum vacuum with:

- magnetic field perturbation
- linearly polarised light beam as a probe
- changes in the polarisation state are the expected signals

Key Ingredients

Ellipticity

$$\psi = \frac{\pi L}{\lambda} \Delta n \sin 2\vartheta$$

- **high magnetic field**
superconducting dipole magnet or high field permanent magnet
- **long optical path**
delay line cavity or very-high Q Fabry-Perot resonator
- **ellipsometer with heterodyne detection for best sensitivity**
periodic change of field amplitude/direction for signal modulation



Present published results - QED

$$\Delta n = 3 A_e B^2$$

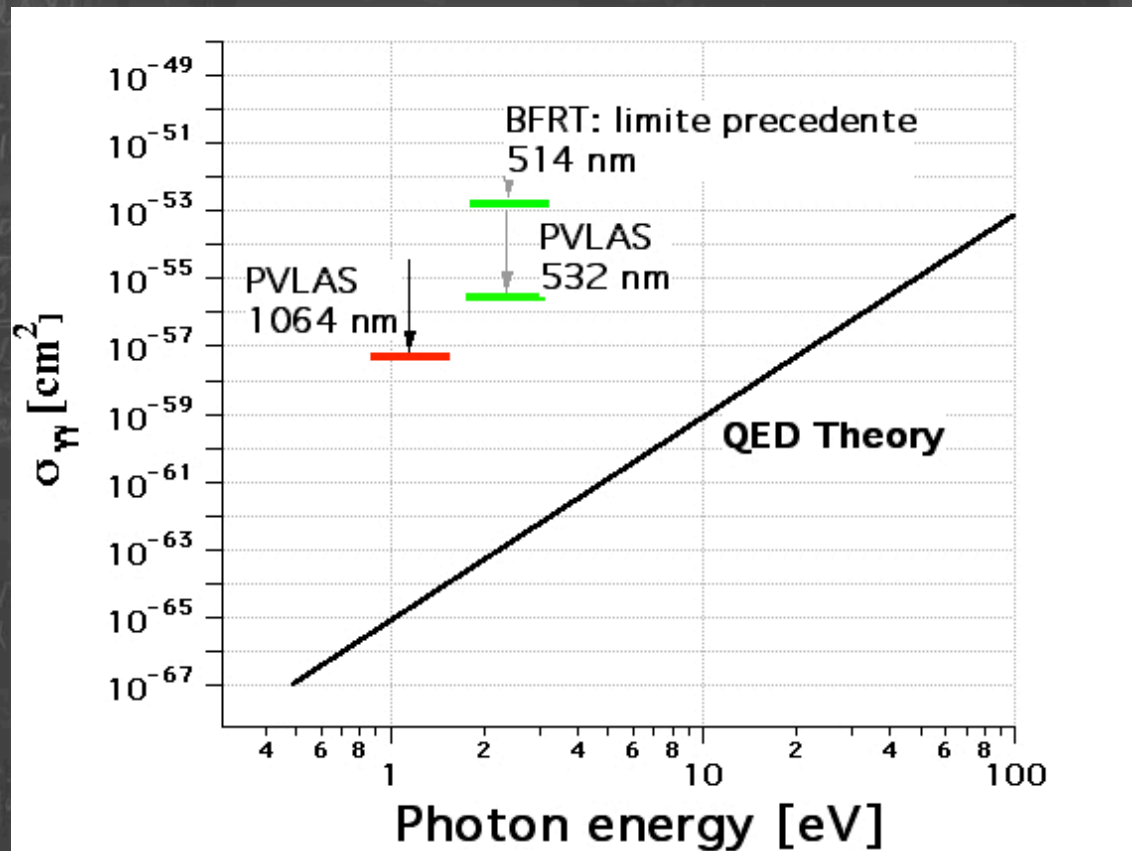
$$\Delta n_{1064} < 4.6 \cdot 10^{-20} @ 1064 \text{ nm}$$

$$\Delta n_{532} < 1.0 \cdot 10^{-19} @ 532 \text{ nm}$$



$$A_e^{(FE)} < 2.9 \cdot 10^{-21} \text{ T}^{-2}$$

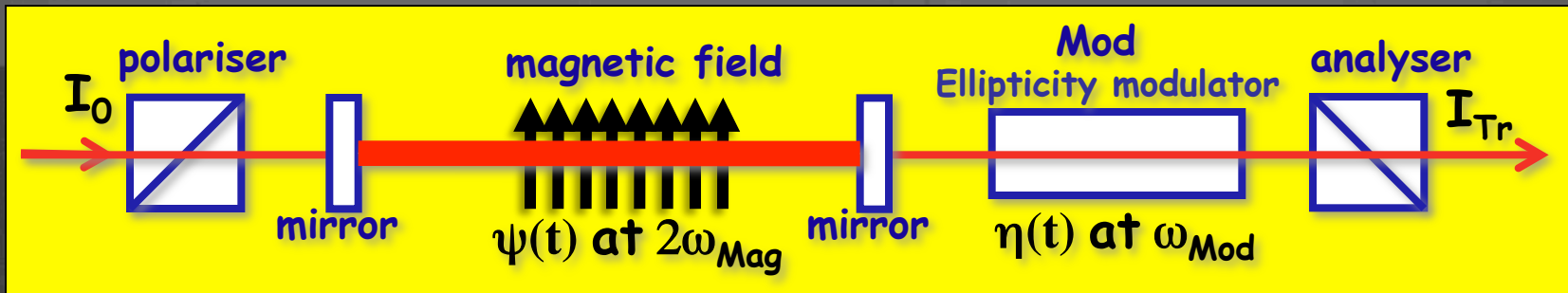
$$A_e^{(QED)} = 1.3 \cdot 10^{-24} \text{ T}^{-2}$$



Bregant et al, PRD 78, 032006 (2008)

$$\sigma_{\gamma\gamma} < 9.5 \cdot 10^{-59} \text{ cm}^2 @ 1064 \text{ nm}$$

$$\sigma_{\gamma\gamma} < 2.7 \cdot 10^{-56} \text{ cm}^2 @ 532 \text{ nm}$$

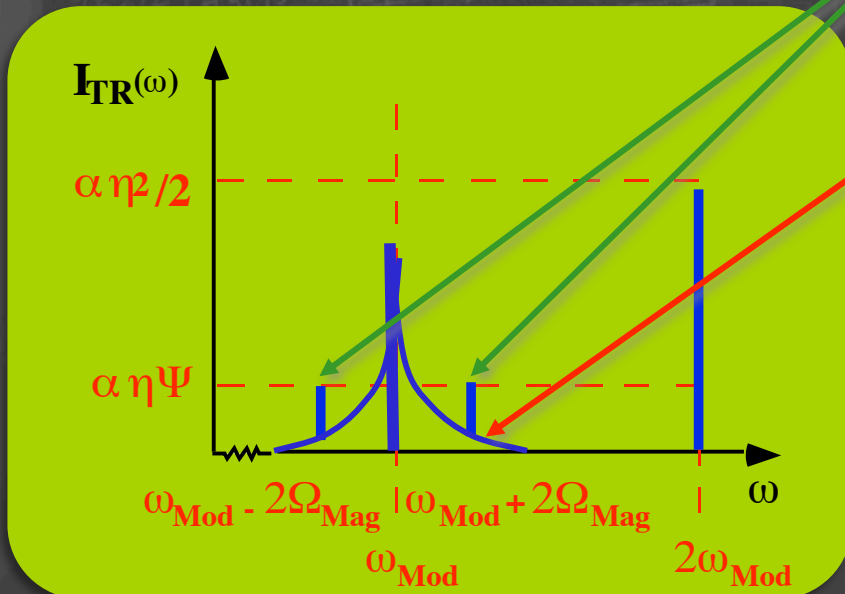


$$I_{Tr} = I_0 \left[\sigma^2 + (\psi(t) + \eta(t) + \beta_s(t))^2 \right]$$

$$= I_0 \left[\sigma^2 + \underbrace{\eta(t)^2}_{\text{noise}} + \underbrace{2\psi(t)\eta(t)}_{\text{signal}} + \underbrace{2\beta_s(t)\eta(t)}_{\text{noise}} + \dots \right]$$

signal

noise



Main frequency components at $\omega_{Mod} \pm 2\omega_{Mag}$ and $2\omega_{Mod}$



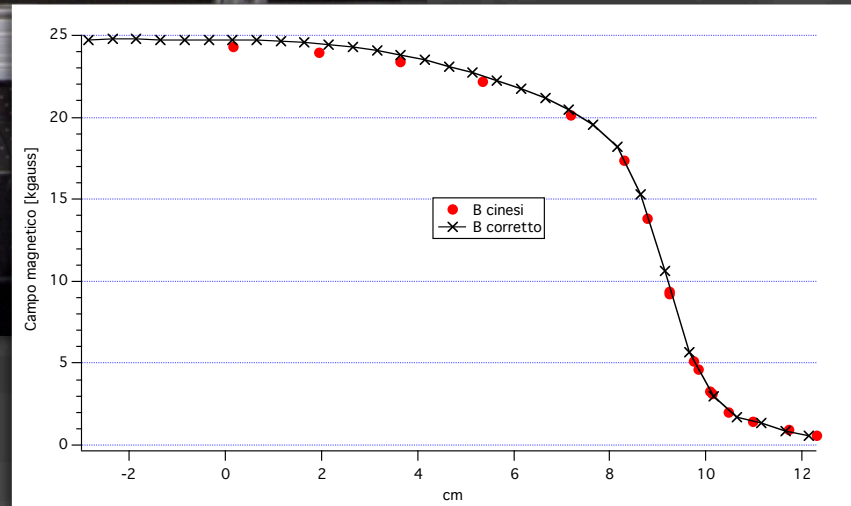
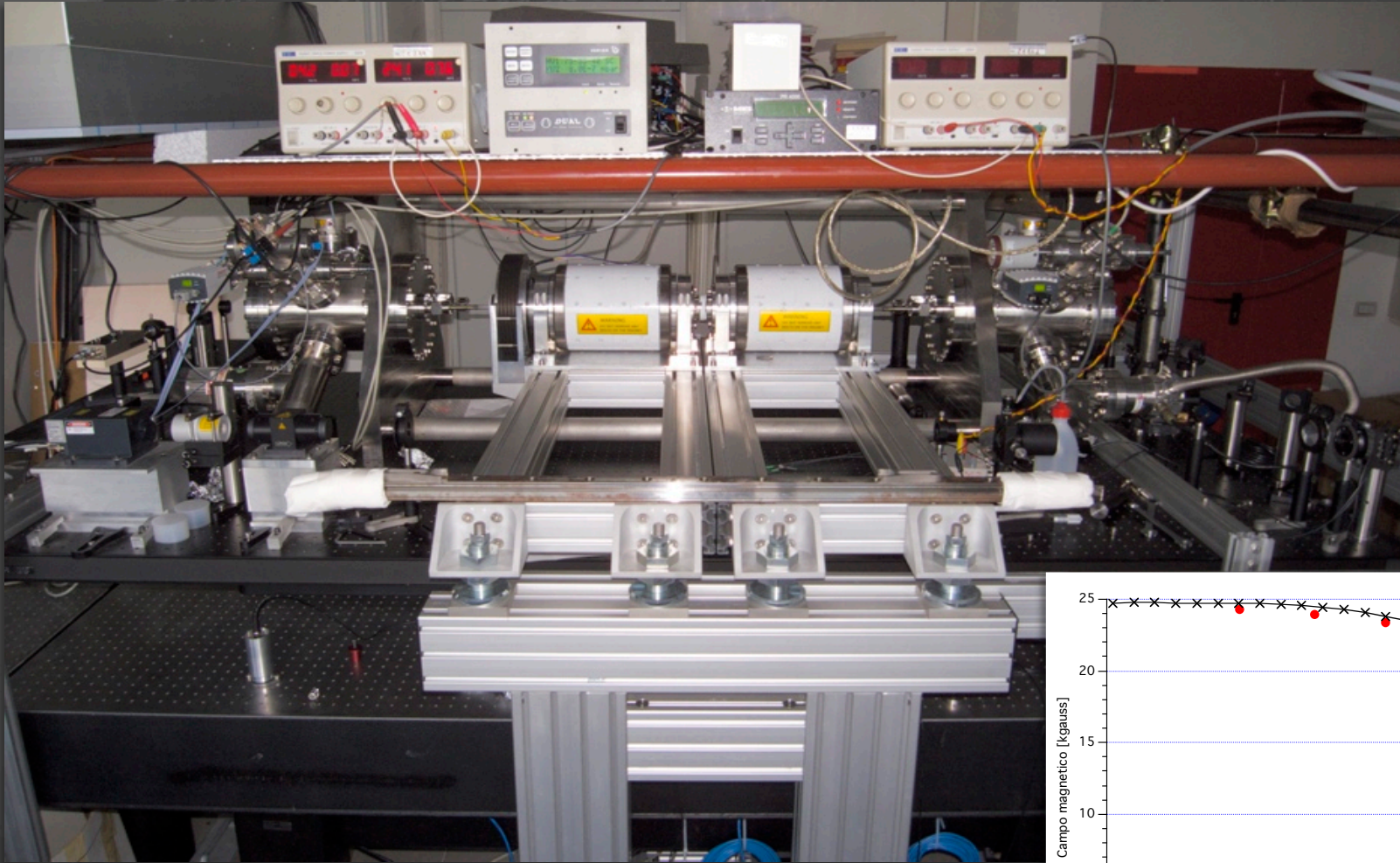
Limitations of the LNL apparatus

- Superconducting magnets produce **stray field** when operated at high fields (saturated iron)
- **Running time limited** due to liquid helium consumption
- Observed **correlation between seismic noise and ellipticity noise**. The Legnaro apparatus is large and therefore difficult to isolate seismically.
- **No zero measurement possibile** with field turned ON.



Test apparatus in Ferrara

Two permanent magnets allow a zero measurement





Fabry Perot

Ferrara test apparatus - High finesse successful

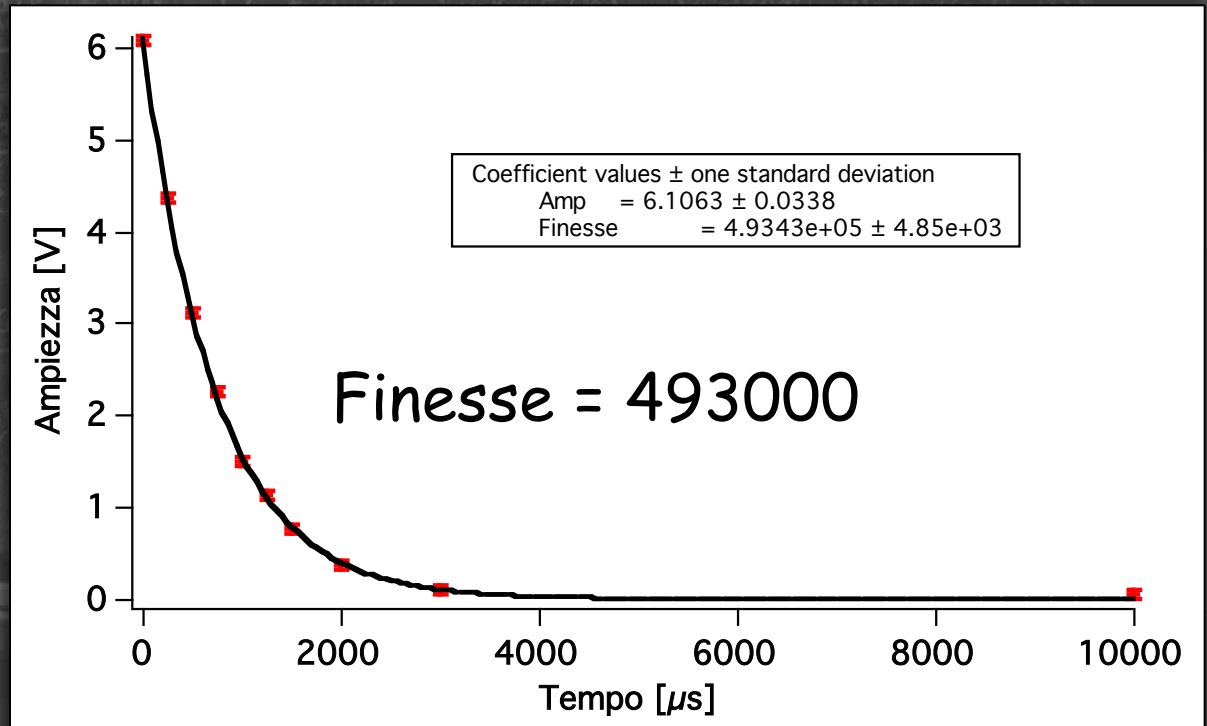
The **Fabry-Perot cavity** is a resonant optical cavity that **increases the effective optical path**. It is composed of **two mirrors placed at a separation d** which is an integer multiple of the light half wavelength. To obtain this condition a laser is phase locked to the cavity using a feedback circuit.

Amplification factor

$$N = \frac{2F}{\pi}$$

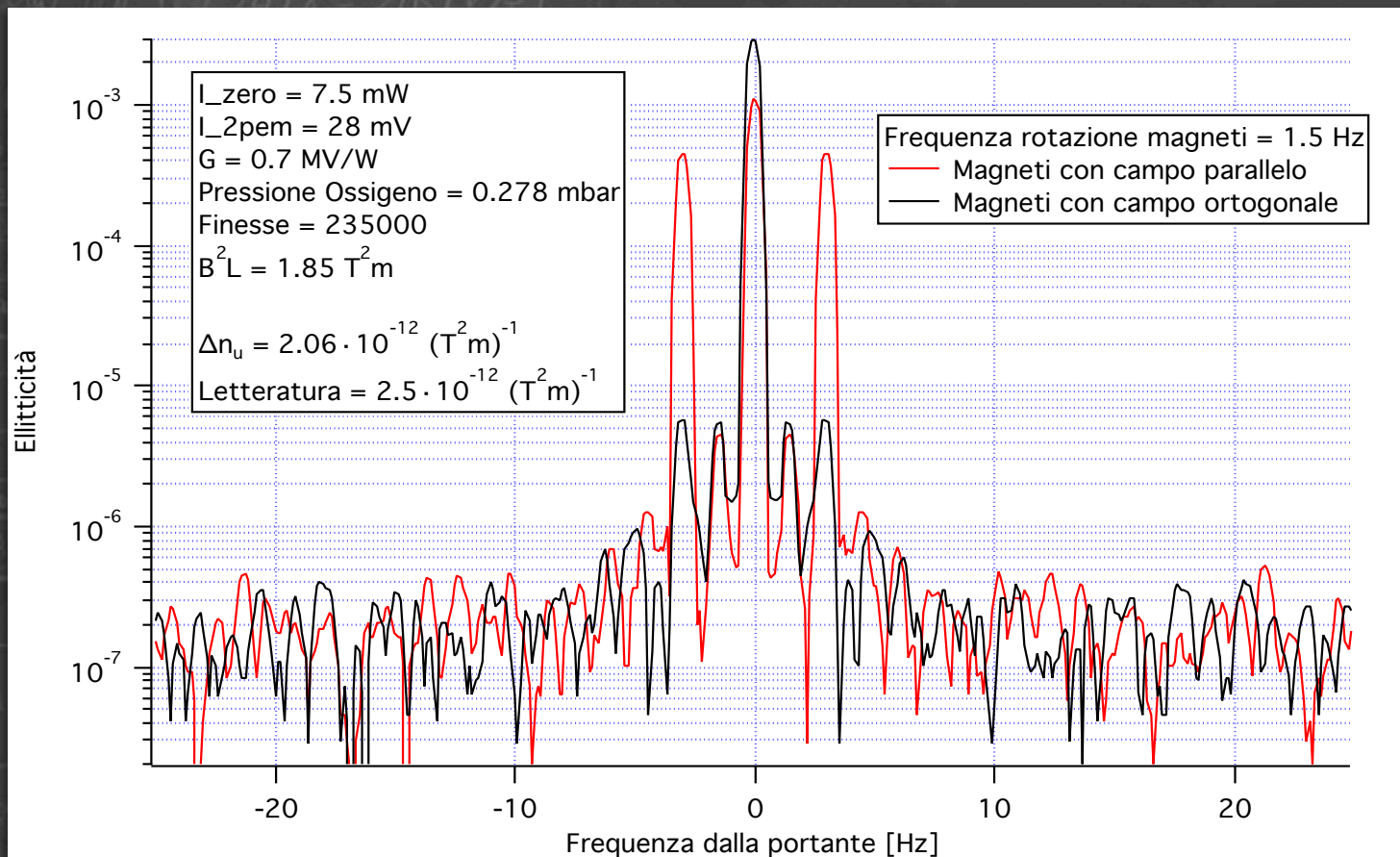
Finesse

$$F = \frac{\pi c \tau}{d}$$





Two magnet configuration



In red, magnets at 0 degrees
In black, magnets at 90 degrees

With a finesse = 245000 and with the magnets perpendicular to each other we demonstrated a reduction of more than a factor 80 of the Cotton Mouton signal



Rotating vs non rotating magnets

Rotating Magnets

No peak

Sensitivity = $3.0 \cdot 10^{-7} \text{ 1}/\sqrt{\text{Hz}}$

Δn sensitivity = $1.8 \cdot 10^{-18} \text{ 1}/\sqrt{\text{Hz}}$

T = 2.3 hours

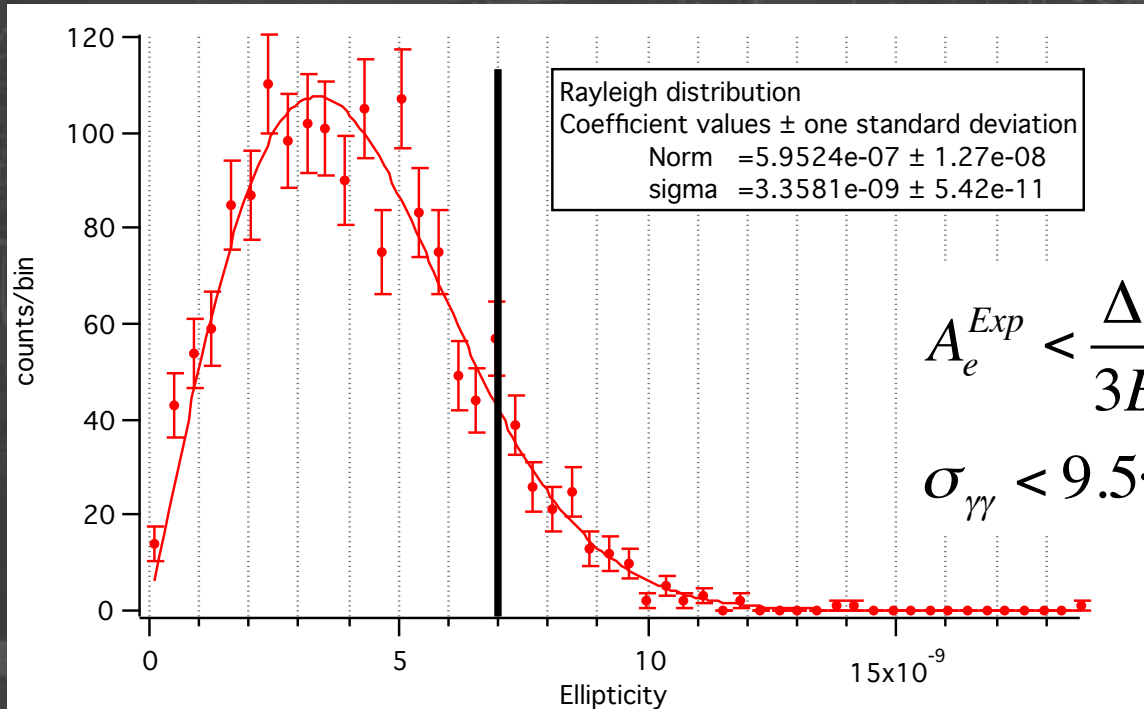
Non rotating Magnets

No peak

Sensitivity = $3.3 \cdot 10^{-7} \text{ 1}/\sqrt{\text{Hz}}$

Δn sensitivity = $1.8 \cdot 10^{-18} \text{ 1}/\sqrt{\text{Hz}}$

T = 4.8 hours



$$A_e^{Exp} < \frac{\Delta n}{3B^2} = \frac{\sigma \lambda}{2FL3B^2} = 2.9 \cdot 10^{-21} \text{ T}^{-2}$$

$$\sigma_{\gamma\gamma} < 9.5 \cdot 10^{-59} \text{ cm}^2 @ 1064 \text{ nm}$$

Ellipticity histograms around $2 \Omega_{Mag}$



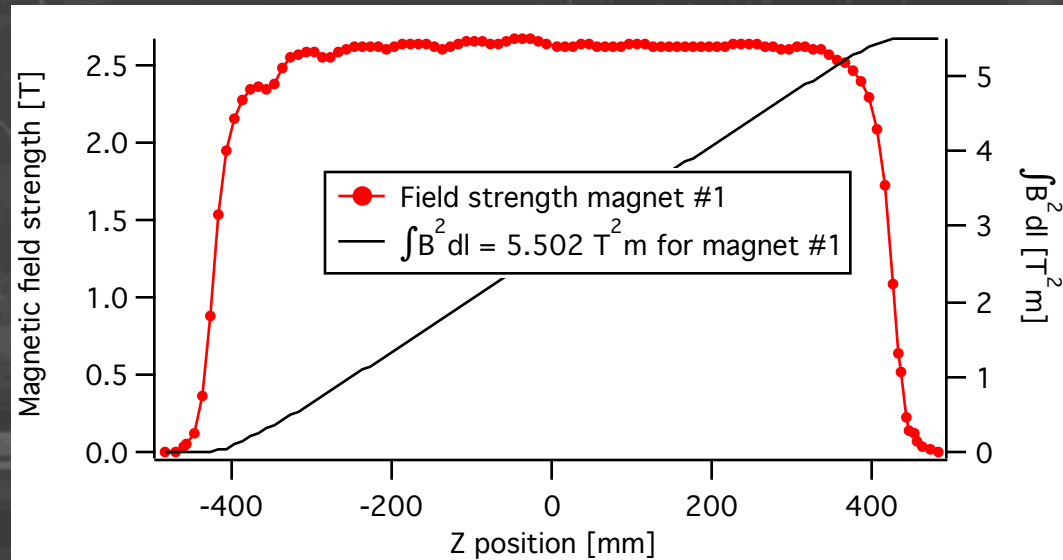
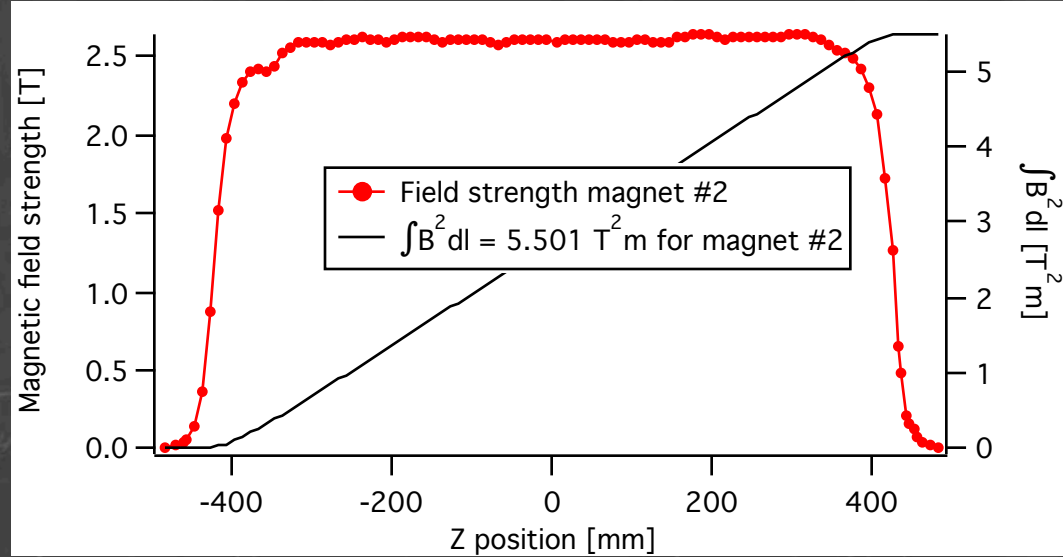
New granite optical bench

Installation in Ferrara clean room





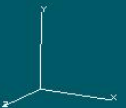
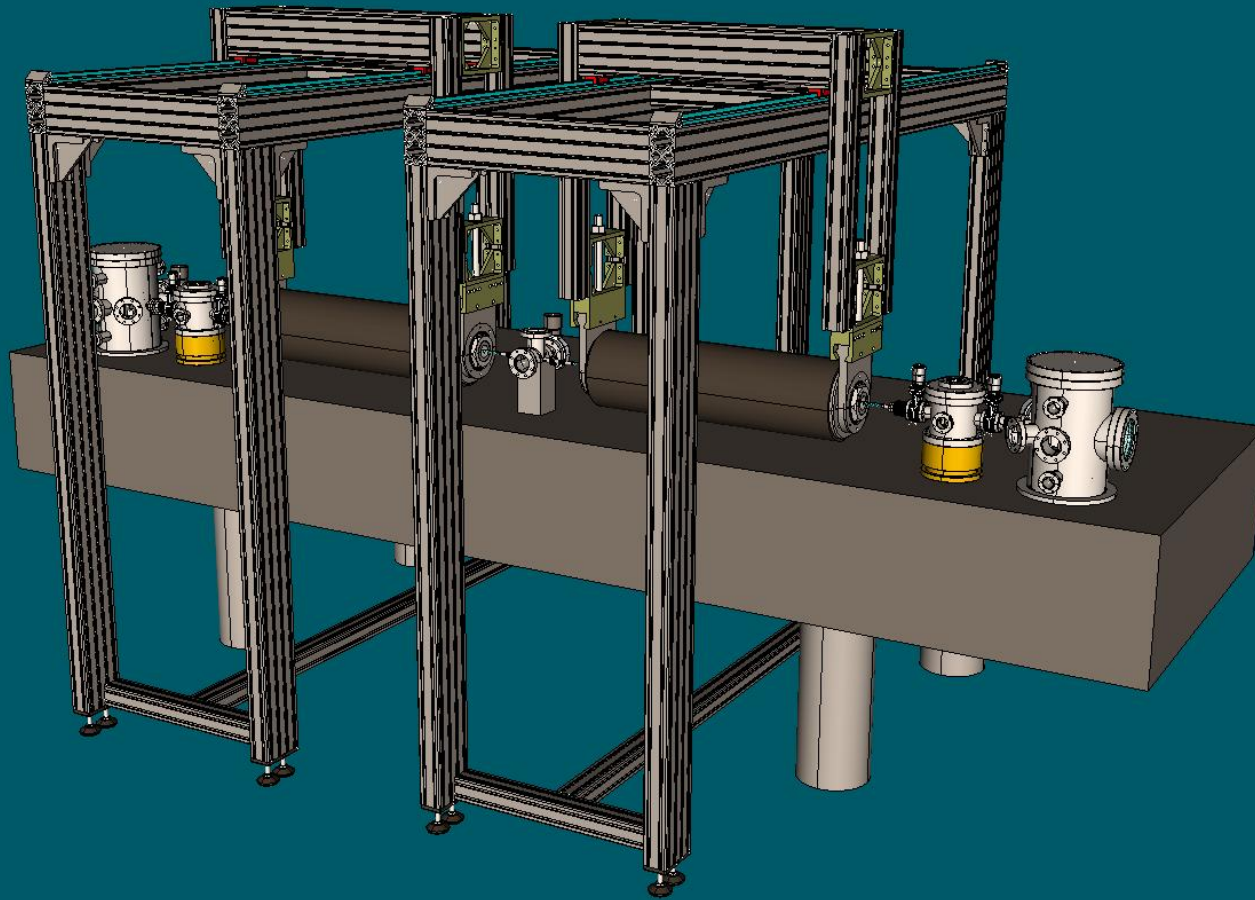
Magnets





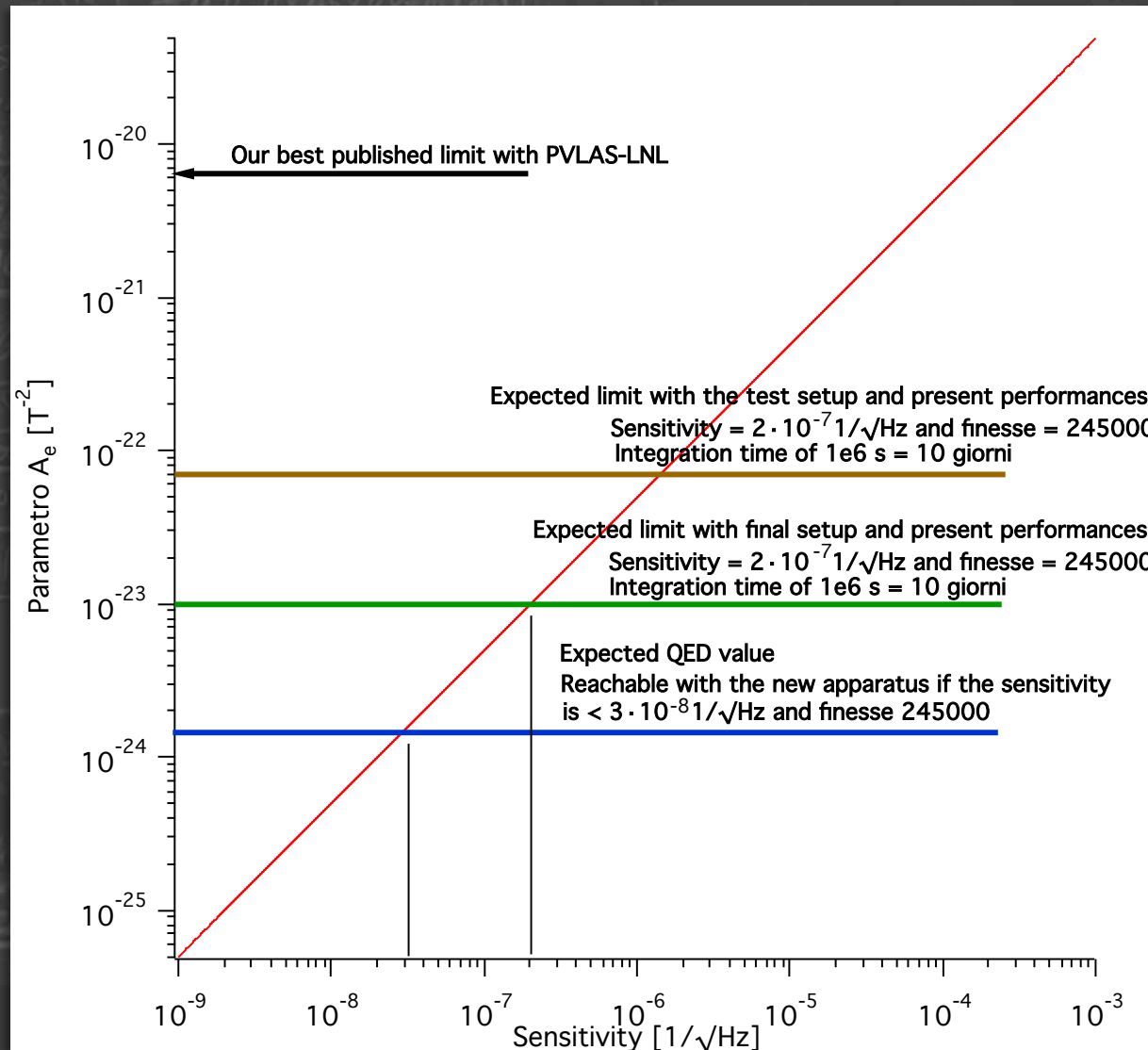
Mechanical Design

B.72843 (cm) / cm 72418 = SIKIVG1
79813 - 200 FEMTIS/15848-10)AD





Parameter space





Other world efforts

Q&A experiment - Taiwan

Sensibilità 10^{-6} $1/\sqrt{\text{Hz}}$, con $F = 30000$, campo 2.3 T, $L = 180$ cm, 1 magnete. Banchi ottici separati. Ellitticità = $2 \cdot 10^{-12}$.

$$T_{\text{QED}} = 2 \cdot 10^{11} \text{ s} = 6500 \text{ anni}$$

BMV experiment - Toulouse

Sensibilità in $\Delta n \approx 5 \cdot 10^{-20}$ $1/\sqrt{\text{Hz}}$ con $F = 529000$, campo impulsato qualche ≈ 3 ms, $115 \text{ T}^2\text{m}$, 1 colpo/12 min \Rightarrow sensibilità efficace = $1.7 \cdot 10^{-17}$ $1/\sqrt{\text{Hz}}$. Ellitticità = $1 \cdot 10^{-10}$

$$T_{\text{QED}} = 1.8 \cdot 10^{13} \text{ s} = 5 \cdot 10^5 \text{ anni}$$

OSQAR experiment - CERN

Sensibilità = ??? Magnete di LHC lungo 15 m, 10 T, Finesse ≈ 1000 , banchi ottici separati. Magnete non modulabile. Ellitticità = $1 \cdot 10^{-11}$. Assumendo sensibilità = 10^{-7} $1/\sqrt{\text{Hz}}$. A che frequenza? Presumibilmente mHz Difficilmente ottenibile

$$T_{\text{QED}} = 10^8 \text{ s} = 3.2 \text{ anni ???}$$

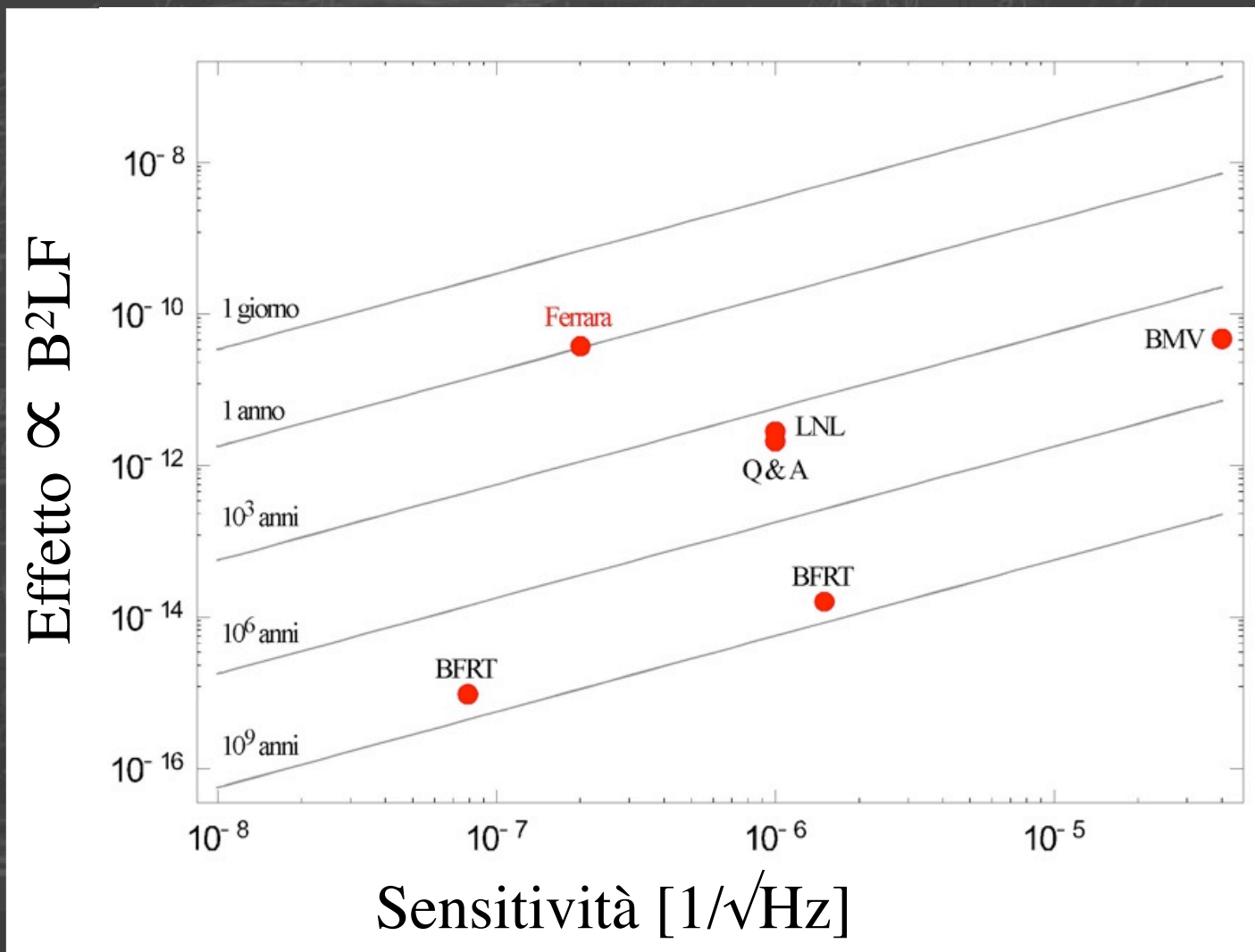
PVLAS - Ferrara

Sensibilità $2 \cdot 10^{-7}$ $1/\sqrt{\text{Hz}}$, con $F = 285000$, campo 2.6 T, $L = 1.8$ m, 2 magneti, $11 \text{ T}^2\text{m}$ Banco unico ottico. Ellitticità = $2.4 \cdot 10^{-11}$

$$T_{\text{QED}} = 7.2 \cdot 10^7 \text{ s} = 2.2 \text{ anni}$$



Measurement time





B. 72843 (Cern)] ^{LNF} ²⁷² ²⁷² ²⁷²
 C. 79593
 √(velocità) = 1.7510⁶ ^{10⁶} ^{10⁶} ^{10⁶}
 B. INFN 3302/98
 1. Apollonio 71516 | 3 m 27 3127 | 74861 | d.
 3388 B. (trazite) 3381 | S. Dip. Mio Ufficio
 10 P. LLHI 7675716 | 164310 | 75848 Mio
 71865 | ore 1430 P. | 7924788 | Bartolo R. 72419
 Centralina 76111 (mit Cern ...) | 2481 Profelsk II
 = 58 1 | IACOPINI (Cern) 79824
 MAIL BOX TS CERN
 73831 Angela Beucelli | 74411
 TS212 | 4.91 10¹⁶ G. B. | Present (CERN) 72843
 Z. etas 164592 | L.S.V. P.
 VACCHI 0039 040 375 6229
 0039 320 92 32 326 Cell
 GUIDO 0039 0532 974 340 | 340 | HOCARDI
 0039 0532 974 299 | 299
 0039 3384 9898 65 | 65

$$E_F = \begin{pmatrix} \cos \alpha - \sin \alpha & 1 + i\eta \\ \sin \alpha \cos \alpha & 0 \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \cdot E(B)$$

$$\begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} (1+i\eta) \cos \alpha & \sin \alpha (1+i\eta) \\ -\sin \alpha (1-i\eta) & \cos \alpha (1-i\eta) \end{pmatrix}$$

$$\begin{pmatrix} \cos^2 \alpha (1-i\eta) + \sin^2 \alpha (1-i\eta) & \sin \alpha \cos \alpha (1+i\eta) - \sin \alpha \cos \alpha (1-i\eta) \\ 1 + \cos^2 \alpha i\eta - \sin^2 \alpha i\eta & \frac{\sin 2\alpha}{2} (1+i\eta) - \frac{\sin 2\alpha}{2} (1-i\eta) \end{pmatrix}$$

$$\begin{pmatrix} \cos \alpha \sin \alpha (1-i\eta) - \sin \alpha \cos \alpha (1-i\eta) & \cos^2 \alpha (1+i\eta) + \sin^2 \alpha (1+i\eta) \end{pmatrix}$$

$$\cos A = \frac{f_a}{r_a} \quad m_a = \frac{f}{r} \cdot m \pi$$

$$f_a m_a = \cos A$$

$$1 - (\cos^2 - \sin^2) i\eta$$

$$= \frac{\sin 2\alpha}{2} i\eta$$

$$= 2\eta$$

Grazie!



Post-Maxwellian models - 3

For $\vartheta \neq 0$

$$\begin{cases} f_{\parallel}^{(pM)}(\vartheta, E_{\gamma}) \propto \eta_1 E_{\gamma}^3 \\ f_{\perp}^{(pM)}(\vartheta, E_{\gamma}) \propto (a\eta_1 + b\eta_2) E_{\gamma}^3 \end{cases} \Rightarrow \text{Always } \neq 0$$

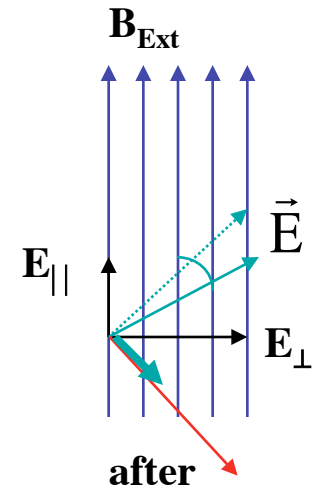
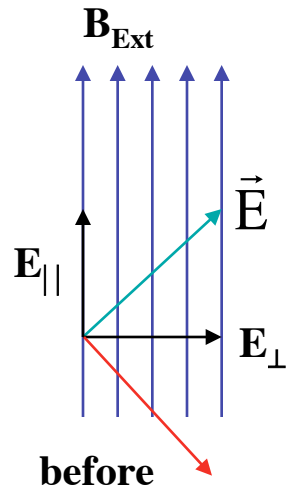
$$\Delta n^{(pM)} = 2\xi(\eta_2 - \eta_1) B_{Ext}^2$$

If $\eta_1 = \eta_2 \Rightarrow \Delta n^{(pM)} = 0$
e.g. Born - Infeld



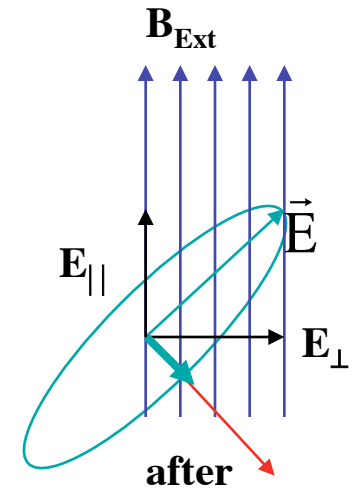
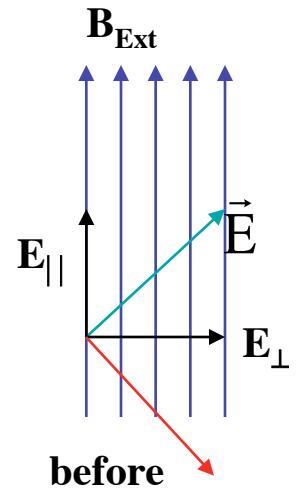
Linear dichroism and birefringence

Dichroism



apparent rotation ϵ

Ellipticity



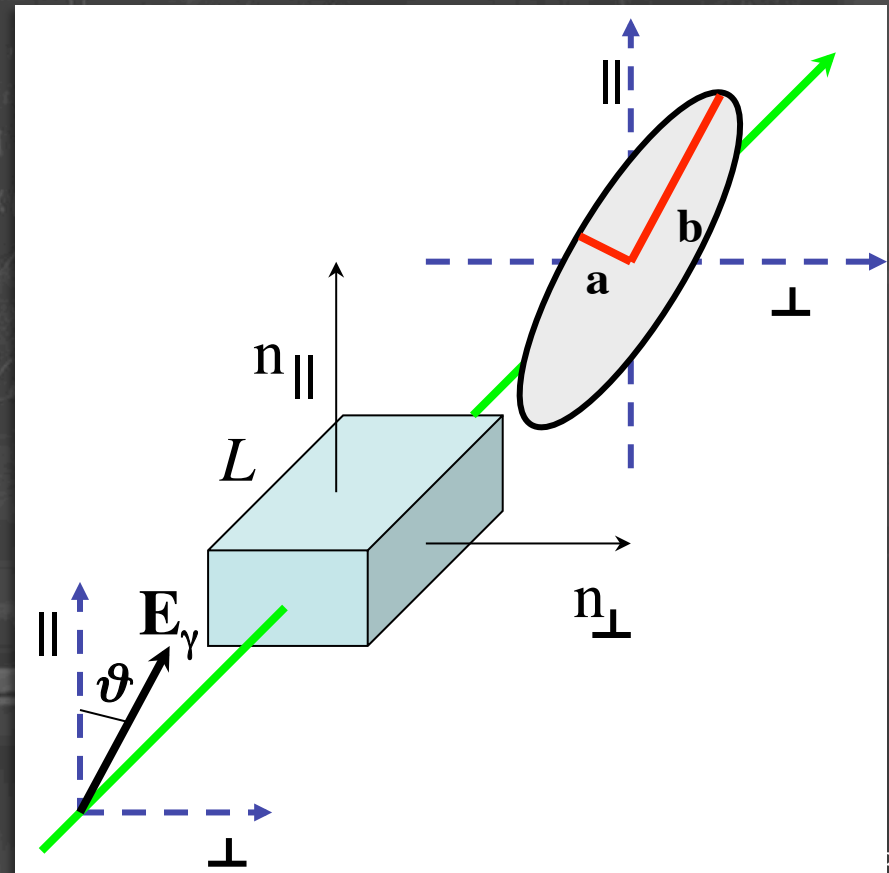
ellipticity ψ



Linear Birefringence

- A birefringent medium has $n_{||} \neq n_{\perp}$
- A linearly polarized light beam propagating through a birefringent medium will acquire an ellipticity ψ

$$\psi = \frac{a}{b} = \frac{\pi L(n_{||} - n_{\perp})}{\lambda} \sin 2\vartheta$$



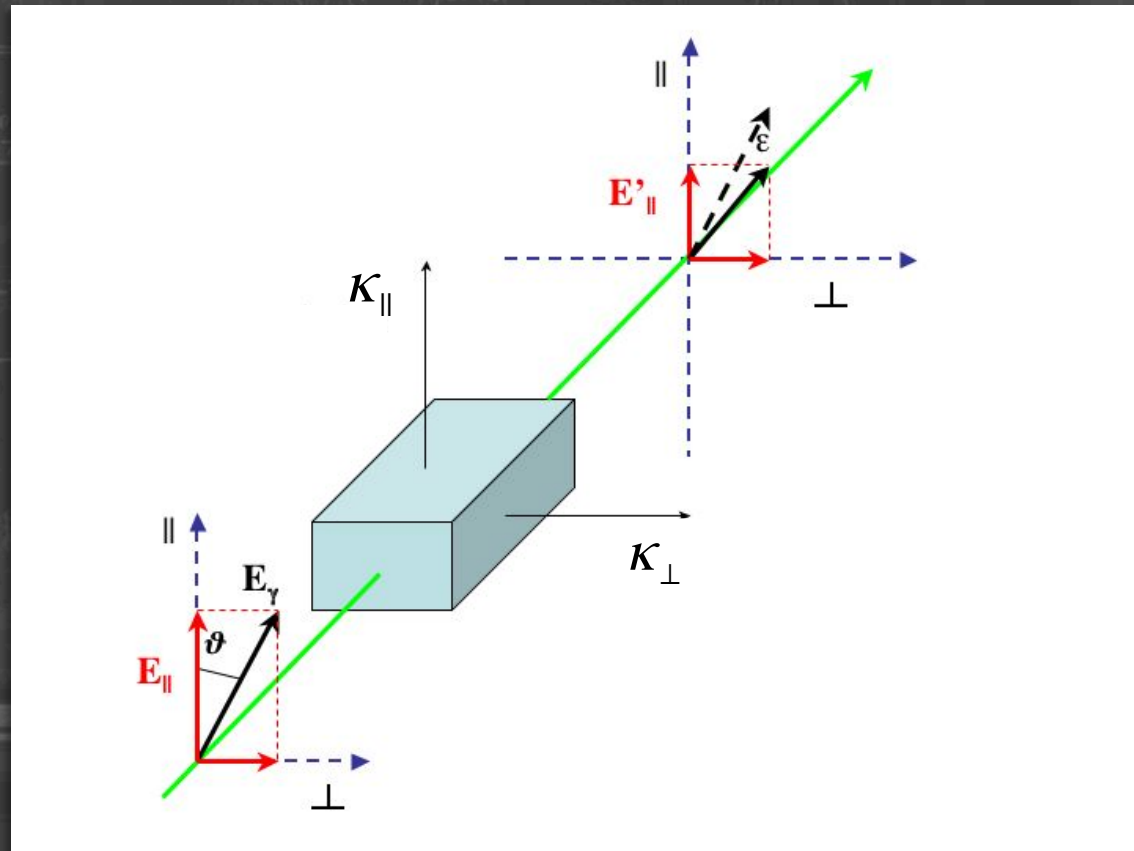


Linear Dichroism

- A dichroic medium has different extinction coefficients: $\kappa_{\parallel} \neq \kappa_{\perp}$
- A linearly polarized light beam propagating through a dichroic medium will acquire an apparent rotation ε

$$\varepsilon = \frac{\pi L(\kappa_{\parallel} - \kappa_{\perp})}{\lambda} \sin 2\vartheta$$

Absorption coefficient $= \frac{2\pi}{\lambda} \kappa$





Axion-like contribution

One can add extra terms [*] to the E-H effective lagrangian to include contributions from hypothetical neutral light particles interacting weakly with two photons

$$L_\phi = \frac{1}{M} \phi (\vec{E}_\gamma \cdot \vec{B}_{ext})$$

pseudoscalar case

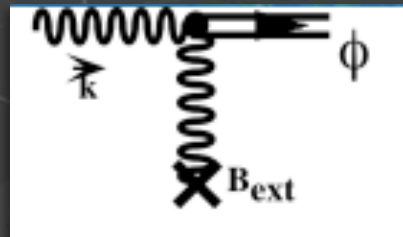
M, M_s are inverse coupling constants

$$L_\sigma = \frac{1}{M_s} \sigma (\vec{B}_\gamma \cdot \vec{B}_{ext})$$

scalar case

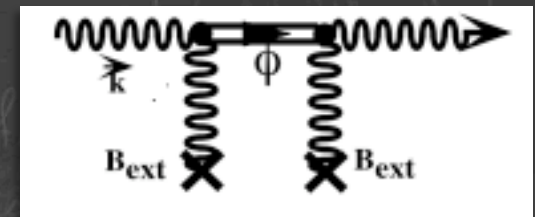
Effects on photon propagation

Absorption



DICHROISM

Dispersion



BIREFRINGENCE



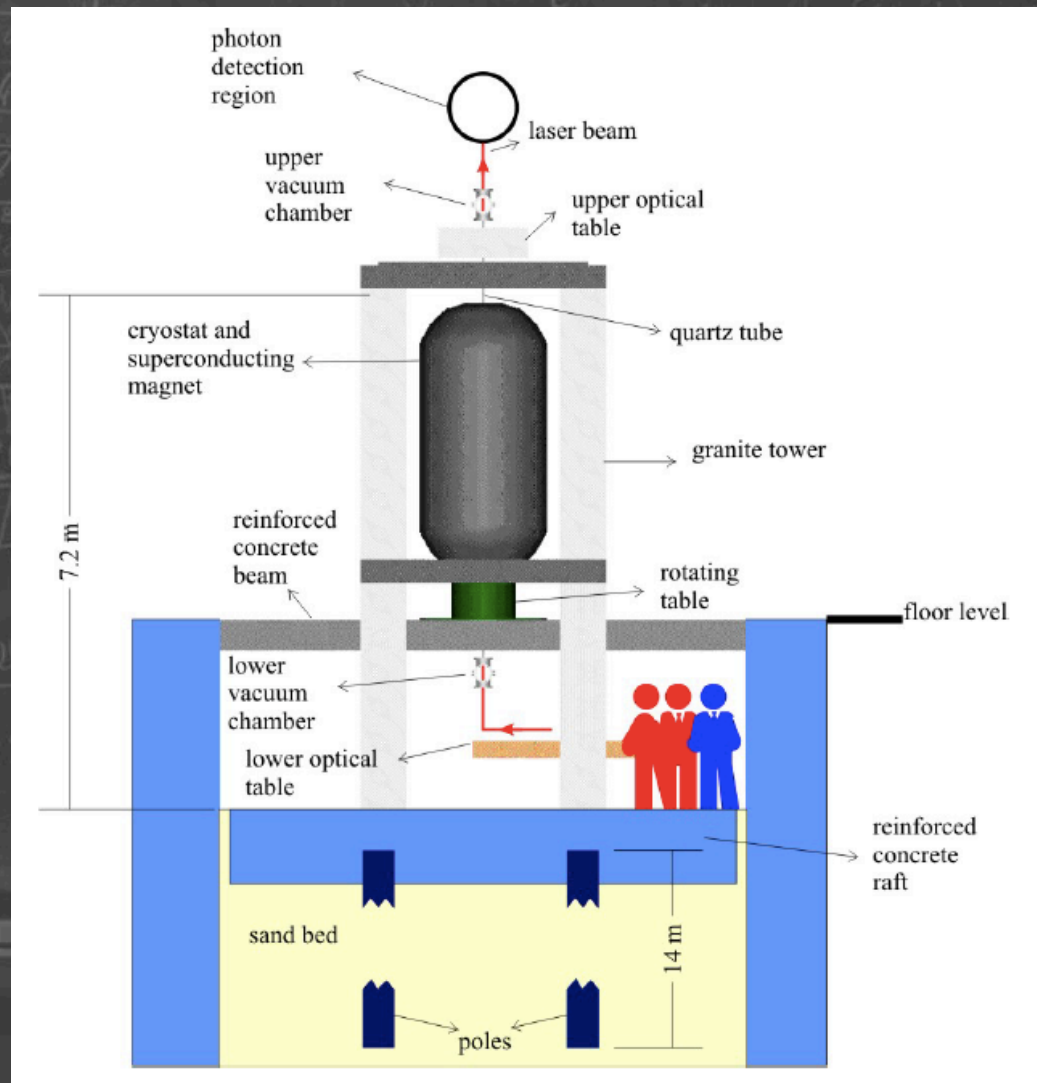
Past - PVLAS at Lab. Nazionali Legnaro

Focused on a general study of the vacuum in the presence of a magnetic field

Polarizzazione del Vuoto con LASer

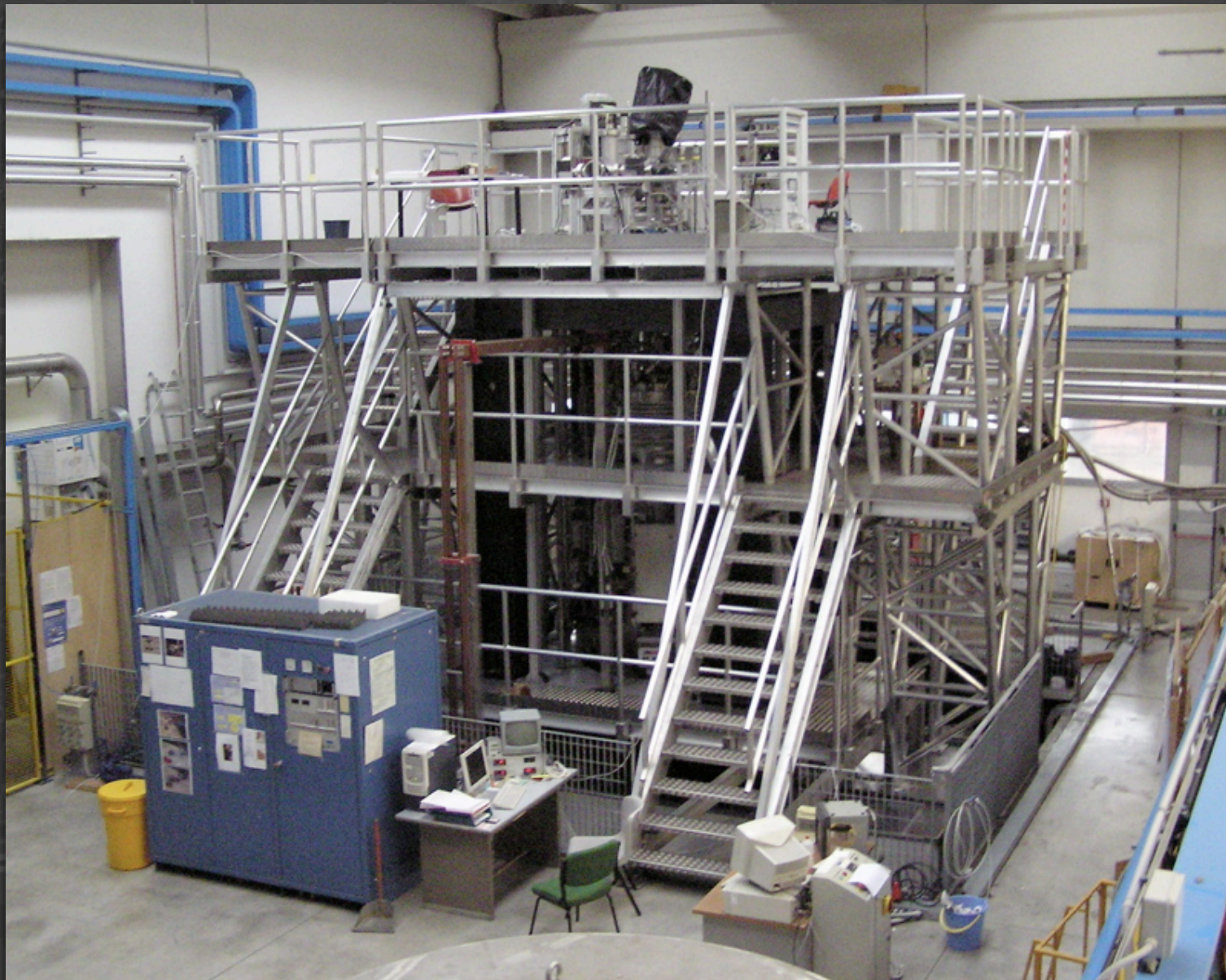
Major improvements compared to previous efforts:

- Resonant FP cavity (6.4 m) for large amplification factor ($> 5 \cdot 10^4$)
- Rotating cryostat allows high modulation frequency (up to 0.4 Hz)
- Large magnetic field (magnet tested up to 7 T)
- Magnetic system mechanically decoupled from optical system



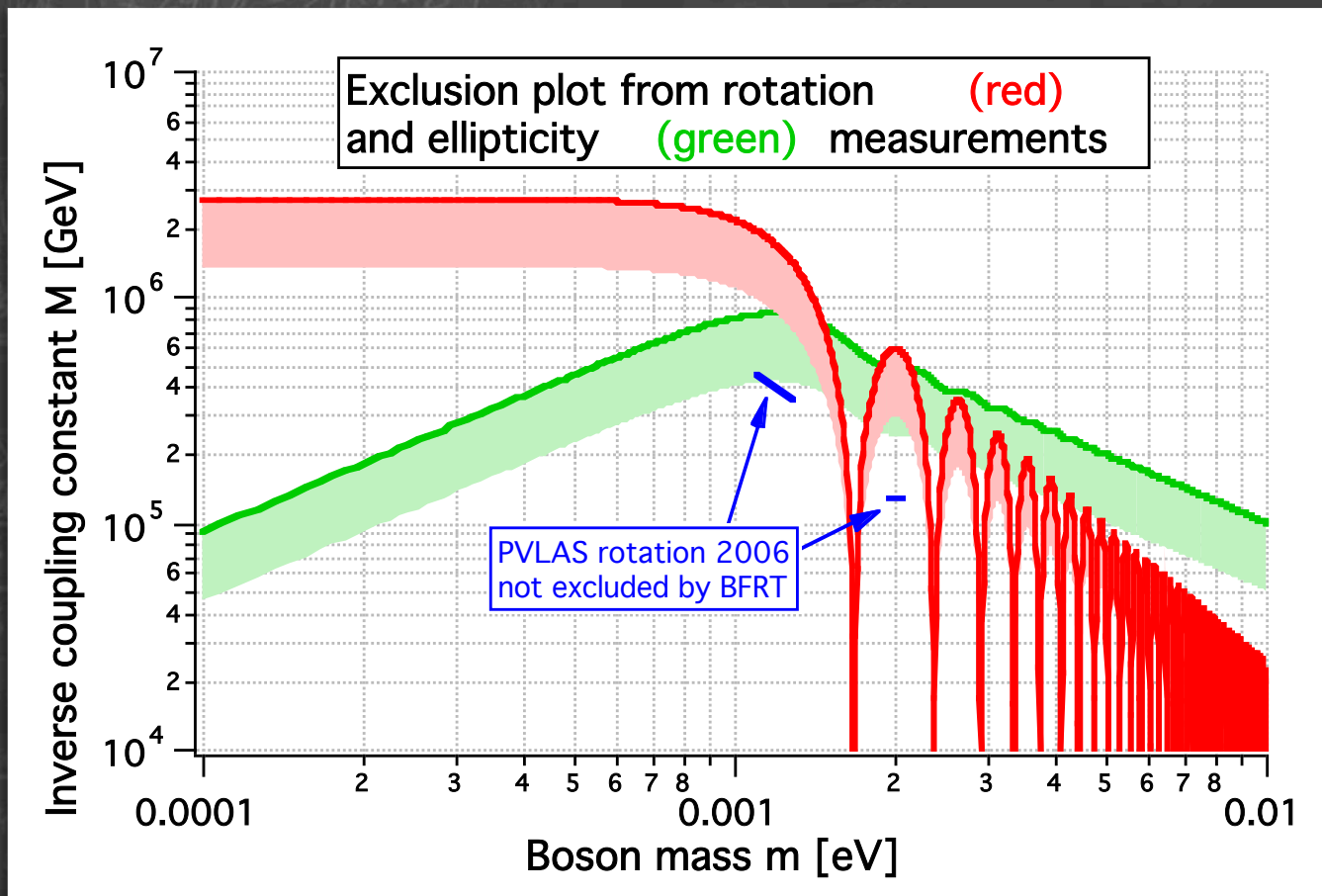


Past - PVLAS at Lab. Nazionali Legnaro





Present published results - ALP

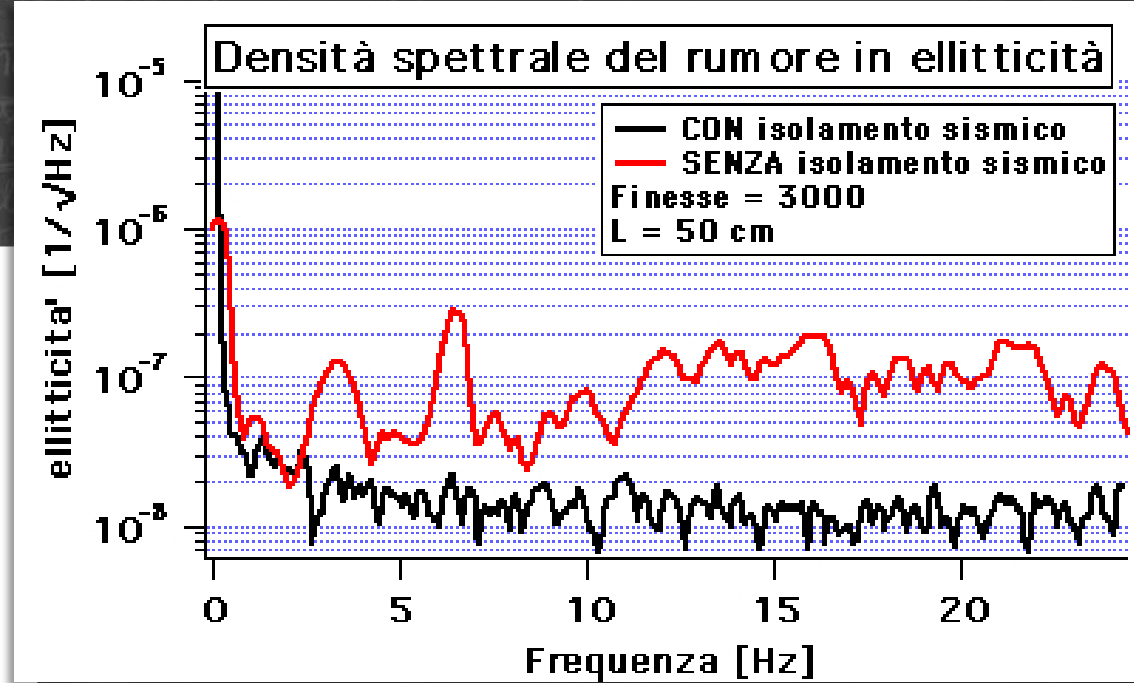
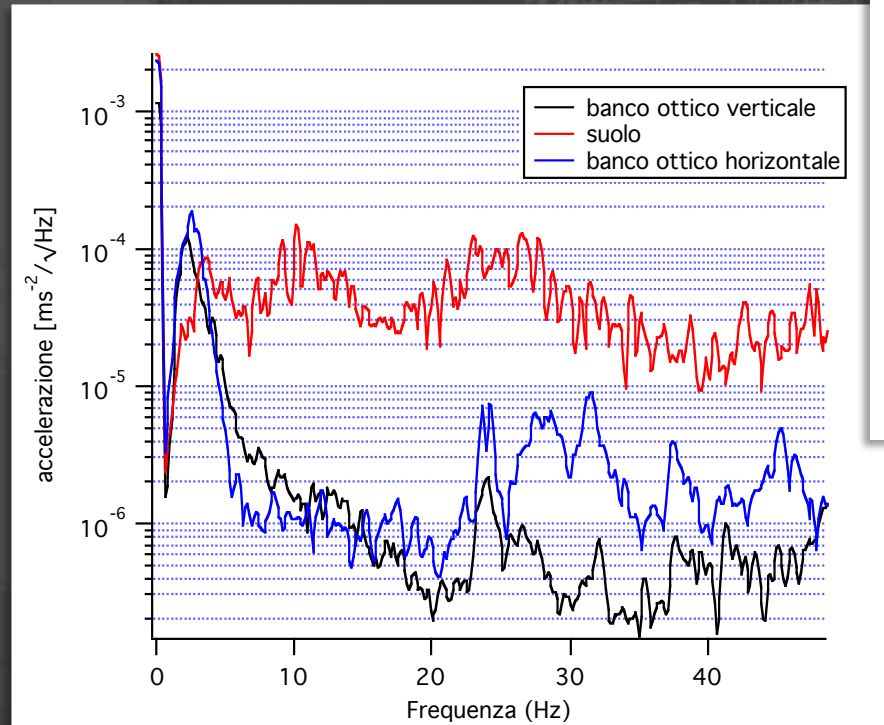


The CAST experiment at CERN has excluded values of $M < 10^{10}$ GeV
Unreachable with present lab techniques



Low finesse - seismic isolation

Compact 50 cm long ellipsometer without magnetic field



Flat noise spectrum above ≈ 5 Hz



High finesse - seismic isolation

High finesse: $F = 414000$

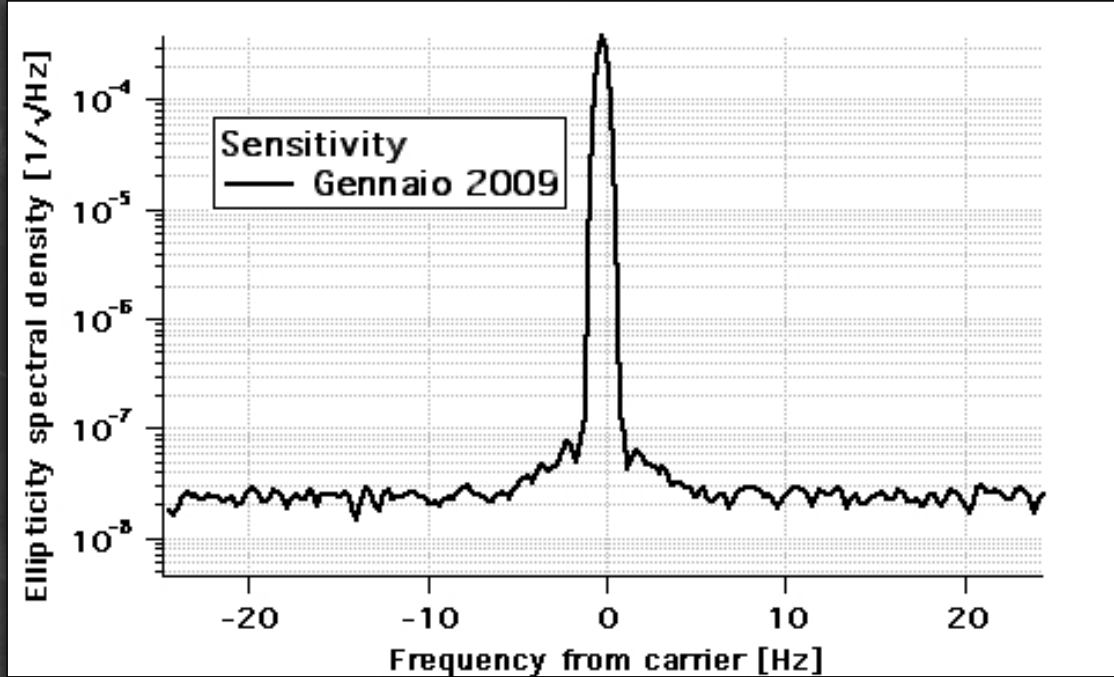
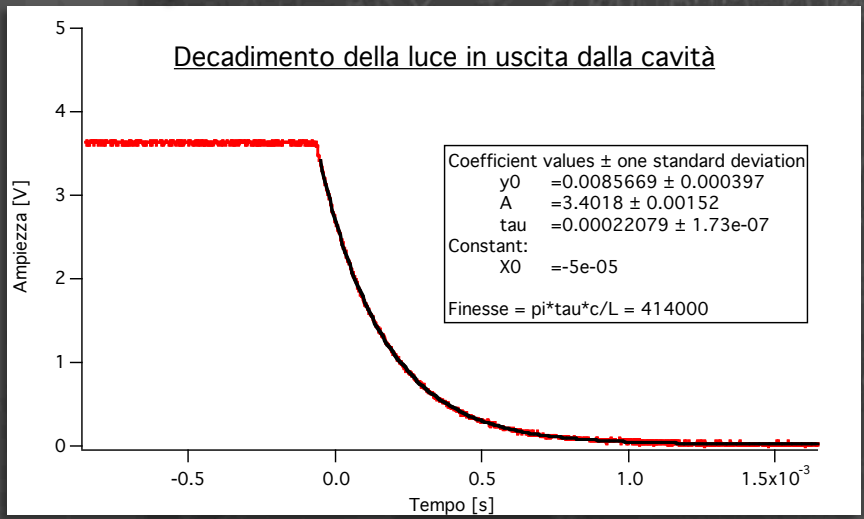
$$L_{\text{eff}} = (2F/\pi)L = 130 \text{ km}$$

Compact 50 cm long ellipsometer without magnetic field

Cavity output power = 25 mW

Laser-cavity coupling = 75%

Cavity transmission = 25%



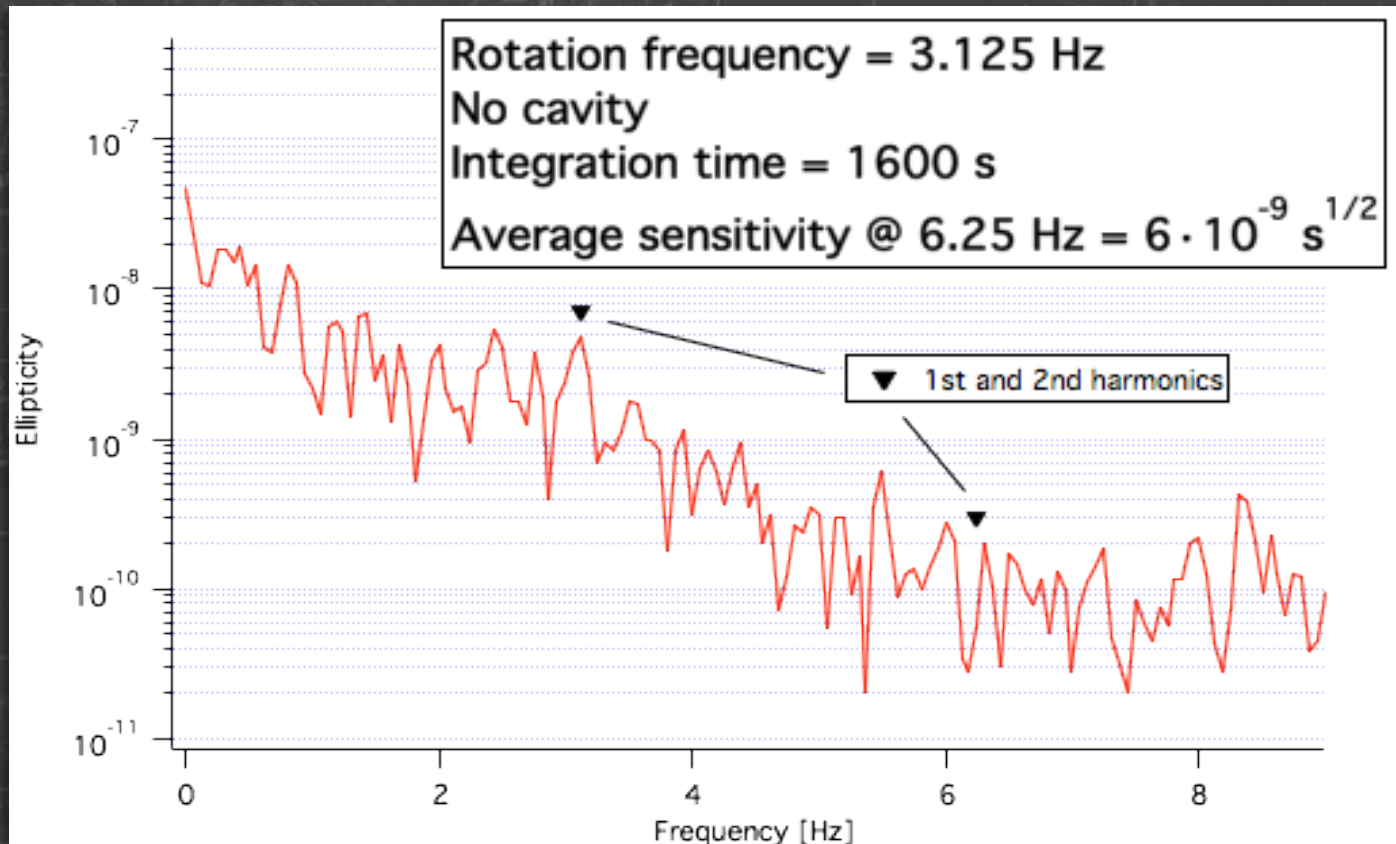
Record sensitivity with a cavity $\Psi = 3 \cdot 10^{-8} \text{ 1}/\sqrt{\text{Hz}}$

Assuming $B = 2.3 \text{ T}$: Sensitivity in $\Delta n / B^2 = 1.5 \cdot 10^{-20} \text{ T}^{-2} \text{ 1}/\sqrt{\text{Hz}}$



Ferrara test apparatus - sensitivity

No cavity - reached expected noise level with rotating magnets



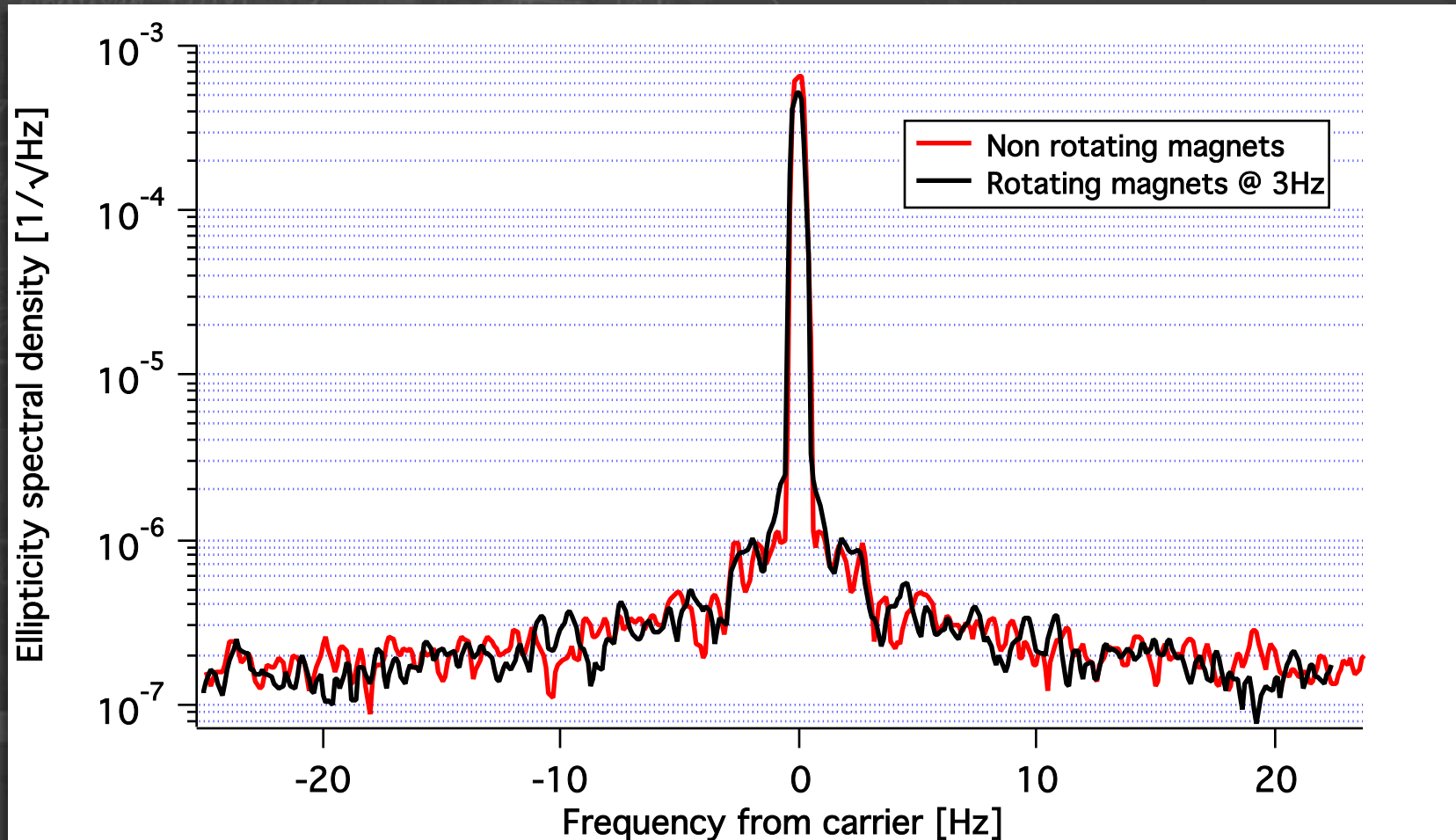
No electronically induced signals in the readout system



Ferrara test apparatus - sensitivity

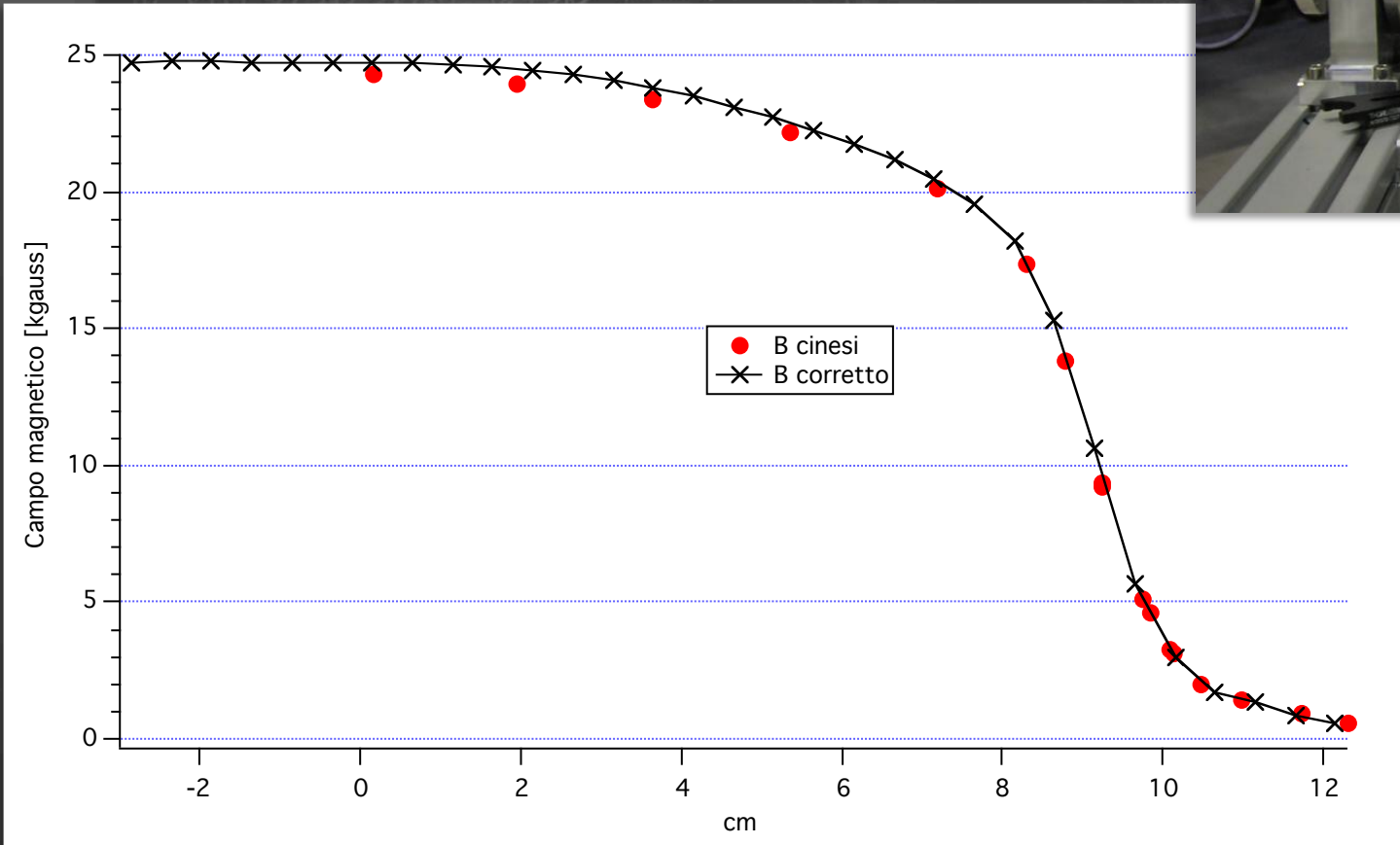
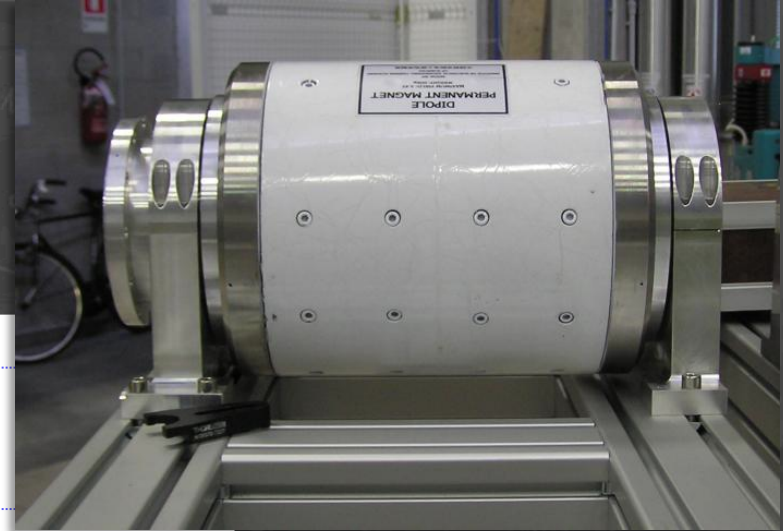
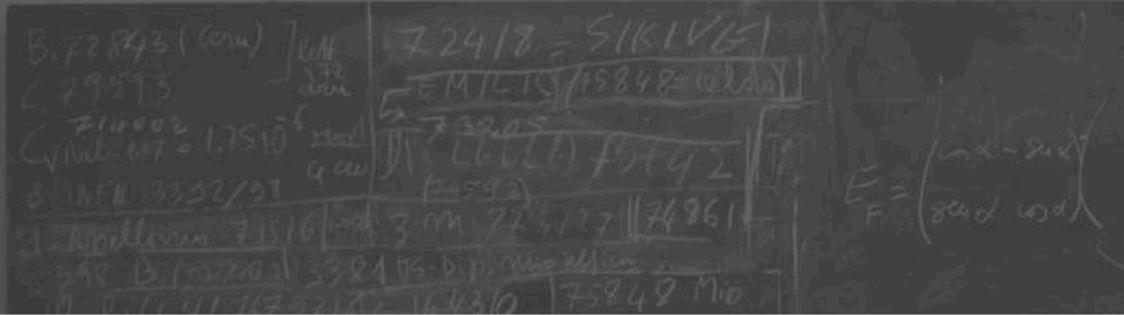
With high-finesse cavity > 400000

Sensitivity worsened - still under study





Permanent Test Magnets





New granite optical bench

Installation in Ferrara clean room





Present - Future

Currently building final apparatus in clean room in Ferrara.

- Magnetic field: 2 x 1 m long magnets with 2.5 T (arrived)
- Magnet support structure (arrived)
- Magnet support mechanics + motor (ordered)
- Optical bench with isolation system (installed)
- Optical enclosures (arriving)
- New laser (2 Watts, 1064 nm) (arrived)
- All optical elements, supports and movements will be non magnetic (arrived)
- Getters will be used as vacuum pumps (arrived)



Some numerical values

Main interest is the Euler-Heisenberg birefringence

- $B = 2.5 \text{ T}$
- $F = 4 \cdot 10^5 \rightarrow \Delta n = 2.5 \cdot 10^{-23} \rightarrow \psi = 3.7 \cdot 10^{-11}$
- $L = 2 \text{ m}$

If we assume a maximum integration time of 10^6 s (= 12 days)



Ellipticity sensitivity of $< 3.7 \cdot 10^{-8} \text{ 1}/\sqrt{\text{Hz}}$
Birefringence sensitivity $< 2.5 \cdot 10^{-20} \text{ 1}/\sqrt{\text{Hz}}$

Present sensitivity in $\Delta n = 1.8 \cdot 10^{-18} \text{ 1}/\sqrt{\text{Hz}}$

$$\text{Shot noise limit} = \sqrt{\frac{e}{2I_0 q}} = 1 \cdot 10^{-9} \frac{1}{\sqrt{\text{Hz}}} \text{ for } I_0 = 100 \text{ mW}$$

(I_0 = output intensity reaching the analyzer)